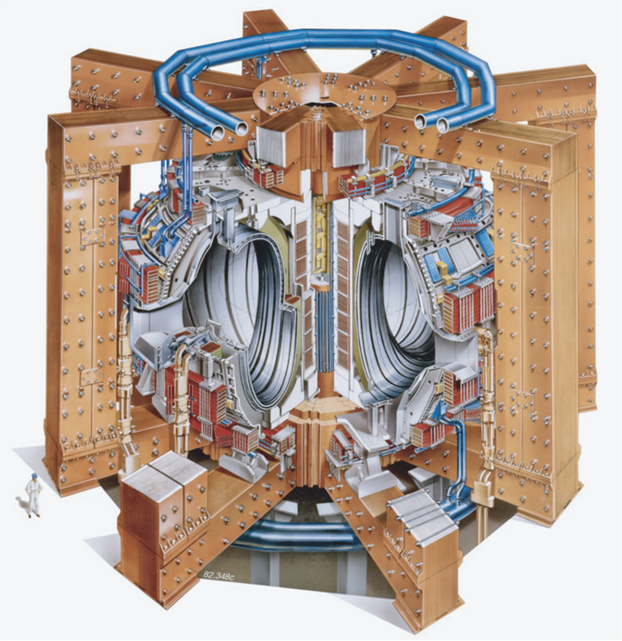

FIGURE 125 A simplified drawing of the Joint European Torus in operation at Culham, showing the large toroidal chamber and the magnets for the plasma confinement (© EFDA-JET).

WHY ARE FUSION REACTORS NOT COMMON YET?

Across the world, for over 50 years, a large number of physicists and engineers have tried to build fusion reactors. Fusion reactors try to copy the mechanism of energy release used by the Sun. The first machine that realized macroscopic energy production through fusion was, in 1991, the Joint European Torus* (JET for short) located in Culham in the United Kingdom. Despite this success, the produced power was still somewhat smaller than the power needed for heating.

Ref. 176 The idea of JET is to produce an extremely hot plasma that is as dense as possible. At high enough temperature and density, fusion takes place; the energy is released as a particle flux that is transformed (like in a fission reactor) into heat and then into electricity. To achieve ignition, JET used the fusion between deuterium and tritium, because this reaction has the largest cross section and energy gain:

$$\mathrm{D} + \mathrm{T} \rightarrow \mathrm{He}^4 + \mathrm{n} + 17.6\,\mathrm{MeV} . \tag{68}$$

Because tritium is radioactive, most research experiments are performed with the far less efficient deuterium–deuterium reactions, which have a lower cross section and a lower energy gain:

$$\begin{aligned} \mathrm{D} + \mathrm{D} &\rightarrow \mathrm{T} + \mathrm{H} + 4\,\mathrm{MeV} \\ \mathrm{D} + \mathrm{D} &\rightarrow \mathrm{He}^3 + \mathrm{n} + 3.3\,\mathrm{MeV} . \end{aligned} \tag{69}$$

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* See www.jet.edfa.org.

Fusion takes place when deuterium and tritium (or deuterium) collide at high energy. The high energy is necessary to overcome the electrostatic repulsion of the nuclei. In other words, the material has to be hot. To release energy from deuterium and tritium, one therefore first needs energy to heat it up. This is akin to the ignition of wood: in order to use wood as a fuel, one first has to heat it with a match.

Following the so-called *Lawson criterion*, rediscovered in 1957 by the English engineer John Lawson, after its discovery by Russian researchers, a fusion reaction releases energy only if the triple product of density n, reaction (or containment) time τ and temperature T exceeds a certain value. Nowadays this criterion is written as Ref. 177

$$n\tau T > 3 \cdot 10^{28}\,\mathrm{s\,K/m^3}\ . \tag{70}$$

In order to realize the Lawson criterion, JET uses temperatures of 100 to 200 MK, particle densities of 2 to $3 \cdot 10^{20}\,\mathrm{m^{-3}}$, and confinement times of 1 s. The temperature in JET is thus much higher than the 15 MK at the centre of the Sun, because the densities and the confinement times are much lower.

Matter at these temperatures is in form of a *plasma*: nuclei and electrons are completely separated. Obviously, it is impossible to pour a 100 MK plasma into a container: the walls would instantaneously evaporate. The only option is to make the plasma float in a vacuum, and to avoid that the plasma touches the container wall. The main challenge of fusion research in the past has been to find a way to keep a hot gas mixture of deuterium and tritium suspended in a chamber so that the gas never touches the chamber walls. The best way is to suspend the gas using a magnetic field. This works because in the fusion plasma, charges are separated, so that they react to magnetic fields. The most successful geometric arrangement was invented by the famous Russian physicists Igor Tamm and Andrei Sakharov: the *tokamak*. Of the numerous tokamaks around the world, JET is the largest and most successful. Its concrete realization is shown in Figure 125. JET manages to keep the plasma from touching the walls for about a second; then the situation becomes unstable: the plasma touches the wall and is absorbed there. After such a *disruption*, the cycle consisting of gas injection, plasma heating and fusion has to be restarted. As mentioned, JET has already achieved ignition, that is the state were more energy is released than is added for plasma heating. However, so far, no sustained commercial energy production is planned or possible, because JET has no attached electrical power generator.

The successor project, *ITER*, an international tokamak built with European, Japanese, US-American and Russian funding, aims to pave the way for commercial energy generation. Its linear reactor size will be twice that of JET; more importantly, ITER plans to achieve 30 s containment time. ITER will use superconducting magnets, so that it will have extremely cold matter at 4 K only a few metres from extremely hot matter at 100 MK. In other words, ITER will be a high point of engineering. The facility is being built in Cadarache in France. Due to its lack of economic sense, ITER has a good chance to be a modern version of the tower of Babylon; but maybe one day, it will start operation.

Like many large projects, fusion started with a dream: scientists spread the idea that fusion energy is safe, clean and inexhaustible. These three statements are still found on every fusion website across the world. In particular, it is stated that fusion reactors are not dangerous, produce much lower radioactive contamination than fission reactors, and use water as basic fuel. 'Solar fusion energy would be as clean, safe and limitless as the Sun.' In reality, the only reason that we do not feel the radioactivity of the Sun is that we are far away from it. Fusion reactors, like the Sun, are highly radioactive. The management of radioactive fusion reactors is much more complex than the management of radioactive fission reactors.

It is true that fusion fuels are almost inexhaustible: deuterium is extracted from water and the tritium – a short-lived radioactive element not found in nature in large quantities – is produced from *lithium*. The lithium must be enriched, but since material is not radioactive, this is not problematic. However, the production of tritium from lithium is a dirty process that produces large amounts of radioactivity. Fusion energy is thus inexhaustible, but not safe and clean.

In summary, of all technical projects ever started by mankind, fusion is by far the most challenging and ambitious. Whether fusion will ever be successful – or whether it ever should be successful – is another issue.

WHERE DO OUR ATOMS COME FROM?

> “The elements were made in less time than you could cook a dish of duck and roast potatoes.”
> George Gamow

People consist of electrons and various nuclei. Where did the nucleosynthesis take place? Many researchers contributed to answering this question. Ref. 178

About three minutes after the big bang, when temperature was around 0.1 MeV, protons and neutrons formed. About seven times as many protons as neutrons were formed, mainly due to their mass difference. Due to the high densities, the neutrons were captured, through the intermediate step of deuterium nuclei, in α particles. The process stopped around 20 minutes after the big bang, when temperatures became too low to allow fusion. After these seventeen minutes, the mass of the universe was split into 75 % of hydrogen, 25 % of helium (both percentages result from the factor 7 between the number of protons and neutrons), and traces of deuterium, lithium and beryllium. This process is called *primordial nucleosynthesis*. No heavier elements were formed, because the temperature fall prevented their accumulation in measurable quantities, and because there are no stable nuclei with 5 or 8 nucleons.

Simulations of primordial nucleosynthesis agree well with the element abundances found in extremely distant, thus extremely old stars. The abundances are deduced from the spectra of these stars. In short, *hydrogen, helium, lithium and beryllium nuclei are formed shortly after ('during') the big bang*. These are the so-called *primordial elements*. Vol. II, page 246

All other nuclei are formed many millions of years after the big bang. In particular, *other light nuclei are formed in stars*. Young stars run hydrogen burning or helium burning; heavier and older stars run neon-burning or even silicon-burning. These latter processes require high temperatures and pressures, which are found only in stars with a mass at least eight times that of the Sun. All these fusion processes are limited by photodissociation and thus will only lead to nuclei up to ^{56}Fe. Ref. 179

Nuclei heavier than iron can only be made by neutron capture. There are *two* main neutron capture processes. The first process is the so-called *s-process* – for 'slow'. The process occurs inside stars, and gradually builds up heavy elements – including the most

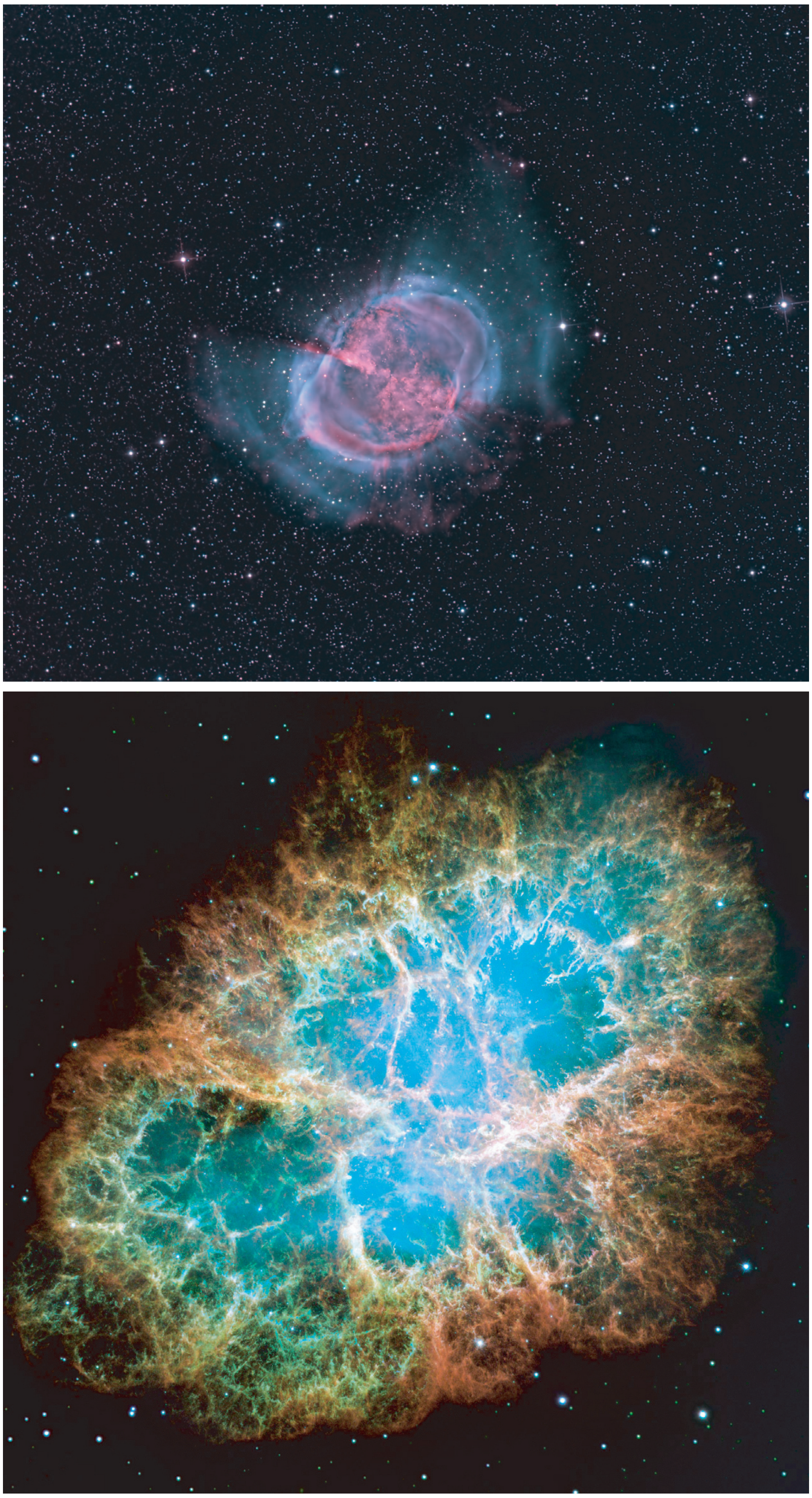

FIGURE 126 Two examples of how exploding stars shoot matter into interstellar space: the Crab nebula M1 and the Dumbbell nebula M27 (courtesy NASA and ESA, © Bill Snyder).

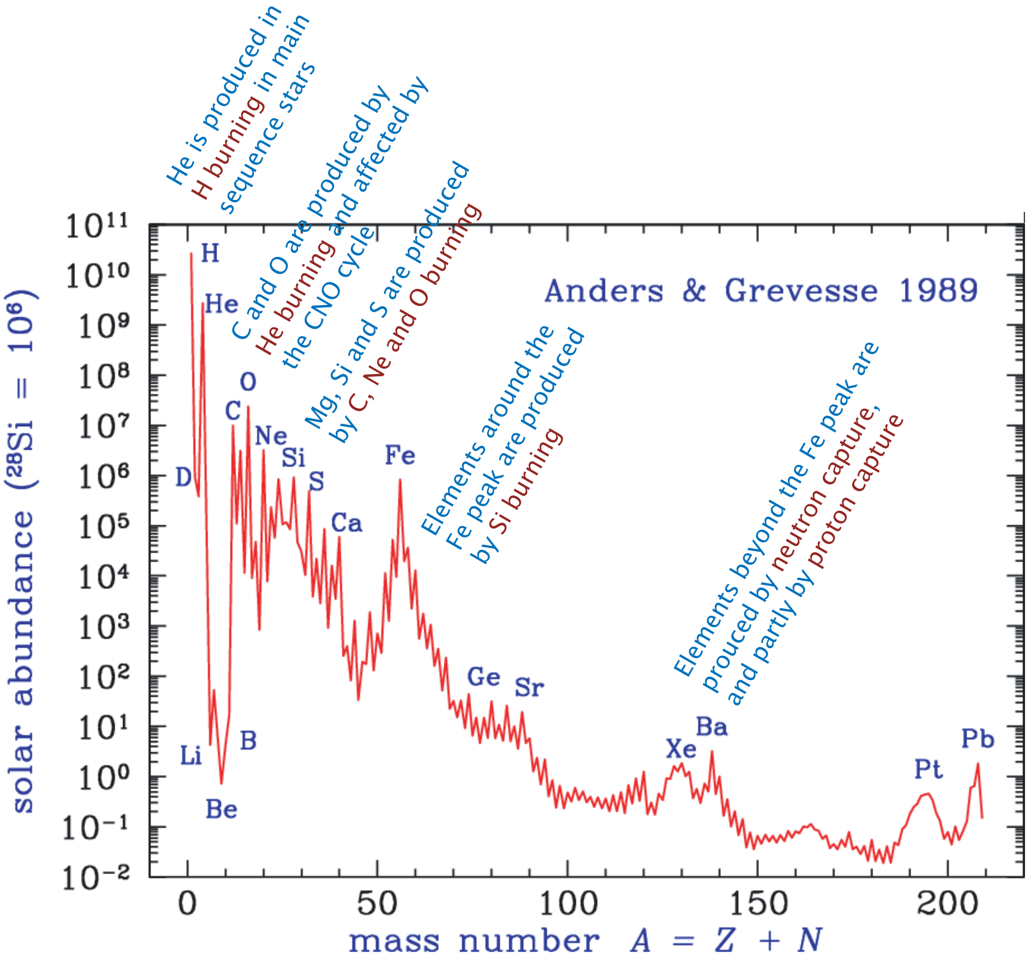

FIGURE 127 The measured nuclide abundances in the solar system and their main production processes (from Ref. 180).

heavy stable nucleus, lead – from neutrons flying around. The second neutron capture process is the rapid *r-process*. For a long time, its was unknown where it occurred; for many decades, it was thought that it takes place in stellar explosions. Recent research points to *neutron star mergers* as the more likely place for the r-process. Such collisions emit material into space. The high neutron flux produces heavy elements. For example, it seems that most *gold* nuclei are synthesized in this way. (The first clear neutron star merger was observed in 2013 with the Hubble space telescope, after it had been detected as a gamma ray burst with the name GRB 130603B. Such an event is also called a *kilonova*, because the emitted energy is between that of a *nova* and of a *supernova*. In October 2017, a further, well-publicized neutron star merger was observed with the help of gravitational wave detectors, of gamma ray burst detectors and of over 70 optical telescopes. It took place at a distance of 130 million light years.) The abundances of the heavy elements in the solar system can be measured with precision, a shown in Figure 127. These data points correspond well with what is expected from the material synthesized by neutron star Ref. 181

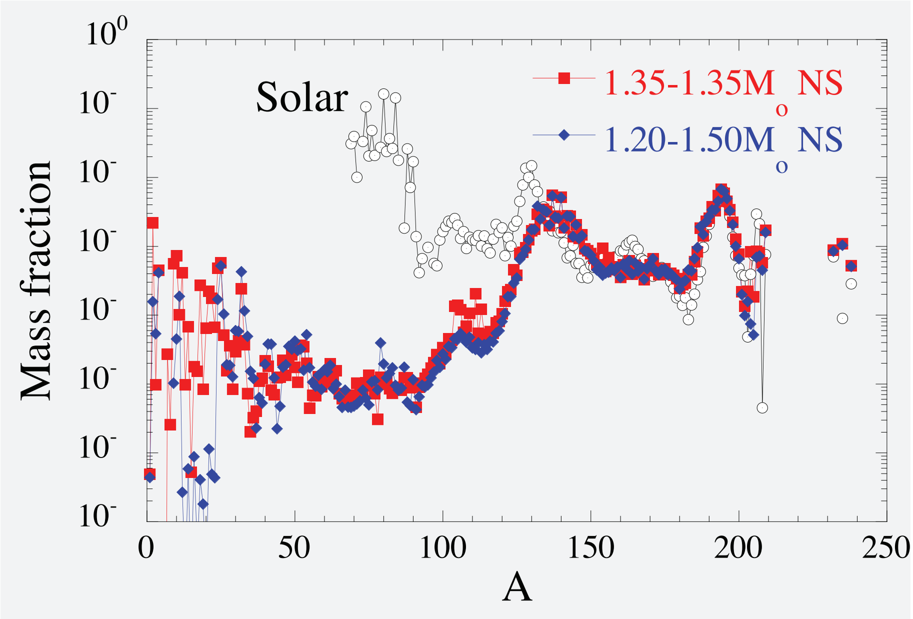

FIGURE 128 The comparison between measured nuclide abundances (dotted circles) in the solar system and the calculated values (red squares, blue diamonds) predicted by neutron star mergers (from reference Ref. 181).

mergers, the most likely candidate for the r-process at present, as illustrated by Figure 128.

A number of other processes, such as *proton capture* and the so-called *equilibrium process*, contributed to the formation of the elements.

In summary, the electrons and protons in our body were made during the big bang; the lighter nuclei, such as carbon or oxygen, in stars; the heavier nuclei in star explosions and neutron star mergers. But how did those nuclei arrive on Earth?

At a certain stage of their life, many stars explode. An exploding star is called a *supernova*. Such a supernova has an important effect: it distributes most of the matter of the star, such as carbon, nitrogen or oxygen, into space. This happens mostly in the form of neutral atoms. (Some elements are also synthesized during the explosion.) Exploding supernovae are thus essential for distributing material into space.

The Sun is a second generation star – as so-called 'population I' star. and the solar system formed from the remnants of a supernova, as did, somewhat later, life on Earth. We all are made of recycled atoms.

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We are recycled stardust. This is the short summary of the extended study by astrophysicists of all the types of stars found in the universe, including their birth, growth, mergers and explosions. The exploration of how stars evolve and then move in galaxies is a fascinating research field, and many aspects are still unknown.

Curiosities about the Sun and the stars

What would happen if the Sun suddenly stopped shining? Obviously, temperatures would fall by several tens of degrees within a few hours. It would rain, and then all water would freeze. After four or five days, all animal life would stop. After a few weeks, the oceans would freeze; after a few months, air would liquefy. Fortunately, this will never happen.

∗ ∗

Not everything about the Sun is known. For example, the neutrino flux from the Sun oscillates with a period of 28.4 days. That is the same period with which the magnetic field of the Sun oscillates. The connection is still being explored.

∗ ∗

The Sun is a fusion reactor. But its effects are numerous. If the Sun were less brighter than it is, evolution would have taken a different course. We would not have eyelids, we would still have more hair, and would have a brighter skin. Can you find more examples?

Challenge 140 e

∗ ∗

Some stars shine like a police siren: their luminosity increases and decreases regularly. Such stars, called *Cepheids*, are important because their period depends on their average (absolute) brightness. Therefore, measuring their period and their brightness on Earth thus allows astronomers to determine their distance.

∗ ∗

The first human-made hydrogen bomb explosion took place the Bikini atoll. Fortunately, none has ever been used on people.

But nature is much better at building bombs. The most powerful nuclear explosions known take place on the surface of neutron stars in X-ray binaries. The matter falling into such a neutron star from the companion star, mostly hydrogen, will heat up until the temperature allows fusion. The resulting explosions can be observed in telescopes as light or X-ray flashes of about 10 s duration; the explosions are millions of times more powerful that those of human-made hydrogen bombs.

∗ ∗

In the 1960s, it was discovered that surface of the Sun oscillates. The surface is covered with standing waves. The amplitude is a few hundred kilometres, the wavelength can be hundred times larger; the typical frequency of the famous p-modes, or trapped acoustic waves, is between 2 and 4 mHz, thus roughly between 8 and 3 minutes. The oscillations are also visible as diameter oscillations of the Sun. This research field is now called *helioseismology*.

∗ ∗

Lithium, beryllium and boron are rare inside stars, because they like to capture protons, and thus change identity. For the same reason, these elements are rare on Earth.

∗ ∗

By chance, the composition ratios between carbon, nitrogen and oxygen inside the Sun are the same as inside the human body.

* *

Nucleosynthesis is mainly regulated by the strong interaction. However, if the electromagnetic interaction would be much stronger or much weaker, stars would either produce too little oxygen or too little carbon, and we would not exist. This famous argument Challenge 141 d is due to Fred Hoyle. Can you fill in the details?

SUMMARY ON STARS AND NUCLEOSYNTHESIS

> “All humans are brothers. We came from the same supernova.”
>
> Allan Sandage

Stars and the Sun burn because of nuclear fusion. The energy liberated in nuclear fusion is due to the strong nuclear interaction that acts between nucleons. When stars have used up their nuclear fuel, they usually explode. In such a supernova explosion, they distribute nuclei into space in the form of dust. Already in the distant past, such dust recollected because of gravity and then formed the Sun, the Earth and, later on, humans.

The nuclear reaction processes behind nucleosynthesis have been studied in great detail. Nucleosynthesis during the big bang formed hydrogen and helium, nucleosynthesis in stars formed the light nuclei, and nucleosynthesis in neutron star mergers and supernovae explosions formed the heavy nuclei.

CHAPTER 7

THE STRONG INTERACTION – INSIDE NUCLEI AND NUCLEONS

Both radioactivity and medical images show that nuclei are composed systems. But quantum theory predicts even more: also protons and neutrons must be composed. There are two reasons: first, nucleons have a finite size, and second, their magnetic moments do not match the value predicted for point particles.

The prediction of components inside protons was confirmed in the late 1960s when Kendall, Friedman and Taylor shot high energy electrons into hydrogen atoms. They (Ref. 182) found that a proton contains *three* constituents with spin 1/2. The experiment was able to 'see' the constituents through large angle scattering of electrons, in the same way that we see objects through large angle scattering of photons. These constituents correspond in number and (most) properties to the so-called *quarks* predicted in 1964 by George (Ref. 183) Zweig and also by Murray Gell-Mann.[**]

Why are there three quarks inside a proton? And how do they interact? The answers are deep and fascinating.

THE FEEBLE SIDE OF THE STRONG INTERACTION

The mentioned deep inelastic scattering experiments show that the interaction keeping the protons together in a nucleus, which was first described by Yukawa Hideki,[***] is

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[**] The physicist George Zweig (b. 1937 Moscow) proposed the quark idea – he called them *aces* – in 1963, with more clarity than Gell-Mann. Zweig stressed the reality of aces, whereas Gell-Mann, in the beginning, did not believe in the existence of quarks. Zweig later moved on to a more difficult field: neurobiology.

Murray Gell-Mann (b. 1929 New York) received the Nobel Prize in Physics in 1969. He is the originator of the term 'quark'. The term has two origins: officially, it is said to be taken from *Finnegans Wake*, a novel by James Joyce; in reality, Gell-Mann took it from a Yiddish and German term meaning 'lean soft cheese' and used figuratively in those languages to mean 'silly idea'. (Ref. 184)

Gell-Mann was the central figure of particle physics in the 20th century; he introduced the concept of strangeness, the renormalization group, the flavour SU(3) symmetry and quantum chromodynamics itself. A disturbing story is that he took the idea, the data, the knowledge, the concepts and even the name of the V−A theory of the weak interaction from the bright physics student George Sudarshan and published it, together with Richard Feynman, as his own. The wrong attribution is still found in many textbooks.

Gell-Mann is also known for his constant battle with Feynman about who deserved to be called the most arrogant physicist of their university. A famous anecdote is the following. Newton's once used a common saying of his time in a letter to Hooke: 'If I have seen further than you and Descartes, it is by standing upon the shoulders of giants.' Gell-Mann is known for saying: 'If I have seen further than others, it is because I am surrounded by dwarfs.'

[***] Yukawa Hideki (b. 1907 Azabu, d. 1981 Kyoto), important physicist specialized in nuclear and particle physics. He received the 1949 Nobel Prize in Physics for his theory of mesons. Yukawa founded the journal

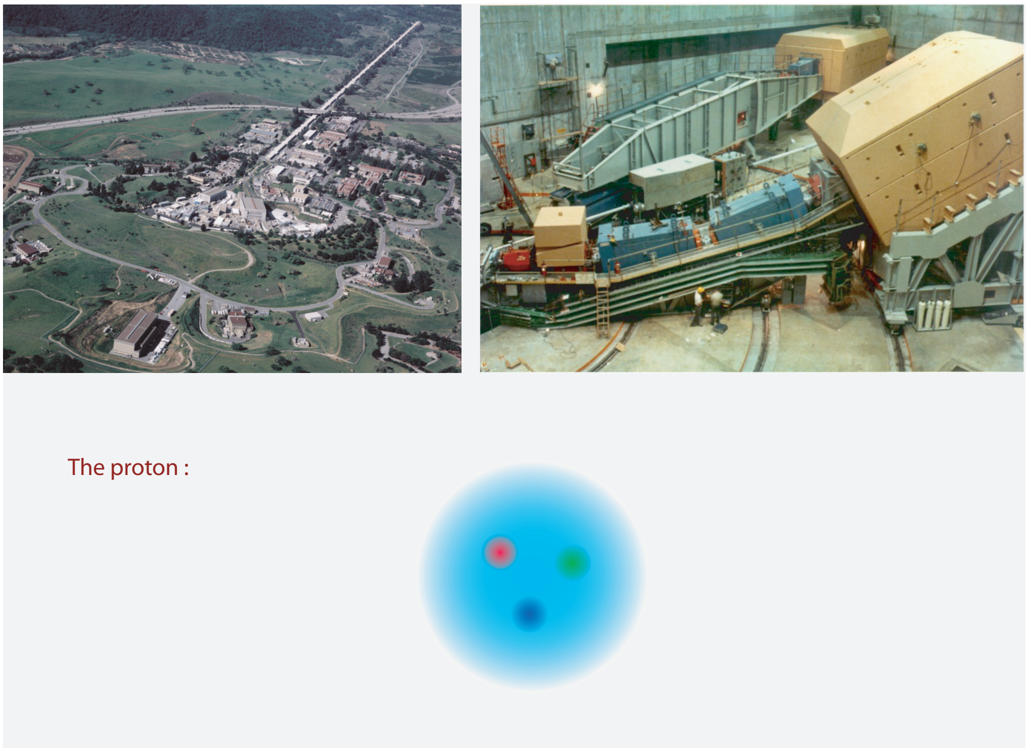

FIGURE 129 Top: SLAC, the electron linear collider, and the detectors used for the deep inelastic electron scattering experiment. Bottom: an artistic illustration of the final result, showing the three scattering centres observed inside the proton.

only a feeble shadow of the interaction that keeps quarks together in a proton. Both interactions are called by the same name. The two cases correspond somewhat to the two cases of electromagnetism found in atomic matter. The clearest example is provided by neon atoms: the strongest and 'purest' aspect of electromagnetism is responsible for the attraction of the electrons to the neon nuclei; its feeble 'shadow', the Van-der-Waals interaction, is responsible for the attraction of neon atoms in liquid neon and for processes like its evaporation and condensation. Both attractions are electromagnetic, but the strengths differ markedly. Similarly, the strongest and 'purest' aspect of the strong interaction leads to the formation of the proton and the neutron through the binding of quarks; the feeble, 'shadow' aspect leads to the formation of nuclei and to α decay. Obviously, most information can be gathered by studying the strongest and 'purest' aspect.

BOUND MOTION, THE PARTICLE ZOO AND THE QUARK MODEL

Deep electron scattering showed that protons are made of interacting constituents. How can one study these constituents?

Physicists are simple people. To understand the constituents of matter, and of protons in particular, they had no better idea than to take all particles they could get hold of and

---

Progress of Theoretical Physics and together with his class mate Tomonaga Shin'ichiro, who also won the prize, he was an example to many scientists in Japan.

FIGURE 130 A typical experiment used to study the quark model: the Proton Synchroton at CERN in Geneva (© CERN).

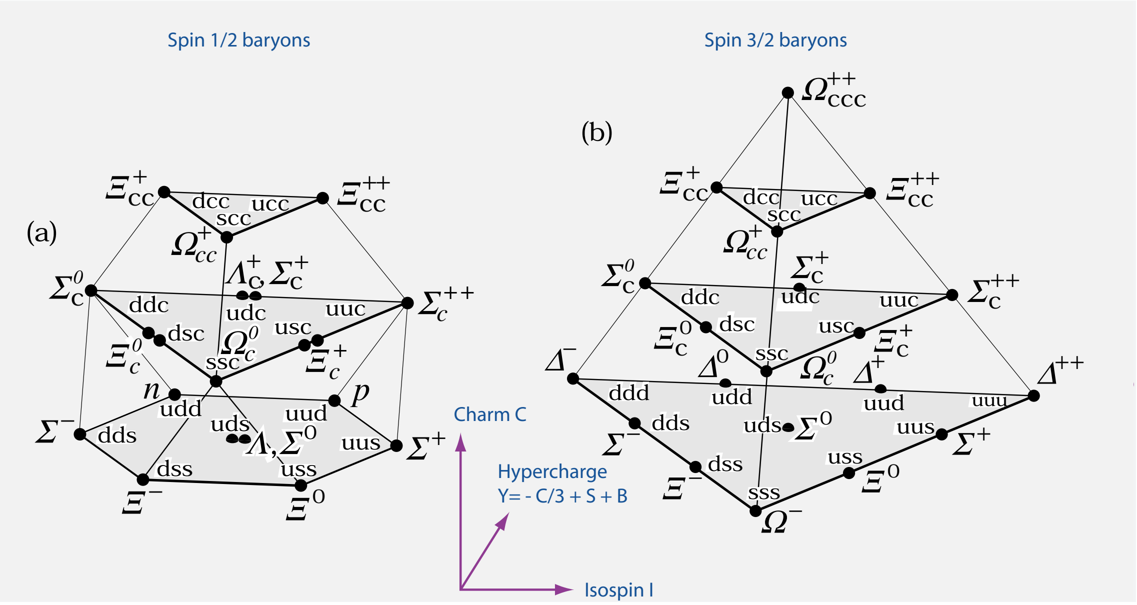

FIGURE 131 The family diagrams for the least massive baryons that can be built as qqq composites of the first four quark types (from Ref. 186).

Ref. 185 to smash them into each other. Many researchers played this game for decades. Obviously, this is a facetious comment; in fact, quantum theory forbids any other method.
Challenge 142 s Can you explain why?

Understanding the structure of particles by smashing them into each other is not simple. Imagine that you want to study how cars are built just by crashing them into

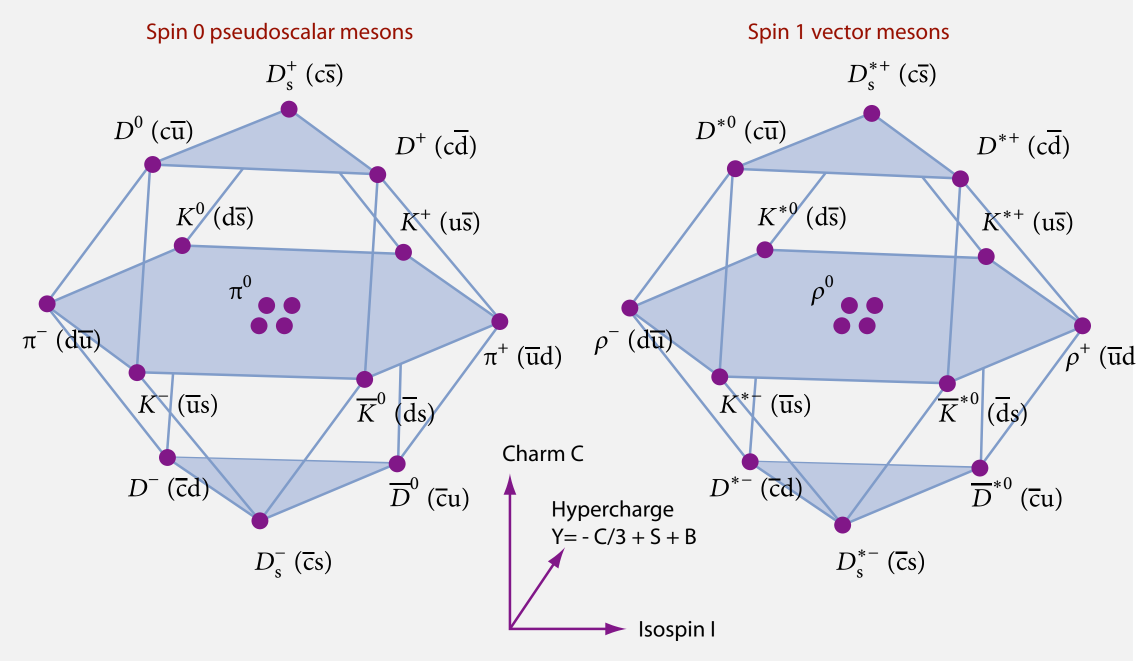

FIGURE 132 The family diagram for the least massive pseudoscalar and vector mesons that can be built as $q\bar{q}$ composites of the first four quark flavours.

each other. Before you get a list of all components, you must perform and study a non-negligible number of crashes. Most give the same result, and if you are looking for a particular part, you might have to wait for a long time. If the part is tightly attached to others, the crashes have to be especially energetic. In addition, the part most likely will be deformed. Compared to car crashes, quantum theory adds the possibility for debris to transform, to react, to bind and to get excited. Therefore the required diligence and patience is even greater for particle crashes than for car crashes. Despite these difficulties, for many decades, researchers have collected an ever increasing number of proton debris,
Page 342 also called *hadrons*. The list, a small part of which is given in Appendix B, is overwhelmingly long; the official full list, several hundred pages of fine print, is found at pdg.web.cern.ch and contains hundreds of hadrons. Hadrons come in two main types: integer spin hadrons are called *mesons*, half-integer spin hadrons are called *baryons*. The proton and the neutron themselves are thus baryons.

Then came the quark model. Using the ingenuity of many experimentalists and theoreticians, the quark model explained the whole meson and baryon catalogue as a consequence of only 6 types of bound quarks. Typically, a large part of the catalogue can be structured in graphs such as the ones given in Figure 132 and Figure 131. These graphs were the beginning of the end of high energy physics. The quark model explained all quantum numbers of the debris, and allowed understanding their mass ratios as well as their decays.

The quark model explained why debris come into two types: all *mesons* consist of a quark and an antiquark and thus have integer spin; all *baryons* consist of three quarks, and thus have half-integer spin. In particular, the proton and the neutron are seen as

TABLE 17 The quarks.

Quark	Mass m (see text)	Spin J parity P	Possible colours; possible weak behaviour	Charge Q, isospin I, strangeness S, charm C, beauty B', topness T	Lepton number L, baryon number B
Down d	4.5 to 5.5 MeV/c^2	$\frac{1}{2}^+$	red, green, blue; singlet, doublet	$-\frac{1}{3}, -\frac{1}{2}, 0, 0, 0, 0$	$0, \frac{1}{3}$
Up u	1.8 to 3.0 MeV/c^2	$\frac{1}{2}^+$	red, green, blue; singlet, doublet	$+\frac{2}{3}, +\frac{1}{2}, 0, 0, 0, 0$	$0, \frac{1}{3}$
Strange s	95(5) MeV/c^2	$\frac{1}{2}^+$	red, green, blue; singlet, doublet	$-\frac{1}{3}, 0, -1, 0, 0, 0$	$0, \frac{1}{3}$
Charm c	1.275(25) GeV/c^2	$\frac{1}{2}^+$	red, green, blue; singlet, doublet	$+\frac{2}{3}, 0, 0, +1, 0, 0$	$0, \frac{1}{3}$
Bottom b	4.18(3) GeV/c^2	$\frac{1}{2}^+$	red, green, blue; singlet, doublet	$-\frac{1}{3}, 0, 0, 0, -1, 0$	$0, \frac{1}{3}$
Top t	173.5(1.4) GeV/c^2	$\frac{1}{2}^+$	red, green, blue; singlet, doublet	$+\frac{2}{3}, 0, 0, 0, 0, +1$	$0, \frac{1}{3}$

combinations of two quark types, called *up* (u) and *down* (d): the proton is a *uud* state, the neutron a *udd* state. The discovery of other hadrons lead to the addition of four additional types of quarks. The quark names are somewhat confusing: they are called *strange* (s), *charm* (c), *bottom* (b) – also called 'beauty' in the old days – and *top* (t) – called 'truth' in the past. The quark types are called *flavours*; in total, there are thus 6 quark flavours in nature.

All quarks have spin one half; they are fermions. Their electric charges are multiples of 1/3 of the electron charge. In addition, quarks carry a strong charge, called, again confusingly, *colour*. In contrast to electromagnetism, which has only positive, negative, and neutral charges, the strong interaction has red, blue, green quarks on one side, and anti-red, anti-blue and anti-green on the other. The neutral state is called 'white'. All baryons, including proton and neutrons, and all mesons are white, in the same way that all atoms are neutral.

The essence of quantum chromodynamics

The theory describing the bound states of quarks is called *quantum chromodynamics*, or QCD. It was formulated in its final form in 1973 by Fritzsch, Gell-Mann and Leutwyler. In the same way that in atoms, electrons and protons are held together by the exchange of Ref. 187

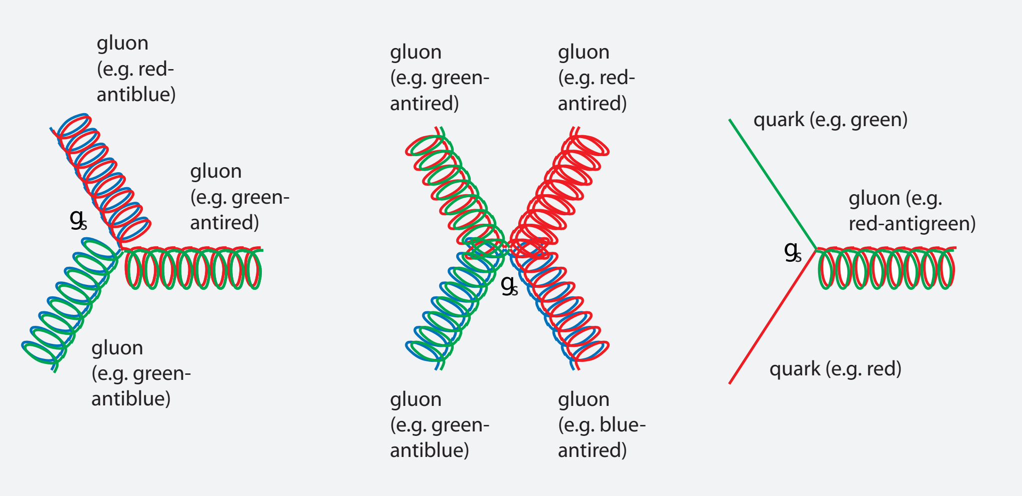

FIGURE 133 The essence of the QCD Lagrangian: the Feynman diagrams of the strong interaction.

virtual photons, in protons, quarks are held together by the exchange of virtual gluons. *Gluons* are the quanta of the strong interaction, and correspond to photons, the quanta of the electromagnetic interactions.

Ref. 188 Quantum chromodynamics describes all motion due to the strong interaction with the three fundamental processes shown in Figure 133: two gluons can scatter, a gluon can emit or absorb another, and a quark can emit or absorb a gluon. In electrodynamics, only the last diagram is possible; in the strong interaction, the first two appear as well. Among others, the first two diagrams are responsible for the confinement of quarks, and thus for the lack of free quarks in nature.

QCD is a *gauge* theory: the fields of the strong interaction show gauge invariance under the Lie group SU(3). We recall that in the case of electrodynamics, the gauge group is U(1), and Abelian, or commutative. In contrast, SU(3) is non-Abelian; QCD is a non-Abelian gauge theory. Non-Abelian gauge theory was invented and popularized by Wolfgang Pauli. It is often incorrectly called *Yang–Mills theory* after the first two physicists who wrote down Pauli's ideas.

Due to the SU(3) gauge symmetry, there are 8 gluons; they are called red-antigreen, blue-antired, etc. Since SU(3) is non-Abelian, gluons interact among themselves, as shown in the first two processes in Figure 133. Out of the three combinations red-antired, blue-antiblue and green-antigreen, only two gluons are linearly independent, thus giving a total of $3^2 - 1 = 8$ gluons.

The coupling strength of the strong interaction, its three fundamental processes in Figure 133, together with its SU(3) gauge symmetry and the observed number of six quarks, completely determine the behaviour of the strong interaction. In particular, they completely determine its Lagrangian density.

THE LAGRANGIAN OF QUANTUM CHROMODYNAMICS*

The Lagrangian density of the strong interaction can be seen as a complicated formulation of the Feynman diagrams of Figure 133. Indeed, the Lagrangian density of quantum chromodynamics is

$$\mathcal{L}_{\mathrm{QCD}} = -\tfrac{1}{4}F^{(a)}_{\mu\nu}F^{(a)\mu\nu} - c^2\sum_q m_q \overline{\psi}^k_q \psi_{qk} + i\hbar c \sum_q \overline{\psi}^k_q \gamma^\mu (D_\mu)_{kl}\psi^l_q \qquad (71)$$

where the gluon field strength and the gauge covariant derivative are

$$F^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu$$

$$(D_\mu)_{kl} = \delta_{kl}\partial_\mu - i\frac{g_s}{2}\sum_a \lambda^a_{k,l} A^a_\mu \,.$$

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We remember from the section on the principle of least action that Lagrangians are always sums of scalar products; this is clearly seen in expression (71). The index $a = 1\dots8$ numbers the eight types of gluons and the index $k = 1, 2, 3$ numbers the three colours, all due to SU(3). The index $q = 1\dots6$ numbers the six quark flavours. The fields $A^a_\mu(x)$ are the eight gluon fields, represented by the coiled lines in Figure 133. The fields $\psi^k_q(x)$ are those of the quarks of flavour q and colour k, represented by the straight line in the figure. The six times three quark fields, like those of any elementary fermion, are 4-component Dirac spinors with masses m_q.**

The Lagrangian (71) is that of a *local* field theory: observables are functions of position. In other words, QCD is similar to quantum electrodynamics and can be compared to experiment in the same way.

The first term of the Lagrangian (71) represents the kinetic energy of the radiation (the gluons), the second or mass term the kinetic energy of the matter particles (the quarks) and the third term the interaction between the two.

The mass term in the Lagrangian is the only term that spoils or *breaks flavour symmetry*, i.e., the symmetry under exchange of quark types. (In particle physics, this symmetry is also called *chiral symmetry*, for historical reasons.) Obviously, the mass term also breaks space-time conformal symmetry.

The interaction term in the Lagrangian thus corresponds to the third diagram in Figure 133. The strength of the strong interaction is described by the *strong coupling constant* g_s. The constant is independent of flavour and colour, as observed in experiment. The Interaction term does not mix different quarks; as observed in experiments, flavour is conserved in the strong interaction, as is baryon number. The strong interaction also conserves spatial parity P and charge conjugation parity C. The strong interaction does not transform matter.

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* This section can be skipped at first reading.

** In their simplest form, the matrices γ_μ can be written as

$$\gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \text{and} \quad \gamma_n = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \text{for } n = 1, 2, 3 \qquad (72)$$

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where the σ^i are the Pauli spin matrices.

In QCD, the eight gluons are massless; also this property is taken from experiment. Therefore no gluon mass term appears in the Lagrangian. It is easy to see that massive gluons would spoil gauge invariance. As mentioned above, in contrast to electromagnetism, where the gauge group U(1) is Abelian, the gauge group SU(3) of the strong interactions is non-Abelian. As a consequence, the colour field itself is charged, i.e., carries colour, and thus the index a appears on the fields A and F. As a result, gluons can interact with each other, in contrast to photons, which pass each other undisturbed. The first two diagrams of Figure 133 are thus reflected in the somewhat complicated definition of the field $F^{(a)}_{\mu\nu}$. In contrast to electrodynamics, the definition has an extra term that is *quadratic* in the fields A; it is described by the so-called structure constants f_{abc} and the interaction strength g_s. The numbers f_{abc} are the structure constants of the SU(3).

Challenge 143 ny

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The behaviour of the gauge transformations and of the gluon field is described by the eight matrices $\lambda^a_{k,l}$. They are a fundamental, 3-dimensional representation of the *generators* of the SU(3) algebra and correspond to the eight gluon types. The matrices λ_a, $a = 1...8$, and the structure constants f_{abc} obey the relations

$$\begin{aligned} [\lambda_a, \lambda_b] &= 2if_{abc}\lambda_c \\ \{\lambda_a, \lambda_b\} &= 4/3\delta_{ab}I + 2d_{abc}\lambda_c \end{aligned} \qquad (73)$$

where I is the unit matrix. The structure constants f_{abc} of SU(3), which are odd under permutation of any pair of indices, and d_{abc}, which are even, have the values

abc	f_{abc}	abc	d_{abc}	abc	d_{abc}
123	1	118	$1/\sqrt{3}$	355	$1/2$
147	$1/2$	146	$1/2$	366	$-1/2$
156	$-1/2$	157	$1/2$	377	$-1/2$
246	$1/2$	228	$1/\sqrt{3}$	448	$-1/(2\sqrt{3})$
257	$1/2$	247	$-1/2$	558	$-1/(2\sqrt{3})$
345	$1/2$	256	$1/2$	668	$-1/(2\sqrt{3})$
367	$-1/2$	338	$1/\sqrt{3}$	778	$-1/(2\sqrt{3})$
458	$\sqrt{3}/2$	344	$1/2$	888	$-1/\sqrt{3}$
678	$\sqrt{3}/2$				

(74)

All other elements vanish. Physically, the structure constants of SU(3) describe the details of the interaction between quarks and gluons and of the interaction between the gluons themselves.

A fundamental 3-dimensional representation of the eight generators λ_a – corresponding to the eight gluon types – is given, for example, by the set of the *Gell-Mann matrices*

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} . \qquad (75)$$

There are eight matrices, one for each gluon type, with 3×3 elements, due to the 3 colours of the strong interaction. There is no ninth gluon, because that gluon would be colourless, or ‘white’.

The Lagrangian is complete only when the 6 quark masses and the coupling constant g_s are included. These values, like the symmetry group SU(3), are not explained by QCD, of course.

Only quarks and gluons appear in the Lagrangian of QCD, because only quarks and gluons interact via the strong force. This can be also expressed by saying that only quarks and gluons carry colour; *colour* is the source of the strong force in the same way that electric charge is the source of the electromagnetic field. In the same way as electric charge, colour charge is *conserved* in all interactions. Electric charge comes in two types, positive and negative; in contrast, colour comes in three types, called *red*, *green* and *blue*. The neutral state, with no colour charge, is called *white*. Protons and neutrons, but also electrons or neutrinos, are thus ‘white’, thus *neutral* for the strong interaction.

In summary, the six quark types interact by exchanging eight gluon types. The interaction is described by the Feynman diagrams of Figure 133, or, equivalently, by the Lagrangian (71). Both descriptions follow from the requirements that the gauge group is SU(3) and that the masses and coupling constants are given. It was a huge amount of work to confirm that all experiments indeed agree with the QCD Lagrangian; various competing descriptions were discarded.

Experimental consequences of the quark model

How can we pretend that quarks and gluons exist, even though they are never found alone? There are a number of arguments in favour.

* *

The quark model explains the non-vanishing magnetic moment of the neutron and explains the magnetic moments μ of the baryons. By describing the proton as a *uud* state and the neutron a *udd* state with no orbital angular momentum and using the precise wave functions, we get

Challenge 144 e

$$\mu_u = \tfrac{1}{5}(4\mu_p + \mu_n) \qquad \text{and} \qquad \mu_d = \tfrac{1}{5}(4\mu_n + \mu_p) . \qquad (76)$$

Assuming that $m_u = m_d$ and that the quark magnetic moment is proportional to their charge, the quark model predicts a ratio of the magnetic moments of the proton and the neutron of

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2} \,. \qquad (77)$$

This prediction differs from measurements only by 3 %. Furthermore, using the same values for the magnetic moment of the quarks, magnetic moment values of over half a dozen of other baryons can be predicted. The results typically deviate from measurements only by around 10 %. In particular, the sign of the resulting baryon magnetic moment is always correctly calculated.

* *

The quark model describes all *quantum numbers* of mesons and baryons. P-parity, C-parity, and the absence of certain meson parities are all reproduced. The observed conservation of electric charge, baryon number, isospin, strangeness etc. is reproduced. Hadron family diagrams such as those shown in Figure 131 and in Figure 132 describe all existing hadron states (of lowest angular momentum) *completely*; the states not listed are not observed. The quark model thus produces a complete and correct classification of all hadrons as bound states of quarks.

* *

The quark model also explains the *mass spectrum* of hadrons. The best predictions are made by QCD lattice calculations. With months of computer time, researchers were able to reproduce the masses of proton and neutron to within a few per cent. Interestingly, if one sets the u and d quark masses to zero, the resulting proton and neutron mass differ from experimental values only by 10 %. The mass of protons and neutrons is almost completely due to the binding, not to the constituents. More details are given below.

Ref. 189
Ref. 190
Page 233

* *

The number of colours of quarks must be taken into account to get correspondence of theory and calculation. For example, the measured decay time of the neutral pion is 83 as. The calculation without colour gives 750 as; if each quark is assumed to appear in 3 colours the value must be divided by 9, and then matches the measurement.

* *

In particle colliders, collisions of electrons and positrons sometimes lead to the production of hadrons. The calculated production rates also fit experiments only if quarks have three colours. In more detail, if one compares the ratio of muon–antimuon production and of hadron production, a simple estimate relates them to their charges:

Challenge 145 s

$$R = \frac{\sum q_{\text{hadrons}}}{\sum q_{\text{muons}}} \qquad (78)$$

Between 2 and 4 GeV, when only three quarks can appear, this argument thus predicts $R = 2$ if colours exist, or $R = 2/3$ if they don't. Experiments yield a value of $R = 2.2$, thus

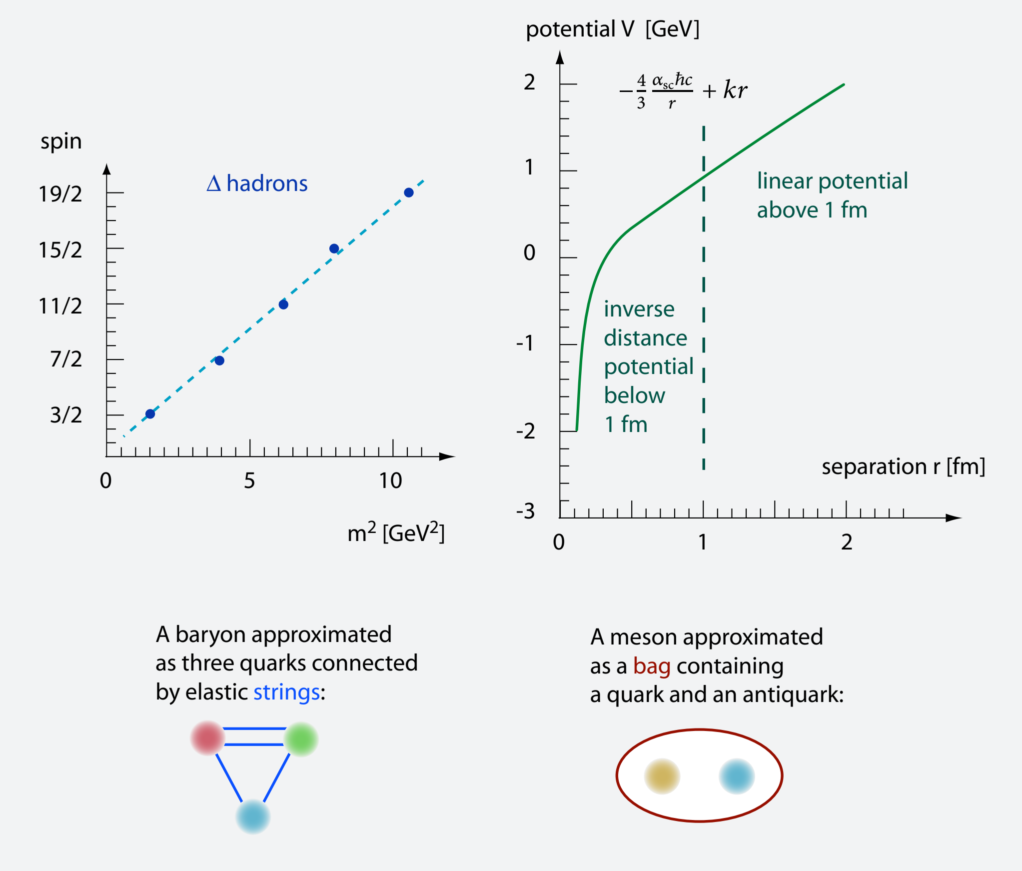

FIGURE 134 Top left: a Regge trajectory, or Chew–Frautschi plot, due to the confinement of quarks. Top right: the quark confinement potential. Bottom: two approximate ways to describe quark confinement: the string model and the bag model of hadrons.

confirming the number of colours. Many other such *branching ratios* can be calculated in this way. They agree with experiments only if the number of colours is three.

CONFINEMENT OF QUARKS – AND ELEPHANTS

Many of the observed hadrons are not part of the diagrams of Figure 131 and Figure 132; these additional hadrons can be explained as *rotational excitations* of the fundamental mesons from those diagrams. As shown by Tullio Regge in 1957, the idea of rotational excitations leads to quantitative predictions. Regge assumed that mesons and baryons are quarks connected by strings, like rubber bands – illustrated in Figure 134 and Figure 135 – and that the force or tension k between the quarks is thus constant over distance.

We assume that the strings, whose length we call $2r_0$, rotate around their centre of mass as rapidly as possible, as shown in Figure 135. Then we have

$$v(r) = c\frac{r}{r_0}\ . \qquad (79)$$

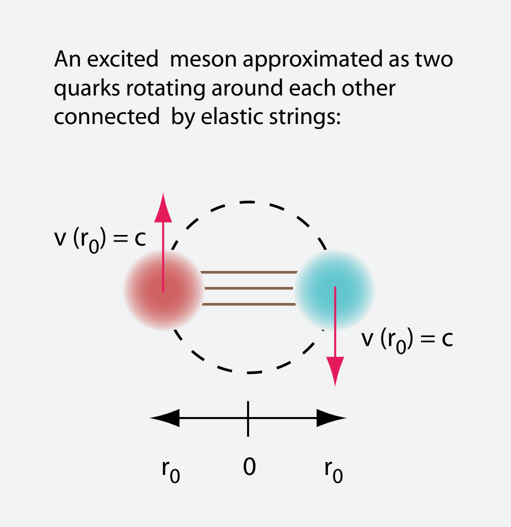

FIGURE 135 Calculating masses of excited hadrons.

The quark masses are assumed negligible. For the total energy this implies the relation

$$E = c^2 m = 2 \int_0^{r_0} \frac{k}{\sqrt{1 - v(r)/c^2}} \mathrm{d}r = k r_0 \pi \tag{80}$$

and for angular momentum the relation

$$J = \frac{2}{\hbar c^2} \int_0^{r_0} \frac{k r v(r)}{\sqrt{1 - v(r)/c^2}} \mathrm{d}r = \frac{k r_0^2}{2\hbar c} \ . \tag{81}$$

Including the spin of the quarks, we thus get

$$J = \alpha_0 + \alpha' m^2 \quad \text{where} \quad \alpha' = \frac{c^3}{2\pi k \hbar} \ . \tag{82}$$

Regge thus deduced a simple expression that relates the mass m of excited hadrons to their total spin J. For bizarre historical reasons, this relation is called a *Regge trajectory*.

The value of the constant α' is predicted to be independent of the quark–antiquark pairing. A few years later, as shown in Figure 134, such linear relations were found in experiments: the *Chew-Frautschi plots*. For example, the three lowest lying states of Δ are the spin 3/2 $\Delta(1232)$ with m^2 of 1.5 GeV^2, the spin 7/2 $\Delta(1950)$ with m^2 of 3.8 GeV^2, and the spin 11/2 $\Delta(2420)$ with m^2 of 5.9 GeV^2. The value of the constant α' is found experimentally to be around 0.93 GeV^{-2} for almost all mesons and baryons, whereas the value for α_0 varies from particle to particle. The quark string tension is thus found to be Ref. 188

$$k = 0.87\,\mathrm{GeV/fm} = 0.14\,\mathrm{MN} \ . \tag{83}$$

In other words, two quarks in a hadron attract each other with a force equal to the weight of two elephants: about 14 tons.

Experiments are thus clear: the observed Chew-Frautschi plots, as well as several other observations not discussed here, are best described by a quark–quark potential that grows, above 1 fm, *linearly* with distance. The slope of the linear potential, the force, has a value equal to the force with which the Earth attracts two elephants. As a result, quarks *never* appear as free particles: quarks are always *confined* in hadrons. This situation is in contrast with QED, where the force between charges goes to zero for large distances; electric charges are thus not confined, but can exist as free particles. At large distances, the electric potential *decreases* in the well-known way, with the inverse of the distance. In contrast, for the strong interaction, experiments lead to a quark potential given by Ref. 185

$$V = -\frac{4}{3}\frac{\alpha_{\text{sc}}\hbar c}{r} + kr \qquad (84)$$

where k is the mentioned 0.87 GeV/fm, α_{sc} is 0.2, and $\hbar c$ is 0.1975 GeV/fm. The quark potential is illustrated in Figure 134.

Even though experiments are clear, theoreticians face a problem. So far, neither the quark-quark potential nor the quark bound states can be deduced from the QCD Lagrangian with a *simple* approximation method. Nevertheless, complicated non-perturbative calculations show that the QCD Lagrangian does predict a force between two coloured particles that levels off at a constant value (corresponding to a linearly increasing potential). These calculations show that the old empirical approximations of hadrons as quarks connected by strings or a quarks in bags, shown in Figure 134, can indeed be deduced from the QCD Lagrangian. However, the calculations are too complex to be summarized in a few lines. Independently, the constant force value has also been reproduced in computer calculations in which one simplifies space-time to a lattice and then approximates QCD by so-called *lattice QCD* or *lattice gauge theory*. Lattice calculations have further reproduced the masses of most mesons and baryons with reasonable accuracy. Using the most powerful computers available, these calculations have given predictions of the mass of the proton and other baryons within a few per cent. Discussing these complex and fascinating calculations lies outside the scope of this text, however. Ref. 192

In fact, the challenge of explaining confinement in simple terms is so difficult that the brightest minds have been unable to solve it yet. This is not a surprise, as its solution probably requires the unification of the interactions and, most probably, also the unification with gravity. We therefore leave this issue for the last part of our adventure.

ASYMPTOTIC FREEDOM

QCD has another property that sets it apart form QED: the behaviour of its coupling with energy. In fact, there are three equivalent ways to describe the strong coupling strength. The first way is the quantity appearing in the QCD Lagrangian, g_s. The second way is often used to define the equivalent quantity $\alpha_s = g_s^2/4\pi$. Both α_s and g_s depend on the energy Q of the experiment. If they are known for one energy, they are known for all of them. Presently, the best experimental value is $\alpha_s(M_Z) = 0.1185 \pm 0.0010$.

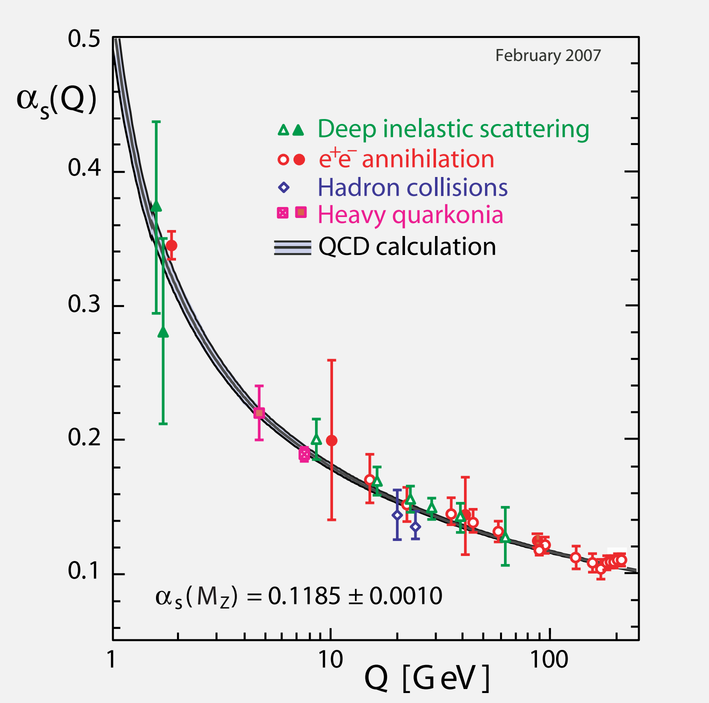

FIGURE 136 The measured and the calculated variation of the strong coupling with energy, showing the precision of the QCD Lagrangian and the asymptotic freedom of the strong interaction (© Siegfried Bethke, updated from Ref. 193).

The energy dependence of the strong coupling can be calculated with the standard renormalization procedures and is expected to be Ref. 186, Ref. 188

$$\alpha_s(Q^2) = \frac{12\pi}{33 - 2n_f} \frac{1}{L} \left(1 - \frac{(918 - 114n_f) \ln L}{(33 - 2n_f)^2 L} + ... \right) \quad \text{where} \quad L = \ln \frac{Q^2}{\Lambda^2(n_f)} \qquad (85)$$

where n_f is the number of quarks with mass below the energy scale Q, thus a number between 3 and 6. (The expression has been expanded to many additional terms with the help of computer algebra.)

The third way to describe the strong coupling is thus the energy parameter $\Lambda(n_f)$. Experiments yield $\Lambda(3) = 230(60)$ GeV, $\Lambda(4) = 180(50)$ GeV and $\Lambda(5) = 120(30)$ GeV.

The accelerator experiments that measure the coupling are extremely involved, and hundreds of people across the world have worked for many years to gather the relevant data. The comparison of QCD and experiment, shown in Figure 136, does not show any contradiction between the two.

Figure 136 and expression (85) illustrate what is called *asymptotic freedom*: α_s decreases at high energies. In other words, at high energies quarks are *freed* from the strong interaction; they behave as free particles.* As a result of asymptotic freedom, in QCD, a perturbation expansion can be used only at energies much larger than Λ. Historically,

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* Asymptotic freedom was discovered in 1972 by Gerard 't Hooft; since he had received the Nobel Prize already, the 2004 Prize was then given to the next people who highlighted it: David Gross, David Politzer and Frank Wilczek, who studied it extensively in 1973.

the discovery of asymptotic freedom was essential to establish QCD as a theory of the strong interaction.

Asymptotic freedom can be understood qualitatively if the situation is compared to QED. The electron coupling increases at small distances, because the screening due to the virtual electron-positron pairs has less and less effect. In QCD, the effective colour coupling also changes at small distances, due to the smaller number of virtual quark-antiquark pairs. However, the gluon properties lead to the opposite effect, an *antiscreening* that is even stronger: in total, the effective strong coupling decreases at small distances.

THE SIZES AND MASSES OF QUARKS

The size of quarks, like that of all elementary particles, is predicted to vanish by QCD, as in all quantum field theory. So far, no experiment has found any effect due to a finite quark size. Measurements show that quarks are surely smaller than 10^{-19} m. No size conjecture has been given by any hypothetical theory. Quarks are assumed point-like, or at most Planck-sized, in all descriptions so far. Ref. 194

We noted in several places that a neutral compound of charged particles is always less massive than its components. But if you look up the mass values for quarks in most tables, the masses of u and d quarks are only of the order of a few MeV/c^2, whereas the proton's mass is $938\,\mathrm{MeV}/c^2$. What is the story here?

It turns out that the definition of the mass is more involved for quarks than for other particles. Quarks are never found as free particles, but only in bound states. As a result, the concept of quark mass depends on the calculation framework one is using.

Due to *asymptotic freedom*, quarks behave almost like free particles only at high energies. The mass of such a 'free' quark is called the *current quark mass*; for the light quarks it is only a few MeV/c^2, as shown in Table 17.

At low energy, for example inside a proton, quarks are *not* free, but must carry along a large amount of energy due to the confinement process. As a result, bound quarks have a much larger effective, so-called *constituent quark mass*, which takes into account this confinement energy. To give an idea of the values, take a proton; the indeterminacy relation for a particle inside a sphere of radius 0.9 fm gives a momentum indeterminacy of around $190\,\mathrm{MeV}/c$. In three dimensions this gives an energy of $\sqrt{3}$ times that value, or an effective, constituent quark mass of about $330\,\mathrm{MeV}/c^2$. Three confined quarks are thus heavier than a proton, whose mass is $938\,\mathrm{MeV}/c^2$; we can thus still say that a compound proton is less massive than its constituents.

In short, the mass of the proton and the neutron is (almost exclusively) the kinetic energy of the quarks inside them, as their rest mass is almost negligible. As Frank Wilczek says, some people put on weight even though they never eat anything heavy. Ref. 195

But also the small current quark mass values for the up, down, strange and charmed quarks that appear in the QCD Lagrangian depend on the calculation framework that is used. The values of Table 17 are those for a renormalization scale of 2 GeV. For half that energy, the mass values increase by 35 %. The heavy quark masses are those used in the so-called $\overline{MS}$ scheme, a particular way to perform perturbation expansions. Ref. 186

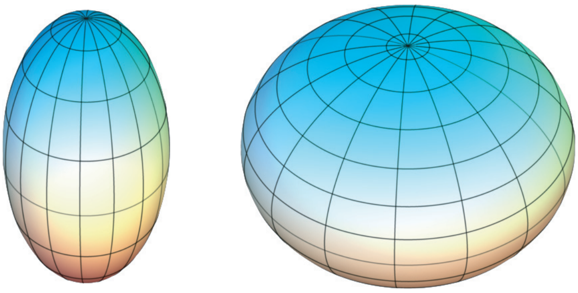


FIGURE 137 An illustration of a *prolate* (left) and an *oblate* (right) ellipsoidal shape (© Sam Derbyshire).

The mass, shape and colour of protons

Ref. 195 Frank Wilczek mentions that one of the main results of QCD, the theory of strong interactions, is to explain mass relations such as

$$m_{\text{proton}} \sim e^{-k/\alpha} m_{\text{Planck}} \quad \text{and} \quad k = 11/2\pi \,, \ \alpha_{\text{unif}} = 1/25 \,. \qquad (86)$$

Here, the value of the coupling constant α_{unif} is taken at the grand unifying energy, a factor of 1000 below the Planck energy. (See the section of grand unification below.) In other words, a general understanding of masses of bound states of the strong interaction, such as the proton, requires almost purely a knowledge of the unification energy and the coupling constant at that energy. The approximate value $\alpha_{\text{unif}} = 1/25$ is an extrapolation from the low energy value, using experimental data. The proportionality factor k in expression (86) is not easy to calculate. It is usually determined on computers using lattice QCD. Page 268

But the mass is not the only property of the proton. Being a cloud of quarks and gluons, it also has a shape. Surprisingly, it took a long time before people started to become interested in this aspect. The proton, being made of two up quarks and one down quark, resembles a ionized H_2^+ molecule, where one electron forms a cloud around two protons. Obviously, the H_2^+ molecule is elongated, or *prolate*, as shown in Figure 137.

Is the proton prolate? There is no spectroscopically measurable asphericity – or quadrupole moment – of the proton. However, the proton has an *intrinsic* quadrupole moment. The quadrupole moments of the proton and of the neutron are predicted to be positive in all known calculation methods, implying an *prolate* shape. Recent measurements at Jefferson Laboratories confirm this prediction. A prolate shape is predicted for all $J = 1/2$ baryons, in contrast to the oblate shape predicted for the $J = 3/2$ baryons. The spin 0 pseudoscalar mesons are predicted to be prolate, whereas the spin 1 vector mesons are expected to be oblate. Ref. 196

The shape of any molecule will depend on whether other molecules surround it. Recent research showed that similarly, both the size and the shape of the proton in nuclei is slightly variable; both seem to depend on the nucleus in which the proton is built-in. Ref. 197

Apart from shapes, molecules also have a colour. The colour of a molecule, like that of any object, is due to the energy absorbed when it is irradiated. For example, the H_2^+ molecule can absorb certain light frequencies by changing to an excited state. Molecules change mass when they absorb light; the excited state is heavier than the ground state.

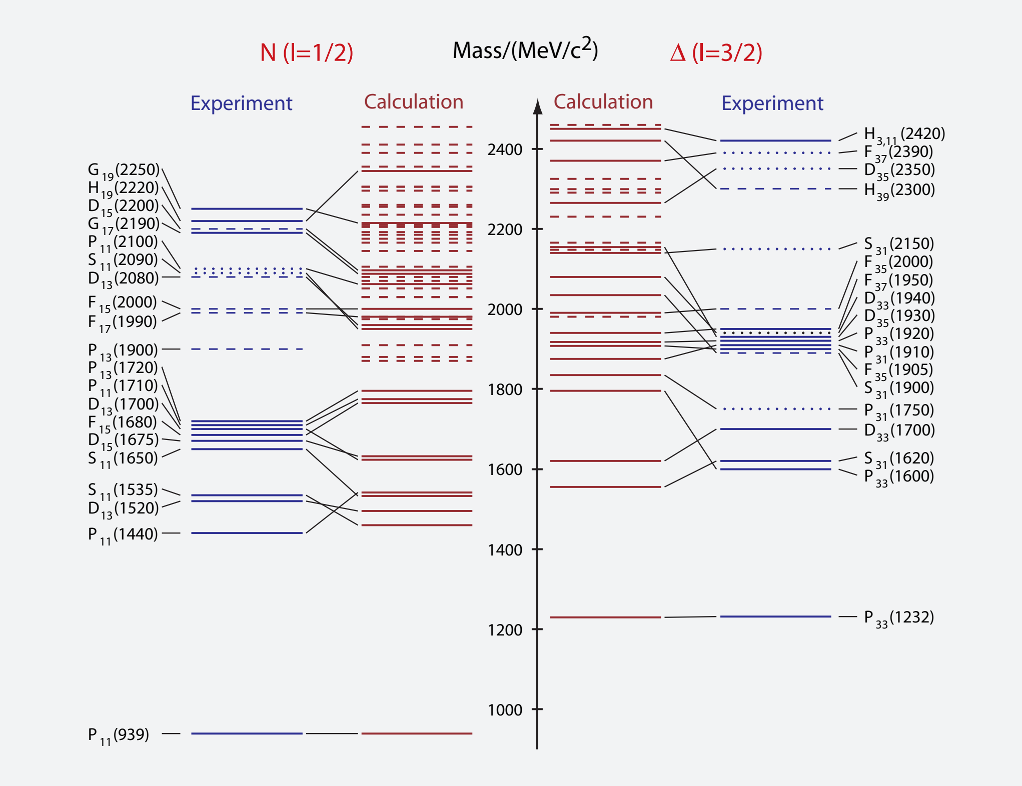


FIGURE 138 The mass spectrum of the excited states of the proton: experimental and calculated values (from Ref. 186).

In the same way, protons and neutrons can be excited. In fact, their excited states have Ref. 186 been studied in detail; a summary, also showing the limitation of the approach, is shown in Figure 138. Many excitations can be explained as excited quarks states, but many more are predicted. The calculated masses agree with observations to within 10 %. The quark model and QCD thus structure and explain a large part of the baryon spectrum; but the agreement is not yet perfect.

Obviously, in our everyday environment the energies necessary to excite nucleons do not appear – in fact, they do not even appear inside the Sun – and these excited states can be neglected. They only appear in particle accelerators and in cosmic rays. In a sense, we can say that in our corner of the universe, the colour of protons usually is not visible.

Curiosities about the strong interaction

In a well-known analogy, QCD can be compared to superconductivity. Table 18 gives an overview of the correspondence.

* *

The computer calculations necessary to extract particle data from the Lagrangian of quantum chromodynamics are among the most complex calculations ever performed. They beat weather forecasts, fluid simulations and the like by orders of magnitude.

TABLE 18 Correspondence between QCD and superconductivity.

QCD	Superconductivity
Quark	magnetic monopole
Colour force non-linearities	Electron–lattice interaction
Chromoelectric flux tube	magnetic flux tube
Gluon-gluon attraction	electron–electron attraction
Glueballs	Cooper pairs
Instability of bare vacuum	instability of bare Fermi surface
Discrete centre symmetry	continuous U(1) symmetry
High temperature breaks symmetry	low temperature breaks symmetry

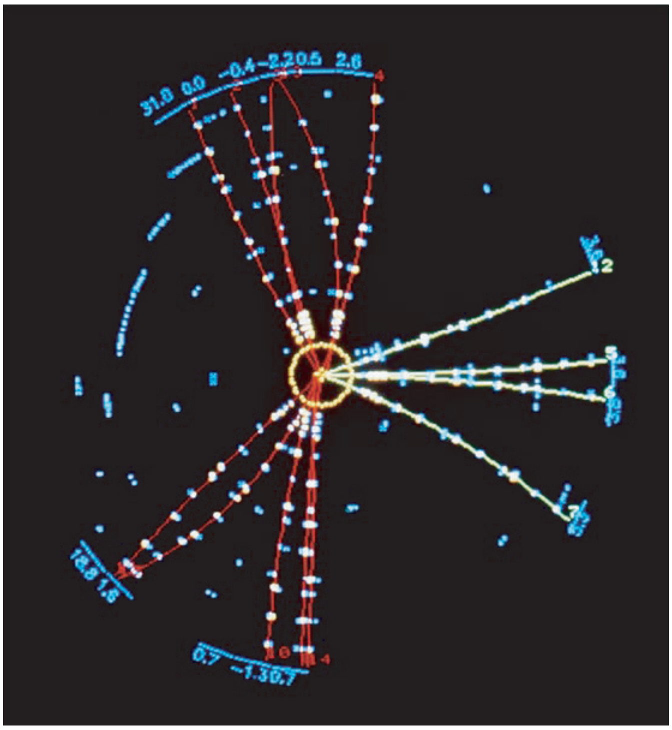


FIGURE 139 A three jet event observed at the PETRA collider in Hamburg in Germany. The event, triggered by an electron-positron collision, allowed detecting the decay of a gluon and measuring its spin (© DESY).

Nobody knows whether this will be necessary also in the future: the race for a simple approximation method for finding solutions is still open.

* *

Even though gluons are massless, like photons and gravitons are, there is *no* colour radiation in nature. Gluons carry colour and couple to themselves; as a result, free gluons were predicted to directly decay into quark–antiquark pairs.

In 1979, the first clear decays of *gluons* have been observed at the PETRA particle collider in Hamburg. The occurrence of certain events, called *gluon jets*, are due to the decay of high-energy gluons into narrow beams of particles. Gluon jets appear in coplanar Ref. 191

three-jet events. The observed rate and the other properties of these events confirmed the predictions of QCD. Experiments at PETRA also determined the spin $S = 1$ of the gluon and the running of the strong coupling constant. The hero of those times was the project manager Gustav-Adolf Voss, who completed the accelerator on budget and six months ahead of schedule.

* *

Something similar to colour radiation, but still stranger, might have been found in 1997. It might be that a scalar meson with a mass of 1.5 GeV/c^2 is a *glueball*. This is a hypothetical meson composed of gluons only. Numerical results from lattice gauge theory seem to confirm the possibility of a glueball in that mass range. The existence of glueballs is hotly debated and still open. Ref. 198

* *

There is a growing consensus that most light scalar mesons below 1 GeV/c^2, are *tetraquarks*. In 2003, experiments provided also candidates for heavier tetraquarks, namely the X(3872), Ds(2317) and Ds(2460). The coming years will show whether this interpretation is correct. Ref. 199

* *

Do particles made of five quarks, so-called *pentaquarks*, exist? So far, they seem to exist only in a few laboratories in Japan, whereas in other laboratories across the world they are not seen. Most researchers do not believe the results any more. Ref. 200

* *

Whenever we look at a periodic table of the elements, we look at a manifestation of the strong interaction. The Lagrangian of the strong interaction describes the origin and properties of the presently known 115 elements.

Nevertheless one central aspect of nuclei is determined by the electromagnetic interaction. Why are there around 115 different elements? Because the electromagnetic coupling constant α is around 1/137.036. In more detail, the answer is the following. If the charge of a nucleus were much higher than around 137, the electric field around nuclei would lead to spontaneous electron–positron pair generation; the generated electron would fall into the nucleus and transform one proton into a neutron, thus inhibiting a larger proton number. The finite number of the elements is thus due to the electromagnetic interaction.

* *

To know more about radioactivity, its effects, its dangers and what a government can do about it, see the English and German language site of the Federal Office for Radiation Protection at www.bfs.de.

* *

From the years 1990 onwards, it has regularly been claimed that extremely poor countries are building nuclear weapons. Why is this highly unlikely? Challenge 146 s

* *

Historically, nuclear reactions provided the first test of the relation $E = c^2\gamma m$. This was achieved in 1932 by Cockcroft and Walton. They showed that by shooting protons into lithium one gets the reaction

$$ {}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{4}\mathrm{Be} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} + 17\,\mathrm{MeV}\ . \qquad (87)$$

The measured energy on the right is exactly the same value that is derived from the differences in total mass of the nuclei on both sides.

* *

A large fraction of researchers say that QCD is defined by *two* parameters. Apart from the coupling constant, they count also the strong CP parameter. Indeed, it might be that the strong interaction violates CP invariance. This violation would be described by a second term in the Lagrangian; its strength would be described by a second parameter, a phase usually called θ_{CP}. However, many high-precision experiments have been performed to search for this effect, and no CP violation in the strong interaction has ever been detected.

A summary of QCD and its open issues

Quantum chromodynamics, the non-Abelian gauge theory based on the Lagrangian with SU(3) symmetry, describes the properties of gluons and quarks, the properties of the proton, the neutron and all other hadrons, the properties of atomic nuclei, the working of the stars and the origin of the atoms inside us and around us. Without the strong interaction, we would not have flesh and blood. And all these aspects of nature follow from a single number, the strong coupling constant, and the SU(3) gauge symmetry.

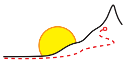
The strong interaction acts only on quarks and gluons. It conserves particle type, colour, electric charge, weak charge, spin, as well as C, P and T parity.

QCD and experiment agree wherever comparisons have been made. QCD is a perfect description of the strong interaction. The limitations of QCD are only conceptual. Like in all of quantum field theory, also in the case of QCD the mathematical form of the Lagrangian is almost uniquely defined by requiring renormalizability, Lorentz invariance and gauge invariance – SU(3) in this case. We say 'almost', because the Lagrangian, despite describing correctly all experiments, contains a few parameters that remain unexplained:

- The number, 6, and the masses m_q of the quarks are not explained by QCD.
- The coupling constant of the strong interaction g_s, or equivalently, α_s or Λ, is unexplained. QCD predicts its energy dependence, but not its absolute value.
- Experimentally, the strong interaction is found to be CP conserving. This is not obvious; the QCD Lagrangian assumes that any possible CP-violating term vanishes, even though there exist CP-violating Lagrangian terms that are Lorentz-invariant, gauge-invariant and renormalizable.
- The properties of space-time, in particular its Lorentz invariance, its continuity and the number of its dimensions are assumed from the outset and are obviously all unexplained in QCD.

- It is also not known how QCD has to be modified in strong gravity, thus in strongly curved space-time.

We will explore ways to overcome these limits in the last part of our adventure. Before we do that, we have a look at the other nuclear interaction observed in nature.



Chapter 8

THE WEAK NUCLEAR INTERACTION AND THE HANDEDNESS OF NATURE

The weirdest interaction in nature is the weak interaction. The weak interaction transforms elementary particles into each other, has radiation particles that have mass, violates parity and treats right and left differently. Fortunately, we do not experience the weak interaction in our everyday life, as its properties violate much of what we normally experience. This contrast makes the weak interaction the most fascinating of the four interactions in nature.

Transformation of elementary particles

Radioactivity, in particular the so-called β decay, is a bizarre phenomenon. Experiments in the 1910s showed that when β sources emit electrons, atoms are *transformed* from one chemical element to another. For example, experiments such as those of Figure 140 show that tritium, an isotope of hydrogen, decays into helium as

$$ {}^{3}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + e^{-} + \overline{\nu}_{\mathrm{e}} \,. \qquad (88)$$

In fact, new elements appear in all β decays. In the 1930s it became clear that the transformation process is due to a neutron in the nucleus changing into a proton (and more):

$$ n \rightarrow p + e^{-} + \overline{\nu}_{\mathrm{e}} \,. \qquad (89)$$

This reaction explains all β decays. In the 1960s, the quark model showed that β decay is in fact due to a down quark changing to an up quark:

$$ d \rightarrow u + e^{-} + \overline{\nu}_{\mathrm{e}} \,. \qquad (90)$$

This reaction explains the transformation of a neutron – a *udd* state – into a proton – a *uud* state. In short, matter particles can transform into each other. We note that this transformation differs from what occurs in other nuclear processes. In fusion, fission or α decay, even though nuclei change, every neutron and every proton retains its nature. In β decay, elementary particles are not immutable. The dream of Democritus and Leucippus about immutable basic building blocks is definitely *not* realized in nature.

Experiments show that quark transformations cannot be achieved with the help of electromagnetic fields, nor with the help of gluon fields, nor with the help of gravitation. There must be another type of radiation in nature, and thus another, fourth in-

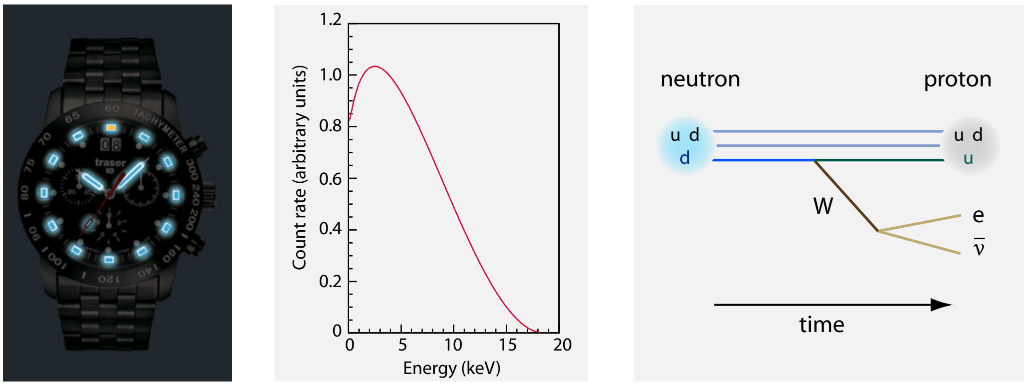


FIGURE 140 β decay (beta decay) in tritium: a modern, tritium-powered illuminated watch, the measured continuous energy spectrum of the emitted electrons from tritium, and the process occurring in the tritium nucleus (© Traser, Katrin collaboration).

teraction. Chemical transformations are also rare, otherwise we would not be running around with a constant chemical composition. The fourth interaction is therefore *weak*. Since the transformation processes were observed in the nucleus first, the interaction was named the *weak nuclear interaction*.

In β decay, the weak nuclear interaction transforms quarks into each other. In fact, the weak nuclear interaction can also transform leptons into each other, such as muons into electrons. But where does the energy released in β decay go to? Measurements in 1911 showed that the energy spectrum of the emitted electron is *continuous*. This is illustrated in Figure 140. How can this be? In 1930, Wolfgang Pauli had the courage and genius to explain this observation with a daring hypothesis: the energy of the decay is split between the electron and a new, truly astonishing particle, the *neutrino* – more precisely, the electron anti-neutrino $\bar{\nu}_e$. In order to agree with data, the neutrino must be uncharged, cannot interact strongly, and must be of very low mass. As a result, neutrinos interact with ordinary matter only extremely rarely, and usually fly through the Earth without being affected. This property makes their detection very difficult, but not impossible; the first neutrino was finally detected in 1952. Later on, it was discovered that there are three types of neutrinos, now called the electron neutrino, the muon neutrino and the tau neutrino, each with its own antiparticle. For the summary of these experimental efforts, see Table 19.

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The weakness of the weak nuclear interaction

From the observation of β decay, and helped by the quark model, physicists quickly concluded that there must be an intermediate particle that carries the weak nuclear interaction, similar to the photon that carries the electromagnetic interaction. This ‘weak radiation’, in contrast to all other types of radiation, consists of *massive* particles.

▷ There are two types of weak radiation particles: the neutral Z boson with a mass of 91.2 GeV – that is the roughly mass of a silver atom – and the electrically charged W boson with a mass of 80.4 GeV.

TABLE 19 The leptons: the three neutrinos and the three charged leptons (antiparticles have opposite charge Q and parity P).

NEUTRINO	MASS m AND DECAY (SEE TEXT)	SPIN J PARITY P	COLOUR; POSSIBLE WEAK BEHAVIOUR	CHARGE Q, ISOSPIN I, STRANGENESS S, CHARM C, BEAUTY B', TOPNESS T	LEPTON NUMBER L, BARYON NUMBER B
Electron neutrino ν_e	$< 2\,\text{eV}/c^2$, oscillates	$\frac{1}{2}^+$	white; singlet, doublet	$0, 0, 0, 0, 0, 0$	$1, 0$
Muon neutrino ν_e	$< 2\,\text{eV}/c^2$, oscillates	$\frac{1}{2}^+$	white; singlet, doublet	$0, 0, 0, 0, 0, 0$	$1, 0$
Tau neutrino ν_e	$< 2\,\text{eV}/c^2$, oscillates	$\frac{1}{2}^+$	white; singlet, doublet	$0, 0, 0, 0, 0, 0$	$1, 0$
Electron e	$0.510\,998\,928(11)\,\text{MeV}/c^2$, stable	$\frac{1}{2}^+$	white; singlet, doublet	$-1, 0, 0, 0, 0, 0$	$1, 0$
Muon μ	$105.658\,3715(35)\,\text{MeV}/c^2$, *c.* 99 % $e\overline{\nu}_e\nu_\mu$	$\frac{1}{2}^+$	white; singlet, doublet	$-1, 0, 0, 0, 0, 0$	$1, 0$
Tau τ	$1.776\,82(16)\,\text{GeV}/c^2$, *c.* 17 % $\mu\overline{\nu}_\mu\nu_\tau$, *c.* 18 % $e\overline{\nu}_e\nu_\tau$	$\frac{1}{2}^+$	white; singlet, doublet	$-1, 0, 0, 0, 0, 0$	$1, 0$

Together, the W and Z bosons are also called the *weak vector bosons*, or the *weak intermediate bosons*.

The masses of the weak vector bosons are so large that free weak radiation exists only for an extremely short time, about 0.1 ys; then the bosons decay. The large mass is the reason that the weak interaction is extremely short range and thus extremely weak. Indeed, any exchange of virtual carrier particles scales with the negative exponential of the intermediate particle's mass. A few additional properties are given in Table 20. In fact, the weak interaction is so weak that neutrinos, particles which interact *only* weakly, have a large probability to fly through the Sun without any interaction.

The existence of a massive charged intermediate vector boson, today called the W, was already deduced and predicted in the 1940s; but theoretical physicists did not accept the idea until the Dutch physicists Martin Veltman and Gerard 't Hooft proved that it was possible to have such a mass without having problems in the rest of the theory. For this proof they later received the Nobel Prize in Physics – after experiments confirmed their prediction.

The existence of an additional massive *neutral* intermediate vector boson, the Z boson, was predicted only much after the W boson, by Salam, Weinberg and Glashow. Experimentally, the Z boson was first observed as a *virtual* particle in 1973 at CERN in Geneva. The discovery was made by looking, one by one, at over 700 000 photographs made at

TABLE 20 The intermediate vector bosons of the weak interaction (the Z boson is its own antiparticle; the W boson has an antiparticle of opposite charge).

Boson	Mass m	Spin J	Colour; weak behaviour	Charge Q, isospin I, strangeness S, charm C, beauty B', topness T	Lepton number L, baryon number B
Z boson	91.1876(21) GeV/c^2	1	white; ‘triplet’	0, 0, 0, 0, 0, 0	0, 0
W boson	80.398(25) GeV/c^2	1	white; ‘triplet’	1, 0, 0, 0, 0, 0	0, 0

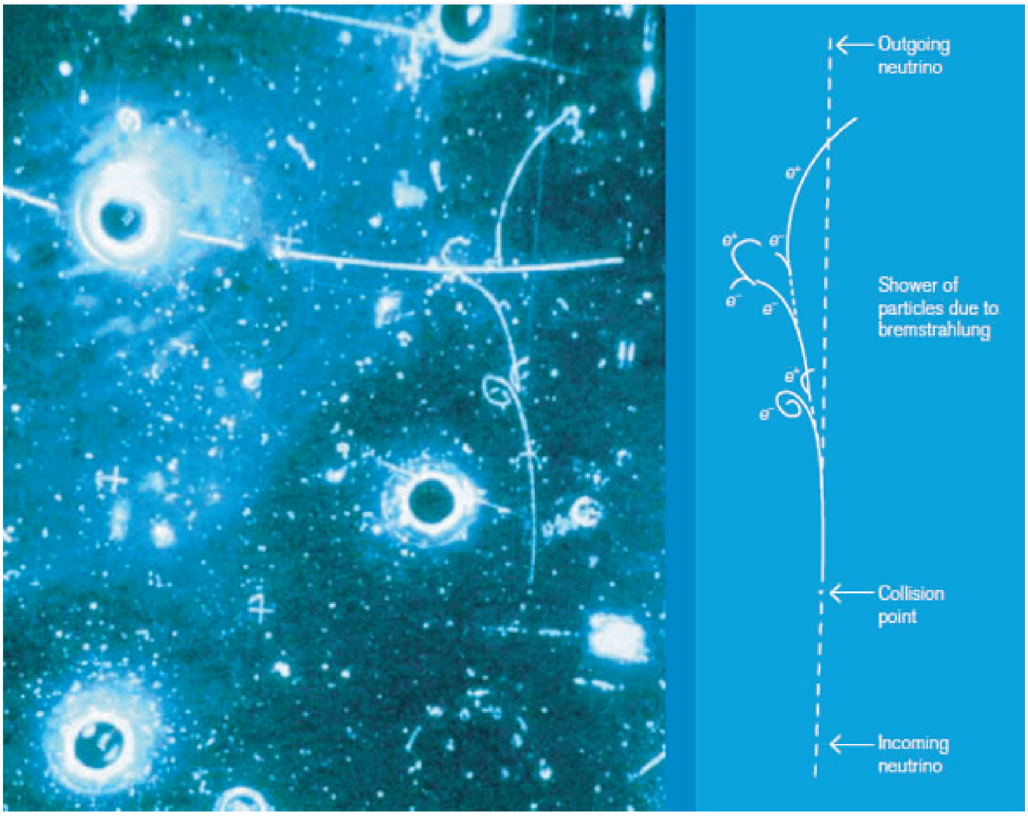

FIGURE 141 The first observation of a virtual Z boson: only neutral weak currents allow that a neutrino collides with an electron in the bubble chamber and leaves again (© CERN).

the Gargamelle bubble chamber. Only a few interesting pictures were found; the most famous one is shown in Figure 141.

In 1983, CERN groups produced and detected the first *real* W and Z bosons. This experiment was a five-year effort by thousands of people working together. The results are

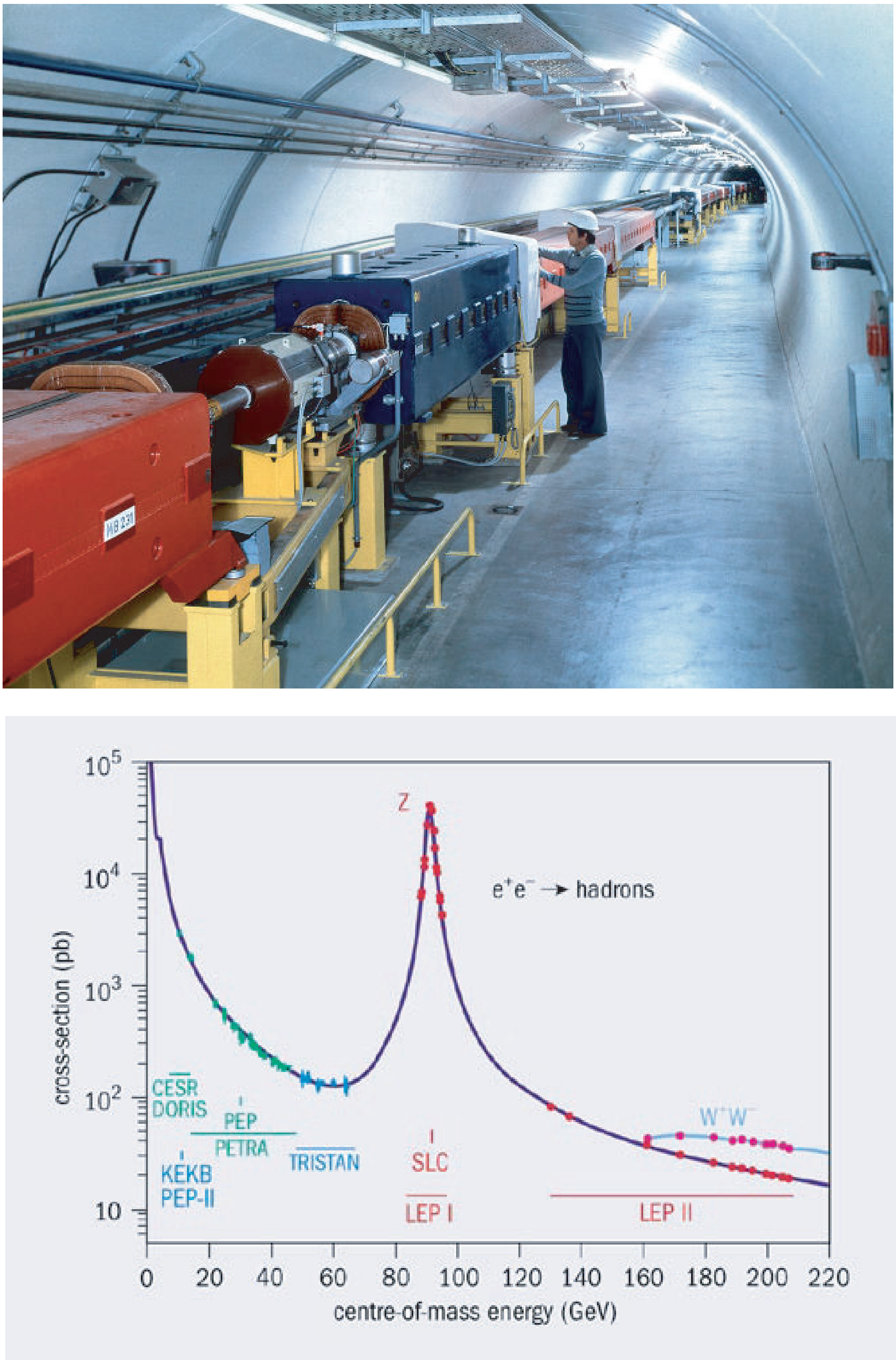

FIGURE 142 Top: the SPS, the proton–antiproton accelerator and collider at CERN, with 7 km circumference, that was used to make the first observations of real W and Z bosons. Bottom: the beautifully simple Z observation made with LEP, the successor machine (© CERN)

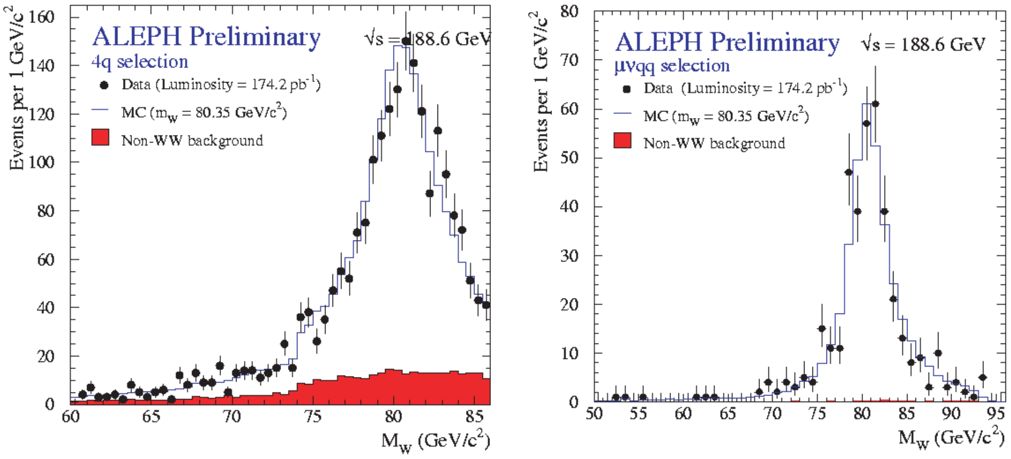

FIGURE 143 A measurement of the W boson mass at LEP (© CERN)

summarized in Table 20. The energetic manager of the project, Carlo Rubbia, whose bad temper made his secretaries leave, on average, after three weeks, and the chief technologist, Simon van der Meer, received the 1984 Nobel Prize in Physics for the discovery. This again confirmed the 'law' of nature that bosons are discovered in Europe and fermions in America. The simplest data that show the Z and W bosons is shown in Figure 142 and Figure 143; both results are deduced from the cross section of electron-positron collisions at LEP, a decade after the original discovery.

In the same way that photons are emitted by accelerated electric charges, W and Z bosons are emitted by accelerated weak charges. Due to the high mass of the W and Z bosons, the required accelerations are very high, so that they are only found in certain nuclear decays and in particle collisions. Nevertheless, the W and Z bosons are, apart from their mass and their weak charge, similar to the photon in most other aspects. In particular, the W and the Z are observed to be elementary. For example, the W gyromagnetic ratio has the value predicted for elementary particles. Ref. 186

DISTINGUISHING LEFT FROM RIGHT

Another weird characteristic of the weak interaction is the non-conservation of parity P under spatial inversion. The weak interaction distinguishes between mirror systems; this is in contrast to everyday life, to gravitation, to electromagnetism, and to the strong interaction. The non-conservation of parity by the weak interaction had been predicted by 1956 by Lee Tsung-Dao and Yang Chen Ning in order to explain the ability of K^0 mesons to decay sometimes into 2 pions, which have even parity, and sometimes into 3 pions, which have odd parity. Ref. 202

Lee and Yang suggested an experiment to Wu Chien-Shiung* The experiment she performed with her team is shown schematically in Figure 144. A few months after the

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* Wu Chien-Shiung (b. 1912 Shanghai, d. 1997 New York) was called 'madame Wu' by her colleagues. She was a bright and driven physicist born in China. She worked also on nuclear weapons; later in life she was president of the American Physical Society.

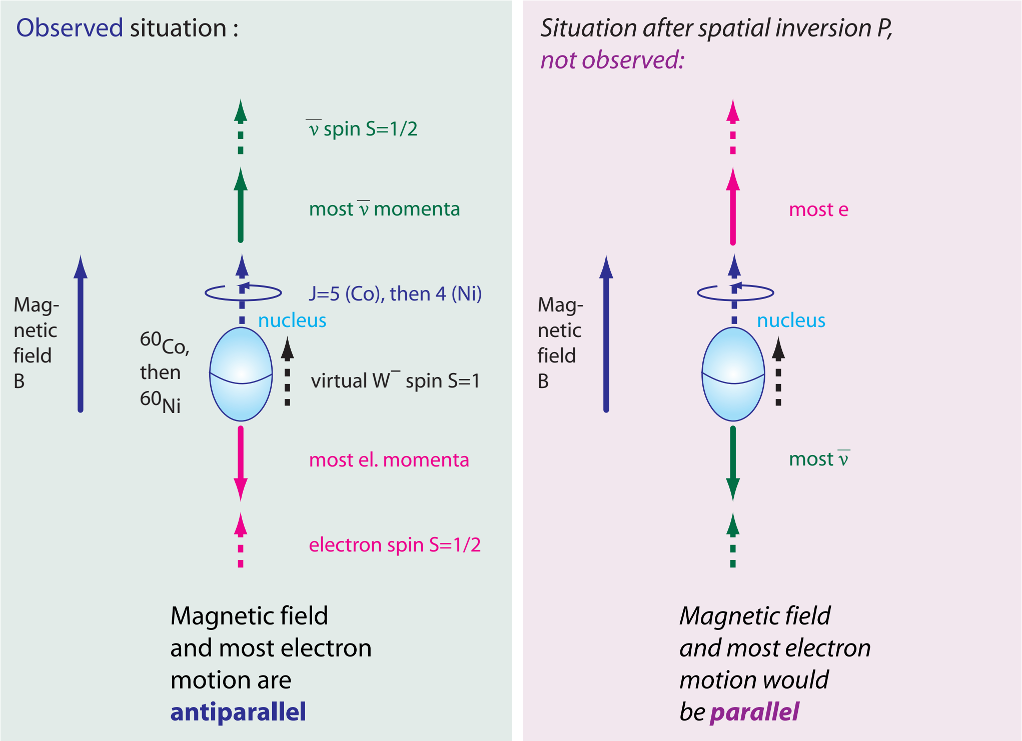

FIGURE 144 The measured behaviour of β decay, and its imagined, but unobserved behaviour under spatial inversion P (corresponding to a mirror reflection plus subsequent rotation by π around an axis perpendicular to the mirror plane).

first meetings with Lee and Yang, Wu and her team found that in the β decay of cobalt nuclei aligned along a magnetic field, the electrons are emitted mostly *against* the spin of the nuclei. In the experiment with inversed parity, the electrons would be emitted *along* the spin direction; however, this case is not observed. Parity is violated. This earned Lee and Yang a Nobel Prize in 1957.

Parity is thus violated in the weak interaction. Parity violation does not only occur in β decay. Parity violation has been found in muon decay and in every other weak process studied so far. In particular, when two electrons collide, those collisions that are mediated by the weak interaction behave differently in a mirror experiment. The number of experiments showing this increases from year to year. In 2004, two polarized beams of electrons – one left-handed and one right-handed – were shot at a matter target and the reflected electrons were counted. The difference was 0.175 parts per million – small, but measurable. The experiment also confirmed the predicted weak charge of −0.046 of the electron. Ref. 203

A beautiful consequence of parity violation is its influence on the *colour* of certain atoms. This prediction was made in 1974 by Bouchiat and Bouchiat. The weak interaction is triggered by the weak charge of electrons and nuclei; therefore, electrons in atoms do not exchange only virtual photons with the nucleus, but also virtual Z particles. The chance for this latter process is extremely small, around 10^{-11} times smaller than for ex- Ref. 204

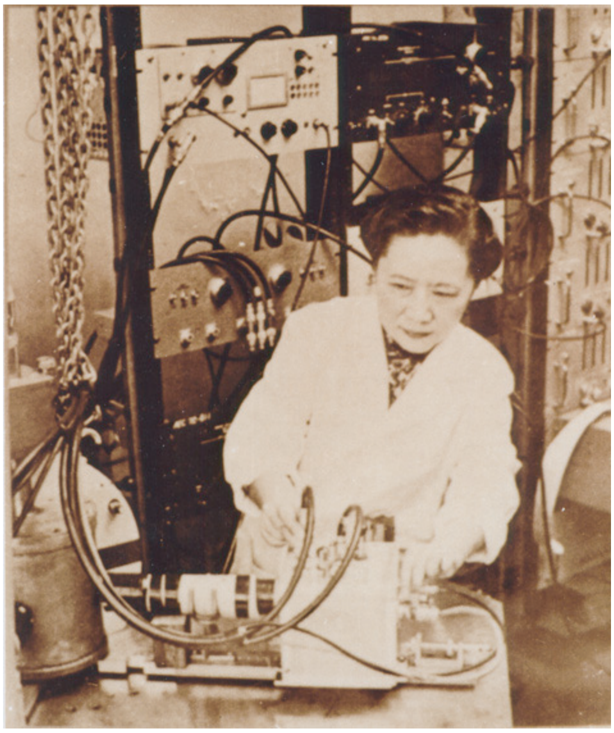

FIGURE 145 Wu Chien-Shiung (1912–1997) at her parity-violation experiment.

change of virtual photons. But since the weak interaction is not parity conserving, this process allows electron transitions which are impossible by purely electromagnetic effects. In 1984, measurements confirmed that certain optical transitions of caesium atoms that are impossible via the electromagnetic interaction, are allowed when the weak interaction is taken into account. Several groups have improved these results and have been able to confirm the calculations based on the weak interaction properties, including the weak charge of the nucleus, to within a few per cent. Ref. 205 Ref. 206

The weak interaction thus allows one to distinguish left from right. Nature contains processes that differ from their mirror version. In short, particle physics has shown that nature is (weakly) left-handed.

The left-handedness of nature is to be taken literally. All experiments confirmed two central statements on the weak interaction that can be already guessed from Figure 144. Challenge 147 e

▷ The weak interaction only couples to left-handed particles and to right-handed antiparticles. Parity is *maximally violated* in the weak interaction.

▷ All neutrinos observed so far are left-handed, and all antineutrinos are right-handed.

This result can only hold if neutrino masses vanish or are negligibly small. These two experimental results fix several aspects of the Lagrangian of the weak interaction.

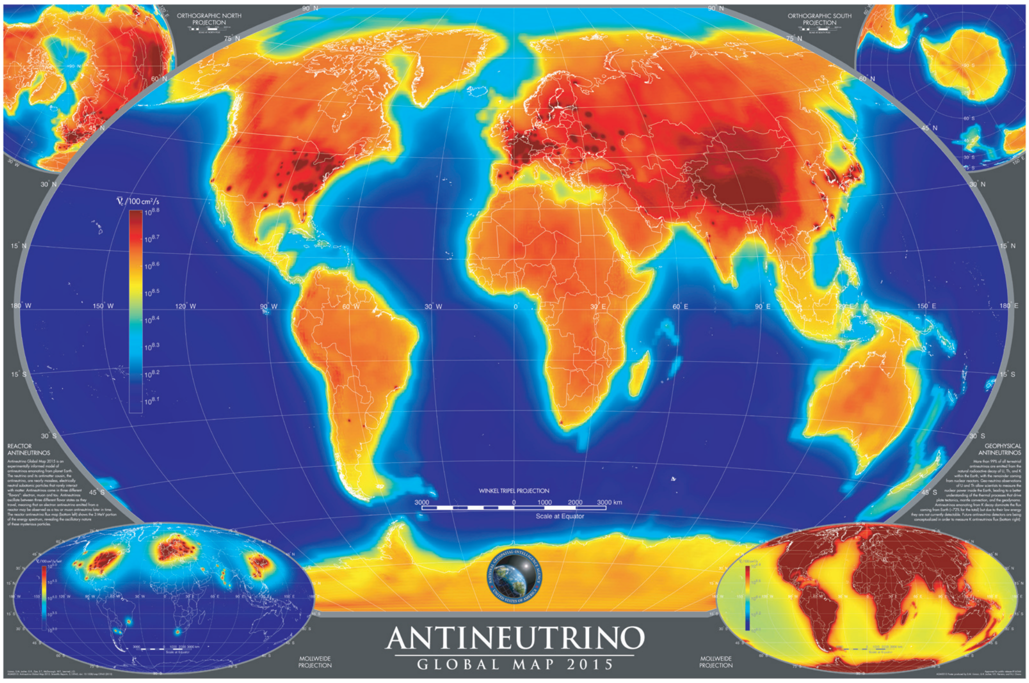

FIGURE 146 A map of the intensity distribution of the $3(2) \cdot 10^{25}$ antineutrinos between 0 and 11 MeV radiated every second from the Earth. Around 99 % of the flux is from natural sources and around 1 % from civil and military nuclear processing plants and reactors. The map is from www.nga.mil, thus cannot be completely trusted about the human sources.

Distinguishing particles and antiparticles, CP violation

In the weak interaction, the observation that only right-handed particles and left-handed antiparticles are affected has an important consequence: it implies a violation of charge conjugation parity C. Observations of muons into electrons shows this most clearly: antimuon decay differs from muon decay. The weak interaction distinguishes particles from antiparticles. Challenge 148 e

▷ Experiments show that C parity, like P parity, is *maximally violated* in the weak interaction.

Also this effect has been confirmed in all subsequent observations ever performed on the weak interaction. But that is not all. In 1964, a now famous observation was made by Val Fitch and James Cronin in the decay of the neutral K mesons.

▷ The weak interaction also violates the combination of parity inversion with particle-antiparticle symmetry, the so-called CP invariance. In contrast to P violation and C violation, which are maximal, CP violation is a tiny effect.

The experiment, shown in Figure 147 earned them the Nobel Prize in 1980. CP violation

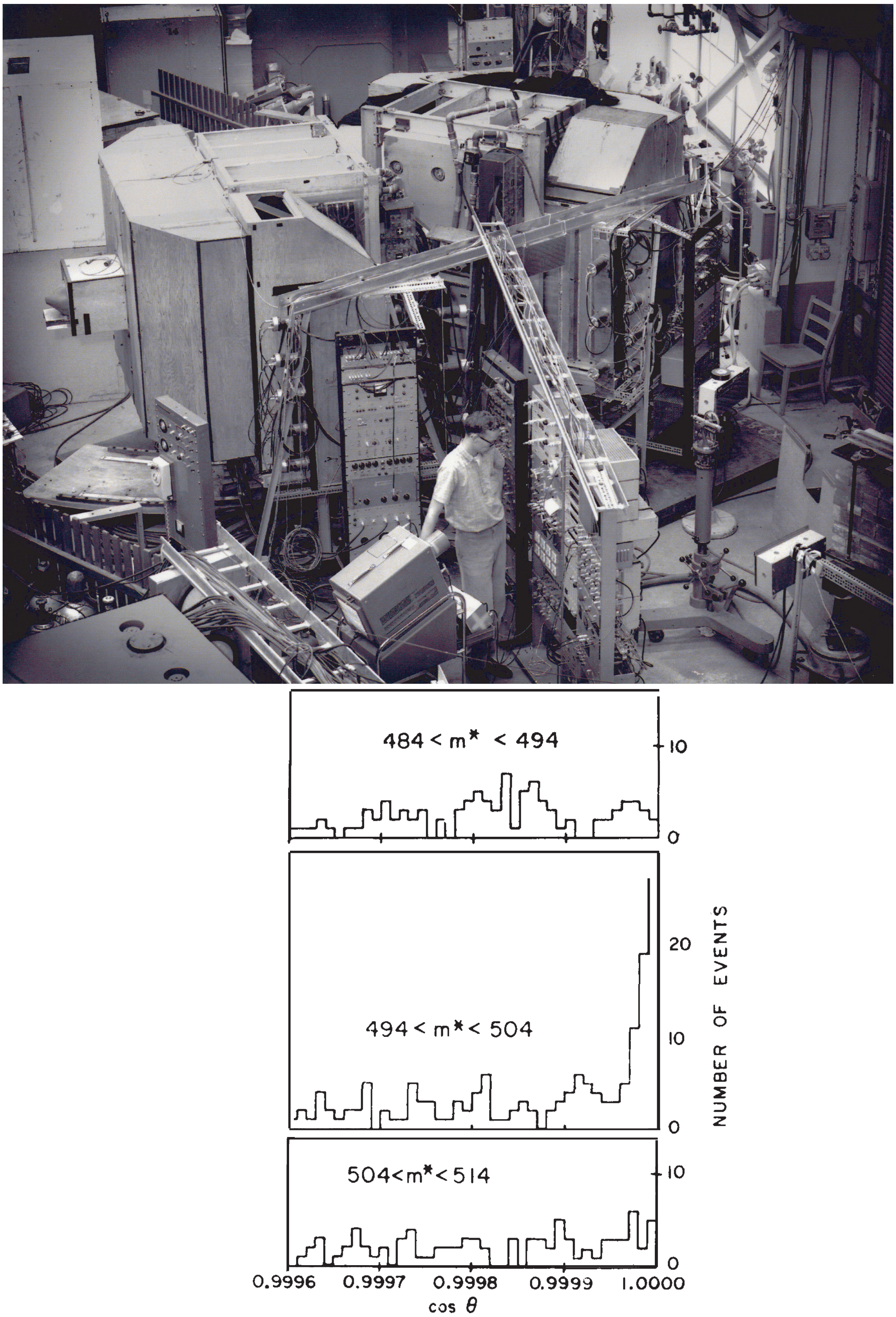

FIGURE 147 Top: the experimental set-up for measuring the behaviour of neutral K meson decay. Bottom: the measured angular dependence; the middle graph shows a peak at the right side that would not appear if CP symmetry would not be violated (© Brookhaven National Laboratory, Nobel Foundation).

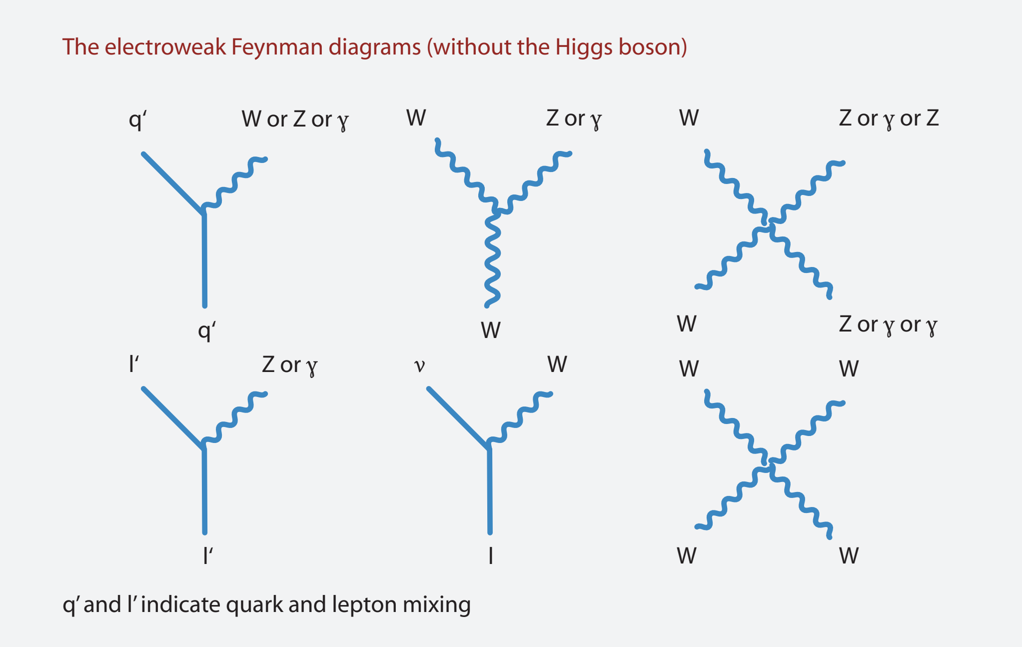

FIGURE 148 The essence of the electroweak interaction Lagrangian.

has also been observed in neutral B mesons, in several different processes and reactions. The search for other manifestations of CP violation, such as in non-vanishing electric dipole moments of elementary particles, is an intense research field. The search is not simple because CP violation is a small effect in an already very weak interaction; this tends to makes experiments large and expensive.

Since the weak interaction violates CP invariance, it also violates motion (or time) reversal T.

▷ But like all gauge theories, the weak interaction does not violate the combined CPT symmetry: it is CPT invariant.

If CPT would be violated, the masses, lifetimes and magnetic moments of particles and antiparticles would differ. That is not observed.

WEAK CHARGE AND MIXINGS

All weak interaction processes can be described by the Feynman diagrams of Figure 148. But a few remarks are necessary. First of all, the W and Z act only on left-handed fermion and on right-handed anti-fermions. Secondly, the weak interaction conserves a so-called *weak charge* T_3, also called *weak isospin*. The three quarks u, c and t, as well as the three neutrinos, have weak isospin $T_3 = 1/2$; the other three quarks and the charged leptons have weak isospin $T_3 = -1/2$. In an idealized, SU(2)-symmetric world, the three vector bosons W^+, W^0, W^- would have weak isospin values $1, 0$ and -1 and be massless. However, a few aspects complicate the issue.

First of all, it turns out that the quarks appearing in Figure 148 are not those of the strong interaction: there is a slight difference, due to *quark mixing*. Secondly, also *neutrinos mix*. And thirdly, the vector bosons are massive and break the SU(2) symmetry of the imagined idealized world; the Lie group SU(2) is not an exact symmetry of the weak interaction, and the famous Higgs boson has mass. We now explore these aspects in this order.

Surprisingly, the weak interaction eigenstates of the quarks are not the same as the mass eigenstates. This discovery by Nicola Cabibbo is described by the so-called Cabibbo–Kobayashi–Maskawa or CKM mixing matrix. The matrix is defined by

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (V_{ij}) \begin{pmatrix} d \\ s \\ b \end{pmatrix} . \tag{91}$$

where, by convention, the states of the +2/3 quarks (u, c, t) are unmixed. In its standard parametrization, the CKM matrix reads

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{92}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and i and j label the generation $(1 \leqslant i, j \leqslant 3)$. In the limit $\theta_{23} = \theta_{13} = 0$, i.e., when only two generations mix, the only remaining parameter is the angle θ_{12}, called the *Cabibbo angle*, which was introduced when only the first two generations of fermions were known. The phase δ_{13}, lying between 0 and 2π, is different from zero in nature, and expresses the fact that CP invariance is violated in the case of the weak interactions. It appears in the third column and shows that CP violation is related to the existence of (at least) three generations.

The CP violating phase δ_{13} is usually expressed with the *Jarlskog invariant*, defined as $J = \sin\theta_{12}\sin\theta_{13}\sin\theta_{23}^2\cos\theta_{12}\cos\theta_{13}\cos\theta_{23}\sin\delta_{13}$. This expression is independent of the definition of the phase angles; it was discovered by Cecilia Jarlskog, an important Swedish particle physicist. Its measured value is $J = 2.96(20) \cdot 10^{-5}$. Ref. 207

The CKM mixing matrix is predicted to be unitary. The unitarity has been confirmed by all experiments so far. The 90 % confidence upper and lower limits for the *magnitude* of the complex CKM matrix V are given by Ref. 186

$$|V| = \begin{pmatrix} 0.97428(15) & 0.2253(7) & 0.00347(16) \\ 0.2252(7) & 0.97345(16) & 0.0410(11) \\ 0.00862(26) & 0.0403(11) & 0.999152(45) \end{pmatrix} . \tag{93}$$

The values have been determined in dozens of experiments by thousands of physicists.

Also neutrinos mix, in the same way as the d, s and b quarks. The determination of the matrix elements is not as complete as for the quark case. This is an intense research field. Like for quarks, also for neutrinos the mass eigenstates and the flavour eigenstates differ. There is a dedicated neutrino mixing matrix, called the *Pontecorvo–Maki–Nakagawa–*

Sakata mixing matrix or *PMNS mixing matrix*, with 4 angles for massive neutrinos (it would have 6 angles if neutrinos were massless). In 2012, the measured matrix values were

$$P = \begin{pmatrix} 0.82 & 0.55 & -0.15 + 0.038i \\ -0.36 + 0.020i & 0.70 + 0.013i & 0.61 \\ 0.44 + 0.026i & -0.45 + 0.017i & 0.77 \end{pmatrix} . \tag{94}$$

Many experiments are trying to measure these parameters with higher precision.

Symmetry breaking – and the lack of electroweak unification

The intermediate W and Z bosons are massive and their masses differ. Thus, the weak interaction does not show a SU(2) symmetry. In addition, electromagnetic and weak processes *mix*.

Beautiful research in the 1960s showed that the mixing of the electromagnetic and the weak interactions can be described by an *'electroweak' coupling constant* g and a *weak mixing angle* θ_W. The mixing angle describes the strength of the breaking of the SU(2) symmetry. Ref. 186

It needs to be stressed that in contrast to what is usually said and written, the weak and the electromagnetic interactions do *not* unify. They have *never* been unified. Despite the incessant use of the term '*electroweak* unification' since several decades, the term is *wrong*. The electromagnetic and the weak interactions are two independent interactions, with two coupling constants, that *mix*. But they do not unify. Even though the Nobel Prize committee used the term 'unification', the relevant Nobel Prize winners confirm that the term is *not correct*.

▷ The electromagnetic and the weak interaction have *not* been unified. Their mixing has been elucidated.

The usual electromagnetic coupling constant e is related to the 'electroweak' coupling g and the mixing angle θ_w by

$$e = g \sin \theta_\mathrm{w} , \tag{95}$$

which at low four-momentum transfers is the fine structure constant with the value 1/137.036. The electroweak coupling constant g also defines the historically defined *Fermi constant* G_F by

$$G_\mathrm{F} = \frac{g^2 \sqrt{2}}{8 M_W^2} . \tag{96}$$

The broken SU(2) symmetry implies that in the *real* world, in contrast to the *ideal* SU(2) world, the intermediate vector bosons are

- the massless, neutral photon, given as $A = B \cos \theta_W + W^3 \sin \theta_W$;
- the massive neutral Z boson, given as $Z = -B \sin \theta_W + W^3 \cos \theta_W$;
- the massive charged W bosons, given as $W^{\pm} = (W^1 \mp i W^2)/\sqrt{2}$.

Together, the mixing of the electromagnetic and weak interactions as well as the breaking of the SU(2) symmetry imply that the electromagnetic coupling e, the weak coupling g and the intermediate boson masses by the impressive relation

$$\left(\frac{m_W}{m_Z}\right)^2 + \left(\frac{e}{g}\right)^2 = 1 \,. \tag{97}$$

The relation is well verified by experiments.

The mixing of the electromagnetic and weak interactions also suggests the existence of a scalar, elementary *Higgs boson*. This prediction, from the year 1963, was made by Peter Higgs and a number of other particles physicists, who borrowed ideas that Yoichiro Nambu and, above all, Philip Anderson introduced in solid state physics. The Higgs boson maintains the unitarity of longitudinal boson scattering at energies of a few TeV and influences the mass of all other elementary particles. In 2012, the Higgs boson has finally been observed in two large experiments at CERN.

Ref. 208

The Lagrangian of the weak and electromagnetic interactions

If we combine the observed properties of the weak interaction mentioned above, namely its observed Feynman diagrams, its particle transforming ability, P and C violation, quark mixing, neutrino mixing and symmetry breaking, we arrive at the full Lagrangian density. It is given by:

$$\begin{aligned}
\mathcal{L}_{\mathrm{E\&W}} = \ & \textstyle\sum_k \overline{\psi}_k (i\partial\!\!\!/ - m_k - \frac{g m_k H}{2 m_W})\psi_k && \Big\} \quad \text{1. fermion mass terms}\\
& -e \textstyle\sum_k q_k \overline{\psi}_k \gamma^\mu \psi_k A_\mu && \Big\} \quad \text{2. e.m. interaction}\\
& -\tfrac{g}{2\sqrt{2}} \textstyle\sum_k \overline{\psi}_k \gamma^\mu (1-\gamma^5)(T^+ W^+_\mu + T^- W^-_\mu)\psi_k && \Big\} \quad \text{3. charged weak currents}\\
& -\tfrac{g}{2\cos\theta_W} \textstyle\sum_k \overline{\psi}_k \gamma^\mu (g^k_V - g^k_A \gamma^5)\psi_k Z_\mu && \Big\} \quad \text{4. neutral weak currents}\\
& -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} && \Big\} \quad \text{5. electromagnetic field}\\
& -\tfrac{1}{2} W^+_{\mu\nu} W^{-\mu\nu} - \tfrac{1}{4} Z_{\mu\nu}Z^{\mu\nu} && \Big\} \quad \text{6. weak W and Z fields}\\
& +m^2_W W^+ W^- + \tfrac{1}{2} m^2_Z Z^2 && \Big\} \quad \text{7. W and Z mass terms}\\
& -gWWA - gWWZ && \Big\} \quad \text{8. cubic interaction}\\
& -\tfrac{g^2}{4}(W^4 + Z^4 + W^2F^2 + Z^2F^2) && \Big\} \quad \text{9. quartic interaction}\\
& +\tfrac{1}{2}(\partial^\mu H)(\partial_\mu H) - \tfrac{1}{2} m^2_H H^2 && \Big\} \quad \text{10. Higgs boson mass}\\
& -\tfrac{g m^2_H}{4 m_W} H^3 - \tfrac{g^2 m^2_H}{32 m^2_W} H^4 && \Big\} \quad \text{11. Higgs self-interaction}\\
& +(g m_W H + \tfrac{g^2}{4} H^2)(W^+_\mu W^{-\mu} + \tfrac{1}{2\cos^2\theta_{\mathrm{w}}} Z_\mu Z^\mu) && \Big\} \quad \text{12. Higgs–W and Z int.}
\end{aligned} \tag{98}$$

The terms in the Lagrangian are easily associated to the Feynman diagrams of Figure 148:

1. this term describes the inertia of every object around us, yields the motion of fermions, and represents the kinetic energy of the quarks and leptons, as it appears in the usual Dirac equation, modified by the so-called *Yukawa coupling* to the Higgs field H and possibly by a Majorana term for the neutrinos (not shown);

2. the second term describes the well-known interaction of matter and electromagnetic radiation, and explains practically all material properties and colours observed in daily life;
3. the term is the so-called *charged weak current interaction*, due to exchange of virtual W bosons, that is responsible for the β decay and for the fact that the Sun is shining;
4. this term is the *neutral weak current interaction*, the '$V - A$ theory' of George Sudarshan, that explains the elastic scattering of neutrinos in matter;
5. this term represents the kinetic energy of photons and yields the evolution of the electromagnetic field in vacuum, thus the basic Maxwell equations;
6. this term represents the kinetic energy of the weak radiation field and gives the evolution of the intermediate W and Z bosons of the weak interaction;
7. this term is the kinetic energy of the vector bosons;
8. this term represents the triple vertex of the self-interaction of the vector boson;
9. this term represents the quadruple vertex of the self-interaction of the vector boson;
10. this term is the kinetic energy of Higgs boson;
11. this term is the self-interaction of the Higgs boson;
12. the last term is expected to represent the interaction of the vector bosons with the Higgs boson that restore unitarity at high energies.

Let us look into the formal details. The quantities appearing in the Lagrangian are:

- The wave functions $\psi_k = (\nu'_k\ l^-_k)$ for leptons and $(u_k\ d'_k)$ for quarks are the left-handed fermion fields of the k-th fermion generation; every component is a spinor. The index $k = 1, 2, 3$ numbers the generation: the value 1 corresponds to (u d $\nu_e e^-$), the second generation is (c s $\nu_\mu \mu^-$) and the third (t b $\nu_\tau \tau^-$). The ψ_k transform as doublets under SU(2); the right handed fields are SU(2) singlets.

  In the doublets, one has
  $$d'_k = \sum_l V_{kl} d_l\ , \tag{99}$$
  where V_{kl} is the Cabibbo–Kobayashi–Maskawa mixing matrix, d'_k are the quark flavour eigenstates and d_k are the quark mass eigenstates. A similar expression holds for the mixing of the neutrinos:
  $$\nu'_k = \sum_l P_{kl} \nu_l\ , \tag{100}$$
  where P_{kl} is the Pontecorvo–Maki–Nakagawa–Sakata mixing matrix, ν'_l the neutrino flavour eigenstates and ν_l the neutrino mass eigenstates.
- For radiation, A^μ and $F^{\mu\nu}$ is the field of the massless vector boson of the electromagnetic field, the *photon* γ.

  $W^\pm_\mu$ are the massive charged gauge vector bosons of the weak interaction; the corresponding particles, W^+ and W^-, are each other's antiparticles.

  Z_μ is the field of the massive neutral gauge vector boson of the weak interactions; the neutral vector boson itself is usually called Z^0.
- H is the field of the neutral scalar Higgs boson H^0, the only elementary scalar particle in the standard model.
- Two charges appear, one for each interaction. The number q_k is the well-known *elec-*

tric charge of the particle ψ_k in units of the positron charge. The number $t_{3L}(k)$ is the *weak isospin*, or *weak charge*, of fermion k, whose value is $+1/2$ for u_k and ν_k and is $-1/2$ for d_k and l_k. These two charges together define the so-called *vector coupling*

$$g_V^k = t_{3L}(k) - 2q_k \sin^2 \theta_W \qquad (101)$$

and the axial coupling

$$g_A^k = t_{3L}(k) \; . \qquad (102)$$

The combination $g_V^k - g_A^k$, or $V - A$ for short, expresses the maximal violation of P and C parity in the weak interaction.

— The operators T^+ and T^- are the weak isospin raising and lowering operators. Their action on a field is given e.g. by $T^+ l_k^- = \nu_k$ and $T^- u_k = d_k$.

We see that the Lagrangian indeed contains all the ideas developed above. The electroweak Lagrangian is essentially *unique*: it could not have a different mathematical form, because both the electromagnetic terms and the weak terms are fixed by the requirements of Lorentz invariance, U(1) and broken SU(2) gauge invariance, permutation symmetry and renormalizability.

The Lagrangian of the weak interaction has been checked and confirmed by thousands of experiments. Many experiments have been designed specifically to probe it to the highest precision possible. In all these cases, no contradictions between observation and theory has ever been found. Even though the last three terms of the Lagrangian are not fully confirmed, this is – most probably – the exact Lagrangian of the weak interaction. Ref. 186

Curiosities about the weak interaction

The weak interaction, with its breaking of parity and its elusive neutrino, exerts a deep fascination on all those who have explored it. Let us explore this fascination a bit more. Ref. 209

* *

The weak interaction is required to have an excess of matter over antimatter. Without the parity violation of the weak interactions, there would be no matter at all in the universe, because all matter and antimatter that appeared in the big bang would have annihilated. The weak interaction prevents the symmetry between matter and antimatter, which is required to have an excess of one over the other in the universe. In short, the parity violation of the weak interaction is a necessary condition for our own existence.

* *

The weak interaction is also responsible for the heat produced inside the Earth. This heat keeps the magma liquid. As a result, the weak interaction, despite its weakness, is responsible for most earthquakes, tsunamis and volcanic eruptions.

* *

The Lagrangian of the weak interaction was clarified by Steven Weinberg, Sheldon

Glashow and Abdus Salam. They received the 1979 Nobel Prize in physics for their work.

Abdus Salam (b. 1926 Santokdas, d. 1996 Oxford) was a physics genius, the greatest Pakistani scientist by far, an example to many scientists across the world, the first muslim science Nobel-Prize winner, and a deeply spiritual man. In his Nobel banquet speech he explained: 'This, in effect, is the faith of all physicists: the deeper we seek, the more is our wonder excited, the more is the dazzlement for our gaze.' Salam often connected his research to the spiritual aspects of Islam. Once he was asked in Pakistani television why he believed in unification of physics. He answered: 'Because god is one!' When the parliament of Pakistan, in one of the great injustices of the twentieth century, declared Ahmadi Muslims to be non-Muslims and thus effectively started a religious persecution, Salam left Pakistan and never returned. The religious persecution continues to this day: on his tombstone in Pakistan, the word 'muslim' has been hammered away, and the internet is full of offensive comments about him by other muslims, even on Wikipedia. Salam was also an important science manager. With support of UNESCO, Salam founded the International Centre for Theoretical Physics and the Third World Academy of Sciences, both in Trieste, in Italy, and attracted there the best scientists from developing countries.

* *

β decay, due to the weak interaction, separates electrons and protons. Finally, in 2005, people have proposed to use this effect to build long-life batteries that could be used in satellites. Future will tell whether the proposals will be successful.

* *

Ref. 211 Every second around 10^{16} neutrinos fly through our body. They have five sources:

— *Solar neutrinos* arrive on Earth at $6 \cdot 10^{14}$ /m^{2}s, with an energy from 0 to 0.42 MeV; they are due to the p-p reaction in the sun; a tiny is due to the ^{8}B reaction and has energies up to 15 MeV.

— *Atmospheric neutrinos* are products of cosmic rays hitting the atmosphere, consist of 2/3 of muon neutrinos and one third of electron neutrinos, and have energies mainly between 100 MeV and 5 GeV.

Page 184 — *Earth neutrinos* from the radioactivity that keeps the Earth warm form a flux of $6 \cdot 10^{10}$ /m^{2}s.

— *Fossil neutrinos* from the big bang, with a temperature of 1.95 K are found in the universe with a density of 300 cm^{-3}, corresponding to a flux of 10^{15} /m^{2}s.

— *Man-made neutrinos* are produced in nuclear reactors (at 4 MeV) and as neutrino beams in accelerators, using pion and kaon decay. A standard nuclear plant produces $5 \cdot 10^{20}$ neutrinos per second. Neutrino beams are produced, for example, at the CERN in Geneva. They are routinely sent 700 km across the Earth to the Gran Sasso laboratory in central Italy, where they are detected. (In 2011, a famous measurement error led some people to believe, incorrectly, that these neutrinos travelled faster than light.)

Neutrinos are mainly created in the atmosphere by cosmic radiation, but also coming directly from the background radiation and from the centre of the Sun. Nevertheless, during our own life – around 3 thousand million seconds – we have only a 10 % chance that one of these neutrinos interacts with one of the $3 \cdot 10^{27}$ atoms of

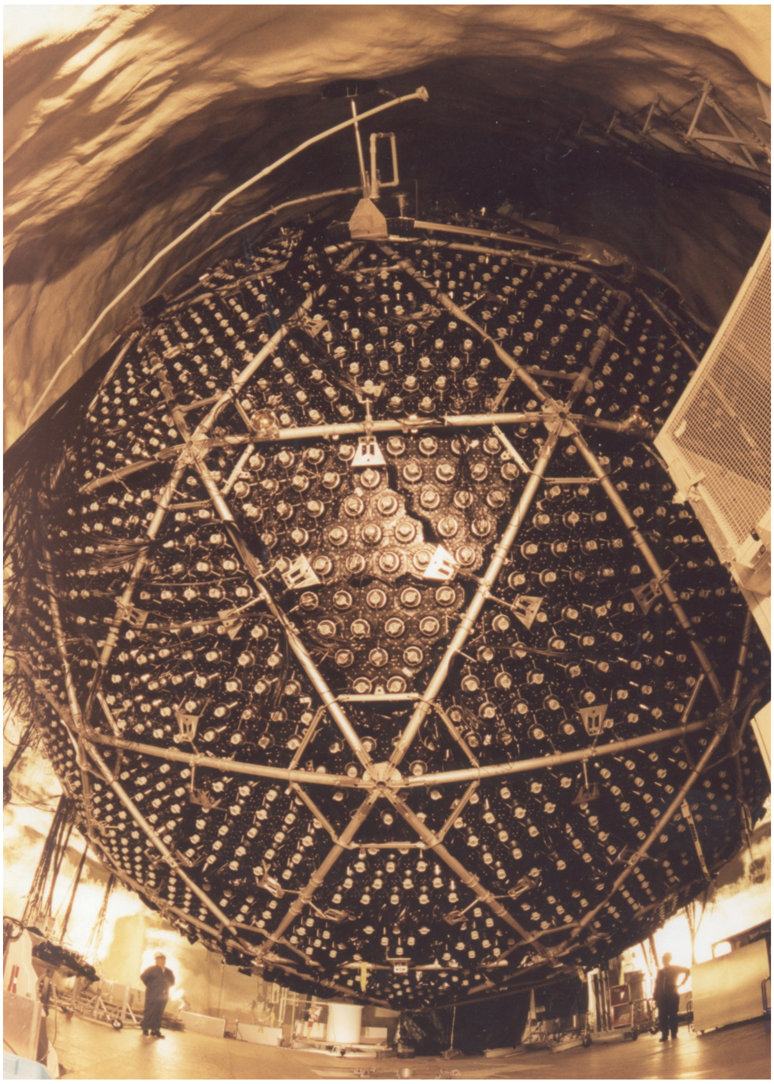

FIGURE 149 A typical underground experiment to observe neutrino oscillations: the Sudbury Neutrino Observatory in Canada (© Sudbury Neutrino Observatory)

our body. The reason is, as usual, that the weak interaction is felt only over distances less than 10^{-17} m, about 1/100th of the diameter of a proton. The weak interaction is indeed weak.

* *

Already in 1957, the great physicist Bruno Pontecorvo imagined that travelling neutrinos could spontaneously change into their own antiparticles. Today, it is known experimentally that travelling neutrinos can change generation, and one speaks of *neutrino oscillations*. Such experiments are carried out in large deep underground caves; the most famous one is shown in Figure 149. In short, experiments show that the weak interaction mixes neutrino types in the same way that it mixes quark types.

* *

Only one type of particles interacts (almost) only weakly: neutrinos. Neutrinos carry no electric charge, no colour charge and almost no gravitational charge (mass). To get an impression of the weakness of the weak interaction, it is usually said that the probability of a neutrino to be absorbed by a lead screen of the thickness of one light-year is less than 50 %. The universe is thus essentially empty for neutrinos. Is there room for bound states of neutrinos circling masses? How large would such a bound state be? Can we imagine bound states, which would be called neutrinium, of neutrinos and antineutrinos circling each other? The answer depends on the mass of the neutrino. Bound states of massless particles do not exist. They could and would decay into two free massless particles.*

Since neutrinos are massive, a neutrino–antineutrino bound state is possible in principle. How large would it be? Does it have excited states? Can they ever be detected? These issues are still open. Challenge 149 ny

The weak interaction is so weak that a neutrino–antineutrino annihilation – which is only possible by producing a massive intermediate Z boson – has never been observed up to this day.

* *

Exploring the mixing of the weak and the electromagnetic interaction led to the prediction of the *Higgs boson*. The fascination of the Higgs boson is underlined by the fact that it is the only fundamental particle that bears the name of a physicist. By the way, the paper by Peter Higgs on the boson named after him is only 79 lines long, and has only five equations. Ref. 210

* *

In the years 1993 and 1994 an intense marketing campaign was carried out across the United States of America by numerous particle physicists. They sought funding for the ‘superconducting supercollider’, a particle accelerator with a circumference of 80 km. This should have been the largest machine ever built, with a planned cost of more than twelve thousand million dollars, aiming at finding the Higgs boson before the Europeans would do so at a fraction of that cost. The central argument brought forward was the following: since the Higgs boson is the basis of particle masses, it was central to US science to know about it first. Apart from the issue of the relevance of the conclusion, the worst is that the premise is wrong.

Page 233 We have seen above that 99 % of the mass of protons, and thus of the universe, is due to quark confinement; this part of mass appears even if the quarks are approximated as massless. The Higgs boson is *not* responsible for the origin of mass itself; it just might shed some light on the issue. Ref. 195 In particular, the Higgs boson does not allow calculating or understanding the mass of any particle. The whole campaign was a classic case of disinformation, and many people involved have shown their lack of honesty.** In the end, the project was stopped, mainly for financial reasons.

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[*] In particular, this is valid for photons bound by gravitation; this state is not possible.

[**] We should not be hypocrites. The supercollider lie is negligible when compared to other lies. The biggest lie in the world is probably the one that states that to ensure its survival, the USA government need to spend more on the military than all other countries in the world combined. This lie is, every single year, around 40 times as big as the once-only supercollider lie. Many other governments devote even larger percentages of

> “Difficile est saturam non scribere.*”
> Juvenal, *Saturae* 1, 30.

* *

There is no generally accepted name for the quantum field theory of the weak interaction. The expression *quantum asthenodynamics* (QAD) – from the Greek word for ‘weak’ – has not been universally adopted.

* *

Do ruminating *cows* move their jaws equally often in clockwise and anticlockwise direction? In 1927, the theoretical physicists Pascual Jordan and Ralph de Laer Kronig published a study showing that in Denmark the two directions are almost equally distributed. Ref. 212 The rumination direction of cows is thus not related to the weak interaction.

* *

Of course, the weak interaction is responsible for radioactive β decay, and thus for part of the radiation background that leads to mutations and thus to biological evolution.

* *

Due to the large toll it placed on society, research in nuclear physics, has almost disappeared from the planet, like poliomyelitis has. Like poliomyelitis, nuclear research is kept alive only in a few highly guarded laboratories around the world, mostly by questionable figures, in order to build dangerous weapons. Only a small number of experiments carried on by a few researchers are able to avoid this involvement and continue to advance the topic.

* *

Interesting aspects of nuclear physics appear when powerful lasers are used. In 1999, a British team led by Ken Ledingham observed laser induced uranium fission in ^{238}U nuclei. In the meantime, this has even be achieved with table-top lasers. The latest feat, in 2003, was the transmutation of ^{129}I to ^{128}I with a laser. This was achieved by focussing Ref. 213 a 360 J laser pulse onto a gold foil; the ensuing plasma accelerates electrons to relativistic speed, which hit the gold and produce high energy γ rays that can be used for the transmutation.

A SUMMARY OF THE WEAK INTERACTION

The weak interaction is described by a non-Abelian gauge theory based on a broken SU(2) gauge group for weak processes. The weak interaction mixes with the unbroken U(1) gauge group of the electrodynamic interaction. This description matches the observed properties of β decay, of particle transformations, of neutrinos and their mixing, of the massive intermediate W and Z bosons, of maximal parity violation, of the heat

---

their gross national product to their own version of this lie. As a result, the defence spending lie is directly responsible for most of the poverty in all the countries that use it.

[*] ‘It is hard not to be satirical.’

production inside the Earth, of several important reactions in the Sun and of the origin of matter in the universe. Even though the weak interaction is weak, it is a bit everywhere.

The weak interaction is described by a Lagrangian. After a century of intense research, the Lagrangian is known in all its details, including, since 2012, the Higgs boson. Theory and experiment agree whenever comparisons have been made.

All remaining limitations of the gauge theory of the weak interaction are only conceptual. Like in all of quantum field theory, also in the case of the weak interaction the mathematical form of the Lagrangian is almost uniquely defined by requiring renormalizability, Lorentz invariance, and (broken) gauge invariance – SU(2) in this case. We say again 'almost', as we did for the case of the strong interaction, because the Lagrangian of the weak and electromagnetic interactions contains a few parameters that remain unexplained:

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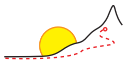
- The two coupling constants g and g' of the weak and the electromagnetic interaction are unexplained. (They define weak mixing angle $\theta_{\text{w}} = \arctan(g'/g)$.)
- The mass $M_Z = 91\,\text{GeV}/c^2$ of the neutral Z boson is unexplained.
- The number $n = 3$ of generations is unexplained.
- The masses of the six leptons and the six quarks are unexplained.
- The four parameters of the Cabibbo–Kobayashi–Maskawa quark mixing matrix and the six parameters of the neutrino mixing matrix are unexplained, including the respective CP violating phases.
- The properties of space-time, in particular its Lorentz invariance, its continuity and the number of its dimensions are assumed from the outset and are obviously all unexplained.
- It is also not known how the weak interaction behaves in strong gravity, thus in strongly curved space-time.

Before exploring how to overcome these limitations, we summarize all results found so far in the so-called *standard model of particle physics*.



CHAPTER 9

THE STANDARD MODEL OF PARTICLE PHYSICS – AS SEEN ON TELEVISION

The expression *standard model of elementary particle physics* stands for the summary of all knowledge about the motion of quantum particles in nature. The standard model can be explained in four tables: the table of the elementary particles, the table of their properties, the table of possible Feynman diagrams and the table of fundamental constants.

The following table lists the known elementary particles found in nature.

TABLE 21 The elementary particles.

Radiation	electromagnetic	weak	strong
	γ	W^+, W^- Z^0	g_1 … g_8
	photon	weak bosons	gluons
	Radiation particles are intermediate vector bosons, thus with spin 1. W^- is the antiparticle of W^+; the photons and the Z^0 are their own antiparticles. Only the $W^\pm$ and Z^0 are massive.		
Matter	generation 1	generation 2	generation 3
Leptons	e ν_e	μ ν_μ	τ ν_τ
Quarks (each in three colours)	d u	s c	t b
	Matter particles are fermions with spin 1/2; all have a corresponding antiparticle. Leptons mix among themselves; so do quarks. All fermions are massive.		
Vacuum state			
Higgs boson	H	Has spin 0 and a mass of 126 GeV.	

The table has not changed much since the mid-1970s, except for the Higgs boson, which has been found in 2012. Assuming that the table is complete, it contains *all* constituents that make up all matter and all radiation in nature. Thus the table lists all constituents – the real 'uncuttables' or 'atoms', as the Greek called them – of material objects and beams of radiation. The elementary particles are the basis for materials science, geology, astronomy, engineering, chemistry, biology, medicine, the neurosciences and psychology. For this reason, the table regularly features in mass tabloids, on television and on the internet.

The full list of elementary particles allows us to put together a full table of particle properties, shown in Table 22. It lists *all* properties of the elementary particles. To save space, colour and weak isospin are not mentioned explicitly. Also the decay modes of the unstable particles are not given in detail; they are found in the standard references.

Ref. 186

The Table 22 on particle properties is fascinating. It allows us to give a *complete* characterization of the intrinsic properties of *any composed object or image.* At the beginning of our study of motion, we were looking for a complete list of the permanent, *intrinsic properties* of moving entities. Now we have it.

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TABLE 22 Elementary particle properties.

PARTICLE	MASS m^a	LIFETIME τ OR ENERGY WIDTH,^{b} MAIN DECAY MODES	ISOSPIN I, SPIN J,^{c} PARITY P, CHARGE PARITY C	CHARGE, ISOSPIN, STRANGENESS,^{c} CHARM, BEAUTY, TOPNESS: $QISCBT$	LEPTON & BARYON NUMBERS LB
Elementary radiation (bosons)					
photon γ	0 $(< 2 \cdot 10^{-54}$ kg)	stable	$I(J^{PC}) = 0, 1(1^{--})$	000000	0, 0
$W^{\pm}$	80.385(15) GeV/c^2	2.085(42) GeV 67.60(27) % hadrons, 32.40(27) % $l^{+}\nu$	$J = 1$	±100000	0, 0
Z	91.1876(21) GeV/c^2	0.265(1) ys = 2.4952(23) GeV/c^2 69.91(6) % hadrons 10.0974(69) % $l^{+}l^{-}$	$J = 1$	000000	0, 0
gluon	0	stable	$I(J^{P}) = 0(1^{-})$	000000	0, 0
Elementary matter (fermions): leptons					
electron e	9.109 382 91(40) · 10^{-31} kg = 81.871 0506(36) pJ/c^2 = 0.510 998 928(11) MeV/c^2 = 0.000 548 579 909 46(22) u gyromagnetic ratio $\mu_e/\mu_B = -1.001\,159\,652\,180\,76(27)$ electric dipole moment^e $d =< 0.87 \cdot 10^{-30} e$ m	$> 13 \cdot 10^{30}$ s	$J = \frac{1}{2}$	−100 000	1, 0

PARTICLE	MASS m^a	LIFETIME τ OR ENERGY WIDTH,^{b} MAIN DECAY MODES	ISOSPIN I, SPIN J,^{c} PARITY P, CHARGE PARITY C	CHARGE, ISOSPIN, STRANGENESS,^{c} CHARM, BEAUTY, TOPNESS: $QISCBT$	LEPTON & BARYON NUMBERS LB
muon μ	0.188 353 109(16) yg	2.196 9811(22) μs 99 % $e^- \bar{\nu}_e \nu_\mu$	$J = \frac{1}{2}$	−100000	1, 0
	$= 105.658\,3715(35)\ \mathrm{MeV}/c^2 \quad = 0.113\,428\,9267(29)\ \mathrm{u}$				
	gyromagnetic ratio $\mu_\mu/(e\hbar/2m_\mu) = -1.001\,165\,9209(6)$				
	electric dipole moment $d = (-0.1 \pm 0.9) \cdot 10^{-21}\,e\,\mathrm{m}$				
tau τ	1.776 82(16) GeV/c^2	290.6(1.0) fs	$J = \frac{1}{2}$	−100000	1, 0
el. neutrino ν_e	$< 2\ \mathrm{eV}/c^2$		$J = \frac{1}{2}$		1, 0
muon neutrino ν_μ	$< 2\ \mathrm{eV}/c^2$		$J = \frac{1}{2}$		1, 0
tau neutrino ν_τ	$< 2\ \mathrm{eV}/c^2$		$J = \frac{1}{2}$		1, 0
Elementary matter (fermions): quarks^f					
up u	1.8 to 3.0 MeV/c^2	see proton	$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$	$+\frac{2}{3}+\frac{1}{2}0000$	$0, \frac{1}{3}$
down d	4.5 to 5.5 MeV/c^2	see proton	$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$	$-\frac{1}{3}-\frac{1}{2}0000$	$0, \frac{1}{3}$
strange s	95(5) MeV/c^2		$I(J^P) = 0(\frac{1}{2}^+)$	$-\frac{1}{3}0{-}1000$	$0, \frac{1}{3}$
charm c	1.275(25) GeV/c^2		$I(J^P) = 0(\frac{1}{2}^+)$	$+\frac{2}{3}00{+}100$	$0, \frac{1}{3}$
bottom b	4.18(17) GeV/c^2	$\tau = 1.33(11)$ ps	$I(J^P) = 0(\frac{1}{2}^+)$	$-\frac{1}{3}000{-}10$	$0, \frac{1}{3}$
top t	173.5(1.4) GeV/c^2		$I(J^P) = 0(\frac{1}{2}^+)$	$+\frac{2}{3}0000{+}1$	$0, \frac{1}{3}$
Elementary boson					
Higgs H	126(1) GeV/c^2	not measured	$J = 0$		

Notes:

a. See also the table of SI prefixes on page 326. About the eV/c^2 mass unit, see page 330.

b. The *energy width* Γ of a particle is related to its lifetime τ by the indeterminacy relation $\Gamma\tau = \hbar$. There is a difference between the *half-life* $t_{1/2}$ and the *lifetime* τ of a particle: they are related by $t_{1/2} = \tau \ln 2$, where $\ln 2 \approx 0.693\,147\,18$; the half-life is thus shorter than the lifetime. The unified *atomic mass unit* u is defined as 1/12 of the mass of a carbon 12 atom at rest and in its ground state. One has $1\ \mathrm{u} = \frac{1}{12} m(^{12}\mathrm{C}) = 1.660\,5402(10)$ yg.

c. To keep the table short, the header does not explicitly mention *colour*, the charge of the strong interactions. This has to be added to the list of basic object properties. Quantum numbers containing the word 'parity' are multiplicative; all others are additive. Time parity T (not to be confused with topness T), better called motion inversion parity, is equal to CP. The isospin I (or I_Z) is defined only for up and down quarks and their composites, such as the proton and the neutron. In the literature one also sees references to the so-called G-parity, defined as $G = (-1)^{IC}$.

The table also does not mention the *weak charge* of the particles. The details on weak charge g, or, more precisely, on the *weak isospin*, a quantum number assigned to all left-handed fermions (and right-handed anti-fermions), but to no right-handed fermion (and no left-handed antifermion), are given in the section on the weak interactions. Page 245

d. 'Beauty' is now commonly called *bottomness*; similarly, 'truth' is now commonly called *topness*. The signs of the quantum numbers S, I, C, B, T can be defined in different ways. In the standard assignment shown here, the sign of each of the non-vanishing quantum numbers is given by the sign of the charge of the corresponding quark.

Ref. 214, Ref. 215 *e*. The electron radius is observed to be less than 10^{-22} m. It is possible to store single electrons in traps for many months.

f. See page 233 for the precise definition and meaning of the quark masses.

The other aim that we formulated at the beginning of our adventure was to find the complete list of all *state properties*. This aim is also achieved, namely by the wave function and the field values due to the various bosons. Were it not for the possibility of space-time curvature, we would be at the end of our exploration.

The main ingredient of the standard model are the Lagrangians of the electromagnetic, the weak and the strong interactions. The combination of the Lagrangians, based on the U(1), SU(3) and broken SU(2) gauge groups, is possible only in one specific way. The Lagrangian can be summarized by the Feynman diagram of Figure 150.

To complete the standard model, we need the coupling constants of the three gauge interactions, the masses of all the particles, and the values of the mixing among quarks and among leptons. Together with all those constants of nature that define the SI system and the number of space-time dimensions, the following table therefore completes the standard model.

TABLE 23 Basic physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.[a]
Constants that define the SI measurement units			
Vacuum speed of light[c]	c	299 792 458 m/s	0
Vacuum permeability[c]	μ_0	$4\pi \cdot 10^{-7}$ H/m	0
		= 1.256 637 061 435 ... µH/m	0
Vacuum permittivity[c]	$\varepsilon_0 = 1/\mu_0 c^2$	8.854 187 817 620 ... pF/m	0
Original Planck constant	h	$6.626\,069\,57(52) \cdot 10^{-34}$ Js	$4.4 \cdot 10^{-8}$
Reduced Planck constant, quantum of action	$\hbar$	$1.054\,571\,726(47) \cdot 10^{-34}$ Js	$4.4 \cdot 10^{-8}$
Positron charge	e	0.160 217 656 5(35) aC	$2.2 \cdot 10^{-8}$
Boltzmann constant	k	$1.380\,6488(13) \cdot 10^{-23}$ J/K	$9.1 \cdot 10^{-7}$
Gravitational constant	G	$6.673\,84(80) \cdot 10^{-11}$ Nm^2/kg^2	$1.2 \cdot 10^{-4}$
Gravitational coupling constant	$\kappa = 8\pi G/c^4$	$2.076\,50(25) \cdot 10^{-43}$ s^2/kg m	$1.2 \cdot 10^{-4}$
Fundamental constants (of unknown origin)			
Number of space-time dimensions		3 + 1	0[b]
Fine-structure constant[d] or	$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$	1/137.035 999 074(44)	$3.2 \cdot 10^{-10}$
e.m. coupling constant	$= g_{\text{em}}(m_e^2 c^2)$	= 0.007 297 352 5698(24)	$3.2 \cdot 10^{-10}$

TABLE 23 (Continued) Basic physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.[a]
Fermi coupling constant[d] or	$G_F/(\hbar c)^3$	$1.166\,364(5)\cdot 10^{-5}\,\mathrm{GeV}^{-2}$	$4.3\cdot 10^{-6}$
weak coupling constant	$\alpha_w(M_Z)=g_w^2/4\pi$	$1/30.1(3)$	$1\cdot 10^{-2}$
Weak mixing angle	$\sin^2\theta_W(\overline{MS})$	$0.231\,24(24)$	$1.0\cdot 10^{-3}$
	$\sin^2\theta_W$ (on shell) $=1-(m_W/m_Z)^2$	$0.2224(19)$	$8.7\cdot 10^{-3}$
Strong coupling constant[d]	$\alpha_s(M_Z)=g_s^2/4\pi$	$0.118(3)$	$25\cdot 10^{-3}$
CKM quark mixing matrix	$\lvert V\rvert$	$\begin{pmatrix} 0.97428(15) & 0.2253(7) & 0.00347(16) \\ 0.2252(7) & 0.97345(16) & 0.0410(11) \\ 0.00862(26) & 0.0403(11) & 0.999152(45) \end{pmatrix}$	
Jarlskog invariant	J	$2.96(20)\cdot 10^{-5}$	
PMNS neutrino mixing m.	P	$\begin{pmatrix} 0.82 & 0.55 & -0.15+0.038i \\ -0.36+0.020i & 0.70+0.013i & 0.61 \\ 0.44+0.026i & -0.45+0.017i & 0.77 \end{pmatrix}$	
Particle masses: see previous table			

a. Uncertainty: standard deviation of measurement errors.

b. Only measured from to 10^{-19} m to 10^{26} m.

c. Defining constant.

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d. All coupling constants depend on the 4-momentum transfer, as explained in the section on renormalization. *Fine-structure constant* is the traditional name for the electromagnetic coupling constant g_{em} in the case of a 4-momentum transfer of $Q^2=m_e^2c^2$, which is the smallest one possible. At higher momentum transfers it has larger values, e.g., $g_{\mathrm{em}}(Q^2=M_W^2c^2)\approx 1/128$. In contrast, the strong coupling constant has lower values at higher momentum transfers; e.g., $\alpha_s(34\,\mathrm{GeV})=0.14(2)$.

In short, with the three tables and the figure, the standard model describes *every* observation ever made in flat space-time. In particular, the standard model includes a minimum action, a maximum speed, electric charge quantization and the least action principle.

SUMMARY AND OPEN QUESTIONS

The standard model of particle physics clearly distinguishes *elementary* from *composed* particles. The standard model provides the full list of properties that characterizes a particle – and thus any moving object and image. These properties are: mass, spin, charge, colour, weak isospin, parity, charge parity, isospin, strangeness, charm, topness, beauty, lepton number and baryon number.

The standard model describes electromagnetic and nuclear interactions as as exchanges of virtual radiation particles. In particular, the standard model describes the three types of radiation that are observed in nature with full precision, using gauge groups. The standard model is based on quantization and conservation of electric charge, weak charge and colour, as well as on a smallest action value $\hbar$ and a maximum energy speed c. As a result, the standard model describes the structure of the atoms, their form-

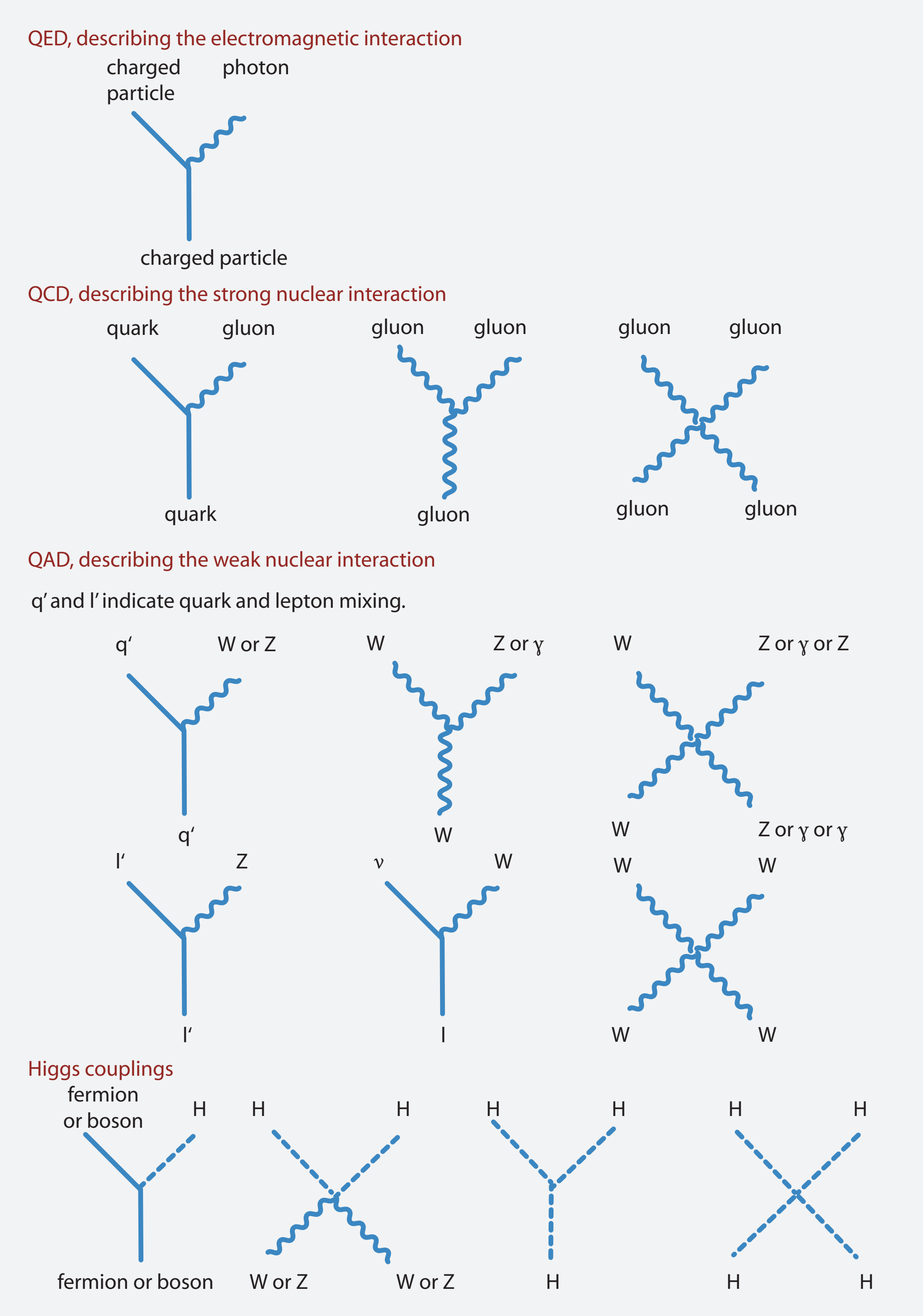

FIGURE 150 The Feynman diagrams of the standard model.

ation in the history of the universe, the properties of matter and the mechanisms of life. Despite the prospect of fame and riches, not one deviation between experiment and the standard model has been found.

In short, the standard model realizes the dream of Leucippus and Democritus, plus a bit more: we know the bricks that compose all of matter and radiation, and in addition we know how they move, interact and transform, in flat space-time, with *perfect* accuracy.

Despite this perfect accuracy, we also know what we still do *not* know:

- We do not know the origin of the coupling constants.
- We do not know why positrons and protons have the same charge.
- We do not know the origin of the masses of the particles.
- We do not know the origin of the mixing and CP violation parameters.
- We do not know the origin of the gauge groups.
- We do not know the origin of the three particle generations.
- We do not know whether the particle concept survives at high energy.
- We do not know what happens in curved space-time.

To study these issues, the simplest way is to explore nature at particle energies that are as high as possible. There are two methods: building large experiments and exploring hypothetical models. Both are useful.

CHAPTER 10

DREAMS OF UNIFICATION

> “Materie ist geronnenes Licht.[**]
> Albertus Magnus”

Is there a common origin to the three particle interactions? We have seen in the preceding chapters that the Lagrangian densities of the three gauge interactions are determined almost uniquely by two types of requirements: to possess a certain gauge symmetry, and to be consistent with space-time, through Lorentz invariance and renormalizability. The search for *unification* of the interactions thus seems to require the identification of the one, unified symmetry of nature. (Do you agree?) Challenge 150 s

Between 1970 and 2015, several conjectures have fuelled the hope to achieve unification through higher symmetry. The most popular were grand unification, supersymmetry, conformal field theory, coupling constant duality and mathematical quantum field theory. We give a short summary of these efforts; we start with the first candidate, which is conceptually the simplest.

GRAND UNIFICATION

At all measured energies up to the year 2016, thus below about 3 TeV, there are no contradictions between the Lagrangian of the standard model and observation. On the other hand, the Lagrangian itself can be conjectured to be a low energy approximation to the unified theory. It should thus be possible – attention, this a belief – to find a unifying symmetry that *contains* the symmetries of the electroweak and strong interactions as subgroups. In this way, the three gauge interactions would be different aspects of a single, ‘unified’ interaction. We can then examine the physical properties that follow from this unifying symmetry and compare them with observation. This approach, called *grand unification*, attempts the unified description of all types of matter. All known elementary particles are seen as fields which appear in a Lagrangian determined by a single gauge symmetry group.

Like for each gauge theory described so far, also the grand unified Lagrangian is fixed by the symmetry group, the representation assignments for each particle, and the corresponding coupling constant. A general search for the grand symmetry group starts with all those (semisimple) Lie groups which contain $U(1)\times SU(2)\times SU(3)$. The smallest groups with these properties are SU(5), SO(10) and E(6); they are defined in Appendix C. Ref. 216

[**] ‘Matter is coagulated light.’ Albertus Magnus (b. *c.* 1192 Lauingen, d. 1280 Cologne) was the most important thinker of his time.

For each of these candidate groups, the predicted consequences of the model can be studied and compared with experiment. Ref. 217

COMPARING PREDICTIONS AND DATA

Grand unification models – also incorrectly called GUTs or *grand unified theories* – make several predictions that can be matched with experiment. First of all, any grand unified model predicts relations between the quantum numbers of quarks and those of leptons. In particular, grand unification explains why the electron charge is exactly the opposite of the proton charge.

Grand unification models predict a value for the weak mixing angle θ_w; this angle is not fixed by the standard model. The most frequently predicted value, Ref. 216

$$\sin^2 \theta_{w,th} = 0.2 \qquad (103)$$

is close to the measured value of

$$\sin^2 \theta_{w,ex} = 0.231(1) , \qquad (104)$$

which is not a good match, but might be correct, in view of the approximations in the prediction.

All grand unified models predict the existence of *magnetic monopoles*, as was shown by Gerard 't Hooft. However, despite extensive searches, no such particles have been found yet. Monopoles are important even if there is only one of them in the whole universe: the existence of a single monopole would imply that electric charge is quantized. If monopoles were found, grand unification would explain why electric charge appears in multiples of a smallest unit. Ref. 218 Ref. 219

Grand unification predicts the existence of heavy intermediate vector bosons, called *X bosons*. Interactions involving these bosons do not conserve baryon or lepton number, but only the difference $B - L$ between baryon and lepton number. To be consistent with data, the X bosons must have a mass of the order of 10^{16} GeV. However, this mass is and always will be outside the range of experiments, so that the prediction cannot be tested directly.

Most spectacularly, the X bosons of grand unification imply that the *proton decays*. This prediction was first made by Pati and Abdus Salam in 1974. If protons decay, means that neither coal nor diamond* – nor any other material – would be for ever. Depending on the precise symmetry group, grand unification predicts that protons decay into pions, electrons, kaons or other particles. Obviously, we know 'in our bones' that the proton lifetime is rather high, otherwise we would die of leukaemia; in other words, the low level of cancer in the world already implies that the lifetime of the proton is larger than about 10^{16} years.

Detailed calculations for the proton lifetime τ_p using the gauge group SU(5) yield the

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* As is well known, diamond is not stable, but metastable; thus diamonds are not for ever, but coal might be, as long as protons do not decay.

Ref. 216 expression

$$\tau_{\mathrm{p}} \approx \frac{1}{\alpha_G^2(M_{\mathrm{X}})} \frac{M_{\mathrm{X}}^4}{M_{\mathrm{p}}^5} \approx 10^{31\pm1}\,\mathrm{a} \tag{105}$$

where the uncertainty is due to the uncertainty of the mass M_{X} of the gauge bosons involved and to the exact decay mechanism. Several large experiments have tried and are still trying to measure this lifetime. So far, the result is simple but clear. Not a single proton decay has ever been observed. The data can be summarized by Ref. 220

$$\begin{aligned} \tau(\mathrm{p} \rightarrow e^+\,\pi^0) &> 5 \cdot 10^{33}\,\mathrm{a} \\ \tau(\mathrm{p} \rightarrow K^+\,\bar{\nu}) &> 1.6 \cdot 10^{33}\,\mathrm{a} \\ \tau(\mathrm{n} \rightarrow e^+\,\pi^-) &> 5 \cdot 10^{33}\,\mathrm{a} \\ \tau(\mathrm{n} \rightarrow K^0\,\bar{\nu}) &> 1.7 \cdot 10^{32}\,\mathrm{a} \end{aligned} \tag{106}$$

These values are higher than the prediction by SU(5) – and SO(10) – models. For other gauge group candidates proton decay measurements require more time.

The state of grand unification

To settle the issue of grand unification definitively, one last prediction of grand unification remains to be checked: the unification of the coupling constants. Most estimates of the grand unification energy are near the Planck energy, the energy at which gravitation starts to play a role even between elementary particles. As grand unification does not take gravity into account, for a long time there was a doubt whether something was lacking in the approach. This doubt changed into certainty when the precision measurements of the coupling constants became available, in 1991, and were put into the diagram of Figure 151. The GUT prediction of the way the constants evolve with energy implies that the three constants do *not* meet at one energy. Simple grand unification by SU(5), SU(10) or E_6 is thus ruled out by experiment. Ref. 221

This state of affairs is changed if *supersymmetry* is taken into account. Supersymmetry is a conjecture on the way to take into account low-energy effects of gravitation in the particle world. Supersymmetry conjectures new elementary particles that change the curves at intermediate energies, so that they all meet at a grand unification energy of about 10^{16} GeV. (The line thicknesses in Figure 151 represent the experimental errors.) The inclusion of supersymmetry also puts the proton lifetime prediction back to a value higher (but not by much) than the present experimental bound and predicts the correct value of the mixing angle. With supersymmetry, one can thus retain the advantages of grand unification (charge quantization, one coupling constant) without being in contradiction with experiments. Page 271

In summary, pure grand unification is in contradiction with experiments. This is not a surprise, as its goal, to unify the description of matter, *cannot* been achieved in this way. Indeed, the unifying gauge group must be introduced, i.e., added, at the very beginning. Adding the group is necessary because grand unification cannot deduce the gauge group from a general principle. Neither does pure grand unification tell us completely

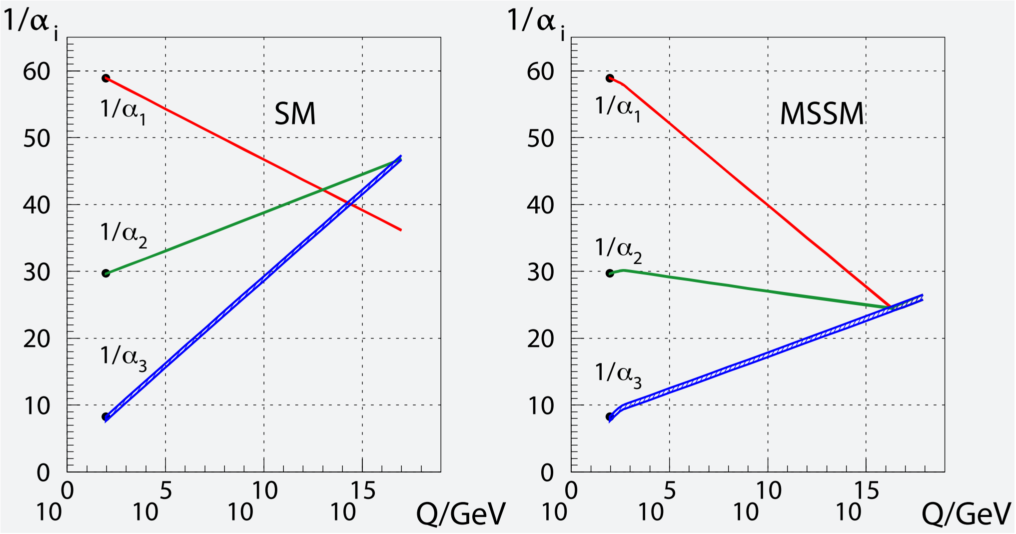

FIGURE 151 The behaviour of the three coupling constants with energy, for simple grand unification (left) and for the minimal supersymmetric model (right); the graph shows the constants $\alpha_1 = \frac{5}{3}\alpha_{\mathrm{em}}/\cos^2\theta_{\mathrm{W}}$ for the electromagnetic interaction (the factor 5/3 appears in GUTs), $\alpha_2 = \alpha_{\mathrm{em}}/\sin^2\theta_{\mathrm{W}}$ for the weak interaction and the strong coupling constant $\alpha_3 = \alpha_s$ (© W. de Boer).

which elementary particles exist in nature. In other words, grand unification only *shifts* the open questions of the standard model to the next level, while keeping most of the open questions unanswered. The name 'grand unification' was wrong from the beginning. We definitively need to continue our adventure.

SEARCHING FOR HIGHER SYMMETRIES

Since we want to reach the top of Motion Mountain, we go on. We have seen in the preceding sections that the main ingredients of the Lagrangian that describes motion are the symmetry properties. We recall that a Lagrangian is just the mathematical name for the concept that measures change. The discovery of the correct symmetry, together with mathematical consistency, usually restricts the possible choices for a Lagrangian down to a limited number, and to one in the best case. This Lagrangian then allows making experimental predictions.

The history of particle physics from 1920 to 1965 has shown that progress was always coupled to the discovery of *larger* symmetries, in the sense that the newly discovered symmetries always included the old ones as a subgroup. Therefore, in the twentieth century, researchers searched for the largest possible symmetry that is consistent with experiments on the one hand and with gauge theories on the other hand. Since grand unification failed, a better approach is to search directly for a symmetry that includes gravity.

SUPERSYMMETRY

In the search for possible symmetries of a Lagrangian describing a gauge theory and gravity, one way to proceed is to find general mathematical theorems which restrict the symmetries that a Lagrangian can possibly have. Ref. 222

Ref. 223 A well-known theorem by Coleman and Mandula states that if the symmetry transformations transform fermions into fermions and bosons into bosons, no quantities other than the following can be conserved:

- the energy momentum tensor $T^{\mu\nu}$, a consequence of the *external* Poincaré space-time symmetry, and
- the internal quantum numbers, all scalars, associated with each gauge group generator – such as electric charge, colour, etc. – and consequences of the *internal* symmetries of the Lagrangian.

But, and here comes a way out, if transformations that *mix* fermions and bosons are considered, new conserved quantities become possible. This family of symmetries includes gravity and came to be known as *supersymmetry*. Its conserved quantities are not scalars but *spinors*. Therefore, *classical* supersymmetry does not exist; it is a purely quantum-mechanical symmetry. The study of supersymmetry has been a vast research field. For example, supersymmetry generalizes gauge theory to super-gauge theory. The possible super-gauge groups have been completely classified.

Supersymmetry can be extended to incorporate gravitation by changing it into a local gauge theory; in that case one speaks of *supergravity*. Supergravity is based on the idea that coordinates can be fermionic as well as bosonic. Supergravity thus makes specific statements on the behaviour of space-time at small distances. Supergravity predicts N additional conserved, spinorial charges. The number N lies between 1 and 8; each value leads to a different candidate Lagrangian. The simplest case is called $N = 1$ supergravity.

In short, supersymmetry is a conjecture to unify matter and radiation at low energies. Many researchers conjectured that supersymmetry, and in particular $N = 1$ supergravity, might be an approximation to reality.

Supersymmetric models have around 100 fundamental constants, in comparison to around 25 for the standard model of particle physics. The precise experimental predictions depend on the values of these constants. Nevertheless, a number of general predictions are possible and can be tested by experiment.

- Supersymmetry predicts partners to the usual elementary particles, called *sparticles*. Fermions are predicted to have boson partners, and vice versa. For example, supersymmetry predicts a *photino* as fermionic partner of the photon, *gluinos* as partner of the gluons, a *selectron* as partner of the electron, etc. However, none of these particles have been observed yet.
- Supersymmetry allows for the unification of the coupling constants in a way compatible with the data, as shown already above. Page 270
- Supersymmetry slows down the proton decay rates predicted by grand unified theories. The slowed-down rates are compatible with observation.
- Supersymmetry predicts electric dipole moments for the neutron and other elementary particles. The largest predicted values for the neutron, $10^{-30}e\,\mathrm{m}$, are in contradiction with observations; the smallest predictions have not yet been reached by experiment. In comparison, the values expected from the standard model are at most $10^{-33}e\,\mathrm{m}$. This is a vibrant experimental research field that can save tax payers from financing an additional large particle accelerator.

However, up to the time of this writing, the year 2016, there was *no* experimental evidence for supersymmetry. In particular, the Large Hadron Collider at CERN in Geneva has not found any hint of supersymmetry. In fact, experiments excluded almost all supersymmetric particle models proposed in the past.

Is supersymmetry an ingredient of the unified theory? The safe answer is: this is unclear. The optimistic answer is: there is still a small chance that supersymmetry can hold in nature. The pessimistic answer is: supersymmetry is a belief system contradicting observations and made up to correct the failings of grand unified theories. The last volume of this adventure will tell which answer is correct.

OTHER ATTEMPTS

If supersymmetry is not successful, it might be that even higher symmetries are required for unification. Therefore, researchers have explored *quantum groups*, *non-commutative space-time*, *conformal symmetry*, *topological quantum field theory*, and other abstract symmetries. None of these approaches led to useful results; neither experimental predictions nor progress towards unification. But two further approaches deserve special mention: duality symmetries and extensions to renormalization.

DUALITIES – THE MOST INCREDIBLE SYMMETRIES OF NATURE

An important discovery of mathematical physics took place in 1977, when Claus Montonen and David Olive proved that the standard concept of symmetry could be expanded dramatically in a different and new way.

The standard class of symmetry transformations, which turns out to be only the first class, acts on fields. This class encompasses gauge symmetries, space-time symmetries, motion reversal, parities, flavour symmetries and supersymmetry.

The second, new class is quite different. If we take the coupling constants of nature, we can imagine that they are members of a continuous space of all possible coupling constants, called the *parameter space*.* Montonen and Olive showed that there are transformations in parameter space that leave nature invariant. These transformations thus form a new, second class of symmetries of nature.

In fact, we already encountered a member of this class: renormalization symmetry. But Olive and Montonen expanded the symmetry class considerably by the discovery of *electromagnetic duality*. Electromagnetic duality is a discrete symmetry exchanging

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$$e \leftrightarrow \frac{4\pi\hbar c}{e} \tag{107}$$

where the right hand side turns out to be the unit of magnetic charge. Electro-magnetic duality thus relates the electric charge e and the magnetic charge m

$$Q_{\mathrm{el}} = me \quad \text{and} \quad Q_{\mathrm{mag}} = ng = 2\pi\hbar c/e \tag{108}$$

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* The space of solutions for all value of the parameters is called the *moduli space*.

and puts them on equal footing. In other words, the transformation exchanges

$$\alpha \leftrightarrow \frac{1}{\alpha} \quad \text{or} \quad \frac{1}{137.04} \leftrightarrow 137.04\,, \tag{109}$$

and thus exchanges weak and strong coupling. In other words, electromagnetic duality relates a regime where particles make sense (the low coupling regime) with one where particles do not make sense (the strong coupling regime). It is the most mind-boggling symmetry ever conceived.

Dualities are among the deepest connections of physics. They contain $\hbar$ and are thus intrinsically quantum. They do not exist in classical physics and thus confirm that quantum theory is more fundamental than classical physics. More clearly stated, dualities are intrinsically non-classical symmetries. Dualities confirm that quantum theory stands on its own.

If one wants to understand the values of unexplained parameters such as coupling constants, an obvious thing to do is to study *all* possible symmetries in parameter space, thus all possible symmetries of the second class, possibly combining them with those of the first symmetry class. Indeed, the combination of duality with supersymmetry is studied in superstring theory. Vol. VI, page 141

These investigations showed that there are several types of dualities, which all are *non-perturbative* symmetries: Ref. 224

— S duality, the generalization of electromagnetic duality for all interactions;
— T duality, also called space-time duality, a mapping between small and large lengths and times following $l \leftrightarrow l_{\rm Pl}^2/l$;*
— infrared dualities.

Despite the fascination of the idea, research into dualities has not led to any experimental prediction. However, the results highlighted a different way to approach quantum field theory. Dualities play an important role in superstring theory, which we will explore later on. Vol. VI, page 141

Collective aspects of quantum field theory

For many decades, mathematicians asked physicists: What is the essence of quantum field theory? Despite intensive research, this question has yet to be answered precisely.

Half of the answer is given by the usual definition found in physics textbooks: QFT is the most general known way to describe quantum mechanically continuous systems with a *finite* number of types of quanta but with an *infinite* number of degrees of freedom. For example, this definition implies that the Lagrangian must be relativistically invariant and must be described by a gauge theory. However, this half of the answer is already sufficient to spell trouble. We will show in the next part of our ascent that space and time have a minimal distance scale and that nature does not have infinite numbers of degrees of freedom. In other words, *quantum field theory is an effective theory*; this is the modern Vol. VI, page 40

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* Space-time duality, the transformation between large and small sizes, leads one to ask whether there is an inside and an outside to particles. We encountered this question already in our study of gloves. We will encounter the issue again below, when we explore eversion and inversion. The issue will be fully clarified only in the last volume of our adventure. Vol. IV, page 112 Page 292

way to say that it is approximate, or more bluntly, that it is wrong. But let us put these issues aside for the time being.

The second, still partly unknown half of the answer would specify which (mathematical) conditions a physical system, i.e., a Lagrangian, actually needs to realize in order to become a quantum field theory. Despite the work of many mathematicians, no complete list of conditions is known yet. But it is known that the list includes at least two conditions. First of all, a quantum field theory must be *renormalizable*. Secondly, a quantum field theory must be *asymptotically free*; in other words, the coupling must go to zero when the energy goes to infinity. This condition ensures that interactions are defined properly. Only a subset of renormalizable Lagrangians obey this condition.

In four dimensions, the *only* known quantum field theories with these two properties are the non-Abelian gauge theories. These Lagrangians have several general aspects which are not directly evident when we arrive at them through the usual way, i.e., by generalizing naive wave quantum mechanics. This standard approach, the historical one, emphasizes the *perturbative* aspects: we think of elementary fermions as field quanta and of interactions as exchanges of virtual bosons, to various orders of perturbation.

On the other hand, all field theory Lagrangians also show two other configurations, apart from particles, which play an important role. These mathematical solutions appear when a non-perturbative point of view is taken; they are *collective* configurations.

— Quantum field theories show solutions which are static and of finite energy, created by non-local field combinations, called *solitons*. In quantum field theories, solitons are usually magnetic monopoles and dyons; also the famous *skyrmions* are solitons. In this approach to quantum field theory, it is assumed that the actual equations of nature are non-linear at high energy. Like in liquids, one then expects stable, localized and propagating solutions, the solitons. These solitons could be related to the observed particles. Vol. I, page 316
— Quantum field theories show self-dual or anti-self dual solutions, called *instantons*. Instantons play a role in QCD, and could also play a role in the fundamental Lagrangian of nature.
— Quantum field theory defines particles and interactions using perturbation expansions. Do particles exist non-perturbatively? When does the perturbation expansion break down? What happens in this case? Despite these pressing issues, no answer has ever been found.

Ref. 225 All these fascinating topics have been explored in great detail by mathematical physicists. This research has deepened the understanding of gauge theories. However, none of the available results has yet helped to approach unification.

Curiosities about unification

From the 1970s onwards, it became popular to draw graphs such as the one of Figure 152. They are found in many books. This approach towards the final theory of motion was inspired by the experimental success of electroweak unification and to the success among theoreticians of the idea of grand unification.

Unfortunately, grand unification contradicts experiment. In fact, as explained above, Page 252 not even the electromagnetic and the weak interactions have been unified. (It took about a decade to brainwash people into believing the contrary; this was achieved by intro-

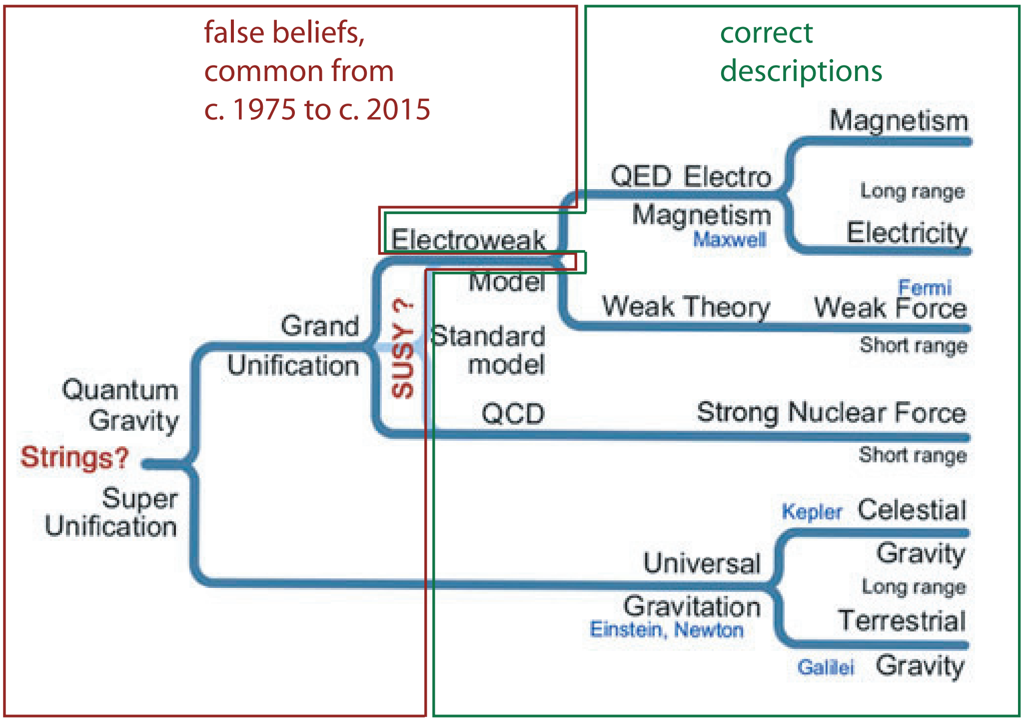

FIGURE 152 Blue and black: a typical graph on unification, as found for years in publications on the topic (© CERN). Red and green: its correct and incorrect parts.

ducing the incorrect term ‘electroweak unification’ instead of a correct term akin to ‘electroweak compatibility’ or ‘electroweak mixing’.) In other words, the larger part of Figure 152 is not correct. Unfortunately, the graph and the story behind it has led most researchers along the wrong path for several decades.

* *

Over the years, the length of this chapter became shorter and shorter. This was due to the large number of unification attempts that were found to be in contradiction with experiment. It is hard to describe the vast amount of effort that has been invested, usually in vain, in the quest for unification of the description of motion.

* *

For a professional and up-to-date introduction into modern particle research, see the summer student lectures that are given at CERN every year. They can be found at cdsweb.cern.ch/collection/SummerStudentLectures?ln=en.

A SUMMARY ON UNIFICATION, MATHEMATICS AND HIGHER SYMMETRIES

The decades of theoretical research since the 1970s have shown:

▷ Mathematical physics is *not* the way to search for unification of the description of motion.

All the searches for unification that were guided by *mathematical ideas* – by mathematical theorems or by mathematical generalizations – have failed. Mathematics is not helpful in this quest. The standard model of particle physics and general relativity remain separate. In addition, the research effort has led to a much more concrete result:

▷ The search for a *higher symmetry* in nature has failed.

Despite thousands of extremely smart people exploring many possible higher symmetries of nature, their efforts have not been successful.

Symmetry considerations are not helpful in the search for unification. Symmetry *simplifies* the description of nature; but symmetry does not *specify* the description. It seems that researchers have fallen into the trap of music theory. Anybody who has learned to play an instrument has heard the statement that 'mathematics is at the basis of music'. Of course, this is nonsense; emotions are at the basis of music. But the incorrect statement about mathematics lurks in the head of every musician. Looking back to research in the twentieth century, it seems that the same has happened to researchers in the field of unification. From these failures we conclude:

▷ Unification requires to extract an underlying *physical* principle.

On the other hand, in the twentieth century, researchers have failed to find such a principle. This failure leads to two questions. In the quest for unification, is there really an alternative to the search for higher symmetry? And did researchers rely on implicit, incorrect assumptions about the structure of particles or of space-time? Before we explore these fascinating issues, we take a break to inspire us.

CHAPTER 11

BACTERIA, FLIES AND KNOTS

> “La première et la plus belle qualité de la nature est le mouvement qui l’agite sans cesse ; mais ce mouvement n’est qu’une suite perpétuelle de crimes ; ce n’est que par des crimes qu’elle le conserve.”
> Donatien de Sade, *Justine, ou les malheurs de la vertu.*[**]

Wobbly entities, in particular jellyfish or amoebas, open up a fresh vision of the world of motion, if we allow being led by the curiosity to study them in detail. We have missed many delightful insights by leaving them aside up to now. In fact, wobbly entities yield surprising connections between shape change and motion that will be of great use in the last part of our mountain ascent. Instead of continuing to look at the smaller and smaller, we now take a second look at everyday motion and its mathematical description.

To enjoy this chapter, we change a dear habit. So far, we always described any general example of motion as composed of the motion of *point particles*. This worked well in classical physics, in general relativity and in quantum theory; we based the approach on the silent assumption that during motion, each point of a complex system can be followed separately. We will soon discover that this assumption is *not* realized at smallest scales. Therefore the most useful description of motion of *extended* bodies uses methods that do not require that body parts be followed piece by piece. We explore these methods in this chapter; doing so is a lot of fun in its own right.

Vol. VI, page 117

If we describe elementary particles as extended entities – as we soon will have to – a particle moving through space is similar to a dolphin swimming through water, or to a bee flying through air, or to a vortex advancing in a liquid. Therefore we explore how this happens.

BUMBLEBEES AND OTHER MINIATURE FLYING SYSTEMS

If a butterfly passes by during our mountain ascent, we can stop a moment to appreciate a simple fact: a butterfly flies, and it is rather small. If we leave some cut fruit in the kitchen until it rots, we observe the even smaller fruit flies (*Drosophila melanogaster*), just about

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[**] ‘The primary and most beautiful of nature’s qualities is motion, which agitates her at all times; but this motion is simply a perpetual consequence of crimes; she conserves it by means of crimes only.’ Donatien Alphonse François de Sade (b. 1740 Paris, d. 1814 Charenton-Saint-Maurice) is the intense writer from whom the term ‘sadism’ was deduced.

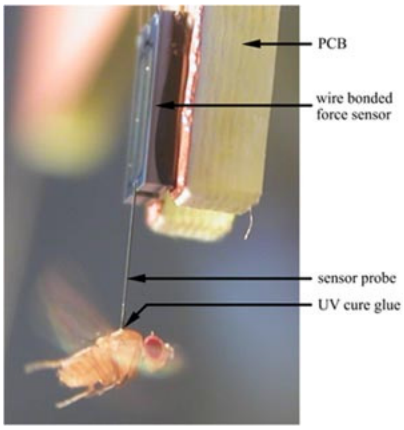

FIGURE 153 A flying fruit fly, tethered to a force-measuring microelectromechanical system (© Bradley Nelson).

FIGURE 154 Vortices around a butterfly wing (© Robert Srygley/Adrian Thomas).

two millimetres in size. Figure 153 shows a fruit fly in flight. If you have ever tried to build small model aeroplanes, or if you even only compare these insects to paper aeroplanes – possibly the smallest man-made flying thing you might have seen – you start to get a feeling for how well evolution has optimized flying insects.

Compared to paper planes, insects also have engines, flapping wings, sensors, navigation systems, gyroscopic stabilizers, landing gear and of course all the features due to life, reproduction and metabolism, built into an incredibly small volume. Evolution really is an excellent engineering team. The most incredible flyers, such as the common house fly (*Musca domestica*), can change flying direction in only 30 ms, using the stabilizers that nature has given them by reshaping the original second pair of wings. Human engineers are getting more and more interested in the technical solutions evolution has developed; Ref. 226 many engineers are trying to achieve similar miniaturization. The topic of miniature flying systems is extremely vast, so that we will pick out only a few examples.

How does a bumblebee (Bombus terrestris) fly? The lift mg generated by a *fixed* wing Vol. I, page 363 (as explained before) follows the empirical relation

$$mg = f\,A\,v^2\,\rho \qquad (110)$$

where A is the surface of the wing, v is the speed of the wing in the fluid of density ρ. The factor f is a pure number, usually with a value between 0.2 and 0.4, that depends Ref. 227 on the angle of the wing and its shape; here we use the average value 0.3. For a Boeing Vol. I, page 38 747, the surface is 511 m^2, the top speed at sea level is 250 m/s; at an altitude of 12 km the density of air is only a quarter of that on the ground, thus only 0.31 kg/m^3. We deduce (correctly) that a Boeing 747 has a mass of about 300 ton. For bumblebees with a speed of 3 m/s and a wing surface of 1 cm^2, we get a lifted mass of about 35 mg, far less than the weight of the bee, namely about 1 g. The mismatch is even larger for fruit flies. In other words, an insect cannot fly if it keeps its wings *fixed*. It could not fly with fixed wings even if it had tiny propellers attached to them! Challenge 151 e

Due to the limitations of fixed wings at small dimensions, insects and small birds must *move* their wings, in contrast to aeroplanes. They must do so not only to take off or to gain height, but also to simply remain airborne in horizontal flight. In contrast, aeroplanes generate enough lift with *fixed* wings. Indeed, if you look at flying animals, such as the ones shown in Figure 155, you note that the larger they are, the less they need to move their wings (at cruising speed).

Can you deduce from equation (110) that birds or insects can fly but people cannot? Conversely, the formula also (partly) explains why human-powered aeroplanes must be so large.* Challenge 152 s

But *how* do insects, small birds, flying fish or bats have to move their wings in order to fly? This is a tricky question and the answer has been uncovered only recently. The main point is that insect wings move in a way to produce eddies at the front edge which in turn thrust the insect upwards. Aerodynamic studies of butterflies – shown in Figure 154 – and studies of enlarged insect models moving in oil instead of in air are exploring the precise way insects make use of vortices. At the same time, more and more ‘mechanical birds’ and ‘model aeroplanes’ that use flapping wings for their propulsion are being built around the world. The field is literally in full swing.** Researchers are especially interested in understanding how vortices allow change of flight direction at the small dimensions typical for insects. Another aim is to reduce the size of flying machines. Ref. 228 Ref. 229

The expression (110) for the lift of fixed wings also shows what is necessary for safe take-off and landing. The lift of all wings *decreases* for smaller speeds. Thus both animals and aeroplanes *increase* their wing surface in these occasions. Many birds also vigorously increase the flapping of wings in these situations. But even strongly flapping, enlarged wings often are insufficient for take-off. Many flying animals, such as swallows, therefore avoid landing completely. For flying animals which do take off from the ground, nature most commonly makes them hit the wings against each other, over their back, so that when the wings separate again, the low pressure between them provides the first lift. This method is used by insects and many birds, including pheasants. As bird watchers

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* Another part of the explanation requires some aerodynamics, which we will not study here. Aerodynamics shows that the power consumption, and thus the resistance of a wing with given mass and given cruise speed, is inversely proportional to the square of the wingspan. Large wingspans with long slender wings are thus of advantage in (subsonic) flying, especially when energy is scarce.

One issue is mentioned here only in passing: why does an aircraft fly? The correct general answer is: *because it deflects air downwards*. How does an aeroplane achieve this? It can do so with the help of a tilted plank, a rotor, flapping wings, or a fixed wing. And when does a fixed wing deflect air downwards? First of all, the wing has to be tilted with respect to the air flow; in addition, the specific cross section of the wing can increase the downward flow. The relation between wing shape and downward flow is a central topic of applied aerodynamics.

** The website www.aniprop.de presents a typical research approach and the sites ovirc.free.fr and www.ornithopter.org give introductions into the way to build such systems for hobbyists.

FIGURE 155 Examples of the three larger wing types in nature, all optimized for rapid flows: turkey vulture (*Cathartes aura*), ruby-throated hummingbird (*Archilochus colubris*) and a dragonfly (© S.L. Brown, Pennsylvania Game Commission/Joe Kosack and nobodythere).

know, pheasants make a loud 'clap' when they take off. The clap is due to the low pressure region thus created.

Both wing use and wing construction depend on size. In fact, there are four types of wings in nature.

1. First of all, all large flying objects, such aeroplanes and large birds, fly using *fixed* wings, except during take-off and landing. This wing type is shown on the left-hand side of Figure 155.
2. Second, common size birds use *flapping* wings. (Hummingbirds can have over 50 wing beats per second.) These first two types of wings have a thickness of about 10 to 15 % of the wing depth. This wing type is shown in the centre of Figure 155.
3. At smaller dimensions, a third wing type appears, the *membrane wing*. It is found in dragonflies and most everyday insects. At these scales, at Reynolds numbers of around 1000 and below, thin *membrane* wings are the most efficient. The *Reynolds number* measures the ratio between inertial and viscous effects in a fluid. It is defined as

$$\mathrm{Re} = \frac{l v \rho}{\eta} \tag{111}$$

   where l is a typical length of the system, v the speed, ρ the density and η the dynamic *viscosity* of the fluid.* A Reynolds number much larger than one is typical for rapid air flow and fast moving water. In fact, the value of the Reynolds number distinguishes a 'rapid' or 'turbulent' flow on the one hand, and a 'slow', 'laminar' or 'viscous' flow on the other. An example of membrane wing is shown on the right-hand side of Figure 155. All the first three wing types are designed for *turbulent* flows.

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* The viscosity is the resistance to flow a fluid poses. It is defined by the force F necessary to move a layer of surface A with respect to a second, parallel one at distance d; in short, the (coefficient of) *dynamic viscosity* is defined as $\eta = d\,F/A\,v$. The unit is 1 kg/s m or 1 Pa s or 1 N s/m^2, once also called 10 P or 10 poise. In other words, given a horizontal tube, the viscosity determines how strong the pump needs to be to pump the fluid through the tube at a given speed. The viscosity of air 20°C is 1.8×10^{-5} kg/s m or 18 μPa s and increases with temperature. In contrast, the viscosity of liquids decreases with temperature. (Why?) The viscosity of water at 0°C is 1.8 mPa s, at 20°C it is 1.0 mPa s (or 1 cP), and at 40°C is 0.66 mPa s. Hydrogen has a viscosity smaller than 10 μPa s, whereas honey has 25 Pa s and pitch 30 MPa s.

Challenge 153 ny

Physicists also use a quantity ν called the *kinematic viscosity*. It is defined with the help of the mass density of the fluid as $\nu = \eta/\rho$ and is measured in m^2/s, once called 10^4 stokes. The kinematic viscosity of water at 20°C is 1 mm^2/s (or 1 cSt). One of the smallest values is that of acetone, with 0.3 mm^2/s; a larger one is glycerine, with 2000 mm^2/s. Gases range between 3 mm^2/s and 100 mm^2/s.

FIGURE 156 The wings of a few types of insects smaller than 1 mm (thrips, *Encarsia*, *Anagrus*, *Dicomorpha*) (HortNET).

4. The fourth type of wings is found at the smallest possible dimensions, for insects smaller than one millimetre; their wings are not membranes at all, but are optimized for viscous air flow. Typical are the cases of thrips and of parasitic wasps, which can be as small as 0.3 mm. All these small insects have wings which consist of a central *stalk* surrounded by hair. In fact, Figure 156 shows that some species of thrips have wings which look like miniature toilet brushes.
5. At even smaller dimensions, corresponding to Reynolds number below 10, nature does not use wings any more, though it still makes use of air transport. In principle, at the smallest Reynolds numbers gravity plays no role any more, and the process of flying merges with that of swimming. However, air currents are too strong compared with the speeds that such a tiny system could realize. No active navigation is then possible any more. At these small dimensions, which are important for the transport through air of spores and pollen, nature uses the air currents for passive transport, making use of special, but fixed shapes.

We summarize: active flying is only possible through shape change. Only two types of shape changes are possible for active flying: that of wings and that of propellers (including turbines). Engineers are studying with intensity how these shape changes have to take place in order to make flying most effective. Interestingly, a similar challenge is posed by swimming.

Ref. 230
Ref. 231

Swimming

Swimming is a fascinating phenomenon. The Greeks argued that the ability of fish to swim is a proof that water is made of atoms. If atoms would not exist, a fish could not advance through it. Indeed, swimming is an activity that shows that matter cannot be continuous. Studying swimming can thus be quite enlightening. But how exactly do fish swim?

Whenever dolphins, jellyfish, submarines or humans *swim*, they take water with their fins, body, propellers, hands or feet and push it backwards. Due to momentum conser-

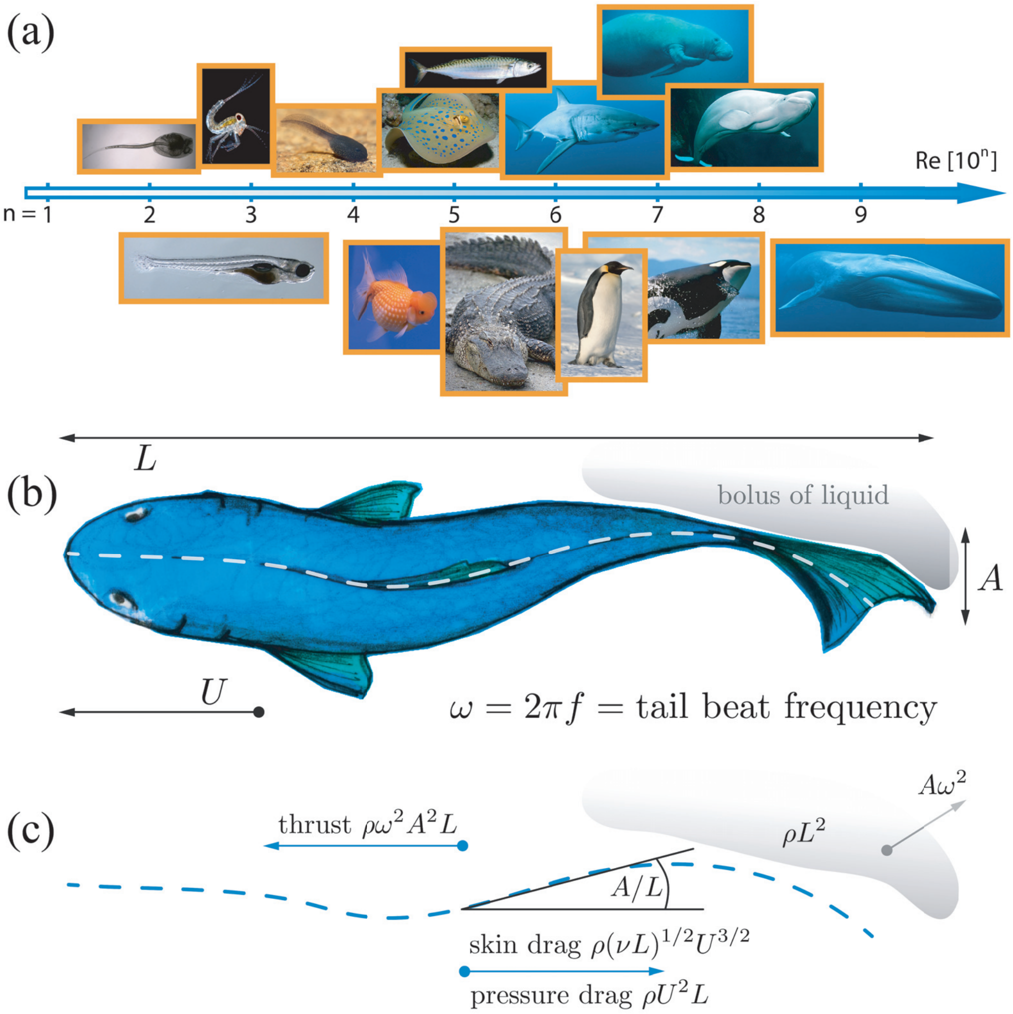

FIGURE 157 A selection of animals using undulatory swimming and the main variables that describe it. Ref. 232 (© L. Mahadevan/Macmillan from)

vation they then move forward.*

Most fish and aquatic mammals swim by bending their bodies and alternating between two extreme deformations. An overview of animals that use this type of swimming – experts call it *undulatory gait* – is given in Figure 157. Ref. 232 For all such swimming, the Reynolds number obeys $\mathrm{Re} = vL/\nu >> 1$; here v is the swimming speed, L the body length and ν the kinematic viscocity. The wide range of Reynolds number values observed for swimming living beings is shown in the graph. The swimming motion is best described by the so-called swimming number $\mathrm{Sw} = \omega AL/\nu$, where ω and A are the circular beat frequency and amplitude. The next graph, Figure 158, shows that for tur-

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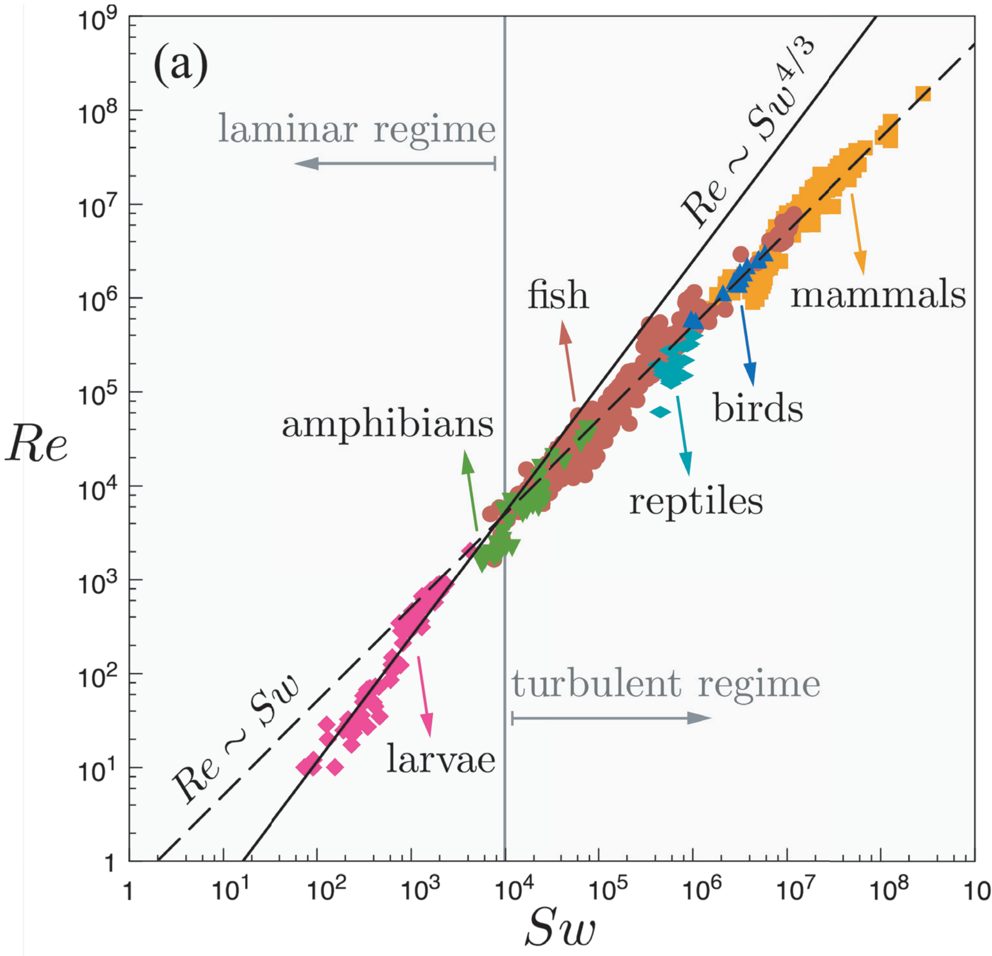
Vol. I, page 89 * Fish could use propellers, as the arguments against wheels we collected at the beginning of our walk do not apply for swimming. But propellers with blood supply would be a weak point in the construction, and thus make fish vulnerable. Therefore, nature has not developed fish with propellers.



Ref. 232

FIGURE 158 Undulatory swimming follows simple scaling rules for organisms that range from a few mm to 30 m in length. The Reynolds number *Re* describes the ratio between inertial effects (the water thrown behind) and the disspiative effects (the friction of the water). The swimming number *Sw* describes the body kinematics. (© L. Mahadevan/Macmillan from)

Challenge 154 e

bulent swimming, the swimming speed v almost exclusively depends on the amplitude and frequency of the undulatory motion; for smaller organisms that swim in the laminar regime, the swimming speed also depends on the length of the organism.

Ref. 234

In short, fish, dolphins, submarines and people swim in the same way that fireworks or rockets fly: by throwing matter behind them, through lift. Lift-based propulsion is the main type of macroscopic swimming. Do all organisms swim in this way? No. Several organisms swim by expelling water jets, for example cephalopods such as squids. And above all, *small* organisms advancing through the molecules of a liquid use a completely different, *microscopic* way of swimming.

Small organisms such as bacteria do *not* have the capacity to propel or accelerate water against their surroundings. Indeed, the water remains attached around a microorganism without ever moving away from it. Physically speaking, in these cases of swimming the kinetic energy of the water is negligible. In order to swim, unicellular beings thus need to

FIGURE 159 A swimming scallop (here from the genus *Chlamys*) (© Dave Colwell).

use other effects. In fact, their only possibility is to change their body shape in controlled ways. Seen from far away, the swimming of microorganisms thus resembles the motion of particles through vacuum: like microorganisms, also particles have nothing to throw behind* them.

A good way to distinguish macroscopic from microscopic swimming is provided by scallops. *Scallops* are molluscs up to a few cm in size; an example is shown in Figure 159. Scallops have a double shell connected by a hinge that they can open and close. If they close it *rapidly*, water is expelled and the mollusc is accelerated; the scallop then can glide for a while through the water. Then the scallop opens the shell again, this time *slowly*, and repeats the feat. When swimming, the larger scallops look like clockwork false teeth. Scallops thus use a macroscopic swimming process.

If we reduce the size of the scallop by a thousand times to the size of a single cell we get a simple result: such a tiny scallop *cannot* swim. The lack of scalability of swimming methods is due to the changing ratio between inertial and dissipative effects at different scales. This ratio is measured by the *Reynolds number Re*. For the scallop the Reynolds number, is defined as $Re = vL/\eta$, where v is the relative speed between swimmer and water, L the length of the swimmer and η is the *kinematic viscosity* of the water, around $1\,\mathrm{mm}^2/\mathrm{s}$. Vol. I, page 429

For the scallop, the Reynolds number is about 100, which shows that when it swims, inertial effects are much more important than dissipative, viscous effects. For a bacterium the Reynolds number is much smaller than 1, so that inertial effects effectively play no role. There is no way to accelerate water *away* from a bacterial-sized scallop, and thus no way to glide. Bacteria *cannot* swim like scallops or people do; bacteria cannot throw water behind them. And this is not the only problem microorganism face when they want to swim.

A well-known mathematical theorem states that no cell-sized being can move if the shape change is the same in the two halves of the motion, i.e., when opening and closing are just the inverse of each other. Such a shape change would simply make it move back and forward. Another mathematical theorem, the so-called *scallop theorem*, that states Ref. 233

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* There is an exception: gliding bacteria move by secreting slime, even though it is still not fully clear why this leads to motion.

that no microscopic system can swim if it uses movable parts with only *one* degree of freedom. Thus it is impossible to move, at cell dimensions, using the method that the scallop uses on centimetre scale.

In order to swim, microorganisms thus need to use a more evolved, *two-dimensional motion* of their shape. Indeed, biologists found that all microorganisms use one of the following four swimming styles:

1. Microorganisms of compact shape of diameter between 20 μm and about 20 mm, use *cilia*. Cilia are hundreds of little hairs on the surface of the organism. Some organisms have cilia across their full surface, other only on part of it. These organisms move the cilia in waves wandering around their surface, and these surface waves make the body advance through the fluid. All children watch with wonder *Paramecium*, the unicellular animal they find under the microscope when they explore the water in which some grass has been left for a few hours. *Paramecium*, which is between 100 μm and 300 μm in size, as well as many plankton species* use cilia for its motion. The cilia and their motion are clearly visible in the microscope. A similar swimming method is even used by some large animals; you might have seen similar waves on the borders of certain ink fish; even the motion of the manta (partially) belongs into this class. Ref. 235 Ciliate motion is an efficient way to change the shape of a body making use of two dimensions and thus avoiding the scallop theorem.
2. Sperm and eukaryote microorganisms whose sizes are in the range between 1 μm and 50 μm swim using an (eukaryote) flagellum.** Flagella, Latin for 'small whips', work like flexible oars. Even though their motion sometimes appears to be just an oscillation, flagella get a kick only during one half of their motion, e.g. at every swing to the left. Flagella are indeed used by the cells like miniature oars. Some cells even twist their flagellum in a similar way that people rotate an arm. Some microorganisms, such as *Chlamydomonas*, even have two flagella which move in the same way Ref. 237 as people move their legs when they perform the breast stroke. Most cells can also change the sense in which the flagellum is kicked, thus allowing them to move either Ref. 238 forward or backward. Through their twisted oar motion, bacterial flagella avoid retracing the same path when going back and forward. As a result, the bacteria avoid the scallop theorem and manage to swim despite their small dimensions. The flexible oar motion they use is an example of a non-adiabatic mechanism; an important fraction of the energy is dissipated.
3. The smallest swimming organisms, bacteria with sizes between 0.2 μm and 5 μm, Ref. 239 swim using *bacterial* flagella. These flagella, also called prokaryote flagella, are different from the ones just mentioned. Bacterial flagella move like turning corkscrews. They are used by the famous *Escherichia coli* bacterium and by all bacteria of the genus *Salmonella*. This type of motion is one of the prominent exceptions to the non-existence of wheels in nature; we mentioned it in the beginning of our walk. Vol. I, page 89 Corkscrew motion is an example of an adiabatic mechanism.

   A Coli bacterium typically has a handful of flagella, each about 30 nm thick and of corkscrew shape, with up to six turns; the turns have a 'wavelength' of 2.3 μm. Each

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* See the www.liv.ac.uk/ciliate website for an overview.

Ref. 236 ** The largest sperm, of 5.8 cm length, are produced by the 1.5 mm sized *Drosophila bifurca* fly, a relative of the famous *Drosophila melanogaster*.

flagellum is turned by a sophisticated rotation motor built into the cell, which the cell can control both in rotation direction and in angular velocity. For Coli bacteria, the range is between 0 and about 300 Hz. Ref. 240

A turning flagellum does not propel a bacterium like a propeller; as mentioned, the velocities involved are much too small, the Reynolds number being only about 10^{-4}. At these dimensions and velocities, the effect is better described by a corkscrew turning in honey or in cork: a turning corkscrew produces a motion against the material around it, in the direction of the corkscrew axis. The flagellum moves the bacterium in the same way that a corkscrew moves the turning hand with respect to the cork.

4. One group of bacteria, the spirochaetes, move as a whole like a cork-screw through water. An example is *Rhodospirillum rubrum*, whose motion can be followed in the video on www.microbiologybytes.com/video/motility.com. These bacteria have an internal motor round an axial filament, that changes the cell shape in a non-symmetrical fashion and yield cork-screw motion. A different bacterium is *Spiroplasma*, a helical bacterium – but not a spirochaete – that changes the cell shape, again in a non-symmetrical fashion, by propagating kink pairs along its body surface. Various other microorganisms move by changing their body shape. Ref. 241

To test your intuition, you may try the following puzzle: is microscopic swimming possible in two spatial dimensions? In four? Challenge 155 s

By the way, still smaller bacteria do not swim at all. Indeed, each bacterium faces a minimum swimming speed requirement: it must outpace diffusion in the liquid it lives in. Slow swimming capability makes no sense; numerous microorganisms therefore do not manage or do not swim at all. Some microorganisms are specialized to move along liquid–air interfaces. In fact, there are many types of interfacial swimming, including macroscopic types, but we do not cover them here. Other microorganisms attach themselves to solid bodies they find in the liquid. Some of them are able to *move* along these solids. The amoeba is an example for a microorganism moving in this way. Also the smallest active motion mechanisms known, namely the motion of molecules in muscles and in cell membranes, work this way. Ref. 242 Page 21

Let us summarize these observations. All known active motion, or self-propulsion, (in flat space) takes place in fluids – be it air or liquids. All active motion requires shape change. Macroscopic swimming works by accelerating the fluid in the direction opposite to the direction of motion. Microscopic swimming works through smart shape change that makes the swimmer advance through the fluid. In order that shape change leads to motion, the environment, e.g. the fluid, must itself consist of moving components always pushing onto the swimming entity. The motion of the swimming entity can then be deduced from the particular shape change it performs. The mathematics of swimming through shape change is fascinating; it deserves to be explored.

Rotation, falling cats and the theory of shape change

At small dimensions, flying and swimming takes place through shape change. In the last decades, the description of shape change has changed from a fashionable piece of research to a topic whose results are both appealing and useful. There are many studies, both experimental and theoretical, about the exact way small systems move in water and

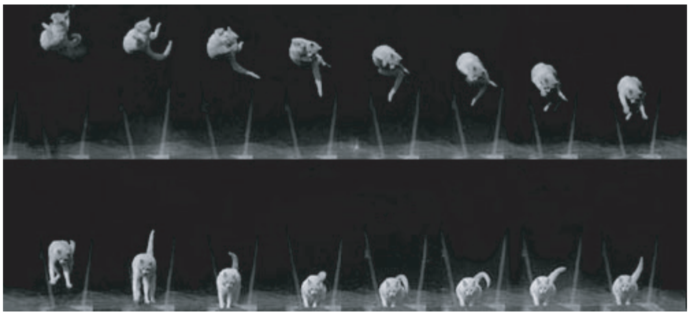

FIGURE 160 Cats can turn themselves, even with *no* initial angular momentum (photographs by Etienne-Jules Marey, 1894).

FIGURE 161 Humans can turn themselves in mid air like cats: see the second, lateral rotation of Artem Silchenko, at the 2006 cliff diving world championship (© World High Diving Federation).

air, about the achievable and achieved efficiency, and much more.

It is not a surprise that organized shape change can lead to *translational motion*. Amoebas, earthworms, caterpillars, snakes, and even human themselves move though shape change.

But shape change can also lead to a *rotation* of a body. In this case, the ideas are not restricted to microscopic systems, but apply at all scales. In particular, the theory of shape change is useful in explaining how falling cats manage to fall always on their feet. Cats are not born with this ability; they have to learn it. But the feat has fascinated people for

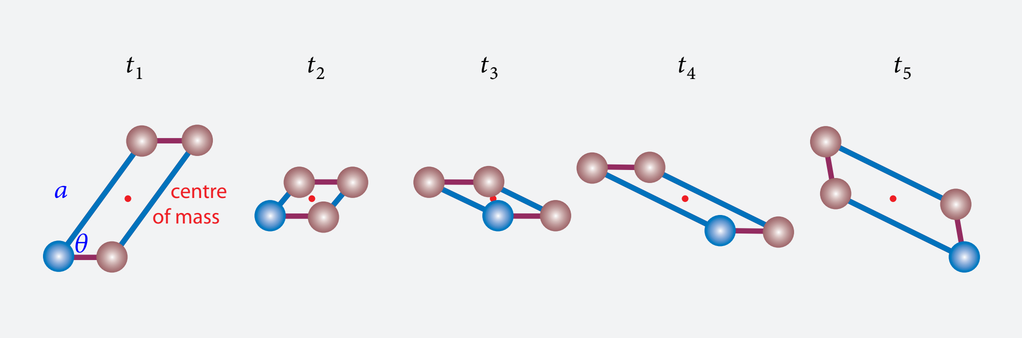

FIGURE 162 The square cat: in free space, or also on perfect ice, a deformable body in the shape of a parallelogram made of four masses and rods that is able to change the body angle θ and two rod lengths a is able to rotate itself around the centre of mass without outside help. One mass and the length-changing rods are coloured to illustrate the motion.

centuries, as shown in the old photograph given in Figure 160. In fact, cats confirm in three dimensions what we already knew for two dimensions: Vol. I, page 120

▷ A deformable body can change its own orientation in space *without* outside help.

This is in strong contrast to translation, for which outside help is always needed. Archimedes famously said: Give me a place to stand, and I'll move the Earth. But to *rotate* the Earth, a place to stand is *not* needed!

Not only cats, also humans can perform the feat: simply observe the second, lateral rotation of the diver in Figure 161. Cosmonauts in space stations and passengers of parabolic 'zero-gravity' flights regularly do the same, as do many artificial satellites sent into space.

In the 1980s, the work by Berry, Wilczek, Zee and ShapereBerry, MichaelWilczek, FrankZee, AnthonyShapere, Alfred showed that all motion due to shape change is described by a gauge theory. The equivalence between the two situations is detailed in Table 24. A simple and beautiful example for these ideas has been given by Putterman and Raz and is illustrated in Figure 162. Imagine four spheres on perfect ice, all of the same mass and size, connected by four rods forming a parallelogram. Now imagine that this parallelogram, using some built-in motors, can change length along one side, called a, and that it can also change the angle θ between the sides. Putterman and Raz call this the *square cat*. The figure shows that the square cat can change its own orientation on the ice while, obviously, keeping its centre of mass at rest. The figure also shows that the change of orientation only works because the two motions that the cat can perform, the stretching and the angle change, do *not* commute. The order in which these deformations occur is essential for achieving the desired rotation. Ref. 243 Ref. 244

The rotation of the square cat occurs in *strokes*; large rotations are achieved by repeating strokes, similar to the situation of swimmers. If the square cat would be swimming in a liquid, the cat could thus rotate itself – though it could not advance.

When the cat rotates itself, each stroke results in a rotation angle that is independent

TABLE 24 The correspondence between shape change and gauge theory.

Concept	Shape change	Gauge theory
System	deformable body	matter–field combination
Gauge freedom	freedom of description of body orientation and position	freedom to define vector potential
Gauge-dependent quantity	shape’s angular orientation and position	vector potential, phase
	orientation and position change along an open path	vector potential and phase change along open path
Gauge transformation	changes angular orientation and position	changes vector potential
Gauge-independent quantities	orientation and position after full stroke	phase difference on closed path, integral of vector potential along a closed path
	deformations	field strengths
Gauge group	e.g. possible rotations SO(3) or motions E(3)	U(1), SU(2), SU(3)

of the speed of the stroke. The same experience can be made when rotating oneself on an office chair by rotating the arm above the head: the chair rotation angle after arm turn is independent of the arm speed. Stroke motion leads to a puzzle: what is the largest angle that a cat can turn in one stroke? Challenge 156 d

Rotation in strokes has a number of important implications. First of all, the number of strokes is a quantity that all observers agree upon: it is observer-invariant. Secondly, the orientation change after a *complete* stroke is also observer-invariant. Thirdly, the orientation change for *incomplete* strokes is observer-dependent: it depends on the way that orientation is defined. For example, if orientation is defined by the direction of the body diagonal through the black mass (see Figure 162), it changes in a certain way during a stroke. If the orientation is defined by the direction of the fixed bar attached to the black mass, it changes in a different way during a stroke. Only when a full stroke is completed do the two values coincide. Mathematicians say that the choice of the definition and thus the value of the orientation is *gauge-dependent*, but that the value of the orientation change at a full stroke is *gauge-invariant*.

In summary, the square cat shows three interesting points. First, already rather simple deformable bodies can change their orientation in space. Secondly, the orientation of a deformable body can only change if the deformations it can perform are *non-commuting*. Thirdly, such deformable bodies are described by *gauge theories*: certain aspects of the bodies are gauge-invariant, others are gauge-dependent. This summary leads to a question: Can we use these ideas to increase our understanding of the gauge theories of the electromagnetic, weak and strong interaction? Shapere and Wilczek say no. We will explore this issue in the next volume. In fact, shape change bears even more surprises.

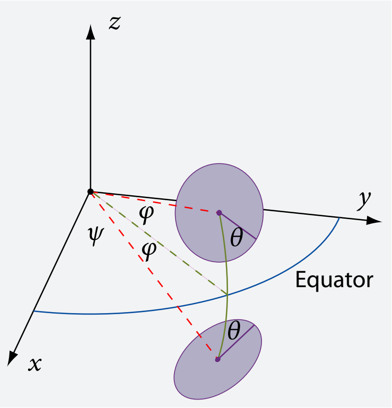

FIGURE 163 Swimming on a curved surface using two discs.

Swimming in curved space

In flat space it is not possible to produce translation through shape change. Only orientation changes are possible. Surprisingly, if space is *curved*, motion does become possible. A simple example was published in 2003 by Jack Wisdom. He found that cyclic changes in the shape of a body can lead to net translation, a rotation of the body, or both. Ref. 245

Indeed, we know from Galilean physics that on a frictionless surface we cannot move, but that we can change orientation. This is true only for a flat surface. On a curved surface, we can use the ability to turn and translate it into motion.

Take two massive discs that lie on the surface of a frictionless, spherical planet, as shown in Figure 163. Consider the following four steps: 1. the disc separation φ is increased by the angle $\Delta\varphi$, 2. the discs are rotated oppositely about their centres by the angle $\Delta\theta$, 3. their separation is decreased by $-\Delta\varphi$, and 4. they are rotated back by $-\Delta\theta$. Due to the conservation of angular momentum, the two-disc system changes its longitude $\Delta\psi$ as Challenge 157 ny

$$\Delta\psi = \frac{1}{2}\gamma^2 \Delta\theta\Delta\varphi \,, \qquad (112)$$

where γ is the angular radius of the discs. This cycle can be repeated over and over. The cycle allows a body, located on the surface of the Earth, to swim along the surface. Unfortunately, for a body of size of one metre, the motion for each swimming cycle is only around 10^{-27} m.

Wisdom showed that the same procedure also works in curved space, thus in the presence of gravitation. The mechanism thus allows a falling body to swim away from the path of free fall. Unfortunately, the achievable distances for everyday objects are negligibly small. Nevertheless, the effect exists.

In other words, there is a way to swim through curved space that looks similar to swimming at low Reynolds numbers, where swimming results of simple shape change. Does this tell us something about fundamental descriptions of motion? The last part of our ascent will tell.

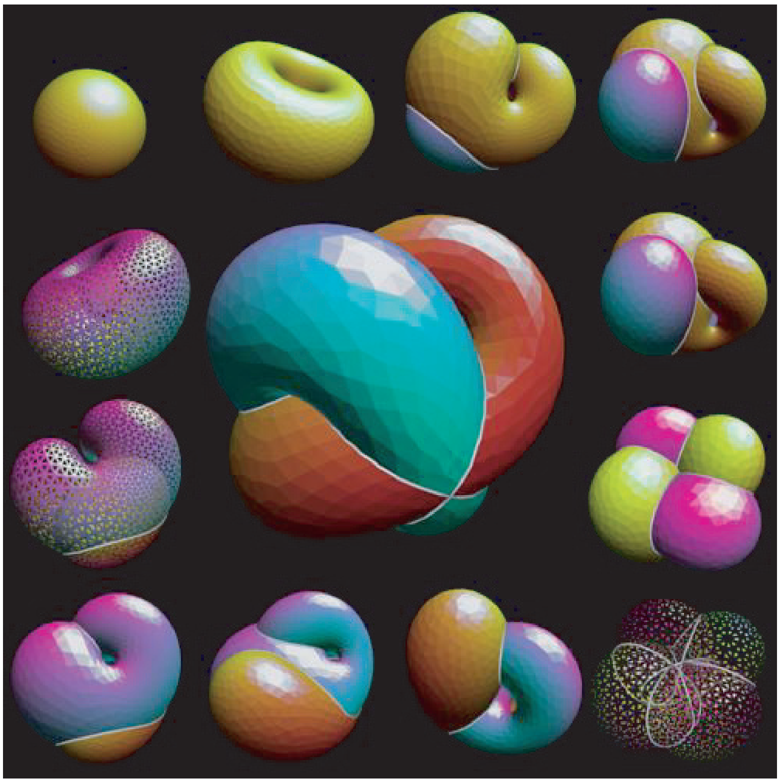

FIGURE 164 A way to turn a sphere inside out, with intermediate steps ordered clockwise (© John Sullivan).

TURNING A SPHERE INSIDE OUT

> “A text should be like a lady's dress; long enough to cover the subject, yet short enough to keep it interesting.”
>
> Anonymous

Exploring the theme of motion of wobbly entities, a famous example cannot be avoided. In 1957, the mathematician Stephen Smale proved that a sphere can be turned inside out. The discovery brought him the Fields medal in 1966, the highest prize for discoveries in mathematics. Mathematicians call his discovery the *eversion* of the sphere. Ref. 246

To understand the result, we need to describe more clearly the rules of mathematical eversion. First of all, it is assumed that the sphere is made of a thin membrane which has the ability to stretch and bend without limits. Secondly, the membrane is assumed to be able to *intersect* itself. Of course, such a ghostly material does not exist in everyday life; but in mathematics, it can be imagined. A third rule requires that the membrane must be deformed in such a way that the membrane is not punctured, ripped nor creased; in short, everything must happen *smoothly* (or differentiably, as mathematicians like to say).

Even though Smale proved that eversion is possible, the first way to actually perform it was discovered by the blind topologist Bernard Morin in 1961, based on ideas of Arnold Shapiro. After him, several additional methods have been discovered. Ref. 247, Ref. 248

Ref. 249 Several computer videos of sphere eversions are now available.* The most famous ones are *Outside in*, which shows an eversion due to William P. Thurston, and *The Optiverse*, which shows the most efficient method known so far, discovered by a team led by John Sullivan and shown in Figure 164.

Why is sphere eversion of interest to physicists? If elementary particles were extended and at the same time were of spherical shape, eversion might be a particle symmetry. To see why, we summarize the effects of eversion on the whole surrounding space, not only on the sphere itself. The final effect of eversion is the transformation

$$(x, y, z) \to \frac{(x, y, -z) R^2}{r^2} \tag{113}$$

where R is the radius of the sphere and r is the length of the coordinate vector (x, y, z), thus $r = \sqrt{x^2 + y^2 + z^2}$. Due to the minus sign in the z-coordinate, eversion differs from inversion, but not by too much. As we will find out in the last part of our adventure, a transformation similar to eversion, space-time duality, is a fundamental symmetry of nature. Vol. VI, page 114

Clouds

Clouds are another important class of wobbly objects. The lack of a definite boundary makes them even more fascinating than amoebas, bacteria or falling cats. We can observe the varieties of clouds from any aeroplane.

Vol. III, page 218 The common cumulus or cumulonimbus in the sky, like all the other meteorological clouds, are vapour and water droplet clouds. Galaxies are clouds of stars. Stars are clouds of plasma. The atmosphere is a gas cloud. Atoms are clouds of electrons. Nuclei are clouds of protons and neutrons, which in turn are clouds of quarks. Comparing different cloud types is illuminating and fun.

Clouds of all types can be described by a shape and a size, even though in theory they have no bound. An *effective* shape and size can be defined by that region in which the cloud density is only, say, 1 % of the maximum density; slightly different procedures can also be used. All clouds are described by *probability densities* of the components making up the cloud. All clouds show *conservation* of the number of their constituents.

Whenever we see a cloud, we can ask why it does not collapse. Every cloud is an aggregate; all aggregates are kept from collapse in only *three* ways: through rotation, through pressure, or through the Pauli principle, i.e., the quantum of action. For example, galaxies are kept from collapsing by rotation. Most stars, the atmosphere and rain clouds are kept from collapsing by gas pressure. Neutron stars, the Earth, atomic nuclei, protons or the electron clouds of atoms are kept apart by the quantum of action. Vol. I, page 257

A rain cloud is a method to keep several thousand tons of water suspended in the air. Can you explain what keeps it afloat, and what else keeps it from continuously diffusing

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* Summaries of the videos can be seen at the website www.geom.umn.edu/docs/outreach/oi, which also has a good pedagogical introduction. Another simple eversion and explanation is given by Erik de Neve on his website www.usefuldreams.org/sphereev.htm. It is even possible to run the eversion film software at home; see the website www.cslub.uwaterloo.ca/~mjmcguff/eversion. Figure 164 is from the website new.math.uiuc.edu/optiverse.

FIGURE 165 A vortex in nature: a waterspout (© Zé Nogueira).

Challenge 158 s

into a thinner and thinner structure?

Vol. IV, page 186

Two rain clouds can merge. So can two atomic electron clouds. So can galaxies. But only atomic clouds are able to *cross* each other. We remember that a normal atom can be inside a Rydberg atom and leave it again without change. In contrast, rain clouds, stars, galaxies or other macroscopic clouds cannot cross each other. When their paths cross, they can only merge or be ripped into pieces. Due to this lack of crossing ability, only microscopic clouds can be counted. In the macroscopic cases, there is no real way to define a 'single' cloud in an accurate way. If we aim for full precision, we are unable to claim that there is more than one rain cloud, as there is no clear-cut boundary between them. Electronic clouds are different. True, in a piece of solid matter we can argue that there is only a single electronic cloud throughout the object; however, when the object is divided, the cloud is divided in a way that makes the original atomic clouds reappear. We thus can speak of 'single' electronic clouds.

If one wants to be strict, galaxies, stars and rain clouds can be seen as made of localized particles. Their cloudiness is only apparent. Could the same be true for electron clouds? And what about space itself? Let us explore some aspects of these questions.

Vortices and the Schrödinger equation

Fluid dynamics is a topic with many interesting aspects. Take the *vortex* that can be observed in any deep, emptying bath tub: it is an extended, one-dimensional 'object', it is

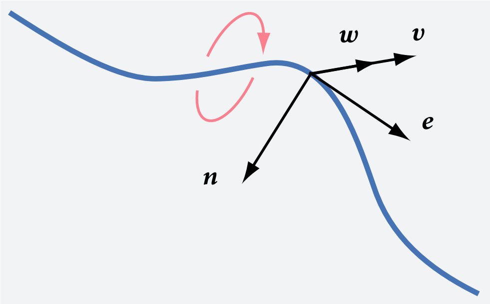

FIGURE 166 The mutually perpendicular tangent $\boldsymbol{e}$, normal $\boldsymbol{n}$, torsion $\boldsymbol{w}$ and velocity $\boldsymbol{v}$ of a vortex in a rotating fluid.

deformable, and it is observed to wriggle around. Larger vortices appear as tornadoes on Earth and on other planets, as *waterspouts*, and at the ends of wings or propellers of all kinds. Smaller, quantized vortices appear in superfluids. An example is shown in Figure 165; also the spectacular fire whirls and fire tornados observed every now and then are vortices. Page 105

Vortices, also called *vortex tubes* or *vortex filaments*, are thus wobbly entities. Now, a beautiful result from the 1960s states that a vortex filament in a rotating liquid is described by the one-dimensional Schrödinger equation. Let us see how this is possible. Ref. 250

Any deformable linear vortex, as illustrated in Figure 166, is described by a continuous set of position vectors $\boldsymbol{r}(t,s)$ that depend on time t and on a single parameter s. The parameter s specifies the relative position along the vortex. At each point on the vortex, there is a unit tangent vector $\boldsymbol{e}(t,s)$, a unit normal curvature vector $\boldsymbol{n}(t,s)$ and a unit torsion vector $\boldsymbol{w}(t,s)$. The three vectors, shown in Figure 166, are defined as usual as

$$\begin{aligned} \boldsymbol{e} &= \frac{\partial \boldsymbol{r}}{\partial s} \,, \\ \kappa \boldsymbol{n} &= \frac{\partial \boldsymbol{e}}{\partial s} \,, \\ \tau \boldsymbol{w} &= -\frac{\partial (\boldsymbol{e} \times \boldsymbol{n})}{\partial s} \,, \end{aligned} \tag{114}$$

where κ specifies the value of the curvature and τ specifies the value of the torsion. In general, both numbers depend on time and on the position along the line.

In the simplest possible case the rotating environment induces a local velocity $\boldsymbol{v}$ for the vortex that is proportional to the curvature κ, perpendicular to the tangent vector $\boldsymbol{e}$ and perpendicular to the normal curvature vector $\boldsymbol{n}$:

$$\boldsymbol{v} = \eta\kappa(\boldsymbol{e} \times \boldsymbol{n}) \,, \tag{115}$$

where η is the so-called *coefficient of local self-induction* that describes the coupling between the liquid and the vortex motion. This is the evolution equation of the vortex. Ref. 250

We now assume that the vortex is deformed only *slightly* from the straight configuration. Technically, we are thus in the *linear* regime. For such a linear vortex, directed along

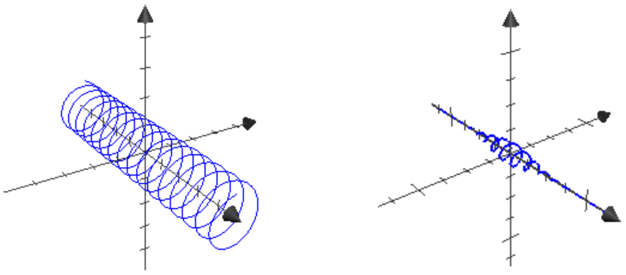

FIGURE 167 Motion of a vortex: the fundamental helical solution and a moving helical ‘wave packet’.

the x-axis, we can write

$$\boldsymbol{r} = (x, y(x,t), z(x,t)) \ . \tag{116}$$

Slight deformations imply $\partial s \approx \partial x$ and therefore

$$\begin{aligned} \boldsymbol{e} &= (1, \frac{\partial y}{\partial x}, \frac{\partial z}{\partial x}) \approx (1,0,0) \ , \\ \kappa\boldsymbol{n} &\approx (0, \frac{\partial^2 y}{\partial x^2}, \frac{\partial^2 z}{\partial x^2}) \ , \text{ and} \\ \boldsymbol{v} &= (0, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}) \ . \end{aligned} \tag{117}$$

We can thus rewrite the evolution equation (115) as

$$(0, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}) = \eta\,(0, -\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 y}{\partial x^2}) \ . \tag{118}$$

This equation is well known; if we drop the first coordinate and introduce complex numbers by setting $\Phi = y + iz$, we can rewrite it as

$$\frac{\partial \Phi}{\partial t} = i\eta \frac{\partial^2 \Phi}{\partial x^2} \ . \tag{119}$$

Ref. 251 This is the one-dimensional Schrödinger equation for the evolution of a free wave function! The complex function Φ specifies the transverse deformation of the vortex. In other words, we can say that the Schrödinger equation in one dimension describes the evolution of the deformation for an almost linear vortex in a rotating liquid. We note that there is no constant $\hbar$ in the equation, as we are exploring a *classical* system.

Schrödinger's equation is linear in Φ. Therefore the fundamental solution is

$$\Phi(x, y, z, t) = a\, \mathrm{e}^{i(\tau x - \omega t)} \quad \text{with} \quad \omega = \eta\tau^2 \quad \text{and} \quad \kappa = a\tau^2 \; . \tag{120}$$

The amplitude a and the wavelength or pitch $b = 1/\tau$ can be freely chosen, as long as the approximation of small deviation is fulfilled; this condition translates as $a \ll b$.* In the present interpretation, the fundamental solution corresponds to a vortex line that is deformed into a *helix*, as shown in Figure 167. The angular speed ω is the rotation speed around the axis of the helix.

Challenge 159 ny A helix moves along the axis with a speed given by

$$v_{\text{helix along axis}} = 2\eta\tau \; . \tag{121}$$

In other words, for extended entities following evolution equation (115), rotation and translation are coupled.** The momentum p can be defined using $\partial\Phi/\partial x$, leading to

$$p = \tau = \frac{1}{b} \; . \tag{122}$$

Momentum is thus inversely proportional to the helix wavelength or pitch, as expected. The energy E is defined using $\partial\Phi/\partial t$, leading to

$$E = \eta\tau^2 = \frac{\eta}{b^2} \; . \tag{123}$$

Energy and momentum are connected by

$$E = \frac{p^2}{2\mu} \quad \text{where} \quad \mu = \frac{1}{2\eta} \; . \tag{124}$$

In other words, a vortex with a coefficient η – describing the coupling between environment and vortex – is thus described by a number μ that behaves like an effective mass. We can also define the (real) quantity $|\Phi| = a$; it describes the *amplitude* of the deformation. (Page 295)

In the Schrödinger equation (119), the second derivative implies that the deformation 'wave packet' has tendency to spread out over space. Can you confirm that the wavelength–frequency relation for a vortex wave group leads to something like the indeterminacy relation (however, without a $\hbar$ appearing explicitly)? Challenge 161 ny

In summary, the complex amplitude Φ for a linear vortex in a rotating liquid behaves like the one-dimensional wave function of a non-relativistic free particle. In addition, we found a suggestion for the reason that complex numbers appear in the Schrödinger equation of quantum theory: they could be due to the intrinsic rotation of an underlying substrate. Is this suggestion correct? We will find out in the last part of our adventure. Vol. VI, page 174

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* The curvature is given by $\kappa = a/b^2$, the torsion by $\tau = 1/b$. Instead of $a \ll b$ one can thus also write $\kappa \ll \tau$.

Challenge 160 ny ** A wave packet moves along the axis with a speed given by $v_{\text{packet}} = 2\eta\tau_0$, where τ_0 is the torsion of the helix of central wavelength.

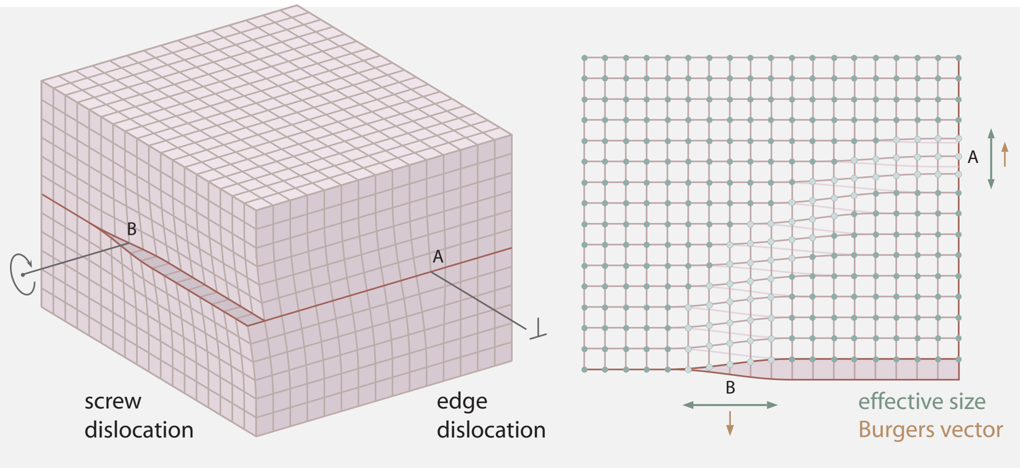

FIGURE 168 The two pure dislocation types, edge and screw dislocations, seen from the outside of a cubic crystal (left) and the mixed dislocation – a quarter of a dislocation loop – joining them in a horizontal section of the same crystal (right) (© Ulrich Kolberg).

Fluid space-time

General relativity shows that space can move and oscillate: space is a wobbly entity. Is space more similar to clouds, to fluids, or to solids?

An intriguing approach to space-time as a fluid was published in 1995 by Ted Jacobson. He explored what happens if space-time, instead of assumed to be continuous, is assumed to be the statistical average of numerous components moving in a disordered fashion. Ref. 252

The standard description of general relativity describes space-time as an entity similar to a flexible mattress. Jacobson studied what happens if the mattress is assumed to be made of a fluid. A fluid is a collection of (undefined) components moving randomly and described by a temperature varying from place to place. Vol. II, page 144

Jacobson started from the Fulling–Davies–Unruh effect and assumed that the local fluid temperature is given by a multiple of the local gravitational acceleration. He also used the proportionality – correct on horizons – between area and entropy. Since the energy flowing through a horizon can be called heat, one can thus translate the expression $\delta Q = T\delta S$ into the expression $\delta E = a\delta A(c^2/4G)$, which describes the behaviour of space-time at horizons. As we have seen, this expression is fully equivalent to general relativity. Page 129 Vol. VI, page 33

In other words, imagining space-time as a fluid is a powerful analogy that allows deducing general relativity. Does this mean that space-time actually *is* similar to a fluid? So far, the analogy is not sufficient to answer the question and we have to wait for the last part of our adventure to settle it. In fact, just to confuse us a bit more, there is an old argument for the opposite statement.

Dislocations and solid space-time

General relativity tells us that space behaves like a deformable mattress; space thus behaves like a solid. There is a second argument that underlines this point and that exerts a continuing fascination. This argument is connected to a famous property of the motion of dislocations.

Dislocations are one-dimensional construction faults in crystals, as shown in Figure 168. A general dislocation is a mixture of the two pure dislocation types: *edge dislocations* and *screw dislocations*. Both are shown in Figure 168.

If one explores how the atoms involved in dislocations can rearrange themselves, one Challenge 162 e finds that edge dislocations can only move perpendicularly to the added plane. In contrast, screw dislocations can move in all directions.* An important case of general, mixed dislocations, i.e., of mixtures of edge and screw dislocations, are closed *dislocation rings*. On such a dislocation ring, the degree of mixture changes continuously from place to place.

Any dislocation is described by its *strength* and by its effective *size*; they are shown, respectively, in red and blue in Figure 168. The *strength* of a dislocation is measured by the so-called *Burgers vector*; it measures the misfits of the crystal around the dislocation. More precisely, the Burgers vector specifies by how much a section of perfect crystal needs to be displaced, after it has been cut open, to produce the dislocation. Obviously, the strength of a dislocation is quantized in multiples of a minimal Burgers vector. In fact, dislocations with large Burgers vectors can be seen as composed of dislocations of minimal Burgers vector, so that one usually studies only the latter.

The size or *width* of a dislocation is measured by an *effective width* w. Also the width is a multiple of the lattice vector. The width measures the size of the deformed region of the crystal around the dislocation. Obviously, the size of the dislocation depends on the elastic properties of the crystal, can take continuous values and is direction-dependent. The width is thus related to the energy content of a dislocation.

A general dislocation can move, though only in directions which are both perpendicular to its own orientation and to its Burgers vector. Screw dislocations are simpler: they can move in any direction. Now, the motion of screw dislocations has a peculiar property. We call c the speed of sound in a pure (say, cubic) crystal. As Frenkel and Kontorowa Ref. 253 found in 1938, when a screw dislocation moves with velocity v, its width w changes as

$$w = \frac{w_0}{\sqrt{1 - v^2/c^2}} . \qquad (125)$$

In addition, the energy of the moving dislocation obeys

$$E = \frac{E_0}{\sqrt{1 - v^2/c^2}} . \qquad (126)$$

A screw dislocation thus cannot move faster than the speed of sound c in a crystal and its width shows a speed-dependent contraction. (Edge dislocations have similar, but more complex behaviour.) The motion of screw dislocations in solids is thus described by the same effects and formulae that describe the motion of bodies in special relativity; the

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* See the uet.edu.pk/dmems/edge_dislocation.htm, uet.edu.pk/dmems/screw_dislocation.htm and uet.edu.pk/dmems/mixed_dislocation.htm web pages to watch a moving dislocation.

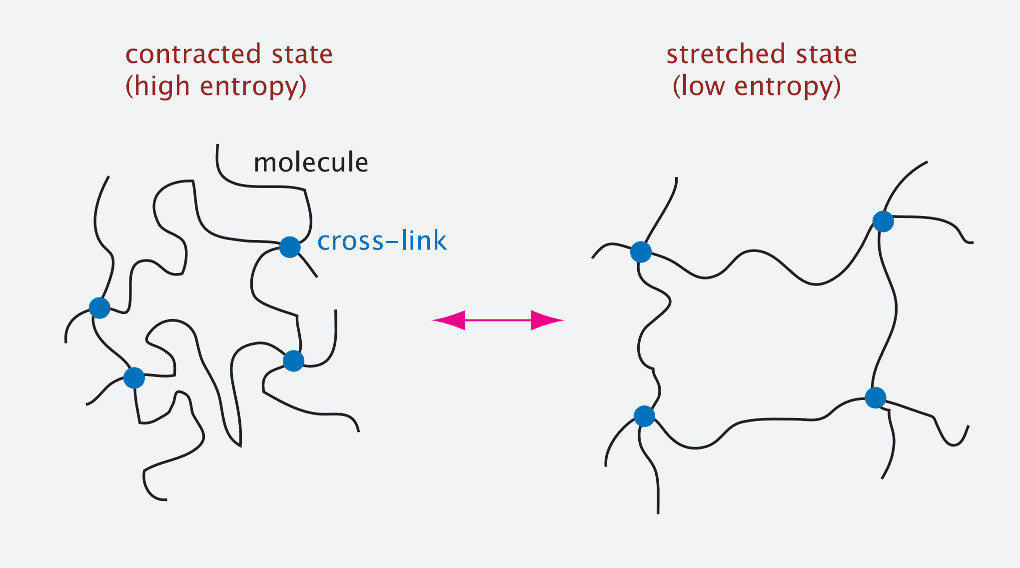

FIGURE 169 An illustration of the relation between polymer configurations and elasticity. The molecules in the stretched situation have fewer possible shape configurations and thus lower entropy; therefore, the material tends back to the contracted situation.

speed of sound is the limit speed for dislocations in the same way that the speed of light is the limit speed for objects.

Does this mean that elementary particles are dislocations of space or even of space-time, maybe even dislocation rings? The speculation is appealing, even though it supposes that space-time is a solid crystal, and thus contradicts the model of space or space-time as a fluid. Worse, we will soon encounter other reasons to reject modelling space-time as a lattice; maybe you can find a few arguments already by yourself. Still, expressions (125) and (126) for dislocations continue to fascinate. Vol. VI, page 76 Challenge 163 s

At this point, we are confused. Space-time seems to be solid and liquid at the same time. Despite this contrast, the discussion somehow gives the impression that there is something waiting to be discovered. But what? We will find out in the last part of our adventure.

Polymers

Ref. 254 The study of polymers is both economically important and theoretically fascinating. Polymers are materials built of long and flexible macromolecules that are sequences of many (‘poly’ in Greek) similar monomers. These macromolecules are thus wobbly entities.

Polymers form *solids*, like rubber or plexiglas, *melts*, like those used to cure teeth, and many kinds of *solutions*, like glues, paints, eggs, or people. Polymer gases are of lesser importance.

All the material properties of polymers, such as their elasticity, their viscosity, their electric conductivity or their unsharp melting point, depend on the number of monomers and the topology of their constituent molecules. In many cases, this dependence can be calculated. Let us explore an example.

If L is the contour length of a free, ideal, unbranched polymer molecule, the *average* end-to-end distance R is proportional to the square root of the length L:

$$R = \sqrt{Ll} \sim \sqrt{L} \quad \text{or} \quad R \sim \sqrt{N} \tag{127}$$

where N is the number of monomers and l is an effective monomer length describing the scale at which the polymer molecule is effectively stiff. R is usually much smaller than L; this means that free, ideal polymer molecules are usually in a *coiled* state.

Obviously, the end-to-end distance R varies from molecule to molecule, and follows a Gaussian distribution for the probability P of a end-to-end distance R:

$$P(R) \sim \mathrm{e}^{\frac{-3R^2}{2Nl^2}} \; . \tag{128}$$

The average end-to-end distance mentioned above is the root-mean-square of this distribution. Non-ideal polymers are polymers which have, like non-ideal gases, interactions with neighbouring molecules or with solvents. In practice, polymers follow the ideal behaviour quite rarely: polymers are ideal only in certain solvents and in melts.

If a polymer is *stretched*, the molecules must rearrange. This changes their entropy and produces an elastic force f that tries to inhibit the stretching. For an ideal polymer, the force is not due to molecular interactions, but is entropic in nature. Therefore the force can be deduced from the *free energy*

$$F \sim -T \ln P(R) \tag{129}$$

of the polymer: the force is then simply given as $f = \partial F(R)/\partial R$. For an ideal polymer, using its probability distribution, the force turns out to be proportional to the stretched length. Thus the spring constant k can be introduced, given by Challenge 164 e

$$k = \frac{f}{R} = \frac{3T}{Ll} \; . \tag{130}$$

We thus deduced a material property, the spring constant k, from the simple idea that polymers are made of long, flexible molecules. The proportionality to temperature T is a result of the entropic nature of the force; the dependence on L shows that longer molecules are more easy to stretch. For a real, non-ideal polymer, the calculation is more complex, but the procedure is the same. Indeed, this is the mechanism at the basis of the elasticity of rubber.

Using the free energy of polymer conformations, we can calculate the material properties of macromolecules in many other situations, such as their reaction to compression, their volume change in the melt, their interactions in solutions, the effect of branched molecules, etc. This is a vast field of knowledge on its own, which we do not pursue here. Modern research topics include the study of knotted polymers and the study of polymer mixtures. Extensive computer calculations and experiments are regularly compared.

Do polymers have some relation to the structure of physical space? The issue is open.

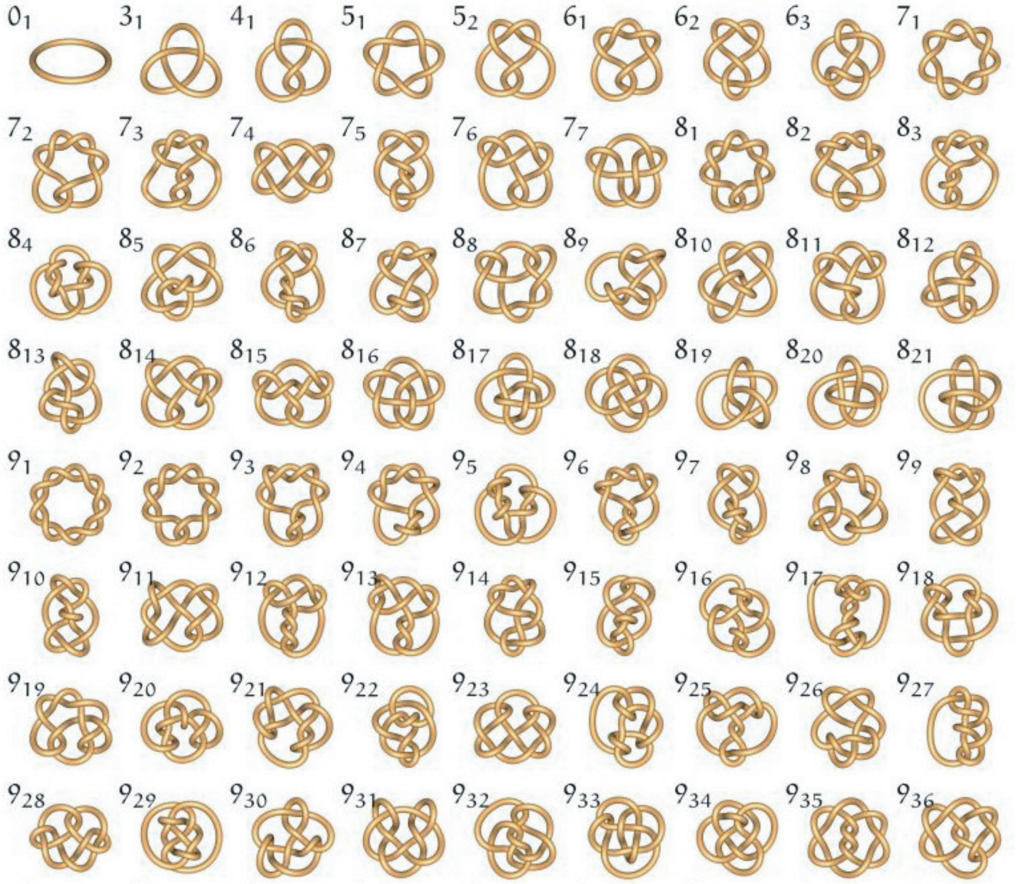

FIGURE 170 The knot diagrams for the simplest prime knots (© Robert Scharein).

It is sure, however, that polymers are often knotted and linked.

KNOTS AND LINKS

> “Don’t touch this, or I shall tie your fingers into knots!”
> (Nasty, but surprisingly efficient child education technique.)

Knots and their generalization are central to the study of wobbly object motion. A *(mathematical) knot* is a closed piece of rubber string, i.e., a string whose ends have been glued together, which cannot be deformed into a circle or a simple loop. The simple loop is also called the *trivial knot*.

Knots are of importance in the context of this chapter as they visualize the limitations of the motion of wobbly entities. In addition, we will discover other reasons to study Ref. 255 knots later on. In this section, we just have a bit of fun.*

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* Beautiful illustrations and detailed information about knots can be found on the Knot Atlas website at katlas.math.toronto.edu and at the KnotPlot website at www.knotplot.com.

In 1949, Schubert proved that every knot can be decomposed in a unique way as sum Ref. 256 of prime knots. Knots thus behave similarly to integers.

If prime knots are ordered by their crossing numbers, as shown in Figure 170, the trivial knot (0_1) is followed by the trefoil knot (3_1) and by the figure-eight knot (4_1). The figure only shows *prime knots*, i.e., knots that cannot be decomposed into two knots that are connected by two parallel strands. In addition, the figure only shows one of the often possible two mirror versions.

Together with the search for invariants, the tabulation of knots – a result of their classification – is a modern mathematical sport. Flat knot diagrams are usually ordered by the minimal number of crossings as done in Figure 170. There is 1 knot with zero, 1 with Ref. 256 three and 1 with four crossings (not counting mirror knots); there are 2 knots with five and 3 with six crossings, 7 knots with seven, 21 knots with eight, 41 with nine, 165 with ten, 552 with eleven, 2176 with twelve, 9988 with thirteen, 46 972 with fourteen, 253 293 with fifteen and 1 388 705 knots with sixteen crossings.

The mirror image of a knot usually, but not always, is different from the original. If you want a challenge, try to show that the trefoil knot, the knot with three crossings, is different from its mirror image. The first mathematical proof was by Max Dehn in 1914.

Antiknots do not exist. An *antiknot* would be a knot on a rope that cancels out the corresponding knot when the two are made to meet along the rope. It is easy to prove that this is impossible. We take an infinite sequence of knots and antiknots on a string, Ref. 257 $K - K + K - K + K - K....$ On the one hand, we could make them disappear in this way $K-K+K-K+K-K... = (K-K)+(K-K)+(K-K)... = 0$. On the other hand, we could do the same thing using $K-K+K-K+K-K... = K+(-K+K)+(-K+K)+(-K+K)... = K$. The only knot K with an antiknot is thus the unknot $K = 0$.*

How do we describe such a knot through the telephone? Mathematicians have spent a lot of time to figure out smart ways to achieve it. The obvious way is to flatten the knot onto a plane and to list the position and the type (below or above) of the crossings. (See Figure 171.) But what is the *simplest* way to describe knots by the telephone? The task is not completely finished, but the end is in sight. Mathematicians do not talk about ‘telephone messages’, they talk about *knot invariants*, i.e., about quantities that do not depend on the precise shape of the knot. At present, the best description of knots use polynomial invariants. Most of them are based on a discovery by Vaughan Jones in 1984. However, though the Jones polynomial allows us to uniquely describe most simple knots, it fails to do so for more complex ones. But the Jones polynomial finally allowed mathematicians to prove that a diagram which is alternating and eliminates nugatory crossings (i.e., if it is ‘reduced’) is indeed one which has minimal number of crossings. The polynomial also allows showing that any two reduced alternating diagrams are related by a sequence of flypes.

In short, the simplest way to describe a knot through the telephone is to give its Kauffman polynomial, together with a few other polynomials.

Since knots are stable in time, a knotted line in three dimensions is equivalent to a knotted surface in space-time. When thinking in higher dimensions, we need to be careful. Every knot (or knotted line) can be untied in four or more dimensions. However,

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* This proof does *not* work when performed with numbers; we would be able to deduce $1 = 0$ by setting K=1. Why is this proof valid with knots but not with numbers? Challenge 165 s

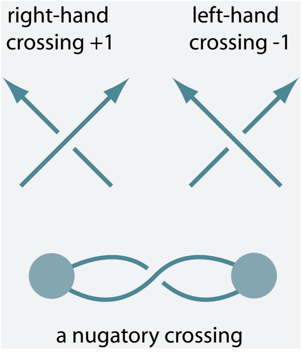

FIGURE 171 Crossing types in knots.

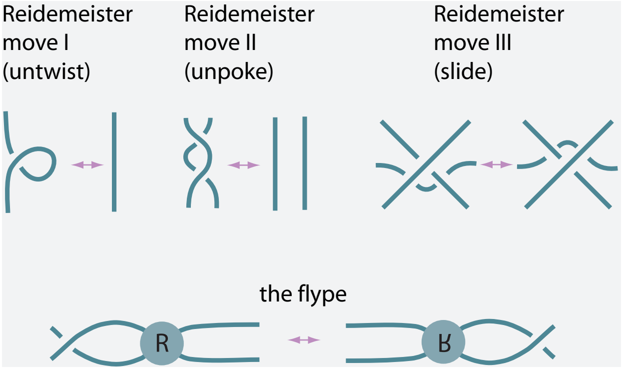

FIGURE 172 The Reidemeister moves and the flype.

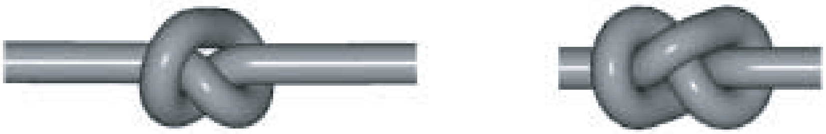

FIGURE 173 A tight open overhand knot and a tight open figure-eight knot (© Piotr Pieranski)

there is no surface *embedded* in four dimensions which has as $t = 0$ slice a knot, and as $t = 1$ slice the circle. Such a surface embedding needs at least five dimensions.

In higher dimensions, knots are thus possible *only* if n-spheres are tied instead of circles; for example, as just said, 2-spheres can be tied into knots in 4 dimensions, 3-spheres in 5 dimensions and so forth.

THE HARDEST OPEN PROBLEMS THAT YOU CAN TELL YOUR GRANDMOTHER

Even though mathematicians have achieved good progress in the classification of knots, surprisingly, they know next to nothing about the *shapes* of knots. Here are a few problems that are still open today:

— This is the simplest unsolved knot problem: Imagine an ideally wobbly rope, that is, a rope that has the same radius everywhere, but whose curvature can be changed as one prefers. Tie a trefoil knot into the rope. By how much do the ends of the rope get nearer? In 2006, there are only numerical estimates for the answer: about 10.1 radiuses. There is no formula yielding the number 10.1. Alternatively, solve the following problem: what is the rope length of a closed trefoil knot? Also in this case, only numerical values are known – about 16.33 radiuses – but no exact formula. The same is valid for any other knot, of course. Challenge 166 r

— For mathematical knots, i.e., *closed* knots, the problem is equally unsolved. For example: the ropelength of the tight trefoil knot is known to be around 16.33 diameters, and that of the figure-eight knot about 21.04 diameters. For beautiful visualizations of Ref. 259

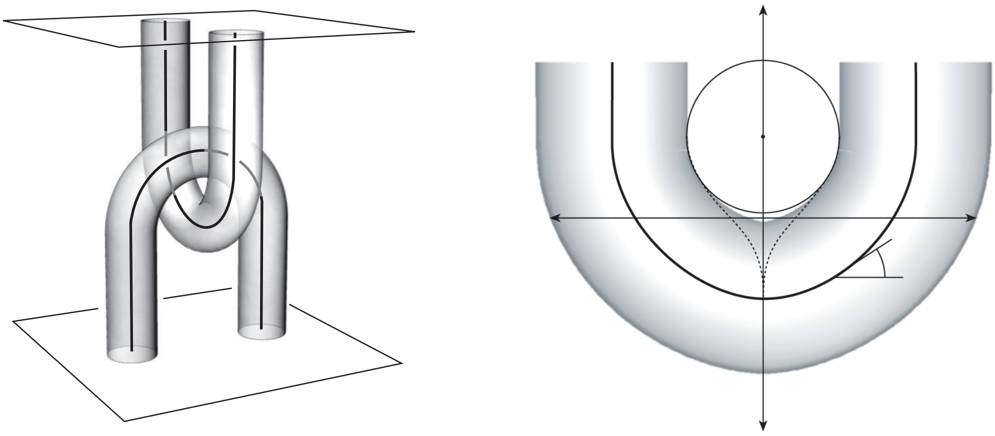

FIGURE 174 The ropelength problem for the simple clasp, and the candidate configuration that probably minimizes ropelength, leaving a gap between the two ropes (© Jason Cantarella).

the tightening process, see the animations on the website www.jasoncantarella.com/movs. But what is the formula giving the ropelength values? Nobody knows, because the precise shape of the trefoil knot – or of any other knot – is unknown. Lou Kauffman has a simple comment for the situation: 'It is a scandal of mathematics!'

- Mathematicians also study more general structures than knots. *Links* are the generalization of knots to several closed strands. *Braids* and *long links* are the generalization of links to *open* strands. Now comes the next surprise, illustrated in Figure 174. Even for two ropes that form a simple *clasp*, i.e., two linked letters 'U', the ropelength problem is unsolved – and there is not even a knot involved! In fact, in 2004, Jason Cantarella and his colleagues have presented a candidate for the shape that minimizes ropelength. Astonishingly, the candidate configuration leaves a small gap between the two ropes, as shown in Figure 174. Ref. 260

In short, the shape of knots is a research topic that has barely taken off. Therefore we have to leave these questions for a future occasion.

Curiosities and fun challenges on knots and wobbly entities

Knots appear rarely in nature. For example, tree branches or roots do not seem to grow many knots during the lifetime of a plant. How do plants avoid this? In other words, why are there no knotted bananas or knotted flower stems in nature? Challenge 167 r

Recent research has also explored how octopusses avoid knots in their arms. It was found that the arms secrete a chemical substance that prevents arms or parts of the arms from sticking together.

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Not only knot, also links can be classified. The simplest links, i.e., the links for which the simplest configuration has the smallest number of crossings, are shown in Figure 175.

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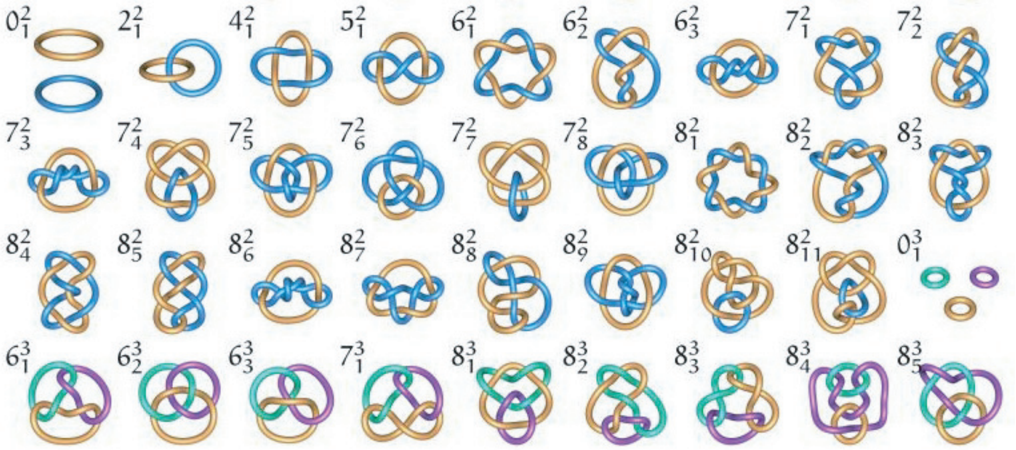


FIGURE 175 The diagrams for the simplest links with two and three components (© Robert Scharein).

FIGURE 176 A hagfish tied into a knot (© Christine Ortlepp).

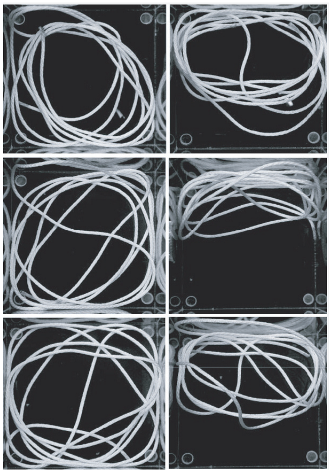


FIGURE 177 How apparent order for long rope coils (left) changes over time when shaking the container (right) (© 2007 PNAS).

The physics of human swimming is fascinating. To learn the details on how to move in order to swim as rapidly as possible, explore the wonderful website coachsci.sdsu.edu/swim by Brent Rushall. He tells how to move the arms, the trunk and the legs and he shows how champions perform these movements. Rushall also tells about the bizarre theories that are aired in the field of swimming, such as the mistaken idea that lift plays a role in human swimming.

* *

A famous type of eel, the *knot fish Myxine glutinosa*, also called hagfish or slime eel, is
Ref. 262 able to make a knot in his body and move this knot from head to tail. Figure 176 shows

an example. The hagfish uses this motion to cover its body with a slime that prevents predators from grabbing it; it also uses this motion to escape the grip of predators, to get rid of the slime after the danger is over, and to push against a prey it is biting in order to extract a piece of meat. All studied knot fish form only left handed trefoil knots, by the way; this is another example of chirality in nature.

* *

Proteins, the molecules that make up many cell structures, are chains of aminoacids. Ref. 261 It seems that very few proteins are knotted, and that most of these form trefoil knots. However, a figure-eight knotted protein has been discovered in 2000 by William Taylor.

* *

One of the most incredible discoveries of recent years is related to knots in DNA molecules. The *DNA molecules* inside cell nuclei can be hundreds of millions of base pairs long; they regularly need to be packed and unpacked. When this is done, often the same happens as when a long piece of rope or a long cable is taken out of a closet.

It is well known that you can roll up a rope and put it into a closet in such a way that it looks orderly stored, but when it is pulled out at one end, a large number of knots Ref. 263 is suddenly found. In 2007, this effects was finally explored in detail. Strings of a few metres in length were put into square boxes and shaken, in order to speed up the effect. The result, shown partly in Figure 177, was astonishing: almost every imaginable knot – up to a certain complexity that depends on the length and flexibility of the string – was formed in this way.

To make a long story short, the tangling also happens to nature when it unpacks DNA in cell nuclei. Life requires that DNA molecules move inside the cell nucleus without hindrance. So what does nature do? Nature takes a simpler approach: when there are unwanted crossings, it cuts the DNA, moves it over and puts the ends together again. In cell nuclei, there are special enzymes, the so-called topoisomerases, which perform this process. The details of this fascinating process are still object of modern research.

* *

The great mathematician Carl-Friedrich Gauß – often written as 'Gauss' in English – was the first person to ask what happens when an electrical current I flows along a wire Ref. 264 A that is *linked* with a wire B. He discovered a beautiful result by calculating the effect of the magnetic field of one wire onto the other:

$$\frac{1}{4\pi I}\int_A \mathrm{d}\boldsymbol{x}_A \cdot \boldsymbol{B}_B = \frac{1}{4\pi}\int_A \mathrm{d}\boldsymbol{x}_A \cdot \int_B \mathrm{d}\boldsymbol{x}_B \times \frac{(\boldsymbol{x}_A - \boldsymbol{x}_B)}{|\boldsymbol{x}_A - \boldsymbol{x}_B|^3} = n \,, \qquad (131)$$

where the integrals are performed along the wires. Gauss found that the number n does not depend on the precise shape of the wires, but only on the way they are linked. Deforming the wires does not change the resulting number n. Mathematicians call such a number a *topological invariant*. In short, Gauss discovered a physical method to calculate a mathematical invariant for links; the research race to do the same for other invariants, and in particular for knots and braids, is still going on today.

In the 1980s, Edward Witten was able to generalize this approach to include the nuc-

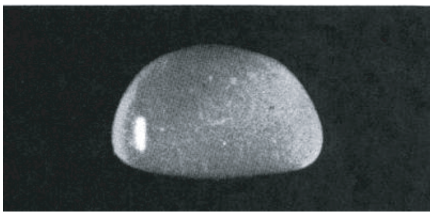


FIGURE 178 A large raindrop falling downwards.

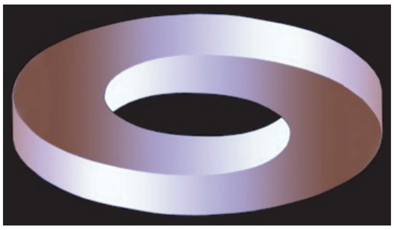


FIGURE 179 Is this possible?

lear interactions, and to define more elaborate knot invariants, a discovery that brought him the Fields medal.

∗ ∗

If we move along a knot and count the crossings where we stay above and subtract the number of crossings where we pass below, we get a number called the *writhe* of the knot. It is not an invariant, but usually a tool in building them. Indeed, the writhe is not necessarily invariant under one of the three Reidemeister moves. Can you see which one, using Figure 172? However, the writhe is invariant under flypes. Challenge 168 e

∗ ∗

Modern knot research is still a topic with many open questions. A recent discovery is the *quasi-quantization of three-dimensional writhe* in tight knots. Many discoveries are still expected in the domain of geometric knot theory. Ref. 265

∗ ∗

Challenge 169 e There are two ways to tie your shoes. Can you find them?

∗ ∗

Challenge 170 s What is the shape of raindrops? Try to picture it. However, use your reason, not your prejudice! By the way, it turns out that there is a maximum size for raindrops, with a value of about 4 mm. The shape of such a large raindrop is shown in Figure 178. Can you imagine where the limit comes from? Ref. 266

For comparison, the drops in clouds, fog or mist are in the range of 1 to 100 μm, with a peak at 10 to 15 μm. In situations where all droplets are of similar size and where light is scattered only once by the droplets, one can observe coronae, glories or fogbows. Vol. III, page 166 Vol. III, page 131

∗ ∗

Challenge 171 s What is the entity shown in Figure 179 – a knot, a braid or a link?

∗ ∗

Challenge 172 d Can you find a way to classify tie knots?

∗ ∗

Challenge 173 s Are you able to find a way to classify the way shoe laces can be threaded?

FIGURE 180 A flying snake, *Chrysopelea paradisii*, performing the feat that gave it its name (QuickTime film © Jake Socha).

* *

Ref. 267 A striking example of how wobbly entities can behave is given in Figure 180. There is indeed a family of snakes that like to jump off a tree and sail through the air to a neighbouring tree. Both the jump and the sailing technique have been studied in recent years. The website www.flyingsnake.org by Jake Socha provides additional films. His fascinating publications tell more about these intriguing reptiles.

* *

Vol. I, page 327 When a plane moves at supersonic speed through humid air, sometimes a conical cloud forms and moves with the plane. How does this cloud differ from the ones studied above?

Challenge 174 e

* *

Challenge 175 ny One of the toughest challenges about clouds: is it possible to make rain on demand? So far, there are almost no positive results. Inventing a method, possibly based on hygroscopic salt injection or with the help of lasers, will be a great help to mankind.

* *

Do knots have a relation to elementary particles? The question is about 150 years old. It was first investigated by William Thomson-Kelvin and Peter Tait in the late nineteenth century. So far, no proof of a relation has been found. Knots might be of importance at Planck scales, the smallest dimensions possible in nature. We will explore how knots and the structure of elementary particles might be related in the last volume of this adventure.

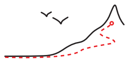
Summary on wobbly objects

Vol. I, page 316 We can sum up the possible motions of extended systems in a few key themes. In earlier chapters we studied waves, solitons and interpenetration. These observations are described by wave equations. In this chapter we explored the way to move through shape change, explored eversion, studied vortices, fluids, polymers, knots and their rearrangement, and explored the motion of dislocations in solids. We found that shape change is described by gauge theory, eversion is described by space-duality, vortices follow the Schrödinger equation, fluids and polymers resemble general relativity and black holes, knot shapes are hard to calculate and dislocations behave relativistically.

The motion of wobbly objects is a neglected topic in textbooks on motion. Research is progressing at full speed; it is expected that many beautiful analogies with traditional physics will be discovered in the near future. For example, in this chapter we have not explored any possible analogy for the motion of light. Similarly, including quantum theory into the description of wobbly bodies' motion remains a fascinating issue for anybody aiming to publish in a new field.

Challenge 176 r

In summary, we found that wobbly entities can reproduce most fields of modern physics. Are there wobbly entities that reproduce *all* of modern physics? We will explore the question in the last volume.



CHAPTER 12

QUANTUM PHYSICS IN A NUTSHELL – AGAIN

Compared to classical physics, quantum theory is definitely more complex. The basic idea however, is simple: in nature there is a smallest change, or a smallest action, with the value $\hbar = 1.1 \cdot 10^{-34}$ Js. The smallest action value leads to all the strange observations made in the microscopic domain, such as wave behaviour of matter, indeterminacy relations, decoherence, randomness in measurements, indistinguishability, quantization of angular momentum, tunnelling, pair creation, decay, particle reactions and virtual particle exchange.

QUANTUM FIELD THEORY IN A FEW SENTENCES

> “Deorum offensae diis curae.”
>
> Voltaire, *Traité sur la tolérance.*

All of quantum theory can be resumed in a few sentences.

▷ In nature, actions smaller than $\hbar = 1.1 \cdot 10^{-34}$ Js are not observed.

The existence of a smallest action in nature directly leads to the main lesson we learned about motion in the quantum part of our adventure:

▷ If something moves, it is made of quantons, or quantum particles.

▷ There are *elementary* quantum particles.

These statements apply to everything, thus to all objects and to all images, i.e., to matter and to radiation. Moving stuff is made of *quantons*. Stones, water waves, light, sound waves, earthquakes, tooth paste and everything else we can interact with is made of moving quantum particles. Experiments show:

▷ All intrinsic object properties observed in nature – such as electric charge, weak charge, colour charge, spin, parity, lepton number, etc., with the only exception of mass – appear as *integer* numbers of a smallest unit; in composed systems they either add or multiply.

Page 261

> ▷ An *elementary quantum particle* or *elementary quanton* is a countable entity, smaller than its own Compton wavelength, described by energy–momentum, mass, spin, C, P and T parity, electric charge, colour, weak isospin, isospin, strangeness, charm, topness, beauty, lepton number and baryon number.

All moving entities are made of elementary quantum particles. To see how deep this result is, you can apply it to all those moving entities for which it is usually forgotten, such as ghosts, spirits, angels, nymphs, daemons, devils, gods, goddesses and souls. You can check yourself what happens when their particle nature is taken into account.

Challenge 177 e

Quantum particles are never at rest, cannot be localized, move probabilistically, behave as particles or as waves, interfere, can be polarized, can tunnel, are indistinguishable, have antiparticles, interact locally, define length and time scales and they limit measurement precision.

Quantum particles come in two types:

> ▷ Matter is composed of *fermions:* quarks and leptons. There are 6 quarks that make up nuclei, and 6 leptons – 3 charged leptons, including the electron, and 3 uncharged neutrinos. Elementary fermions have spin 1/2 and obey the Pauli exclusion principle.
>
> ▷ Radiation is due to the three gauge interactions and is composed of *bosons*: photons, the weak vector bosons and the 8 gluons. These elementary bosons all have spin 1.

In our adventure, we wanted to know what matter and interactions are. Now we know: they are due to elementary quantum particles. The exploration of motion inside matter, including particle reactions and virtual particle exchange, showed us that matter is made of a *finite number* of elementary quantum particles. Experiments show:

> ▷ In flat space, elementary particles interact in one of three ways: there is the *electromagnetic interaction*, the *strong nuclear interaction* and the *weak nuclear interaction*.
>
> ▷ The three interactions are *exchanges of virtual bosons*.
>
> ▷ The three interactions are described by the gauge symmetries U(1), SU(3) and a broken, i.e., approximate SU(2) symmetry.

The three gauge symmetries fix the Lagrangian of every physical system in flat space-time. The most simple description of the Lagrangians is with the help of Feynman diagrams and the gauge groups.

> ▷ In all interactions, energy, momentum, angular momentum, electric charge, colour charge, CPT parity, lepton number and baryon number are conserved.

The list of conserved quantities implies:

▷ Quantum *field* theory is the part of quantum physics that includes the description of *particle transformations*.

The possibility of particle transformations – including particle reactions, particle emission and particle absorption – results from the existence of a minimum action and of a maximum speed in nature. Emission of light, radioactivity, the burning of the Sun and the history of the composite matter we are made of are due to particle transformations. Vol. IV, page 31

Due to the possibility of particle transformations, quantum field theory introduces a limit for the localization of particles. In fact, any object of mass m can be localized only within intervals of the Compton wavelength

$$\lambda_\mathrm{C} = \frac{h}{mc} = \frac{2\pi\hbar}{mc} , \tag{132}$$

where c is the speed of light. At the latest at this distance we have to abandon the classical description and use quantum field theory. If we approach the Compton wavelength, particle transformations become so important that classical physics and even simple quantum theory are not sufficient.

Quantum electrodynamics is the quantum field description of electromagnetism. It includes and explains all particle transformations that involve photons. The Lagrangian of QED is determined by the electromagnetic gauge group U(1), the requirements of space-time (Poincaré) symmetry, permutation symmetry and renormalizability. The latter requirement follows from the continuity of space-time. Through the effects of virtual particles, QED describes electromagnetic decay, lamps, lasers, pair creation, Unruh radiation for accelerating observers, vacuum energy and the Casimir effect, i.e., the attraction of neutral conducting bodies. Particle transformations due to quantum electrodynamics also introduce corrections to classical electrodynamics; among others, particle transformations produce small departures from the superposition principle for electromagnetic fields, including the possibility of photon-photon scattering.

The theory of *weak nuclear interaction* describes parity violation, quark mixings, neutrino mixings, massive vector bosons and the Higgs field for the breaking of the weak SU(2) gauge symmetry. The weak interaction explains a large part of radioactivity, including the heat production inside the Earth, and describes processes that make the Sun shine.

Quantum chromodynamics, the field theory of the strong nuclear interaction, describes all particle transformations that involve gluons. At fundamental scales, the strong interaction is mediated by eight elementary gluons. At larger, femtometre scales, the strong interaction effectively acts through the exchange of spin 0 pions, is strongly attractive, and leads to the formation of atomic nuclei. The strong interaction determines nuclear fusion and fission. Quantum chromodynamics, or QCD, explains the masses of mesons and baryons through their description as bound quark states. Page 200

By including particle transformations, quantum field theory provides a common basis of concepts and descriptions to materials science, nuclear physics, chemistry, biology, medicine and to most of astronomy. For example, the same concepts allow us to answer questions such as why water is liquid at room temperature, why copper is red, why the rainbow is coloured, why the Sun and the stars continue to shine, why there are about

110 elements, where a tree takes the material to make its wood and why we are able to move our right hand at our own will. Quantum theory explains the origin of material properties and the origin of the properties of life.

Quantum field theory describes all material properties, be they mechanical, optical, electric or magnetic. It describes all waves that occur in materials, such as sound and phonons, magnetic waves and magnons, light, plasmons, and all localized excitations. Quantum field theory also describes collective effects in matter, such as superconductivity, semiconductor effects and superfluidity. Finally, quantum field theory describes all interactions between matter and radiation, from colour to antimatter creation.

Quantum field theory also clarifies that the particle description of nature, including the conservation of particle number – defined as the difference between particles and antiparticles – follows from the possibility to describe interactions *perturbatively*. A perturbative description of nature is possible only at low energies. At extremely high energies, higher than those observed in experiments, the situation is expected to change and non-perturbative effects should come into play. These situations will be explored in the next volume.

ACHIEVEMENTS IN PRECISION

Classical physics is unable to predict *any* property of matter. Quantum field theory predicts all properties of matter, and to the full number of digits – sometimes thirteen – that can be measured today. The precision is usually *not* limited by the inaccuracy of theory, it is limited by the measurement accuracy. In other words, the agreement between quantum field theory and experiment is only limited by the amount of money one is willing to spend. Table 25 shows some predictions of classical physics and of quantum field theory. The predictions are deduced from the properties of nature collected in the millennium list, which is given in the next section.

TABLE 25 Selected comparisons between classical physics, quantum theory and experiment.

OBSERVABLE	CLASSICAL PREDICTION	PREDICTION OF QUANTUM THEORY[a]	MEASUREMENT	COST ESTIMATE[b]
Simple motion of bodies				
Indeterminacy	0	$\Delta x \Delta p \geqslant \hbar/2$	$(1 \pm 10^{-2})\,\hbar/2$	10 k€
Matter wavelength	none	$\lambda p = 2\pi\hbar$	$(1 \pm 10^{-2})\,\hbar$	10 k€
Tunnelling rate in α decay	0	$1/\tau$ is finite	$(1 \pm 10^{-2})\,\tau$	5 k€
Compton wavelength	none	$\lambda_{\mathrm{c}} = h/m_{\mathrm{e}}c$	$(1 \pm 10^{-3})\,\lambda$	20 k€
Pair creation rate	0	σE	agrees	100 k€
Radiative decay time in hydrogen	none	$\tau \sim 1/n^3$	(1 ± 10^{-2})	5 k€
Smallest angular momentum	0	$\hbar/2$	$(1 \pm 10^{-6})\,\hbar/2$	10 k€
Casimir effect/pressure	0	$p = (\pi^2 \hbar c)/(240 r^4)$	(1 ± 10^{-3})	30 k€

TABLE 25 (Continued) Selected comparisons between classical physics, quantum theory and experiment.

OBSERVABLE	CLASSICAL PREDICTION	PREDICTION OF QUANTUM THEORY[a]	MEASUREMENT	COST ESTIMATE[b]
Colours of objects				
Spectrum of hot objects	diverges	$\lambda_{\mathrm{max}} = hc/(4.956\,kT)$	$(1 \pm 10^{-4})\ \Delta\lambda$	10 k€
Lamb shift	none	$\Delta\lambda = 1057.86(1)\,\mathrm{MHz}$	$(1 \pm 10^{-6})\,\Delta\lambda$	50 k€
Rydberg constant	none	$R_\infty = m_e c\alpha^2/2h$	$(1 \pm 10^{-9})\,R_\infty$	50 k€
Stefan–Boltzmann constant	none	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	$(1 \pm 3 \cdot 10^{-8})\,\sigma$	20 k€
Wien's displacement constant	none	$b = \lambda_{\mathrm{max}} T$	$(1 \pm 10^{-5})\,b$	20 k€
Refractive index of water	none	1.34	a few %	1 k€
Photon-photon scattering	0	from QED: finite	agrees	50 M€
Laser radiation exists	no	yes	agrees	10€
Particle and interaction properties				
Electron gyromagnetic ratio	1 or 2	2.002 319 304 3(1)	2.002 319 304 3737(82)	30 M€
Z boson mass	none	$m_Z^2 = m_W^2(1 + \sin\theta_W^2)$	$(1 \pm 10^{-3})\,m_Z$	100 M€
Proton mass	none	$(1 \pm 5\,\%)\,m_p$	m_p =1.67 yg	1 M€
Proton lifetime	≈ 1 μs	∞	$> 10^{35}$ a	100 M€
Chemical reaction rate	0	from QED	correct within errors	2 k€
Composite matter properties				
Atom lifetime	≈ 1 μs	∞	$> 10^{20}$ a	1 €
Molecular size	none	from QED	within 10^{-3}	20 k€
Von Klitzing constant	∞	$h/e^2 = \mu_0 c/2\alpha$	$(1 \pm 10^{-7})\,h/e^2$	1 M€
AC Josephson constant	0	$2e/h$	$(1 \pm 10^{-6})\,2e/h$	5 M€
Heat capacity of metals at 0 K	25 J/K	0	$< 10^{-3}$ J/K	10 k€
Heat capacity of diatomic gas at 0 K	25 J/K	0	$< 10^{-3}$ J/K	10 k€
Water density	none	1000.00 kg/m^3 at 4°C	agrees	10 k€
Minimum electr. conductivity	0	$G = 2e^2/\hbar$	$G(1 \pm 10^{-3})$	3 k€
Ferromagnetism	none	exists	exists	2 €
Superfluidity	none	exists	exists	200 k€
Bose–Einsein condensation	none	exists	exists	2 M€

TABLE 25 (Continued) Selected comparisons between classical physics, quantum theory and experiment.

Observable	Classical prediction	Prediction of quantum theory[a]	Measurement	Cost estimate[b]
Superconductivity (metal)	none	exists	exists	100 k€
Superconductivity (high T)	none	*none yet*	exists	100 k€

a. All these predictions are calculated from the fundamental quantities given in the millennium list.

b. Sometimes the cost for the calculation of the prediction is higher than that of the experimental observation. (Can you spot the examples?) The sum of the two is given. Challenge 178 s

We notice that the values predicted by quantum theory do not differ from the measured ones. In contrast, classical physics does not allow us to calculate any of the observed values. This shows the progress that quantum physics has brought in the description of nature.

In short, in the microscopic domain quantum theory is in *perfect* correspondence with nature; despite prospects of fame and riches, despite the largest number of researchers ever, no contradiction with observation has been found yet. But despite this impressive agreement, there still are *unexplained* observations; they form the so-called *millennium list*.

WHAT IS UNEXPLAINED BY QUANTUM THEORY AND GENERAL RELATIVITY?

The material gathered in this quantum part of our mountain ascent, together with the earlier summary of general relativity, allows us to describe *all* observed phenomena connected to motion. For the first time, there are no known differences between theory and practice. Vol. II, page 287

Despite the precision of the description of nature, some things are missing. Whenever we ask 'why?' about an observation and continue doing so after each answer, we arrive at one of the *unexplained* properties of nature listed in Table 26. The table lists all issues about fundamental motion that were unexplained in the year 2000, so that we can call it the *millennium list* of open problems.

TABLE 26 The millennium list: *everything* the standard model and general relativity *cannot* explain; thus, also the list of the *only* experimental data available to test the final, unified description of motion.

Observable	Property unexplained since the year 2000
Local quantities unexplained by the standard model: particle properties	
$\alpha = 1/137.036(1)$	the low energy value of the electromagnetic coupling or fine structure constant
α_w or θ_w	the low energy value of the weak coupling constant or the value of the weak mixing angle

TABLE 26 (Continued) *Everything* the standard model and general relativity *cannot* explain.

Observable	Property unexplained since the year 2000
α_s	the value of the strong coupling constant at one specific energy value
m_q	the values of the 6 quark masses
m_l	the values of 6 lepton masses
m_W	the value of the mass of the W vector boson
m_H	the value of the mass of the scalar Higgs boson
$\theta_{12}, \theta_{13}, \theta_{23}$	the value of the three quark mixing angles
δ	the value of the CP violating phase for quarks
$\theta^{\nu}_{12}, \theta^{\nu}_{13}, \theta^{\nu}_{23}$	the value of the three neutrino mixing angles
$\delta^{\nu}, \alpha_1, \alpha_2$	the value of the three CP violating phases for neutrinos
$3 \cdot 4$	the number of fermion generations and of particles in each generation
J, P, C, etc.	the origin of all quantum numbers of each fermion and each boson
Concepts unexplained by the standard model	
$c, \hbar, k$	the origin of the invariant Planck units of quantum field theory
$3 + 1$	the number of dimensions of physical space and time
SO(3,1)	the origin of Poincaré symmetry, i.e., of spin, position, energy, momentum
Ψ	the origin and nature of wave functions
$S(n)$	the origin of particle identity, i.e., of permutation symmetry
Gauge symmetry	the origin of the gauge groups, in particular:
U(1)	the origin of the electromagnetic gauge group, i.e., of the quantization of electric charge, of the vanishing of magnetic charge, and of minimal coupling
SU(2)	the origin of weak interaction gauge group, its breaking and P violation
SU(3)	the origin of strong interaction gauge group and its CP conservation
Renorm. group	the origin of renormalization properties
$\delta W = 0$	the origin of the least action principle in quantum theory
$W = \int L_{SM}\, dt$	the origin of the Lagrangian of the standard model of particle physics
Global quantities unexplained by general relativity and cosmology	
0	the observed flatness, i.e., vanishing curvature, of the universe
$1.2(1) \cdot 10^{26}$ m	the distance of the horizon, i.e., the ‘size’ of the universe (if it makes sense)
$\rho_{de} = \Lambda c^4/(8\pi G) \approx 0.5\,\mathrm{nJ/m^3}$	the value and nature of the observed vacuum energy density, dark energy or cosmological constant
$(5 \pm 4) \cdot 10^{79}$	the number of baryons in the universe (if it makes sense), i.e., the average visible matter density in the universe
ρ_{dm}	the density and nature of dark matter
$f_0(1, ..., c.\ 10^{90})$	the initial conditions for c. 10^{90} particle fields in the universe (if or as long as they make sense), including the homogeneity and isotropy of matter distribution, and the density fluctuations at the origin of galaxies
Concepts unexplained by general relativity and cosmology	
c, G	the origin of the invariant Planck units of general relativity

TABLE 26 (Continued) *Everything* the standard model and general relativity *cannot* explain.

Observable	Property unexplained since the year 2000
$\mathrm{R} \times \mathrm{S}^3$	the observed topology of the universe
$G^{\mu\nu}$	the origin and nature of curvature, the metric and horizons
$\delta W = 0$	the origin of the least action principle in general relativity
$W = \int L_{\mathrm{GR}} \mathrm{d}t$	the origin of the Lagrangian of general relativity

The millennium list has several notable aspects. First of all, neither quantum mechanics nor general relativity explain any property unexplained in the other field. The two theories do not help each other; the unexplained parts of both fields simply add up. Secondly, both in quantum theory and in general relativity, motion still remains the change of position with time. In short, so far, we did not achieve our goal: we still do not understand motion! We are able to describe motion with full precision, but we still do not know what it is. Our basic questions remain: What are time and space? What is mass? What is charge and what are the other properties of objects? What are fields? Why are all the electrons the same?

Page 316 We also note that the millennium list of open questions, Table 26, contains extremely *different* concepts. This means that at this point of our walk there is *a lot* we do not understand. Finding the answers will require effort.

On the other hand, the millennium list of unexplained properties of nature is also *short*. The description of nature that our adventure has produced so far is concise and precise. No discrepancies from experiments are known. In other words, we have a good description of motion *in practice*. Going further is unnecessary if we only want to improve measurement precision. Simplifying the above list is mainly important from the *conceptual* point of view. For this reason, the study of physics at university often stops at this point. However, as the millennium list shows, even though we have *no* known discrepancies with experiments, we are *not* at the top of Motion Mountain.

THE PHYSICS CUBE

Page 8 Another review of the progress and of the open issues of physics, already given in the introduction, is shown in Figure 181: the *physics cube*. From the lowest corner of the cube, representing Galilean physics and related topics from everyday life, three edges – labelled c, G and $\hbar$, e, k – lead to classical gravity, special relativity and quantum theory. Each constant implies a limit to motion; in the corresponding theory, this *one* limit is taken into account, thus improving the precision of the description. From these second level theories, similar edges lead upwards to general relativity, quantum field theory and quantum theory with gravity. Each of these third level theories takes into account *two* of the limits and thus improves precision even more.* The present volume completes the third level of precision. We stress that each theory in the second and third level is *exact*,

---

* Of course, Figure 181 gives a simplified view of the history of physics. A more precise diagram might use different arrows for $\hbar$ (with k) and e, making the figure a four-dimensional cube. However, not all of its corners would have dedicated theories (can you confirm this?). Also the weak and the strong coupling constants might have to be added. The diagram would be far less appealing. And most of all, the conclusions mentioned in the text would not change. Challenge 179 e

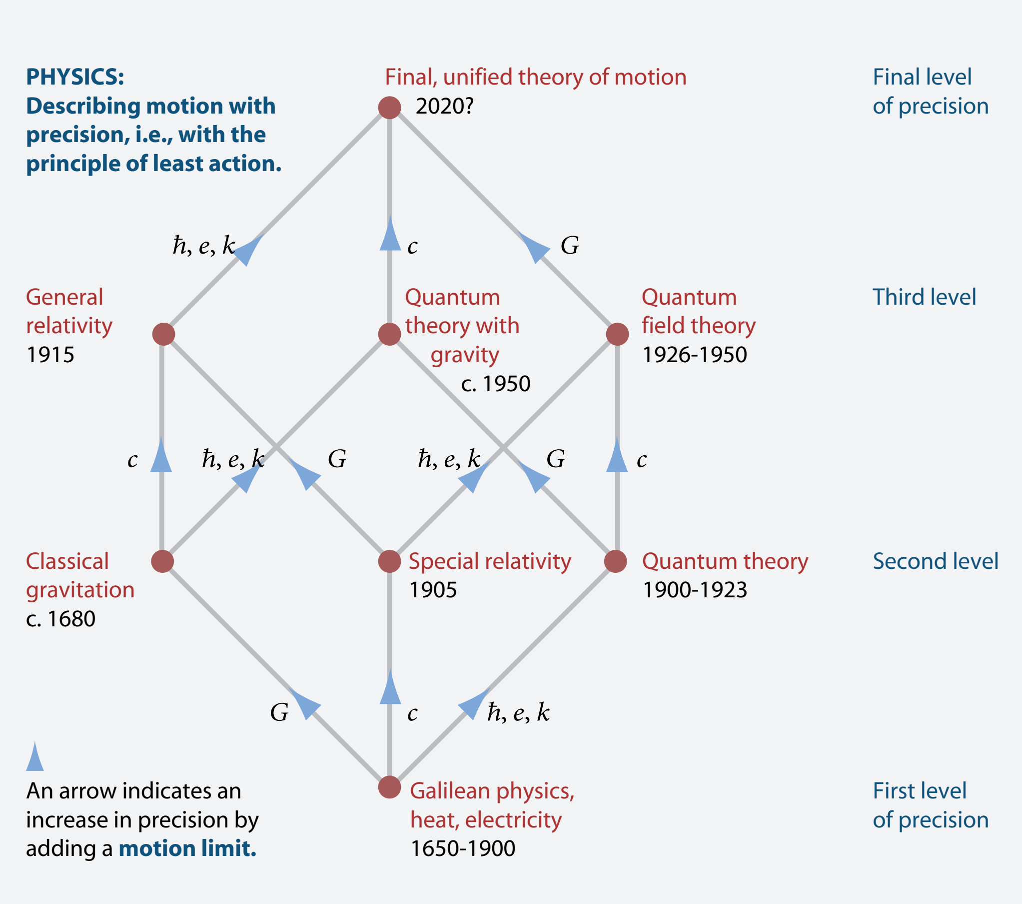


FIGURE 181 A simplified history of the description of motion in physics, by giving the limits to motion included in each description. The arrows show which constant of nature needs to be added and taken into account to reach the next level of precision. (The electric charge e is taken to represent all three discrete gauge charges.)

though only *in its domain*. And even though the limits of each domain are obvious, no differences between experiment and theory are known.

From the third level theories, the edges lead to the last missing corner: the (unified) theory of motion that takes into account *all limits* of nature. Only this theory is a complete and unified description of nature. Since we already know all limits to motion, in order to arrive at the last level, we do not need new experiments. We do not need new knowledge. We only have to advance, in the right direction, with careful thinking. And we can start from three different points. This is the topic of the last volume of our adventure.

THE INTENSE EMOTIONS DUE TO QUANTUM FIELD THEORY AND GENERAL RELATIVITY

It is sometimes deemed chic to pretend that the adventure is over at the stage we have just reached,* the third level of Figure 181. The reasoning given is as follows. If we change Page 316 the values of the unexplained constants in the millennium list of Table 26 only ever so Ref. 270 slightly, nature would look completely different from what it does. Indeed, these consequences have been studied in great detail; an overview of the connections is given in the following table.

TABLE 27 A selection of the consequences of changing the properties of nature.

Observable	Change	Result
Local quantities, from quantum theory		
α_{em}	smaller:	Only short lived, smaller and hotter stars; no Sun.
	larger:	Darker Sun, animals die of electromagnetic radiation, too much proton decay, no planets, no stellar explosions, no star formation, no galaxy formation.
	+60 %:	Quarks decay into leptons.
	+200 %:	Proton-proton repulsion makes nuclei impossible.
α_{w}	−50 %:	Carbon nucleus unstable.
	very weak:	No hydrogen, no p-p cycle in stars, no C-N-O cycle.
	+2 %:	No protons from quarks.
	$G_F m_e^2 \neq \sqrt{G m_e^2}$:	Either no or only helium in the universe.
	much larger:	No stellar explosions, faster stellar burning.
α_{s}	−9 %:	No deuteron, stars far less bright.
	−1 %:	No C resonance, no life.
	+3.4 %:	Diproton stable, faster star burning.
	much larger:	Carbon unstable, heavy nuclei unstable, widespread leukaemia.
n-p mass difference	larger:	Neutron decays in proton inside nuclei; no elements.
	smaller:	Free neutron not unstable, all protons into neutrons during big bang; no elements.
	smaller than m_e:	Protons would capture electrons, no hydrogen atoms, star life much shorter.
m_{l} changes:		

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* Actually this attitude is not new. Only the arguments have changed. Maybe the greatest physicist ever, Ref. 269 James Clerk Maxwell, already fought against this attitude over a hundred years ago: ‘The opinion seems to have got abroad that, in a few years, all great physical constants will have been approximately estimated, and that the only occupation which will be left to men of science will be to carry these measurements to another place of decimals. [...] The history of science shows that even during that phase of her progress in which she devotes herself to improving the accuracy of the numerical measurement of quantities with which she has long been familiar, she is preparing the materials for the subjugation of new regions, which would have remained unknown if she had been contented with the rough methods of her early pioneers.’

TABLE 27 (Continued) A selection of the consequences of changing the properties of nature.

Observable	Change	Result
e-p mass ratio	much different:	No molecules.
	much smaller:	No solids.
3 generations	6-8:	Only helium in nature.
	>8:	No asymptotic freedom and confinement.
Global quantities, from general relativity		
horizon size	much smaller:	No people.
baryon number	very different:	No smoothness .
	much higher:	No solar system.
Initial condition changes:		
Moon mass	smaller:	Small Earth magnetic field; too much cosmic radiation; widespread child skin cancer.
Moon mass	larger:	Large Earth magnetic field; too little cosmic radiation; no evolution into humans.
Sun's mass	smaller:	Too cold for the evolution of life.
Sun's mass	larger:	Sun too short lived for the evolution of life.
Jupiter mass	smaller:	Too many comet impacts on Earth; extinction of animal life.
Jupiter mass	larger:	Too little comet impacts on Earth; no Moon; no dinosaur extinction.
Oort cloud object number	smaller:	No comets; no irregular asteroids; no Moon; still dinosaurs.
galaxy centre distance	smaller:	Irregular planet motion; supernova dangers.
initial cosmic speed	+0.1 %:	1000 times faster universe expansion.
	−0.0001 %:	Universe recollapses after 10 000 years.
vacuum energy density	change by 10^{-55}:	No flatness.
3 + 1 dimensions	different:	No atoms, no planetary systems.
Local structures, from quantum theory		
permutation symmetry	none:	No matter.
Lorentz symmetry	none:	No communication possible.
U(1)	different:	No Huygens principle, no way to *see* anything.
SU(2)	different:	No radioactivity, no Sun, no life.
SU(3)	different:	No stable quarks and nuclei.
Global structures, from general relativity		

TABLE 27 (Continued) A selection of the consequences of changing the properties of nature.

Observable	Change	Result
topology	other:	Unknown; possibly correlated γ ray bursts or star images at the antipodes.

Note. Some researchers speculate that the whole of Table 27 can be condensed into a single sentence: if any parameter in nature is changed, the universe would either have too many or too few black holes. However, the proof of this condensed summary is not complete yet. But it is a beautiful hypothesis.

Ref. 271
Challenge 180 r

The effects of changing nature that are listed in Table 27 lead us to a profound experience: even the tiniest changes in the properties of nature are incompatible with our existence. What does this experience mean? Answering this question too rapidly is dangerous. Many have fallen into one of several traps:

- The first trap is to deduce, incorrectly, that the unexplained numbers and other properties from the millennium list do not need to or even cannot be explained, i.e., deduced from more general principles.
- The second trap is to deduce, incorrectly, that the universe has been *created* or *designed*.
- The third trap is to deduce, incorrectly, that the universe is *designed for people*.
- The fourth trap is to deduce, incorrectly, that the universe is *one of many*.

All these traps are irrational and incorrect beliefs. All these beliefs have in common that they have no factual basis, that they discourage further search and that they sell many books.

The first trap is due to a combination of pessimism and envy; it is a type of wishful thinking. But wishful thinking has no place in the study of motion. The second trap works because many physicists incorrectly speak of *fine tuning* in nature. Many researchers succumb to the belief in 'creation' and are unable to steer clear from the logical errors contained in it. We discussed them earlier on. The third trap, is often, again incorrectly, called the *anthropic principle*. The name is a mistake, because we saw that the anthropic principle is indistinguishable both from the simian principle and from the simple request that statements be based on observations. Around 2000, the third trap has even become fashionable among frustrated particle theorists. The fourth trap, the belief in *multiple universes*, is a minority view, but sells many books. Most people that hold this view are found in institutions. And that is indeed where they belong.

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Stopping our adventure, our mountain ascent, with an incorrect belief at the present stage is not different from doing so directly at the beginning. Such a choice has been taken in various societies that lacked the passion for rational investigation, and still is taken in circles that discourage the use of reason among their members. Looking for beliefs instead of looking for answers means to give up our ascent while pretending to have reached the top. Every such case is a tragedy, sometimes a small one, sometimes a larger one.

In fact, Table 27 purveys only one message: all evidence implies that we are only a *tiny part* of the universe, but that we are *linked* with all other aspects of it. Due to our small size and due to all the connections with our environment, any imagined tiny change would make us disappear, like a water droplet is swept away by large wave. Our walk has repeatedly reminded us of this smallness and dependence, and overwhelmingly does so again at this point.

In our adventure, accepting the powerful message of Table 27 is one of the most awe-inspiring, touching and motivating moments. It shows clearly how vast the universe is. It also shows how much we are dependent on many different and distant aspects of nature. Having faced this powerful experience, everybody has to make up his or her own mind on whether to proceed with the adventure or not. Of course, there is no obligation to do so. Challenge 181 s

WHAT AWAITS US?

Assuming that you have decided to continue the adventure, it is natural to ask what awaits you. The shortness of the millennium list of unexplained aspects of nature, given in Table 26, means that *no additional experimental data* are available as check of the final description of nature. Everything we need to arrive at the final description of motion will be deduced from the experimental data given in the millennium list, and from nothing else. In other words, future experiments will *not* help us – except if they change something in the millennium list. Accelerator experiments might do this with the particle list or astronomical experiments with the topology issue. Fantasy provides no limits; fortunately, nature does. Page 316

The lack of new experimental data means that to continue the walk is a *conceptual* adventure only. Nevertheless, storms rage near the top of Motion Mountain. We have to walk keeping our eyes open, without any other guidance except our reason. This is not an adventure of action, but an adventure of the mind. And it is a fascinating one, as we shall soon find out. To provide an impression of what awaits us, we rephrase the remaining issues in five simple challenges.

1 – What determines *colours*? In other words, what relations of nature fix the famous fine structure constant? Like the hero of Douglas Adams' books, physicists know the answer to the greatest of questions: it is 137.036. But they do not know the question.

2 – What fixes the contents of a *teapot*? It is given by its size to the third power. But why are there only three dimensions? Why is the tea content limited in this way?

3 – Was Democritus *right*? Our adventure has confirmed his statement up to this point: nature is indeed well described by the concepts of *particle* and of *vacuum*. At large scales, relativity has added a horizon, and at small scales, quantum field theory added vacuum energy and pair creation. Nevertheless, both theories *assume* the existence of particles and the existence of space-time, and neither *predicts* them. Even worse, both theories completely fail to predict the existence of *any* of the properties either of space-time – such as its dimensionality – or of particles – such as their masses and other quantum numbers. A lot is missing.

4 – Was Democritus *wrong*? It is often said that the standard model has only about twenty unknown parameters; this common mistake negates about 10^{93} initial conditions! To get an idea of the problem, we simply estimate the number N of possible states of all

particles in the universe by

$$N = n\,v\,d\,p\,f \tag{133}$$

where n is the number of particles, v is the number of variables (position, momentum, spin), d is the number of different values each of them can take (limited by the maximum of 61 decimal digits), p is the number of visible space-time points (about 10^{183}) and f is a factor expressing how many of all these initial conditions are actually independent of each other. We thus get the following number of possible states of all particles in the universe:

$$N = 10^{92} \cdot 8 \cdot 10^{61} \cdot 10^{183} \cdot f = 10^{336} \cdot f \tag{134}$$

from which the 10^{93} initial conditions have to be explained. But no explanation is known. Worse, there is also the additional problem that we know nothing whatsoever about f. Its value could be 0, if all data were interdependent, or 1, if none were. Even worse, above we noted that initial conditions cannot be defined for the universe at all; thus f should be undefined and not be a number at all! Whatever the case, we need to understand how all the visible particles acquire their present 10^{93} states.

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5 – Were our efforts up to this point *in vain*? Quite at the beginning of our walk we noted that in classical physics, space and time are defined using matter, whereas matter is defined using space-time. Hundred years of general relativity and of quantum theory, including dozens of geniuses, have not solved this oldest paradox of all. The issue is still open at this point of our walk, as you might want to check by yourself.

Challenge 182 e

The answers to these five challenges define the goal of our adventure: the top of Motion Mountain. Answering the five challenges means to know *everything* about motion. It means to find, finally, the answer to the question that drove us here:

▷ What is motion?

In short, our quest for the unravelling of the essence of motion gets really interesting from this point onwards!

> “That is why Leucippus and Democritus, who say that the atoms move always in the void and the unlimited, must say what movement is, and in what their natural motion consists.”
>
> Aristotle, *Treaty of the Heaven*

Ref. 272

Appendix A

UNITS, MEASUREMENTS AND CONSTANTS

Measurements are comparisons with standards. Standards are based on *units*. Many different systems of units have been used throughout the world. Most of these standards confer power to the organization in charge of them. Such power can be misused; this is the case today, for example in the computer industry, and was so in the distant past. The solution is the same in both cases: organize an independent and global standard. For measurement units, this happened in the eighteenth century: in order to avoid misuse by authoritarian institutions, to eliminate problems with differing, changing and irreproducible standards, and – this is not a joke – to simplify tax collection and to make it more just, a group of scientists, politicians and economists agreed on a set of units. It is called the *Système International d'Unités*, abbreviated *SI*, and is defined by an international treaty, the 'Convention du Mètre'. The units are maintained by an international organization, the 'Conférence Générale des Poids et Mesures', and its daughter organizations, the 'Commission Internationale des Poids et Mesures' and the 'Bureau International des Poids et Mesures' (BIPM). All Ref. 273 originated in the times just before the French revolution.

SI UNITS

All SI units are built from seven *base units*. Their simplest definitions, translated from French into English, are the following ones, together with the dates of their formulation and a few comments:

- 'The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.' (1967) The 2019 definition is equivalent, but much less clear.*
- 'The *metre* is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.' (1983) The 2019 definition is equivalent, but much less clear.*
- 'The *kilogram*, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,15 \cdot 10^{-34}$ when expressed in the unit $\mathrm{J \cdot s}$, which is equal to $\mathrm{kg \cdot m^2 \cdot s^{-1}}$.' (2019)*
- 'The *ampere*, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,634 \cdot 10^{-19}$ when expressed in the unit C, which is equal to $\mathrm{A \cdot s}$.' (2019)* This definition is equivalent to: One ampere is $6.241\,509\,074... \cdot 10^{18}$ elementary charges per second.
- 'The *kelvin*, symbol K, is the SI unit of thermodynamic temperature. It is defined by

taking the fixed numerical value of the Boltzmann constant k to be $1.380649 \cdot 10^{-23}$ when expressed in the unit $\mathrm{J \cdot K^{-1}}$.' (2019)*

- 'The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \cdot 10^{23}$ elementary entities.' (2019)*
- 'The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ hertz and has a radiant intensity in that direction of (1/683) watt per steradian.' (1979) The 2019 definition is equivalent, but much less clear.*

We note that both time and length units are defined as certain properties of a standard example of motion, namely light. In other words, also the Conférence Générale des Poids et Mesures makes the point that the observation of motion is a *prerequisite* for the definition and construction of time and space. *Motion is the fundament of every observation and of all measurement.* By the way, the use of light in the definitions had been proposed already in 1827 by Jacques Babinet.**

From these basic units, all other units are defined by multiplication and division. Thus, all SI units have the following properties:

- SI units form a system with *state-of-the-art precision*: all units are defined with a precision that is higher than the precision of commonly used measurements. Moreover, the precision of the definitions is regularly being improved. The present relative uncertainty of the definition of the second is around 10^{-14}, for the metre about 10^{-10}, for the kilogram about 10^{-9}, for the ampere 10^{-7}, for the mole less than 10^{-6}, for the kelvin 10^{-6} and for the candela 10^{-3}.
- SI units form an *absolute* system: all units are defined in such a way that they can be reproduced in every suitably equipped laboratory, independently, and with high precision. This avoids as much as possible any error or misuse by the standard-setting organization. In fact, the SI units are as now as near as possible to Planck's natural units, which are presented below. In practice, the SI is now an international standard defining the numerical values of the seven constants $\Delta\nu_{\mathrm{Cs}}$, c, $\hbar$, e, k, N_{A} and K_{cd}. After over 200 years of discussions, the CGPM has little left to do.
- SI units form a *practical* system: the base units are quantities of everyday magnitude. Frequently used units have standard names and abbreviations. The complete list includes the seven base units just given, the supplementary units, the derived units and the admitted units.

The *supplementary* SI units are two: the unit for (plane) angle, defined as the ratio of arc length to radius, is the *radian* (rad). For solid angle, defined as the ratio of the subtended area to the square of the radius, the unit is the *steradian* (sr).

The *derived* units with special names, in their official English spelling, i.e., without capital letters and accents, are:

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[*] The symbols of the seven units are s, m, kg, A, K, mol and cd. The full offical definitions are found at www.bipm.org. For more details about the levels of the caesium atom, consult a book on atomic physics. The Celsius scale of temperature θ is defined as: $\theta/°\mathrm{C} = T/\mathrm{K} - 273.15$; note the small difference with the number appearing in the definition of the kelvin. In the definition of the candela, the frequency of the light corresponds to 555.5 nm, i.e., green colour, around the wavelength to which the eye is most sensitive. Ref. 274

[**] Jacques Babinet (1794–1874), French physicist who published important work in optics.

Name	Abbreviation
hertz	$\mathrm{Hz} = 1/\mathrm{s}$
pascal	$\mathrm{Pa} = \mathrm{N/m^2} = \mathrm{kg/m\,s^2}$
watt	$\mathrm{W} = \mathrm{kg\,m^2/s^3}$
volt	$\mathrm{V} = \mathrm{kg\,m^2/As^3}$
ohm	$\Omega = \mathrm{V/A} = \mathrm{kg\,m^2/A^2s^3}$
weber	$\mathrm{Wb} = \mathrm{Vs} = \mathrm{kg\,m^2/As^2}$
henry	$\mathrm{H} = \mathrm{Vs/A} = \mathrm{kg\,m^2/A^2s^2}$
lumen	$\mathrm{lm} = \mathrm{cd\,sr}$
becquerel	$\mathrm{Bq} = 1/\mathrm{s}$
sievert	$\mathrm{Sv} = \mathrm{J/kg} = \mathrm{m^2/s^2}$

Name	Abbreviation
newton	$\mathrm{N} = \mathrm{kg\,m/s^2}$
joule	$\mathrm{J} = \mathrm{Nm} = \mathrm{kg\,m^2/s^2}$
coulomb	$\mathrm{C} = \mathrm{As}$
farad	$\mathrm{F} = \mathrm{As/V} = \mathrm{A^2s^4/kg\,m^2}$
siemens	$\mathrm{S} = 1/\Omega$
tesla	$\mathrm{T} = \mathrm{Wb/m^2} = \mathrm{kg/As^2} = \mathrm{kg/Cs}$
degree Celsius	°C (see definition of kelvin)
lux	$\mathrm{lx} = \mathrm{lm/m^2} = \mathrm{cd\,sr/m^2}$
gray	$\mathrm{Gy} = \mathrm{J/kg} = \mathrm{m^2/s^2}$
katal	$\mathrm{kat} = \mathrm{mol/s}$

The *admitted* non-SI units are *minute*, *hour*, *day* (for time), *degree* $1^\circ = \pi/180\,\mathrm{rad}$, *minute* $1' = \pi/10\,800\,\mathrm{rad}$, *second* $1'' = \pi/648\,000\,\mathrm{rad}$ (for angles), *litre*, and *tonne*. All other units are to be avoided.

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called *prefixes*:*

Power	Name	
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	Mega	M
10^{9}	Giga	G
10^{12}	Tera	T
10^{15}	Peta	P

Power	Name	
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Power	Name	
10^{18}	Exa	E
10^{21}	Zetta	Z
10^{24}	Yotta	Y
unofficial:		
10^{27}	Xenta	X
10^{30}	Wekta	W
10^{33}	Vendekta	V
10^{36}	Udekta	U

Power	Name	
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y
Ref. 275		
10^{-27}	xenno	x
10^{-30}	weko	w
10^{-33}	vendeko	v
10^{-36}	udeko	u

- SI units form a *complete* system: they cover in a systematic way the full set of observables of physics. Moreover, they fix the units of measurement for all other sciences as well.
- SI units form a *universal* system: they can be used in trade, in industry, in commerce,

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[*] Some of these names are invented (yocto to sound similar to Latin *octo* 'eight', zepto to sound similar to Latin *septem*, yotta and zetta to resemble them, exa and peta to sound like the Greek words ἑξάκις and πεντάκις for 'six times' and 'five times', the unofficial ones to sound similar to the Greek words for nine, ten, eleven and twelve); some are from Danish/Norwegian (atto from *atten* 'eighteen', femto from *femten* 'fifteen'); some are from Latin (from *mille* 'thousand', from *centum* 'hundred', from *decem* 'ten', from *nanus* 'dwarf'); some are from Italian (from *piccolo* 'small'); some are Greek (micro is from μικρός 'small', deca/deka from δέκα 'ten', hecto from ἑκατόν 'hundred', kilo from χίλιοι 'thousand', mega from μέγας 'large', giga from γίγας 'giant', tera from τέρας 'monster').

Translate: I was caught in such a traffic jam that I needed a microcentury for a picoparsec and that my car's fuel consumption was two tenths of a square millimetre.

Challenge 183 e

at home, in education and in research. They could even be used by extraterrestrial civilizations, if they existed.

- SI units form a *self-consistent* system: the product or quotient of two SI units is also an SI unit. This means that in principle, the same abbreviation, e.g. 'SI', could be used for every unit.

The SI units are not the only possible set that could fulfil all these requirements, but they are the only existing system that does so.*

The meaning of measurement

Every measurement is a comparison with a standard. Therefore, any measurement requires *matter* to realize the standard (even for a speed standard), and *radiation* to achieve the comparison. The concept of measurement thus assumes that matter and radiation exist and can be clearly separated from each other. Challenge 184 e

Every measurement is a comparison. Measuring thus implies that space and time exist, and that they differ from each other.

Every measurement produces a measurement result. Therefore, every measurement implies the *storage* of the result. The process of measurement thus implies that the situation before and after the measurement can be distinguished. In other terms, every measurement is an *irreversible* process.

Every measurement is a process. Thus every measurement takes a certain amount of time and a certain amount of space.

All these properties of measurements are simple but important. Beware of anybody who denies them.

Planck's natural units

Since the exact form of many equations depends on the system of units used, theoretical physicists often use unit systems optimized for producing simple equations. The chosen units and the values of the constants of nature are related. In microscopic physics, the system of *Planck's natural units* is frequently used. They are defined by setting $c = 1$, $\hbar = 1$, $G = 1$, $k = 1$, $\varepsilon_0 = 1/4\pi$ and $\mu_0 = 4\pi$. Planck units are thus defined from combinations of fundamental constants; those corresponding to the fundamental SI units are given in Table 29.** The table is also useful for converting equations written in natural units back to SI units: just substitute every quantity X by X/X_{Pl}. Challenge 185 e

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* Apart from international units, there are also *provincial* units. Most provincial units still in use are of Roman origin. The mile comes from *milia passum*, which used to be one thousand (double) strides of about 1480 mm each; today a nautical mile, once defined as minute of arc on the Earth's surface, is defined as exactly 1852 m. The inch comes from *uncia/onzia* (a twelfth – now of a foot). The pound (from *pondere* 'to weigh') is used as a translation of *libra* – balance – which is the origin of its abbreviation lb. Even the habit of counting in dozens instead of tens is Roman in origin. These and all other similarly funny units – like the system in which all units start with 'f', and which uses furlong/fortnight as its unit of velocity – are now officially defined as multiples of SI units.

** The natural units x_{Pl} given here are those commonly used today, i.e., those defined using the constant $\hbar$, and not, as Planck originally did, by using the constant $h = 2\pi\hbar$. The electromagnetic units can also be defined with other factors than $4\pi\varepsilon_0$ in the expressions: for example, using $4\pi\varepsilon_0\alpha$, with the *fine-structure constant* α, gives $q_{\mathrm{Pl}} = e$. For the explanation of the numbers between brackets, see below. Vol. IV, page 196

TABLE 29 Planck's (uncorrected) natural units.

NAME	DEFINITION	VALUE
Basic units		
the Planck length	$l_{\mathrm{Pl}} = \sqrt{\hbar G/c^3}$	$= 1.616\,0(12) \cdot 10^{-35}\,\mathrm{m}$
the Planck time	$t_{\mathrm{Pl}} = \sqrt{\hbar G/c^5}$	$= 5.390\,6(40) \cdot 10^{-44}\,\mathrm{s}$
the Planck mass	$m_{\mathrm{Pl}} = \sqrt{\hbar c/G}$	$= 21.767(16)\,\mu\mathrm{g}$
the Planck current	$I_{\mathrm{Pl}} = \sqrt{4\pi\varepsilon_0 c^6/G}$	$= 3.479\,3(22) \cdot 10^{25}\,\mathrm{A}$
the Planck temperature	$T_{\mathrm{Pl}} = \sqrt{\hbar c^5/Gk^2}$	$= 1.417\,1(91) \cdot 10^{32}\,\mathrm{K}$
Trivial units		
the Planck velocity	$v_{\mathrm{Pl}} = c$	$= 0.3\,\mathrm{Gm/s}$
the Planck angular momentum	$L_{\mathrm{Pl}} = \hbar$	$= 1.1 \cdot 10^{-34}\,\mathrm{Js}$
the Planck action	$S_{\mathrm{aPl}} = \hbar$	$= 1.1 \cdot 10^{-34}\,\mathrm{Js}$
the Planck entropy	$S_{\mathrm{ePl}} = k$	$= 13.8\,\mathrm{yJ/K}$
Composed units		
the Planck mass density	$\rho_{\mathrm{Pl}} = c^5/G^2\hbar$	$= 5.2 \cdot 10^{96}\,\mathrm{kg/m^3}$
the Planck energy	$E_{\mathrm{Pl}} = \sqrt{\hbar c^5/G}$	$= 2.0\,\mathrm{GJ} = 1.2 \cdot 10^{28}\,\mathrm{eV}$
the Planck momentum	$p_{\mathrm{Pl}} = \sqrt{\hbar c^3/G}$	$= 6.5\,\mathrm{Ns}$
the Planck power	$P_{\mathrm{Pl}} = c^5/G$	$= 3.6 \cdot 10^{52}\,\mathrm{W}$
the Planck force	$F_{\mathrm{Pl}} = c^4/G$	$= 1.2 \cdot 10^{44}\,\mathrm{N}$
the Planck pressure	$p_{\mathrm{Pl}} = c^7/G\hbar$	$= 4.6 \cdot 10^{113}\,\mathrm{Pa}$
the Planck acceleration	$a_{\mathrm{Pl}} = \sqrt{c^7/\hbar G}$	$= 5.6 \cdot 10^{51}\,\mathrm{m/s^2}$
the Planck frequency	$f_{\mathrm{Pl}} = \sqrt{c^5/\hbar G}$	$= 1.9 \cdot 10^{43}\,\mathrm{Hz}$
the Planck electric charge	$q_{\mathrm{Pl}} = \sqrt{4\pi\varepsilon_0 c\hbar}$	$= 1.9\,\mathrm{aC} = 11.7\,\mathrm{e}$
the Planck voltage	$U_{\mathrm{Pl}} = \sqrt{c^4/4\pi\varepsilon_0 G}$	$= 1.0 \cdot 10^{27}\,\mathrm{V}$
the Planck resistance	$R_{\mathrm{Pl}} = 1/4\pi\varepsilon_0 c$	$= 30.0\,\Omega$
the Planck capacitance	$C_{\mathrm{Pl}} = 4\pi\varepsilon_0\sqrt{\hbar G/c^3}$	$= 1.8 \cdot 10^{-45}\,\mathrm{F}$
the Planck inductance	$L_{\mathrm{Pl}} = (1/4\pi\varepsilon_0)\sqrt{\hbar G/c^7}$	$= 1.6 \cdot 10^{-42}\,\mathrm{H}$
the Planck electric field	$E_{\mathrm{Pl}} = \sqrt{c^7/4\pi\varepsilon_0\hbar G^2}$	$= 6.5 \cdot 10^{61}\,\mathrm{V/m}$
the Planck magnetic flux density	$B_{\mathrm{Pl}} = \sqrt{c^5/4\pi\varepsilon_0\hbar G^2}$	$= 2.2 \cdot 10^{53}\,\mathrm{T}$

The natural units are important for another reason: whenever a quantity is sloppily called 'infinitely small (or large)', the correct expression is 'as small (or as large) as the corresponding corrected Planck unit'. As explained throughout the text, and especially in the final part, this substitution is possible because almost all Planck units provide, within a correction factor of order 1, the extremal value for the corresponding observable – some an upper and some a lower limit. Unfortunately, these correction factors are not yet widely known. The exact extremal value for each observable in nature is obtained

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when G is substituted by $4G$ and $4\pi\varepsilon_0$ by $4\pi\varepsilon_0\alpha$ in all Planck quantities. These extremal values, or *corrected Planck units*, are the *true natural units*. To exceed the extremal values is possible only for some extensive quantities. (Can you find out which ones?) Challenge 186 s

Other unit systems

A central aim of research in high-energy physics is the calculation of the strengths of all interactions; therefore it is not practical to set the gravitational constant G to unity, as in the Planck system of units. For this reason, high-energy physicists often only set $c = \hbar = k = 1$ and $\mu_0 = 1/\varepsilon_0 = 4\pi$,* leaving only the gravitational constant G in the equations.

In this system, only one fundamental unit exists, but its choice is free. Often a standard length is chosen as the fundamental unit, length being the archetype of a measured quantity. The most important physical observables are then related by

$$\begin{aligned}
1/[l^2] = \; & [E]^2 = [F] = [B] = [E_{\text{electric}}] \, , \\
1/[l] = \; & [E] = [m] = [p] = [a] = [f] = [I] = [U] = [T] \, , \\
1 = \; & [v] = [q] = [e] = [R] = [S_{\text{action}}] = [S_{\text{entropy}}] = \hbar = c = k = [\alpha] \, , \\
[l] = \; & 1/[E] = [t] = [C] = [L] \quad \text{and} \\
[l]^2 = \; & 1/[E]^2 = [G] = [P]
\end{aligned} \tag{135}$$

where we write $[x]$ for the unit of quantity x. Using the same unit for time, capacitance and inductance is not to everybody's taste, however, and therefore electricians do not use this system.**

Often, in order to get an impression of the energies needed to observe an effect under study, a standard energy is chosen as fundamental unit. In particle physics the most common energy unit is the *electronvolt* (eV), defined as the kinetic energy acquired by an electron when accelerated by an electrical potential difference of 1 volt ('protonvolt' would be a better name). Therefore one has $1\,\text{eV} = 1.6 \cdot 10^{-19}\,\text{J}$, or roughly

$$1\,\text{eV} \approx \tfrac{1}{6}\,\text{aJ} \tag{136}$$

which is easily remembered. The simplification $c = \hbar = 1$ yields $G = 6.9 \cdot 10^{-57}\,\text{eV}^{-2}$ and allows one to use the unit eV also for mass, momentum, temperature, frequency, time and length, with the respective correspondences $1\,\text{eV} \equiv 1.8 \cdot 10^{-36}\,\text{kg} \equiv 5.4 \cdot 10^{-28}\,\text{Ns} \equiv 242\,\text{THz} \equiv 11.6\,\text{kK}$ and $1\,\text{eV}^{-1} \equiv 4.1\,\text{fs} \equiv 1.2\,\mu\text{m}$. Challenge 187 e

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* Other definitions for the proportionality constants in electrodynamics lead to the Gaussian unit system often used in theoretical calculations, the Heaviside–Lorentz unit system, the electrostatic unit system, and the electromagnetic unit system, among others. Ref. 276

** In the list, l is length, E energy, F force, E_{electric} the electric and B the magnetic field, m mass, p momentum, a acceleration, f frequency, I electric current, U voltage, T temperature, v speed, q charge, R resistance, P power, G the gravitational constant.

The web page www.chemie.fu-berlin.de/chemistry/general/units_en.html provides a tool to convert various units into each other.

Researchers in general relativity often use another system, in which the *Schwarzschild radius* $r_{\text{S}} = 2Gm/c^2$ is used to measure masses, by setting $c = G = 1$. In this case, mass and length have the *same* dimension, and $\hbar$ has the dimension of an area.

To get some feeling for the unit eV, the following relations are useful. Room temperature, usually taken as 20°C or 293 K, corresponds to a kinetic energy per particle of 0.025 eV or 4.0 zJ. The highest particle energy measured so far belongs to a cosmic ray Ref. 277 with an energy of $3 \cdot 10^{20}$ eV or 48 J. Down here on the Earth, an accelerator able to produce an energy of about 105 GeV or 17 nJ for electrons and antielectrons has been built, and one able to produce an energy of 14 TeV or 2.2 μJ for protons will be finished soon. Both are owned by CERN in Geneva and have a circumference of 27 km.

The lowest temperature measured up to now is 280 pK, in a system of rhodium Ref. 278 nuclei held inside a special cooling system. The interior of that cryostat may even be the coolest point in the whole universe. The kinetic energy per particle corresponding to that temperature is also the smallest ever measured: it corresponds to 24 feV or $3.8\,\mathrm{vJ} = 3.8 \cdot 10^{-33}$ J. For isolated particles, the record seems to be for neutrons: kinetic energies as low as 10^{-7} eV have been achieved, corresponding to de Broglie wavelengths of 60 nm.

Curiosities and fun challenges about units

The Planck length is roughly the de Broglie wavelength $\lambda_B = h/mv$ of a man walking Ref. 279 comfortably ($m = 80$ kg, $v = 0.5$ m/s); this motion is therefore aptly called the 'Planck stroll.'

* *

The Planck mass is equal to the mass of about 10^{19} protons. This is roughly the mass of a human embryo at about ten days of age.

* *

The most precisely measured quantities in nature are the frequencies of certain milli- Ref. 280 second pulsars, the frequency of certain narrow atomic transitions, and the Rydberg constant of *atomic* hydrogen, which can all be measured as precisely as the second is defined. The caesium transition that defines the second has a finite line width that limits the achievable precision: the limit is about 14 digits.

* *

The most precise clock ever built, using microwaves, had a stability of 10^{-16} during a Ref. 281 running time of 500 s. For longer time periods, the record in 1997 was about 10^{-15}; but Ref. 282 values around 10^{-17} seem within technological reach. The precision of clocks is limited for short measuring times by noise, and for long measuring times by drifts, i.e., by systematic effects. The region of highest stability depends on the clock type; it usually lies between 1 ms for optical clocks and 5000 s for masers. Pulsars are the only type of clock for which this region is not known yet; it certainly lies at more than 20 years, the time elapsed at the time of writing since their discovery.

* *

The shortest times measured are the lifetimes of certain 'elementary' particles. In par- Ref. 283 ticular, the lifetime of certain D mesons have been measured at less than 10^{-23} s. Such times are measured using a bubble chamber, where the track is photographed. Can you

Challenge 188 s estimate how long the track is? (This is a trick question – if your length cannot be observed with an optical microscope, you have made a mistake in your calculation.)

* *

The longest times encountered in nature are the lifetimes of certain radioisotopes, over 10^{15} years, and the lower limit of certain proton decays, over 10^{32} years. These times are thus much larger than the age of the universe, estimated to be fourteen thousand million years. Ref. 284

* *

There is a unit for the spicy heat of chili peppers, officially called the *pungency*. The pungency is due to an organic compound called *capsaicin*. If you multiply by 16 the capsaicin concentration in parts per million, you get the *Scoville heat unit* for chili peppers. A few extreme chili varieties exceed the value of 2 million Scoville units.

PRECISION AND ACCURACY OF MEASUREMENTS

Measurements are the basis of physics. Every measurement has an *error*. Errors are due to lack of precision or to lack of accuracy. *Precision* means how well a result is reproduced when the measurement is repeated; *accuracy* is the degree to which a measurement corresponds to the actual value.

Lack of precision is due to accidental or *random errors*; they are best measured by the *standard deviation*, usually abbreviated σ; it is defined through

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 , \qquad (137)$$

where $\bar{x}$ is the average of the measurements x_i. (Can you imagine why $n-1$ is used in the formula instead of n?) Challenge 189 s

For most experiments, the distribution of measurement values tends towards a normal distribution, also called *Gaussian distribution*, whenever the number of measurements is increased. The distribution, shown in Figure 182, is described by the expression

$$N(x) \approx \mathrm{e}^{-\frac{(x-\bar{x})^2}{2\sigma^2}} . \qquad (138)$$

The square σ^2 of the standard deviation is also called the *variance*. For a Gaussian distribution of measurement values, 2.35σ is the full width at half maximum. Challenge 190 e

Lack of accuracy is due to *systematic errors*; usually these can only be estimated. This estimate is often added to the random errors to produce a *total experimental error*, sometimes also called *total uncertainty*. The *relative* error or uncertainty is the ratio between the error and the measured value. Ref. 285

For example, a professional measurement will give a result such as 0.312(6) m. The number between the parentheses is the standard deviation σ, in units of the last digits. As above, a Gaussian distribution for the measurement results is assumed. Therefore, a value of 0.312(6) m implies that the actual value is expected to lie Challenge 191 e

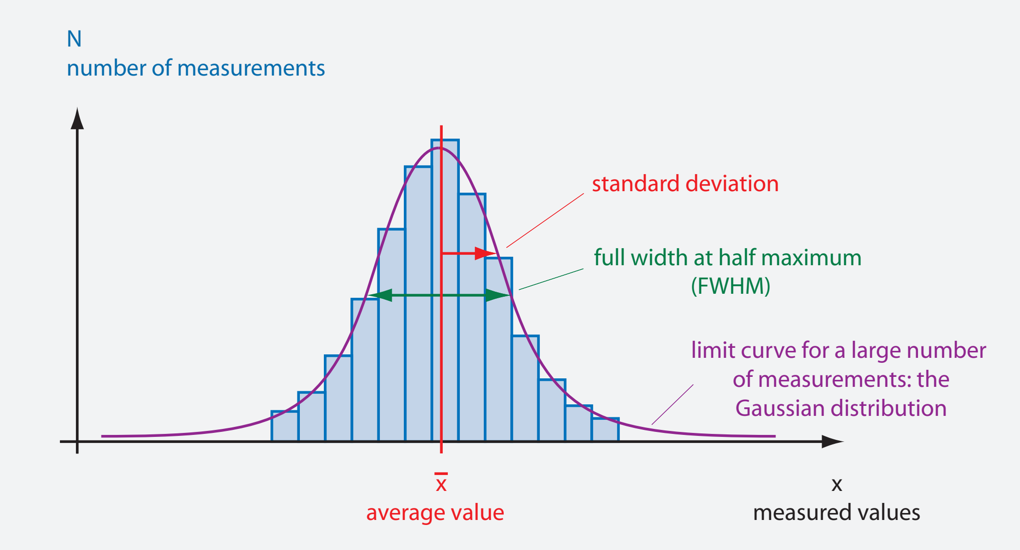

FIGURE 182 A precision experiment and its measurement distribution. The precision is high if the width of the distribution is narrow; the accuracy is high if the centre of the distribution agrees with the actual value.

- within 1σ with 68.3 % probability, thus in this example within 0.312 ± 0.006 m;
- within 2σ with 95.4 % probability, thus in this example within 0.312 ± 0.012 m;
- within 3σ with 99.73 % probability, thus in this example within 0.312 ± 0.018 m;
- within 4σ with 99.9937 % probability, thus in this example within 0.312 ± 0.024 m;
- within 5σ with 99.999 943 % probability, thus in this example within 0.312 ± 0.030 m;
- within 6σ with 99.999 999 80 % probability, thus within 0.312 ± 0.036 m;
- within 7σ with 99.999 999 999 74 % probability, thus within 0.312 ± 0.041 m.

However, these numbers are much too precise and should be taken with a grain of salt.

Note that standard deviations have one digit; you must be a world expert to use two, and a fool to use more. If no standard deviation is given, a (1) is assumed. As a result, among professionals, 1 km and 1000 m are *not* the same length!

What happens to the errors when two measured values A and B are added or subtracted? If the all measurements are independent – or uncorrelated – the standard deviation of the sum *and* that of difference is given by $\sigma = \sqrt{\sigma_A^2 + \sigma_B^2}$. For both the product or ratio of two measured and uncorrelated values C and D, the result is $\rho = \sqrt{\rho_C^2 + \rho_D^2}$, where the ρ terms are the *relative* standard deviations.

LIMITS TO PRECISION

What are the limits to accuracy and precision? There is no way, even in principle, to measure a length x to a *precision* higher than about 61 digits, because in nature, the ratio between the largest and the smallest measurable length is $\Delta x/x > l_{\text{Pl}}/d_{\text{horizon}} = 10^{-61}$. (Is this ratio valid also for force or for volume?) In the final volume of our text, studies of clocks and metre bars strengthen this theoretical limit.

Challenge 192 e

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But it is not difficult to deduce more stringent practical limits. No imaginable machine can measure quantities with a higher precision than measuring the diameter of the Earth within the smallest length ever measured, about 10^{-19} m; that is about 26 digits of precision. Using a more realistic limit of a 1000 m sized machine implies a limit of 22 digits. If, as predicted above, time measurements really achieve 17 digits of precision, then they are nearing the practical limit, because apart from size, there is an additional practical restriction: cost. Indeed, an additional digit in measurement precision often means an additional digit in equipment cost.

PHYSICAL CONSTANTS

In physics, general observations are deduced from more fundamental ones. As a consequence, many measurements can be deduced from more fundamental ones. The most fundamental measurements are those of the physical constants.

The following tables give the world's best values of the most important physical constants and particle properties – in SI units and in a few other common units – as published in the standard references. The values are the world averages of the best measurements made up to the present. As usual, experimental errors, including both random and estimated systematic errors, are expressed by giving the standard deviation in the last digits. In fact, behind each of the numbers in the following tables there is a long story which is worth telling, but for which there is not enough room here.

Ref. 286

Ref. 287

In principle, *all* quantitative properties of matter can be calculated with quantum theory – more precisely, equations of the standard model of particle – and a set of *basic* physical constants that are given in the next table. For example, the colour, density and elastic properties of any material can be predicted, in principle, in this way.

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TABLE 30 Basic physical constants.

Quantity	Symbol	Value in SI units	Uncert.[a]
Constants that define the SI measurement units			
Vacuum speed of light [c]	c	299 792 458 m/s	0
Original Planck constant [c]	h	$6.626\,070\,15 \cdot 10^{-34}$ Js	0
Reduced Planck constant, quantum of action	$\hbar$	$1.054\,571\,817\ldots \cdot 10^{-34}$ Js	0
Positron charge [c]	e	0.160 217 6634 aC	0
Boltzmann constant [c]	k	$1.380\,649 \cdot 10^{-23}$ J/K	0
Avogadro's number	N_A	$6.022\,140\,76 \cdot 10^{23}$ 1/mol	0
Constant that *should* define the SI measurement units			
Gravitational constant	G	$6.674\,30(15) \cdot 10^{-11}$ Nm^2/kg^2	$2.2 \cdot 10^{-5}$
Other fundamental constants			
Number of space-time dimensions		3 + 1	0 [b]
Fine-structure constant [d] or	$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$	1/137.035 999 084(21)	$1.5 \cdot 10^{-10}$
e.m. coupling constant	$= g_{em}(m_e^2c^2)$	= 0.007 297 352 5693(11)	$1.5 \cdot 10^{-10}$
Fermi coupling constant [d] or	$G_F/(\hbar c)^3$	$1.166\,3787(6) \cdot 10^{-5}$ GeV^{-2}	$5.1 \cdot 10^{-7}$
weak coupling constant	$\alpha_w(M_Z) = g_w^2/4\pi$	1/30.1(3)	$1 \cdot 10^{-2}$

TABLE 30 (Continued) Basic physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.[a]
Strong coupling constant[d]	$\alpha_s(M_Z) = g_s^2/4\pi$	0.1179(10)	$8.5 \cdot 10^{-3}$
Weak mixing angle	$\sin^2\theta_W(\overline{MS})$	0.231 22(4)	$1.7 \cdot 10^{-4}$
	$\sin^2\theta_W$ (on shell) $= 1 - (m_W/m_Z)^2$	0.222 90(30)	$1.3 \cdot 10^{-3}$
CKM quark mixing matrix	$\lvert V \rvert$	$\begin{pmatrix} 0.97383(24) & 0.2272(10) & 0.00396(9) \\ 0.2271(10) & 0.97296(24) & 0.04221(80) \\ 0.00814(64) & 0.04161(78) & 0.999100(34) \end{pmatrix}$	
Jarlskog invariant	J	$3.08(18) \cdot 10^{-5}$	
PMNS neutrino mixing m.	$\lvert P \rvert$	$\begin{pmatrix} 0.82(2) & 0.55(4) & 0.150(7) \\ 0.37(13) & 0.57(11) & 0.71(7) \\ 0.41(13) & 0.59(10) & 0.69(7) \end{pmatrix}$	
Electron mass	m_e	$9.109\,383\,7015(28) \cdot 10^{-31}$ kg	$3.0 \cdot 10^{-10}$
		$5.485\,799\,090\,65(16) \cdot 10^{-4}$ u	$2.9 \cdot 10^{-11}$
		0.510 998 950 00(15) MeV	$3.0 \cdot 10^{-10}$
Muon mass	m_μ	$1.883\,531\,627(42) \cdot 10^{-28}$ kg	$2.2 \cdot 10^{-8}$
		105.658 3755(23) MeV	$2.2 \cdot 10^{-8}$
Tau mass	m_τ	1.776 82(12) GeV/c^2 $6.8 \cdot 10^{-5}$	
El. neutrino mass	m_{ν_e}	< 2 eV/c^2	
Muon neutrino mass	m_{ν_μ}	< 2 eV/c^2	
Tau neutrino mass	m_{ν_τ}	< 2 eV/c^2	
Up quark mass	u	$21.6(+0.49/-0.26)$ MeV/c^2	
Down quark mass	d	$4.67(+0.48/-0.17)$ MeV/c^2	
Strange quark mass	s	$93(+11/-5)$ MeV/c^2	
Charm quark mass	c	1.27(2) GeV/c^2	
Bottom quark mass	b	4.18(3) GeV/c^2	
Top quark mass	t	172.9(0.4) GeV/c^2	
Photon mass	γ	$< 2 \cdot 10^{-54}$ kg	
W boson mass	$W^\pm$	80.379(12) GeV/c^2	
Z boson mass	Z^0	91.1876(21) GeV/c^2	
Higgs mass	H	125.10(14) GeV/c^2	
Gluon mass	$g_{1...8}$	*c.* 0 MeV/c^2	

a. Uncertainty: standard deviation of measurement errors.

b. Measured from to 10^{-19} m to 10^{26} m.

c. Defining constant.

d. All coupling constants depend on the 4-momentum transfer, as explained in the section on renormalization. *Fine-structure constant* is the traditional name for the electromagnetic coupling constant g_{em} in the case of a 4-momentum transfer of $Q^2 = m_e^2 c^2$, which is the smallest one possible. At higher momentum transfers it has larger values, e.g., $g_{em}(Q^2 = M_W^2 c^2) \approx 1/128$. In contrast, the strong coupling constant has lover values at higher momentum transfers; e.g., $\alpha_s(34\,\text{GeV}) = 0.14(2)$.

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Why do all these basic constants have the values they have? For any basic constant *with a dimension*, such as the quantum of action $\hbar$, the numerical value has only historical meaning. It is $1.054 \cdot 10^{-34}$ Js because of the SI definition of the joule and the second. The question why the value of a *dimensional* constant is not larger or smaller therefore always requires one to understand the origin of some *dimensionless* number giving the ratio between the constant and the corresponding *natural unit* that is defined with c, G, k, N_A and $\hbar$. Details and values for the natural units are given in the dedicated section.

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In other words, understanding the sizes of atoms, people, trees and stars, the duration of molecular and atomic processes, or the mass of nuclei and mountains, implies understanding the ratios between these values and the corresponding natural units. The key to understanding nature is thus the understanding of all measurement ratios, and thus of all dimensionless constants. This quest, including the understanding of the fine-structure constant α itself, is completed only in the final volume of our adventure.

The basic constants yield the following useful high-precision observations.

TABLE 31 Derived physical constants.

Quantity	Symbol	Value in SI units	Uncert.
Vacuum permeability	μ_0	1.256 637 062 12(19) μH/m	$1.5 \cdot 10^{-10}$
Vacuum permittivity	$\varepsilon_0 = 1/\mu_0 c^2$	8.854 187 8128(13) pF/m	$1.5 \cdot 10^{-10}$
Vacuum impedance	$Z_0 = \sqrt{\mu_0/\varepsilon_0}$	376.730 313 668(57) Ω	$1.5 \cdot 10^{-10}$
Loschmidt's number at 273.15 K and 101 325 Pa	N_L	$2.686\,780\,111... \cdot 10^{25}$ 1/m³	0
Faraday's constant	$F = N_A e$	96 485.332 12... C/mol	0
Universal gas constant	$R = N_A k$	8.314 462 618... J/(mol K)	0
Molar volume of an ideal gas at 273.15 K and 101 325 Pa	$V = RT/p$	22.413 969 54... l/mol	0
Rydberg constant [a]	$R_\infty = m_e c \alpha^2/2h$	10 973 731.568 160(21) m⁻¹	$1.9 \cdot 10^{-12}$
Conductance quantum	$G_0 = 2e^2/h$	77.480 917 29... μS	0
Magnetic flux quantum	$\varphi_0 = h/2e$	2.067 833 848... fWb	0
Josephson frequency ratio	$2e/h$	483.597 8484... THz/V	0
Von Klitzing constant	$h/e^2 = \mu_0 c/2\alpha$	25 812.807 45... Ω	0
Bohr magneton	$\mu_B = e\hbar/2m_e$	9.274 010 0783(28) yJ/T	$3.0 \cdot 10^{-10}$
Classical electron radius	$r_e = e^2/4\pi\varepsilon_0 m_e c^2$	2.817 940 3262(13) fm	$4.5 \cdot 10^{-10}$
Compton wavelength	$\lambda_C = h/m_e c$	2.426 310 238 67(73) pm	$3.0 \cdot 10^{-10}$
of the electron	$\lambdabar_c = \hbar/m_e c = r_e/\alpha$	0.386 159 267 96(12) pm	$3.0 \cdot 10^{-10}$
Bohr radius [a]	$a_\infty = r_e/\alpha^2$	52.917 721 0903(80) pm	$1.5 \cdot 10^{-10}$
Quantum of circulation	$h/2m_e$	3.636 947 5516(11) cm²/s	$3.0 \cdot 10^{-10}$
Specific positron charge	e/m_e	175.882 001 076(55) GC/kg	$3.0 \cdot 10^{-10}$
Cyclotron frequency of the electron	$f_c/B = e/2\pi m_e$	27.992 489 872(9) GHz/T	$3.0 \cdot 10^{-10}$
Electron magnetic moment	μ_e	−9.284 764 7043(28) yJ/T	$3.0 \cdot 10^{-10}$
	μ_e/μ_B	−1.001 159 652 181 28(18)	$1.7 \cdot 10^{-13}$
	μ_e/μ_N	$-1\,838.281\,971\,88(11) \cdot 10^3$	$6.0 \cdot 10^{-11}$

TABLE 31 (Continued) Derived physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.
Electron g-factor	g_e	−2.002 319 304 362 56(35)	$1.7 \cdot 10^{-13}$
Muon–electron mass ratio	m_μ/m_e	206.768 2830(46)	$2.2 \cdot 10^{-8}$
Muon magnetic moment	μ_μ	$-4.490\,448\,30(10) \cdot 10^{-26}$ J/T	$2.2 \cdot 10^{-8}$
Muon g-factor	g_μ	−2.002 331 8418(13)	$6.3 \cdot 10^{-10}$
Atomic mass unit	$1\,\mathrm{u} = m_{^{12}\mathrm{C}}/12$	$1.660\,539\,066\,60(50) \cdot 10^{-27}$ kg	$3.0 \cdot 10^{-10}$
Proton mass	m_p	$1.672\,621\,923\,69(51) \cdot 10^{-27}$ kg	$3.1 \cdot 10^{-10}$
		1.007 276 466 621(53) u	$5.3 \cdot 10^{-11}$
		938.272 088 16(29) MeV	$3.1 \cdot 10^{-10}$
Proton–electron mass ratio	m_p/m_e	1 836.152 673 43(11)	$6.0 \cdot 10^{-11}$
Specific proton charge	e/m_p	$9.578\,833\,1560(29) \cdot 10^{7}$ C/kg	$3.1 \cdot 10^{-10}$
Proton Compton wavelength	$\lambda_{C,p} = h/m_p c$	1.321 409 855 39(40) fm	$3.1 \cdot 10^{-10}$
Nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,7461(15) \cdot 10^{-27}$ J/T	$3.1 \cdot 10^{-10}$
Proton magnetic moment	μ_p	$1.410\,606\,797\,36(60) \cdot 10^{-26}$ J/T	$4.2 \cdot 10^{-10}$
	μ_p/μ_B	$1.521\,032\,202\,30(46) \cdot 10^{-3}$	$3.0 \cdot 10^{-10}$
	μ_p/μ_N	2.792 847 344 63(82)	$2.9 \cdot 10^{-10}$
Proton gyromagnetic ratio	$\gamma_p = 2\mu_p/h$	42.577 478 518(18) MHz/T	$4.2 \cdot 10^{-10}$
Proton g factor	g_p	5.585 694 6893(16)	$2.9 \cdot 10^{-10}$
Neutron mass	m_n	$1.674\,927\,498\,04(95) \cdot 10^{-27}$ kg	$5.7 \cdot 10^{-10}$
		1.008 664 915 95(43) u	$4.8 \cdot 10^{-10}$
		939.565 420 52(54) MeV	$5.7 \cdot 10^{-10}$
Neutron–electron mass ratio	m_n/m_e	1 838.683 661 73(89)	$4.8 \cdot 10^{-10}$
Neutron–proton mass ratio	m_n/m_p	1.001 378 419 31(49)	$4.9 \cdot 10^{-10}$
Neutron Compton wavelength	$\lambda_{C,n} = h/m_n c$	1.319 590 905 81(75) fm	$5.7 \cdot 10^{-10}$
Neutron magnetic moment	μ_n	$-0.966\,236\,51(23) \cdot 10^{-26}$ J/T	$2.4 \cdot 10^{-7}$
	μ_n/μ_B	$-1.041\,875\,63(25) \cdot 10^{-3}$	$2.4 \cdot 10^{-7}$
	μ_n/μ_N	−1.913 042 73(45)	$2.4 \cdot 10^{-7}$
Stefan–Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	56.703 744 19... $\mathrm{nW/m^2K^4}$	0
Wien's displacement constant	$b = \lambda_{max} T$	2.897 771 955... mmK	0
		58.789 257 57... GHz/K	0
Electron volt	eV	0.160 217 6634... aJ	0
Bits to entropy conversion const.	$k \ln 2$	10^{23} bit = 0.956 994... J/K	0
TNT energy content		3.7 to 4.0 MJ/kg	$4 \cdot 10^{-2}$

a. For infinite mass of the nucleus.

Some useful properties of our local environment are given in the following table.

TABLE 32 Astronomical constants.

QUANTITY	SYMBOL	VALUE
Tropical year 1900 [a]	a	31 556 925.974 7 s
Tropical year 1994	a	31 556 925.2 s
Mean sidereal day	d	$23^h 56' 4.090\,53''$
Average distance Earth–Sun [b]		149 597 870.691(30) km
Astronomical unit [b]	AU	149 597 870 691 m
Light year, based on Julian year [b]	al	9.460 730 472 5808 Pm
Parsec	pc	30.856 775 806 Pm = 3.261 634 al
Earth's mass	$M_{♁}$	$5.973(1) \cdot 10^{24}$ kg
Geocentric gravitational constant	GM	$3.986\,004\,418(8) \cdot 10^{14}\ \mathrm{m^3/s^2}$
Earth's gravitational length	$l_{♁} = 2GM/c^2$	8.870 056 078(16) mm
Earth's equatorial radius [c]	$R_{♁\mathrm{eq}}$	6378.1366(1) km
Earth's polar radius [c]	$R_{♁\mathrm{p}}$	6356.752(1) km
Equator–pole distance [c]		10 001.966 km (average)
Earth's flattening [c]	$e_{♁}$	1/298.25642(1)
Earth's av. density	$\rho_{♁}$	$5.5\ \mathrm{Mg/m^3}$
Earth's age	$T_{♁}$	4.54(5) Ga = 143(2) Ps
Earth's normal gravity	g	$9.806\,65\ \mathrm{m/s^2}$
Earth's standard atmospher. pressure	p_0	101 325 Pa
Moon's radius	$R_{☾\mathrm{v}}$	1738 km in direction of Earth
Moon's radius	$R_{☾\mathrm{h}}$	1737.4 km in other two directions
Moon's mass	$M_{☾}$	$7.35 \cdot 10^{22}$ kg
Moon's mean distance [d]	$d_{☾}$	384 401 km
Moon's distance at perigee [d]		typically 363 Mm, historical minimum 359 861 km
Moon's distance at apogee [d]		typically 404 Mm, historical maximum 406 720 km
Moon's angular size [e]		average 0.5181° = 31.08 ′, minimum 0.49°, maximum 0.55°
Moon's average density	$\rho_{☾}$	$3.3\ \mathrm{Mg/m^3}$
Moon's surface gravity	$g_{☾}$	$1.62\ \mathrm{m/s^2}$
Moon's atmospheric pressure	$p_{☾}$	from 10^{-10} Pa (night) to 10^{-7} Pa (day)
Jupiter's mass	$M_{♃}$	$1.90 \cdot 10^{27}$ kg
Jupiter's radius, equatorial	$R_{♃}$	71.398 Mm
Jupiter's radius, polar	$R_{♃}$	67.1(1) Mm
Jupiter's average distance from Sun	$D_{♃}$	778 412 020 km
Jupiter's surface gravity	$g_{♃}$	$24.9\ \mathrm{m/s^2}$
Jupiter's atmospheric pressure	$p_{♃}$	from 20 kPa to 200 kPa
Sun's mass	$M_{\odot}$	$1.988\,43(3) \cdot 10^{30}$ kg
Sun's gravitational length	$2GM_{\odot}/c^2$	2.953 250 08(5) km
Heliocentric gravitational constant	$GM_{\odot}$	$132.712\,440\,018(8) \cdot 10^{18}\ \mathrm{m^3/s^2}$

TABLE 32 (Continued) Astronomical constants.

QUANTITY	SYMBOL	VALUE
Sun's luminosity	$L_\odot$	384.6 YW
Solar equatorial radius	$R_\odot$	695.98(7) Mm
Sun's angular size		0.53° average; minimum on fourth of July (aphelion) 1888″, maximum on fourth of January (perihelion) 1952″
Sun's average density	$\rho_\odot$	$1.4\ \mathrm{Mg/m^3}$
Sun's average distance	AU	149 597 870.691(30) km
Sun's age	$T_\odot$	4.6 Ga
Solar velocity around centre of galaxy	$v_{\odot\mathrm{g}}$	220(20) km/s
Solar velocity against cosmic background	$v_{\odot\mathrm{b}}$	370.6(5) km/s
Sun's surface gravity	$g_\odot$	$274\ \mathrm{m/s^2}$
Sun's lower photospheric pressure	$p_\odot$	15 kPa
Distance to Milky Way's centre		8.0(5) kpc = 26.1(1.6) kal
Milky Way's age		13.6 Ga
Milky Way's size		*c.* 10^{21} m or 100 kal
Milky Way's mass		10^{12} solar masses, *c.* $2 \cdot 10^{42}$ kg
Most distant galaxy cluster known	SXDF-XCLJ 0218-0510	$9.6 \cdot 10^{9}$ al

a. Defining constant, from vernal equinox to vernal equinox; it was once used to define the second. (Remember: π seconds is about a nanocentury.) The value for 1990 is about 0.7 s less, corresponding to a slowdown of roughly 0.2 ms/a. (Watch out: why?) There is even an empirical formula for the change of the length of the year over time.

Challenge 193 s

Ref. 288

b. The truly amazing precision in the average distance Earth–Sun of only 30 m results from time averages of signals sent from Viking orbiters and Mars landers taken over a period of over twenty years. Note that the International Astronomical Union distinguishes the average distance Earth–Sun from the *astronomical unit* itself; the latter is defined as a fixed and exact length. Also the *light year* is a unit defined as an exact number by the IAU. For more details, see www.iau.org/public/measuring.

c. The shape of the Earth is described most precisely with the World Geodetic System. The last edition dates from 1984. For an extensive presentation of its background and its details, see the www.wgs84.com website. The International Geodesic Union refined the data in 2000. The radii and the flattening given here are those for the 'mean tide system'. They differ from those of the 'zero tide system' and other systems by about 0.7 m. The details constitute a science in itself.

d. Measured centre to centre. To find the precise position of the Moon at a given date, see the www.fourmilab.ch/earthview/moon_ap_per.html page. For the planets, see the page www.fourmilab.ch/solar/solar.html and the other pages on the same site.

e. Angles are defined as follows: 1 degree = 1° = π/180 rad, 1 (first) minute = 1′ = 1°/60, 1 second (minute) = 1″ = 1′/60. The ancient units 'third minute' and 'fourth minute', each 1/60th of the preceding, are not in use any more. ('Minute' originally means 'very small', as it still does in modern English.)

Challenge 194 s

Some properties of nature at large are listed in the following table. (If you want a challenge, can you determine whether any property of the universe itself is listed?)

TABLE 33 Cosmological constants.

QUANTITY	SYMBOL	VALUE
Cosmological constant	Λ	$c.\ 1 \cdot 10^{-52}\,\mathrm{m}^{-2}$
Age of the universe [a]	t_0	$4.333(53) \cdot 10^{17}\,\mathrm{s} = 13.8(0.1) \cdot 10^{9}\,\mathrm{a}$
(determined from space-time, via expansion, using general relativity)		
Age of the universe [a]	t_0	over $3.5(4) \cdot 10^{17}\,\mathrm{s} = 11.5(1.5) \cdot 10^{9}\,\mathrm{a}$
(determined from matter, via galaxies and stars, using quantum theory)		
Hubble parameter [a]	H_0	$2.3(2) \cdot 10^{-18}\,\mathrm{s}^{-1} = 0.73(4) \cdot 10^{-10}\,\mathrm{a}^{-1}$
		$= h_0 \cdot 100\,\mathrm{km/s\,Mpc} = h_0 \cdot 1.0227 \cdot 10^{-10}\,\mathrm{a}^{-1}$
Reduced Hubble parameter [a]	h_0	$0.71(4)$
Deceleration parameter [a]	$q_0 = -(\ddot{a}/a)_0/H_0^2$	$-0.66(10)$
Universe's horizon distance [a]	$d_0 = 3ct_0$	$40.0(6) \cdot 10^{26}\,\mathrm{m} = 13.0(2)\,\mathrm{Gpc}$
Universe's topology		trivial up to $10^{26}\,\mathrm{m}$
Number of space dimensions		3, for distances up to $10^{26}\,\mathrm{m}$
Critical density	$\rho_\mathrm{c} = 3H_0^2/8\pi G$	$h_0^2 \cdot 1.878\,82(24) \cdot 10^{-26}\,\mathrm{kg/m^3}$
of the universe		$= 0.95(12) \cdot 10^{-26}\,\mathrm{kg/m^3}$
(Total) density parameter [a]	$\Omega_0 = \rho_0/\rho_\mathrm{c}$	$1.02(2)$
Baryon density parameter [a]	$\Omega_{\mathrm{B}0} = \rho_{\mathrm{B}0}/\rho_\mathrm{c}$	$0.044(4)$
Cold dark matter density parameter [a]	$\Omega_{\mathrm{CDM}0} = \rho_{\mathrm{CDM}0}/\rho_\mathrm{c}$	$0.23(4)$
Neutrino density parameter [a]	$\Omega_{\nu 0} = \rho_{\nu 0}/\rho_\mathrm{c}$	0.001 to 0.05
Dark energy density parameter [a]	$\Omega_{\mathrm{X}0} = \rho_{\mathrm{X}0}/\rho_\mathrm{c}$	$0.73(4)$
Dark energy state parameter	$w = p_\mathrm{X}/\rho_\mathrm{X}$	$-1.0(2)$
Baryon mass	m_b	$1.67 \cdot 10^{-27}\,\mathrm{kg}$
Baryon number density		$0.25(1)\,/\mathrm{m}^3$
Luminous matter density		$3.8(2) \cdot 10^{-28}\,\mathrm{kg/m^3}$
Stars in the universe	n_s	$10^{22\pm1}$
Baryons in the universe	n_b	$10^{81\pm1}$
Microwave background temperature [b]	T_0	$2.725(1)\,\mathrm{K}$
Photons in the universe	n_γ	10^{89}
Photon energy density	$\rho_\gamma = \pi^2 k^4/15 T_0^4$	$4.6 \cdot 10^{-31}\,\mathrm{kg/m^3}$
Photon number density		$410.89\,/\mathrm{cm}^3$ or $400\,/\mathrm{cm}^3 (T_0/2.7\,\mathrm{K})^3$
Density perturbation amplitude	$\sqrt{S}$	$5.6(1.5) \cdot 10^{-6}$
Gravity wave amplitude	$\sqrt{T}$	$< 0.71\sqrt{S}$
Mass fluctuations on 8 Mpc	σ_8	$0.84(4)$
Scalar index	n	$0.93(3)$
Running of scalar index	$\mathrm{d}n/\mathrm{d}\ln k$	$-0.03(2)$
Planck length	$l_\mathrm{Pl} = \sqrt{\hbar G/c^3}$	$1.62 \cdot 10^{-35}\,\mathrm{m}$

TABLE 33 (Continued) Cosmological constants.

QUANTITY	SYMBOL	VALUE
Planck time	$t_{\mathrm{Pl}} = \sqrt{\hbar G/c^5}$	$5.39 \cdot 10^{-44}$ s
Planck mass	$m_{\mathrm{Pl}} = \sqrt{\hbar c/G}$	21.8 µg
Instants in history [a]	t_0/t_{Pl}	$8.7(2.8) \cdot 10^{60}$
Space-time points inside the horizon [a]	$N_0 = (R_0/l_{\mathrm{Pl}})^3 \cdot (t_0/t_{\mathrm{Pl}})$	$10^{244\pm1}$
Mass inside horizon	M	$10^{54\pm1}$ kg

[a] The index 0 indicates present-day values.

[b] The radiation originated when the universe was 380 000 years old and had a temperature of about 3000 K; the fluctuations ΔT_0 which led to galaxy formation are today about 16 ± 4 µK = $6(2) \cdot 10^{-6}\,T_0$. Vol. II, page 231

USEFUL NUMBERS

Ref. 289

π	3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510$_5$
e	2.71828 18284 59045 23536 02874 71352 66249 77572 47093 69995$_9$
γ	0.57721 56649 01532 86060 65120 90082 40243 10421 59335 93992$_3$
ln 2	0.69314 71805 59945 30941 72321 21458 17656 80755 00134 36025$_5$
ln 10	2.30258 50929 94045 68401 79914 54684 36420 76011 01488 62877$_2$
$\sqrt{10}$	3.16227 76601 68379 33199 88935 44432 71853 37195 55139 32521$_6$

APPENDIX B

COMPOSITE PARTICLE PROPERTIES

The following table lists the most important composite particles. The list has not changed much recently, mainly because of the vast progress that was achieved already in the middle of the twentieth century. In principle, using the standard model of particle physics, *all* properties of composite matter and radiation can be deduced. In particular, all properties of objects encountered in everyday life follow. (Can you explain how the size of an apple follows from the standard model?) The most important examples of composites are grouped in the following table.

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Challenge 195 s

TABLE 34 Properties of selected composites.

COMPOSITE	MASS m, QUANTUM NUMBERS[a]	LIFETIME τ, MAIN DECAY MODES	SIZE (DIAM.)
Mesons (hadrons, bosons) (selected from over 130 known types)			
Pion $\pi^0(u\bar{u} - d\bar{d})/\sqrt{2}$	134.976 4(6) MeV/c^2 $I^G(J^{PC}) = 1^-(0^{-+})$, $S = C = B = 0$	84(6) as, 2γ 98.798(32) %	~ 1 fm
Pion $\pi^+(u\bar{d})$	139.569 95(35) MeV/c^2 $I^G(J^P) = 1^-(0^-)$, $S = C = B = 0$	26.030(5) ns, $\mu^+\nu_\mu$ 99.987 7(4) %	~ 1 fm
Kaon K^0_S	$m_{K^0_S}$	89.27(9) ps	~ 1 fm
Kaon K^0_L	$m_{K^0_S}$ + 3.491(9) μeV/c^2	51.7(4) ns	~ 1 fm
Kaon $K^\pm$ $(u\bar{s}, \bar{u}s)$	493.677(16) MeV/c^2	12.386(24) ns, $\mu^+\nu_\mu$ 63.51(18) % $\pi^+\pi^0$ 21.16(14) %	~ 1 fm
Kaon K^0 $(\mathrm{d\bar{s}})$ (50 % K_S, 50 % K_L)	497.672(31) MeV/c^2	n.a.	~ 1 fm
All kaons $K^\pm, K^0, K^0_S, K^0_L$:	$I(J^P) = \frac{1}{2}(0^-)$, $S = \pm1$, $B = C = 0$		
Baryons (hadrons, fermions) (selected from over 100 known types)			
Proton p or N^+ (uud)	1.672 621 58(13) yg = 1.007 276 466 88(13) u = 938.271 998(38) MeV/c^2 $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$, $S = 0$ gyromagnetic ratio μ_p/μ_N = 2.792 847 337(29) electric dipole moment $d = (-4 \pm 6) \cdot 10^{-26} e\,\mathrm{m}$	$\tau_{\text{total}} > 1.6 \cdot 10^{25}$ a, $\tau(p \rightarrow e^+\pi^0) > 5.5 \cdot 10^{32}$ a	0.89(1) fm Ref. 290

TABLE 34 (Continued) Properties of selected composites.

Composite	Mass m, quantum numbers[a]	Lifetime τ, main decay modes	Size (diam.)
	electric polarizability $\alpha_e = 12.1(0.9)\cdot 10^{-4}\,\mathrm{fm}^3$		
	magnetic polarizability $\alpha_m = 2.1(0.9)\cdot 10^{-4}\,\mathrm{fm}^3$		
Neutron[b] n or N^0 (udd)	1.674 927 16(13) yg	887.0(2.0) s, $pe^-\bar{\nu}_e$ 100 %	~ 1 fm
	= 1.008 664 915 78(55) u = 939.565 330(38) MeV/c^2		
	$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$, $S = 0$		
	gyromagnetic ratio $\mu_n/\mu_N = -1.913\,042\,72(45)$		
	electric dipole moment $d_n = (-3.3 \pm 4.3)\cdot 10^{-28} e\,\mathrm{m}$		
	electric polarizability $\alpha = 0.98(23)\cdot 10^{-3}\,\mathrm{fm}^3$		
Omega Ω^- (sss)	1672.43(32) MeV/c^2	82.2(1.2) ps,	~ 1 fm
		ΛK^- 67.8(7) %,	
		$\Xi^0\pi^-$ 23.6(7) %	
	gyromagnetic ratio $\mu_\Omega/\mu_N = -1.94(22)$		
composite radiation: glueballs			
glueball candidate $f_0(1500)$, status unclear	1503(11) MeV	full width 120(19) MeV	~ 1 fm
	$I^G(J^{PC}) = 0^+(0^{++})$		
Atoms (selected from 114 known elements with over 2000 known nuclides) Ref. 291			
Hydrogen (^{1}H) [lightest]	1.007 825 032(1) u = 1.6735 yg		2 · 53 pm
Antihydrogen[c]	1.007 u = 1.67 yg		2 · 53 pm
Helium (^{4}He) [smallest]	4.002 603250(1) u = 6.6465 yg		2 · 31 pm
Carbon (^{12}C)	12 u = 19.926 482(12) yg		2 · 77 pm
Bismuth (^{209}Bi*) [shortest living and rarest]	209 u	0.1 ps Ref. 292	
Tantalum (^{180m}Ta) [second longest living radioactive]	180 u	$> 10^{15}$ a Ref. 293	
Bismuth (^{209}Bi) [longest living radioactive]	209 u	$1.9(2)10^{19}$ a Ref. 292	
Francium (^{223}Fr) [largest]	223 u	22 min	2 · 0.28 nm
Oganesson (^{289}Og) [heaviest]	294 u	0.9 ms	
Molecules[d] (selected from over 10^7 known types)			
Hydrogen (H_2)	~ 2 u	$> 10^{25}$ a	
Water (H_2O)	~ 18 u	$> 10^{25}$ a	
ATP (adenosinetriphosphate)	507 u	$> 10^{10}$ a	*c.* 3 nm
Human Y chromosome	$70\cdot 10^6$ base pairs	$> 10^6$ a	*c.* 50 mm (uncoiled)

TABLE 34 (Continued) Properties of selected composites.

Composite	Mass m, quantum numbers[a]	Lifetime τ, main decay modes	Size (diam.)
Other composites			
Blue whale nerve cell	~ 1 kg	~ 50 a	20 m
Cell (red blood)	0.1 ng	7 plus 120 days	~ 10 µm
Cell (sperm)	10 pg	not fecundated: ~ 5 d	length 60 µm, head 3 µm × 5 µm
Cell (ovule)	1 µg	fecundated: over 4000 million years	~ 120 µm
Cell (*E. coli*)	1 pg	4000 million years	body: 2 µm
Apple	0.1 kg	4 weeks	0.1 m
Adult human	35 kg < m < 350 kg	$\tau \approx 2.5 \cdot 10^9$ s Ref. 294 ≈ 600 million breaths ≈ 2 500 million heartbeats < 122 a, 60 % H_2O and 40 % dust	~ 1.7 m
Heaviest living thing: colony of aspen trees	$6.6 \cdot 10^6$ kg	> 130 a	> 4 km
Larger composites	See the table on page 260 in volume I.		

Page 263 Notes (see also the notes of Table 9):
a. The charge parity C is defined only for certain neutral particles, namely those that are different from their antiparticles. For neutral mesons, the charge parity is given by $C = (-1)^{L+S}$, where L is the orbital angular momentum.
P is the parity under space inversion $\boldsymbol{r} \rightarrow -\boldsymbol{r}$. For mesons, it is related to the orbital angular momentum L through $P = (-1)^{L+1}$.
The electric polarizability, defined on page 72 in volume III, is predicted to vanish for all elementary particles.
G-parity is defined only for mesons and given by $G = (-1)^{L+S+I} = (-1)^{I}C$.
b. Neutrons bound in nuclei have a lifetime of at least 10^{20} years.
c. The first *anti-atoms*, made of antielectrons and antiprotons, were made in January 1996 at CERN in Geneva. All properties of antimatter checked so far are consistent with theoretical predictions. Ref. 296
d. The number of existing molecules is several orders of magnitude larger than the number of molecules that have been analysed and named.

The most important matter composites are the *atoms*. Their size, structure and interactions determine the properties and colour of everyday objects. Atom types, also called *elements* in chemistry, are most usefully set out in the so-called *periodic table*, which groups together atoms with similar properties in rows and columns. It is given in Table 35 and results from the various ways in which protons, neutrons and electrons can combine to form aggregates.

Comparable to the periodic table of the atoms, there are tables for the mesons (made of two quarks) and the baryons (made of three quarks). Neither the meson nor the baryon table is included here; they can both be found in the *Review of Particle Physics* at pdg.web.cern.ch. In fact, the baryon table still has a number of vacant spots. The missing baryons are extremely heavy and short-lived (which means expensive to make and detect), and their discovery is not expected to yield deep new insights.

TABLE 35 The periodic table of the elements, with their atomic numbers. Light blue: nonmetals, orange: alkali metals, green: alkaline earth metals, grey: transition metals, dark blue: basic metals, light orange: semimetals, yellow: halogens, brown: noble gases, red: lanthanoids, dark red: actinoids, black: no data.

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	I	II	IIIa	IVa	Va	VIa	VIIa		VIIIa		Ia	IIa	III	IV	V	VI	VII	VIII
Period																		
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	* *	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
Lanthanoids			*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinoids			* *	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

More elaborate periodic tables can be found on the chemlab.pc.maricopa.edu/periodic website. The most beautiful of them all can be found on page 60. The *atomic number* gives the number of protons (and electrons) found in an atom of a given element. This number determines the chemical behaviour of an element. Most – but not all – elements up to 92 are found on Earth; the others can be produced in laboratories. The highest element discovered is element 118. In a famous case of research fraud, a scientist in the 1990s tricked two whole research groups into claiming to have made and observed elements 116 and 118. Both elements were independently made and observed later on.

Ref. 297 Nowadays, extensive physical and chemical data are available for every element. Photographs of the pure elements are shown in Figure 19. Elements in the same *group* behave similarly in chemical reactions. The *periods* define the repetition of these similarities. Page 61

The elements of group 1 are the *alkali metals* (though the exceptional hydrogen is a gas), those of group 2 are the alkaline earth metals. Also *actinoids*, *lanthanoids* are metals, as are the elements of groups 3 to 12, which are called *transition* or *heavy metals*. The elements of group 16 are called *chalkogens*, i.e., ore-formers; group 17 are the *halogens*, i.e., the salt-formers, and group 18 are the inert *noble gases*, which form (almost) no chemical compounds. The groups 13, 14 and 15 contain metals, semimetals, the only room-temperature liquid – bromine – and a few gases and non-metals; these groups have no special name. Groups 1 and 13 to 17 are central for the chemistry of life; in fact, 96 % of living matter is made of C, O, N, H;* almost 4 % of P, S, Ca, K, Na, Cl; trace elements such as Mg, V, Cr, Mn, Fe, Co, Ni, Cu, Zn, Cd, Pb, Sn, Li, Mo, Se, Si, I, F, As, B form the rest. Over 30 elements are known to be essential for animal life. The full list is not yet known; candidate elements to extend this list are Al, Br, Ge and W.

Many elements exist in versions with different numbers of neutrons in their nucleus, and thus with different mass; these various *isotopes* – so called because they are found at the *same place* in the periodic table – behave identically in chemical reactions. There are over 2000 of them. Ref. 291, Ref. 298

TABLE 36 The elements, with their atomic number, average mass, atomic radius and main properties.

NAME	SYMBOL	AT. N.	AVER. MASS[a] IN U (ERROR), LONGEST LIFETIME	ATOMIC[e] RADIUS IN PM	MAIN PROPERTIES, (NAMING)[h] DISCOVERY DATE AND USE
Actinium[b]	Ac	89	(227.0277(1)) 21.77(2) a	(188)	Highly radioactive metallic rare Earth (Greek *aktis* ray) 1899, used as alpha-emitting source.
Aluminium	Al	13	26.981 538 (8) stable	118c, 143m	Light metal (Latin *alumen* alum) 1827, used in machine construction and living beings.
Americium[b]	Am	95	(243.0614(1)) 7.37(2) ka	(184)	Radioactive metal (Italian *America* from Amerigo) 1945, used in smoke detectors.

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* The ‘average formula’ of life is approximately $C_5H_{40}O_{18}N$.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

NAME	SYM-BOL	AT. N.	AVER. MASS[a] IN U (ERROR), LONGEST LIFETIME	ATOMIC[e] RADIUS IN PM	MAIN PROPERTIES, (NAMING)[h] DISCOVERY DATE AND USE
Antimony	Sb	51	121.760(1)[f] stable	137c, 159m, 205v	Toxic semimetal (via Arabic from Latin *stibium*, itself from Greek, Egyptian for one of its minerals) antiquity, colours rubber, used in medicines, constituent of enzymes.
Argon	Ar	18	39.948(1)[f] stable	(71n)	Noble gas (Greek *argos* inactive, from *anergos* without energy) 1894, third component of air, used for welding and in lasers.
Arsenic	As	33	74.921 60(2) stable	120c, 185v	Poisonous semimetal (Greek *arsenikon* tamer of males) antiquity, for poisoning pigeons and doping semiconductors.
Astatine[b]	At	85	(209.9871(1)) 8.1(4) h	(140)	Radioactive halogen (Greek *astatos* unstable) 1940, no use.
Barium	Ba	56	137.327(7) stable	224m	Earth-alkali metal (Greek *bary* heavy) 1808, used in vacuum tubes, paint, oil industry, pyrotechnics and X-ray diagnosis.
Berkelium[b]	Bk	97	(247.0703(1)) 1.4(3) ka	n.a.	Made in lab, probably metallic (Berkeley, US town) 1949, no use because rare.
Beryllium	Be	4	9.012 182(3) stable	106c, 113m	Toxic Earth-alkali metal (Greek *beryllos*, a mineral) 1797, used in light alloys, in nuclear industry as moderator.
Bismuth	Bi	83	208.980 40(1) stable	170m, 215v	Diamagnetic metal (Latin via German *weisse Masse* white mass) 1753, used in magnets, alloys, fire safety, cosmetics, as catalyst, nuclear industry.
Bohrium[b]	Bh	107	(264.12(1)) 0.44 s[g]	n.a.	Made in lab, probably metallic (after Niels Bohr) 1981, found in nuclear reactions, no use.
Boron	B	5	10.811(7)[f] stable	83c	Semimetal, semiconductor (Latin *borax*, from Arabic and Persian for brilliant) 1808, used in glass, bleach, pyrotechnics, rocket fuel, medicine.
Bromine	Br	35	79.904(1) stable	120c, 185v	Red-brown liquid (Greek *bromos* strong odour) 1826, fumigants, photography, water purification, dyes, medicines.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

NAME	SYMBOL	AT. N.	AVER. MASS[a] IN U (ERROR), LONGEST LIFETIME	ATOMIC[e] RADIUS IN PM	MAIN PROPERTIES, (NAMING)[h] DISCOVERY DATE AND USE
Cadmium	Cd	48	112.411(8)[f] stable	157m	Heavy metal, cuttable and screaming (Greek *kadmeia*, a zinc carbonate mineral where it was discovered) 1817, electroplating, solder, batteries, TV phosphors, dyes.
Caesium	Cs	55	132.905 4519(2) stable	273m	Alkali metal (Latin *caesius* sky blue) 1860, getter in vacuum tubes, photoelectric cells, ion propulsion, atomic clocks.
Calcium	Ca	20	40.078(4)[f] stable	197m	Earth-alkali metal (Latin *calcis* chalk) antiquity, pure in 1880, found in stones and bones, reducing agent, alloying.
Californium[b]	Cf	98	(251.0796(1)) 0.90(5) ka	n.a.	Made in lab, probably metallic, strong neutron emitter (Latin *calor* heat and *fornicare* have sex, the land of hot sex :-) 1950, used as neutron source, for well logging.
Carbon	C	6	12.0107(8)[f] stable	77c	Makes up coal and diamond (Latin *carbo* coal) antiquity, used to build most life forms.
Cerium	Ce	58	140.116(1)[f] stable	183m	Rare Earth metal (after asteroid Ceres, Roman goddess) 1803, cigarette lighters, incandescent gas mantles, glass manufacturing, self-cleaning ovens, carbon-arc lighting in the motion picture industry, catalyst, metallurgy.
Chlorine	Cl	17	35.453(2)[f] stable	102c, 175v	Green gas (Greek *chloros* yellow-green) 1774, drinking water, polymers, paper, dyes, textiles, medicines, insecticides, solvents, paints, rubber.
Chromium	Cr	24	51.9961(6) stable	128m	Transition metal (Greek *chromos* colour) 1797, hardens steel, makes steel stainless, alloys, electroplating, green glass dye, catalyst.
Cobalt	Co	27	58.933 195(5) stable	125m	Ferromagnetic transition metal (German *Kobold* goblin) 1694, part of vitamin B_{12}, magnetic alloys, heavy-duty alloys, enamel dyes, ink, animal nutrition.
Copernicium[b]	Cn	112	(285) 34 s[g]	n.a.	Made in lab, 1996, no use.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

Name	Symbol	At. n.	Aver. mass[a] in u (error), longest lifetime	Atomic[e] radius in pm	Main properties, (naming)[h] discovery date and use
Copper	Cu	29	63.546(3)[f] stable	128m	Red metal (Latin *cuprum* from Cyprus island) antiquity, part of many enzymes, electrical conductors, bronze, brass and other alloys, algicides, etc.
Curium[b]	Cm	96	(247.0704(1)) 15.6(5) Ma	n.a.	Highly radioactive, silver-coloured (after Pierre and Marie Curie) 1944, used as radioactivity source.
Darmstadtium[b]	Ds	110	(271) 1.6 min[g]	n.a.	Made in lab (after the German city) 1994, no use.
Dubnium[b]	Db	105	(262.1141(1)) 34(5) s	n.a.	Made in lab in small quantities, radioactive (Dubna, Russian city) 1967, no use (once known as hahnium).
Dysprosium	Dy	66	162.500(1)[f] stable	177m	Rare Earth metal (Greek *dysprositos* difficult to obtain) 1886, used in laser materials, as infrared source material, and in nuclear industry.
Einsteinium[b]	Es	99	(252.0830(1)) 472(2) d	n.a.	Made in lab, radioactive (after Albert Einstein) 1952, no use.
Erbium	Er	68	167.259(3)[f] stable	176m	Rare Earth metal (Ytterby, Swedish town) 1843, used in metallurgy and optical fibres.
Europium	Eu	63	151.964(1)[f] stable	204m	Rare Earth metal (named after the continent) 1901, used in red screen phosphor for TV tubes.
Fermium[b]	Fm	100	(257.0901(1)) 100.5(2) d	n.a.	Made in lab (after Enrico Fermi) 1952, no use.
Flerovium[b]	Fl	114	(289)	2.7 s[g]	1999, no use.
Fluorine	F	9	18.998 4032(5) stable	62c, 147v	Gaseous halogen (from fluorine, a mineral, from Greek *fluo* flow) 1886, used in polymers and toothpaste.
Francium[b]	Fr	87	(223.0197(1)) 22.0(1) min	(278)	Radioactive metal (from France) 1939, no use.
Gadolinium	Gd	64	157.25(3)[f] stable	180m	Rare-earth metal (after Johan Gadolin) 1880, used in lasers and phosphors.
Gallium	Ga	31	69.723(1) stable	125c, 141m	Almost liquid metal (Latin for both the discoverer's name and his nation, France) 1875, used in optoelectronics.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

Name	Symbol	At. n.	Aver. mass[a] in u (error), longest lifetime	Atomic[e] radius in pm	Main properties, (naming)[h] discovery date and use
Germanium	Ge	32	72.64(1) stable	122c, 195v	Semiconductor (from Germania, as opposed to gallium) 1886, used in electronics.
Gold	Au	79	196.966 569(4) stable	144m	Heavy noble metal (Sanskrit *jval* to shine, Latin aurum) antiquity, electronics, jewels.
Hafnium	Hf	72	178.49(2)[c] stable	158m	Metal (Latin for Copenhagen) 1923, alloys, incandescent wire.
Hassium[b]	Hs	108	(277) 16.5 min[g]	n.a.	Radioactive element (Latin form of German state Hessen) 1984, no use .
Helium	He	2	4.002 602(2)[f] stable	(31n)	Noble gas (Greek *helios* Sun) where it was discovered 1895, used in balloons, stars, diver's gas and cryogenics.
Holmium	Ho	67	164.930 32(2) stable	177m	Metal (Stockholm, Swedish capital) 1878, alloys.
Hydrogen	H	1	1.007 94(7)[f] stable	30c	Reactive gas (Greek for water-former) 1766, used in building stars and universe.
Indium	In	49	114.818(3) stable	141c, 166m	Soft metal (Greek *indikon* indigo) 1863, used in solders and photocells.
Iodine	I	53	126.904 47(3) stable	140c, 198v	Blue-black solid (Greek *iodes* violet) 1811, used in photography.
Iridium	Ir	77	192.217(3) stable	136m	Precious metal (Greek *iris* rainbow) 1804, electrical contact layers.
Iron	Fe	26	55.845(2) stable	127m	Metal (Indo-European *ayos* metal, Latin ferrum) antiquity, used in metallurgy.
Krypton	Kr	36	83.798(2)[f] stable	(88n)	Noble gas (Greek *kryptos* hidden) 1898, used in lasers.
Lanthanum	La	57	138.905 47(7)[c,f] stable	188m	Reactive rare Earth metal (Greek *lanthanein* to be hidden) 1839, used in lamps and in special glasses.
Lawrencium[b]	Lr	103	(262.110 97(1)) 3.6(3) h	n.a.	Appears in reactions (after Ernest Lawrence) 1961, no use.
Lead	Pb	82	207.2(1)[c,f] stable	175m	Poisonous, malleable heavy metal (Latin *plumbum*) antiquity, used in car batteries, radioactivity shields, paints.
Lithium	Li	3	6.941(2)[f] stable	156m	Light alkali metal with high specific heat (Greek *lithos* stone) 1817, used in batteries, anti-depressants, alloys, nuclear fusion and many chemicals.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

Name	Symbol	At. n.	Aver. mass[a] in u (error), longest lifetime	Atomic[e] radius in pm	Main properties, (naming)[h] discovery date and use
Livermorium[b]	Lv	116	(293)	61 ms[g]	False discovery claim from 1999, correct claim from 2000, no use.
Lutetium	Lu	71	174.967(1)[f] stable	173m	Rare-earth metal (Latin *Lutetia* for Paris) 1907, used as catalyst.
Magnesium	Mg	12	24.3050(6) stable	160m	Light common alkaline Earth metal (from Magnesia, a Greek district in Thessalia) 1755, used in alloys, pyrotechnics, chemical synthesis and medicine, found in chlorophyll.
Manganese	Mn	25	54.938 045(5) stable	126m	Brittle metal (Italian *manganese*, a mineral) 1774, used in alloys, colours amethyst and permanganate.
Meitnerium[b]	Mt	109	(268.1388(1)) 0.070 s[g]	n.a.	Appears in nuclear reactions (after Lise Meitner) 1982, no use.
Mendelevium[b]	Md	101	(258.0984(1)) 51.5(3) d	n.a.	Appears in nuclear reactions (after Дмитрии Иванович Менделеев Dmitriy Ivanovich Mendeleyev) 1955, no use.
Mercury	Hg	80	200.59(2) stable	157m	Liquid heavy metal (Latin god Mercurius, Greek *hydrargyrum* liquid silver) antiquity, used in switches, batteries, lamps, amalgam alloys.
Molybdenum	Mo	42	95.94(2)[f] stable	140m	Metal (Greek *molybdos* lead) 1788, used in alloys, as catalyst, in enzymes and lubricants.
Moscovium[b]	Mc	115	(288)	8 ms[g]	2004 (Moscow), no use.
Neodymium	Nd	60	144.242(3)[c,f] stable	182m	(Greek *neos* and *didymos* new twin) 1885, used in magnets.
Neon	Ne	10	20.1797(6)[f] stable	(36n)	Noble gas (Greek *neos* new) 1898, used in lamps, lasers and cryogenics.
Neptunium[b]	Np	93	(237.0482(1)) 2.14(1) Ma	n.a.	Radioactive metal (planet Neptune, after Uranus in the solar system) 1940, appears in nuclear reactors, used in neutron detection and by the military.
Nickel	Ni	28	58.6934(2) stable	125m	Metal (German *Nickel* goblin) 1751, used in coins, stainless steels, batteries, as catalyst.
Nihonium[b]	Nh	113	(284)	0.48 s[g]	2003 (Nihon is Japan in Japanese), no use.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

NAME	SYMBOL	AT. N.	AVER. MASS[a] IN U (ERROR), LONGEST LIFETIME	ATOMIC[e] RADIUS IN PM	MAIN PROPERTIES, (NAMING)[h] DISCOVERY DATE AND USE
Niobium	Nb	41	92.906 38(2) stable	147m	Ductile metal (Greek Niobe, mythical daughter of Tantalos) 1801, used in arc welding, alloys, jewellery, superconductors.
Nitrogen	N	7	14.0067(2)[f] stable	70c, 155v	Diatomic gas (Greek for nitre-former) 1772, found in air, in living organisms, Viagra, fertilizers, explosives.
Nobelium[b]	No	102	(259.1010(1)) 58(5) min	n.a.	(after Alfred Nobel) 1958, no use.
Oganesson[b]	Og	118	(294)	0.9 ms[g]	False discovery claim in 1999, correct claim from 2006 (after Yuri Oganessian), no use.
Osmium	Os	76	190.23(3)[f] stable	135m	Heavy metal (from Greek *osme* odour) 1804, used for fingerprint detection and in very hard alloys.
Oxygen	O	8	15.9994(3)[f] stable	66c, 152v	Transparent, diatomic gas (formed from Greek to mean 'acid former') 1774, used for combustion, blood regeneration, to make most rocks and stones, in countless compounds, colours auroras red.
Palladium	Pd	46	106.42(1)[f] stable	138m	Heavy metal (from asteroid Pallas, after the Greek goddess) 1802, used in alloys, white gold, catalysts, for hydride storage.
Phosphorus	P	15	30.973 762(2) stable	109c, 180v	Poisonous, waxy, white solid (Greek *phosphoros* light bearer) 1669, fertilizers, glasses, porcelain, steels and alloys, living organisms, bones.
Platinum	Pt	78	195.084(9) stable	139m	Silvery-white, ductile, noble heavy metal (Spanish *platina* little silver) pre-Columbian, again in 1735, used in corrosion-resistant alloys, magnets, furnaces, catalysts, fuel cells, cathodic protection systems for large ships and pipelines; being a catalyst, a fine platinum wire glows red hot when placed in vapour of methyl alcohol, an effect used in hand warmers.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

Name	Symbol	At. n.	Aver. mass[a] in u (error), longest lifetime	Atomic[e] radius in pm	Main properties, (naming)[h] discovery date and use
Plutonium	Pu	94	(244.0642(1)) 80.0(9) Ma	n.a.	Extremely toxic alpha-emitting metal (after the planet) synthesized 1940, found in nature 1971, used as nuclear explosive, and to power space equipment, such as satellites and the measurement equipment brought to the Moon by the Apollo missions.
Polonium	Po	84	(208.9824(1)) 102(5) a	(140)	Alpha-emitting, volatile metal (from Poland) 1898, used as thermoelectric power source in space satellites, as neutron source when mixed with beryllium; used in the past to eliminate static charges in factories, and on brushes for removing dust from photographic films.
Potassium	K	19	39.0983(1) stable	238m	Reactive, cuttable light metal (German *Pottasche*, Latin *kalium* from Arabic *quilyi*, a plant used to produce potash) 1807, part of many salts and rocks, essential for life, used in fertilizers, essential to chemical industry.
Praeseodymium	Pr	59	140.907 65(2) stable	183m	White, malleable rare Earth metal (Greek *praesos didymos* green twin) 1885, used in cigarette lighters, material for carbon arcs used by the motion picture industry for studio lighting and projection, glass and enamel dye, darkens welder's goggles.
Promethium[b]	Pm	61	(144.9127(1)) 17.7(4) a	181m	Radioactive rare Earth metal (from the Greek mythical figure of Prometheus) 1945, used as β source and to excite phosphors.
Protactinium	Pa	91	(231.035 88(2)) 32.5(1) ka	n.a.	Radioactive metal (Greek *protos* first, as it decays into actinium) 1917, found in nature, no use.
Radium	Ra	88	(226.0254(1)) 1599(4) a	(223)	Highly radioactive metal (Latin *radius* ray) 1898, no use any more; once used in luminous paints and as radioactive source and in medicine.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

Name	Symbol	At. n.	Aver. mass[a] in u (error), longest lifetime	Atomic[e] radius in pm	Main properties, (naming)[h] discovery date and use
Radon	Rn	86	(222.0176(1)) 3.823(4) d	(130n)	Radioactive noble gas (from its old name 'radium emanation') 1900, no use (any more), found in soil, produces lung cancer .
Rhenium	Re	75	186.207(1)[c] stable	138m	TRansition metal (Latin *rhenus* for Rhine river) 1925, used in filaments for mass spectrographs and ion gauges, superconductors, thermocouples, flash lamps, and as catalyst.
Rhodium	Rh	45	102.905 50(2) stable	135m	White metal (Greek *rhodon* rose) 1803, used to harden platinum and palladium alloys, for electroplating, and as catalyst.
Roentgenium[b]	Rg	111	(272.1535(1)) 1.5 ms[g]	n.a.	Made in lab (after Conrad Roentgen) 1994, no use.
Rubidium	Rb	37	85.4678(3)[f] stable	255m	Silvery-white, reactive alkali metal (Latin *rubidus* red) 1861, used in photocells, optical glasses, solid electrolytes.
Ruthenium	Ru	44	101.107(2)[f] stable	134m	White metal (Latin *Rhuthenia* for Russia) 1844, used in platinum and palladium alloys, superconductors, as catalyst; the tetroxide is toxic and explosive.
Rutherfordium[b]	Rf	104	(261.1088(1)) 1.3 min[g]	n.a.	Radioactive transactinide (after Ernest Rutherford) 1964, no use.
Samarium	Sm	62	150.36(2)[c,f] stable	180m	Silver-white rare Earth metal (from the mineral samarskite, after Wassily Samarski) 1879, used in magnets, optical glasses, as laser dopant, in phosphors, in high-power light sources.
Scandium	Sc	21	44.955 912(6) stable	164m	Silver-white metal (from Latin *Scansia* Sweden) 1879, the oxide is used in high-intensity mercury vapour lamps, a radioactive isotope is used as tracer.
Seaborgium[b]	Sg	106	266.1219(1) 21 s[g]	n.a.	Radioactive transurane (after Glenn Seaborg) 1974, no use.
Selenium	Se	34	78.96(3)[f] stable	120c, 190v	Red or black or grey semiconductor (Greek *selene* Moon) 1818, used in xerography, glass production, photographic toners, as enamel dye.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

NAME	SYMBOL	AT. N.	AVER. MASS[a] IN U (ERROR), LONGEST LIFETIME	ATOMIC[e] RADIUS IN PM	MAIN PROPERTIES, (NAMING)[h] DISCOVERY DATE AND USE
Silicon	Si	14	28.0855(3)[f] stable	105c, 210v	Grey, shiny semiconductor (Latin *silex* pebble) 1823, Earth's crust, electronics, sand, concrete, bricks, glass, polymers, solar cells, essential for life.
Silver	Ag	47	107.8682(2)[f] stable	145m	White metal with highest thermal and electrical conductivity (Latin *argentum*, Greek *argyros*) antiquity, used in photography, alloys, to make rain.
Sodium	Na	11	22.989 769 28(2) stable	191m	Light, reactive metal (Arabic *souwad* soda, Egyptian and Arabic *natrium*) component of many salts, soap, paper, soda, salpeter, borax, and essential for life.
Strontium	Sr	38	87.62(1)[f] stable	215m	Silvery, spontaneously igniting light metal (Strontian, Scottish town) 1790, used in TV tube glass, in magnets, and in optical materials.
Sulphur	S	16	32.065(5)[f] stable	105c, 180v	Yellow solid (Latin) antiquity, used in gunpowder, in sulphuric acid, rubber vulcanization, as fungicide in wine production, and is essential for life; some bacteria use sulphur instead of oxygen in their chemistry.
Tantalum	Ta	73	180.947 88(2) stable	147m	Heavy metal (Greek Tantalos, a mythical figure) 1802, used for alloys, surgical instruments, capacitors, vacuum furnaces, glasses.
Technetium[b]	Tc	43	(97.9072(1)) 6.6(10) Ma	136m	Radioactive (Greek *technetos* artificial) 1939, used as radioactive tracer and in nuclear technology.
Tellurium	Te	52	127.60(3)[f] stable	139c, 206v	Brittle, garlic-smelling semiconductor (Latin *tellus* Earth) 1783, used in alloys and as glass component.
Tennessine[b]	Ts	117	(294)	78 ms[g]	2010 (Tennessee), no use.
Terbium	Tb	65	158.925 35(2) stable	178m	Malleable rare Earth metal (Ytterby, Swedish town) 1843, used as dopant in optical material.
Thallium	Tl	81	204.3833(2) stable	172m	Soft, poisonous heavy metal (Greek *thallos* branch) 1861, used as poison and for infrared detection.

TABLE 36 (Continued) The elements, with their atomic number, average mass, atomic radius and main properties.

Name	Symbol	At. n.	Aver. mass[a] in u (error), longest lifetime	Atomic[e] radius in pm	Main properties, (naming)[h] discovery date and use
Thorium	Th	90	232.038 06(2)[d,f] 14.0(1) Ga	180m	Radioactive (Nordic god Thor, as in 'Thursday') 1828, found in nature, heats Earth, used as oxide in gas mantles for campers, in alloys, as coating, and in nuclear energy.
Thulium	Tm	69	168.934 21(2) stable	175m	Rare Earth metal (Thule, mythical name for Scandinavia) 1879, found in monazite, used in lasers and radiation detectors.
Tin	Sn	50	118.710(7)[f] stable	139c, 210v, 162m	Grey metal that, when bent, allows one to hear the 'tin cry' (Latin *stannum*) antiquity, used in paint, bronze and superconductors.
Titanium	Ti	22	47.867(1) stable	146m	Metal (Greek hero Titanos) 1791, alloys, fake diamonds.
Tungsten	W	74	183.84(1) stable	141m	Heavy, highest-melting metal (Swedish *tung sten* heavy stone, German name *Wolfram*) 1783, lightbulbs.
Uranium	U	92	238.028 91(3)[d,f] $4.468(3) \cdot 10^9$ a	156m	Radioactive and of high density (planet Uranus, after the Greek sky god) 1789, found in pechblende and other minerals, used for nuclear energy.
Vanadium	V	23	50.9415(1) stable	135m	Metal (Vanadis, scandinavian goddess of beauty) 1830, used in steel.
Xenon	Xe	54	131.293(6)[f] stable	(103n) 200v	Noble gas (Greek *xenos* foreign) 1898, used in lamps and lasers.
Ytterbium	Yb	70	173.04(3)[f] stable	174m	Malleable heavy metal (Ytterby, Swedish town) 1878, used in superconductors.
Yttrium	Y	39	88.905 85(2) stable	180m	Malleable light metal (Ytterby, Swedish town) 1794, used in lasers.
Zinc	Zn	30	65.409(4) stable	139m	Heavy metal (German *Zinke* protuberance) antiquity, iron rust protection.
Zirconium	Zr	40	91.224(2)[f] stable	160m	Heavy metal (from the mineral zircon, after Arabic *zargum* golden colour) 1789, chemical and surgical instruments, nuclear industry.

a. The atomic mass unit is defined as $1\,\mathrm{u} = \frac{1}{12} m(^{12}\mathrm{C})$, making $1\,\mathrm{u} = 1.660\,5402(10)\,\mathrm{yg}$. For elements found on Earth, the *average* atomic mass for the naturally occurring isotope mixture is given, with the error in the last digit in brackets. For elements not found on Earth, the mass of

Ref. 298

the *longest living* isotope is given; as it is not an average, it is written in brackets, as is customary in this domain.

b. The element is not found on Earth because of its short lifetime.

c. The element has at least one radioactive isotope.

d. The element has no stable isotopes.

e. Strictly speaking, the *atomic radius* does not exist. Because atoms are clouds, they have no boundary. Several approximate definitions of the 'size' of atoms are possible. Usually, the radius is defined in such a way as to be useful for the estimation of distances between atoms. This distance is different for different bond types. In the table, radii for metallic bonds are labelled m, radii (Ref. 299) for (single) covalent bonds with carbon c, and Van der Waals radii v. Noble gas radii are labelled (Ref. 299) n. Note that values found in the literature vary by about 10 %; values in brackets lack literature references.

The covalent radius can be up to 0.1 nm smaller than the metallic radius for elements on the (lower) left of the periodic table; on the (whole) right side it is essentially equal to the metallic radius. In between, the difference between the two decreases towards the right. Can you explain why? (Challenge 196 s) By the way, ionic radii differ considerably from atomic ones, and depend both on the ionic charge and the element itself.

All these values are for atoms in their ground state. Excited atoms can be hundreds of times larger than atoms in the ground state; however, excited atoms do not form solids or chemical compounds.

f. The isotopic composition, and thus the average atomic mass, of the element varies depending on the place where it was mined or on subsequent human treatment, and can lie outside the values given. For example, the atomic mass of commercial lithium ranges between 6.939 and 6.996 u. (Ref. 291) The masses of isotopes are known in atomic mass units to nine or more significant digits, and usually with one or two fewer digits in kilograms. The errors in the atomic mass are thus mainly due to the variations in isotopic composition. (Ref. 298)

g. The lifetime errors are asymmetric or not well known.

h. Extensive details on element names can be found on elements.vanderkrogt.net.

APPENDIX C

ALGEBRAS, SHAPES AND GROUPS

Mathematicians are fond of generalizing concepts. One of the most generalized concepts of all is the concept of *space*. Understanding mathematical definitions and generalizations means learning to think with precision. The appendix of the previous, fourth volume provided a simple introduction to the types of *spaces* that are of importance in physics; this appendix provides an introduction to the *algebras* that are of importance in physics.

ALGEBRAS

The term *algebra* is used in mathematics with three different, but loosely related, meanings. First, it denotes a part of mathematics, as in 'I hated algebra at school'. Secondly, it denotes a set of formal rules that are obeyed by abstract objects, as in the expression 'tensor algebra'. Finally – and this is the only meaning used here – an algebra denotes a *specific* type of mathematical structure.

Intuitively, an algebra is a set of vectors with a vector multiplication defined on it. More precisely, a (unital, associative) algebra is a vector space (over a field K) that is also a (unital) ring. (The concept is due to Benjamin Peirce (b. 1809 Salem, d. 1880 Cambridge), father of Charles Sanders Peirce.) A ring is a set for which an addition and a multiplication is defined – like the integers. Thus, in an algebra, there are (often) *three* types of multiplications:

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— the (main) algebraic multiplication: the product of two vectors x and y is another vector $z = xy$;

— the scalar multiplication: the c-fold multiple of a vector x is another vector $y = cx$;

— if the vector space is a inner product space, the scalar product: the scalar product of two algebra elements (vectors) x and y is a scalar $c = x \cdot y$;

A precise definition of an algebra thus only needs to define properties of the (main) multiplication and to specify the number field K. An *algebra* is defined by the following axioms

$$\begin{aligned} x(y+z) = xy + xz \quad , \quad (x+y)z = xz + yz & \quad \textit{distributivity of multiplication} \\ c(xy) = (cx)y = x(cy) & \quad \textit{bilinearity} \end{aligned} \qquad (139)$$

for all vectors x, y, z and all scalars $c \in \mathbb{K}$. To stress their properties, algebras are also called *linear* algebras.

For example, the set of all linear transformations of an n-dimensional linear space (such as the translations on a plane, in space or in time) is a linear algebra, if the composition is taken as multiplication. So is the set of observables of a quantum mechanical system.*

An *associative algebra* is an algebra whose multiplication has the additional property that

$$x(yz) = (xy)z \quad \textit{associativity} . \tag{141}$$

Most algebras that arise in physics are associative** and unital. Therefore, in mathematical physics, a linear unital associative algebra is often simply called an algebra.

The set of multiples of the unit 1 of the algebra is called the *field of scalars* scal(A) of the algebra A. The field of scalars is also a subalgebra of A. The field of scalars and the scalars themselves behave in the same way.

We explore a few examples. The set of all polynomials in one variable (or in several Challenge 198 e variables) forms an algebra. It is commutative and infinite-dimensional. The constant polynomials form the field of scalars.

The set of $n \times n$ matrices, with the usual operations, also forms an algebra. It is n^2-dimensional. Those diagonal matrices (matrices with all off-diagonal elements equal to zero) whose diagonal elements all have the same value form the field of scalars. How is Challenge 199 ny the scalar product of two matrices defined?

The set of all real-valued functions over a set also forms an algebra. Can you specify Challenge 200 s the multiplication? The constant functions form the field of scalars.

A *star algebra*, also written $*$-*algebra*, is an algebra over the *complex* numbers for

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* *Linear transformations* are mappings from the vector space to itself, with the property that sums and scalar multiples of vectors are transformed into the corresponding sums and scalar multiples of the transformed Challenge 197 s vectors. Can you specify the set of all linear transformations of the plane? And of three-dimensional space? And of Minkowski space?

All linear transformations transform some special vectors, called *eigenvectors* (from the German word *eigen* meaning 'self') into multiples of themselves. In other words, if T is a transformation, e a vector, and

$$T(e) = \lambda e \tag{140}$$

where λ is a scalar, then the vector e is called an *eigenvector* of T, and λ is associated *eigenvalue*. The set of all eigenvalues of a transformation T is called the *spectrum* of T. Physicists did not pay much attention to these Vol. IV, page 88 mathematical concepts until they discovered quantum theory. Quantum theory showed that observables are transformations in Hilbert space, because any measurement interacts with a system and thus transforms it. Quantum-mechanical experiments also showed that a measurement result for an observable must be an eigenvalue of the corresponding transformation. The state of the system after the measurement is given by Vol. IV, page 157 the eigenvector corresponding to the measured eigenvalue. Therefore every expert on motion must know what an eigenvalue is.

** Note that a non-associative algebra does not possess a matrix representation.

which there is a mapping $* : A \rightarrow A,\ x \mapsto x^*$, called an *involution*, with the properties

$$\begin{aligned}(x^*)^* &= x\\(x+y)^* &= x^* + y^*\\(cx)^* &= \bar{c}x^* \quad \text{for all} \quad c \in \mathbb{C}\\(xy)^* &= y^*x^*\end{aligned} \tag{142}$$

valid for all elements x, y of the algebra A. The element x^* is called the *adjoint* of x. Star algebras are the main type of algebra used in quantum mechanics, since quantum-mechanical observables form a $*$-algebra.

A C$*$-*algebra* is a Banach algebra over the complex numbers with an involution $*$ (a function that is its own inverse) such that the norm $\|x\|$ of an element x satisfies

$$\|x\|^2 = x^*\ x\ . \tag{143}$$

(A Banach algebra is a complete normed algebra; an algebra is *complete* if all Cauchy sequences converge.) In short, C$*$-*algebra* is a nicely behaved algebra whose elements form a continuous set and a complex vector space. The name C comes from 'continuous functions'. Indeed, the *bounded* continuous functions form such an algebra, with a properly defined norm. Can you find it?

Challenge 201 s

Every C$*$-algebra (pronounced 'Cee-star') contains a space of Hermitean elements (which have a real spectrum), a set of normal elements, a multiplicative group of unitary elements and a set of positive elements (with non-negative spectrum). In quantum theory, a physical system is described by a C$*$-algebra, and its Hermitean elements are the observables.

We should mention one important type of algebra used in mathematics. A *division algebra* is an algebra for which the equations $ax = b$ and $ya = b$ are uniquely solvable in x or y for all b and all $a \neq 0$. Obviously, all type of continuous numbers must be division algebras. Division algebras are thus one way to generalize the concept of a number. One of the important results of modern mathematics states that (finite-dimensional) division algebras can *only* have dimension 1, like the reals, dimension 2, like the complex numbers, dimension 4, like the quaternions, or dimension 8, like the octonions. There is thus no way to generalize the concept of (continuous) 'number' to other dimensions.

And now for some fun. Imagine a ring A which contains a number field K as a subring (or 'field of scalars'). If the ring multiplication is defined in such a way that a general ring element multiplied with an element of K is the same as the scalar multiplication, then A is a vector space, and thus an algebra – *provided* that every element of K commutes with every element of A. (In other words, the subring K must be *central*.)

For example, the quaternions $\mathbb{H}$ are a four-dimensional real division algebra, but although $\mathbb{H}$ is a two-dimensional complex vector space, it is *not* a complex algebra, because i does not commute with j (one has $ij = -ji = k$). In fact, there are no finite-dimensional complex division algebras, and the only finite-dimensional real associative division algebras are $\mathbb{R}$, $\mathbb{C}$ and $\mathbb{H}$.

Now, if you are not afraid of getting a headache, think about this remark: every K-algebra is also an algebra over its field of scalars. For this reason, some mathematicians prefer to define an (associative) K-algebra simply as a ring which contains K as a central subfield.

In physics, it is the algebras related to symmetries which play the most important role. We study them next.

LIE ALGEBRAS

A Lie algebra is special type of algebra (and thus of vector space). Lie algebras are the most important type of non-associative algebras. A vector space L over the field ℝ (or ℂ) with an additional binary operation [,], called *Lie multiplication* or the *commutator*, is called a real (or complex) *Lie algebra* if this operation satisfies

$$\begin{aligned} [X,Y] &= -[Y,X] && \textit{antisymmetry} \\ [aX+bY,Z] &= a[X,Z]+b[Y,Z] && \textit{(left-)linearity} \\ [X,[Y,Z]]+[Y,[Z,X]]+[Z,[X,Y]] &= 0 && \textit{Jacobi identity} \end{aligned} \qquad (144)$$

Challenge 202 e

for all elements $X, Y, Z \in L$ and for all $a, b \in \mathbb{R}$ (or ℂ). (Lie algebras are named after Sophus Lie.) The first two conditions together imply bilinearity. A Lie algebra is called *commutative* if $[X,Y] = 0$ for all elements X and Y. The *dimension* of the Lie algebra is the dimension of the vector space. A subspace N of a Lie algebra L is called an *ideal** if $[L,N] \subset N$; any ideal is also a *subalgebra*. A *maximal ideal* M which satisfies $[L,M] = 0$ is called the *centre* of L.

A Lie algebra is called a *linear* Lie algebra if its elements are linear transformations of another vector space V (intuitively, if they are 'matrices'). It turns out that every finite-dimensional Lie algebra is isomorphic to a linear Lie algebra. Therefore, there is no loss of generality in picturing the elements of finite-dimensional Lie algebras as matrices.

Page 371

The name 'Lie algebra' was chosen because the *generators*, i.e., the infinitesimal elements of every Lie group, form a Lie algebra. Since all important symmetries in nature form Lie groups, Lie algebras appear very frequently in physics. In mathematics, Lie algebras arise frequently because from any associative finite-dimensional algebra (in which the symbol · stands for its multiplication) a Lie algebra appears when we define the *commutator* by

$$[X,Y] = X \cdot Y - Y \cdot X \; . \qquad (145)$$

(This fact gave the commutator its name.) Lie algebras are non-associative in general; but the above definition of the commutator shows how to build one from an associative algebra.

Since Lie algebras are vector spaces, the elements T_i of a *basis* of the Lie algebra always obey a relation of the form:

$$[T_i, T_j] = \sum_k c_{ij}^k T_k \; . \qquad (146)$$

The numbers c_{ij}^k are called the *structure constants* of the Lie algebra. They depend on

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Challenge 203 ny

* Can you explain the notation $[L,N]$? Can you define what a maximal ideal is and prove that there is only one?

the choice of basis. The structure constants determine the Lie algebra completely. For example, the algebra of the Lie group SU(2), with the three generators defined by $T_a = \sigma^a/2i$, where the σ^a are the Pauli spin matrices, has the structure constants $C_{abc} = \varepsilon_{abc}$.[*]

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Classification of Lie algebras

Finite-dimensional Lie algebras are classified as follows. Every finite-dimensional Lie algebra is the (semidirect) sum of a semisimple and a solvable Lie algebra.

A Lie algebra is called *solvable* if, well, if it is not semisimple. Solvable Lie algebras have not yet been classified completely. They are not important in physics.

A *semisimple* Lie algebra is a Lie algebra which has no non-zero solvable ideal. Other equivalent definitions are possible, depending on your taste:

— a semisimple Lie algebra does not contain non-zero Abelian ideals;
— its Killing form is non-singular, i.e., non-degenerate;
— it splits into the direct sum of non-Abelian simple ideals (this decomposition is unique);
— every finite-dimensional linear representation is completely reducible;
— the one-dimensional cohomology of g with values in an arbitrary finite-dimensional g-module is trivial.

Finite-dimensional semisimple Lie algebras have been completely classified. They decompose uniquely into a direct sum of *simple* Lie algebras. Simple Lie algebras can be complex or real.

The simple finite-dimensional *complex* Lie algebras all belong to four infinite classes and to five exceptional cases. The infinite classes are also called *classical*, and are: A_n for $n \geqslant 1$, corresponding to the Lie groups SL$(n+1)$ and their compact 'cousins' SU$(n+1)$; B_n for $n \geqslant 1$, corresponding to the Lie groups SO$(2n+1)$; C_n for $n \geqslant 1$, corresponding to the Lie groups Sp$(2n)$; and D_n for $n \geqslant 4$, corresponding to the Lie groups SO$(2n)$. Thus A_n is the algebra of all skew-Hermitean matrices; B_n and D_n are the algebras of the

---

[*] Like groups, Lie algebras can be represented by matrices, i.e., by linear operators. Representations of Lie algebras are important in physics because many continuous symmetry groups are Lie groups.

The *adjoint representation* of a Lie algebra with basis $a_1 ... a_n$ is the set of matrices ad(a) defined for each element a by

$$[a, a_j] = \sum_c \mathrm{ad}(a)_{cj} a_c \;. \tag{147}$$

The definition implies that $\mathrm{ad}(a_i)_{jk} = c^k_{ij}$, where c^k_{ij} are the structure constants of the Lie algebra. For a real Lie algebra, all elements of ad(a) are real for all $a \in L$.

Note that for any Lie algebra, a scalar product can be defined by setting

$$X \cdot Y = \mathrm{Tr}(\, \mathrm{ad}X \cdot \mathrm{ad}Y\,) \;. \tag{148}$$

This scalar product is symmetric and bilinear. (Can you show that it is independent of the representation?) The corresponding bilinear form is also called the *Killing form*, after the mathematician Wilhelm Killing (b. 1847 Burbach, d. 1923 Münster), the discoverer of the 'exceptional' Lie groups. The Killing form is invariant under the action of any automorphism of the Lie algebra L. In a given basis, one has

$$X \cdot Y = \mathrm{Tr}(\,(\mathrm{ad}X) \cdot (\mathrm{ad}Y)) = c^i_{lk} c^k_{si} x^l y^s = g_{ls} x^l y^s \tag{149}$$

where $g_{ls} = c^i_{lk} c^k_{si}$ is called the *Cartan metric tensor* of L.

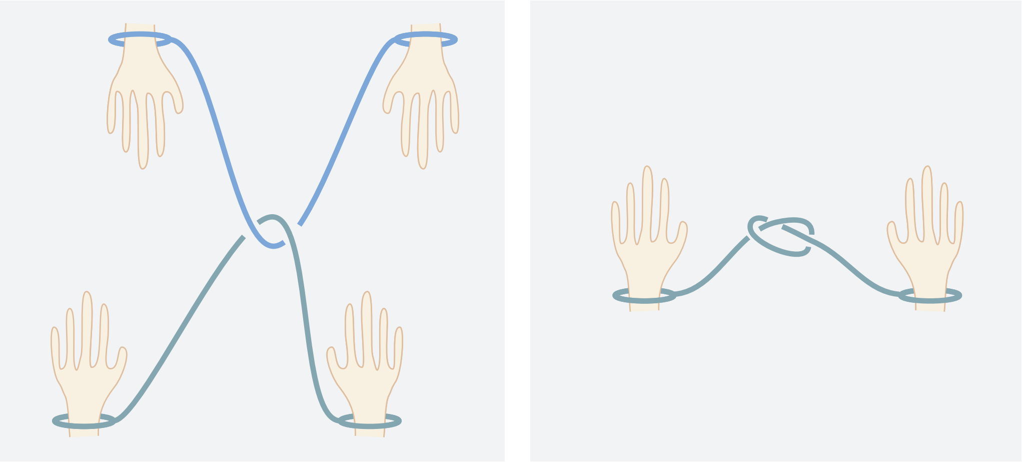

FIGURE 183 Which of the two situations can be untied without cutting?

symmetric matrices; and C_n is the algebra of the traceless matrices.

The exceptional Lie algebras are G_2, F_4, E_6, E_7, E_8. In all cases, the index gives the number of so-called *roots* of the algebra. The dimensions of these algebras are $A_n : n(n+2)$; B_n and $C_n : n(2n+1)$; $D_n : n(2n-1)$; $G_2 : 14$; $F_4 : 32$; $E_6 : 78$; $E_7 : 133$; $E_8 : 248$.

The simple and finite-dimensional *real* Lie algebras are more numerous; their classification follows from that of the complex Lie algebras. Moreover, corresponding to each complex Lie group, there is always one compact real one. Real Lie algebras are not so important in fundamental physics. Ref. 300

Of the large number of *infinite-dimensional* Lie algebras, only one is important in physics, the *Poincaré algebra*. A few other such algebras only appeared in failed attempts for unification.

TOPOLOGY – WHAT SHAPES EXIST?

> “Topology is group theory.
> The Erlangen program”

In a simplified view of topology that is sufficient for physicists, only one type of entity can possess shape: manifolds. *Manifolds* are generalized examples of pullovers: they are locally flat, can have holes and boundaries, and can often be turned inside out.

Pullovers are subtle entities. For example, can you turn your pullover inside out while your hands are tied together? (A friend may help you.) Challenge 204 s By the way, the same feat is also possible with your trousers, while your feet are tied together. Certain professors like to demonstrate this during topology lectures – of course with a carefully selected pair of underpants.

Ref. 301 Another good topological puzzle, the handcuff puzzle, is shown in Figure 183. Which of the two situations can be untied without cutting the ropes? Challenge 205 s

For a mathematician, pullovers and ropes are everyday examples of manifolds, and the operations that are performed on them are examples of deformations. Let us look at some more precise definitions. In order to define what a manifold is, we first need to define the concept of topological space.

TOPOLOGICAL SPACES

> “En Australie, une mouche qui marche au plafond se trouve dans le même sens qu’une vache chez nous.”
>
> Philippe Geluck, *La marque du chat*.

Ref. 302 The study of shapes requires a good definition of a set made of ‘points’. To be able to talk about shape, these sets must be structured in such a way as to admit a useful concept of ‘neighbourhood’ or ‘closeness’ between the elements of the set. The search for the most general type of set which allows a useful definition of neighbourhood has led to the concept of topological space. There are two ways to define a topology: one can define the concept of *open set* and then define the concept of *neighbourhood* with their help, or the other way round. We use the second option, which is somewhat more intuitive.

A *topological space* is a finite or infinite set X of elements, called *points*, together with the neighbourhoods for each point. A *neighbourhood* N of a point x is a collection of subsets Y_x of X with the properties that

- x is in every Y_x;
- if N and M are neighbourhoods of x, so is $N \cap M$;
- anything containing a neighbourhood of x is itself a neighbourhood of x.

The choice of the subsets Y_x is free. The subsets Y_x for all points x, chosen in a particular definition, contain a neighbourhood for each of their points; they are called *open sets*. (A neighbourhood and an open set usually differ, but all open sets are also neighbourhoods. Neighbourhoods of x can also be described as subsets of X that contain an open set that contains x.)

One also calls a topological space a ‘set with a topology’. In effect, a *topology* specifies the systems of ‘neighbourhoods’ of every point of the set. ‘Topology’ is also the name of the branch of mathematics that studies topological spaces.

For example, the real numbers together with all open intervals form the *usual* topology of $\mathbb{R}$. Mathematicians have generalized this procedure. If one takes *all* subsets of $\mathbb{R}$ – or any other basis set – as open sets, one speaks of the *discrete* topology. If one takes *only* the full basis set and the empty set as open sets, one speaks of the *trivial* or *indiscrete* topology.

The concept of topological space allows us to define continuity. A mapping from one topological space X to another topological space Y is *continuous* if the inverse image of every open set in Y is an open set in X. You may verify that this condition is not satisfied by a real function that makes a jump. You may also check that the term ‘inverse’ is necessary in the definition; otherwise a function with a jump would be continuous, as such a function may still map open sets to open sets.* Challenge 206 e

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* The Cauchy–Weierstass definition of continuity says that a real function $f(x)$ is continuous at a point a if (1) f is defined on an open interval containing a, (2) $f(x)$ tends to a limit as x tends to a, and (3) the

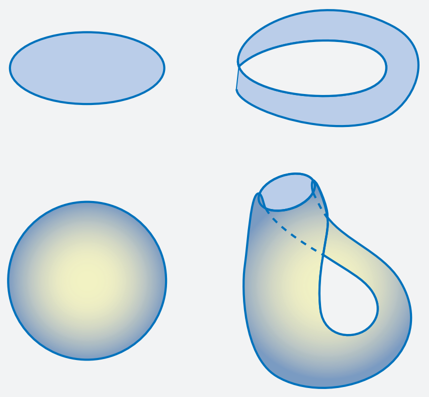

FIGURE 184 Examples of orientable and non-orientable manifolds of two dimensions: a disc, a Möbius strip, a sphere and a Klein bottle.

We thus need the concept of topological space, or of neighbourhood, if we want to express the idea that there are no jumps in nature. We also need the concept of topological space in order to be able to define limits.

Of the many special kinds of topological spaces that have been studied, one type is particularly important. A *Hausdorff space* is a topological space in which for any two points x and y there are disjoint open sets U and V such that x is in U and y is in V. A Hausdorff space is thus a space where, no matter how 'close' two points are, they can always be separated by open sets. This seems like a desirable property; indeed, non-Hausdorff spaces are rather tricky mathematical objects. (At Planck energy, it seems that vacuum appears to behave like a non-Hausdorff space; however, at Planck energy, vacuum is not really a space at all. So non-Hausdorff spaces play no role in physics.) A special case of Hausdorff space is well-known: the manifold.

MANIFOLDS

In physics, the most important topological spaces are differential manifolds. Loosely speaking, a *differential manifold* – physicists simply speak of a *manifold* – is a set of points that looks like $\mathbb{R}^n$ under the microscope – at small distances. For example, a sphere and a torus are both two-dimensional differential manifolds, since they look locally like a plane. Not all differential manifolds are that simple, as the examples of Figure 184 show.

A differential manifold is called *connected* if any two points can be joined by a path lying in the manifold. (The term has a more general meaning in topological spaces. But the notions of connectedness and pathwise connectedness coincide for differential manifolds.) We focus on connected manifolds in the following discussion. A manifold is called *simply connected* if every loop lying in the manifold can be contracted to a point. For example, a sphere is simply connected. A connected manifold which is not simply connected is called *multiply connected*. A torus is multiply connected.

Manifolds can be *non-orientable*, as the well-known Möbius strip illustrates. Non-orientable manifolds have only one surface: they do not admit a distinction between

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limit is $f(a)$. In this definition, the continuity of f is defined using the intuitive idea that the real numbers form the basic model of a set that has no gaps. Can you see the connection with the general definition given above?

Challenge 207 ny

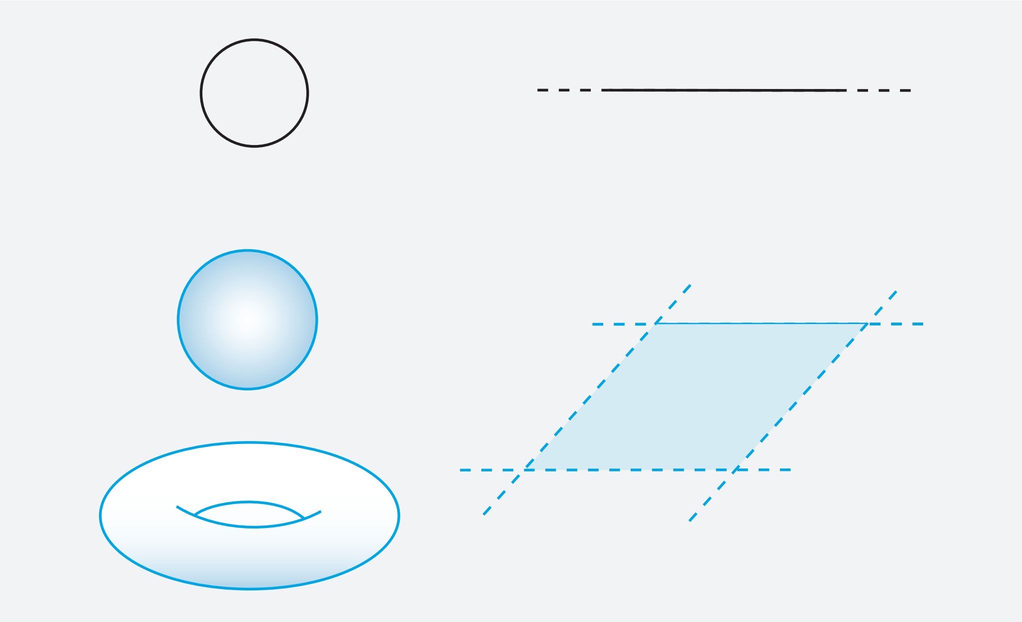

FIGURE 185 Compact (left) and non-compact (right) manifolds of various dimensions.

Challenge 208 e

front and back. If you want to have fun, cut a paper Möbius strip into two along a centre line. You can also try this with paper strips with different twist values, and investigate the regularities.

In two dimensions, closed manifolds (or surfaces), i.e., surfaces that are compact and without boundary, are always of one of three types:

- The simplest type are spheres with n attached handles; they are called *n-tori* or *surfaces of genus n*. They are orientable surfaces with *Euler characteristic* $2 - 2n$.
- The *projective planes* with n handles attached are non-orientable surfaces with Euler characteristic $1 - 2n$.
- The Klein bottles with n attached handles are non-orientable surfaces with Euler characteristic $-2n$.

Therefore Euler characteristic and orientability describe compact surfaces up to homeomorphism (and if surfaces are smooth, then up to diffeomorphism). Homeomorphisms are defined below.

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The two-dimensional compact manifolds or surfaces with boundary are found by removing one or more discs from a surface in this list. A compact surface can be embedded in $\mathbb{R}^3$ if it is orientable or if it has non-empty boundary.

In physics, the most important manifolds are space-time and Lie groups of observables. We study Lie groups below. Strangely enough, the topology of space-time is not known. For example, it is unclear whether or not it is simply connected. Obviously, the reason is that it is difficult to observe what happens at large distances form the Earth. However, a similar difficulty appears near Planck scales.

If a manifold is imagined to consist of rubber, connectedness and similar global properties are not changed when the manifold is deformed. This fact is formalized by saying

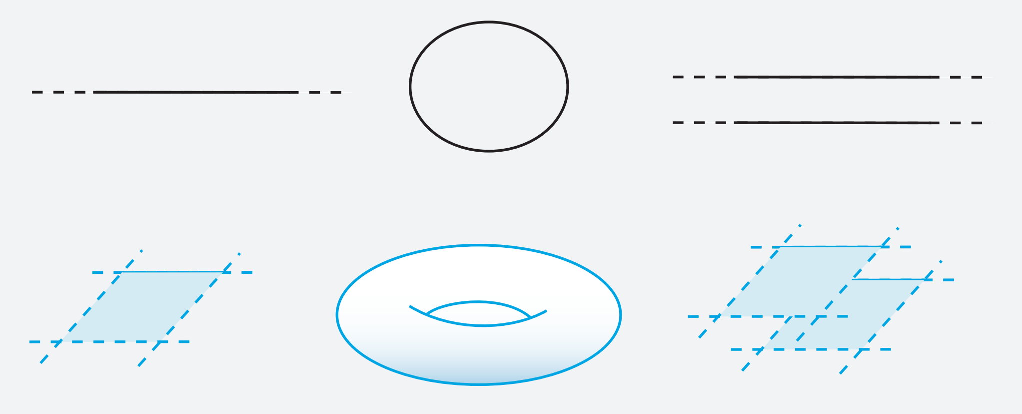

FIGURE 186 Simply connected (left), multiply connected (centre) and disconnected (right) manifolds of one (above) and two (below) dimensions.

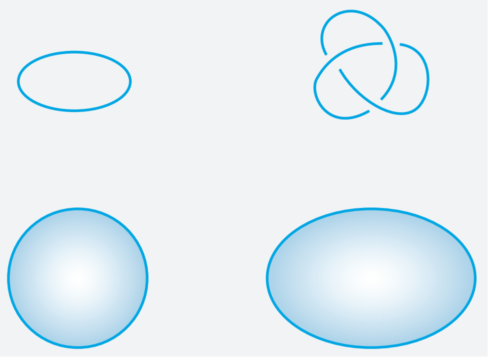

FIGURE 187 Examples of homeomorphic pairs of manifolds.

that two manifolds are *homeomorphic* (from the Greek words for 'same' and 'shape') if between them there is a continuous, one-to-one and onto mapping with a continuous inverse. The concept of homeomorphism is somewhat more general than that of rubber deformation, as can be seen from Figure 187. If the mapping and the manifolds are differentiable, one says that the two manifolds are *diffeomorphic*.

Holes, homotopy and homology

Only 'well-behaved' manifolds play a role in physics: namely those which are orientable and connected. In addition, the manifolds associated with observables, are always compact. The main non-trivial characteristic of connected compact orientable manifolds is that they contain 'holes' (see Figure 188). It turns out that a proper description of the holes of manifolds allows us to distinguish between all different, i.e., non-homeomorphic, types of manifold.

There are three main tools to describe holes of manifolds and the relations among them: homotopy, homology and cohomology. These tools play an important role in the study of gauge groups, because any gauge group defines a manifold.

In other words, through homotopy and homology theory, mathematicians can *clas-*

FIGURE 188 The first four two-dimensional compact connected orientable manifolds: 0-, 1-, 2- and 3-tori.

sify manifolds. Given two manifolds, the properties of the holes in them thus determine whether they can be deformed into each other.

Physicists are now extending these results of standard topology. Deformation is a classical idea which assumes continuous space and time, as well as arbitrarily small action. In nature, however, quantum effects cannot be neglected. It is speculated that quantum effects can transform a physical manifold into one with a *different* topology: for example, a torus into a sphere. Can you find out how this can be achieved? Challenge 209 d

Topological changes of physical manifolds happen via objects that are generalizations of manifolds. An *orbifold* is a space that is locally modelled by $\mathbb{R}^n$ modulo a finite group. Examples are the tear-drop or the half-plane. Orbifolds were introduced by Satake Ichiro in 1956; the name was coined by William Thurston. Orbifolds are heavily studied in string theory.

TYPES AND CLASSIFICATION OF GROUPS

Vol. I, page 272 We introduced mathematical *groups* early on because groups, especially symmetry groups, play an important role in many parts of physics, from the description of solids, molecules, atoms, nuclei, elementary particles and forces up to the study of shapes, cycles and patterns in growth processes.

Group theory is also one of the most important branches of modern mathematics, and is still an active area of research. One of the aims of group theory is the *classification* of all groups. This has been achieved only for a few special types. In general, one distinguishes between finite and infinite groups. Finite groups are better understood.

Every *finite* group is isomorphic to a subgroup of the symmetric group S_N, for some number N. Examples of finite groups are the crystalline groups, used to classify crystal structures, or the groups used to classify wallpaper patterns in terms of their symmetries. The symmetry groups of Platonic and many other regular solids are also finite groups.

Finite groups are a complex family. Roughly speaking, a general (finite) group can be seen as built from some fundamental bricks, which are groups themselves. These fundamental bricks are called *simple* (finite) groups. One of the high points of twentieth-century mathematics was the classification of the finite simple groups. It was a collaborative effort that took around 30 years, roughly from 1950 to 1980. The complete list of finite simple groups consists of Ref. 303

1) the *cyclic groups* Z_p of prime group order;

2) the *alternating groups* A_n of degree n at least five;

3) the *classical linear groups*, $\mathrm{PSL}(n;q)$, $\mathrm{PSU}(n;q)$, $\mathrm{PSp}(2n;q)$ and $\mathrm{P}\Omega^{\varepsilon}(n;q)$;

4) the *exceptional* or *twisted groups* of Lie type ${}^3\mathrm{D}_4(q)$, $\mathrm{E}_6(q)$, ${}^2\mathrm{E}_6(q)$, $\mathrm{E}_7(q)$, $\mathrm{E}_8(q)$, $\mathrm{F}_4(q)$, ${}^2\mathrm{F}_4(2^n)$, $\mathrm{G}_2(q)$, ${}^2\mathrm{G}_2(3^n)$ and ${}^2\mathrm{B}_2(2^n)$;

5) the 26 *sporadic groups*, namely M_{11}, M_{12}, M_{22}, M_{23}, M_{24} (the Mathieu groups), J_1,

J_2, J_3, J_4 (the Janko groups), Co_1, Co_2, Co_3 (the Conway groups), HS, Mc, Suz (the Co_1 'babies'), Fi_{22}, Fi_{23}, Fi'_{24} (the Fischer groups), $F_1 = M$ (the Monster), F_2, F_3, F_5, He ($= F_7$) (the Monster 'babies'), Ru, Ly, and ON.

The classification was finished in the 1980s after over 10 000 pages of publications. The proof is so vast that a special series of books has been started to summarize and explain it. The first three families are infinite. The last family, that of the sporadic groups, is the most peculiar; it consists of those finite simple groups which do not fit into the other families. Some of these sporadic groups might have a role in particle physics: possibly even the largest of them all, the so-called *Monster* group. This is still a topic of research. (The Monster group has about $8.1 \cdot 10^{53}$ elements; more precisely, its order is 808 017 424 794 512 875 886 459 904 961 710 757 005 754 368 000 000 000 or $2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$.)

Of the *infinite* groups, only those with some finiteness condition have been studied. It is only such groups that are of interest in the description of nature. Infinite groups are divided into *discrete* groups and *continuous* groups. Discrete groups are an active area of mathematical research, having connections with number theory and topology. Continuous groups are divided into *finitely generated* and *infinitely generated* groups. Finitely generated groups can be finite-dimensional or infinite-dimensional.

The most important class of finitely generated continuous groups are the Lie groups.

LIE GROUPS

In nature, the Lagrangians of the fundamental forces are invariant under gauge transformations and under continuous space-time transformations. These symmetry groups are examples of Lie groups, which are a special type of infinite continuous group. They are named after the great mathematician Sophus Lie (b. 1842 Nordfjordeid, d. 1899 Kristiania). His name is pronounced like 'Lee'.

A (real) *Lie group* is an infinite symmetry group, i.e., a group with infinitely many elements, which is also an analytic manifold. Roughly speaking, this means that the elements of the group can be seen as points on a smooth (hyper-) surface whose shape can be described by an analytic function, i.e., by a function so smooth that it can be expressed as a power series in the neighbourhood of every point where it is defined. The points of the Lie group can be multiplied according to the group multiplication. Furthermore, the coordinates of the product have to be analytic functions of the coordinates of the factors, and the coordinates of the inverse of an element have to be analytic functions of the coordinates of the element. In fact, this definition is unnecessarily strict: it can be proved that a Lie group is just a topological group whose underlying space is a finite-dimensional, locally Euclidean manifold.

A *complex Lie group* is a group whose manifold is complex and whose group operations are holomorphic (instead of analytical) functions in the coordinates.

In short, a Lie group is a well-behaved manifold in which points can be multiplied (and technicalities). For example, the circle $\boldsymbol{T} = \{z \in \mathbb{C} : |z| = 1\}$, with the usual complex multiplication, is a real Lie group. It is Abelian. This group is also called S^1, as it is the one-dimensional sphere, or U(1), which means 'unitary group of one dimension'. The other one-dimensional Lie groups are the multiplicative group of non-zero real numbers and its subgroup, the multiplicative group of positive real numbers.

So far, in physics, only *linear* Lie groups have played a role – that is, Lie groups which act as linear transformations on some vector space. (The cover of SL(2,ℝ) or the complex compact torus are examples of non-linear Lie groups.) The important linear Lie groups for physics are the Lie subgroups of the general linear group GL(N,K), where K is a number field. This is defined as the set of all non-singular, i.e., invertible, N×N real, complex or quaternionic matrices. All the Lie groups discussed below are of this type.

Every *complex* invertible matrix A can be written in a unique way in terms of a unitary matrix U and a Hermitean matrix H:

$$A = U\mathrm{e}^{H} \; . \tag{150}$$

Challenge 210 s

(H is given by $H = \frac{1}{2}\ln A^{\dagger}A$, and U is given by $U = A\mathrm{e}^{-H}$.)

The simple Lie groups U(1) and SO(2,ℝ) and the Lie groups based on the real and complex numbers are Abelian (see Table 37); all others are non-Abelian.

Lie groups are manifolds. Therefore, in a Lie group one can define the distance between two points, the tangent plane (or tangent space) at a point, and the notions of integration and differentiations. Because Lie groups are manifolds, Lie groups have the same kind of structure as the objects of Figures 184, 185 and 186. Lie groups can have any number of dimensions. Like for any manifold, their global structure contains important information; let us explore it.

CONNECTEDNESS

It is not hard to see that the Lie groups SU(N) are simply connected for all $\mathrm{N} = 2, 3 \ldots$; they have the topology of a 2N-dimensional sphere. The Lie group U(1), having the topology of the 1-dimensional sphere, or circle, is multiply connected.

The Lie groups SO(N) are *not* simply connected for any $\mathrm{N} = 2, 3 \ldots$. In general, SO(N,K) is connected, and GL(N,ℂ) is connected. All the Lie groups SL(N,K) are connected; and SL(N,ℂ) is simply connected. The Lie groups Sp(N,K) are connected; Sp(2N,ℂ) is simply connected. Generally, all semi-simple Lie groups are connected.

The Lie groups O(N,K), SO(N,M,K) and GL(N,ℝ) are not connected; they contain two connected components.

Note that the Lorentz group is not connected: it consists of four separate pieces. Like the Poincaré group, it is not compact, and neither is any of its four pieces. Broadly speaking, the non-compactness of the group of space-time symmetries is a consequence of the non-compactness of space-time.

COMPACTNESS

A Lie group is *compact* if it is closed and bounded when seen as a manifold. For a given parametrization of the group elements, the Lie group is compact if all parameter ranges are closed and finite intervals. Otherwise, the group is called *non-compact*. Both compact and non-compact groups play a role in physics. The distinction between the two cases is important, because representations of compact groups can be constructed in the same simple way as for finite groups, whereas for non-compact groups other methods have to be used. As a result, physical observables, which always belong to a representation of a symmetry group, have different properties in the two cases: if the symmetry group is compact, observables have *discrete* spectra; otherwise they do not.

All groups of internal gauge transformations, such as U(1) and SU(n), form compact groups. In fact, field theory *requires* compact Lie groups for gauge transformations. The only compact Lie groups are the torus groups T^n, O(n), U(n), SO(n) and SU(n), their double cover Spin(n) and the Sp(n). In contrast, SL(n,$\mathbb{R}$), GL(n,$\mathbb{R}$), GL(n,$\mathbb{C}$) and all others are not compact.

Besides being manifolds, Lie groups are obviously also groups. It turns out that most of their group properties are revealed by the behaviour of the elements which are very close (as points on the manifold) to the identity.

Every element of a compact and connected Lie group has the form exp(A) for some A. The elements A arising in this way form an algebra, called the *corresponding Lie algebra*. For any linear Lie group, every element of the connected subgroup can be expressed as a finite product of exponentials of elements of the corresponding Lie algebra. Mathematically, the vector space defined by the Lie algebra is tangent to the manifold defined by the Lie group, at the location of the unit element. In short, Lie algebras express the local properties of Lie groups near the identity. That is the reason for their importance in physics.

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TABLE 37 Properties of the most important real and complex Lie groups.

Lie group	Description	Properties[a]	Lie algebra	Description of Lie algebra	Dimension
1. Real groups					real
$\mathbb{R}^n$	Euclidean space with addition	Abelian, simply connected, not compact; $\pi_0 = \pi_1 = 0$	$\mathbb{R}^n$	Abelian, thus Lie bracket is zero; not simple	n
$\mathbb{R}^\times$	non-zero real numbers with multiplication	Abelian, not connected, not compact; $\pi_0 = \mathbb{Z}_2$, no π_1	$\mathbb{R}$	Abelian, thus Lie bracket is zero	1
$\mathbb{R}^{>0}$	positive real numbers with multiplication	Abelian, simply connected, not compact; $\pi_0 = \pi_1 = 0$	$\mathbb{R}$	Abelian, thus Lie bracket is zero	1
$\mathrm{S}^1 = \mathbb{R}/\mathbb{Z} = \mathrm{U}(1) = \mathrm{T} = \mathrm{SO}(2) = \mathrm{Spin}(2)$	complex numbers of absolute value 1, with multiplication	Abelian, connected, not simply connected, compact; $\pi_0 = 0$, $\pi_1 = \mathbb{Z}$	$\mathbb{R}$	Abelian, thus Lie bracket is zero	1
$\mathbb{H}^\times$	non-zero quaternions with multiplication	simply connected, not compact; $\pi_0 = \pi_1 = 0$	$\mathbb{H}$	quaternions, with Lie bracket the commutator	4

TABLE 37 (Continued) Properties of the most important real and complex Lie groups.

Lie group	Description	Properties[a]	Lie algebra	Description of Lie algebra	Dimension
S^3	quaternions of absolute value 1, with multiplication, also known as Sp(1); topologically a 3-sphere	simply connected, compact; isomorphic to SU(2), Spin(3) and to double cover of SO(3); $\pi_0 = \pi_1 = 0$	$\mathrm{Im}(\mathbb{H})$	quaternions with zero real part, with Lie bracket the commutator; simple and semi-simple; isomorphic to real 3-vectors, with Lie bracket the cross product; also isomorphic to su(2) and to so(3)	3
$\mathrm{GL}(n, \mathbb{R})$	general linear group: invertible n-by-n real matrices	not connected, not compact; $\pi_0 = \mathbb{Z}_2$, no π_1	$\mathrm{M}(n, \mathbb{R})$	n-by-n matrices, with Lie bracket the commutator	n^2
$\mathrm{GL}^+(n, \mathbb{R})$	n-by-n real matrices with positive determinant	simply connected, not compact; $\pi_0 = 0$, for $n = 2$: $\pi_1 = \mathbb{Z}$, for $n \geq 2$: $\pi_1 = \mathbb{Z}_2$; $\mathrm{GL}^+(1, \mathbb{R})$ isomorphic to $\mathbb{R}^{>0}$	$\mathrm{M}(n, \mathbb{R})$	n-by-n matrices, with Lie bracket the commutator	n^2
$\mathrm{SL}(n, \mathbb{R})$	special linear group: real matrices with determinant 1	simply connected, not compact if $n > 1$; $\pi_0 = 0$, for $n = 2$: $\pi_1 = \mathbb{Z}$, for $n \geq 2$: $\pi_1 = \mathbb{Z}_2$; $\mathrm{SL}(1, \mathbb{R})$ is a single point, $\mathrm{SL}(2, \mathbb{R})$ is isomorphic to SU(1, 1) and $\mathrm{Sp}(2, \mathbb{R})$	$\mathrm{sl}(n, \mathbb{R}) = \mathrm{A}_{n-1}$	n-by-n matrices with trace 0, with Lie bracket the commutator	$n^2 - 1$
$\mathrm{O}(n, \mathbb{R}) = \mathrm{O}(n)$	orthogonal group: real orthogonal matrices; symmetry of hypersphere	not connected, compact; $\pi_0 = \mathbb{Z}_2$, no π_1	$\mathrm{so}(n, \mathbb{R})$	skew-symmetric n-by-n real matrices, with Lie bracket the commutator; $\mathrm{so}(3, \mathbb{R})$ is isomorphic to su(2) and to $\mathbb{R}^3$ with the cross product	$n(n-1)/2$

TABLE 37 (Continued) Properties of the most important real and complex Lie groups.

Lie group	Description	Properties[a]	Lie algebra	Description of Lie algebra	Dimension
$\mathrm{SO}(n,\mathbb{R}) = \mathrm{SO}(n)$	special orthogonal group: real orthogonal matrices with determinant 1	connected, compact; for $n \geqslant 2$ not simply connected; $\pi_0 = 0$, for $n = 2$: $\pi_1 = \mathbb{Z}$, for $n \geq 2$: $\pi_1 = \mathbb{Z}_2$	$\mathrm{so}(n,\mathbb{R}) = \mathrm{B}_{\frac{n-1}{2}}$ or $\mathrm{D}_{\frac{n}{2}}$	skew-symmetric n-by-n real matrices, with Lie bracket the commutator; for $n = 3$ and $n \geqslant 5$ simple and semisimple; SO(4) is semisimple but not simple	$n(n-1)/2$
$\mathrm{Spin}(n)$	spin group; double cover of $\mathrm{SO}(n)$; Spin(1) is isomorphic to $\mathbb{Q}_2$, Spin(2) to S^1	simply connected for $n \geqslant 3$, compact; for $n = 3$ and $n \geqslant 5$ simple and semisimple; for $n > 1$: $\pi_0 = 0$, for $n > 2$: $\pi_1 = 0$	$\mathrm{so}(n,\mathbb{R})$	skew-symmetric n-by-n real matrices, with Lie bracket the commutator	$n(n-1)/2$
$\mathrm{Sp}(2n,\mathbb{R})$	symplectic group: real symplectic matrices	not compact; $\pi_0 = 0$, $\pi_1 = \mathbb{Z}$	$\mathrm{sp}(2n,\mathbb{R}) = \mathrm{C}_n$	real matrices A that satisfy $JA + A^T J = 0$ where J is the standard skew-symmetric matrix;[b] simple and semisimple	$n(2n+1)$
$\mathrm{Sp}(n)$ for $n \geqslant 3$	compact symplectic group: quaternionic $n \times n$ unitary matrices	compact, simply connected; $\pi_0 = \pi_1 = 0$	$\mathrm{sp}(n)$	n-by-n quaternionic matrices A satisfying $A = -A^*$, with Lie bracket the commutator; simple and semisimple	$n(2n+1)$
$\mathrm{U}(n)$	unitary group: complex $n \times n$ unitary matrices	not simply connected, compact; it is *not* a complex Lie group/algebra; $\pi_0 = 0$, $\pi_1 = \mathbb{Z}$; isomorphic to S^1 for $n = 1$	$\mathrm{u}(n)$	n-by-n complex matrices A satisfying $A = -A^*$, with Lie bracket the commutator	n^2
$\mathrm{SU}(n)$	special unitary group: complex $n \times n$ unitary matrices with determinant 1	simply connected, compact; it is *not* a complex Lie group/algebra; $\pi_0 = \pi_1 = 0$	$\mathrm{su}(n)$	n-by-n complex matrices A with trace 0 satisfying $A = -A^*$, with Lie bracket the commutator; for $n \geqslant 2$ simple and semisimple	$n^2 - 1$
2. Complex groups[c]					complex

TABLE 37 (Continued) Properties of the most important real and complex Lie groups.

LIE GROUP	DESCRIPTION	PROPERTIES[a]	LIE ALGEBRA	DESCRIPTION OF LIE ALGEBRA	DIMENSION
$\mathbb{C}^n$	group operation is addition	Abelian, simply connected, not compact; $\pi_0 = \pi_1 = 0$	$\mathbb{C}^n$	Abelian, thus Lie bracket is zero	n
$\mathbb{C}^\times$	non-zero complex numbers with multiplication	Abelian, not simply connected, not compact; $\pi_0 = 0$, $\pi_1 = \mathbb{Z}$	$\mathbb{C}$	Abelian, thus Lie bracket is zero	1
$\mathrm{GL}(n, \mathbb{C})$	general linear group: invertible n-by-n complex matrices	simply connected, not compact; $\pi_0 = 0$, $\pi_1 = \mathbb{Z}$; for $n = 1$ isomorphic to $\mathbb{C}^\times$	$\mathrm{M}(n, \mathbb{C})$	n-by-n matrices, with Lie bracket the commutator	n^2
$\mathrm{SL}(n, \mathbb{C})$	special linear group: complex matrices with determinant 1	simply connected; for $n \geqslant 2$ not compact; $\pi_0 = \pi_1 = 0$; $\mathrm{SL}(2, \mathbb{C})$ is isomorphic to $\mathrm{Spin}(3, \mathbb{C})$ and $\mathrm{Sp}(2, \mathbb{C})$	$\mathrm{sl}(n, \mathbb{C})$	n-by-n matrices with trace 0, with Lie bracket the commutator; simple, semisimple; $\mathrm{sl}(2, \mathbb{C})$ is isomorphic to $\mathrm{su}(2, \mathbb{C}) \otimes \mathbb{C}$	$n^2 - 1$
$\mathrm{PSL}(2, \mathbb{C})$	projective special linear group; isomorphic to the Möbius group, to the restricted Lorentz group $\mathrm{SO}^+(3, 1, \mathbb{R})$ and to $\mathrm{SO}(3, \mathbb{C})$	not compact; $\pi_0 = 0$, $\pi_1 = \mathbb{Z}_2$	$\mathrm{sl}(2, \mathbb{C})$	2-by-2 matrices with trace 0, with Lie bracket the commutator; $\mathrm{sl}(2, \mathbb{C})$ is isomorphic to $\mathrm{su}(2, \mathbb{C}) \otimes \mathbb{C}$	3
$\mathrm{O}(n, \mathbb{C})$	orthogonal group: complex orthogonal matrices	not connected; for $n \geqslant 2$ not compact; $\pi_0 = \mathbb{Z}_2$, no π_1	$\mathrm{so}(n, \mathbb{C})$	skew-symmetric n-by-n complex matrices, with Lie bracket the commutator	$n(n-1)/2$

TABLE 37 (Continued) Properties of the most important real and complex Lie groups.

LIE GROUP	DESCRIPTION	PROPERTIES[a]	LIE ALGEBRA	DESCRIPTION OF LIE ALGEBRA	DIMENSION
$SO(n, \mathbb{C})$	special orthogonal group: complex orthogonal matrices with determinant 1	for $n \geqslant 2$ not compact; not simply connected; $\pi_0 = 0$, for $n = 2$: $\pi_1 = \mathbb{Z}$, for $n \geq 2$: $\pi_1 = \mathbb{Z}_2$; non-Abelian for $n > 2$, $SO(2, \mathbb{C})$ is Abelian and isomorphic to $\mathbb{C}^\times$	$so(n, \mathbb{C})$	skew-symmetric n-by-n complex matrices, with Lie bracket the commutator; for $n = 3$ and $n \geqslant 5$ simple and semisimple	$n(n-1)/2$
$Sp(2n, \mathbb{C})$	symplectic group: complex symplectic matrices	not compact; $\pi_0 = \pi_1 = 0$	$sp(2n, \mathbb{C})$	complex matrices that satisfy $JA + A^T J = 0$ where J is the standard skew-symmetric matrix;[b] simple and semi-simple	$n(2n+1)$

[a] The *group of components* π_0 of a Lie group is given; the order of π_0 is the number of components of the Lie group. If the group is trivial (0), the Lie group is connected. The *fundamental group* π_1 of a connected Lie group is given. If the group π_1 is trivial (0), the Lie group is simply connected. This table is based on that in the Wikipedia, at en.wikipedia.org/wiki/Table_of_Lie_groups.

[b] The standard skew-symmetric matrix J of rank $2n$ is $J_{kl} = \delta_{k,n+l} - \delta_{k+n,l}$.

[c] Complex Lie groups and Lie algebras can be viewed as real Lie groups and real Lie algebras of twice the dimension.

MATHEMATICAL CURIOSITIES AND FUN CHALLENGES

Challenge 211 ny A theorem of topology says: you cannot comb a hairy football. Can you prove it?

* *

Topology is fun. If you want to laugh for half an hour, fix a modified pencil, as shown in Figure 189, to a button hole and let people figure out how to get it off again.

* *

There are at least six ways to earn a million dollars with mathematical research. The Clay Mathematics Institute at www.claymath.org offered such a prize for major advances in seven topics:

- proving the Birch and Swinnerton–Dyer conjecture about algebraic equations;
- proving the Poincaré conjecture about topological manifolds;
- solving the Navier–Stokes equations for fluids;

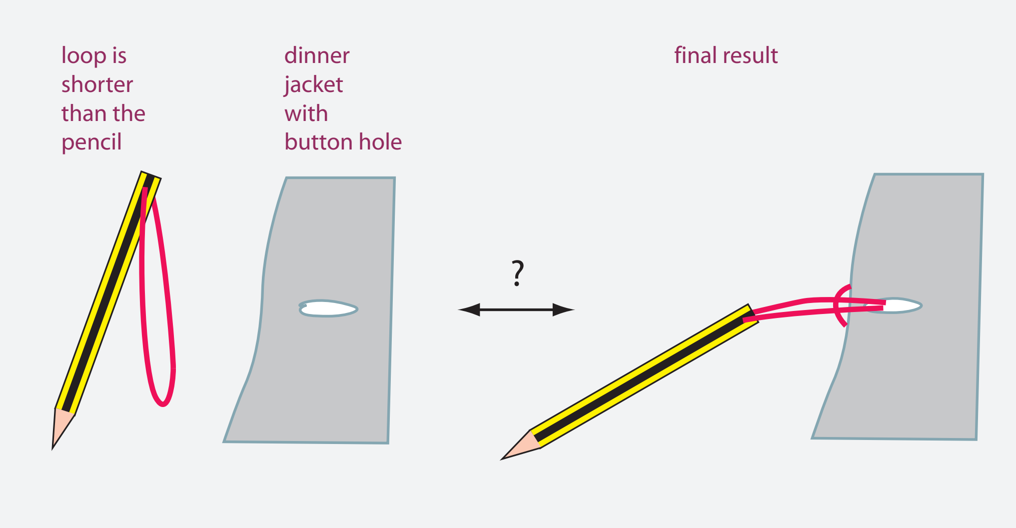

FIGURE 189 A well-known magic dexterity trick: make your friend go mad by adding a pencil to his dinner jacket.

— finding criteria distinguishing P and NP numerical problems;
— proving the Riemann hypothesis stating that the non-trivial zeros of the zeta function lie on a line;
— proving the Hodge conjectures;
— proving the connection between Yang–Mills theories and a mass gap in quantum field theory. Vol. VI, page 345

The Poincaré conjecture was solved in 2002 by Grigori Perelman; on each of the other six topics, substantial progress can buy you a house.

CHALLENGE HINTS AND SOLUTIONS

Challenge 1, page 10: Do not hesitate to be demanding and strict. The next edition of the text will benefit from it.

Challenge 2, page 17: A *virus* is an example. It has no own metabolism. By the way, the ability of some viruses to form crystals is *not* a proof that they are not living beings, in contrast to what is often said. Apart from viruses, also *prions*, *viroids* and *virusoids* are examples of non-living systems that can reproduce. Another, quite different border example between life and non-living matter is provided by the *tardigrades*. These little animals, about 1 mm in size, can loose all their water, remain in this dry – or ''dead'' – state for years, and then start living again when a drop of water is added to them.

Challenge 3, page 18: The navigation systems used by flies are an example.

Challenge 6, page 22: The thermal energy kT is about 4 zJ and a typical relaxation time is 0.1 ps.

Challenge 10, page 30: The argument is correct.

Challenge 8, page 30: This is not possible at present. If you know a way, publish it. It would help a sad single mother who has to live without financial help from the father, despite a lawsuit, as it was yet impossible to decide which of the two candidates is the right one.

Challenge 9, page 30: Also identical twins count as different persons and have different fates. Imprinting in the womb is different, so that their temperament will be different. The birth experience will be different; this is the most intense experience of every human, strongly determining his fears and thus his character. A person with an old father is also quite different from that with a young father. If the womb is not that of his biological mother, a further distinction of the earliest and most intensive experiences is given.

Challenge 11, page 30: Be sure to publish your results.

Challenge 12, page 31: Yes, but only very young ones. Why?

Challenge 13, page 31: The reason of animal symmetry is simple: without symmetry, they would not be able to move in a *straight* line.

Challenge 14, page 31: Life's chemicals are synthesized inside the body; the asymmetry has been inherited along the generations. The common asymmetry thus shows that all life has a common origin.

Challenge 15, page 31: Well, men are more similar to chimpanzees than to women. More seriously, the above data, even though often quoted, are wrong. Newer measurements by Roy Britten in 2002 have shown that the difference in genome between humans and chimpanzees is about 5 % (See R. J. Britten, *Divergence between samples of chimpanzee and human DNA sequences is 5 %, counting indels*, Proceedings of the National Academy of Sciences **99**, pp. 13633–13635, 15th of October, 2002.) In addition, though the difference between man and woman is smaller than one whole chromosome, the large size of the X chromosome, compared with the small size of the Y chromosome, implies that men have about 3 % less genetic material than women. However, all men have an X chromosome as well. That explains that still other measurements suggest that all humans share a pool of at least 99.9 % of common genes.

Challenge 18, page 34: Chemical processes, including diffusion and reaction rates, are strongly temperature dependent. They affect the speed of motion of the individual and thus its chance of survival. Keeping temperature in the correct range is thus important for evolved life forms.

Challenge 19, page 34: The first steps are not known at all. Subsequent processes that added the complexity of cells are better understood.

Challenge 20, page 34: Since all the atoms we are made of originate from outer space, the answer is yes. But if one means that biological cells came to Earth from space, the answer is no, as most cells do not like vacuum. The same is true for DNA.

In fact, life and reproduction are properties of complex systems. In other words, asking whether life comes from outer space is like asking: 'Could car insurance have originated in outer space?'

Challenge 23, page 40: Haven't you tried yet? Physics is an experimental science.

Challenge 31, page 48: Exponential decays occur when the probability of decay is constant over time. For humans, this is not the case. Why not?

Challenge 32, page 51: There are no non-physical processes: anything that can be observed is a physical process. Consciousness is due to processes in the brain, thus inside matter; thus it is a quantum process. At body temperature, coherence has lifetimes much smaller than the typical thought process.

Challenge 33, page 52: Radioactive dating methods can be said to be based on the nuclear interactions, even though the detection is again electromagnetic.

Challenge 34, page 53: All detectors of light can be called relativistic, as light moves with maximal speed. Touch sensors are not relativistic following the usual sense of the word, as the speeds involved are too small. The energies are small compared to the rest energies; this is the case even if the signal energies are attributed to electrons only.

Challenge 35, page 53: The noise is due to the photoacoustic effect; the periodic light periodically heats the air in the jam glass at the blackened surface and thus produces sound. See M. Euler, *Kann man Licht hören?*, Physik in unserer Zeit **32**, pp. 180–182, 2001.

Challenge 36, page 55: It implies that neither resurrection nor reincarnation nor eternal life are possible.

Challenge 39, page 65: The ethanol disrupts the hydrogen bonds between the water molecules so that, on average, they can get closer together. A video of the experiment is found at www.youtube.com/watch?v=LUW7a7H-KuY.

Challenge 42, page 66: You get an intense yellow colour due to the formation of lead iodide (PbI_2).

Challenge 44, page 73: The usual way to pack oranges on a table is the densest way to pack spheres.

Challenge 45, page 74: Just use a paper drawing. Draw a polygon and draw it again at latter times, taking into account how the sides grow over time. You will see by yourself how the faster growing sides disappear over time.

Challenge 47, page 84: With a combination of the methods of Table 7, this is indeed possible. In fact, using cosmic rays to search for unknown chambers in the pyramids has been already done in the 1960s. The result was that no additional chambers exist. Ref. 304

Challenge 49, page 88: For example, a heavy mountain will push down the Earth's crust into the mantle, makes it melt on the bottom side, and thus lowers the position of the top.

Challenge 50, page 88: These developments are just starting; the results are still far from the original one is trying to copy, as they have to fulfil a second condition, in addition to being a ‘copy’ of original feathers or of latex: the copy has to be cheaper than the original. That is often a much tougher request than the first.

Challenge 51, page 88: About 0.2 m.

Challenge 53, page 89: Since the height of the potential is always finite, walls can always be overcome by tunnelling.

Challenge 54, page 90: The lid of a box can never be at rest, as is required for a tight closure, but is always in motion, due to the quantum of action.

Challenge 56, page 90: The unit of thermal conductance is $T\pi^2k^2/3\hbar$, where T is temperature and k is the Boltzmann constant.

Challenge 57, page 91: Extremely slender structures are not possible for two reasons: First, because structures built of homogeneous materials do not to achieve such ratios; secondly, the bending behaviour of plants is usually not acceptable in human-built structures.

Challenge 58, page 93: The concentrations and can be measured from polar ice caps, by measuring how the isotope concentration changes over depth. Both in evaporation and in condensation of water, the isotope ratio depends on the temperature. The measurements in Antarctica and in Greenland coincide, which is a good sign of their trustworthiness.

Challenge 59, page 94: In the summer, tarmac is soft.

Challenge 62, page 113: The one somebody else has thrown away. Energy costs about 10 cents/kWh. For new lamps, the fluorescent lamp is the best for the environment, even though it is the least friendly to the eye and the brain, due to its flickering.

Challenge 63, page 118: This old dream depends on the precise conditions. How flexible does the display have to be? What lifetime should it have? The newspaper like display is many years away and maybe not even possible.

Challenge 64, page 118: There is only speculation on the answer; the tendency of most researchers is to say no.

Challenge 65, page 118: The challenge here is to find a cheap way to deflect laser beams in a controlled way. Cheap lasers are already available.

Challenge 66, page 118: No, as it is impossible because of momentum conservation and because of the no-cloning theorem.

Challenge 67, page 119: There are companies trying to sell systems based on quantum cryptology; but despite the technical interest, the commercial success is questionable.

Challenge 68, page 119: I predicted since the year 2000 that mass-produced goods using this technology (at least 1 million pieces sold) will not be available before 2025.

Challenge 69, page 119: Maybe, but for extremely high prices.

Challenge 70, page 120: The set-up is affordable: it uses a laser at 3.39 μm, a detector and some optics on a tripod. Sensitivity to alcohol absorption is excellent. Only future will tell.

Challenge 72, page 125: For example, you could change gravity between two mirrors.

Challenge 73, page 125: As usual in such statements, either group or phase velocity is cited, but not the corresponding energy velocity, which is always below c.

Challenge 75, page 128: Echoes do not work once the speed of sound is reached and do not work well when it is approached. Both the speed of light and that of sound have a finite value. Moving with a mirror still gives a mirror image. This means that the speed of light cannot be reached. If it cannot be reached, it must be the same for all observers.

Challenge 76, page 129: Mirrors do not usually work for matter; in addition, if they did, matter, because of its rest energy, would require much higher acceleration values.

Challenge 79, page 131: The classical radius of the electron, which is the size at which the field energy would make up the full electron mass, is about 137 times smaller, thus much smaller, than the Compton wavelength of the electron.

Challenge 80, page 133: The overhang can have any value whatsoever. There is no limit. Taking the indeterminacy relation into account introduces a limit as the last brick or card must not allow the centre of gravity, through its indeterminacy, to be over the edge of the table.

Challenge 81, page 133: A larger charge would lead to a field that spontaneously generates electron positron pairs, the electron would fall into the nucleus and reduce its charge by one unit.

Challenge 83, page 133: The Hall effect results from the deviation of electrons in a metal due to an applied magnetic field. Therefore it depends on their speed. One gets values around 1 mm. Inside atoms, one can use Bohr's atomic model as approximation.

Challenge 84, page 133: The steps are due to the particle nature of electricity and all other moving entities.

Challenge 85, page 134: If we could apply the Banach–Tarski paradox to vacuum, it seems that Vol. I, page 57 we could split, without any problem, one ball of vacuum into *two* balls of vacuum, each with the same volume as the original. In other words, one ball with vacuum energy E could not be distinguished from two balls of vacuum energy $2E$.

We used the Banach–Tarski paradox in this way to show that chocolate (or any other matter) Vol. I, page 333 possesses an intrinsic length. But it is *not* clear that we can now deduce that the vacuum has an intrinsic length. Indeed, the paradox cannot be applied to vacuum for *two* reasons. First, there indeed is a maximum energy and minimum length in nature. Secondly, there is no place in nature without vacuum energy; so there is no place were we could put the second ball. We thus do not know why the Banach–Tarski paradox for vacuum cannot be applied, and thus cannot use it to deduce the existence of a minimum length in vacuum.

It is better to argue in the following way for a minimum length in vacuum. If there were no intrinsic length cut-off, the vacuum energy would be infinite. Experiments however, show that it is finite.

Challenge 86, page 134: Mud is a suspension of sand; sand is not transparent, even if made of clear quartz, because of the scattering of light at the irregular surface of its grains. A suspension cannot be transparent if the index of refraction of the liquid and the suspended particles is different. It is never transparent if the particles, as in most sand types, are themselves not transparent.

Challenge 87, page 134: The first answer is probably no, as composed systems cannot be smaller than their own compton wavelength; only elementary systems can. However, the universe is not a system, as it has no environment. As such, its length is not a precisely defined concept, as an environment is needed to measure and to define it. (In addition, gravity must be taken into account in those domains.) Thus the answer is: in those domains, the question makes no sense.

Challenge 88, page 134: Methods to move on perfect ice from mechanics:

- if the ice is perfectly flat, rest is possible only in one point – otherwise you oscillate around that point, as shown in challenge 26;
- do nothing, just wait that the higher centrifugal acceleration at body height pulls you away;
- to rotate yourself, just rotate your arm above your head;
- throw a shoe or any other object away;
- breathe in vertically, breathing out (or talking) horizontally (or vice versa);
- wait to be moved by the centrifugal acceleration due to the rotation of the Earth (and its oblateness);

- jump vertically repeatedly: the Coriolis acceleration will lead to horizontal motion;
- wait to be moved by the Sun or the Moon, like the tides are;
- 'swim' in the air using hands and feet;
- wait to be hit by a bird, a flying wasp, inclined rain, wind, lava, earthquake, plate tectonics, or any other macroscopic object (all objects pushing count only as one solution);
- wait to be moved by the change in gravity due to convection in Earth's mantle;
- wait to be moved by the gravitation of some comet passing by;
- counts only for kids: spit, sneeze, cough, fart, pee; or move your ears and use them as wings.

Note that gluing your tongue is not possible on perfect ice.

Challenge 89, page 135: Methods to move on perfect ice using thermodynamics and electrodynamics:

- use the radio/TV stations nearby to push you around;
- use your portable phone and a mirror;
- switch on a pocket lamp, letting the light push you;
- wait to be pushed around by Brownian motion in air;
- heat up one side of your body: black body radiation will push you;
- heat up one side of your body, e.g. by muscle work: the changing airflow or the evaporation will push you;
- wait for one part of the body to be cooler than the other and for the corresponding black body radiation effects;
- wait for the magnetic field of the Earth to pull on some ferromagnetic or paramagnetic metal piece in your clothing or in your body;
- wait to be pushed by the light pressure, i.e. by the photons, from the Sun or from the stars, maybe using a pocket mirror to increase the efficiency;
- rub some polymer object to charge it electrically and then move it in circles, thus creating a magnetic field that interacts with the one of the Earth.

Note that perfect frictionless surfaces do not melt.

Challenge 90, page 135: Methods to move on perfect ice using general relativity:

- move an arm to emit gravitational radiation;
- deviate the cosmic background radiation with a pocket mirror;
- wait to be pushed by gravitational radiation from star collapses;
- wait for the universe to contract.

Challenge 91, page 135: Methods to move on perfect ice using quantum effects:

- wait for your wave function to spread out and collapse at the end of the ice surface;
- wait for the pieces of metal in the clothing to attract to the metal in the surrounding through the Casimir effect;
- wait to be pushed around by radioactive decays in your body.

Challenge 92, page 135: Methods to move on perfect ice using materials science, geophysics and astrophysics:

- be pushed by the radio waves emitted by thunderstorms and absorbed in painful human joints;
- wait to be pushed around by cosmic rays;
- wait to be pushed around by the solar wind;
- wait to be pushed around by solar neutrinos;

- wait to be pushed by the transformation of the Sun into a red giant;
- wait to be hit by a meteorite.

Challenge 93, page 135: A method to move on perfect ice using self-organization, chaos theory, and biophysics:

- wait that the currents in the brain interact with the magnetic field of the Earth by controlling your thoughts.

Challenge 94, page 135: Methods to move on perfect ice using quantum gravity:

- accelerate your pocket mirror with your hand;
- deviate the Unruh radiation of the Earth with a pocket mirror;
- wait for proton decay to push you through the recoil.

Challenge 96, page 142: This is a trick question: if you can say why, you can directly move to the last volume of this adventure and check your answer. The gravitational potential changes the phase of a wave function, like any other potential does; but the reason *why* this is the case will only become clear in the last volume of this series.

Challenge 101, page 144: No. Bound states of massless particles are always unstable.

Challenge 102, page 146: This is easy only if the black hole size is inserted into the entropy bound by Bekenstein. A simple deduction of the black hole entropy that includes the factor 1/4 is not yet at hand; more on this in the last volume.

Challenge 103, page 146: An entropy limit implies an information limit; only a given information can be present in a given region of nature. This results in a memory limit.

Challenge 104, page 146: In natural units, the exact expression for entropy is $S = 0.25A$. If each Planck area carried one bit (degree of freedom), the entropy would be $S = \ln W = \ln(2^A) = A \ln 2 = 0.693A$. This close to the exact value.

Challenge 108, page 152: The universe has about 10^{22} stars; the Sun has a luminosity of about 10^{26} W; the total luminosity of the visible matter in the universe is thus about 10^{48} W. A gamma-ray burster emits up to $3 \cdot 10^{47}$ W.

Challenge 114, page 154: They are carried away by the gravitational radiation.

Challenge 120, page 158: No system is known in nature which emits or absorbs only one graviton at a time. This is another point speaking against the existence of gravitons.

Challenge 124, page 167: Two stacked foils show the same effect as one foil of the same total thickness. Thus the surface plays no role.

Challenge 126, page 172: The electron is held back by the positive charge of the nucleus, if the number of protons in the nucleus is sufficient, as is the case for those nuclei we are made of.

Challenge 128, page 180: The half-time $t_{1/2}$ is related to the life-time τ by $t_{1/2} = \tau \ln 2$.

Challenge 129, page 181: The number is small compared with the number of cells. However, it is possible that the decays are related to human ageing.

Challenge 131, page 184: By counting decays and counting atoms to sufficient precision.

Challenge 133, page 185: The radioactivity necessary to keep the Earth warm is low; lava is only slightly more radioactive than usual soil.

Challenge 134, page 197: There is no way to conserve both energy and momentum in such a decay.

Challenge 135, page 197: The combination of high intensity X-rays and UV rays led to this effect.

Challenge 139, page 209: The nuclei of nitrogen and carbon have a high electric charge which strongly repels the protons.

Challenge 141, page 218: See the paper by C.J. Hogan mentioned in Ref. 270.

Challenge 142, page 221: Touching something requires getting near it; getting near means a small time and position indeterminacy; this implies a small wavelength of the probe that is used for touching; this implies a large energy.

Challenge 145, page 228: The processes are electromagnetic in nature, thus electric charges give the frequency with which they occur.

Challenge 146, page 237: Designing a nuclear weapon is not difficult. University students can do it, and even have done so a few times. The first students who did so were two physics graduates in 1964, as told on www.guardian.co.uk/world/2003/jun/24/usa.science. It is not hard to conceive a design and even to build it. By far the hardest problem is getting or making the nuclear material. That requires either an extensive criminal activity or a vast technical effort, with numerous large factories, extensive development, and coordination of many technological activities. Most importantly, such a project requires a large financial investment, which poor countries cannot afford without great sacrifices for all the population. The problems are thus not technical, but financial.

Challenge 150, page 268: In 2008, an estimated 98 % of all physicists agreed. Time will tell whether they are right.

Challenge 152, page 280: A mass of 100 kg and a speed of 8 m/s require 43 m^2 of wing surface.

Challenge 155, page 287: The issue is a red herring. The world has three dimensions.

Challenge 156, page 290: The largest rotation angle $\Delta\varphi$ that can be achieved in one stroke C is found by maximizing the integral

$$\Delta\varphi = -\int_C \frac{a^2}{a^2+b^2}\,\mathrm{d}\theta \tag{151}$$

Since the path C in shape space is closed, we can use Stokes' theorem to transform the line integral to a surface integral over the surface S enclosed by C in shape space:

$$\Delta\varphi = \int_S \frac{2ab^2}{(a^2+b^2)^2}\,\mathrm{d}a\,\mathrm{d}\theta\;. \tag{152}$$

The maximum angle is found by noting that θ can vary at most between 0 and π, and that a can vary at most between 0 and ∞. This yields

$$\Delta\varphi_{\max} = \int_{\theta=0}^{\pi}\int_{a=0}^{\infty} \frac{2ab^2}{(a^2+b^2)^2}\,\mathrm{d}a\,\mathrm{d}\theta = \pi\;. \tag{153}$$

Challenge 158, page 294: A cloud is kept afloat and compact by convection currents. Clouds without convection can often be seen in the summer: they diffuse and disappear. The details of the internal and external air currents depend on the cloud type and are a research field on its own.

Challenge 163, page 300: Lattices are not isotropic, lattices are not Lorentz invariant.

Challenge 165, page 303: The infinite sum is not defined for numbers; however, it is defined for a knotted string.

Challenge 166, page 304: The research race for the solution is ongoing, but the goal is still far.

Challenge 167, page 305: This is a simple but hard question. Find out!

Challenge 170, page 308: Large raindrops are pancakes with a massive border bulge. When the size increases, e.g. when a large drop falls through vapour, the drop splits, as the central membrane is then torn apart.

Challenge 171, page 308: It is a drawing; if it is interpreted as an image of a three-dimensional object, it either does not exist, or is not closed, or is an optical illusion of a torus.

Challenge 172, page 308: See T. Fink & Y. Mao, *The 85 Ways to Tie a Tie*, Broadway Books, 2000.

Challenge 173, page 308: See T. Clarke, *Laces high*, Nature Science Update 5th of December, 2002, or www.nature.com/nsu/021202/021202-4.html.

Challenge 176, page 310: In fact, nobody has even tried to do so yet. It may also be that the problem makes no sense.

Challenge 178, page 316: Most macroscopic matter properties fall in this class, such as the change of water density with temperature.

Challenge 180, page 322: Before the speculation can be fully tested, the relation between particles and black holes has to be clarified first.

Challenge 181, page 323: Never expect a correct solution for personal choices. Do what you yourself think and feel is correct.

Challenge 186, page 330: Planck limits can be exceeded for extensive observables for which many particle systems can exceed single particle limits, such as mass, momentum, energy or electrical resistance.

Challenge 188, page 332: Do not forget the relativistic time dilation.

Challenge 189, page 332: The formula with $n - 1$ is a better fit. Why?

Challenge 193, page 339: The slowdown goes *quadratically* with time, because every new slowdown adds to the old one!

Challenge 194, page 340: No, only properties of parts of the universe are listed. The universe itself has no properties, as shown in the last volume. Vol. VI, page 112

Challenge 195, page 342: The gauge coupling constants, via the Planck length, determine the size of atoms, the strength of chemical bonds and thus the size of all things.

Challenge 196, page 357: Covalent bonds tend to produce full shells; this is a smaller change on the right side of the periodic table.

Challenge 197, page 359: The solution is the set of all two by two matrices, as each two by two matrix specifies a linear transformation, if one defines a transformed point as the product of the point and this matrix. (Only multiplication with a fixed matrix can give a linear transformation.) Can you recognize from a matrix whether it is a rotation, a reflection, a dilation, a shear, or a stretch along two axes? What are the remaining possibilities?

Challenge 200, page 359: The (simplest) product of two functions is taken by point-by-point multiplication.

Challenge 201, page 360: The norm $\|f\|$ of a real function f is defined as the supremum of its absolute value:

$$\|f\| = \sup_{x\in\mathrm{R}} |f(x)| \; . \tag{154}$$

In simple terms: the maximum value taken by the absolute of the function is its norm. It is also called 'sup'-norm. Since it contains a supremum, this norm is only defined on the subspace of *bounded* continuous functions on a space X, or, if X is compact, on the space of all continuous functions (because a continuous function on a compact space must be bounded).

Challenge 204, page 363: Take out your head, then pull one side of your pullover over the corresponding arm, continue pulling it over the over arm; then pull the other side, under the first, to the other arm as well. Put your head back in. Your pullover (or your trousers) will be inside out.

Challenge 205, page 363: Both can be untied.

Challenge 209, page 368: The transformation from one manifold to another with different topology can be done with a tiny change, at a so-called *singular point*. Since nature shows a minimum action, such a tiny change cannot be avoided.

Challenge 210, page 370: The product $M^{\dagger}M$ is Hermitean, and has positive eigenvalues. Thus H is uniquely defined and Hermitean. U is unitary because $U^{\dagger}U$ is the unit matrix.

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FIGURE 190 The wonderful origin of human life (© W.C.M. Weijmar Schultz).

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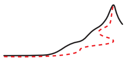

MOTION MOUNTAIN

The Adventure of Physics – Vol. V

Motion inside Matter – Pleasure, Technology and Stars

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Christoph Schiller, PhD Université Libre de Bruxelles, is a physicist and physics popularizer. He wrote this book for his children and for all students, teachers and readers interested in physics, the science of motion.