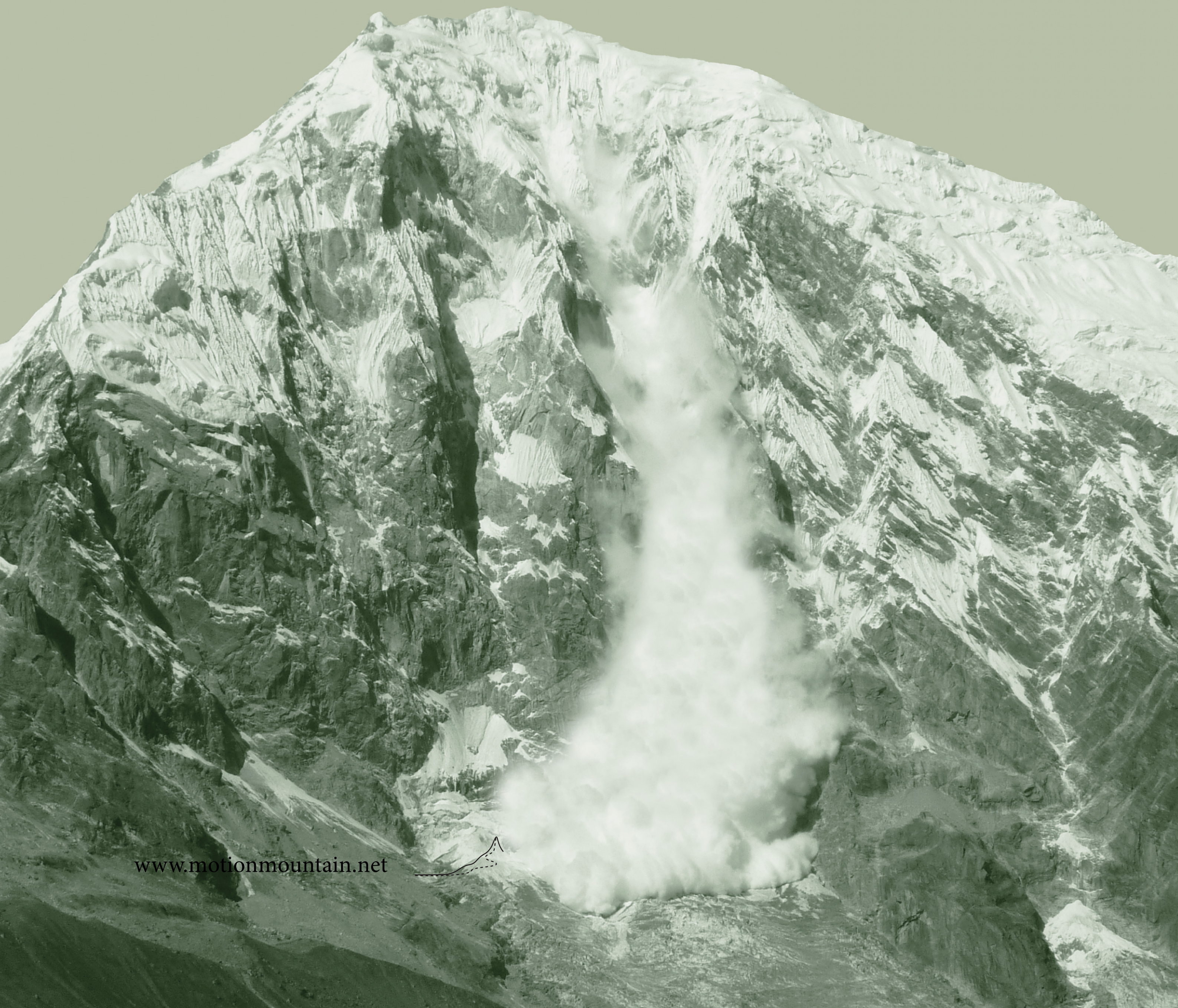
Christoph Schiller

MOTION MOUNTAIN

THE ADVENTURE OF PHYSICS – VOL. I

FALL, FLOW AND HEAT



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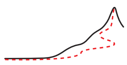


Christoph Schiller

Motion Mountain

The Adventure of Physics
Volume I

Fall, Flow and Heat

Edition 31, available as free pdf
with films at www.motionmountain.net

Editio trigesima prima.

Thirty-first edition.

To Britta, Esther and Justus Aaron

τῷ ἐμοὶ δαὶμονι

Die Menschen stärken, die Sachen klären.

PREFACE

> Primum movere, deinde docere.*
>
> Antiquity

This book series is for anybody who is curious about motion in nature. How do things, people, animals, images and empty space move? The answer leads to many adventures; this volume presents the best ones about *everyday* motion. Carefully observing everyday motion allows us to deduce six essential statements: everyday motion is continuous, conserved, relative, reversible, mirror-invariant – and lazy. Yes, nature is indeed lazy: in every motion, it minimizes change. This text explores how these six results are deduced and how they fit with all those observations that seem to contradict them.

In the structure of modern physics, shown in Figure 1, the results on everyday motion form the major part of the starting point at the bottom. The present volume is the first of a six-volume overview of physics. It resulted from a threefold aim I have pursued since 1990: to present motion in a way that is simple, up to date and captivating.

In order to be *simple*, the text focuses on concepts, while keeping mathematics to the necessary minimum. Understanding the concepts of physics is given precedence over using formulae in calculations. The whole text is within the reach of an undergraduate.

In order to be *up to date*, the text is enriched by the many gems – both theoretical and empirical – that are scattered throughout the scientific literature.

In order to be *captivating*, the text tries to startle the reader as much as possible. Reading a book on general physics should be like going to a magic show. We watch, we are astonished, we do not believe our eyes, we think, and finally we understand the trick. When we look at nature, we often have the same experience. Indeed, every page presents at least one surprise or provocation for the reader to think about. Numerous interesting challenges are proposed.

The motto of the text, *die Menschen stärken, die Sachen klären*, a famous statement on pedagogy, translates as: 'To fortify people, to clarify things.' Clarifying things – and adhering only to the truth – requires courage, as changing the habits of thought produces fear, often hidden by anger. But by overcoming our fears we grow in strength. And we experience intense and beautiful emotions. All great adventures in life allow this, and exploring motion is one of them. Enjoy it.

Christoph Schiller

* 'First move, then teach.' In modern languages, the mentioned type of *moving* (the heart) is called *motivating*; both terms go back to the same Latin root.

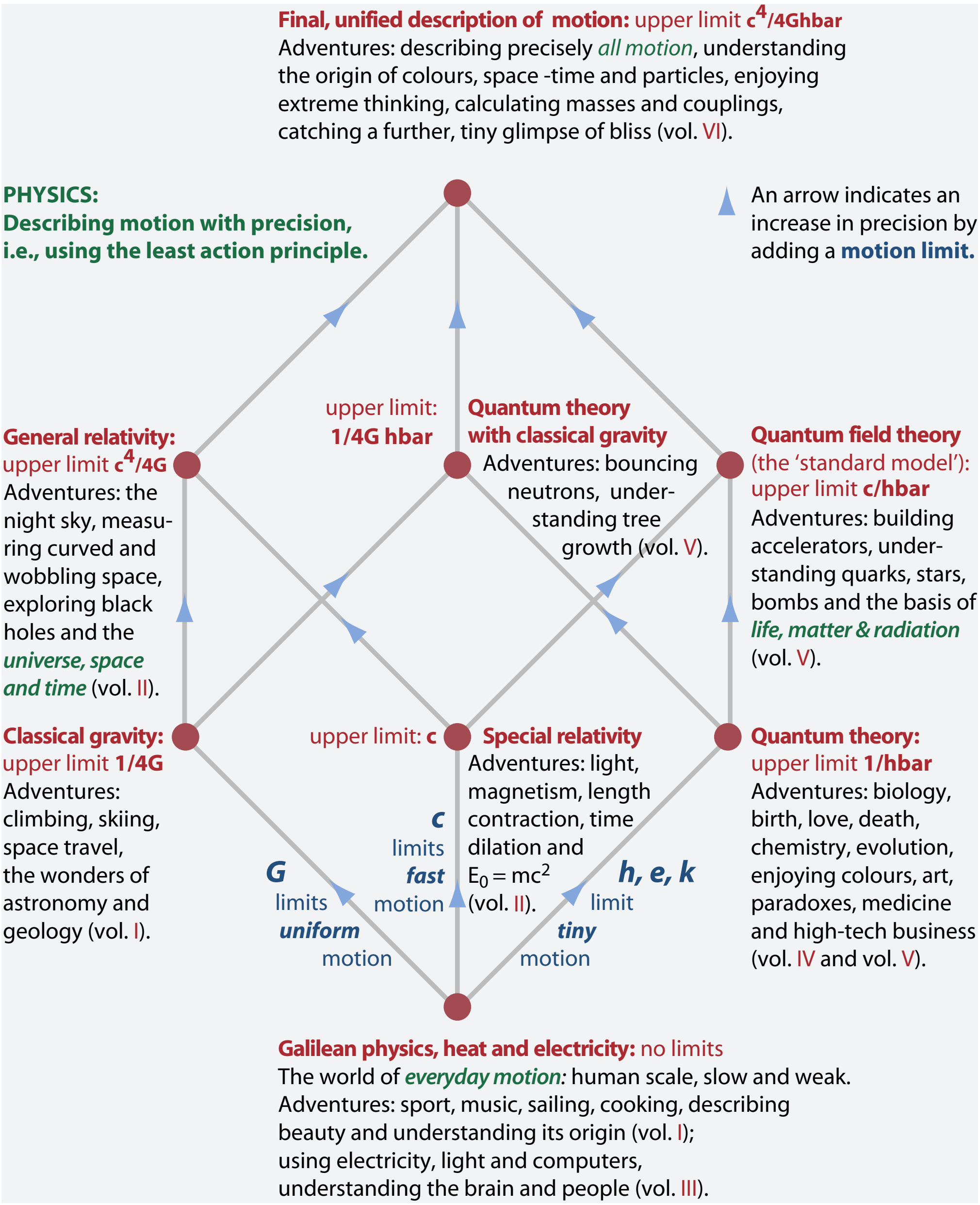


FIGURE 1 A complete map of physics, the science of motion, as first proposed by Matvei Bronshtein (b. 1907 Vinnytsia, d. 1938 Leningrad). The Bronshtein cube starts at the bottom with everyday motion, and shows the connections to the fields of modern physics. Each connection increases the precision of the description and is due to a limit to motion that is taken into account. The limits are given for uniform motion by the gravitational constant G, for fast motion by the speed of light c, and for tiny motion by the Planck constant h, the elementary charge e and the Boltzmann constant k.

Using this book

Marginal notes refer to bibliographic references, to other pages or to challenge solutions. In the colour edition, marginal notes, pointers to footnotes and links to websites are typeset in green. Over time, links on the internet tend to disappear. Most links can be recovered via www.archive.org, which keeps a copy of old internet pages. In the free pdf edition of this book, available at www.motionmountain.net, all green pointers and links are clickable. The pdf edition also contains all films; they can be watched directly in Adobe Reader.

Solutions and hints for *challenges* are given in the appendix. Challenges are classified as easy (e), standard student level (s), difficult (d) and research level (r). Challenges for which no solution has yet been included in the book are marked (ny).

Advice for learners

Learning allows us to discover what kind of person we can be. Learning widens knowledge, improves intelligence and provides a sense of achievement. Therefore, learning from a book, especially one about nature, should be efficient and enjoyable. Avoid bad learning methods like the plague! Do not use a marker, a pen or a pencil to highlight or underline text on paper. It is a waste of time, provides false comfort and makes the text unreadable. Add notes and comments instead! And do not learn from a screen. In particular, do not learn from videos, from games or from a smartphone. All games and almost all videos are drugs for the brain. Smartphones are drug dispensers that make people addicted and prevent learning. Learn from paper – at your speed, and allow your mind to wander! Nobody marking paper or looking at a screen is learning efficiently.

In my experience as a pupil and teacher, one learning method never failed to transform unsuccessful pupils into successful ones: if you read a text for study, summarize every section you read, *in your own words and images, aloud*. If you are unable to do so, read the section again. Repeat this until you can clearly summarize what you read in your own words and images, aloud. And *enjoy* the telling aloud! You can do this alone or with friends, in a room or while walking. If you do this with everything you read, you will reduce your learning and reading time significantly; you will enjoy learning from good texts much more and hate bad texts much less. Masters of the method can use it even while listening to a lecture, in a low voice, thus avoiding to ever take notes.

Advice for teachers

A teacher likes pupils and likes to lead them into exploring the field he or she chose. His or her enthusiasm is the key to job satisfaction. If you are a teacher, before the start of a lesson, picture, feel and tell yourself how you enjoy the topic of the lesson; then picture, feel and tell yourself how you will lead each of your pupils into enjoying that topic as much as you do. Do this exercise consciously, every day. You will minimize trouble in your class and maximize your teaching success.

This book is not written with exams in mind; it is written to make teachers and students *understand* and *enjoy* physics, the science of motion.

Feedback

The latest pdf edition of this text is and will remain free to download from the internet. I would be delighted to receive an email from you at fb@motionmountain.net, especially on the following issues:

Challenge 1 s
— What was unclear and should be improved?
— What story, topic, riddle, picture or film did you miss?

Also help on the specific points listed on the www.motionmountain.net/help.html web page is welcome. All feedback will be used to improve the next edition. You are welcome to send feedback by mail or by sending in a pdf with added yellow notes, to provide illustrations or photographs, or to contribute to the errata wiki on the website. If you would like to translate a chapter of the book in your language, please let me know.

On behalf of all readers, thank you in advance for your input. For a particularly useful contribution you will be mentioned – if you want – in the acknowledgements, receive a reward, or both.

Support

Your donation to the charitable, tax-exempt non-profit organisation that produces, translates and publishes this book series is welcome. For details, see the web page www.motionmountain.net/donation.html. The German tax office checks the proper use of your donation. If you want, your name will be included in the sponsor list. Thank you in advance for your help, on behalf of all readers across the world.

The paper edition of this book is available, either in colour or in black and white, from www.amazon.com, in English and in certain other languages. And now, enjoy the reading.

CONTENTS

Fall, Flow and Heat

In our quest to learn how things move,
the experience of hiking and other motion
leads us to introduce the concepts of
velocity, time, length, mass and temperature.
We learn to use them to *measure change*
and find that nature minimizes it.
We discover how to float in free space,
why we have legs instead of wheels,
why disorder can never be eliminated,
and why one of the most difficult open issues
in science is the flow of water through a tube.

Chapter 1

WHY SHOULD WE CARE ABOUT MOTION?

> “All motion is an illusion.”
>
> Zeno of Elea**

Wham! The lightning striking the tree nearby violently disrupts our quiet forest walk and causes our hearts to suddenly beat faster. In the top of the tree we see the fire start and fade again. The gentle wind moving the leaves around us helps to restore the calmness of the place. Nearby, the water in a small river follows its complicated way down the valley, reflecting on its surface the ever-changing shapes of the clouds.

Motion is everywhere: friendly and threatening, terrible and beautiful. It is fundamental to our human existence. We need motion for growing, for learning, for thinking, for remaining healthy and for enjoying life. We use motion for walking through a forest, for listening to its noises and for talking about all this. Like all animals, we rely on motion to get food and to survive dangers. Like all living beings, we need motion to reproduce, to breathe and to digest. Like all objects, motion keeps us warm.

Motion is the most fundamental observation about nature at large. It turns out that *everything* that happens in the world is some type of motion. There are no exceptions. Motion is such a basic part of our observations that even the origin of the word is lost in the darkness of Indo-European linguistic history. The fascination of motion has always made it a favourite object of curiosity. By the fifth century BCE in ancient Greece, its study had been given a name: *physics*. Ref. 1

Motion is also important to the human condition. What can we know? Where does the world come from? Who are we? Where do we come from? What will we do? What should we do? What will the future bring? What is death? Where does life lead? All these questions are about motion. And the study of motion provides answers that are both deep and surprising.

Ref. 2 Motion is mysterious. Though found everywhere – in the stars, in the tides, in our eyelids – neither the ancient thinkers nor myriads of others in the 25 centuries since then have been able to shed light on the central mystery: *what is motion?* We shall discover that the standard reply, ‘motion is the change of place in time’, is correct, but inadequate. Just recently a full answer has finally been found. This is the story of the way to find it.

Motion is a part of human experience. If we imagine human experience as an island, then destiny, symbolized by the waves of the sea, carried us to its shore. Near the centre of

** Zeno of Elea (*c.* 450 BCE), one of the main exponents of the Eleatic school of philosophy.

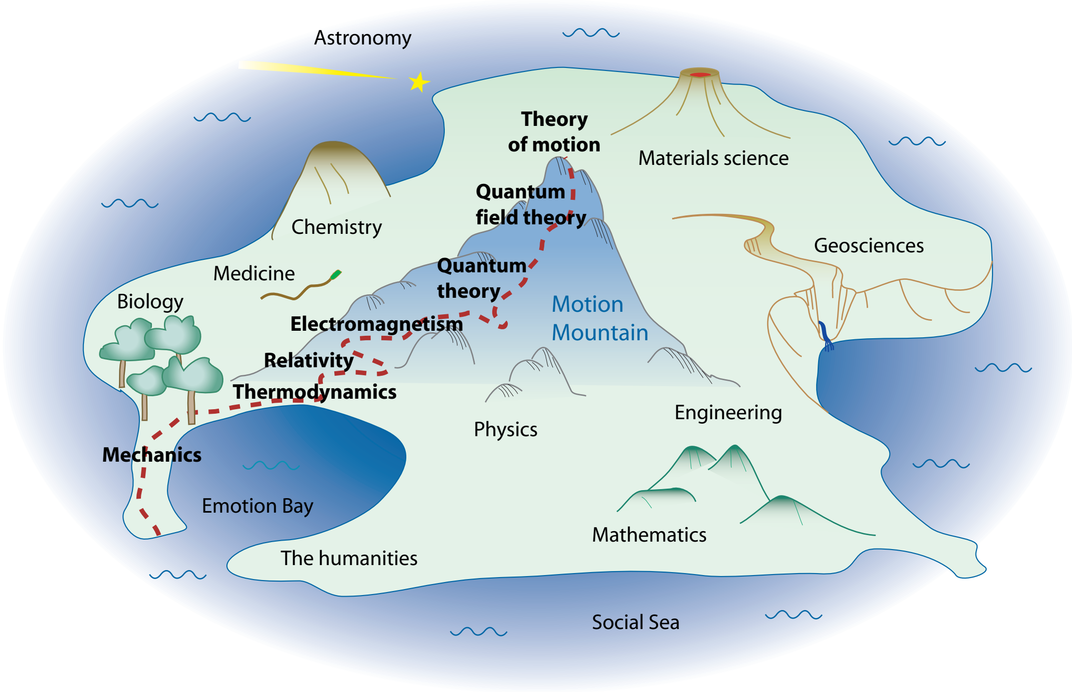


FIGURE 2 Experience Island, with Motion Mountain and the trail to be followed.

the island an especially high mountain stands out. From its top we can see over the whole landscape and get an impression of the relationships between all human experiences, and in particular between the various examples of motion. This is a guide to the top of what I have called Motion Mountain (see Figure 2; a less symbolic and more exact version is given in Figure 1). The hike is one of the most beautiful adventures of the human mind. The first question to ask is:

Page 8

Does motion exist?

> *Das Rätsel* gibt es nicht. Wenn sich eine Frage überhaupt stellen läßt, so *kann* sie beantwortet werden.*
>
> Ludwig Wittgenstein, *Tractatus*, 6.5

To sharpen the mind for the issue of motion's existence, have a look at Figure 3 or Figure 4 and follow the instructions. In all cases the figures seem to rotate. You can experience similar effects if you walk over cobblestone pavement that is arranged in arched patterns or if you look at the numerous motion illusions collected by Kitaoka Akiyoshi at www.ritsumei.ac.jp/~akitaoka. How can we make sure that real motion is different from these or other similar illusions?

Ref. 3 Ref. 4 Challenge 2 s

Many scholars simply argued that motion does not exist at all. Their arguments deeply influenced the investigation of motion over many centuries. For example, the Greek

Ref. 5

* '*The riddle* does not exist. If a question can be put at all, it *can* also be answered.'

FIGURE 3 Illusions of motion: look at the figure on the left and slightly move the page, or look at the white dot at the centre of the figure on the right and move your head back and forward.

philosopher Parmenides (born *c.* 515 BCE in Elea, a small town near Naples) argued that since nothing comes from nothing, change cannot exist. He underscored the *permanence* of nature and thus consistently maintained that all change and thus all motion is an illusion. Ref. 6

Heraclitus (*c.* 540 to *c.* 480 BCE) held the opposite view. Plato describes Heraclitus as making the famous statement πάντα ῥεῖ 'panta rhei' or 'everything flows'.* He saw change as the essence of nature, in contrast to Parmenides. These two equally famous opinions induced many scholars to investigate in more detail whether in nature there are *conserved* quantities or whether *creation* is possible. We will uncover the answer later on; until then, you might ponder which option you prefer. Challenge 3 s

Parmenides' collaborator Zeno of Elea (born *c.* 500 BCE) argued so intensely against motion that some people still worry about it today. In one of his arguments he claims – in simple language – that it is impossible to slap somebody, since the hand first has to travel halfway to the face, then travel through half the distance that remains, then again so, and so on; the hand therefore should never reach the face. Zeno's argument focuses on the relation between *infinity* and its opposite, finitude, in the description of motion. In modern quantum theory, a related issue is a subject of research up to this day. Ref. 7

Zeno also stated that by looking at a moving object at a *single* instant of time, one cannot maintain that it moves. He argued that at a single instant of time, there is no difference between a moving and a resting body. He then deduced that if there is no difference at a single time, there cannot be a difference for longer times. Zeno therefore questioned whether motion can clearly be distinguished from its opposite, *rest*. Indeed, in the history of physics, thinkers switched back and forward between a positive and a negative answer. It was this very question that led Albert Einstein to the development of general relativity, one of the high points of our journey. In our adventure, we will explore all known differences between motion and rest. Eventually, we will dare to ask whether single instants of time do exist at all. Answering this question is essential for reaching the top of Motion Mountain.

When we explore quantum theory, we will discover that motion is indeed – to a certain extent – an illusion, as Parmenides claimed. More precisely, we will show that motion is observed only due to the limitations of the human condition. We will find that we experience motion only because

* Appendix A explains how to read Greek text.

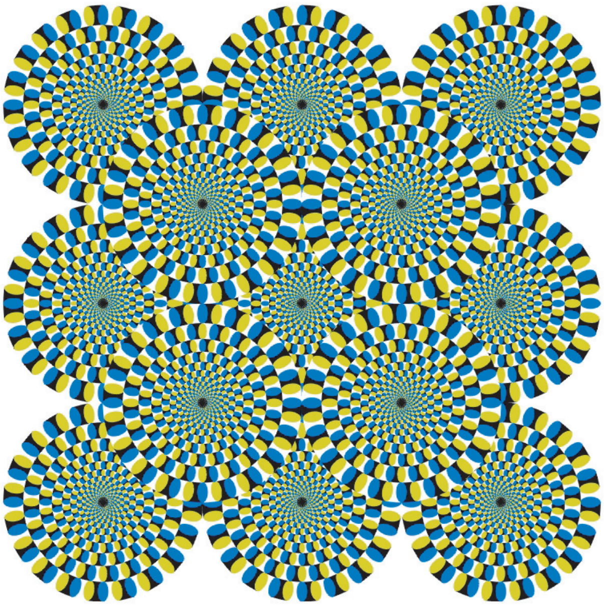


FIGURE 4 Zoom this image to large size or approach it closely in order to enjoy its apparent motion (© Michael Bach after the discovery of Kitaoka Akiyoshi).

— we have a finite size,
— we are made of a large but finite number of atoms,
— we have a finite but moderate temperature,
— we move much more slowly than the speed of light,
— we live in three dimensions,
— we are large compared with a black hole of our own mass,
— we are large compared with our quantum mechanical wavelength,
— we are small compared with the universe,
— we have a working but limited memory,
— we are forced by our brain to approximate space and time as continuous entities, and
— we are forced by our brain to approximate nature as made of different parts.

If any one of these conditions were not fulfilled, we would not observe motion; motion, then, would not exist! If that were not enough, note that none of the conditions requires human beings; they are equally valid for many animals and machines. Each of these conditions can be uncovered most efficiently if we start with the following question:

How should we talk about motion?

> “Je hais le mouvement, qui déplace les lignes,
> Et jamais je ne pleure et jamais je ne ris.”
> Charles Baudelaire, *La Beauté*.*

Like any science, the approach of physics is twofold: we advance with *precision* and with *curiosity*. Precision is the extent to which our description matches observations. Curios-

* Charles Baudelaire (b. 1821 Paris, d. 1867 Paris) *Beauty*: ‘I hate movement, which changes shapes, and never do I weep and never do I laugh.’ *Beauty*.

Ref. 8

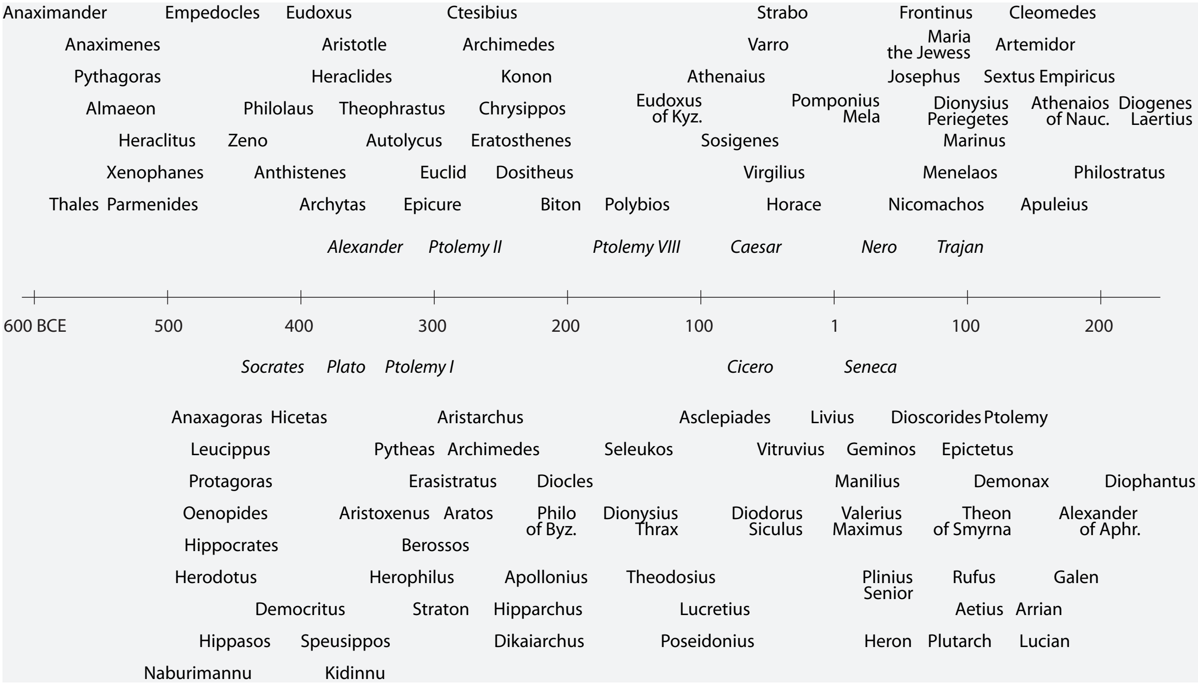


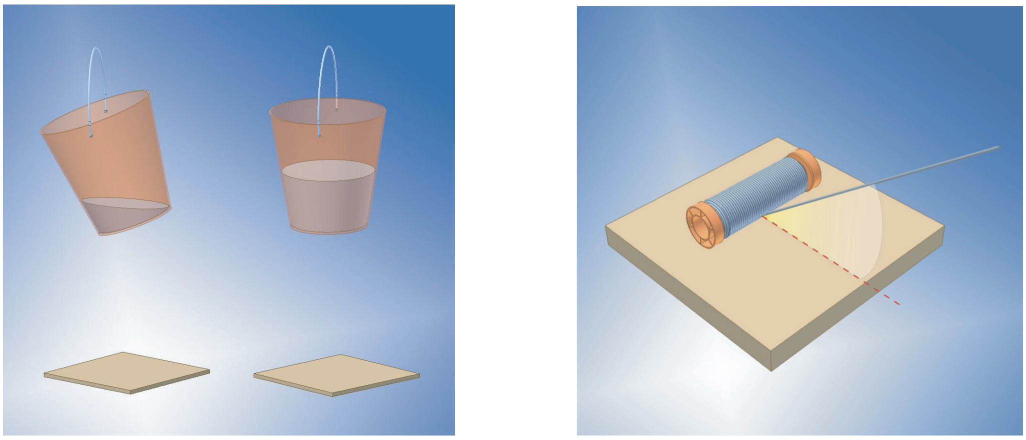
FIGURE 5 A timeline of scientific and *political* personalities in antiquity. The last letter of the name is aligned with the year of death. For example, Maria the Jewess is the inventor of the bain-marie process and Thales is the first mathematician and scientist known by name.

ity is the passion that drives all scientists. Precision makes meaningful communication possible, and curiosity makes it worthwhile. Take an eclipse, a beautiful piece of music
Ref. 9 or a feat at the Olympic games: the world is full of fascinating examples of motion.

If you ever find yourself talking about motion, whether to understand it more precisely or more deeply, you are taking steps up Motion Mountain. The examples of Figure 6 make the point. An empty bucket hangs vertically. When you fill the bucket with a certain amount of water, it does not hang vertically any more. (Why?) If you continue adding water, it starts to hang vertically again. How much water is necessary for this last
Challenge 4 s transition? The second illustration in Figure 6 is for the following puzzle. When you pull a thread from a reel in the way shown, the reel will move either forwards or backwards, depending on the angle at which you pull. What is the limiting angle between the two possibilities?

High precision means going into fine details. Being attuned to details actually *increases* the pleasure of the adventure.* Figure 7 shows an example. The higher we get on Motion Mountain, the further we can see and the more our curiosity is rewarded. The views offered are breathtaking, especially from the very top. The path we will follow – one of the many possible routes – starts from the side of biology and directly enters the
Ref. 10 forest that lies at the foot of the mountain.

Challenge 6 s * Distrust anybody who wants to talk you *out* of investigating details. He is trying to deceive you. Details are important. Be vigilant also during *this* journey.



Challenge 5 s

FIGURE 6 How much water is required to make a bucket hang vertically? At what angle does the reel (drawn incorrectly, with too small rims) change direction of motion when pulled along with the thread? (© Luca Gastaldi).

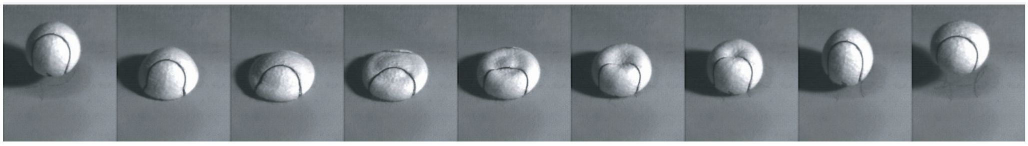


FIGURE 7 An example of how precision of observation can lead to the discovery of new effects: the deformation of a tennis ball during the *c.* 6 ms of a fast bounce (© International Tennis Federation).

Intense curiosity drives us to go straight to the limits: understanding motion requires exploration of the largest distances, the highest velocities, the smallest particles, the strongest forces, the highest precision and the strangest concepts. Let us begin.

What are the types of motion?

> Every movement is born of a desire for change.
> Antiquity

A good place to obtain a general overview on the types of motion is a large library. Table 1 shows the results. The domains in which motion, movements and moves play a role are indeed varied. Already the earliest researchers in ancient Greece – listed in Figure 5 – had the suspicion that all types of motion, as well as many other types of change, are related. Three categories of change are commonly recognized:

1. *Transport*. The only type of change we call motion in everyday life is material transport, such as a person walking, a leaf falling from a tree, or a musical instrument playing. Transport is the change of position or orientation of objects, fluids included. To a large extent, the behaviour of people also falls into this category.
2. *Transformation*. Another category of change groups observations such as the dissolution of salt in water, the formation of ice by freezing, the rotting of wood, the cooking of food, the coagulation of blood, and the melting and alloying of metals.

TABLE 1 Content of books about motion found in a public library.

Motion topics	Motion topics
motion pictures and digital effects	motion as therapy for cancer, diabetes, acne and depression
motion perception Ref. 11	motion sickness
motion for fitness and wellness	motion for meditation
motion control and training in sports and singing	motion ability as health check
perpetual motion	motion in dance, music and other performing arts
motion as proof of various gods Ref. 12	motion of planets, stars and angels Ref. 13
economic efficiency of motion	the connection between motional and emotional habits
motion as help to overcome trauma	motion in psychotherapy Ref. 14
locomotion of insects, horses, animals and robots	motion of cells and plants
collisions of atoms, cars, stars and galaxies	growth of multicellular beings, mountains, sunspots and galaxies
motion of springs, joints, mechanisms, liquids and gases	motion of continents, bird flocks, shadows and empty space
commotion and violence	motion in martial arts
motions in parliament	movements in art, sciences and politics
movements in watches	movements in the stock market
movement teaching and learning	movement development in children Ref. 15
musical movements	troop movements Ref. 16
religious movements	bowel movements
moves in chess	cheating moves in casinos Ref. 17
connection between gross national product and citizen mobility	

These changes of colour, brightness, hardness, temperature and other material properties are all transformations. Transformations are changes not visibly connected with transport. To this category, a few ancient thinkers added the emission and absorption of light. In the twentieth century, these two effects were proven to be special cases of transformations, as were the newly discovered appearance and disappearance of matter, as observed in the Sun and in radioactivity. *Mind change*, such as change of mood, of health, of education and of character, is also (mostly) a type of transformation. Ref. 18

Ref. 19

3. *Growth*. This last and especially important category of change, is observed for animals, plants, bacteria, crystals, mountains, planets, stars and even galaxies. In the nineteenth century, changes in the population of systems, *biological evolution*, and in the twentieth century, changes in the size of the universe, *cosmic evolution*, were added to this category. Traditionally, these phenomena were studied by separate sciences. Independently they all arrived at the conclusion that growth is a combination of transport and transformation. The difference is one of complexity and of time scale.

FIGURE 8 An example of transport, at Mount Etna (© Marco Fulle).

At the beginnings of modern science during the Renaissance, only the study of transport was seen as the topic of physics. Motion was equated to transport. The other two domains were neglected by physicists. Despite this restriction, the field of enquiry remains large, covering a large part of Experience Island.
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Early scholars differentiated types of transport by their origin. Movements such as those of the legs when walking were classified as *volitional*, because they are controlled by one's will, whereas movements of external objects, such as the fall of a snowflake, which cannot be influenced by will-power, were classified as *passive*. Young humans, especially young male humans, spend considerable time in learning elaborate volitional movements. An example is shown in Figure 10.

The complete distinction between passive and volitional motion is made by children by the age of six, and this marks a central step in the development of every human towards a precise description of the environment.* From this distinction stems the historical but now outdated definition of physics as the science of motion of non-living things.

The advent of machines forced scholars to rethink the distinction between volitional and passive motion. Like living beings, machines are self-moving and thus mimic volitional motion. However, careful observation shows that every part in a machine is moved by another, so their motion is in fact passive. Are living beings also machines? Are human actions examples of passive motion as well? The accumulation of observations in the last 100 years made it clear that volitional movement** indeed has the same physical prop-

* Failure to pass this stage completely can result in a person having various strange beliefs, such as believing in the ability to influence roulette balls, as found in compulsive players, or in the ability to move other bodies by thought, as found in numerous otherwise healthy-looking people. An entertaining and informative account of all the deception and self-deception involved in creating and maintaining these beliefs is given by James Randi, *The Faith Healers*, Prometheus Books, 1989. A professional magician, he presents many similar topics in several of his other books. See also his www.randi.org website for more details.
** The word 'movement' is rather modern; it was imported into English from the old French and became popular only at the end of the eighteenth century. It is never used by Shakespeare.

FIGURE 9 Transport, growth and transformation (© Philip Plisson).

FIGURE 10 One of the most difficult volitional movements known, performed by Alexander Tsukanov, the first man able to do this: jumping from one ultimate wheel to another (© Moscow State Circus).

erties as passive movement in non-living systems. A distinction between the two types of motion is thus unnecessary. Of course, from the emotional viewpoint, the differences are important; for example, *grace* can only be ascribed to volitional movements. Ref. 20

Since passive and volitional motion have the same properties, through the study of motion of non-living objects we can learn something about the human condition. This is most evident when touching the topics of determinism, causality, probability, infinity, time, love and death, to name but a few of the themes we will encounter during our adventure.

In the nineteenth and twentieth centuries other classically held beliefs about motion fell by the wayside. Extensive observations showed that all transformations and all growth phenomena, including behaviour change and evolution, are also examples of transport. In other words, over 2 000 years of studies have shown that the ancient classification of observations was useless:

▷ All change is transport.

And

▷ Transport and motion are the same.

In the middle of the twentieth century the study of motion culminated in the experimental confirmation of an even more specific idea, previously articulated in ancient Greece:

▷ Every type of change is due to the motion of particles.

It takes time and work to reach this conclusion, which appears only when we relentlessly pursue higher and higher precision in the description of nature. The first five parts of this adventure retrace the path to this result. (Do you agree with it?) Challenge 7 s

The last decade of the twentieth century again completely changed the description of motion: the particle idea turns out to be limited and wrong. This recent result, reached through a combination of careful observation and deduction, will be explored in the last part of our adventure. But we still have some way to go before we reach that result, just below the summit of our journey.

In summary, history has shown that classifying the various types of motion is not productive. Only by trying to achieve maximum precision can we hope to arrive at the fundamental properties of motion. *Precision, not classification, is the path to follow.* As Ernest Rutherford said jokingly: 'All science is either physics or stamp collecting.'

In order to achieve precision in our description of motion, we need to select specific examples of motion and study them fully in detail. It is intuitively obvious that the most precise description is achievable for the *simplest* possible examples. In everyday life, this is the case for the motion of any non-living, solid and rigid body in our environment, such as a stone thrown through the air. Indeed, like all humans, we learned to throw objects long before we learned to walk. Ref. 21 Throwing is one of the first physical experiments we performed by ourselves. The importance of throwing is also seen from the terms derived from it: in Latin, words like *subject* or 'thrown below', *object* or 'thrown in front', and *interjection* or 'thrown in between'; in Greek, the act of throwing led to terms like *symbol* or 'thrown together', *problem* or 'thrown forward', *emblem* or 'thrown into', and – last but not least – *devil* or 'thrown through'. And indeed, during our early childhood, by throwing stones, toys and other objects until our parents feared for every piece of the household, we explored the perception and the properties of motion. We do the same here.

> “Die Welt ist unabhängig von meinem Willen.* ”
> Ludwig Wittgenstein, *Tractatus*, 6.373

* 'The world is independent of my will.'

Perception, permanence and change

> Only wimps specialize in the general case; real scientists pursue examples.
> Adapted by Michael Berry from a remark by Beresford Parlett

Human beings enjoy perceiving. Perception starts before birth, and we continue enjoying it for as long as we can. That is why television or videos, even when devoid of content, are so successful. During our walk through the forest at the foot of Motion Mountain we cannot avoid perceiving. Perception is first of all the ability to *distinguish*. We use the basic mental act of distinguishing in almost every instant of life; for example, during childhood we first learned to distinguish familiar from unfamiliar observations. This is possible in combination with another basic ability, namely the capacity to *memorize* experiences. Memory gives us the ability to experience, to talk and thus to explore nature. Perceiving, classifying and memorizing together form *learning*. Without any one of these three abilities, we could not study motion.

Children rapidly learn to distinguish *permanence* from *variability*. They learn to *recognize* human faces, even though a face never looks exactly the same each time it is seen. From recognition of faces, children extend recognition to all other observations. Recognition works pretty well in everyday life; it is nice to recognize friends, even at night, and even after many beers (not a challenge). The act of recognition thus always uses a form of *generalization*. When we observe, we always have some general idea in our mind. Let us specify the main ones.

Sitting on the grass in a clearing of the forest at the foot of Motion Mountain, surrounded by the trees and the silence typical of such places, a feeling of calmness and tranquillity envelops us. We are thinking about the essence of perception. Suddenly, something moves in the bushes; immediately our eyes turn and our attention focuses. The nerve cells that detect motion are part of the most ancient part of our brain, shared with birds and reptiles: the brain stem. Then the cortex, or modern brain, takes over to analyse the type of motion and to identify its origin. Watching the motion across our field of vision, we observe two invariant entities: the fixed landscape and the moving animal. After we recognize the animal as a deer, we relax again.

Ref. 22

How did we distinguish, in case of Figure 11, between landscape and deer? Perception involves several processes in the eye and in the brain. An essential part for these processes is motion, as is best deduced from the flip film shown in the lower left corners of these pages. Each image shows only a rectangle filled with a mathematically random pattern. But when the pages are scanned in rapid succession, you discern a shape – a square – moving against a fixed background. At any given instant, the square cannot be distinguished from the background; there is no visible object at any given instant of time. Nevertheless it is easy to perceive its motion.* Perception experiments such as this one have been performed in many variations. Such experiments showed that detecting a moving square against a random background is nothing special to humans; flies have the same ability, as do, in fact, all animals that have eyes.

Ref. 23

* The human eye is rather good at detecting motion. For example, the eye can detect motion of a point of light even if the change of angle is smaller than that which can be distinguished in a fixed image. Details of this and similar topics for the other senses are the domain of perception research.

Ref. 11

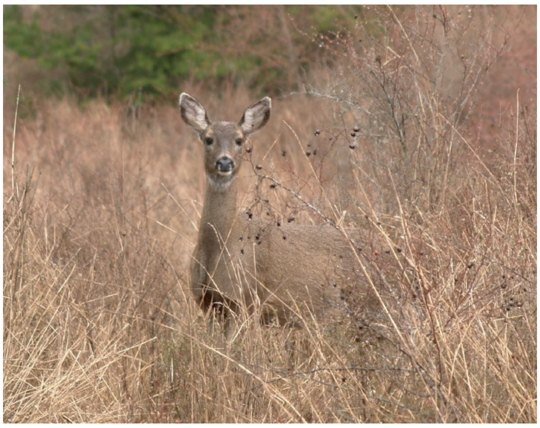


FIGURE 11 How do we distinguish a deer from its environment? (© Tony Rodgers).

The flip film in the lower left corner, like many similar experiments, illustrates two central attributes of motion. First, motion is perceived only if an *object* can be distinguished from a *background* or *environment*. Many motion illusions focus on this point.* Second, motion is required to *define* both the object and the environment, and to distinguish them from each other. In fact, the concept of space is – among others – an abstraction of the idea of background. The background is extended; the moving entity is localized. Does this seem boring? It is not; just wait for a second.

We call a localized entity of investigation that can change or move a *physical system* – or simply a system. A system is a recognizable, thus permanent part of nature. Systems can be objects – also called 'physical bodies' – or radiation. Therefore, images, which are made of radiation, are aspects of physical systems, but not themselves physical systems.
Challenge 8 s
These connections are summarized in Table 2. Now, are holes physical systems?

In other words, we call the set of localized aspects that remain invariant or permanent during motion, such as size, shape, colour etc., taken together, a (physical) *object* or a (physical) *body*. We will tighten the definition shortly, to distinguish objects from images.

We note that to specify permanent moving objects, we need to distinguish them from the environment. In other words, right from the start, we experience motion as a *relative* process; it is perceived in relation and in opposition to the environment.

The conceptual distinction between localized, isolable objects and the extended environment is important. True, it has the appearance of a circular definition. (Do you agree?)
Challenge 9 s
Page 438
Indeed, this issue will keep us busy later on. On the other hand, we are so used to our ability to isolate local systems from the environment that we take it for granted. However, as we will discover later on in our walk, this distinction turns out to be logically
Vol. VI, page 85
and experimentally impossible!** The reason for this impossibility will turn out to be fascinating. To discover the impossibility, we note, as a first step, that apart from mov-

* The topic of motion perception is full of interesting aspects. An excellent introduction is chapter 6 of the beautiful text by Donald D. Hoffman, *Visual Intelligence – How We Create What We See*, W.W. Norton & Co., 1998. His collection of basic motion illusions can be experienced and explored on the associated www.cogsci.uci.edu/~ddhoff website.

** Contrary to what is often read in popular literature, the distinction *is* possible in quantum theory. It becomes impossible only when quantum theory is unified with general relativity.

TABLE 2 Family tree of the basic physical concepts.

motion

the basic type of change: a system changing position relative to a background

parts/systems	*relations*	*background*
permanent		measurable
bounded	produce boundaries	unbounded
have shapes	produce shapes	extended

objects	*radiation*	*states*	*interactions*	*phase space*	*space-time*
impenetrable	penetrable	global	local	composed	simple

The corresponding aspects:

mass	intensity	instant	source	dimension	curvature
size	colour	position	domain	distance	topology
charge	image	momentum	strength	volume	distance
spin	appearance	energy	direction	subspaces	area
etc.	etc.	etc.	etc.	etc.	etc.

world – nature – universe – cosmos

the collection of all parts, relations and backgrounds

ing entities and the permanent background, we also need to describe their relations. The necessary concepts are summarized in Table 2.

> Wisdom is one thing: to understand the thought which steers all things through all things.
> Heraclitus of Ephesus

Ref. 24

Does the world need states?

> Das Feste, das Bestehende und der Gegenstand sind Eins. Der Gegenstand ist das Feste, Bestehende; die Konfiguration ist das Wechselnde, Unbeständige.*
> Ludwig Wittgenstein, *Tractatus*, 2.027 – 2.0271

What distinguishes the various patterns in the lower left corners of this text? In everyday life we would say: the situation or configuration of the involved entities. The situation somehow describes all those aspects that can differ from case to case. It is customary to call the list of all *variable* aspects of a set of objects their *(physical) state of motion*, or simply their *state*. How is the state characterized?

* 'The fixed, the existent and the object are one. The object is the fixed, the existent; the configuration is the changing, the variable.'

The configurations in the lower left corners differ first of all in *time*. Time is what makes opposites possible: a child is in a house and the same child is outside the house. Time describes and resolves this type of contradiction. But the state not only distinguishes situations in time: the state contains *all* those aspects of a *system* – i.e., of a group of objects – that set it apart from all *similar* systems. Two similar objects can differ, at each instant of time, in their

— position,
— velocity,
— orientation, or
— angular velocity.

These properties determine the state and pinpoint the *individuality* of a physical system among *exact copies* of itself. Equivalently, the state describes the relation of an object or a system with respect to its environment. Or, again, equivalently:

▹ The *state* describes all aspects of a system that depend on the observer.

The definition of state is not boring at all – just ponder this: Does the *universe* have a state? And: is the list of state properties just given *complete*? Challenge 10 s

In addition, physical systems are described by their permanent, *intrinsic properties*. Some examples are

— mass,
— shape,
— colour,
— composition.

Intrinsic properties do not depend on the observer and are independent of the state of the system. They are permanent – at least for a certain time interval. Intrinsic properties also allow to distinguish physical systems from each other. And again, we can ask: What is the *complete* list of intrinsic properties in nature? And does the universe have intrinsic properties? Challenge 11 s

The various aspects of objects and of their states are called (physical) *observables*. We will refine this rough, preliminary definition in the following.

Describing nature as a collection of permanent entities and changing states is the starting point of the study of motion. Every observation of motion requires the distinction of permanent, intrinsic properties – describing the objects that move – and changing states – describing the way the objects move. Without this distinction, there is no motion. Without this distinction, there is not even a way to *talk* about motion.

Using the terms just introduced, we can say

▹ Motion is the change of state of permanent objects.

The exact separation between those aspects belonging to the object, the permanent *intrinsic properties*, and those belonging to the state, the varying *state properties*, depends on the precision of observation. For example, the length of a piece of wood is not permanent; wood shrinks and bends with time, due to processes at the molecular level. To be precise, the length of a piece of wood is not an aspect of the object, but an aspect of its state. Precise observations thus *shift* the distinction between the object and its state;

the distinction itself does not disappear – at least not in the first five volumes of our adventure.

At the end of the twentieth century, neuroscience discovered that the distinction between changing states and permanent objects is not only made by scientists and engineers. Also nature makes the distinction. In fact, nature has hard-wired the distinction into the brain! Using the output signals from the visual cortex that processes what the eyes observe, the adjacent parts on the *upper* side of the human brain – the *dorsal stream* – process the *state* of the objects that are seen, such their distance and motion, whereas the adjacent parts on the *lower* side of the human brain – the *ventral stream* – process *intrinsic properties*, such as shape, colours and patterns.

In summary, states are indeed required for the description of motion. So are permanent, intrinsic properties. In order to proceed and to achieve a *complete* description of motion, we thus need a complete description of the possible states and a complete description of the intrinsic properties of objects. The first approach that attempts this is called Galilean physics; it starts by specifying our *everyday* environment and the motion in it as precisely as possible.

Galilean physics in six interesting statements

The study of everyday motion, Galilean physics, is already worthwhile in itself: we will uncover many results that are in contrast with our usual experience. For example, if we recall our own past, we all have experienced how important, delightful or unwelcome *surprises* can be. Nevertheless, the study of everyday motion shows that there are *no* surprises in nature. Motion, and thus the world, is *predictable* or *deterministic*.

The main surprise of our exploration of motion is that there are no surprises in nature. Nature is predictable. In fact, we will uncover six aspects of the predictability of everyday motion:

1. *Continuity.* We know that eyes, cameras and measurement apparatus have a finite resolution. All have a smallest distance they can observe. We know that clocks have a smallest time they can measure. Despite these limitations, in everyday life all movements, their states, as well as space and time themselves, are *continuous*.
2. *Conservation.* We all observe that people, music and many other things in motion stop moving after a while. The study of motion yields the opposite result: motion never stops. In fact, three aspects of motion do not change, but are *conserved*: momentum, angular momentum and energy are conserved, separately, in all examples of motion. No exception to these three types of conservation has ever been observed. (In contrast, mass is often, but not always conserved.) In addition, we will discover that conservation implies that motion and its properties are the same at all places and at all times: motion is *universal*.
3. *Relativity.* We all know that motion differs from rest. Despite this experience, careful study shows that there is no intrinsic difference between the two. Motion and rest depend on the observer. Motion is *relative*. And so is rest. This is the first step towards understanding the theory of relativity.
4. *Reversibility.* We all observe that many processes happen only in one direction. For example, spilled milk never returns into the container by itself. Despite such obser-

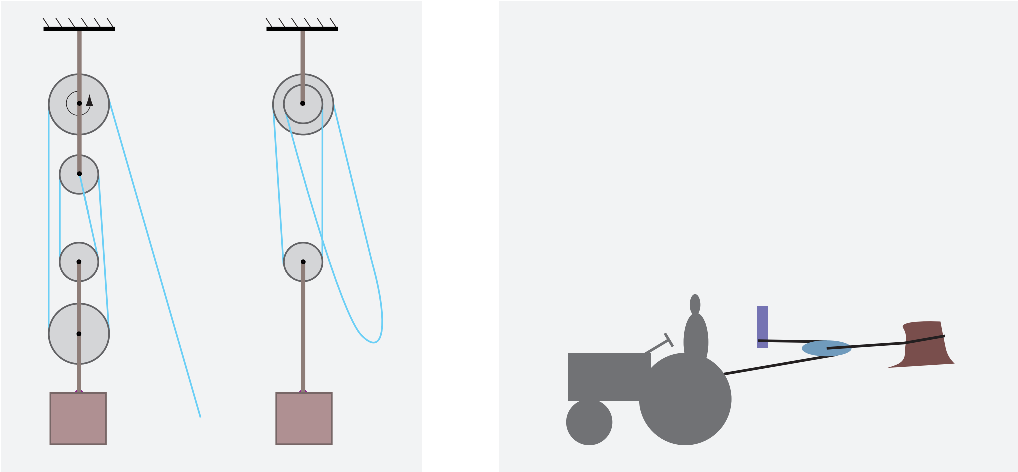


FIGURE 12 A block and tackle and a differential pulley (left) and a farmer (right).

vations, the study of motion will show us that all everyday motion is *reversible*. Physicists call this the invariance of everyday motion under *motion reversal*. Sloppily, but incorrectly, one sometimes speaks of ‘time reversal’.

5. *Mirror invariance*. Most of us find scissors difficult to handle with the left hand, have difficulties to write with the other hand, and have grown with a heart on the left side. Despite such observations, our exploration will show that everyday motion is *mirror-invariant* (or *parity-invariant*). Mirror processes are always possible in everyday life.
6. *Change minimization*. We all are astonished by the many observations that the world offers: colours, shapes, sounds, growth, disasters, happiness, friendship, love. The variation, beauty and complexity of nature is amazing. We will confirm that all observations are due to motion. And despite the appearance of complexity, all motion is simple. Our study will show that all observations can be summarized in a simple way: Nature is lazy. All motion happens in a way that *minimizes change*. Change can be measured, using a quantity called ‘action’, and nature keeps it to a minimum. Situations – or states, as physicists like to say – evolve by minimizing change. Nature is lazy.

These six aspects are essential in understanding motion in sport, in music, in animals, in machines or among the stars. This first volume of our adventure will be an exploration of such movements. In particular, we will confirm, against all appearances of the contrary, the mentioned six key properties in all cases of everyday motion.

Curiosities and fun challenges about motion*

In contrast to most animals, sedentary creatures, like plants or sea anemones, have no legs and cannot move much; for their self-defence, they developed *poisons*. Examples of such plants are the stinging nettle, the tobacco plant, digitalis, belladonna and poppy;

* Sections entitled ‘curiosities’ are collections of topics and problems that allow one to check and to expand the usage of concepts already introduced.

poisons include caffeine, nicotine, and curare. Poisons such as these are at the basis of most medicines. Therefore, most medicines exist essentially because plants have no legs.

* *

A man climbs a mountain from 9 a.m. to 1 p.m. He sleeps on the top and comes down the next day, taking again from 9 a.m. to 1 p.m. for the descent. Is there a place on the Challenge 12 s path that he passes at the same time on the two days?

* *

Every time a soap bubble bursts, the motion of the surface during the burst is the same, Challenge 13 s even though it is too fast to be seen by the naked eye. Can you imagine the details?

* *

Challenge 14 s Is the motion of a ghost an example of motion?

* *

Challenge 15 s Can something stop moving? How would you show it?

* *

Challenge 16 s Does a body moving forever in a straight line show that nature or space is infinite?

* *

What is the length of rope one has to pull in order to lift a mass by a height h with a block Challenge 17 s and tackle with four wheels, as shown on the left of Figure 12? Does the farmer on the right of the figure do something sensible?

In the past, block and tackles were important in many machines. Two particularly Ref. 25 useful versions are the *differential* block and tackle, also called a differential pulley, which is easy to make, and the *Spanish burton*, which has the biggest effect with the smallest Challenge 18 e number of wheels. There is even a so-called *fool's tackle*. Enjoy their exploration.

All these devices are examples of the *golden rule of mechanics*: what you gain in force, you loose in displacement. Or, equivalently: force times displacement – also called (physical) *work* – remains unchanged, whatever mechanical device you may use. This is one example of conservation that is observed in everyday motion.

* *

Challenge 19 s Can the universe move?

* *

To talk about precision with precision, we need to measure precision itself. How would Challenge 20 s you do that?

* *

Challenge 21 s Would we observe motion if we had no memory?

* *

Challenge 22 s What is the lowest speed you have observed? Is there a lowest speed in nature?

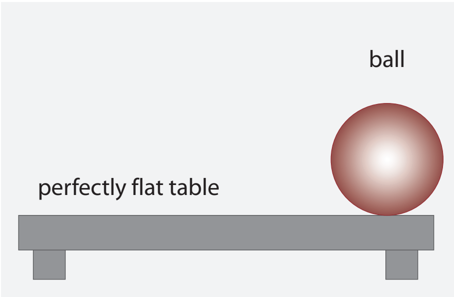


FIGURE 13 What happens?

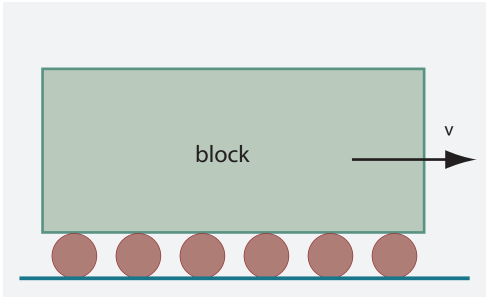


FIGURE 14 What is the speed of the rollers? Are other roller shapes possible?

* *

According to legend, Sissa ben Dahir, the Indian inventor of the game of *chaturanga* or chess, demanded from King Shirham the following reward for his invention: he wanted one grain of wheat for the first square, two for the second, four for the third, eight for the fourth, and so on. How much time would all the wheat fields of the world take to produce the necessary grains? Challenge 23 s

* *

When a burning candle is moved, the flame lags behind the candle. How does the flame behave if the candle is inside a glass, still burning, and the glass is accelerated? Challenge 24 s

* *

A good way to make money is to build motion detectors. A motion detector is a small box with a few wires. The box produces an electrical signal whenever the box moves. What types of motion detectors can you imagine? How cheap can you make such a box? How precise? Challenge 25 d

* *

A perfectly frictionless and spherical ball lies near the edge of a perfectly flat and horizontal table, as shown in Figure 13. What happens? In what time scale? Challenge 26 d

* *

You step into a closed box without windows. The box is moved by outside forces unknown to you. Can you determine *how* you are moving from inside the box? Challenge 27 s

* *

When a block is rolled over the floor over a set of cylinders, as shown in Figure 14, how are the speed of the block and that of the cylinders related? Challenge 28 s

* *

Ref. 18 Challenge 29 s
Do you dislike formulae? If you do, use the following three-minute method to change the situation. It is worth trying it, as it will make you enjoy this book much more. Life is short; as much of it as possible, like reading this text, should be a pleasure.

1. Close your eyes and recall an experience that was *absolutely marvellous*, a situation when you felt excited, curious and positive.
2. Open your eyes for a second or two and look at page 280 – or any other page that contains many formulae.
3. Then close your eyes again and return to your marvellous experience.
4. Repeat the observation of the formulae and the visualization of your memory – steps 2 and 3 – three more times.

Then leave the memory, look around yourself to get back into the here and now, and test yourself. Look again at page 280. How do you feel about formulae now?

∗ ∗

In the sixteenth century, Niccolò Tartaglia* proposed the following problem. Three young couples want to cross a river. Only a small boat that can carry two people is available. The men are extremely jealous, and would never leave their brides with another man. How many journeys across the river are necessary? (Challenge 30 s)

∗ ∗

Cylinders can be used to roll a flat object over the floor, as shown in Figure 14. The cylinders keep the object plane always at the same distance from the floor. What cross-sections *other* than circular, so-called *curves of constant width*, can a cylinder have to realize the same feat? How many examples can you find? Are objects different than cylinders possible? (Challenge 31 s)

∗ ∗

Hanging pictures on a wall is not easy. First puzzle: what is the best way to hang a picture on one nail? The method must allow you to move the picture in horizontal position after the nail is in the wall, in the case that the weight is not equally distributed. (Challenge 32 s) Second puzzle: Can you hang a picture on a wall – this time with a long rope – over two nails in such a way that pulling either nail makes the picture fall? (Ref. 26) And with three nails? And n nails? (Challenge 33 s)

First summary on motion

Motion, the change of position of physical systems, is the most fundamental observation in nature. Everyday motion is predictable and deterministic. Predictability is reflected in six aspects of motion: continuity, conservation, reversibility, mirror-invariance, relativity and minimization. Some of these aspects are valid for *all* motion, and some are valid only for *everyday* motion. Which ones, and why? We explore this now. (Challenge 34 d)

* Niccolò Fontana Tartaglia (1499–1557), was an important Renaissance mathematician.

Chapter 2

FROM MOTION MEASUREMENT TO CONTINUITY

> Physic ist wahrlich das eigentliche Studium des Menschen.**
>
> Georg Christoph Lichtenberg

The simplest description of motion is the one we all, like cats or monkeys, use throughout our everyday life: *only one thing can be at a given spot at a given time.* This general description can be separated into three assumptions: matter is *impenetrable* and *moves*, time is made of *instants*, and space is made of *points*. Without these three assumptions (do you agree with them?) it is not even possible to define velocity. We thus need points embedded in continuous space and time to talk about motion. This description of nature is called *Galilean physics*, or sometimes *Newtonian physics*.

Challenge 35 s

Galileo Galilei (1564–1642), Tuscan professor of mathematics, was the central founder of modern physics. He became famous for advocating the importance of observations as checks of statements about nature. By requiring and performing these checks throughout his life, he was led to continuously increase the accuracy in the description of motion. For example, Galileo studied motion by measuring change of position with a self-constructed stopwatch. Galileo's experimental aim was to measure all that is measurable about motion. His approach changed the speculative description of ancient Greece into the experimental physics of Renaissance Italy.***

After Galileo, the English alchemist, occultist, theologian, physicist and politician Isaac Newton (1643–1727) continued to explore with vigour the idea that different types of motion have the same properties, and he made important steps in constructing the concepts necessary to demonstrate this idea.****

Above all, the explorations and books by Galileo popularized the fundamental experimental statements on the properties of speed, space and time.

** 'Physics truly is the proper study of man.' Georg Christoph Lichtenberg (b. 1742 Ober-Ramstadt, d. 1799 Göttingen) was an important physicist and essayist.

*** The best and most informative book on the life of Galileo and his times is by Pietro Redondi (see the section on page 335). Galileo was born in the year the pencil was invented. Before his time, it was impossible to do paper and pencil calculations. For the curious, the www.mpiwg-berlin.mpg.de website allows you to read an original manuscript by Galileo.

**** Newton was born a year after Galileo died. For most of his life Newton searched for the philosopher's stone. Newton's hobby, as head of the English mint, was to supervise personally the hanging of counterfeiters. About Newton's lifelong infatuation with alchemy, see the books by Dobbs. A misogynist throughout his life, Newton believed himself to be chosen by god; he took his Latin name, *Isaacus Neuutonus*, and formed the anagram *Jeova sanctus unus*. About Newton and his importance for classical mechanics, see the text by Clifford Truesdell.

Ref. 27 Ref. 28

FIGURE 15 Galileo Galilei (1564–1642).

FIGURE 16 Some speed measurement devices: an anemometer, a tachymeter for inline skates, a sports radar gun and a Pitot–Prandtl tube in an aeroplane (© Fachhochschule Koblenz, Silva, Tracer, Wikimedia).

What is velocity?

> “There is nothing else like it.
>
> Jochen Rindt*”

Velocity fascinates. To physicists, not only car races are interesting, but any moving entity is. Therefore, physicists first measure as many examples as possible. A selection of measured speed values is given in Table 3. The units and prefixes used are explained in Page 453 detail in Appendix B. Some speed measurement devices are shown in Figure 16.

Everyday life teaches us a lot about motion: objects can overtake each other, and they can move in different directions. We also observe that velocities can be added or changed smoothly. The precise list of these properties, as given in Table 4, is summarized by mathematicians in a special term; they say that velocities form a *Euclidean vector space*.** More Page 81 details about this strange term will be given shortly. For now we just note that in describing nature, mathematical concepts offer the most accurate vehicle.

When velocity is assumed to be a Euclidean vector, it is called *Galilean* velocity. Velocity is a profound concept. For example, velocity does not need space and time measurements to be defined. Are you able to find a means of measuring velocities without

* Jochen Rindt (1942–1970), famous Austrian Formula One racing car driver, speaking about speed.
** It is named after Euclid, or Eukleides, the great Greek mathematician who lived in Alexandria around 300 bce. Euclid wrote a monumental treatise of geometry, the Στοιχεῖα or *Elements*, which is one of the milestones of human thought. The text presents the whole knowledge of geometry of that time. For the first time, Euclid introduces two approaches that are now in common use: all statements are deduced from a small number of basic *axioms* and for every statement a *proof* is given. The book, still in print today, has been the reference geometry text for over 2000 years. On the web, it can be found at aleph0.clarku.edu/~djoyce/java/elements/elements.html.

TABLE 3 Some measured velocity values.

Observation	Velocity
Growth of deep sea manganese crust	80 am/s
Can you find something slower?	Challenge 36 s
Stalagmite growth	0.3 pm/s
Lichen growth	down to 7 pm/s
Typical motion of continents	10 mm/a = 0.3 nm/s
Human growth during childhood, hair growth	4 nm/s
Tree growth	up to 30 nm/s
Electron drift in metal wire	1 µm/s
Sperm motion	60 to 160 µm/s
Speed of light at Sun's centre Ref. 29	1 mm/s
Ketchup motion	1 mm/s
Slowest speed of light measured in matter on Earth Ref. 30	0.3 m/s
Speed of snowflakes	0.5 m/s to 1.5 m/s
Signal speed in human nerve cells Ref. 31	0.5 m/s to 120 m/s
Wind speed at 1 and 12 Beaufort (light air and hurricane)	<1.5 m/s, >33 m/s
Speed of rain drops, depending on radius	2 m/s to 8 m/s
Fastest swimming fish, sailfish (*Istiophorus platypterus*)	22 m/s
2009 Speed sailing record over 500 m (by trimaran Hydroptère)	26.4 m/s
Fastest running animal, cheetah (*Acinonyx jubatus*)	30 m/s
Speed of air in throat when sneezing	42 m/s
Fastest throw: a cricket ball thrown with baseball technique while running	50 m/s
Freely falling human, depending on clothing	50 to 90 m/s
Fastest bird, diving *Falco peregrinus*	60 m/s
Fastest badminton smash	70 m/s
Average speed of oxygen molecule in air at room temperature	280 m/s
Speed of sound in dry air at sea level and standard temperature	330 m/s
Speed of the equator	434 m/s
Cracking whip's end	750 m/s
Speed of a rifle bullet	1 km/s
Speed of crack propagation in breaking silicon	5 km/s
Highest macroscopic speed achieved by man – the *Helios II* satellite	70.2 km/s
Speed of Earth through universe	370 km/s
Average speed (and peak speed) of lightning tip	600 km/s (50 Mm/s)
Highest macroscopic speed measured in our galaxy Ref. 32	$0.97 \cdot 10^8$ m/s
Speed of electrons inside a colour TV tube	$1 \cdot 10^8$ m/s
Speed of radio messages in space	299 792 458 m/s
Highest ever measured group velocity of light	$10 \cdot 10^8$ m/s
Speed of light spot from a lighthouse when passing over the Moon	$2 \cdot 10^9$ m/s
Highest *proper* velocity ever achieved for electrons by man	$7 \cdot 10^{13}$ m/s
Highest possible velocity for a light spot or a shadow	no limit

TABLE 4 Properties of everyday – or Galilean – velocity.

Velocities can	Physical property	Mathematical name	Definition
Be distinguished	distinguishability	element of set	Vol. III, page 285
Change gradually	continuum	real vector space	Page 80, Vol. V, page 364
Point somewhere	direction	vector space, dimensionality	Page 80
Be compared	measurability	metricity	Vol. IV, page 236
Be added	additivity	vector space	Page 80
Have defined angles	direction	Euclidean vector space	Page 81
Exceed any limit	infinity	unboundedness	Vol. III, page 286

Challenge 37 d

measuring space and time? If so, you probably want to skip to the next volume, jumping 2000 years of enquiries. If you cannot do so, consider this: whenever we measure a quantity we assume that everybody is able to do so, and that everybody will get the same result. In other words, we define *measurement* as a comparison with a standard. We thus implicitly assume that such a standard exists, i.e., that an example of a 'perfect' velocity can be found. Historically, the study of motion did not investigate this question first, because for many centuries nobody could find such a standard velocity. You are thus in good company.

How is velocity measured in everyday life? Animals and people estimate their velocity in two ways: by estimating the frequency of their own movements, such as their steps, or by using their eyes, ears, sense of touch or sense of vibration to deduce how their own position changes with respect to the environment. But several animals have additional capabilities: certain snakes can determine speeds with their infrared-sensing organs, others with their magnetic field sensing organs. Still other animals emit sounds that create echoes in order to measure speeds to high precision. Other animals use the stars to navigate. A similar range of solutions is used by technical devices. Table 5 gives an overview.

Velocity is not always an easy subject. Physicists like to say, provokingly, that what cannot be measured does not exist. Can you measure your own velocity in empty interstellar space?

Challenge 38 s

Velocity is of interest to both engineers and evolution scientist. In general, self-propelled systems are faster the larger they are. As an example, Figure 17 shows how this applies to the cruise speed of flying things. In general, cruise speed scales with the sixth root of the weight, as shown by the trend line drawn in the graph. (Can you find out why?) By the way, similar *allometric scaling* relations hold for many other properties of moving systems, as we will see later on.

Challenge 39 d

Some researchers have specialized in the study of the lowest velocities found in nature: they are called geologists. Do not miss the opportunity to walk across a landscape while listening to one of them.

Ref. 33

Challenge 40 e

Velocity is a profound subject for an additional reason: we will discover that all its seven properties of Table 4 are only approximate; *none* is actually correct. Improved experiments will uncover exceptions for every property of Galilean velocity. The failure of

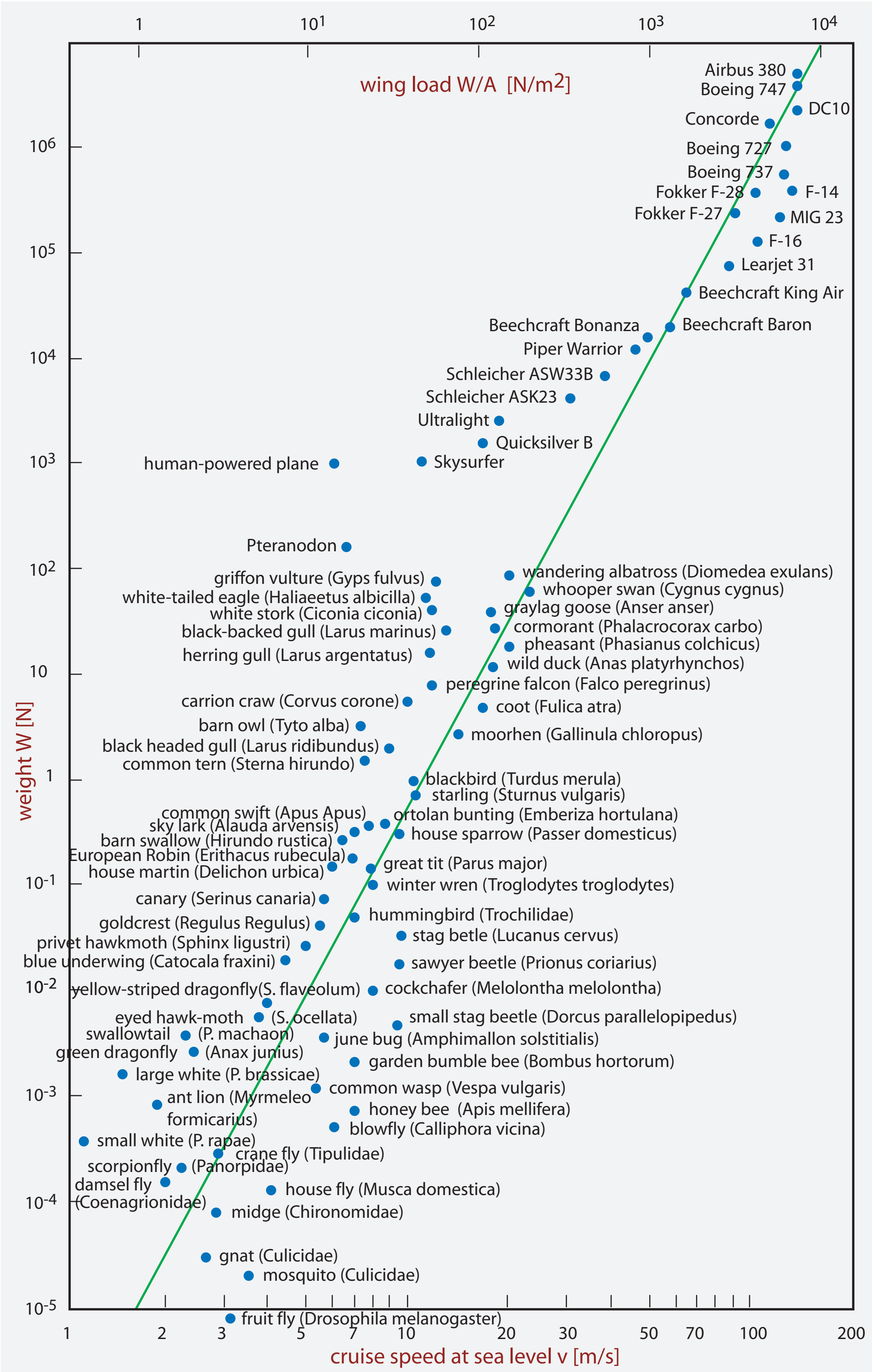


Figure 17 How wing load and sea-level cruise speed scales with weight in flying objects, compared with the general trend line (after a graph © Henk Tennekes).

TABLE 5 Speed measurement devices in biological and engineered systems.

Measurement	Device	Range
Own running speed in insects, mammals and humans	leg beat frequency measured with internal clock	0 to 33 m/s
Own car speed	tachymeter attached to wheels	0 to 150 m/s
Predators and hunters measuring prey speed	vision system	0 to 30 m/s
Police measuring car speed	radar or laser gun	0 to 90 m/s
Bat measuring own and prey speed at night	doppler sonar	0 to 20 m/s
Sliding door measuring speed of approaching people	doppler radar	0 to 3 m/s
Own swimming speed in fish and humans	friction and deformation of skin	0 to 30 m/s
Own swimming speed in dolphins and ships	sonar to sea floor	0 to 20 m/s
Diving speed in fish, animals, divers and submarines	pressure change	0 to 5 m/s
Water predators and fishing boats measuring prey speed	sonar	0 to 20 m/s
Own speed relative to Earth in insects	often none (grasshoppers)	n.a.
Own speed relative to Earth in birds	visual system	0 to 60 m/s
Own speed relative to Earth in aeroplanes or rockets	radio goniometry, radar	0 to 8000 m/s
Own speed relative to air in insects and birds	filiform hair deflection, feather deflection	0 to 60 m/s
Own speed relative to air in aeroplanes	Pitot–Prandtl tube	0 to 340 m/s
Wind speed measurement in meteorological stations	thermal, rotating or ultrasound anemometers	0 to 80 m/s
Swallows measuring prey speed	visual system	0 to 20 m/s
Bats measuring prey speed	sonar	0 to 20 m/s
Macroscopic motion on Earth	Global Positioning System, Galileo, Glonass	0 to 100 m/s
Pilots measuring target speed	radar	0 to 1000 m/s
Motion of stars	optical Doppler effect	0 to 1000 km/s
Motion of star jets	optical Doppler effect	0 to 200 Mm/s

the last three properties of Table 4 will lead us to special and general relativity, the failure of the middle two to quantum theory and the failure of the first two properties to the unified description of nature. But for now, we'll stick with Galilean velocity, and continue with another Galilean concept derived from it: time.

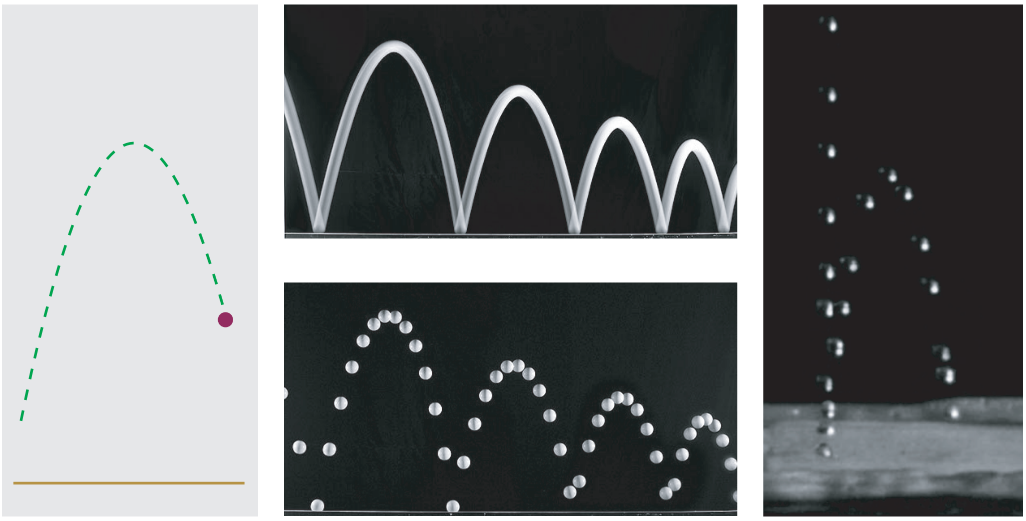


FIGURE 18 A typical path followed by a stone thrown through the air – a parabola – with photographs (blurred and stroboscopic) of a table tennis ball rebounding on a table (centre) and a stroboscopic photograph of a water droplet rebounding on a strongly hydrophobic surface (right, © Andrew Davidhazy, Max Groenendijk).

> Without the concepts *place*, *void* and *time*, change cannot be. [...] It is therefore clear [...] that their investigation has to be carried out, by studying each of them separately.
>
> Aristotle* *Physics*, Book III, part 1.

What is time?

> Time is an accident of motion.
>
> Theophrastus**

> Time does not exist in itself, but only through the perceived objects, from which the concepts of past, of present and of future ensue.
>
> Lucretius,*** *De rerum natura*, lib. 1, v. 460 ss.

In their first years of life, children spend a lot of time throwing objects around. The term 'object' is a Latin word meaning 'that which has been thrown in front.' Developmental psychology has shown experimentally that from this very experience children extract the concepts of time and space. Adult physicists do the same when studying motion at university.

Ref. 21

When we throw a stone through the air, we can define a *sequence* of observations. Figure 18 illustrates how. Our memory and our senses give us this ability. The sense of

* Aristotle (b. 384/3 Stageira, d. 322 BCE Euboea), important Greek philosopher and scientist, founder of the *Peripatetic school* located at the Lyceum, a gymnasium dedicated to Apollo Lyceus.
** Theophrastus of Eresos (c. 371 – c. 287) was a revered Lesbian philosopher, and successor of Aristoteles at the Lyceum.
*** Titus Lucretius Carus (*c.* 95 to *c.* 55 BCE), Roman scholar and poet.

TABLE 6 Selected time measurements.

Observation	Time
Shortest measurable time	10^{-44} s
Shortest time ever measured	10 ys
Time for light to cross a typical atom	0.1 to 10 as
Shortest laser light pulse produced so far	200 as
Period of caesium ground state hyperfine transition	108.782 775 707 78 ps
Beat of wings of fruit fly	1 ms
Period of pulsar (rotating neutron star) PSR 1913+16	0.059 029 995 271(2) s
Human 'instant'	20 ms
Shortest lifetime of living being	0.3 d
Average length of day 400 million years ago	79 200 s
Average length of day today	86 400.002(1) s
From birth to your 1000 million seconds anniversary	31.7 a
Age of oldest living tree	4600 a
Use of human language	0.2 Ma
Age of Himalayas	35 to 55 Ma
Age of oldest rocks, found in Isua Belt, Greenland and in Porpoise Cove, Hudson Bay	3.8 Ga
Age of Earth	4.6 Ga
Age of oldest stars	13.8 Ga
Age of most protons in your body	13.8 Ga
Lifetime of tantalum nucleus ^{180m}Ta	10^{15} a
Lifetime of bismuth ^{209}Bi nucleus	$1.9(2) \cdot 10^{19}$ a

hearing registers the various sounds during the rise, the fall and the landing of the stone. Our eyes track the location of the stone from one point to the next. All observations have their place in a sequence, with some observations preceding them, some observations simultaneous to them, and still others succeeding them. We say that observations are perceived to happen at various *instants* – also called 'points in time' – and we call the sequence of all instants *time*.

An observation that is considered the smallest part of a sequence, i.e., not itself a sequence, is called an *event*. Events are central to the definition of time; in particular, starting or stopping a stopwatch are events. (But do events really exist? Keep this question in the back of your head as we move on.) Challenge 41 s

Sequential phenomena have an additional property known as stretch, extension or duration. Some measured values are given in Table 6.* *Duration* expresses the idea that sequences *take* time. We say that a sequence takes time to express that other sequences can take place in parallel with it.

How exactly is the concept of time, including sequence and duration, deduced from observations? Many people have looked into this question: astronomers, physicists,

* A year is abbreviated a (Latin 'annus').

watchmakers, psychologists and philosophers. All find:

▷ Time is deduced by comparing motions.

This is even the case for children and animals. Beginning at a very young age, they Ref. 21 develop the concept of 'time' from the comparison of motions in their surroundings. Grown-ups take as a standard the motion of the Sun and call the resulting type of time *local time*. From the Moon they deduce a *lunar calendar*. If they take a particular village clock on a European island they call it the *universal time coordinate* (UTC), once known as 'Greenwich mean time.'* Astronomers use the movements of the stars and call the result *ephemeris time* (or one of its successors). An observer who uses his personal watch calls the reading his *proper time*; it is often used in the theory of relativity.

Not every movement is a good standard for time. In the year 2000, an Earth rotation Page 456 did not take 86 400 seconds any more, as it did in the year 1900, but 86 400.002 seconds. Can you deduce in which year your birthday will have shifted by a whole day from the Challenge 43 s time predicted with 86 400 seconds?

All methods for the definition of time are thus based on comparisons of motions. In order to make the concept as precise and as useful as possible, a *standard* reference motion is chosen, and with it a standard sequence and a standard duration is defined. The device that performs this task is called a *clock*. We can thus answer the question of the section title:

▷ Time is what we read from a clock.

Note that all definitions of time used in the various branches of physics are equivalent to this one; no 'deeper' or more fundamental definition is possible.** Note that the word 'moment' is indeed derived from the word 'movement'. Language follows physics in this case. Astonishingly, the definition of time just given is final; it will never be changed, not even at the top of Motion Mountain. This is surprising at first sight, because many books have been written on the nature of time. Instead, they should investigate the nature of motion!

▷ Every clock reminds us that in order to understand time, we need to understand motion.

But this is the aim of our walk anyhow. We are thus set to discover all the secrets of time as a side result of our adventure.

Time is not only an aspect of observations, it is also a facet of personal experience. Even in our innermost private life, in our thoughts, feelings and dreams, we experience sequences and durations. Children learn to relate this internal experience of time with

* Official UTC is used to determine the phase of the power grid, phone and internet companies' bit streams and the signal to the GPS system. The latter is used by many navigation systems around the world, especially in ships, aeroplanes and mobile phones. For more information, see the www.gpsworld.com website. The time-keeping infrastructure is also important for other parts of the modern economy. Can you spot the Challenge 42 s most important ones?

Ref. 34 ** The oldest clocks are sundials. The science of making them is called *gnomonics*.

TABLE 7 Properties of Galilean time.

Instants of time	Physical property	Mathematical name	Definition
Can be distinguished	distinguishability	element of set	Vol. III, page 285
Can be put in order	sequence	order	Vol. V, page 364
Define duration	measurability	metricity	Vol. IV, page 236
Can have vanishing duration	continuity	denseness, completeness	Vol. V, page 364
Allow durations to be added	additivity	metricity	Vol. IV, page 236
Don't harbour surprises	translation invariance	homogeneity	Page 238
Don't end	infinity	unboundedness	Vol. III, page 286
Are equal for all observers	absoluteness	uniqueness	

external observations, and to make use of the sequential property of events in their actions. Studies of the origin of psychological time show that it coincides – apart from its lack of accuracy – with clock time.* Every living human necessarily uses in his daily life the concept of time as a combination of sequence and duration; this fact has been checked in numerous investigations. For example, the term 'when' exists in all human languages. Ref. 36

Time is a concept *necessary* to distinguish between observations. In any sequence of observations, we observe that events succeed each other smoothly, apparently without end. In this context, 'smoothly' means that observations that are not too distant tend to be not too different. Yet between two instants, as close as we can observe them, there is always room for other events. Durations, or *time intervals*, measured by different people with different clocks agree in everyday life; moreover, all observers agree on the order of a sequence of events. Time is thus *unique* in everyday life. One also says that time is *absolute* in everyday life.

Time is necessary to distinguish between observations. For this reason, all observing devices that distinguish between observations, from brains to dictaphones and video cameras, have internal clocks. In particular, all animal brains have internal clocks. These brain clocks allow their users to distinguish between present, recent and past data and observations.

When Galileo studied motion in the seventeenth century, there were as yet no stopwatches. He thus had to build one himself, in order to measure times in the range between a fraction and a few seconds. Can you imagine how he did it? Challenge 44 s

If we formulate with precision all the properties of time that we experience in our daily life, we are lead to Table 7. This concept of time is called *Galilean time*. All its properties can be expressed simultaneously by describing time with the help of *real numbers*. In fact, real numbers have been constructed by mathematicians to have exactly the same properties as Galilean time, as explained in the chapter on the brain. In the case of Ga- Vol. III, page 295

* The brain contains numerous clocks. The most precise clock for short time intervals, the internal interval timer of the brain, is more accurate than often imagined, especially when trained. For time periods between a few tenths of a second, as necessary for music, and a few minutes, humans can achieve timing accuracies of a few per cent. Vol. V, page 42 Ref. 35

lilean time, every instant of time can be described by a real number, often abbreviated t. The duration of a sequence of events is then given by the difference between the time values of the final and the starting event.

We will have quite some fun with Galilean time in this part of our adventure. However, hundreds of years of close scrutiny have shown that *every single* property of Galilean time listed in Table 7 is approximate, and none is strictly correct. This story is told in the rest of our adventure.

Clocks

> "The most valuable thing a man can spend is time."
>
> Theophrastus

A *clock* is a moving system whose position can be read.

There are many types of clocks: stopwatches, twelve-hour clocks, sundials, lunar clocks, seasonal clocks, etc. A few are shown in Figure 19. Most of these clock types are
Ref. 37 also found in plants and animals, as shown in Table 8.

Ref. 38 Interestingly, there is a strict rule in the animal kingdom: large clocks go slow. How this happens is shown in Figure 20, another example of an *allometric scaling 'law'*.

A clock is a moving system whose position can be read. Of course, a *precise* clock is a system moving as regularly as possible, with as little outside disturbance as possible. Clockmakers are experts in producing motion that is as regular as possible. We
Page 181 will discover some of their tricks below. We will also explore, later on, the limits for the
Vol. V, page 45 precision of clocks.

Is there a perfect clock in nature? Do clocks exist at all? We will continue to study these questions throughout this work and eventually reach a surprising conclusion. At this point, however, we state a simple intermediate result: since clocks do exist, somehow
Challenge 45 s there is in nature an intrinsic, natural and *ideal* way to measure time. Can you see it?

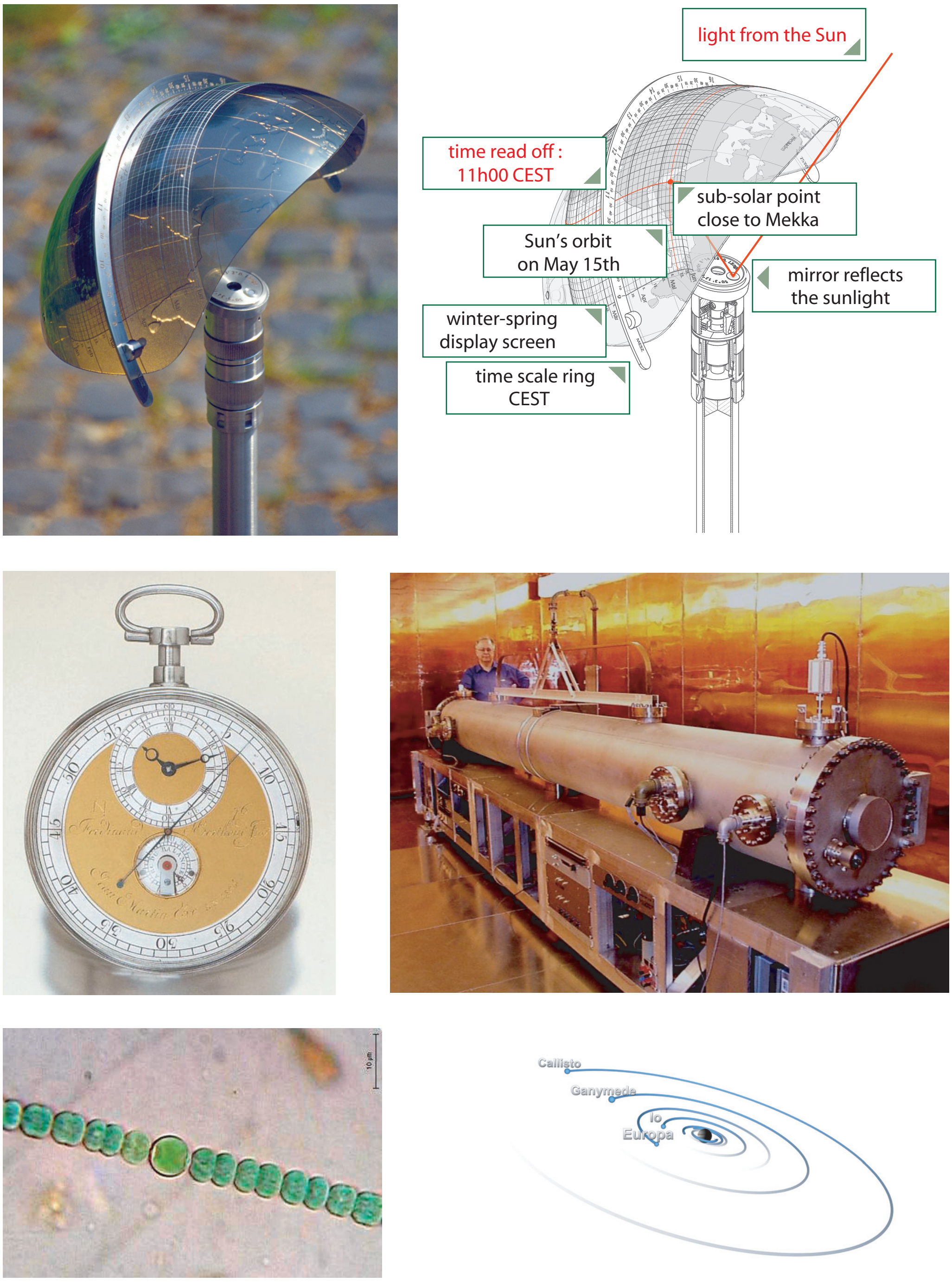


FIGURE 19 Different types of clocks: a high-tech sundial (size c. 30 cm), a naval pocket chronometer (size c. 6 cm), a caesium atomic clock (size c. 4 m), a group of cyanobacteria and the Galilean satellites of Jupiter (© Carlo Heller at www.heliosuhren.de, Anonymous, INMS, Wikimedia, NASA).

TABLE 8 Examples of biological rhythms and clocks.

Living being	Oscillating system	Period
Sand hopper (*Talitrus saltator*)	knows in which direction to flee from the position of the Sun or Moon	circadian
Human (*Homo sapiens*)	gamma waves in the brain	0.023 to 0.03 s
	alpha waves in the brain	0.08 to 0.13 s
	heart beat	0.3 to 1.5 s
	delta waves in the brain	0.3 to 10 s
	blood circulation	30 s
	cellular circhoral rhythms	1 to 2 ks
	rapid-eye-movement sleep period	5.4 ks
	nasal cycle	4 to 9 ks
	growth hormone cycle	11 ks
	suprachiasmatic nucleus (SCN), circadian hormone concentration, temperature, etc.; leads to jet lag	90 ks
	skin clock	circadian
	monthly period	2.4(4) Ms
	built-in aging	3.2(3) Gs
Common fly (*Musca domestica*)	wing beat	30 ms
Fruit fly (*Drosophila melanogaster*)	wing beat for courting	34 ms
Most insects (e.g. wasps, fruit flies)	winter approach detection (diapause) by length of day measurement; triggers metabolism changes	yearly
Algae (*Acetabularia*)	Adenosinetriphosphate (ATP) concentration	
Moulds (e.g. *Neurospora crassa*)	conidia formation	circadian
Many flowering plants	flower opening and closing	circadian
Tobacco plant	flower opening clock; triggered by length of days, discovered in 1920 by Garner and Allard	annual
Arabidopsis	circumnutation	circadian
	growth	a few hours
Telegraph plant (*Desmodium gyrans*)	side leaf rotation	200 s
Forsythia europaea, *F. suspensa*, *F. viridissima*, *F. spectabilis*	Flower petal oscillation, discovered by Van Gooch in 2002	5.1 ks

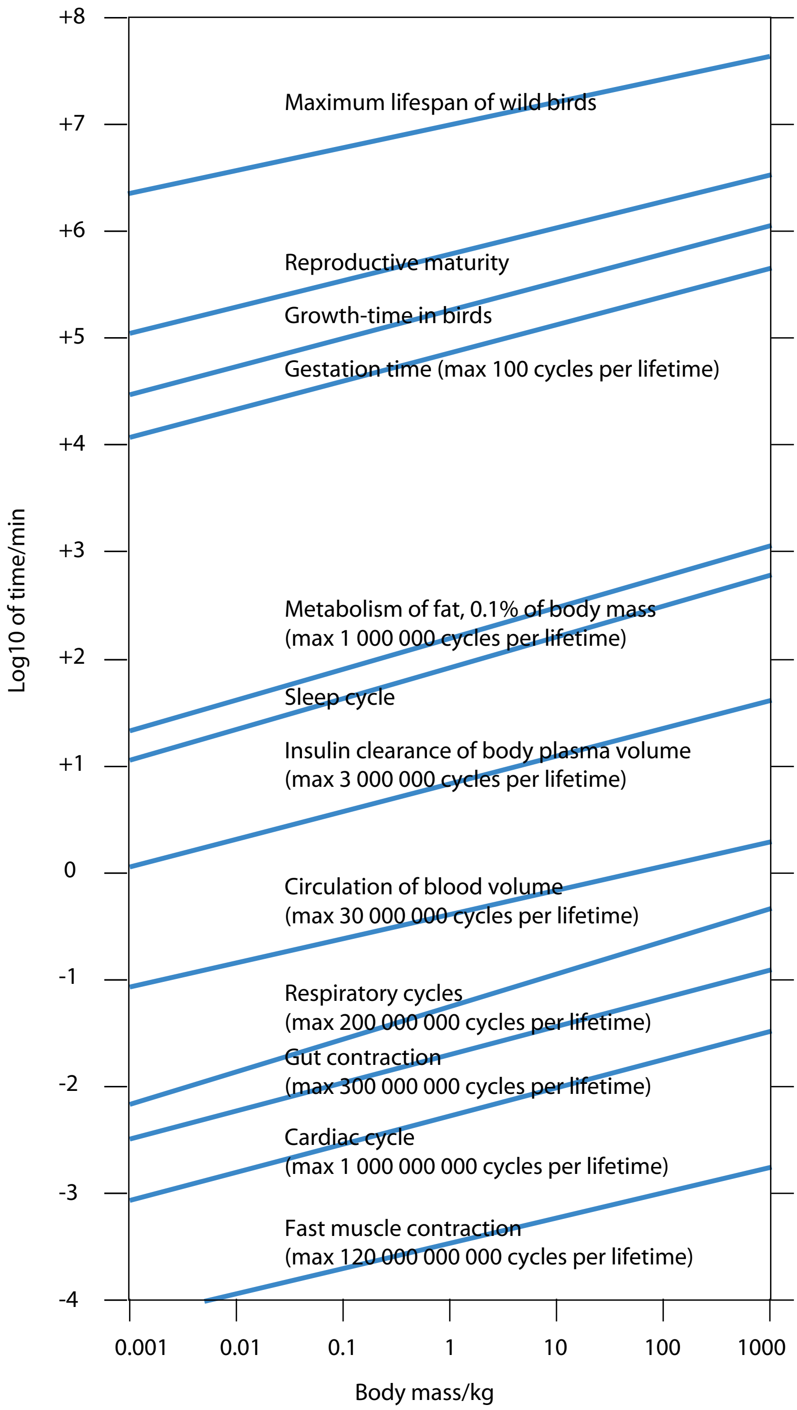


FIGURE 20 How biological rhythms scale with size in mammals: all scale more or less with a quarter power of the mass (after data from the EMBO and Enrique Morgado).

Why do clocks go clockwise?

Challenge 46 s

> What time is it at the North Pole now?

Most rotational motions in our society, such as athletic races, horse, bicycle or ice skating races, turn anticlockwise.* Mathematicians call this the positive rotation sense. Every supermarket leads its guests anticlockwise through the hall. Why? Most people are right-handed, and the right hand has more freedom at the outside of a circle. Therefore thousands of years ago chariot races in stadia went anticlockwise. As a result, all stadium races still do so to this day, and that is why runners move anticlockwise. For the same reason, helical stairs in castles are built in such a way that defending right-handers, usually from above, have that hand on the outside.

On the other hand, the clock imitates the shadow of sundials; obviously, this is true on the northern hemisphere only, and only for sundials on the ground, which were the most common ones. (The old trick to determine south by pointing the hour hand of a horizontal watch to the Sun and halving the angle between it and the direction of 12 o'clock does not work on the southern hemisphere – but there you can determine north in this way.) So every clock implicitly continues to state on which hemisphere it was invented. In addition, it also tells us that sundials on walls came in use much later than those on the floor.

Does time flow?

> Wir können keinen Vorgang mit dem 'Ablauf der Zeit' vergleichen – diesen gibt es nicht –, sondern nur mit einem anderen Vorgang (etwa dem Gang des Chronometers).**
>
> Ludwig Wittgenstein, *Tractatus*, 6.3611

> Si le temps est un fleuve, quel est son lit?***

The expression 'the flow of time' is often used to convey that in nature change follows after change, in a steady and continuous manner. But though the hands of a clock 'flow', time itself does not. Time is a concept introduced specially to describe the flow of events around us; it does not itself flow, it *describes* flow. Time does not advance. Time is neither linear nor cyclic. The idea that time flows is as hindering to understanding nature as is the idea that mirrors exchange right and left. Vol. III, page 90

The misleading use of the incorrect expression 'flow of time' was propagated first by some flawed Greek thinkers and then again by Newton. And it still continues. Aristotle, careful to think logically, pointed out its misconception, and many did so after him. Nevertheless, expressions such as 'time reversal', the 'irreversibility of time', and the much-abused 'time's arrow' are still common. Just read a popular science magazine Ref. 39

* Notable exceptions are most, but not all, Formula 1 races.
** 'We cannot compare any process with 'the passage of time' – there is no such thing – but only with another process (say, with the working of a chronometer).'
*** 'If time is a river, what is his bed?'

Challenge 47 e chosen at random. The fact is: time cannot be reversed, only motion can, or more precisely, only velocities of objects; time has no arrow, only motion has; it is not the flow of time that humans are unable to stop, but the motion of all the objects in nature. Incredibly, there are even books written by respected physicists that study different types Ref. 40 of 'time's arrows' and compare them with each other. Predictably, no tangible or new result is extracted.

▷ Time does *not* flow. Only bodies flow.

Time has no direction. Motion has. For the same reason, colloquial expressions such as 'the start (or end) of time' should be avoided. A motion expert translates them straight away into 'the start (or end) of motion'.

What is space?

> "The introduction of numbers as coordinates [...] is an act of violence [...]."
> Hermann Weyl, *Philosophie der Mathematik und Naturwissenschaft.**

Whenever we distinguish two objects from each other, such as two stars, we first of all distinguish their positions. We distinguish positions with our senses of sight, touch, proprioception and hearing. Position is therefore an important aspect of the physical state of an object. A position is taken by only one object at a time. Positions are limited. The set of all available positions, called *(physical) space*, acts as both a container and a background.

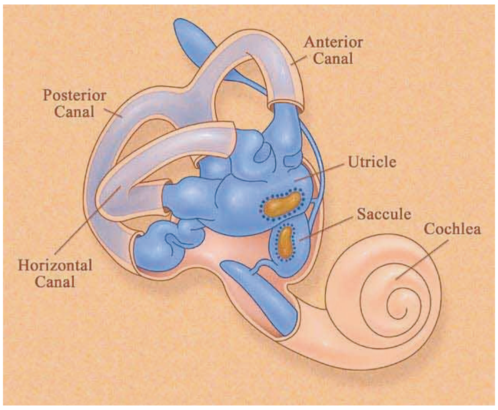
Closely related to space and position is *size*, the set of positions an object occupies. Small objects occupy only subsets of the positions occupied by large ones. We will discuss size in more detail shortly. Page 52

How do we deduce space from observations? During childhood, humans (and most higher animals) learn to bring together the various *perceptions* of space, namely the visual, the tactile, the auditory, the kinaesthetic, the vestibular etc., into one self-consistent set of experiences and description. The result of this learning process is a certain concept of space in the brain. Indeed, the question 'where?' can be asked and answered in all languages of the world. Being more precise, adults derive space from distance measurements. The concepts of length, area, volume, angle and solid angle are all deduced with their help. Geometers, surveyors, architects, astronomers, carpet salesmen and producers of metre sticks base their trade on distance measurements.

▷ Space is formed from all the position and distance relations between objects using metre sticks.

Humans developed metre sticks to specify distances, positions and sizes as accurately as possible.

* Hermann Weyl (1885–1955) was one of the most important mathematicians of his time, as well as an important theoretical physicist. He was one of the last universalists in both fields, a contributor to quantum theory and relativity, father of the term 'gauge' theory, and author of many popular texts.



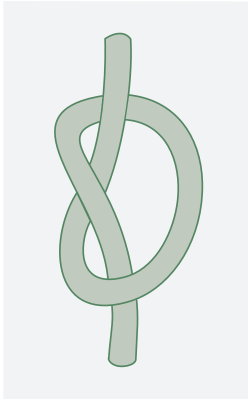


FIGURE 21 Two proofs that space has three dimensions: the vestibular labyrinth in the inner ear of mammals (here a human) with three canals and a knot (© Northwestern University).

Metre sticks work well only if they are straight. But when humans lived in the jungle, there were no straight objects around them. No straight rulers, no straight tools, nothing. Today, a cityscape is essentially a collection of straight lines. Can you describe how humans achieved this? Challenge 48 s

Once humans came out of the jungle with their newly built metre sticks, they collected a wealth of results. The main ones are listed in Table 9; they are easily confirmed by personal experience. In particular, objects can take positions in an apparently *continuous* manner: there indeed are more positions than can be counted.* Size is captured by defining the distance between various positions, called *length*, or by using the field of view an object takes when touched, called its *surface area*. Length and area can be measured with the help of a metre stick. (Selected measurement results are given in Table 10; some length measurement devices are shown in Figure 23.) The length of objects is independent of the person measuring it, of the position of the objects and of their orientation. In daily life the sum of angles in any triangle is equal to two right angles. There are no limits to distances, lengths and thus to space.

Experience shows us that space has three dimensions; we can define sequences of positions in precisely three independent ways. Indeed, the inner ear of (practically) all vertebrates has three semicircular canals that sense the body's acceleration in the three dimensions of space, as shown in Figure 21.** Similarly, each human eye is moved by three pairs of muscles. (Why three?) Challenge 49 s Another proof that space has three dimensions is provided by shoelaces: if space had more than three dimensions, shoelaces would not be useful, because knots exist only in three-dimensional space. But why does space have three dimensions? This is one of the most difficult question of physics. We leave it open for the time being.

It is often said that thinking in four dimensions is impossible. That is wrong. Just try. For example, can you confirm that in four dimensions knots are impossible? Challenge 50 s

Like time intervals, length intervals can be described most precisely with the help

* For a definition of uncountability, see page 288 in Volume III.

** Note that saying that space has three dimensions *implies* that space is continuous; the mathematician and philosopher Luitzen Brouwer (b. 1881 Overschie, d. 1966 Blaricum) showed that dimensionality is only a useful concept for continuous sets.

TABLE 9 Properties of Galilean space.

Points, or positions in space	Physical property	Mathematical name	Definition
Can be distinguished	distinguishability	element of set	Vol. III, page 285
Can be lined up if on one line	sequence	order	Vol. V, page 364
Can form shapes	shape	topology	Vol. V, page 363
Lie along three independent directions	possibility of knots	3-dimensionality	Page 81, Vol. IV, page 235
Can have vanishing distance	continuity	denseness, completeness	Vol. V, page 364
Define distances	measurability	metricity	Vol. IV, page 236
Allow adding translations	additivity	metricity	Vol. IV, page 236
Define angles	scalar product	Euclidean space	Page 81
Don't harbour surprises	translation invariance	homogeneity	
Can beat any limit	infinity	unboundedness	Vol. III, page 286
Defined for all observers	absoluteness	uniqueness	Page 52

of *real numbers*. In order to simplify communication, standard *units* are used, so that everybody uses the same numbers for the same length. Units allow us to explore the general properties of *Galilean space* experimentally: space, the container of objects, is continuous, three-dimensional, isotropic, homogeneous, infinite, Euclidean and unique – or 'absolute'. In mathematics, a structure or mathematical concept with all the properties just mentioned is called a three-dimensional *Euclidean space*. Its elements, *(mathematical) points*, are described by three real parameters. They are usually written as

$$(x, y, z) \tag{1}$$

and are called *coordinates*. They specify and order the location of a point in space. (For the precise definition of Euclidean spaces, see below.)

Page 81

What is described here in just half a page actually took 2000 years to be worked out, mainly because the concepts of 'real number' and 'coordinate' had to be discovered first. The first person to describe points of space in this way was the famous mathematician and philosopher René Descartes*, after whom the coordinates of expression (1) are named *Cartesian*.

Like time, space is a *necessary* concept to describe the world. Indeed, space is automatically introduced when we describe situations with many objects. For example, when many spheres lie on a billiard table, we cannot avoid using space to describe the relations between them. There is no way to avoid using spatial concepts when talking about nature.

Even though we need space to talk about nature, it is still interesting to ask why this is possible. For example, since many length measurement methods do exist – some are

* René Descartes or Cartesius (b. 1596 La Haye, d. 1650 Stockholm), mathematician and philosopher, author of the famous statement 'je pense, donc je suis', which he translated into 'cogito ergo sum' – I think therefore I am. In his view, this is the only statement one can be sure of.

FIGURE 22 René Descartes (1596–1650).

listed in Table 11 – and since they all yield consistent results, there must be a *natural* or *ideal* way to measure distances, sizes and straightness. Can you find it? Challenge 51 s

As in the case of time, each of the properties of space just listed has to be checked. And again, careful observations will show that each property is an approximation. In simpler and more drastic words, *all* of them are wrong. This confirms Weyl's statement at the beginning of this section. In fact, his statement about the violence connected with the introduction of numbers is told by every forest in the world. The rest of our adventure will show this.

> “Μέτρον ἄριστον.*”
>
> Cleobulus

Are space and time absolute or relative?

In everyday life, the concepts of Galilean space and time include two opposing aspects; the contrast has coloured every discussion for several centuries. On the one hand, space and time express something invariant and permanent; they both act like big *containers* for all the objects and events found in nature. Seen this way, space and time have an existence of their own. In this sense one can say that they are fundamental or *absolute*. On the other hand, space and time are tools of description that allow us to talk about relations between objects. In this view, they do not have any meaning when separated from objects, and only result from the relations between objects; they are derived, relational or *relative*. Which of these viewpoints do you prefer? The results of physics have alternately favoured one viewpoint or the other. We will repeat this alternation throughout our adventure, until we find the solution. And obviously, it will turn out to be a third option. Challenge 52 e Ref. 41

Size – why length and area exist, but volume does not

A central aspect of objects is their size. As a small child, under school age, every human learns how to use the properties of size and space in their actions. As adults seeking precision, with the definition of *distance* as the difference between coordinates allows us to define *length* in a reliable way. It took hundreds of years to discover that this is *not* the case. Several investigations in physics and mathematics led to complications.

The physical issues started with an astonishingly simple question asked by Lewis

* 'Measure is the best (thing).' Cleobulus (Κλεοβουλος) of Lindos, (*c.* 620–550 BCE) was another of the proverbial seven sages.

TABLE 10 Some measured distance values.

Observation	Distance
Galaxy Compton wavelength	10^{-85} m (calculated only)
Planck length, the shortest measurable length	10^{-35} m
Proton diameter	1 fm
Electron Compton wavelength	2.426 310 215(18) pm
Smallest air oscillation detectable by human ear	11 pm
Hydrogen atom size	30 pm
Size of small bacterium	0.2 μm
Wavelength of visible light	0.4 to 0.8 μm
Radius of sharp razor blade	5 μm
Point: diameter of smallest object visible with naked eye	20 μm
Diameter of human hair (thin to thick)	30 to 80 μm
Record diameter of hailstone	20 cm
Total length of DNA in each human cell	2 m
Longest human throw with any object, using a boomerang	427 m
Highest human-built structure, Burj Khalifa	828 m
Largest living thing, the fungus *Armillaria ostoyae*	3 km
Largest spider webs in Mexico	*c.* 5 km
Length of Earth's Equator	40 075 014.8(6) m
Total length of human blood vessels (rough estimate)	$4to16 \cdot 10^4$ km
Total length of human nerve cells (rough estimate)	$1.5to8 \cdot 10^5$ km
Average distance to Sun	149 597 870 691(30) m
Light year	9.5 Pm
Distance to typical star at night	10 Em
Size of galaxy	1 Zm
Distance to Andromeda galaxy	28 Zm
Most distant visible object	125 Ym

Richardson:* How long is the western coastline of Britain?

Following the coastline on a map using an odometer, a device shown in Figure 24, Richardson found that the length l of the coastline depends on the scale s (say 1 : 10 000 or 1 : 500 000) of the map used:

$$l = l_0 \, s^{0.25} \tag{2}$$

(Richardson found other exponentials for other coasts.) The number l_0 is the length at scale 1 : 1. The main result is that the larger the map, the longer the coastline. What would happen if the scale of the map were increased even beyond the size of the original? The length would increase beyond all bounds. Can a coastline really have *infinite* length? Yes, it can. In fact, mathematicians have described many such curves; nowadays, they

* Lewis Fray Richardson (1881–1953), English physicist and psychologist.

FIGURE 23 Three mechanical (a vernier caliper, a micrometer screw, a moustache) and three optical (the eyes, a laser meter, a light curtain) length and distance measurement devices (© www.medien-werkstatt.de, Naples Zoo, Keyence, and Leica Geosystems).

are called *fractals*. An infinite number of them exist, and Figure 25 shows one example.*

* Most of these curves are *self-similar*, i.e., they follow scaling 'laws' similar to the above-mentioned. The term 'fractal' is due to the mathematician Benoît Mandelbrot and refers to a strange property: in a certain sense, they have a non-integral number D of dimensions, despite being one-dimensional by construction. Mandelbrot saw that the non-integer dimension was related to the exponent e of Richardson by $D = 1 + e$,
Ref. 42 thus giving $D = 1.25$ in the example above. The number D varies from case to case. Measurements yield a value $D = 1.14$ for the land frontier of Portugal, $D = 1.13$ for the Australian coast and $D = 1.02$ for the South African coast.

TABLE 11 Length measurement devices in biological and engineered systems.

Measurement	Device	Range
Humans		
Measurement of body shape, e.g. finger distance, eye position, teeth distance	muscle sensors	0.3 mm to 2 m
Measurement of object distance	stereoscopic vision	1 to 100 m
Measurement of object distance	sound echo effect	0.1 to 1000 m
Animals		
Measurement of hole size	moustache	up to 0.5 m
Measurement of walking distance by desert ants	step counter	up to 100 m
Measurement of flight distance by honey bees	eye	up to 3 km
Measurement of swimming distance by sharks	magnetic field map	up to 1000 km
Measurement of prey distance by snakes	infrared sensor	up to 2 m
Measurement of prey distance by bats, dolphins, and hump whales	sonar	up to 100 m
Measurement of prey distance by raptors	vision	0.1 to 1000 m
Machines		
Measurement of object distance by laser	light reflection	0.1 m to 400 Mm
Measurement of object distance by radar	radio echo	0.1 to 50 km
Measurement of object length	interferometer	0.5 μm to 50 km
Measurement of star, galaxy or quasar distance	intensity decay	up to 125 Ym
Measurement of particle size	accelerator	down to 10^{-18} m

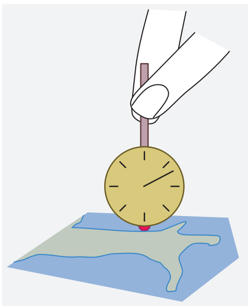



FIGURE 24 A curvimeter or odometer (photograph © Frank Müller).

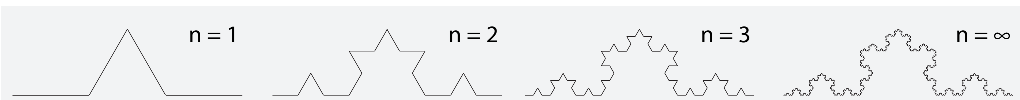


FIGURE 25 An example of a fractal: a self-similar curve of *infinite* length (far right), and its construction.

Challenge 53 e Can you construct another?

Length has other strange properties. The mathematician Giuseppe Vitali was the first to discover that it is possible to cut a line segment of length 1 into pieces that can be reas-

sembled – merely by shifting them in the direction of the segment – into a line segment of length 2. Are you able to find such a division using the hint that it is only possible using infinitely many pieces?
Challenge 54 d

To sum up

▷ Length exists. But length is well defined only for lines that are straight or nicely curved, but not for intricate lines, or for lines that can be cut into infinitely many pieces.

We therefore avoid fractals and other strangely shaped curves in the following, and we take special care when we talk about infinitely small segments. These are the central assumptions in the first five volumes of this adventure, and we should never forget them! We will come back to these assumptions in the last part of our adventure.

In fact, all these problems pale when compared with the following problem. Commonly, area and volume are defined using length. You think that it is easy? You're wrong, as well as being a victim of prejudices spread by schools around the world. To define area and volume with precision, their definitions must have two properties: the values must be *additive*, i.e., for finite and infinite sets of objects, the total area and volume must be the sum of the areas and volumes of each element of the set; and the values must be *rigid*, i.e., if we cut an area or a volume into pieces and then rearrange the pieces, the value must remain the same. Are such definitions possible? In other words, do such concepts of volume and area exist?

For areas in a plane, we proceed in the following standard way: we define the area A of a rectangle of sides a and b as $A = ab$; since any polygon can be rearranged into a rectangle with a finite number of straight cuts, we can then define an area value for all polygons. Subsequently, we can define the area for nicely curved shapes as the limit of the sum of infinitely many polygons. This method is called *integration*; it is introduced in detail in the section on physical action.
Challenge 55 s
Page 251

However, integration does not allow us to define area for arbitrarily bounded regions. (Can you imagine such a region?) For a complete definition, more sophisticated tools are needed. They were discovered in 1923 by the famous mathematician Stefan Banach.* He proved that one can indeed define an area for any set of points whatsoever, even if the border is not nicely curved but extremely complicated, such as the fractal curve previously mentioned. Today this generalized concept of area, technically a 'finitely additive isometrically invariant measure,' is called a *Banach measure* in his honour. Mathematicians sum up this discussion by saying that since in two dimensions there is a Banach measure, there is a way to define the concept of area – an additive and rigid measure – for any set of points whatsoever.** In short,
Challenge 56 s

▷ Area exists. Area is well defined for plane and other nicely behaved surfaces,

* Stefan Banach (b. 1892 Krakow, d. 1945 Lvov), important mathematician.

** Actually, this is true only for sets on the plane. For curved surfaces, such as the surface of a sphere, there are complications that will not be discussed here. In addition, the problems mentioned in the definition of length of fractals also reappear for area if the surface to be measured is not flat. A typical example is the area of the human lung: depending on the level of details examined, the area values vary from a few up to over a hundred square metres.

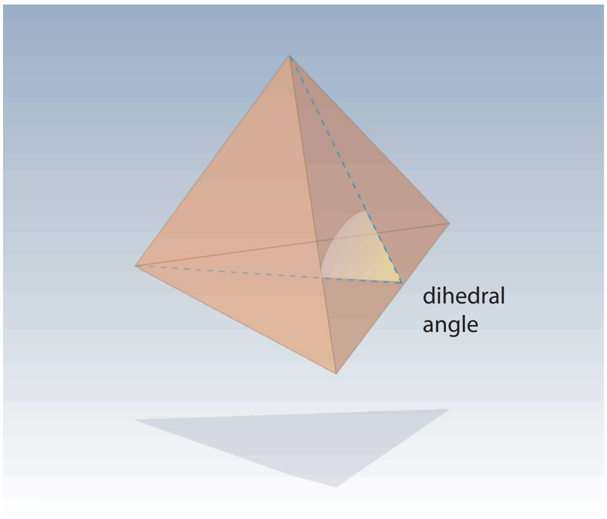


FIGURE 26 A polyhedron with one of its dihedral angles (© Luca Gastaldi).

but not for intricate shapes.

What is the situation in *three* dimensions, i.e., for volume? We can start in the same way as for area, by defining the volume V of a rectangular polyhedron with sides a, b, c as $V = abc$. But then we encounter a first problem: a general polyhedron cannot be cut into a cube by straight cuts! The limitation was discovered in 1900 and 1902 by Max Dehn.* He found that the possibility depends on the values of the edge angles, or dihedral angles, as the mathematicians call them. (They are defined in Figure 26.) If one ascribes to every edge of a general polyhedron a number given by its length l times a special function $g(\alpha)$ of its dihedral angle α, then Dehn found that the sum of all the numbers for all the edges of a solid does not change under dissection, provided that the function fulfils $g(\alpha + \beta) = g(\alpha) + g(\beta)$ and $g(\pi) = 0$. An example of such a strange function g is the one assigning the value 0 to any rational multiple of π and the value 1 to a basis set of irrational multiples of π. The values for all other dihedral angles of the polyhedron can then be constructed by combination of rational multiples of these basis angles. Using this
Challenge 57 s function, you may then deduce for yourself that a cube cannot be dissected into a regular tetrahedron because their respective Dehn invariants are different.**

Despite the problems with Dehn invariants, a rigid and additive concept of volume for polyhedra does exist, since for all polyhedra and, in general, for all 'nicely curved' shapes, the volume can be defined with the help of integration.

Now let us consider general shapes and general cuts in three dimensions, not just the 'nice' ones mentioned so far. We then stumble on the famous *Banach–Tarski theorem*
Ref. 43 (or paradox). In 1924, Stefan Banach and Alfred Tarski*** proved that it is possible to

* Max Dehn (b. 1878 Hamburg, d. 1952 Black Mountain), mathematician, student of David Hilbert.
** This is also told in the beautiful book by M. Aigler & G. M. Ziegler, *Proofs from the Book*, Springer Verlag, 1999. The title is due to the famous habit of the great mathematician Paul Erdős to imagine that all beautiful mathematical proofs can be assembled in the 'book of proofs'.
*** Alfred Tarski (b. 1902 Warsaw, d. 1983 Berkeley), influential mathematician.

FIGURE 27 Straight lines found in nature: cerussite (picture width approx. 3 mm, © Stephan Wolfsried) and selenite (picture width approx. 15 m, © Arch. Speleoresearch & Films/La Venta at www.laventa.it and www.naica.com.mx).

cut one sphere into five pieces that can be recombined to give *two* spheres, each the size of the original. This counter-intuitive result is the Banach–Tarski theorem. Even worse, another version of the theorem states: take any two sets not extending to infinity and containing a solid sphere each; then it is always possible to dissect one into the other with a *finite* number of cuts. In particular it is possible to dissect a pea into the Earth, or vice versa. Size does not count!* In short, volume is thus not a useful concept at all!

The Banach–Tarski theorem raises two questions: first, can the result be applied to gold or bread? That would solve many problems. Second, can it be applied to empty space? In other words, are matter and empty space continuous? Both topics will be explored later in our walk; each issue will have its own, special consequences. For the moment, we eliminate this troubling issue by restricting our interest – again – to smoothly curved shapes (and cutting knives). With this restriction, volumes of matter and of empty space do behave nicely: they are additive and rigid, and show no paradoxes.** Indeed, the cuts required for the Banach–Tarski paradox are not smooth; it is not possible to perform them with an everyday knife, as they require (infinitely many) infinitely sharp bends performed with an infinitely sharp knife. Such a knife does not exist. Nevertheless, we keep in the back of our mind that the size of an object or of a piece of empty space is a tricky quantity – and that we need to be careful whenever we talk about it.

Challenge 58 s

In summary,

▷ Volume only exists as an approximation. Volume is well-defined only for regions with smooth surfaces. Volume does not exist in general, when infinitely sharp cuts are allowed.

We avoid strangely shaped volumes, surfaces and curves in the following, and we take special care when we talk about infinitely small entities. We can talk about length, area and volume *only* with this restriction. This avoidance is a central assumption in the first

* The proof of the result does not need much mathematics; it is explained beautifully by Ian Stewart in *Paradox of the spheres*, New Scientist, 14 January 1995, pp. 28–31. The proof is based on the axiom of choice, which is presented later on. The Banach–Tarski paradox also exists in four dimensions, as it does in any higher dimension. More mathematical detail can be found in the beautiful book by Stan Wagon.

Vol. III, page 286
Ref. 44

** Mathematicians say that a so-called *Lebesgue measure* is sufficient in physics. This countably additive isometrically invariant measure provides the most general way to define a volume.

FIGURE 28 A photograph of the Earth – seen from the direction of the Sun (NASA).

five volumes of this adventure. Again: we should never forget these restrictions, even though they are not an issue in everyday life. We will come back to the assumptions at the end of our adventure.

What is straight?

When you see a solid object with a straight edge, it is a 99 %-safe bet that it is man-made. Of course, there are exceptions, as shown in Figure 27.* The largest crystals ever Ref. 46 found are 18 m in length. But in general, the contrast between the objects seen in a city – buildings, furniture, cars, electricity poles, boxes, books – and the objects seen in a forest – trees, plants, stones, clouds – is evident: in the forest no object is straight or flat, whereas in the city most objects are.

Page 479 Any forest teaches us the origin of straightness; it presents tall tree trunks and rays of daylight entering from above through the leaves. For this reason we call a line *straight* if it touches either a plumb-line or a light ray along its whole length. In fact, the two definitions are equivalent. Can you confirm this? Can you find another definition? Obviously, Challenge 60 s we call a surface *flat* if for any chosen orientation and position the surface touches a plumb-line or a light ray along its whole extension.

Challenge 59 ny * Why do crystals have straight edges? Another example of straight lines in nature, unrelated to atomic Page 429 structures, is the well-known Irish geological formation called the Giant's Causeway. Other candidates that might come to mind, such as certain bacteria which have (almost) square or (almost) triangular shapes are Ref. 45 not counter-examples, as the shapes are only approximate.

FIGURE 29 A model illustrating the hollow Earth theory, showing how day and night appear (© Helmut Diehl).

In summary, the concept of straightness – and thus also of flatness – is defined with the help of bodies or radiation. In fact, all spatial concepts, like all temporal concepts, require motion for their definition.

A hollow Earth?

Space and straightness pose subtle challenges. Some strange people maintain that all humans live on the *inside* of a sphere; they call this the *hollow Earth model*. They claim that Ref. 47 the Moon, the Sun and the stars are all near the centre of the hollow sphere, as illustrated in Figure 29. They also explain that light follows curved paths in the sky and that when conventional physicists talk about a distance r from the centre of the Earth, the real hol- Challenge 61 s low Earth distance is $r_{he} = R^2_{Earth}/r$. Can you show that this model is wrong? Roman Sexl* used to ask this question to his students and fellow physicists.

The answer is simple: if you think you have an argument to show that the hollow Earth model is wrong, you are mistaken! There is *no way* of showing that such a view is wrong. It is possible to explain the horizon, the appearance of day and night, as well as the Challenge 62 e satellite photographs of the round Earth, such as Figure 28. To explain what happened during a flight to the Moon is also fun. A consistent hollow Earth view is fully *equivalent* to the usual picture of an infinitely extended space. We will come back to this problem Vol. II, page 285 in the section on general relativity.

* Roman Sexl, (1939–1986), important Austrian physicist, author of several influential textbooks on gravitation and relativity.

Curiosities and fun challenges about everyday space and time

Challenge 63 s

How does one measure the speed of a gun bullet with a stopwatch, in a space of $1\,\mathrm{m}^3$, without electronics? Hint: the same method can also be used to measure the speed of light.

* *

For a striking and interactive way to zoom through all length scales in nature, from the Planck length to the size of the universe, see the website www.htwins.net/scale2/.

* *

Challenge 64 s

What is faster: an arrow or a motorbike?

* *

Challenge 65 s

Why are manholes always round?

* *

Ref. 48

Apart from the speed of light, another speed is important in nature: the speed of ear growth. It was determined in published studies which show that on average, for old men, age t and ear circumference e are related by $e = t0.51\,\mathrm{mm/a} + 88.1\,\mathrm{mm}$. In the units of the expression, 'a', from Latin 'annus', is the international abbreviation for 'year'.

* *

Challenge 66 e

Can you show to a child that the sum of the angles in a triangle equals two right angles? What about a triangle on a sphere or on a saddle?

* *

Do you own a glass whose height is larger than its maximum circumference?

* *

Challenge 67 e

A gardener wants to plant nine trees in such a way that they form ten straight lines with three trees each. How does he do it?

* *

Challenge 68 d

How fast does the grim reaper walk? This question is the title of a publication in the British Medial Journal from the year 2011. Can you imagine how it is answered?

* *

Time measurements require periodic phenomena. Tree rings are traces of the seasons. Glaciers also have such traces, the *ogives*. Similar traces are found in teeth. Do you know more examples?

* *

A man wants to know how many stairs he would have to climb if the escalator in front of him, which is running upwards, were standing still. He walks up the escalator and counts 60 stairs; walking down the same escalator with the same speed he counts 90 stairs. What

FIGURE 30 At what height is a conical glass half full?

Challenge 69 s is the answer?

* *

You have two hourglasses: one needs 4 minutes and one needs 3 minutes. How can you use them to determine when 5 minutes are over? Challenge 70 e

* *

You have two fuses of different length that each take one minute to burn. You are not allowed to bend them nor to use a ruler. How do you determine that 45 s have gone by? Now the tougher puzzle: How do you determine that 10 s have gone by with a single fuse? Challenge 71 e

* *

You have three wine containers: a full one of 8 litres, an empty one of 5 litres, and another empty one of 3 litres. How can you use them to divide the wine evenly into two? Challenge 72 e

* *

Challenge 73 s How can you make a hole in a postcard that allows you to step through it?

* *

What fraction of the height of a conical glass, shown in Figure 30, must be filled to make the glass half full? Challenge 74 s

* *

Challenge 75 s How many pencils are needed to draw a line as long as the Equator of the Earth?

* *

Challenge 76 e Can you place five equal coins so that each one touches the other four? Is the stacking of two layers of three coins, each layer in a triangle, a solution for six coins? Why?

What is the smallest number of coins that can be laid flat on a table so that every coin is touching exactly three other coins? Challenge 77 e

* *

Challenge 78 e Can you find three crossing points on a chessboard that lie on an equilateral triangle?

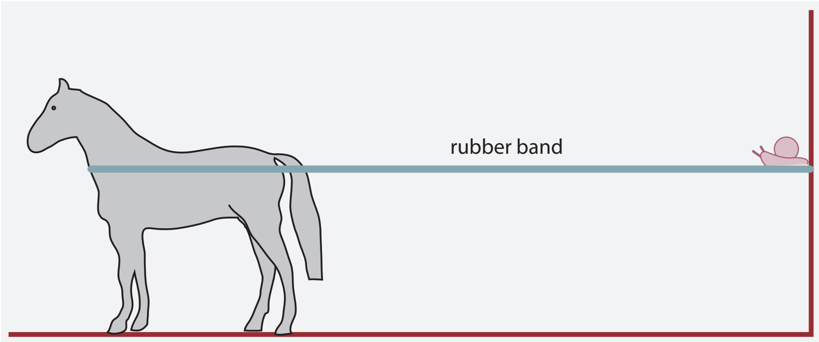


FIGURE 31 Can the snail reach the horse once it starts galloping away?

* *

The following bear puzzle is well known: A hunter leaves his home, walks 10 km to the South and 10 km to the West, shoots a bear, walks 10 km to the North, and is back home. What colour is the bear? You probably know the answer straight away. Now comes the harder question, useful for winning money in bets. The house could be on several *additional* spots on the Earth; where are these less obvious spots from which a man can have *exactly* the same trip (forget the bear now) that was just described and be at home again?

Challenge 79 s

* *

Imagine a rubber band that is attached to a wall on one end and is attached to a horse at the other end, as shown in Figure 31. On the rubber band, near the wall, there is a snail. Both the snail and the horse start moving, with typical speeds – with the rubber being infinitely stretchable. Can the snail reach the horse?

Challenge 80 s

* *

For a mathematician, 1 km is the same as 1000 m. For a physicist the two are different! Indeed, for a physicist, 1 km is a measurement lying between 0.5 km and 1.5 km, whereas 1000 m is a measurement between 999.5 m and 1000.5 m. So be careful when you write down measurement values. The professional way is to write, for example, 1000(8) m to mean 1000 ± 8 m, i.e., a value that lies between 992 and 1008 m with a probability of 68.3 %.

Page 459

* *

Imagine a black spot on a white surface. What is the colour of the line separating the spot from the background? This question is often called Peirce's puzzle.

Challenge 81 s

* *

Also bread is an (approximate) fractal, though an irregular one. The fractal dimension of bread is around 2.7. Try to measure it!

Challenge 82 s

* *

Challenge 83 e

How do you find the centre of a beer mat using paper and pencil?

* *

Challenge 84 s How often in 24 hours do the hour and minute hands of a clock lie on top of each other? For clocks that also have a second hand, how often do all three hands lie on top of each other?

* *

Challenge 85 s How often in 24 hours do the hour and minute hands of a clock form a right angle?

* *

Challenge 86 s How many times in twelve hours can the two hands of a clock be *exchanged* with the result that the new situation shows a *valid* time? What happens for clocks that also have a third hand for seconds?

* *

Challenge 87 s How many minutes does the Earth rotate in one minute?

* *

Challenge 88 s What is the highest speed achieved by throwing (with and without a racket)? What was the projectile used?

* *

Challenge 89 s A rope is put around the Earth, on the Equator, as tightly as possible. The rope is then lengthened by 1 m. Can a mouse slip under the rope? The original, tight rope is lengthened by 1 mm. Can a child slip under the rope?

* *

Challenge 90 s Jack was rowing his boat on a river. When he was under a bridge, he dropped a ball into the river. Jack continued to row in the same direction for 10 minutes after he dropped the ball. He then turned around and rowed back. When he reached the ball, the ball had floated 600 m from the bridge. How fast was the river flowing?

* *

Challenge 91 e Adam and Bert are brothers. Adam is 18 years old. Bert is twice as old as at the time when Adam was the age that Bert is now. How old is Bert?

* *

Challenge 92 s 'Where am I?' is a common question; 'When am I?' is almost never asked, not even in other languages. Why?

* *

Challenge 93 s Is there a smallest time interval in nature? A smallest distance?

* *

Challenge 94 s Given that you know what straightness is, how would you characterize or define the curvature of a curved line using numbers? And that of a surface?

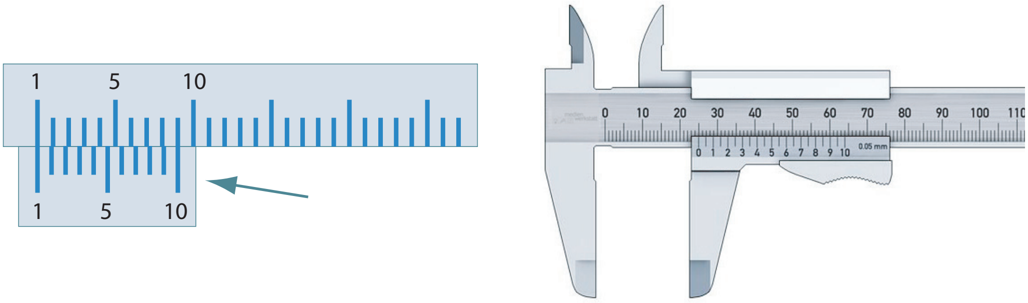


FIGURE 32 A 9-to-10 vernier/nonius/clavius and a 19-to-20 version (in fact, a 38-to-40 version) in a caliper (© www.medien-werkstatt.de).

* *

Challenge 95 s What is the speed of your eyelid?

* *

The surface area of the human body is about 400 m^2. Can you say where this large number comes from? Challenge 96 s

* *

How does a *vernier* work? It is called *nonius* in other languages. The first name is derived from a French military engineer* who did not invent it, the second is a play of words on the Latinized name of the Portuguese inventor of a more elaborate device** and the Latin word for 'nine'. In fact, the device as we know it today – shown in Figure 32 – was designed around 1600 by Christophonius Clavius,*** the same astronomer whose studies were the basis of the Gregorian calendar reform of 1582. Are you able to design a vernier/nonius/clavius that, instead of increasing the precision tenfold, does so by an arbitrary factor? Is there a limit to the attainable precision? Challenge 97 s

* *

Page 55 Fractals in three dimensions bear many surprises. Let us generalize Figure 25 to three dimensions. Take a regular tetrahedron; then glue on every one of its triangular faces a smaller regular tetrahedron, so that the surface of the body is again made up of many equal regular triangles. Repeat the process, glueing still smaller tetrahedrons to these new (more numerous) triangular surfaces. What is the shape of the final fractal, after an infinite number of steps? Challenge 98 s

* *

Motoring poses many mathematical problems. A central one is the following *parallel parking* challenge: what is the shortest distance d from the car in front necessary to leave a parking spot without using reverse gear? (Assume that you know the geometry of your Challenge 99 s

* Pierre Vernier (1580–1637), French military officer interested in cartography.
** Pedro Nuñes or Peter Nonnius (1502–1578), Portuguese mathematician and cartographer.
*** Christophonius Clavius or Schlüssel (1537–1612), Bavarian astronomer, one of the main astronomers of his time.

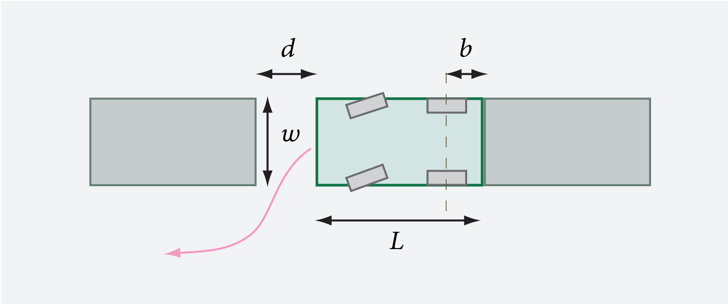


FIGURE 33 Leaving a parking space.

TABLE 12 The exponential notation: how to write small and large numbers.

Number	Exponential notation	Number	Exponential notation
1	10^0		
0.1	10^{-1}	10	10^1
0.2	$2 \cdot 10^{-1}$	20	$2 \cdot 10^1$
0.0324	$3.24 \cdot 10^{-2}$	32.4	$3.24 \cdot 10^1$
0.01	10^{-2}	100	10^2
0.001	10^{-3}	1000	10^3
0.000 1	10^{-4}	10 000	10^4
0.000 056	$5.6 \cdot 10^{-5}$	56 000	$5.6 \cdot 10^4$
0.000 01	10^{-5} etc.	100 000	10^5 etc.

car, as shown in Figure 33, and its smallest outer turning radius R, which is known for every car.) Next question: what is the smallest gap required when you are allowed to manoeuvre back and forward as often as you like? Now a problem to which no solution seems to be available in the literature: How does the gap depend on the number, n, of times you use reverse gear? (The author had offered 50 euro for the first well-explained solution; the winning solution by Daniel Hawkins is now found in the appendix.)

Challenge 100 s

Challenge 101 s

* *

Scientists use a special way to write large and small numbers, explained in Table 12.

* *

Ref. 49

In 1996 the smallest experimentally probed distance was 10^{-19} m, achieved between quarks at Fermilab. (To savour the distance value, write it down without the exponent.) What does this measurement mean for the continuity of space?

Challenge 102 s

* *

Zeno, the Greek philosopher, discussed in detail what happens to a moving object at a given instant of time. To discuss with him, you decide to build the fastest possible shutter for a photographic camera that you can imagine. You have all the money you want. What

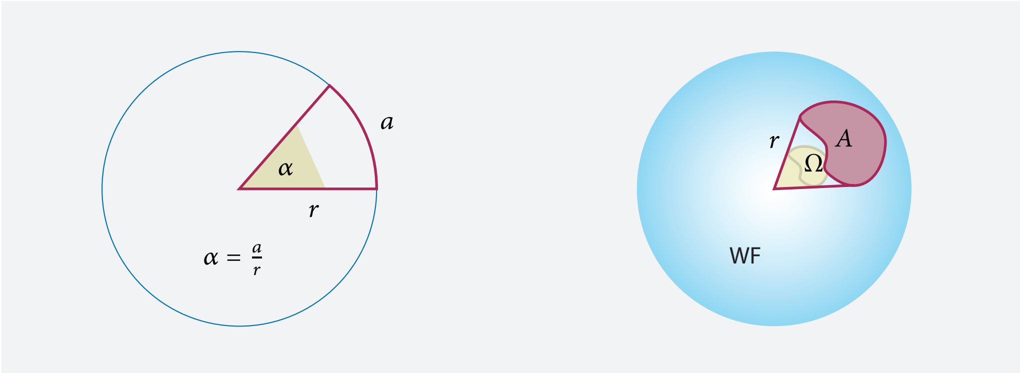


FIGURE 34 The definition of plane and solid angles.

Challenge 103 s is the shortest shutter time you would achieve?

* *

Can you prove Pythagoras' theorem by geometrical means alone, without using coordinates? (There are more than 30 possibilities.) Challenge 104 s

* *

Page 59 Why are most planets and moons, including ours, (almost) spherical (see, for example, Figure 28)? Challenge 105 s

* *

A rubber band connects the tips of the two hands of a clock. What is the path followed by the mid-point of the band? Challenge 106 s

* *

There are two important quantities connected to angles. As shown in Figure 34, what is usually called a *(plane) angle* is defined as the ratio between the lengths of the arc and the radius. A right angle is $\pi/2$ *radian* (or $\pi/2$ rad) or 90°.

The *solid angle* is the ratio between area and the square of the radius. An eighth of a sphere is $\pi/2$ *steradian* or $\pi/2$ sr. (Mathematicians, of course, would simply leave out the steradian unit.) As a result, a small solid angle shaped like a cone and the angle of the cone tip are *different*. Can you find the relationship? Challenge 107 s

* *

The definition of angle helps to determine the size of a firework display. Measure the time T, in seconds, between the moment that you see the rocket explode in the sky and the moment you hear the explosion, measure the (plane) angle α – pronounced 'alpha' – of the ball formed by the firework with your hand. The diameter D is

$$D \approx \frac{6\,\mathrm{m}}{\mathrm{s}\,^\circ} T \alpha\,. \tag{3}$$

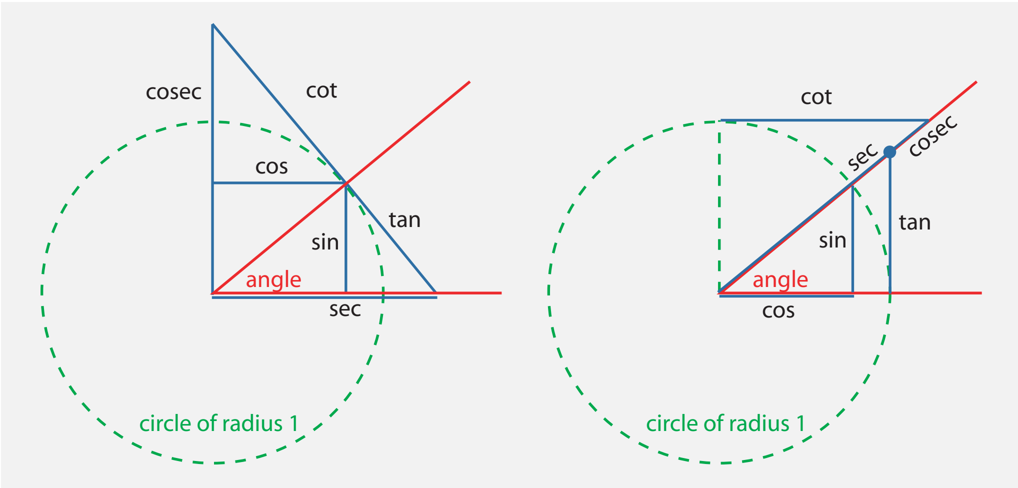


FIGURE 35 Two equivalent definitions of the sine, cosine, tangent, cotangent, secant and cosecant of an angle.

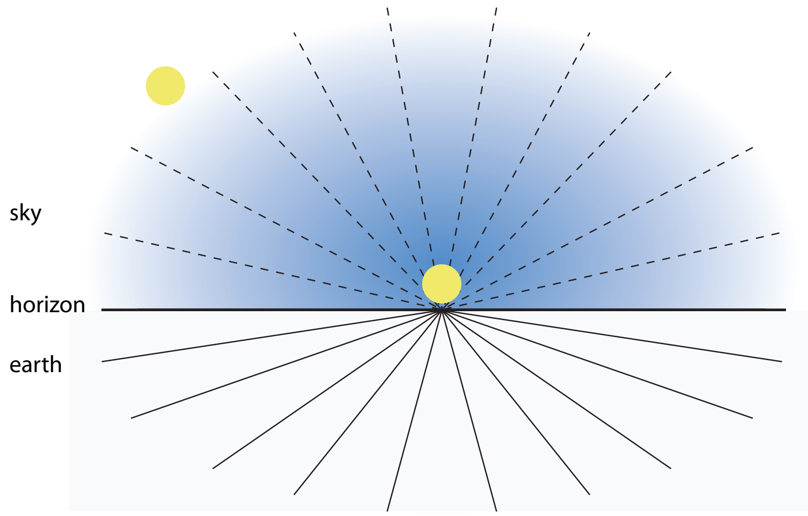


FIGURE 36 How the apparent size of the Moon and the Sun changes during a day.

Challenge 108 e
Why? For more information about fireworks, see the cc.oulu.fi/~kempmp website. By the way, the angular distance between the knuckles of an extended fist are about 3°, 2° and 3°, the size of an extended hand 20°. Can you determine the other angles related to your hand?
Challenge 109 s

* *

Using angles, the *sine*, *cosine*, *tangent*, *cotangent*, *secant* and *cosecant* can be defined, as shown in Figure 35. You should remember this from secondary school. Can you confirm
Challenge 110 e
that $\sin 15° = (\sqrt{6} - \sqrt{2})/4$, $\sin 18° = (-1 + \sqrt{5})/4$, $\sin 36° = \sqrt{10 - 2\sqrt{5}}/4$, $\sin 54° =$

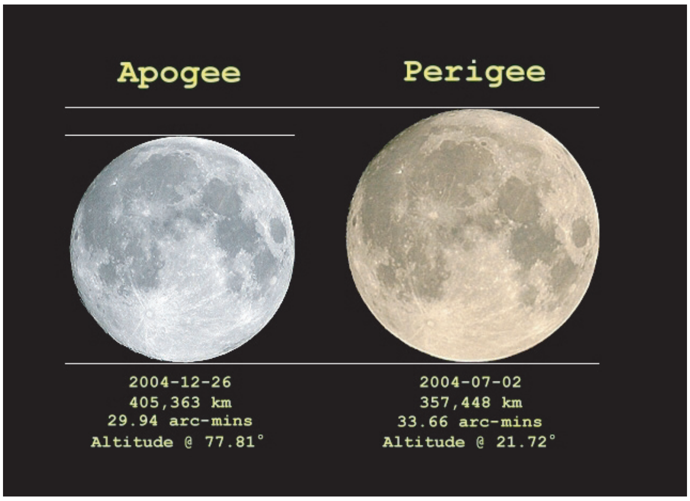


FIGURE 37 How the size of the Moon actually changes during its orbit (© Anthony Ayiomamitis).

$(1+\sqrt{5})/4$ and that $\sin 72° = \sqrt{10+2\sqrt{5}}/4$? Can you show also that

$$\frac{\sin x}{x} = \cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}... \quad (4)$$

Challenge 111 e is correct?

* *

Measuring angular size with the eye only is tricky. For example, can you say whether the Moon is larger or smaller than the nail of your thumb at the end of your extended arm? Angular size is not an intuitive quantity; it requires measurement instruments. (Challenge 112 e)

A famous example, shown in Figure 36, illustrates the difficulty of estimating angles. Both the Sun and the Moon seem larger when they are on the horizon. In ancient times, Ptolemy explained this so-called *Moon illusion* by an unconscious apparent distance change induced by the human brain. Indeed, the Moon illusion disappears when you look at the Moon through your legs. (Ref. 50) In fact, the Moon is even *further away* from the observer when it is just above the horizon, and thus its image is *smaller* than it was a few hours earlier, when it was high in the sky. Can you confirm this? (Challenge 113 s)

The Moon's angular size changes even more due to another effect: the orbit of the Moon round the Earth is elliptical. An example of the consequence is shown in Figure 37.

* *

Galileo also made mistakes. In his famous book, the *Dialogues*, he says that the curve formed by a thin chain hanging between two nails is a parabola, i.e., the curve defined by $y = x^2$. That is not correct. What is the correct curve? You can observe the shape (approximately) in the shape of suspension bridges. (Challenge 114 d)

* *

Draw three circles, of different sizes, that touch each other, as shown in Figure 38. Now

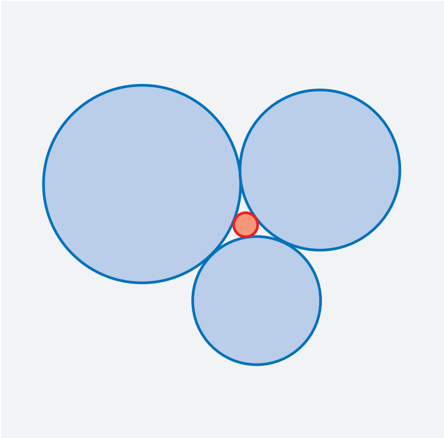


FIGURE 38 A famous puzzle: how are the four radii related?

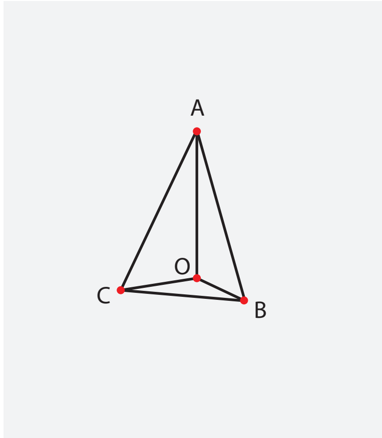


FIGURE 39 What is the area ABC, given the other three areas and three right angles at O?

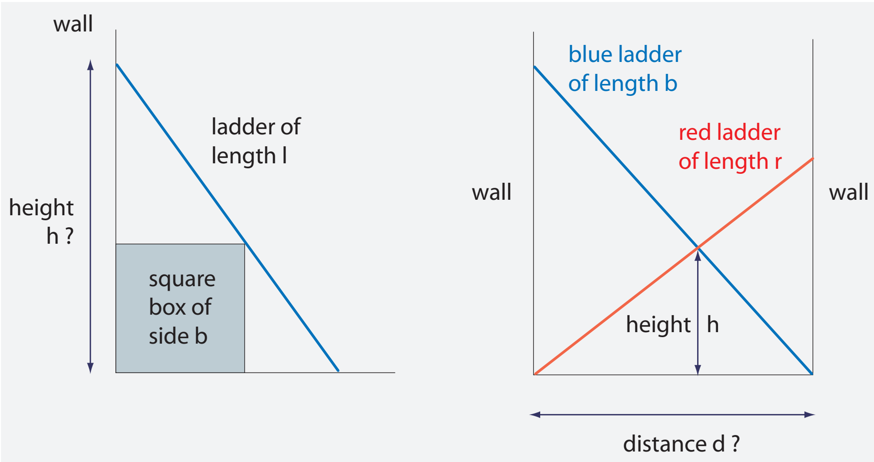


FIGURE 40 Two ladder puzzles: a moderately difficult (left) and a difficult one (right).

draw a fourth circle in the space between, touching the outer three. What simple relation
Challenge 115 s do the inverse radii of the four circles obey?

* *

Take a tetrahedron OABC whose triangular sides OAB, OBC and OAC are rectangular in O, as shown in Figure 39. In other words, the edges OA, OB and OC are all perpendicular to each other. In the tetrahedron, the areas of the triangles OAB, OBC and OAC are
Challenge 116 s respectively 8, 4 and 1. What is the area of triangle ABC?

* *

Ref. 51 There are many puzzles about ladders. Two are illustrated in Figure 40. If a 5 m ladder is put against a wall in such a way that it just touches a box with 1 m height and depth,
Challenge 117 s how high does the ladder reach? If two ladders are put against two facing walls, and if

FIGURE 41 Anticrepuscular rays - where is the Sun in this situation? (© Peggy Peterson)

the lengths of the ladders and the height of the crossing point are known, how distant
Challenge 118 d are the walls?

* *

With two rulers, you can add and subtract numbers by lying them side by side. Are you
Challenge 119 s able to design rulers that allow you to multiply and divide in the same manner?

* *

How many days would a year have if the Earth turned the other way with the same ro-
Challenge 120 s tation frequency?

* *

Challenge 121 s The Sun is hidden in the spectacular situation shown in Figure 41 Where is it?

* *

A slightly different, but equally fascinating situation – and useful for getting used to perspective drawing – appears when you have a lighthouse in your back. Can you draw the
Challenge 122 e rays you see in the sky up to the horizon?

* *

Two cylinders of equal radius intersect at a right angle. What is the value of the intersec-
Challenge 123 s tion volume? (First make a drawing.)

* *

Two sides of a hollow cube with side length 1 dm are removed, to yield a tunnel with square opening. Is it true that a cube with edge length 1.06 dm can be made to pass
Challenge 124 s through the hollow cube with side length 1 dm?

* *

Ref. 52 Could a two-dimensional universe exist? Alexander Dewdney imagined such a universe in great detail and wrote a well-known book about it. He describes houses, the transportation system, digestion, reproduction and much more. Can you explain why a two-

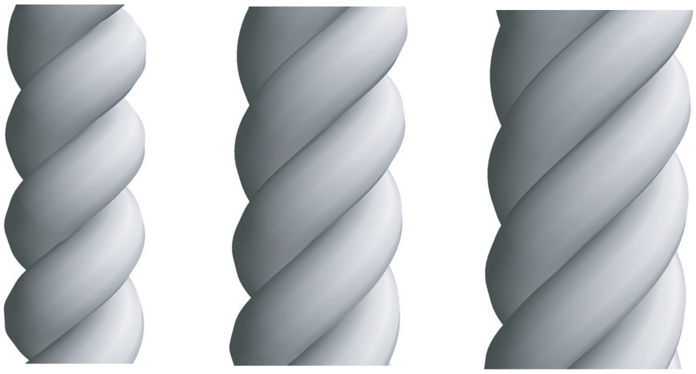


FIGURE 42 Ideal configurations of ropes made of two, three and four strands. In the ideal configuration, the specific pitch angle relative to the equatorial plane – 39.4°, 42.8° and 43.8°, respectively – leads to zero-twist structures. In these ideal configurations, the rope will neither rotate in one nor in the other direction under vertical strain (© Jakob Bohr).

Challenge 125 d dimensional universe is impossible?

* *

Ropes are wonderful structures. They are flexible, they are helically woven, but despite this, they do not unwind or twist, they are almost inextensible, and their geometry depends little on the material used in making them. What is the origin of all these properties? Ref. 53

Laying rope is an old craft; it is based on a purely geometric result: among all possible helices of n strands of given length laid around a central structure of fixed radius, there is one helix for which the number of turns is *maximal*. For purely geometric reasons, ropes with that specific number of turns and the corresponding inner radius have the mentioned properties that make ropes so useful. The geometries of ideal ropes made of two, three and four strands are shown in Figure 42.

* *

Challenge 126 s Some researchers are investigating whether time could be two-dimensional. Can this be?

* *

Other researchers are investigating whether space could have more than three dimensions. Can this be? Challenge 127 s

* *

One way to compare speeds of animals and machines is to measure them in 'body lengths per second'. The click beetle achieves a value of around 2000 during its jump phase, certain Archaea (bacteria-like) cells a value of 500, and certain hummingbirds 380. These are the record-holders so far. Cars, aeroplanes, cheetahs, falcons, crabs, and all other motorized systems are much slower. Ref. 54

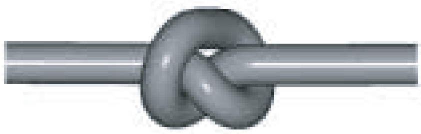


FIGURE 43 An open research problem: What is the ropelength of a tight knot? (© Piotr Pieranski, from Ref. 55)

* *

Challenge 128 e Why is the cross section of a tube usually circular?

* *

What are the dimensions of an open rectangular water tank that contains 1 m^3 of water and uses the smallest amount of wall material? Challenge 129 e

* *

Draw a square consisting of four equally long connecting line segments hinged at the vertices. Such a structure may be freely deformed into a rhombus if some force is applied. How many additional line interlinks of the same length must be supplemented to avoid this freedom and to prevent the square from being deformed? The extra line interlinks must be in the same plane as the square and each one may only be pegged to others at the endpoints. Challenge 130 s

* *

Area measurements can be difficult. In 2014 it became clear that the area of the gastrointestinal tract of adult health humans is between 30 and 40 m^2. For many years, the mistaken estimate for the area was between 180 and 300 m^2.

* *

If you never explored plane geometry, do it once in your life. An excellent introduction is Claudi Alsina & Roger B. Nelsen, *Icons of Mathematics: An Exploration of Twenty Key Images*, MAA Press, 2011. This is a wonderful book with many simple and surprising facts about geometry that are never told in school or university. You will enjoy it.

* *

Triangles are full of surprises. Together, Leonhard Euler, Charles Julien Brianchon and Jean Victor Poncelet discovered that in any triangle, nine points lie on the same circle: the midpoints of the sides, the feet of the altitude lines, and the midpoints of the altitude segments connecting each vertex to the orthocenter. Euler also discovered that in every triangle, the orthocenter, the centroid, the circumcenter and the center of the nine-point-circle lie on the same line, now called the Euler line.

For the most recent recearch results on plane triangles, see the wonderful *Encyclopedia of Triangle Centers*, available at faculty.evansville.edu/ck6/encyclopedia/ETC.html.

* *

Here is a simple challenge on length that nobody has solved yet. Take a piece of ideal rope: of constant radius, ideally flexible, and completely slippery. Tie a tight knot into it, as shown in Figure 43. By how much did the two ends of the rope come closer together?

Challenge 131 r

Summary about everyday space and time

Motion defines speed, time and length. Observations of everyday life and precision experiments are conveniently and precisely described by describing velocity as a vector, space as three-dimensional set of points, and time as a one-dimensional real line, also made of points. These three definitions form the everyday, or *Galilean*, description of our environment.

Modelling velocity, time and space as *continuous* quantities is precise and convenient. The modelling works during most of the adventures that follows. However, this common model of space and time *cannot* be confirmed by experiment. For example, no experiment can check distances larger than 10^{25} m or smaller than 10^{-25} m; the continuum model is likely to be incorrect at smaller scales. We will find out in the last part of our adventure that this is indeed the case.

CHAPTER 3

HOW TO DESCRIBE MOTION – KINEMATICS

> La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi agli occhi (io dico l'universo) … Egli è scritto in lingua matematica.**
>
> Galileo Galilei, *Il saggiatore* VI.

Experiments show that the properties of motion, time and space are extracted from the environment both by children and animals. This extraction has been confirmed for cats, dogs, rats, mice, ants and fish, among others. They all find the same results.

First of all, *motion is change of position with time*. This description is illustrated by rapidly flipping the lower left corners of this book, starting at page 242. Each page simulates an instant of time, and the only change that takes place during motion is in the position of the object, say a stone, represented by the dark spot. The other variations from one picture to the next, which are due to the imperfections of printing techniques, can be taken to simulate the inevitable measurement errors.

Stating that 'motion is the change of position with time' is *neither* an explanation *nor* a definition, since both the concepts of time and position are deduced from motion itself. It is only a *description* of motion. Still, the statement is useful, because it allows for high precision, as we will find out by exploring gravitation and electrodynamics. After all, precision is our guiding principle during this promenade. Therefore the detailed description of changes in position has a special name: it is called *kinematics*.

The idea of change of positions implies that the object can be *followed* during its motion. This is not obvious; in the section on quantum theory we will find examples where this is impossible. But in everyday life, objects can always be tracked. The set of all positions taken by an object over time forms its *path* or *trajectory*. The origin of this concept
Ref. 56 is evident when one watches fireworks or again the flip film in the lower left corners starting at page 242.

In everyday life, animals and humans agree on the Euclidean properties of velocity, space and time. In particular, this implies that a trajectory can be described by specifying three numbers, three *coordinates* (x, y, z) – one for each dimension – as continuous

** Science is written in this huge book that is continuously open before our eyes (I mean the universe) … It is written in mathematical language.

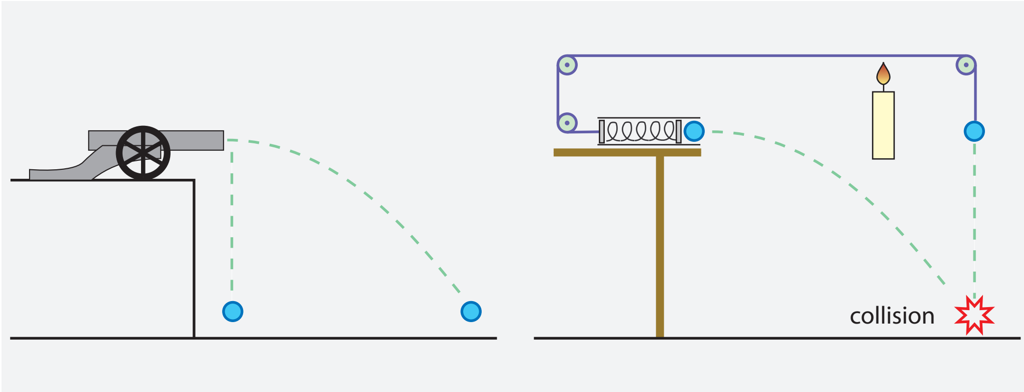


FIGURE 44 Two ways to test that the time of free fall does not depend on horizontal velocity.

Vol. III, page 289 functions of time t. (Functions are defined in detail later on.) This is usually written as

$$\boldsymbol{x} = \boldsymbol{x}(t) = (x(t), y(t), z(t)) . \tag{5}$$

For example, already Galileo found, using stopwatch and ruler, that the height z of any thrown or falling stone changes as

$$z(t) = z_0 + v_{z0}\,(t - t_0) - \tfrac{1}{2} g\,(t - t_0)^2 \tag{6}$$

where t_0 is the time the fall starts, z_0 is the initial height, v_{z0} is the initial velocity in the vertical direction and $g = 9.8\,\mathrm{m/s^2}$ is a constant that is found to be the same, within about one part in 300, for all falling bodies on all points of the surface of the Earth. Ref. 57 Where do the value $9.8\,\mathrm{m/s^2}$ and its slight variations come from? A preliminary answer will be given shortly, but the complete elucidation will occupy us during the larger part of this hike.

The special case with no initial velocity is of great interest. Like a few people before him, Galileo made it clear that g is the same for all bodies, if air resistance can be neglected. Page 202 He had many arguments for this conclusion; can you find one? And of course, his famous experiment at the leaning tower in Pisa confirmed the statement. (It is a *false* Ref. 58 urban legend that Galileo never performed the experiment. He did it.)

Equation (6) therefore allows us to determine the depth of a well, given the time a Challenge 132 s stone takes to reach its bottom. The equation also gives the speed v with which one hits the ground after jumping from a tree, namely

$$v = \sqrt{2gh} . \tag{7}$$

A height of 3 m yields a velocity of 27 km/h. The velocity is thus proportional only to the square root of the height. Does this mean that one's strong fear of falling results from an Challenge 133 s overestimation of its actual effects?

Galileo was the first to state an important result about free fall: the motions in the horizontal and vertical directions are *independent*. He showed that the time it takes for

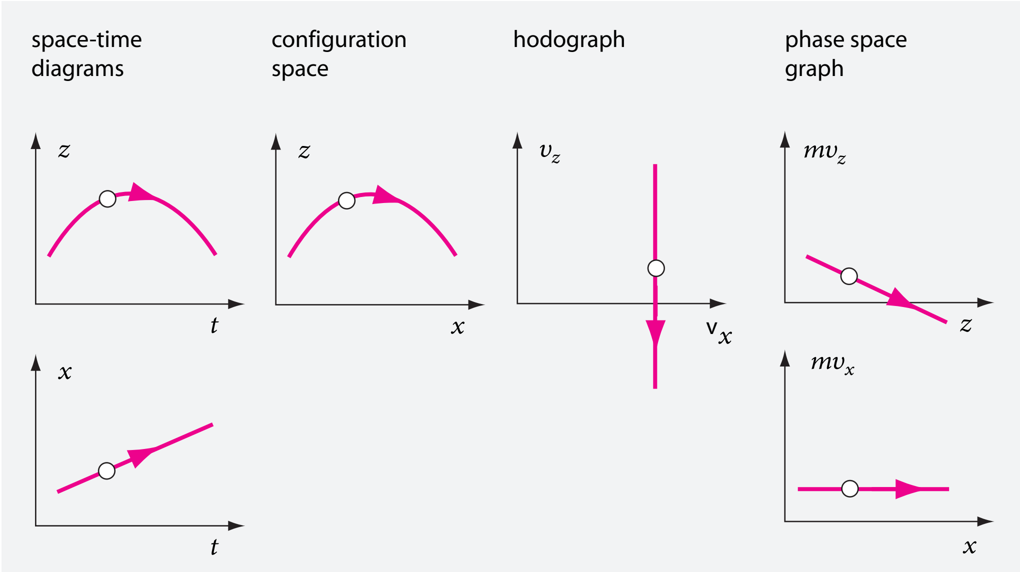


FIGURE 45 Various types of graphs describing the same path of a thrown stone.

a cannon ball that is shot exactly horizontally to fall is *independent* of the strength of the gunpowder, as shown in Figure 44. Many great thinkers did not agree with this statement even after his death: in 1658 the Academia del Cimento even organized an experiment to check this assertion, by comparing the flying cannon ball with one that simply fell vertically. Can you imagine how they checked the simultaneity? Figure 44 shows how you can check this at home. In this experiment, whatever the spring load of the cannon, the two bodies will always collide in mid-air (if the table is high enough), thus proving the assertion.

Ref. 59

Challenge 134 s

In other words, a flying cannon ball is not accelerated in the horizontal direction. Its horizontal motion is simply unchanging – as long as air resistance is negligible. By extending the description of equation (6) with the two expressions for the horizontal coordinates x and y, namely

$$x(t) = x_0 + v_{x0}(t - t_0)$$
$$y(t) = y_0 + v_{y0}(t - t_0) , \quad (8)$$

a *complete* description for the path followed by thrown stones results. A path of this shape is called a *parabola*; it is shown in Figures 18, 44 and 45. (A parabolic shape is also used for light reflectors inside pocket lamps or car headlights. Can you show why?)

Page 40

Challenge 135 s

Ref. 60

Physicists enjoy generalizing the idea of a path. As Figure 45 shows, a path is a trace left in a diagram by a moving object. Depending on what diagram is used, these paths have different names. Space-time diagrams are useful to make the theory of relativity accessible. The *configuration space* is spanned by the coordinates of all particles of a system. For many particles, it has a high number of dimensions and plays an important role in self-organization. The difference between chaos and order can be described as a difference in the properties of paths in configuration space. *Hodographs*, the paths in 'velocity

Page 415

space', are used in weather forecasting. The phase space diagram is also called *state space diagram*. It plays an essential role in thermodynamics.

Throwing, jumping and shooting

The kinematic description of motion is useful for answering a whole range of questions.

* *

What is the upper limit for the long jump? The running peak speed world record in 2019 was over 12.5 m/s ≈ 45 km/h by Usain Bolt, and the 1997 women's record was 11 m/s ≈ 40 km/h. However, male long jumpers never run much faster than about 9.5 m/s. How much extra jump distance could they achieve if they could run full speed? How could they achieve that? In addition, long jumpers take off at angles of about 20°, as they are not able to achieve a higher angle at the speed they are running. How much would they gain if they could achieve 45°? Is 45° the optimal angle?

Ref. 61 Ref. 62 Ref. 63 Challenge 136 s

* *

Why was basketball player Dirk Nowitzki so successful? His trainer Holger Geschwindner explained him that a throw is most stable against mistakes when it falls into the basket at around 47 degrees from the horizontal. He further told Nowitzki that the ball flies in a plane, and that therefore the arms should also move in that plane only. And he explained that when the ball leaves the hand, it should roll over the last two fingers like a train moves on rails. Using these criteria to check and to improve Nowitzki's throws, he made him into one of the best basket ball throwers in the world.

* *

What do the athletes Usain Bolt and Michael Johnson, the last two world record holders on the 200 m race at time of this writing, have in common? They were tall, athletic, and had many fast twitch fibres in the muscles. These properties made them good sprinters. A last difference made them world-class sprinters: they had a flattened spine, with almost no S-shape. This abnormal condition saves them a little bit of time at every step, because their spine is not as flexible as in usual people. This allows them to excel at short-distance races.

* *

Athletes continuously improve speed records. Racing horses do not. Why? For racing horses, breathing rhythm is related to gait; for humans, it is not. As a result, racing horses cannot change or improve their technique, and the speed of racing horses is essentially the same since it is measured.

* *

What is the highest height achieved by a human throw of any object? What is the longest distance achieved by a human throw? How would you clarify the rules? Compare the results with the record distance with a crossbow, 1, 871.8 m, achieved in 1988 by Harry Drake, the record distance with a footbow, 1854.4 m, achieved in 1971 also by Harry Drake, and with a hand-held bow, 1, 222.0 m, achieved in 1987 by Don Brown.

Challenge 137 s

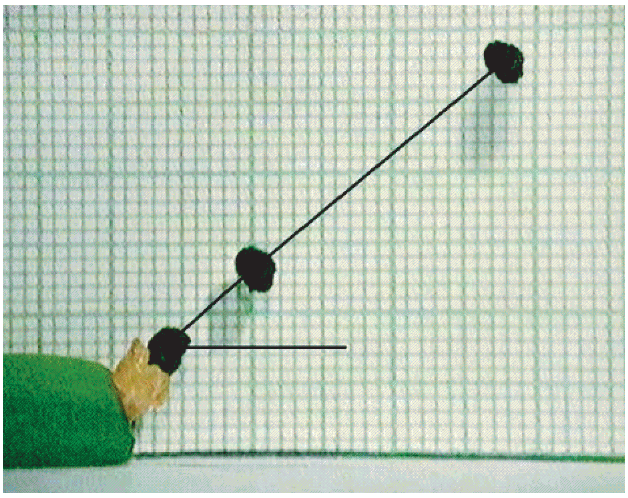


FIGURE 46 Three superimposed images of a frass pellet shot away by a caterpillar inside a rolled-up leaf (© Stanley Caveney).

∗ ∗

Challenge 138 s How can the speed of falling rain be measured using an umbrella? The answer is important: the same method can also be used to measure the speed of light, as we will find out Vol. II, page 17 later. (Can you guess how?)

∗ ∗

When a dancer jumps in the air, how many times can he or she rotate around his or her Challenge 139 s vertical axis before arriving back on earth?

∗ ∗

Numerous species of moth and butterfly caterpillars shoot away their frass – to put it Ref. 64 more crudely: their faeces – so that its smell does not help predators to locate them. Stanley Caveney and his team took photographs of this process. Figure 46 shows a caterpillar (yellow) of the skipper *Calpodes ethlius* inside a rolled up green leaf caught in the act. Given that the record distance observed is 1.5 m (though by another species, *Epargyreus* Challenge 140 s *clarus*), what is the ejection speed? How do caterpillars achieve it?

∗ ∗

What is the horizontal distance one can reach by throwing a stone, given the speed and Challenge 141 s the angle from the horizontal at which it is thrown?

∗ ∗

Challenge 142 s What is the maximum numbers of balls that could be juggled at the same time?

∗ ∗

Is it true that rain drops would kill if it weren’t for the air resistance of the atmosphere? Challenge 143 s What about hail?

∗ ∗

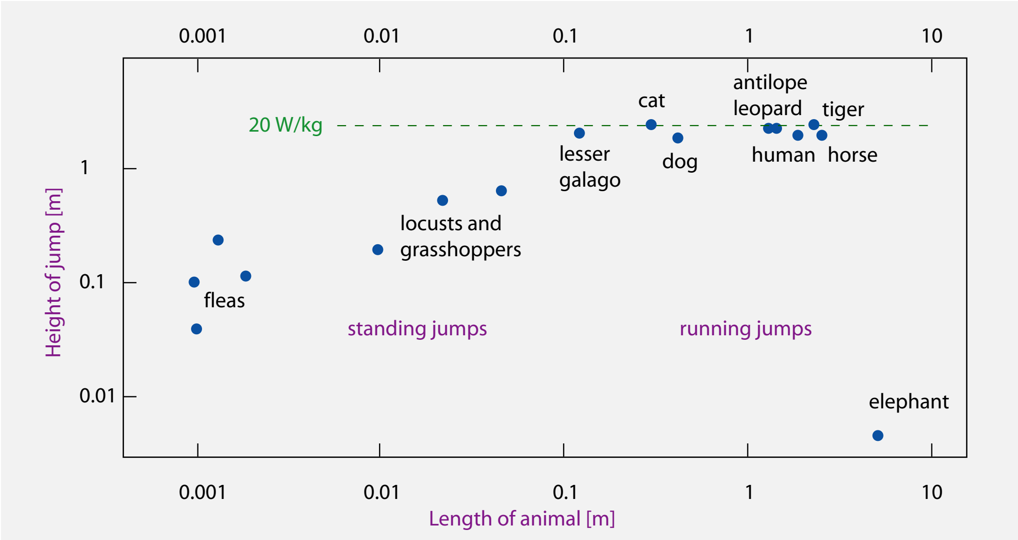


FIGURE 47 The height achieved by jumping land animals.

Challenge 144 s Are bullets, fired into the air from a gun, dangerous when they fall back down?

* *

Challenge 145 s Police finds a dead human body at the bottom of cliff with a height of 30 m, at a distance of 12 m from the cliff. Was it suicide or murder?

* *

Ref. 65 Challenge 146 s All land animals, regardless of their size, achieve jumping heights of at most 2.2 m, as shown in Figure 47. The explanation of this fact takes only two lines. Can you find it?

The last two issues arise because the equation (6) describing free fall does not hold in all cases. For example, leaves or potato crisps do not follow it. As Galileo already knew, this is a consequence of air resistance; we will discuss it shortly. Because of air resistance, the path of a stone is not a parabola.

Challenge 147 s In fact, there are other situations where the path of a falling stone is not a parabola, even without air resistance. Can you find one?

Enjoying vectors

Physical quantities with a defined direction, such as speed, are described with three numbers, or three components, and are called *vectors*. Learning to calculate with such multi-component quantities is an important ability for many sciences. Here is a summary.

Vectors can be pictured by small arrows. Note that vectors do not have specified points at which they start: two arrows with same direction and the same length are the *same* vector, even if they start at different points in space. Since vectors behave like arrows, vectors can be added and they can be multiplied by numbers. For example, stretching an arrow $\boldsymbol{a} = (a_x, a_y, a_z)$ by a number c corresponds, in component notation, to the vector $c\boldsymbol{a} = (ca_x, ca_y, ca_z)$.

In precise, mathematical language, a vector is an element of a set, called *vector space*, in which the following properties hold for all vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ and for all numbers c and d:

$$c(\boldsymbol{a}+\boldsymbol{b}) = c\boldsymbol{a}+c\boldsymbol{b} \quad , \quad (c+d)\boldsymbol{a} = c\boldsymbol{a}+d\boldsymbol{a} \quad , \quad (cd)\boldsymbol{a} = c(d\boldsymbol{a}) \quad \text{and} \quad 1\boldsymbol{a} = \boldsymbol{a} \; . \tag{9}$$

Examples of vector spaces are the set of all *positions* of an object, or the set of all its
Challenge 148 s possible velocities. Does the set of all rotations form a vector space?

All vector spaces allow defining a unique *null vector* and a unique *negative vector* for
Challenge 149 e each vector.

In most vector spaces of importance when describing nature the concept of *length* – specifying the 'magnitude' – of a vector can be introduced. This is done via an intermediate step, namely the introduction of the scalar product of two vectors. The product is called 'scalar' because its result is a scalar; a *scalar* is a number that is the same for all observers; for example, it is the same for observers with different orientations.* The *scalar product* between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is a number that satisfies

$$\begin{aligned} \boldsymbol{aa} &\geqslant 0 \, , \\ \boldsymbol{ab} &= \boldsymbol{ba} \, , \\ (\boldsymbol{a}+\boldsymbol{a}')\boldsymbol{b} &= \boldsymbol{ab}+\boldsymbol{a}'\boldsymbol{b} \, , \\ \boldsymbol{a}(\boldsymbol{b}+\boldsymbol{b}') &= \boldsymbol{ab}+\boldsymbol{ab}' \quad \text{and} \\ (c\boldsymbol{a})\boldsymbol{b} &= \boldsymbol{a}(c\boldsymbol{b}) = c(\boldsymbol{ab}) \, . \end{aligned} \tag{10}$$

This definition of a scalar product is not unique; however, there is a *standard* scalar product. In Cartesian coordinate notation, the standard scalar product is given by

$$\boldsymbol{ab} = a_x b_x + a_y b_y + a_z b_z \; . \tag{11}$$

If the scalar product of two vectors vanishes the two vectors are *orthogonal*, at a right
Challenge 150 e angle to each other. (Show it!) Note that one can write either $\boldsymbol{ab}$ or $\boldsymbol{a} \cdot \boldsymbol{b}$ with a central dot.

The *length* or *magnitude* or *norm* of a vector can then be defined as the square root of the scalar product of a vector with itself: $a = \sqrt{\boldsymbol{aa}}$. Often, and also in this text, lengths are written in *italic* letters, whereas vectors are written in **bold** letters. The magnitude is often written as $a = \sqrt{\boldsymbol{a}^2}$. A vector space with a scalar product is called an *Euclidean* vector space.

The scalar product is especially useful for specifying directions. Indeed, the scalar product between two vectors encodes the angle between them. Can you deduce this im-
Challenge 151 s portant relation?

* We mention that in mathematics, a scalar is a *number*; in physics, a scalar is an *invariant* number, i.e., a number that is the same for all observers. Likewise, in mathematics, a vector is an element of a vector space; in physics, a vector is an *invariant* element of a vector space, i.e., a quantity whose coordinates, when observed by different observers, change like the components of velocity.

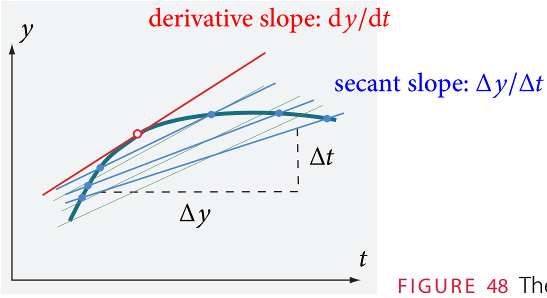


FIGURE 48 The derivative in a point as the limit of secants.

What is rest? What is velocity?

In the Galilean description of nature, motion and rest are opposites. In other words, a body is at rest when its position, i.e., its coordinates, do not change with time. In other words, (Galilean) *rest* is defined as

$$\boldsymbol{x}(t) = \text{const} . \qquad (12)$$

We recall that $\boldsymbol{x}(t)$ is the abbreviation for the three coordinates $(x(t), y(t), z(t))$. Later we will see that this definition of rest, contrary to first impressions, is not much use and will have to be expanded. Nevertheless, any definition of rest implies that non-resting objects can be distinguished by comparing the rapidity of their displacement. Thus we can define the *velocity* $\boldsymbol{v}$ of an object as the change of its position $\boldsymbol{x}$ with time t. This is usually written as

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} . \qquad (13)$$

In this expression, valid for each coordinate separately, d/dt means 'change with time'. We can thus say that velocity is the *derivative* of position with respect to time. The *speed* v is the name given to the magnitude of the velocity $\boldsymbol{v}$. Thus we have $v = \sqrt{\boldsymbol{v}\boldsymbol{v}}$. Derivatives are written as fractions in order to remind the reader that they are derived from the idea of slope. The expression

$$\frac{\mathrm{d}s}{\mathrm{d}t} \quad \text{is meant as an abbreviation of} \quad \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} , \qquad (14)$$

a shorthand for saying that the *derivative at a point* is the limit of the secant slopes in the neighbourhood of the point, as shown in Figure 48. This definition implies the working rules

Challenge 152 e

$$\frac{\mathrm{d}(s+r)}{\mathrm{d}t} = \frac{\mathrm{d}s}{\mathrm{d}t} + \frac{\mathrm{d}r}{\mathrm{d}t} , \quad \frac{\mathrm{d}(cs)}{\mathrm{d}t} = c\frac{\mathrm{d}s}{\mathrm{d}t} , \quad \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} , \quad \frac{\mathrm{d}(sr)}{\mathrm{d}t} = \frac{\mathrm{d}s}{\mathrm{d}t}r + s\frac{\mathrm{d}r}{\mathrm{d}t} , \qquad (15)$$

c being any number. This is all one ever needs to know about derivatives in physics. Quantities such as dt and ds, sometimes useful by themselves, are called *differentials*.

FIGURE 49 Gottfried Wilhelm Leibniz (1646–1716).

These concepts are due to Gottfried Wilhelm Leibniz.* Derivatives lie at the basis of all calculations based on the continuity of space and time. Leibniz was the person who made it possible to describe and use velocity in physical formulae and, in particular, to use the idea of velocity at a given point in time or space for calculations.

The definition of velocity assumes that it makes sense to take the limit $\Delta t \rightarrow 0$. In other words, it is assumed that *infinitely small* time intervals do exist in nature. The definition of velocity with derivatives is possible only because both space and time are described by sets which are *continuous*, or in mathematical language, *connected and complete*. In the rest of our walk we shall not forget that from the beginning of classical physics, *infinities* are present in its description of nature. The infinitely small is part of our definition of velocity. Indeed, differential calculus can be defined as the study of infinity and its uses. We thus discover that the appearance of infinity does not automatically render a description impossible or imprecise. In order to remain precise, physicists use only the smallest two of the various possible types of infinities. Their precise definition and an overview of other types are introduced later on. Vol. III, page 288

The appearance of infinity in the usual description of motion was first criticized in his famous ironical arguments by Zeno of Elea (around 445 BCE), a disciple of Parmenides. Ref. 66 In his so-called third argument, Zeno explains that since at every instant a given object occupies a part of space corresponding to its size, the notion of velocity at a given instant makes no sense; he provokingly concludes that therefore motion does not exist. Nowadays we would not call this an argument against the *existence* of motion, but against its usual *description*, in particular against the use of infinitely divisible space and time. (Do you agree?) Challenge 153 e Nevertheless, the description criticized by Zeno actually works quite well in everyday life. The reason is simple but deep: in daily life, changes are indeed continuous.

Large changes in nature are made up of many small changes. This property of nature is not obvious. For example, we note that we have (again) tacitly assumed that the path of an object is not a fractal or some other badly behaved entity. In everyday life this is correct; in other domains of nature it is not. The doubts of Zeno will be partly rehabilitated later in our walk, and increasingly so the more we proceed. Vol. VI, page 65 The rehabilitation is only partial, as the final solution will be different from that which he envisaged; on the other hand,

* Gottfried Wilhelm Leibniz (b. 1646 Leipzig, d. 1716 Hannover), lawyer, physicist, mathematician, philosopher, diplomat and historian. He was one of the great minds of mankind; he invented the differential calculus (before Newton) and published many influential and successful books in the various fields he explored, among them *De arte combinatoria*, *Hypothesis physica nova*, *Discours de métaphysique*, *Nouveaux essais sur l'entendement humain*, the *Théodicée* and the *Monadologia*.

TABLE 13 Some measured acceleration values.

Observation	Acceleration
What is the lowest you can find?	Challenge 154 s
Back-acceleration of the galaxy M82 by its ejected jet	$10\,\mathrm{fm/s^2}$
Acceleration of a young star by an ejected jet	$10\,\mathrm{pm/s^2}$
Fathoumi Acceleration of the Sun in its orbit around the Milky Way	$0.2\,\mathrm{nm/s^2}$
Deceleration of the Pioneer satellites, due to heat radiation imbalance	$0.8\,\mathrm{nm/s^2}$
Centrifugal acceleration at Equator due to Earth's rotation	$33\,\mathrm{mm/s^2}$
Electron acceleration in household electricity wire due to alternating current	$50\,\mathrm{mm/s^2}$
Acceleration of fast underground train	$1.3\,\mathrm{m/s^2}$
Gravitational acceleration on the Moon	$1.6\,\mathrm{m/s^2}$
Minimum deceleration of a car, by law, on modern dry asphalt	$5.5\,\mathrm{m/s^2}$
Gravitational acceleration on the Earth's surface, depending on location	$9.8 \pm 0.3\,\mathrm{m/s^2}$
Standard gravitational acceleration	$9.806\,65\,\mathrm{m/s^2}$
Highest acceleration for a car or motorbike with engine-driven wheels	$15\,\mathrm{m/s^2}$
Space rockets at take-off	20 to $90\,\mathrm{m/s^2}$
Acceleration of cheetah	$32\,\mathrm{m/s^2}$
Gravitational acceleration on Jupiter's surface	$25\,\mathrm{m/s^2}$
Flying fly (*Musca domestica*)	*c.* $100\,\mathrm{m/s^2}$
Acceleration of thrown stone	*c.* $120\,\mathrm{m/s^2}$
Acceleration that triggers air bags in cars	$360\,\mathrm{m/s^2}$
Fastest leg-powered acceleration (by the froghopper, *Philaenus spumarius*, an insect)	$4\,\mathrm{km/s^2}$
Tennis ball against wall	$0.1\,\mathrm{Mm/s^2}$
Bullet acceleration in rifle	$2\,\mathrm{Mm/s^2}$
Fastest centrifuges	$0.1\,\mathrm{Gm/s^2}$
Acceleration of protons in large accelerator	$90\,\mathrm{Tm/s^2}$
Acceleration of protons inside nucleus	$10^{31}\,\mathrm{m/s^2}$
Highest possible acceleration in nature	$10^{52}\,\mathrm{m/s^2}$

the doubts about the idea of 'velocity at a point' will turn out to be well-founded. For the moment though, we have no choice: we continue with the basic assumption that in nature changes happen smoothly.

Why is velocity necessary as a concept? Aiming for precision in the description of motion, we need to find the complete list of aspects necessary to specify the state of an object. The concept of velocity is obviously on this list.

Acceleration

Continuing along the same line, we call *acceleration* $\boldsymbol{a}$ of a body the change of velocity $\boldsymbol{v}$ with time, or

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} . \qquad (16)$$

Acceleration is what we feel when the Earth trembles, an aeroplane takes off, or a bicycle goes round a corner. More examples are given in Table 13. Acceleration is the time derivative of velocity. Like velocity, acceleration has both a magnitude and a direction. In short, acceleration, like velocity, is a vector quantity. As usual, this property is indicated by the use of a **bold** letter for its abbreviation.

In a usual car, or on a motorbike, we can *feel* being accelerated. (These accelerations are below $1g$ and are therefore harmless.) We feel acceleration because some part inside us is moved against some other part: acceleration deforms us. Such a moving part can be, for example, some small part inside our ear, our stomach inside the belly, or simply our limbs against our trunk. All acceleration sensors, including those listed in Table 14 or those shown in Figure 50, whether biological or technical, work in this way.

Acceleration is felt. Our body is deformed and the sensors in our body detect it, for example in amusement parks. Higher accelerations can have stronger effects. For example, when accelerating a sitting person in the direction of the head at two or three times the value of usual gravitational acceleration, eyes stop working and the sight is greyed out, because the blood cannot reach the eye any more. Between 3 and $5g$ of continuous acceleration, or 7 to $9g$ of short time acceleration, consciousness is lost, because the brain does not receive enough blood, and blood may leak out of the feet or lower legs. High acceleration in the direction of the feet of a sitting person can lead to haemorrhagic strokes in the brain. The people most at risk are jet pilots; they have special clothes that send compressed air onto the pilot's bodies to avoid blood accumulating in the wrong places. (Ref. 67)

Can you think of a situation where you are accelerated but do *not* feel it? (Challenge 155 s)

Velocity is the time derivative of position. Acceleration is the second time derivative of position. Higher derivatives than acceleration can also be defined, in the same manner. They add little to the description of nature, because – as we will show shortly – neither these higher derivatives nor even acceleration itself are useful for the description of the state of motion of a system. (Challenge 156 s)

From objects to point particles

> Wenn ich den Gegenstand kenne, so kenne ich auch sämtliche Möglichkeiten seines Vorkommens in Sachverhalten.*
>
> Ludwig Wittgenstein, *Tractatus*, 2.0123

One aim of the study of motion is to find a complete and precise description of both states and objects. With the help of the concept of space, the description of objects can be refined considerably. In particular, we know from experience that all objects seen in daily life have an important property: they can be divided into *parts*. Often this observation is (Challenge 157 e)

* 'If I know an object, then I also know all the possibilities of its occurrence in atomic facts.'

TABLE 14 Some acceleration sensors.

Measurement	Sensor	Range
Direction of gravity in plants (roots, trunk, branches, leaves)	statoliths in cells	0 to 10 m/s^2
Direction and value of accelerations in mammals	the utricle and saccule in the inner ear (detecting linear accelerations), and the membranes in each semicircular canal (detecting rotational accelerations)	0 to 20 m/s^2
Direction and value of acceleration in modern step counters for hikers	piezoelectric sensors	0 to 20 m/s^2
Direction and value of acceleration in car crashes	airbag sensor using piezoelectric ceramics	0 to 2000 m/s^2

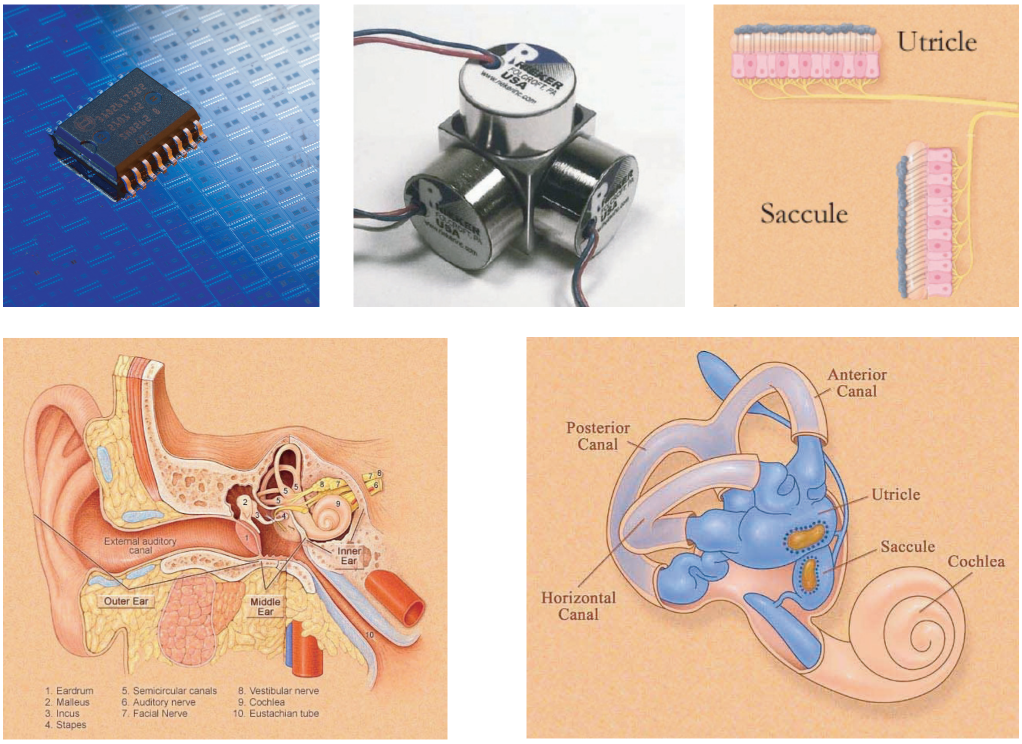


FIGURE 50 Three accelerometers: a one-axis piezoelectric airbag sensor, a three-axis capacitive accelerometer, and the utricle and saccule near the three semicircular canals inside the human ear (© Bosch, Rieker Electronics, Northwestern University).

expressed by saying that all objects, or bodies, have two properties. First, they are made out of *matter*,* defined as that aspect of an object responsible for its impenetrability, i.e., the property preventing two objects from being in the same place. Secondly, bodies

Ref. 68

* Matter is a word derived from the Latin 'materia', which originally meant 'wood' and was derived via intermediate steps from 'mater', meaning 'mother'.

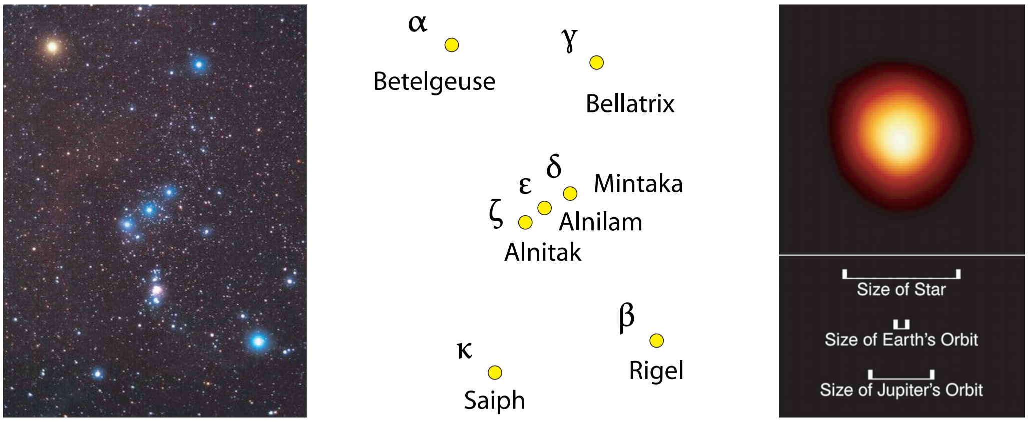


FIGURE 51 Orion in natural colours (© Matthew Spinelli) and Betelgeuse (ESA, NASA).

have a certain form or *shape*, defined as the precise way in which this impenetrability is distributed in space.

In order to describe motion as accurately as possible, it is convenient to start with those bodies that are as simple as possible. In general, the smaller a body, the simpler it is. A body that is so small that its parts no longer need to be taken into account is called a *particle*. (The older term *corpuscle* has fallen out of fashion.) Particles are thus idealized small stones. The extreme case, a particle whose size is *negligible* compared with the dimensions of its motion, so that its position is described completely by a *single* triplet of coordinates, is called a *point particle* or a *point mass* or a *mass point*. In equation (6), the stone was assumed to be such a point particle.

Do point-like objects, i.e., objects smaller than anything one can measure, exist in daily life? Yes and no. The most notable examples are the stars. At present, angular sizes as small as 2 μrad can be measured, a limit given by the fluctuations of the air in the atmosphere. In space, such as for the Hubble telescope orbiting the Earth, the angular limit is due to the diameter of the telescope and is of the order of 10 nrad. Practically all stars seen from Earth are smaller than that, and are thus effectively 'point-like', even when seen with the most powerful telescopes.

As an exception to the general rule, the size of a few large or nearby stars, mostly of red giant type, can be measured with special instruments.* Betelgeuse, the higher of the two shoulders of Orion shown in Figure 51, Mira in Cetus, Antares in Scorpio, Aldebaran in Taurus and Sirius in Canis Major are examples of stars whose size has been measured;
Ref. 69 they are all less than two thousand light years from Earth. For a comparison of dimensions, see Figure 52. Of course, like the Sun, also all other stars have a finite size, but one
Challenge 158 s cannot prove this by measuring their dimension in photographs. (True?)

* The website stars.astro.illinois.edu/sow/sowlist.html gives an introduction to the different types of stars. The www.astro.wisc.edu/~dolan/constellations website provides detailed and interesting information about constellations.

For an overview of the planets, see the beautiful book by Kenneth R. Lang & Charles A. Whitney, *Vagabonds de l'espace – Exploration et découverte dans le système solaire*, Springer Verlag, 1993. Amazingly beautiful pictures of the stars can be found in David Malin, *A View of the Universe*, Sky Publishing and Cambridge University Press, 1993.

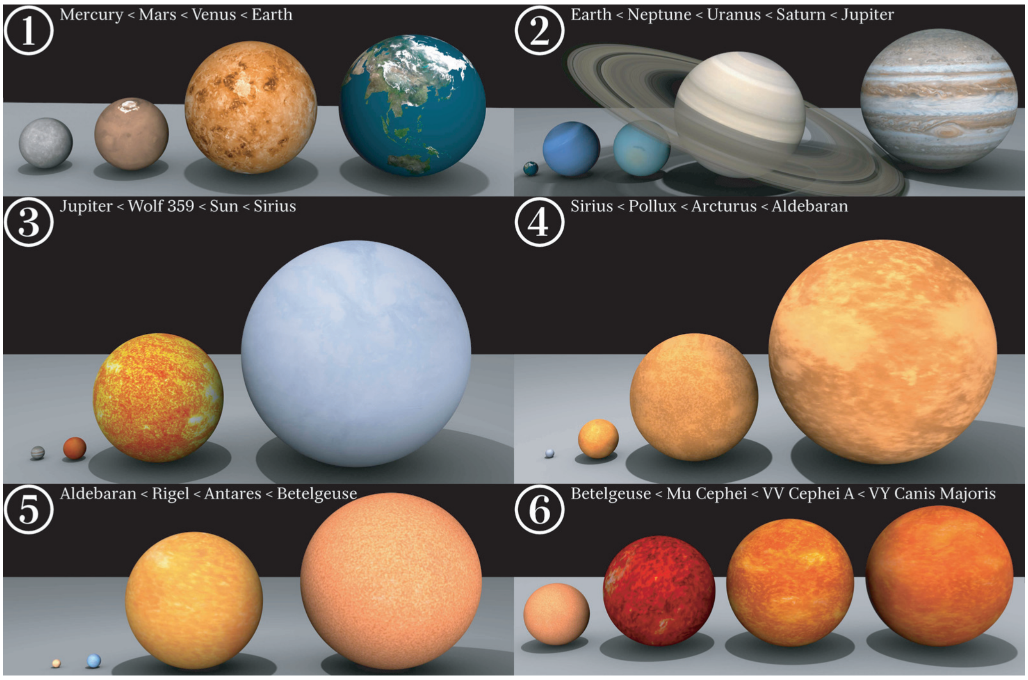


FIGURE 52 A comparison of star sizes (© Dave Jarvis).

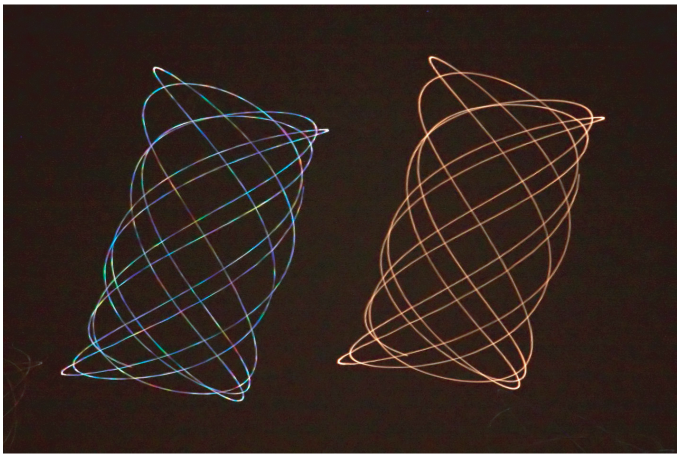


FIGURE 53 Regulus and Mars, photographed with an exposure time of 10 s on 4 June 2010 with a wobbling camera, show the difference between a point-like star that twinkles and an extended planet that does not (© Jürgen Michelberger).

Challenge 159 e

The difference between 'point-like' and finite-size sources can be seen with the naked eye: at night, stars twinkle, but planets do not. (Check it!) A beautiful visualization is shown in Figure 53. This effect is due to the turbulence of air. Turbulence has an effect on the almost point-like stars because it deflects light rays by small amounts. On the other hand, air turbulence is too weak to lead to the twinkling of sources of larger angular size, such as planets or artificial satellites,* because the deflection is averaged out in this case.

* A *satellite* is an object circling a planet, like the Moon; an *artificial satellite* is a system put into orbit by humans, like the Sputniks.

An object is *point-like for the naked eye* if its angular size is smaller than about
Challenge 160 s $2' = 0.6$ mrad. Can you estimate the size of a 'point-like' dust particle? By the way, an object is *invisible* to the naked eye if it is point-like *and* if its luminosity, i.e., the intensity of the light from the object reaching the eye, is below some critical value. Can you estimate whether there are any man-made objects visible from the Moon, or from the space
Challenge 161 s shuttle?

The above definition of 'point-like' in everyday life is obviously misleading. Do proper, real point particles exist? In fact, is it at all possible to show that a particle has a vanishing size? In the same way, we need to ask and check whether points in space do exist. Our walk will lead us to the astonishing result that all the answers to these ques-
Challenge 162 s tions are negative. Can you imagine why? Do not be disappointed if you find this issue difficult; many brilliant minds have had the same problem.

However, many particles, such as electrons, quarks or photons are point-like for all practical purposes. Once we know how to describe the motion of point particles, we can also describe the motion of extended bodies, rigid or deformable: we assume that they are made of parts. This is the same approach as describing the motion of an animal as a whole by combining the motion of its various body parts. The simplest description, the *continuum approximation*, describes extended bodies as an infinite collection of point particles. It allows us to understand and to predict the motion of milk and honey, the motion of the air in hurricanes and of perfume in rooms. The motion of fire and all other gaseous bodies, the bending of bamboo in the wind, the shape changes of chewing
Ref. 70 gum, and the growth of plants and animals can also be described in this way.

All observations so far have confirmed that the motion of large bodies can be described to full precision as the result of the motion of their parts. All machines that humans ever built are based on this idea. A description that is even more precise than the
Vol. IV, page 15 continuum approximation is given later on. Describing body motion with the motion of body parts will guide us through the first five volumes of our mountain ascent; for example, we will understand life in this way. Only in the final volume will we discover that, at a fundamental scale, this decomposition is impossible.

Legs and wheels

The parts of a body determine its shape. Shape is an important aspect of bodies: among other things, it tells us how to count them. In particular, living beings are always made of a single body. This is not an empty statement: from this fact we can deduce that animals cannot have large wheels or large propellers, but only legs, fins, or wings. Why?

Living beings have only one surface; simply put, they have only one piece of skin.
Vol. V, page 365 Mathematically speaking, animals are *connected*. This is often assumed to be obvious, and
Ref. 71 it is often mentioned that the blood supply, the nerves and the lymphatic connections to a rotating part would get tangled up. However, this argument is not so simple, as Figure 54 shows. The figure proves that it is indeed possible to rotate a body continuously against a second one, without tangling up the connections. Three dimensions of space allow *tethered rotation*. Can you find an example for this kind of motion, often called *tethered*
Challenge 163 s *rotation*, in your own body? Are you able to see how many cables may be attached to the
Challenge 164 s rotating body of the figure without hindering the rotation?

Despite the possibility of animals having rotating parts, the method of Figure 54 or

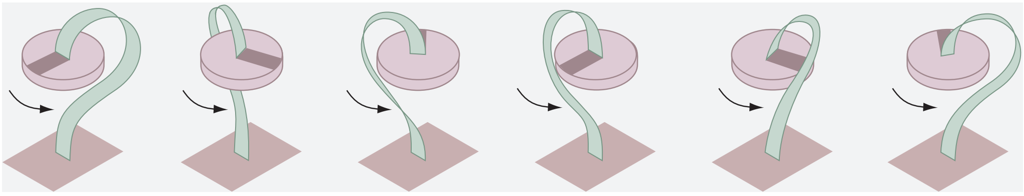


FIGURE 54 Tethered rotation: How an object can rotate continuously without tangling up the connection to a second object.

FIGURE 55 Tethered rotation: the continuous rotation of an object attached to its environment (QuickTime film © Jason Hise).

Figure 55 still cannot be used to make a practical wheel or propeller. Can you see why? Challenge 165 s Therefore, evolution had no choice: it had to avoid animals with (large) parts rotating around axles. That is the reason that propellers and wheels do not exist in nature. Of course, this limitation does not rule out that living bodies move by rotation as a whole: Ref. 72 tumbleweed, seeds from various trees, some insects, several spiders, certain other animals, children and dancers occasionally move by rolling or rotating as a whole.

Large single bodies, and thus all large living beings, can thus only move through *de-* Ref. 73 *formation* of their shape: therefore they are limited to walking, running, jumping, rolling, gliding, crawling, flapping fins, or flapping wings. Moving a leg is a common way to deform a body.

Ref. 74 Extreme examples of leg use in nature are shown in Figure 56 and Figure 57. The most extreme example of rolling spiders – there are several species – are *Cebrennus villosus* and Ref. 75 live in the sand in Morocco. They use their legs to accelerate the rolling, they can steer the rolling direction and can even roll uphill slopes of 30 % – a feat that humans are

FIGURE 56 Legs and 'wheels' in living beings: the red millipede *Aphistogoniulus erythrocephalus* (15 cm body length), a gecko on a glass pane (15 cm body length), an amoeba *Amoeba proteus* (1 mm size), the rolling shrimp *Nannosquilla decemspinosa* (2 cm body length, 1.5 rotations per second, up to 2 m, can even roll slightly uphill slopes) and the rolling caterpillar *Pleurotya ruralis* (can only roll downhill, to escape predators), (© David Parks, Marcel Berendsen, Antonio Guillén Oterino, Robert Full, John Brackenbury / Science Photo Library).

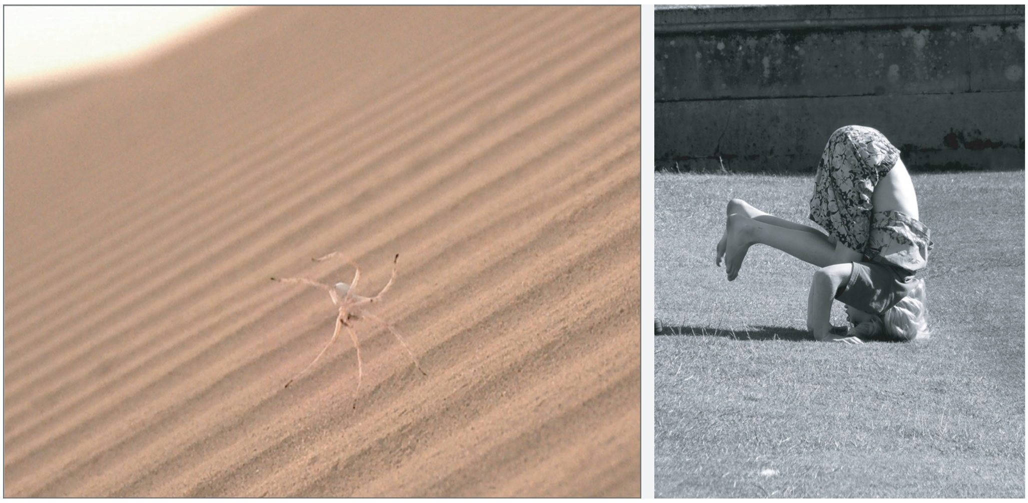


FIGURE 57 Two of the rare lifeforms that are able to roll *uphill* also on steep slopes: the desert spider *Cebrennus villosus* and *Homo sapiens* (© Ingo Rechenberg, Karva Javi).

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unable to perform. Films of the rolling motion can be found at www.bionik.tu-berlin.de.* Walking on water is shown in Figure 127 on page 170; examples of wings are given later on, as are the various types of deformations that allow swimming in water.

In contrast, *systems of several bodies*, such as bicycles, pedal boats or other machines, can move *without* any change of shape of their components, thus enabling the use of axles with wheels, propellers and other rotating devices.**

In short, whenever we observe a construction in which some part is turning continuously (and without the 'wiring' of Figure 54) we know immediately that it is an artefact: it is a machine, not a living being (but built by one). However, like so many statements about living creatures, this one also has exceptions.

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The distinction between one and two bodies is poorly defined if the whole system is made of only a few molecules. This happens most clearly inside bacteria. Organisms such as *Escherichia coli*, the well-known bacterium found in the human gut, or bacteria from the *Salmonella* family, all swim using flagella. *Flagella* are thin filaments, similar to tiny hairs that stick out of the cell membrane. In the 1970s it was shown that each flagellum, made of one or a few long molecules with a diameter of a few tens of nanometres, does in fact turn about its axis.

Ref. 76
Ref. 77
Bacteria are able to rotate their flagella in both clockwise and anticlockwise directions, can achieve more than 1000 turns per second, and can turn all their flagella in perfect synchronization. These wheels are so tiny that they do not need a mechanical connection; Figure 58 shows a number of motor models found in bacteria. The motion and the construction of these amazing structures are shown in more details in the films Figure 59 and Figure 60.

In summary, wheels actually do exist in living beings, albeit only tiny ones. The growth and motion of these wheels are wonders of nature. Macroscopic wheels in living beings are not possible, though rolling motion is.

Curiosities and fun challenges about kinematics

Challenge 167 s
What is the biggest wheel ever made?

* *

Challenge 168 s
A football is shot, by a goalkeeper, with around 30 m/s. Use a video to calculate the distance it should fly and compare it with the distances realized in a soccer match. Where does the difference come from?

* *

A train starts to travel at a constant speed of 10 m/s between two cities A and B, 36 km

* Rolling is also known for the Namibian wheel spiders of the *Carparachne* genus; films of their motion can be found on the internet.

** Despite the disadvantage of not being able to use rotating parts and of being restricted to one piece only, nature's moving constructions, usually called animals, often outperform human-built machines. As an example, compare the size of the smallest flying systems built by evolution with those built by humans. (See, e.g., pixelito.reference.be.) There are two reasons for this discrepancy. First, nature's systems have integrated repair and maintenance systems. Second, nature can build large structures inside containers with small openings. In fact, nature is very good at what people do when they build sailing ships inside glass bottles. The human body is full of such examples; can you name a few?
Challenge 166 s

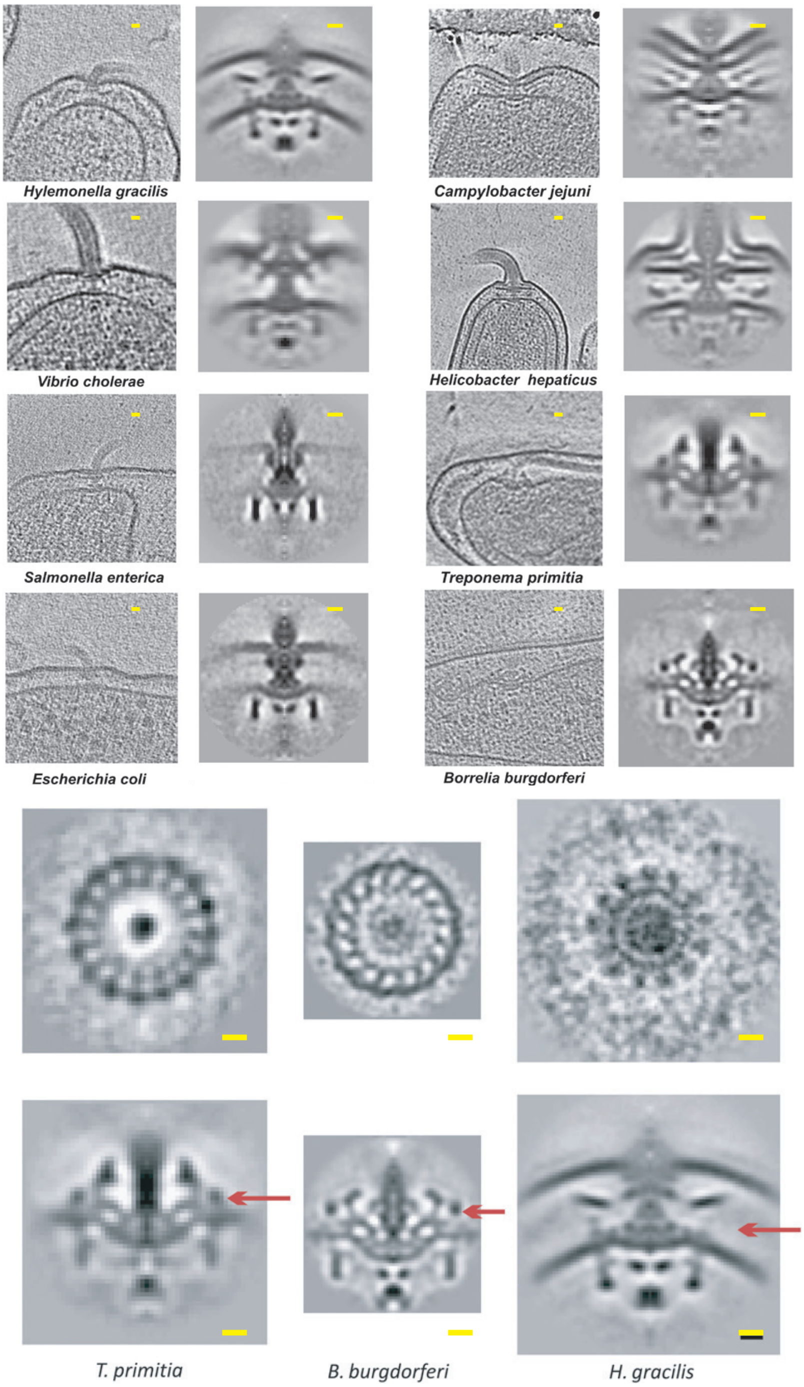


FIGURE 58 Some types of flagellar motors found in nature; the images are taken by cryotomography. All yellow scale bars are 10 nm long (© S. Chen & al., EMBO Journal, Wiley & Sons).

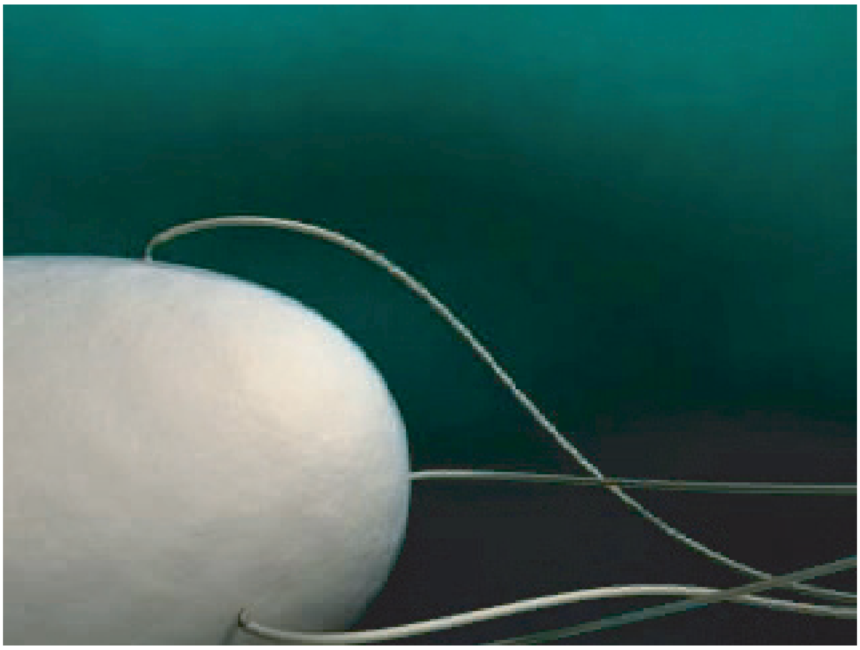


FIGURE 59 The rotational motion of a bacterial flagellum, and its reversal (QuickTime film © Osaka University).

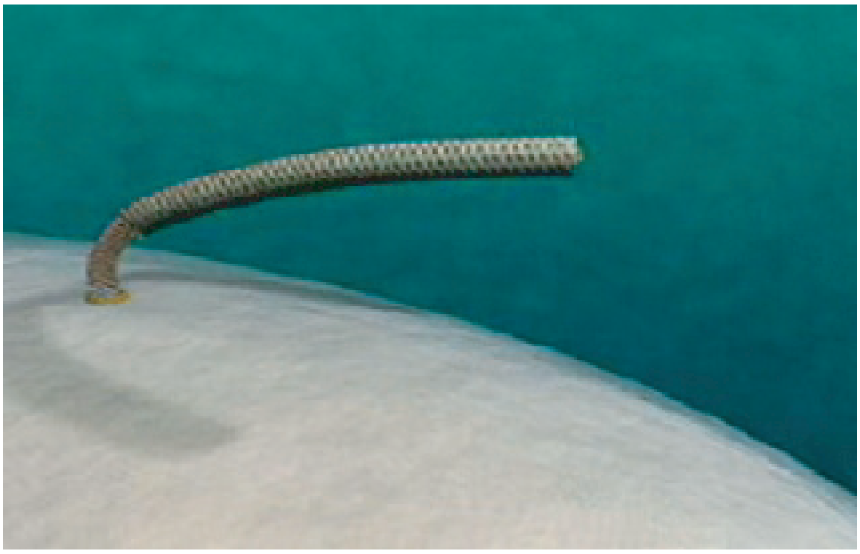


FIGURE 60 The growth of a bacterial flagellum, showing the molecular assembly (QuickTime film © Osaka University).

apart. The train will take one hour for the journey. At the same time as the train, a fast dove starts to fly from A to B, at 20 m/s. Being faster than the train, the dove arrives at B first. The dove then flies back towards A; when it meets the train, it turns back again, to city B. It goes on flying back and forward until the train reaches B. What distance did the dove cover?

Challenge 169 e

FIGURE 61 Are comets, such as the beautiful comet McNaught seen in 2007, images or bodies? How can you show it? (And why is the tail curved?) (© Robert McNaught)

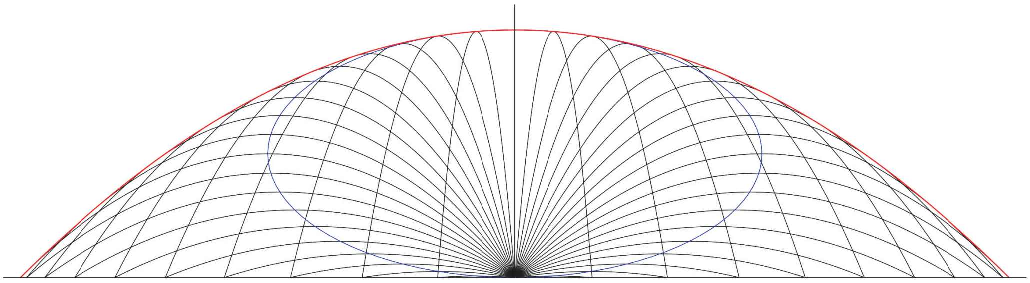


FIGURE 62 The parabola of safety around a cannon, shown in red. The highest points of all trajectories form an ellipse, shown in blue. (© Theon)

* *

Figure 62 illustrates that around a cannon, there is a line outside which you cannot be hit. Already in the 17th century, Evangelista Torricelli showed, without algebra, that the line is a parabola, and called it the *parabola of safety*. Can you show this as well? Can you confirm that the highest points of all trajectories lie on an ellipse? The parabola of safety also appears in certain water fountains. Challenge 170 e

* *

Balance a pencil vertically (tip upwards!) on a piece of paper near the edge of a table. How can you pull out the paper without letting the pencil fall? Challenge 171 e

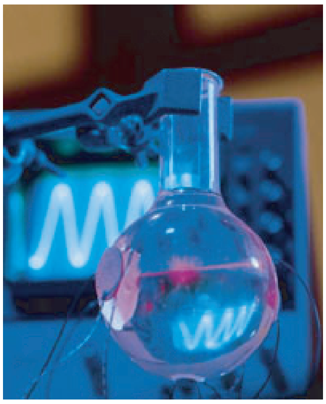


FIGURE 63 Observation of sonoluminescence with a simple set-up that focuses ultrasound in water (© Detlef Lohse).

* *

Challenge 172 e

Is a return flight by aeroplane – from a point A to B and back to A – faster if the wind blows or if it does not?

* *

The level of acceleration that a human can survive depends on the duration over which one is subjected to it. For a tenth of a second, $30\,g = 300\,\mathrm{m/s^2}$, as generated by an ejector seat in an aeroplane, is acceptable. (It seems that the record acceleration a human has survived is about $80\,g = 800\,\mathrm{m/s^2}$.) But as a rule of thumb it is said that accelerations of $15\,g = 150\,\mathrm{m/s^2}$ or more are fatal.

* *

The highest *microscopic* accelerations are observed in particle collisions, where values up to $10^{35}\,\mathrm{m/s^2}$ are achieved. The highest *macroscopic* accelerations are probably found in the collapsing interiors of *supernovae*, exploding stars which can be so bright as to be visible in the sky even during the daytime. A candidate on Earth is the interior of collapsing bubbles in liquids, a process called *cavitation*. Cavitation often produces light, an effect discovered by Frenzel and Schultes in 1934 and called *sonoluminescence*. (See Figure 63.) It appears most prominently when air bubbles in water are expanded and contracted by underwater loudspeakers at around 30 kHz and allows precise measurements of bubble motion. At a certain threshold intensity, the bubble radius changes at 1500 m/s in as little as a few μm, giving an acceleration of several $10^{11}\,\mathrm{m/s^2}$.

Ref. 78

Ref. 79

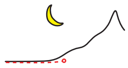
* *

Legs are easy to build. Nature has even produced a millipede, *Illacme plenipes*, that has 750 legs. The animal is 3 to 4 cm long and about 0.5 mm wide. This seems to be the record so far. In contrast to its name, no millipede actually has a thousand legs.

Summary of kinematics

The description of everyday motion of mass points with three coordinates as $(x(t), y(t), z(t))$ is simple, precise and complete. This description of paths is the basis of kinematics. As a consequence, space is described as a three-dimensional Euclidean space and velocity and acceleration as Euclidean vectors.

The description of motion with paths assumes that the motion of objects can be *followed* along their paths. Therefore, the description often does not work for an important case: the motion of images.



CHAPTER 4

FROM OBJECTS AND IMAGES TO CONSERVATION

Walking through a forest we observe two rather different types of motion: we see the breeze move the leaves, and at the same time, on the ground, we see their shadows move. Shadows are a simple type of image. Both objects and images are able to move; both change position over time. Running tigers, falling snowflakes, and material ejected by volcanoes, but also the shadow following our body, the beam of light circling the tower of a lighthouse on a misty night, and the rainbow that constantly keeps the same apparent distance from us are examples of motion.

Ref. 80

Both objects and images differ from their environment in that they have *boundaries* defining their size and shape. But everybody who has ever seen an animated cartoon knows that images can move in more surprising ways than objects. Images can change their size and shape, they can even change colour, a feat only a few objects are able to perform.** Images can appear and disappear without a trace, multiply, interpenetrate, go backwards in time and defy gravity or any other force. Images, even ordinary shadows, can move faster than light. Images can float in space and keep the same distance from approaching objects. Objects can do almost none of this. In general, the 'laws of cartoon physics' are rather different from those in nature. In fact, the motion of images does not seem to follow any rules, in contrast to the motion of objects. We feel the need for precise criteria allowing the two cases to be distinguished.

Ref. 82

Making a clear distinction between images and objects is performed using the same method that children or animals use when they stand in front of a mirror for the first time: they try to *touch* what they see. Indeed,

▷ If we are able to touch what we see – or more precisely, if we are able to move it with a collision – we call it an *object*, otherwise an *image*.***

** Excluding very slow changes such as the change of colour of leaves in the Autumn, in nature only certain crystals, the octopus and other cephalopods, the chameleon and a few other animals achieve this. Of man-made objects, television, computer displays, heated objects and certain lasers can do it. Do you know more examples? An excellent source of information on the topic of colour is the book by K. NASSAU, *The Physics and Chemistry of Colour – the fifteen causes of colour*, J. Wiley & Sons, 1983. In the popular science domain, the most beautiful book is the classic work by the Flemish astronomer MARCEL G. J. MINNAERT, *Light and Colour in the Outdoors*, Springer, 1993, an updated version based on his wonderful book series, *De natuurkunde van 't vrije veld*, Thieme & Cie, 1937. Reading it is a must for all natural scientists. On the web, there is also the – simpler, but excellent – webexhibits.org/causesofcolour website.

Challenge 173 s

Ref. 81

*** One could propose including the requirement that objects may be rotated; however, this requirement, surprisingly, gives difficulties in the case of atoms, as explained on page 85 in Volume IV.

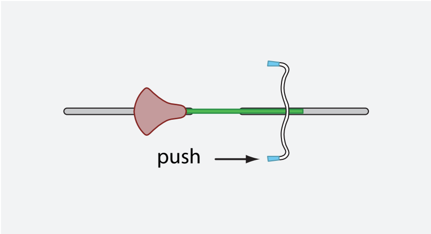


FIGURE 64 In which direction does the bicycle turn?

Vol. IV, page 139 Images cannot be touched, but objects can. Images cannot hit each other, but objects can. And as everybody knows, touching something means feeling that it resists movement. Certain bodies, such as butterflies, pose little resistance and are moved with ease, others, such as ships, resist more, and are moved with more difficulty.

▷ The resistance to motion – more precisely, to change of motion – is called *inertia*, and the difficulty with which a body can be moved is called its *(inertial) mass*.

Images have neither inertia nor mass.

Summing up, for the description of motion we must distinguish bodies, which can be touched and are impenetrable, from images, which cannot and are not. Everything visible is either an object or an image; there is no third possibility. (Do you agree?) Challenge 174 s If the object is so far away that it cannot be touched, such as a star or a comet, it can be difficult to decide whether one is dealing with an image or an object; we will encounter this difficulty repeatedly. For example, how would you show that comets – such as the beautiful example of Figure 61 – are objects and not images, as Galileo (falsely) claimed?

Challenge 175 s

In the same way that objects are made of *matter*, images are made of *radiation*. Images are the domain of shadow theatre, cinema, television, computer graphics, belief systems and drug experts. Ref. 83 Photographs, motion pictures, ghosts, angels, dreams and many hallucinations are images (sometimes coupled with brain malfunction). To understand images, we need to study radiation (plus the eye and the brain). However, due to the importance of objects – after all, we are objects ourselves – we study the latter first.

Motion and contact

> Democritus affirms that there is only one type of movement: That resulting from collision.
> Aetius, *Opinions*.

Ref. 84

When a child rides a unicycle, she or he makes use of a general rule in our world: one body acting on another puts it in motion. Indeed, in about six hours, anybody can learn to ride and enjoy a unicycle. As in all of life's pleasures, such as toys, animals, women, machines, children, men, the sea, wind, cinema, juggling, rambling and loving, something pushes something else. Thus our first challenge is to describe the transfer of motion due to contact – and to collisions – in more precise terms.

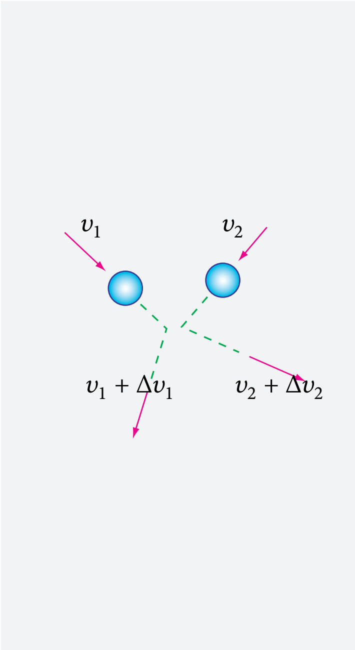


FIGURE 65 Collisions define mass.

FIGURE 66 The standard kilogram (until 2019) (© BIPM).

Contact is not the only way to put something into motion; a counter-example is an apple falling from a tree or one magnet pulling another. Non-contact influences are more fascinating: nothing is hidden, but nevertheless something mysterious happens. Contact motion seems easier to grasp, and that is why one usually starts with it. However, despite this choice, non-contact interactions cannot be avoided. Our choice to start with contact will lead us to a similar experience to that of riding a bicycle. (See Figure 64.) If we ride a bicycle at a sustained speed and try to turn left by pushing the right-hand steering bar, we will turn *right*. By the way, this surprising effect, also known to motorbike riders, obviously works only above a certain minimum speed. Can you determine what this
Challenge 176 s speed is? Be careful! Too strong a push will make you fall.

Something similar will happen to us as well; despite our choice of contact motion, the rest of our walk will rapidly force us to study non-contact interactions.

What is mass?

> Δός μοί (φησι) ποῦ στῶ καὶ κινῶ τὴν γῆν.
> Da ubi consistam, et terram movebo.*
>
> Archimedes

When we push something we are unfamiliar with, such as when we kick an object on the street, we automatically pay attention to the same aspect that children explore when

* 'Give me a place to stand, and I'll move the Earth.' Archimedes (*c.* 283–212), Greek scientist and engineer.
Ref. 85 This phrase is attributed to him by Pappus. Already Archimedes knew that the distinction used by lawyers between movable and immovable objects made no sense.

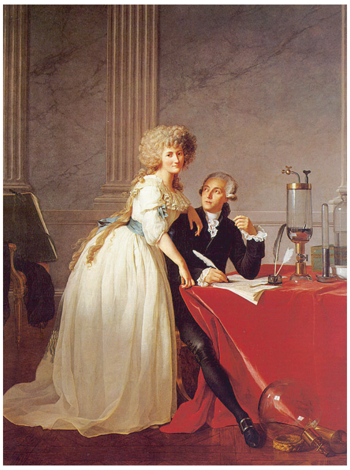


FIGURE 67 Antoine Lavoisier (1743–1794) and his wife.

they stand in front of a mirror for the first time, or when they see a red laser spot for the first time. They check whether the unknown entity can be pushed or caught, and they pay attention to how the unknown object moves under their influence. All these are collision experiments. The high-precision version of any collision experiment is illustrated in Figure 65. Repeating such experiments with various pairs of objects, we find:

▷ A *fixed* quantity m_i can be ascribed to every object i, determined by the relation

$$\frac{m_2}{m_1} = -\frac{\Delta v_1}{\Delta v_2} \qquad (17)$$

where Δv is the velocity change produced by the collision. The quantity m_i is called the *mass* of the object i.

The more difficult it is to move an object, the higher the mass value. In order to have mass values that are common to everybody, the mass value for one particular, selected object has to be fixed in advance. Until 2019, there really was one such special object in the world, shown in Figure 66; it was called the *standard kilogram*. It was kept with great care in a glass container in Sèvres near Paris. Until 2019, the standard kilogram determined the value of the mass of every other object in the world. The standard kilogram was touched only once every few years because otherwise dust, humidity, or scratches would change its mass. For example, the standard kilogram was *not* kept under vacuum, because this would lead to outgassing and thus to changes in its mass. All the care did not avoid the stability issues though, and in 2019, the kilogram unit has been redefined using the fundamental constants G (indirectly, via the caesium transition frequency), c and $\hbar$ that are shown in Figure 1. Since that change, everybody can produce his or her own standard kilogram in the laboratory – provided that sufficient care is used.

The *mass* thus measures *the difficulty of getting something moving.* High masses are harder to move than low masses. Obviously, only objects have mass; images don't. (By

Ref. 68 the way, the word 'mass' is derived, via Latin, from the Greek μαζα – bread – or the

FIGURE 68 Christiaan Huygens (1629–1695).

Hebrew 'mazza' – unleavened bread. That is quite a change in meaning.)

Experiments with everyday life objects also show that throughout any collision, the sum of all masses is *conserved*:

$$\sum_i m_i = \text{const} . \quad (18)$$

The principle of conservation of mass was first stated by Antoine-Laurent Lavoisier.* Conservation of mass also implies that the mass of a composite system is the sum of the mass of the components. In short, *mass is also a measure for the quantity of matter.*

In a famous experiment in the sixteenth century, for several weeks Santorio Santorio (Sanctorius) (1561–1636), a friend of Galileo, lived with all his food and drink supply, and also his toilet, on a large balance. He wanted to test mass conservation. How did the measured weight change with time? Challenge 177 s

Various cult leaders pretended and still pretend that they can produce matter out of nothing. This would be an example of non-conservation of mass. How can you show that all such leaders are crooks? Challenge 178 s

Momentum and mass

The definition of mass can also be given in another way. We can ascribe a number m_i to every object i such that for collisions free of outside interference the following sum is unchanged *throughout* the collision:

$$\sum_i m_i \boldsymbol{v}_i = \text{const} . \quad (19)$$

The product of the velocity $\boldsymbol{v}_i$ and the mass m_i is called the (linear) *momentum* of the body. The sum, or *total momentum* of the system, is the same before and after the collision; momentum is a *conserved* quantity.

* Antoine-Laurent Lavoisier (b. 1743 Paris , d. 1794 Paris), chemist and genius. Lavoisier was the first to understand that combustion is a reaction with oxygen; he discovered the components of water and introduced mass measurements into chemistry. A famous story about his character: When he was (unjustly) sentenced to the guillotine during the French revolution, he decided to use the situation for a scientific experiment. He announced that he would try to blink his eyes as frequently as possible after his head was cut off, in order to show others how long it takes to lose consciousness. Lavoisier managed to blink eleven times. It is unclear whether the story is true or not. It is known, however, that it could be true. Indeed, after a decapitation without pain or shock, a person can remain conscious for up to half a minute. Ref. 86

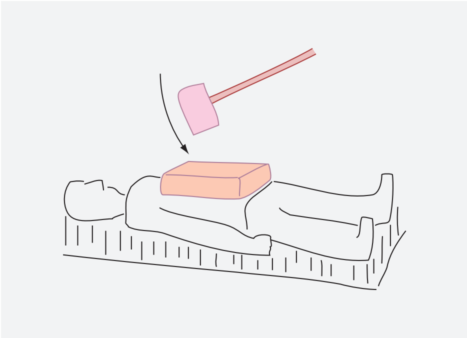


FIGURE 69 Is this dangerous?

▷ Momentum conservation defines mass.

The two conservation principles (18) and (19) were first stated in this way by the important physicist Christiaan Huygens:* *Momentum and mass are conserved in the everyday motion of objects.* In particular, neither quantity can be defined for the motion of images. Some typical momentum values are given in Table 15.

Momentum conservation implies that when a moving sphere hits a resting one of the same mass and without loss of energy, a simple rule determines the angle between the directions the two spheres take after the collision. Can you find this rule? It is particularly useful when playing billiards. We will find out later that the rule is *not* valid for speeds near that of light.

Challenge 179 s
Vol. II, page 67

Another consequence of momentum conservation is shown in Figure 69: a man is lying on a bed of nails with a large block of concrete on his stomach. Another man is hitting the concrete with a heavy sledgehammer. As the impact is mostly absorbed by the concrete, there is no pain and no danger – unless the concrete is missed. Why?

Challenge 180 s

The above definition (17) of mass has been generalized by the physicist and philosopher Ernst Mach** in such a way that it is valid even if the two objects interact without contact, as long as they do so along the line connecting their positions.

▷ The mass ratio between two bodies is defined as a negative inverse acceleration ratio, thus as

$$\frac{m_2}{m_1} = -\frac{a_1}{a_2} , \qquad (20)$$

* Christiaan Huygens (b. 1629 's Gravenhage, d. 1695 Hofwyck) was one of the main physicists and mathematicians of his time. Huygens clarified the concepts of mechanics; he also was one of the first to show that light is a wave. He wrote influential books on probability theory, clock mechanisms, optics and astronomy. Among other achievements, Huygens showed that the Orion Nebula consists of stars, discovered Titan, the moon of Saturn, and showed that the rings of Saturn consist of rock. (This is in contrast to Saturn itself, whose density is lower than that of water.)

** Ernst Mach (1838 Chrlice–1916 Vaterstetten), Austrian physicist and philosopher. The *mach* unit for aeroplane speed as a multiple of the speed of sound in air (about 0.3 km/s) is named after him. He also studied the basis of mechanics. His thoughts about mass and inertia influenced the development of general relativity and led to Mach's principle, which will appear later on. He was also proud to be the last scientist denying – humorously, and against all evidence – the existence of atoms.

TABLE 15 Some measured momentum values.

Observation	Momentum
Images	0
Momentum of a green photon	$1.2 \cdot 10^{-27}$ Ns
Average momentum of oxygen molecule in air	10^{-26} Ns
X-ray photon momentum	10^{-23} Ns
γ photon momentum	10^{-17} Ns
Highest particle momentum in accelerators	1 fNs
Highest possible momentum of a single elementary particle – the Planck momentum	6.5 Ns
Fast billiard ball	3 Ns
Flying rifle bullet	10 Ns
Box punch	15 to 50 Ns
Comfortably walking human	80 Ns
Lion paw strike	*c.* 0.2 kNs
Whale tail blow	*c.* 3 kNs
Car on highway	40 kNs
Impact of meteorite with 2 km diameter	100 TNs
Momentum of a galaxy in galaxy collision	up to 10^{46} Ns

where a is the acceleration of each body during the interaction.

This definition of mass has been explored in much detail in the physics community, mainly in the nineteenth century. A few points sum up the results:

— The definition of mass *implies* the conservation of total momentum $\sum mv$. Momentum conservation is *not* a separate principle. Conservation of momentum cannot be checked experimentally, because mass is defined in such a way that the momentum conservation holds.
— The definition of mass *implies* the equality of the products $m_1 a_1$ and $-m_2 a_2$. Such products are called *forces*. The equality of acting and reacting forces is not a separate principle; mass is defined in such a way that the principle holds.
— The definition of mass is *independent* of whether contact is involved or not, and whether the accelerations are due to electricity, gravitation, or other interactions.* Since the interaction does not enter the definition of mass, mass values defined with the help of the electric, nuclear or gravitational interaction all agree, as long as momentum is conserved. All known interactions conserve momentum. For some unfortunate historical reasons, the mass value measured with the electric or nuclear interactions is called the 'inertial' mass and the mass measured using gravity is called

Page 119
Challenge 181 s
* As mentioned above, only *central* forces obey the relation (20) used to define mass. Central forces act between the centre of mass of bodies. We give a precise definition later. However, since all fundamental forces are central, this is not a restriction. There seems to be one notable exception: magnetism. Is the definition of mass valid in this case?

TABLE 16 Some measured mass values.

Observation	Mass
Probably lightest known object: neutrino	*c.* $2 \cdot 10^{-36}$ kg
Mass increase due to absorption of one green photon	$4.1 \cdot 10^{-36}$ kg
Lightest known charged object: electron	$9.109\,381\,88(72) \cdot 10^{-31}$ kg
Atom of argon	39.962 383 123(3) u = 66.359 1(1) yg
Lightest object ever weighed (a gold particle)	0.39 ag
Human at early age (fertilized egg)	10^{-8} g
Water adsorbed on to a kilogram metal weight	10^{-5} g
Planck mass	$2.2 \cdot 10^{-5}$ g
Fingerprint	10^{-4} g
Typical ant	10^{-4} g
Water droplet	1 mg
Honey bee, *Apis mellifera*	0.1 g
Euro coin	7.5 g
Blue whale, *Balaenoptera musculus*	180 Mg
Heaviest living things, such as the fungus *Armillaria ostoyae* or a large Sequoia *Sequoiadendron giganteum*	10^{6} kg
Heaviest train ever	$99.7 \cdot 10^{6}$ kg
Largest ocean-going ship	$400 \cdot 10^{6}$ kg
Largest object moved by man (Troll gas rig)	$687.5 \cdot 10^{6}$ kg
Large antarctic iceberg	10^{15} kg
Water on Earth	10^{21} kg
Earth's mass	$5.98 \cdot 10^{24}$ kg
Solar mass	$2.0 \cdot 10^{30}$ kg
Our galaxy's visible mass	$3 \cdot 10^{41}$ kg
Our galaxy's estimated total mass	$2 \cdot 10^{42}$ kg
virgo supercluster	$2 \cdot 10^{46}$ kg
Total mass visible in the universe	10^{54} kg

the 'gravitational' mass. As it turns out, this artificial distinction makes no sense; this becomes especially clear when we take an observation point that is *far away* from all the bodies concerned.

— The definition of mass requires observers at rest or in inertial motion.

By measuring the masses of bodies around us we can explore the science and art of experiments. An overview of mass measurement devices is given in Table 18 and Figure 71. Some measurement results are listed in Table 16.

By measuring mass values around us we confirm the main properties of mass. First of all, mass is *additive* in everyday life, as the mass of two bodies combined is equal to the sum of the two separate masses. Furthermore, mass is *continuous*; it can seemingly take any positive value. Finally, mass is *conserved* in everyday life; the mass of a system,

TABLE 17 Properties of mass in everyday life.

Masses	Physical property	Mathematical name	Definition
Can be distinguished	distinguishability	element of set	Vol. III, page 285
Can be ordered	sequence	order	Vol. IV, page 224
Can be compared	measurability	metricity	Vol. IV, page 236
Can change gradually	continuity	completeness	Vol. V, page 364
Can be added	quantity of matter	additivity	Page 81
Beat any limit	infinity	unboundedness, openness	Vol. III, page 286
Do not change	conservation	invariance	m = const
Do not disappear	impenetrability	positivity	$m \geqslant 0$

defined as the sum of the mass of all constituents, does not change over time if the system is kept isolated from the rest of the world. Mass is not only conserved in collisions but also during melting, evaporation, digestion and all other everyday processes.

All the properties of everyday mass are summarized in Table 17. Later we will find that several of the properties are only approximate. High-precision experiments show deviations.* However, the definition of mass remains unchanged throughout our adventure.

The definition of mass through momentum conservation implies that when an object falls, the Earth is accelerated upwards by a tiny amount. If we could measure this tiny amount, we could determine the mass of the Earth. Unfortunately, this measurement is impossible. Can you find a better way to determine the mass of the Earth? Challenge 182 s

The definition of mass and momentum allows answering the question of Figure 70. A brick hangs from the ceiling; a second thread hangs down from the brick, and you can pull it. How can you tune your pulling method to make the upper thread break? The lower one? Challenge 183 e

Summarizing Table 17, the mass of a body is thus most precisely described by a *positive real number*, often abbreviated m or M. This is a direct consequence of the impenetrability of matter. Indeed, a *negative* (inertial) mass would mean that such a body would move in the opposite direction of any applied force or acceleration. Such a body could not be kept in a box; it would break through any wall trying to stop it. Strangely enough, negative mass bodies would still fall downwards in the field of a large positive mass (though more slowly than an equivalent positive mass). Are you able to confirm this? However, a small positive mass object would float away from a large negative-mass body, as you can easily deduce by comparing the various accelerations involved. A positive and a negative mass of the same value would remain at a constant distance and spontaneously accelerate away along the line connecting the two masses. Note that both energy and momentum are conserved in all these situations.** Negative-mass bodies have never been observed. Antimatter, which will be discussed later, also has positive mass.

Challenge 184 e
Challenge 185 e
Vol. II, page 72
Vol. IV, page 192

* For example, in order to define mass we must be able to *distinguish* bodies. This seems a trivial requirement, but we discover that this is not always possible in nature.

** For more curiosities, see R. H. Price, *Negative mass can be positively amusing*, American Journal of

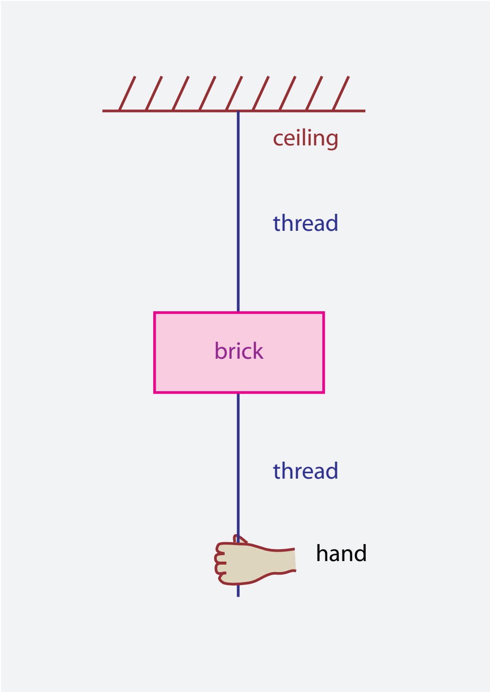


FIGURE 70 Depending on the way you pull, either the upper of the lower thread snaps. What are the options?

TABLE 18 Some mass sensors.

Measurement	Sensor	Range
Precision scales	balance, pendulum, or spring	1 pg to 10^3 kg
Particle collision	speed	below 1 mg
Sense of touch	pressure sensitive cells	1 mg to 500 kg
Doppler effect on light reflected off the object	interferometer	1 mg to 100 g
Cosmonaut body mass measurement device	spring frequency	around 70 kg
Truck scales	hydraulic balance	10^3 to $60 \cdot 10^3$ kg
Ship weight	water volume measurement	up to $500 \cdot 10^6$ kg

Physics **61**, pp. 216–217, 1993. Negative mass particles in a box would heat up a box made of positive mass while traversing its walls, and accelerating, i.e., losing energy, at the same time. They would allow one to build a *perpetuum mobile* of the second kind, i.e., a device circumventing the second principle of thermodynamics. Moreover, such a system would have no thermodynamic equilibrium, because its energy could decrease forever. The more one thinks about negative mass, the more one finds strange properties contradicting observations. By the way, what is the range of possible mass values for tachyons?

Page 110
Challenge 186 e
Challenge 187 s

FIGURE 71 Mass measurement devices: a vacuum balance used in 1890 by Dmitriy Ivanovich Mendeleyev, a modern laboratory balance, a device to measure the mass of a cosmonaut in space and a truck scales (© Thinktank Trust, Mettler-Toledo, NASA, Anonymous).

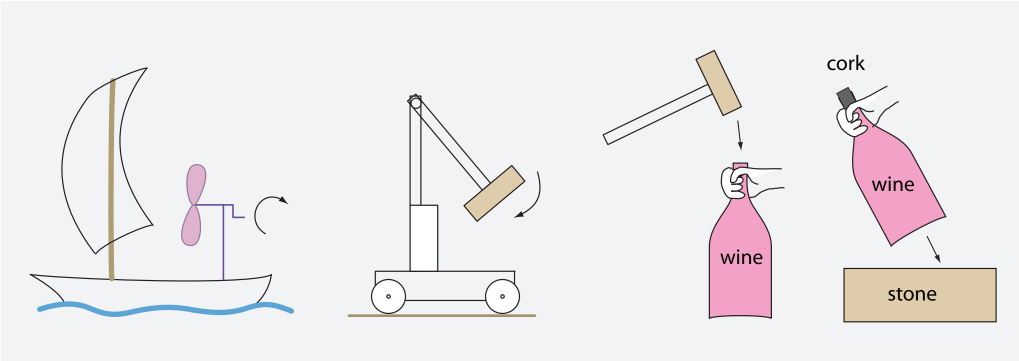
Is motion eternal? – Conservation of momentum

> “Every body continues in the state of rest or of uniform motion in a straight line except in so far as it doesn’t.”
>
> Arthur Eddington*

The product $\boldsymbol{p} = m\boldsymbol{v}$ of mass and velocity is called the *momentum* of a particle; it describes the tendency of an object to keep moving during collisions. The larger it is, the harder it is to stop the object. Like velocity, momentum has a direction and a magnitude: it is a vector. In French, momentum is called ‘quantity of motion’, a more appropriate term. In the old days, the term ‘motion’ was used instead of ‘momentum’, for example by Newton. The conservation of momentum, relation (19), therefore expresses the conservation of motion during interactions.

Momentum is an *extensive quantity*. That means that it can be said that it *flows* from one body to the other, and that it can be *accumulated* in bodies, in the same way that water flows and can be accumulated in containers. Imagining momentum as something

* Arthur Eddington (1882–1944), British astrophysicist.



Challenge 188 s FIGURE 72 What happens in these four situations?

Ref. 87 that can be *exchanged* between bodies in collisions is always useful when thinking about the description of moving objects.

Momentum is conserved. That explains the limitations you might experience when being on a perfectly frictionless surface, such as ice or a polished, oil covered marble: you cannot propel yourself forward by patting your own back. (Have you ever tried to put a cat on such a marble surface? It is not even able to stand on its four legs. Neither are humans. Can you imagine why?) Momentum conservation also answers the puzzles of Figure 72. Challenge 189 s

The conservation of momentum and mass also means that teleportation ('beam me up') is impossible in nature. Can you explain this to a non-physicist? Challenge 190 s

Momentum conservation implies that momentum can be imagined to be like an invisible *fluid*. In an interaction, the invisible fluid is transferred from one object to another. In such transfers, the amount of fluid is always constant.

Momentum conservation implies that motion never stops; it is only *exchanged*. On the other hand, motion often 'disappears' in our environment, as in the case of a stone dropped to the ground, or of a ball left rolling on grass. Moreover, in daily life we often observe the creation of motion, such as every time we open a hand. How do these examples fit with the conservation of momentum?

It turns out that apparent momentum disappearance is due to the microscopic aspects of the involved systems. A muscle only *transforms* one type of motion, namely that of the electrons in certain chemical compounds* into another, the motion of the fingers. The working of muscles is similar to that of a car engine transforming the motion of electrons in the fuel into motion of the wheels. Both systems need fuel and get warm in the process.

We must also study the microscopic behaviour when a ball rolls on grass until it stops. The apparent disappearance of motion is called *friction*. Studying the situation carefully, we find that the grass and the ball heat up a little during this process. *During friction, visible motion is transformed into heat.* A striking observation of this effect for a bicycle is shown below, in Figure 273. Later, when we discover the structure of matter, it will become clear that heat is the disorganized motion of the microscopic constituents of every material. When the microscopic constituents all move in the same direction, the object as a whole moves; when they oscillate randomly, the object is at rest, but is warm. Page 384

Ref. 88 * The fuel of most processes in animals usually is adenosine triphosphate (ATP).

Heat is a form of motion. Friction thus only seems to be disappearance of motion; in fact it is a transformation of ordered into unordered motion.

Page 395 Despite momentum conservation, *macroscopic* perpetual motion does not exist, since friction cannot be completely eliminated.* Motion is eternal only at the microscopic scale. In other words, the disappearance and also the spontaneous appearance of motion in everyday life is an illusion due to the limitations of our senses. For example, the motion proper of every living being exists before its birth, and stays after its death. The same happens with its energy. This result is probably the closest one can get to the idea of everlasting life from evidence collected by observation. It is perhaps less than a coincidence that energy used to be called *vis viva*, or 'living force', by Leibniz and many others.

Since motion is conserved, it has no origin. Therefore, at this stage of our walk we cannot answer the fundamental questions: Why does motion exist? What is its origin? The end of our adventure is nowhere near.

More conservation – energy

When collisions are studied in detail, a second conserved quantity turns up. Experiments show that in the case of perfect, or elastic collisions – collisions without friction – the following quantity, called the *kinetic energy* T of the system, is also conserved:

$$T = \sum_i \tfrac{1}{2} m_i \boldsymbol{v}_i^2 = \text{const} . \quad (21)$$

Kinetic energy is the ability that a body has to induce change in bodies it hits. Kinetic energy thus depends on the mass and on the square of the speed v of a body. The full name 'kinetic energy' was introduced by Gustave-Gaspard Coriolis.** Some measured energy values are given in Table 19.

* Some funny examples of past attempts to built a *perpetual motion machine* are described in Stanislav Michel, *Perpetuum mobile*, VDI Verlag, 1976. Interestingly, the idea of eternal motion came to Europe from India, via the Islamic world, around the year 1200, and became popular as it opposed the then standard view that all motion on Earth disappears over time. See also the web.archive.org/web/20040812085618/http://www.geocities.com/mercutio78_99/pmm.html and the www.lhup.edu/~dsimanek/museum/unwork.htm websites. The conceptual mistake made by eccentrics and used by crooks is always the same: the hope of overcoming friction. (In fact, this applied only to the perpetual motion machines of the second kind; those of the first kind – which are even more in contrast with observation – even try to generate energy from nothing.)

If the machine is well constructed, i.e., with little friction, it can take the little energy it needs for the sustenance of its motion from very subtle environmental effects. For example, in the Victoria and Albert Museum in London one can admire a beautiful clock powered by the variations of air pressure over time. Ref. 89

Low friction means that motion takes a long time to stop. One immediately thinks of the motion of the planets. In fact, there *is* friction between the Earth and the Sun. (Can you guess one of the mechanisms?) Challenge 191 s But the value is so small that the Earth has already circled around the Sun for thousands of millions of years, and will do so for quite some time more.

** Gustave-Gaspard Coriolis (b. 1792 Paris, d. 1843 Paris) was engineer and mathematician. He introduced the modern concepts of 'work' and of 'kinetic energy', and explored the Coriolis effect discovered by Page 138 Laplace. Coriolis also introduced the factor 1/2 in the kinetic energy T, in order that the relation $\mathrm{d}T/\mathrm{d}v = p$ would be obeyed. (Why?) Challenge 192 s

The experiments and ideas mentioned so far can be summarized in the following definition:

▷ (Physical) *energy* is the measure of the ability to generate motion.

A body has a lot of energy if it has the ability to move many other bodies. Energy is a number; energy, in contrast to momentum, has no direction. The total momentum of two equal masses moving with opposite velocities is zero; but their total energy is not, and it increases with velocity. Energy thus also measures motion, but in a different way than momentum. Energy measures motion in a more global way.

An equivalent definition is the following:

▷ Energy is the ability to perform work.

Here, the physical concept of work is just the precise version of what is meant by work in everyday life. As usual, (physical) *work* is the product of force and distance in direction of the force. In other words, work is the *scalar product* of force and distance. Physical work is a quantity that describes the effort of pushing of an object along a distance. As a result, in physics, work is a form of energy.

Another, equivalent definition of energy will become clear shortly:

▷ Energy is what can be transformed into heat.

Energy is a word taken from ancient Greek; originally it was used to describe character, and meant 'intellectual or moral vigour'. It was taken into physics by Thomas Young (1773–1829) in 1807 because its literal meaning is 'force within'. (The letters E, W, A and several others are also used to denote energy.)

Both energy and momentum measure how systems change. Momentum tells how systems change *over distance*: momentum is action (or change) divided by distance. Momentum is needed to compare motion here and there.

Energy measures how systems change *over time*: energy is action (or change) divided by time. Energy is needed to compare motion now and later.

Do not be surprised if you do not grasp the difference between momentum and energy straight away: physicists took about a century to figure it out! So you are allowed to take some time to get used to it. Indeed, for many decades, English physicists insisted on using the same term for both concepts; this was due to Newton's insistence that – no joke – the existence of god implied that energy was the same as momentum. Leibniz, instead, knew that energy increases with the square of the speed and proved Newton wrong. In 1722, Willem Jacob 's Gravesande even showed the difference between energy and momentum experimentally. He let metal balls of different masses fall into mud from different heights. By comparing the size of the imprints he confirmed that Newton was wrong both with his physical statements and his theological ones.

Ref. 90

One way to explore the difference between energy and momentum is to think about the following challenges. Which running man is more difficult to stop? One of mass m running at speed v, or one with mass $m/2$ and speed $2v$, or one with mass $m/2$ and speed $\sqrt{2}\,v$? You may want to ask a rugby-playing friend for confirmation.

Challenge 193 e

TABLE 19 Some measured energy values.

Observation	Energy
Average kinetic energy of oxygen molecule in air	6 zJ
Green photon energy	0.37 aJ
X-ray photon energy	1 fJ
γ photon energy	1 pJ
Highest particle energy in accelerators	0.1 µJ
Kinetic energy of a flying mosquito	0.2 µJ
Comfortably walking human	20 J
Flying arrow	50 J
Right hook in boxing	50 J
Energy in torch battery	1 kJ
Energy in explosion of 1 g TNT	4.1 kJ
Energy of 1 kcal	4.18 kJ
Flying rifle bullet	10 kJ
One gram of fat	38 kJ
One gram of gasoline	44 kJ
Apple digestion	0.2 MJ
Car on highway	0.3 to 1 MJ
Highest laser pulse energy	1.8 MJ
Lightning flash	up to 1 GJ
Planck energy	2.0 GJ
Small nuclear bomb (20 ktonne)	84 TJ
Earthquake of magnitude 7	2 PJ
Largest nuclear bomb (50 Mtonne)	210 PJ
Impact of meteorite with 2 km diameter	1 EJ
Yearly machine energy use	420 EJ
Rotation energy of Earth	$2 \cdot 10^{29}$ J
Supernova explosion	10^{44} J
Gamma-ray burst	up to 10^{47} J
Energy content $E = c^2 m$ of Sun's mass	$1.8 \cdot 10^{47}$ J
Energy content of Galaxy's central black hole	$4 \cdot 10^{53}$ J

Another distinction between energy and momentum is illustrated by athletics: the *real* long jump world record, almost 10 m, is still kept by an athlete who in the early twentieth century ran with two weights in his hands, and then threw the weights behind him at the moment he took off. Can you explain the feat? Challenge 194 s

When a car travelling at 100 m/s runs head-on into a parked car of the same kind and make, which car receives the greatest damage? What changes if the parked car has its brakes on? Challenge 195 s

To get a better feeling for energy, here is an additional aspect. The world consumption of energy by human machines (coming from solar, geothermal, biomass, wind, nuclear,

FIGURE 73 Robert Mayer (1814–1878).

hydro, gas, oil, coal, or animal sources) in the year 2000 was about 420 EJ,* for a world population of about 6000 million people. To see what this energy consumption means, we translate it into a personal power consumption; we get about 2.2 kW. The watt W is the unit of power, and is simply defined as 1 W = 1 J/s, reflecting the definition of *(physical) power* as energy used per unit time. The precise wording is: power is energy flowing per time through a defined closed surface. See Table 20 for some power values found in nature, and Table 21 for some measurement devices.

Ref. 91

As a working person can produce mechanical work of about 100 W, the average human energy consumption corresponds to about 22 humans working 24 hours a day. In particular, if we look at the energy consumption in First World countries, the average inhabitant there has machines working for him or her that are equivalent to several hundred 'servants'. Machines do a lot of good. Can you point out some of these machines?

Challenge 196 s

Kinetic energy is thus not conserved in everyday life. For example, in non-elastic collisions, such as that of a piece of chewing gum hitting a wall, kinetic energy is lost. *Friction* destroys kinetic energy. At the same time, friction produces heat. It was one of the important conceptual discoveries of physics that *total* energy is conserved if one includes the discovery that heat is a form of energy. Friction is thus a process transforming kinetic energy, i.e., the energy connected with the motion of a body, into heat. On a microscopic scale, *energy is always conserved*.

Any example for the non-conservation of energy is only apparent. ** Indeed, without energy conservation, the concept of time would not be definable! We will show this important connection shortly.

In summary, in addition to mass and momentum, everyday linear motion also conserves energy. To discover the last conserved quantity, we explore another type of motion: rotation.

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* For the explanation of the abbreviation E, see Appendix B.

** In fact, the conservation of energy was stated in its full generality in public only in 1842, by Julius Robert Mayer. He was a medical doctor by training, and the journal *Annalen der Physik* refused to publish his paper, as it supposedly contained 'fundamental errors'. What the editors called errors were in fact mostly – but not only – contradictions of their prejudices. Later on, Helmholtz, Thomson-Kelvin, Joule and many others acknowledged Mayer's genius. However, the first to have stated energy conservation in its modern form was the French physicist Sadi Carnot (1796–1832) in 1820. To him the issue was so clear that he did not publish the result. In fact he went on and discovered the second 'law' of thermodynamics. Today, energy conservation, also called the first 'law' of thermodynamics, is one of the pillars of physics, as it is valid in all its domains.

TABLE 20 Some measured power values.

Observation	Power
Radio signal from the Galileo space probe sending from Jupiter	10 zW
Power of flagellar motor in bacterium	0.1 pW
Power consumption of a typical cell	1 pW
sound power at the ear at hearing threshold	2.5 pW
CR-R laser, at 780 nm	40-80 mW
Sound output from a piano playing fortissimo	0.4 W
Dove (0.16 kg) basal metabolic rate	0.97 W
Rat (0.26 kg) basal metabolic rate	1.45 W
Pigeon (0.30 kg) basal metabolic rate	1.55 W
Hen (2.0 kg) basal metabolic rate	4.8 W
Incandescent light bulb light output	1 to 5 W
Dog (16 kg) basal metabolic rate	20 W
Sheep (45 kg) basal metabolic rate	50 W
Woman (60 kg) basal metabolic rate	68 W
Man (70 kg) basal metabolic rate	87 W
Incandescent light bulb electricity consumption	25 to 100 W
A human, during one work shift of eight hours	100 W
Cow (400 kg) basal metabolic rate	266 W
One horse, for one shift of eight hours	300 W
Steer (680 kg) basal metabolic rate	411 W
Eddy Merckx, the great bicycle athlete, during one hour	500 W
Metric horse power power unit ($75\,\mathrm{kg} \cdot 9.81\,\mathrm{m/s^2} \cdot 1\,\mathrm{m/s}$)	735.5 W
British horse power power unit	745.7 W
Large motorbike	100 kW
Electrical power station output	0.1 to 6 GW
World's electrical power production in 2000 Ref. 91	450 GW
Power used by the geodynamo	200 to 500 GW
Limit on wind energy production Ref. 92	18 to 68 TW
Input on Earth surface: Sun's irradiation of Earth Ref. 93	0.17 EW
Input on Earth surface: thermal energy from inside of the Earth	32 TW
Input on Earth surface: power from tides (i.e., from Earth's rotation)	3 TW
Input on Earth surface: power generated by man from fossil fuels	8 to 11 TW
Lost from Earth surface: power stored by plants' photosynthesis	40 TW
World's record laser power	1 PW
Output of Earth surface: sunlight reflected into space	0.06 EW
Output of Earth surface: power radiated into space at 287 K	0.11 EW
Peak power of the largest nuclear bomb	5 YW
Sun's output	384.6 YW
Maximum power in nature, $c^5/4G$	$9.1 \cdot 10^{51}$ W

TABLE 21 Some power sensors.

Measurement	Sensor	Range
Heart beat as power meter	deformation sensor and clock	75 to 2 000 W
Fitness power meter	piezoelectric sensor	75 to 2 000 W
Electricity meter at home	rotating aluminium disc	20 to 10 000 W
Power meter for car engine	electromagnetic brake	up to 1 MW
Laser power meter	photoelectric effect in semiconductor	up to 10 GW
Calorimeter for chemical reactions	temperature sensor	up to 1 MW
Calorimeter for particles	light detector	up to a few μJ/ns

FIGURE 74 Some power measurement devices: a bicycle power meter, a laser power meter, and an electrical power meter (© SRAM, Laser Components, Wikimedia).

The cross product, or vector product

The discussion of rotation is easiest if we introduce an additional way to multiply vectors. This new product between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is called the *cross product* or *vector product* $\boldsymbol{a} \times \boldsymbol{b}$.

The result of the vector product is another *vector*; thus it differs from the *scalar* product, whose result is a scalar, i.e., a number. The result of the vector product is that vector

— that is orthogonal to both vectors to be multiplied,
— whose orientation is given by the *right-hand rule*, and
— whose length is given by the surface area of the parallelogram spanned by the two vectors, i.e., by $ab \sin \sphericalangle(\boldsymbol{a}, \boldsymbol{b})$.

The definition implies that the cross product vanishes if and only if the vectors are par-

Challenge 197 e allel. From the definition you can also show that the vector product has the properties

$$\begin{aligned}
&\boldsymbol{a}\times\boldsymbol{b} = -\boldsymbol{b}\times\boldsymbol{a}\,, \quad \boldsymbol{a}\times(\boldsymbol{b}+\boldsymbol{c}) = \boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{a}\times\boldsymbol{c}\,,\\
&\lambda\boldsymbol{a}\times\boldsymbol{b} = \lambda(\boldsymbol{a}\times\boldsymbol{b}) = \boldsymbol{a}\times\lambda\boldsymbol{b}\,, \quad \boldsymbol{a}\times\boldsymbol{a} = \boldsymbol{0}\,,\\
&\boldsymbol{a}(\boldsymbol{b}\times\boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{c}\times\boldsymbol{a}) = \boldsymbol{c}(\boldsymbol{a}\times\boldsymbol{b})\,, \quad \boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{ac})\boldsymbol{b}-(\boldsymbol{ab})\boldsymbol{c}\,,\\
&(\boldsymbol{a}\times\boldsymbol{b})(\boldsymbol{c}\times\boldsymbol{d}) = \boldsymbol{a}(\boldsymbol{b}\times(\boldsymbol{c}\times\boldsymbol{d})) = (\boldsymbol{ac})(\boldsymbol{bd})-(\boldsymbol{bc})(\boldsymbol{ad})\,,\\
&(\boldsymbol{a}\times\boldsymbol{b})\times(\boldsymbol{c}\times\boldsymbol{d}) = ((\boldsymbol{a}\times\boldsymbol{b})\boldsymbol{d})\boldsymbol{c}-((\boldsymbol{a}\times\boldsymbol{b})\boldsymbol{c})\boldsymbol{d}\,,\\
&\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})+\boldsymbol{b}\times(\boldsymbol{c}\times\boldsymbol{a})+\boldsymbol{c}\times(\boldsymbol{a}\times\boldsymbol{b}) = \boldsymbol{0}\,.
\end{aligned} \tag{22}$$

The vector product exists only in vector spaces with *three* dimensions. We will explore Vol. IV, page 234 more details on this connection later on.

The vector product is useful to describe systems that *rotate* – and (thus) also systems with magnetic forces. The motion of an orbiting body is always perpendicular both to the axis and to the line that connects the body with the axis. In rotation, axis, radius and velocity form a right-handed set of mutually orthogonal vectors. This connection lies at the origin of the vector product.

Challenge 198 e Confirm that the best way to calculate the vector product $\boldsymbol{a}\times\boldsymbol{b}$ component by component is given by the symbolic determinant

$$\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{e}_x & a_x & b_x \\ \boldsymbol{e}_y & a_y & b_y \\ \boldsymbol{e}_z & a_z & b_z \end{vmatrix} \quad \text{or, sloppily} \quad \boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} + & - & + \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}\,. \tag{23}$$

These symbolic determinants are easy to remember and easy to perform, both with letters and with numerical values. (Here, $\boldsymbol{e}_x$ is the unit basis vector in the x direction.) Written out, the symbolic determinants are equivalent to the relation

$$\boldsymbol{a}\times\boldsymbol{b} = (a_y b_z - b_y a_z, b_x a_z - a_x b_z, a_x b_y - b_x a_y) \tag{24}$$

which is harder to remember, though.

Challenge 199 e Show that the *parallelepiped* spanned by three arbitrary vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ has the volume $V = \boldsymbol{c}\,(\boldsymbol{a}\times\boldsymbol{b})$. Show that the *pyramid* or *tetrahedron* formed by the same three Challenge 200 e vectors has one sixth of that volume.

Rotation and angular momentum

Rotation keeps us alive. Without the change of day and night, we would be either fried or frozen to death, depending on our location on our planet. But rotation appears in many other settings, as Table 22 shows. A short exploration of rotation is thus appropriate.

All objects have the ability to rotate. We saw before that a body is described by its reluctance to move, which we called mass; similarly, a body also has a reluctance to turn. This quantity is called its *moment of inertia* and is often abbreviated Θ – pronounced ‘theta’. The speed or rate of rotation is described by *angular velocity*, usually abbreviated ω – pronounced ‘omega’. A few values found in nature are given in Table 22.

The observables that describe rotation are similar to those describing linear motion,

TABLE 22 Some measured rotation frequencies.

OBSERVATION	ANGULAR VELOCITY $\omega = 2\pi/T$
Galactic rotation	$2\pi \cdot 0.14 \cdot 10^{-15}$/ s $= 2\pi/(220 \cdot 10^6$ a)
Average Sun rotation around its axis	$2\pi \cdot 3.8 \cdot 10^{-7}$/ s = 2π / 30 d
Typical lighthouse	$2\pi \cdot 0.08$/ s
Pirouetting ballet dancer	$2\pi \cdot 3$/ s
Ship's diesel engine	$2\pi \cdot 5$/ s
Helicopter rotor	$2\pi \cdot 5.3$/ s
Washing machine	up to $2\pi \cdot 20$/ s
Bacterial flagella	$2\pi \cdot 100$/ s
Fast CD recorder	up to $2\pi \cdot 458$/ s
Racing car engine	up to $2\pi \cdot 600$/ s
Fastest turbine built	$2\pi \cdot 10^3$/ s
Fastest pulsars (rotating stars)	up to at least $2\pi \cdot 716$/ s
Ultracentrifuge	$> 2\pi \cdot 3 \cdot 10^3$/ s
Dental drill	up to $2\pi \cdot 13 \cdot 10^3$/ s
Technical record	$2\pi \cdot 333 \cdot 10^3$/ s
Proton rotation	$2\pi \cdot 10^{20}$/ s
Highest possible, Planck angular velocity	$2\pi \cdot 10^{35}$/ s

as shown in Table 24. Like mass, the moment of inertia is defined in such a way that the sum of *angular momenta* L – the product of moment of inertia and angular velocity – is conserved in systems that do not interact with the outside world:

$$\sum_i \Theta_i \boldsymbol{\omega}_i = \sum_i \boldsymbol{L}_i = \text{const} . \quad (25)$$

In the same way that the conservation of linear momentum defines mass, the conservation of angular momentum defines the moment of inertia. Angular momentum is a concept introduced in the 1730s and 1740s by Leonhard Euler and Daniel Bernoulli.

The moment of inertia can be related to the mass and shape of a body. If the body is imagined to consist of small parts or mass elements, the resulting expression is

$$\Theta = \sum_n m_n r_n^2 , \quad (26)$$

where r_n is the distance from the mass element m_n to the axis of rotation. Can you confirm the expression? Therefore, the moment of inertia of a body depends on the chosen axis of rotation. Can you confirm that this is so for a brick?

Challenge 201 e
Challenge 202 s

In contrast to the case of mass, there is *no* conservation of the moment of inertia. In fact, the value of the moment of inertia depends both on the direction and on the

TABLE 23 Some measured angular momentum values.

Observation	Angular momentum
Smallest observed value in nature, $\hbar/2$, in elementary matter particles (fermions)	$0.53 \cdot 10^{-34}$ Js
Spinning top	$5 \cdot 10^{-6}$ Js
CD (compact disc) playing	*c.* 0.029 Js
Walking man (around body axis)	*c.* 4 Js
Dancer in a pirouette	5 Js
Typical car wheel at 30 m/s	10 Js
Typical wind generator at 12 m/s (6 Beaufort)	10^4 Js
Earth's atmosphere	1 to $2 \cdot 10^{26}$ Js
Earth's oceans	$5 \cdot 10^{24}$ Js
Earth around its axis	$7.1 \cdot 10^{33}$ Js
Moon around Earth	$2.9 \cdot 10^{34}$ Js
Earth around Sun	$2.7 \cdot 10^{40}$ Js
Sun around its axis	$1.1 \cdot 10^{42}$ Js
Jupiter around Sun	$1.9 \cdot 10^{43}$ Js
Solar System around Sun	$3.2 \cdot 10^{43}$ Js
Milky Way	10^{68} Js
All masses in the universe	0 (within measurement error)

TABLE 24 Correspondence between linear and rotational motion.

Quantity	Linear motion		Rotational motion	
State	time	t	time	t
	position	$\boldsymbol{x}$	angle	$\boldsymbol{\varphi}$
	momentum	$\boldsymbol{p} = m\boldsymbol{v}$	angular momentum	$\boldsymbol{L} = \Theta\boldsymbol{\omega}$
	energy	$mv^2/2$	energy	$\Theta\omega^2/2$
Motion	velocity	$\boldsymbol{v}$	angular velocity	$\boldsymbol{\omega}$
	acceleration	$\boldsymbol{a}$	angular acceleration	$\boldsymbol{\alpha}$
Reluctance to move	mass	m	moment of inertia	Θ
Motion change	force	$m\boldsymbol{a}$	torque	$\Theta\boldsymbol{\alpha}$

location of the axis used for its definition. For each axis direction, one distinguishes an *intrinsic* moment of inertia, when the axis passes through the centre of mass of the body, from an *extrinsic* moment of inertia, when it does not.* In the same way, we distinguish

* Extrinsic and intrinsic moment of inertia are related by

$$\Theta_{ext} = \Theta_{int} + md^2 , \tag{27}$$

where d is the distance between the centre of mass and the axis of extrinsic rotation. This relation is called *Steiner's parallel axis theorem*. Are you able to deduce it?

Challenge 203 s

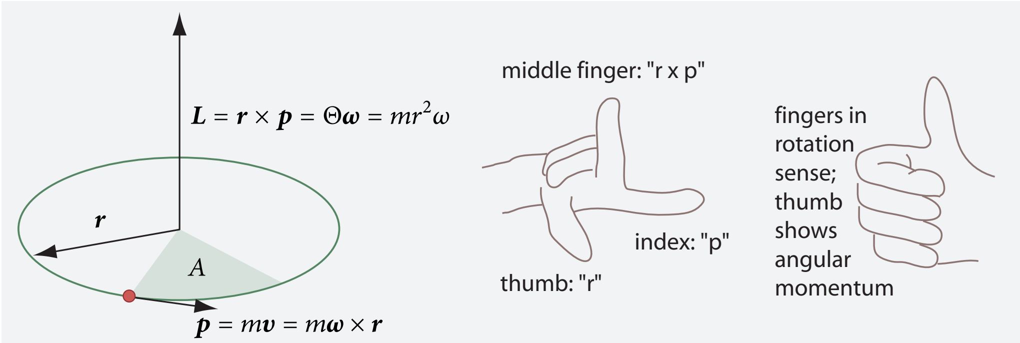

FIGURE 75 Angular momentum and other quantities for a point particle in circular motion, and the two versions of the right-hand rule.

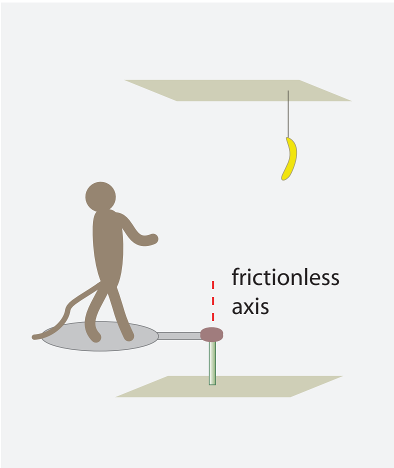

FIGURE 76 Can the ape reach the banana?

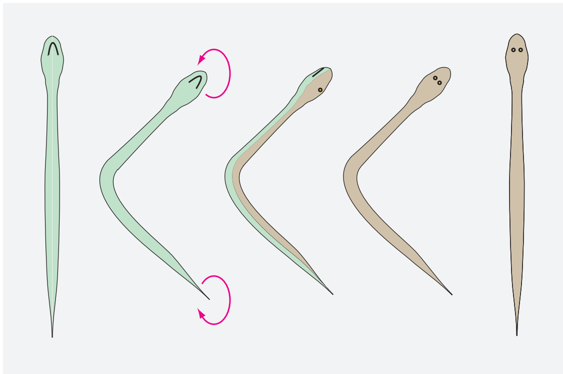

FIGURE 77 How a snake turns itself around its axis.

intrinsic and extrinsic angular momenta. (By the way, the *centre of mass* of a body is that imaginary point which moves straight during vertical fall, even if the body is rotating. Challenge 204 s Can you find a way to determine its location for a specific body?)

We now define the *rotational energy* as

$$E_{\text{rot}} = \tfrac{1}{2}\,\Theta\,\omega^2 = \frac{L^2}{2\Theta}\;. \qquad (28)$$

The expression is similar to the expression for the kinetic energy of a particle. For rotating objects with a fixed shape, rotational energy is conserved.

Can you guess how much larger the rotational energy of the Earth is compared with Challenge 205 s the yearly electricity usage of humanity? In fact, if you could find a way to harness the Earth's rotational energy, you would become famous.

Every object that has an orientation also has an intrinsic angular momentum. (What Challenge 206 s about a sphere?) Therefore, *point* particles do *not* have intrinsic angular momenta – at least in classical physics. (This statement will change in quantum theory.) The *extrinsic* angular momentum $\boldsymbol{L}$ of a point particle is defined as

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} \tag{29}$$

where $\boldsymbol{p}$ is the momentum of the particle and $\boldsymbol{r}$ the position vector. The angular momentum thus points along the rotation axis, following the right-hand rule, as shown in Figure 75. A few values observed in nature are given in Table 23. The definition implies that the angular momentum can also be determined using the expression

Challenge 207 e

$$L = \frac{2A(t)m}{t} , \tag{30}$$

where $A(t)$ is the area *swept* by the position vector $\boldsymbol{r}$ of the particle during time t. For example, by determining the swept area with the help of his telescope, Johannes Kepler discovered in the year 1609 that each planet orbiting the Sun has an angular momentum value that is *constant* over time.

A physical body can rotate simultaneously about *several* axes. The film of Figure 108 shows an example: The top rotates around its body axis and around the vertical at the same time. A detailed exploration shows that the exact rotation of the top is given by the *vector sum* of these two rotations. To find out, 'freeze' the changing rotation axis at a specific time. Rotations thus are a type of vectors.

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Challenge 208 e
Page 161

As in the case of linear motion, rotational energy and angular momentum are not always conserved in the macroscopic world: rotational energy can change due to friction, and angular momentum can change due to external forces (torques). But for *closed* (undisturbed) systems, both angular momentum and rotational energy are always conserved. In particular, on a microscopic scale, most objects are undisturbed, so that conservation of rotational energy and angular momentum usually holds on microscopic scales.

Angular momentum is conserved. This statement is valid for any axis of a physical system, *provided* that external forces (torques) play no role. To make the point, Jean-Marc Lévy-Leblond poses the problem of Figure 76. Can the ape reach the banana without leaving the plate, assuming that the plate on which the ape rests can turn around the axis without any friction?

Ref. 2
Challenge 209 s

We note that many effects of rotation are the same as for acceleration: both acceleration and rotation of a car pushed us in our seats. Therefore, many sensors for rotation are the same as the acceleration sensors we explored above. But a few sensors for rotation are fundamentally new. In particular, we will meet the gyroscope shortly.

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Page 141

On a frictionless surface, as approximated by smooth ice or by a marble floor covered by a layer of oil, it is impossible to move forward. In order to move, we need to push *against* something. Is this also the case for rotation?

Surprisingly, it is possible to turn even *without* pushing against something. You can check this on a well-oiled rotating office chair: simply rotate an arm above the head. After each turn of the hand, the orientation of the chair has changed by a small amount. Indeed, conservation of angular momentum and of rotational energy do *not* prevent bodies from changing their orientation. Cats learn this in their youth. After they have learned