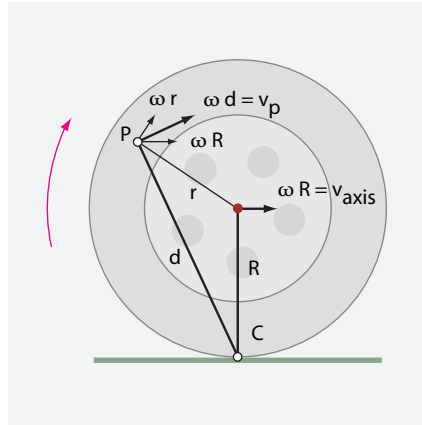
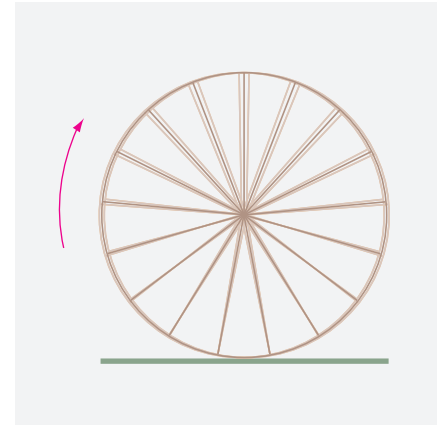




**FIGURE 78** The velocities and unit vectors for a rolling wheel.



**FIGURE 79** A simulated photograph of a rolling wheel with spokes.

the trick, if cats are dropped legs up, they can turn themselves in such a way that they always land feet first. Snakes also know how to rotate themselves, as [Figure 77](#) shows. Also humans have the ability: during the Olympic Games you can watch board divers and gymnasts perform similar tricks. Rotation thus differs from translation in this important aspect. (Why?)

Ref. 94

Challenge 210 d

### ROLLING WHEELS

Rotation is an interesting phenomenon in many ways. A rolling wheel does *not* turn around its axis, but around its point of contact. Let us show this.

A wheel of radius  $R$  is *rolling* if the speed of the axis  $v_{\text{axis}}$  is related to the angular velocity  $\omega$  by

$$\omega = \frac{v_{\text{axis}}}{R}. \quad (31)$$

For any point  $P$  on the wheel, with distance  $r$  from the axis, the velocity  $v_P$  is the sum of the motion of the axis and the motion around the axis. [Figure 78](#) shows that  $v_P$  is orthogonal to  $d$ , the distance between the point  $P$  and the contact point of the wheel. The figure also shows that the length ratio between  $v_P$  and  $d$  is the same as between  $v_{\text{axis}}$  and  $R$ . As a result, we can write

Challenge 211 e

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{d}, \quad (32)$$

which shows that a rolling wheel does indeed rotate about its point of contact with the ground.

Surprisingly, when a wheel rolls, some points on it move *towards* the wheel's axis, some stay at a *fixed* distance and others move *away* from it. Can you determine where these various points are located? Together, they lead to an interesting pattern when a rolling wheel with spokes, such as a bicycle wheel, is photographed, as shown in [Figure 79](#).

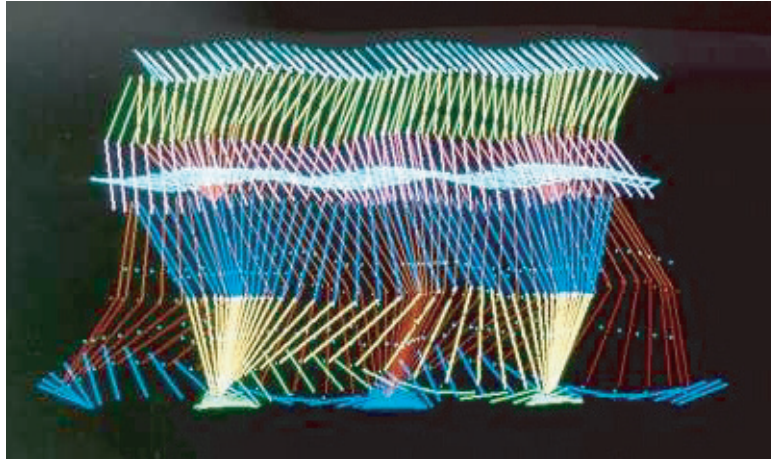
Challenge 212 s

Ref. 95

Ref. 96

With these results you can tackle the following beautiful challenge. When a turning bicycle wheel is deposited on a slippery surface, it will slip for a while, then slip and roll,





**FIGURE 80** The measured motion of a walking human (© Ray McCoy).

Challenge 213 d

and finally roll only. How does the final speed depend on the initial speed and on the friction?

#### HOW DO WE WALK AND RUN?

“ Golf is a good walk spoiled.

”  
The Allens

Ref. 20

Why do we move our arms when walking or running? To save energy or to be graceful? In fact, whenever a body movement is performed with as little energy as possible, it is both natural and graceful. This correspondence can indeed be taken as the actual definition of grace. The connection is common knowledge in the world of dance; it is also a central aspect of the methods used by actors to learn how to move their bodies as beautifully as possible.

To convince yourself about the energy savings, try walking or running with your arms fixed or moving in the opposite direction to usual: the effort required is considerably higher. In fact, when a leg is moved, it produces a torque around the body axis which has to be counterbalanced. The method using the least energy is the swinging of arms, as depicted in Figure 80. Since the arms are lighter than the legs, they must move further from the axis of the body, to compensate for the momentum; evolution has therefore moved the attachment of the arms, the shoulders, farther apart than those of the legs, the hips. Animals on two legs but without arms, such as penguins or pigeons, have more difficulty walking; they have to move their whole torso with every step.

Ref. 97

Measurements show that all walking animals follow

$$v_{\text{max walking}} = (2.2 \pm 0.2 \text{ m/s}) \sqrt{l/m} . \quad (33)$$

Indeed, walking, the moving of one leg after the other, can be described as a concatenation of (inverted) pendulum swings. The pendulum length is given by the leg length  $l$ . The typical time scale of a pendulum is  $t \sim \sqrt{l/g}$ . The maximum speed of walking then

becomes  $v \sim l/t \sim \sqrt{gl}$ , which is, up to a constant factor, the measured result.

Which muscles do most of the work when walking, the motion that experts call *gait*?  
 Ref. 98 In 1980, Serge Gracovetsky found that in human gait a large fraction of the power comes from the muscles along the *spine*, not from those of the legs. (Indeed, people without legs are also able to walk. However, a number of muscles in the legs must work in order to walk normally.) When you take a step, the lumbar muscles straighten the spine; this automatically makes it turn a bit to one side, so that the knee of the leg on that side automatically comes forward. When the foot is moved, the lumbar muscles can relax, and then straighten again for the next step. In fact, one can experience the increase in tension in the *back* muscles when walking without moving the arms, thus confirming where the human engine, the so-called *spinal engine* is located.

Challenge 214 e

Human legs differ from those of apes in a fundamental aspect: humans are able to *run*. In fact the whole human body has been optimized for running, an ability that no other primate has. The human body has shed most of its hair to achieve better cooling, has evolved the ability to run while keeping the head stable, has evolved the right length of arms for proper balance when running, and even has a special ligament in the back that works as a shock absorber while running. In other words, running is the most human of all forms of motion.

#### CURIOSITIES AND FUN CHALLENGES ABOUT MASS, CONSERVATION AND ROTATION

“It is a mathematical fact that the casting of this pebble from my hand alters the centre of gravity of the universe.”

Thomas Carlyle,\* *Sartor Resartus III*.

Challenge 216 e

A cup with water is placed on a weighing scale, as shown in Figure 81. How does the mass result change if you let a piece of metal attached to a string hang into the water?

\* \*

Challenge 217 e

Take ten coins of the same denomination. Put nine of them on a table and form a closed loop with them of any shape you like, as shown in Figure 82. (The nine coins thus look like a section of pearl necklace where the pearls touch each other.) Now take the tenth coin and let it roll around the loop, thus without ever sliding it. How many turns does this last coin make during one round?

\* \*

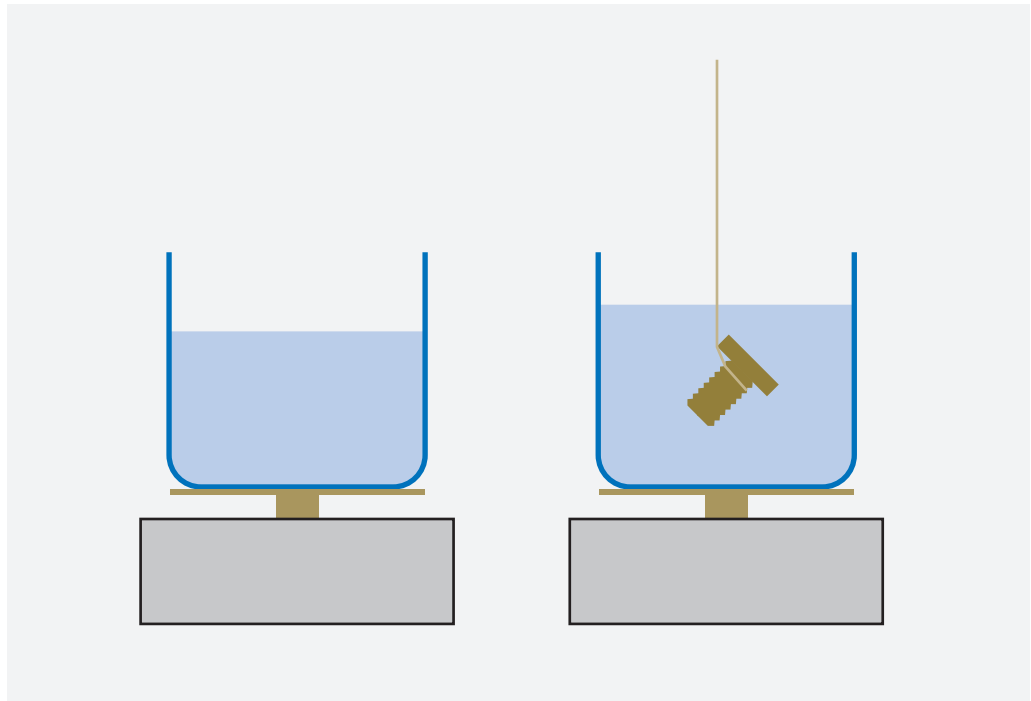
Challenge 218 e

Conservation of momentum is best studied by playing and exploring billiards, snooker or pool. The best introduction is provided by the trickshot films found across the internet. Are you able to use momentum conservation to deduce ways for improving your billiards game?

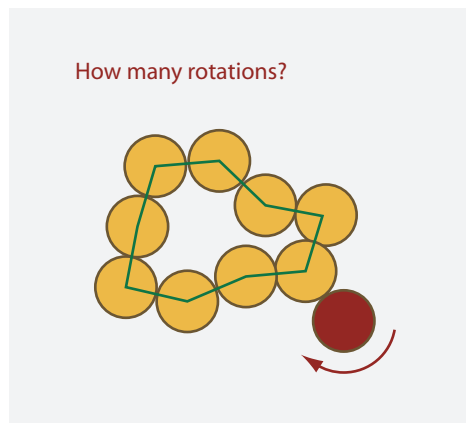
Another way to explore momentum conservation is to explore the ball-chain, or ball collision pendulum, that was invented by Edme Mariotte. Decades later, Newton claimed it as his, as he often did with other people's results. Playing with the toy is fun – and explaining its behaviour even more. Indeed, if you lift and let go three balls on one side,

Challenge 215 s

\* Thomas Carlyle (1797–1881), Scottish essayist. Do you agree with the quotation?



**FIGURE 81** How does the displayed weight value change when an object hangs into the water?



**FIGURE 82** How many rotations does the tenth coin perform in one round?

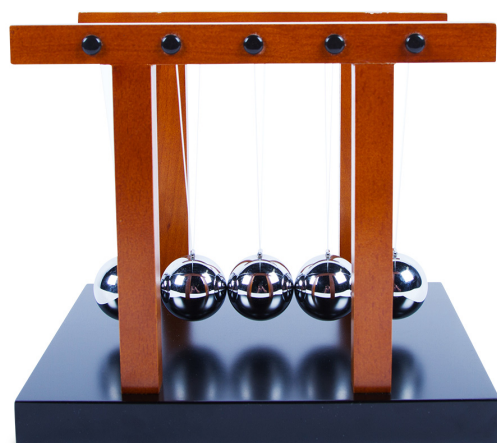
you will see three balls departing on the other side; for the explanation of this behaviour the conservation of momentum and energy conservation are *not* sufficient, as you should be able to find out. Are you able to build a high-precision ball-chain?

Challenge 219 d

\* \*

Challenge 220 s There is a well-known way to experience 81 sunrises in just 80 days. How?

\* \*



**FIGURE 83** The ball-chain or cradle invented by Mariotte allows exploring momentum conservation, energy conservation, and the difficulties of precision manufacturing (© [www.questacon.edu.au](http://www.questacon.edu.au)).

Ref. 99 Walking is a source of many physics problems. When climbing a mountain, the most energy-effective way is not always to follow the steepest ascent; indeed, for steep slopes, zig-zagging is more energy efficient. Why? And can you estimate the slope angle at which this will happen?

Challenge 221 s

\* \*

Challenge 222 e Asterix and his friends from the homonymous comic strip, fear only one thing: that the sky might fall down. Is the sky an object? An image?

\* \*

Ref. 100 Death is a physical process and thus can be explored. In general, animals have a *lifespan*  $T$  that scales with fourth root of their mass  $M$ . In other terms,  $T = M^{1/4}$ . This is valid from bacteria to insects to blue whales. Animals also have a power consumption per mass, or *metabolic rate* per mass, that scales with the *inverse* fourth root. We conclude that death occurs for all animals when a certain fixed energy consumption per mass has been achieved. This is indeed the case; death occurs for most animals when they have consumed around 1 GJ/kg. (But quite a bit later for humans.) This surprisingly simple result is valid, *on average*, for all known animals.

Note that the argument is only valid when *different* species are compared. The dependence on mass is *not* valid when specimen of the same species are compared. (You cannot live longer by eating less.)

In short, animals die after they metabolized 1 GJ/kg. In other words, once we ate all the calories we were designed for, we die.

\* \*

Challenge 223 s A car at a certain speed uses 7 litres of gasoline per 100 km. What is the combined air and rolling resistance? (Assume that the engine has an efficiency of 25 %.)

\* \*

A cork is attached to a thin string a metre long. The string is passed over a long rod

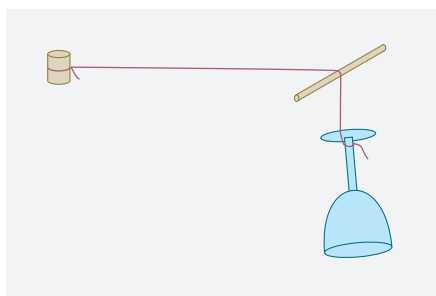


FIGURE 84 Is it safe to let the cork go?

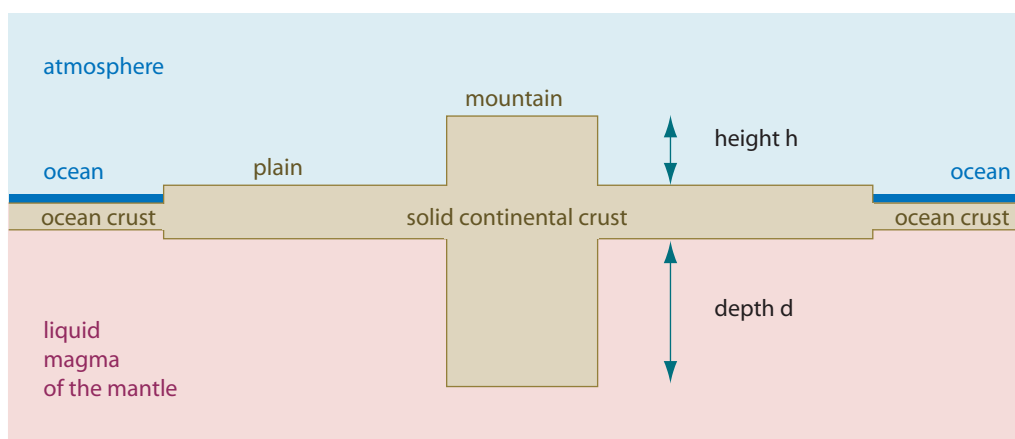


FIGURE 85 A simple model for continents and mountains.

Challenge 224 s held horizontally, and a wine glass is attached at the other end. If you let go the cork in Figure 84, nothing breaks. Why not? And what happens exactly?

\* \*

Ref. 101 In 1907, Duncan MacDougalls, a medical doctor, measured the weight of dying people, in the hope to see whether death leads to a mass change. He found a sudden decrease between 10 and 20 g at the moment of death. He attributed it to the soul exiting the body.

Challenge 225 s Can you find a more satisfying explanation?

\* \*

It is well known that the weight of a one-year-old child depends on whether it wants to be carried or whether it wants to reach the floor. Does this contradict mass conservation?

Challenge 226 e

\* \*

Challenge 227 s The Earth's crust is less dense (2.7 kg/l) than the Earth's mantle (3.1 kg/l) and floats on it. As a result, the lighter crust below a mountain ridge must be much deeper than below a plain. If a mountain rises 1 km above the plain, how much deeper must the crust be below it? The simple block model shown in Figure 85 works fairly well; first, it explains why, near mountains, measurements of the deviation of free fall from the vertical line

lead to so much lower values than those expected without a deep crust. Later, sound measurements confirmed directly that the continental crust is indeed thicker beneath mountains.

\* \*

Challenge 228 e All homogeneous cylinders roll down an inclined plane in the same way. True or false? And what about spheres? Can you show that spheres roll faster than cylinders?

\* \*

Challenge 229 s Which one rolls faster: a soda can filled with liquid or a soda can filled with ice? (And how do you make a can filled with ice?)

\* \*

Challenge 230 e Take two cans of the same size and weight, one full of ravioli and one full of peas. Which one rolls faster on an inclined plane?

\* \*

Another difference between matter and images: matter smells. In fact, the nose is a matter sensor. The same can be said of the tongue and its sense of taste.

\* \*

Challenge 231 e Take a pile of coins. You can push out the coins, starting with the one at the bottom, by shooting another coin over the table surface. The method also helps to visualize two-dimensional momentum conservation.

\* \*

Ref. 102 In early 2004, two men and a woman earned £ 1.2 million in a single evening in a London casino. They did so by applying the formulae of Galilean mechanics. They used the method pioneered by various physicists in the 1950s who built various small computers that could predict the outcome of a roulette ball from the initial velocity imparted by the croupier. In the case of Britain, the group added a laser scanner to a smartphone that measured the path of a roulette ball and predicted the numbers where it would arrive. In this way, they increased the odds from 1 in 37 to about 1 in 6. After six months of investigations, Scotland Yard ruled that they could keep the money they won.

In fact around the same time, a few people earned around 400 000 euro over a few weeks by using the same method in Germany, but with no computer at all. In certain casinos, machines were throwing the roulette ball. By measuring the position of the zero to the incoming ball with the naked eye, these gamblers were able to increase the odds of the bets they placed during the last allowed seconds and thus win a considerable sum purely through fast reactions.

\* \*

Challenge 232 s Does the universe rotate?

\* \*

The toy of Figure 86 shows interesting behaviour: when a number of spheres are lifted

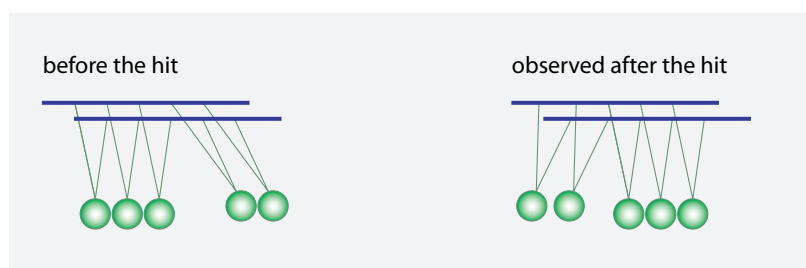


FIGURE 86 A well-known toy.

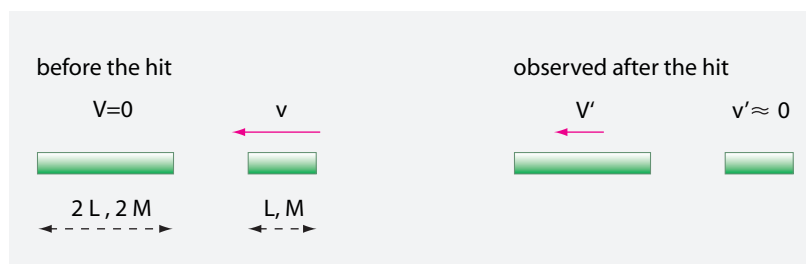


FIGURE 87 An elastic collision that seems not to obey energy conservation.

and dropped to hit the resting ones, the same number of spheres detach on the other side, whereas the previously dropped spheres remain motionless. At first sight, all this seems to follow from energy and momentum conservation. However, energy and momentum conservation provide only two equations, which are insufficient to explain or determine the behaviour of five spheres. Why then do the spheres behave in this way? And why do they all swing in phase when a longer time has passed?

Challenge 233 d

\* \*

A surprising effect is used in home tools such as hammer drills. We remember that when a small ball elastically hits a large one at rest, both balls move after the hit, and the small one obviously moves faster than the large one. Despite this result, when a short cylinder hits a long one of the same diameter and material, but with a length that is some *integer* multiple of that of the short one, something strange happens. After the hit, the small cylinder remains almost at rest, whereas the large one moves, as shown in Figure 87. Even though the collision is elastic, conservation of energy seems not to hold in this case. (In fact this is the reason that demonstrations of elastic collisions in schools are always performed with spheres.) What happens to the energy?

Ref. 103

Challenge 234 d

\* \*

Is the structure shown in Figure 88 possible?

\* \*

Does a wall get a stronger jolt when it is hit by a ball rebounding from it or when it is hit by a ball that remains stuck to it?

Challenge 235 s

\* \*



FIGURE 88 Is this possible?

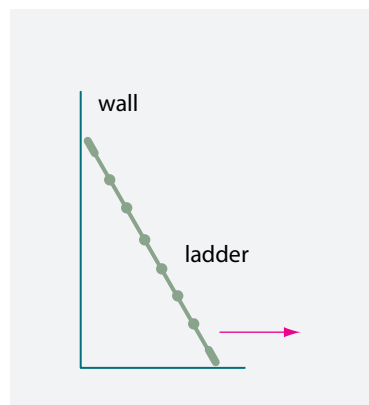


FIGURE 89 How does the ladder fall?

Housewives know how to extract a cork of a wine bottle using a cloth or a shoe. Can you imagine how? They also know how to extract the cork with the cloth if the cork has fallen inside the bottle. How?

Challenge 236 s

\* \*

The sliding ladder problem, shown schematically in Figure 89, asks for the detailed motion of the ladder over time. The problem is more difficult than it looks, even if friction is not taken into account. Can you say whether the lower end always touches the floor, or if is lifted into the air for a short time interval?

Challenge 237 s

\* \*

A homogeneous ladder of length 5 m and mass 30 kg leans on a wall. The angle is  $30^\circ$ ; the static friction coefficient on the wall is negligible, and on the floor it is 0.3. A person of mass 60 kg climbs the ladder. What is the maximum height the person can climb before the ladder starts sliding? This and many puzzles about ladders can be found on [www.mathematische-basteleien.de/leiter.htm](http://www.mathematische-basteleien.de/leiter.htm).

\* \*

A common fly on the stern of a 30 000 ton ship of 100 m length tilts it by less than the diameter of an atom. Today, distances that small are easily measured. Can you think of at least two methods, one of which should not cost more than 2000 euro?

Ref. 104  
Challenge 238 s

\* \*

Is the image of three stacked spinning tops shown in Figure 90 a true photograph, showing a real observation, or is it the result of digital composition, showing an impossible situation?

Challenge 239 ny

\* \*

How does the kinetic energy of a rifle bullet compare to that of a running man?

Challenge 240 s





**FIGURE 90** Is this a possible situation or is it a fake photograph?  
(© Wikimedia)

\* \*

**Challenge 241 s** What happens to the size of an egg when one places it in a jar of vinegar for a few days?

\* \*

**Challenge 242 s** What is the amplitude of a pendulum oscillating in such a way that the absolute value of its acceleration at the lowest point and at the return point are equal?

\* \*

**Challenge 243 d** Can you confirm that the value of the acceleration of a drop of water falling through mist is  $g/7$ ?

\* \*

**Challenge 244 s** You have two hollow spheres: they have the same weight, the same size and are painted in the same colour. One is made of copper, the other of aluminium. Obviously, they fall with the same speed and acceleration. What happens if they both roll down a tilted plane?

\* \*

**Challenge 245 s** What is the shape of a rope when rope jumping?

\* \*

**Challenge 246 s** How can you determine the speed of a rifle bullet with only a scale and a metre stick?

\* \*

**Challenge 247 e** Why does a gun make a hole in a door but cannot push it open, in exact contrast to what a finger can do?

\* \*

**Challenge 248 s** What is the curve described by the midpoint of a ladder sliding down a wall?

\* \*



**FIGURE 91** A commercial clock that needs no special energy source, because it takes its energy from the environment (© Jaeger-LeCoultre).

Challenge 249 s

A high-tech company, see [www.enocean.com](http://www.enocean.com), sells electric switches for room lights that have no cables and no power cell (battery). You can glue such a switch to the centre of a window pane. How is this possible?

\* \*

Challenge 250 s

For over 50 years now, a famous Swiss clockmaker is selling table clocks with a rotating pendulum that need no battery and no manual rewinding, as they take up energy from the environment. A specimen is shown in [Figure 91](#). Can you imagine how this clock works?

\* \*

Challenge 251 s

Ship lifts, such as the one shown in [Figure 92](#), are impressive machines. How does the weight of the lift change when the ship enters?

\* \*

Challenge 252 e

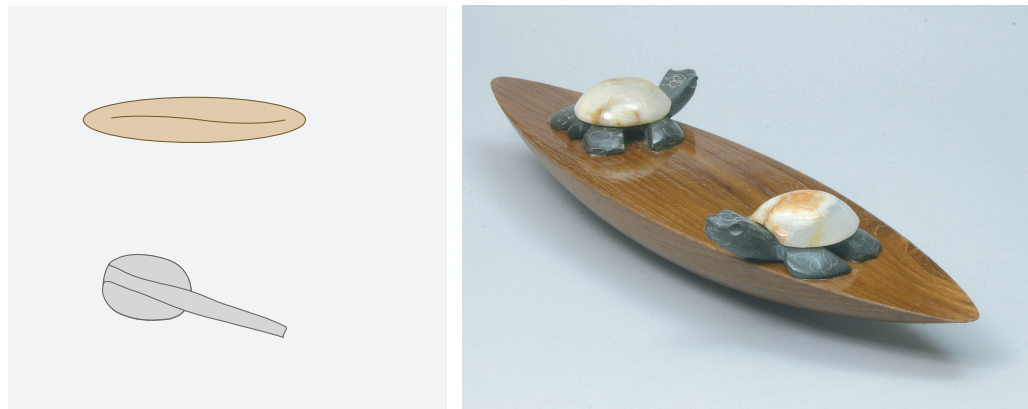
How do you measure the mass of a ship?

\* \*

All masses are measured by comparing them, directly or indirectly, to the *standard kilogram* in Sèvres near Paris. For a number of years, there was serious doubt that the stand-



**FIGURE 92** The spectacular ship lift at Strépy-Thieux in Belgium. What engine power is needed to lift a ship, if the right and left lifts were connected by ropes or by a hydraulic system? (© Jean-Marie Hoornaert)



**FIGURE 93** The famous Celtic wobble stone – above and right – and a version made by bending a spoon – bottom left (© Ed Keath).

ard kilogram was losing weight, possibly through outgassing, with an estimated rate of around  $0.5 \mu\text{g/a}$ . This was an awkward situation, and there has been a vast, worldwide effort to find a better definition of the kilogram. Such an improved definition had to be simple, precise, and make trips to Sèvres unnecessary. Finally, an alternative was found and defined in 2019.

\* \*

Which engine is more efficient: a moped or a human on a bicycle?

\* \*

Both mass and moment of inertia can be defined and measured both with and without contact. Can you do so?

Challenge 253 e

\* \*

**Figure 93** shows the so-called *Celtic wobble stone*, also called *anagyre* or *rattleback*, a stone that starts rotating on a plane surface when it is put into up-and-down oscillation. The size can vary between a few centimetres and a few metres. By simply bending a spoon one can realize a primitive form of this strange device, if the bend is not completely symmetrical. The rotation is always in the same direction. If the stone is put into rotation in the wrong direction, after a while it stops and starts rotating in the other sense! Can you explain the effect that seems to contradict the conservation of angular momentum?

Ref. 103

Challenge 254 d

\* \*

A beautiful effect, the *chain fountain*, was discovered in 2013 by Steve Mould. Certain chains, when flowing out of a container, first shoot up in the air. See the video at [www.youtube.com/embed/\\_dQJBBklpQQ](https://www.youtube.com/embed/_dQJBBklpQQ) and the story of the discovery at [stevemould.com](http://stevemould.com). Can you explain the effect to your grandmother?

Challenge 255 ny



## SUMMARY ON CONSERVATION IN MOTION

“The gods are not as rich as one might think:  
what they give to one, they take away from the  
other.”

Antiquity

We have encountered four conservation principles that are valid for the motion of all closed systems in everyday life:

- conservation of total linear momentum,
- conservation of total angular momentum,
- conservation of total energy,
- conservation of total mass.

None of these conservation principles applies to the motion of images. These principles thus allow us to distinguish objects from images.

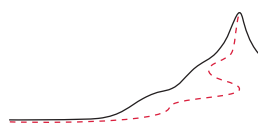
The conservation principles are among the great results in science. They limit the surprises that nature can offer: conservation means that linear momentum, angular momentum, and mass–energy can neither be created from nothing, nor can they disappear into nothing. Conservation limits creation. The quote below the section title expresses this idea.

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Later on we will find out that these results could have been deduced from three simple observations: closed systems behave the same independently of where they are, in what direction they are oriented and of the time at which they are set up. In more abstract terms, physicists like to say that all conservation principles are consequences of the *invariances*, or *symmetries*, of nature.

Later on, the theory of special relativity will show that energy and mass are conserved only when taken together. Many adventures still await us.





## CHAPTER 5

# FROM THE ROTATION OF THE EARTH TO THE RELATIVITY OF MOTION

“Eppur si muove!”

Anonymous\*\*

Is the Earth rotating? The search for definite answers to this question gives an interesting cross-section of the history of classical physics. In the fourth century, in ancient Greece, Hicetas and Philolaus, already stated that the Earth rotates. Then, in the year 265 BCE, Aristarchus of Samos was the first to explore the issue in detail. He had measured the parallax of the Moon (today known to be up to  $0.95^\circ$ ) and of the Sun (today known to be  $8.8'$ ).\*\*\* The *parallax* is an interesting effect; it is the angle describing the difference between the directions of a body in the sky when seen by an observer on the surface of the Earth and when seen by a hypothetical observer at the Earth's centre. (See Figure 94.) Aristarchus noticed that the Moon and the Sun *wobble* across the sky, and this wobble has a period of 24 hours. He concluded that the Earth rotates. It seems that Aristarchus received death threats for his conclusion.

Challenge 256 e

Challenge 257 s

Aristarchus' observation yields an even more powerful argument than the trails of the stars shown in Figure 95. Can you explain why? (And how do the trails look at the most populated places on Earth?)

Ref. 136

Challenge 258 s

Experiencing Figure 95 might be one reason that people dreamt and still dream about reaching the poles. Because the rotation and the motion of the Earth make the poles extremely cold places, the adventure of reaching them is not easy. Many tried unsuccessfully. A famous crook, Robert Peary, claimed to have reached the North Pole in 1909. (In fact, Roald Amundsen reached both the South and the North Pole first.) Among others, Peary claimed to have taken a picture there, but that picture, which went round the world, turned out to be one of the proofs that he had not been there. Can you imagine how?

Challenge 259 e

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If the Earth rotates instead of being at rest, said the unconvinced, the speed at the equator has the substantial value of  $0.46 \text{ km/s}$ . How did Galileo explain why we do not feel or notice this speed?

Measurements of the aberration of light also show the rotation of the Earth; it can be detected with a telescope while looking at the stars. The *aberration* is a change of the

\*\* ‘And yet she moves’ is the sentence about the Earth attributed, most probably incorrectly, to Galileo since the 1640s. It is true, however, that at his trial he was forced to publicly retract the statement of a moving Earth to save his life. For more details of this famous story, see the section on page 335.

\*\*\* For the definition of the concept of angle, see page 67, and for the definition of the measurement units for angle see Appendix B.

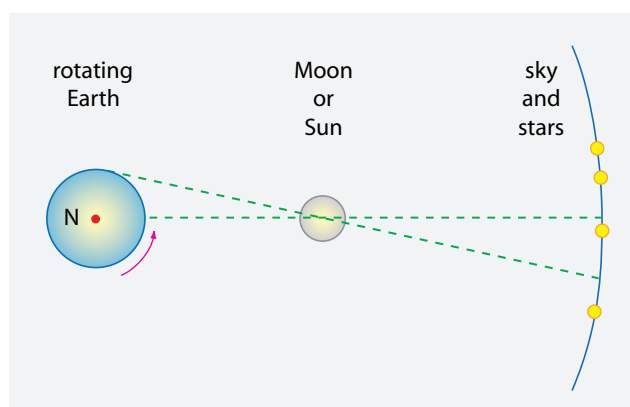


FIGURE 94 The parallax – not drawn to scale.

expected light direction, which we will discuss shortly. At the Equator, Earth rotation adds an angular deviation of  $0.32'$ , changing sign every 12 hours, to the aberration due to the motion of the Earth around the Sun, about  $20.5'$ . In modern times, astronomers have found a number of additional proofs for the rotation of the Earth, but none is accessible to the man on the street.

Also the measurements showing that the Earth is not a sphere, but is *flattened* at the poles, confirmed the rotation of the Earth. Figure 96 illustrates the situation. Again, however, this eighteenth-century measurement by Maupertuis\* is not accessible to everyday observation.

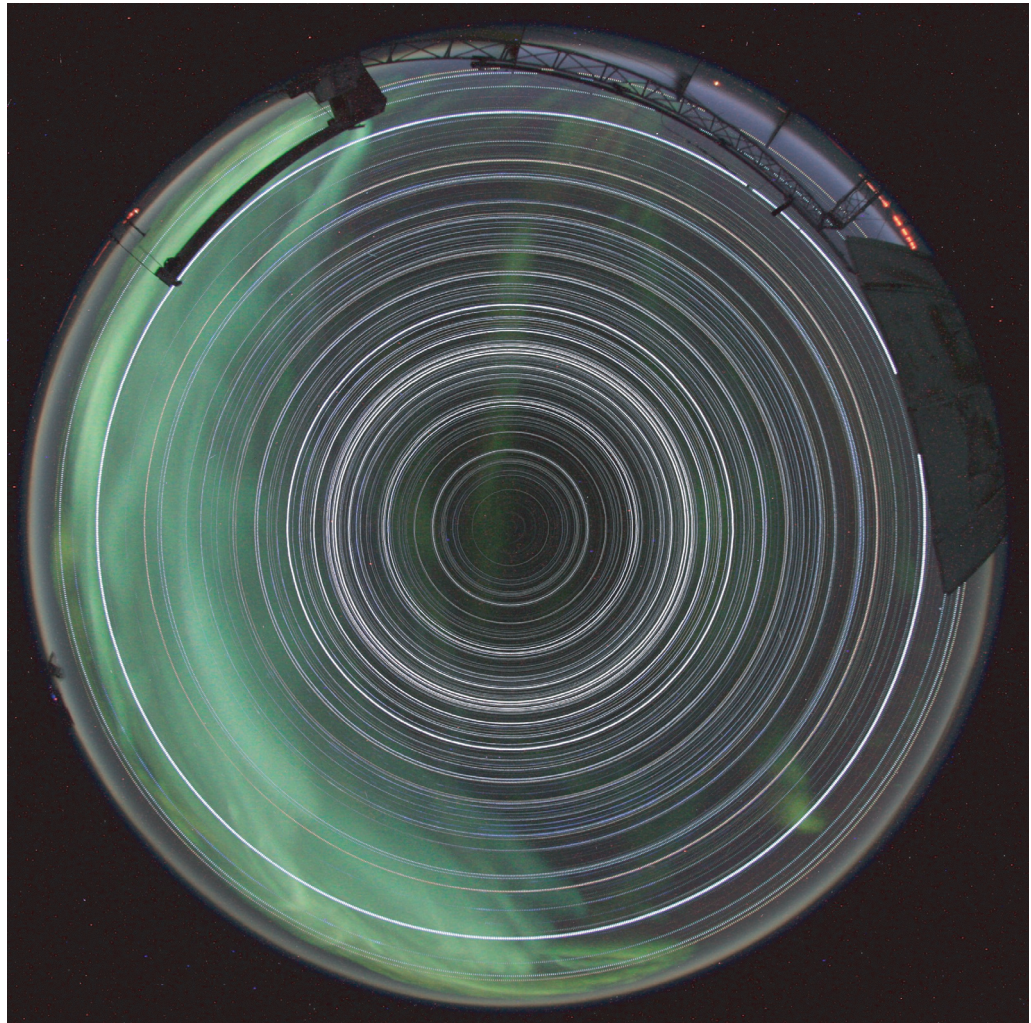
Then, in the years 1790 to 1792 in Bologna, Giovanni Battista Guglielmini (1763–1817) finally succeeded in measuring what Galileo and Newton had predicted to be the simplest proof for the Earth's rotation. On the rotating Earth, *objects do not fall vertically*, but are slightly deviated to the east. This deviation appears because an object keeps the larger horizontal velocity it had at the height from which it started falling, as shown in Figure 97. Guglielmini's result was the first non-astronomical proof of the Earth's rotation. The experiments were repeated in 1802 by Johann Friedrich Benzenberg (1777–1846). Using metal balls which he dropped from the Michaelis tower in Hamburg – a height of 76 m – Benzenberg found that the deviation to the east was 9.6 mm. Can you confirm that the value measured by Benzenberg almost agrees with the assumption that the Earth turns once every 24 hours? There is also a much smaller deviation towards the Equator, not measured by Guglielmini, Benzenberg or anybody after them up to this day; however, it completes the list of effects on free fall by the rotation of the Earth.

Both deviations from vertical fall are easily understood if we use the result (described below) that falling objects describe an ellipse around the centre of the rotating Earth. The elliptical shape shows that the path of a thrown stone does not lie on a plane for an

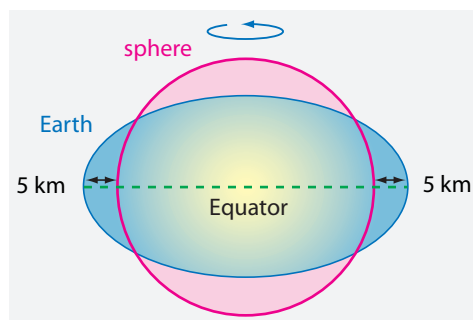
Challenge 260 d

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\* Pierre Louis Moreau de Maupertuis (1698–1759), physicist and mathematician, was one of the key figures in the quest for the principle of least action, which he named in this way. He was also the founding president of the Berlin Academy of Sciences. Maupertuis thought that the principle reflected the maximization of goodness in the universe. This idea was thoroughly ridiculed by Voltaire in his *Histoire du Docteur Akakia et du natif de Saint-Malo*, 1753. (Read it at [gallica.bnf.fr/ark:/12148/bpt6k6548988f.texteImage](http://gallica.bnf.fr/ark:/12148/bpt6k6548988f.texteImage).) Maupertuis performed his measurement of the Earth to distinguish between the theory of gravitation of Newton and that of Descartes, who had predicted that the Earth is elongated at the poles, instead of flattened.



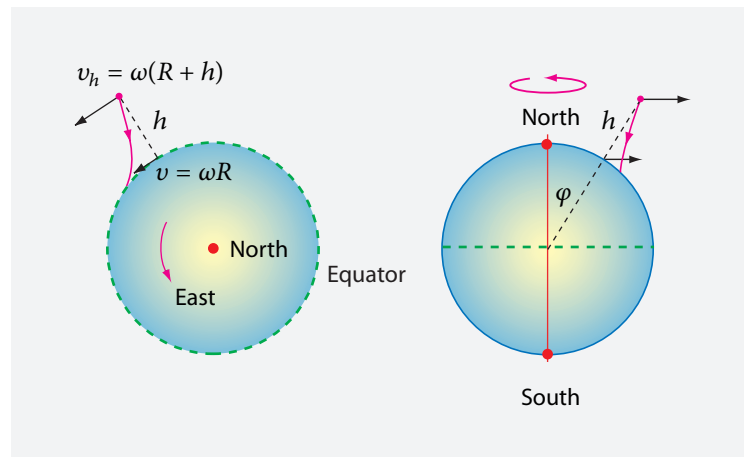
**FIGURE 95** The motion of the stars during the night, observed on 1 May 2012 from the South Pole, together with the green light of an aurora australis (© Robert Schwartz).



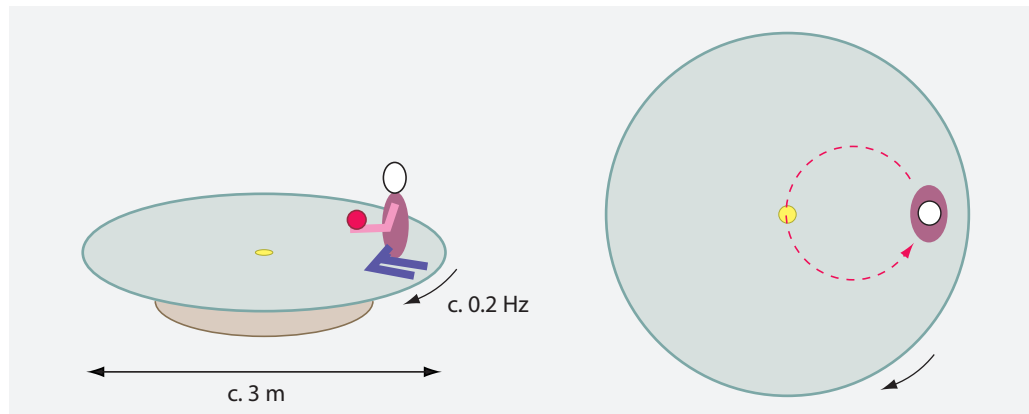
**FIGURE 96** Earth's deviation from spherical shape due to its rotation (exaggerated).

observer standing on Earth; for such an observer, the exact path of a stone thus cannot





**FIGURE 97** The deviations of free fall towards the east and towards the Equator due to the rotation of the Earth.

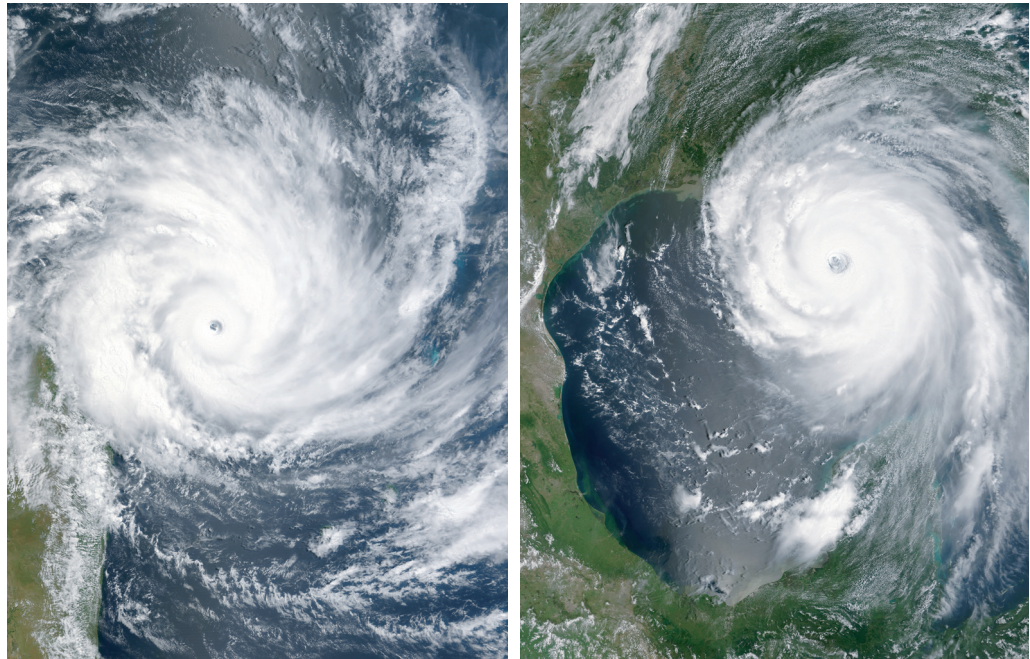


**FIGURE 98** A typical carousel allows observing the Coriolis effect in its most striking appearance: if a person lets a ball roll with the proper speed and direction, the ball is deflected so strongly that it comes back to her.

be drawn on a flat piece of paper!

Ref. 106 In 1798, Pierre Simon Laplace\* explained how bodies move on the rotating Earth and showed that they feel an apparent force. In 1835, Gustave-Gaspard Coriolis then reformulated and simplified the description. Imagine a ball that rolls over a table. For a person on the floor, the ball rolls in a straight line. Now imagine that the table rotates. For the person on the floor, the ball still rolls in a straight line. But for a person on the rotating table, the ball traces a *curved* path. In short, any object that travels in a rotating background is subject to a transversal acceleration. The acceleration, discovered by Laplace, is nowadays called *Coriolis acceleration* or *Coriolis effect*. On a rotating background, travelling objects deviate from the straight line. The best way to understand the Coriolis effect is to exper-

\* Pierre Simon Laplace (b. 1749 Beaumont-en-Auge, d. 1827 Paris), important mathematician. His famous treatise *Traité de mécanique céleste* appeared in five volumes between 1798 and 1825. He was the first to propose that the Solar System was formed from a rotating gas cloud, and one of the first people to imagine and explore black holes.



**FIGURE 99** Cyclones, with their low pressure centre, differ in rotation sense between the southern hemisphere, here cyclone Larry in 2006, and the northern hemisphere, here hurricane Katrina in 2005. (Courtesy NOAA)

Ref. 107

ience it yourself; this can be done on a carousel, as shown in Figure 98. Watching films on the internet on the topic is also helpful. You will notice that on a rotating carousel it is not easy to hit a target by throwing or rolling a ball.

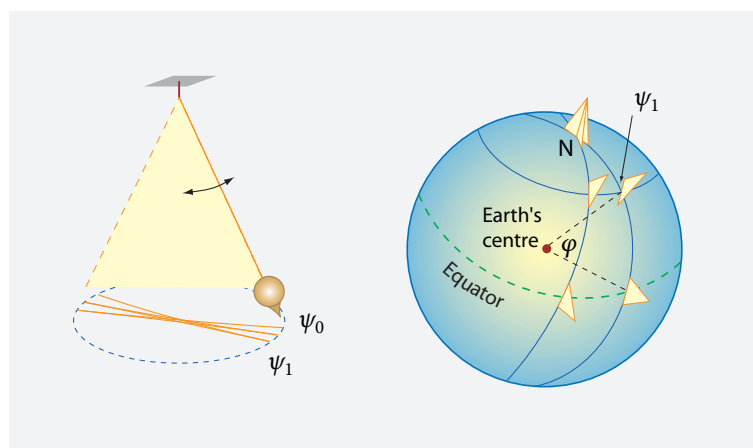
Challenge 261 s

Also the Earth is a rotating background. On the northern hemisphere, the rotation is anticlockwise. As the result, any moving object is slightly deviated to the right (while the magnitude of its velocity stays constant). On Earth, like on all rotating backgrounds, the *Coriolis acceleration*  $\mathbf{a}_C$  results from the change of distance to the rotation axis. Can you deduce the analytical expression for the Coriolis effect, namely  $\mathbf{a}_C = -2\boldsymbol{\omega} \times \mathbf{v}$ ?

Ref. 108

Ref. 109

On Earth, the Coriolis acceleration generally has a small value. Therefore it is best observed either in large-scale or high-speed phenomena. Indeed, the Coriolis acceleration determines the handedness of many large-scale phenomena with a spiral shape, such as the directions of cyclones and anticyclones in meteorology – as shown in Figure 99 – the general wind patterns on Earth and the deflection of ocean currents and tides. These phenomena have opposite handedness on the northern and the southern hemisphere. Most beautifully, the Coriolis acceleration explains why icebergs do not follow the direction of the wind as they drift away from the polar caps. The Coriolis acceleration also plays a role in the flight of cannon balls (that was the original interest of Coriolis), in satellite launches, in the motion of sunspots and even in the motion of electrons in molecules. All these Coriolis accelerations are of opposite sign on the northern and southern hemispheres and thus prove the rotation of the Earth. For example, in the First World War, many naval guns missed their targets in the southern hemisphere because the engineers had compensated them for the Coriolis effect in the northern hemisphere.



**FIGURE 100** The turning motion of a pendulum showing the rotation of the Earth.

Ref. 110 Only in 1962, after several earlier attempts by other researchers, Asher Shapiro was the first to verify that the Coriolis effect has a tiny influence on the direction of the vortex formed by the water flowing out of a bath-tub. Instead of a normal bath-tub, he had to use a carefully designed experimental set-up because, contrary to an often-heard assertion, no such effect can be seen in a real bath-tub. He succeeded only by carefully eliminating all disturbances from the system; for example, he waited 24 hours after the filling of the reservoir (and never actually stepped in or out of it!) in order to avoid any left-over motion of water that would disturb the effect, and built a carefully designed, completely rotationally-symmetric opening mechanism. Others have repeated the experiment in the southern hemisphere, finding opposite rotation direction and thus confirming the result. In other words, the handedness of usual bath-tub vortices is *not* caused by the rotation of the Earth, but results from the way the water starts to flow out. (A number of crooks in Quito, a city located on the Equator, show gullible tourists that the vortex in a sink changes when crossing the Equator line drawn on the road.) But let us go on with the story about the Earth's rotation.

Ref. 110

In 1851, the physician-turned-physicist Jean Bernard Léon Foucault (b. 1819 Paris, d. 1868 Paris) performed an experiment that removed all doubts and rendered him world-famous practically overnight. He suspended a 67 m long pendulum\* in the Panthéon in Paris and showed the astonished public that the direction of its swing changed over time, rotating slowly. To anybody with a few minutes of patience to watch the change of direction, the experiment proved that the Earth rotates. If the Earth did not rotate, the swing of the pendulum would always continue in the same direction. On a rotating Earth, in Paris, the direction changes to the right, in clockwise sense, as shown in Figure 100. The swing direction does not change if the pendulum is located at the Equator, and it changes to the left in the southern hemisphere.\*\* A modern version of the

Challenge 262 d

Ref. 111

\* Why was such a long pendulum necessary? Understanding the reasons allows one to repeat the experiment at home, using a pendulum as short as 70 cm, with the help of a few tricks. To observe Foucault's effect with a simple set-up, attach a pendulum to your office chair and rotate the chair slowly. Several pendulum animations, with exaggerated deviation, can be found at [commons.wikimedia.org/wiki/Foucault\\_pendulum](https://commons.wikimedia.org/wiki/Foucault_pendulum).

\*\* The discovery also shows how precision and genius go together. In fact, the first person to observe the

pendulum can be observed via the web cam at [pendelcam.kip.uni-heidelberg.de](http://pendelcam.kip.uni-heidelberg.de); high speed films of the pendulum's motion during day and night can also be downloaded at [www.kip.uni-heidelberg.de/oefw/pendel/zeitraffer/](http://www.kip.uni-heidelberg.de/oefw/pendel/zeitraffer/).

The time over which the orientation of the pendulum's swing performs a full turn – the *precession time* – can be calculated. Study a pendulum starting to swing in the North–South direction and you will find that the precession time  $T_{\text{Foucault}}$  is given by

$$T_{\text{Foucault}} = \frac{23 \text{ h } 56 \text{ min}}{\sin \varphi} \quad (34)$$

where  $\varphi$  is the latitude of the location of the pendulum, e.g.  $0^\circ$  at the Equator and  $90^\circ$  at the North Pole. This formula is one of the most beautiful results of Galilean kinematics.\*

Foucault was also the inventor and namer of the *gyroscope*. He built the device, shown in Figure 101 and Figure 102, in 1852, one year after his pendulum. With it, he again demonstrated the rotation of the Earth. Once a gyroscope rotates, the axis stays fixed in space – but only when seen from distant stars or galaxies. (By the way, this is not the same as talking about absolute space. Why?) For an observer on Earth, the axis direction changes regularly with a period of 24 hours. Gyroscopes are now routinely used in ships and in aeroplanes to give the direction of north, because they are more precise and more reliable than magnetic compasses. The most modern versions use laser light running in circles instead of rotating masses.\*\*

In 1909, Roland von Eötvös measured a small but surprising effect: due to the rotation of the Earth, the weight of an object depends on the direction in which it moves. As a result, a balance in rotation around the vertical axis does not stay perfectly horizontal: the balance starts to oscillate slightly. Can you explain the origin of the effect?

In 1910, John Hagen published the results of an even simpler experiment, proposed by Louis Poincaré in 1851. Two masses are put on a horizontal bar that can turn around a vertical axis, a so-called *isotomeograph*. Its total mass was 260 kg. If the two masses are slowly moved towards the support, as shown in Figure 103, and if the friction is kept low enough, the bar rotates. Obviously, this would not happen if the Earth were not rotating. Can you explain the observation? This little-known effect is also useful for winning bets between physicists.

In 1913, Arthur Compton showed that a closed tube filled with water and some small floating particles (or bubbles) can be used to show the rotation of the Earth. The device is called a *Compton tube* or *Compton wheel*. Compton showed that when a horizontal tube filled with water is rotated by  $180^\circ$ , something happens that allows one to prove that the Earth rotates. The experiment, shown in Figure 104, even allows measuring the latitude of the point where the experiment is made. Can you guess what happens?

Another method to detect the rotation of the Earth using light was first realized in 1913 Georges Sagnac:\*\*\* he used an *interferometer* to produce bright and dark fringes

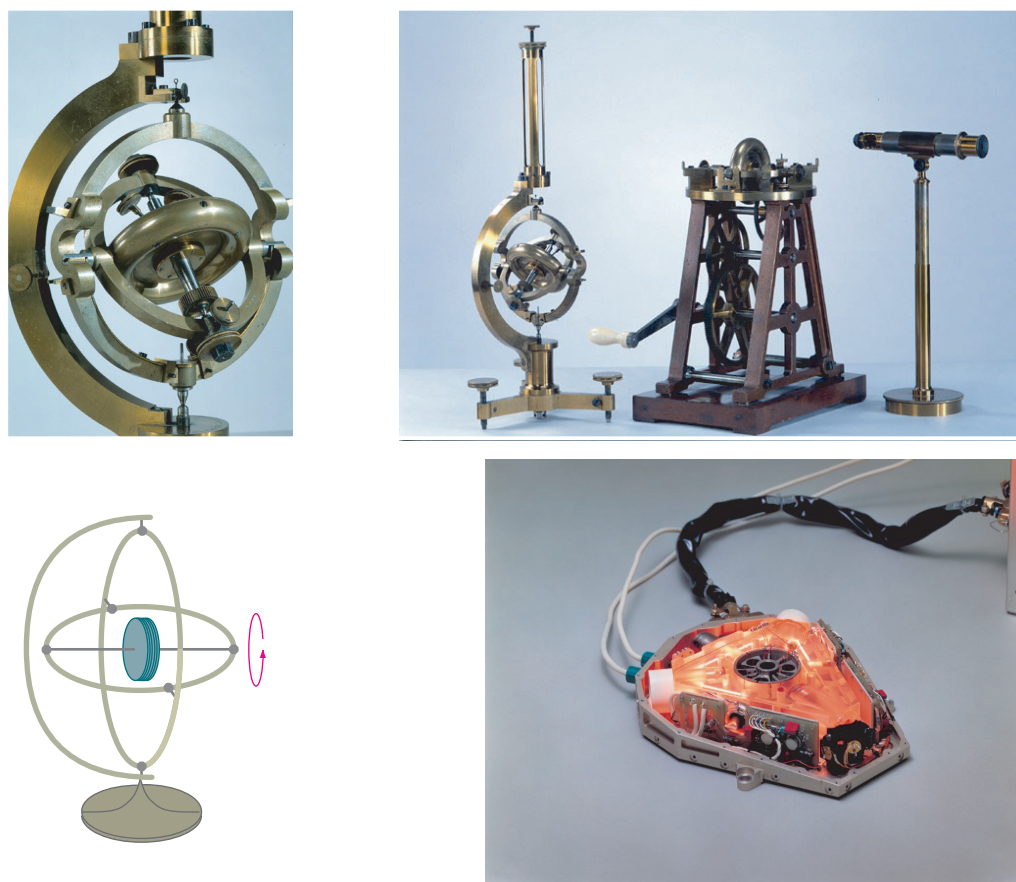
effect was Vincenzo Viviani, a student of Galileo, as early as 1661! Indeed, Foucault had read about Viviani's work in the publications of the Academia dei Lincei. But it took Foucault's genius to connect the effect to the rotation of the Earth; nobody had done so before him.

\* The calculation of the period of Foucault's pendulum assumes that the precession rate is constant during a rotation. This is only an approximation (though usually a good one).

\*\* Can you guess how rotation is detected in this case?

\*\*\* Georges Sagnac (b. 1869 Périgueux, d. 1928 Meudon-Bellevue) was a physicist in Lille and Paris, friend





**FIGURE 101** The gyroscope: the original system by Foucault with its freely movable spinning top, the mechanical device to bring it to speed, the optical device to detect its motion, the general construction principle, and a modern (triangular) ring laser gyroscope, based on colour change of rotating laser light instead of angular changes of a rotating mass (© CNAM, JAXA).

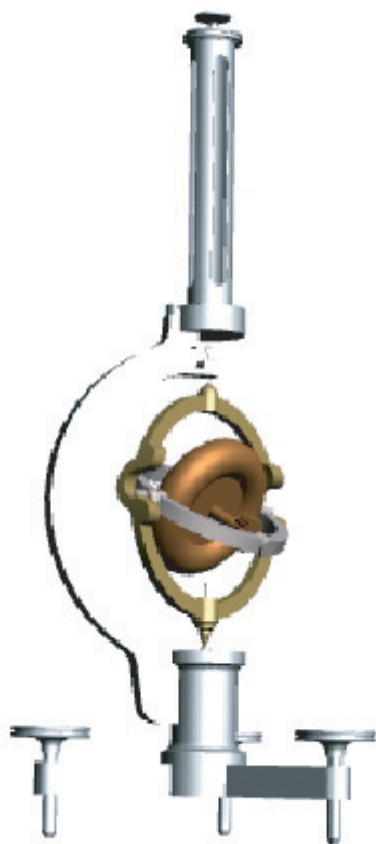
Vol. IV, page 56 of light with two light beams, one circulating in clockwise direction, and the second circulating in anticlockwise direction. The interference fringes are *shifted* when the whole system rotates; the faster it rotates, the larger the shift. A modern, high-precision version of the experiment, which uses lasers instead of lamps, is shown in Figure 105. (More details on interference and fringes are found in volume III.) Sagnac also determined the relationship between the fringe shift and the details of the experiment. The rotation of a complete ring interferometer with angular frequency (vector)  $\Omega$  produces a fringe shift of angular phase  $\Delta\phi$  given by

$$\Delta\phi = \frac{8\pi \Omega a}{c \lambda} \quad (35)$$

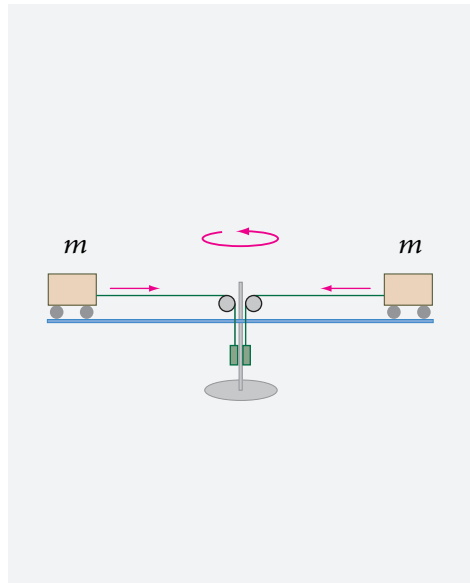
where  $a$  is the area (vector) enclosed by the two interfering light rays,  $\lambda$  their wavelength and  $c$  the speed of light. The effect is now called the *Sagnac effect* after its discoverer. It had

of the Curies, Langevin, Perrin, and Borel. Sagnac also deduced from his experiment that the speed of light was independent of the speed of its source, and thus confirmed a prediction of special relativity.

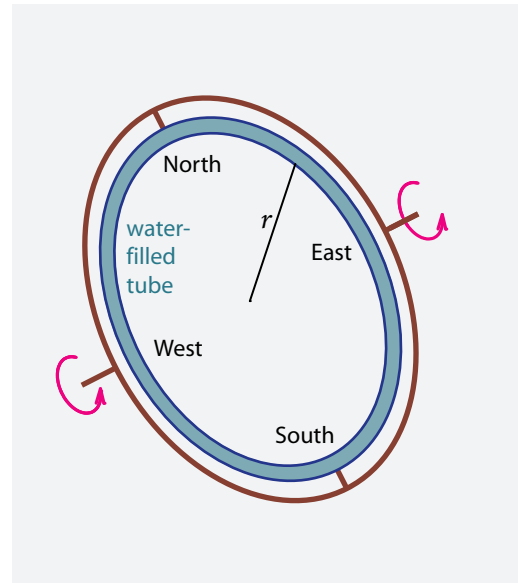




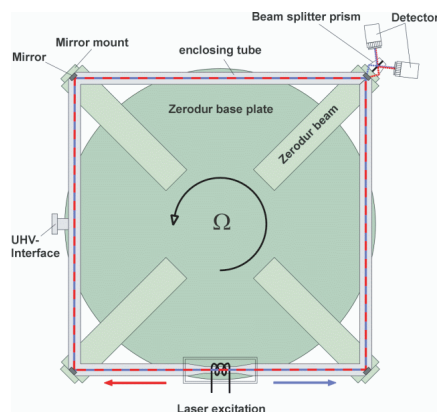
**FIGURE 102** A three-dimensional model of Foucault's original gyroscope: in the pdf version of this text, the model can be rotated and zoomed by moving the cursor over it (© Zach Joseph Espiritu).



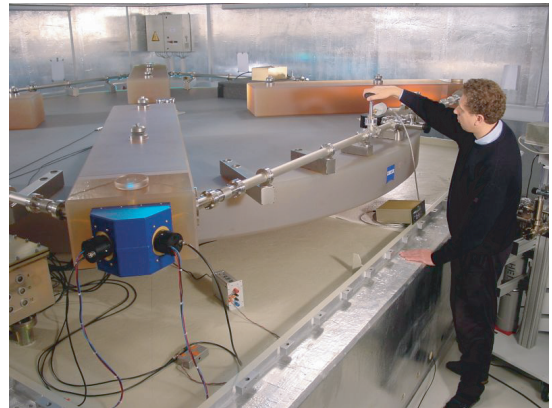
**FIGURE 103** Showing the rotation of the Earth through the rotation of an axis.



**FIGURE 104** Demonstrating the rotation of the Earth with water.



**FIGURE 105** A modern precision ring laser interferometer (© Bundesamt für Kartographie und Geodäsie, Carl Zeiss).



Ref. 115 already been predicted 20 years earlier by Oliver Lodge.\* Today, Sagnac interferometers are the central part of laser gyroscopes – shown in Figure 101 – and are found in every passenger aeroplane, missile and submarine, in order to measure the changes of their motion and thus to determine their actual position.

A part of the fringe shift is due to the rotation of the Earth. Modern high-precision Sagnac interferometers use ring lasers with areas of a few square metres, as shown in

\* Oliver Lodge (b. 1851, Stoke, d. on-Trent-1940, Wiltshire) was a physicist and spiritualist who studied electromagnetic waves and tried to communicate with the dead. A strange but influential figure, his ideas are often cited when fun needs to be made of physicists; for example, he was one of those (rare) physicists who believed that at the end of the nineteenth-century physics was complete.

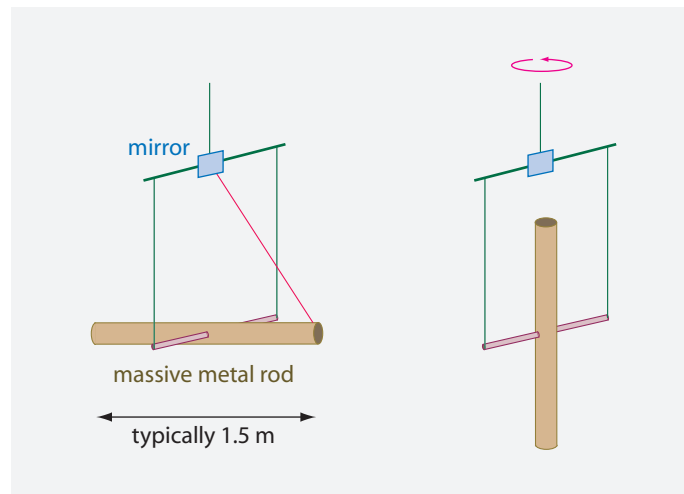


FIGURE 106 Observing the rotation of the Earth in two seconds.

Figure 105. Such a ring interferometer is able to measure variations in the rotation rates of the Earth of less than one part per million. Indeed, over the course of a year, the length of a day varies irregularly by a few milliseconds, mostly due to influences from the Sun or the Moon, due to weather changes and due to hot magma flows deep inside the Earth.\* But also earthquakes, the El Niño effect in the climate and the filling of large water dams have effects on the rotation of the Earth. All these effects can be studied with such high-precision interferometers; they can also be used for research into the motion of the soil due to lunar tides or earthquakes, and for checks on the theory of special relativity.

Ref. 116

Finally, in 1948, Hans Bucka developed the simplest experiment so far to show the rotation of the Earth. A metal rod allows anybody to detect the rotation of the Earth after only a few seconds of observation, using the set-up of Figure 106. The experiment can be easily performed in class. Can you guess how it works?

Ref. 117

Challenge 270 s

In summary: all observations show that the Earth surface rotates at 464 m/s at the Equator, a larger value than that of the speed of sound in air, which is about 340 m/s at usual conditions. The rotation of the Earth also implies an acceleration, at the Equator, of  $0.034 \text{ m/s}^2$ . We are in fact *whirling* through the universe.

### HOW DOES THE EARTH ROTATE?

Is the rotation of the Earth, the length of the day, *constant* over geological time scales? That is a hard question. If you find a method leading to an answer, publish it! (The same is true for the question of whether the length of the year is constant.) Only a few methods are known, as we will find out shortly.

Ref. 118

The rotation of the Earth is not even constant during a human lifespan. It varies by a few parts in  $10^8$ . In particular, on a 'secular' time scale, the length of the day increases by about 1 to 2 ms per century, mainly because of the friction by the Moon and the melting of the polar ice caps. This was deduced by studying historical astronomical observations

\* The growth of leaves on trees and the consequent change in the Earth's moment of inertia, already thought of in 1916 by Harold Jeffreys, is way too small to be seen, as it is hidden by larger effects.

Ref. 119 of the ancient Babylonian and Arab astronomers. Additional ‘decadic’ changes have an amplitude of 4 or 5 ms and are due to the motion of the liquid part of the Earth’s core. (The centre of the Earth’s core is solid; this was discovered in 1936 by the Danish seismologist Inge Lehmann (1888–1993); her discovery was confirmed most impressively by two British seismologists in 2008, who detected shear waves of the inner core, thus confirming Lehmann’s conclusion. There is a liquid core around the solid core.)

The seasonal and biannual changes of the length of the day – with an amplitude of 0.4 ms over six months, another 0.5 ms over the year, and 0.08 ms over 24 to 26 months – are mainly due to the effects of the *atmosphere*. In the 1950s the availability of precision measurements showed that there is even a 14 and a 28 day period with an amplitude of 0.2 ms, due to the Moon. In the 1970s, when *wind oscillations* with a length scale of about 50 days were discovered, they were also found to alter the length of the day, with an amplitude of about 0.25 ms. However, these last variations are quite irregular.

Ref. 120 Also the oceans influence the rotation of the Earth, due to the tides, ocean currents, wind forcing, and atmospheric pressure forcing. Further effects are due to the ice sheet variations and due to water evaporation and rainfalls. Last but not least, flows in the interior of the Earth, both in the mantle and in the core, change the rotation. For example, earthquakes, plate motion, post-glacial rebound and volcanic eruptions all influence the rotation.

But why does the Earth rotate at all? The rotation originated in the rotating gas cloud at the origin of the Solar System. This connection explains that the Sun and all planets, except two, turn around their axes in the same direction, and that they also all orbit the Sun in that same direction. But the complete story is outside the scope of this text.

Ref. 121 The rotation around its axis is not the only motion of the Earth; it performs other motions as well. This was already known long ago. In 128 BCE, the Greek astronomer Hipparchos discovered what is today called the (*equinoctial*) *precession*. He compared a measurement he made himself with another made 169 years before. Hipparchos found that the Earth’s axis points to different stars at different times. He concluded that the sky was moving. Today we prefer to say that the axis of the Earth is moving. (Why?) During a period of 25 800 years the axis draws a cone with an opening angle of 23.5°. This motion, shown in Figure 107, is generated by the tidal forces of the Moon and the Sun on the equatorial bulge of the Earth that result from its flattening. The Sun and the Moon try to align the axis of the Earth at right angles to the Earth’s path; this torque leads to the precession of the Earth’s axis.

Challenge 271 e

Precession is a motion common to all rotating systems: it appears in planets, spinning tops and atoms. (Precession is also at the basis of the surprise related to the suspended wheel shown on page 244.) Precession is most easily seen in spinning tops, be they suspended or not. An example is shown in Figure 108; for atomic nuclei or planets, just imagine that the suspending wire is missing and the rotating body less flattened. On the Earth, precession leads to the upwelling of deep water in the equatorial Atlantic Ocean and regularly changes the ecology of algae.

In addition, the axis of the Earth is not even fixed relative to the Earth’s surface. In 1884, by measuring the exact angle above the horizon of the celestial North Pole, Friedrich Küstner (1856–1936) found that the axis of the Earth *moves* with respect to the Earth’s crust, as Bessel had suggested 40 years earlier. As a consequence of Küstner’s discovery, the International Latitude Service was created. The *polar motion* Küstner dis-

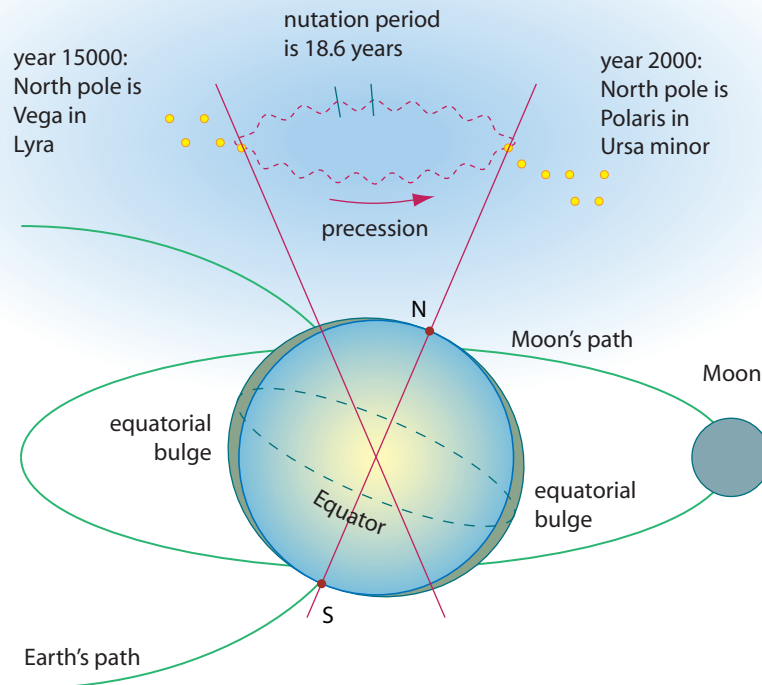


FIGURE 107 The precession and the nutation of the Earth's axis.

covered turned out to consist of three components: a small linear drift – not yet understood – a yearly elliptical motion due to seasonal changes of the air and water masses, and a circular motion\* with a period of about 1.2 years due to fluctuations in the pressure at the bottom of the oceans. In practice, the North Pole moves with an amplitude of about 15 m around an average central position, as shown in Figure 109. Short-term variations of the North Pole position, due to local variations in atmospheric pressure, to weather changes and to the tides, have also been measured. The high precision of the GPS system is possible only with the help of the exact position of the Earth's axis; and only with this knowledge can artificial satellites be guided to Mars or other planets.

Ref. 122

Ref. 123

The details of the motion of the Earth have been studied in great detail. Table 25 gives an overview of the knowledge and precision that is available today.

In 1912, the meteorologist and geophysicist Alfred Wegener (1880–1930) discovered an even larger effect. After studying the shapes of the continental shelves and the geological layers on both sides of the Atlantic, he conjectured that the continents *move*, and that they are all fragments of a single continent that broke up 200 million years ago.\*\*

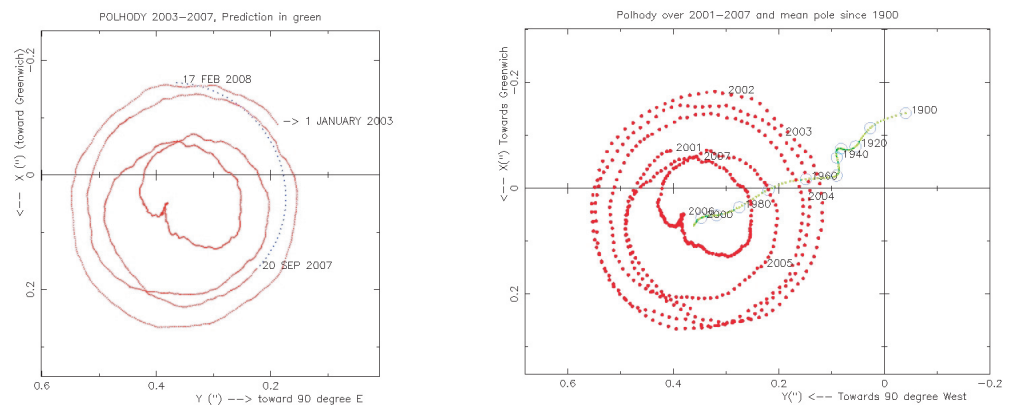
\* The circular motion, a wobble, was predicted by the great Swiss mathematician Leonhard Euler (1707–1783). In a disgusting story, using Euler's and Bessel's predictions and Küstner's data, in 1891 Seth Chandler claimed to be the discoverer of the circular component.

\*\* In this old continent, called Gondwanaland, there was a huge river that flowed westwards from Chad





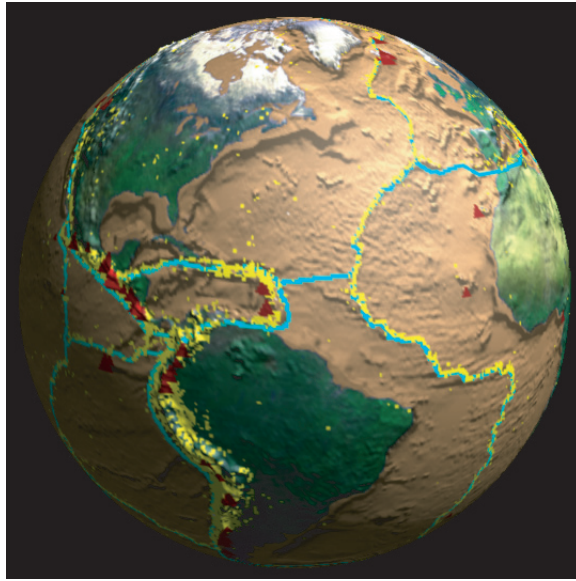
**FIGURE 108**  
Precession of  
a suspended  
spinning top  
(mpg film  
© Lucas  
Barbosa)



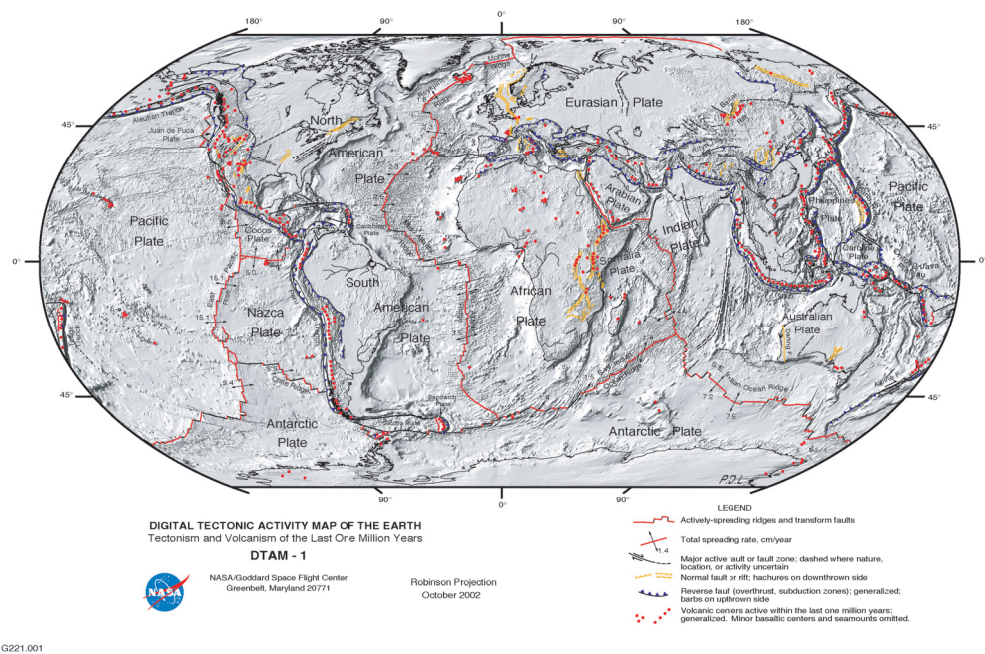
**FIGURE 109** The motion of the North Pole – roughly speaking, the Earth’s effective *polhode* – from 2003 to 2007, including the prediction until 2008 (left) and the average position since 1900 (right) – with 0.1 arcsecond being around 3.1 m on the surface of the Earth – not showing the diurnal and semidiurnal variations of a fraction of a millisecond of arc due to the tides (from [hpiers.obspm.fr/eop-pc](http://hpiers.obspm.fr/eop-pc)).

Even though at first derided across the world, Wegener’s discoveries were correct. Modern satellite measurements, shown in [Figure 110](#), confirm this model. For example, the American continent moves away from the European continent by about 23 mm every

to Guayaquil in Ecuador. After the continent split up, this river still flowed to the west. When the Andes appeared, the water was blocked, and many millions of years later, it reversed its flow. Today, the river still flows eastwards: it is called the Amazon River.

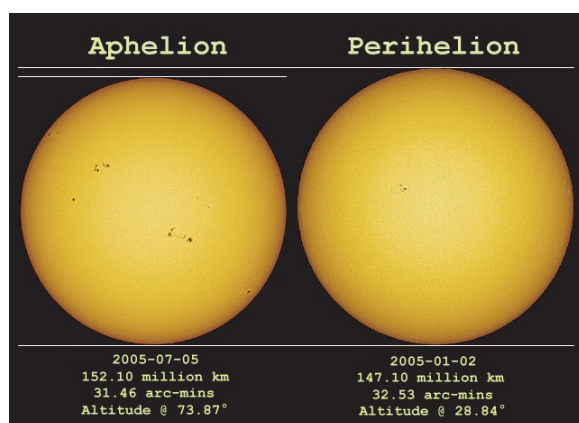


**FIGURE 110** The continental plates are the objects of tectonic motion (HoloGlobe project, NASA).



**FIGURE 111** The tectonic plates of the Earth, with the relative speeds at the boundaries. (© NASA)

year, as shown in **Figure 111**. There are also speculations that this velocity may have been much higher at certain periods in the past. The way to check this is to look at the magnetization of sedimental rocks. At present, this is still a hot topic of research. Following the modern version of the model, called *plate tectonics*, the continents (with a density of



**FIGURE 112** The angular size of the Sun changes due to the elliptical motion of the Earth (© Anthony Ayiomamitis).



**FIGURE 113** Friedrich Wilhelm Bessel (1784–1846).

Page 126  
Vol. III, page 223  
Ref. 124

$2.7 \cdot 10^3 \text{ kg/m}^3$ ) float on the fluid mantle of the Earth (with a density of  $3.1 \cdot 10^3 \text{ kg/m}^3$ ) like pieces of cork on water, and the convection inside the mantle provides the driving mechanism for the motion.

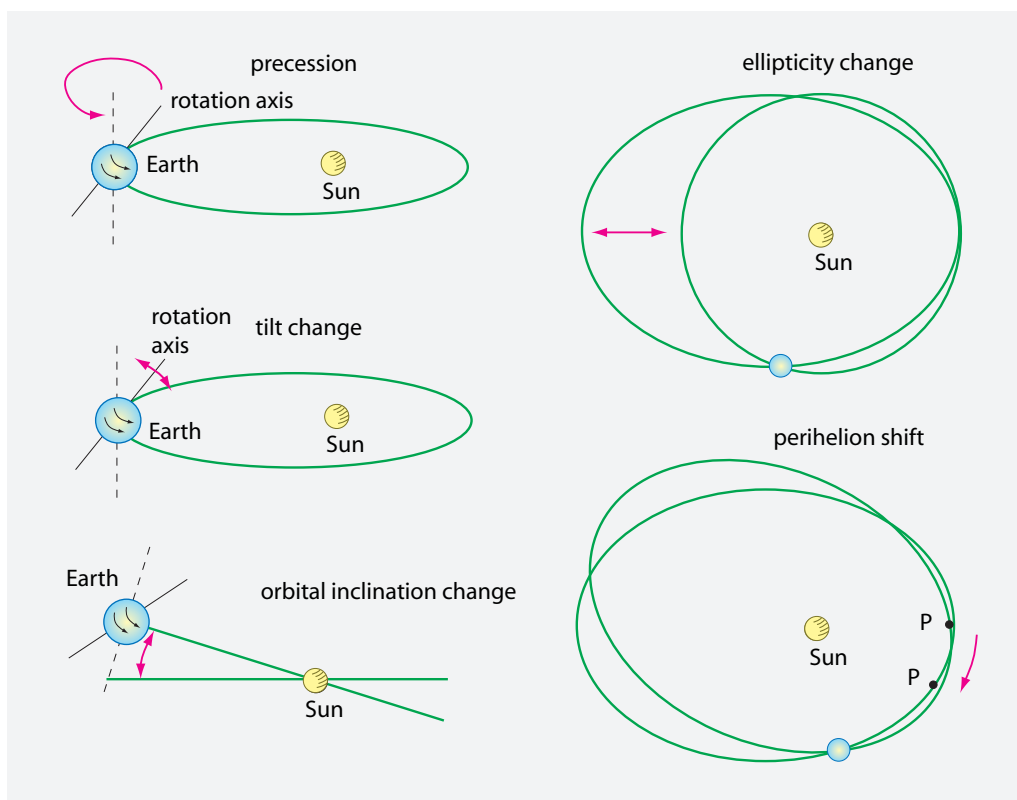
### DOES THE EARTH MOVE?

Also the centre of the Earth is not at rest in the universe. In the third century BCE Aristarchus of Samos maintained that the Earth turns around the Sun. Experiments such as that of Figure 112 confirm that the orbit is an ellipse. However, a fundamental difficulty of the heliocentric system is that the stars look the same all year long. How can this be, if the Earth travels around the Sun? The distance between the Earth and the Sun has been known since the seventeenth century, but it was only in 1837 that Friedrich Wilhelm Bessel\* became the first person to observe the *parallax* of a star. This was a result of extremely careful measurements and complex calculations: he discovered the *Bessel functions* in order to realize it. He was able to find a star, 61 Cygni, whose apparent position changed with the month of the year. Seen over the whole year, the star describes a small ellipse in the sky, with an opening of  $0.588''$  (this is the modern value). After care-

\* Friedrich Wilhelm Bessel (1784–1846), Westphalian astronomer who left a successful business career to dedicate his life to the stars, and became the foremost astronomer of his time.

TABLE 25 Modern measurement data about the motion of the Earth (from [hpiers.obspm.fr/eop-pc](http://hpiers.obspm.fr/eop-pc)).

| OBSERVABLE                                       | SYMBOL                                                     | VALUE                                                       |
|--------------------------------------------------|------------------------------------------------------------|-------------------------------------------------------------|
| Mean angular velocity of Earth                   | $\Omega$                                                   | 72.921 150(1) $\mu\text{rad/s}$                             |
| Nominal angular velocity of Earth (epoch 1820)   | $\Omega_N$                                                 | 72.921 151 467 064 $\mu\text{rad/s}$                        |
| Conventional mean solar day (epoch 1820)         | $d$                                                        | 86 400 s                                                    |
| Conventional sidereal day                        | $d_{\text{si}}$                                            | 86 164.090 530 832 88 s                                     |
| Ratio conv. mean solar day to conv. sidereal day | $k = d/d_{\text{si}}$                                      | 1.002 737 909 350 795                                       |
| Conventional duration of the stellar day         | $d_{\text{st}}$                                            | 86 164.098 903 691 s                                        |
| Ratio conv. mean solar day to conv. stellar day  | $k' = d/d_{\text{st}}$                                     | 1.002 737 811 911 354 48                                    |
| General precession in longitude                  | $p$                                                        | 5.028 792(2) $''/\text{a}$                                  |
| Obliquity of the ecliptic (epoch 2000)           | $\varepsilon_0$                                            | 23° 26' 21.4119 $''$                                        |
| Küstner-Chandler period in terrestrial frame     | $T_{\text{KC}}$                                            | 433.1(1.7) d                                                |
| Quality factor of the Küstner-Chandler peak      | $Q_{\text{KC}}$                                            | 170                                                         |
| Free core nutation period in celestial frame     | $T_{\text{F}}$                                             | 430.2(3) d                                                  |
| Quality factor of the free core nutation         | $Q_{\text{F}}$                                             | $2 \cdot 10^4$                                              |
| Astronomical unit                                | AU                                                         | 149 597 870.691(6) km                                       |
| Sidereal year (epoch 2000)                       | $a_{\text{si}}$                                            | 365.256 363 004 d<br>= 365 d 6 h 9 min 9.76 s               |
| Tropical year                                    | $a_{\text{tr}}$                                            | 365.242 190 402 d<br>= 365 d 5 h 48 min 45.25 s             |
| Mean Moon period                                 | $T_{\text{M}}$                                             | 27.321 661 55(1) d                                          |
| Earth's equatorial radius                        | $a$                                                        | 6 378 136.6(1) m                                            |
| First equatorial moment of inertia               | $A$                                                        | $8.0101(2) \cdot 10^{37} \text{ kg m}^2$                    |
| Longitude of principal inertia axis $A$          | $\lambda_A$                                                | $-14.9291(10)^\circ$                                        |
| Second equatorial moment of inertia              | $B$                                                        | $8.0103(2) \cdot 10^{37} \text{ kg m}^2$                    |
| Axial moment of inertia                          | $C$                                                        | $8.0365(2) \cdot 10^{37} \text{ kg m}^2$                    |
| Equatorial moment of inertia of mantle           | $A_{\text{m}}$                                             | $7.0165 \cdot 10^{37} \text{ kg m}^2$                       |
| Axial moment of inertia of mantle                | $C_{\text{m}}$                                             | $7.0400 \cdot 10^{37} \text{ kg m}^2$                       |
| Earth's flattening                               | $f$                                                        | 1/298.25642(1)                                              |
| Astronomical Earth's dynamical flattening        | $h = (C - A)/C$                                            | 0.003 273 794 9(1)                                          |
| Geophysical Earth's dynamical flattening         | $e = (C - A)/A$                                            | 0.003 284 547 9(1)                                          |
| Earth's core dynamical flattening                | $e_{\text{f}}$                                             | 0.002 646(2)                                                |
| Second degree term in Earth's gravity potential  | $J_2 = -(A + B - 1.082 635 9(1) \cdot 10^{-3} 2C)/(2MR^2)$ |                                                             |
| Secular rate of $J_2$                            | $dJ_2/dt$                                                  | $-2.6(3) \cdot 10^{-11} / \text{a}$                         |
| Love number (measures shape distortion by tides) | $k_2$                                                      | 0.3                                                         |
| Secular Love number                              | $k_{\text{s}}$                                             | 0.9383                                                      |
| Mean equatorial gravity                          | $g_{\text{eq}}$                                            | 9.780 3278(10) $\text{m/s}^2$                               |
| Geocentric constant of gravitation               | $GM$                                                       | $3.986 004 418(8) \cdot 10^{14} \text{ m}^3/\text{s}^2$     |
| Heliocentric constant of gravitation             | $GM_{\odot}$                                               | $1.327 124 420 76(50) \cdot 10^{20} \text{ m}^3/\text{s}^2$ |
| Moon-to-Earth mass ratio                         | $\mu$                                                      | 0.012 300 038 3(5)                                          |



**FIGURE 114** Changes in the Earth's motion around the Sun, as seen from different observers outside the orbital plane.

fully eliminating all other possible explanations, he deduced that the change of position was due to the motion of the Earth around the Sun, when seen from distant stars. From the size of the ellipse he determined the distance to the star to be 105 Pm, or 11.1 light years.

Challenge 272 s

Bessel had thus managed, for the first time, to measure the distance of a star. By doing so he also proved that the Earth is not fixed with respect to the stars in the sky. The motion of the Earth was not a surprise. It confirmed the result of the mentioned aberration of light, discovered in 1728 by James Bradley\* and to be discussed below. When seen from the sky, the Earth indeed revolves around the Sun.

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With the improvement of telescopes, other motions of the Earth were discovered. In 1748, James Bradley announced that there is a small regular *change* of the precession, which he called *nutation*, with a period of 18.6 years and an angular amplitude of  $19.2''$ . Nutation occurs because the plane of the Moon's orbit around the Earth is not exactly

\* James Bradley (b. 1693 Sherborne, d. 1762 Chalford), was an important astronomer. He was one of the first astronomers to understand the value of precise measurement, and thoroughly modernized the Greenwich observatory. He discovered, independently of Eustachio Manfredi, the aberration of light, and showed with it that the Earth moves. In particular, the discovery allowed him to measure the speed of light and confirm the value of 0.3 Gm/s. He later discovered the nutation of the Earth's axis.



Challenge 273 e

the same as the plane of the Earth's orbit around the Sun. Are you able to confirm that this situation produces nutation?

Ref. 125

Astronomers also discovered that the  $23.5^\circ$  tilt – or *obliquity* – of the Earth's axis, the angle between its intrinsic and its orbital angular momentum, actually changes from  $22.1^\circ$  to  $24.5^\circ$  with a period of 41 000 years. This motion is due to the attraction of the Sun and the deviations of the Earth from a spherical shape. In 1941, during the Second World War, the Serbian astronomer Milutin Milankovitch (1879–1958) retreated into solitude and explored the consequences. In his studies he realized that this 41 000 year period of the obliquity, together with an average period of 22 000 years due to precession,\* gives rise to more than 20 *ice ages* in the last 2 million years. This happens through stronger or weaker irradiation of the poles by the Sun. The changing amounts of melted ice then lead to changes in average temperature. The last ice age had its peak about 20 000 years ago and ended around 11 800 years ago; the next is still far away. A spectacular confirmation of the relation between ice age cycles and astronomy came through measurements of oxygen isotope ratios in ice cores and sea sediments, which allow the average temperature over the past million years to be tracked. [Figure 115](#) shows how closely the temperature follows the changes in irradiation due to changes in obliquity and precession.

The Earth's orbit also changes its *eccentricity* with time, from completely circular to slightly oval and back. However, this happens in very complex ways, not with periodic regularity, and is due to the influence of the large planets of the solar system on the Earth's orbit. The typical time scale is 100 000 to 125 000 years.

In addition, the Earth's orbit changes in *inclination* with respect to the orbits of the other planets; this seems to happen regularly every 100 000 years. In this period the inclination changes from  $+2.5^\circ$  to  $-2.5^\circ$  and back.

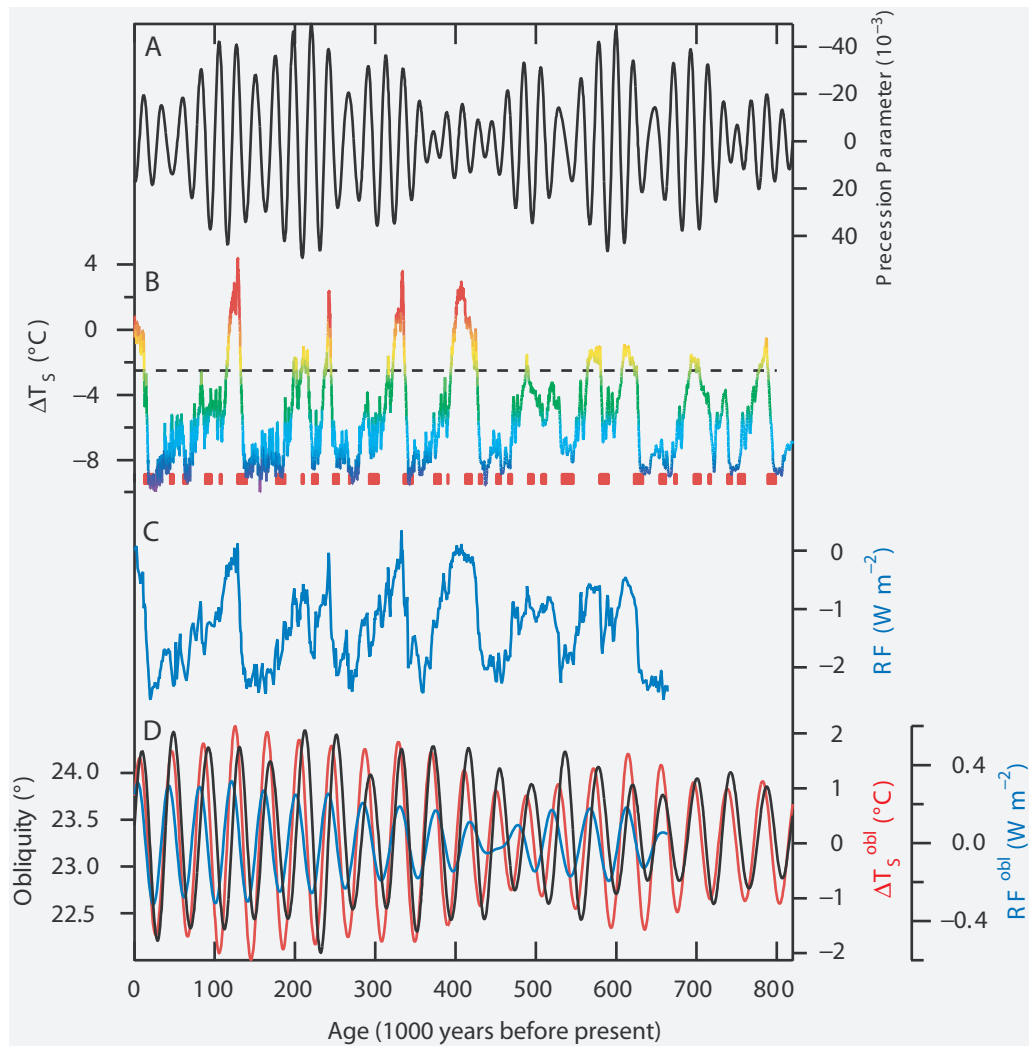
Even the direction in which the ellipse points changes with time. This so-called *perihelion shift* is due in large part to the influence of the other planets; a small remaining part will be important in the chapter on general relativity. The perihelion shift of Mercury was the first piece of data confirming Einstein's theory.

Obviously, the length of the year also changes with time. The measured variations are of the order of a few parts in  $10^{11}$  or about 1 ms per year. However, knowledge of these changes and of their origins is much less detailed than for the changes in the Earth's rotation.

Ref. 126

The next step is to ask whether the Sun itself moves. Indeed it does. Locally, it moves with a speed of 19.4 km/s towards the constellation of Hercules. This was shown by William Herschel in 1783. But globally, the motion is even more interesting. The diameter of the galaxy is at least 100 000 light years, and we are located 26 000 light years from the centre. (This has been known since 1918; the centre of the galaxy is located in the direction of Sagittarius.) At our position, the galaxy is 1 300 light years thick; presently, we are 68 light years 'above' the centre plane. The Sun, and with it the Solar System, takes about 225 million years to turn once around the galactic centre, its orbital velocity being around 220 km/s. It seems that the Sun will continue moving away from the galaxy plane until it is about 250 light years above the plane, and then move back, as shown in [Figure 116](#). The oscillation period is estimated to be around 62 million years, and has been suggested as

\* In fact, the 25 800 year precession leads to three insolation periods, of 23 700, 22 400 and 19 000 years, due to the interaction between precession and perihelion shift.

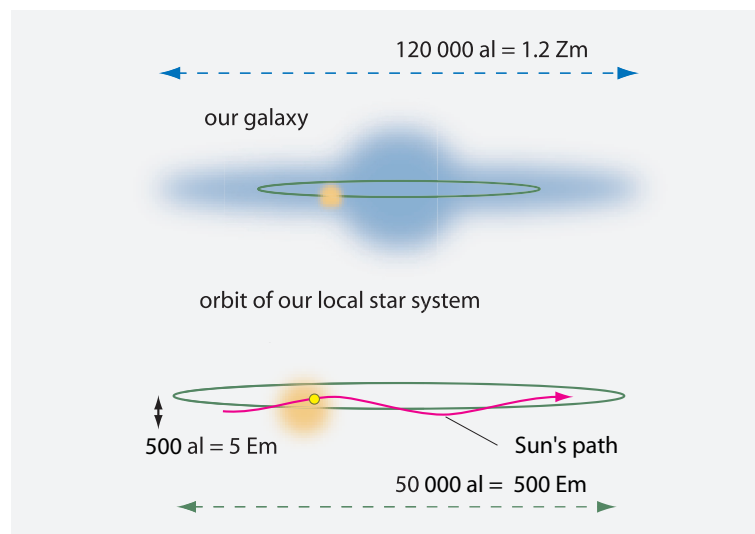


**FIGURE 115** Modern measurements showing how Earth's precession parameter (black curve A) and obliquity (black curve D) influence the average temperature (coloured curve B) and the irradiation of the Earth (blue curve C) over the past 800 000 years: the obliquity deduced by Fourier analysis from the irradiation data RF (blue curve D) and the obliquity deduced by Fourier analysis from the temperature (red curve D) match the obliquity known from astronomical data (black curve D); sharp cooling events took place whenever the obliquity rose while the precession parameter was falling (marked red below the temperature curve) (© Jean Jouzel/Science from Ref. 125).

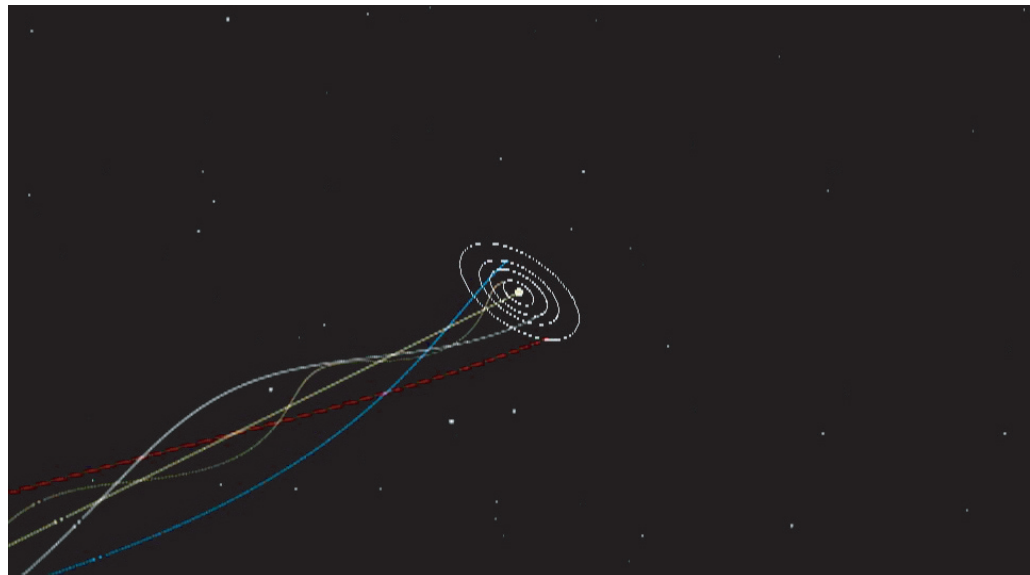
the mechanism for the mass extinctions of animal life on Earth, possibly because some gas cloud or some cosmic radiation source may be periodically encountered on the way. The issue is still a hot topic of research.

The motion of the Sun around the centre of the Milky Way implies that the planets of the Solar System can be seen as forming helices around the Sun. Figure 117 illustrates their helical path.

We turn around the galaxy centre because the formation of galaxies, like that of planetary systems, always happens in a whirl. By the way, can you confirm from your own



**FIGURE 116** The motion of the Sun around the galaxy.



**FIGURE 117** The helical motion of the first four planets around the path traced by the Sun during its travel around the centre of the Milky Way. Brown: Mercury, white: Venus, blue: Earth, red: Mars. (QuickTime film © Rhys Taylor at [www.rhysy.net](http://www.rhysy.net)).

**Challenge 274 s** observation that our galaxy itself rotates?

Finally, we can ask whether the galaxy itself moves. Its motion can indeed be observed because it is possible to give a value for the motion of the Sun through the universe, defining it as the motion against the background radiation. This value has been measured to be 370 km/s. (The velocity of the *Earth* through the background radiation of course depends on the season.) This value is a combination of the motion of the Sun around the galaxy centre and of the motion of the galaxy itself. This latter motion is due to the

Ref. 127



**FIGURE 118** Driving through snowflakes shows the effects of relative motion in three dimensions. Similar effects are seen when the Earth speeds through the universe. (© Neil Provo at [neilprovo.com](http://neilprovo.com)).

gravitational attraction of the other, nearby galaxies in our local group of galaxies.\*

In summary, the Earth really moves, and it does so in rather complex ways. As Henri Poincaré would say, if we are in a given spot today, say the Panthéon in Paris, and come back to the same spot tomorrow at the same time, we are in fact 31 million kilometres away. This state of affairs would make time travel extremely difficult even if it were possible (which it is not); whenever you went back to the past, you would have to get to the old spot exactly!

#### IS VELOCITY ABSOLUTE? – THE THEORY OF EVERYDAY RELATIVITY

Why don't we feel all the motions of the Earth? The two parts of the answer were already given in 1632. First of all, as Galileo explained, we do not feel the accelerations of the Earth because the effects they produce are too small to be detected by our senses. Indeed, many of the mentioned accelerations do induce measurable effects only in high-precision experiments, e.g. in atomic clocks.

But the second point made by Galileo is equally important: it is impossible to feel the high speed at which we are moving. We do not feel translational, unaccelerated motions because this is impossible *in principle*. Galileo discussed the issue by comparing the observations of two observers: one on the ground and another on the most modern means of unaccelerated transportation of the time, a ship. Galileo asked whether a man on the

\* This is roughly the end of the ladder. Note that the expansion of the universe, to be studied later, produces no motion.

Challenge 275 e

ground and a man in a ship moving at constant speed experience (or ‘feel’) anything different. Einstein used observers in trains. Later it became fashionable to use travellers in rockets. (What will come next?) Galileo explained that only *relative* velocities between bodies produce effects, not the absolute values of the velocities. For the senses and for all measurements we find:

- ▷ There is no difference between constant, undisturbed motion, however rapid it may be, and rest. This is called *Galileo’s principle of relativity*.

Indeed, in everyday life we feel motion only if the means of transportation trembles – thus if it accelerates – or if we move against the air. Therefore Galileo concludes that two observers in straight and undisturbed motion against each other cannot say who is ‘really’ moving. Whatever their relative speed, neither of them ‘feels’ in motion.\*

*Rest is relative.* Or more clearly: rest is an observer-dependent concept. This result of Galilean physics is so important that Poincaré introduced the expression ‘theory of relativity’ and Einstein repeated the principle explicitly when he published his famous theory of special relativity. However, these names are awkward. Galilean physics is also a theory of relativity! The relativity of rest is common to *all* of physics; it is an essential aspect of motion.

In summary, undisturbed or uniform motion has no observable effect; only *change* of motion does. Velocity cannot be felt; acceleration can. As a result, every physicist can deduce something simple about the following statement by Wittgenstein:

---

\* In 1632, in his *Dialogo*, Galileo writes: ‘Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal: jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that, you will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship’s motion is common to all the things contained in it, and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted.’ (Translation by Stillman Drake)



Daß die Sonne morgen aufgehen wird, ist eine Hypothese; und das heißt: wir *wissen* nicht, ob sie aufgehen wird.\*

Challenge 276 s The statement is *wrong*. Can you explain why Wittgenstein erred here, despite his strong desire not to?

### IS ROTATION RELATIVE?

When we turn rapidly, our arms lift. Why does this happen? How can our body detect whether we are rotating or not? There are two possible answers. The first approach, promoted by Newton, is to say that there is an absolute space; whenever we rotate against this space, the system reacts. The other answer is to note that whenever the arms lift, the stars also rotate, and in exactly the same manner. In other words, our body detects rotation because we move against the average mass distribution in space.

The most cited discussion of this question is due to Newton. Instead of arms, he explored the water in a rotating bucket. In a rotating bucket, the water surface forms a concave shape, whereas the surface is flat for a non-rotating bucket. Newton asked why this is the case. As usual for philosophical issues, Newton's answer was guided by the mysticism triggered by his father's early death. Newton saw absolute space as a mystical and religious concept and was not even able to conceive an alternative. Newton thus saw rotation as an *absolute* type of motion. Most modern scientists have fewer personal problems and more common sense than Newton; as a result, today's consensus is that rotation effects are due to the mass distribution in the universe:

▷ Rotation is relative.

A number of high-precision experiments confirm this conclusion; thus it is also part of Einstein's theory of relativity.

### CURIOSITIES AND FUN CHALLENGES ABOUT ROTATION AND RELATIVITY

Challenge 277 e When travelling in the train, you can test Galileo's statement about the everyday relativity of motion. Close your eyes and ask somebody to turn you around several times: are you able to say in which direction the train is running?

\* \*

Challenge 278 s A good bathroom scales, used to determine the weight of objects, does not show a constant weight when you step on it and stay motionless. Why not?

\* \*

Challenge 279 s If a gun located at the Equator shoots a bullet vertically, where does the bullet fall?

\* \*

Challenge 280 s Why are most rocket launch sites as near as possible to the Equator?

---

\* 'That the Sun will rise to-morrow, is an hypothesis; and that means that we do not *know* whether it will rise.' This well-known statement is found in Ludwig Wittgenstein, *Tractatus*, 6.36311.

\* \*

At the Equator, the speed of rotation of the Earth is 464 m/s, or about Mach 1.4; the latter number means that it is 1.4 times the speed of sound. This supersonic motion has two intriguing consequences.

First of all, the rotation speed determines the size of typical weather phenomena. This size, the so-called *Rossby radius*, is given by the speed of sound (or some other typical speed) divided by twice the local rotation speed, multiplied by the radius of the Earth. At moderate latitudes, the Rossby radius is about 2000 km. This is a sizeable fraction of the Earth's radius, so that only a few large weather systems are present on Earth at any specific time. If the Earth rotated more slowly, the weather would be determined by short-lived, local flows and have no general regularities. If the Earth rotated more rapidly, the weather would be much more violent – as on Jupiter – but the small Rossby radius implies that large weather structures have a huge lifetime, such as the red spot on Jupiter, which lasted for several centuries. In a sense, the rotation of the Earth has the speed that provides the most interesting weather.

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The other consequence of the value of the Earth's rotation speed concerns the thickness of the atmosphere. Mach 1 is also, roughly speaking, the thermal speed of air molecules. This speed is sufficient for an air molecule to reach the characteristic height of the atmosphere, about 6 km. On the other hand, the speed of rotation  $\Omega$  of the Earth determines its departure  $h$  from sphericity: the Earth is flattened, as we saw above. Roughly speaking, we have  $gh = \Omega^2 R^2 / 2$ , or about 12 km. (This is correct to within 50 %, the actual value is 21 km.) We thus find that the speed of rotation of the Earth implies that its flattening is comparable to the thickness of the atmosphere.

\* \*

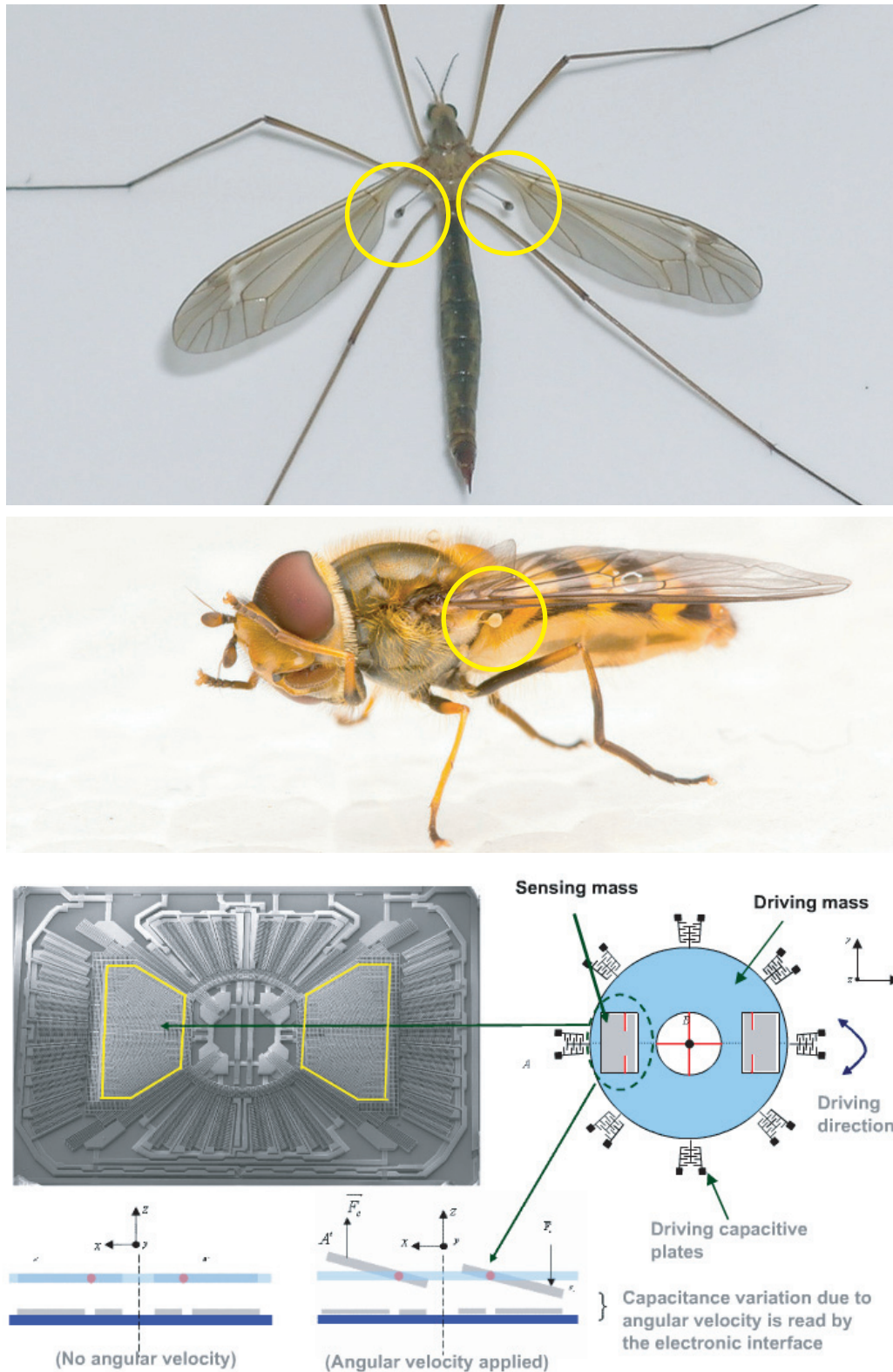
Challenge 281 e

The Coriolis effect influences rivers and their shores. This surprising connection was made in 1860 by Karl Ernst von Baer who found that in Russia, many rivers flowing north in lowlands had right shores that are steep and high, and left shores that are low and flat. (Can you explain the details?) He also found that rivers in the southern hemisphere show the opposite effect.

\* \*

The Coriolis effect saves lives and helps people. Indeed, it has an important application for navigation systems; the typical uses are shown in [Figure 119](#). Insects use vibrating masses to stabilize their orientation, to determine their direction of travel and to find their way. Most two-winged insects, or diptera, use *vibrating halteres* for navigation: in particular, bees, house-flies, hover-flies and crane flies use them. Other insects, such as moths, use *vibrating antennae* for navigation. Cars, satellites, mobile phones, remote-controlled helicopter models, and computer games also use tiny vibrating masses as orientation and navigation sensors, in exactly the same way as insects do.

In all these navigation applications, one or a few tiny masses are made to vibrate; if the system to which they are attached turns, the change of orientation leads to a Coriolis effect. The effect is measured by detecting the ensuing change in geometry; the change, and thus the signal strength, depends on the angular velocity and its direction. Such orientation sensors are therefore called *vibrating Coriolis gyroscopes*. Their development



**FIGURE 119** The use of the Coriolis effect in insects – here a crane fly and a hovering fly – and in micro-electromechanic systems (size about a few mm); all provide navigation signals to the systems to which they are attached (© Pinzo, Sean McCann, ST Microelectronics).

and production is a sizeable part of high-tech business – and of biological evolution.

\* \*

Challenge 282 e A wealthy and quirky customer asked his architect to plan and build a house whose four walls all faced south. How did the architect realize the request?

\* \*

Ref. 128 Would travelling through interplanetary space be healthy? People often fantasize about long trips through the cosmos. Experiments have shown that on trips of long duration, cosmic radiation, bone weakening, muscle degeneration and psychological problems are the biggest dangers. Many medical experts question the viability of space travel lasting longer than a couple of years. Other dangers are rapid sunburn, at least near the Sun, and exposure to the vacuum. So far only one man has experienced vacuum without protection. He lost consciousness after 14 seconds, but survived unharmed.

\* \*

Challenge 283 s In which direction does a flame lean if it burns inside a jar on a rotating turntable?

\* \*

Challenge 284 s Galileo's principle of everyday relativity states that it is impossible to determine an absolute velocity. It is equally impossible to determine an absolute position, an absolute time and an absolute direction. Is this correct?

\* \*

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Challenge 285 s Does *centrifugal acceleration* exist? Most university students go through the shock of meeting a teacher who says that it doesn't because it is a 'fictitious' quantity, in the face of what one experiences every day in a car when driving around a bend. Simply ask the teacher who denies it to define 'existence'. (The definition physicists usually use is given later on.) Then check whether the definition applies to the term and make up your own mind.

Whether you like the term 'centrifugal acceleration' or avoid it by using its negative, the so-called *centripetal acceleration*, you should know how it is calculated. We use a simple trick. For an object in circular motion of radius  $r$ , the magnitude  $v$  of the velocity  $\mathbf{v} = d\mathbf{x}/dt$  is  $v = 2\pi r/T$ . The vector  $\mathbf{v}$  behaves over time exactly like the position of the object: it rotates continuously. Therefore, the magnitude  $a$  of the centrifugal/centripetal acceleration  $\mathbf{a} = d\mathbf{v}/dt$  is given by the corresponding expression, namely  $a = 2\pi v/T$ . Eliminating  $T$ , we find that the centrifugal/centripetal acceleration  $a$  of a body rotating at speed  $v$  at radius  $r$  is given by

$$a = \frac{v^2}{r} = \omega^2 r . \quad (36)$$

This is the acceleration we feel when sitting in a car that goes around a bend.

\* \*

Rotational motion holds a little surprise for anybody who studies it carefully. Angular momentum is a quantity with a magnitude and a direction. However, it is *not* a vector,

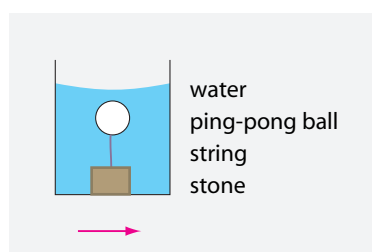


FIGURE 120 How does the ball move when the jar is accelerated in direction of the arrow?

Challenge 286 e

Challenge 287 e

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as any mirror shows. The angular momentum of a body circling in a plane parallel to a mirror behaves in a different way from a usual arrow: its mirror image is not reflected if it points towards the mirror! You can easily check this for yourself. For this reason, angular momentum is called a *pseudo-vector*. (Are rotations pseudo-vectors?) The fact has no important consequences in classical physics; but we have to keep it in mind for later, when we explore nuclear physics.

\* \*

Challenge 288 s

What is the best way to transport a number of full coffee or tea cups while at the same time avoiding spilling any precious liquid?

\* \*

Challenge 289 s

A ping-pong ball is attached by a string to a stone, and the whole is put under water in a jar. The set-up is shown in Figure 120. Now the jar is accelerated horizontally, for example in a car. In which direction does the ball move? What do you deduce for a jar at rest?

\* \*

Challenge 290 s

The Moon recedes from the Earth by 3.8 cm a year, due to friction. Can you find the mechanism responsible for the effect?

\* \*

What are earthquakes? *Earthquakes* are large examples of the same process that make a door squeak. The continental plates correspond to the metal surfaces in the joints of the door.

Earthquakes can be described as energy sources. The Richter scale is a direct measure of this energy. The *Richter magnitude*  $M_s$  of an earthquake, a pure number, is defined from its energy  $E$  in joule via

$$M_s = \frac{\log(E/1 \text{ J}) - 4.8}{1.5}. \quad (37)$$

The strange numbers in the expression have been chosen to put the earthquake values as near as possible to the older, qualitative Mercalli scale (now called EMS98) that classifies the intensity of earthquakes. However, this is not fully possible; the most sensitive instruments today detect earthquakes with magnitudes of  $-3$ . The highest value ever measured



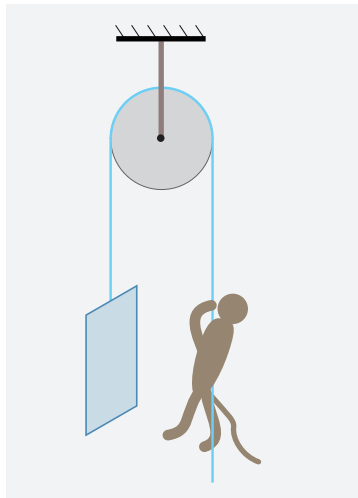


FIGURE 121 What happens when the ape climbs?

Challenge 291 s was a Richter magnitude of 10, in Chile in 1960. Magnitudes above 12 are probably impossible. Can you show why?

\* \*

Challenge 292 ny What is the motion of the point on the surface of the Earth that has Sun in its zenith – i.e., vertically above it – when seen on a map of the Earth during one day? And day after day?

\* \*

Challenge 293 s Can it happen that a satellite dish for geostationary TV satellites focuses the sunshine onto the receiver?

\* \*

Challenge 294 s Why is it difficult to fire a rocket from an aeroplane in the direction opposite to the motion of the plane?

\* \*

Challenge 295 s An ape hangs on a rope. The rope hangs over a wheel and is attached to a mass of equal weight hanging down on the other side, as shown in Figure 121. The rope and the wheel are massless and frictionless. What happens when the ape climbs the rope?

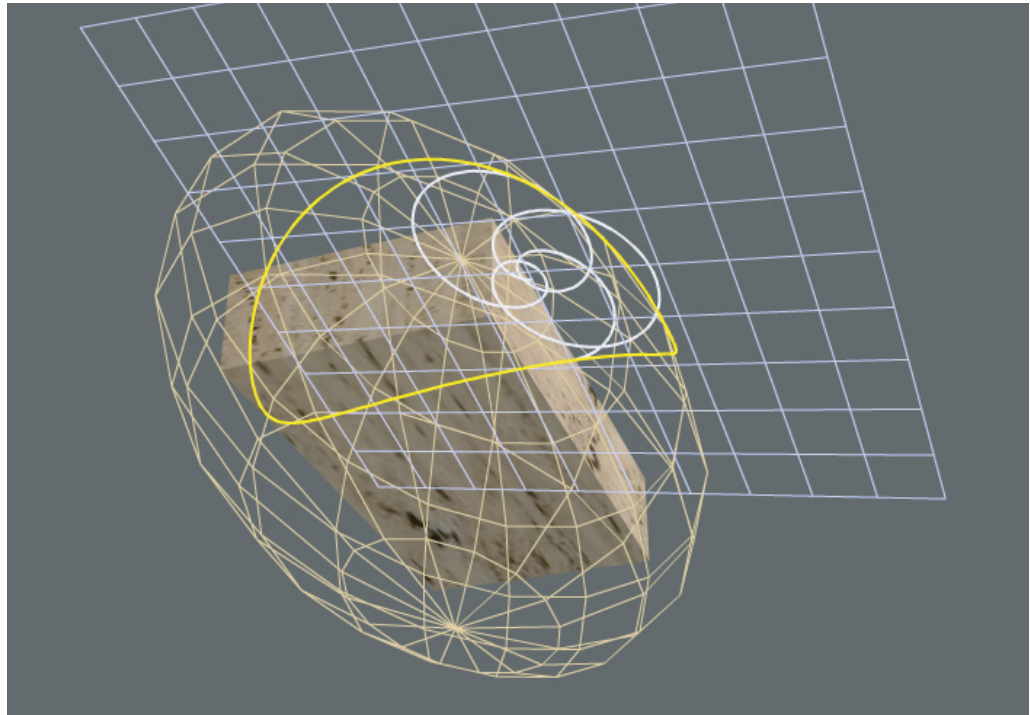
\* \*

Challenge 296 s Can a water skier move with a higher speed than the boat pulling him?

\* \*

Challenge 297 d You might know the ‘Dynabee’, a hand-held gyroscopic device that can be accelerated to high speed by proper movements of the hand. How does it work?

\* \*



**FIGURE 122** The motion of the angular velocity of a tumbling brick. The tip of the angular velocity vector moves along the yellow curve, the *polhode*. It moves together with the tumbling object, as does the elliptical mesh representing the energy ellipsoid; the *herpolhode* is the white curve and lies in the plane bright blue cross pattern that represents the invariable plane (see text). The animation behind the screenshot illustrates the well-known nerd statement: *the polhode rolls without slipping on the herpolhode lying in the invariable plane*. The full animation is available online at [www.ialms.net/sim/3d-rigid-body-simulation/](http://www.ialms.net/sim/3d-rigid-body-simulation/). Any rotating, irregular, free rigid body follows such a motion. For the Earth, which is rotating and irregular, but neither fully free nor fully rigid, this description is only an approximation. (© Svetoslav Zabunov)

Challenge 298 s

It is possible to make a spinning top with a metal paper clip. It is even possible to make one of those tops that turn onto their head when spinning. Can you find out how?

\* \*

Challenge 299 s

The moment of inertia of a body depends on the shape of the body; usually, the angular momentum and the angular velocity do not point in the same direction. Can you confirm this with an example?

\* \*

Challenge 300 s

What is the moment of inertia of a homogeneous sphere?

\* \*

Challenge 301 s

The complete moment of inertia of a rigid body is determined by the values along its three principal axes. These values are all equal for a sphere and for a cube. Does it mean that it is impossible to distinguish a sphere from a cube by their inertial behaviour?

\* \*

Here is some mathematical fun about the rotation of a free rigid body. Even for a free rotating rigid body, such as a brick rotating in free space, the angular velocity is, in general, *not* constant: the brick *tumbles* while rotating. In this motion, both energy and angular momentum are constant, but the angular velocity is not. In particular, not even the direction of angular velocity is constant; in other words, the north pole changes with time. In fact, the north pole changes with time both for an observer on the body and for an observer in free space. How does the north pole, the end point of the angular velocity vector, move?

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Challenge 302 e

The moment of inertia is a tensor and can thus be represented by an ellipsoid. In addition, the motion of a free rigid body is described, in the angular velocity space, by the kinetic energy ellipsoid – because its rotation energy is constant. When a free rigid body moves, the energy ellipsoid – not the moment of inertia ellipsoid – *rolls* on the invariable plane that is perpendicular to the initial (and constant) angular momentum of the body. This is the mathematical description of the tumbling motion. The curve traced by the angular velocity, or the extended north pole of the body, on the invariable plane is called the *herpolhode*. It is an involved curve that results from two superposed conical motions. For an observer on the rotating body, another curve is more interesting. The pole of the rotating body traces a curve on the energy ellipsoid – which is itself attached to the body. This curve is called the *polhode*. The polhode is a closed curve in three dimensions, the herpolhode is an open curve in two dimensions. The curves were named by Louis Poincaré and are especially useful for the description of the motion of rotating irregularly shaped bodies. For a complete visualization, see the excellent website [www.ialms.net/sim/3d-rigid-body-simulation/](http://www.ialms.net/sim/3d-rigid-body-simulation/).

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Challenge 303 e

The polhode is a circle only if the rigid body has rotational symmetry; the pole motion is then called *precession*; we encountered it above. As shown in Figure 109, the measured polhode of the Earth is not of the expected shape; this irregularity is due to several effects, among them the non-rigidity of our planet.

Even though the angular momentum of a freely tumbling brick is fixed in space, it is not fixed in the body frame. Can you confirm this? In the body frame, the end of the angular momentum of a tumbling brick moves along still another curve, given by the intersection of a sphere and an ellipsoid, as Jacques Binet pointed out.

\* \*

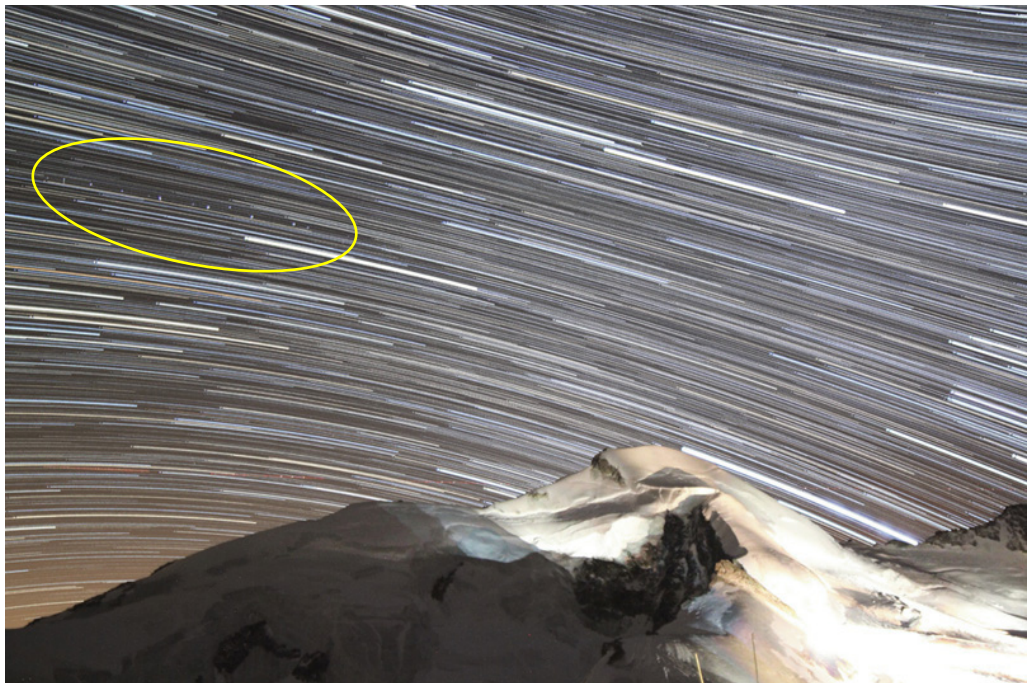
Challenge 304 s

Is it true that the Moon in the first quarter in the northern hemisphere looks like the Moon in the last quarter in the southern hemisphere?

\* \*

An impressive confirmation that the Earth is a sphere can be seen at sunset, if we turn, against our usual habit, our back to the Sun. On the eastern sky we can then see the impressive rise of the Earth's shadow. We can admire the vast shadow rising over the whole horizon, clearly having the shape of a segment of a huge circle. Figure 124 shows an example. In fact, more precise investigations show that it is not the shadow of the Earth alone, but the shadow of its ionosphere.

\* \*



**FIGURE 123** Long exposures of the stars at night – one when facing north, above the Gemini telescope in Hawaii, and one above the Alps that includes the celestial equator, with the geostationary satellites on it (© Gemini Observatory/AURA, Michael Kunze).





**FIGURE 124** The shadow of the Earth – here a panoramic photograph taken at the South Pole – shows that the Earth is round (© Ian R. Rees).

**Challenge 305 s** How would **Figure 123** look if taken at the Equator?

\* \*

Precision measurements show that not all planets move in exactly the same plane. Mercury shows the largest deviation. In fact, no planet moves exactly in an ellipse, nor even in a plane around the Sun. Almost all of these effects are too small and too complex to explain here.

\* \*

**Challenge 306 e**

Since the Earth is round, there are many ways to drive from one point on the Earth to another along a circle segment. This freedom of choice has interesting consequences for volleyballs and for men watching women. Take a volleyball and look at its air inlet. If you want to move the inlet to a different position with a simple rotation, you can choose the rotation axis in many different ways. Can you confirm this? In other words, when we look in a given direction and then want to look in another, the eye can accomplish this change in different ways. The option chosen by the human eye had already been studied by medical scientists in the eighteenth century. It is called *Listing's 'law'*.<sup>\*</sup> It states that all axes that nature chooses lie in one plane. Can you imagine its position in space? Many men have a real interest that this mechanism is strictly followed; if not, looking at women on the beach could cause the muscles moving their eyes to get knotted up.

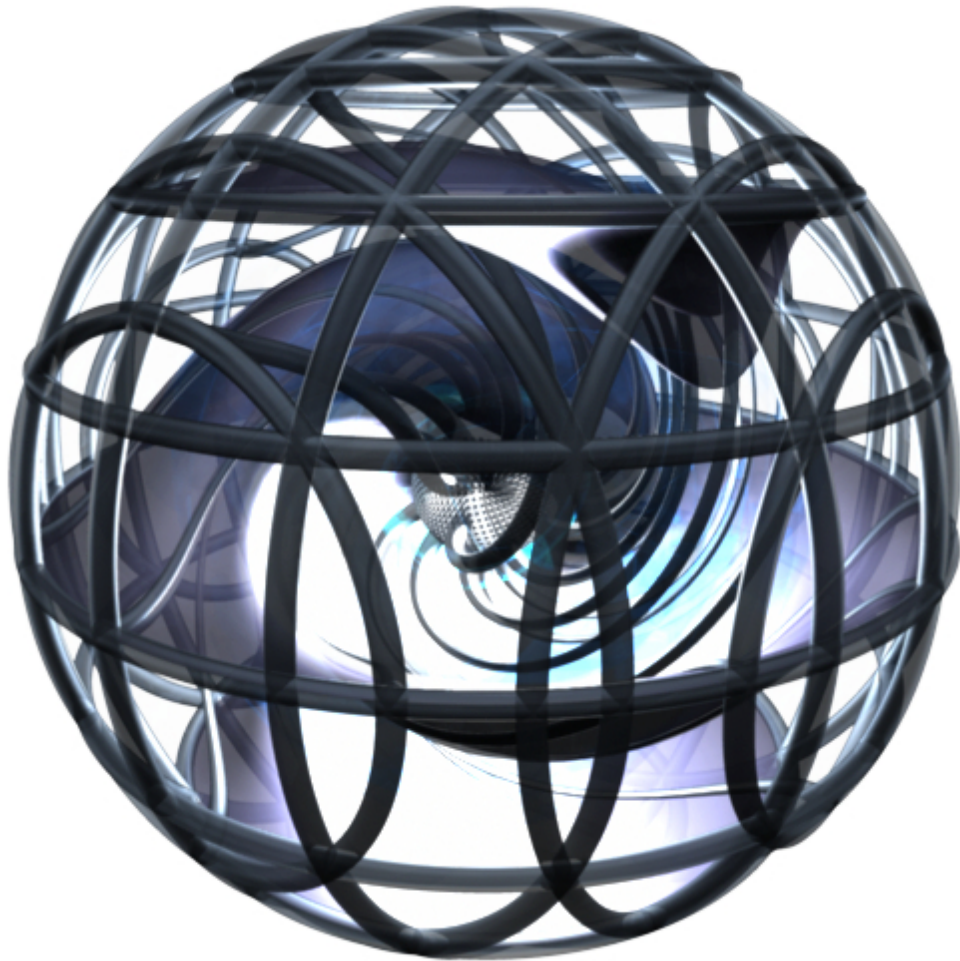
**Challenge 307 s**

\* \*

Imagine to cut open a soft mattress, glue a steel ball into it, and glue the mattress together again. Now imagine that we use a magnetic field to rotate the steel ball glued inside. Intu-

<sup>\*</sup> If you are interested in learning in more detail how nature and the eye cope with the complexities of three dimensions, see the [schorlab.berkeley.edu/vilis/whatisLL.htm](http://schorlab.berkeley.edu/vilis/whatisLL.htm) and [www.physpharm.fmd.uwo.ca/undergrad/llconsequencesweb/ListingsLaw/perceptual1.htm](http://www.physpharm.fmd.uwo.ca/undergrad/llconsequencesweb/ListingsLaw/perceptual1.htm) websites.





**FIGURE 125** A steel ball glued in a mattress can rotate forever. (QuickTime film © Jason Hise).

itively, we think that the ball can only be rotated by a finite angle, whose value would be limited by the elasticity of the mattress. But in reality, the steel ball can be rotated *infinitely often!* This surprising possibility is a consequence of the tethered rotation shown in **Figure 54** and **Figure 55**. Such a continuous rotation in a mattress is shown in **Figure 125**. And despite its fascination, nobody has yet realized the feat. Can you?

Page 90  
Challenge 308 r

#### LEGS OR WHEELS? – AGAIN

The acceleration and deceleration of standard wheel-driven cars is never much greater than about  $1g = 9.8 \text{ m/s}^2$ , the acceleration due to gravity on our planet. Higher accelerations are achieved by motorbikes and racing cars through the use of suspensions that divert weight to the axes and by the use of spoilers, so that the car is pushed downwards



**FIGURE 126** A basilisk lizard (*Basiliscus basiliscus*) running on water, with a total length of about 25 cm, showing how the propulsive leg pushes into the water (© TERRA).

with more than its own weight. Modern spoilers are so efficient in pushing a car towards the track that racing cars could race on the roof of a tunnel without falling down.

Through the use of special tyres the downward forces produced by aerodynamic effects are transformed into grip; modern racing tyres allow forward, backward and sideways accelerations (necessary for speed increase, for braking and for turning corners) of about 1.1 to 1.3 times the load. Engineers once believed that a factor 1 was the theoretical limit and this limit is still sometimes found in textbooks; but advances in tyre technology, mostly by making clever use of interlocking between the tyre and the road surface as in a gear mechanism, have allowed engineers to achieve these higher values. The highest accelerations, around 4 *g*, are achieved when part of the tyre melts and glues to the surface. Special tyres designed to make this happen are used for dragsters, but high-performance radio-controlled model cars also achieve such values.

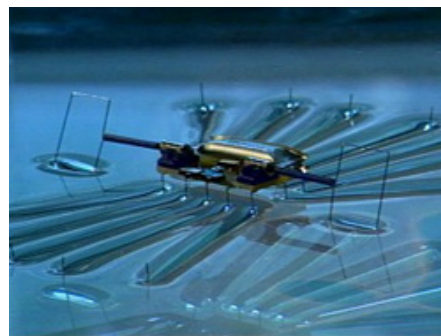
How do wheels compare to using legs? High jump athletes can achieve peak accelerations of about 2 to 4 *g*, cheetahs over 3 *g*, bushbabies up to 13 *g*, locusts about 18 *g*, and fleas have been measured to accelerate about 135 *g*. The maximum acceleration known for animals is that of click beetles, a small insect able to accelerate at over  $2000 \text{ m/s}^2 = 200 \text{ g}$ , about the same as an airgun pellet when fired. Legs are thus definitively more efficient accelerating devices than wheels – a cheetah can easily beat any car or motorbike – and evolution developed legs, instead of wheels, to improve the chances of an animal in danger getting to safety.

In short, legs *outperform* wheels. But there are other reasons for using legs instead of wheels. (Can you name some?) For example, legs, unlike wheels, allow walking on water. Most famous for this ability is the *basilisk*,\* a lizard living in Central America and shown in Figure 126. This reptile is up to 70 cm long and has a mass of up to 500 g. It looks like a miniature *Tyrannosaurus rex* and is able to run over water surfaces on its hind legs. The motion has been studied in detail with high-speed cameras and by measurements using aluminium models of the animal's feet. The experiments show that the feet slapping on the water provides only 25 % of the force necessary to run above water; the other 75 % is provided by a pocket of compressed air that the basilisks create between their feet and the water once the feet are inside the water. In fact, basilisks mainly walk on air. (Both

\* In the Middle Ages, the term 'basilisk' referred to a mythical monster supposed to appear shortly before the end of the world. Today, it is a small reptile in the Americas.



**FIGURE 127** A water strider, total size about 10 mm (© Charles Lewallen).



**FIGURE 128** A water walking robot, total size about 20 mm (© AIP).

Ref. 131 effects used by basilisks are also found in fast canoeing.) It was calculated that humans are also able to walk on water, provided their feet hit the water with a speed of 100 km/h using the simultaneous physical power of 15 sprinters. Quite a feat for all those who ever did so.

There is a second method of walking and running on water; this second method even allows its users to remain immobile on top of the water surface. This is what water striders, insects of the family *Gerridae* with an overall length of up to 15 mm, are able to do (together with several species of spiders), as shown in Figure 127. Like all insects, the water strider has six legs (spiders have eight). The water strider uses the back and front legs to hover over the surface, helped by thousands of tiny hairs attached to its body. The hairs, together with the surface tension of water, prevent the strider from getting wet. If you put shampoo into the water, the water strider sinks and can no longer move.

Ref. 132 The water strider uses its large middle legs as oars to advance over the surface, reaching speeds of up to 1 m/s doing so. In short, water striders actually row over water. The same mechanism is used by the small robots that can move over water and were developed by Metin Sitti and his group, as shown in Figure 128.

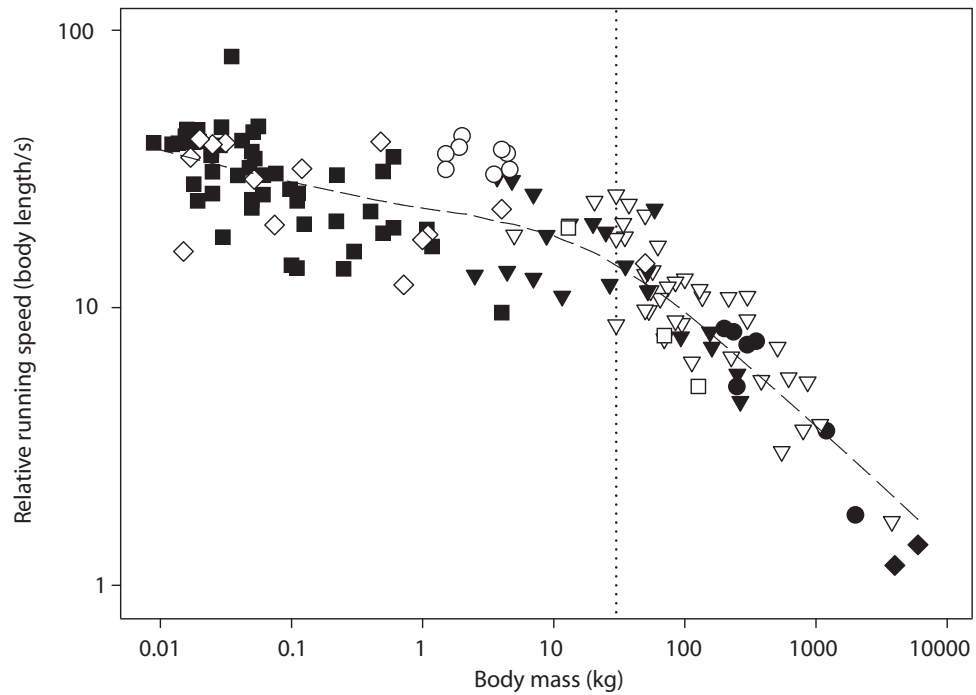
Robot design is still in its infancy. No robot can walk or even run as fast as the animal system it tries to copy. For two-legged robots, the most difficult kind, the speed record is around 3.5 leg lengths per second. In fact, there is a race going on in robotics departments: each department tries to build the first robot that is faster, either in metres per second or in leg lengths per second, than the original four-legged animal or two-legged human. The difficulties of realizing this development goal show how complicated walking motion is and how well nature has optimized living systems.

Challenge 310 s Legs pose many interesting problems. Engineers know that a staircase is comfortable to walk only if for each step the depth  $l$  plus twice the height  $h$  is a constant:  $l + 2h = 0.63 \pm 0.02$  m. This is the so-called *staircase formula*. Why does it hold?

Challenge 311 s Most animals have an *even* number of legs. Do you know an exception? Why not? In fact, one can argue that no animal has less than four legs. Why is this the case?

Challenge 312 e On the other hand, all animals with two legs have legs side by side, whereas most systems with two wheels have them one behind the other. Why is this not the other way round?

Legs are very efficient actuators. As Figure 129 shows, most small animals can run



**FIGURE 129** The graph shows how the relative running speed changes with the mass of terrestrial mammal species, for 142 different species. The graph also illustrates how the running performance changes above 30 kg. Filled squares show Rodentia; open squares show Primata; filled diamonds Proboscidae; open diamonds Marsupialia; filled triangles Carnivora; open triangles Artiodactyla; filled circles Perissodactyla; open circles Lagomorpha (© José Iriarte-Díaz/JEB).

Ref. 133 with about 25 body lengths per second. For comparison, almost no car achieves such a speed. Only animals that weigh more than about 30 kg, including humans, are slower.

Ref. 134 Legs also provide simple distance rulers: just count your steps. In 2006, it was discovered that this method is used by certain ant species, such as *Cataglyphis fortis*. They can count to at least 25 000, as shown by Matthias Wittlinger and his team. These ants use the ability to find the shortest way back to their home even in structureless desert terrain.

Ref. 135 Why do 100 m sprinters run faster than ordinary people? A thorough investigation shows that the speed  $v$  of a sprinter is given by

$$v = f L_{\text{stride}} = f L_c \frac{F_c}{W}, \quad (38)$$

where  $f$  is the frequency of the legs,  $L_{\text{stride}}$  is the stride length,  $L_c$  is the contact length – the length that the sprinter advances during the time the foot is in contact with the floor –  $W$  the weight of the sprinter, and  $F_c$  the average force the sprinter exerts on the floor during contact. It turns out that the frequency  $f$  is almost the same for all sprinters; the only way to be faster than the competition is to increase the stride length  $L_{\text{stride}}$ . Also the contact length  $L_c$  varies little between athletes. Increasing the stride length thus requires that the athlete hits the ground with strong strokes. This is what athletic training for

sprinters has to achieve.

### SUMMARY ON GALILEAN RELATIVITY

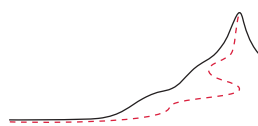
The Earth rotates. The acceleration is so small that we do not feel it. The speed of rotation is large, but we do not feel it, because there is *no way* to do so.

Undisturbed or inertial motion *cannot* be felt or measured. It is thus impossible to distinguish motion from rest. The distinction between rest and motion depends on the observer: *Motion of bodies is relative*. That is why the soil below our feet seems so stable to us, even though it moves with high speed across the universe.

Since motion is relative, speed values depend on the observer. Later on, we will discover that one example of motion in nature has a speed value that is *not* relative: the motion of light. But we continue first with the study of motion transmitted over distance, without the use of any contact at all.







## CHAPTER 6

# MOTION DUE TO GRAVITATION

“Caddi come corpo morto cade.”  
Dante, *Inferno*, c. V, v. 142.\*\*

The first and main method to generate motion without any contact that we discover in our environment is *height*. Waterfalls, snow, rain, the ball of your favourite game and falling apples all rely on it. It was one of the fundamental discoveries of physics that height has this property because there is an interaction between every body and the Earth. *Gravitation* produces an *acceleration* along the line connecting the centres of gravity of the body and the Earth. Note that in order to make this statement, it is necessary to realize that the Earth is a body in the same way as a stone or the Moon, that this body is finite and that therefore it has a centre and a mass. Today, these statements are common knowledge, but they are by no means evident from everyday personal experience.

Challenge 313 s

In several myths about the creation or the organization of the world, such as the biblical one or the Indian one, the Earth is not an object, but an imprecisely defined entity, such as an island floating or surrounded by water with unclear boundaries and unclear method of suspension. Are you able to convince a friend that the Earth is round and not flat? Can you find another argument apart from the roundness of the Earth's shadow when it is visible on the Moon, shown in [Figure 134](#)?

### GRAVITATION AS A LIMIT TO UNIFORM MOTION

A productive way to define gravitation, or gravity for short, appears when we note that no object around us moves along a straight line. In nature, there is a *limit* to steady, or constant motion:

- ▷ Gravity prevents uniform motion, i.e., it prevents constant and straight motion.

In nature, we *never* observe bodies moving at constant speed along a straight line. Speaking with the vocabulary of kinematics: Gravity introduces an *acceleration* for every physical body. The gravitation of the objects in the environment curves the path of a body, changes its speed, or both. This limit has two aspects. First, gravity prevents unlimited uniform motion:

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\*\* ‘I fell like dead bodies fall.’ Dante Alighieri (1265, Firenze–1321, Ravenna), the powerful Italian poet.

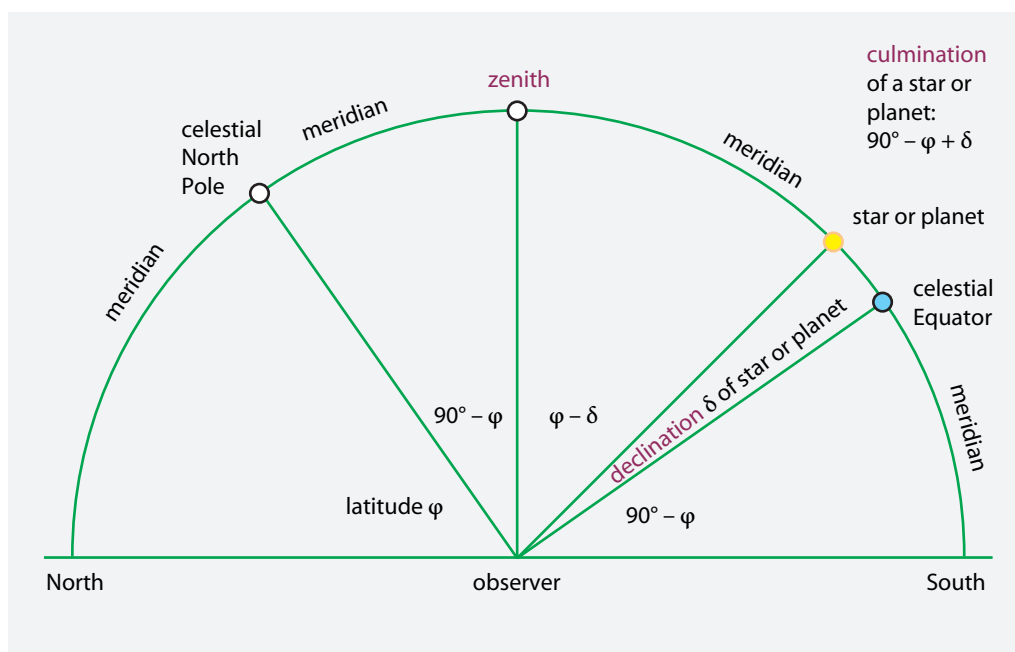


FIGURE 130 Some important concepts when observing the stars and at night.

- ▷ Motion cannot be straight for ever. Motion is not boundless.

We will learn later what this means for the universe as a whole. Secondly,

- ▷ Motion cannot be constant and straight even during short time intervals.

In other words, if we measure with sufficient precision, we will always find deviations from uniform motion. (Physicist also say that motion is never inertial.) These limits apply no matter how *powerful* the motion is and how *free* the moving body is from external influence. In nature, gravitation prevents the steady, uniform motion of atoms, billiard balls, planets, stars and even galaxies.

Gravitation is the first limitation to motion that we discover in nature. We will discover two additional limits to motion later on in our walk. These three fundamental limits are illustrated in Figure 1. To achieve a *precise* description of motion, we need to take each limit of motion into account. This is our main aim in the rest of our adventure.

Gravity affects *all* bodies, even if they are distant from each other. How exactly does gravitation affect two bodies that are far apart? We ask the experts for measuring distant objects: the astronomers.

### GRAVITATION IN THE SKY

The gravitation of the Earth forces the Moon in an orbit around it. The gravitation of the Sun forces the Earth in an orbit around it and sets the length of the year. Similarly, the gravitation of the Sun determines the motion of all the other planets across the sky. We



**FIGURE 131** 'Planet' means 'wanderer'. This composed image shows the retrograde motion of planet Mars across the sky – the Pleiades star cluster is at the top left – when the planet is on the other side of the Sun. The pictures were taken about a week apart and superimposed. The motion is one of the many examples that are fully explained by universal gravitation (© Tunc Tezel).

usually imagine to be located at the centre of the Sun and then say that the planets 'orbit the Sun'. The Sun thus prevents the planets from moving in straight lines and forces them into orbits. How can we check this?

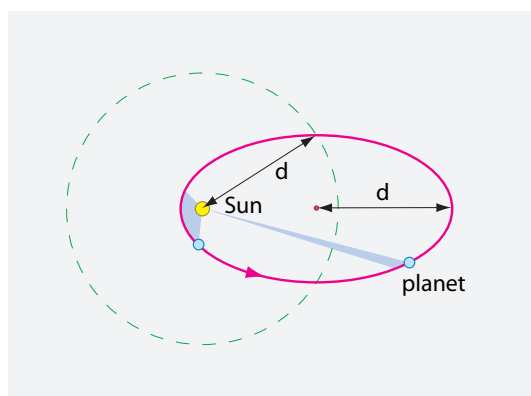
Page 222 First of all, looking at the sky at night, we can check that the planets always stay within the *zodiac*, a narrow stripe across the sky. The centre line of the zodiac gives the path of the Sun and is called the *ecliptic*, since the Moon must be located on it to produce an eclipse. This shows that planets move (approximately) in a single, common plane.\*

Page 223 To learn more about the motion in the sky, astronomers have performed numerous measurements of the movements of the Moon and the planets. The most industrious of all was Tycho Brahe,\*\* who organized an industrial-scale search for astronomical facts sponsored by his king. His measurements were the basis for the research of his young assistant, the Swabian astronomer Johannes Kepler\*\*\* who found the first precise

\* The apparent height of the ecliptic changes with the time of the year and is the reason for the changing seasons. Therefore seasons are a gravitational effect as well.

\*\* Tycho Brahe (b. 1546 Scania, d. 1601 Prague), famous astronomer, builder of Uraniaborg, the astronomical castle. He consumed almost 10 % of the Danish gross national product for his research, which produced the first star catalogue and the first precise position measurements of planets.

\*\*\* Johannes Kepler (1571 Weil der Stadt–1630 Regensburg) studied Protestant theology and became a teacher of mathematics, astronomy and rhetoric. He helped his mother to defend herself successfully in a trial where she was accused of witchcraft. His first book on astronomy made him famous, and he became assistant to Tycho Brahe and then, at his teacher's death, the Imperial Mathematician. He was the first to



**FIGURE 132** The motion of a planet around the Sun, showing its semimajor axis  $d$ , which is also the spatial average of its distance from the Sun.

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description of planetary motion. This is not an easy task, as the observation of [Figure 131](#) shows. In his painstaking research on the movements of the planets in the zodiac, Kepler discovered several ‘laws’, i.e., patterns or rules. The motion of all the planets follow the same rules, confirming that the Sun determines their orbits. The three main ones are as follows:

1. Planets move on ellipses with the Sun located at one focus (1609).
2. Planets sweep out equal areas in equal times (1609).
3. All planets have the same ratio  $T^2/d^3$  between the orbit duration  $T$  and the semimajor axis  $d$  (1619).

Kepler’s results are illustrated in [Figure 132](#). The sheer work required to deduce the three ‘laws’ was enormous. Kepler had no calculating machine available. The calculation technology he used was the recently discovered logarithms. Anyone who has used tables of logarithms to perform calculations can get a feeling for the amount of work behind these three discoveries.

Finally, in 1684, all observations by Kepler about planets and stones were condensed into an astonishingly simple result by the English physicist Robert Hooke and a few others:\*

- ▷ Every body of mass  $M$  attracts any other body towards its centre with an acceleration whose magnitude  $a$  is given by

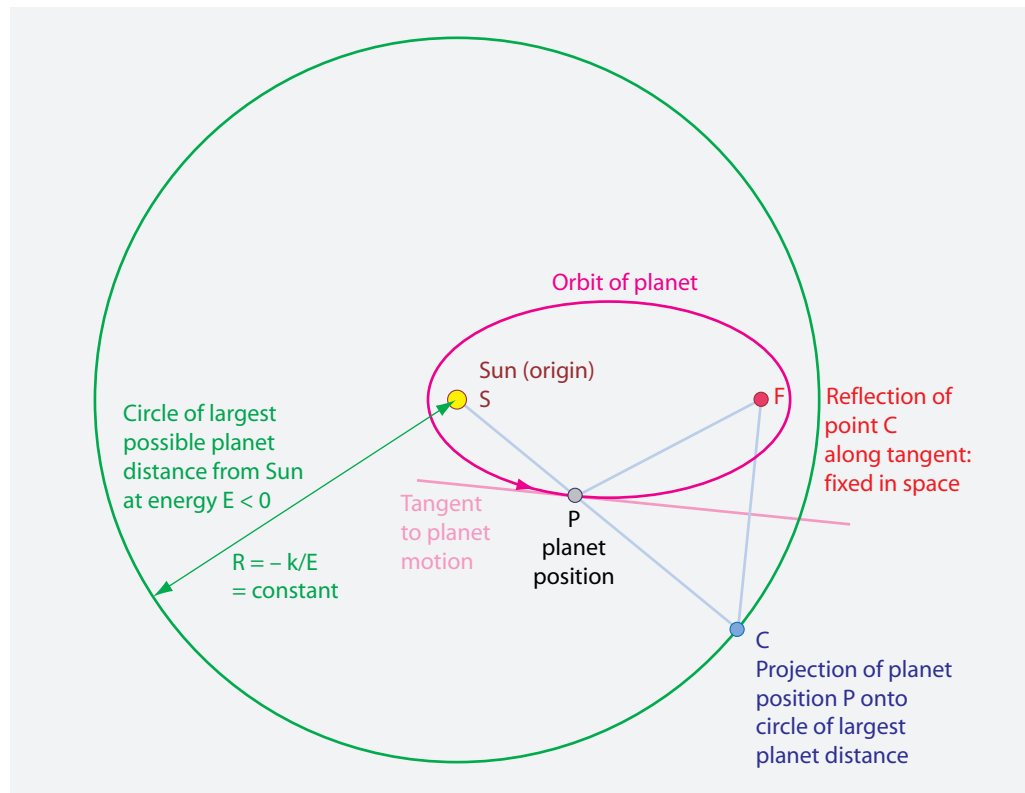
$$a = G \frac{M}{r^2} \quad (39)$$

where  $r$  is the centre-to-centre distance of the two bodies.

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use mathematics in the description of astronomical observations, and introduced the concept and field of ‘celestial physics’.

\* Robert Hooke (1635–1703), important English physicist and secretary of the Royal Society. Apart from discovering the inverse square relation and many others, such as Hooke’s ‘law’, he also wrote the *Micrographia*, a beautifully illustrated exploration of the world of the very small.



**FIGURE 133** The proof that a planet moves in an ellipse (magenta) around the Sun, given an inverse square distance relation for gravitation. The proof – detailed in the text – is based on the relation  $SP+PF=R$ . Since  $R$  is constant, the orbit is an ellipse

This is called *universal gravitation*, or the *universal ‘law’ of gravitation*, because it is valid both in the sky and on Earth, as we will see shortly. The proportionality constant  $G$  is called the *gravitational constant*; it is one of the fundamental constants of nature, like the speed of light  $c$  or the quantum of action  $\hbar$ . More about  $G$  will be said shortly.

The effect of gravity thus decreases with increasing distance; the effect depends on the inverse distance squared of the bodies under consideration. If bodies are small compared with the distance  $r$ , or if they are spherical, expression (39) is correct as it stands; for non-spherical shapes the acceleration has to be calculated separately for each part of the bodies and then added together.

Why is the usual planetary orbit an ellipse? The simplest argument is given in **Figure 133**. We know that the acceleration due to gravity varies as  $a = GM/r^2$ . We also know that an orbiting body of mass  $m$  has a constant energy  $E < 0$ . We then can draw, around the Sun, the circle with radius  $R = -GMm/E$ , which gives the largest distance that a body with energy  $E$  can be from the Sun. We now project the planet position  $P$  onto this circle, thus constructing a position  $C$ . We then reflect  $C$  along the tangent to get a position  $F$ . This last position  $F$  is fixed in space and time, as a simple argument shows. (Can you find it?) As a result of the construction, the distance sum  $SP+PF$  is constant in time, and given by the radius  $R = -GMm/E$ . Since the distance sum is constant, the

Page 182

Ref. 147

Challenge 314 s



orbit is an ellipse, because an ellipse is precisely the curve that appears when this sum is constant. (Remember that an ellipse can be drawn with a piece of rope in this way.) Point  $F$ , like the Sun, is a focus of the ellipse. The construction thus shows that the motion of a planet defines two foci and follows an elliptical orbit defined by these two foci. In short, we have deduced the first of Kepler's 'laws' from the expression of universal gravitation.

The second of Kepler's 'laws', about equal swept areas, implies that planets move faster when they are near the Sun. It is a simple way to state the conservation of angular momentum. What does the third 'law' state?

Challenge 315 e

Can you confirm that also the second and third of Kepler's 'laws' follow from Hooke's expression of universal gravity? Publishing this result – which was obvious to Hooke – was one of the achievements of Newton. Try to repeat this achievement; it will show you not only the difficulties, but also the possibilities of physics, and the joy that puzzles give.

Challenge 316 s

Newton solved these puzzles with geometric drawings – though in quite a complex manner. It is well known that Newton was not able to write down, let alone handle, differential equations at the time he published his results on gravitation. In fact, Newton's notation and calculation methods were poor. (Much poorer than yours!) The English mathematician Godfrey Hardy\* used to say that the insistence on using Newton's integral and differential notation, rather than the earlier and better method, still common today, due to his rival Leibniz – threw back English mathematics by 100 years.

Ref. 28

To sum up, Kepler, Hooke and Newton became famous because they brought order to the description of planetary motion. They showed that all motion due to gravity follows from the same description, the inverse square distance. For this reason, the inverse square distance relation  $a = GM/r^2$  is called the *universal* law of gravity. Achieving this unification of motion description, though of small practical significance, was widely publicized. The main reasons were the age-old prejudices and fantasies linked with astrology.

In fact, the inverse square distance relation explains many additional phenomena. It explains the motion and shape of the Milky Way and of the other galaxies, the motion of many weather phenomena, and explains why the Earth has an atmosphere but the Moon does not. (Can you explain this?)

Challenge 317 s

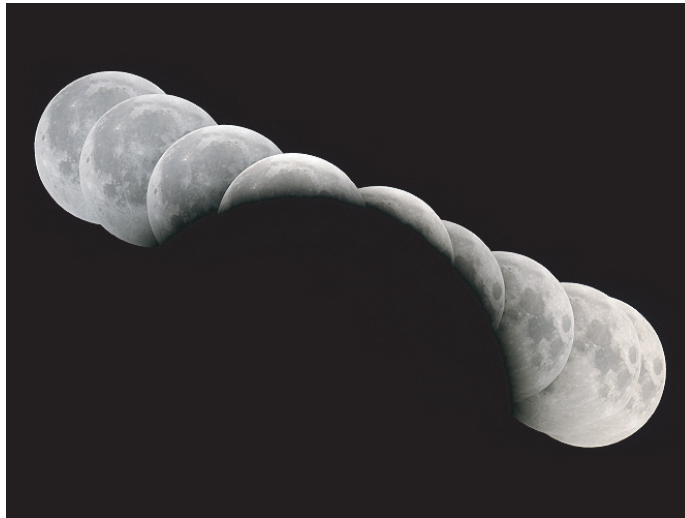
## GRAVITATION ON EARTH

This inverse square dependence of gravitational acceleration is often, but incorrectly, called Newton's 'law' of gravitation. Indeed, the occultist and physicist Isaac Newton proved more elegantly than Hooke that the expression agreed with all astronomical and terrestrial observations. Above all, however, he organized a better public relations campaign, in which he falsely claimed to be the originator of the idea.

Ref. 137

Newton published a simple proof showing that the description of astronomical gravitation also gives the correct description for stones thrown through the air, down here on 'father Earth'. To achieve this, he compared the acceleration  $a_m$  of the Moon with that of stones  $g$ . For the ratio between these two accelerations, the inverse square relation predicts a value  $g/a_m = d_m^2/R^2$ , where  $d_m$  the distance of the Moon and  $R$  is the radius of the Earth. The Moon's distance can be measured by triangulation, comparing

\* Godfrey Harold Hardy (1877–1947) was an important number theorist, and the author of the well-known *A Mathematician's Apology*. He also 'discovered' the famous Indian mathematician Srinivasa Ramanujan, and brought him to Britain.



**FIGURE 134** How to compare the radius of the Earth with that of the Moon during a partial lunar eclipse (© Anthony Ayiomamitis).

the position of the Moon against the starry background from two different points on Earth.\* The result is  $d_m/R = 60 \pm 3$ , depending on the orbital position of the Moon, so that an average ratio  $g/a_m = 3.6 \cdot 10^3$  is predicted from universal gravity. But both accelerations can also be measured directly. At the surface of the Earth, stones are subject to an acceleration due to gravitation with magnitude  $g = 9.8 \text{ m/s}^2$ , as determined by measuring the time that stones need to fall a given distance. For the Moon, the definition of acceleration,  $a = dv/dt$ , in the case of circular motion – roughly correct here – gives  $a_m = d_m(2\pi/T)^2$ , where  $T = 2.4 \text{ Ms}$  is the time the Moon takes for one orbit around the Earth.\*\* The measurement of the radius of the Earth\*\*\* yields  $R = 6.4 \text{ Mm}$ , so that

\* The first precise – but not the first – measurement was achieved in 1752 by the French astronomers Lalande and La Caille, who simultaneously measured the position of the Moon seen from Berlin and from Le Cap.

\*\* This expression for the centripetal acceleration is deduced easily by noting that for an object in circular motion, the magnitude  $v$  of the velocity  $\mathbf{v} = d\mathbf{x}/dt$  is given as  $v = 2\pi r/T$ . The drawing of the vector  $\mathbf{v}$  over time, the so-called *hodograph*, shows that it behaves exactly like the position of the object. Therefore the magnitude  $a$  of the acceleration  $\mathbf{a} = d\mathbf{v}/dt$  is given by the corresponding expression, namely  $a = 2\pi v/T$ .

\*\*\* This is the hardest quantity to measure oneself. The most surprising way to determine the Earth's size is the following: watch a sunset in the garden of a house, with a stopwatch in hand, such as the one in your mobile phone. When the last ray of the Sun disappears, start the stopwatch and run upstairs. There, the Sun is still visible; stop the stopwatch when the Sun disappears again and note the time  $t$ . Measure the height difference  $h$  between the two eye positions where the Sun was observed. The Earth's radius  $R$  is then given by  $R = kh/t^2$ , with  $k = 378 \cdot 10^6 \text{ s}^2$ .

There is also a simple way to measure the distance to the Moon, once the size of the Earth is known. Take a photograph of the Moon when it is high in the sky, and call  $\theta$  its zenith angle, i.e., its angle from the vertical above you. Make another photograph of the Moon a few hours later, when it is just above the horizon. On this picture, unlike the common optical illusion, the Moon is smaller, because it is further away. With a sketch the reason becomes immediately clear. If  $q$  is the ratio of the two angular diameters, the Earth–Moon distance  $d_m$  is given by the relation  $d_m^2 = R^2 + (2Rq \cos \theta / (1 - q^2))^2$ . Enjoy finding its derivation from the sketch.

Another possibility is to determine the size of the Moon by comparing it with the size of the Earth in a lunar eclipse, as shown in Figure 134. The distance to the Moon is then computed from its angular size, about  $0.5^\circ$ .

Challenge 318 s

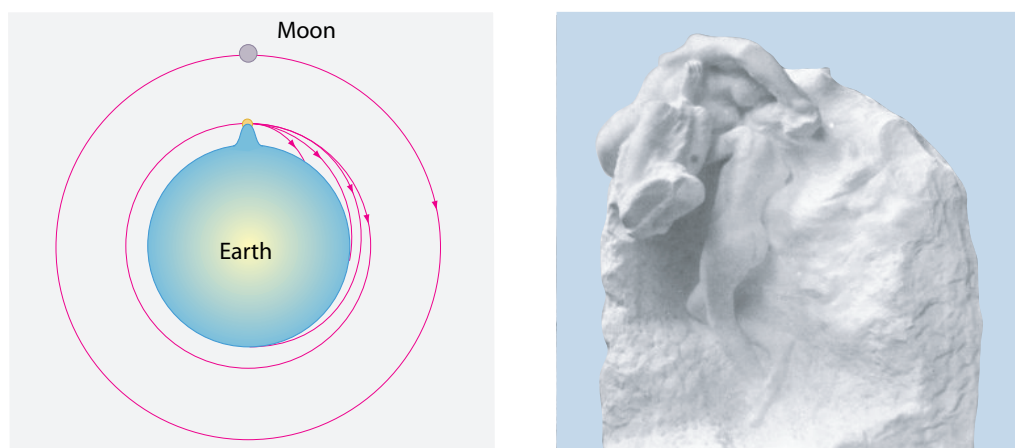
Ref. 138

Challenge 319 s

Ref. 139

Page 69

Challenge 320 s



**FIGURE 135** A physicist's and an artist's view of the fall of the Moon: a diagram by Christiaan Huygens (not to scale) and a marble statue by Auguste Rodin.

the average Earth–Moon distance is  $d_m = 0.38$  Gm. One thus has  $g/a_m = 3.6 \cdot 10^3$ , in agreement with the above prediction. With this famous ‘Moon calculation’ we have thus shown that the inverse square property of gravitation indeed describes both the motion of the Moon and that of stones. You might want to deduce the value of the product  $GM$  for Earth.

Challenge 321 s

Universal gravitation thus describes all motion due to gravity – both on Earth and in the sky. This was an important step towards the unification of physics. Before this discovery, from the observation that on the Earth all motion eventually comes to rest, whereas in the sky all motion is eternal, Aristotle and many others had concluded that motion in the *sublunar* world has *different* properties from motion in the *translunar* world. Several thinkers had criticized this distinction, notably the philosopher and rector of the University of Paris, Jean Buridan.\* The Moon calculation was the most important result showing this distinction to be wrong. This is the reason for calling Hooke's expression (39) the *universal* gravitation.

Ref. 140

Universal gravitation allows us to answer another old question. Why does the Moon not fall from the sky? Well, the preceding discussion showed that *fall* is motion due to gravitation. Therefore the Moon actually *is* falling, with the peculiarity that instead of falling *towards* the Earth, it is continuously falling *around* it. Figure 135 illustrates the idea. The Moon is continuously missing the Earth.\*\*

The Moon is not the only object that falls around the Earth. Figure 137 shows another.

\* Jean Buridan (c. 1295 to c. 1366) was also one of the first modern thinkers to discuss the rotation of the Earth about an axis.

\*\* Another way to put it is to use the answer of the Dutch physicist Christiaan Huygens (1629–1695): the Moon does not fall from the sky because of the centrifugal acceleration. As explained on page 161, this explanation is often out of favour at universities.

Ref. 141  
Challenge 322 d

There is a beautiful problem connected to the left side of the figure: Which points on the surface of the Earth can be hit by shooting from a mountain? And which points can be hit by shooting horizontally?



**FIGURE 136** A precision second pendulum, thus about 1 m in length; almost at the upper end, the vacuum chamber that compensates for changes in atmospheric pressure; towards the lower end, the wide construction that compensates for temperature variations of pendulum length; at the very bottom, the screw that compensates for local variations of the gravitational acceleration, giving a final precision of about 1 s per month (© Erwin Sattler OHG).



**FIGURE 137** The man in orbit feels no weight, the blue atmosphere, which is not, does (NASA).

TABLE 26 Some measured values of the acceleration due to gravity.

| P L A C E | V A L U E                  |
|-----------|----------------------------|
| Poles     | 9.83 m/s <sup>2</sup>      |
| Trondheim | 9.8215243 m/s <sup>2</sup> |
| Hamburg   | 9.8139443 m/s <sup>2</sup> |
| Munich    | 9.8072914 m/s <sup>2</sup> |
| Rome      | 9.8034755 m/s <sup>2</sup> |
| Equator   | 9.78 m/s <sup>2</sup>      |
| Moon      | 1.6 m/s <sup>2</sup>       |
| Sun       | 273 m/s <sup>2</sup>       |

### PROPERTIES OF GRAVITATION: $G$ AND $g$

Page 77 Gravitation implies that the path of a stone is not a parabola, as stated earlier, but actually an *ellipse* around the centre of the Earth. This happens for exactly the same reason that  
 Page 177 the planets move in ellipses around the Sun. Are you able to confirm this statement?

Universal gravitation allows us to understand the puzzling acceleration value  $g = 9.8 \text{ m/s}^2$  we encountered in equation (6). The value is due to the relation

$$g = GM_{\text{Earth}}/R_{\text{Earth}}^2. \quad (40)$$

The expression can be deduced from equation (39), universal gravity, by taking the Earth to be spherical. The everyday acceleration of gravity  $g$  thus results from the size of the Earth, its mass, and the universal constant of gravitation  $G$ . Obviously, the value for  $g$  is almost constant on the surface of the Earth, as shown in Table 26, because the Earth is almost a sphere. Expression (40) also explains why  $g$  gets smaller as one rises above the Earth, and the deviations of the shape of the Earth from sphericity explain why  $g$  is different at the poles and higher on a plateau. (What would  $g$  be on the Moon? On Mars? On Jupiter?)

Challenge 323 s

By the way, it is possible to devise a simple machine, other than a yo-yo, that slows down the effective acceleration of gravity by a known amount, so that one can measure its value more easily. Can you imagine this machine?

Challenge 324 s

Note that 9.8 is roughly  $\pi^2$ . This is *not* a coincidence: the metre has been chosen in such a way to make this (roughly) correct. The period  $T$  of a swinging pendulum, i.e., a



Challenge 325 s back and forward swing, is given by\*

$$T = 2\pi\sqrt{\frac{l}{g}}, \quad (41)$$

where  $l$  is the length of the pendulum, and  $g = 9.8 \text{ m/s}^2$  is the gravitational acceleration. (The pendulum is assumed to consist of a compact mass attached to a string of negligible mass.) The oscillation time of a pendulum depends only on the length of the string and on  $g$ , thus on the planet it is located on.

Challenge 327 e If the metre had been defined such that  $T/2 = 1 \text{ s}$ , the value of the normal acceleration  $g$  would have been exactly  $\pi^2 \text{ m/s}^2 = 9.869\,604\,401\,09 \text{ m/s}^2$ . Indeed, this was the first proposal for the definition of the metre; it was made in 1673 by Huygens and repeated in 1790 by Talleyrand, but was rejected by the conference that defined the metre because variations in the value of  $g$  with geographical position, temperature-induced variations of the length of a pendulum and even air pressure variations induce errors that are too large to yield a definition of useful precision. (Indeed, all these effects must be corrected in pendulum clocks, as shown in Figure 136.)

Ref. 142 Finally, the proposal was made to define the metre as  $1/40\,000\,000$  of the circumference of the Earth through the poles, a so-called *meridian*. This proposal was almost identical to – but much more precise than – the pendulum proposal. The meridian definition of the metre was then adopted by the French national assembly on 26 March 1791, with the statement that ‘a meridian passes under the feet of every human being, and all meridians are equal’. (Nevertheless, the distance from the Equator to the poles is not exactly 10 Mm; that is a strange story. One of the two geographers who determined the size of the first metre stick was dishonest. The data he gave for his measurements – the general method of which is shown in Figure 138 – was fabricated. Thus the first official metre stick in Paris was shorter than it should be.)

Continuing our exploration of the gravitational acceleration  $g$ , we can still ask: Why does the Earth have the mass and size it has? And why does  $G$  have the value it has? The first question asks for a history of the Solar System; it is still unanswered and is topic of research. The second question is addressed in Appendix B.

If gravitation is indeed universal, and if all objects really attract each other, attraction should also occur between any two objects of everyday life. Gravity must also work *side-ways*. This is indeed the case, even though the effects are extremely small. Indeed, the effects are so small that they were measured only long after universal gravity had predicted them. On the other hand, measuring this effect is the only way to determine the gravitational constant  $G$ . Let us see how to do it.

\* Formula (41) is noteworthy mainly for all that is missing. The period of a pendulum does *not* depend on the mass of the swinging body. In addition, the period of a pendulum does *not* depend on the amplitude. (This is true as long as the oscillation angle is smaller than about  $15^\circ$ .) Galileo discovered this as a student, when observing a chandelier hanging on a long rope in the dome of Pisa. Using his heartbeat as a clock he found that even though the amplitude of the swing got smaller and smaller, the time for the swing stayed the same.

Challenge 326 s A leg also moves like a pendulum, when one walks normally. Why then do taller people tend to walk faster? Is the relation also true for animals of different size?



FIGURE 138 The measurements that lead to the definition of the metre (© Ken Alder).

We note that measuring the gravitational constant  $G$  is also the only way to determine the mass of the *Earth*. The first to do so, in 1798, was the English physicist Henry Cavendish; he used the machine, ideas and method of John Michell who died when attempting the experiment. Michell and Cavendish\* called the aim and result of their experiments ‘weighing the Earth’.

The idea of Michell was to suspended a horizontal handle, with two masses at the end, at the end of a long metal wire. He then approached two additional large masses at the two ends of the handle, avoiding any air currents, and measured how much the handle rotated. Figure 139 shows how to repeat this experiment in your basement, and Figure 140 how to perform it when you have a larger budget.

The value the gravitational constant  $G$  found in more elaborate versions of the Michell–Cavendish experiments is

$$G = 6.7 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2 = 6.7 \cdot 10^{-11} \text{ m}^3/\text{kg s}^2. \quad (42)$$

Cavendish’s experiment was thus the first to confirm that gravity also works sideways. The experiment also allows deducing the mass  $M$  of the Earth from its radius  $R$  and the relation  $g = GM/R^2$ . Therefore, the experiment also allows to deduce the average density of the Earth. Finally, as we will see later on, this experiment proves, if we keep in mind that the speed of light is finite and invariant, that space is curved. All this is achieved with this simple set-up!

\* Henry Cavendish (b. 1731 Nice, d. 1810 London) was one of the great geniuses of physics; rich, autistic, misogynist, unmarried and solitary, he found many rules of nature, but never published them. Had he done so, his name would be much more well-known. John Michell (1724–1793) was church minister, geologist and amateur astronomer.



**FIGURE 139** Home experiments that allow determining the gravitational constant  $G$ , weighing the Earth, proving that gravity also works sideways and thus showing that gravity curves space. Top left and right: a torsion balance made of foam and lead, with pétanque (boules) masses as fixed masses; centre right: a torsion balance made of wood and lead, with stones as fixed masses; bottom: a time sequence showing how the stones do attract the lead (© John Walker).

Ref. 143

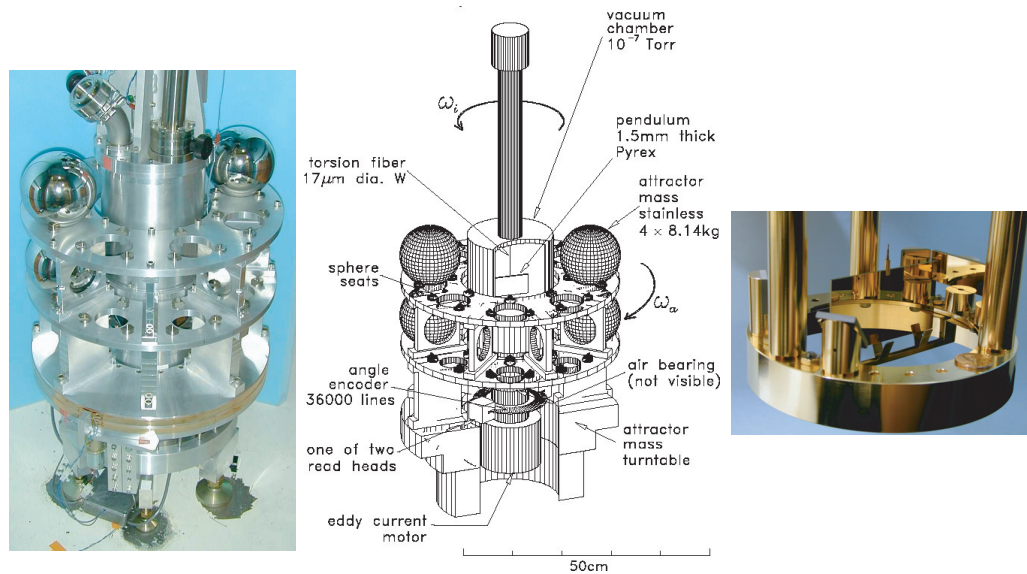
Challenge 329 e

Cavendish found a mass density of the Earth of 5.5 times that of water. This was a surprising result because rock only has 2.8 times the density of water. What is the origin of the large density value?

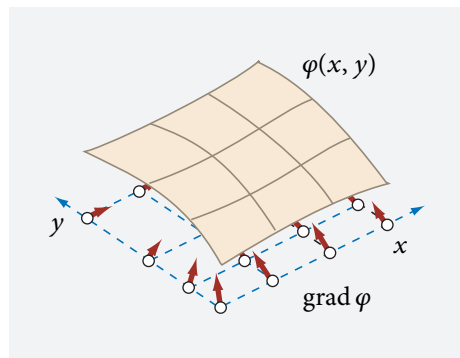
We note that  $G$  has a small value. Above, we mentioned that gravity limits motion. In fact, we can write the expression for universal gravitation in the following way:

$$\frac{ar^2}{M} = G > 0 \quad (43)$$

Gravity prevents uniform motion. In fact, we can say more: *Gravitation is the smallest*



**FIGURE 140** A modern precision torsion balance experiment to measure the gravitational constant, performed at the University of Washington (© Eöt-Wash Group).



**FIGURE 141** The potential and the gradient, visualized for two spatial dimensions.

*possible effect of the environment on a moving body.* All other effects come on top of gravity. However, it is not easy to put this statement in a simple formula.

Challenge 330 d

Gravitation between everyday objects is weak. For example, two average people 1 m apart feel an acceleration towards each other that is less than that exerted by a common fly when landing on the skin. Therefore we usually do not notice the attraction to other people. When we notice it, it is much stronger than that. The measurement of  $G$  thus proves that gravitation cannot be the true cause of people falling in love, and also that erotic attraction is not of gravitational origin, but of a different source. The physical basis for love will be studied later in our walk: it is called *electromagnetism*.

Challenge 331 s

Vol. III, page 15

### THE GRAVITATIONAL POTENTIAL

Gravity has an important property: all effects of gravitation can also be described by another observable, namely the (*gravitational*) *potential*  $\varphi$ . We then have the simple relation

that the acceleration is given by the *gradient* of the potential

$$\mathbf{a} = -\nabla\varphi \quad \text{or} \quad \mathbf{a} = -\text{grad } \varphi . \quad (44)$$

The gradient is just a learned term for ‘slope along the steepest direction’. The gradient is defined for any point on a slope, is large for a steep one and small for a shallow one. The gradient points in the direction of steepest ascent, as shown in Figure 141. The gradient is abbreviated  $\nabla$ , pronounced ‘nabla’, and is mathematically defined through the relation  $\nabla\varphi = (\partial\varphi/\partial x, \partial\varphi/\partial y, \partial\varphi/\partial z) = \text{grad } \varphi$ .<sup>\*</sup> The minus sign in (44) is introduced by convention, in order to have higher potential values at larger heights. In everyday life, when the spherical shape of the Earth can be neglected, the gravitational potential is given by

$$\varphi = gh . \quad (45)$$

The potential  $\varphi$  is an interesting quantity; with a single number at every position in space we can describe the vector aspects of gravitational acceleration. It automatically gives that gravity in New Zealand acts in the opposite direction to gravity in Paris. In addition, the potential suggests the introduction of the so-called *potential energy*  $U$  by setting

$$U = m\varphi \quad (46)$$

and thus allowing us to determine the change of *kinetic energy*  $T$  of a body falling from a point 1 to a point 2 via

$$T_1 - T_2 = U_2 - U_1 \quad \text{or} \quad \frac{1}{2}m_1\mathbf{v}_1^2 - \frac{1}{2}m_2\mathbf{v}_2^2 = m\varphi_2 - m\varphi_1 . \quad (47)$$

In other words, the *total energy*, defined as the sum of kinetic and potential energy, is *conserved* in motion due to gravity. This is a characteristic property of gravitation. Gravity conserves energy and momentum.

Page 110

Not all accelerations can be derived from a potential; systems with this property are called *conservative*. Observation shows that accelerations due to friction are not conservative, but accelerations due to electromagnetism are. In short, we can either say that gravity can be described by a potential, or say that it conserves energy and momentum. Both mean the same. When the non-spherical shape of the Earth can be neglected, the potential energy of an object at height  $h$  is given by

$$U = mgh . \quad (48)$$

To get a feeling of how much energy this is, answer the following question. A car with mass 1 Mg falls down a cliff of 100 m. How much water can be heated from freezing point to boiling point with the energy of the car?

Challenge 332 s

<sup>\*</sup> In two or more dimensions slopes are written  $\partial\varphi/\partial z$  – where  $\partial$  is still pronounced ‘d’ – because in those cases the expression  $d\varphi/dz$  has a slightly different meaning. The details lie outside the scope of this walk.



### THE SHAPE OF THE EARTH

Universal gravity also explains why the Earth and most planets are (almost) spherical. Since gravity increases with decreasing distance, a liquid body in space will always try to form a spherical shape. Seen on a large scale, the Earth is indeed liquid. We also know that the Earth is cooling down – that is how the crust and the continents formed. The sphericity of smaller solid objects encountered in space, such as the Moon, thus means that they used to be liquid in older times.

The Earth is thus not flat, but roughly spherical. Therefore, the top of two tall buildings is further apart than their base. Can this effect be measured?

Challenge 333 s

Sphericity considerably simplifies the description of motion. For a spherical or a point-like body of mass  $M$ , the potential  $\varphi$  is

Challenge 334 e

$$\varphi = -G \frac{M}{r} . \quad (49)$$

Challenge 336 s

Ref. 144

A potential considerably simplifies the description of motion, since a potential is additive: given the potential of a point particle, we can calculate the potential and then the motion around any other irregularly shaped object.\* Interestingly, the number  $d$  of dimensions of space is coded into the potential  $\varphi$  of a spherical mass: the dependence of  $\varphi$  on the radius  $r$  is in fact  $1/r^{d-2}$ . The exponent  $d - 2$  has been checked experimentally to extremely high precision; no deviation of  $d$  from 3 has ever been found.

Ref. 145

Ref. 146

Challenge 337 ny

The concept of potential helps in understanding the *shape* of the Earth in more detail. Since most of the Earth is still liquid when seen on a large scale, its surface is always horizontal with respect to the direction determined by the combination of the accelerations of gravity and rotation. In short, the Earth is *not* a sphere. It is not an ellipsoid either. The mathematical shape defined by the equilibrium requirement is called a *geoid*. The geoid shape, illustrated in Figure 142, differs from a suitably chosen ellipsoid by at most 50 m. Can you describe the geoid mathematically? The geoid is an excellent approximation to the actual shape of the Earth; sea level differs from it by less than 20 metres. The differences can be measured with satellite radar and are of great interest to geologists and geographers. For example, it turns out that the South Pole is nearer to the equatorial

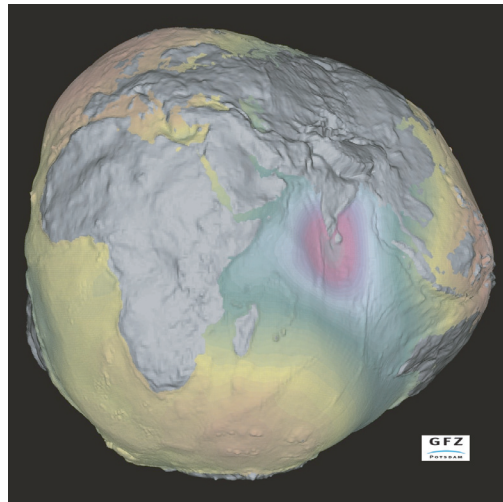
\* Alternatively, for a general, extended body, the potential is found by requiring that the *divergence* of its gradient is given by the mass (or charge) density times some proportionality constant. More precisely, we have

$$\Delta\varphi = 4\pi G\rho \quad (50)$$

Challenge 335 e

where  $\rho = \rho(\mathbf{x}, t)$  is the mass volume density of the body and the so-called *Laplace operator*  $\Delta$ , pronounced ‘delta’, is defined as  $\Delta f = \nabla\nabla f = \partial^2 f/\partial x^2 + \partial^2 f/\partial y^2 + \partial^2 f/\partial z^2$ . Equation (50) is called the *Poisson equation* for the potential  $\varphi$ . It is named after Siméon-Denis Poisson (1781–1840), eminent French mathematician and physicist. The positions at which  $\rho$  is not zero are called the *sources* of the potential. The so-called source term  $\Delta\varphi$  of a function is a measure for how much the function  $\varphi(\mathbf{x})$  at a point  $\mathbf{x}$  differs from the average value in a region around that point. (Can you show this, by showing that  $\Delta\varphi \sim \bar{\varphi} - \varphi(\mathbf{x})$ ?) In other words, the Poisson equation (50) implies that the actual value of the potential at a point is the same as the average value around that point minus the mass density multiplied by  $4\pi G$ . In particular, in the case of empty space the potential at a point is equal to the average of the potential around that point.

Often the concept of *gravitational field* is introduced, defined as  $\mathbf{g} = -\nabla\varphi$ . We avoid this in our walk because we will discover that, following the theory of relativity, gravity is not due to a field at all; in fact, even the concept of gravitational potential turns out to be only an approximation.



**FIGURE 142** The shape of the Earth, with exaggerated height scale  
(© GeoForschungsZentrum Potsdam).

plane than the North Pole by about 30 m. This is probably due to the large land masses in the northern hemisphere.

Page 137

Above we saw how the inertia of matter, through the so-called ‘centrifugal force’, increases the radius of the Earth at the Equator. In other words, the Earth is *flattened* at the poles. The Equator has a radius  $a$  of 6.38 Mm, whereas the distance  $b$  from the poles to the centre of the Earth is 6.36 Mm. The precise flattening  $(a - b)/a$  has the value  $1/298.3 = 0.0034$ . As a result, the top of Mount Chimborazo in Ecuador, even though its height is only 6267 m above sea level, is about 20 km farther away from the centre of the Earth than the top of Mount Sagarmatha\* in Nepal, whose height above sea level is 8850 m. The top of Mount Chimborazo is in fact the point on the surface most distant from the centre of the Earth.

Appendix B

The shape of the Earth has another important consequence. If the Earth stopped rotating (but kept its shape), the water of the oceans would flow from the Equator to the poles; all of Europe would be under water, except for the few mountains of the Alps that are higher than about 4 km. The northern parts of Europe would be covered by between 6 km and 10 km of water. Mount Sagarmatha would be over 11 km above sea level. We would also walk inclined. If we take into account the resulting change of shape of the Earth, the numbers come out somewhat smaller. In addition, the change in shape would produce extremely strong earthquakes and storms. As long as there are none of these effects, we can be *sure* that the Sun will indeed rise tomorrow, despite what some philosophers pretended.

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### DYNAMICS – HOW DO THINGS MOVE IN VARIOUS DIMENSIONS?

The concept of potential is a powerful tool. If a body can move only along a – straight or curved – line, the concepts of kinetic and potential energy are sufficient to determine completely the way the body moves.

\* Mount Sagarmatha is sometimes also called Mount Everest.



**FIGURE 143** The change of the moon during the month, showing its libration (QuickTime film © Martin Elsässer)

In fact, motion in *one dimension* follows directly from energy conservation. For a body moving along a given curve, the speed at every instant is given by energy conservation.

If a body can move in *two dimensions* – i.e., on a flat or curved surface – *and* if the forces involved are *internal* (which is always the case in theory, but not in practice), the conservation of angular momentum can be used. The full motion in two dimensions thus follows from energy and angular momentum conservation. For example, all properties of free fall follow from energy and angular momentum conservation. (Are you able to show this?) Again, the potential is essential.

Challenge 338 s

In the case of motion in *three dimensions*, a more general rule for determining motion is necessary. If more than *two spatial dimensions* are involved conservation is insufficient to determine how a body moves. It turns out that general motion follows from a simple principle: the time average of the difference between kinetic and potential energy must be as small as possible. This is called the *least action principle*. We will explain the details of this calculation method later. But again, the potential is the main ingredient in the calculation of change, and thus in the description of any example of motion.

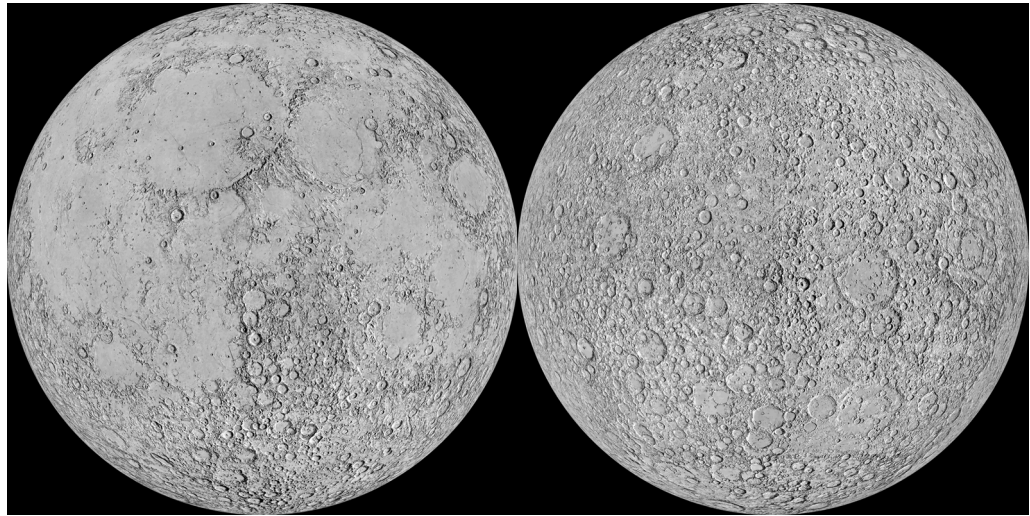
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For simple gravitational motions, motion is two-dimensional, in a plane. Most three-dimensional problems are outside the scope of this text; in fact, some of these problems are so hard that they still are subjects of research. In this adventure, we will explore three-dimensional motion only for selected cases that provide important insights.

### THE MOON

How long is a day on the Moon? The answer is roughly 29 Earth-days. That is the time that it takes for an observer on the Moon to see the Sun again in the same position in the sky.

One often hears that the Moon always shows the same side to the Earth. But this is



**FIGURE 144** High resolution maps (not photographs) of the near side (left) and far side (right) of the moon, showing how often the latter saved the Earth from meteorite impacts (courtesy USGS).

wrong. As one can check with the naked eye, a given feature in the centre of the face of the Moon at full Moon is not at the centre one week later. The various motions leading to this change are called *librations*; they are shown in the film in [Figure 143](#). The motions appear mainly because the Moon does not describe a circular, but an elliptical orbit around the Earth and because the axis of the Moon is slightly inclined, when compared with that of its rotation around the Earth. As a result, only around 45 % of the Moon's surface is permanently hidden from Earth.

The first photographs of the hidden area of the Moon were taken in the 1960s by a Soviet artificial satellite; modern satellites provided exact maps, as shown in [Figure 144](#). (Just zoom into the figure for fun.) The hidden surface is much more irregular than the visible one, as the hidden side is the one that intercepts most asteroids attracted by the Earth. Thus the gravitation of the Moon helps to deflect asteroids from the Earth. The number of animal life extinctions is thus reduced to a small, but not negligible number. In other words, the gravitational attraction of the Moon has saved the human race from extinction many times over.\*

The trips to the Moon in the 1970s also showed that the Moon originated from the Earth itself: long ago, an object hit the Earth almost tangentially and threw a sizeable fraction of material up into the sky. This is the only mechanism able to explain the large size of the Moon, its low iron content, as well as its general material composition.

The Moon is receding from the Earth at 3.8 cm a year. This result confirms the old deduction that the tides slow down the Earth's rotation. Can you imagine how this measurement was performed? Since the Moon slows down the Earth, the Earth also changes shape due to this effect. (Remember that the shape of the Earth depends on its speed of

\* The web pages [www.minorplanetcenter.net/iau/lists/Closest.html](http://www.minorplanetcenter.net/iau/lists/Closest.html) and [InnerPlot.html](http://InnerPlot.html) give an impression of the number of objects that almost hit the Earth every year. Without the Moon, we would have many additional catastrophes.

Challenge 339 e

Ref. 148

Ref. 149

Challenge 340 s

rotation.) These changes in shape influence the tectonic activity of the Earth, and maybe also the drift of the continents.

Ref. 150 The Moon has many effects on animal life. A famous example is the midge *Clunio*, which lives on coasts with pronounced tides. *Clunio* spends between six and twelve weeks as a larva, sure then hatches and lives for only one or two hours as an adult flying insect, during which time it reproduces. The midges will only reproduce if they hatch during the low tide phase of a *spring tide*. Spring tides are the especially strong tides during the full and new moons, when the solar and lunar effects combine, and occur only every 14.8 days. In 1995, Dietrich Neumann showed that the larvae have two built-in clocks, a circadian and a circalunar one, which together control the hatching to precisely those few hours when the insect can reproduce. He also showed that the circalunar clock is synchronized by the brightness of the Moon at night. In other words, the larvae monitor the Moon at night and then decide when to hatch: they are the smallest known astronomers.

Ref. 151 If insects can have circalunar cycles, it should come as no surprise that women also have such a cycle; however, in this case the precise origin of the cycle length is still unknown and a topic of research.

Ref. 152 The Moon also helps to stabilize the tilt of the Earth's axis, keeping it more or less fixed relative to the plane of motion around the Sun. Without the Moon, the axis would change its direction irregularly, we would not have a regular day and night rhythm, we would have extremely large climate changes, and the evolution of life would have been impossible. Without the Moon, the Earth would also rotate much faster and we would have much less clement weather. The Moon's main remaining effect on the Earth, the precession of its axis, is responsible for the ice ages.  
Ref. 153  
Page 146

Ref. 154 The orbit of the Moon is still a topic of research. It is still not clear why the Moon orbit is at a  $5^\circ$  to the ecliptic and how the orbit changed since the Moon formed. Possibly, the collision that led to the formation of the Moon tilted the rotation axis of the Earth and the original Moon; then over thousands of millions of years, both axes moved in complicated ways towards the ecliptic, one more than the other. During this evolution, the distance to the Moon is estimated to have increased by a factor of 15.

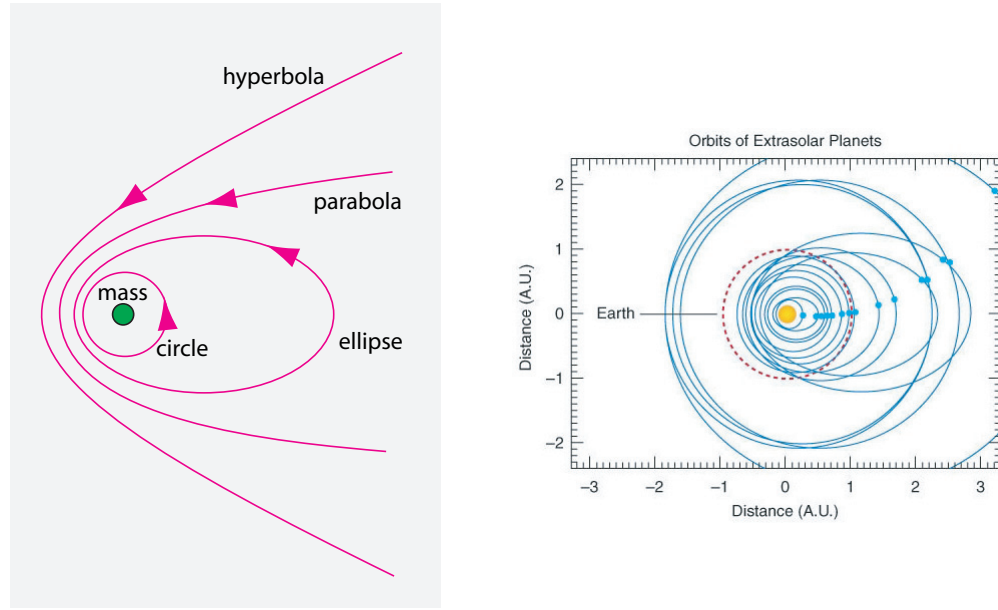
### ORBITS – CONIC SECTIONS AND MORE

Challenge 341 e The path of a body continuously orbiting another under the influence of gravity is an *ellipse* with the central body at one focus. A circular orbit is also possible, a circle being a special case of an ellipse. Single encounters of two objects can also be *parabolas* or *hyperbolas*, as shown in Figure 145. Circles, ellipses, parabolas and hyperbolas are collectively known as *conic sections*. Indeed each of these curves can be produced by cutting a cone with a knife. Are you able to confirm this?

If orbits are mostly ellipses, it follows that comets *return*. The English astronomer Edmund Halley (1656–1742) was the first to draw this conclusion and to predict the return of a comet. It arrived at the predicted date in 1756, after his death, and is now named after him. The period of Halley's comet is between 74 and 80 years; the first recorded sighting was 22 centuries ago, and it has been seen at every one of its 30 passages since, the last time in 1986.

Depending on the initial energy and the initial angular momentum of the body with





**FIGURE 145** The possible orbits, due to universal gravity, of a small mass around a *single* large mass (left) and a few recent examples of measured orbits (right), namely those of some extrasolar planets and of the Earth, all drawn around their respective central star, with distances given in astronomical units (© Geoffrey Marcy).

Challenge 342 s

respect to the central planet, paths are either *elliptic*, *parabolic* or *hyperbolic*. Can you determine the conditions for the energy and the angular momentum needed for these paths to appear?

In practice, parabolic orbits do not exist in nature. (Some comets seem to approach this case when moving around the Sun; but almost all comets follow elliptical paths – as long as they are far from other planets.). Hyperbolic paths do exist; artificial satellites follow them when they are shot towards a planet, usually with the aim of changing the direction of the satellite's journey across the Solar System.

Why does the inverse square 'law' lead to conic sections? First, for two bodies, the total angular momentum  $L$  is a constant:

$$L = mr^2\dot{\phi} = mr^2\left(\frac{d\phi}{dt}\right) \quad (51)$$

and therefore the motion lies in a plane. Also the energy  $E$  is a constant

$$E = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}m\left(r\frac{d\phi}{dt}\right)^2 - G\frac{mM}{r}. \quad (52)$$

Challenge 343 e Together, the two equations imply that

$$r = \frac{L^2}{Gm^2M} \frac{1}{1 + \sqrt{1 + \frac{2EL^2}{G^2m^3M^2}} \cos \varphi}. \quad (53)$$

Now, any curve defined by the general expression

$$r = \frac{C}{1 + e \cos \varphi} \quad \text{or} \quad r = \frac{C}{1 - e \cos \varphi} \quad (54)$$

is an ellipse for  $0 < e < 1$ , a parabola for  $e = 1$  and a hyperbola for  $e > 1$ , one focus being at the origin. The quantity  $e$ , called the *eccentricity*, describes how squeezed the curve is. In other words, a body in orbit around a central mass follows a conic section.

In all orbits, also the heavy mass moves. In fact, both bodies orbit around the common centre of mass. Both bodies follow the same type of curve – ellipse, parabola or hyperbola – but the sizes of the two curves differ.

Challenge 344 e

If more than two objects move under mutual gravitation, many additional possibilities for motions appear. The classification and the motions are quite complex. In fact, this so-called *many-body problem* is still a topic of research, both for astronomers and for mathematicians. Let us look at a few observations.

Page 152

When several planets circle a star, they also attract each other. Planets thus do not move in perfect ellipses. The largest deviation is a perihelion shift, as shown in [Figure 114](#). It is observed for Mercury and a few other planets, including the Earth. Other deviations from elliptical paths appear during a single orbit. In 1846, the observed deviations of the motion of the planet Uranus from the path predicted by universal gravity were used to predict the existence of another planet, Neptune, which was discovered shortly afterwards.

Page 106

We have seen that mass is always positive and that gravitation is thus always attractive; there is *no antigravity*. Can gravity be used for *levitation* nevertheless, using more than two bodies? Yes; there are two examples.\* The first are the geostationary satellites, which are used for easy transmission of television and other signals from and towards Earth.

Challenge 345 s

Challenge 346 d

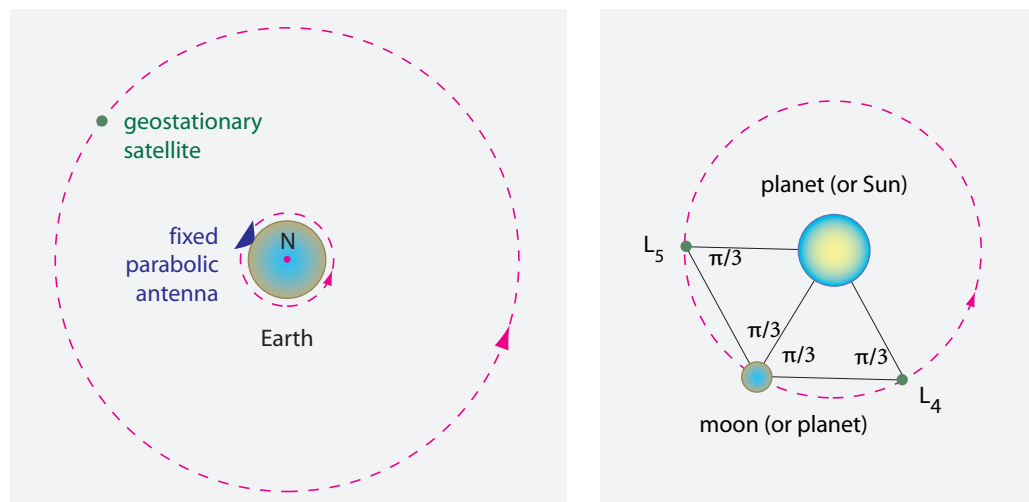
The *Lagrangian libration points* are the second example. Named after their discoverer, these are points in space near a two-body system, such as Moon–Earth or Earth–Sun, in which small objects have a stable equilibrium position. An overview is given in [Figure 147](#). Can you find their precise position, remembering to take rotation into account? There are three additional Lagrangian points on the Earth–Moon line (or Sun–planet line). How many of them are stable?

Ref. 156

There are thousands of asteroids, called *Trojan asteroids*, at and around the Lagrangian points of the Sun–Jupiter system. In 1990, a Trojan asteroid for the Mars–Sun system was discovered. Finally, in 1997, an ‘almost Trojan’ asteroid was found that follows the Earth on its way around the Sun (it is only transitional and follows a somewhat more complex orbit). This ‘second companion’ of the Earth has a diameter of 5 km. Similarly, on the



**FIGURE 146** Geostationary satellites, seen here in the upper left quadrant, move against the other stars and show the location of the celestial Equator. (MP4 film © Michael Kunze)

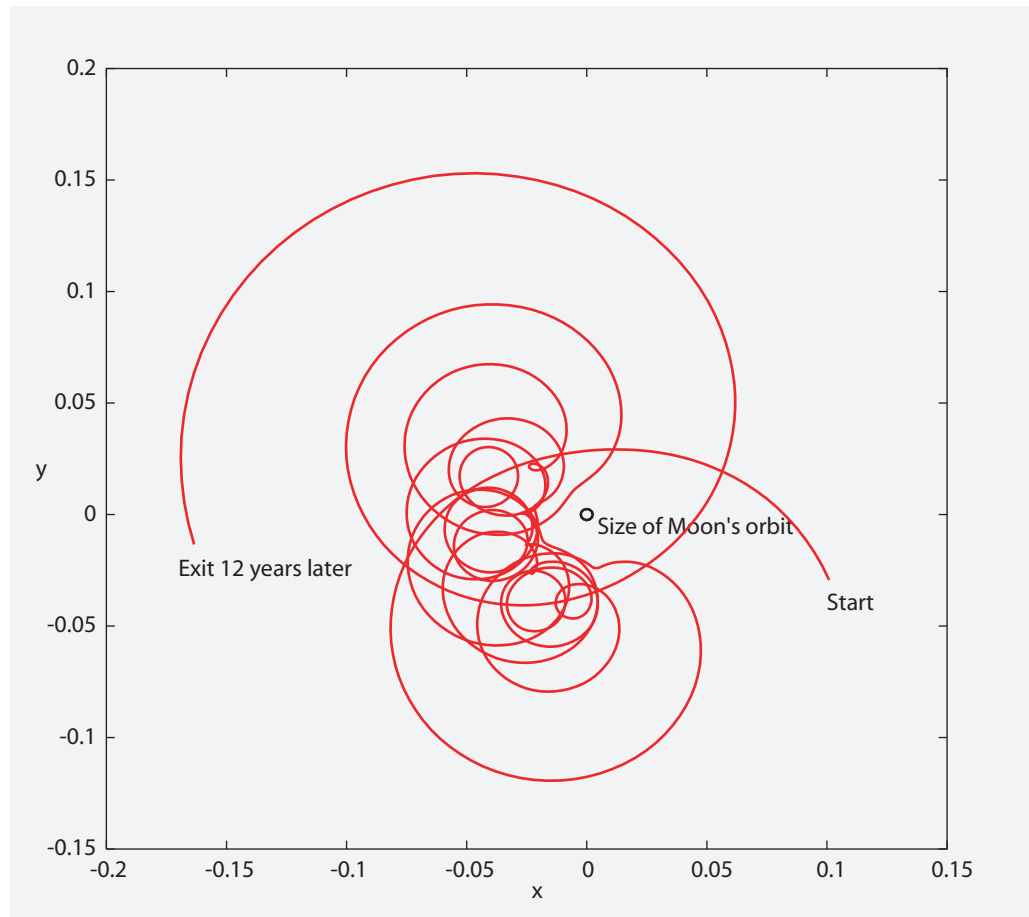


**FIGURE 147** Geostationary satellites (left) and the main stable Lagrangian points (right).

main Lagrangian points of the Earth–Moon system a high concentration of dust has been observed.

Ref. 155 Astronomers know that many other objects follow irregular orbits, especially asteroids. For example, asteroid 2003 YN107 followed an irregular orbit, shown in Figure 148, that accompanied the Earth for a number of years.

To sum up, the single equation  $\mathbf{a} = -GM\mathbf{r}/r^3$  correctly describes a large number of phenomena in the sky. The first person to make clear that this expression describes



**FIGURE 148** An example of irregular orbit, partly measured and partly calculated, due to the gravitational attraction of *several* masses: the orbit of the temporary Earth quasi-satellite 2003 YN107 in geocentric coordinates. This asteroid, with a diameter of 20(10) m, became orbitally trapped near the Earth around 1995 and remained so until 2006. The black circle represents the Moon's orbit around the Earth. (© Seppo Mikkola).

*everything* happening in the sky was Pierre Simon Laplace in his famous treatise *Traité de mécanique céleste*. When Napoleon told him that he found no mention about the creator in the book, Laplace gave a famous, one sentence summary of his book: *Je n'ai pas eu besoin de cette hypothèse*. 'I had no need for this hypothesis.' In particular, Laplace studied the stability of the Solar System, the eccentricity of the lunar orbit, and the eccentricities of the planetary orbits, always getting full agreement between calculation and measurement.

These results are quite a feat for the simple expression of universal gravitation; they also explain why it is called 'universal'. But how *accurate* is the formula? Since astronomy allows the most precise measurements of gravitational motion, it also provides the most stringent tests. In 1849, Urbain Le Verrier concluded after intensive study that there was only one known example of a discrepancy between observation and universal gravity, namely one observation for the planet Mercury. (Nowadays a few more are known.) The



FIGURE 149 Tides at Saint-Valéry en Caux on 20 September 2005 (© Gilles Régnier).

point of least distance to the Sun of the orbit of planet Mercury, its *perihelion*, rotates around the Sun at a rate that is slightly less than that predicted: he found a tiny difference, around  $38''$  per century. (This was corrected to  $43''$  per century in 1882 by Simon Newcomb.) Le Verrier thought that the difference was due to a planet between Mercury and the Sun, *Vulcan*, which he chased for many years without success. Indeed, Vulcan does not exist. The correct explanation of the difference had to wait for Albert Einstein.

Ref. 157

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### TIDES

Ref. 158 Why do physics texts always talk about tides? Because, as general relativity will show, tides prove that space is curved! It is thus useful to study them in a bit more detail. Figure 149 how striking tides can be. Gravitation explains the sea tides as results of the attraction of the ocean water by the Moon and the Sun. Tides are interesting; even though the amplitude of the tides is only about 0.5 m on the open sea, it can be up to 20 m at special places near the coast. Can you imagine why? The *soil* is also lifted and lowered by the Sun and the Moon, by about 0.3 m, as satellite measurements show. Even the *atmosphere* is subject to tides, and the corresponding pressure variations can be filtered out from the weather pressure measurements.

Challenge 347 s

Ref. 57

Ref. 159

Tides appear for any *extended* body moving in the gravitational field of another. To understand the origin of tides, picture a body in orbit, like the Earth, and imagine its components, such as the segments of Figure 150, as being held together by springs. Universal gravity implies that orbits are slower the more distant they are from a central body. As a result, the segment on the outside of the orbit would like to be slower than the cent-



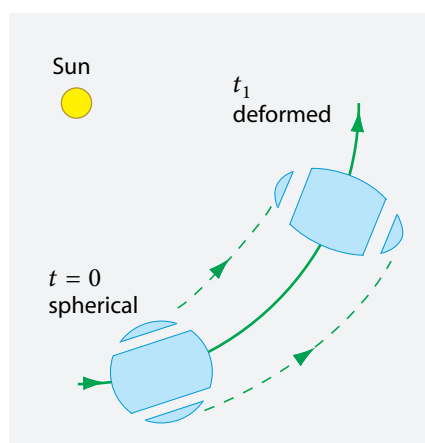


FIGURE 150 Tidal deformations due to gravity.

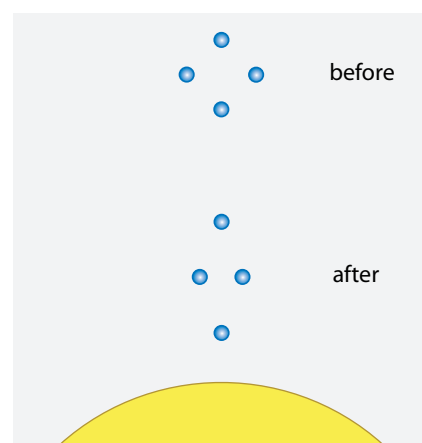


FIGURE 151 The origin of tides.

ral one; but it is *pulled* by the rest of the body through the springs. In contrast, the inside segment would like to orbit more rapidly but is *retained* by the others. Being slowed down, the inside segments want to fall towards the Sun. In sum, both segments feel a pull away from the centre of the body, limited by the springs that stop the deformation. Therefore, *extended bodies are deformed in the direction of the field inhomogeneity*.

For example, as a result of tidal forces, the Moon always has (roughly) the same face to the Earth. In addition, its radius in direction of the Earth is larger by about 5 m than the radius perpendicular to it. If the inner springs are too weak, the body is torn into pieces; in this way a *ring* of fragments can form, such as the asteroid ring between Mars and Jupiter or the rings around Saturn.

Let us return to the Earth. If a body is surrounded by water, it will form bulges in the direction of the applied gravitational field. In order to measure and compare the strength of the tides from the Sun and the Moon, we reduce tidal effects to their bare minimum. As shown in Figure 151, we can study the deformation of a body due to gravity by studying the arrangement of four bodies. We can study the free fall case because orbital motion and free fall are equivalent. Now, gravity makes some of the pieces approach and others diverge, depending on their relative positions. The figure makes clear that the strength of the deformation – water has no built-in springs – depends on the change of gravitational acceleration with distance; in other words, the *relative* acceleration that leads to the tides is proportional to the derivative of the gravitational acceleration.

Page 453

Using the numbers from Appendix B, the gravitational accelerations from the Sun and the Moon measured on Earth are

$$\begin{aligned} a_{\text{Sun}} &= \frac{GM_{\text{Sun}}}{d_{\text{Sun}}^2} = 5.9 \text{ mm/s}^2 \\ a_{\text{Moon}} &= \frac{GM_{\text{Moon}}}{d_{\text{Moon}}^2} = 0.033 \text{ mm/s}^2 \end{aligned} \quad (55)$$

and thus the attraction from the Moon is about 178 times weaker than that from the Sun.

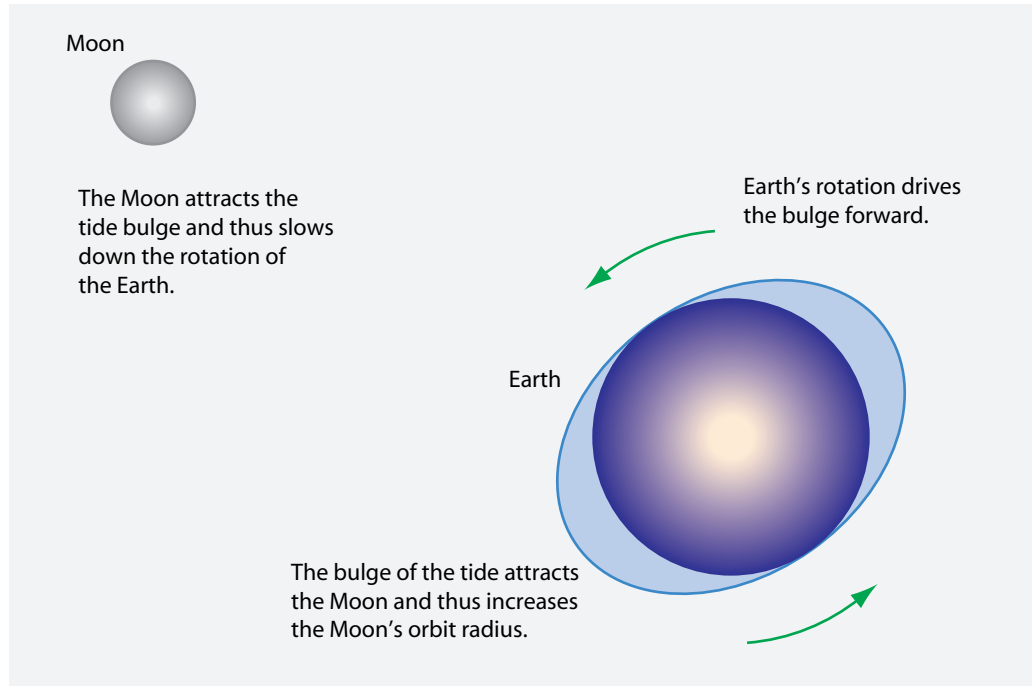


FIGURE 152 The Earth, the Moon and the friction effects of the tides (not to scale).

When two nearby bodies fall near a large mass, the relative acceleration is proportional to their distance, and follows  $da = (da/dr)dr$ . The proportionality factor  $da/dr = \nabla a$ , called the *tidal acceleration* (gradient), is the true measure of tidal effects. Near a large spherical mass  $M$ , it is given by

$$\frac{da}{dr} = -\frac{2GM}{r^3} \quad (56)$$

which yields the values

$$\begin{aligned} \frac{da_{\text{Sun}}}{dr} &= -\frac{2GM_{\text{Sun}}}{d_{\text{Sun}}^3} = -0.8 \cdot 10^{-13} / \text{s}^2 \\ \frac{da_{\text{Moon}}}{dr} &= -\frac{2GM_{\text{Moon}}}{d_{\text{Moon}}^3} = -1.7 \cdot 10^{-13} / \text{s}^2 . \end{aligned} \quad (57)$$

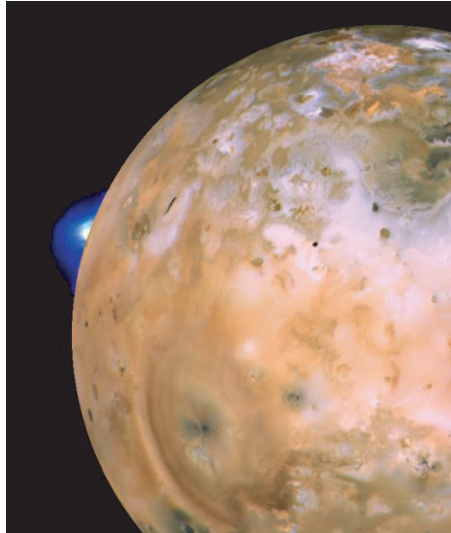
In other words, despite the much weaker pull of the Moon, its tides are predicted to be over *twice as strong* as the tides from the Sun; this is indeed observed. When Sun, Moon and Earth are aligned, the two tides add up; these so-called *spring tides* are especially strong and happen every 14.8 days, at full and new moon.

Tides lead to a pretty puzzle. Moon tides are much stronger than Sun tides. This implies that the Moon is much denser than the Sun. Why?

Tides also produce *friction*, as shown in Figure 152. The friction leads to a slowing of the Earth's rotation. Nowadays, the slowdown can be measured by precise clocks (even

Challenge 348 e

Challenge 349 s



**FIGURE 153** A spectacular result of tides: volcanism on Io (NASA).

Ref. 116 though short time variations due to other effects, such as the weather, are often larger).  
 Vol. II, page 231 The results fit well with fossil results showing that 400 million years ago, in the Devonian  
 Challenge 350 s period, a year had 400 days, and a day about 22 hours. It is also estimated that 900 million  
 years ago, each of the 481 days of a year were 18.2 hours long. The friction at the basis of  
 this slowdown also results in an increase in the distance of the Moon from the Earth by  
 about 3.8 cm per year. Are you able to explain why?

As mentioned above, the tidal motion of the soil is also responsible for the triggering of *earthquakes*. Thus the Moon can have also dangerous effects on Earth. (Unfortunately, knowing the mechanism does not allow predicting earthquakes.) The most fascinating example of tidal effects is seen on Jupiter's satellite Io. Its tides are so strong that they induce intense volcanic activity, as shown in Figure 153, with eruption plumes as high as 500 km. If tides are even stronger, they can destroy the body altogether, as happened to the body between Mars and Jupiter that formed the planetoids, or (possibly) to the moons that led to Saturn's rings.

Vol. II, page 136 In summary, tides are due to relative accelerations of nearby mass points. This has an  
 important consequence. In the chapter on general relativity we will find that time multi-  
 Vol. II, page 190 plied by the speed of light plays the same role as length. Time then becomes an additional  
 dimension, as shown in Figure 154. Using this similarity, two free particles moving in the  
 same direction correspond to parallel lines in space-time. Two particles falling side-by-  
 side also correspond to parallel lines. Tides show that such particles approach each other.  
 In other words, tides imply that parallel lines approach each other. But parallel lines can  
 approach each other *only* if space-time is curved. In short, tides imply *curved* space-time  
 and space. This simple reasoning could have been performed in the eighteenth century;  
 however, it took another 200 years and Albert Einstein's genius to uncover it.

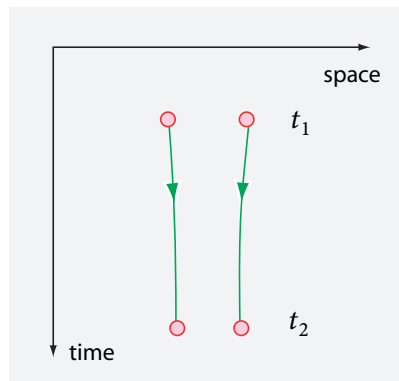


FIGURE 154 Particles falling side-by-side approach over time.

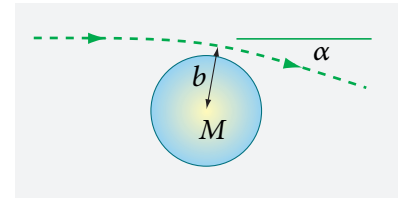


FIGURE 155 Masses bend light.

### CAN LIGHT FALL?

“Die Maxime, jederzeit selbst zu denken, ist die Aufklärung.”  
Immanuel Kant\*

Vol. II, page 15

Towards the end of the seventeenth century people discovered that light has a finite velocity – a story which we will tell in detail later. An entity that moves with infinite velocity cannot be affected by gravity, as there is no time to produce an effect. An entity with a finite speed, however, should feel gravity and thus fall.

Ref. 160

Does its speed increase when light reaches the surface of the Earth? For almost three centuries people had no means of detecting any such effect; so the question was not investigated. Then, in 1801, the Prussian astronomer Johann Soldner (1776–1833) was the first to put the question in a different way. Being an astronomer, he was used to measuring stars and their observation angles. He realized that light passing near a massive body would be *deflected* due to gravity.

Challenge 351 ny

Soldner studied a body on a hyperbolic path, moving with velocity  $c$  past a spherical mass  $M$  at distance  $b$  (measured from the centre), as shown in Figure 155. Soldner deduced the deflection angle

$$\alpha_{\text{univ. grav.}} = \frac{2}{b} \frac{GM}{c^2}. \quad (58)$$

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The value of the angle is largest when the motion is just grazing the mass  $M$ . For light deflected by the mass of the Sun, the angle turns out to be at most a tiny  $0.88'' = 4.3 \mu\text{rad}$ . In Soldner's time, this angle was too small to be measured. Thus the issue was forgotten. Had it been pursued, general relativity would have begun as an experimental science, and not as the theoretical effort of Albert Einstein! Why? The value just calculated is *different* from the measured value. The first measurement took place in 1919;\*\* it found the correct dependence on the distance, but found a deflection of up to  $1.75''$ , exactly double that of expression (58). The reason is not easy to find; in fact, it is due to the curvature of space,

\* The maxim to think at all times for oneself is the enlightenment.

Challenge 352 s

\*\* By the way, how would you measure the deflection of light near the bright Sun?

as we will see. In summary, light can fall, but the issue hides some surprises.

### MASS: INERTIAL AND GRAVITATIONAL

Mass describes how an object interacts with others. In our walk, we have encountered two of its aspects. *Inertial mass* is the property that keeps objects moving and that offers resistance to a change in their motion. *Gravitational mass* is the property responsible for the acceleration of bodies nearby (the active aspect) or of being accelerated by objects nearby (the passive aspect). For example, the active aspect of the mass of the Earth determines the surface acceleration of bodies; the passive aspect of the bodies allows us to weigh them in order to measure their mass using distances only, e.g. on a scale or a balance. The gravitational mass is the basis of *weight*, the difficulty of lifting things.\*

Is the gravitational mass of a body equal to its inertial mass? A rough answer is given by the experience that an object that is difficult to move is also difficult to lift. The simplest experiment is to take two bodies of different masses and let them fall. If the acceleration is the same for all bodies, inertial mass is equal to (passive) gravitational mass, because in the relation  $ma = \nabla(GMm/r)$  the left-hand  $m$  is actually the inertial mass, and the right-hand  $m$  is actually the gravitational mass.

Already in the seventeenth century Galileo had made widely known an even older argument showing without a single experiment that the gravitational acceleration is indeed *the same* for all bodies. If larger masses fell more rapidly than smaller ones, then the following paradox would appear. Any body can be seen as being composed of a large fragment attached to a small fragment. If small bodies really fell less rapidly, the small fragment would slow the large fragment down, so that the complete body would have to fall *less* rapidly than the larger fragment (or break into pieces). At the same time, the body being larger than its fragment, it should fall *more* rapidly than that fragment. This is obviously impossible: all masses must fall with the same acceleration.

Ref. 161

Many accurate experiments have been performed since Galileo's original discussion. In all of them, the independence of the acceleration of free fall from mass and material composition has been confirmed with the precision they allowed. In other words, experiments confirm:

▷ Gravitational mass and inertial mass are *equal*.

What is the origin of this mysterious equality?

Page 100

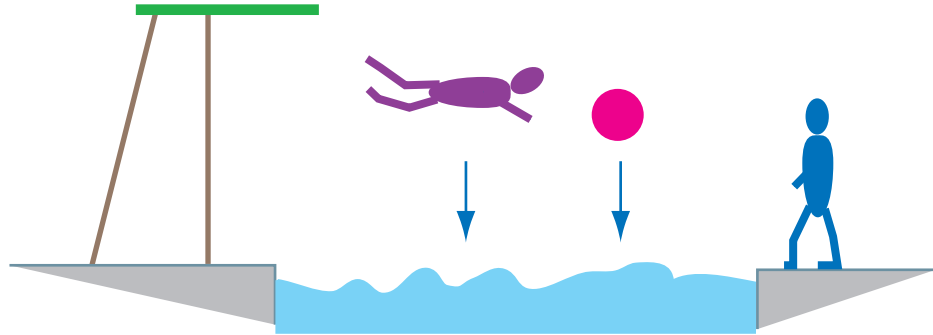
The equality of gravitational and inertial mass is not a mystery at all. Let us go back to the definition of mass as a negative inverse acceleration ratio. We mentioned that the physical origin of the accelerations does not play a role in the definition because the origin does not appear in the expression. In other words, the value of the mass is by definition independent of the interaction. That means in particular that inertial mass, based on and measured with the electromagnetic interaction, and gravitational mass are identical *by definition*.

The best proof of the equality of inertial and gravitational mass is illustrated in [Figure 156](#): it shows that the two concepts only differ by the viewpoint of the observer. Inertial mass and gravitational mass describe the same observation.

Challenge 353 ny

\* What are the weight values shown by a balance for a person of 85 kg juggling three balls of 0.3 kg each?





**FIGURE 156** The falling ball is in inertial motion for a falling observer and in gravitational motion for an observer on the ground. Therefore, inertial mass is the same as gravitational mass.

We also note that we have not defined a separate concept of ‘passive gravitational mass’. (This concept is sometimes found in research papers.) The mass being accelerated by gravitation is the inertial mass. Worse, there is no way to define a ‘passive gravitational mass’ that differs from inertial mass. Try it! All methods that measure a passive gravitational mass, such as weighing an object, cannot be distinguished from the methods that determine inertial mass from its reaction to acceleration. Indeed, all these methods use the same non-gravitational mechanisms. Bathroom scales are a typical example.

Challenge 354 s

Indeed, if the ‘passive gravitational mass’ were different from the inertial mass, we would have strange consequences. Not only is it hard to distinguish the two in an experiment; for those bodies for which it were different we would get into trouble with energy conservation.

Challenge 355 e

In fact, also assuming that (‘active’) ‘gravitational mass’ differs from inertial mass gets us into trouble. How could ‘gravitational mass’ differ from inertial mass? Would the difference depend on relative velocity, time, position, composition or on mass itself? No. Each of these possibilities contradicts either energy or momentum conservation.

Challenge 356 s

In summary, it is no wonder that all measurements confirm the equality of all mass types: there is no other option – as Galileo pointed out. The lack of other options is due to the fundamental equivalence of all mass definitions:

- ▷ Mass ratios are acceleration ratios.

Vol. II, page 157

The topic is usually rehashed in general relativity, with no new results, because the definition of mass remains the same. Gravitational and inertial masses remain equal. In short:

- ▷ Mass is a unique property of each body.

Another, deeper issue remains, though. What is the *origin* of mass? Why does it exist? This simple but deep question cannot be answered by classical physics. We will need some patience to find out.

### CURIOSITIES AND FUN CHALLENGES ABOUT GRAVITATION

“Fallen ist weder gefährlich noch eine Schande;  
Liegen bleiben ist beides.\*”  
Konrad Adenauer

Cosmonauts on the International Space Station face two challenges: cancer-inducing cosmic radiation and the lack of gravity. The lack of gravity often leads to orientation problems and nausea in the first days, the so-called *space sickness* and *motion sickness*. When these disappear, the muscles start to reduce in volume by a few % per month, bones get weaker every week, the immune system is on permanent alarm state, blood gets pumped into the head more than usual and produces round ‘baby’ faces – easily seen on television – and strong headaches, legs loose blood and get thinner, body temperature permanently increases by over one degree Celsius, the brain gets compressed by the blood and spinal fluid, and the eyesight deteriorates, because also the eyes get compressed. When cosmonauts return to Earth after six months in space, they have weak bones and muscles, and they are unable to walk and stand. They need a day or two to learn to do so again. Later, they often get hernias, and because of the bone reduction, kidney stones. Other health issues are likely to exist; but they have been kept confidential by cosmonauts in order to maintain their image and their chances for subsequent missions.

\* \*

Gravity on the Moon is only one sixth of that on the Earth. Why does this imply that it is difficult to walk quickly and to run on the Moon (as can be seen in the TV images recorded there)?

\* \*

Challenge 357 e

Understand and explain the following statement: a beam balance measures mass, a spring scale measures weight.

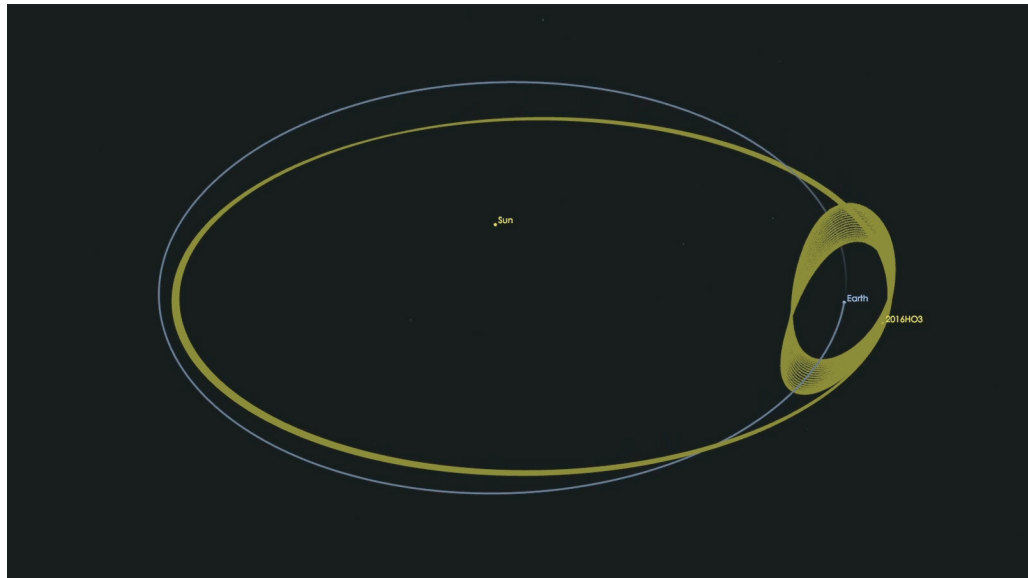
\* \*

Does the Earth have other satellites apart from the Moon and the artificial satellites shot into orbit up by rockets? Yes. The Earth has a number of mini-satellites and a large number of quasi-satellites. An especially long-lived quasi-satellite, an asteroid called 2016 HO3, has a size of about 60 m and was discovered in 2016. As shown in [Figure 157](#), it orbits the Earth and will continue to do so for another few hundred years, at a distance from 40 to 100 times that of the Moon.

\* \*

---

\* ‘Falling is neither dangerous nor a shame; to keep lying is both.’ Konrad Adenauer (b. 1876 Köln, d. 1967 Rhöndorf), West German Chancellor.



**FIGURE 157** The calculated orbit of the quasi-satellite 2016 HO3, a temporary companion of the Earth (courtesy NASA).

Show that a sphere bouncing – without energy loss – down an inclined plane hits the plane in spots whose distances increase by a constant amount at every bounce.

\* \*

Is the acceleration due to gravity constant over time? Not really. Every day, it is estimated that  $10^8$  kg of material fall onto the Earth in the form of meteorites and asteroids. (Examples can be seen in [Figure 158](#) and [Figure 159](#).) Nevertheless, it is unknown whether the mass of the Earth increases with time (due to the collection of meteorites and cosmic dust) or decreases (due to gas loss). If you find a way to settle the issue, publish it.

\* \*

Incidentally, discovering objects hitting the Earth is not at all easy. Astronomers like to point out that an asteroid as large as the one that led to the extinction of the dinosaurs could hit the Earth without any astronomer noticing in advance, if the direction is slightly unusual, such as from the south, where few telescopes are located.

\* \*

Several humans have survived free falls from aeroplanes for a thousand metres or more, even though they had no parachute. A minority of them even did so without any harm at all. How was this possible?

Challenge 358 s

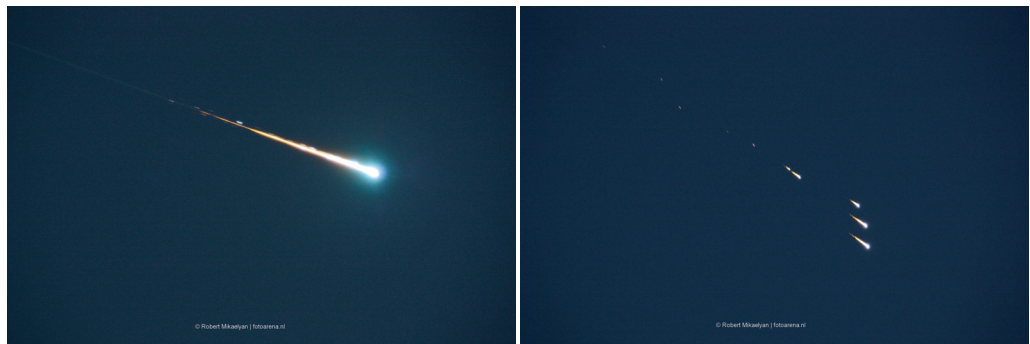
\* \*

Imagine that you have twelve coins of identical appearance, of which one is a forgery. The forged one has a different mass from the eleven genuine ones. How can you decide which is the forged one and whether it is lighter or heavier, using a simple balance only



**FIGURE 158** A composite photograph of the Perseid meteor shower that is visible every year in mid August. In that month, the Earth crosses the cloud of debris stemming from comet Swift–Tuttle, and the source of the meteors appears to lie in the constellation of Perseus, because that is the direction in which the Earth is moving in mid August. The effect and the picture are thus similar to what is seen on the windscreen when driving by car while it is snowing. (© Brad Goldpaint at [goldpaintphotography.com](http://goldpaintphotography.com)).

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**FIGURE 159** Two photographs, taken about a second apart, showing a meteor break-up (© Robert Mikaelyan).

Challenge 359 e three times?

You have nine identically-looking spheres, all of the same mass, except one, which is heavier. Can you determine which one, using the balance only two times?

\* \*

For a physicist, *antigravity* is repulsive gravity – it does not exist in nature. Nevertheless,

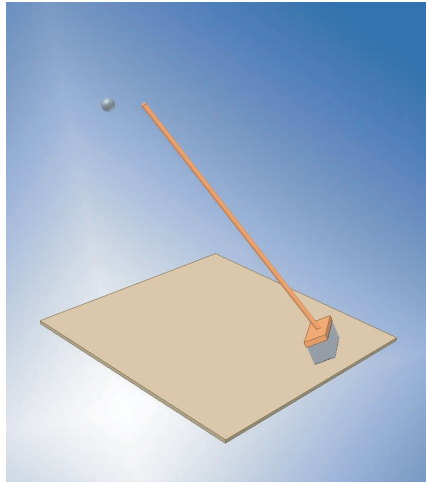


FIGURE 160 Brooms fall more rapidly than stones (© Luca Gastaldi).

the term ‘antigravity’ is used incorrectly by many people, as a short search on the internet shows. Some people call any effect that *overcomes* gravity, ‘antigravity’. However, this definition implies that tables and chairs are antigravity devices. Following the definition, most of the wood, steel and concrete producers are in the antigravity business. The internet definition makes absolutely no sense.

\* \*

Challenge 360 s What is the cheapest way to switch gravity off for 25 seconds?

\* \*

Do all objects on Earth fall with the same acceleration of  $9.8 \text{ m/s}^2$ , assuming that air resistance can be neglected? No; every housekeeper knows that. You can check this by yourself. As shown in Figure 160, a broom angled at around  $35^\circ$  hits the floor before a stone, as the sounds of impact confirm. Are you able to explain why?

Challenge 361 s

\* \*

Also bungee jumpers are accelerated more strongly than  $g$ . For a bungee cord of mass  $m$  and a jumper of mass  $M$ , the maximum acceleration  $a$  is

$$a = g \left( 1 + \frac{m}{8M} \left( 4 + \frac{m}{M} \right) \right). \quad (59)$$

Challenge 362 s Can you deduce the relation from Figure 161?

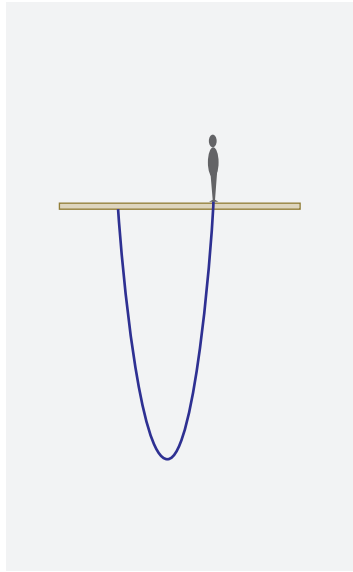
\* \*

Challenge 363 s Guess: What is the mass of a ball of cork with a radius of 1 m?

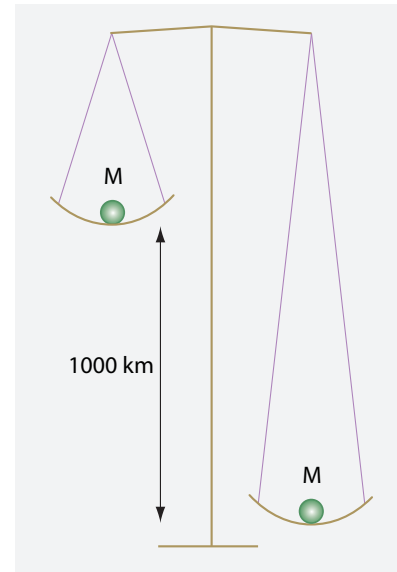
\* \*

Challenge 364 s Guess: One thousand 1 mm diameter steel balls are collected. What is the mass?





**FIGURE 161** The starting situation for a bungee jumper.



**FIGURE 162** An honest balance?

\* \*

Challenge 365 s

How can you use your observations made during your travels with a bathroom scale to show that the Earth is not flat?

\* \*

Challenge 366 s

Both the Earth and the Moon attract bodies. The centre of mass of the Earth–Moon system is 4800 km away from the centre of the Earth, quite near its surface. Why do bodies on Earth still fall towards the centre of the Earth?

\* \*

Challenge 367 e

Page 202

Does every spherical body fall with the same acceleration? No. If the mass of the object is comparable to that of the Earth, the distance decreases in a different way. Can you confirm this statement? **Figure 162** shows a related puzzle. What then is wrong about Galileo's argument about the constancy of acceleration of free fall?

\* \*

Vol. II, page 136

What is the fastest speed that a human can achieve making use of gravitational acceleration? There are various methods that try this; a few are shown in **Figure 163**. Terminal speed of free falling skydivers can be even higher, but no reliable record speed value exists. The last word is not spoken yet, as all these records will be surpassed in the coming years. It is important to require normal altitude; at stratospheric altitudes, speed values can be four times the speed values at low altitude.

\* \*

It is easy to put a mass of a kilogram onto a table. Twenty kilograms is harder. A thousand



**FIGURE 163** Reducing air resistance increases the terminal speed: left, the 2007 speed skiing world record holder Simone Origone with 69.83 m/s and right, the 2007 speed world record holder for bicycles on snow Éric Barone with 61.73 m/s (© Simone Origone, Éric Barone).

Challenge 368 s is impossible. However,  $6 \cdot 10^{24}$  kg is easy. Why?

\* \*

Page 199 The friction between the Earth and the Moon slows down the rotation of both. The Moon stopped rotating millions of years ago, and the Earth is on its way to doing so as well. When the Earth stops rotating, the Moon will stop moving away from Earth. How far will the Moon be from the Earth at that time? Afterwards however, even further in the future, the Moon will move back towards the Earth, due to the friction between the Earth–Moon system and the Sun. Even though this effect would only take place if the Sun burned for ever, which is known to be false, can you explain it?

Challenge 369 ny

Challenge 370 s

\* \*

When you run towards the east, you *lose weight*. There are two different reasons for this: the ‘centrifugal’ acceleration increases so that the force with which you are pulled down diminishes, and the Coriolis force appears, with a similar result. Can you estimate the size of the two effects?

Challenge 371 ny

\* \*

Laboratories use two types of ultracentrifuges: *preparative* ultracentrifuges isolate viruses, organelles and biomolecules, whereas *analytical* ultracentrifuges measure the shape and mass of macromolecules. The fastest commercially available models achieve 200 000 rpm, or 3.3 kHz, and a centrifugal acceleration of  $10^6 \cdot g$ .

\* \*

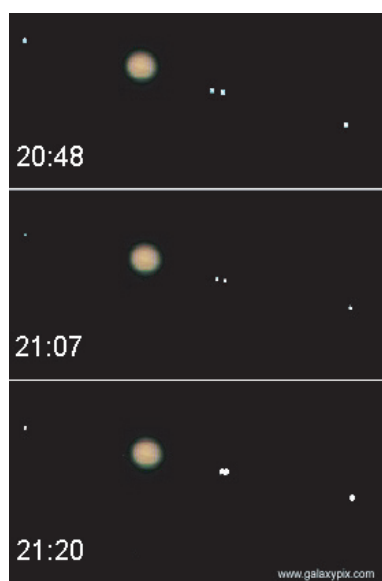


FIGURE 164 The four satellites of Jupiter discovered by Galileo and their motion (© Robin Scagell).

Challenge 372 s

What is the relation between the time a stone takes falling through a distance  $l$  and the time a pendulum takes swinging through half a circle of radius  $l$ ? (This problem is due to Galileo.) How many digits of the number  $\pi$  can one expect to determine in this way?

\* \*

Challenge 373 s

Ref. 162

Why can a spacecraft accelerate through the *slingshot effect* when going round a planet, despite momentum conservation? It is speculated that the same effect is also the reason for the few exceptionally fast stars that are observed in the galaxy. For example, the star HE0457-5439 moves with 720 km/s, which is much higher than the 100 to 200 km/s of most stars in the Milky Way. It seems that the role of the accelerating centre was taken by a black hole.

\* \*

Ref. 163

Challenge 374 s

The orbit of a planet around the Sun has many interesting properties. What is the hodograph of the orbit? What is the hodograph for parabolic and hyperbolic orbits?

\* \*

The *Galilean satellites* of Jupiter, shown in Figure 164, can be seen with small amateur telescopes. Galileo discovered them in 1610 and called them the *Medicean satellites*. (Today, they are named, in order of increasing distance from Jupiter, as Io, Europa, Ganymede and Callisto.) They are almost mythical objects. They were the first bodies found that obviously did not orbit the Earth; thus Galileo used them to deduce that the Earth is not at the centre of the universe. The satellites have also been candidates to be the first *standard clock*, as their motion can be predicted to high accuracy, so that the 'standard time' could be read off from their position. Finally, due to this high accuracy, in 1676, the speed of light was first measured with their help, as told in the section on

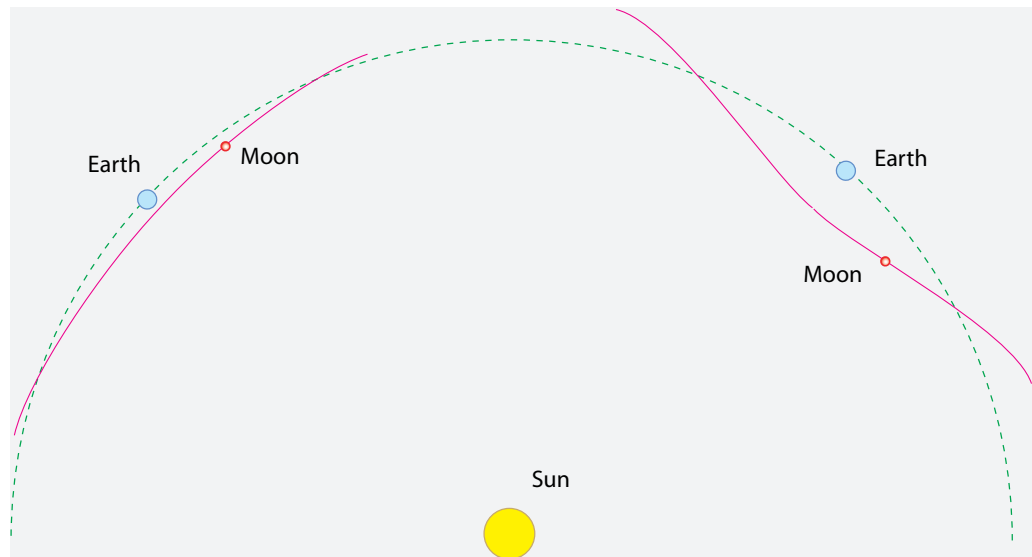


FIGURE 165 Which of the two Moon paths is correct?

Vol. II, page 16 special relativity.

\* \*

Challenge 375 s

A simple, but difficult question: if all bodies attract each other, why don't or didn't all stars fall towards each other? Indeed, the inverse square expression of universal gravity has a limitation: it does not allow one to make sensible statements about the matter in the universe. Universal gravity predicts that a homogeneous mass distribution is unstable; indeed, an inhomogeneous distribution is observed. However, universal gravity does not predict the average mass density, the darkness at night, the observed speeds of the distant galaxies, etc. In fact, 'universal' gravity does not explain or predict a single property of the universe. To do this, we need general relativity.

Vol. II, page 211

\* \*

Ref. 164

The acceleration  $g$  due to gravity at a depth of 3000 km is  $10.05 \text{ m/s}^2$ , over 2 % more than at the surface of the Earth. How is this possible? Also, on the Tibetan plateau,  $g$  is influenced by the material below it.

\* \*

Challenge 376 s

When the Moon circles the Sun, does its path have sections *concave* towards the Sun, as shown at the right of Figure 165, or not, as shown on the left? (Independent of this issue, both paths in the diagram disguise that the Moon path does *not* lie in the same plane as the path of the Earth around the Sun.)

\* \*

You can prove that objects *attract each other* (and that they are not only attracted by the Earth) with a simple experiment that anybody can perform at home, as described on the [www.fourmilab.ch/gravitation/foobar](http://www.fourmilab.ch/gravitation/foobar) website.

\* \*

It is instructive to calculate the *escape velocity* from the Earth, i.e., that velocity with which a body must be thrown so that it never falls back. It turns out to be around 11 km/s. (This was called the *second cosmic velocity* in the past; the *first cosmic velocity* was the name given to the lowest speed for an orbit, 7.9 km/s.) The exact value of the escape velocity depends on the latitude of the thrower, and on the direction of the throw. (Why?)

Challenge 377 e

What is the escape velocity from the Solar System? (It was once called the *third cosmic velocity*.) By the way, the escape velocity from our galaxy is over 500 km/s. What would happen if a planet or a system were so heavy that the escape velocity from it would be larger than the speed of light?

Challenge 378 s

\* \*

What is the largest asteroid one can escape from by jumping?

Challenge 379 s

\* \*

For bodies of irregular shape, the centre of gravity of a body is *not* the same as the centre of mass. Are you able to confirm this? (Hint: Find and use the simplest example possible.)

Challenge 380 s

\* \*

Can gravity produce repulsion? What happens to a small test body on the inside of a large C-shaped mass? Is it pushed towards the centre of mass?

Challenge 381 ny

\* \*

A heavily disputed argument for the equality of inertial and gravitational mass was given by Chubykalo, and Vlaev. The total kinetic energy  $T$  of two bodies circling around their common centre of mass, like the Earth and the Moon, is given by  $T = GmM/2R$ , where the two quantities  $m$  and  $M$  are the *gravitational* masses of the two bodies and  $R$  their distance. From this expression, in which the inertial masses do *not* appear on the right side, they deduce that the inertial and gravitational mass must be proportional to each other. Can you see how? Is the reasoning correct?

Ref. 165

Challenge 382 s

\* \*

The *shape* of the Earth is not a sphere. As a consequence, a plumb line usually does not point to the centre of the Earth. What is the largest deviation in degrees?

Ref. 166  
Challenge 383 ny

\* \*

Owing to the slightly flattened shape of the Earth, the source of the Mississippi is about 20 km nearer to the centre of the Earth than its mouth; the water effectively runs uphill. How can this be?

Challenge 384 s

\* \*

If you look at the sky every day at 6 a.m., the Sun's position varies during the year. The result of photographing the Sun on the same film is shown in [Figure 166](#). The curve, called the analemma, is due to two combined effects: the inclination of the Earth's axis and the elliptical shape of the Earth's orbit around the Sun. The top and the (hidden) bottom points of the analemma correspond to the solstices. How does the analemma look





**FIGURE 166** The analemma over Delphi, taken between January and December 2002  
(© Anthony Ayiomamitis).

**Challenge 385 s** if photographed every day at local noon? Why is it not a straight line pointing exactly south?

\* \*

The constellation in which the Sun stands at noon (at the centre of the time zone) is supposedly called the 'zodiacal sign' of that day. Astrologers say there are twelve of them, namely Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpius, Sagittarius, Capricornus, Aquarius and Pisces and that each takes (quite precisely) a twelfth of a year or a twelfth of the ecliptic. Any check with a calendar shows that at present, the midday Sun is never in the zodiacal sign during the days usually connected to it. The relation has shifted by about a month since it was defined, due to the precession of the Earth's axis. A check with a map of the star sky shows that the twelve constellations do not have the same length and that on the ecliptic there are fourteen of them, not twelve. There is *Ophiuchus* or *Serpentarius*, the serpent bearer constellation, between Scorpius and Sagittarius, and *Cetus*, the whale, between Aquarius and Pisces. In fact, not a single astronomical statement about zodiacal signs is correct. To put it clearly, astrology, in contrast to its name, is *not* about stars. (In German, the word 'Strolch', meaning 'rogue' or 'scoundrel', is derived from the word 'astrologer'.)

\* \*

For a long time, it was thought that there is no additional planet in our Solar System outside Neptune and Pluto, because their orbits show no disturbances from another body. Today, the view has changed. It is known that there are only eight planets: Pluto is not

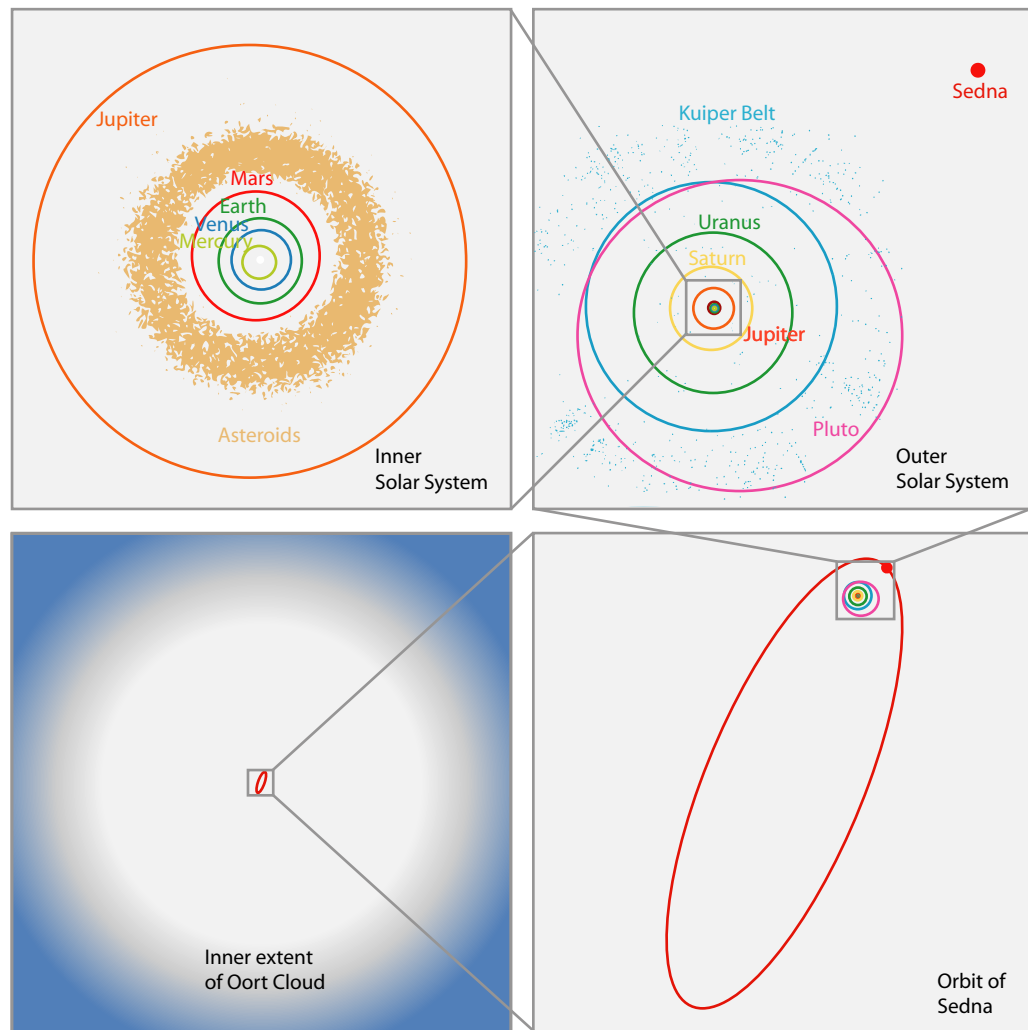
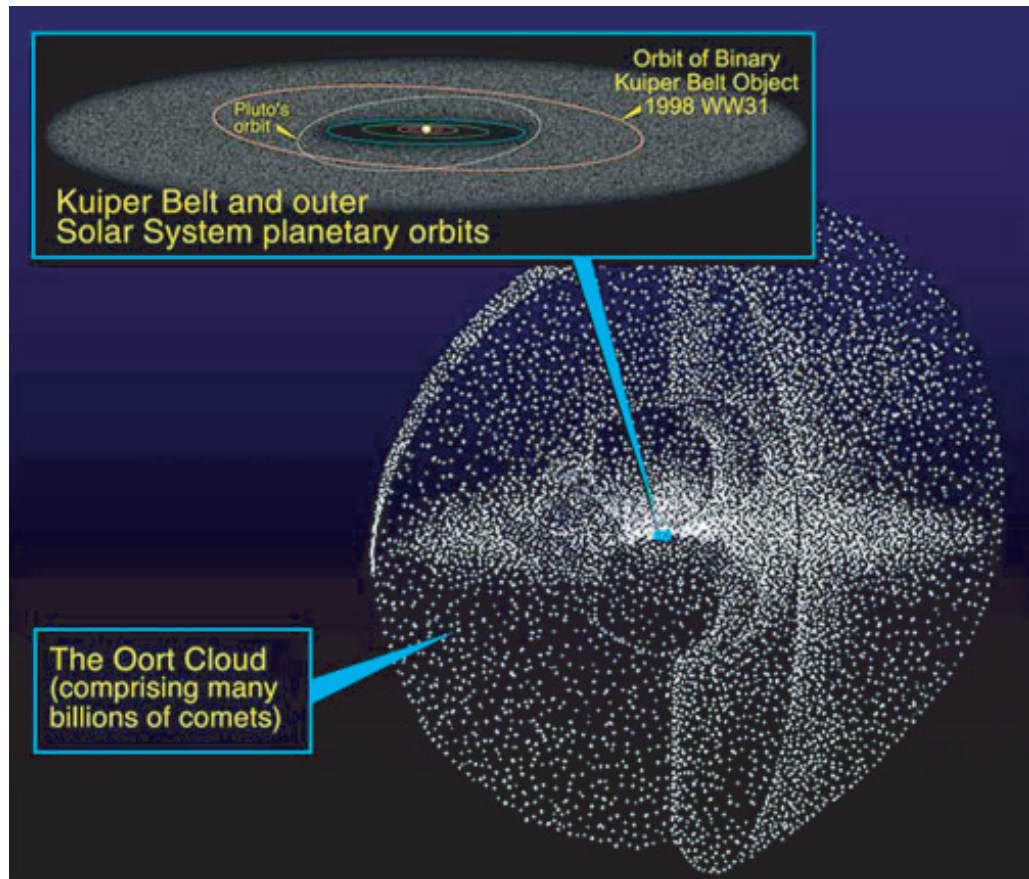


FIGURE 167 The orbit of Sedna in comparison with the orbits of the planets in the Solar System (NASA).

a planet, but the first of a set of smaller objects in the so-called *Kuiper belt*. Kuiper belt objects are regularly discovered; over 1000 are known today.

Ref. 169 In 2003, two major Kuiper objects were discovered; one, called *Sedna*, is almost as large as Pluto, the other, called *Eris*, is even larger than Pluto and has a moon. Both have strongly elliptical orbits (see Figure 167). Since Pluto and Eris, like the asteroid Ceres, have cleaned their orbit from debris, these three objects are now classified as *dwarf planets*.

Outside the Kuiper belt, the Solar System is surrounded by the so-called *Oort cloud*. In contrast to the flattened Kuiper belt, the Oort cloud is spherical in shape and has a radius of up to 50 000 AU, as shown in Figure 167 and Figure 168. The Oort cloud consists of a huge number of icy objects consisting of mainly of water, and to a lesser degree, of methane and ammonia. Objects from the Oort cloud that enter the inner Solar System become comets; in the distant past, such objects have brought water onto the Earth.



**FIGURE 168** The Kuiper belt, containing mainly planetoids, and the Oort cloud orbit, containing comets, around the Solar System (NASA, JPL, Donald Yeoman).

\* \*

In astronomy new examples of motion are regularly discovered even in the present century. Sometimes there are also false alarms. One example was the alleged fall of *mini comets* on the Earth. They were supposedly made of a few dozen kilograms of ice, hitting the Earth every few seconds. It is now known not to happen.

Ref. 170

\* \*

Universal gravity allows only elliptical, parabolic or hyperbolic orbits. It is impossible for a small object approaching a large one to be captured. At least, that is what we have learned so far. Nevertheless, all astronomy books tell stories of capture in our Solar System; for example, several outer satellites of Saturn have been captured. How is this possible?

Challenge 386 s

\* \*

How would a tunnel have to be shaped in order that a stone would fall through it without touching the walls? (Assume constant density.) If the Earth did not rotate, the tunnel

Challenge 387 s would be a straight line through its centre, and the stone would fall down and up again, in an oscillating motion. For a rotating Earth, the problem is much more difficult. What is the shape when the tunnel starts at the Equator?

\* \*

Challenge 388 e The International Space Station circles the Earth every 90 minutes at an altitude of about 380 km. You can see where it is from the website [www.heavens-above.com](http://www.heavens-above.com). By the way, whenever it is just above the horizon, the station is the third brightest object in the night sky, superseded only by the Moon and Venus. Have a look at it.

\* \*

Challenge 389 s Is it true that the centre of mass of the Solar System, its barycentre, is always inside the Sun? Even though the Sun or a star move very little when planets move around them, this motion can be detected with precision measurements making use of the Doppler effect for light or radio waves. Jupiter, for example, produces a speed change of 13 m/s in the Sun, the Earth 1 m/s. The first planets outside the Solar System, around the pulsar PSR1257+12 and around the normal G-type star Pegasi 51, were discovered in this way, in 1992 and 1995. In the meantime, several thousand so-called exoplanets have been discovered with this and other methods. Some have even masses comparable to that of the Earth. This research also showed that exoplanets are more numerous than stars, and that earth-like planets are rare.

Vol. II, page 31

\* \*

Challenge 390 d Not all points on the Earth receive the same number of daylight hours during a year. The effects are difficult to spot, though. Can you find one?

\* \*

Challenge 391 s Can the phase of the Moon have a measurable effect on the human body, for example through tidal effects?

\* \*

Challenge 392 s There is an important difference between the heliocentric system and the old idea that all planets turn around the Earth. The heliocentric system states that certain planets, such as Mercury and Venus, can be *between* the Earth and the Sun at certain times, and *behind* the Sun at other times. In contrast, the geocentric system states that they are always in between. Why did such an important difference not immediately invalidate the geocentric system? And how did the observation of phases, shown in Figure 169 and Figure 170, invalidate the geocentric system?

\* \*

Ref. 171 The strangest reformulation of the description of motion given by  $\mathbf{ma} = \nabla U$  is the almost absurd looking equation

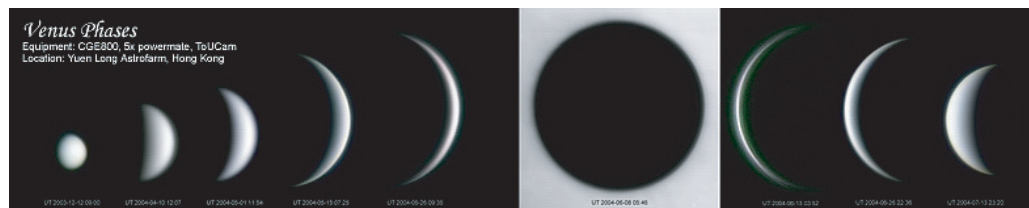
$$\nabla v = d\mathbf{v}/ds \quad (60)$$

Challenge 393 s where  $s$  is the motion path length. It is called the *ray form* of the equation of motion. Can you find an example of its application?





**FIGURE 169** The phases of the Moon and of Venus, as observed from Athens in summer 2007 (© Anthony Ayiomamitis).



**FIGURE 170** Universal gravitation also explains the observations of Venus, the evening and morning star. In particular, universal gravitation, and the elliptical orbits it implies, explains its phases and its change of angular size. The pictures shown here were taken in 2004 and 2005. The observations can easily be made with a binocular or a small telescope (© Wah!; film available at [apod.nasa.gov/apod/ap060110.html](http://apod.nasa.gov/apod/ap060110.html)).

\* \*

Challenge 394 s

Seen from Neptune, the size of the Sun is the same as that of Jupiter seen from the Earth at the time of its closest approach. True?

\* \*

Ref. 172

Challenge 395 s

The gravitational acceleration for a particle inside a spherical shell is zero. The vanishing of gravity in this case is independent of the particle shape and its position, and independent of the thickness of the shell. Can you find the argument using Figure 171? This works only because of the  $1/r^2$  dependence of gravity. Can you show that the result does not



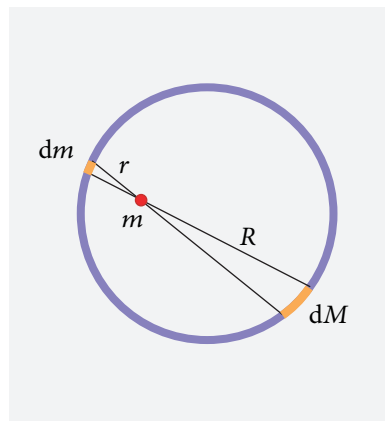


FIGURE 171 The vanishing of gravitational force inside a spherical shell of matter.

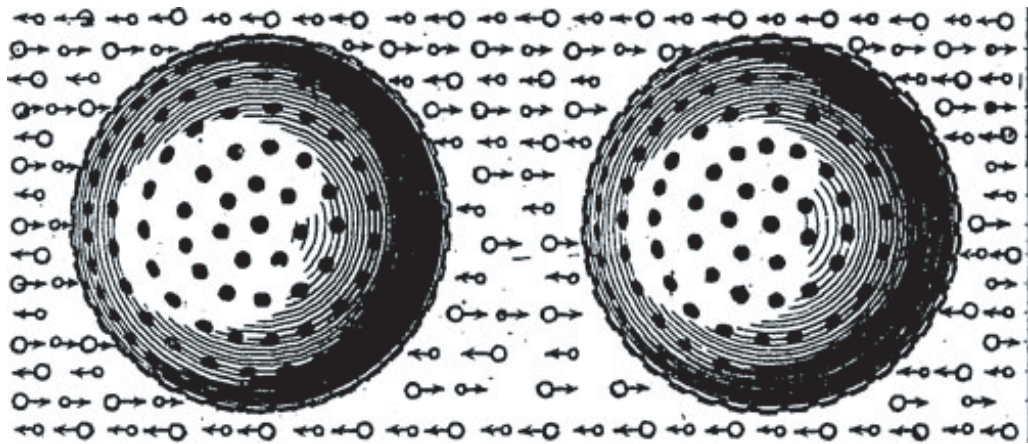


FIGURE 172 Le Sage's own illustration of his model, showing the smaller density of 'ultramondane corpuscles' between the attracting bodies and the higher density outside them (© Wikimedia)

hold for non-spherical shells? Note that the vanishing of gravity inside a spherical shell usually does not hold if other matter is found outside the shell. How could one eliminate the effects of outside matter?

Challenge 396 s

\* \*

Ref. 173

What is gravity? This simple question has a long history. In 1690, Nicolas Fatio de Duillier and in 1747, Georges-Louis Le Sage proposed an explanation for the  $1/r^2$  dependence. Le Sage argued that the world is full of small particles – he called them 'corpuscules ultra-mondains' – flying around randomly and hitting all objects. Single objects do not feel the hits, since they are hit continuously and randomly from all directions. But when two objects are near to each other, they produce shadows for part of the flux to the other body, resulting in an attraction, as shown in Figure 172. Can you show that such an attraction has a  $1/r^2$  dependence?

Challenge 397 e

Challenge 398 e

However, Le Sage's proposal has a number of problems. First, the argument only works if the collisions are inelastic. (Why?) However, that would mean that all bodies

Ref. 2 would heat up with time, as Jean-Marc Lévy-Leblond explains. Secondly, a moving body in free space would be hit by more or faster particles in the front than in the back; as a result, the body should be decelerated. Finally, gravity would depend on size, but in a strange way. In particular, three bodies lying on a line should *not* produce shadows, as no such shadows are observed; but the naive model predicts such shadows.

Despite all criticisms, the idea that gravity is due to particles has regularly resurfaced in physics research ever since. In the most recent version, the hypothetical particles are called *gravitons*. On the other hand, no such particles have ever been observed. We will understand the origin of gravitation in the final part of our mountain ascent.

\* \*

Challenge 399 ny For which bodies does gravity decrease as you approach them?

\* \*

Challenge 400 s Could one put a satellite into orbit using a cannon? Does the answer depend on the direction in which one shoots?

\* \*

Two old computer users share experiences. 'I threw my Pentium III and Pentium IV out of the window.' 'And?' 'The Pentium III was faster.'

\* \*

Challenge 401 s How often does the Earth rise and fall when seen from the Moon? Does the Earth show phases?

\* \*

Challenge 402 ny What is the weight of the Moon? How does it compare with the weight of the Alps?

\* \*

Challenge 403 s If a star is made of high-density material, the speed of a planet orbiting near to it could be greater than the speed of light. How does nature avoid this strange possibility?

\* \*

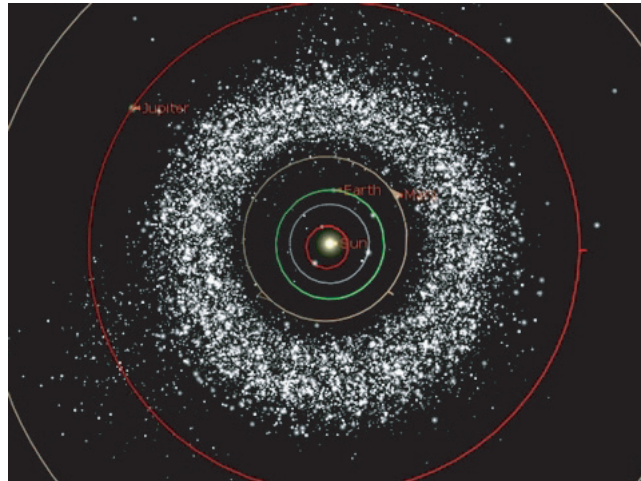
Ref. 174  
Page 424 What will happen to the Solar System in the future? This question is surprisingly hard to answer. The main expert of this topic, French planetary scientist Jacques Laskar, simulated a few hundred million years of evolution using computer-aided calculus. He found that the planetary orbits are stable, but that there is clear evidence of chaos in the evolution of the Solar System, at a small level. The various planets influence each other in subtle and still poorly understood ways. Effects in the past are also being studied, such as the energy change of Jupiter due to its ejection of smaller asteroids from the Solar System, or the energy gains of Neptune. There is still a lot of research to be done in this field.

\* \*

One of the open problems of the Solar System is the description of planet distances discovered in 1766 by Johann Daniel Titius (1729–1796) and publicized by Johann Elert

**TABLE 27** An unexplained property of nature: planet distances from the Sun and the values resulting from the Titius–Bode rule.

| PLANET         | $n$       | PREDICTED | MEASURED   |
|----------------|-----------|-----------|------------|
| DISTANCE IN AU |           |           |            |
| Mercury        | $-\infty$ | 0.4       | 0.4        |
| Venus          | 0         | 0.7       | 0.7        |
| Earth          | 1         | 1.0       | 1.0        |
| Mars           | 2         | 1.6       | 1.5        |
| Planetoids     | 3         | 2.8       | 2.2 to 3.2 |
| Jupiter        | 4         | 5.2       | 5.2        |
| Saturn         | 5         | 10.0      | 9.5        |
| Uranus         | 6         | 19.6      | 19.2       |
| Neptune        | 7         | 38.8      | 30.1       |
| Pluto          | 8         | 77.2      | 39.5       |



**FIGURE 173** The motion of the planetoids compared to that of the planets (Shockwave animation © Hans-Christian Greier)

Bode (1747–1826). Titius discovered that planetary distances  $d$  from the Sun can be approximated by

$$d = a + 2^n b \quad \text{with} \quad a = 0.4 \text{ AU}, \quad b = 0.3 \text{ AU} \quad (61)$$

where distances are measured in astronomical units and  $n$  is the number of the planet. The resulting approximation is compared with observations in [Table 27](#).

Interestingly, the last three planets, as well as the planetoids, were discovered *after* Bode's and Titius' deaths; the rule had successfully predicted Uranus' distance, as well as that of the planetoids. Despite these successes – and the failure for the last two planets – nobody has yet found a model for the formation of the planets that explains Titius' rule. The large satellites of Jupiter and of Uranus have regular spacing, but not according

**TABLE 28** The orbital periods known to the Babylonians.

| B O D Y | P E R I O D |
|---------|-------------|
| Saturn  | 29 a        |
| Jupiter | 12 a        |
| Mars    | 687 d       |
| Sun     | 365 d       |
| Venus   | 224 d       |
| Mercury | 88 d        |
| Moon    | 29 d        |

to the Titius–Bode rule.

Explaining or disproving the rule is one of the challenges that remain in classical mechanics. Some researchers maintain that the rule is a consequence of scale invariance, others maintain that it is an accident or even a red herring. The last interpretation is also suggested by the non-Titius–Bode behaviour of practically all extrasolar planets. The issue is not closed.

\* \*

Around 3000 years ago, the Babylonians had measured the orbital times of the seven celestial bodies that move across the sky. Ordered from longest to shortest, they wrote them down in [Table 28](#). Six of the celestial bodies are visible in the beautiful [Figure 174](#).

The Babylonians also introduced the week and the division of the day into 24 hours. They dedicated every one of the 168 hours of the week to a celestial body, following the order of [Table 28](#). They also dedicated the whole day to that celestial body that corresponds to the first hour of that day. The first day of the week was dedicated to Saturn; the present ordering of the other days of the week then follows from [Table 28](#). This story was told by Cassius Dio (c. 160 to c. 230). Towards the end of Antiquity, the ordering was taken up by the Roman empire. In Germanic languages, including English, the Latin names of the celestial bodies were replaced by the corresponding Germanic gods. The order Saturday, Sunday, Monday, Tuesday, Wednesday, Thursday and Friday is thus a consequence of both the astronomical measurements and the astrological superstitions of the ancients.

\* \*

In 1722, the great mathematician Leonhard Euler made a mistake in his calculation that led him to conclude that if a tunnel, or better, a deep hole were built from one pole of the Earth to the other, a stone falling into it would arrive at the Earth's centre and then immediately turn and go back up. Voltaire made fun of this conclusion for many years. Can you correct Euler and show that the real motion is an oscillation from one pole to the other, and can you calculate the time a pole-to-pole fall would take (assuming homogeneous density)?

Challenge 405 s

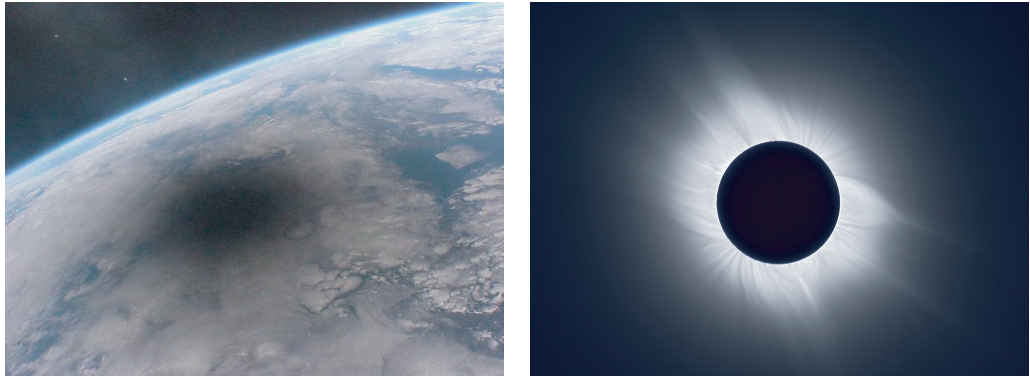
What would be the oscillation time for an arbitrary straight surface-to-surface tunnel



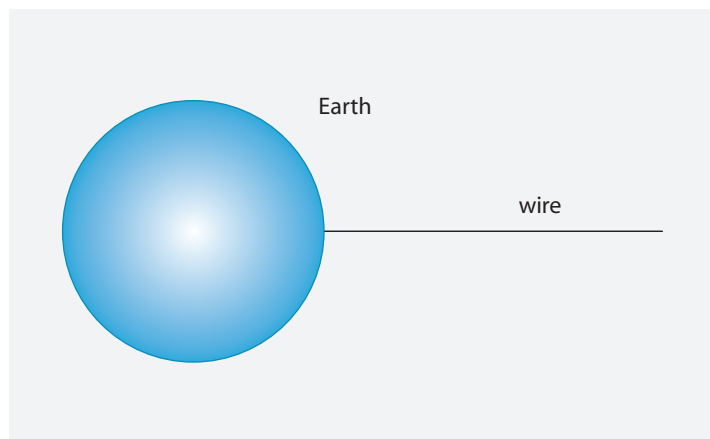


**FIGURE 174** These are the six celestial bodies that are visible at night with the naked eye and whose positions vary over the course of the year. The nearly vertical line connecting them is the *ecliptic*, the narrow stripe around it the *zodiac*. Together with the Sun, the seven celestial bodies were used to name the days of the week. (© Alex Cherney)





**FIGURE 175** The solar eclipse of 11 August 1999, photographed by Jean-Pierre Haigneré, member of the Mir 27 crew, and the (enhanced) solar eclipse of 29 March 2006 (© CNES and Laurent Laveder/PixHeaven.net).



**FIGURE 176** A wire attached to the Earth's Equator.

Challenge 406 s of length  $l$ , thus *not* going from pole to pole?

Ref. 178

Challenge 407 s

The previous challenges circumvented the effects of the Earth's rotation. The topic becomes much more interesting if rotation is included. What would be the shape of a tunnel so that a stone falling through it never touches the wall?

\* \*

Figure 175 shows a photograph of a solar eclipse taken from the Russian space station *Mir* and a photograph taken at the centre of the shadow from the Earth. Indeed, a global view of a phenomenon can be quite different from a local one. What is the speed of the shadow?

Challenge 408 s

\* \*

In 2005, satellite measurements have shown that the water in the Amazon river presses down the land up to 75 mm more in the season when it is full of water than in the season when it is almost empty.

Ref. 179

\* \*

Imagine that a wire existed that does not break. How long would such a wire have to be so that, when attached to the Equator, it would stand upright in the air, as shown in [Figure 176](#)? Could one build an elevator into space in this way?

Challenge 409 s

\* \*

Usually there are roughly two tides per day. But there are places, such as on the coast of Vietnam, where there is only *one* tide per day. See [www.jason.oceanobs.com/html/applications/marees/marees\\_m2k1\\_fr.html](http://www.jason.oceanobs.com/html/applications/marees/marees_m2k1_fr.html). Why?

Challenge 410 ny

\* \*

It is sufficient to use the concept of centrifugal force to show that the rings of Saturn cannot be made of massive material, but must be made of separate pieces. Can you find out how?

Challenge 411 s

\* \*

Why did Mars lose its atmosphere? Nobody knows. It has recently been shown that the solar wind is too weak for this to happen. This is one of the many open riddles of the solar system.

\* \*

All bodies in the Solar System orbit the Sun in the same direction. All? No; there are exceptions. One intriguing asteroid that orbits the Sun near Jupiter in the wrong direction was discovered in 2015: it has a size of 3 km. For an animation of its astonishing orbit, opposite to all Trojan asteroids, see [www.astro.uwo.ca/~wiegert](http://www.astro.uwo.ca/~wiegert).

\* \*

The observed motion due to gravity can be shown to be the *simplest* possible, in the following sense. If we measure change of a falling object with the expression  $\int mv^2/2 - mgh \, dt$ , then a constant acceleration due to gravity *minimizes* the change in every example of fall. Can you confirm this?

Page 253

Challenge 412 e

\* \*

Motion due to gravity is fun: think about roller coasters. If you want to know more about how they are built, visit [www.vekoma.com](http://www.vekoma.com).

\* \*

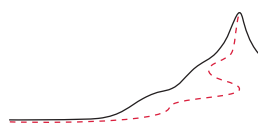
Gravity and air friction lead to interesting effects. What is the shape of a rope hanging from a helicopter when the helicopter is flying? What is the shape when there is a weight at the end of the rope? A small parachute? (The internet has the answer.)

“The scientific theory I like best is that the rings of Saturn are made of lost airline luggage.”  
Mark Russel

### SUMMARY ON GRAVITATION

Spherical bodies of mass  $M$  attract other bodies at a distance  $r$  by inducing an acceleration towards them given by  $a = GM/r^2$ . This expression, *universal gravitation*, describes snowboarders, skiers, paragliders, athletes, couch potatoes, pendula, stones, canons, rockets, tides, eclipses, planet shapes, planet motion and much more. Universal gravitation is the first example of a unified description: it describes *how everything falls*. By the acceleration it produces, gravitation limits the appearance of uniform motion in nature.





## CHAPTER 7

# CLASSICAL MECHANICS, FORCE AND THE PREDICTABILITY OF MOTION

All those types of motion in which the only permanent property of a body is mass define the field of *mechanics*. The same name is given also to the experts studying the field. We can think of mechanics as the athletic part of physics.\*\* Both in athletics and in mechanics only lengths, times and masses are measured – and of interest at all.

More specifically, our topic of investigation so far is called *classical* mechanics, to distinguish it from *quantum* mechanics. The main difference is that in classical physics arbitrary small values are assumed to exist, whereas this is not the case in quantum physics. Classical mechanics is often also called *Galilean physics* or *Newtonian physics*\*\*\*.

Classical mechanics states that motion is *predictable*: it thus states that there are no surprises in motion. Is this correct in all cases? Is predictability valid in the presence of friction? Of free will? Are there really no surprises in nature? These issues merit a discussion; they will accompany us for a stretch of our adventure.

We know that there is more to the world than gravity. Simple observations make this point: *floors* and *friction*. Neither can be due to gravity. Floors do not fall, and thus are not described by gravity; and friction is not observed in the skies, where motion is purely due to gravity\*\*\*\*. Also on Earth, friction is unrelated to gravity, as you might want to check yourself. There must be another interaction responsible for friction. We shall study it in the third volume. But a few issues merit a discussion right away.

Challenge 413 e

\*\* This is in contrast to the actual origin of the term ‘mechanics’, which means ‘machine science’. It derives from the Greek μηχανή, which means ‘machine’ and even lies at the origin of the English word ‘machine’ itself. Sometimes the term ‘mechanics’ is used for the study of the motion of *solid* bodies only, excluding, e.g., hydrodynamics. This use fell out of favour in physics in the twentieth century.

\*\*\* The basis of classical mechanics, the description of motion using only space and time, is called *kinematics*. An example is the description of free fall by  $z(t) = z_0 + v_0(t - t_0) - \frac{1}{2}g(t - t_0)^2$ . The other, main part of classical mechanics is the description of motion as a consequence of interactions between bodies; it is called *dynamics*. An example of dynamics is the formula of universal gravity. The distinction between kinematics and dynamics can also be made in relativity, thermodynamics and electrodynamics.

\*\*\*\* This is not completely correct: in the 1980s, the first case of gravitational friction was discovered: the emission of gravity waves. We discuss it in detail in the chapter on general relativity. The discovery does not change the main point, however.



**FIGURE 177** The parabola shapes formed by accelerated water beams show that motion in everyday life is predictable (© Oase GmbH).

**TABLE 29** Some force values in nature.

| OBSERVATION                                                         | FORCE                 |
|---------------------------------------------------------------------|-----------------------|
| Value measured in a magnetic resonance force microscope             | 820 zN                |
| Force needed to rip a DNA molecule apart by pulling at its two ends | 600 pN                |
| Maximum force exerted by human bite                                 | 2.1 kN                |
| Force exerted by quadriceps                                         | up to 3 kN            |
| Typical peak force exerted by sledgehammer                          | 5 kN                  |
| Force sustained by 1 cm <sup>2</sup> of a good adhesive             | up to 10 kN           |
| Force needed to tear a good rope used in rock climbing              | 30 kN                 |
| Maximum force measurable in nature                                  | $3.0 \cdot 10^{43}$ N |

**SHOULD ONE USE FORCE? POWER?**

“The direct use of physical force is so poor a solution [...] that it is commonly employed only by small children and great nations.”  
David Friedman

Everybody has to take a stand on this question, even students of physics. Indeed, many types of forces are used and observed in daily life. One speaks of muscular, gravitational, psychic, sexual, satanic, supernatural, social, political, economic and many others. Physicists see things in a simpler way. They call the different types of forces observed between objects *interactions*. The study of the details of all these interactions will show that, in everyday life, they are of electrical or gravitational origin.

For physicists, all change is due to motion. The term force then also takes on a more



restrictive definition. (*Physical*) force is defined as the *change of momentum with time*, i.e., as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} . \quad (62)$$

A few measured values are listed in [Figure 29](#).

[Ref. 87](#) A horse is running so fast that the hooves touch the ground only 20 % of the time.  
[Challenge 414 s](#) What is the load carried by its legs during contact?

Force is the *change* of momentum. Since momentum is conserved, we can also say that force measures the *flow* of momentum. As we will see in detail shortly, whenever a force accelerates a body, momentum flows into it. Indeed, momentum can be imagined to be some invisible and intangible substance. Force measures how much of this substance flows into or out of a body per unit time.

[Ref. 87](#)

▷ Force is momentum flow.

The conservation of momentum is due to the conservation of this liquid. Like any liquid, momentum flows through a surface.

Using the Galilean definition of linear momentum  $\mathbf{p} = m\mathbf{v}$ , we can rewrite the definition of force (for constant mass) as

$$\mathbf{F} = m\mathbf{a} , \quad (63)$$

where  $\mathbf{F} = \mathbf{F}(t, \mathbf{x})$  is the force acting on an object of mass  $m$  and where  $\mathbf{a} = \mathbf{a}(t, \mathbf{x}) = d\mathbf{v}/dt = d^2\mathbf{x}/dt^2$  is the acceleration of the same object, that is to say, its change of velocity.\* The expression states in precise terms that force is what changes the *velocity* of masses. The quantity is called ‘force’ because it corresponds in many, but *not* all aspects to everyday muscular force. For example, the more force is used, the further a stone can be thrown. Equivalently, the more momentum is pumped into a stone, the further it can be thrown. As another example, the concept of *weight* describes the flow of momentum due to gravity.

▷ Gravitation constantly pumps momentum into massive bodies.

[Challenge 415 s](#) Sand in an hourglass is running, and the hourglass is on a scale. Is the weight shown on the scale larger, smaller or equal to the weight when the sand has stopped falling?

Forces are measured with the help of deformations of bodies. Everyday force values can be measured by measuring the extension of a spring. Small force values, of the order of 1 nN, can be detected by measuring the deflection of small levers with the help of a reflected laser beam.

\* This equation was first written down by the mathematician and physicist Leonhard Euler (b. 1707 Basel, d. 1783 St. Petersburg) in 1747, 20 years after the death of Newton, to whom it is usually and falsely ascribed. It was Euler, one of the greatest mathematicians of all time (and not Newton), who first understood that this definition of force is useful in *every* case of motion, whatever the appearance, be it for point particles or extended objects, and be they rigid, deformable or fluid bodies. Surprisingly and in contrast to frequently-made statements, equation (63) is even correct in relativity.

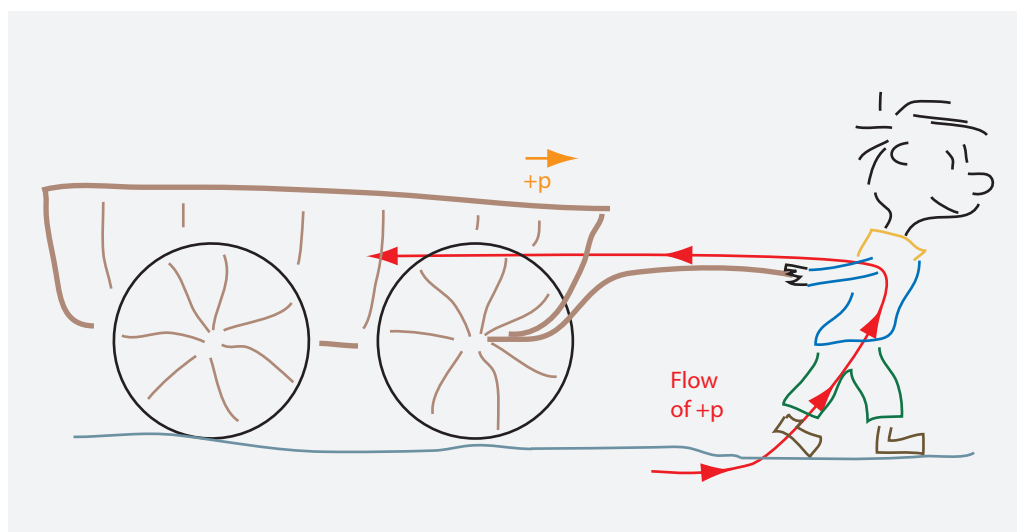


FIGURE 178 The pulling child pumps momentum into the chariot. In fact, some momentum flows back to the ground due to dynamic friction (not drawn).

However, whenever the concept of force is used, it should be remembered that *physical force is different from everyday force or everyday effort*. Effort is probably best approximated by the concept of (*physical*) *power*, usually abbreviated  $P$ , and defined (for constant force) as

$$P = \frac{dW}{dt} = Fv \quad (64)$$

in which (*physical*) *work*  $W$  is defined as  $W = Fs$ , where  $s$  is the distance along which the force acts. Physical work is a form of energy, as you might want to check. Work, as a form of energy, has to be taken into account when the conservation of energy is checked.

Challenge 416 s

Challenge 417 d

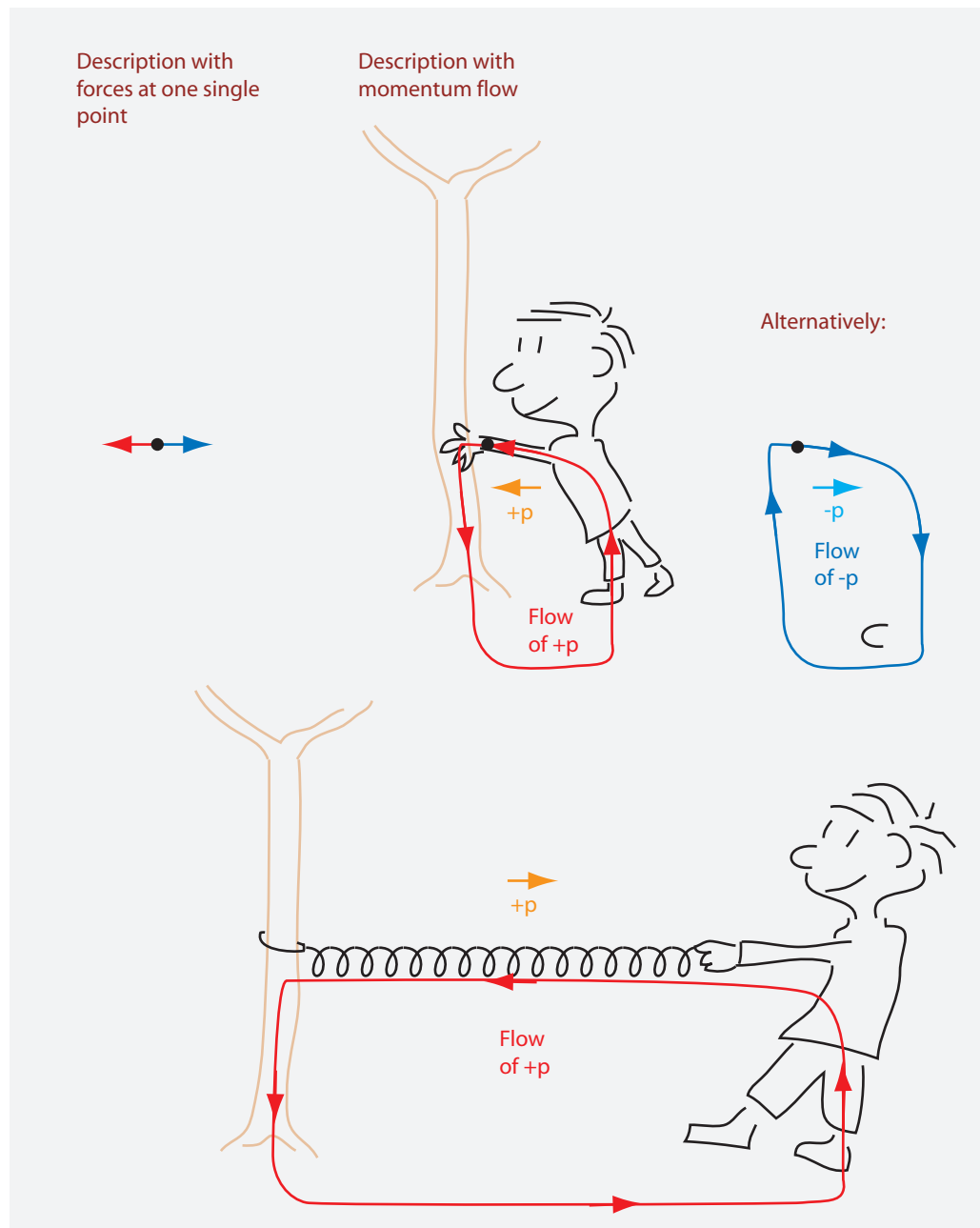
Challenge 418 s

With the definition of work just given you can solve the following puzzles. What happens to the electricity consumption of an escalator if you walk on it instead of standing still? What is the effect of the definition of power for the salary of scientists? A man who walks carrying a heavy rucksack is hardly doing any work; why then does he get tired?

Ref. 180

When students in exams say that the force acting on a thrown stone is least at the highest point of the trajectory, it is customary to say that they are using an incorrect view, namely the so-called *Aristotelian view*, in which force is proportional to velocity. Sometimes it is even said that they are using a different concept of *state* of motion. Critics then add, with a tone of superiority, how wrong all this is. This is an example of intellectual disinformation. Every student knows from riding a bicycle, from throwing a stone or from pulling an object that increased *effort* results in increased speed. The student is right; those theoreticians who deduce that the student has a mistaken concept of *force* are wrong. In fact, instead of the *physical* concept of force, the student is just using the *everyday* version, namely effort. Indeed, the effort exerted by gravity on a flying stone is least at the highest point of the trajectory. Understanding the difference between physical force and everyday effort is the main hurdle in learning mechanics.\*

\* This stepping stone is so high that many professional physicists do not really take it themselves; this is



**FIGURE 179** The two equivalent descriptions of situations with zero net force, i.e., with a closed momentum flow. Compression occurs when momentum flow and momentum point in the same direction; extension occurs when momentum flow and momentum point in opposite directions.

confirmed by the innumerable comments in papers that state that physical force is defined using mass, and, at the same time, that mass is defined using force (the latter part of the sentence being a fundamental mistake).

Often the flow of momentum, equation (62), is not recognized as the definition of force. This is mainly due to an everyday observation: there seem to be forces without any associated acceleration or change in momentum, such as in a string under tension or in water at high pressure. When one pushes against a tree, as shown in Figure 179, there is no motion, yet a force is applied. If force is momentum flow, where does the momentum go? It flows into the slight deformations of the arms and the tree. In fact, when one starts pushing and thus deforming, the associated momentum change of the molecules, the atoms, or the electrons of the two bodies can be observed. After the deformation is established a continuous and equal flow of momentum is going on in both directions.

Ref. 87 Because force is net momentum flow, the concept of force is not really needed in the description of motion. But sometimes the concept is practical. This is the case in everyday life, where it is useful in situations where net momentum values are small or negligible. For example, it is useful to define pressure as force per area, even though it is actually a momentum flow per area. At the microscopic level, momentum alone suffices for the description of motion.

In the section title we asked about the usefulness of force and power. Before we can answer conclusively, we need more arguments. Through its definition, the concepts of force and power are distinguished clearly from ‘mass’, ‘momentum’, ‘energy’ and from each other. But where do forces originate? In other words, which effects in nature have the capacity to accelerate bodies by pumping momentum into objects? Table 30 gives an overview.

### FORCES, SURFACES AND CONSERVATION

We saw that force is the change of momentum. We also saw that momentum is conserved. How do these statements come together? The answer is the same for all conserved quantities. We imagine a closed surface that is the boundary of a volume in space. Conservation implies that the conserved quantity enclosed *inside* the surface can only change by *flowing through* that surface.\*

All conserved quantities in nature – such as energy, linear momentum, electric charge, and angular momentum – can only change by flowing through surfaces. In particular, whenever the momentum of a body changes, this happens through a surface. Momentum change is due to momentum flow. In other words, the concept of force always implies a surface through which momentum flows.

\* Mathematically, the conservation of a quantity  $q$  is expressed with the help of the volume density  $\rho = q/V$ , the current  $I = q/t$ , and the flow or flux  $\mathbf{j} = \rho\mathbf{v}$ , so that  $j = q/At$ . Conservation then implies

$$\frac{dq}{dt} = \int_V \frac{\partial \rho}{\partial t} dV = - \int_{A=\partial V} \mathbf{j} d\mathbf{A} = -I \quad (65)$$

or, equivalently,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0. \quad (66)$$

This is the *continuity equation* for the quantity  $q$ . All this only states that a conserved quantity in a closed volume  $V$  can only change by flowing through the surface  $A$ . This is a typical example of how complex mathematical expressions can obfuscate the simple physical content.



FIGURE 180 Friction-based processes (courtesy Wikimedia).

▷ Force is the flow of momentum through a surface.

Ref. 299 This point is essential in understanding physical force. Every force requires a surface for its definition.

Challenge 419 e To refine your own concept of force, you can search for the relevant surface when a rope pulls a chariot, or when an arm pushes a tree, or when a car accelerates. It is also helpful to compare the definition of force with the definition of power: both quantities are flows through surfaces. As a result, we can say:

▷ A *motor* is a momentum pump.

### FRICTION AND MOTION

Every example of motion, from the motion that lets us choose the direction of our gaze to the motion that carries a butterfly through the landscape, can be put into one of the two left-most columns of Table 30. Physically, those two columns are separated by the following criterion: in the first class, the acceleration of a body can be in a different direction from its velocity. The second class of examples produces only accelerations that are exactly *opposed* to the velocity of the moving body, as seen from the frame of reference



TABLE 30 Selected processes and devices changing the motion of bodies.

| SITUATIONS THAT CAN<br>LEAD TO ACCELERATION | SITUATIONS THAT<br>ONLY LEAD TO<br>DECELERATION | MOTORS AND<br>ACTUATORS |
|---------------------------------------------|-------------------------------------------------|-------------------------|
| <i>piezoelectricity</i>                     |                                                 |                         |
| quartz under applied voltage                | thermoluminescence                              | walking piezo tripod    |
| <i>collisions</i>                           |                                                 |                         |
| satellite in planet encounter               | car crash                                       | rocket motor            |
| growth of mountains                         | meteorite crash                                 | swimming of larvae      |
| <i>magnetic effects</i>                     |                                                 |                         |
| compass needle near magnet                  | electromagnetic braking                         | electromagnetic gun     |
| magnetostriction                            | transformer losses                              | linear motor            |
| current in wire near magnet                 | electric heating                                | galvanometer            |
| <i>electric effects</i>                     |                                                 |                         |
| rubbed comb near hair                       | friction between solids                         | electrostatic motor     |
| bombs                                       | fire                                            | muscles, sperm flagella |
| cathode ray tube                            | electron microscope                             | Brownian motor          |
| <i>light</i>                                |                                                 |                         |
| levitating objects by light                 | light bath stopping atoms                       | (true) light mill       |
| solar sail for satellites                   | light pressure inside stars                     | solar cell              |
| <i>elasticity</i>                           |                                                 |                         |
| bow and arrow                               | trouser suspenders                              | ultrasound motor        |
| bent trees standing up again                | pillow, air bag                                 | bimorphs                |
| <i>osmosis</i>                              |                                                 |                         |
| water rising in trees                       | salt conservation of food                       | osmotic pendulum        |
| electro-osmosis                             |                                                 | tunable X-ray screening |
| <i>heat &amp; pressure</i>                  |                                                 |                         |
| freezing champagne bottle                   | surfboard water resistance                      | hydraulic engines       |
| tea kettle                                  | quicksand                                       | steam engine            |
| barometer                                   | parachute                                       | air gun, sail           |
| earthquakes                                 | sliding resistance                              | seismometer             |
| attraction of passing trains                | shock absorbers                                 | water turbine           |
| <i>nuclei</i>                               |                                                 |                         |
| radioactivity                               | plunging into the Sun                           | supernova explosion     |
| <i>biology</i>                              |                                                 |                         |
| bamboo growth                               | decreasing blood vessel<br>diameter             | molecular motors        |
| <i>gravitation</i>                          |                                                 |                         |
| falling                                     | emission of gravity waves                       | pulley                  |

Challenge 420 e of the braking medium. Such a resisting force is called *friction*, *drag* or a *damping*. All examples in the second class are types of friction. Just check. Some examples of processes based on friction are given in Figure 180.

Challenge 421 s Here is a puzzle on cycling: does side wind brake – and why?

Friction can be so strong that all motion of a body against its environment is made impossible. This type of friction, called *static friction* or *sticking friction*, is common and important: without it, turning the wheels of bicycles, trains or cars would have no effect. Without static friction, wheels driven by a motor would have no grip. Similarly, not a single screw would stay tightened and no hair clip would work. We could neither run nor walk in a forest, as the soil would be more slippery than polished ice. In fact not only our own motion, but all *voluntary motion* of living beings is *based* on friction. The same is the case for all self-moving machines. Without static friction, the propellers in ships, aeroplanes and helicopters would not have any effect and the wings of aeroplanes would produce no lift to keep them in the air. (Why?)

Challenge 422 s

In short, static friction is necessary whenever we or an engine want to move against the environment.

### FRICITION, SPORT, MACHINES AND PREDICTABILITY

Once an object moves through its environment, it is hindered by another type of friction; it is called *dynamic friction* and acts between all bodies in relative motion.\* Without dynamic friction, falling bodies would always rebound to the same height, without ever coming to a stop; neither parachutes nor brakes would work; and even worse, we would have no memory, as we will see later.

Ref. 181

All motion examples in the second column of Table 30 include friction. In these examples, macroscopic energy is not conserved: the systems are *dissipative*. In the first column, macroscopic energy is constant: the systems are *conservative*.

The first two columns can also be distinguished using a more abstract, mathematical criterion: on the left are accelerations that can be derived from a potential, on the right, decelerations that can not. As in the case of gravitation, the description of any kind of motion is much simplified by the use of a potential: at every position in space, one needs only the single value of the potential to calculate the trajectory of an object, instead of the three values of the acceleration or the force. Moreover, the magnitude of the velocity of an object at any point can be calculated directly from energy conservation.

The processes from the second column *cannot* be described by a potential. These are the cases where it is best to use force if we want to describe the motion of the system. For example, the friction or drag force  $F$  due to *wind resistance* of a body is *roughly* given by

$$F = \frac{1}{2} c_w \rho A v^2 \quad (67)$$

where  $A$  is the area of its cross-section and  $v$  its velocity relative to the air,  $\rho$  is the density of air. The *drag coefficient*  $c_w$  is a pure number that depends on the shape of the moving

\* There might be one exception. Recent research suggest that maybe in certain crystalline systems, such as tungsten bodies on silicon, under ideal conditions gliding friction can be extremely small and possibly even vanish in certain directions of motion. This so-called *superlubrication* is presently a topic of research.

Ref. 182

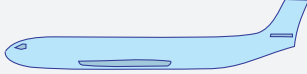
|                                                                                   |                             |               |
|-----------------------------------------------------------------------------------|-----------------------------|---------------|
|  | typical passenger aeroplane | $c_W = 0.03$  |
|  | typical sports car or van   | $c_W = 0.44$  |
|                                                                                   | modern sedan                | $c_W = 0.28$  |
|  | dolphin and penguin         | $c_W = 0.035$ |
|  | soccer ball                 |               |
|                                                                                   | turbulent (above c. 10 m/s) | $c_W = 0.2$   |
|                                                                                   | laminar (below c. 10 m/s)   | $c_W = 0.45$  |

FIGURE 181 Shapes and air/water resistance.

object. A few examples are given in Figure 181. The formula is valid for all fluids, not only for air, below the speed of sound, as long as the drag is due to turbulence. This is usually the case in air and in water. (At very low velocities, when the fluid motion is not turbulent but laminar, drag is called *viscous* and follows an (almost) linear relation with speed.) You may check that drag, or aerodynamic resistance *cannot* be derived from a potential.\*

The drag coefficient  $c_W$  is a measured quantity. Calculating drag coefficients with computers, given the shape of the body and the properties of the fluid, is one of the most difficult tasks of science; the problem is still not solved. An aerodynamic car has a value between 0.25 and 0.3; many sports cars share with vans values of 0.44 and higher, and racing car values can be as high as 1, depending on the amount of force that is used to keep the car fastened to the ground. The lowest known values are for dolphins and penguins.\*\*

Wind resistance is also of importance to humans, in particular in athletics. It is estimated that 100 m sprinters spend between 3 % and 6 % of their power overcoming drag. This leads to varying sprint times  $t_w$  when wind of speed  $w$  is involved, related by the expression

$$\frac{t_0}{t_w} = 1.03 - 0.03 \left( 1 - \frac{wt_w}{100 \text{ m}} \right)^2, \quad (68)$$

\* Such a statement about friction is correct only in three dimensions, as is the case in nature; in the case of a single dimension, a potential can *always* be found.

\*\* It is unclear whether there is, in nature, a smallest possible value for the drag coefficient.

The topic of aerodynamic shapes is also interesting for fluid bodies. They are kept together by *surface tension*. For example, surface tension keeps the wet hairs of a soaked brush together. Surface tension also determines the shape of rain drops. Experiments show that their shape is spherical for drops smaller than 2 mm diameter, and that larger rain drops are *lens* shaped, with the flat part towards the bottom. The usual tear shape is *not* encountered in nature; something vaguely similar to it appears during drop detachment, but *never* during drop fall.

Page 361

Challenge 423 e

Ref. 184

Challenge 424 s

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Ref. 183

where the more conservative estimate of 3 % is used. An opposing wind speed of  $-2$  m/s gives an increase in time of  $0.13$  s, enough to change a potential world record into an ‘only’ excellent result. (Are you able to deduce the  $c_w$  value for running humans from the formula?)

Challenge 425 ny

Likewise, parachuting exists due to wind resistance. Can you determine how the speed of a falling body, with or without a parachute, changes with time, assuming *constant* shape and drag coefficient?

Challenge 426 s

Ref. 185

In contrast, static friction has different properties. It is proportional to the force pressing the two bodies together. Why? Studying the situation in more detail, sticking friction is found to be proportional to the actual contact area. It turns out that putting two solids into contact is rather like turning Switzerland upside down and putting it onto Austria; the area of contact is much smaller than that estimated macroscopically. The important point is that the area of actual contact is proportional to the *normal* force, i.e., the force component that is perpendicular to the surface. The study of what happens in that contact area is still a topic of research; researchers are investigating the issues using instruments such as atomic force microscopes, lateral force microscopes and triboscopes. These efforts resulted in computer hard discs which last longer, as the friction between the disc and the reading head is a central quantity in determining the lifetime.

All forms of friction are accompanied by an increase in the temperature of the moving body. The reason became clear after the discovery of atoms. Friction is not observed in few – e.g. 2, 3, or 4 – particle systems. Friction only appears in systems with *many* particles, usually millions or more. Such systems are called *dissipative*. Both the temperature changes and friction itself are due to the motion of large numbers of microscopic particles against each other. This motion is not included in the Galilean description. When it is included, friction and energy loss disappear, and potentials can then be used throughout. Positive accelerations – of microscopic magnitude – then also appear, and motion is found to be conserved.

In short, all motion is conservative on a microscopic scale. On a microscopic scale it is thus possible and most practical to describe *all* motion without the concept of force.\*

The moral of the story is twofold: First, one should use force and power only in one situation: in the case of friction, and only when one does not want to go into the details.\*\* Secondly, friction is not an obstacle to predictability. Motion remains predictable.

\* The first scientist who eliminated force from the description of nature was Heinrich Rudolf Hertz (b. 1857 Hamburg, d. 1894 Bonn), the famous discoverer of electromagnetic waves, in his textbook on mechanics, *Die Prinzipien der Mechanik*, Barth, 1894, republished by Wissenschaftliche Buchgesellschaft, 1963. His idea was strongly criticized at that time; only a generation later, when quantum mechanics quietly got rid of the concept for good, did the idea become commonly accepted. (Many have speculated about the role Hertz would have played in the development of quantum mechanics and general relativity, had he not died so young.) In his book, Hertz also formulated the principle of the straightest path: particles follow geodesics. This same description is one of the pillars of general relativity, as we will see later on.

\*\* But the cost is high; in the case of human relations the evaluation should be somewhat more discerning, as research on violence has shown.

Ref. 186

“Et qu’avons-nous besoin de ce moteur, quand l’étude réfléchie de la nature nous prouve que le mouvement perpétuel est la première de ses lois ?\*  
Donatien de Sade *Justine, ou les malheurs de la vertu*.

### COMPLETE STATES – INITIAL CONDITIONS

“Quid sit futurum cras, fuge quaerere ...\*\*  
Horace, *Odi*, lib. I, ode 9, v. 13.

Let us continue our exploration of the predictability of motion. We often describe the motion of a body by specifying the time dependence of its position, for example as

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}_0(t - t_0) + \frac{1}{2}\mathbf{a}_0(t - t_0)^2 + \frac{1}{6}\mathbf{j}_0(t - t_0)^3 + \dots \quad (69)$$

The quantities with an index 0, such as the starting position  $\mathbf{x}_0$ , the starting velocity  $\mathbf{v}_0$ , etc., are called *initial conditions*. Initial conditions are necessary for any description of motion. Different physical systems have different initial conditions. Initial conditions thus specify the *individuality* of a given system. Initial conditions also allow us to distinguish the present situation of a system from that at any previous time: initial conditions specify the *changing aspects* of a system. Equivalently, they summarize the *past* of a system.

Page 27

Initial conditions are thus precisely the properties we have been seeking for a description of the *state* of a system. To find a complete description of states we thus need only a complete description of initial conditions, which we can thus rightly call also *initial states*. It turns out that for gravitation, as for all other microscopic interactions, there is *no* need for initial acceleration  $\mathbf{a}_0$ , initial jerk  $\mathbf{j}_0$ , or higher-order initial quantities. In nature, acceleration and jerk depend only on the properties of objects and their environment; they do not depend on the past. For example, the expression  $a = GM/r^2$  of universal gravity, giving the acceleration of a small body near a large one, does not depend on the past, but only on the environment. The same happens for the other fundamental interactions, as we will find out shortly.

Page 87

The *complete state* of a moving mass point is thus described by specifying its position and its momentum at all instants of time. Thus we have now achieved a *complete* description of the *intrinsic properties* of point objects, namely by their mass, and of their *states of motion*, namely by their momentum, energy, position and time. For *extended rigid* objects we also need orientation and angular momentum. This is the full list for rigid objects; no other state observables are needed.

Challenge 427 s

Challenge 428 s

Can you specify the necessary state observables in the cases of extended elastic bodies and of fluids? Can you give an example of an intrinsic property that we have so far missed?

\* ‘And whatfor do we need this motor, when the reasoned study of nature proves to us that perpetual motion is the first of its laws?’

Ref. 85 \*\* ‘What future will be tomorrow, never ask ...’ Horace is Quintus Horatius Flaccus (65–8 BCE), the great Roman poet.



The set of all possible states of a system is given a special name: it is called the *phase space*. We will use the concept repeatedly. Like any space, it has a number of dimensions.

Challenge 429 s Can you specify this number for a system consisting of  $N$  point particles?

Challenge 430 s It is interesting to recall an older challenge and ask again: does the universe have initial conditions? Does it have a phase space?

Challenge 431 s Given that we now have a description of both properties and states for point objects, extended rigid objects and deformable bodies, can we predict all motion? Not yet. There are situations in nature where the motion of an object depends on characteristics other than its mass; motion can depend on its colour (can you find an example?), on its temperature, and on a few other properties that we will soon discover. And for each intrinsic property there are state observables to discover. Each additional intrinsic property is the basis of a field of physical enquiry. Speed was the basis for mechanics, temperature is the basis for thermodynamics, charge is the basis for electrodynamics, etc. We must therefore conclude that as yet we do not have a complete description of motion.

“An optimist is somebody who thinks that the future is uncertain.”  
Anonymous

### DO SURPRISES EXIST? IS THE FUTURE DETERMINED?

“Die Ereignisse der Zukunft *können* wir nicht aus den gegenwärtigen erschließen. Der Glaube an den Kausalnexus ist ein Aberglaube.\*”  
Ludwig Wittgenstein, *Tractatus*, 5.1361

“Freedom is the recognition of necessity.”  
Friedrich Engels (1820–1895)

Challenge 432 e If, after climbing a tree, we jump down, we cannot halt the jump in the middle of the trajectory; once the jump has begun, it is unavoidable and determined, like all passive motion. However, when we begin to move an arm, we can stop or change its motion from a hit to a caress. Voluntary motion does not seem unavoidable or predetermined. Which of these two cases is the general one?

Let us start with the example that we can describe most precisely so far: the fall of a body. Once the gravitational potential  $\varphi$  acting on a particle is given and taken into account, we can use the expression

$$\mathbf{a}(\mathbf{x}) = -\nabla\varphi = -GM\mathbf{r}/r^3, \quad (70)$$

and we can use the state at a given time, given by initial conditions such as

$$\mathbf{x}(t_0) \quad \text{and} \quad \mathbf{v}(t_0), \quad (71)$$

to determine the motion of the particle in advance. Indeed, with these two pieces of information, we can calculate the complete trajectory  $\mathbf{x}(t)$ .

\* ‘We cannot infer the events of the future from those of the present. Belief in the causal nexus is superstition.’ Our adventure, however, will confirm the everyday observation that this statement is wrong.

An equation that has the potential to predict the course of events is called an *evolution equation*. Equation (70), for example, is an evolution equation for the fall of the object. (Note that the term ‘evolution’ has different meanings in physics and in biology.) An evolution equation embraces the observation that not all types of change are observed in nature, but only certain specific cases. Not all imaginable sequences of events are observed, but only a limited number of them. In particular, equation (70) embraces the idea that from one instant to the next, falling objects change their motion based on the gravitational potential acting on them.

Evolution equations do not exist only for motion due to gravity, but for motion due to all forces in nature. Given an evolution equation and initial state, the whole motion of a system is thus *uniquely fixed*, a property of motion often called *determinism*. For example, astronomers can calculate the position of planets with high precision for thousands of years in advance.

Let us carefully distinguish determinism from several similar concepts, to avoid misunderstandings. Motion can be deterministic and at the same time be *unpredictable in practice*. The unpredictability of motion can have four origins:

1. an impracticably large number of particles involved, including situations with friction,
2. insufficient information about initial conditions, and
3. the mathematical complexity of the evolution equations,
4. strange shapes of space-time.

For example, in case of the *weather* the first three conditions are fulfilled at the same time. It is hard to predict the weather over periods longer than about a week or two. (In 1942, Hitler made once again a fool of himself across Germany by requesting a precise weather forecast for the following twelve months.) Despite the difficulty of prediction, weather change is still deterministic. As another example, near *black holes* all four origins apply together. We will discuss black holes in the section on general relativity. Despite being unpredictable, motion is deterministic near black holes.

Motion can be both deterministic and time *random*, i.e., with different outcomes in similar experiments. A roulette ball’s motion is deterministic, but it is also random.\* As we will see later, quantum systems fall into this category, as do all examples of irreversible motion, such as a drop of ink spreading out in clear water. Also the fall of a die is both deterministic and random. In fact, studies on how to predict the result of a die throw with the help of a computer are making rapid progress; these studies also show how to throw a die in order to increase the odds to get a specific desired result. In all such cases the randomness and the irreproducibility are only apparent; they disappear when the description of states and initial conditions in the microscopic domain are included. In short, determinism does not contradict (*macroscopic*) *irreversibility*. However, on the microscopic scale, deterministic motion is always reversible.

A final concept to be distinguished from determinism is *acausality*. Causality is the requirement that a cause must precede the effect. This is trivial in Galilean physics, but

\* Mathematicians have developed a large number of tests to determine whether a collection of numbers may be called *random*; roulette results pass all these tests – in honest casinos only, however. Such tests typically check the equal distribution of numbers, of pairs of numbers, of triples of numbers, etc. Other tests are the  $\chi^2$  test, the Monte Carlo test(s), and the gorilla test.

Challenge 433 s

becomes of importance in special relativity, where causality implies that the speed of light is a limit for the spreading of effects. Indeed, it seems impossible to have deterministic motion (of matter and energy) which is *acausal*, in other words, faster than light. Can you confirm this? This topic will be looked at more deeply in the section on special relativity.

Saying that motion is ‘deterministic’ means that it is fixed in the future *and also in the past*. It is sometimes stated that predictions of *future* observations are the crucial test for a successful description of nature. Owing to our often impressive ability to influence the future, this is not necessarily a good test. Any theory must, first of all, describe *past* observations correctly. It is our lack of freedom to change the past that results in our lack of choice in the description of nature that is so central to physics. In this sense, the term ‘initial condition’ is an unfortunate choice, because in fact, initial conditions summarize the *past* of a system.\* The central ingredient of a deterministic description is that all motion can be reduced to an evolution equation plus one specific state. This state can be either initial, intermediate, or final. Deterministic motion is uniquely specified into the past and into the future.

Challenge 434 s

To get a clear concept of determinism, it is useful to remind ourselves why the concept of ‘time’ is introduced in our description of the world. We introduce time because we observe first that we are able to define sequences in observations, and second, that unrestricted change is impossible. This is in contrast to films, where one person can walk through a door and exit into another continent or another century. In nature we do not observe metamorphoses, such as people changing into toasters or dogs into toothbrushes. We are able to introduce ‘time’ only because the sequential changes we observe are extremely restricted. If nature were not reproducible, time could not be used. In short, determinism expresses the observation that *sequential changes are restricted to a single possibility*.

Since determinism is connected to the use of the concept of time, new questions arise whenever the concept of time changes, as happens in special relativity, in general relativity and in theoretical high energy physics. There is a lot of fun ahead.

In summary, every description of nature that uses the concept of time, such as that of everyday life, that of classical physics and that of quantum mechanics, is intrinsically and inescapably deterministic, since it connects observations of the past and the future, *eliminating* alternatives. In short,

▷ The use of time implies determinism, and vice versa.

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When drawing metaphysical conclusions, as is so popular nowadays when discussing quantum theory, one should never forget this connection. Whoever uses clocks but denies determinism is nurturing a split personality!\*\* The future is determined.

\* The problems with the term ‘initial conditions’ become clear near the big bang: at the big bang, the universe has no past, but it is often said that it has initial conditions. This contradiction will be explored later in our adventure.

\*\* That can be a lot of fun though.