

been changed for the purpose of this assignment and for other assignments in future. I had the girls explore with light and used their own words and findings to help them understand more about how and why light works.

– **Light leaves a source in all directions.**

For this idea the girls and I went to a room where I could close the door and there were no windows. I then placed a lamp with no lamp shade in the center of the room on a table and had each girl sit on her own bed in the room. I turned off the room light and asked the girls what they noticed about the light.

Lucie: “The light shines up and down, up towards the ceiling and down towards the ground so we can see both.”

Ava: “The light in the center is brighter than the top and bottom”

Me: “what do you mean by that Ava?”

Ava: “The light in the middle of the light bulb is brighter than it is on the ceiling or the floor or the walls.”

Me: So where is the light shining?

Ruby: “Everywhere.”

Me: Everywhere?

A: Well everywhere in the room its shining.

Me: Where is the light shining from. Does it start at the walls and move in, is it just here, does it have a place where it starts?”

Lucie: “The light is coming from the light on the lamp. And it shines in all of the room”

So our evidence that the light leaves a source in all directions was as follows:

-Light could be seen in all corners of the room.

-Light could be seen on the floor and ceiling of the room.

-The light came from the lamp (source) and then filled the room in all areas.

- Light can be envisioned as rays traveling in straight lines

For this idea the girls and I went outside the house to where a light post was shining light onto a wall of the barn and onto the ground. We then examined our shadows to see what we could find out about how light works. The following are observations made by the girls that allowed us to make connections to the idea that light can be envisioned as rays traveling in straight lines. They will be serving as evidence for this idea

-“When an object blocks the light it casts a shadow.”

-“Even if we move or change our spots in the light, our shadows always fall the same way. So the light is traveling in only that one way.”

-“If we stand outside the light and look we can see the edges of where the light shines because the light stand tunnels -focuses- the light into one spot, like a flashlight.

-When one girl stood in the light I handed a long yard stick to the other girls and asked them to make a series of lines from different positions ie. make a line with the yard stick from Lucie’s hand to the same spot on Shadow Lucie’s hand. The girls did so many times and ultimately decided that since the shadows blocked light in a straight line, that the light must be going in a straight line as well.

-For someone to see something, light has to travel to the person’s eyes.

For the purpose of our project it is important to note that I changed the order in which these experiments took place. I first worked with the girls on the idea for someone to see something, light has to travel to the person’s eyes. We then explored light bounces off of objects such as someone’s nose in many different directions.

For this experiment I had the girls return to the room in which we found out that light leaves a source in all directions. In this room I changed the lights multiple times and asked the girls questions; while doing so I had the girls also switch between covering and closing their eyes and opening their eyes. The following are a list of what was changed and what the girls’ answers were.

-Lucie covers and closes her eyes, but Ava and Ruby do not. I ask “who can see me? Raise your hand” Lucie is the only girl who does not raise her hand. I ask her: “why

couldn't you see me?" Lucie responds that she couldn't see me because she "can't see with her eyes closed."

-Next, I had all three girls cover their eyes and asked if they could see me, the lights were on at this time. All three girls reported they could not see me.

- I then turned off the lights and had all three girls open their eyes and asked if they could see me. All three girls reported that no, they could not see me. When asked why Ava responded " it's too dark to see you."

-I then turned on the light and asked the girls if they could see me with their eyes opened. All three girls said yes. When I asked them what they thought their eyes needed to see they responded "Light!" When I asked if they only needed to open their eyes to see they said "No, because we opened our eyes with no lights on and couldn't see." So I asked them again what was needed in order for eyes to see, the girls told me you must open your eyes and also have the lights on which means that our eyes need light to see.

- Light bounces off of objects such as someone's nose in many different directions.

Since the girls had already made the connection that light is needed to see, and that it must travel to their eyes to see, this project was fairly easy. I turned off the light in the room and shined a flash light at one object at a time and asked questions as to what the girls saw. I then turned on the light and asked more questions to the girls. The following serves as our evidence for this idea.

- When the light is shined on one object such as Lucie's teddy bear the girls could all see it. When I asked why that was the girls said " because our eyes can only see what the light sends back to us, we can only see what is in the light, because our eyes need light to see."

-I then turned on the light and asked what the girls could see, they said "everything." When I asked why that was they said it was "because the lights are on." I asked if this meant the light was traveling to their eyes like earlier and they said "Yes!"

-I then pointed to my nose and asked everyone who could see it to raise their hand. All three girls raised their hand.

-I asked the girls how they could see my nose and they said because the light was on

it and coming back to their eyes. I asked how each girl could see it from where they were standing. Can the light reach all of those spots or does the light only bounce off of something in one direction?

After some thought and deliberation as well as Lucie shining a flashlight at my nose, the girls came to a conclusion. In order for all of them to see my nose in different spots, my nose must be bouncing back light to all of their eyes. This means that light bounces off of objects in many different directions.

-Two kinds of shadows

For this part of the experiment I had the girls return outside to the light post. I then asked a series of questions about shadows, as well as where shadows could be seen. The following is what was discovered and decided upon as evidence.

Me: What is a shadow?

Lucie: "A shadow is where something blocks the light from traveling any farther and reflects the light only backwards."

Me: What does a shadow look like?

Ava: "Shadows are darker than the parts where the light shines through. You can't see them as much."

-I then stood with my back to the light so my back would be lit up and my front would be shadowed. In front of me a shadow was cast on the ground. I asked the girls to come touch where they saw a shadow. All three touched the shadow on the ground.

- I then asked the girls again what shadows look like and the same answer was given. So I asked the girls to look for any other areas on me or near me that looked like a shadow. After some deliberation and much circling Ava exclaimed "Your face and Belly are dark, they don't have light!" I asked if this counted as a shadow, Ava said "Yes." The other two girls were unconvinced. So I asked Ava if she could explain what she had found.

-Ava explained that "(her) back is blocking the light from reaching to her front. A shadow is when something blocks the light and doesn't let it go forwards. So (her) front is in shadow cause the light can't reach there!"

-The girls then decided that in fact there were two kinds of shadows, one on the ground and one on the back of the object casting the shadow.

Physics Student, Fall, 2015

The students generate many ideas about light and shadows through such conversations, often exploring well beyond what they might have undertaken if simply given a set of directions in class. Also teaching in a non-threatening environment with friends or family members can enhance a prospective teacher's confidence. As one student reflected, *"During this experience I learned that teaching science is not as scary as I initially thought."*

IV. Using Central Ideas to Develop an Explanation for Intriguing Phenomena

So far we have developed two central ideas based on evidence about the nature of light:

Light leaves most sources in many directions.

Light can be envisioned as rays traveling in straight lines.

These ideas form an *initial conceptual model of light*. These are ideas that most people find easy to understand; they even are evident in the art of young children, who typically draw a sun as a circle with straight rays streaming out from the circle. Figure 1.10 is an example.



FIG. 1.10 Child's drawing with with rays from the Sun.

You can use these two central ideas to explain intriguing phenomena such as what happens when light passes through a tiny pinhole.

A. Exploring pinhole phenomena

Question 1.9 What happens when light passes through a tiny pinhole and shines on a screen?

There are many ways to explore what happens when light passes through a tiny pinhole. One way involves making a *pinhole camera* with materials typically available at home. This is called a camera but no photograph is made as no film is used in this exploration.

Equipment for each student: a toilet paper or paper towel roll, square piece of wax paper, square piece of aluminum foil, two rubber bands. Equipment for each small group: a push pin. A meter stick (or yard stick) and ruler also will be helpful in developing an explanation of what one sees when looking through such a pinhole camera at a bright light bulb in a dark room.

In a remote learning situation, students might have access to a paper cup or a cardboard box with a open end such as a snack box whose bottom can be made light tight with duct tape. Possible substitutions for the wax paper include a translucent cereal box liner or plastic vegetable bag from a grocery store to cover the open end. A chocolate bar liner might be a source of aluminum foil if a student has an empty paper roll with both open ends.

A student described making such a pinhole camera as follows:

First I laid the wax paper over one opening of the toilet paper roll and snugly wrapped the edges down and secured the wax paper with a rubber band. I then laid the aluminum foil on the other end of the toilet paper roll and secured the edges of the aluminum foil with a rubber band. I took a push pin and made a single hole in the middle of the aluminum foil. I held the camera up so that the light from the light bulb on the table could shine through the pinhole in the aluminum foil and the wax paper faced me.

Physics student, Spring 2016

- Make a pinhole camera as described above.
- When you look through a pinhole camera at a bright light bulb in a dark room, what do you think you will see on the wax paper screen? Record your prediction and your reasoning in the *Before* section of your physics notebook page.
- After making a pinhole camera, darken the room and turn on a light bulb without a shade. Look at the light bulb through the pinhole camera. Hold the camera away from your face and point the aluminum foil end at the bulb as shown in Fig. 1.11. Move your hand around until you clearly see something on the wax paper screen. You may need to move closer or farther from the lamp.
- Also move your head from side to side while holding the camera steady. Does this change what you see on the wax paper screen? If so, how?



FIG. 1.11 Looking at a light bulb through a pinhole camera made out of a paper towel roll.

- Record what you are doing in the *During* section of your notebook page in both words and a sketch of the set up. Include a sketch of what you are seeing on the wax paper screen.
- Complete your entry on your physics notebook page before reading an example of student work about exploring pinhole phenomena.

1. *Example of student work about exploring pinhole phenomena*

Figure 1.12 shows what one sees when looking at a bright light bulb through a pinhole camera in a dark room.

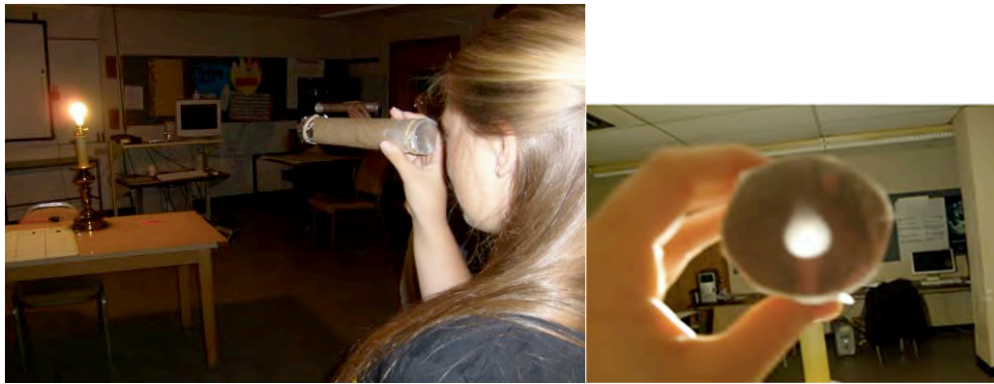


FIG 1.12 What one sees when looking at a light bulb through a pinhole camera.

A student described the surprising effect visible on the wax paper screen: “On the wax paper, I observed the projection of the light bulb but it looked like it was upside down!”

The student drew the sketch in Figure 1.13 to represent the set up and what was visible on the wax paper screen.

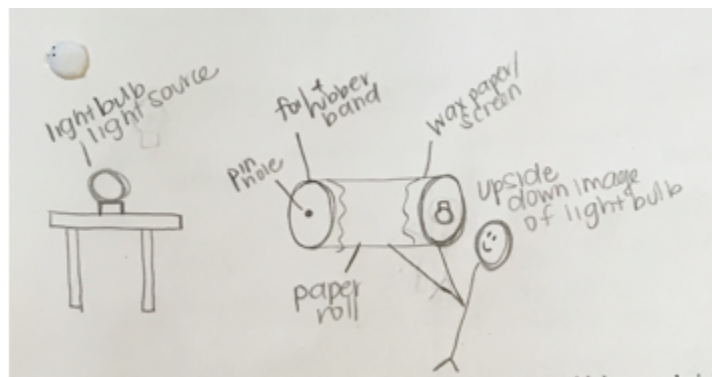


FIG. 1.13 Student's sketch of exploring pinhole phenomena.

Sketches are free form. This student wanted to show both the pinhole in the aluminum foil at one end of the roll and the upside down projection of the light bulb on the wax paper screen at the other end of the roll and so has shown both ends of the roll, although one would not see that in a 3-dimensional view. Labels include the light bulb, identified as the light source, the pinhole, foil and rubber bands holding the foil in place on one end of the paper roll, the wax paper screen on the other end of the paper roll, and the upside

down image of the light bulb.

B. Explaining pinhole phenomena

After observing *what is happening...* and *how something is happening*, one likely will be asking a *why* question such as Question 1.10:

Question 1.10 Why are you seeing what you are seeing when looking at a bright light bulb through a pinhole camera in a dark room?

- Discuss with your group members some possible explanations for what you are seeing on the wax paper screen.
- Sketching what is happening can help suggest and evaluate ideas. Get a large white board or piece of chart paper and work together to sketch the ideas you have generated.
- Every so often step back and ask yourselves:
 - What you are doing?
 - Why are you doing that?
 - How is that helping you?

(*Alan Schoenfeld (1992) suggests this questioning sequence to help people help themselves stay aware of whether what they are doing is likely to be useful in solving a complex mathematics problem.)

- Draw the light bulb and its *projection* (what you are seeing on the wax paper screen).
- Omit drawing the camera itself; just draw the light bulb and its projection roughly parallel to each other and some distance apart.
- How can you use your *conceptual model of light* (the two central ideas developed so far) to explain why you are seeing what you are seeing on the wax paper screen?

How, for example, are light rays leaving the light source? Draw some rays.

How are some of these rays traveling from the light bulb to its projection on the screen?

- How can you use your *physical model of light rays* (meter stick or yard stick) traveling from one place to another to think about how light is getting from the top of the light bulb to where the top of the projected bulb is on the wax paper screen?
- Use your physical model of light rays to draw multiple rays traveling from various parts of the light source to where you are envisioning that they land on the screen to form the projection of the light bulb you are seeing.

In particular, draw rays leaving the top of the light bulb, the middle of the light bulb, and the bottom of the light bulb and traveling through the pinhole to form the projection of the bulb on the wax paper screen.

- Where is the pinhole located through which you are envisioning the light rays traveling from the source to the screen?

Represent the pinhole with an open circle and add the rest of the pinhole camera to your sketch.

- Do not erase the sketch that you have been drawing. This likely is quite messy. Admire it as a document representing some of your thought processes in developing this explanation of why you are seeing what you are seeing when light from a source passes through a pinhole to a screen.
- A careful drawing with straight lines showing how one is envisioning light rays traveling from one place to another is called a *ray diagram*.

On a new large white board or piece of chart paper, draw a careful ray diagram of this situation by using the meter stick to draw straight lines representing how you are inferring light rays leave the light bulb, travel through a tiny pinhole, and form a projection of the light bulb on the wax paper screen.

- In the “After” section of your physics notebook page, draw a careful ray diagram by using a ruler to draw straight lines to represent what happens when light passes through a tiny pinhole and shines on a screen.
- Also record new central ideas, the evidence that supports these claims, and a

rationale that explains clearly why this happens.

- Also write a reflection about what you have learned.
- What are you still wondering?

Complete your entries on your physics notebook page before reading a student's example response and some nuances about exploring, representing, and explaining pinhole phenomena.

1. Student's example explanation of pinhole phenomena

In (Question 1.3 and 1.5), we discussed how light goes everywhere and that light rays travel in straight lines. With these ideas in mind, we inferred that all of the light rays that travel through the pinhole are traveling in straight lines. Because not all of these lines can travel straight forward, some of the lines end up traveling diagonally. As a result, the top of the light bulb gives off light rays that travel diagonally straight so that they travel through the pinhole and this diagonal line ends up projecting onto the bottom of the screen of the wax paper as shown by the blue line in (Fig. 1.14).

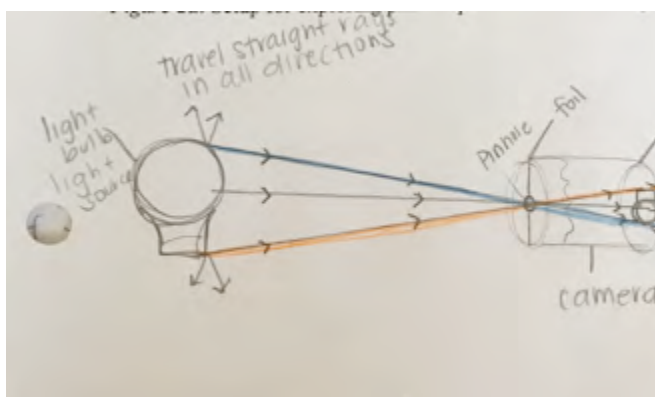


FIG. 1.14 Student's ray diagram representing explanation of pinhole phenomena.

The light rays from the center of the light bulb travel straight forward through the pinhole and project the image of the center of the light bulb on the screen of wax paper.

The orange line in Fig. (1.14) shows that the light rays given off of the bottom of the light

source travel in a straight line that is diagonal and projects onto the top of the screen of wax paper.

There are many light rays given off of the light bulb between the top and the bottom that all travel through the pinhole and project the image of the light bulb to appear upside down on the screen of wax paper.

Physics student, Spring 2016

The separate discussion of rays from the top, middle, and bottom of the light source is helpful when explaining pinhole phenomena. Also helpful is tracing each line with one's finger while discussing each ray if one is talking about the ray diagram with someone just learning about these phenomena.

2. *Some nuances in representing and explaining pinhole phenomena*

Note that in the ray diagram in Fig. 1.14, the student has represented the pinhole as an open circle, where the lines representing the top, middle, and bottom light rays cross in the diagram. A filled-in circle would not be appropriate in a ray diagram representing pinhole phenomena because this would represent a solid barrier through which light rays could not pass. The student also used a ruler when representing light rays traveling in straight lines and put arrows on the lines to indicate the direction that the student was envisioning the light rays to be traveling.

Also note that in Fig. 1.14 the student has drawn some short rays that are leaving the top of bulb but are not heading toward the pinhole. This represents a further refinement of the first central idea about how light leaves a source.

Children's drawings of the Sun typically show rays leaving the Sun in many directions as in Fig. 1.10. Each ray, however, seems to leave a point on the source in only one direction, straight outward. Such single rays are arrayed around the circle representing the Sun. This conception of how light leaves a source prompts an issue in explaining pinhole phenomena: if rays only leave in a perpendicular direction from a source, how can any rays from near the top of the bulb be traveling at an angle, diagonally downward, toward the pinhole?

One way to address this issue is to cover the light bulb with aluminum foil or a dark cloth so that light only shines out from a hole near the top of the bulb. Does light shine from the hole only in one direction perpendicular to the surface there? Or are the ceiling, floor, walls of the room and people's faces still lit, at least dimly? If so, this suggests

an additional refinement to the first central idea: **light leaves a point on most sources in many directions**. This is an example of an aspect of the nature of science articulated in the US Next Generation Science Standards that *science is open to revision in light of new evidence* (NGSS, Lead States, 2013, Appendix H, <https://www.nextgenscience.org/resources/ngss-appendices>).

The student who drew Fig. 1.14 seems to be grappling with this issue in the language used: *the top of the light bulb gives off light rays that travel diagonally straight so that they travel through the pinhole*. This student's language also illustrates a nuance that sometimes confuses students: the meaning of the word *straight*. For some people, *straight* only means *horizontal* rather than the more general meaning of *without any bends*. This student invented the phrases *straight forward* to mean *traveling horizontally without bends*, as in the center gray rays, and *diagonally straight* to mean *traveling at an angle without bends*, as in the top blue rays and the bottom orange rays. Such elaboration is not required in this course; simply discussing light as envisioned as rays traveling in straight lines is sufficient.

It is important to realize that nothing happens within the pinhole. This explanation envisions the light rays as simply passing through the pinhole in a straight line on their way to where they are going, some light rays from the top of the bulb traveling straight through the pinhole to near the bottom of the screen, some light rays from the middle of the bulb traveling straight through the pinhole to near the middle of the screen, and some light rays from the bottom of the bulb traveling straight through the pinhole to near the top of the screen. The light rays are envisioned as simply passing through the pinhole on their way to separate places on the screen.

Sometimes students use the word *flipped* to refer to the upside down projection that they see on the screen. This word is not appropriate here, however, because it suggests that something happened to the rays to cause a flip but nothing happens to flip the rays that form the projection. The rays forming the upside down projection are envisioned as simply continuing to travel in straight lines from the source through the pinhole toward the screen. One can see the upside down bulb because the aluminum foil blocks other rays from traveling to the screen.

A confusion also can occur over the use of the word *projection*. Does the phrase *top of the projection* refer to what one sees near the top of the screen (which is the projection of the bottom of the bulb) OR does the phrase *top of the projection* refer to the projection of the top of the bulb (which occurs near the bottom of the screen)? To avoid confusion, it is important to use the cumbersome but clear phrase “where these light rays form the projection of the top of the bulb near the bottom of the screen.” The word *projection*

is preferred over the word *image* here as the word *image* in physics typically refers to different light phenomena involving mirrors and lenses.

Like adults, children typically are surprised and intrigued by seeing an upside down light bulb when they look through a pinhole camera. As indicated by their drawings of the sun, children already know the key idea for understanding pinhole phenomena, that light can be envisioned as rays traveling in straight lines.

When discussing pinhole phenomena, with both children and adults, it is helpful to use a finger to trace the line on a ray diagram while discussing how rays from a particular part of the source are traveling:

Some light rays from the top of the light bulb are traveling straight through the pinhole to form the projection of the top of the light bulb near the bottom of the screen,

Some light rays from the middle of the light bulb are traveling straight through the pinhole to form the projection of the middle of the light bulb near the middle of the screen,

Some light rays from the bottom of the light bulb are traveling straight through the pinhole to form the projection of the bottom of the light bulb near the top of the screen and

Some light rays from all up and down the light bulb are traveling straight through the pinhole to form the rest of that upside down projection of the light bulb on the screen.

C. Exploring a critical issue

Missing from the discussion so far is a critical issue, how is information about what is happening getting to an observer?

Question 1.11 How does someone see this projection on the screen?

- Look again at what you see when looking at a bright light bulb through a pinhole camera.
- Hold the camera steady while moving your head from side to side. Does your view of the projection change? If so, how? What does this imply about how light travels through translucent materials such as the wax paper screen?
- Also consider what has to happen for someone to see something: How did the light

rays get from the projection on the wax paper screen to your eye?

- Where on your ray diagram would you represent your eye looking at the projection on the screen?
- How would you represent on the ray diagram what is happening when you see the projection on the screen?

Complete your entry on your physics notebook page and Table 1 (continued). Then write a summary of what you have learned before reading example student work explaining how one sees pinhole phenomena.

TABLE I.1 Explorations of light phenomena (continued)

TABLE I.1 Explorations of light phenomena (continued)			
Description of Exploration	Evidence Observed	Central Ideas	Vocabulary
		Light rays emerge from translucent material (such as wax paper) in many different directions	translucent
		For someone to see something, light has to get to the person's eyes.	

1. Example of student work about how one sees the projection on the screen

A student reported, “I was still able to see the projection on the wax paper when I would move my head from side to side” and offered an explanation “We are able to see the image even when we move our head because the light rays travel in many directions as they emerge from the translucent wax paper so some of them still travel to our eyes.”

This student was invoking two additional central ideas developed in class:

Light rays emerge from translucent material (such as wax paper) in many different directions. Some of those directions are toward the eye.

For someone to see something, light has to get to the person's eye.

The student added an eye to the ray diagram, with straight lines from the projection to the eye to represent light leaving the projection and traveling to her eye as shown in Fig. 1.15.

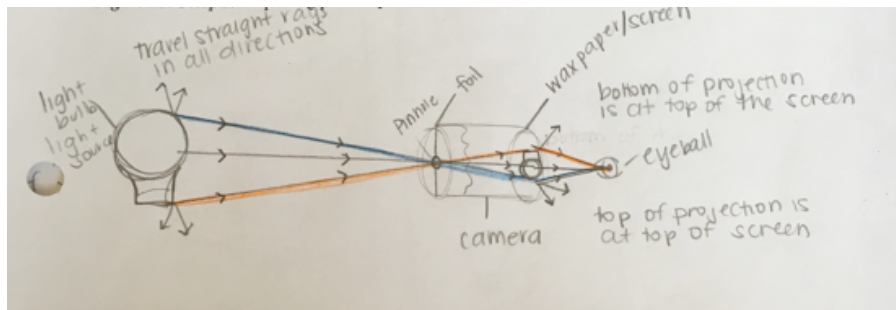


FIG. 1.15. Ray diagram for pinhole camera that includes light getting to viewer's eye.

The student explained the complete ray diagram as follows:

The orange line represents light rays traveling from the bottom of the light bulb through the pinhole to the top of the screen and then some of the rays travel in a straight line to the eyeball.

The middle grey line represents the light rays that travel from the center of the light bulb through the pinhole, creating the image of the middle of the bulb on the screen, then some travel to the eyeball.

The blue line represents the light rays that travel straight from the top of the light bulb diagonally down through the pinhole and then project the image of the top of the light bulb onto the bottom of the screen of wax paper. Some of the blue rays then travel ...to the eyeball.

The notes added near the top and bottom on the right side of this diagram illustrate the confusion of the meaning of the word *projection* – does this refer to what one is seeing? (bottom of the bulb near the top of the screen) or to the entity itself (the top of the projection refers to what is at the top of the screen)? In this course, avoid the phrases

“bottom of projection” and “top of projection” and use instead the clear phrase “where these light rays form the upside down projection of the top of the bulb near the bottom of the screen.”

D. Exploring variables affecting pinhole phenomena

Another type of question focuses upon identifying what is important to notice in a situation:

Question 1.12 What variables affect what one is seeing on the screen?

- Discuss with your group members ways you can change the pinhole camera you made and ways that you are using it.
- Also consider making new ones such as comparing what happens when one uses an empty toilet paper roll, paper towel roll, or holiday wrapping paper roll.
- Another possibility is to make an “open” pinhole device without an enclosing body. Make a pinhole holder by cutting out an area within a piece of cardboard, taping a “cover” of Aluminum foil over the area, and poking a hole in the Al foil. Make a firm screen by taping a piece of copy paper to a piece of cardboard the same size. Then hold the pinhole holder in one hand and the firm screen in the other hand in front of a lamp without a shade until you see the upside down projection. Then observe how the projection changes as you change how you hold the pinhole holder and screen with respect to the lamp.
- Explore and report on some of these possibilities in Table I.2

TABLE I.2 Variables in exploration of pinhole phenomena	
Variable	Observation

- In the “After” section of your physics notebook page, record two new central ideas, the evidence that supports these claims, and a rationale that explains how the evidence supports the ideas and why these are important.
- Also write a reflection about what you have learned.
- What are you still wondering?

Complete your entry on your physics notebook page and write a summary of what you have learned before reading examples of student work of exploring variables that affect pinhole phenomena

1. *Example of student work about variables that affect pinhole phenomena.*

How big is the projection? How bright is it? How sharp is it? How many projections does one see? Do distance from the light source, length of the roll, size of the pinhole, shape of the light fixture, or number of pinholes matter? As shown in Fig. 1.16, a student included the following variables in reporting findings: distance from light, multiple pinholes, size of camera roll, larger pinhole, and light fixtures of different shapes:

Variables in Exploration of Pinhole Phenomena	
Variable	Observation
distance from light	the image is larger when closer
multiple pinholes	the image shows up twice w/ 2 pinholes
size of camera roll	image was larger w/ more length
larger pinhole	image is larger + more fuzzy
light fixtures of diff shape	shape of image changes with different light fixtures

* not flipped, = projection is displayed upside down

FIG. 1.16 Student's table of variables in exploring pinhole phenomena.

This student then described these findings using “projected image” rather than the more technically preferred “projection.” The multiple examples illustrate well, however, the many ways even young students can enjoy exploring pinhole phenomena:

When the distance between the camera and the light bulb increases, the projected image of the light bulb on the wax paper screen becomes smaller. When there are multiple pinholes in the aluminum foil, there will be multiple projected images of

light bulbs on the screen of wax paper. When the length of the paper roll of the camera increases, the projected image of the light bulb gets larger. When the pinhole is made into a larger hole, the image becomes larger and fuzzier, it is more difficult to tell if the image is still upside down or not.

When the camera is used to view light bulbs of different shapes, the projected image of the light bulb on the wax paper screen will change with the light fixture. For example, in class we looked at a light bulb that was similar to a tear drop shape and our projected image looked similar to an upside down teardrop. When I used the camera on a spherical light fixture, the projected image looked like an upside down circle. Some other questions that I have is: is this how our pupils work? How did people discover the pinhole phenomena?

Physics Student, Spring 2016

In this course, we are not exploring what happens within the eye, just getting light to the eye. If you are wondering what happens next, after the light enters the eye, consult a book or internet website about how eyes work such as https://nei.nih.gov/kids/about_the_eye.

The eye's pupil acts like a pinhole with a variable diameter that controls how much light enters the eye. The projections on the retina are upside down and are turned 'right side up' by your brain. Some additional information about how the eye works is at: http://www.mind.ilstu.edu/curriculum/vision_science_intro/vision_science_intro.php

Some historical information about pinhole cameras is at <https://jongrepstad.com/pinhole-photography/pinhole-photography-history-images-cameras-formulas/>

E. Exploring pinhole phenomena with friends and/or family members

Question 1.13 What happens when exploring pinhole phenomena with a friend or family member

- Continue exploring pinhole phenomena outside of class with a friend or family

member.

- Complete your entry on your physics notebook page and write a summary of what you have learned before reading examples of student work exploring pinhole phenomena with friends and/or family members .

1. Examples of student explorations of pinhole phenomena with friends and/or family

A student explored pinhole phenomena with a cousin:

I showed the pinhole phenomena with my 15 year old cousin. Before seeing the projection he guessed that the projection on the wax paper would be the shape of the pinhole. When he saw the projection his exact words were “what the fudge?”

It took Z. a moment to realize what he saw was an upside down light bulb. When asked why he thought the projection was upside down he said “I have no idea.... I don’t even know how it is projecting the stupid light bulb!”

I then asked Z. what he knew about light; after some discussion he and I came to the conclusion that light travels in all directions and that light travels in straight lines called rays.

I asked Z. to draw on a whiteboard a representation of what he saw, much like what I myself did in class with my group. Z. did so, and I then asked him to use a ruler and his pen to draw rays leaving the light bulb. At first Z. drew only light rays leaving in vertical and horizontal lines. I then asked Z. to tell me again how light travels. Z. said that it travels in all directions in straight lines. I asked him to show me lines that traveled in all directions, so Z. drew more lines leaving the bulb in all directions much like the lines above.

I then posed the question to Z. of how light was getting through the tube and onto the screen. Something seemed to click with Z. then because he erased his original drawing and drew a new bulb with about 10 rays some of which went through the pinhole and onto the screen. He then told me that since the rays that he drew at the top of the bulb “would continue traveling on their downward path and would be projected onto the bottom of the screen. The lights from the bottom of my bulb will travel towards the top and be projected there. Is that right?”

I was honestly amazed at how quickly he picked up on the information. I told him

he was right and showed him my own ray diagram and he said that it made sense. He then helped me find ways to alter the experiment, which are shown in table 2.

From my experience working with and teaching Z. I realized how frustrating it can be to work with older students. I do not mean for this to sound bad, because in some ways it was refreshing to work with an older student. Yet, as I worked with Z. I found that I am so accustomed to working with younger children, these children need a certain amount of guidance, more so than he did. Z. did need my help to understand but much of what he was able to figure out on his own was frustrating and interesting to me. His mind works so quickly and he needed only a slight push in the right direction to understand. Knowing this it was hard to hold my tongue and not explain everything to him before allowing his mind to make the connections first.

Physics student, Fall 2015

Another student chose to explore pinhole phenomena with a roommate:

I explored pinhole cameras with one of my roommates. I began by asking her what she knew about light (such as the direction it travels, whether or not it travels in a straight line). To my surprise she already knew that light traveled in all directions and in a straight line. I told her to keep that in mind.

I then showed her the pinhole camera I had made in class and explained what she would be doing with the light bulb and the camera. Before turning on the light bulb I asked her what she thought would happen. She said that she thought the light bulb would appear on the wax paper.

I then turned on the light bulb and asked her to look and tell me what she saw. She was puzzled to find that the light bulb that appeared on the wax paper was actually upside down. She could not tell me why this happened.

I gave her a piece of paper and pencil and asked her to draw the entire set up including what she had seen. I then asked her again to tell me what she knew about light. When she said that light traveled in a straight line I told her to stop and think about that.

She thought for a few seconds and then told me that light was entering the pinhole camera in a straight line. I encouraged this by giving her a ruler to add some straight light rays to her picture. After drawing straight lines for a few seconds she connected the top of the light bulb to the bottom of the light bulb on the camera with a straight line. She did the same for the bottom of the light bulb. I asked her to tell me what this meant. She explained that the ray coming from the top of the light bulb travels in a straight line through the pin hole to the bottom of the screen and the ray coming

from the bottom of the light bulb does the same, except it appears at the top of the screen.

I learned that being patient and providing my roommate with adequate thinking time (private reasoning time), she was able to tell me exactly what had happened. I prompted her throughout the experiment with questions instead of telling her the answers. I also encouraged her to build off of what she already knew (for example: that light travels in a straight line).

Physics student, Spring 2014

This concludes this example of using central ideas based on evidence to explain intriguing phenomena.

V. Developing Mathematical Representations of Pinhole Phenomena

Pinhole phenomena can provide useful tools in estimating quantities that are otherwise hard to measure. To do so, we need to develop ways to represent pinhole phenomena mathematically, both geometrically and algebraically.

A. Representing pinhole phenomena geometrically

The visual display provided by a ray diagram suggests describing pinhole phenomena geometrically. First we simplify the diagram, next name and label angles, and then interpret geometric aspects of two triangles evident in the refined ray diagram.

Question 1.14 How can you describe pinhole phenomena geometrically?

The ray diagram in Fig. 1.14 includes realistic sketches of the light bulb, the upside-down projection, and the pinhole camera as well as lines representing light rays inferred to be leaving the source and traveling straight through the pinhole to the screen.

This realistic portrayal of what we can actually see (the light bulb, camera, and projection on the screen) was helpful in using our conceptual model for light in explaining the upside-down projection of the light bulb observed on the screen: We could infer that light was leaving points on this source in many directions and envision that some light rays were traveling in straight lines:

- from the top of the light bulb straight through the pinhole to form the projection of the top of the light bulb near the bottom of the screen,
- from the middle of the bulb traveling straight through the pinhole to form the projection of the middle of the light bulb near the middle of the screen, and
- from the bottom of the bulb traveling straight through the pinhole to form the projection of the bottom of the light bulb near the top of the screen.

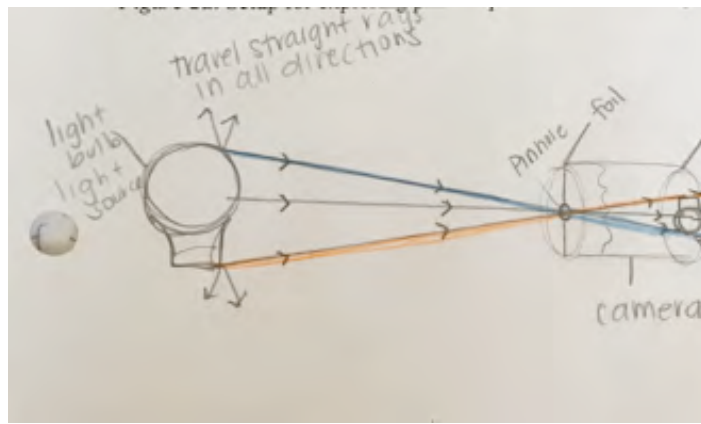


FIG. 1.14 (repeated) Student's ray diagram representing explanation of pinhole phenomena.

A more abstract stylized ray diagram, however, can focus attention on the envisioned light rays without the details of source, camera, and screen as shown in Fig. 1.17:

- Instead of a drawing of a light bulb, a vertical line represents the light source.
- Instead of a drawing of an upside-down projection of the light bulb on the screen, a parallel shorter vertical line represents this projection.
- Instead of an open circle, the pinhole is represented by an opening between short vertical lines above and below the point where the lines representing envisioned light rays cross. These short vertical lines represent the opaque end of the camera through which a pin was pushed to make the pinhole

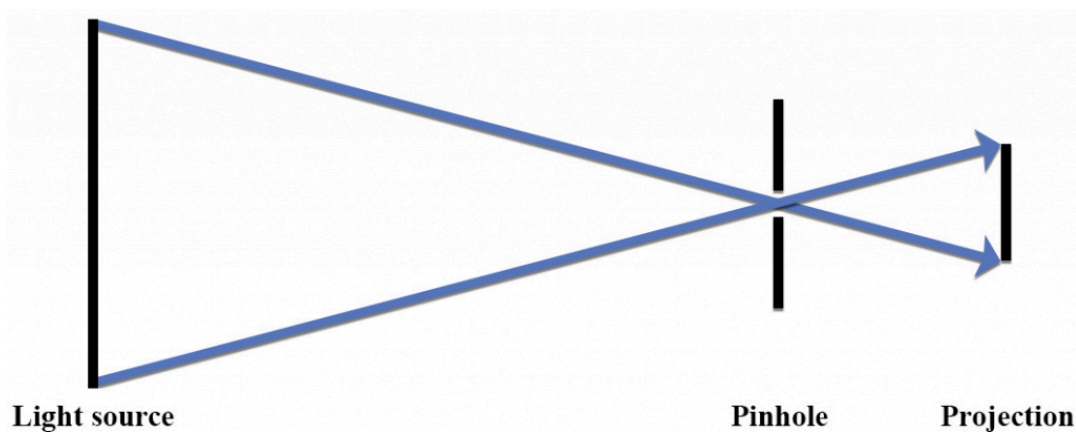


FIG 1.17 Stylized ray diagram representing pinhole phenomena.

In Fig. 1.17, vertical black parallel lines represent the source and projection rather than sketches of the bulb and upside down projection. An opening between two small black vertical lines represents the open pinhole. The blue straight lines represent rays of light inferred to be traveling from the top and bottom of the bulb to the bottom and top of the screen where they form the upside down projection of the light bulb. The rest of the details of the source and the camera are not shown. The description of what is happening is the same:

- We are envisioning rays of light from the *top* of the vertical line representing the light bulb traveling straight through the opening representing the pinhole to form the projection of the top of the light bulb near the *bottom* of the screen.
- We are envisioning rays of light from the *bottom* of the light bulb traveling straight through the opening representing the pinhole to form a projection of the bottom of the light bulb near the *top* of the screen.

By using straight parallel vertical lines to represent the source and the projection, Fig. 1.17 simplifies the ray diagram so that two triangles become evident.

- What do these two triangles represent?
- How are these two triangles related?

One way to describe a ray diagram geometrically is to begin by naming the vertices of the triangles. In Fig. 1.18 for example, the vertices of the triangle on the left are labeled A, B, C, whereas the vertices of the triangle on the right are labeled F, E, C. (Later the letter D will represent the distance between line AB and vertex C, so D is omitted in naming vertices here.)

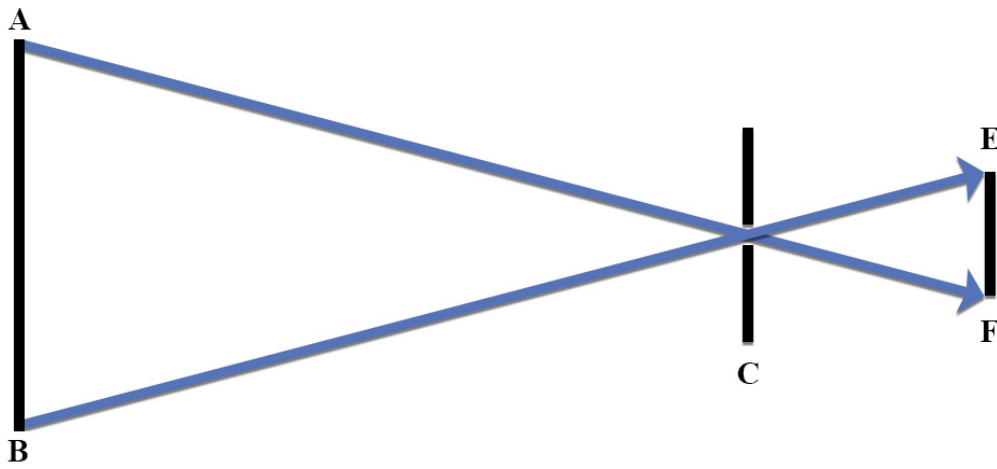


FIG. 1.18 Triangles ACB and FCE.

- How are the angles in these triangles related?
 - What does it mean, for example, for two angles to be *corresponding*? Which angle in triangle FCE corresponds to Angle A in triangle ACB? Which angle in triangle FCE corresponds to Angle B in triangle ACB? Which angle in triangle FCE corresponds to Angle C in triangle ACB?
 - What does it mean for two angles to be *congruent*?
 - When are two angles congruent if formed by intersecting lines?

Triangles with three congruent corresponding angles are called *similar triangles*.

- Are triangles ACB and FCE similar triangles? If so, which are the three congruent corresponding angles?
- If triangles ACB and FCE are similar triangles, how are corresponding lengths related?

If you need to learn about similar triangles, see www.mathopenref.com/similartriangles.html and <http://www.mathopenref.com/similartrianglesparts.html>

For information about vertical angles, see <http://www.mathopenref.com/anglesvertical.html>

For information about congruent angles created by parallel lines and a transversal

(sometimes called *alternate interior angles*), see <http://www.mathopenref.com/anglesalternateinterior.html>

Complete writing your own response to Question 1.14 before reading an example of student work and some nuances in representing pinhole phenomena geometrically.

1. *Example of student work representing pinhole phenomena geometrically*

A student drew the ray diagram in Figure 1.19 at the close of a class session in which the students developed and then used mathematical representations of pinhole phenomena to estimate a quantity. This student drew single, double, and triple lines to indicate clearly the congruent corresponding angles of the two triangles formed in the ray diagram:

Single lines indicate that angle c of the large triangle on the left corresponds to and is congruent with angle c of the small triangle on the right

Double lines indicate that angle a corresponds to and is congruent with angle f .

Triple lines indicate that angle b corresponds to and is congruent with angle e .

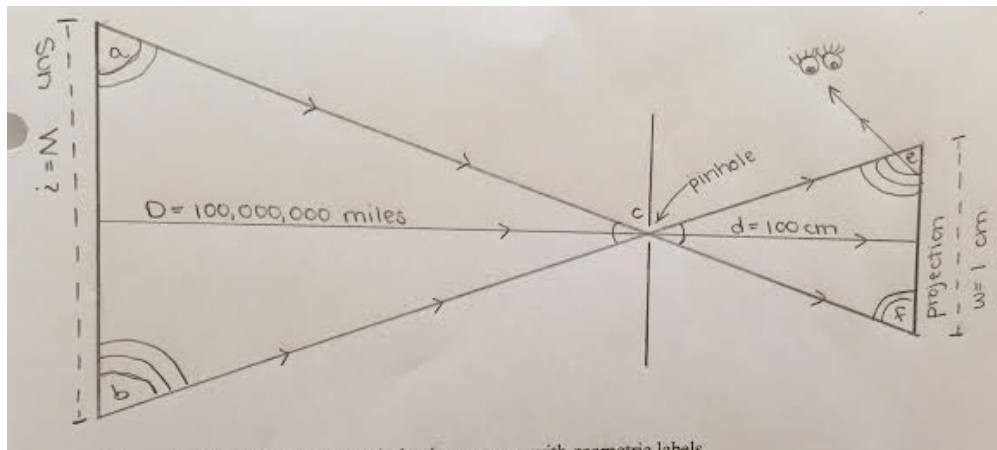


FIG. 1.19 Student's ray diagram showing corresponding congruent angles.

The student described this ray diagram geometrically:

(The figure above) is a ray diagram consisting of two triangles that share a point. I have labeled the vertices of each of the triangles a b c , c e f , noticing that their shared vertex is c ...We can conclude that the two triangles are similar because of two

different properties. The first property, vertical angles property, states that angles acb and ecf are congruent. Next, the alternate interior angles property allows us to confirm that two different sets of angles are congruent: angles abc and fec as well as angles bac and efc . The two triangles above, written acb and ecf , are similar because each of their corresponding angles is congruent. We know this because of the AAA, or Angle Angle Angle, theorem. Because these triangles are similar, we can set up a proportion...

Physics student, Spring 2016

(This student also included some numerical information on this ray diagram that refers to the activity involving pinhole cameras discussed below under section VI.)

2. Some nuances in representing pinhole phenomena geometrically

In writing about the ray diagram shown in Fig. 1.19, the student invoked several geometric ideas:

- Angles formed by two intersecting lines are called *vertical angles* (as occurs at the pinhole C in Fig. 1.19) and are congruent, that is, they have the same measure; if one is 30 degrees, the other also is 30 degrees.
- Some angles formed by a line (a *transversal*) intersecting two other lines are called *alternate interior angles* (such as angles a and f; also angles b and e in Fig. 1.19).

Alternate interior angles are congruent *only if* the two other lines are *parallel*. If the two other lines are not parallel, the angles will still be alternate interior angles but they will not have the same measure, that is, they will not be congruent.

- Two triangles are similar if all three corresponding angles are congruent:

Angle a is congruent with Angle f because they are alternate interior angles formed by a transversal crossing two parallel lines.

Angle b is congruent with Angle e because they are alternate interior angles formed by a transversal crossing two parallel lines.

Angle c of the big triangle is congruent with Angle c of the little triangle because they are vertical angles

- Corresponding lengths of similar triangles are proportional.

In naming the triangles “triangle acb” and “triangle ecf” this student did not follow the convention to name similar triangles so that the names of the corresponding angles are in the same position: the position of Angle a in the name “triangle acb” would correspond to the position of Angle f in name “triangle fce”; the position of Angle c in the name “triangle acb” would correspond to the position of Angle c in the name “triangle fce”; and the position of Angle b in the name “triangle acb” would correspond to the position of Angle e in the name “triangle fce”.

In referring to “angles abc and fec as well as angles bac and efc”, however, this student has followed the convention in that congruent angles a and f are in the same position, congruent angles b and e are in the same position, as are the angle c’s.

Note that this student used lower case letters, rather than capital letters, to identify angles on the ray diagram and then named each angle by listing three lower case letters with the angle letter in the middle. In this course, capital letters placed outside the vertex as in Fig. 1.18 are preferred in labeling angles of triangles such as Angle A and in naming triangles, as in the statement “Triangle ABC is similar to Triangle FEC”.

The ray diagram in Fig. 1.19 clearly represents the congruent corresponding angles in these two triangles. Note that the student has used the space between two short vertical lines to represent the pinhole as well as a ruler to make straight lines. The line with arrows drawn from the projection to the viewer’s eyes is ok in that light all the way along the projections bounces off the screen to the viewer’s eyes. However, starting this line higher, at the vertex of Angle e, would better represent rays *continuously* traveling from the bottom of the source straight through the pinhole to form the projection of the bottom of the light bulb near the top of the screen and then travel to the eye.

Note that the eye is shown to the left of the screen. This represents a scenario in which the student was looking at a projection on a screen where the light forming the projection reflected off an opaque screen back to the eye as in Fig. 1.21 below rather than passing through a translucent wax paper screen as in Fig. 1.15 above.

B. Representing pinhole phenomena algebraically

The visual display provided by a ray diagram also suggests describing pinhole phenomena algebraically. First, we name and label lengths as well as angles, identify ratios of corresponding lengths that are equal, and consider nuances in representing pinhole phenomena algebraically.

Question 1.15 How can you represent pinhole phenomena algebraically?

Representing pinhole phenomena algebraically requires naming aspects of pinhole phenomena that you can measure and use in a calculation. Identifying such quantities involves thinking about the *variables* in a situation: what, for example, affects the size of the projection that you see?

- What can you do to make the projection bigger? Smaller?
 - What happens to the projection if you move the camera closer to the source? Farther from the source?
 - What happens to the projection if you make the camera longer? Shorter?
 - What happens to the projection if you choose a higher light source? Lower?
- Give names to those variables and draw a ray diagram that includes identifying those variables with symbols, in addition to labeling angles A, B, C, E and F.

Include, for example, a horizontal line from the middle of the vertical line representing the light source, AB, to the middle of the vertical line representing the projection to EF.

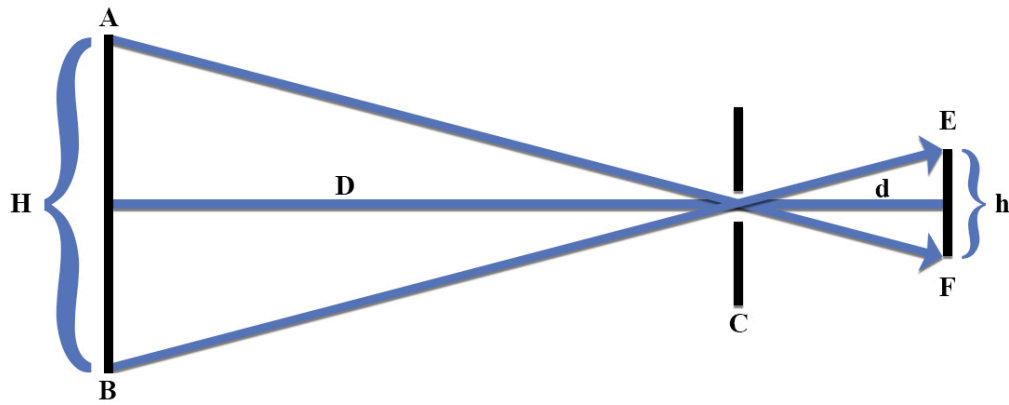


FIG. 1.20 Ray diagram representing pinhole phenomena.

Figure 1.20, for example, includes a line representing light rays leaving the middle of the source, traveling straight through the pinhole, to form the projection of the middle of the source on the middle of the screen. This line also can represent two variables, the length, *Distance D*, between the light source and the pinhole, and the length, *distance d*, between the pinhole and the projection on the screen. The ray diagram in Fig. 1.20 also identifies *H* as the *Height* of the source *AB* and *h* as the *height* of the projection *EF*.

Note that the symbols *H*, *D*, *h* and *d* represent *lengths* in the two triangles whereas the symbols *A*, *B*, *C*, *E*, and *F* represent angles.

- How are lengths related in similar triangles?

If two triangles are similar, their corresponding lengths are proportional.

- How would you express the relationship of corresponding lengths in similar triangles algebraically with symbols?
- How would you express the algebraic relationship of corresponding lengths in similar triangles with words?

Complete writing your own response to Question 1.15 before reading an example of student work and some nuances in representing pinhole phenomena algebraically.

1. Example of student work representing pinhole phenomena algebraically

A student summarized the mathematical relationship among the lengths of similar triangles as follows:

The property of similar triangles states that triangles can be different sizes as long as they have the same shape. The same shape depends on corresponding angles and proportionate sides. In the figure we can see the relevant vertices of the triangles and proportionate sides where “H” is the height of the bulb and “d” is the distance from foil to wax paper. The Height of the bulb over the Distance from the bulb to the foil is equal to the height of the projection over the distance from the pinhole foil to the wax paper.

$$\frac{\text{Height of the bulb}}{\text{Distance from the bulb to pinhole}} = \frac{\text{height of the projection}}{\text{distance from the pinhole to the projection}}$$

Physics student, Spring 2014

This student understood the basic property of similar triangles, that they have the same shape but may be different sizes and that this means that the ratios of corresponding lengths are equal.

2. Nuances in representing pinhole phenomena algebraically

There are several aspects of representing pinhole phenomena algebraically that need attention. These include choices made in using language, naming variables, forming ratios, justifying the equal sign, deciding on an appropriate algebraic representation to use, and using one’s sense-making skills to envision and monitor what one is doing.

(a) *Using language.* Part of learning physics is learning to use language precisely. What words, for example, describe “H/D”? Colloquially this often is referred to as “H over D” or “height over distance”. The use of the word “over” refers to how one expresses this relationship in writing: one writes the H “over” the D. Such language can be confusing for someone who does not infer the process represented by the word ‘over’ and the line

drawn between the H and D. A more precise version would be to refer to “H divided by D” or “height divided by distance”. This articulates the mathematical process involved, division. However, a more informative version would be to refer to “the ratio formed by comparing H to D” or “the ratio formed by comparing the height of something to some relevant distance, such as “the ratio formed by comparing the height of the light source to its distance from the pinhole.”

(b) *Naming variables.* In developing an algebraic description of a phenomenon, one has to give names and symbols to the variables that matter, in this case how high the light bulb and its projection are, and how far they each are from the pinhole. *Height* and *Distance* seem the natural names for these variables with H and D used for the height of the light source and its distance from the pinhole in the large triangle and h and d used for the height of the projection and its distance from the pinhole in the smaller triangle.

The horizontal line representing two distances here, however, would typically be described as the heights of triangle ABC and triangle FEC. Should one give a name to this variable that refers to what it is representing, distance from the pinhole, or a name to this variable that refers to its geometric role in the diagram, height of a triangle?

To avoid this confusion, sometimes we have used *length* to refer to the vertical lines, but the lower-case l looks like a one and $L/D = l/d$ is also confusing. Note in Fig. 1.19, that this student had used the letters W and w to represent the *width* of a circular light source and its projection where use of a D and d might have been expected for representing their *diameters*, but D and d were already in use for representing *distances* from the pinhole.

The advantage of using the name *height* for the variables represented by the vertical lines AB and EF is that this connects the mathematical description to the objects the lines are representing, the heights of the light bulb and its projection.

An essential aspect of developing a mathematical representation of a phenomena is that after deciding on the names and symbols one wants to use, be sure to state clearly what each variable and its symbol represent.

(c) *Forming ratios.* When considering two similar triangles, it can be helpful to form a ratio with variables in one triangle and set this equal to a ratio formed with the corresponding variables in the other triangle. The equation with ratios that this student stated, for example, $H/D = h/d$, compares two lengths of the large triangle, representing the height of the light source and its distance from the pinhole, and sets this equal to a comparison of the two corresponding lengths of the small triangle, representing the height of the projection and its distance from the pinhole.

It is important to be able to express the equality of such ratios with words as well as symbols: With $H/D = h/d$, one is equating “how the height of the light source compares to the light source’s distance from the pinhole” with “how the height of the projection compares to the projection’s distance from the pinhole.”

One also has the choice of in what order to make such a comparison. With $D/H = d/h$, for example, one is equating “how the light source’s distance from the pinhole compares to the light source’s height” with “how the projection’s distance from the pinhole compares to the projection’s height.”

In both cases, this approach has the advantage that the units used within the same triangle will match. In this case of comparing one length to another length, with both lengths measured in the same units, the ratios will be dimensionless. This means that, if convenient, one can use units for the lengths in the large triangle that differ from the units for the lengths in the small triangle.

Sometimes, however, one might want to compare the same variable in the two triangles, setting the ratio of the heights, for example, equal to the ratio of the distances: $H/h = D/d$. In this case, one is setting equal “how the light source’s height compares to the projection’s height” with “how the light source’s distance from the pinhole compares to the projection’s distance from the pinhole.” One also could set this equation up as: $h/H = d/D$. Here one is setting equal “how the projection’s height compares to the light source’s height” with “how the distance of the projection from the pinhole compares to the distance of the light source from the pinhole.”

Although cumbersome, keeping in mind the word description of what these ratios represent can prevent randomly setting up an equation that may look appropriate: $h/D = H/d$ but is not justified in this context.

(d) *Justifying the equal sign.* When setting one algebraic expression equal to another in this course, it is important to justify the equal sign explicitly. For example, one student wrote:

In the figure, there are two triangles seen, triangle ABC and triangle FCE. The two triangles are similar because they have three sets of congruent angles. The first set of congruent angles are vertical angles, in which Angle ACB \approx Angle FCE. The second set of congruent angles are alternate interior angles, in which Angle CAB \approx Angle CFE.

The third set of congruent angles are alternate interior angles, in which Angle CBA \approx Angle CEF.

Since the two triangles have congruent angles, they are therefore similar triangles

because of the AAA theorem. The corresponding sides of similar triangles are proportional, which is why we are able to set up a proportion comparing the two triangles.

Physics student, Spring 2016

Note that this student has not used the convention in naming the triangles. Triangle ABC puts angle C in the third position but Triangle FCE puts angle C in the second position. An alternative would be Triangle ABC is similar to Triangle FEC or Triangle ACB is similar to Triangle FCE. The student has used the conventions in naming angles so that the angle of interest is the middle letter: Angle ACB and Angle FCE are both referring to the angle where the two lines cross at C. Angle CAB refers to the angle at A and Angle CFE refers to the angle at F, a set of alternate interior angles at A and F. Angle CBA refers to the angle at B and Angle CEF refers to the angle at E, the other set of alternate interior angles.

(e) *Choosing an appropriate algebraic representation.* If $H/D = h/d$, $D/H = d/h$, $H/h = D/d$ and $h/H = d/D$, which set of equal ratios should one use?

If you can measure three of the variables and need to calculate the fourth, any of these will work. However, some are easier to use than others.

Solving for an unknown that is in the numerator for many students is easier than solving for an unknown in the denominator. If asked to find the height of a projection, h , for example, choose a version with h in the numerator: $H/D = h/d$ or $h/H = d/D$. Then isolate h by multiplying both sides by the variable in the denominator:

$$h = d\left(\frac{H}{D}\right) \text{ or } h = H\left(\frac{d}{D}\right)$$

Note: Avoid using \times to represent multiplication in an algebraic equation as this can be misinterpreted as representing an unknown. Use parentheses or a dot:

$$h = H \bullet \frac{d}{D} \text{ to represent multiplication.}$$

f) *Using sense-making skills to envision what an equation is doing.* People in this course typically differ in their experiences and comfort in setting up and solving algebraic equations. If you are feeling some anxiety after reading this section, it is important to give yourself time to connect each symbol with what it means, not only by writing out its definition and labeling it on your ray diagram but also by visualizing what the symbol represents within an equation.

When solving a pinhole problem, for example, write h = height of the projection and

place an h next to the vertical line representing the projection on your ray diagram; then step back and visualize the size of the upside down projection you saw on the wax paper screen of the pinhole camera. Do the same for the d , H , and D .

The goal is to become familiar enough with the symbols that when you see $H/D = h/d$, your mind visualizes what those ratios mean:

- you can “see a comparison in your head of the big Height of the light bulb with the big Distance from the light bulb to the pinhole of your pinhole camera;
- you can “see” a comparison in your head of the little height of the projection with the little distance from the pinhole to the wax paper screen of the pinhole camera;
- and you can “see” that those comparisons seem about equal;
- that, for example, the big Height of the light bulb is about a tenth of the size of the big Distance from the light bulb to the pinhole in the pinhole camera in your hand
- and that the little height of the upside down light bulb projection seems about a tenth of the size of the little distance from the pinhole in the aluminum foil end of the tube to the wax paper screen.

Also use words to tell yourself a story about what the equation is doing, that the equation is comparing heights to distances in two similar triangles and if the triangles are similar, the comparisons of heights to distances are equal even when the sizes of the triangles are very different. Visualizing what the equation is doing is key to clarifying in your mind whether you have set up the equation in a way that makes sense.

(g) Using sense-making skills to monitor what one is doing. In addition to visualizing what an equation is doing, it is important to use sense-making skills to monitor what you and your group members are doing. In solving a complex problem, it is easy to get focused on some detail that in the end is not needed. To make productive progress, it is important to step back for a moment to ponder:

- What are we doing?
- Why are we doing that?
- How will that help us?*

This is particularly important to do periodically when engaged in a multi-step endeavor, such as designing an experiment, collecting data, and using the data and one’s knowledge

of physical phenomena to estimate a quantity that can not be directly measured such as the diameter of the Sun.

*These questions are suggested by Alan Schoenfeld (1992) as meta-cognitive checks when engaged in problem-solving.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: MacMillan

(h) *Using one's knowledge of geometrical and algebraic representations of pinhole phenomena to explore ways to estimate quantities one wants to know.* Suppose you want to know, for example, how high something is, like a tree. How would you use a pinhole device to find that out? What would you need to measure and do if you used the equation $H/D = h/d$? You can practice such approaches in a dark room by using data from your exploration of variables affecting pinhole phenomena (Question 1.12). What would you need to measure if you wanted to estimate the height of the lightbulb? If $H = D(h/d)$ and you substitute your measurements in this equation, how does your estimate compare with an actual measurement of the height of the bulb?

VI. Using Mathematical Representations to Estimate an Interesting Quantity

Pinhole phenomena can provide a way to estimate a quantity that one can not directly measure.

A. Using pinhole phenomena to estimate the diameter of the Sun

Question 1.16 How can you use pinhole phenomena to estimate the diameter of the Sun?

How big do you think the Sun is? It would not be possible to travel to the Sun and measure its diameter directly. Using pinhole phenomena, however, provides a way to estimate this quantity from Earth.

- In the *Before* section of a notebook page, record your initial estimate of the size of the Sun.

It looks about the same size as the Moon in the sky. Both seem small compared to the size of the Earth from which we are viewing them. However both are far away and things far away look smaller than they are. How big do you think the Sun's diameter might be?

- How could you use pinhole phenomena here on Earth to estimate the Sun's diameter?
- A good way to start designing an exploration is to draw something that will help you think about what to do. For example, draw a ray diagram representing pinhole phenomena.
 - Which line on your ray diagram could represent the diameter of the Sun?
 - Which line could represent a projection of the Sun here on Earth?

- What equipment would you need to make such a projection of the Sun?
- How would you use this equipment?

Figure 1.21 shows one approach.

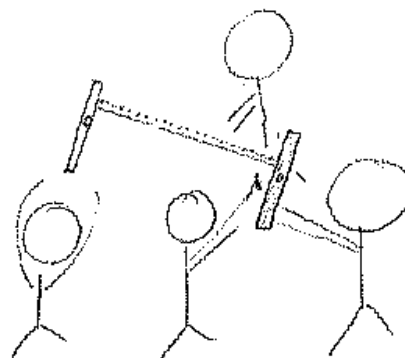
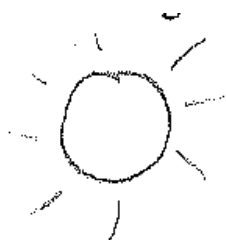


FIG. 1.21 Student drawing to illustrate using pinhole phenomena to estimate the diameter of the Sun.

Equipment: One way to estimate the diameter of the sun involves using a meter stick, some aluminum foil, a piece of cardboard to support the aluminum foil, scissors to cut a hole in the center of the cardboard, tape to tape the aluminum foil in place, a pin to make a pinhole in the foil, a white paper screen, a piece of cardboard to which to tape the white paper screen to keep it firm, a pencil to trace the Sun's projection on the white paper screen, and a sunny day.

[In a remote learning situation, students may not have access to a meter stick or yard stick but they likely have a ruler and shoestrings. After tying the shoestrings together to make a long string, use the ruler to measure where to tie a knot on the long string to measure 100 centimeters (or 3 feet if the ruler only has inches marked). Keep the string taut when establishing the distance between the pinhole and the screen.]

- In the *During* section of your physics notebook page, record what you do to measure the projection of the Sun by using such equipment.

Also draw a ray diagram to represent this setup and identify on the diagram each relevant quantity with a word or phrase and symbol.

In addition, state and justify an algebraic equation that represents the mathematical relationship among these quantities. Solve that equation for the diameter of the Sun.

Also create a table, Table I.3, that organizes the relevant information such as the name of each quantity identified, its symbol, its numerical value, and status as measured, provided, chosen, or calculated.

- In the *After* section of your physics notebook page, record the claim you are making about the diameter of the Sun, the evidence that you have used to support that claim, and the rationale that explains how you have interpreted the measurement you made to calculate an estimate of the Sun's diameter.
- In the *Reflection* section of your physics notebook page, comment upon your experiences in using mathematical representations of pinhole phenomena to estimate an interesting quantity in this way.

What have you learned not only about the Sun but also about the scientific process of designing explorations, using geometrical reasoning, writing and justifying algebraic equations, estimating quantities, and considering whether the estimate is reasonable?

- Also record on your physics notebook page, what you are still wondering.

Complete your entry on your physics notebook page and write a summary of what you have learned before reading an example of student work in estimating the diameter of the Sun. Also read about nuances in using mathematical representations of pinhole phenomena. In addition, you may find interesting reflections about using pinhole phenomena to estimate the Sun's diameter with friends and/or family members and some thoughts about the nature of science in this context.

1. *Example of student work in estimating the diameter of the Sun*

A student described the exploration process as follows:

First create a camera using foil, a piece of paper, cardboard, a push pin, a meter stick, and a sunny day. First cut a square out of the middle of the cardboard and

cover that opening with the foil. Poke a hole in the middle of the foil with a push pin.

To carry out the experiment, hold the screen up so that it faces the sun. Another person will hold the foil and cardboard between the sun and the screen and direct the sun rays that are traveling through the pinhole to project the upside-down image of the sun onto the screen. A third person will hold a meter stick with one end touching the screen and one end touching the cardboard with foil. A fourth person will trace the projected upside-down image of the sun that is on the paper.

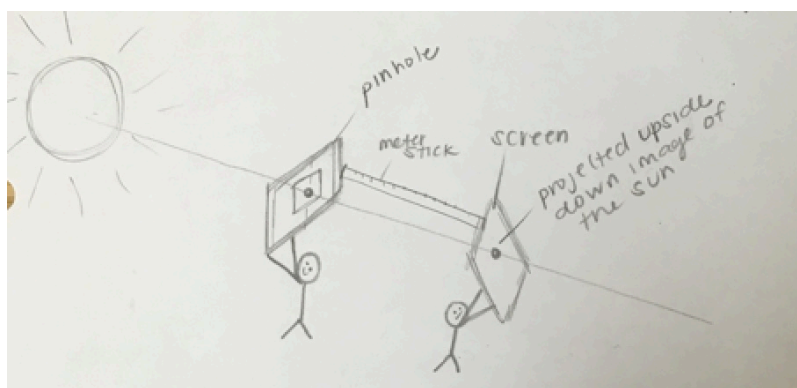


FIG. 1.22. Sketch of estimating the sun's diameter with pinhole phenomena.

Physics student, Spring 2016

The student sketched this exploration process as shown in Fig. 1.22. In this class, we referred to the diameter of the sun as its *Width*, represented by the symbol capital W , in order to avoid confusion with the symbol D representing the distance from the sun to the pinhole. We referred to the diameter of the Sun's projection as its *width*, represented by the lower-case symbol w in order to avoid confusion with the symbol d representing the distance of the screen from the pinhole. Note that in this activity the Sun's rays should be shown as bouncing off the opaque screen rather than shining through as in the previous pinhole activity shown in Fig. 1.15.

This student described her group's interpretative process as follows:

The group will measure the diameter of the traced projection onto the screen. This was estimated to be about one centimeter (1 cm). This will be variable w which represents the width of the projection.

We know that the distance from the pinhole to the projection is one-meter-long (1

m) since we used a meter stick to separate the two. We will convert the one meter (1 m) to one hundred centimeters (100cm) so that the units are overall easier to work with. We will label this distance as variable d .

We are given the distance from the sun to the earth and our camera, which is about one-hundred million miles (100,000,000 mi) and will label this as variable D .

We want to find out the width of the sun so we will label this as variable W .

Since we know that this experiment creates triangles that are similar, we can set up a proportion using our known and unknown variables to determine the width of the sun.

Physics Student, Spring 2016

This student drew a ray diagram and wrote the relevant mathematics as shown in Fig. 1.23.

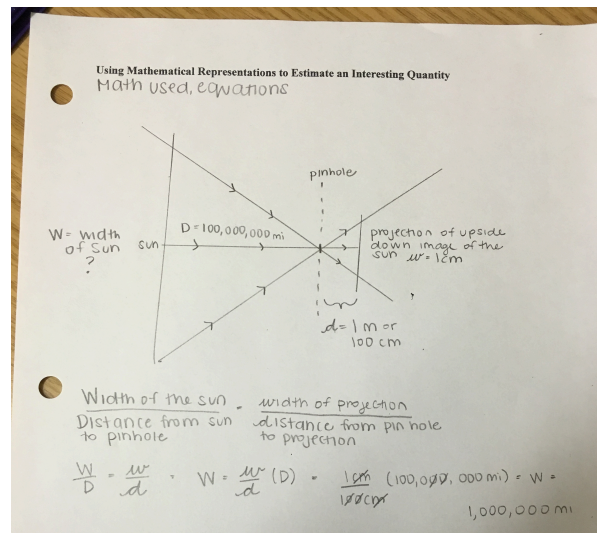


Figure 1.23. Student ray diagram and relevant mathematics for estimating the diameter of the Sun.

The student labeled the diagram well:

- the long vertical line on the left as representing the Sun, with the symbol W defined as the width of the Sun, and $?$ to indicate that this was the unknown
- the horizontal line from the “Sun” to the pinhole as the distance D , equal to about 100,000,000 miles
- the dotted line representing the foil with a pinhole in the middle as the pinhole
- the short horizontal line from the pinhole to the projection as d , equal to 1 meter or

100 centimeters

- the short vertical line on the right as representing the projection of upside down image of the Sun, with the symbol lower case w , equal to 1 centimeter.

Fig. 1.23 clearly shows the representation of pinhole phenomena by this student in estimating the diameter of the Sun. There are, however, several minor issues to consider in this otherwise excellent ray diagram. An open circle, rather than a solid line, would better represent the pinhole through which light rays are envisioned moving. A solid rather than dotted line would better represent a solid surface that blocks other rays from traveling through the pinhole. The rays from the Sun bounced off the opaque screen back to the eyes as shown in Fig. 1.19 rather than continuing through a translucent wax paper screen as in Fig. 1.14.

This student wrote out the relevant ratios in words first before representing them with symbols:

$$\begin{aligned} \frac{\text{Width of the sun}}{\text{Distance from Sun to pinhole}} &= \frac{\text{width of projection}}{\text{distance from pinhole to projection}} \\ \frac{W}{D} &= \frac{w}{d} \\ W &= \left(\frac{w}{d}\right)D \end{aligned}$$

Writing out an equation in words as well as in symbols is important to do when teaching in order to be sure that students are following the reasoning.

The justification for the equal sign was that the triangle formed by the rays from the Sun and the pinhole is similar to the triangle formed by the pinhole and rays from the Sun inside the camera, and that the ratios of corresponding lengths of similar triangles are equal.

The student solved for the unknown, $W = (w/d) (D)$,

substituted numerical values, $W = \frac{1 \text{ cm}}{100 \text{ cm}} (100,000,000 \text{ miles})$

and calculated the estimate for the sun's diameter, 1,000,000 miles.

In this case, the students did not need to convert miles to centimeters because the ratio w/d was dimensionless, $1 \text{ cm}/100 \text{ cm} = 1/100$.

The distance of the sun to the pinhole was given as 100,000,000 miles (rather than about 93,000,000 miles) so that calculating the estimate would be easy and the attention would be on the magnitude of this estimate rather than on details involving a more accurate value for the distance to the sun. What is one hundredth of a hundred million miles? One million miles! If you can envision what the equation is representing, one can calculate this estimate in one's head!

The student also reported the status of the variables, as shown in Fig. 1.24 below.

Mathematical Representation of Pinhole Phenomena			
Quantity	Symbol	Value with Units	Comment: Is this a measured number? Information provided? Part of the equipment design? A calculated value?
distance from pinhole to screen	d	1 meter = 100 cm	measured during experiment with meter stick
width (diameter) of projection	w	1 cm	measured the outline of the projection onto the screen
Distance from Sun to pinhole	D	100,000,000 mi	given
Width (diameter) of Sun	W	1,000,000 mi	found through math and proportions

FIG. 1.24 Student's report of the status of variables in estimate of the diameter of the Sun.

In designing and carrying out a mathematical estimation, it is useful to consider what each symbol represents. The value represented by the symbol d , for example, the distance of the projection from the pinhole, was *chosen* as part of the design of the exploration by the availability of a convenient tool, a meter stick of length 100 centimeters.

The diameter of the projection, w , 1 centimeter, was *measured* during the exploration. Some students' tracing of the projection had a diameter of 0.9 cm; others 1.1 cm; many 1.0 cm. Using 1.0 cm seems a reasonable value given the variation in ways small groups held the pinhole and screen a meter stick length apart.

The distance D of the Sun from the Earth was *an approximate value provided by the instructor* based on an accepted value of about 93 million miles. This approximation made calculating the estimate easy so the students' focus would be on the phenomena and not on detailed calculations.

The diameter of the Sun, W , was the *unknown*, estimated by calculating its value in the equation based upon the mathematics describing pinhole phenomena.

Is a diameter of one million miles a sensible result for an estimate of the diameter of the Sun?

What sense-making strategies can you use to consider the reasonableness of this result?

2. *Some nuances in using mathematical representations of pinhole phenomena*

Students often use an equal sign to represent “the next step is” to solve for the unknown variable. That is best represented by an empty space between two equations as the ratio w/d does not equal W .

After writing the equal ratios in words and the equivalent mathematical statement in symbols, this student appropriately solved for the unknown, W , with $W = (w/d) (D)$ before substituting numerical values. This is a process that is very important to do. Substituting numerical values before solving for the unknown often leads to calculation errors.

By solving an equation for the unknown before substituting numerical values, one can readily check whether the dimensions of both sides match:

If $(W \text{ in length units}) = (w \text{ in length units}/d \text{ in length units}) (D \text{ in length units})$, the w/d will be dimensionless and the result will be length units on the left and length units on the right.

By solving an equation for the unknown before substituting numerical values, one also can consider whether the mathematics processes indicated are conceptually appropriate:

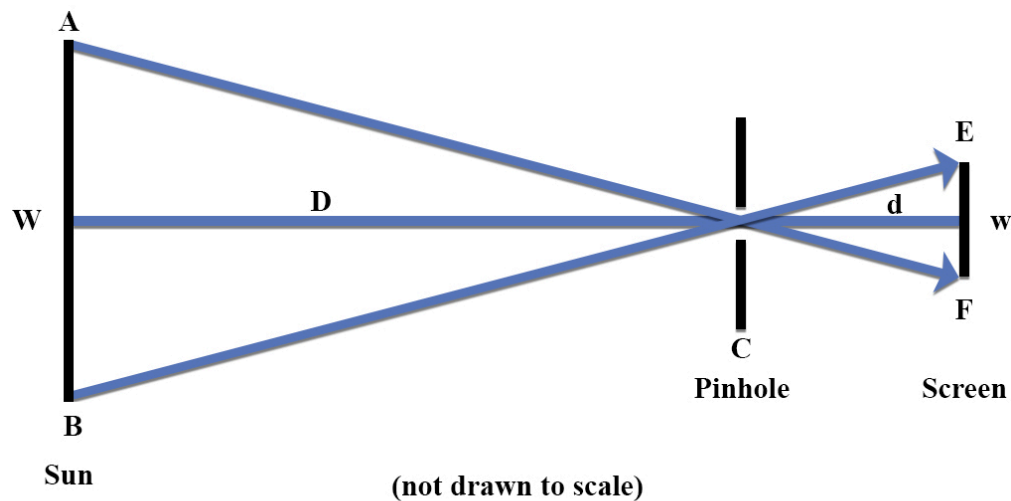
$W = (w/d) D$ indicates that first one is finding a ratio of two quantities (w/d) and using that to find out how much of the distance D in the other triangle, the other width W would be. If the little width w is half the length of the little distance d for one triangle, for example, then the big width W would be one half of the big distance D . Such visualizing of what the equation is doing is a useful way to check whether the equation one has derived makes sense.

This form of the equation, $W = (w/d) (D)$, also makes clear that the ratio w/d is dimensionless ($w/d = 1 \text{ centimeter}/100 \text{ centimeters} = 1/100$) so there is no need to convert between centimeters and miles when substituting values. This is an example of a detail that can consume a lot of time, if one just does what one thinks one is supposed to do, converting centimeters to miles, without monitoring whether it makes sense to do so. It was necessary to convert 1 meter to 100 centimeters to use centimeters for both

quantities in the ratio of w/d in order to obtain the dimensionless ratio of $1/100$ but it was not necessary to convert centimeters to miles.

We used the approximate value of 100,000,000 miles for the distance between the Earth and the Sun (the distance D of the source from the pinhole). This made the calculation easy, as $1/100$ of 100 million miles is 1 million miles. Although the Sun looks small in the sky, its diameter is very large!

A “not to scale” note would be helpful, given that rays of light from the Sun are coming from so far away that they are nearly parallel as they reach the earth rather than coming in at such a large angle. Such a “not drawn to scale” also would be appropriate given that the 1 cm projection is shown as larger than the 100 centimeter distance to the pinhole. A more realistic ray diagram for this situation would be similar to Fig. 1.25:



D = Distance of sun from pinhole d = Distance of projection from pinhole

FIG. 1.25 Ray diagram representing pinhole phenomena with an object very far away

You can use your knowledge of pinhole phenomena to generate and solve pinhole math problems. To make up such a problem, decide on a scenario and specify three of the four variables involved: height of the object, H ; its distance to the pinhole, D ; height of its projection on the screen, h ; and its distance to the pinhole, d .

In this course, the goal in solving a pinhole math problem is not the “answer.” The goal is to build your ability to help someone else understand what to do and why. Start by helping the learner to understand what is happening by describing the scenario verbally with words and visually with a sketch. Next review the physics involved by

stating what the relevant central ideas are. Also draw a careful ray diagram and use it and the central ideas to explain why the projection is upside down. Then describe the ray diagram mathematically, being clear about what each symbol represents, why the two triangles are similar, and how the lengths of similar triangles are related. Finally write the equation in both words and symbols, solve for the unknown in symbols, substitute values and calculate the answer. Be sure to also discuss why that answer seems reasonable. In facilitating a conversation with someone about pinhole phenomena, ask questions rather than tell answers throughout this process. In responding to homework problems in this course, however, follow the format provided here:

Solving a Pinhole Math Problem

- a. **Describe** the scenario in words
- b. **Make a sketch**
- c. **Review what you know** about this phenomenon (the relevant central ideas)
- d. **Draw a careful ray diagram.** Include:
 - light source
 - object and projection
 - eye
 - 3 example rays from light source (from top, middle, bottom of object) that pass through the pinhole to form the projection
 - light rays traveling to eye
 - gap representing pinhole (or small circle)
 - arrows to show direction light rays are traveling
- e. **Tell the ‘story’ of the ray diagram** with the relevant central ideas to **explain** why the projection is upside down. Describe what happens to each example ray separately.
- f. **Label angles and sides of triangles.** Define symbols with words and record any numbers you know or can estimate.
- g. **Justify that the triangles are similar.**
- h. **Write an equation** that relates the sides of the triangles using words to describe the quantities.
- i. **Rewrite your equation using your symbols.**
- j. **Algebraically isolate the unknown quantity.**
- k. **Plug-in numbers and calculate answer.**
- l. **Check answer.** Does the number you get from the calculation seem reasonable?

Explain.

3. *Using pinhole phenomena to estimate the Sun's diameter with friends and/or family members*

Question 1.17 What happens when estimating the diameter of the Sun with friends or family members?

A student reflected upon engaging her sister and mother in estimating the diameter of the Sun as follows:

To explore pinhole phenomena, I involved my younger sister to perform the same experiment we did in class. I first explained what we did and she stated that she has never heard of it before. Because she said she has never heard of pinhole phenomena, I was excited to show her. I used the same materials and made my own pinhole on a sheet of tinfoil. My mom also helped by holding the meter stick. My mom also said she has never heard of it. My mom is a third-grade teacher so she was excited to try it out.

During the experiment, I asked them how big they thought the projection on the white sheet was. Like I did, they thought it looked like about one centimeter wide in diameter. My sister asked, "How can someone use such a small reflection of the sun to help find the diameter of the actual sun since the sun is so large and this is so small?" Because my mom and sister don't have as much of a math background as I do, I was very careful about what questions I asked.

I started out by giving them a hint and asked, "Have you heard of similar triangles and corresponding angles?" I could tell my mom was a little lost because math isn't her strong suit. Much like my sister, she is a lot better at literature. My sister said yes! I told them the distance from the sun is 100,000,000 miles, so by using the three dimensions, I asked, "How can we use equal fractions to find the missing dimension (diameter of sun)?"

My sister yelled, "Ratios!" My mom seemed a little lost so I then drew similar triangles a lot like we did in class and labeled each dimension we knew. I explained how the sides and angles were proportionate, which is how we could create equal ratios. I asked them if our units needed to be the same and my sister said yes. As a group, we found the same results as we did in class.

My sister took calculus in high school and my mom doesn't know much about math. Having those two different types of "students" that were at either end of the spectrum, gave me the opportunity to practice for when I am trying to explain to two students where one is at a higher level than the other. I wanted my sister to explain the concept to my mom because I know that some students learn better when they explain things to other students.

Through this experience, I learned that when you have two students where one understands the concept faster than the other, it's important to let both students talk but also let the one student who understands, explain what they know to the other student. This helps them learn from each other.

I also learned to make sure I give time for the student that doesn't understand to ask any questions they have and not let the other student do all the work. I know it's important to make sure you leave a lesson knowing that both students understand the process.

The last thing I learned was how to ask questions to give a hint when the student is completely lost without giving them the answer. I understand how frustrating it is when the teacher asked questions you have no idea what the answer is. It's sometimes better to give them a nudge when they are completely lost in the beginning and let them do the rest of the work once they understand how they are going to find the outcome.

Physics student, Fall 2016

4. Some thoughts about the nature of science in this context:

This completes an example of the process outlined for each unit:

Students first identify resources by reflecting upon what they already know about a topic, such as the nature of light.

Next, they develop some central ideas based on evidence, such as light leaves a point on most sources in many directions and light can be envisioned as rays traveling in straight lines.

They can use those ideas to explain an interesting phenomenon, such as why the projection of a light bulb seen through a pinhole camera is upside down.

Then they develop mathematical representations of the phenomenon, such as an equation stating that the ratio of the height of the light source to the distance of the

source from the pinhole is equal to the ratio of the height of the projection to the distance of the projection from the pinhole.

Finally, they use those mathematical representations to estimate an interesting quantity, such as the diameter of the Sun. It is not possible for anyone to travel to the Sun and use a measuring device to directly measure its diameter. An estimate is feasible, however, using simple equipment, some additional information from a reliable source, and the scientific process developed here.

This process illustrates the strength of scientific ways of knowing. One may be able to estimate a quantity of interest through a similar process of identifying resources, developing central ideas based on evidence, using these ideas to explain a puzzling phenomenon, figuring out ways to represent the phenomenon mathematically, and using those mathematical tools to estimate the quantity one wants to know.

Several students recognized that this estimation process might be useful in other contexts such as answering the question: ‘how big is the Moon’? Note that making the decision to go there, or not, or to fund creating a settlement on the Moon, or not, is a question of a different kind, one that involves cultural issues and societal values as well as the technical and scientific knowledge needed.

Many students also experienced some of the human aspects of science in their surprise that the projection was upside down, frustration in making sense of the mathematics, and perhaps pleasure in persisting through to understanding what seemed for many to be an initially confusing experience.

The process involved in estimating the diameter of the Sun assumes that the ratio of small distances here on Earth can be compared to the ratio of very large distances in the solar system. This is an example of an aspect of the nature of science articulated in the *US Next Generation Science Standards* that *scientific knowledge assumes an order and consistency in natural systems*. Third to fifth grade students, for example, should understand that *basic laws of nature are the same everywhere in the universe* (NGSS, Lead States, 2013, Appendix H) <https://www.nextgenscience.org/resources/ngss-appendices>).

VII. Developing Additional Central Ideas Based on Evidence

When you look in a mirror, swim in a pool, or enjoy playing with a prism, you are experiencing a variety of light phenomena. In this section, you will develop additional central ideas about reflection, refraction, and dispersion. Then you will use these central ideas to explain an intriguing phenomenon: seeing rainbows in the sky.

A. Exploring reflection phenomena

The word *reflection* has many meanings in many different contexts. In physics, the word *reflection* has a specific meaning. *Reflection* refers to one aspect of what happens when light shines on smooth or rough surfaces.

Question 1.18 What happens when light shines upon a smooth surface?

Equipment: To explore reflection phenomena, obtain a flashlight, flat mirror, table, two straight sticks such as meter sticks, yard sticks, rulers or pencils, a dark room and at least three people with whom to collaborate. Clean the surface of the mirror so that the light reflects from the mirror rather than from dust on the surface.[In a remote learning situation, a student working alone with a clear mirror and two rulers, can place the mirror on a table next to a wall, lean one end of a ruler against the wall, place the other end of that ruler on or next to the mirror, place one end of the other ruler next to the first ruler on or next to the mirror, and hold the flashlight so that it is shining on the mirror at the same angle as the ruler. Move that ruler (and flashlight) at different angles to see how the reflection of the light moves up and down on the wall. Note the angle of the ruler when the reflection on the wall is on the top of the first ruler.]

- Record the **Topic** of this exploration on your physics notebook page.
- What happens when a flashlight shines on a flat mirror in a dark room?

Note your initial ideas in the **Before** section of the physics notebook page documenting your exploration of reflection phenomena.

- What can you find out about reflection phenomena by playing with the flashlight and mirror? Keep track of your explorations in the **During** section of your notebook page.
- Next explore reflection in a particular way: place the flat mirror on the table. Ask group members to each stand on one side of the table as shown in Fig. 1.26

Person 1 is the person holding the flashlight.

Person 2 is the person standing at the other end of the table.

Person 3 is standing at one side of the table. (If there is a fourth person, Person 4 is standing at the other side of the table.)

- Persons 1 and 2 start the activity while Persons 3 and 4 report what happens. After a while switch positions so that everyone experiences what happens in each location.
- How can Person 1 aim the flashlight at the mirror so that light shines on Person 2's face at the other end of the table?

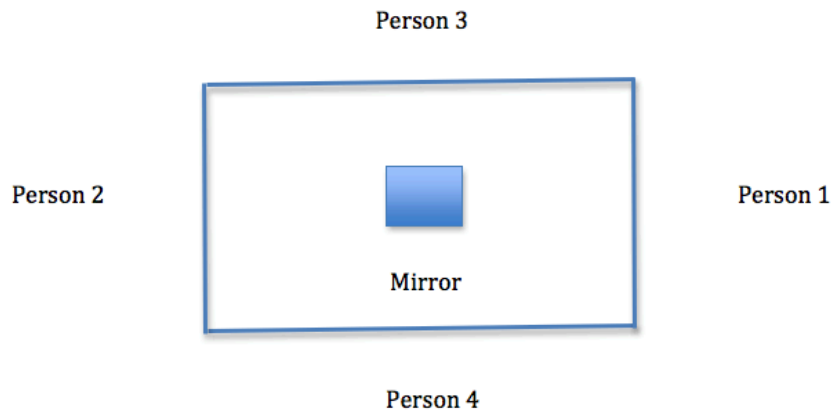


FIG. 1.26 Who can see the light from the flashlight in the mirror?

- When Person 2's face is lit, who can see light from the flashlight shining on the mirror?

Person 1 with the flashlight?

Person 2 at the other end of the table?

Person 3 at one side of the table

Person 4 at the other side of the table?

- If Person 2 goes up on tiptoes or crouches down low, what does Person 1 need to do to keep the light reflecting off the mirror onto Person 2's face?
- Can you see light rays traveling from the flashlight to the mirror?
Can you see light rays traveling from the mirror to Person 2's face?
- To envision light rays traveling from the flashlight to the mirror and from the mirror to Person 2's face, use a physical model for light rays:
 - Hold one meter stick so that it models light rays traveling from the flashlight down to the mirror.
 - Hold the other meter stick so that it models light rays traveling from the mirror up to Person 2's face.
 - Make sure the two meter sticks are meeting on the mirror at the spot where Person 2 sees the beam of light reflected in the mirror.
- The meter sticks make angles with the flat mirror. How do these angles compare?
- Make a sketch showing one meter stick modeling light rays traveling from flashlight to mirror and the other meter stick modeling light rays traveling from the mirror to Person 2's face.
- Focus first on the meter stick modeling light rays traveling from the flashlight to the mirror. The meter stick and the mirror on the table are modeling the angle that one can envision *incident light rays* making with the mirror. This angle is called the *angle of incidence*.

About how big is this angle of incidence? (about 15°? 30°? 45°? 60°? 75°?)

- Now look at the meter stick modeling light rays traveling from the mirror to Person 2's face. This meter stick and mirror on the table are modeling the angle that one can envision *reflected light rays* making with the mirror. This angle is called the *angle of reflection*.

About how big is this angle of reflection? (about 15°? 30°? 45°? 60°? 75°?)

- Draw a ray diagram that represents what you infer is happening when light from the flashlight reflects off the mirror into Person 2's face.
- Label the angle of incidence and the angle of reflection. How are these angles related?

Question 1.19 What happens when light shines upon a rough surface?

- Now shine the flashlight on the table rather than on the mirror.

Who can see the light from the flashlight shining on the table?

Person 1 with the flashlight?

Person 2 at the other end of the table?

Person 3 at one side of the table

Person 4 at the other side of the table?

- How does what you see here or do not see here compare with what you saw or did not see when the flash light was aimed at the mirror rather than the table?
- Although the table may seem smooth, it is a rough surface compared to the mirror. What are the similarities and differences in what you observe when a flashlight is aimed at the mirror or at the table?
- Under the ***During*** section of your physics notebook page, record your observations and interpret these results.
- Use sketches of light rays reflecting from smooth and rough surfaces to explain these differences.
- Discuss your findings and formulate central ideas about how light is reflected from smooth and rough surfaces. In the ***After*** section of the physics notebook page, report these ideas, the evidence on which they are based, and the rationale that connects the evidence to these claims and notes their importance.

To complete this exploration of reflection phenomena:

- Using the everyday meaning of the word, reflect upon your experiences in exploring the nature of reflection phenomena.
- What are you still wondering?
- Add the central ideas about reflection from smooth and rough surfaces to Table I.1:

TABLE I.1 (continued) Explorations of light phenomena: Reflection

TABLE I.1 (continued) Explorations of light phenomena: Reflection			
Sketch of set up Ray diagram	Evidence	Central Idea	Vocabulary
		<p>Light rays reflect from smooth surfaces in a regular way, where the angle of reflection equals the angle of incidence.</p> <p>Light rays bounce from rough surfaces in many different directions.:</p>	

Complete entries on your physics notebook page and Table I.1. Then write a summary of what you have learned about reflection phenomena before reading an example of student work in response to questions 1.18 and 1.19.

1. Example of student work about reflection phenomena

As shown in Fig. 1.27, a student added to Table 1.1 about reflection phenomena ray diagrams for reflection from smooth and rough surfaces. For evidence to support the two new central ideas stated, the student wrote: “Using the meter sticks to show where the light rays are going we can see that the light hits the mirror and then reflects off of it at the same angle it hits at.”

Sketch of set up Ray diagram	Evidence	Powerful Idea	Vocabulary
	<p>Using the ruler sticks to show where the light rays are going we can see that the light hits the mirror and then reflects off of it at the same angle it hit at.</p>	<p>Light rays reflect from smooth surfaces in a regular way, where the angle of reflection equals the angle of incidence.</p> <p>Light rays bounce from rough surfaces in many different directions.</p>	<p>Reflection</p> <p>Angle of Incidence</p> <p>Angle of Reflection</p>

FIG. 1.27 Student's addition about reflection in Table 1.1

Light rays reflect from smooth surfaces in a regular way, where the angle of reflection equals the angle of incidence. In this experiment we used two surfaces; the smooth surface of a mirror; and the rough wood surface of the table. We found that when the light rays traveling in straight lines from the flashlight to the mirror hit the mirror's surface, the light is reflected. We can see this when looking at the face of the person on the opposite side of the table; light is hitting her face. The angles of the line of incidence and the line of reflection appear to be the same. For any one particular light ray the angle of incidence equals the angle of reflection.

Light rays bounce from rough surfaces in many directions. As demonstrated in the ray diagram, any one single ray of light that hits the rough surface will still have an equal angle of incidence and angle of reflection but individual rays of light hit the surface at a different angle.

Physics student, Fall 2015

When shining on a rough surface, the light from a flashlight can be envisioned as rays traveling in the same direction but that direction will form different angles of incidence with different parts of a bumpy surface, so that the rays bounce off at many different angles of reflection.

2. Some nuances in explaining reflection phenomena

Defining the angles of incidence and reflection with respect to the mirror is a choice made here because the angle between the meter stick representing the light rays and the mirror is visually easy for children to perceive and compare.

Physics texts typically define a different set of angles, however, for the angle of incidence and angle of reflection. Draw a line perpendicular to the mirror at the point that the light beam shines on the mirror. This perpendicular line is called the *normal* line. As shown in Fig. 1.28, physicists define the angle of incidence as the angle between the incident beam and the normal to the surface. Physicists define the angle of reflection as the angle between the reflected beam and the normal to the surface.

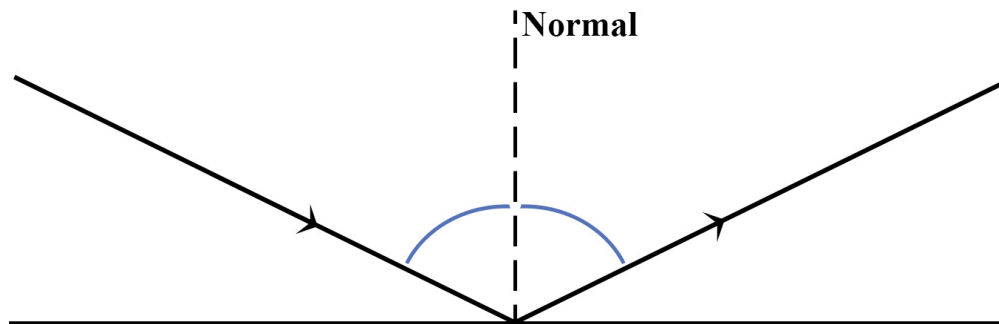


FIG. 1.28 Angles defined with respect to a normal line rather than to the mirror.

If the surface of the flat mirror were clean, only Person 2 should have been able to see the light from the flashlight in the mirror. Persons 1, 3, and 4 would not have been able to see light in the mirror if all of the light rays incident on the mirror were being reflected at the same angle toward Person 2's face.

If the surface of the mirror were dusty, however, some of the light rays may have bounced off the dust in directions toward the other members of the group and they may have reported seeing the light in the mirror. If the classroom still uses chalk on a blackboard, one can clap an eraser near the mirror and “see” a beam of light from the flashlight to the mirror and from the mirror to Person 2's face as some light bounces off the chalk dust in many directions and makes such beams visible.

When the flashlight was aimed at the table surface, all members of the group should have been able to see light on the table as some light rays were being reflected in many directions by the rough surface of the table as shown in Fig. 1.29.

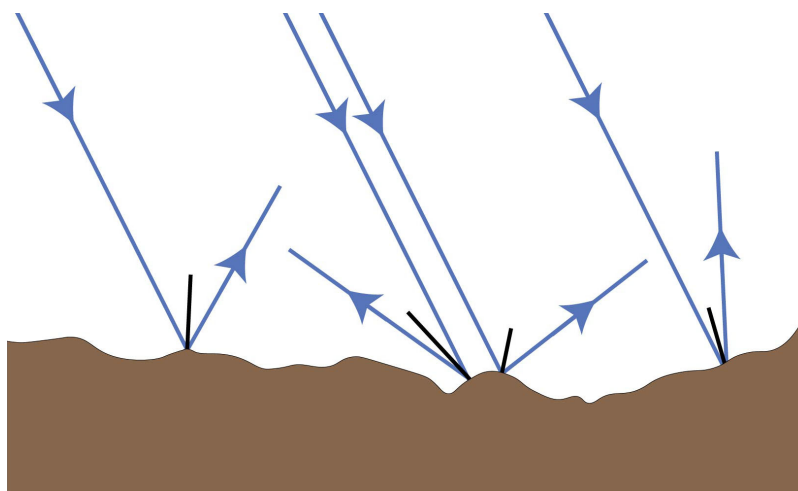


FIG. 1.29 Light rays reflecting in different directions from bumps in a rough surface.

There are several ways to think about what happens when light shines on rough surfaces. One way is to notice what happens when a group of people look at someone speaking in a room. Who can see that person's nose? Can someone by the window see the speaker's nose? What about someone over in a corner? Someone on the other side of the room? One can infer that light is bouncing off that person's nose in many different directions if all of these people can see that person's nose,

This visual representation of a model of light in which light rays bounce off a surface with the angle of incidence equal to the angle of reflection suggests a visual experience you may have had:

- What happens when a basketball player dribbles a ball down a court? How does the ball bounce off the floor? Or when a tennis player practices by hitting a ball hard against a wall with a forehand shot and the ball rebounds to be hit again backhand?
- Try this by rolling a basketball at an angle toward a wall. How does the basketball rebound from the wall?

Make a sketch of how balls rebound when bouncing at an angle off a floor or wall.

- If such a sketch is analogous to a diagram showing how light rays reflect from a mirror, what does this suggest about the nature of light?

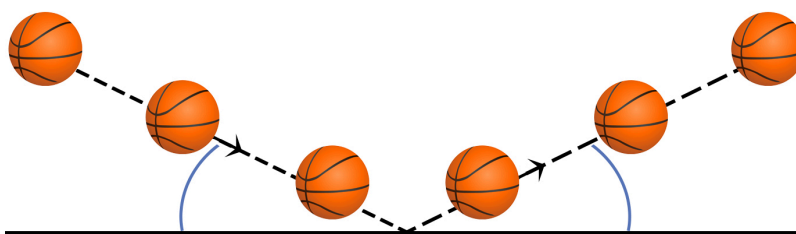


FIG. 1.30 A ball rebounding from a surface.

As shown in Fig. 1.30, a ball thrown against a wall rebounds in a way that suggests **another physical model for light**: envision light rays as streams of particles traveling in straight lines and reflecting from a surface with angle of incidence equal to angle of reflection. When this physical model for light is used, the particles are called *photons*. Such streams of photons can be envisioned as rays of light reflecting from a surface with the angle of reflection equal to the angle of incidence. Thus our initial physical model for light rays traveling in straight lines, a meter or yard stick, can now be elaborated as a stream of photons traveling in straight lines from a source.

Question 1.20 How well do different materials reflect light?

Sunlight shines on many different materials such as water, sand, soil, asphalt pavement, grass, trees, people... How each material responds depends upon the material's *properties*. Such properties include how well the material reflects light, its *reflectivity*.

Equipment: Use a digital light probe connected to a computer or calculator to show a graph of light intensity versus time as you move the probe over various materials. Or use a free app on a cell phone, search for LUX light meter.

Try whatever is nearby as well as white paper and various colors including black, wax paper, aluminum foil, paper towel, cardboard, and dark cloth.

- What can you find out about materials' reflectivity with the light probe or light meter? In the **Before** section of a physics notebook page, record your questions and your predictions.
- Begin by playing with the light probe or light meter to see what you can find out

with this device.

If using a light probe: you may need to reduce the highest value on the vertical axis of the graph on the computer screen if you do not have very bright lights in the room. Click on the top number on the vertical axis and reduce it until the signals from the probe are high enough on the graph to be seen easily.

How can you use it and the various materials to make a graph that looks like the letter W? a sharp W? a rounded one? a letter M? sharp or rounded M's? What other letters can you make on the graph on the screen?

Trace one of the graphs with your finger and tell the story of making a line that rises steeply or not so steeply. How do you make a line that descends steeply? Not so steeply?

- Compare how light reflects from a variety of materials, such as paper of different colors, aluminum foil, wax paper, cardboard, dark and light cloth...
- How does the distance from the material affect the intensity reading?
- If using a light probe: one way to keep it the same distance away from each material is to use a rubber band to hold a ruler next to the light probe. Attach the ruler so that it is extending a certain distance (say 5 cm) beyond the end of the light probe as shown in Fig. 1.31.

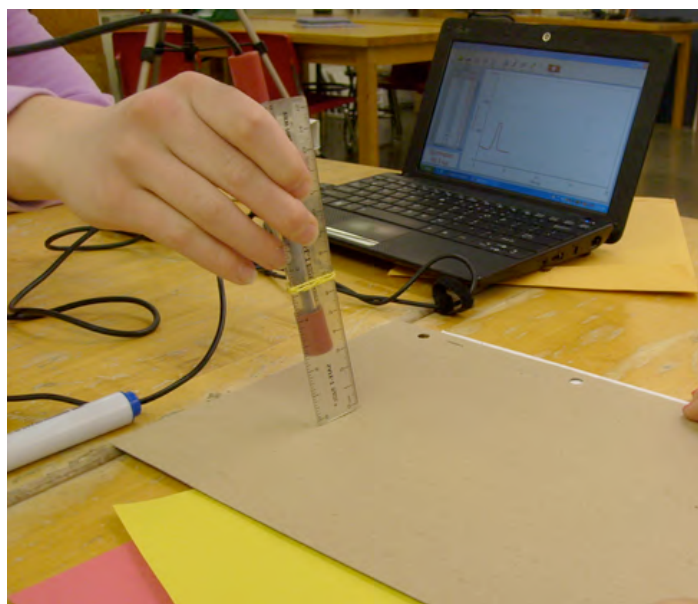


FIG. 1.31 Using a light probe connected to a computer to compare the reflectivity of various materials.

- In the **During** section of your physics notebook page, record your findings. If using a light probe: draw a picture of the graph and indicate the parts of the graph that represent the different materials that you tested. Note any vocabulary that is new to you.
- Discuss your findings and formulate relevant central ideas. In the **After** section of the physics notebook page, report these ideas and the evidence on which they are based.
- Write a rationale that explains how the evidence supports the ideas and why these are important.
- Also reflect upon this exploration such as what connections can you make to other experiences? How might you use what you learned in your own classroom?
- What have you learned and what are you still wondering?
- Complete documenting your exploration and writing a summary before looking at an example of student work and nuances about exploring the property of reflectivity.

3. *Example of student work about the property of reflectivity*

Materials differ in the property of reflectivity, how well they reflect light. Corollary: Materials differ in how well they absorb light. *To measure the light reflectivity of different materials we took a light sensor to the surface of different materials and recorded them on a computer. We used the program LoggerLite which records and graphs the amount of light being reflected off any particular surface. We used different materials like foil, dark cloth, white paper, and even our clothing and nothing at all; we tried just pointing the sensor directly at the light.*

In Figure (1.32) below you can see the different levels of reflectivity of light. The furthest point to the left that I circled represents when we were covering the end of the sensor with our hand; it was reflecting no light at all.

The next point is circled in close to the top of the graph; this spot in the graph represents us pointing the sensor straight at the lights above us. There was quite a bit of reaction because we were pointing the sensor at the direct light.

The next circle that I circled to focus on is close to the bottom of the graph. This spot isn't as low as when we were covering the end of the sensor, but we were shining the sensor at our hand, which wasn't very reflective.

The next circled spot in the graph represents when we shined the sensor at a piece of tin foil. The graph went very high telling us that this material was very reflective.

The last circled section of the graph represents when we shined the sensor on a piece of navy blue fabric. We held it here for a few seconds and the line leveled out, telling us that the material wasn't reflective and that this was a consistent reading.

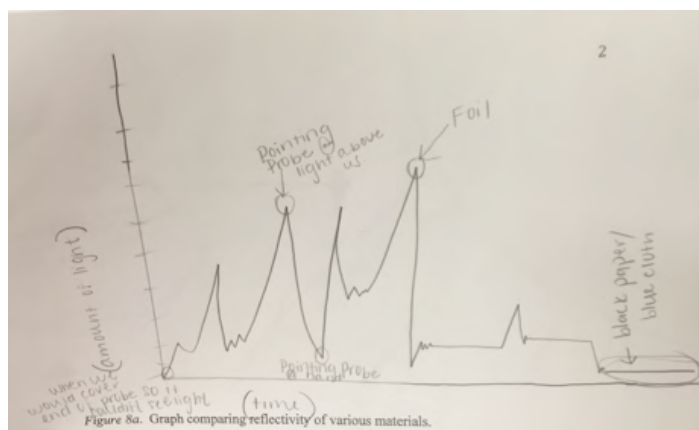


FIG. 1.32 Student graph from exploring reflectivity of various materials.

This student labeled both axes: “amount of light” on the vertical axis and “time” on the horizontal axis. The student used small circles to direct attention to different parts of the sketched graph, based on the graph produced by the computer as the students moved the light probe from one material to another. The student described the circle at the origin as “when we would cover end of probe so it couldn’t see light.” The first labeled peak was “pointing probe at light above us;” the second labeled peak as “foil;” and the low horizontal line as “black paper/blue cloth.”

4. Some nuances about exploring the property of reflectivity

Taking a photograph of the graph on the computer screen is helpful for retaining accurate information. Sketching the graph is helpful for noting important aspects, as this student did, such as the meaning of peaks and flat lines.

The peaks in Fig. 1.32 represent pointing the probe briefly at a reflective material; the flat lines represent pointing the probe at a material for more than a moment. A more careful graph would have perpendicular axes, with the same labels as those used by the computer program and with the values represented by the vertical marks noted. One is puzzled by how the intensity of light reflected by the foil could have been more than

the intensity of the incoming light, perhaps identification of these peaks was switched later during preparation of this report. Or perhaps this group was near a window and additional light from the Sun was shining on the foil. However, this graph clearly conveys the information that the intensity of reflected light varies with materials.

- How can you use your knowledge of reflectivity to design a solar oven? Which two of the above materials would you choose to use?

See http://static.lawrencehallofscience.org/diy_sun_science/downloads/diy_ss_cook_solar_oven.pdf and <https://climatekids.nasa.gov/smores/> for examples.

This is a context within which students can participate in *engineering design* as advocated in the Next Generation Science Standards (NGSS Lead States, 2013). See Appendix I at <https://www.nextgenscience.org/resources/ngss-appendices>. Engineering design involves defining a problem, developing solutions, and optimizing the solution (see pages 3-6).

B. Exploring refraction phenomena

When light shines on a surface, light *reflects* from the surface as just discussed but sometimes some of the light does not. Some of the light *refracts* instead as it keeps on traveling through the surface into a new *medium*. The word medium refers to the substance through which the light is traveling, such as air or water.

Question 1.21 What happens when light travels from one medium into another medium, such as from air into water or from water into air?

Equipment: To explore refraction phenomena, use a pencil or stirring rod, a tray, two paper cups, a pen, and access to some water.

- What happens if you drop something into a pail of water? How does it look different if at all? What if you are swimming under water and look up through the water at something overhead?

Note some of your experiences with looking at objects in water in the **Before** section of the physics notebook page documenting your exploration of refraction phenomena.

- Make a large dot in a paper cup about a third of the way down from the top. Place the cup with the dot on a tray. Fill the other cup with water and put it on the tray.
- Bend down until you can just see the dot in the cup. Then bend a little more so that

you can no longer see the dot as shown in Figure 1.33. What do you think will happen when water is poured into the cup and covers the dot?

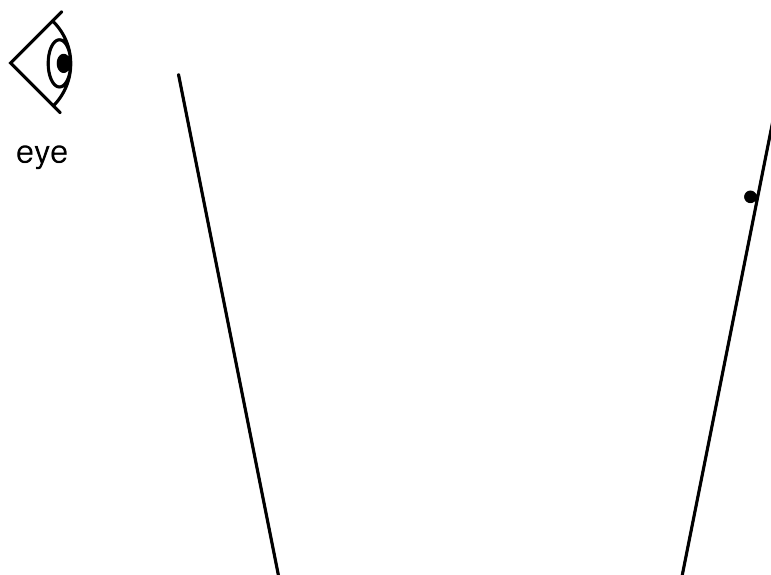


FIG. 1.33 An observer's eye is just below the point where the person can see the dot in the cup.

Light rays bouncing off the dot in the direction toward the eye are blocked from reaching the eye by the side of the cup near its rim.

- Have someone pour water into the cup until the water flows above the dot. What happens?
Record what you observe in the **During** section of your physics notebook page.
- Why do you see what you see? Discuss with your group members some ideas about why someone would see the dot appear to move in the way it does when water is poured into the cup and covers the dot.
- Sometimes when one is puzzled by a phenomenon, it helps to think about a similar phenomenon. Put a pencil or stirrer in a glass or tub of water as in Fig. 1.34. What do you see?

Does the pencil appear higher or lower in the water? Does the dot appear to be higher or lower in the water in the cup?

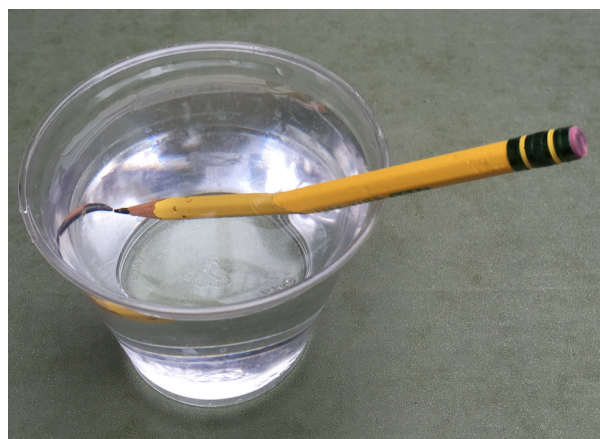


FIG. 1.34 Pencil appearing bent in a glass of water.

- How could you draw rays representing light traveling from the dot to the eye so that the eye would perceive the light coming from the “apparent dot” rather than from the real dot?
- Discuss various possibilities with your group members. How can you use the technique of drawing rays to represent a straight line path of light in this scenario?
- As you work together generating ways to think about explaining why the dot appears to be moving up as you pour water into the cup, step back occasionally and ask yourselves,
 - What are we doing?
 - Why are we doing this?
 - How is this helping us?
- One way to think about something puzzling is to make a sketch of what is happening. On a large white board, draw an outline of the cup without water, dot, and observer’s eye just below where the observer could see the dot as in Fig. 1.33

Also draw an outline of the cup with water, dot, and observer’s eye in the same position. Use a *dashed line* to outline where the dot *appears to be* in the cup with water.

- How is light apparently traveling from the apparent dot to the observer’s eye?
 - Place a physical model of light rays, a ruler, on your drawing in a way that represents the path of light rays appearing to travel from the apparent dot

toward the eye.

- Which part of that path would represent light rays apparently traveling from the apparent dot toward the observer's eye?
 - Draw a dashed line to represent the apparent path of light rays from the apparent dot.
 - Which part of that path would represent light rays actually traveling to the observer's eye?
 - Draw a solid line to represent the actual path of light rays to the eye from the end of the apparent path from the apparent dot.
 - Interpret the point where the dashed line and solid line meet. What is happening at that point? How do the actual light rays get to that point?
 - Draw a solid line from the actual dot to the point where the dashed line representing the apparent path of light from the apparent dot meets the solid line representing the actual path of light to the eye.
 - Where is this point?
- Describe in words this envisioned path of light rays when they travel from the actual dot to the surface of the water and then to the observer's eye.
 - Note any vocabulary that is new to you.
 - Discuss your findings and formulate a relevant central idea. In the **After** section of the physics notebook page, report this idea and the evidence on which it is based.
 - Write a rationale that explains how the evidence supports this central idea and why this is important
 - Also reflect upon this exploration such as what helped you learn during this exploration and how might you use this in your own classroom?
 - What are you still wondering?
 - Write a summary of what you have learned about refraction phenomena that states a central idea about refraction and explains why the dot seems to move up in the cup when covered with water.
 - Add this idea about refraction to Table I.1:

TABLE I.1 (continued) Explorations of Light Phenomena: Refraction

TABLE I.1 (continued) Explorations of light phenomena: Refraction			
Sketch of set up Ray diagram	Evidence	Central Idea	Vocabulary
		Light rays refract (bend, change direction) at the surface when light moves from one medium into another such as from air to water or from water to air	Refraction

- Complete your entries on your physics notebook page and the continuation of Table I.1. Write a summary of what you have learned about refraction phenomena before reading an example of student work. Then:
- Share what you have learned about refraction by engaging a friend or family member in the dot-in-the-cup activity. Describe what your learner asked, said, did and found in exploring this example of refraction phenomena. Also reflect upon what you learned from this experience in learning and teaching science.
- In addition, read a student’s description of engaging a friend in exploring refraction phenomena as well as some thoughts about the nature of science exemplified by these explorations.

1. Example of student work about exploring refraction phenomena

A student added to Table 1.1 about explorations of refraction phenomena as shown in Fig. 1.35. The sketch is labeled “can’t see dot” before the water covers it and “can see dot” after the water is up to the top of the cup. The ray diagram shows the “original” dot with a solid line showing the path the light traveled and the “perceived dot” with a dotted line to show the light’s apparent path.

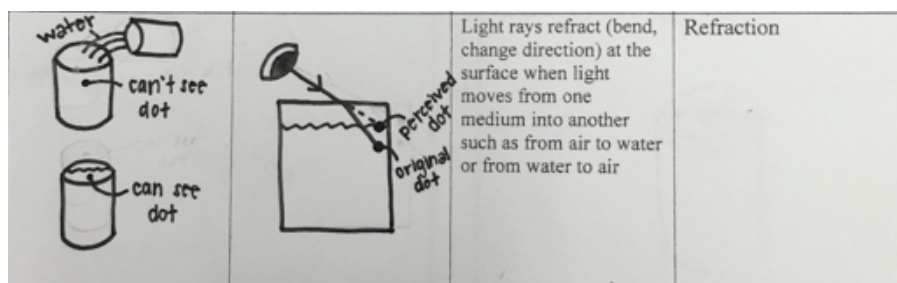


FIG. 1.35. Student's addition to Table 1.1. about refraction phenomena.

Note that in Fig. 1.35, the bend is shown occurring at the top of the cup rather than at the top of the water line. Figure 1.36 presents a diagram of a dot in a cup without and with water where the bend is drawn at the water line.

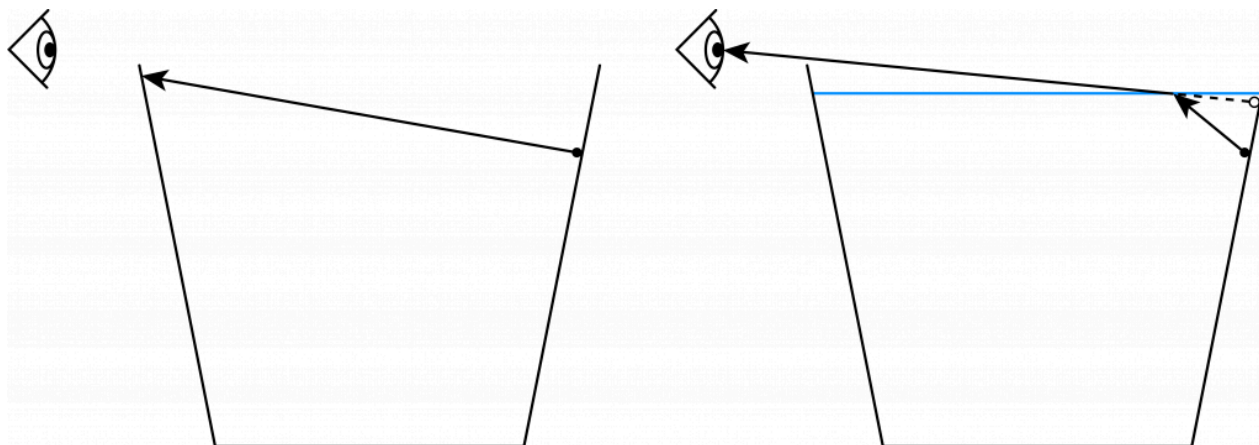


FIG. 1.36. Diagram of the dot in a cup without and with water as seen by an observer.

A student stated a central idea about refraction phenomena and used it to explain this phenomenon as follows:

Light rays refract (bend, change direction) at the surface when light moves from one medium into another such as from air to water or from water to air. In this experiment students had two cups. One cup was filled with water, the second cup was empty and had a dot drawn on one inside wall. Students were asked to crouch in front of the cup to where they could just see the dot; then they were told to go just lower than that. A second student poured the water from cup 1 into cup 2 and asked the students what they saw. As the student watched the cup being filled, water rose to the top. When the water rose

above the dot on the wall of the cup, the crouching student could see the dot once more, even though the student had not moved.

The dot gave the appearance of moving up the wall of the cup. Light was traveling to the dot the entire time it was in the cup. That is why the students could see the dot when they were standing up and looking in the cup and when they first crouched down...However when the student crouched down farther the light leaving the dot was not able to reach the student's eyes. As water rose over the dot, the light that was leaving the dot was refracted, meaning that the light ray traveled in a straight line under the water, but when it reached the surface of the water and moved into another medium (from water to air) the light rays bent traveling in many directions, one of which happened to be to the student's eyes.

Physics student, Fall 2015

What the student seemed to mean in the last sentence is that light rays traveling from the dot to the surface of the water bend as they move from the water into the air; each ray bends in a particular direction, depending upon the direction and angle at which the ray had been traveling in the water when it moved through the surface of the water to the air; some of these rays happen to bend in just the right way that they travel from the dot to the surface of the water and then, as they enter the air, bend in the direction toward the student's eyes.

2. Nuances about exploring refraction phenomena

A good place to start in thinking about why the dot seems to be moving up when covered with water in the cup is with the conceptual model of light developed so far: the central ideas that light can be envisioned as rays moving in straight lines and that for someone to see something, light has to get to the person's eyes. Also relevant is the central idea that light rays bounce off a rough surface in many directions.

It is important to remember that thinking about light rays is simply a way to envision how light could be moving through the water and the air to get to the observer's eye when the dot appears to be moving up as water is covering the dot in the cup.

Because the dot can be seen from many different positions when looking down at the dot in the water, the dot can be considered a rough surface. Note that the source of the light somewhere up in the air above the cup of water has not been included here. Thinking