

budget.

7. **Use your budget to make decisions and adjust for any changes** Your budget is a changeable document. Add to it when you wish, refer to it when special purchases are to be made. Keeping your budget up to date helps accommodate changes in income and expenses.

 VIDEO

[Creating a Budget \(https://openstax.org/r/creating_budget\)](https://openstax.org/r/creating_budget)

In this section, we will focus on income and expenses. One of the easiest ways to manage a budget is to create a table, with one column containing income sources, another with income values, a third with expense categories, and a last containing expenses. An example is shown in [Table 6.1](#).

Income Source	Amount	Expense	Amount
Full-time job	\$3,565	Rent	\$975
Uber	\$185	Car Payment	\$355
TOTAL	\$3,750	Student Loan	\$418
		Electric	\$76
		Food	\$400
		Gasoline	\$250
		Car Insurance	\$165
		Clothing	\$100
		Entertainment	\$100
		TOTAL	\$2,839

Table 6.1 Table with Budget

WHO KNEW?

Gross Pay and Take Home Pay

If you've ever had a paycheck, you know that taxes are taken out of your pay before you get your check. This amount of money varies from state to state, and sometimes even city to city. For a person making \$50,000 per year gross salary in Salt Lake City, Utah, take home pay is about 75.6% of gross salary. In Detroit, Michigan, take home pay is about 74.5% of gross salary. Lakeland, Florida, take home pay is about 80.5% of gross salary. These also change based on how much a person earns! Before choosing a place to live, it makes sense to determine how much deductions from pay will impact your income.

EXAMPLE 6.49

Creating a Budget

Heather has graduated college and currently works as a nurse for a rural medical group. Her net monthly income from that job is \$3,765.40. She also works part-time on the weekends, earning another \$672.00 per month. Her monthly expenses are rent at \$1,050, car payments at \$489, student loan payments at \$728, car insurance at \$139, utilities at \$130, clothing at \$150, entertainment (going out with friends, Netflix, Amazon Prime, movies) at \$300, credit card debt at

\$200, food at \$360, and gasoline at \$275. Create her budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

 Solution

Step 1: To begin, we create the table with appropriate headings.

Income Source	Amount	Expense	Amount

Step 2: Her income categories are her nursing job, with \$3,765.40 per month, and her part-time job, with \$672.00 per month. Entering these into the table, we have the following.

Income Source	Amount	Expense	Amount
Nursing	\$3,765.40		
Part-time	\$672.00		

Step 3: Her monthly expenses are listed above. Entering the categories and the amount for each of those expenses, the table is now

Income Source	Amount	Expense	Amount
Nursing	\$3,765.40	Rent	\$1,050
Part-time	\$672.00	Car Payment	\$489
		Student Loan	\$728
		Car Insurance	\$139
		Utilities	\$130
		Clothing	\$150
		Entertainment	\$300
		Credit Card	\$200
		Food	\$400
		Gasoline	\$250

Step 4: Totaling the income and expenses, we see that her total income is \$4,437.40 per month, and her total expenses are \$3,836 per month. Comparing these, we see that Heather has \$601.40 in excess income per month. This provides a cushion in her budget.

> YOUR TURN 6.49

1. Mateo works as a union electrician in a suburban area. Monthly, his take home pay is \$3,375. He sometimes does small side jobs for family or friends, and averages about \$300 per month from these little jobs. His monthly expenses are his mortgage at \$986.78, truck payments at \$589.00, truck insurance at \$312, utilities at \$167, clothing at \$150, entertainment at \$400, credit card debt at \$325, food at \$470, and gasoline at \$375. Create Mateo's budget in a table, compare the total income to total expenses, and determine how much excess income per month he has or how much he falls short by each month.

EXAMPLE 6.50

Creating a Budget

Carol is working in a dental lab, creating dentures and bridges. Monthly her take home pay is \$2,816 (based on \$22 per hour minus payroll taxes). She also receives \$320 per month in child support for her one daughter. Her monthly expenses are rent at \$700, car payments at \$229, student loan payments at \$250, car insurance at \$119, health insurance at \$225, utilities at \$80, clothing at \$75, entertainment at \$200, food at \$275, and gasoline at \$275. Create Carol's budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

✓ Solution

Step 1: To begin, we create the table with appropriate headings.

Income Source	Amount	Expense	Amount

Step 2: Her income categories are from work, \$2,816, and child support, \$320, per month. Entering these into the table, we have the following.

Income Source	Amount	Expense	Amount
Job	\$2,816.00		
Child support	\$320.00		

Step 3: Her monthly expenses are listed above. Entering the categories and the amount for each of those expenses, the table is now

Income Source	Amount	Expense	Amount
Job	\$2,816.00	Rent	\$700
Child support	\$320.00	Car Payment	\$229
		Student Loan	\$250
		Car Insurance	\$119
		Utilities	\$80

Income Source	Amount	Expense	Amount
		Health insurance	\$225
		Clothing	\$75
		Entertainment	\$200
		Food	\$275
		Gasoline	\$275

Step 4: Totaling the income and expenses, we see that her total income is \$3,136.00 per month, and her total expenses are \$2,428.00 per month. Comparing these, we see that Carol has \$708.00 in excess income per month. This is the cushion in her budget.

> YOUR TURN 6.50

1. Maddy works as a mechanical engineer, making \$6,093.75 monthly after payroll taxes. Her monthly expenses are her mortgage at \$1,452.89, car payments at \$627.38, car insurance at \$179.00, health insurance at \$265.00, utilities at \$320, clothing at \$150, entertainment at \$400, credit card debt at \$450, food at \$370, and gasoline at \$175. Create Maddy's budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

Using the budget process, we can make decisions on adding expenses to the budget. To do so, check the cushion of the budget to see if there is room in the budget for the new expense.

EXAMPLE 6.51

Adding to an Existing Budget

In the example above, Carol had excess income of \$708.00. She looks up the cost of before-school care for her daughter. She finds that, monthly, the cost would be \$252.00 per month. Is this an affordable program for Carol? Add this expense to her budget table.

✓ Solution

She can afford this, as the cost for the before school program is \$252.00 and she had extra income of \$708.00. Adding this to her budget, her budget table is now

Income Source	Amount	Expense	Amount
Job	\$2,816.00	Rent	\$700
Child support	\$320.00	Car Payment	\$229
		Student Loan	\$250
		Car Insurance	\$119
		Utilities	\$80

Income Source	Amount	Expense	Amount
		Health insurance	\$225
		Clothing	\$75
		Entertainment	\$200
		Food	\$275
		Gasoline	\$275
		Before-school care	\$252

Now, she has \$456.00 in excess income per month.

> YOUR TURN 6.51

1. Recall Heather's budget from Example 6.49. She decides she wants to buy her own home, which would increase her expenses. Instead of \$1,050.00 in rent, she would pay \$1,240.00 for her mortgage. Her utilities costs would increase to \$295.00 per month. Add these to Heather's budget to determine if the changes are affordable.

The 50-30-20 Budget Philosophy

It isn't clear, obvious, or easy to decide how much of your income to allocate to various categories of expenses. Many people pay their bills and then consider all the leftover money to be spending money. However, when developing your own budget, you may want to follow the **50-30-20 budget philosophy**, which provides a basic guideline for how your income could be allocated. Fifty percent of your budget is allotted to your needs, 30% of your budget is allotted to pay for your wants, and 20% of your budget is allotted for savings and debt service (paying off your debts).

Knowing what expenses are necessary and what expenses are wants is important, since wants and needs are often confused. The following are **necessary expenses** that represent basic living requirements and debt services. This list isn't complete: mortgage/rent, utilities, car, car insurance, health care, groceries, gasoline, child care (for working parents), and minimum debt payments. The 50-30-20 budget philosophy suggests that 50%, or half, your income go to these necessities.

Wants, though, are things you could live without but still wish to have, such as Amazon Prime, restaurant dinners, coffee from Starbucks, vacation trips, and hobby costs. Even a gym membership or that new laptop are wants. Creating the room to afford these wants is important to our mental health. Not budgeting for things we want will negatively impact our quality of life.

The remaining 20% should be set aside, either in retirement funds, stocks, other investments, an emergency fund (recommendations are that an emergency fund have 3 months of income), and perhaps extra spent to pay down debt. This 20% is very useful for addressing those unexpected costs, such as repairs or replacement of items that no longer work. Without budgeting this cushion, any expense that is a surprise can cause us to miss necessary payments.

The list of necessary expenses was not complete. There are other expenses that could be included.

Necessary Expenses and Expenses that are Wants

For some people, an expense will be necessary while the same expense for someone else will be a want. A good example of this is internet service. Many people consider internet service as a need, especially those who work from home or who are not able to leave their homes. One could also call internet service a need if they have children in school. For others, internet service is a want. If a person's job doesn't require them to be online, if they are not in school, if they do not have kids, then internet service can be dropped. There are public options for internet service. One could even use their phone as a hot spot.

Cars often fall into the category of need, but could also fall into the want category, depending on where and how you

live. Bikes, public transportation, and walking are all options that could replace a car. This would then remove the cost of gasoline and car insurance.

Another consideration when deciding if an item on your budget is a need or a want is about your choices and priorities. A car is a need for many. But the need for a car is not the same as the need for a specific car. If you choose to buy a car with payments that exceed your budgeted amount for the car, then that car is a want. The amount you exceed the budget now belongs in the want category.

The same can be said for housing. If you want an apartment that costs \$1,250 per month, but your budget only allows for an apartment that costs \$900, then \$350 of the rent is a want.

The point of that is to carefully consider if an expense is a need as opposed to a want.

When your expenses exceed your income, you may want to change how you budget your income to line up with these guidelines. This may mean cutting back, finding less-expensive living arrangements, finding a less-expensive (and more fuel-efficient) car, or sacrificing some specialty groceries. Using these guidelines keeps your financial life manageable.

Better still, they can guide you as you begin your life after graduation.

EXAMPLE 6.52

Evaluate a Budget Using 50-30-20 model

In the example above, after Carol added before school care for her daughter to the budget, her budget was as shown below. Evaluate Carol's budget using the 50-30-20 budget philosophy.

Income Source	Amount	Expense	Amount
Job	\$2,816.00	Rent	\$700
Child support	\$320.00	Car Payment	\$229
		Student Loan	\$250
		Car Insurance	\$119
		Utilities	\$80
		Health insurance	\$225
		Clothing	\$75
		Entertainment	\$200
		Food	\$275
		Gasoline	\$275
		Before-school care	\$252

Solution

Carol's total income is \$3,136.00. Applying the 50-30-20 budget philosophy to this income requires the calculation of each of those percentages.

For the necessities, Carol should budget 50% of her income, or $0.5 \times \$3,136.00 = \$1,568.00$.

For her wants, she should budget 30% of her income, or $0.3 \times \$3,136.00 = \940.80 .

For savings and extra debt service, she should budget 20% of her income, or $0.2 \times \$3,136.00 = \627.20 .

In her budget, her necessities include all expenses except for entertainment. These expenses total \$2,480, which exceeds the suggested budget amount of \$1568.00. To follow the guidelines, Carol would have to cut back on these necessities.

For her wants, she spends \$200.00 on entertainment, which is well below the suggested budget amount of \$940.80. If she modifies how much she spends on needs, she may be able to increase the spending on her wants.

Her excess income is \$456.00, which is below what she should be saving and using to pay down extra debt. If she does adjust how much she spends on needs, she could increase the amount for savings.

> YOUR TURN 6.52

1. Recall Heather's budget from Example 6.49, before she thought of moving. That budget is below. Evaluate Heather's budget using the 50-30-20 budget philosophy.

Income Source	Amount	Expense	Amount
Nursing	\$3,765.40	Rent	\$1,050
Part-time	\$672.00	Car Payment	\$489
		Student Loan	\$728
		Car Insurance	\$139
		Utilities	\$130
		Clothing	\$150
		Entertainment	\$300
		Credit Card	\$200
		Food	\$400
		Gasoline	\$250

EXAMPLE 6.53

Creating a Budget Based on the 50-30-20 Budget Philosophy

Carmen is about to graduate and has been offered a job at a bank as a data scientist. She estimates her monthly take home pay to be \$5,662.50. Apply the 50-30-20 philosophy to that monthly income. How should Carmen use this information?

✓ Solution

Step 1. To apply the 50-30-20 budget philosophy to Carmen's income, she needs to calculate 50%, 30%, and 20% of her income. Fifty percent of her income is $0.5 \times \$5,662.50 = \$2,831.25$. Thirty percent of her income is $0.3 \times \$5,662.50 = \$1,698.75$. Twenty percent of her income is $0.2 \times \$5,662.50 = \$1,132.50$.

Step 2. She would then budget \$2,831.25 for her needs, \$1,698.75 for her wants, and \$1,132.50 for savings and debt service.

Step 3. When choosing where to live, what to eat, and what to drive, she should make choices that keep those costs, combined with her debt service costs, gasoline, and utilities, below \$2,831.25. This means she will have to make

decisions about what her priorities are.

Step 4. She should then figure out what she wants to do with her money, and stay within the limits, that is, keep those costs below \$1,698.75.

Step 5. Finally, she can begin building her savings with the remaining \$1,132.50.

YOUR TURN 6.53

1. Elijah has finished an apprenticeship and is about to start his first job as an HVAC (heating, ventilation, and air conditioning) tech. He estimates that his net monthly income will be \$3,263.44. Apply the 50-30-20 budget philosophy to his income to set guidelines for Elijah's budget. How should Elijah use this information?

EXAMPLE 6.54

Using the 50-30-20 Budget Philosophy to Analyze Affordability

Steve is thinking of moving out of his family's home. He currently works at a full-time job making \$18 per hour, which will give him, approximately, a net annual income of \$29,180 (working 40 hours per week for 52 weeks per year). He has student debt that he pays off at \$218.00 per month, and already owns a car that he pays \$162.00 per month for.

1. Apply the 50-30-20 budget philosophy to Steve's income.
2. If he follows the budget, how much does he have, after paying his car payment and student loan, to spend on necessities.
3. If he follows the budget, how much will he set aside for wants? For savings?
4. Discuss the affordability of moving out, based on Steve's budget.

Solution

Before the 50-30-20 philosophy can be applied, Steve's monthly income needs to be determined. This is found by dividing his annual income by 12. This gives $\$29,180/12 = \$2,431.67$. This will be used for his monthly budget.

1. To apply the 50-30-20 philosophy to Steve's income, find 50%, 30%, and 20% of his monthly income.
Needs (50%): $50\% \text{ of his income is } 0.50 \times \$2,431.67 = \$1,215.83$.
Wants (30%): $0.30 \times \$2,431.67 = \729.50
Savings (20%): $0.20 \times \$2,431.67 = \486.33
2. The total for Steve's needs is \$1,215.83. From this, he already pays \$218.00 for his student loans, and \$162.00 for his car payment. Together that is \$380.00. Subtracting from the amount he should budget for his needs, he can spend \$835.83 on other needs.
3. Steve budgeted \$729.50 for wants, and \$486.33 for savings and other debt servicing.
4. Steve will have other needs to pay for, including rent, utilities, food, health care, gasoline, and car insurance. It is difficult to imagine Steve being able to afford to move out, unless he reallocates money that he would want to save, or use for entertainment and other wants, or takes on another job. Even if Steve uses all the money that the 50-30-20 budget sets aside for savings, he still only has \$1,322.16 to spend on those necessities. It does not appear he can afford to move out.

YOUR TURN 6.54

Fran wants to take a new job but will have to move to an area with a higher cost of living. With her current income, she can use the 50-30-20 budget philosophy. The new job will have a net pay of \$43,700 annually. She will still have to pay her car payment of \$295.00, her student loans that cost \$264.00 per month, and her outstanding credit card debt, on which she pays \$200 per month.

1. Apply the 50-30-20 budget philosophy to Fran's new income.
2. If she follows the budget, how much does she have, after paying her credit card debt, car payment and student loan, to spend on necessities.
3. If she follows the budget, how much will she set aside for wants? For savings?

4. Discuss the affordability of changing jobs and moving, based on Fran's budget.

VIDEO

50-30-20 Budget Philosophy (https://openstax.org/r/50-30-20_budgeting_rule)

Check Your Understanding

26. What is a budget?
27. What are necessary expenses?
28. David gathers his paystubs and bills from the past 6 months. His income, after taxes, is \$3,450 per month. His rent, utilities included, is \$925. His car payments are \$178.54 per month, his car insurance is \$129.49 per month, his credit cards cost him \$117.00 per month, he spends \$195 per month on gas, his food costs are \$290 per month. He also spends \$21.99 on Amazon prime, \$49.99 on his internet bill, and \$400 per month going out. Create David's monthly budget, including totals, based on that information.
29. Using David's Budget from Exercise 28, how much income does he have per month after accounting for his expenses?
30. Apply the 50-30-20 budget philosophy to David's budget.
31. Evaluate David's budget with respect to the 50-30-20 budget philosophy.



SECTION 6.5 EXERCISES

In the following exercises, categorize each expense as a necessary expense or an expense that is a want.

1. Rent
2. Dinner at a restaurant.
3. Car payment
4. New game system
5. Gym membership
6. Electric bill
7. Heating bill
8. Phone bill
9. Netflix
10. Student Loan Payment
11. Explain how a necessary expense for one person could be a want expense for another person.
12. Explain how a necessary expense may be partly a necessary expense and partially a want expense.

In the following exercises, create the budget, including totals and how much the income exceeds or falls short of the expenses, based on the information given.

13. Per month: paychecks = \$3,680, consulting = \$900, Mortgage = \$1,198.00, Utilities = \$376, Cell phone = \$67.50, Car payments = \$627.85, Car insurance = \$183.50, Student loans = \$833, Food = \$450, Gasoline = \$275, Internet = \$69, Dining out = \$250, Credit cards = \$375, entertainment = \$300
14. Per month: paychecks = \$2,750, child support = \$500, Mortgage = \$945.50, Utilities = \$195, Cell phone = \$37.50, Car payments = \$298.23, Car insurance = \$163.50, Student loans = \$438, Food = \$250, Gasoline = \$175, Internet = \$49, Netflix = \$15, After school care = \$711, Credit cards = \$150, entertainment = \$150
15. Per month: paychecks = \$4,385, Rent = \$1095, Utilities = \$165, Cell phone = \$67.50, Car payments = \$467.35, Car insurance = \$243.75, Student loans = \$1,150, Food = \$325, Gasoline = \$260, Internet = \$99, Netflix = \$15, Amazon = \$23, Gym membership = \$49, entertainment = \$650
16. Per month: paychecks = \$3,460, Gig job = \$173, Rent = \$895, Utilities = \$165, Car payments = \$195.80, Car insurance = \$123.30, Food = \$265, Gasoline = \$185, Internet = \$39, Hulu = \$15, Amazon = \$23, Credit cards = \$97.60, Entertainment = \$600

In the following exercises, determine the amount of money that should be allocated to each of the three categories of the 50-30-20 budget philosophy guidelines.

17. Referring to Exercise 13: Monthly income = \$4,580.00
18. Referring to Exercise 14: Monthly income = \$3,250.00

19. Referring to Exercise 15: Monthly income = \$4,385.00

20. Referring to Exercise 16: Monthly income = \$3,633.00

In the following exercises, evaluate the given budget with respect to the 50-30-20 budget philosophy guidelines.

21. The budget and 50-30-20 rule from exercises 13 and 17.

22. The budget and 50-30-20 rule from exercises 14 and 18.

23. The budget and 50-30-20 rule from exercises 15 and 19.

24. The budget and 50-30-20 rule from exercises 16 and 20.

For the following exercises, Kiera and Logan sit down to make their budget. Kiera works full time as a mental health counselor and sells kids toys on her own. Logan works as a branch manager at a local bank and works part-time at the nearby bar. They collect their financial document to work out their budget. Kiera's paychecks from her job as a mental health counselor, after taxes and per month, total \$3,021. Logan's paychecks from the bank, after taxes and per month, total \$3,827. Kiera's income from toy sales for the last 3 months were \$140, \$87, and \$475. Logan's take-home pay from the bartending job for the last 3 months were \$540, \$310, and \$449.

25. Determine how much income Kiera and Logan have per month.

26. Apply the 50-30-20 budget philosophy to their income.

For the following exercises, Kiera and Logan gather their bills from the last 6 months. Their fixed expenses, with costs, are rent for \$1,350, Kiera's car payment for \$275, Logan's car payment of \$380, student loans (they each have students loans) for \$934, car insurance for \$289, internet service for \$39, Netflix for \$15, Amazon Prime for \$24, gym membership for \$99, and cell phones for \$250. The variable cost expenses, and their average costs for the last 6 months, are utilities for \$370, gasoline for \$500, food for \$475, clothing for \$225, and miscellaneous entertainment expenses for \$535. They always pay off their credit card bill and carry no balance.

27. Create their budget, using the income from Exercise 25.

28. Categorize each expense as a need or a want. Find the total for each, along with remaining income.

29. Compare their budget to the guidelines from the 50-30-20 budget from Exercise 27.

30. Determine if Kiera and Logan can afford to buy a new computer, which would cost \$330 per month for the next 6 months.

In the following exercises, [the Federal Paycheck Calculator \(https://openstax.org/r/smartasset\)](https://openstax.org/r/smartasset) was used to estimate monthly take-home pay. The annual salary, before taxes and deductions, is provided. Then, the monthly take-home pay after taxes and deductions is given (which means the monthly take-home pay is not just the annual salary divided by 12!). In each case, apply the 50-30-20 budget philosophy to the monthly take-home income. Note: These are based on living in Indianapolis, Indianapolis, unmarried and with no dependents.

31. Annual salary: \$30,000. Monthly take home: \$1,938

32. Annual salary: \$40,000.00. Monthly take home: \$2,564

33. Annual salary: \$50,000. Monthly take home: \$3,144

34. Annual salary: \$70,000. Monthly take home: \$4,229

35. Annual salary: \$100,000. Monthly take home: \$5,840

36. Annual salary: 150,000. Monthly take home: \$8,506

In the following exercises, [the Federal Paycheck Calculator \(https://openstax.org/r/smartasset\)](https://openstax.org/r/smartasset) was used to estimate monthly take-home pay. The hourly pay, before taxes and deductions, is provided. Then, the monthly take-home pay after taxes and deductions is given (which means the monthly take-home pay is not just the hourly pay times 174 hours!). In each case, apply the 50-30-20 budget philosophy to the monthly take-home income. Note: These are based on living in Tempe, Arizona, unmarried and with no dependents.

37. Hourly pay: \$12.15 (minimum wage in Tempe, Arizona as of September 2022). Monthly take home: \$1,698

38. Hourly pay: \$15.00. Monthly take home: \$2,083

39. Hourly pay: \$17.50. Monthly take home: \$2,421

40. Hourly pay: \$19.75. Monthly take home: \$2,725

41. Hourly pay: \$25.00. Monthly take home: \$3,369

42. Hourly pay: \$35.00. Monthly take home: \$4,547

6.6 Methods of Savings



Figure 6.8 Money wisely invested grows over time. (credit: "Stack of Cash" by Janak Raja/Flickr, Public Domain Mark 1.0)

Learning Objectives

After completing this section, you should be able to:

1. Distinguish various basic forms of savings plans.
2. Compute return on investment for basic forms of savings plans.
3. Compute payment to reach a financial goal.

The stock market crash of 1929 led to the Great Depression, a decade-long global downturn in productivity and employment. A state of shock swept through the United States; the damage to people's lives was immeasurable. Americans no longer trusted established financial institutions. By October 1931, the banking industry's biggest challenge was restoring confidence to the American public. In the next 10 years, the federal government would impose strict regulations and guidelines on the financial industry. The Emergency Banking Act of 1933 created the Federal Deposit Insurance Corporation (FDIC), which insures bank deposits. The new federal guidelines helped ease suspicions among the general public about the banking industry. Gradually, things returned to normal, and today we have more investment instruments, many insured through the FDIC, than ever before.

In this section, we will first look at the different types of savings accounts and proceed to discuss the various types of investments. There is some overlap, but we will try to differentiate among these financial instruments. Saving money should be a goal of every adult, but it can also be a difficult goal to attain.

Distinguish Various Basic Forms of Savings Plans

There are at least three types of savings accounts. Traditional savings accounts, certificates of deposit (CDs), and money market accounts are three main savings account vehicles.

Savings Account

A savings account is probably the most well-known type of investment, and for many people it is their first experience with a bank. A **savings account** is a deposit account, held at a bank or other financial institution, which bears some interest on the deposited money. Savings accounts are intended as a place to save money for emergencies or to achieve short-term goals. They typically pay a low interest rate, but there is virtually no risk involved, and they are insured by the FDIC for up to \$250,000.

Savings accounts have some strengths. They are highly flexible. Generally, there are no limitations on the number of withdrawals allowed and no limit on how much you can deposit. It is not unusual, however, that a savings account will have a minimum balance in order for the bank to pay maintenance costs. If your account should dip below the minimum, there are usually fees attached.

? WHO KNEW?

Many banks are covered by FDIC insurance. The FDIC is the Federal Deposit Insurance Corporation and is an independent agency created by the U.S. Congress. One of its purposes is to provide insurance for deposits in banks, including savings accounts. Be aware, not all banks are FDIC insured. The FDIC insures up to \$250,000 for a savings account, so you do not want your balance to exceed that federally insured limit.

Having your savings account at the same bank as your checking account does offer a real advantage. For example, if your checking account is approaching its lower limit, you can transfer funds from your savings account and avoid any bank fees. Similarly, if you have an excess of funds in your checking account, you can transfer funds to your savings account and earn some interest. Checking accounts rarely pay interest.

**PEOPLE IN MATHEMATICS**

J.P. Morgan

J.P. Morgan was a wealthy banker around the turn of the 20th century. His business interests included railroads and the steel industry. However, it was in 1907 that a financial crisis, caused by poor banking decisions and followed by such great distrust in the banking system that a frenzy of withdrawals from banks occurred, that J.P. Morgan and other wealthy bankers lent from their own funds to help stabilize and save the system.

There are some weaknesses to savings accounts. Primarily, it is because savings accounts earn very low interest rates. This means they are not the best way to grow your money. Experts, though, recommend keeping a savings account balance to cover 3 to 6 months of living expenses in case you should lose your job, have a sudden medical expense, or other emergency.

Around tax time, you will receive a 1099-INT form stating the amount of interest earned on your savings, which is the amount that must be reported when you file your tax return. A **1099 form** is a tax form that reports earnings that do not come from your employer, including interest earned on savings accounts. These 1099 forms have the suffix INT to indicate that the income is interest income.

Savings accounts earn interest, and those earnings can be found using the interest formulas from previous sections. The final value of these accounts is sometimes called the future value of the account.

EXAMPLE 6.55**Single Deposit in a Savings Account**

Violet deposits \$4,520.00 in a savings account bearing 1.45% interest compounded annually. If she does not add to or withdraw any of that money, how much will be in the account after 3 years?

✓ Solution

To find the compound interest, use the formula from [Compound Interest](#), $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the amount in the account after t years, with initial deposit (or principal) of P , at an annual interest rate, in decimal form, of r , compounded n times per year. Violet has a principal of \$4,520.00, which will earn an interest of $r = 0.0145$, compounded yearly (so $n = 1$), for $t = 3$ years. Substituting and calculating, we find that Violet's account will be worth

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= \$4,520.00\left(1 + \frac{0.0145}{1}\right)^{1 \times 3} \\ &= \$4,520.00(1.0145)^3 \\ &= \$4,719.48 \end{aligned}$$

Or, Violet will have \$4,719.48 after 3 years.

> YOUR TURN 6.55

1. Brian deposits \$5,600 in a savings account that yields 1.23% interest compounded annually. If he leaves that deposit in the account and adds nothing new to the account, what will the account be worth in 5 years?

? WHO KNEW?

Banks have not always offered interest on savings accounts. An 1836 publication from Indiana noted that banks in other states allow small interest on deposits. It specifically says that in these other states, these deposits are what business transactions are based upon. And that giving interest would encourage deposits, and thus increase the business that banks can do.

[Journal of the House of Representatives of the State of Indiana \(https://openstax.org/r/onepage\)](https://openstax.org/r/onepage)

Certificates of Deposit, or CDs

We discussed **certificates of deposit** (CDs) in earlier sections. CDs differ from savings accounts in a few ways. First, the investment lasts for a fixed period of time, agreed to when the money is invested in the CD. These time periods often range from 6 months to 5 years. Money from the CD cannot be withdrawn (without penalty) until the investment period is up. Also, money cannot be added to an existing CD.

Certificates of deposit have features similar to savings accounts. They are insured by the FDIC. They are entirely safe. They do, though, offer a better interest rate. The trade-off is that once the money is invested in a CD, that money is unavailable until the investment period ends.

EXAMPLE 6.56**5-Year CD**

Silvio deposits \$10,000 in a CD that yields 2.17% compounded semiannually for 5 years. How much is the CD worth after 5 years?

✓ Solution

This also uses the compound interest formula from [Compound Interest](#), $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Substituting the values $P = \$10,000$, $r = 0.0217$, $n = 2$ (semiannually means twice per year), and $t = 5$, we find the account will be worth

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= \$10,000.00\left(1 + \frac{0.0217}{2}\right)^{2 \times 5} \\ &= \$10,000.00(1.01085)^{10} \\ &= \$11,239.53 \end{aligned}$$

The CD will be worth \$11,239.53 after 5 years.

> YOUR TURN 6.56

1. Denise deposits \$3,500 in a CD bearing 2.23% interest compounded quarterly for 3 years. How much will Denise's CD be worth after those 3 years?

Money Market Account

A **money market account** is similar to a savings account, except the number of transactions (withdrawals and transfers) is generally limited to six each month. Money market accounts typically have a minimum balance that must be maintained. If the balance in the account drops below the minimum, there is likely to be a penalty. Money market accounts offer the flexibility of checks and ATM cards. Finally, the interest rate on a money market account is typically

higher than the interest rate on a savings account.

EXAMPLE 6.57

Single Deposit to a Money Market Account

Marietta opens a money market account, and deposits \$2,500.00 in the account. It bears 1.76% interest compounded monthly. If she makes no other transactions on the account, how much will be in the account after 4 years?

Solution

This, once again, uses the compound interest formula from [Compound Interest](#): $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Substituting the values $P = \$2,500$, $r = 0.0176$, $n = 12$, and $t = 4$, we find the account will be worth

$$\begin{aligned} A &= P\left(1 + \frac{r}{n}\right)^{nt} \\ &= \$2,500.00\left(1 + \frac{0.0176}{12}\right)^{12 \times 4} \\ &= \$2,500.00(1.0014\overline{6})^{48} \\ &= \$2,682.20 \end{aligned}$$

The money market account will be worth \$2,682.20 after 4 years.

YOUR TURN 6.57

1. Chuck opens a money market account, and deposits \$8,500.00 in the account. It bears 1.83% interest compounded quarterly. If he leaves makes no other transactions on the account, how much will be in the account after 3 years?

Return on Investment

If we want to compare the profitability of different investments, like savings accounts versus other investment tools, we need a measure that evens the playing field. Such a measure is **return on investment**.

FORMULA

The return on investment, often denoted ROI, is the percent difference between the initial investment, P , and the final value of the investment, FV , or $\text{ROI} = \frac{FV - P}{P}$, expressed as a percentage.

 The length of time of the investment is not considered in ROI.

EXAMPLE 6.58

Calculating Return on Investment

1. Determine the return on investment for the 5-year CD from [Example 6.56](#). Round the percentage to two decimal places.
2. Determine the return on investment for the money market account from [Example 6.57](#). Round the percentage to two decimal places.

Solution

1. The initial deposit in the CD was \$10,000, so $P = \$10,000$. The value at the end of 5 years was \$11,239.53. so $FV = \$11,239.53$. Substituting and computing we find the return on investment.

$$\begin{aligned}
 \text{ROI} &= \frac{FV - P}{P} \\
 &= \frac{\$11,239.53 - \$10,000}{\$10,000} \\
 &= \frac{\$1,239.53}{\$10,000} \\
 &= 0.123953
 \end{aligned}$$

The ROI is 12.40%.

2. The initial deposit in the money market was \$2,500, so $P = \$2,500$. The value at the end of 4 years was \$2,682.20. so $FV = \$2,682.20$. Substituting and computing we find the return on investment.

$$\begin{aligned}
 \text{ROI} &= \frac{FV - P}{P} \\
 &= \frac{\$2,682.20 - \$2,500}{\$2,500} \\
 &= \frac{\$182.20}{\$2,500} \\
 &= 0.07288
 \end{aligned}$$

The ROI is 7.29%.

> YOUR TURN 6.58

1. The amount of \$13,000 is invested in a savings account. After 10 years the account has \$15,250.00. Find the return on investment for this account.
2. The amount of \$6,500 is deposited in a money market account. After 7 years, the account has \$7,358.00. Find the return on investment for this account.

▶ VIDEO

[Return on Investment, ROI \(https://openstax.org/r/This_video\)](https://openstax.org/r/This_video)

Annuities as Savings

In [Compound Interest](#), we talked about the future value of a single deposit. In reality, people often open accounts that allow them to add deposits, or *payments*, to the account at regular intervals. This agrees with the 50-30-20 budget philosophy, where some income is saved every month. When a deposit is made at the end of each compounding period, such a savings account is called an **ordinary annuity**.

The formula for the future value of an ordinary annuity is $FV = pmt \times \frac{(1 + r/n)^{n \times t} - 1}{r/n}$, where FV is the future value of the annuity, pmt is the payment, r is the annual interest rate (in decimal form), n is the number of compounding periods per year, and t is the number of years.

 It is important to note that the number of deposits per year and the number of periods per year are the same.

 Another form of annuity is the annuity due, which has deposits at the start of each compounding period. This other annuity type has different formulas and is not addressed in this text.

EXAMPLE 6.59

Future Value of an Ordinary Annuity

Jill has an account that bears 3.75% interest compounded monthly. She decides to deposit \$250.00 each month, at the

end of the compounding period, into this account. What is the future value of this account, after 8 years?

 Solution

These are regular payments into an account bearing compound interest. She is depositing them at the end of each compounding period. This makes this an ordinary annuity. Substituting the values $pmt = 250$, $r = 0.0375$, $n = 12$, and $t = 8$ into the formula, we find the future value of the account.

$$\begin{aligned}
 FV &= pmt \times \frac{(1 + r/n)^{n \times t} - 1}{r/n} \\
 &= 250 \times \frac{(1 + 0.0375/12)^{12 \times 8} - 1}{0.0375/12} \\
 &= 250 \times \frac{(1.003125)^{96} - 1}{0.003125} \\
 &= 250 \times \frac{1.34922752406 - 1}{0.003125} \\
 &= 250 \times \frac{0.34922752406}{0.003125} \\
 &= 250 \times 111.752807699 \\
 &= 27,938.202
 \end{aligned}$$

The account, after 8 years, will contain \$27,938.20.

 YOUR TURN 6.59

1. Kelly invests \$525 every third month, at the end of the compounding period, into an account bearing 3.89% interest compounded quarterly. How much will be in the account after 15 years?

 WHO KNEW?

Setting Savings Account Interest Rates

There are a number of factors that contribute to the amount a bank gives for savings accounts. The interest rate reflects how much the bank values deposits. It also reflects the money that the bank will earn when they lend out money. Finally, interest rates are impacted by the Federal Reserve Bank. When the Fed raises interest rates, so do banks.



PEOPLE IN MATHEMATICS

The Federal Reserve Chairperson

The Federal Reserve Board monitors the risks in the financial system to help ensure a healthy economy for individuals, companies, and communities. The Board oversees the 12 regional reserve banks. The Chairperson of the Federal Reserve Board testifies to Congress twice per year, meets with the secretary of the Treasury, chairs the Federal Open Market Committee, and is the face of federal monetary policy. Currently, the Fed Chair is Jerome Powell, who has served since 2018.

EXAMPLE 6.60

Saving for College

When Yusef was born, Rita and George began to save for Yusef's college years by investing \$2,500 each year in a savings account bearing 3.4% interest compounded annually. How much will they have saved after 18 years?

✓ **Solution**

To find the future value of the account, we use the ordinary annuity formula $FV = pmt \times \frac{(1 + r/n)^{n \times t} - 1}{r/n}$. The payment is \$2,500, rate is 0.034, the number of compounding periods is 1, and the number of years is 18. Substituting these values and computing, we have

$$\begin{aligned}
 FV &= pmt \times \frac{(1 + r/n)^{n \times t} - 1}{r/n} \\
 &= \$2,500 \times \frac{(1 + 0.034/1)^{1 \times 18} - 1}{0.034/1} \\
 &= \$2,500 \times \frac{(1.034)^{18} - 1}{0.034} \\
 &= \$2,500 \times \frac{1.82544897331 - 1}{0.034} \\
 &= \$2,500 \times 24.2779109798 \\
 &= \$60,694.77
 \end{aligned}$$

After saving for 18 years, Rita and George will have \$60,694.77 for Yusef's college.

> **YOUR TURN 6.60**

1. Bemnet saves \$280 per month in a savings account bearing 3.11% interest compounded monthly. After 20 years, how much does Bemnet have in the account?

 TECH CHECK

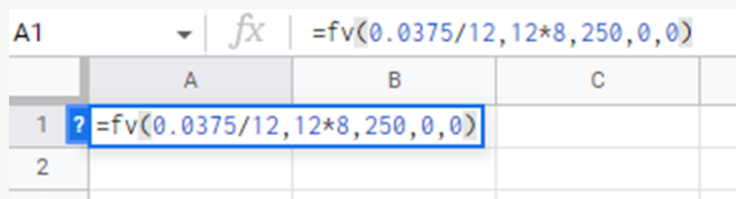
Google Sheets offers a function to calculate the future value of an ordinary annuity. To get Google Sheets to calculate the future value, you use the following:

`=fv(rate,number_of_periods, payment, present_value, end_or_beginning).`

To explain, the rate is the rate per compounding period. From our formula, that is r/n . Also, the number of periods must be entered. From our formula, that is $n \times t$. The payment is the amount deposited each period. Present value is 0 if we begin with no money and rely only on the payments to be made. However, if some money is available to put in the account before the payments start, that amount, an initial deposit, would be the value of PV . Finally, for an ordinary annuity, enter 0 for end or beginning. Using the values for Jill, the payment amount is \$250, $r = 0.0375$, $n = 12$, $t = 8$, and that there is no initial deposit, $PV = 0$, the Google Sheets formula is

`=fv(0.0375/12,12*8,250,0,0).`

Figure 6.9 shows the formula in Google Sheets.



A1	fx =fv(0.0375/12, 12*8, 250, 0, 0)			
	A	B	C	
1	=fv(0.0375/12, 12*8, 250, 0, 0)			
2				

Figure 6.9 Google Sheets formula

Hitting the enter key shows the payment value (Figure 6.10).

A1	∇	fx	$=fv(0.0375/12, 12*8, 250, 0, 0)$		
	A	B	C		
1	$-\$27,938.20$				
2					
3					

Figure 6.10 Payment value

Notice that the future value is negative, since it is a payment leaving an account.

VIDEO

Future Value Using Google Sheets (https://openstax.org/r/cell_references)

Compute Payment to Reach a Financial Goal

The formula used to get the future value of an ordinary annuity is useful, finding out what the final amount in the account will be. However, that isn't how planning works. To plan, we need to know how much to put into the ordinary annuity each compounding period in order to reach a goal. Fortunately, that formula exists.

FORMULA

The formula for the amount that needs to be deposited per period, pmt , of an ordinary annuity to reach a specified goal, FV , is $pmt = \frac{FV \times (r/n)}{(1 + r/n)^{n \times t} - 1}$, where r is the annual interest rate (in decimal form), n is the number of periods per year, and t is the number of years.

With this formula, it is possible to plan the amount to be saved.

EXAMPLE 6.61

Saving for a Car

Yaroslava wants to save in order to buy a car, in 3 years, without taking out a loan. She determines that she'll need \$35,500 for the purchase. If she deposits money into an ordinary annuity that yields 4.25% interest compounded monthly, how much will she need to deposit each month?

Solution

Yaroslava has a goal and needs to know the payments to make to reach the goal. Her goal is $FV = \$35,500$, with an interest rate $r = 0.0425$, compounded per month so $n = 12$, and for 3 years, making $t = 3$. Substituting into the formula, Yaroslava finds the necessary payment.

$$\begin{aligned}
 pmt &= \frac{FV \times (r/n)}{(1 + r/n)^{n \times t} - 1} \\
 &= \frac{35,500 \times (0.0425/12)}{(1 + 0.0425/12)^{12 \times 3} - 1} \\
 &= \frac{35,500 \times (0.003541\overline{6})}{(1.003541\overline{6})^{36} - 1} \\
 &= \frac{125.7291\overline{6}}{0.13572901696} \\
 &= 926.325
 \end{aligned}$$

To reach her goal, Yaroslava would need to deposit \$926.33 in her account each month.



This has been rounded up, so that the deposits don't fall short of the goal. However, some round off using the

standard rounding rules: if the last digit is 1, 2, 3, or 4, the number is rounded down; if the last digit is 5, 6, 7, 8, or 9 the number is rounded up.

> YOUR TURN 6.61

1. Chione decides to put new siding on her house. She finds that it will cost about \$27,800. She decides to begin saving for the purchase so that she doesn't take on debt to side the house. How much would Chione need to deposit each quarter in an ordinary annuity that yields 5.16% compounded quarterly for 5 years?

TECH CHECK

Google Sheets offers a function to calculate the payment necessary to reach a goal using ordinary annuities. To get Google Sheets to calculate the payment, you use the following:

`=pmt(rate,number_of_periods, present_value, future_value, end_or_beginning).`

To explain, the rate is per compounding period. From our formula, that is r/n . Also, the number of periods must be entered. From our formula, that is $n \times t$. The present value is the amount of money that the account begins with. If we begin with no money and rely only on the payments to be made, then this number is 0. However, if some money is available to put in the account before the payments start, that amount, an initial deposit, would be the value of PV . Next, enter the future value, FV . Finally, for an ordinary annuity, enter 0 for end or beginning. Using the values for Yaroslava, $r = 0.0425$, $n = 12$, $t = 3$, and that there is no initial deposit, $PV = 0$, the Google Sheets formula is

`=pmt(0.0425/12,12*3,0,35500,0).`

Figure 6.11 shows the formula in Google Sheets.

The screenshot shows a Google Sheets spreadsheet. The formula bar at the top displays `=pmt(0.0425/12,12*3,0,35500,0)`. In the spreadsheet grid, cell A1 contains the same formula. The columns are labeled A, B, C, and D. The rows are numbered 1, 2, and 3. Cell A1 is highlighted with a blue border.

	A	B	C	D
1	<code>=pmt(0.0425/12,12*3,0,35500,0)</code>			
2				
3				

Figure 6.11 Google Sheets formula

Hitting the enter key shows the payment value (Figure 6.12).

The screenshot shows the same Google Sheets spreadsheet as Figure 6.11, but now the result of the formula is displayed. Cell A1 now contains the value `-$926.32`. The formula bar still shows the original formula. The columns are labeled A, B, C, and D. The rows are numbered 1 and 2. Cell A1 is highlighted with a blue border.

	A	B	C	D
1	<code>-\$926.32</code>			
2				

Figure 6.12 Payment value

Notice that the payment is negative, since it is a payment leaving an account. Additionally, the payment is \$926.32. We rounded that up, but Google Sheets rounded off.

Check Your Understanding

32. What is a savings account?
33. How does a CD differ from a savings account?
34. Which is more flexible, a CD or a money market account? Why?
35. If \$7,500 is deposited in a 4-year CD earning 3.28% interest compounded monthly, how much is in the account after 4 years?

36. If the initial deposit in an account is \$10,000 and the account is worth \$12,560 after 7 years, what is the return on investment?
37. Find the future value of an account if \$450.00 per quarter is invested in a savings account bearing 3.5% interest compounded quarterly for 10 years.
38. How much must be deposited per quarter in an account bearing 2.98% interest compounded quarterly if the account is to be worth \$300,000 after 25 years?



SECTION 6.6 EXERCISES

1. Which account has the greatest flexibility, savings, certificate of deposit, or money market?
2. Why are interest rates on savings accounts, CDs, and money market accounts low?
3. Which of savings accounts, certificates of deposit, and money market accounts, allow for transactions?
4. How does number of years impact the return on investment?

In the following exercises, find the future value of the account based on the information given.

5. The amount of \$3,000 deposited in a CD bearing 2.6% compounded semi-annually for 3 years.
6. The amount of \$1,500 deposited in a money market account bearing 3.11% interest compounded monthly for 10 years.
7. The amount of \$8,450 deposited in a savings account bearing 1.75% interest compounded monthly for 2 years.
8. The amount of \$10,500 deposited in a savings account bearing 1.35% interest compounded quarterly for 20 years.
9. The amount of \$24,800 deposited in a money market account bearing 2.53% interest compounded semi-annually for 13 years.
10. The amount of \$16,400 deposited in a CD bearing 2.55% interest compounded quarterly for 18 years.

In the following exercises, find the return on investment based on the specified exercise. Round to two decimal places.

11. Account from Exercise 5.
12. Account from Exercise 6.
13. Account from Exercise 7.
14. Account from Exercise 8.

In the following exercises, find the future value of the ordinary annuities based on the payment, interest rate, compounding periods and length of time given.

15. The amount of \$150 deposited monthly in an account bearing 4.22% interest compounded monthly for 20 years.
16. The amount of \$500 deposited semi-annually in an account bearing 3.62% interest compounded semi-annually for 30 years.
17. The amount of \$250 deposited quarterly in an account bearing 3.61% interest compounded quarterly for 25 years.
18. The amount of \$250 deposited monthly in an account bearing 3.09% interest compounded monthly for 40 years.
19. The amount of \$1,500 deposited annually in an account bearing 3.34% interest compounded annually for 10 years.
20. The amount of \$1400 deposited semi-annually in an account bearing 2.78% interest compounded semi-annually for 30 years.

In the following exercises, find the payment per period necessary to reach a specified future value based on the given interest rate, compounding periods per year, and number of years. Recall, the number of payments per year and the number of compounding periods per year are the same.

21. Future value of \$1,000,000 from an account bearing 3.94% interest compounded monthly for 40 years.
22. Future value of \$500,000 from an account bearing 2.11% interest compounded quarterly for 30 years.
23. Future value of \$750,000 from an account bearing 3.27% interest compounded monthly for 25 years.
24. Future value of \$300,000 from an account bearing 3.59% interest compounded semiannually for 35 years.
25. Future value of \$1,000,000 from an account bearing 3.62% interest compounded annually for 25 years.
26. Future value of \$600,000 from an account bearing 4.02% interest compounded quarterly for 30 years.
27. Dina deposits \$3,000 in a 5-year CD that bears 3.25% interest compounded quarterly. What is the CD worth after those 5 years?

28. Timothy deposits \$1,200 in a savings account that bears 1.85% interest compounded monthly. If Timothy does not deposit or withdraw money from the account how much is in Timothy's account after 3 years?
29. Leslie deposits \$13,000 in a money market account that bears 2.55% interest compounded semi-annually. If Leslie does not withdraw or deposit money into the account, how much is in Leslie's account after 6 years?
30. Jennifer deposits \$8,500 in a 3-year CD bearing 2.71% interest compounded annually. How much is Jennifer's CD worth after those 3 years?
31. Yasmin has analyzed her budget and decides to deposit \$425 per month in an account bearing 3.99% interest compounded monthly. How much will be in the account after 20 years? After 30 years? After 40 years?
32. Brad applied the 50-30-20 budget philosophy to his income and decides that he can afford \$380 per month for savings. He finds an account bearing 3.47% interest compounded monthly. How much will he have in the account after 25 years? 30 years? 35 years?
33. Ashliegh wants to save for an early retirement. She thinks she needs \$1,250,000 to retire at the age of 55, which is 30 years from now. How much must she deposit per month in an account bearing 3.48% interest compounded monthly to reach her goal?
34. Colin plans out the next 38 years of his life. In order to retire in 38 years (age 65) with \$1,450,000, how much should he deposit quarterly in an account bearing 4.21% interest compounded quarterly to reach his goal?

In the following exercises, different savings strategies will be compared.

35. Sam is 23 years old and has just landed her first post-college job. She creates a budget, and using the 50-30-20 budget philosophy, she sees she should save or pay down debt with \$650. She decides to apply \$300 per month to long-term savings. She finds an account bearing 3.75% interest compounded monthly. Sam begins investing \$300 per month in that account on her 24th birthday. How much will be in the account at age 65 (41 years)?
36. Sam decides instead to delay investing in the account until her 35th birthday. How much will be in the account at age 65 (30 years)?
37. Sam decides to deposit the \$300 per month until she turns 35 years old (11 years). She will then stop investing the \$300 monthly, and just allow the money to earn interest until her 65th birthday (30 more years). How much will be in her account on her 65th birthday? Hint: First, compute the *FV* of the deposits. Then use that *FV* as the principal for a single deposit into an account bearing 3.75% interest compounded monthly.
38. Compare the results of the three investment strategies.

In the following exercises, different savings strategies will be compared.

39. Dahlia is 22 years old and has just landed a banking job. She creates a budget, and using the 50-30-20 budget philosophy, she sees she should save or pay down debt with \$400. She decides to apply \$250 per month to long-term savings. She finds an account bearing 6.2% interest compounded monthly. Dahlia begins investing \$250 per month in that account on her 23rd birthday. How much will be in the account on her 68th birthday (45 years)?
40. Dahlia decides instead to delay investing in the account until her 34th birthday. How much will be in the account on her 68th birthday (34 years)?
41. Dahlia decides to deposit the \$250 per month until her 34th birthday (11 years). She will then stop investing the \$250 monthly, and just allow the money to earn interest until her 68th birthday (34 more years). How much will be in her account on her 65th birthday? Hint: First compute the *FV* of the deposits. Then use that *FV* as the principal for a single deposit into an account bearing 6.2% interest compounded monthly.
42. Compare the results of the three investment strategies.

6.7 Investments



Figure 6.13 Stocks are bought and sold to improve investment values. (credit: modification of work "FT ringing the Closing Bell at the NYSE" by Financial Times/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Distinguish between basic forms of investments including stocks, bonds, and mutual funds.
2. Understand what bonds are and how bond investments work.
3. Understand how stocks are purchased and gain or lose value.
4. Read and derive information from a stock table.
5. Define a mutual fund and how to invest.
6. Compute return on investment for basic forms of investments.
7. Compute future value of investments.
8. Compute payment to reach a financial goal.
9. Identify and distinguish between retirement savings accounts.

You can save your money in a safe or a vault (or worse, under the mattress!), but that money does not grow. It would be hard to save enough for retirement that way. What can be done to increase the value of the money you already have?

The answer is to invest it. Use the money that you have to earn more money back. For instance, as we saw in [Methods of Savings](#), you can save it in a bank. Or, to reach loftier goals, invest in something more likely to grow, such as stocks.

A great example of this is Apple stock. Anyone who bought stock in Apple Inc. (formerly Apple Computer, Inc.) in 1997 and held onto the shares earned a lot of money. [To be more specific, \\$100 worth of Apple shares bought in 1980, when it was first sold to the public, was valued at \\$67,564 in 2019, or 676 times more! \(\[https://openstax.org/r/much_aspx\]\(https://openstax.org/r/much_aspx\)\)](#) Perhaps you have heard a story like that, of an investment opportunity taken that paid off, or the story of an investment opportunity missed. But such stories are the exceptions.

In this section, we'll investigate bonds, stocks, and mutual funds and their comparative strengths and weaknesses. We close the section with a discussion of retirement savings accounts.

Distinguish Between Basic Forms of Investments

Bonds, stocks, and mutual funds tend to offer higher returns, but to varying degrees, come with higher risks. Stocks and mutual funds also vary in how much they earn. Their predicted rates of return on investment are not guaranteed, but educated guesses based on market trends and historical performance.

We will use the methods and formulas we learned earlier to evaluate these forms of investment.

Bonds

Bonds are issued from big companies and from governments. Selling bonds is an alternative to an institution taking a loan from a bank. The funds from the selling of bonds are often used for large projects, like funding the building of a new highway or hospital.

Bonds are considered a conservative investment. They are bought for what is known as the **issue price**. The interest is fixed (does not change) at the time of purchase and is based on the issue price of the bond. The interest rate is often referred to as the coupon rate; the interest paid is often called the coupon yield. The interest paid is often higher than savings accounts and the risk is exceptionally low. The bond is for a fixed length of time. The end of this time is the

maturity date of the bond.

There are several types of bonds:

- **Treasury bonds** are issued by the federal government.
- **Municipal bonds** are issued by state and local governments.
- **Corporate bonds** are issued by major corporations.

There are other types of bonds available, but they are beyond the scope of this section.

?? WHO KNEW?

Trading Bonds

Bonds are often part of larger investment portfolios. These bonds may be traded. However, the interest paid is based on the price when the bond was bought (the issue price). These bonds can be bought and sold for more or less money than the issue price. If the bond is bought for more than the issue price, the interest is still paid on the issue price, not on the purchase price when the trade was made. This means the actual return on the bond decreases. If the bond is bought for less than the issue price, the return on the bond goes up.

▶ VIDEO

[Bonds \(https://openstax.org/r/investing_basics_bonds\)](https://openstax.org/r/investing_basics_bonds)

EXAMPLE 6.62

Bond Investment

Muriel purchases a \$3,000 bond with a maturity of 4 years at a fixed coupon rate of 5.5% paid annually. How much is Muriel paid each year, and how much does she receive on the maturity date?

✓ Solution

The coupon rate is 5.5%. 5.5% of her bond value is $0.055 \times \$3,000 = \165 . After year 1, Muriel receives \$165. She receives \$165 after years 2 and 3 also. In year 4, when the bond matures, Muriel receives \$3,165, or the interest and the initial investment, or principal.

> YOUR TURN 6.62

1. Maureen invests \$5,000 in a bond with a maturity date in 5 years at a fixed coupon rate of 4.75%. How much is Maureen paid each year and how much does she receive on the maturity date?

Stocks

Stocks are part ownership in a company. They come in units called **shares**. The performance and earnings of stocks is not guaranteed, which makes them riskier than any other investment discussed earlier. However, they can offer higher return on investment than the other investments. Their value grows in two ways. They offer **dividends**, which is a portion of the profit made by the company. And the price per share can increase based on how others see that value of the company changing. If the value of the company drops, or the company folds, the money invested in the stock also drops.

Most stock transactions are executed through a broker. Brokers' commissions can be a percentage of value of the trades made or a flat fee. There are full-service brokers who charge higher commission rates, but they also offer financial advice and perform the research that you may not have the time or the expertise to do on your own. A discount broker only executes the stock transactions, buying or selling, so they charge lower rates than full-service brokers. There are also brokers that offer commission-free trading.

An important thing to remember is that stocks might provide a very large return on investment, but the trade-off is the risk associated with owning stocks.

? WHO KNEW?**Chapter 11 Bankruptcy and Stocks**

In the fall of 2022, the parent company of Regal Theaters, named Cineworld, filed for Chapter 11 bankruptcy. According to news articles, the bankruptcy was necessitated due to its heavy debt load. Generally, a company can file for Chapter 11 bankruptcy to allow them time to reorganize and restructure debts. When this happens, the company, after the Chapter 11 process is over, offers new stock. This makes the previous stock worthless. However, the company may allow an exchange of old stock for a discounted amount of the new stock. This in effect reduces (maybe vastly) the wealth held by those who owned the original stock.

EXAMPLE 6.63**Buying Stock in Company ABC**

Haniah buys stock in the ABC company, investing a total of \$13,000. She expects the stock to grow, through stock price increase and reinvestment of dividends, by 12.3% per year and compounded annually. If she leaves that money invested, how much will the stocks be worth in 20 years?

✓ Solution

Calculating this is a compound interest calculation, if Haniah's assumption about the stock's performance is correct. If so, then the principal is \$13,000, the rate is 0.123, the number of compounding periods per year is 1, and the time is 20 years. Substituting into the compound interest formula from [Methods of Savings](#), and computing, we have

$A = P\left(1 + \frac{r}{n}\right)^{nt} = \$13,000\left(1 + \frac{0.123}{1}\right)^{1 \times 20} = \$13,000 \times 10.1764223996 = \$132,293.49$. After 20 years, her stock is now worth \$132,293.49.

> YOUR TURN 6.63

1. Rixie deposits \$23,000 in the stock of DEF company. She assumes the stock value to grow though stock price increase and reinvestment of dividends by 13.8% compounded annually. How much will her stock be worth in 12 years?

? WHO KNEW?**Risk and Volkswagen**

The question of risk hovers over every investment. How risky can it get? Volkswagen seems to be a rather safe investment. But in 2015, Volkswagen's stock tumbled 30% over a few days when it was revealed that the company had installed software that altered the emission performance of some of their diesel engines. Volkswagen's hope was that lower emissions would bolster US sales of some of their diesel models. This was a drastic drop, and many investors lost a lot of money. However, the stock has come back since then. This was mild compared to the 65% drop in the Martha Stewart Living Omnimedia stocks.

**PEOPLE IN MATHEMATICS****Warren Buffett**

Warren Buffett is an investment legend. He began his career as an investment salesman in the 1950s. He formed Buffett associates in 1956. In 1965, he was in control of Berkshire Hathaway, which began as a merger between two textile companies. In his role there, he began to invest in a variety of companies. It is now a conglomerate holding company, and fully owns GEICO, Duracell, Dairy Queen, and other large companies.

His investment philosophy involves finding stocks and bonds from companies that have high intrinsic worth compared to their stock or bond prices. This means he focuses not on the supply and demand side of stock investing, but instead on the company's worth in total. Using this philosophy, he has become one of the world's most successful investors.

Reading Stock Tables

Information about particular stocks is contained in **stock tables**. This information includes how much the stock is selling for, and its high and low values form the past year (52 weeks). In a newspaper, the stock table may look like this:

52-Week High Low		Stock	SYM	Div	Yld %	P/E	Vol 100s	High	Low	Close	Net Chg
41.66	18.90	McDonald's	MCD	.72	2.9	12	7588	25.73	23.87	25.42	+0.31
22.60	13.20	Monsanto	MON	.52	2.4	55	15474	21.86	21.48	21.64	-0.29
17.05	8.30	Motorola	MOT	.16	1.7	dd	16149	10.57	8.88	10.43	+0.14
31.75	22.99	Mueller	MLI	-	-	16	1564	29.32	27.03	27.11	-0.02

Table 6.2 excerpt from a stock table, 2008

The symbols and abbreviations are defined here:

52-week High	52-week Low	The highest and lowest price of the stock over the past 52 weeks
Stock	SYM	The name of the company and the symbol used for trading
	Annual DIV	The current annual dividend per share
	Yld %	Percent yield is = $\frac{\text{annual dividend}}{\text{share price}} \times 100$
	P/E	Price to earnings ratio, share price divided by earnings per share over past year (dd indicates loss)
	Vol 100s	The number of shares traded yesterday in 100s
High	Low	The highest and lowest prices at which stocks traded yesterday
	Close	The price at which the stock traded at the close of the market yesterday
	Net Chg	Net change; change in price from market close 2 days ago to yesterday's close

The formulas for yield and price to earnings is a good way to measure how much the stock returns per share. Their values are calculated in the stock table, but deserve attention here.

FORMULA

The price to earnings ratio of a stock, P/E, is $P/E = \frac{\text{Share Price}}{\text{Dividend}}$. The percent yield for a stock, Yld%, is $Yld\% = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\%$.

It should be noted that the price of a stock increases and decreases every moment, and so these value change as the share price changes.

EXAMPLE 6.64

Computing Percent Yield

1. Find the percent yield for a stock with a price of \$30.69 and an annual dividend of \$1.48.
2. Find the percent yield for a stock with a price of \$62.25 and an annual dividend of \$1.76.

Solution

1. Substituting the values for price, \$30.69, and annual dividend, \$1.48, we find the percent yield for the stock to be

$$\text{Yld}\% = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\% = \frac{\$1.48}{\$30.69} \times 100\% = 4.82\%$$
2. Substituting the values for price, \$62.25, and annual dividend, \$1.76, we find the percent yield for the stock to be

$$\text{Yld}\% = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\% = \frac{\$1.76}{\$62.25} \times 100\% = 2.83\%$$

YOUR TURN 6.64

1. Find the percent yield for a stock with a price of \$37.40 and an annual dividend of \$1.60.
2. Find the percent yield for a stock with a price of \$73.22 and an annual dividend of \$2.41.

The stock table information is now, and has been, available online, from websites such as cnn.com/markets, markets.businessinsider.com/stocks, and marketwatch.com. The same information is available from these sites as from the newspaper listings, but are often accessed one stock at a time. [Figure 6.14](#) shows the stock table for Lowe's on September 7, 2022.

Lowe's Cos.

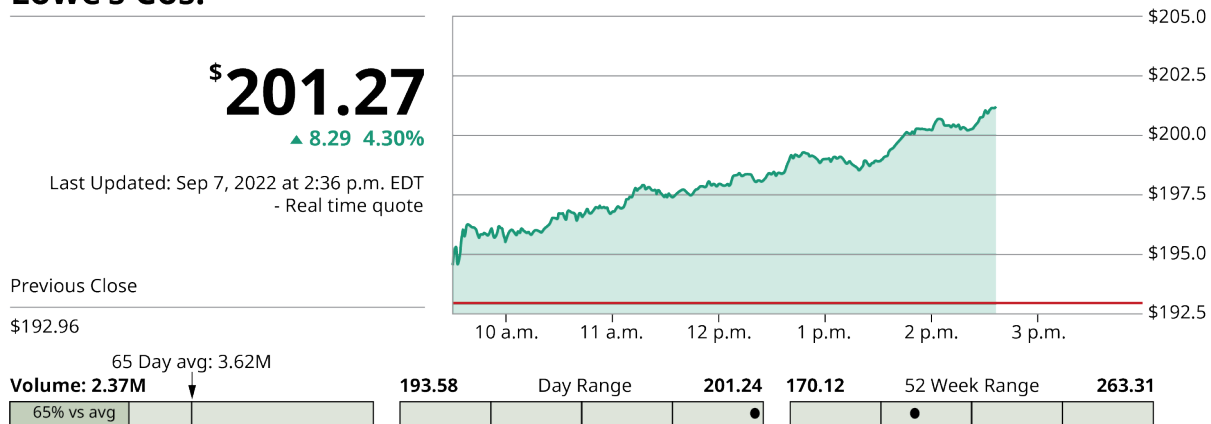


Figure 6.14 Key data for Lowe's stock 9/7/2022 (data source: marketwatch.com)

Other key data is further down on the website, and is shown in [Figure 6.15](#), below.

Key Data

Open \$193.71	Day Range 193.58–201.44
52 Week Range 170.12–263.31	Market Cap \$119.77B
Shares Outstanding 620.7M	Public Float 620.17M
Beta 1.14	Rev. Per Employee \$280.57K
P/E Ratio 15.79	EPS \$12.69
Yield 2.10%	Dividend \$1.05
Ex-Dividend Date Oct 18, 2022	Short Interest 11.09M 08/15/22
% of Float Shorted 1.79%	Average Volume 3.62M

Performance

5 Day	3.74% 
1 Month	-0.01% 
3 Month	4.65% 
YTD	-22.08% 
1 Year	-1.32% 

Figure 6.15 Key data for Lowe's stock 9/7/2022 (data source: marketwatch.com)

Notice that the 52-week high and low are now shown as the 52-week range. However, you get additional information, including the stock performance over the past 5 days, past month, past 3 months, the year to date (YTD), and over the past year. You can also read the number of shares outstanding, the expected date for the dividend (EX-DIVIDEND DATE), and importantly for the P/E ratio, the earning per share (EPS).

EXAMPLE 6.65**Reading an Online Stock Table**

Consider the stock table ([Figure 6.16](#)), and answer the questions based on the table.

McDonald's Corp.

\$258.87

▲4.48 1.76%

Last Updated: Sep 7, 2022 2:49 p.m. EDT
- Real time quotePrevious Close
\$254.39

Key Data

Open \$255.14	Day Range 254.96–259.82	P/R Ratio 31.89	EPS \$8.12
52 Week Range 217.68–271.15	Market Cap \$187.16B	Yield 2.13%	Dividend \$1.38
Shares Outstanding 735.72M	Public Float 734.72M	Ex-Dividend Date Aug 31, 2022	Short Interest 5.53M 08/15/22
Beta 0.76	Rev. Per Employee \$117.97K	% of Float Shorted 0.75%	Average Volume 2.47M

Performance

5 Day	2.61%
1 Month	-0.95%
3 Month	5.40%
YTD	-3.43%
1 Year	8.49%

Figure 6.16 Key data for McDonald's stock 9/7/2022 (data source: marketwatch.com)

1. What is the current price for McDonald's Corp on this date?
2. What is the 52-wk high? 52-wk low?
3. When is the dividend expected?
4. What is its yield?
5. What is the earnings per share?

✓ **Solution**

1. Looking at the table, the current price of a share is \$258.87.
2. The high was \$271.15, and the low was \$217.68.
3. August 31, 2022
4. 2.13%
5. The EPS value is \$8.12.

> **YOUR TURN 6.65**

Consider the stock table below, and answer the questions based on the table.

Intel Corp.

\$30.67

▲ 0.31 1.02%

Last Updated: Sep 7, 2022 2:54 p.m. EDT
- Real time quote

Previous Close

\$30.36

65 Day avg: 39.01M

Volume: 23.89M

61% vs avg

30.05

Day Range

30.68

30.05

52 Week Range

56.28

Key Data

Open

\$30.42

Day Range

30.05–30.68

P/E Ratio

6.57

EPS

\$4.66

52 Week Range

30.05–56.28

Market Cap

\$124.66B

Yield

4.77%

Dividend

\$0.37

Shares Outstanding

4.11B

Public Float

4.1B

Ex-Dividend Date

Aug 4, 2022

Short Interest

63.38M 08/15/22

Beta

1.21

Rev. Per Employee

\$606.06K

% of Float Shorted

1.54%

Average Volume

39.01M

Performance

5 Day

■ -3.92%

YTD

■ -40.45%

1 Month

■ -13.29%

1 Year

■ -42.75%

3 Month

■ -25.61%

Key data for Intel stock 9/7/2022 (data source: marketwatch.com)

1. What is the current price for Intel Corp on this date?
2. What is the 52-wk high? 52-wk low?
3. When is the dividend expected?
4. What is its yield?
5. What is the earnings per share?

As mentioned, stocks earn money in two ways, through dividends and increase in share price.

EXAMPLE 6.66

Dividends Paid

Darma owns 150 shares of stock in the GDW company. This quarter, GDW is paying \$0.87 per share in dividends. How much will Darma earn in dividends this quarter?

✓ Solution

Each share pays \$0.87, so Darma earns $150 \times \$0.87 = \130.50 .

 YOUR TURN 6.66

1. Yulia owns 300 shares of stock in YYZ company. It pays \$1.12 per share this quarter. How much did Yulia earn this quarter on stock in YYZ?

EXAMPLE 6.67**Stock Price Increases**

Vincent buys 100 stocks in the REM company for \$21.87 per share. One year later, he sells those 100 shares for \$29.15 per share.

1. How much money did Vincent make?
2. What was his return on investment for that one year?

 Solution

1. Vincent spent \$21.87 per share to buy the stock. The total he spent on the stock was $\$21.87 \times 100 = \$2,187.00$. When he sold the stock, the price was \$29.15, so he received $\$29.15 \times 100 = \$2,915.00$. He made $\$2,915.00 - \$2,187.00 = \$728.00$.
2. His return on investment was $\frac{\text{Earnings}}{\text{Original Price}} = \frac{\$728}{\$2,187} = 33.29\%$.

 YOUR TURN 6.67

Ginny buys 200 shares of stock in UUK company for \$9.76 per share. At the end of the year, she sells those stocks for \$10.02 per share.

1. How much money did Ginny make?
2. What was her return on investment for that one year?

 VIDEO

[Reading Stock Summary Online \(https://openstax.org/r/Reading_Stock\)](https://openstax.org/r/Reading_Stock)

Mutual Funds

A **mutual fund** is a collection of investments that are all bundled together. When you buy shares of a mutual fund, your money is pooled with the assets of other investors. This pooled money is invested in stocks, bonds, money market instruments, and other assets. Mutual funds are typically operated by professional money managers who allocate the fund's assets and attempt to produce capital gains or income for the fund's investors.

A key benefit of mutual funds is that they allow small or individual investors to invest in professionally managed portfolios of equities, bonds, and other securities. This means each shareholder participates proportionally in the gains or losses of the fund. The performance of a mutual fund is usually stated as how much the mutual fund's total value has increased or decreased. Since there are many different investments inside the mutual fund, the risk is reduced significantly, compared to direct ownership of stocks. Even so, mutual funds historically perform well and can earn more than 10% annually.

The investments that make up a mutual fund are structured and maintained to match stated investment objectives, which are specified in its **prospectus**. A prospectus is a pamphlet or brochure that provides information about the mutual fund. Before buying shares of a mutual fund, consult its prospectus, consider its goals and strategies to see if they match your goals and values and also research any associated fees.

 VIDEO

[Mutual Funds \(https://openstax.org/r/investing_basics_mutual_funds\)](https://openstax.org/r/investing_basics_mutual_funds)

EXAMPLE 6.68**Investing in a Mutual Fund**

Kaitlyn has analyzed her \$12,862.50 quarterly budget using the 50-30-20 budget philosophy, and sees she should be saving or paying down debt with \$2,572.50 per quarter. She decides to invest \$1,300 quarterly a mutual fund that reports an average return of 11.62% over the 18-year life of the mutual fund. Assuming that this interest rate continues, and is compounded quarterly, how much will her mutual fund account be worth after 5 years?

✓ Solution

Kaitlyn's plan is an ordinary annuity, and so the future value of her account can be found using the formula

$FV = pmt \times \frac{(1 + r/n)^n \times t - 1}{r/n}$, with a payment of \$1,300, a rate of 0.1162, number of compounding periods 4, after 5 years. Substituting these values into the formula and calculating, we find

$$\begin{aligned} FV &= pmt \times \frac{(1 + r/n)^n \times t - 1}{r/n} \\ &= \$1,300 \times \frac{(1.02905)^{20} - 1}{0.02905} \\ &= \$1,300 \times \frac{0.773084935178}{0.032905} \\ &= \$1,300 \times 26.6122189784 \\ &= \$34,595.88 \end{aligned}$$

Kaitlyn's mutual fund will be worth \$34,595.88 after 5 years.

> YOUR TURN 6.68

1. Aidan decides to invest \$3,200 annually in a mutual fund. He expects the fund to have a 10.8% interest rate compounded annually. How much will Aidan's mutual fund account have after 15 years?

EXAMPLE 6.69**Investing in a Mutual Fund to Reach a Goal**

Kaitlyn wants to retire with \$1,500,000 in her mutual fund account. She will invest for 35 years. The mutual fund reports an average return of 11.62% over the 18-year-long life of the mutual fund. Assuming that this interest rate continues, and is compounded quarterly, how much will she need to pay annually into her mutual fund to reach her goal?

✓ Solution

Kaitlyn's plan is an ordinary annuity, and so the payment to reach her goal can be found using the formula

$pmt = \frac{FV \times (r/n)}{(1 + r/n)^n \times t - 1}$, with a FV , or goal, of \$1,500,000, a rate of 0.1162, for 35 years. Substituting these values into the formula and calculating, we find

$$\begin{aligned} pmt &= \frac{FV \times (r/n)}{(1 + r/n)^n \times t - 1} \\ &= \frac{\$1,500,000 \times (0.1162/4)}{(1 + 0.1162/4)^{140} - 1} \\ &= \frac{\$174,300}{26.0558103113} \\ &= \$6,689.49 \end{aligned}$$

Kaitlyn needs to invest \$6,689.49 per year (or \$557.46 per month) into the mutual fund to reach \$1,500,000 in 35 years.

> YOUR TURN 6.69

1. How much does Aidan need to invest annually in his mutual fund to reach a goal of \$1,000,000 in 40 years. He expects the fund to have a 10.8% interest rate compounded annually.

Return on Investment

As in [Methods of Savings](#), the formula for return on investment is $ROI = \frac{FV - P}{P}$. As indicated before, this formula does not take into account how long the investment took to reach its current value. It depends only on the initial value, P , and the value at the end of the investment, FV .

EXAMPLE 6.70

Return on Investment for a Bond

Recall [Example 6.62](#), in which Muriel purchased a \$3,000 bond with a maturity of 4 years at a fixed coupon rate of 5.5% paid annually. What was Muriel's return on investment?

Solution

Each year, Muriel received \$165. She received this money four times, so earned a total of \$660. This represents $FV - P$, or just the earnings. Using that we find that the ROI is $ROI = \frac{660}{3000} = 0.22$, or 22%.

YOUR TURN 6.70

1. Maureen invests \$5,000 in a bond with a maturity date in 5 years at a fixed coupon rate of 4.75%. What is Maureen's return on investment?

As mentioned, the ROI does not address the length of time of the investment. A good way to do that is to equate the ROI to an account bearing interest that is compounded annually.

The annual return is the average annual rate, or the annual percentage yield (APY) that would result in the same amount were the interest paid once a year.

FORMULA

The formula for annual return is $\text{annual return} = \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1$, where t = the number of years, FV = new value, and P = starting principal.

We apply this to the previous example.

EXAMPLE 6.71

Annual Return on Investment for a Bond

Recall [Example 6.70](#), in which Muriel purchased a \$3,000 bond with a maturity of 4 years at a fixed coupon rate of 5.5% paid annually. What was Muriel's annual return on investment? Interpret this as compound interest.

Solution

Muriel earned a total of \$660. This represents $FV - P$, or just the earnings. The starting principal was \$3,000. The value at the end of 4 years was $\$3,000 + \$660 = \$3,660$. The time of the investment was 4 years. Using that we find that the annual return is $\text{annual return} = \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1 = \left(\frac{\$3,660}{\$3,000}\right)^{\left(\frac{1}{4}\right)} - 1 = 1.22^{\frac{1}{4}} - 1 = 1.050969 = 0.050969$, or 5.10%. The 5.5% bond earned the equivalent of 5.10% compounded annually.

YOUR TURN 6.71

1. Maureen invests \$5,000 in a bond with a maturity date in 5 years at a fixed coupon rate of 4.75%. What is Maureen's annual return on investment?

In [Example 6.71](#) and Your Turn, the annual return was lower than the interest rate of the investment. This is because the

interest from a bond is simple interest, but annual yield equates to compounded annually.

EXAMPLE 6.72

Return on Investment for Stock in Company ABC

Haniah buys stock in the ABC company, investing a total of \$13,000. After 20 years, the stock is worth \$132,293.49, including reinvestment of dividends.

1. What is Haniah's return on investment?
2. What is Haniah's annual return?

Solution

1. To calculate Haniah's return on investment, substitute \$13,000 for P and \$132,293.49 for FV in the formula $ROI = \frac{FV-P}{P}$ and calculate. Doing so we find Haniah's return on investment to be

$$ROI = \frac{FV-P}{P} = \frac{\$132,293.49 - \$13,000}{\$13,000} = 9.176422, \text{ or } 917.64\%$$

2. To calculate Haniah's annual return, substitute \$13,000 for P and \$132,293.49 for FV in the formula

$$\text{annual return} = \left(\frac{FV}{P} \right)^{\left(\frac{1}{t} \right)} - 1 \text{ and calculate. Doing so we find her annual return to be}$$

$$\text{annual return} = \left(\frac{FV}{P} \right)^{\left(\frac{1}{t} \right)} - 1 = \left(\frac{\$132,293.49}{\$13,000} \right)^{\left(\frac{1}{20} \right)} - 1 = 0.1230, \text{ or } 12.3\%$$

YOUR TURN 6.72

Rixie deposits \$23,000 in the stock of DEF company. After 12 years, her stock is worth \$108,501.30.

1. What is Rixie's return on investment?
2. What is Rixie's annual return?

You should see that the annual return is equal to the annual compounded interest that was assumed for the stocks.

Compute Payment to Reach a Financial Goal

As in [Methods of Savings](#), determining the payment necessary to reach a financial goal uses the payment formula for an ordinary annuity, $pmt = \frac{FV \times (r/n)}{(1 + r/n)^n \times t - 1}$. If dealing with mutual funds or stocks, an assumed annual interest rate, compounded, will be used. This value is often determined through research and informed speculation.

EXAMPLE 6.73

Richard is saving for new siding for his home. He and his partner believe they will need \$37,500 in 10 years to pay for the siding. How much should they invest yearly in a mutual fund they believe will have an annual interest rate of 12%, compounded annually, in order to reach their goal?

Solution

The necessary annual payment is found using the function $pmt = \frac{FV \times (r/n)}{(1 + r/n)^n \times t - 1}$ with $FV = 37,500$, $r = 0.12$, and $n = 1$. Substituting and calculating, we find the annual payment should be

$$\begin{aligned} pmt &= \frac{FV \times (r/n)}{(1 + r/n)^n \times t - 1} \\ &= \frac{\$37,500 \times (0.12/1)}{(1 + 0.12/1)^1 \times 10 - 1} \\ &= \frac{\$4,500}{(1.12)^{10} - 1} \\ &= \frac{\$4,500}{2.10584820834} \\ &= \$2,136.91 \end{aligned}$$

> YOUR TURN 6.73

1. Pete and Erin want to save for their child's college. They think they will need \$90,000 in 14 years. How much should they invest annually in a mutual fund they believe will yield 9.5% compounded annually?

Retirement Savings Plans

We close this section by investigating the three main forms of retirement savings accounts: traditional individual retirement accounts (IRAs), Roth IRAs, and 401(k) accounts. Each has distinct characteristics that are suited to different investors' needs.

Individual Retirement Accounts

A **traditional IRA** lets you contribute up to an amount set by the government, which may change from year to year. For example, the maximum contribution for 2022 is \$6,000; \$7,000 over age 50. Anyone is eligible to contribute to a traditional IRA, regardless of your income level. Your money grows tax-deferred, but withdrawals after age 59½ are taxed at current rates. Traditional IRAs also allow you to use the contribution itself as a deduction on a current year tax return.

Roth IRAs allow contributions at the same levels as traditional IRAs, with a maximum \$6,000 for 2022; \$7,000 over age 50. However, to be eligible to make contributions, your earned income must be below a certain level. A Roth IRA allows after-tax contributions. In other words, the contribution itself is not tax-deductible, as it is with the traditional IRA. However, your money grows tax-free. If you make no withdrawals until you are age 59½, there are no penalties. IRAs pay a modest interest rate.

In either case, IRA deposits have to be from earned income, which in effect means if your earned income is over \$6,000 (\$7,000) then you can deposit the maximum.

EXAMPLE 6.74

Comparing Roth IRAs to Traditional IRAs

Which type of IRA, Roth or traditional, has an income limit for its use?

✓ Solution

Roth IRAs require income to be below a certain limit.

> YOUR TURN 6.74

1. Which type of IRA, Roth or traditional, allow deposits before tax, but have earnings that are taxed after the age of 59 ½?

? WHO KNEW?

In 2022, the maximum that can be added to a Roth IRA was \$6,000 for those under 50 years of age. For those over 50 years of age, the maximum that can be added to a Roth IRA is \$7,000. However, to qualify for a Roth IRA in fall of 2022, a single person's modified adjusted gross income (MAGI) must be below \$129,000. Then, if a single person's income is between \$129,000 and \$144,000, the maximum contribution is reduced from the limit for incomes below \$129,000. For a married couples filing a joint tax return those values are \$204,000 to \$214,000.

401(k) Accounts

Your employer may offer a retirement account to you. These are often in the form of a **401(k)** account. There are traditional and Roth 401(k) accounts, which differ in how they are taxed, much as with other IRAs. In the traditional 401(k) plans, the money is deposited before tax is assessed, which means you do not pay taxes on this money. However, that means when money is withdrawn, it is taxed. These accounts are similar to mutual funds, in that the money is invested in a wide range of assets, spreading the risk.

One of the perks some employers offer is to match some amount of your contributions to the 401(k) plan. For instance,

they may match your deposits up to 5% of your income. This is an instant 100% return on the money that was matched.

 VIDEO

[401\(k\) Accounts \(https://openstax.org/r/401\(k\)s\)](https://openstax.org/r/401(k)s)

EXAMPLE 6.75

Matching 401(k) Deposit

Alice signs up for her employer-based 401(k). The employer matches any 401(k) contribution up to 6% of the employee salary. Alice's annual salary is \$51,600.

1. What is the most money that Alice can deposit that will be fully matched by the company?
2. How much total will be deposited into Alice's account if she deposits the full 6%?
3. How much return does Alice earn if she deposits exactly 6% in her 401(k)?

 Solution

1. The employer will match up to 6% of any employee's salary. 6% of Alice's salary is $0.06 \times \$51,600 = \$3,096$. So Alice can deposit up to \$3,096 and receive that amount in matching funds in her account.
2. Alice's contribution plus the company's contribution is $\$3,096 + \$3,096 = \$6,192$, which is the total that is deposited into Alice's account.
3. She earns a 100% return on the day she deposits her \$3,096.

YOUR TURN 6.75

Jameis signs up for his employer-based 401(k). The employer matches any 401(k) contribution up to 7.5% of the employee salary. Jameis' annual salary is \$72,800.

1. What is the most money that Jamie can deposit that will be fully matched by the company?
2. How much total will be deposited into Jameis' account if he deposits the full 7.5%?
3. How much return does Jameis earn if he deposits exactly 7.5% in her 401(k)?

401(k) plans with matching funds provide great value, as their rates of return are high compared to savings accounts, and are less risky than stocks since such funds invest across many investment vehicles. The next example demonstrates the power of constant deposits into a 401(k) plan that has some employer match.

EXAMPLE 6.76

Constant Deposits into a 401(k) Plan

Dejean begins depositing \$300 per month from his paycheck each month in his employer-based 401(k) account. The employer matches this deposit as it falls below their matching threshold. Dejean expects the return to average 10% per year, compounded annually.

1. How much will Dejean's account be worth if he keeps making those payments for 30 years?
2. What will his account be worth without the matching funds?

 Solution

1. This is a form of an ordinary annuity, so the formula $FV = pmt \times \frac{(1 + r/n)^n \times t - 1}{r/n}$ will be used. The company matches Dejean's full deposit, so each month \$600 will be deposited. He is assuming the money will compound annually, so the amount deposited each year is needed as the value of pmt. For the year, he will deposit $12 \times \$600 = \$7,200$. The rate is 0.1, the number of compounding periods is 1, and the number of years is 30. Substituting and calculating, the value of Dejean's account after 30 years will be

$$\begin{aligned}
 FV &= pmt \times \frac{(1 + r/n)^n \times t - 1}{r/n} \\
 &= \$7,200 \times \frac{(1 + 0.1/1)^1 \times 30 - 1}{0.1/1} \\
 &= \$7,200 \times \frac{(1.1)^{30} - 1}{0.1} \\
 &= \$7,200 \times 164.494022689 \\
 &= \$1,184,356.96
 \end{aligned}$$

2. This is a form of an ordinary annuity, so the formula $FV = pmt \times \frac{(1 + r/n)^n \times t - 1}{r/n}$ will be used but the deposit is now only \$300 per month without the matching funds. For the year, he will deposit $12 \times \$300 = \$3,600$. The rate is 0.1, the number of compounding periods is 1, and the number of years is 30. Substituting and calculating, the value of Dejean's account after 30 years will be

$$\begin{aligned}
 FV &= pmt \times \frac{(1 + r/n)^n \times t - 1}{r/n} \\
 &= \$3,600 \times \frac{(1 + 0.1/1)^1 \times 30 - 1}{0.1/1} \\
 &= \$3,600 \times \frac{(1.1)^{30} - 1}{0.1} \\
 &= \$3,600 \times 164.494022689 \\
 &= \$592,178.48
 \end{aligned}$$

> YOUR TURN 6.76

Crystal begins depositing \$450 per month from her paycheck each month in her employer-based 401(k) account. The employer matches \$350 of each deposit. Crystal expects the return to average 9% per year, compounded annually.

1. How much will Crystal's account be worth if she keeps making those payments for 25 years?
2. What will her account be worth without the matching funds?

Check Your Understanding

39. Which investment has the highest risk?
40. What is different about who can use a Roth IRA versus a traditional IRA??
41. How do mutual funds reduce risk?
42. How much in earnings does a 10-year bond with issue price \$5,000 that pays 4% interest annually is generated?
43. If an individual makes \$115,000 annually, is the person eligible for a Roth IRA?
44. An employer 401(k) matches up to 4% of income by employees. Merisol's annual salary is \$87,500. If Merisol wants to deposit \$4,400 annually in the 401(k), how much will be matched?
45. A person owns 300 shares of stock. It pays \$.38 per share this quarter. How much does the share owner earn from the stock that quarter?
46. David purchased stocks for \$3,500. The stocks paid dividends by reinvesting them in the stock. After 3 years, he sold the socks for \$4,650. What was David's annual return on that investment?
47. How much must be deposited annually in a mutual fund that is expected to bear 12.5% interest compounded annually if the account is to be worth \$750,000 after 25 years?



SECTION 6.7 EXERCISES

1. What is the maturity date for a bond?
2. What is the issue price for a bond?
3. Stock investments increase in value in what two ways?

4. Which is the least risky of stocks, bonds, mutual funds, CDs?
5. Which type of individual retirement account allows pre-tax deposits?
6. What are the limits on contributions to individual retirement accounts in 2022?
7. Of bonds, stocks, mutual funds, CDs, and money market accounts, which do not allow for withdrawal until a certain time period has passed?
8. Which type of IRA allows the account to grow tax free, provided no withdrawals are made until after the age of $59\frac{1}{2}$?
9. Which of bonds, mutual funds, and CDs are professionally managed?
10. Why do mutual funds and IRAs have relatively low risk?

For the bonds with the given properties, find a. the amount paid each year and b. the total amount earned with the bond.

11. Issue price of \$10,000 pays 3.5% annually, matures in 5 years.
12. Issue price of \$3,400 pays 2.75% annually, matures in 10 years.
13. Issue price of \$1,000 pays 2.8% annually, matures in 5 years.
14. Issue price of \$5,000 pays 3.75% annually, matures in 15 years.

In the following exercises, find: a. the return on investment and b. the annual return for the bond described. Round to two decimal places.

15. Issue price of \$10,000 pays 3.5% annually, matures in 5 years.
16. Issue price of \$3,400 pays 2.75% annually, matures in 10 years.
17. Issue price of \$1,000 pays 2.8% annually, matures in 5 years.
18. Issue price of \$5,000 pays 3.75% annually, matures in 15 years.
19. 40 shares of stock are owned. The dividend per share is \$0.38 for a quarter. How much was earned in dividends on this stock this quarter?
20. 150 shares of stock are owned. The dividend per share is \$0.78 for a quarter. How much was earned in dividends on this stock this quarter?
21. 100 shares of stock are owned. The dividend per share is \$0.18 for a quarter. How much was earned in dividends on this stock this quarter?
22. 250 shares of stock are owned. The dividend per share is \$0.41 for a quarter. How much was earned in dividends on this stock this quarter?

For the following exercises, 70 shares of stock were purchased for \$31.50 per share 5 years ago. Over the 5 years, the total of all dividends earned from this stock was \$6.34 per share. The stock is sold for \$34.83.

23. How much was earned with dividends and share price increase combined?
24. What was the return on investment for these stocks?
25. What was the annual return for these stocks?

For the following exercises, 10 shares of stock were purchased for \$18.91 per share 3 years ago. Over the 3 years, the total of all dividends earned from this stock was \$3.18 per share. The stock is sold for \$22.01.

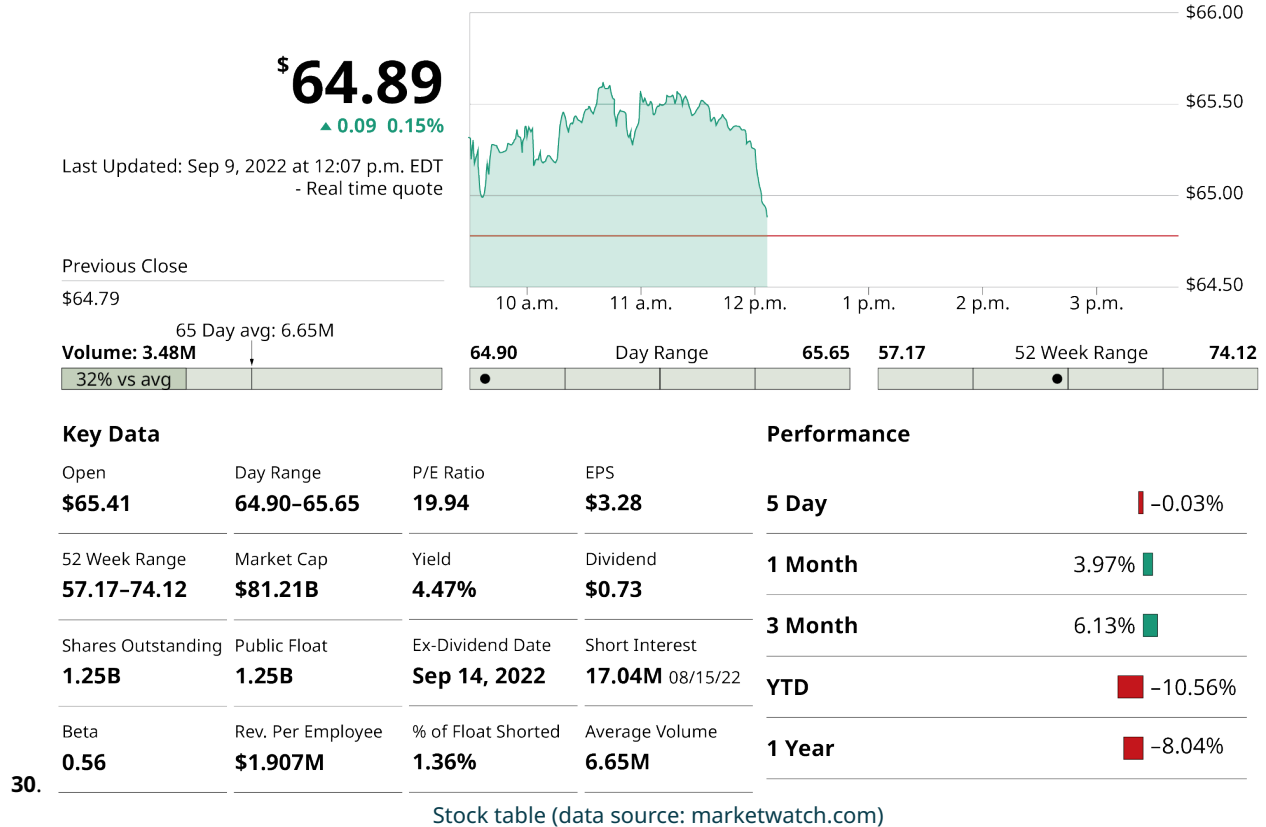
26. How much was earned with dividends and share price increase combined?
27. What was the return on investment for these stocks?
28. What was the annual return for these stocks?

For the following exercises, use the given stock table to answer the following:

- a. What was the 52-week low?
- b. What was the dividend?
- c. What is its year-to-date performance?
- d. What is its yield?



Stock table (data source: marketwatch.com)



Stock table (data source: marketwatch.com)

In the following exercises, find the future value of the mutual fund or IRA with the given annual deposit, the duration of the investment, and the assumed annual, compounded, percentage rate.

31. IRA, annual deposit = \$6,000, 25 years, assumed percentage rate of 11.3%
32. IRA, annual deposit = \$4,800, 15 years, assumed percentage rate of 9.7%

33. Mutual fund, annual deposit = \$7,500, 35 years, assumed percentage rate of 10%

34. Mutual fund, annual deposit = \$12,000, 40 years, assumed percentage rate of 9%

In the following exercises, find the annual deposit necessary, into an IRA or mutual fund, to reach the stated financial goal, given the assumed annual, compounded, interest rate, and the duration of the deposits. Convert that annual deposit to a monthly amount.

35. Mutual fund, goal = \$1,000,000, 25 years, assumed percentage rate of 11.3%

36. Mutual fund, goal = \$100,000, 15 years, assumed percentage rate of 9.7%

37. IRA, goal = \$750,000, 35 years, assumed percentage rate of 10%

38. IRA, goal = \$1,750,000, 40 years, assumed percentage rate of 9%

39. Francis's employer matches 401(k) contributions up to 7% of annual salary. She makes \$98,500. What is the maximum amount that the company will match for Francis?

40. Miles' employer matches 401(k) contributions up to 4% of annual salary. He makes \$38,500. What is the maximum amount that the company will match for Miles?

41. Ila's employer matches 401(k) contributions, up to 4% of salary. She makes \$49,000. She wants to deposit \$3,000 of her own earnings per year (\$250 per month). How much, including her employer's matching funds, will be deposited into Ila's account each year?

42. Georgia works at a company that matches 401(k) contributions up to 5.5% of salary. Georgia wants to deposit \$6,000 of her own earnings per year (\$500 per month) in her 401(k). If she makes \$65,000 annually, how much, including employer contributions, will be deposited in Georgia's account annually?

In the following exercises, Cheryl's company offers a 401(k) account to all employees. The company will match employee contributions up to 6% of the employee's salary. She earns \$81,000 per year. Cheryl decides to deposit, or contribute, \$10,000 annually in the 401(k). She expects a return of 9.5% per year.

43. How much will the company match?

44. What is her total contribution per year?

45. If she deposits into the account for 25 years, how much will her 401(k) be worth?

In the following exercises, Lavanya's company offers a 401(k) account to all employees. Her annual salary is \$58,000. The company will match employee contributions up to 7% of the employee's salary. Lavanya decides to deposit \$3,200 annually in the 401(k). She expects a return of 8.5% per year.

46. How much will the company match?

47. What is her total contribution per year?

48. If she deposits into the account for 20 years, how much will her 401(k) be worth?

In the following exercises, Ruslana wants to use her 401(k) to save \$1,400,000 when she retires in 36 years. She assumes the plan will yield 11% compounded annually. Her company will match contributions up to 5% of annual salary. Her salary is \$48,000.

49. How much total will need to be added to her account each year to reach her goal?

50. What is 5% of Ruslana's salary?

51. The company will match up to 5% of Ruslana's salary, which means half the payment (up to 5% of salary) will be contributed by the company. How much is half the necessary payment?

52. Does the answer to Exercise 51 exceed the result from Exercise 50?

53. If the answer to Exercise 52 is no, then the company contributes half the deposit in the 401(k). How much will the company contribute annually to the 401(k) if this is the case?

54. If the answer to Exercise 52 is yes, then the company contributes only the 5% match of Ruslana's salary. In this case, how much does the company contribute?

55. How much will Ruslana need to contribute per year, after the employer contribution?

56. Divide the answer to Exercise 55 by 12 to find the monthly contribution Ruslana will make.

In the following exercises, Remy wants to use his 401(k) to save \$1,750,000 when he retires in 30 years. He assumes the plan will yield 10.4% compounded annually. His company will match contributions up to 6% of annual salary. His salary is \$78,000.

57. How much total will need to be added to his account each year to reach his goal?

58. What is 6% of Remy's salary?

59. The company will match up to 6% of Remy's salary, which means half the payment (up to 6% of salary) will be contributed by the company. How much is half the necessary payment?

60. Does the answer to Exercise 59 exceed the result from Exercise 58?

61. If the answer to Exercise 60 is no, then the company contributes half the deposit in the 401(k). How much will

the company contribute annually to the 401(k) if this is the case?

62. If the answer to Exercise 60 is yes, then the company contributes only the 6% of Remy's salary. In this case, how much does the company contribute?
63. How much will Remy need to contribute per year, after the employer contribution?
64. Divide the answer to Exercise 63 by 12 to find the monthly contribution Remy will make.

6.8 The Basics of Loans



Figure 6.17 Loans are contracts that allow people to buy now but require them to pay more. (credit: "Closing" by Tim Pierce/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Describe various reasons for loans.
2. Describe the terminology associated with loans.
3. Understand how credit scoring works.
4. Calculate the payment necessary to pay off a loan.
5. Read an amortization table.
6. Determine the cost to finance for a loan.

New car envy is real. Some people look at a new car and feel that they too should have a new car. The search begins. They find the model they want, in the color they want, with the features they want, and then they look at the price. That's often the point where the new car fever breaks and the reality of borrowing money to purchase the car enters the picture. This borrowing takes the form of a loan.

In this section, we look at the basics of loans, including terminology, credit scores, payments, and the cost of borrowing money.

Reasons for Loans

Even if you want a new car because you need one, or if you need a new computer since your current one no longer runs as fast or smoothly as you would like, or you need a new chimney because the one on your house is crumbling, it's likely you do not have that cost in cash. Those are very large purchases. How do you buy that if you don't have the cash? You borrow the money.

And for helping you with your purchase, the company or bank charges you interest.

Loans are taken out to pay for goods or services when a person does not have the cash to pay for the goods or services. We are most familiar with loans for the big purchases in our lives, such as cars, homes, and a college education. Loans are also taken out to pay for repairs, smaller purchases, and home goods like furniture and computers.

Loans can come from a bank, or from the company selling the goods or providing the service. The borrower agrees to

pay back more than the amount borrowed. So there is a cost to borrowing that should be considered when deciding on a purchase bought with credit or a borrowed money.

Even using a credit card is a form of a loan.

Essentially, a loan can be obtained for just about any purchase, large or small, that has a cost beyond a person's cash on hand.

The Terminology of Loans

There are many words and acronyms that get used in relation to loans. A few are below.

APR is the annual percentage rate. It is the annual interest paid on the money that was borrowed. The **principal** is the total amount of the loan, or that has been financed. A **fixed interest rate** loan has an interest rate that does not change during the life of the loan. A **variable interest rate** loan has an interest rate that may change during the life of the loan. The **term** of the loan is how long the borrower has to pay the loan back. An **installment loan** is a loan with a fixed period, and the borrower pays a fixed amount per period until the loan is paid off. The periods are almost uniformly monthly. **Loan amortization** is the process used to calculate how much of each payment will be applied to principal and how much is applied to interest. **Revolving credit**, also known as open-end credit, is how most credit cards work but is also a kind of loan account. (We will learn about credit cards in [Credit Cards](#)) You can use up to some specified value, called the limit, any way you want, and as long as you pay the issuer of the credit according to their terms, you can keep borrowing from this account.

These and other terminologies can be researched further at [Forbes \(https://openstax.org/r/terminologies\)](https://openstax.org/r/terminologies).

? WHO KNEW?

Credit Scores

Not everyone pays the same interest for the same loan. One person might get an APR of 2.9% while another pays 6.9%. These rates are based on your credit score.

Data about you and your credit is collected by three credit bureaus—Experian, Equifax, and TransUnion. They calculate your score using one of two main models: FICO and Vantage Score. The score they develop is based on the following categories:

- **Payment History:** Making your payments on time and not missing payments is by far the most important factor. All three credit types—revolving, installment, and open—contribute to this factor.
- **Credit Utilization or Amount Owed:** How much do you owe on your credit card accounts? This category is concerned with the ratio of how much you owe on revolving credit accounts relative to your available credit, also known as your credit utilization ratio.
This is the only category that depends solely on your revolving credit accounts.
- **Length of Credit History:** This is the average age of your credit history, including the age of the oldest and newest accounts. All three types of credit accounts play a role in this category.
- **Credit Mix:** This number represents the different types of credit accounts you have, such as credit cards, car loan, mortgages, and whether you are successful managing both revolving and installment accounts.
- **New Credit:** Have you recently opened a new account or applied for new credit? Lenders want to know how much new credit you are taking on. So, if you are planning to buy a car and make another large purchase with a credit card, you may want to space these purchases out.

If you have done well in these categories, your credit score will be high, and you will qualify for lower interest rates because you are not perceived as being a risky investment. However, if you do poorly in these categories, your score will be low and you will pay higher interest rates since you present a greater risk.

Check out [this nerdwallet article on credit scores \(https://openstax.org/r/how_to_improve\)](https://openstax.org/r/how_to_improve) to learn more!

▶ VIDEO

[Credit Scores Explained \(https://openstax.org/r/credit_scores_explained\)](https://openstax.org/r/credit_scores_explained)

? WHO KNEW?**Where Do Interest Rates Come From?**

There are many factors that impact your interest rate beyond your credit score. Banks have the authority to set their own rates, so competition between banks impacts interest rates. Bank A doesn't want to charge interest rates that are too high, since borrowers will find banks with better rates. Banks also don't want to charge too little interest.

The too little interest is more involved than the too high. The bank needs to make a profit on its loans. Deposits at the bank are used by the bank to generate loans. The bank has to pay those depositors interest. The bank must charge more for loans they give than they pay to people with deposits in the bank.

Banks also borrow money from each other. These loans have an interest rate, and once more, the bank making a loan must make a profit. More directly, they must charge more for loans they give than they pay for loans they take. In the United States, banks may also borrow from the Federal Reserve, which also charges an interest rate, which is also called the discount rate.

This is where a bank's prime rate comes from. A bank's prime rate is the interest rate it will give to its very best customers, which means most customers will pay more than the bank's prime rate. To confuse the issue, there is also the *Wall Street Journal's* prime rate. It is the average of the prime rates charged by individual banks. The *Wall Street Journal* surveys several banks to generate this value.

Banks also increase the interest rate charged to customers based on both the credit risk presented by the customer, and the risk associated with what the loan will be used for.

Calculating Loan Payments

Loan payments are made up of two components. One component is the interest that accrued during the payment period. The other component is part of the principal. This should remind you of partial payments from [Simple Interest](#).

Over the course of the loan, the amount of principal remaining to be paid decreases. The interest you pay in a month is based on the remaining principal, just as in the partial payments of [Simple Interest](#).

FORMULA

The amount of interest, I , to be paid for one period of a loan with remaining principal P is $I = P \times \frac{r}{n}$, where r is the interest rate in decimal form and n is the number of payments in a year (most often $n = 12$). Since the interest is for the one period, the time is 1 and does not impact the calculation. Note, interest paid to lenders is always rounded up to the next penny.

EXAMPLE 6.77**Interest for a Monthly Payment of a Loan**

Find the interest to be paid for the period on loans with the following remaining principal and given annual interest rate. Each period is a month.

1. Remaining principal is \$13,450, interest rate is 6.75%
2. Remaining principal is \$8,460, interest rate is 5.99%

✓ Solution

1. Substituting \$13,450 for the remaining principal P , 0.0675 for r , and $n = 12$ since the period is a month into the formula, we find that the interest to be paid this period is $I = P \times \frac{r}{n} = \$13,450 \times \frac{0.0675}{12} = \75.66 .
2. Substituting \$8,460 for the remaining principal P , 0.0599 for r , and $n = 12$ since the period is a month into the formula, we find that the interest to be paid this period is $I = P \times \frac{r}{n} = \$8,450 \times \frac{0.0599}{12} = \42.18 .

> YOUR TURN 6.77

Find the interest to be paid for the period on loans with the following remaining principal and given annual interest rate. Each period is a month.

1. Remaining principal is \$56,945, interest rate is 7.5%
2. Remaining principal is \$25,850, interest rate is 2.9%

The payment of the loan has to be such that the principal of the loan is paid off with the last payment. In any period, the amount of interest is defined by the formula above, but changes from period to period since the principal is decreasing with each payment. The trick is knowing how much principal should be paid each payment so that the loan is paid off at the stated time. Fortunately, that is found using the following formula.

FORMULA

The payment, pmt , per period to pay down a loan with beginning principal P is $pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$, where r is the annual interest rate in decimal form, t is the number of years of the loan, and n is the number of payments per year (typically, loans are paid monthly making $n = 12$).

Note, payment to lenders is always rounded up to the next penny.

 Often, the formula takes the form $pmt = \frac{P \times (r) \times (1 + r)^n}{(1 + r)^n - 1}$, where r is the interest rate per period (annual rate divided by the number of periods per year), and n is the total number of payments to be made.

EXAMPLE 6.78

Calculating the Payment for a Loan

In the following, calculate the payment necessary to pay off the loan with the given details. The payments are monthly.

1. A car loan taken out for \$28,500 at an annual interest rate of 3.99% for 5 years.
2. A home loan taken out for \$136,700 and an annual interest rate of 5.75% for 15 years.

Solution

1. The loan is for \$28,500, which is the principal. The rate is 3.99%, so $r = 0.0399$. The term of the loan is 5 years, so $t = 5$. Monthly payments means $n = 12$. Substituting these values for P , r , n , and t into the formula

$pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$ and calculating, we find the payment for the loan.

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1} \\
 &= \frac{\$28,500 \times (0.0399/12) \times (1 + 0.0399/12)^{12 \times 5}}{(1 + 0.0399/12)^{12 \times 5} - 1} \\
 &= \frac{\$28,500 \times (0.003325) \times (1.003325)^{60}}{(1.003325)^{60} - 1} \\
 &= \frac{\$115.6470437}{0.220388273} \\
 &= \$524.75
 \end{aligned}$$

The monthly payment needed is \$524.75.

2. The loan is for \$136,000, which is the principal P . The rate is 5.75% so $r = 0.0575$. The term of the loan is 15 years, so $n = 15$. Monthly payments mean $n = 12$. Substituting these values for P , r , n , and t into the formula

$pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$ and calculating, we find the payment for the loan.

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$136,700 \times (0.0575/12) \times (1 + 0.0575/12)^{12 \times 15}}{(1 + 0.0575/12)^{12 \times 15} - 1} \\
 &= \frac{\$28,500 \times (0.0047917) \times (1.0047917)^{180}}{(1.003325)^{180} - 1} \\
 &= \frac{\$1,548.600986}{1.364201118} \\
 &= \$1,135.18
 \end{aligned}$$

The monthly payment needed is \$1,135.18.

> YOUR TURN 6.78

In the following, calculate the payment necessary to pay off the loan with the given details. The payments are monthly.

1. A home improvement loan taken out for \$17,950 at an annual interest rate of 7.5% for 10 years.
2. A solar panel installation loan taken out for \$33,760 and an annual interest rate of 4.3% for 20 years.

TECH CHECK

Using Google Sheets to Calculate Loan Payments

Google Sheets has a formula to calculate the monthly payment necessary to pay off a loan with a specified interest rate and term. The formula is the PMT formula. It uses the principal P , interest rate r , and t years. Follow these steps

Open a Google worksheet and click on any cell.

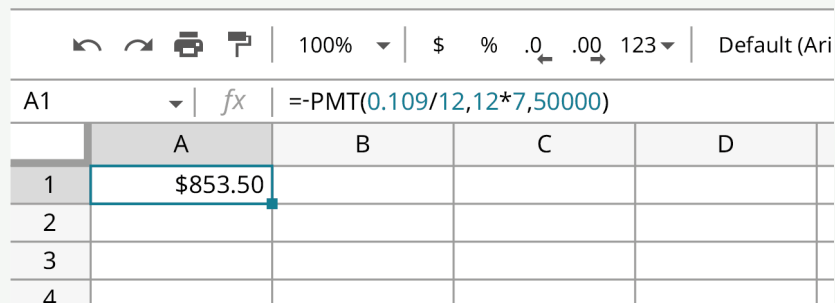
Type `=PMT(r/12,12*t,P)`.

Hit the enter key.

The cell displays the payment.

Please note the negative sign. Since Google Sheets is a spreadsheet program, it sees the payment as funds leaving the account, and so they are, by default, negative. The negative sign in front of the formula makes the result positive.

For example, for a \$50,000 loan at 10.9% interest for 7 years, you would type `=PMT(0.109/12,12*7,50000)`. The formula and the result are shown in [Figure 6.18](#).



The screenshot shows a Google Sheets interface. At the top, there is a toolbar with icons for undo, redo, print, and a zoom dropdown set to 100%. Next to it are currency and decimal formatting options (\$, %, .0, .00, 123) and a 'Default (Ari)' dropdown. Below the toolbar, the formula bar shows the formula `=PMT(0.109/12,12*7,50000)` entered into cell A1. The spreadsheet grid below has columns labeled A, B, C, and D, and rows numbered 1, 2, 3, and 4. Cell A1 contains the result '\$853.50'.

	A	B	C	D
1	\$853.50			
2				
3				
4				

Figure 6.18 Google Sheets formula

Alternatively, you can also use an online calculator to find monthly payments for a loan, such as the one at [Caluculator.net \(https://openstax.org/r/payment_calculator\)](https://openstax.org/r/payment_calculator)

Reading Amortization Tables

An **amortization table** or amortization schedule is a table that provides the details of the periodic payments for a loan where the payments are applied to both the principal and the interest. The principal of the loan is paid down over the life of the loan. Typically, the payments each period are equal. Importantly, one of the columns will show how much of each payment is used for interest, another column shows how much is applied to the outstanding principal, and another column shows the remaining principal or balance [Figure 6.19](#).

Amortization Schedule Calculator					
Loan Amount	\$5,000.00				
Interest Rate (annual)	8.00%				
Term (Years)	5				
Monthly Payment	\$101.38				
Month	Payment	Principal	Interest	Total Interest	Balance
					\$5,000.00
1	\$101.38	\$68.05	\$33.33	\$33.33	\$4,931.95
2	\$101.38	\$68.50	\$32.88	\$66.21	\$4,863.45
3	\$101.38	\$68.96	\$32.42	\$98.64	\$4,794.49
4	\$101.38	\$69.42	\$31.96	\$130.60	\$4,725.07
5	\$101.38	\$69.88	\$31.50	\$162.10	\$4,655.19
6	\$101.38	\$70.35	\$31.03	\$193.13	\$4,584.84
7	\$101.38	\$70.82	\$30.57	\$223.70	\$4,514.03
8	\$101.38	\$71.29	\$30.09	\$253.79	\$4,442.74
9	\$101.38	\$71.76	\$29.62	\$283.41	\$4,370.97
10	\$101.38	\$72.24	\$29.14	\$312.55	\$4,298.73
11	\$101.38	\$72.72	\$28.66	\$341.21	\$4,226.01
12	\$101.38	\$73.21	\$28.17	\$369.38	\$4,152.80
13	\$101.38	\$73.70	\$27.69	\$397.07	\$4,079.10
14	\$101.38	\$74.19	\$27.19	\$424.26	\$4,004.91
15	\$101.38	\$74.68	\$26.70	\$450.96	\$3,930.23
16	\$101.38	\$75.18	\$26.20	\$477.16	\$3,855.05
17	\$101.38	\$75.68	\$25.70	\$502.86	\$3,779.37
18	\$101.38	\$76.19	\$25.20	\$528.06	\$3,703.18
19	\$101.38	\$76.69	\$24.69	\$552.75	\$3,626.49
20	\$101.38	\$77.21	\$24.18	\$576.92	\$3,549.28
21	\$101.38	\$77.72	\$23.66	\$600.59	\$3,471.56
22	\$101.38	\$78.24	\$23.14	\$623.73	\$3,393.33
23	\$101.38	\$78.76	\$22.62	\$646.35	\$3,314.57
24	\$101.38	\$79.28	\$22.10	\$668.45	\$3,235.28
25	\$101.38	\$79.81	\$21.57	\$690.02	\$3,155.47
26	\$101.38	\$80.35	\$21.04	\$711.05	\$3,075.12
27	\$101.38	\$80.88	\$20.50	\$731.55	\$2,994.24

Figure 6.19 Amortization table

EXAMPLE 6.79

Reading from an Amortization Table

Using the partial amortization table ([Figure 6.23](#)), answer the following questions.

Amortization Schedule Calculator					
Loan Amount		\$10,000.00			
Interest Rate (annual)		4.75%			
Term (Years)		20			
Monthly Payment		\$64.62			
Month	Payment	Principal	Interest	Total Interest	Balance
					\$10,000.00
1	\$64.62	\$25.04	\$39.58	\$39.58	\$9,974.96
2	\$64.62	\$25.14	\$39.48	\$79.07	\$9,949.82
3	\$64.62	\$25.24	\$39.38	\$118.45	\$9,924.59
4	\$64.62	\$25.34	\$39.28	\$157.74	\$9,899.25
5	\$64.62	\$25.44	\$39.18	\$196.92	\$9,873.81
6	\$64.62	\$25.54	\$39.08	\$236.01	\$9,848.27
7	\$64.62	\$25.64	\$38.98	\$274.99	\$9,822.63
8	\$64.62	\$25.74	\$38.88	\$313.87	\$9,796.89
9	\$64.62	\$25.84	\$38.78	\$352.65	\$9,771.05
10	\$64.62	\$25.95	\$38.68	\$391.33	\$9,745.10
11	\$64.62	\$26.05	\$38.57	\$429.90	\$9,719.05
12	\$64.62	\$26.15	\$38.47	\$468.37	\$9,692.90
13	\$64.62	\$26.25	\$38.37	\$506.74	\$9,666.65
14	\$64.62	\$26.36	\$38.26	\$545.00	\$9,640.29
15	\$64.62	\$26.46	\$38.16	\$583.16	\$9,613.83
16	\$64.62	\$26.57	\$38.05	\$621.22	\$9,587.26
17	\$64.62	\$26.67	\$37.95	\$659.17	\$9,560.59
18	\$64.62	\$26.78	\$37.84	\$697.01	\$9,533.81
19	\$64.62	\$26.88	\$37.74	\$734.75	\$9,506.92
20	\$64.62	\$26.99	\$37.63	\$772.38	\$9,479.93
21	\$64.62	\$27.10	\$37.52	\$809.91	\$9,452.84
22	\$64.62	\$27.20	\$37.42	\$847.32	\$9,425.63
23	\$64.62	\$27.31	\$37.31	\$884.63	\$9,398.32
24	\$64.62	\$27.42	\$37.20	\$921.83	\$9,370.90
25	\$64.62	\$27.53	\$37.09	\$958.93	\$9,343.37
26	\$64.62	\$27.64	\$36.98	\$995.91	\$9,315.73
27	\$64.62	\$27.75	\$36.87	\$1,032.79	\$9,287.98

Figure 6.20 Amortization table

1. What is the loan amount (principal), the interest rate, and the term of the loan?
2. How much is the monthly payment?
3. How much remaining balance is there after the payment in month 15?
4. How much was the interest in payment 10?
5. What is the total of the interest paid after payment 18?
6. What happens to the amount paid in interest each month?

 Solution

1. Reading the values at the top of the table, we see the principal is \$10,000, the interest rate is 4.75%, and has a term of 20 years.
2. The monthly payment is listed below the term of the loan, and is \$64.62.

3. \$9,613.83
4. \$38.68
5. \$697.01
6. The amount paid to interest decreases each month.

> YOUR TURN 6.79

Using the partial amortization table below, answer the following questions.

Amortization Schedule Calculator					
Loan Amount	\$34,700.00				
Interest Rate (annual)	3.79%				
Term (Years)	6				
Monthly Payment	\$539.57				
Month	Payment	Principal	Interest	Total Interest	Balance
					\$34,700.00
1	\$539.57	\$429.98	\$109.59	\$109.59	\$34,270.02
2	\$539.57	\$431.34	\$108.24	\$217.83	\$33,838.68
3	\$539.57	\$432.70	\$106.87	\$324.70	\$33,405.98
4	\$539.57	\$434.07	\$105.51	\$430.21	\$32,971.92
5	\$539.57	\$435.44	\$104.14	\$534.35	\$32,536.48
6	\$539.57	\$436.81	\$102.76	\$637.11	\$32,099.66
7	\$539.57	\$438.19	\$101.38	\$738.49	\$31,661.47
8	\$539.57	\$439.58	\$100.00	\$838.49	\$31,221.90
9	\$539.57	\$440.96	\$98.61	\$937.10	\$30,780.93
10	\$539.57	\$442.36	\$97.22	\$1,034.31	\$30,338.57
11	\$539.57	\$443.75	\$95.82	\$1,130.13	\$29,894.82
12	\$539.57	\$445.16	\$94.42	\$1,224.55	\$29,449.66
13	\$539.57	\$446.56	\$93.01	\$1,317.56	\$29,003.10
14	\$539.57	\$447.97	\$91.60	\$1,409.16	\$28,555.13
15	\$539.57	\$449.39	\$90.19	\$1,499.35	\$28,105.74
16	\$539.57	\$450.81	\$88.77	\$1,588.12	\$27,654.93
17	\$539.57	\$452.23	\$87.34	\$1,675.46	\$27,202.70
18	\$539.57	\$453.66	\$85.92	\$1,761.38	\$26,749.05
19	\$539.57	\$455.09	\$84.48	\$1,845.86	\$26,293.95
20	\$539.57	\$456.53	\$83.05	\$1,928.90	\$25,837.42
21	\$539.57	\$457.97	\$81.60	\$2,010.51	\$25,379.45
22	\$539.57	\$459.42	\$80.16	\$2,090.66	\$24,920.04
23	\$539.57	\$460.87	\$78.71	\$2,169.37	\$24,459.17
24	\$539.57	\$462.32	\$77.25	\$2,246.62	\$23,996.84
25	\$539.57	\$463.78	\$75.79	\$2,322.41	\$23,533.06
26	\$539.57	\$465.25	\$74.33	\$2,396.74	\$23,067.81
27	\$539.57	\$466.72	\$72.86	\$2,469.59	\$22,601.09

Amortization table

1. What is the loan amount (principal), the interest rate, and the term of the loan?
2. How much is the monthly payment?

3. How much remaining balance is there after the payment in month 20?
4. How much was the interest in payment 5?
5. What is the total of the interest paid after payment 24?
6. What happens to the amount paid to the principal each month?

 VIDEO

[Reading an Amortization Table \(https://openstax.org/r/Amortization_Table\)](https://openstax.org/r/Amortization_Table)

Cost of Finance

There are often costs associated with a loan beyond the interest being paid. The **cost of finance** of a loan is the sum of all costs, fees, interest, and other charges paid over the life of the loan.

EXAMPLE 6.80

Cost of Financing a Personal Loan

Irena signed for a loan of \$15,000 at 6.33% for 5 years. When she took out the loan, Irena paid a \$750 origination fee. Over the course of the loan, she pays \$2,537.96 in interest. What was her cost to finance the loan?

Solution

The cost of finance is the sum of all interest and any fees paid for the loan. The fees paid were \$750.00 and the interest was \$2,537.96. Her cost of finance for this loan was \$3,287.96.

YOUR TURN 6.80

1. Samantha takes out a \$28,000 loan for 6 years at 5.5% interest. When she took out the loan, she paid \$4,937.18 in interest, an origination fee of \$1,400 and an application fee of \$100. What was her cost of finance for this loan?

Check Your Understanding

48. Give three reasons for taking out a loan.
49. What is an installment loan?
50. In an installment loan what happens to the amount of the payment that is applied to interest in later payments compared to earlier payments?
51. An installment loan is paid monthly. The remaining principal is \$5,498. If the interest rate is 12.9%, how much interest is paid that month?
52. A business loan is taken out for \$250,000 for 15 years at 6.9% interest. What are the payments for the loan?
53. Consider the amortization table below. What is the balance after the eleventh payment?
54. The total interest paid on a loan is \$38,519.50. The origination fee was \$300 while the processing fee was \$250. An additional fee was charged to the borrowers, amounting to another \$475. How much was the cost of financing for this loan?

Amortization Schedule Calculator					
Loan Amount	\$45,000.00				
Interest Rate (annual)	13.90%				
Term (Years)	10				
Monthly Payment	\$696.00				
Month	Payment	Principal	Interest	Total Interest	Balance
					\$45,000.00
1	\$696.00	\$174.75	\$521.25	\$521.25	\$44,825.25
2	\$696.00	\$176.77	\$519.23	\$1,040.48	\$44,648.48
3	\$696.00	\$178.82	\$517.18	\$1,557.65	\$44,469.67
4	\$696.00	\$180.89	\$515.11	\$2,072.76	\$44,288.78
5	\$696.00	\$182.98	\$513.01	\$2,585.77	\$44,105.79
6	\$696.00	\$185.10	\$510.89	\$3,096.66	\$43,920.69
7	\$696.00	\$187.25	\$508.75	\$3,605.41	\$43,733.44
8	\$696.00	\$189.42	\$506.58	\$4,111.99	\$43,544.03
9	\$696.00	\$191.61	\$504.38	\$4,616.38	\$43,352.41
10	\$696.00	\$193.83	\$502.17	\$5,118.54	\$43,158.58
11	\$696.00	\$196.08	\$499.92	\$5,618.46	\$42,962.51
12	\$696.00	\$198.35	\$497.65	\$6,116.11	\$42,764.16
13	\$696.00	\$200.64	\$495.35	\$6,611.46	\$42,563.52
14	\$696.00	\$202.97	\$493.03	\$7,104.49	\$42,360.55
15	\$696.00	\$205.32	\$490.68	\$7,595.17	\$42,155.23
16	\$696.00	\$207.70	\$488.30	\$8,083.47	\$41,947.53
17	\$696.00	\$210.10	\$485.89	\$8,569.36	\$41,737.43
18	\$696.00	\$212.54	\$483.46	\$9,052.82	\$41,524.89
19	\$696.00	\$215.00	\$481.00	\$9,533.81	\$41,309.89
20	\$696.00	\$217.49	\$478.51	\$10,012.32	\$41,092.40
21	\$696.00	\$220.01	\$475.99	\$10,488.31	\$40,872.39
22	\$696.00	\$222.56	\$473.44	\$10,961.74	\$40,649.84
23	\$696.00	\$225.14	\$470.86	\$11,432.60	\$40,424.70
24	\$696.00	\$227.74	\$468.25	\$11,900.86	\$40,196.96
25	\$696.00	\$230.38	\$465.61	\$12,366.47	\$39,966.58
26	\$696.00	\$233.05	\$462.95	\$12,829.42	\$39,733.53
27	\$696.00	\$235.75	\$460.25	\$13,289.67	\$39,497.78



SECTION 6.8 EXERCISES

- How do fixed interest rates and variable interest rates differ?
- What is revolving credit?
- What is loan amortization?
- What are the two components of a loan payment?
- Name three details that are presented on an amortization schedule.

For the following exercises, calculate the interest due for the monthly installment payment given the remaining principal and interest rate.

- Remaining principal = \$21,872.99, interest rate = 13.9%

7. Remaining principal = \$2,845.43, interest rate = 4.99%
8. Remaining principal = \$78,913.76, interest rate = 2.9%
9. Remaining principal = \$6,445.22, interest rate = 5.65%

In the following exercises, calculate the monthly payment for the loan.

10. Principal = \$8,600, annual interest rate = 6.75%, term is 5 years
11. Principal = \$19,400, annual interest rate = 2.25%, term is 6 years
12. Principal = \$11,870, annual interest rate = 3.59%, term is 3 years
13. Principal = \$41,900, annual interest rate = 8.99%, term is 15 years
14. Principal = \$26,150, annual interest rate = 11.1%, term is 7 years
15. Principal = \$46,350, annual interest rate = 2.9%, term is 6 years
16. Principal = \$175,800, annual interest rate = 4.73%, term is 25 years
17. Principal = \$225,000, annual interest rate = 5.06%, term is 30 years

In the following exercises, use the amortization table below to answer the questions.

Amortization Schedule Calculator					
Loan Amount		\$19,250.00			
Interest Rate (annual)		4.75%			
Term (Years)		5			
Monthly Payment		\$361.07			
Month	Payment	Principal	Interest	Total Interest	Balance
					\$19,250.00
1	\$361.07	\$284.87	\$76.20	\$76.20	\$18,965.13
2	\$361.07	\$286.00	\$75.07	\$151.27	\$18,679.13
3	\$361.07	\$287.13	\$73.94	\$225.21	\$18,391.99
4	\$361.07	\$288.27	\$72.80	\$298.01	\$18,103.73
5	\$361.07	\$289.41	\$71.66	\$369.67	\$17,814.32
6	\$361.07	\$290.56	\$70.52	\$440.18	\$17,523.76
7	\$361.07	\$291.71	\$69.36	\$509.55	\$17,232.05
8	\$361.07	\$292.86	\$68.21	\$577.76	\$16,939.19
9	\$361.07	\$294.02	\$67.05	\$644.81	\$16,645.17
10	\$361.07	\$295.18	\$65.89	\$710.70	\$16,349.99
11	\$361.07	\$296.35	\$64.72	\$775.42	\$16,053.64
12	\$361.07	\$297.52	\$63.55	\$838.96	\$15,756.11
13	\$361.07	\$298.70	\$62.37	\$901.33	\$15,457.41
14	\$361.07	\$299.88	\$61.19	\$962.51	\$15,157.53
15	\$361.07	\$301.07	\$60.00	\$1,022.51	\$14,856.46
16	\$361.07	\$302.26	\$58.81	\$1,081.32	\$14,554.19
17	\$361.07	\$303.46	\$57.61	\$1,138.93	\$14,250.73
18	\$361.07	\$304.66	\$56.41	\$1,195.34	\$13,946.07
19	\$361.07	\$305.87	\$55.20	\$1,250.54	\$13,640.20
20	\$361.07	\$307.08	\$53.99	\$1,304.54	\$13,333.12
21	\$361.07	\$308.29	\$52.78	\$1,357.31	\$13,024.83
22	\$361.07	\$309.51	\$51.56	\$1,408.87	\$12,715.32
23	\$361.07	\$310.74	\$50.33	\$1,459.20	\$12,404.58
24	\$361.07	\$311.97	\$49.10	\$1,508.30	\$12,092.61
25	\$361.07	\$313.20	\$47.87	\$1,556.17	\$11,779.40
26	\$361.07	\$314.44	\$46.63	\$1,602.80	\$11,464.96
27	\$361.07	\$315.69	\$45.38	\$1,648.18	\$11,149.27
28	\$361.07	\$316.94	\$44.13	\$1,692.31	\$10,832.33

18. What was the loan amount, or starting principal?
19. What was the interest rate?
20. What is the term of the loan?
21. What are the monthly payments?
22. What is the remaining principal after payment 17?
23. How much of payment 27 was for interest?
24. How much of payment 19 was for principal?
25. How much total interest was paid after payment 22?

In the following exercises, use the amortization table for payments 131–152 of a mortgage below to answer the questions.

Amortization Schedule Calculator					
Loan Amount		\$145,750.00			
Interest Rate (annual)		5.41%			
Term (Years)		20			
Monthly Payment		\$995.20			
Month	Payment	Principal	Interest	Total Interest	Balance
					\$145,750.00
1	\$995.20	\$338.11	\$657.09	\$657.09	\$145,411.89
131	\$995.20	\$606.77	\$388.44	\$70,174.09	\$85,552.69
132	\$995.20	\$609.50	\$385.70	\$70,559.79	\$84,943.19
133	\$995.20	\$612.25	\$382.95	\$70,942.74	\$84,330.94
134	\$995.20	\$615.01	\$380.19	\$71,322.93	\$83,715.93
135	\$995.20	\$617.78	\$377.42	\$71,700.35	\$83,098.15
136	\$995.20	\$620.57	\$374.63	\$72,074.99	\$82,477.59
137	\$995.20	\$623.37	\$371.84	\$72,446.82	\$81,854.22
138	\$995.20	\$626.18	\$369.03	\$72,815.85	\$81,228.04
139	\$995.20	\$629.00	\$366.20	\$73,182.05	\$80,599.05
140	\$995.20	\$631.83	\$363.37	\$73,545.42	\$79,967.21
141	\$995.20	\$634.68	\$360.52	\$73,905.94	\$79,332.53
142	\$995.20	\$637.54	\$357.66	\$74,263.60	\$78,694.99
143	\$995.20	\$640.42	\$354.78	\$74,618.38	\$78,054.57
144	\$995.20	\$643.31	\$351.90	\$74,970.28	\$77,411.26
145	\$995.20	\$646.21	\$349.00	\$75,319.27	\$76,765.06
146	\$995.20	\$649.12	\$346.08	\$75,665.35	\$76,115.94
147	\$995.20	\$652.05	\$343.16	\$76,008.51	\$75,463.89
148	\$995.20	\$654.99	\$340.22	\$76,348.73	\$74,808.91
149	\$995.20	\$657.94	\$337.26	\$76,685.99	\$74,150.97
150	\$995.20	\$660.90	\$334.30	\$77,020.29	\$73,490.06
151	\$995.20	\$663.88	\$331.32	\$77,351.61	\$72,826.18
152	\$995.20	\$666.88	\$328.32	\$77,679.93	\$72,159.30

26. What was the loan amount, or starting principal?
27. What was the interest rate?
28. What is the term of the loan?
29. What are the monthly payments?
30. What is the remaining principal after payment 140?
31. How much of the principal has been paid off after payment 151?
32. How much of payment 138 was for principal?
33. How much total interest was paid after payment 135?

In the following exercises find the cost to finance for the loan.

34. Total interest paid is \$94,598.36, origination fee was \$450, processing fee was \$300, commission fee was \$1,457.50.
35. Total interest paid was \$3,209.34, origination fee was \$100, processing fee was \$200.
36. Total interest paid was \$8,295.50, fees were \$875.
37. Total interest paid was \$56,114.90, origination fee was \$1,230, and filing fee was \$250.
38. Kylie takes out a \$16,780 loan from her credit union for a new car. The loan's term is 4 years with an interest rate of 6.77%. What are Kylie's monthly payments?
39. Crissy and Jonesy take out a \$13,200 loan for repairs to their roof. The loan is for 10 years at 7.15% interest. What are their payments for this loan?

Below are two amortization tables. They are for loans with the same principal and interest rate, but different terms, the first a 20-year term and the second a 30-year term. The tables show the last payments of each loan. Use those two tables for the following exercises.

Amortization Schedule Calculator					
Loan Amount		\$184,900.00			
Interest Rate (annual)		4.78%			
Term (Years)		20			
Monthly Payment		\$1,197.90			
Month	Payment	Principal	Interest	Total Interest	Balance
					\$184,900.00
1	\$1,197.90	\$461.38	\$736.52	\$736.52	\$184,438.62
219	\$1,197.90	\$1,097.58	\$100.32	\$101,526.39	\$24,086.51
220	\$1,197.90	\$1,101.95	\$95.94	\$101,622.33	\$22,984.55
221	\$1,197.90	\$1,106.34	\$91.56	\$101,713.89	\$21,878.21
222	\$1,197.90	\$1,110.75	\$87.15	\$101,801.04	\$20,767.46
223	\$1,197.90	\$1,115.18	\$82.72	\$101,883.76	\$19,652.28
224	\$1,197.90	\$1,119.62	\$78.28	\$101,962.04	\$18,532.66
225	\$1,197.90	\$1,124.08	\$73.82	\$102,035.86	\$17,408.59
226	\$1,197.90	\$1,128.55	\$69.34	\$102,105.21	\$16,280.03
227	\$1,197.90	\$1,133.05	\$64.85	\$102,170.06	\$15,146.98
228	\$1,197.90	\$1,137.56	\$60.34	\$102,230.39	\$14,009.42
229	\$1,197.90	\$1,142.09	\$55.80	\$102,286.20	\$12,867.32
230	\$1,197.90	\$1,146.64	\$51.25	\$102,337.45	\$11,720.68
231	\$1,197.90	\$1,151.21	\$46.69	\$102,384.14	\$10,569.47
232	\$1,197.90	\$1,155.80	\$42.10	\$102,426.24	\$9,413.67
233	\$1,197.90	\$1,160.40	\$37.50	\$102,463.74	\$8,253.27
234	\$1,197.90	\$1,165.02	\$32.88	\$102,496.61	\$7,088.24
235	\$1,197.90	\$1,169.66	\$28.23	\$102,524.85	\$5,918.58
236	\$1,197.90	\$1,174.32	\$23.58	\$102,548.42	\$4,744.26
237	\$1,197.90	\$1,179.00	\$18.90	\$102,567.32	\$3,565.26
238	\$1,197.90	\$1,183.70	\$14.20	\$102,581.52	\$2,381.56
239	\$1,197.90	\$1,188.41	\$9.49	\$102,591.01	\$1,193.15
240	\$1,197.90	\$1,193.15	\$4.75	\$102,595.76	\$0.00

Amortization Schedule Calculator					
Loan Amount	\$184,900.00				
Interest Rate (annual)	4.78%				
Term (Years)	30				
Monthly Payment	\$967.87				
Month	Payment	Principal	Interest	Total Interest	Balance
					\$184,900.00
1	\$967.87	\$231.35	\$736.52	\$736.52	\$184,668.65
339	\$967.87	\$886.82	\$81.05	\$162,669.98	\$19,461.29
340	\$967.87	\$890.35	\$77.52	\$162,747.50	\$18,570.94
341	\$967.87	\$893.90	\$73.97	\$162,821.47	\$17,677.04
342	\$967.87	\$897.46	\$70.41	\$162,891.89	\$16,779.58
343	\$967.87	\$901.03	\$66.84	\$162,958.72	\$15,878.55
344	\$967.87	\$904.62	\$63.25	\$163,021.97	\$14,973.93
345	\$967.87	\$908.23	\$59.65	\$163,081.62	\$14,065.70
346	\$967.87	\$911.84	\$56.03	\$163,137.65	\$13,153.86
347	\$967.87	\$915.48	\$52.40	\$163,190.04	\$12,238.38
348	\$967.87	\$919.12	\$48.75	\$163,238.79	\$11,319.26
349	\$967.87	\$922.78	\$45.09	\$163,283.88	\$10,396.47
350	\$967.87	\$926.46	\$41.41	\$163,325.30	\$9,470.01
351	\$967.87	\$930.15	\$37.72	\$163,363.02	\$8,539.86
352	\$967.87	\$933.86	\$34.02	\$163,397.03	\$7,606.01
353	\$967.87	\$937.57	\$30.30	\$163,427.33	\$6,668.43
354	\$967.87	\$941.31	\$26.56	\$163,453.89	\$5,727.12
355	\$967.87	\$945.06	\$22.81	\$163,476.71	\$4,782.06
356	\$967.87	\$948.82	\$19.05	\$163,495.76	\$3,833.24
357	\$967.87	\$952.60	\$15.27	\$163,511.02	\$2,880.64
358	\$967.87	\$956.40	\$11.47	\$163,522.50	\$1,924.24
359	\$967.87	\$960.21	\$7.66	\$163,530.16	\$964.03
360	\$967.87	\$964.03	\$3.84	\$163,534.00	\$0.00

40. How much total interest was paid for the 20-year loan? The 30-year loan? Calculate the difference.
41. What was the payment for the 20-year loan? The 30-year loan? Calculate the difference.
42. Compare the two loans. Why would the 30-year be preferable over the 20-year loan? Why would the 20-year loan be preferable to the 30-year loan?
- In the following exercises, compare payments per month and interest rates.
43. A loan for \$20,000 is taken out for 5 years. Find the payment if the interest rate is:
- 2.9%
 - 3.9%
 - 4.9%
 - 5.9%
 - Did the payments increase by the same amount for each 1% jump in interest rate? Describe the pattern.
44. A loan for \$50,000 is taken out for 7 years. Find the payment if the interest rate is:
- 3.5%
 - 4.5%
 - 5.5%
 - 6.5%
 - Did the payments increase by the same amount for each 1% jump in interest rate?

6.9 Understanding Student Loans



Figure 6.21 College is often paid for through loans. (credit: "Sidewalk Scene in front of Science Building" by IMCBerea College Wikimedia Commons, CC- BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Describe how to obtain a student loan.
2. Distinguish between federal and private student loans and state distinctions.
3. Understand the limits on student loans.
4. Summarize the standard prepayment plan.
5. Understand student loan consolidation.
6. Summarize and describe benefits or drawbacks of other repayment plans.
7. Summarize possible courses of action if a student loan defaults.

Obtaining a Student Loan

All college students are eligible to apply for a loan regardless of their financial situation or credit rating. Federal student loans do not require a co-signer or a credit check. Most students do not have a credit history when they begin college, and the federal government is aware of this. However, private loans will generally require a co-signer as well as a credit check. The co-signer will assume responsibility for paying off the loan if the student cannot make the payments.

The first step in applying for student loans is to fill out the **FAFSA** (Free Application for Student Aid). FAFSA determines financial need and what type of loan the student is qualified to obtain. For students who are still dependents on their parent's taxes, the parents also fill out the FAFSA, as their wealth and income impacts what the dependent student is eligible for. Students who cannot demonstrate financial need will also be helped by applying with FAFSA, as it will help guide them to the type of loan most appropriate. The FAFSA must be submitted each year.

 *The FAFSA deadline is the spring of the student's next academic year. The deadline is often in March. Do not allow this deadline to pass.*

As soon as an offer letter from the college is received, the student should start the application process. The college will determine the loan amount needed. Also, there are limits on the amount a student can borrow. There are both yearly limits and aggregate limits. See the table later in this section that outlines the loan limits per school year and in the aggregate.

If the student receives a direct subsidized loan, there is a limit on the eligibility period. The time limit on eligibility depends on the college program into which the student enrolls. The school publishes how long a program is expected to take. The eligibility period is 150% of that published time. For example, if a student is enrolled in a 4-year program, such as a bachelor's degree program, their eligibility period is 6 years, as $1.50(4) = 6$. Therefore, the student may receive direct

subsidized loans for a period of 6 years.

Types and Features of Student Loans

Once a tuition statement is received, and all the non-loan awards are analyzed that are applicable to the costs of college (such as scholarships and grants), there still may be quite of bit of an expense to attend college. This difference between what college will cost (including tuition, room and board, books, computers) and the non-loan awards received is the **college funding gap**.

EXAMPLE 6.81

College Funding Gap

Ishraq receives her award and tuition letter from the college she wants to attend. Her tuition, fees, books, and room and board all come to \$24,845 for the year. Her non-loan awards include an instant scholarship from the school for \$7,500, a scholarship she earned for enrolling in a STEM program for \$3,750, and a \$1,000 scholarship from her church. What is Ishraq's college funding gap?

Solution

Her awards total to \$12,250. Her cost to attend is \$24,845. Her college funding gap is then $\$24,845 - \$12,250 = \$12,595$. She will need to find \$12,595 in funding.

YOUR TURN 6.81

1. Yuan-Teng receives his award and tuition letter from the college he wants to attend. His tuition, fees, books, and room and board all come to \$34,845 for the year. His non-loan awards include an instant scholarship from the school for \$17,500, three scholarships he earned from sources he found online, which total \$5,600, and a \$2,000 scholarship from his employer. What is Yuan-Teng's college funding gap?

There are several loan types, which basically break down into four broad categories: subsidized loans, unsubsidized loans, PLUS loans, and private loans. These loans are meant to fill the college funding gap.

Federal **subsidized loans** are backed by the U.S. Department of Education. These loans are intended for undergraduate students who can demonstrate financial need. Subsidized federal loans, including Stafford loans, defer payments until the student has graduated. During the deferment, the government pays the interest while the student is enrolled at least half-time. These loans are generally made directly to students. However, there are restrictions on how the money can be used. It can only be used for tuition, room and board, computers, books, fees, and college-related expenses. Interest rates are not based on the financial markets but determined by Congress. Federal loans are backed by the Department of Education.

Federal **unsubsidized loans**, including unsubsidized Stafford loans, are available for undergraduate and graduate students who cannot demonstrate financial need. If the student meets the program requirements, they are automatically approved. The student is not required to pay these loans during their time in college (enrolled at least half-time). However, the interest rate is generally higher and there is no deferment period, as with subsidized loans. Interest begins accruing as soon as the money is disbursed.

 *The immediate accrual of interest means the balance of the loan grows as the student attends school. A loan that was for \$10,000 can grow past \$13,000 over five years of college. Some advisors tell students to pay the interest portion of the loan while it is deferred to prevent this growth of debt.*

Parent Loans for Undergraduate Students (PLUS) are federal loans made directly to parents. They are available even if parents are not deemed financially needy. A credit check is performed and approval is not automatic. The limit to what parents can borrow from a PLUS loan each year is still the college funding gap, but the aggregate of the PLUS loans does not have a limit. This means the PLUS loan can cover whatever is left in the funding gap once all other aid and loans are applied. Payments do not begin until the student is out of school, but interest begins to accrue the moment funds are disbursed. Because the parents take out the loan, the parents are responsible for paying back the loan.

Private student loans are backed by a bank or credit institution and require a credit check, and interest rates are variable. As private loans are not subsidized by the government, no one pays the interest but the borrower. The student

does not have to start repaying the loan until after graduation, but interest starts to accrue immediately. This loan has fewer repayment options, more fees and penalties, and the loan cannot be discharged through bankruptcy. Many students need a co-signer to acquire a private loan. Like PLUS loans, private student loans can cover whatever is left in the funding gap once all other aid and loans are applied

Student loans, in general, have a term of 10 years, that is, the loans are paid back over 10 years. This can vary, but 10 years is the standard.

?? WHO KNEW?

School-Channel Loans and Direct-to-Consumer Loans

Private loans can fall into one of two categories: school-channel loans and direct to consumer loans. **School-channel loans** are disbursed directly to the school. The school verifies the loan does not exceed the cost to attend school. **Direct-to-consumer loans** do not have the verification process. Those proceeds are sent directly to the borrower. They are processed more quickly, but often have higher interest rates.

▶ VIDEO

[Types of Student Loans \(https://openstax.org/r/Student_Loans\)](https://openstax.org/r/Student_Loans)

Limits on Student Loans

As mentioned earlier, there are limits to how much a student can borrow, per year and in total. The following table shows a general breakdown of the amounts the federal government and private lenders will lend. Amounts are based on level of need and whether the student is a dependent or an independent student. Independent students include those who are at least 24 years old, married, a professional, a graduate student, a veteran, a member of the armed forces, an emancipated minor, or an orphan. The amounts shown are as of this writing in 2022.

Year	Dependent Students Maximum Amounts	Independent Students Maximum Amounts
First-Year Undergraduate	\$5,500 but no more than \$3,500 may be in subsidized loans	\$9,500 but no more than \$3,500 may be in subsidized loans
Second-Year Undergraduate	\$6,500 but no more than \$4,500 may be in subsidized loans	\$10,500 but no more than \$4,500 in subsidized loans
Third Year and Additional Years	\$7,500 but no more than \$5,500 may be in subsidized loans	\$12,500 but no more than \$5,500 may be in subsidized loans
Graduate and Professional	Not applicable	\$20,500 in unsubsidized loans
Limits	\$31,000 but no more than \$23,000 in subsidized loans	\$57,500 for undergraduates but no more than \$23,000 in subsidized. \$138,500 for graduate or professional but no more than \$65,500 may be subsidized loans.

Check out this [Edvisors page on the limits of student borrowing \(https://openstax.org/r/loan_limits\)](https://openstax.org/r/loan_limits) to learn more!

EXAMPLE 6.82**Loan for Year 5 of College**

Efraim is a dependent undergraduate student enrolled in a biology program. He's about to attend for the fifth year. In year 1 he took out \$5,000 in federal subsidized and unsubsidized loans, in year 2 he took out \$6,400 in federal subsidized and unsubsidized loans, in years 3 and 4, he took out the maximum federal subsidized and unsubsidized loans amounts. He needs federal subsidized and unsubsidized loans for his fifth year of school. How much can he obtain in federal subsidized and unsubsidized student loans?

 Solution

The sum of his previous loans is $\$5,000 + \$6,400 + \$7,500 + \$7,500 = \$26,400$. The limit for federal subsidized and unsubsidized loans is \$31,000, so in year 5 he can get student loans in the amount of $\$31,000 - \$26,400 = \$4,600$.

 YOUR TURN 6.82

1. Tiana is an independent college student entering the sixth and final year of her engineering program. In the previous years, she took out student loans of \$5,500, \$8,470, \$10,000, \$7,890, and \$11,900. She needs a loan to finish her final year. How much in federal subsidized and unsubsidized student loans can she obtain?

Putting this all together, we have a way to determine the student loans needed for a student to attend college.

- First, determine the funding gap. If the student or family can cover the gap, then no loans are necessary.
- Second, determine how much in federal subsidized and unsubsidized student loans can be taken out. If the total federal loans available is more than the funding gap, no other loans are needed.
- Third, if the federal subsidized and unsubsidized loans do not cover the gap, PLUS and private student loans can be taken out to cover the remainder of the gap.

At each step, if the student and family can cover some or all of the gap, they can do so without taking out a loan.

EXAMPLE 6.83**College Funding Gap and PLUS and Private Student Loans**

Olivia receives her award and tuition letter from the college she wants to attend. Her tuition, fees, books, and room and board all come to \$44,845 for her second year. Her non-loan awards include an instant scholarship from the school for \$13,500, a scholarship she earned for enrolling in an engineering program for \$5,750, and a \$2,000 scholarship from her parent's workplace. For her first year, what is Olivia's college funding gap? How much can Olivia borrow in federal subsidized and unsubsidized student loans? Once Olivia takes out her maximum subsidized and unsubsidized federal student loans, how much will have to be paid for using PLUS and private student loans?

 Solution

Her awards total to \$21,250. Her cost to attend is \$44,845. Her college funding gap is then $\$44,845 - \$21,250 = \$23,595$. The maximum in federal student loans that Olivia can borrow is \$6,500 in year 2. The remaining funding gap is $\$23,595 - \$6,500 = \$17,095$. Private student loans, PLUS loans, or other sources must be used to cover this gap.

 YOUR TURN 6.83

1. Makenzy receives her award and tuition letter from the college she wants to attend. Her tuition, fees, books, and room and board all come to \$39,200 for her third year. Her non-loan awards include an instant scholarship from the school for \$19,500, three scholarships she earned from sources she found online which total \$3,850, and a \$5,000 scholarship from her employer. What is Makenzy's college funding gap? How much can Makenzy borrow in federal subsidized and unsubsidized student loans? Once she takes out her maximum subsidized and unsubsidized federal student loans, how much will Makenzy and her family have to pay using PLUS and private student loans?

Student Loan Interest Rates

Student loans are first and foremost loans. Students will pay them back and will pay interest. In the fall of 2022, the federal student loan interest rate was 4.99%. Private student loans rates ranged between 3.22% and 13.95%. Finding the lowest interest rate you can helps with the payments, and especially helps if the loan is not federally subsidized. Remember, if the loan is not federally subsidized, the student is on the hook for the interest that is accumulating with the loan.

Interest Accrual

The interest on student loans begins as soon as the loan is disbursed (paid to the borrower). When the loan is federally subsidized, the government pays that interest for the student. This means the loan for a subsidized loan of \$3,000 is still a loan for \$3,000 when the student graduates. However, if the loan is not federally subsidized, the student is responsible for the interest that accrues on the loan. The \$3,000 loan from year 1 of college is now a loan for more due to that added interest. The interest on that loan grew while the student was in college. The formula for growth of the loan's balance is the same as compound interest formula from [Compound Interest](#), $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

EXAMPLE 6.84

Denise takes out unsubsidized student loan, in August, in her first year of college for \$2,000. She manages an interest rate of 8%. She graduates after her fifth year of college, in May. She does not pay the interest on the loan during her time in college. What is the balance of her first year loan in May of her graduation year?

Solution

The principal of the loan is \$2,000. Her interest rate is 8%. Since student loans are typically paid monthly, there are 12 periods per year. Since the time she has had the loan is not in years, we will use the number of months for the value of nt in the formula. She has had the loan for 4 years and 9 months, meaning 57 period have passed. Substituting those values into the formula and calculating, we find her balance in May of her graduating year is

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = \$2,000\left(1 + \frac{0.08}{12}\right)^{57} = \$2,000(1.06)^{57} = \$2,920.89.$$

YOUR TURN 6.84

1. Priya takes out unsubsidized student loan, in August, in her first year of college for \$2,000. She manages an interest rate of 7%. She graduates after her sixth year of college, in May. She does not pay the interest on the loan during her time in college. What is the balance of her first year loan in May of her graduation year?

The Standard Repayment Plan

There are various repayment plans available. The one most likely to apply to a student loan is the **standard repayment plan**, which is available to everyone. Borrowers pay a fixed amount monthly so the loan is paid in full within 10 years. Consolidated loans, discussed later in this section, also qualify for the standard repayment plan, and may allow the payoff period to range from 10 to 30 years. Direct subsidized and unsubsidized loans, PLUS loans, and federal Stafford loans are eligible.

Since these are loans, they are paid back with interest. As with most installment loans, their payments are due monthly. The formula for paying back these loans is the same as the formula used for paying loans in [The Basics of Loans](#):

$$pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}.$$

Using that formula, we can calculate how much the payment is for a student loan. Remember that all loan payments are rounded up to the next penny.

EXAMPLE 6.85

Standard Repayment Plan

Find the payment for the following student loans using the standard repayment plan:

1. Loan is \$3,500, interest is 4.99%
2. Loan is for \$6,200, interest is 6.75%

 Solution

1. The principal is $P = \$3,500$ and the rate is $r = 0.0499$. Since this is the standard repayment plan, there are $n = 12$ payments per year for 10 years. Substituting those values into $pmt = \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1}$ and calculating gives a monthly payment of

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$3,500 \times (0.0499/12) \times (1 + 0.0499/12)^{12 \times 10}}{(1 + 0.0499/12)^{12 \times 10} - 1} \\
 &= \frac{\$3,500 \times (0.004158\bar{3}) \times (1.004158\bar{3})^{120}}{(1.004158\bar{3})^{120} - 1} \\
 &= \frac{\$23.9469911276}{0.645370131872} \\
 &= \$37.11
 \end{aligned}$$

2. The principal is $P = \$6,200$ and the rate is $r = 0.0675$. Since this is the standard repayment plan, there are $n = 12$ payments per year for 10 years. Substituting those values into $pmt = \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1}$ and calculating gives a monthly payment of

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$6,200 \times (0.0675/12) \times (1 + 0.0675/12)^{12 \times 10}}{(1 + 0.0675/12)^{12 \times 10} - 1} \\
 &= \frac{\$6,200 \times (0.005625) \times (1.005625)^{120}}{(1.005625)^{120} - 1} \\
 &= \frac{68.3662233426}{0.960321816275} \\
 &= \$71.20
 \end{aligned}$$

 YOUR TURN 6.85

Find the payment for the following student loans using the standard repayment plan:

1. Loan is \$7,800, interest is 4.99%
2. Loan is for \$11,450, interest is 7.75%

EXAMPLE 6.86

Standard Repayment Plan for an Unsubsidized Loan

Erson has a balance of \$8,132.55 when he starts paying off the 8.6% unsubsidized student loan he took out in his third year. How much are his payments if the term for his loan is the standard 10 years?

 Solution

Using the payment formula, $pmt = \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1}$, with $P = \$8,132.55$, $r = 0.086$, $t = 10$ and $n = 12$, we calculate that his monthly payment will be

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$8,132.55 \times (0.086/12) \times (1 + 0.086/12)^{12 \times 10}}{(1 + 0.086/12)^{12 \times 10} - 1} \\
 &= \frac{\$8,132.55 \times (0.0071\bar{6}) \times (1.0071\bar{6})^{120}}{(1.0071\bar{6})^{120} - 1} \\
 &= \frac{137.310962384}{1.35592393158} \\
 &= \$101.27
 \end{aligned}$$

> YOUR TURN 6.86

1. Kathleen has an unsubsidized student loan from graduate school. She begins to pay it back in the September after she earned her doctorate. The balance at that point was \$22,666.21. The interest rate on the loan is 6.7%. How much are her payments if she pays it off in the standard of 10 years?

Student Loan Consolidation

When a student graduates, they may have multiple different student loans. Keeping track of them and paying them off separately can be a burden. Instead, these loans can be consolidated into a single loan. If they are federal loans the combination is called **federal consolidation**. Combining private loans is often referred to as **refinancing**. Refinancing, or **private consolidation**, can be used to combine both private and federal student loans. Be aware that consolidated federal loans may still be subject to the rules and protections that govern subsidized loans. Refinancing loans, private or federal, are no longer subject to those rules and guidelines. Check out this [Experian article about consolidation and refinancing \(https://openstax.org/r/my_student_loans\)](https://openstax.org/r/my_student_loans) for more detail.

In consolidation of federal direct student loans, the interest rate is the weighted average of the interest rates on the subsidized loans. This means the interest rate remains the same. However, if the term is extended, then the student will pay back more over time than if they did not extend the loan term.

In refinancing, it is possible to obtain a lower interest rate on the student loans, which may lower how much is paid per month and lower the total paid back over time. These monthly payments are calculated using the same formula as for any other loan payment, $pmt = \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1}$. The term of the refinanced loan may also be changed, which would also impact the payment per month.

In either case, consolidating or refinancing, the monthly financial burden on the student can decrease. However, if the term is extended, the total amount repaid may increase.

EXAMPLE 6.87

Federal Loan Consolidation and Interest Rates

Ernest has four federal student loans that he wants to consolidate. He combines them into one loan. What is the maximum Ernest can reduce the interest rate by?

✓ Solution

Consolidating subsidized loans has no impact on the interest rate of the loans, so the maximum that the interest rate can be reduced is 0%.

> YOUR TURN 6.87

1. Ryann has five federal student loans that he wishes to consolidate. He wants to extend the term to 15 years. How does this impact Ryann's interest rate? The total Ryann pays back?

EXAMPLE 6.88**Payments for Consolidated Student Loans**

Brianna consolidates her student loans, some federal and some private, into a single refinanced student loan with a principal of \$27,800. The interest rate that Brianna received was 8.375%. If Brianna's new term is 15 years, how much are her payments per month?

✓ Solution

The principal is $P = \$27,800$ and the rate is $r = 0.08375$. Since the payments are monthly, $n = 12$. The loan term is for 15 years, so $t = 15$ years. Substituting those values into $pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$ and calculating gives a monthly payment of

$$\begin{aligned} pmt &= \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1} \\ &= \frac{\$27,800 \times (0.08375/12) \times (1 + 0.08375/12)^{12 \times 15}}{(1 + 0.08375/12)^{12 \times 15} - 1} \\ &= \frac{\$6,200 \times (0.0069791\bar{6}) \times (1.0069791\bar{6})^{180}}{(1.0069791\bar{6})^{180} - 1} \\ &= \frac{\$678.477992464}{2.49693370968} \\ &= \$271.73 \end{aligned}$$

> YOUR TURN 6.88

1. Brian consolidates his federal student loans into one loan at 4.99%. The consolidated balance is \$20,340, and he extends the term to 12 years. What are Brian's monthly payments for the consolidated loan?

Other Repayment Plans

There are various other repayment plans available to students. Plans other than the standard repayment plan typically require the student to meet certain criteria. The following plans are independent of student income, but may make early payments easier.

- **Graduated repayment plans** are plans where the amount of payments gradually increases so that the loan is paid off in 10 years, or within 10 to 30 years for consolidated loans. Payments start off small and increase approximately every 2 years. Almost all loan types are eligible, including direct subsidized and unsubsidized loans, Stafford loans, PLUS loans, and consolidated loans.
- **Extended repayment plans** are available to the direct loan borrower if the outstanding direct loans are over \$30,000. The payments, fixed or graduated, are designed so that the loans are satisfied within 25 years. Eligible loans include both direct subsidized or unsubsidized loans, Stafford loans, PLUS loans, and consolidated loans.

If student earnings are such that the standard, graduated, or extended repayment plans are unaffordable, then one can make payments that are based on their **discretionary income**. Discretionary income is federally defined to be the difference between (adjusted) gross income and 150% of the poverty guideline for location and family size. This discretionary income then depends on where one lives (contiguous United States or Hawaii or Alaska) and how many dependents one has. If married, a spouse's income will be included in the adjusted gross income. Understanding discretionary income is necessary to understand how income driven payments plans work.

EXAMPLE 6.89**Discretionary Income**

1. The poverty guideline for a single person living in Arkansas, is \$12,000. If Harriet is a single person in Arkansas with a (adjusted) gross income of \$23,500, what is her discretionary income?
2. For California, the poverty guideline for a person with four people in the household is \$27,750. If such a person has a (adjusted) gross income of \$48,600, what is their discretionary income?

✓ **Solution**

1. The poverty guideline for a single person in Arkansas is \$12,000. 150% of that guideline value is $1.5 \times \$12,000 = \$18,000$. The gross income that Harriet makes over that \$18,000 is her discretionary income. That gross income is \$23,500, so her discretionary income is $\$23,500 - \$18,000 = \$5,500$.
2. The poverty guideline for a household of four in California is \$27,750. 150% of that guideline value is $1.5 \times \$27,750 = \$41,625$. The gross income that Harriet makes over that \$41,625 is her discretionary income. That gross income is \$48,600, so her discretionary income is $\$48,600 - \$41,625 = \$6,975$.

> **YOUR TURN 6.89**

1. The poverty guideline for two-person household in Hawaii is \$21,060. If Jamie lives in a two-person household and has a gross income of \$36,900, what is his discretionary income?

The following plans all depend on discretionary income.

- **Pay As You Earn (PAYE) repayment plans** have monthly payments that are 10% of discretionary income based on a student's updated income and family size. If a borrower files a joint tax return, their spouse's income and debt may also be considered. Eligible loans include direct subsidized and unsubsidized loans, PLUS loans made to students, and some consolidated loans. These loans are forgiven (student does not pay the remaining balance) after 20 years of monthly payments if they were direct federal student loans.
- **Revised Pay As You Earn (REPAYE) repayment plans** have payment amounts that are based on income and family size and calculated as 10% of discretionary income. Eligible loans include direct subsidized and unsubsidized loans, direct PLUS loans, and some consolidated loans. These loans are forgiven, that is, the student does not pay more, after 20 or 25 years, provided they were direct federal student loans.
- **Income-Based Repayment (IBR) plans** sound like a few of the others and there are similarities in all of them. The payments are either 10% or 15% of discretionary income, but this plan is meant for those with a relatively high debt. Every year, income and family size must be updated, and payments are calculated based on those figures. Eligible loans include direct subsidized and unsubsidized loans, Stafford loans, and PLUS loans made to students, but not PLUS loans made to parents.
- **Income-Contingent Repayment (ICR) plans** have payments that are either 20% of discretionary income or whatever would be paid if a student were on a fixed payment plan for more than 12 years, whichever is less. Eligible loans include direct subsidized and unsubsidized, PLUS loans to students, and consolidated loans.

There are many similarities among these repayment plans, and it is easy to misunderstand the nuances of each. Therefore, be careful entering into any type of repayment contract until you fully understand all the details and repercussions of the plan you choose. For more detail, see [this nerdwallet article "Income-Driven Repayment: Is It Right for You?"](https://openstax.org/r/driven_repayment_right) to learn more (https://openstax.org/r/driven_repayment_right)!

EXAMPLE 6.90

Payments for a REPAYE Program

Warren qualifies for a REPAYE payment plan. His gross income is \$32,700. He is single and live in Montana, so the federal poverty guideline for Warren is \$12,000.

1. What is Warren's discretionary income?
2. Under the REPAYE plan, he pays 10% of his discretionary income, but monthly. How much are Warren's REPAYE payments?

✓ **Solution**

1. The poverty guideline for Warren is \$12,000. 150% of that guideline value is $1.5 \times \$12,000 = \$18,000$. The gross income that Warren makes over that \$18,000 is his discretionary income. That gross income is \$32,700, so his discretionary income is $\$32,700 - \$18,000 = \$14,700$.
2. 10% of Warren's discretionary income is $0.1 \times \$14,700 = \$1,470$. He pays monthly, so his monthly payments are \$1,470 divided by 12, or \$122.50 per month.

> YOUR TURN 6.90

Lauren, a Missouri resident, qualifies for a REPAYE payment plan. Her gross income is \$39,200. She has one child, so there are two people in her household. The federal poverty guideline for Lauren is \$18,310.

1. What is Lauren's discretionary income?
2. Under the REPAYE plan, she pays 10% of her discretionary income, but monthly. How much are Lauren's REPAYE payments?

Using income to determine payments initially seems excellent. However, if there is no forgiveness at the end of the loan, then the income driven payment plans can cause problems. For one, your payment may not be sufficient to cover the interest rate of your loans. In that case, your loan balance actually *increases* as you make your payments. Eventually, you are paying for not only your original loan balance, but interest that has been growing and compounding over time. Also, if the loan term is extended, you may pay more, perhaps a lot more, money over time. You may find yourself in the position of paying these loans for decades.

With those possible drawbacks, great care must be taken to avoid large problems down the line.

▶ VIDEO

[Repayment Plans \(https://openstax.org/r/student_loan_repayment_plans\)](https://openstax.org/r/student_loan_repayment_plans)

Student Loan Default and Consequences

The first day a payment is late, the account becomes **delinquent**. After 90 days, this delinquency is reported to the credit bureaus, and goes into **default**. This is serious, as now a credit score is affected, meaning that it will be harder to buy a car, a home, get a credit card, or a cell phone. Even renting an apartment may be a task not easily overcome. The default rate for students who do not complete their degree is three times higher than for students who do.

Further, defaulting on a student loan may mean that the borrower loses eligibility for repayment plans, as the balance and any unpaid interest may become due immediately, and any tax refunds may be withheld and applied to the loan, and wages may be garnished. One should immediately contact your loan servicer and try to make other arrangements for repayment if this situation becomes apparent, as different repayment plans are available, if actions are taken quickly.

There are several options that may be open to avoid defaulting. One is called **rehabilitation**, or is the process in which a borrower may bring a student loan out of default by adhering to specified repayment requirements, and the other is consolidation. Certain criteria must be met to enter these programs.

Both of these options are detailed, including the criteria required for eligibility, on the [studentaid.gov loan management page \(https://openstax.org/r/default_get_out\)](https://openstax.org/r/default_get_out).

Professionals advise hiring an attorney if one of these paths is chosen.

Check Your Understanding

55. When is the FAFSA typically due?
56. If the college has a program that it advertises will take 6 years, how long is the eligibility period?
57. What is a student's funding gap?
58. What is a main difference between subsidized federal loans and unsubsidized loans?
59. What is the limit on the total in federal subsidized and unsubsidized loans that can be borrowed for an undergraduate degree?
60. If a student takes out a private student loan for \$9,800 at 5.99% interest but does not begin paying it back for 66 months, how much is then owed on the loan?
61. How long is the standard repayment plan for student loans?
62. How much of a student's discretionary income is paid for student loans under the REPAYE payment plan?



SECTION 6.9 EXERCISES

1. What form must be filled out before applying for a student loan?

2. What two issues limit the amount of a federal subsidized or unsubsidized student loan a student can take out?
3. What is the limit to the number of years a student can take out a student loan?
4. What types of student loans have deferred payments and do not accrue interest?
5. Which type of student loan is made directly to parents?
6. Which types of student loans require a credit check?
7. What is the limit on the total amount borrowed in federal subsidized and unsubsidized student loans for an independent undergraduate student?
8. What is the limit on subsidized student loans in the third year of an undergraduate program if the student is a dependent student?
9. What is the maximum allowed to be borrowed on a PLUS loan?
10. What is the maximum allowed to be borrowed on a private student loan?
11. What is the current interest rate on federal subsidized loans?
12. What feature does a graduated payment plan have?
13. If a student is struggling with paying back their student loans, what can the student do?
14. When a student consolidates federal subsidized and unsubsidized loans, does the interest rate change? Does the term of the loan change?
15. If a person with federal subsidized and unsubsidized loans enters a pay as you earn program (PAYE), how long will the person pay on their student loans?
16. How long is it between the time a person is delinquent on a student loan and being in default on the student loan?
17. What is discretionary income?
18. What are the two remedies if a borrower default on their student loans?
19. Toni's total cost to attend college including all fees, books, and room and board is \$18,450. If Toni receives \$5,500 in an instant scholarship from the college and has \$2,175 in scholarships, what is Toni's funding gap?
20. Arthur's total cost to attend college including all fees, books, and room and board is \$31,500. If Arthur receives \$13,500 in an instant scholarship from the college and has \$3,475 in private scholarships, what is Arthur's funding gap?
21. Amelia is an independent student in her fourth year of an urban studies undergraduate degree. What is the maximum she can borrow in federal subsidized and unsubsidized student loans that year?
22. Yaroslav is a dependent student in his second year of an undergraduate psychology program. What is the maximum that he can borrow in federal subsidized and unsubsidized student loans that year?
23. Toni's total cost to attend her third year of undergraduate school, including all fees, books, and room and board, is \$18,450. Toni receives \$5,500 in an instant scholarship from the college and has \$2,175 in scholarships. She is a dependent student. After receiving the maximum federal student loans (subsidized and unsubsidized), and without her parents taking out a PLUS loan, how much will she need to borrow in private student loans to cover the funding gap?
24. Arthur's total cost to attend his second year of his undergraduate program in data science, including all fees, books, and room and board, is \$31,500. Arthur receives \$13,500 in an instant scholarship from the college and has \$3,475 in private scholarships. He is a dependent student. After receiving the maximum federal student loans (subsidized and unsubsidized), and without his parents taking out a PLUS loan, how much will he need to borrow in private student loans to cover the funding gap?
25. Nantumbwe takes out a private student loan for \$7,950 in her second year of her undergraduate health administration program. She begins paying her loan after 39 months. If her interest rate was 8.75%, how much is her balance when she begins to pay off her loan?
26. Cameron takes out a private student loan for \$11,385 in their third year of their undergraduate communications program. They begin paying their loan after 27 months. If Cameron's interest rate was 9.125%, how much is their balance when they begin to pay off the loan?

27. Farzana refinances her loans for a reduced interest rate before she begins to pay them off. The total of her refinanced loan is \$39,447.50. The new interest rate is 6.99%. How much are Farzana's payments if she has a 10-year term on this loan?
28. Carly refinances her loans for a reduced interest rate before she begins to pay them off. The total of her refinanced loan is \$61,332.00. The new interest rate is 7.45%. How much are Carly's payments if she has a 10-year term on this loan?
29. Andrew consolidates his federal student loans to make paying them easier. They total \$27,582.00. The consolidated loan is at 4.99% interest. How much are Andrew's payments if his new term is 15 years?
30. Manuel consolidates his federal student loans to make paying them easier. They total \$19,470.00. The consolidated loan is at 4.99% interest. How much are Manuel's payments if his term is 10 years?
31. Steven has four members in his Georgia household and makes \$53,400 annually. The federal poverty guideline for him is \$27,750. What is Steven's discretionary income?
32. Akash has three members in his New Jersey household and makes \$38,120 annually. The federal poverty guideline for him is \$23,030. What is Akash's discretionary income?
33. Kateryna has two members in her Iowa household and earns \$42,500 annually. The federal poverty guideline for her is \$18,310. She qualifies for a REPAYE repayment plan. What is the maximum monthly payment Kateryna would pay?
34. Keirstin lives by herself in her New Mexico home and earns \$28,530 annually. The federal poverty guideline for her is \$13,590. She qualifies for a REPAYE repayment plan. What is the maximum monthly payment Keirstin would pay?

6.10 Credit Cards



Figure 6.22 Credit cards are both a convenience and a danger. (credit: "Credit Cards" by Sean MacEntee/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Apply for a credit card armed with basic knowledge.
2. Distinguish between three basic types of credit cards.
3. Compare and contrast the benefits and drawbacks of credit cards.
4. Read and understand the basic parts of a credit card statement.
5. Compute interest, balance due, and minimum payment due for a credit card.

It can be difficult to get along these days without at least one credit card. Most hotels and rental car agencies require that a credit card is used. There are even a number of retailers and restaurants that no longer accept cash. They make online purchasing easier. And nothing contributes more to a good credit rating than a solid history of making credit card payments on time.

Being granted a credit card is a privilege. Used unwisely that privilege can become a curse and the privilege may be withdrawn. In this section, we will talk about the different types of credit cards and their advantages and disadvantages.

The more knowledge a cardholder has about the credit card industry, the better able credit accounts can be managed, and that knowledge may cause major adjustments to a cardholder's lifestyle.

All credit cards are not equal, but they all represent consumers borrowing money, usually from a bank, to pay for needs and “wants.” As such, they are a type of loan, and your repayment may include interest. (You might want to review Section 6.8, which discusses loans and repayment plans.)

There are many institutions and credit cards to choose from. Use caution as you shop around for a credit card that suits you. Your top concern is likely the interest rates on purchases and cash advances. But be careful to also read the small print regarding charges for late payments, and other fees such as an **annual fee**, where the credit card charges you (the cardholder) a fee each year for the privilege of using the cards. Many cards charge no such fee, but there are many that charge modest to heavy fees. Make sure to understand rules for **reward programs**, where the credit card issuer grants benefits based on one's spending. Finally, once one applies for and is granted a credit card, pay attention to the **credit limit** the bank offers. Once a company is owed that much money, use of the card for purchases should be curtailed until some of the debt is paid off.

 *The interest rate will not matter if the balance is paid every month. When the balance is paid every month, there is NO INTEREST charged.*

Types of Credit Cards

There are basically three types of credit cards: bank-issued credit cards, store-issued credit cards, and travel/entertainment credit cards. We will look at all three and explain the good and the bad qualities of each.

Bank-Issued Credit Cards

Perhaps the most widely used credit card type is the **bank-issued credit card**, like Visa or MasterCard (and even American Express and Discover cards). These types of cards are an example of revolving credit, meaning that additional credit is extended before the previous balance is paid—but only up to the assigned credit limit. Bank-issued cards are considered the most convenient, as they can be used to purchase anything, including apparel, furniture, groceries, fuel for automobiles, meals, hotel bills, and so on, just as if paying with cash. The interest rates on bank-issued credit cards are usually lower than those for other credit cards we'll discuss, and the credit limits are generally higher. Currently, bank-issued cards have an average 20.09% APR.

Store-Issued Credit Cards

Store-issued credit cards are issued by retailers. One can hardly walk into a store these days without being offered a discount on purchases if one applies for the store credit card. These cards can only be used in that store or family of stores that issues the card. However, if a store credit card is associated with Visa, MasterCard, or American Express, then the card might be used the same way that the bank-issued cards are used. This is called cobranding. The logo of the bank-issued card will be present on the store card. Many stores offer both types. Like other credit cards, they may come with an annual fee.

Store credit cards usually charge higher interest rates than bank-issued cards. Currently, store credit cards have an APR (annual percentage rate) of 24.15%. Any rewards offered by store credit cards are usually limited to purchases made in their own store, and it typically takes longer to accumulate enough rewards or points to redeem them, whereas cobranding credit cards offer opportunities to earn rewards on all purchases, regardless of whether purchases are made in the issuing store or not.

Store credit cards usually offer lower credit limits, at least in the beginning. After being proved to be a responsible credit card owner, credit limits can be raised. Nevertheless, store credit cards are a good choice for those new to the credit card industry. If on-time payments are consistently made, it is an excellent way to get started building a credit history.

Travel/Entertainment Cards, or Charge Cards

This is the third type of credit card. The **travel and entertainment cards**, also known as **charge cards**, first and foremost offer very high limits or unlimited credit, but they must be paid in full every month. They generally charge high annual fees and impose expensive penalties should a payment be late. On the other hand, they typically have longer grace periods and offer many and various kinds of rewards.

Check out this [nerdwallet article about the differences between a charge card and a credit card \(https://openstax.org/r/cards_charge_cards\)](https://openstax.org/r/cards_charge_cards).

 *An interest rate will greatly depend on credit score. Responsible use of credit cards will increase a credit score. See*

the WHO KNEW? from [The Basics of Loans](#).

▶ VIDEO

[Choosing a Credit Card \(https://openstax.org/r/Credit_Card\)](https://openstax.org/r/Credit_Card)

?? WHO KNEW?

Top Travel and Entertainment Cards

At one time, Diner's Club was the premier entertainment card. To be accepted into the Diner's Club and be rewarded with a charge card meant one was special. Today, an American Express card is held with the same reverence as Diner's Club was in the past. It was a privilege to own one of these cards. There is a lot of competition going on to supplant Diner's Club, as shown by the [Chase Sapphire Preferred Card's \(https://openstax.org/r/Chase_Sapphire\)](https://openstax.org/r/Chase_Sapphire) travel rewards and benefits.

Another thing worth mentioning here and something that appears to be an unusual offering is part of the American Express Gold and higher-level cards. Cardholders can actually open a savings account, buy a CD, or apply for a personal or business loan from American Express. The company boasts a higher interest rate than what is paid on traditional savings accounts and CDs.

EXAMPLE 6.91

Comparing Credit Cards

1. Which type of credit card is paid off every month so has no interest to be paid, but comes with high fees?
2. Which type of credit cards are the most widely accepted?
3. Which type of credit cards are the most limited?

✓ Solution

1. Charge cards are to be paid off completely each month.
2. Bank-issued credit cards are the most flexible to use, because they are not limited to which retailers or service providers accept them.
3. Store issues credit cards are the most limited, since they only work in that family of stores.

> YOUR TURN 6.91

1. Which type of credit card typically have the highest interest rates?
2. What should be considered other than interest rate when selecting a card?
3. Which type of cards typically have high annual fees?

Credit Card Statements

Cardholders usually receive monthly statements and have 21 days to pay the minimum amount due. The statements itemize and summarize activity on the credit card for that statement's **billing period**. The billing period for a credit card is generally a month long, but typically does not start and end on the first and last days of the month. The statement will include the current balance, interest rate, the minimum payment due, and the due date. Be aware, different companies produce statements that are laid out differently. The information will be clearly labeled though.

The due date is a top concern. Missing a due date is one of the worst things a cardholder can do financially, and this is by far the biggest downfall of owning a credit card. Not only is the cardholder subject to late fees, but when a payment is late more than once there is a high probability that the cardholder will be negatively reported to the credit bureaus, which can quickly erode a credit score. [Figure 6.23](#) shows an excerpt from an actual statement from a Chase Bank Visa card, based on the current \$668.25 balance.

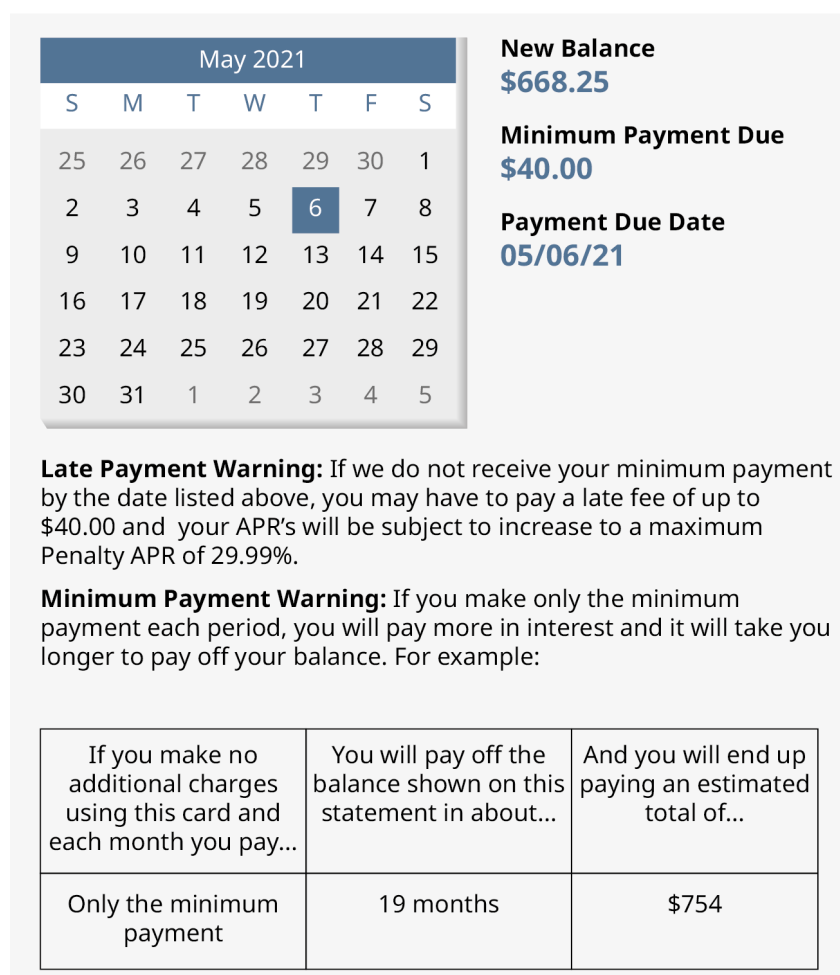


Figure 6.23 Credit card statement

Specifically pay attention to the late payment penalty and minimum payment warning statements. Stating that if no other purchases are made and you continue making only the minimum payment, it will take 19 months to pay off the balance and you will pay \$754.00. You can't say you were not warned.

It is critical that you examine your statement every month because it is always a possibility that your account may have been compromised. If you should notice fraudulent charges on your statement, notifying the credit card company is often enough to have those charges researched by the company and removed. The card with the fraudulent charges will be canceled and a new card with a new account number will be sent to you.

EXAMPLE 6.92

Reading a Credit Card Statement

On the credit card statement [Figure 6.24](#), identify

1. The balance due
2. The minimum required payment
3. The length of time it takes to pay off the balance by paying the minimum payments and without charging more to the card
4. The interest rate for purchases


YourBank

Your Credit Card Account Statement

PAYMENT INFORMATION

New Balance:	\$ 3663.23
Your Minimum Payment:	\$ 36.63
Your Minimum Payment Due Date:	May 24, 2015

Estimated time to pay
The estimated time to pay your New Balance in full if you pay only the Minimum Payment each month is 2 year(s) and 4 month(s).

SUMMARY OF YOUR ACCOUNT

Previous Statement Balance:	\$ 2654.48
Payments:	\$-2654.48
New Purchases:	\$ 1957.24
Balance Transfers and Access Cheques:	\$ 1200.00
Cash Advances	\$ 500.00
Interest:	\$ 0.00
Fees:	\$ 5.99
Subtotal	\$ 3663.23
Your New Balance:	\$ 3663.23
Credit Limit:	\$ 9000.00
Cash Advance Limit:	\$ 500.00
Credit Available:	\$ 5336.77
Cash Advance Limit:	\$ 0.00
Statement Closing Date:	April 30, 2015
Days in Statement Period:	30
Annual Interest Rate for Purchases:	19.99%
Annual Interest Rate for Balance Transf. and Access Cheques:	2.50%
Annual Interest Rate for Cash Advances:	19.99%

Statement Period:
 From April 01, 2015
 To April 30, 2015

 Account Number:
 9999 99XX XXXX 1234

 Primary Cardholder:
 Yourname Yoursurname

Ways to pay:
 Online Banking
 Telephone Banking
 ATM
 Pie-Authorized Payment
 By Mail To:
 YourBank
 P.O. Box 1234 Section Z
 BankCity, NY, 98456

Contact Information:
www.websitename.com
 Customer Service/Lost or Stolen
 1-888-123-4567
 TTY/TDD
 1-888-123-4567


YourBank

Payment Slip

PAYEE FIRST NAME LAST NAME
 PAYEE ADDRESS:
 99 STREET NAME
 CITY NAME, ZIP CODE
 STATE, COUNTRY

Account Number: **9999 99XX XXXX 123**
 Your New Balance: **\$ 3663.23**
 Your Minimum Payment: **\$ 36.63**
 Your Minimum Payment Due Date: **May 24, 2015**

 Amount you're paying: **\$ 3663.23**

999 999 999 99999 999 9999 99XX XXXX 1234 XX

Figure 6.24 Credit card account statement

✓ **Solution**

1. The balance due is under the payment information heading and is \$3,663.23.
2. The minimum payment due is also under the payment information heading, and is \$36.63.
3. The time to pay off the balance using only minimum payments is below the payment information, and says it takes 2 years and 4 months to pay off the balance.
4. The interest rate for purchases is toward the bottom of the statement. It is 19.99%.

> YOUR TURN 6.92

Referring to the statement above, answer the following:

1. What is the statement period?
2. What is the credit limit?
3. How much in fees were charged?

▶ VIDEO

[Reading Credit Card Statements \(https://openstax.org/r/Reading_Credit\)](https://openstax.org/r/Reading_Credit)

Compute Interest, Balance Due, and Minimum Payment Due for a Credit Card

Computing all of these values depends on understanding and computing the average daily balance on a credit card. Once that is known, the interest, balance due, and minimum payment can be found.

Above all else, if you pay off the entire balance each month, interest is not charged.

Average Daily Balance

Most credit card companies compute interest using the **average daily balance** method.

To find the average daily balance on your credit card, determine the balance on the card each day of the billing period (often that month), and take the average. One process to find that average daily balance follows these steps:

1. Start with a list of transactions with their dates and amounts.
2. For each day that had transactions, find the total of the transactions for the day. Expenditures are treated as positive values, payments are treated as negative values.
3. Create a table containing each day with a different balance. The balance is the previous balance plus the day's total transactions.
4. Add a column for the number of days those balances until the balance changed.
5. Add a column that contains the balances multiplied by the number of days until the balance changed.
6. Find the sum of that last column.
7. Divide the sum by the number of days in the billing period (often the number of days in the month). This is the average daily balance.

EXAMPLE 6.93

Computing Average Daily Balance

The billing cycle goes from May 1 to May 31. The balance at the start of the billing cycle is \$450.21. The list of transactions on the card is below.

Date	Activity	Amount
1-May	Billing Date Balance	\$450.21
10-May	Payment	\$120.00
15-May	Groceries	\$83.43
26-May	Auto Parts	\$45.12
26-May	Restaurant	\$85.34
30-May	Shoes	\$98.23

Find the average daily balance for the credit card during the month of May.

✓ **Solution**

To find the average daily balance, we use the following steps.

1. Start with a list of transactions with their dates and amounts.
This list is provided.
2. For each day that had transactions, find the total of the transactions for the day.
The only day with more than one transaction was May 26. The sum of those transactions is \$130.46. Treating the payment as a negative value, the daily transaction amounts are

Date	Amount
1-May	\$450.21
10-May	-\$120.00
15-May	\$83.43
26-May	\$130.46
30-May	\$98.23

3. Create a table containing each day with a different balance.
The new table with dates that had different balances is below.

Date	Balance
1-May	\$450.21
10-May	\$330.21
15-May	\$413.64
26-May	\$544.10
30-May	\$642.33

4. Now, add a column for the number of days those balances until the balance changed. The days until the balance changes is found by finding the difference in the dates. For instance, from May 15 to May 26 was 11. Adding that column to the table we have

Date	Balance	Days Until Balance Changes
1-May	\$450.21	9
10-May	\$330.21	5
15-May	\$413.64	11
26-May	\$544.10	4
30-May	\$642.33	2

The last entry was 2 since there are 31 days in May.

5. Add a column that contains the balances multiplied by the number of days until the balance changed. We create the

column and multiply the values.

Date	Balance	Days Until Balance Changes	Balance Times Days
1-May	\$450.21	9	\$4,051.89
10-May	\$330.21	5	\$1,651.05
15-May	\$413.64	11	\$4,550.04
26-May	\$544.10	4	\$2,176.40
30-May	\$642.33	2	\$1,284.66

- Find the sum of that last column. Adding that last column we have a sum of \$13,714.04.
- There are 31 days in May, so divide the sum by 31, which gives an average of \$442.39, which is the average daily balance.

> YOUR TURN 6.93

- The billing cycle goes from June 1 to June 30. The previous month's balance is \$563.80. The transactions are in the table below.

Date	Activity	Amount
1-Jun	Balance	\$563.80
2-Jun	Gasoline	\$47.50
2-Jun	Groceries	\$63.42
15-Jun	Movie	\$38.75
15-Jun	Payment	\$250.00
27-Jun	Pharmacy	\$31.21
28-Jun	Auto fuel	\$48.00

Find the average daily balance for this credit card.

Calculating the Interest for a Credit Card

The interest charged for a credit card is based on the daily interest rate of the card, the number of days in the billing cycle, and the average daily balance on the card.

FORMULA

The interest charge, I , for a credit card during a billing cycle is $I = \frac{\text{ADB} \times r \times d}{365}$, where ADB is the average daily balance, r is the annual percentage rate, and d is the number of days in the billing cycle. As before, interest is

rounded up to the next penny.

EXAMPLE 6.94

Calculating Interest for a Credit Card Billing Cycle

Compute the interest charged for the credit card based on the given average daily balance (ABD), annual interest rate, and number of days in the billing cycle.

1. ABD = \$2,765.00, annual interest rate 13.99%, billing cycle of 30 days
2. ABD = \$789.30, annual interest rate 17.99%, billing cycle of 31 days
3. ABD = \$1,037.85, annual interest rate 11.99%, billing cycle of 28 days

Solution

1. Substituting \$2,765.00 for ABD, 0.1399 for r and 30 for d and calculating, we find the interest charge to be

$$I = \frac{\text{ABD} \times r \times d}{365} = \frac{\$2,765.00 \times 0.1399 \times 30}{365} = \$31.80.$$
2. Substituting \$789.30 for ABD, 0.1799 for r and 31 for d and calculating, we find the interest charge to be

$$I = \frac{\text{ABD} \times r \times d}{365} = \frac{\$789.30 \times 0.1799 \times 31}{365} = \$12.06.$$
3. Substituting \$1,037.85 for ABD, 0.1199 for r and 28 for d and calculating, we find the interest charge to be

$$I = \frac{\text{ABD} \times r \times d}{365} = \frac{\$1,037.85 \times 0.1199 \times 28}{365} = \$9.55.$$

YOUR TURN 6.94

Compute the interest charged for the credit card based on the given average daily balance (ABD), annual interest rate, and number of days in the billing cycle.

1. ABD = \$2,135.00, annual interest rate 12.9%, billing cycle of 30 days
2. ABD = \$1,589.63, annual interest rate 9.99%, billing cycle of 31 days
3. ABD = \$6,803.41, annual interest rate 14.9%, billing cycle of 28 days

WHO KNEW?

Credit Cards Charge Stores Fees

The interest you pay is not the only way a credit card company generates revenue. It also charges fees to the retailers, online stores, and service providers that allow the consumer, you, to use your credit card to pay them. These are called processing fees. Currently they typically range from 2.87% to 4.35% of each transaction. That means if you use your credit card at a store and spend \$100.00, the store will have to pay the credit card company somewhere between \$2.87 and \$4.35.

One type of processing fee is the interchange fee. Mastercard charges the vendor 1.35% of the sale, plus an additional percentage up to 3.25%, and a fixed \$0.01 fee for each transaction.

Added to that is an assessment fee. This fee is 0.14% for Visa cards.

Calculating the Balance of a Credit Card

The **balance**, or sometimes balance due, on a credit card is the previous balance, plus all expenses, minus all payments and credits, plus the interest on the card. As stated before, if the card was paid off, there is no interest to be paid.

EXAMPLE 6.95

Calculating the Balance of a Credit Card

Find the balance on the credit card with the given interest charge and balance before interest was charged. The cards

were not paid off previously.

1. Balance before interest is \$708.50, interest charge is \$8.15
2. Balance before interest is \$1,395.10, interest charge is \$21.32

✓ **Solution**

1. Adding the balance before interest to the interest charge, we find the balance to be \$716.65.
2. Adding the balance before interest to the interest charge, we find the balance to be \$1,416.42.

> **YOUR TURN 6.95**

Find the balance on the credit card with the given interest charge and balance before interest was charged. The cards were not paid off previously.

1. Balance before interest is \$560.00, interest charge is \$6.44
2. Balance before interest is \$3,218.00, interest charge is \$49.17

The next example puts all those steps together.

EXAMPLE 6.96

Find Balance Due from Transactions and Interest Rate

Kaylen's credit card charges 16.9% annual interest. His current billing period is from November 1 to November 30. The balance on November 1 was \$1,845.23. Use Kaylen's following transactions to determine his balance due at the end of the billing cycle.

Date	Activity	Amount
1-Nov	Billing Date Balance	\$1,845.23
3-Nov	Groceries	\$78.50
4-Nov	Tablet	\$159.00
4-Nov	Online Game Purchase	\$39.99
4-Nov	Restaurant	\$47.10
10-Nov	Payment	\$300.00
13-Nov	Gasoline	\$58.75
13-Nov	Clothing	\$135.00
18-Nov	Gift	\$30.00
18-Nov	Restaurant	\$21.75
28-Nov	Gasoline	\$43.79

✓ **Solution**

The first step is to find Kaylen's average daily balance. To find the average daily balance, we use the following steps.

1. Start with a list of transactions with their dates and amounts.

This list is provided.

- For each day that had transactions, find the total of the transactions for the day. The days with more than one transaction were Nov. 4, Nov. 13, and Nov. 18. Treating the payment on November 10 as a negative value, the daily transaction amounts are

Date	Amount
1-Nov	\$1,845.23
3-Nov	\$78.50
4-Nov	\$246.09
10-Nov	-\$300.00
13-Nov	\$193.75
18-Nov	\$51.75
28-Nov	\$43.79

- Create a table containing each day with a different balance. The new table with dates that had different balances is below.

Date	Balance
1-Nov	\$1,845.23
3-Nov	\$1,923.73
4-Nov	\$2,169.82
10-Nov	\$1,869.82
13-Nov	\$2,063.57
18-Nov	\$2,115.32
28-Nov	\$2,159.11

- Now, add a column for the number of days those balances until the balance changed. The days until the balance changes is found by finding the difference in the dates. For instance, from May 15 to May 26 was 11. Adding that column to the table we have

The last entry was 3 since there are 30 days in November (the 28th, 29th, and 30th).

Date	Balance	Days Until Balance Changes
1-Nov	\$1,845.23	2
3-Nov	\$1,923.73	1

Date	Balance	Days Until Balance Changes
4-Nov	\$2,169.82	6
10-Nov	\$1,869.82	3
13-Nov	\$2,063.57	5
18-Nov	\$2,115.32	10
28-Nov	\$2,159.11	3

5. Add a column that contains the balances multiplied by the number of days until the balance changed. We create the column and multiply the values.

Date	Balance	Days Until Balance Changes	Balance Times Days
1-Nov	\$1,845.23	2	\$3,690.46
3-Nov	\$1,923.73	1	\$1,923.73
4-Nov	\$2,169.82	6	\$13,018.92
10-Nov	\$1,869.82	3	\$5,609.46
13-Nov	\$2,063.57	5	\$10,317.85
18-Nov	\$2,115.32	10	\$21,153.20
28-Nov	\$2,159.11	3	\$6,477.33

6. Find the sum of that last column. Adding that last column we have a sum of \$62,190.95.
 7. There are 30 days in November, so divide the sum by 30, which gives an average of \$2,073.03, which is the average daily balance.

With the average daily balance, we can determine the interest that is charged for November. Substituting ADB = \$2,073.03, $r = 0.169$, and $d = 30$ into the formula $I = \frac{\text{ADB} \times r \times d}{365}$ and calculating, we find the interest to be

$$I = \frac{\text{ADB} \times r \times d}{365} = \frac{\$2,073.03 \times 0.169 \times 30}{365} = \$28.80.$$

This interest is added to the final balance from the table in step 3, \$2,159.11, which yields a balance due of \$2,101.83.

> YOUR TURN 6.96

1. Angel's credit card charges 16.9% annual interest. His current billing period is from August 1 to August 31. The balance on August 1 was \$982.45. Use Angel's following transactions to determine his balance due at the end of the billing cycle.

Date	Activity	Amount
1-Aug	Billing Date Balance	\$982.45
5-Aug	Food	\$125.31
13-Aug	Payment	\$500.00
14-Aug	Gasoline	\$51.65
14-Aug	Pizza	\$36.99
14-Aug	Shoes	\$89.45
19-Aug	Electric bill	\$178.34
21-Aug	Internet	\$36.99
21-Aug	Food	\$93.45
30-Aug	Gasoline	\$43.18

Minimum Payment Due

The **minimum payment** due is the smallest required amount to be paid on a credit card to avoid late fees and penalties, such as an increased interest rate. The calculations for this may differ from card to card. They also depend in the balance of the credit card. General guidelines for minimum payment due are:

- For larger balances (usually over \$1,000), the minimum payment will be some percentage of the balance due.
- For moderate balances (between \$25 and \$1,000), the minimum would be a specified dollar amount. \$25 seems to be a common value.
- If the balance is small (under \$25 for instance), then the minimum payment is the balance.

Those are just guidelines. Individual cards may vary in these values.

Minimum payments should only be paid if money is short in a given month. The length of time to pay off a credit card using minimum payments is quite long, and results in paying a lot of interest. It is strongly discouraged.

EXAMPLE 6.97

Calculate the Minimum Payment Due

The FYA credit card company has the following minimum payment policy. For balances over \$1,000, the minimum payment is 2.5% of the balance due plus fees, but not interest. For balances between \$500.00 and \$999.99, the minimum payment is \$50.00. For balances \$499.99 and under, the minimum payment is \$25.00 or the balance due, whichever is smaller.

In the following, calculate the minimum payment due given the credit card minimum payment policy, the balance due and fees charged.

1. Balance due is \$1,309.00, no fees
2. Balance due is \$265.50, \$35 in fees
3. Balance due is \$784.90, no fees

Solution

1. The balance is over \$1,000, so the minimum payment is 2.5% of the balance due plus fees. 2.5% of the balance due

is $0.025 \times \$1,309.00 = \32.73 . Since there are no fees, the minimum payment due is \$32.73.

2. The balance is under \$499.99, so the minimum payment due is \$25.00.
3. The balance is between \$500.00 and \$999.00, so the minimum payment due is \$50.00.

> YOUR TURN 6.97

The CLH credit card company has the following minimum payment policy. For balances over \$1,000, the minimum payment is 1% of the balance due plus interest and fees. For balances between \$25.00 and \$999.99, the minimum payment is \$25.00 plus any fees. For balances under \$25.00, the minimum payment is the balance due plus any fees. In the following, calculate the minimum payment due given the credit card minimum payment policy, the balance due and fees charged.

1. Balance due is \$2,308.00, billing cycle interest is \$24.39, no fees
2. Balance due is \$265.50, \$59.00 in fees
3. Balance due is \$19.90, no fees

Check out this [nerdwallet article about minimum payments \(https://openstax.org/r/minimum_payment\)](https://openstax.org/r/minimum_payment) for more!

Check Your Understanding

63. What are the three main types of credit cards?
64. Which type of credit card typically has the highest interest rate?
65. Name three important pieces of information that are included in a credit card statement.
66. A credit card has an average daily balance of \$1,428.50 during a 31-day billing cycle. Find the interest if the interest rate is 15.9%.
67. The billing cycle goes from June 15 to July 14, or 30 days. The balance at the start of the billing cycle is \$3,825.50. The list of transactions on the card is below. Find the average daily balance.

Date	Activity	Amount
15-Jun	Billing Date Balance	\$3,825.50
20-Jun	Food	\$125.31
20-Jun	Clothing	\$345.00
29-Jun	Payment	\$750.00
1-Jul	Restaurant	\$94.80
6-Jul	Gasoline	\$49.75
6-Jul	Home Repair Supplies	\$683.94
6-Jul	Internet	\$49.99
10-Jul	Cell Phone	\$85.00
10-Jul	Food	\$175.24

68. Find the balance due on a credit card statement if ADB = \$487.65, the annual interest rate is 16.8%, the billing cycle was 30 days, and the balance at the end of the billing cycle, before interest is added, was \$689.47.



SECTION 6.10 EXERCISES

1. Name three main criteria to choose a credit card by.
2. Which type of credit card is most convenient to use?
3. Which type of credit card comes with no preset spending limit?
4. How much interest does a credit card holder pay if they pay off the balance every month?
5. Which type of credit card comes with the highest annual fees?

Use the following credit card statement for the following exercises.

September 2022							New Balance \$2,144.89
S	M	T	W	T	F	S	
28	29	30	31	1	2	3	Minimum Payment Due \$50.00
4	5	6	7	8	9	10	
11	12	13	14	15	16	17	Payment Due Date 09/26/22
18	19	20	21	22	23	24	
25	26	27	28	29	30	1	
2	3	4	5	6	7	8	

Late Payment Warning: If we do not receive your minimum payment by the date listed above, you may have to pay a late fee of up to \$39.00.

Minimum Payment Warning: If you make only the minimum payment each period, you will pay more in interest and it will take you longer to pay off your balance. For example:

If you make no additional charges using this card and each month you pay...	You will pay off the balance shown on this statement in about...	And you will end up paying an estimated total of...
Only the minimum payment	8 years	\$3,837
\$76	3 years	\$2,738 (Savings = \$1,099)

Previous Balance	\$1,937.30
Payment, Credits	-\$300.00
Purchases	+\$478.31
Cash Advances	\$0.00
Balance Transfers	\$0.00
Fees Charged	\$0.00
Interest Charged	+\$29.28
New Balance	\$2,144.89

Opening/Closing Date	08/02/22 - 09/01/22
Credit Access Line	\$7,000
Available Credit	\$4,855
Cash Access Line	\$350
Available for Cash	\$350

Past Due Amount	\$0.00
Balance over the Credit Access Line	\$0.00

6. What is the minimum payment due?
 7. How long will it take to pay the balance if the minimum only is paid and no new purchases are made?
 8. How much interest was charged in this billing cycle?
 9. What is the balance on the credit card?
 10. What is the credit limit on this card?
 11. What is the late payment fee for this card?
 12. When is the payment due?
13. The billing cycle for a credit card goes from April 1 to April 30. The balance at the start of the billing cycle is \$1,598.00. The list of transactions on the card is below. Find the average daily balance for the billing cycle.

Date	Activity	Amount
1-Apr	Billing Date Balance	\$1,598.00
9-Apr	Gasoline	\$51.24
9-Apr	Food	\$105.56
9-Apr	Payment	\$675.00
13-Apr	Camping Trip	\$229.75
21-Apr	Gasoline	\$38.45
22-Apr	Gifts	\$148.88
22-Apr	Food	\$49.75
30-Apr	Gym Payment	\$74.99

14. The billing cycle for a credit card goes from September 1 to September 30. The balance at the start of the billing cycle is \$384.25. The list of transactions on the card is below. Find the average daily balance for the billing cycle.

Date	Activity	Amount
1-Sep	Billing Date Balance	\$384.25
2-Sep	Food	\$94.54
5-Sep	Gasoline	\$25.65
5-Sep	Internet	\$39.99
6-Sep	Payment	\$380.00
9-Sep	Insurance	\$174.52
16-Sep	Food	\$83.54
16-Sep	Day Care	\$350.00
20-Sep	Tires	\$2,337.56
21-Sep	Child Clothing	\$27.65
21-Sep	Gasoline	\$31.00
28-Sep	Television	\$299.95

15. The billing cycle for a credit card goes from October 10 to November 9. The balance at the start of the billing cycle is \$930.50. The list of transactions on the card is below. Find the average daily balance for the billing cycle.

Date	Activity	Amount
10-Oct	Billing Date Balance	\$930.50
11-Oct	Clothing	\$350.00
14-Oct	Computer	\$865.84
17-Oct	Food	\$106.51
21-Oct	Payment	\$700.00
21-Oct	Restaurant	\$134.52
21-Oct	Hotel	\$387.56
30-Oct	Hockey Game	\$76.47
5-Nov	Memorabilia	\$150.00
5-Nov	Restaurant	\$94.45
6-Nov	Gasoline	\$49.19

16. The billing cycle for a credit card goes from February 15 to March 16 during a non-leap year. The balance at the start of the billing cycle is \$292.82. The list of transactions on the card is below. Find the average daily balance for the billing cycle.

Date	Activity	Amount
15-Feb	Billing Date Balance	\$292.82
21-Feb	Food	\$64.57
22-Feb	Gasoline	\$31.50
22-Feb	Food	\$71.94
28-Feb	Insurance	\$133.25
28-Feb	Payment	\$100.00
3-Mar	Gasoline	\$26.61
12-Mar	School Trip	\$300.00

In the following exercises, compute the interest charged for the credit card based on the given average daily balance (ADB), annual interest rate, and number of days in the billing cycle.

17. ADB = \$350.00, annual interest rate 14.9%, billing cycle of 30 days.
18. ADB = \$4,312.00, annual interest rate 9.99%, billing cycle of 31 days.
19. ADB = \$563.38, annual interest rate 17.9%, billing cycle of 30 days.
20. ADB = \$1,043.53, annual interest rate 11.9%, billing cycle of 31 days.

In the following exercises, find the balance on the credit card with the given interest charge and balance before interest was charged. The cards were not paid off previously.

21. Balance before interest is \$1630.00, interest charge is \$16.48.
 22. Balance before interest is \$621.00, interest charge is \$7.81.
 23. Balance before interest is \$1,380.00, interest charge is \$15.35
 24. Balance before interest is \$2,774.00, interest charge is \$44.05.
25. Alsaggr's credit card charges 11.9% annual interest. His current billing period is from April 1 to April 30. The balance on April 1 was \$1,598.00. Use Alsaggr's following transactions to determine his balance due at the end of the billing cycle.

Date	Activity	Amount
1-Apr	Billing Date Balance	\$1,598.00
9-Apr	Gasoline	\$51.24
9-Apr	Food	\$105.56
9-Apr	Payment	\$675.00
13-Apr	Camping Trip	\$229.75
21-Apr	Gasoline	\$38.45
22-Apr	Gifts	\$148.88
22-Apr	Food	\$49.75
30-Apr	Gym Payment	\$74.99

26. Marisa's credit card charges 8.9% annual interest. Her current billing period is from September 1 to September 30. The balance on September 1 was \$384.25. Use Marisa's following transactions to determine her balance due at the end of the billing cycle.

Date	Activity	Amount
1-Sep	Billing Date Balance	\$384.25
2-Sep	Food	\$94.54
5-Sep	Gasoline	\$25.65
5-Sep	Internet	\$39.99
9-Sep	Insurance	\$174.52
16-Sep	Food	\$83.54
16-Sep	Day Care	\$350.00
20-Sep	Tires	\$2,337.56
21-Sep	Child Clothing	\$27.65

Date	Activity	Amount
21-Sep	Gasoline	\$31.00
28-Sep	Television	\$299.95

27. Haley's credit card charges 18.9% annual interest. Her current billing period is from October 10 to November 9. The balance on October 10 was \$930.50. Use Haley's following transactions to determine her balance due at the end of the billing cycle.

Date	Activity	Amount
10-Oct	Billing Date Balance	\$930.50
11-Oct	Clothing	\$350.00
14-Oct	Computer	\$865.84
17-Oct	Food	\$106.51
21-Oct	Payment	\$700.00
21-Oct	Restaurant	\$134.52
21-Oct	Hotel	\$387.56
30-Oct	Hockey Game	\$76.47
5-Nov	Memorabilia	\$150.00
5-Nov	Restaurant	\$94.45
6-Nov	Gasoline	\$49.19

28. Pavly's credit card charges 10.9% annual interest. His current billing period is from February 15 to March 16 in a non-leap year. The balance on February 15 was \$292.82. Use Pavly's following transactions to determine his balance due at the end of the billing cycle.

Date	Activity	Amount
15-Feb	Billing Date Balance	\$292.82
21-Feb	Food	\$64.57
22-Feb	Gasoline	\$31.50
22-Feb	Food	\$71.94
28-Feb	Insurance	\$133.25
28-Feb	Payment	\$100.00

Date	Activity	Amount
3-Mar	Gasoline	\$26.61
12-Mar	School Trip	\$300.00

6.11 Buying or Leasing a Car



Figure 6.25 The choice to lease or buy is based on cost and other concerns. (credit: “Car Insurance” by Pictures of Money/ Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Evaluate basics of car purchasing.
2. Compute purchase payments and identify the related interest cost.
3. Evaluate the basics of leasing a car.
4. Identify and contrast the pros and cons of purchasing versus leasing a car.
5. Investigate the types of car insurance.
6. Solve application problems involving owning and maintaining a car.

There are people who don't need a car and won't purchase one. But for many people, whether or not to have a car is not a question. Having a car is a basic necessity for these people.

Obtaining a car can be daunting. The models, the features, the additional costs, and finding funding are all steps that need to be taken. One of the big decisions is whether to buy the car or to lease the car. This section will address some of the issues associated with each option.

The Basics of Car Purchasing

The biggest questions you will answer before purchasing a car are, what do you want and what do you need?

Does it have to be new? Does it have to be a make and model you are familiar with? Does it have to have assisted driving? What other details are important to you? For a new vehicle, every feature beyond standard features comes with additional cost, which leads to the question that constrains all of your decisions about a car. How much can you afford to spend on a car?

What you can afford must include insurance costs (discussed later in this section) and maintenance and upkeep. Once you have this in mind, you can search for a car that matches, as closely as possible, what you want and can afford. Most, if not all, dealers have websites that you can search through to identify the car you want. If new cars are not affordable, used cars cost less but come with the wear and tear of use.

The sticker price of the car, called the manufacturer's suggested retail price (MSRP), or the negotiated price you arrive at, isn't the end of the cost to buying a car. There are many fees that accompany the purchase of the car, and perhaps even sales tax. These include but aren't necessarily limited to the following:

- the **title and registration fee**, which includes registering your car with the state, getting the license plate, and assigning the title of the car to the lender. This cannot be avoided.
- a **destination fee**, which covers the cost of delivering the vehicle to the dealer
- a **documentation fee**, sometimes referred to a processing fee of handling fee, is the cost of all the paperwork the dealer did to get you the car
- a **dealer preparation fee**, which is for washing the car and other preparation of that sort. You should try to negotiate that out of the cost of the dealer tries to charge for that
- **extended warranties** and maintenance plans, which help cover some of the costs of caring for the car.
- Sales tax.

You could pay for these immediately, but they are often added to the financing of the car, meaning they become part of the principal of your loan.

EXAMPLE 6.98

Total Cost to Purchase a Car

Nichole negotiates with her car dealership so that the price is \$21,800. She needs to pay the 6.75% sales tax on the car. Other fees are \$31.00 for title and registration, \$1,000 in destination fees, and a documentation fee of \$175. What is the total cost of Nichole's car?

Solution

We add the car's sales cost, sales tax and all other fees to arrive at this value. The sales tax is 6.75% on the price she negotiated, so is $\$21,800 \times 0.0675 = \$1,471.50$. Adding these up, we have a total cost of $\$21,800 + \$1,471.50 + \$31.00 + \$1,000 + \$175 = \$24,477.50$.

YOUR TURN 6.98

1. Luther negotiates for the price of his car, reaching agreement at \$28,975. He needs to pay 8% in sales tax, 2.1% in ownership tax, a \$950 destination fee, processing fees totaling \$370, and registration fee of \$617. What is the total cost for Luther's purchase?

One way to bring down payments on a car is to provide a **down payment** or a trade in. This is money applied to the purchase price before financing happens. Be warned, the sales tax applies to the full purchase price! If you reduce the amount financed, the payments respond by going down. This often becomes part of the negotiating process.

EXAMPLE 6.99

Total Cost to Purchase a Car with Down Payment

Sophia negotiates a \$19,800 price for her new car. The sales tax is 9.5% in her area, and the dealership charges her \$300 in documentation fees. Her title, plates, and registration come to \$321.50. The dealership adds to this a destination fee of \$1,100. If she places a down payment of \$5,000 on the car, what is the total she will finance for the car?

Solution

The price was \$19,800. The sales tax of 9.5% is based on this number. The sales tax comes to $\$19,800 \times 0.095 = \$1,881$. Adding all the fees to the price and the sales tax brings the total cost of the car to $\$19,800 + \$1,881 + \$300 + \$321.50 + \$1,100 = \$23,402.50$. Her down payment is applied to this number, so the \$5,000 is subtracted from \$23,402.50. The subtraction yields the amount to be financed, which is \$18,402.50.

YOUR TURN 6.99

1. Carlos buys a car with a negotiated price of \$36,250. The sales tax in his region is 6.5%. The dealer charges a

\$1,200 destination fee and a \$450 documentation fee. He must pay for title, registration, and license plates, which come to \$21.50. If he has a \$7,500 down payment, how much will he have to finance?

When purchasing a car, the total cost to obtain the car is not the only factor in your monthly price. You will also pay an interest rate for the loan you obtain. The interest rate you will get is dependent on your credit score (see [The Basics of Loans](#)). But you can choose from different lenders. The dealership will likely offer to finance your car loan. Frequently, dealerships offer special financing with very low rates. This is to help move inventory, and may indicate their desire to make sales. This might make negotiating easier. Even if the dealership offers financing, check with your bank or credit union to determine the interest rates they are offering. To reduce your payments, choose the lowest rate you can find.

Purchase Payments and Interest

Whether or not you buy a new car or a used car, if you finance the purchase, you are taking out a loan. The interest rates available for used cars are frequently higher than those for new cars. These loan payments work exactly the same way as other loans do as far as payments are concerned. The payment function comes from [The Basics of Loans](#). The difference between financing a new car or a used car is that financing a new car typically comes with a lower interest rate and a longer term than financing a used car.

FORMULA

The payment, pmt , per period to pay off a loan with beginning principal P is $pmt = \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1}$, where r is the annual interest rate in decimal form, t is the term in years, and n is the number of payments per year (typically, loans are paid monthly making $n = 12$).

Note, payment to lenders is always rounded up to the next penny.

 Often, the formula takes the form $pmt = \frac{P \times (r) \times (1 + r)^n}{(1 + r)^n - 1}$, where r is the interest rate per period (annual rate divided by the number of periods per year), and n is the total number of payments to be made.

EXAMPLE 6.100

New Car Payments

In the following, calculate the monthly payment using the given total to be financed, the interest rate, and the term of the car loan.

1. Total to be financed is \$31,885, interest rate is 2.9%, for 5 years.
2. Total to be financed is \$22,778, interest rate is 4.5%, for 6 years.

Solution

1. The amount to be financed is the principal, P , which is \$31,885. The rate r is 0.029, and the term is $t = 5$ years. These are monthly payments, so $n = 12$. Substituting and calculating, we find the monthly payment to be

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$31,885 \times (0.029/12) \times (1 + 0.029/12)^{12 \times 5}}{(1 + 0.029/12)^{12 \times 5} - 1} \\
 &= \frac{\$31,885 \times (0.00241\bar{6}) \times (1.00241\bar{6})^{60}}{(1.00241\bar{6})^{60} - 1} \\
 &= \frac{\$31,885 \times (0.00241\bar{6}) \times (1.15583736592)}{1.15583736592 - 1} \\
 &= \frac{\$89.0635298302}{0.15583736592} \\
 &= \$571.52
 \end{aligned}$$

2. The amount to be financed is the principal, P , which is \$22,778. The rate r is 0.045, and the term is $t = 6$ years. These are monthly payments, so $n = 12$. Substituting and calculating, we find the monthly payment to be

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$22,778 \times (0.045/12) \times (1 + 0.045/12)^{12 \times 6}}{(1 + 0.045/12)^{12 \times 6} - 1} \\
 &= \frac{\$22,778 \times (0.00375) \times (1.00375)^{72}}{(1.00375)^{72} - 1} \\
 &= \frac{\$22,778 \times (0.00375) \times (1.3093031051)}{1.3093031051 - 1} \\
 &= \frac{\$111.837397673}{0.3093031051} \\
 &= \$361.58
 \end{aligned}$$

YOUR TURN 6.100

In the following, calculate the monthly payment using the given total to be financed, the interest rate, and the term of the car loan.

1. Total to be financed is \$18,325, interest rate is 6.75%, for 4 years.
2. Total to be financed is \$41,633, interest rate is 3.9%, for 6 years.

EXAMPLE 6.101

Used Car Payments

Calculate the monthly payment for the used car if the total to be financed is \$16,990, the interest rate is 7.5%, and the loan term is 3 years.

Solution

The amount to be financed is the principal, P , which is \$16,990. The rate r is 0.075, and the term is $t = 3$ years. These are monthly payments, so $n = 12$. Substituting and calculating, we find the monthly payment to be

$$\begin{aligned}
 pmt &= \frac{P \times (r/n) \times (1 + r/n)^n \times t}{(1 + r/n)^n \times t - 1} \\
 &= \frac{\$16,990 \times (0.075/12) \times (1 + 0.075/12)^{12 \times 3}}{(1 + 0.075/12)^{12 \times 3} - 1} \\
 &= \frac{\$16,990 \times (0.00625) \times (1.00625)^{36}}{(1.00625)^{36} - 1} \\
 &= \frac{\$16,990 \times (0.00375) \times (1.25144613551)}{1.25144613551 - 1} \\
 &= \frac{\$132.887936515}{0.25144613551} \\
 &= \$528.50
 \end{aligned}$$

YOUR TURN 6.101

1. Calculate the monthly payment for a car loan that has \$21,845 being financed at an interest rate of 6.3% for 4 years.

The Basics of Leasing a Car

Leasing a car is an alternative to purchasing a car. It is still a loan, and acts like one in many respects. They typically last either 24 months or 36 months, though other terms are available. Leases also come with mileage limits, frequently 10,000, 12,000, or 15,000 miles per year. When the lease is over, the car is returned to the dealer. At that time there may be fees that have to be paid, such as for damage to the car or for extra miles driven over the limit.

There are two components to lease costs. One is the monthly payment for the lease. The other is the fees for leasing. These often are paid before the lease is complete. These include:

- a down payment, which is your initial payment that is applied to the price of the car. It reduces the amount you finance, much the same as when you purchase a car. It is recommended that this be negotiated away.
- the **acquisition fee**, sometimes called the bank fee. This is the money charge for the company to set up the lease. It is essentially a paperwork fee. It is not likely that this can be negotiated.
- a **security deposit**, which might be required. It is about the same as 1 month's payment for the lease. The deposit is returned to you if the car is in good shape at the end. This can be negotiated away.
- **disposition fees**, which cover the cost the company will incur when they take your car back and are typically between \$200 and \$450.
- the title, registration, and license fees, just as with the purchase of a car.
- sales tax, which will likely be applied. The sales tax only covers the depreciated portion of the car (more on depreciation later) in many states. Since this depends on the state in which the car is leased, you should determine the sales tax rules for where you lease the car.

As you can imagine, this can come to a fairly high dollar amount.

EXAMPLE 6.102

Cost to Obtain a Lease

Donna wants to lease a Subaru Outback in Eden, New York. Find the total cost of obtaining her lease if there is no down payment, \$175.00 in acquisition fees, a security deposit of \$300.00, \$350.00 in disposition fees, \$102.50 in title and registration fees, and sales tax of \$3,536.05.

Solution

Adding these values together we find the total cost is \$4,463.55.

> YOUR TURN 6.102

1. RJ wants to lease a Tahoe in Salt Lake City, Utah. Find the total cost of obtaining his lease if he has a \$5,000.00 down payment, \$225 in acquisition fees, a security deposit of \$750.00, \$500.00 in disposition fees, and sales tax of \$4,092.



PEOPLE IN MATHEMATICS

Zollie Frank

Zollie Frank and Armund Shoen founded one of the original leasing companies, Four Wheels, in 1939. Their company leased automobiles to corporations. They began by leasing 5 cars to the Petrolager pharmaceutical company in year one. This saved Petrolager money and provided a steady cash flow to the Four Wheels business. In year two, the number of cars leased to Petrolager was 75. Their new idea was to lease cars directly to companies for one year. Previously, such companies might pay for mileage, gas, and a partial down payment. Sadly, the salesmen who were being so helped often left the company before the car was paid for, and so the company lost the down payment money.

The lease was for \$45 per month per car for one year.

You have some obligations when you lease a car. You must keep the car in good condition, cleaned, maintained, and free of anything more than minor damage. If the car is in poor condition when the car is returned, you will be responsible for the cost to bring the car to an acceptable condition. You are also expected to keep the mileage under its limit. If you go over, you will pay 10 to 25 cents per mile over.

Lease Payments

Lease payments are similar to regular loan payments, but have some other details. Calculating a lease payment involves knowing the following values:

- **The price of the car.** This is the cost you would pay for the car after applying all discounts, incentives, and

negotiations.

- **Residual Value.** This is the manufacturer's estimate of the car's value after a set period of time. The residual value is expressed as a percentage of the manufacturer's suggested retail price (MSRP).
- **Months.** This is the length of the lease. Most leases are either 24- or 36-month leases, but other terms are available.
- **Monthly Depreciation.** The monthly depreciation is the difference between the price of the car and the residual value, divided by the number of months of the lease, and represents the monthly loss of value of the car while it's being used.
- **Money Factor (MF).** This is the interest rate, but expressed in a different way for a lease. Converting from the money factor to the annual percentage rate (APR) is done by multiplying the MF by 2400. Naturally, converting an APR to a MF is done by dividing the APR by 2400.

FORMULA

The monthly depreciation for a car, MD, is $MD = \frac{P-R}{n}$, P is the price paid for the car, R is the residual value of the car, and n is the number of months of the lease.

The annual percentage rate for a lease is $APR = 2400 \times MF$, where MF is the money factor of the lease. The MF for a lease is $MF = APR/2,400$.

EXAMPLE 6.103

Monthly Depreciation of a Car

1. The purchase price of a car is \$25,000. Its residual price is \$14,500. What is its monthly depreciation for a 36-month lease?
2. The purchase price of a car is \$30,000. Its residual price is \$18,600. What is its monthly depreciation for a 24-month lease?

Solution

1. The monthly depreciation formula is $MD = \frac{P-R}{n}$. Substituting \$25,000 for P , \$14,500 for R , and 36 for n , we find MD to be $MD = \frac{P-R}{n} = \frac{\$25,000 - \$14,500}{36} = \frac{\$10,500}{36} = \$291.67$.
2. The monthly depreciation formula is $MD = \frac{P-R}{n}$. Substituting \$30,000 for P , \$18,600 for R , and 24 for n , we find MD to be $MD = \frac{P-R}{n} = \frac{\$30,000 - \$18,600}{24} = \frac{\$11,400}{24} = \$475.00$.

YOUR TURN 6.103

1. The purchase price of a car is \$27,500. Its residual price is \$17,875.00. What is its monthly depreciation for a 24-month lease?

EXAMPLE 6.104

Converting Between APR and MF

1. Find the annual percentage rate if the money factor is 0.00001875.
2. Find the money factor if the APR is 6.25%.

Solution

1. The APR is the money factor times 2400, so $APR = 2400 \times MF = 2400 \times 0.00001875 = 0.045$. Expressed as a percentage, the APR is 4.5%.
2. The MF is the APR divided by 2400, so $MF = APR/2400 = 0.0625/2400 = 0.000026041\bar{7}$.

YOUR TURN 6.104

1. Find the annual percentage rate if the money factor is $0.00001208\bar{3}$.

- Find the money factor if the APR is 7.9%.

Once the values above are found, the payment for the lease can be calculated.

FORMULA

The payment, PMT , for a lease is $PMT = \frac{(P - R)}{n} + (P + R) \times MF$, where P is the price paid for the car, R is the residual value of the car, n is the number of months of the lease, and MF is the money factor for the lease.

EXAMPLE 6.105

Calculating Car Lease Payments

Calculate the lease payments for car with the following price, residual price, length of lease, and money factor or APR.

- Price is \$28,344, residual price is \$18,140.16, 24-month lease, money factor is 0.000025.
- Price is \$22,500, residual price is \$13,050, 36-month lease, APR is 7.5%.

Solution

- Substituting the values $P = \$28,344$, $R = \$18,140.16$, $n = 24$ and $MF = 0.000025$ into the formula and calculating, the monthly lease payment is

$$\begin{aligned} PMT &= \frac{(P - R)}{n} + (P + R) \times MF \\ &= \frac{(\$28,344 - \$18,140.16)}{24} + (\$28,344 + \$18,140.16) \times 0.000025 \\ &= \frac{\$10,203.84}{24} + (\$46,484.16) \times 0.000025 \\ &= \$426.33 \end{aligned}$$

- Given the APR, we find the MF which is $MF = APR/2400 = 0.075/2400 = 0.00003125$. Substituting the values $P = \$28,344$, $R = \$18,140.16$, $n = 24$ and the MF into the formula and calculating, the monthly lease payment is

$$\begin{aligned} PMT &= \frac{(P - R)}{n} + (P + R) \times MF \\ &= \frac{(\$22,500 - \$13,050)}{36} + (\$22,500 + \$13,050) \times 0.00003125 \\ &= \frac{\$9,450}{36} + (\$35,550) \times 0.000025 \\ &= \$263.62 \end{aligned}$$

YOUR TURN 6.105

Calculate the lease payments for car with the following price, residual price, length of lease, and money factor or APR.

- Price is \$38,750, residual price is \$18,140.16, 36-month lease, money factor is 0.000035.
- Price is \$45,600, residual price is \$21,312.50, 24-month lease, APR is 11.7%.

Comparing Purchasing and Leasing

When deciding to buy or lease a car, the differences between the two options should be carefully evaluated. The following is a list of points of comparison between the two.

- The payments for a lease are likely less than the payment for purchasing.
- When leasing, you get a new car after the lease term is over, typically 24 or 36 months. Buying the car means the same car is driven until it is re-sold and a new one bought. Essentially, leasing a car is equivalent to renting a car.
- The leased car is new, so all warranties are in force and you drive the car during its best years. When the car is purchased, it may be kept past its warranties and may be driven until it is quite old.
- Each time you lease a new car, all the fees and taxes must be paid again. When buying a car, these fees are only paid once.
- Leasing contracts carry restrictions on the mileage you can drive per year, and going over incurs more cost at the

end of the lease. Buying the car means no such mileage limits.

- When leasing, you are obligated to keep the vehicle in good condition and maintained according to the dealer's schedule. Some dealerships will even pay for oil changes over the life of the lease. When the car is purchased, the upkeep schedule is the choice of the owner.
- When a car is purchased and kept for long enough, the warranty expires and the owner is responsible for all maintenance items and repairs. The warranty for a car won't expire during the lease term.
- When a new car is purchased and the loan is paid off the car is still owned by the buyer and may be traded in when a new car is to be purchased. When leasing, the car is returned to the dealer when the lease term is over.

When deciding between the two, you are choosing between these features. If you aren't willing to drive an older car or deal with the upkeep that accompanies an older car, you may want to lease. This means you will need to pay those beginning costs each time the lease is up. If you want to own the car after the payments are over, then you may want to buy a car. This means you are paying for all the upkeep after the warranties expire, but you have no limits on mileage and own the car at the end. It really depends on your preferences.

EXAMPLE 6.106

Lease or Buy

In the following, determine if a lease or purchase of a car is better.

1. Joyce is concerned with large repairs and does not want to deal with them.
2. Maurice prefers to drive newer cars.

Solution

1. Since Joyce does not want to deal with repairs, so leasing would be a better choice. This way, the warranty covers most of the big repairs that could need to be done.
2. Since Maurice likes to drive new cars, leasing is a better option, since he will lease a new car every 2 to 3 years.

YOUR TURN 6.106

1. Christopher does a lot of driving, averaging over 20,000 miles per year.
2. John eventually wants to not make car payments.

Car Insurance

Car insurance is meant to cover costs associated with accidents involving cars. Most states (all except New Hampshire and Virginia) require some insurance. Without insurance, the state may not let you get a license for your car or register your car. Your state's requirements can be hard to follow. Fortunately, insurance companies and brokers will make sure your insurance is sufficient for your state and will warn you if you try to not meet the requirements. Of course, they may offer more than what is sufficient, so it is your responsibility to determine how much coverage you want, as long as the minimum insurance requirements are met. The cost of insurance should be accounted for when evaluating the affordability of buying or leasing a car.

Whether your car is leased or owned, you do need insurance. This contributes to the cost of having the vehicle. Leasing or owning makes no difference to the insurance company you choose, because they are insuring you based on what you are driving, your driving record, and other information about you including where you live and your age. These insurance policies have many components that address different costs that can come from auto accidents. This may make details confusing, and you may not realize what you are paying for until you must use it. Here is a brief outline of the different components of auto insurance, many of which are required by the state that issues your driver's license.

- **Liability insurance** is mandatory coverage in most states. Liability insurance covers property damage and injuries to others should you be found legally responsible for an accident. You are required to have the minimum amount of coverage, as determined by your state, in both areas.
- **Collision insurance** is insurance covering the damage caused to your car if involved in an accident with another vehicle.
- **Comprehensive insurance** is an extra level of coverage if involved in an accident with another vehicle and covers other things like theft, vandalism, fire, or weather events as outlined in your policy. There is a deductible assigned to each type of insurance, an amount that you pay out of pocket before your comprehensive coverage takes effect. Comprehensive insurance is often required if you lease or finance the purchase of a vehicle.

- **Uninsured or underinsured motorist insurance:** If you are hit by an uninsured or underinsured motorist, this insurance will help pay medical bills and damage to your car.
- **Medical payments insurance** is mandatory in some states and helps pay for medical costs associated with an accident, regardless of who is at fault.
- **Personal injury protection insurance** is coverage for certain medical bills and other expenses due to a car accident. Other covered expenses may include loss of income or childcare, depending on your policy.
- **Gap insurance** is designed to cover the gap between what is owed on the car and what the car is worth in the event your car is a total loss.
- **Rental reimbursement insurance** is coverage for a rental car while your car is under repair resulting from an accident.

You can also purchase other special insurance policies, such as classic car insurance, new car replacement insurance, and sound system replacement insurance, to name a few. It is important that you determine exactly what you need, as insurance policies can be expensive and vary according to your age, driving history, and where you live.

EXAMPLE 6.107

Types of Insurance

1. Which component of insurance pays if you are in an accident with a motorist without insurance?
2. Which component of insurance pays for the remaining principal owed on your car in the case of a total loss?

Solution

1. Uninsured motorist insurance covers accidents with those who have no insurance.
2. Gap insurance will cover the gap between what is owed on the car and what it is worth if an accident results in a total loss.

YOUR TURN 6.107

1. Which component of insurance covers medical bills in the event of an accident?
2. Which component of insurance pays for costs to others if you have an accident and are found legally responsible for those damages?

EXAMPLE 6.108

Monthly Cost of Owning a Car

If your car payment is \$287.50 per month and your car insurance is \$930 every 6 months, what is the cost of the car per month when accounting for the insurance?

Solution

The cost of the car including insurance is the monthly payment, \$287.50, plus the monthly cost of the insurance. The insurance cost per month is $\$930/6 = \155 since the insurance cost is for every 6 months. Adding those the cost with insurance is \$442.50.

YOUR TURN 6.108

1. If your car payment is \$410.86 per month and your car insurance is \$2,190 per year, what is the cost of the car per month when accounting for the insurance?

Maintaining a Car

Cars are not a buy it and forget it item. They require upkeep, which adds to the cost of owning the car. Tires, brakes, and wipers need replacing. Oil changes, inspections, so many things other than gasoline. Below is a list of some maintenance requirements for cars, along with cost and roughly how often they should happen.

Maintenance	Frequency	Cost Range
New Tires	Every 5 years	\$25–\$300 per tire
Oil Change	Every 3,000–6,000 miles	\$35–\$75
Wipers	Every 6–12 months	\$20–\$40
Inspection	Annual	\$10–\$50
Brake pads	10,000–20,000 miles	\$200–\$300
Air Filter	15,000–30,000 miles	\$35–\$80

When designing a budget, these expected costs should be accounted for. Extra money per month should be saved in addition to this budget category, to handle unanticipated, and perhaps very costly, repairs.

EXAMPLE 6.109

Estella needs to budget for her car maintenance. She expects to buy new tires each 4 years, which will cost her \$480 to replace them all. Oil changes near her cost \$49.99, and she believes she will get one every 4 months. Her inspection costs \$15 per year. Wipers for her car cost \$95 for all three and she anticipates changing them every year. She drives less than 30,000 miles per year, so she plans to replace the air filter once per year. The air filter for her car costs \$57.50. How much should she budget per month to cover these costs?

Solution

Her yearly costs are the wipers, inspection, and tires, which total \$167.50. Tires will be bought every 5 years, so per year she should budget \$96. Her oil changes, which will happen three times per year, cost \$49.99 each, so she'll spend \$149.97 for the year on oil changes. Adding these up, her yearly budget should include \$413.47 for maintenance. Dividing by 12 gives the monthly budget for maintenance, which is \$34.46 (rounded up to the next penny).

YOUR TURN 6.109

1. Natalie needs to budget for her car maintenance. She expects to buy new tires each 5 years, which will cost her \$390 to replace them all. Oil changes near her cost \$59.99, and she believes she will get one every 3 months. Her inspection costs \$25 per year. Wipers for her car cost \$115 for all three and she anticipates changing them every year. She drives less than 30,000 miles per year, so she plans to replace the air filter once per year. The air filter for her car costs \$46.25. How much should she budget per month to cover these costs?

Check Your Understanding

69. What is destination fee?
70. What is a title and registration fee?
71. Calculate the monthly payment if the total to be financed is \$34,570, the interest rate is 3.5%, for 5 years.
72. What might be different for a used car loan and a new car loan?
73. What fees are similar between leasing a car and buying a car?
74. If the cost of a car is \$30,000, and the residual value of the car after 3 years is \$18,000, what is the monthly depreciation for the car?
75. If the money factor for a lease is 0.000015625, what is the annual interest rate?
76. For people who do not mind driving an older car or maintaining a car, which is preferable, a lease or purchasing?

77. What does collision insurance cover?



SECTION 6.11 EXERCISES

1. What are the two main questions to answer before buying a car?
2. What is a documentation fee?
3. What is a dealer preparation fee?
4. What is a down payment?
5. How long do leases typically last?
6. Can you drive a leased car any mileage?
7. What are three fees associated with leasing a car?
8. What is the monthly depreciation for a leased car?
9. How are the money factor and the annual percentage rate related?
10. Name two advantages to leasing over buying a car.
11. Name two advantages to buying a car over leasing a car.
12. What does liability insurance cover?
13. What does collision insurance cover?
14. Many expenses associated with a car can be anticipated. Name three maintenance expenses that can be anticipated.

In the following exercises, find the total cost to purchase the car.

15. Alexia negotiates a purchase price of \$17,850 for her new car. The sales tax in her area is 6.5%. Her license, plates, and registration come to \$285.00. The dealership charges her a \$600 destination fee and a \$150 processing fee. How much will she finance in total for the car?
16. Stephanie negotiates a purchase price of \$25,670 for her new car. The sales tax in her area is 8.0%. Her license, plates, and registration come to \$389.00. The dealership charges her a \$700 destination fee and a \$345 processing fee. How much will she finance in total for the car?
17. Matthew negotiates a purchase price of \$35,100 for her new car. The sales tax in his area is 7.25%. His license, plates, and registration come to \$325.00. The dealership charges him a \$900 destination fee and a \$125 processing fee. How much will he finance in total for the car?
18. Madisyn negotiates a purchase price of \$45,800 for her new car. The sales tax in her area is 7.25%. Her license, plates, and registration come to \$199.00. The dealership charges her a \$1,000 destination fee and a \$275 processing fee. How much will she finance in total for the car?

In the following exercises, calculate the car payment based on the total financed and the interest rate.

19. Total to be financed is \$36,775, interest rate is 2.75%, for 6 years.
20. Total to be financed is \$29,350, interest rate is 3.9%, for 5 years.
21. Total to be financed is \$27,180, interest rate is 1.99%, for 7 years.
22. Total to be financed is \$15,489, interest rate is 6.75%, for 4 years.

In the following exercises, find the total cost to obtain the lease.

23. A \$2,000.00 down payment, \$120 in acquisition fees, a security deposit of \$350.00, \$200.00 in disposition fees, and sales tax of \$2,860.
24. No down payment, \$260 in acquisition fees, a security deposit of \$450.00, \$400.00 in disposition fees, and sales tax of \$3,155.00.
25. A \$4,000 down payment, \$360 in acquisition fees, a security deposit of \$900.00, \$1,000.00 in disposition fees, and sales tax of \$4,275.
26. A \$7,000.00 down payment, \$225 in acquisition fees, a security deposit of \$800.00, \$675.00 in disposition fees, and sales tax of \$3,673.

In the following exercises, find the monthly depreciation of the car.

27. The purchase price of a car is \$34,000. Its residual price is \$23,500. What is its monthly depreciation for a 24-month lease?

28. The purchase price of a car is \$23,500. Its residual price is \$11,750. What is its monthly depreciation for a 36-month lease?

In the following exercises find the APR based on the MF.

29. $MF = 0.00004125$
 30. $MF = 0.000046458\overline{3}$

In the following exercises find the MF based on the APR.

31. 8.75%
 32. 6.25%

In the following exercises, find the lease payment based on the given information.

33. Price is \$41,700, residual price is \$27,105, 24-month lease, money factor is 0.000025.
 34. Price is \$22,165, residual price is \$12,855.70, 24-month lease, money factor is 0.0000275.
 35. Price is \$30,650, residual price is \$16,857.50, 36-month lease, APR is 8.25%.
 36. Price is \$24,800, residual price is \$14,384, 36-month lease, APR is 5.85%.
37. Sara needs to budget for her car maintenance. She expects to buy new tires each 3 years, which will cost her \$540 to replace them all. Oil changes near her cost \$39.99, and she believes she will get one every 4 months. Her inspection costs \$20 per year. Wipers for her car cost \$115 for all three and she anticipates changing them every year. She drives less than 30,000 miles per year, so she plans to replace the air filter once per year. The air filter for her car costs \$36.25. She plans to get new brake pads every year, which cost her \$267. How much should she budget per month to cover these costs?
38. Mwibeleca needs to budget for his car maintenance. He expects to buy new tires each 3 years, which will cost her \$560 to replace them all. Oil changes near his cost \$69.99, and he believes he will get one every 4 months. His inspection costs \$25 per year. Brake pads will cost \$215 each year. Wipers for his car cost \$143.50 for all three and he anticipates changing them every year. He drives less than 30,000 miles per year, so he plans to replace the air filter once per year. The air filter for his car costs \$62.88. How much should Mwibeleca budget per month to cover these costs?
39. John will either lease or buy a car. The total cost to purchase the car is \$35,830, and he would finance the car for 5 years at 2.99%. If he leases, he would pay \$3,287.50 for the lease, and then his payments would be based on a price of \$32,750, a residual price of \$20,305, 36 months, with a money factor of 0.00001725. Compare the payments to purchase the car to the payments of the lease plus the lease cost divided by 36.
40. Zachary will either lease or buy a car. The total cost to purchase the car is \$22,945, and he would finance the car for 6 years at 1.99%. If he leases, he would pay \$2,387.75 for the lease, and then his payments would be based on a price of \$21,350, a residual price of \$12,390.30, 36 months, with a money factor of 0.000018375. Compare the payments to purchase the car to the payments of the lease plus the lease cost divided by 36.

6.12 Renting and Homeownership



Figure 6.26 Purchasing a home is a big investment, while renting is a lower cost alternative. (credit: "New construction, new development house for sale" by jongorey/houseandhammer.com, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Evaluate advantages and disadvantages of renting.
2. Evaluate advantages and disadvantages of homeownership.
3. Calculate the monthly payment for a mortgage and related interest cost.
4. Read and interpret an amortization schedule.
5. Solve application problems involving affordability of a mortgage.

After renting an apartment for 10 years, you realize that it may be time to purchase a home. Your job is stable, and you could use more space. It is time to investigate becoming a homeowner. What are the things that you must consider, and what is the financial benefit of owning as opposed to renting? This section is about the advantages, disadvantages, and costs of homeownership as opposed to renting.

Advantages and Disadvantages of Renting

When renting, you will likely sign a **lease**, which is a contract between a renter and a landlord. A **landlord** is the person or company that owns property that is rented. The lease will detail your responsibilities, restrictions on activities, deposits, fees, maintenance, repairs, and rent during the term of the lease. It also defines what your landlord can, and cannot, do with the property while you occupy the property.

Like leasing a car, there are advantages to renting but also some disadvantages. Some advantages are:

- Lower cost.
- Short-term commitment.
- Little to no maintenance cost. The landlord pays for or performs most maintenance.
- You need not stay at end of lease. Once the lease term is over (the lease is up), you are not obligated to stay.
- If renting in an apartment complex, there may be a pool, gym, or community room for renters to use.

Of course, there are disadvantage too:

- No tax incentives.
- Housing cost is not fixed. When the lease is up, the rent can change.
- No equity. When you are done living in a rental, you have built no value.
- Restrictions on occupants. There may be a limit on how many can live in the apartment.
- Restrictions on decorating. The property is not yours, so any decorating or improvements need landlord permission.
- Limits on pets. Permission for pets, and their number and type, will be set forth in the lease.
- May not be able to remain when lease term is over. The landlord can, at the end of your lease, invite you to leave.
- The building may be sold, and the new landlord may institute changes to the lease when the previous lease expires.

Renting has fees to be paid at the start of the lease. Typically, when you rent, you will pay first and last months' rent and a security deposit. A security deposit is a sum of money that the landlord holds until the renter leaves the rental property. The deposit will cover repairs for damage to the apartment during the renter's stay but may be returned if the apartment is in good condition. If your landlord runs a credit check on you, the landlord may charge you for that.

Advantages of Buying a Home

The advantages to buying a home mirror the disadvantages of renting, and the disadvantages of home ownership mirror the advantages of renting.

Some advantages to buying a home are:

- There are tax incentives. The interest you pay for your mortgage (more on that later) is deductible on your federal income tax.
- There are no restrictions on pets or occupants, unless laws in your area specify limits for homes.
- You can redecorate any way you wish, limited only by the laws in your area.
- Once your mortgage is set with a fixed-interest rate, your housing cost is fixed.
- Your home grows equity, that is, the difference between what you owe and what the house is worth grows. You can use the equity to secure loans, and you recover the equity (and more if you're fortunate) when you sell the house.
- As long as you pay your mortgage and maintain the home to the standards of your community, you can stay as long as you wish.

Some disadvantages to home ownership are:

- The cost is higher than renting. Mortgages and associated costs are typically higher than rent for a similar living space.
- The owner is responsible for upkeep, maintenance, and repairs. These can be extremely costly.
- The owner cannot walk away from the property. It can be sold, but simply leaving the property, especially if not paid off yet, has serious consequences.

The big question of affordability looms large over the decision to rent or buy. Renting, strictly from an affordability viewpoint, comes with much less initial outlay and smaller commitment. If you do not have sufficient income to regularly save for possibly expensive repairs, or your credit isn't quite as good as it needs to be, then renting may be the best choice. Of course, even if you can afford to buy a home, you may choose to rent based on the comparative advantages.

VIDEO

[Rent or Buy \(https://openstax.org/r/Rent_or_Buy\)](https://openstax.org/r/Rent_or_Buy)

Buying a home really involves two buyers. You and the mortgage company. The mortgage company has interest in the home, as they are providing the funds for the home. They want to protect their investment, and many fees are about the bank as much as the buyer. They fund a mortgage based on the value they assign the property. Not you. This means they will want some certainty that the home is sound, and you are a good investment.

WHO KNEW?

Closing Costs

When a home is bought, there are many costs that need to be paid at the time of purchase, which are lumped under the term closing costs. At the start of 2022, the average closing costs for a single-family home exceeded \$6,800. These costs include:

- The appraisal fee, which is what is paid to someone to establish the home's worth. The value of the home to the bank may differ from what the home is listed for, or what an app tells you the home is worth. It may run approximately \$350.
- The home inspection fee. The inspection should reveal any problems with the house that will need to be fixed either before or after you obtain the home.
- The title search. This is a records search to insure there are no issues with who actually owns the property. It can cost about 0.5% to 1% of the amount you are financing.
- Prepaid taxes. You will need to pay about 6 months of taxes at the time of purchase.
- The credit report fee. This is a fee for checking your credit. You might pay \$25 or more for this.
- The origination fee. This is the price the mortgage company charges you to cover the costs of creating the mortgage. This could be 0.5% to 1% (or more) of the amount you are borrowing.
- The application fee. This is just a processing fee and could come to several hundred dollars.
- The underwriting fee. This covers the cost of verifying your financial qualifications. It could be a flat fee, or some

small percentage of the amount financed. Such as 0.5% or 1%.

- Attorney fees. If you use an attorney, you will have to pay the attorney.
- State of local fees. This may include a filing fee charged by the county or municipality in which you reside.

That's a long list, and it is not even complete. When buying, be prepared to see these costs. It can be surprising. But in the end, you will have equity in the home, which means when you sell your home, you will get some of your money back.

VIDEO

[Closing Costs \(https://openstax.org/r/closing_costs\)](https://openstax.org/r/closing_costs)

In the end, you must weigh your options and carefully consider your priorities in choosing to rent or buy a home.

Mortgages

Some people will purchase a home or condo with cash, but the majority of people will apply for a mortgage. A **mortgage** is a long-term loan and the property itself is the security. The bank decides the minimum down payment (with your input), the payment schedule, the duration of the loan, whether the loan can be assumed by another party, and the penalty for late payments. The title of the home belongs to the bank.

Since a mortgage is a loan, everything about loans from [The Basics of Loans](#) holds true, including the formula for the payments.

Monthly Mortgage Payments

The formula to calculate your monthly payments of principal and interest uses APR as the annual interest rate.

FORMULA

The payment, pmt , per month to pay down a mortgage with beginning principal P is

$pmt = \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1}$, where r is the annual interest rate in decimal form and t is the number of years of the payment.

Note, payment to lenders is always rounded up to the next penny.

To find the total amount of your payments over the life of the loan, multiply your monthly payments by the number of payments.

EXAMPLE 6.110

30-Year Mortgage at 4.8% Interest

Evan buys a house. His 30-year mortgage comes to \$132,650 with 4.8% interest. Find Evan's monthly payments.

✓ Solution

Using the information above, $P = \$132,650$, $r = 0.048$ and $t = 30$. Substituting those values into the formula

$pmt = \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1}$ and calculating, we find the payment is

$$\begin{aligned}
 pmt &= \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1} \\
 &= \frac{\$132,650 \times (0.048/12) \times (1 + 0.048/12)^{12 \times 30}}{(1 + 0.048/12)^{12 \times 30} - 1} \\
 &= \frac{\$132,650 \times (0.004) \times (1.004)^{360}}{(1.004)^{360} - 1} \\
 &= \frac{\$2,233.07781448}{3.20858992551} \\
 &= \$695.97
 \end{aligned}$$

His mortgage payment is \$695.97.

YOUR TURN 6.110

1. Paulo buys a house. His 20-year mortgage comes to \$153,899 with 4.21% interest. Find Evan's monthly payments.

To find the total amount of your payments over the life of the loan, multiply your monthly payments by the number of payments. This can be useful information, but not too many people reach the end of their mortgage. They tend to move before the mortgage is paid off.

FORMULA

The total paid, T , on an t year mortgage with monthly payments pmt is $T = pmt \times 12 \times t$.

EXAMPLE 6.111

30-Year Mortgage at 5.35% Interest

Cassandra buys a house. Her 30-year mortgage comes to \$99,596 with 5.35% interest. If Cassandra pays off the mortgage over those 30 years, how much will she have paid in total?

Solution

To find the total paid over the life of the mortgage, use the formula $T = pmt \times 12 \times t$. To calculate this, the payment must be found. Using the information above, $P = \$99,596$, $r = 0.0535$ and $t = 30$. Substituting those values into the

formula $pmt = \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1}$ and calculating, we find the payment is

$$\begin{aligned} pmt &= \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1} \\ &= \frac{\$99,596 \times (0.0535/12) \times (1 + 0.0535/12)^{12 \times 30}}{(1 + 0.048/12)^{12 \times 30} - 1} \\ &= \frac{\$99,596 \times (0.004458\bar{3}) \times (1.004458\bar{3})^{360}}{(1.004458\bar{3})^{360} - 1} \\ &= \frac{\$2,202.45911222}{3.96013414693} \\ &= \$556.16 \end{aligned}$$

Using the mortgage payment of \$556.16 and $t = 30$ years in the formula $T = pmt \times 12 \times t$, the total that Cassandra will pay for the mortgage is \$200,217.60.

YOUR TURN 6.111

1. Arthur buys a house. Their 15-year mortgage comes to \$225,879 with 4.91% interest. If Arthur pays off the mortgage over those 15 years, how much will they have paid in total?

With the principal of the mortgage and how much total is paid over the life of the mortgage, the cost of financing can be found by subtracting the principal of the mortgage from the total paid over the life of the mortgage.

FORMULA

The cost of financing a mortgage, CoF, is $\text{CoF} = T - P$ where P is the mortgage's starting principal and T is the total

paid over the life of the mortgage.

EXAMPLE 6.112

30-Year Mortgage at 5.35% Interest

Cassandra buys a house. Her 30-year mortgage comes to \$99,596 with 5.35% interest. What was Cassandra's cost of financing?

✓ Solution

In [Example 6.111](#), we found that the total Cassandra will pay for the \$99,569 mortgage is \$200,217.60. Subtracting those we find the cost of financing $\text{CoF} = T - P = \$200,217.60 - \$99,596 = \$100,621.60$.

> YOUR TURN 6.112

1. Arthur buys a house. Their 15-year mortgage comes to \$225,879 with 4.91% interest. What was Arthur's cost of financing?

? WHO KNEW?

Private Mortgage Insurance (PMI)

When you purchase a home, you will have to pay a down payment. This means you have money tied to the property, which lenders believe makes you less likely to walk away from a property. The amount of the down payment will be decided between you and the mortgage company. However, if your down payment is less than 20% of the property value, you will be required to pay private mortgage insurance (PMI). This is insurance you pay for so that the mortgage company is protected if *you* default on the loan. It often comes to between 0.5% and 2.25% of the original loan amount. It increases your monthly payment. Once you reach 20% of the loan value, you can request that the PMI be dropped. Even if you do not request cancelling the PMI, it will eventually and automatically be dropped.

For more, see [this article on ways to get rid of PMI \(https://openstax.org/r/mortgage_insurance\)](https://openstax.org/r/mortgage_insurance).

Reading and Interpreting Amortization Tables

Amortization tables were addressed in [The Basics of Loans](#). They are most frequently encountered when analyzing mortgages.

The amortization table for a 30-year mortgage is quite long, containing 360 rows. A full table will not be reproduced here. We can, though, read information from a portion of an amortization table.

EXAMPLE 6.113

Amortization Table for a 30-Year, \$165,900 Mortgage

[Figure 6.27](#) shows a portion of an amortization table for a 30-year, \$165,900 mortgage. Use that table to answer the following questions.

1. What is the interest rate?
2. How much are the payments?
3. How much of payment 175 goes to principal?
4. How much of payment 180 goes to interest?
5. What's the remaining balance on the mortgage after payment 170?

Amortization Schedule Calculator					
Loan Amount		\$165,900.00			
Interest Rate (annual)		5.61%			
Term (Years)		30			
Monthly Payment		\$953.44			
Month	Payment	Principal	Interest	Total Interest	Balance
167	\$953.44	\$385.77	\$567.67	\$114,366.57	\$121,041.52
168	\$953.44	\$387.57	\$565.87	\$114,932.44	\$120,653.95
169	\$953.44	\$389.39	\$564.06	\$115,496.50	\$120,264.56
170	\$953.44	\$391.21	\$562.24	\$116,058.73	\$119,873.35
171	\$953.44	\$393.04	\$560.41	\$116,619.14	\$119,480.32
172	\$953.44	\$394.87	\$558.57	\$117,177.71	\$119,085.44
173	\$953.44	\$396.72	\$556.72	\$117,734.44	\$118,688.73
174	\$953.44	\$398.57	\$554.87	\$118,289.31	\$118,290.15
175	\$953.44	\$400.44	\$553.01	\$118,842.31	\$117,889.71
176	\$953.44	\$402.31	\$551.13	\$119,393.45	\$117,487.41
177	\$953.44	\$404.19	\$549.25	\$119,942.70	\$117,083.22
178	\$953.44	\$406.08	\$547.36	\$120,490.06	\$116,677.14
179	\$953.44	\$407.98	\$545.47	\$121,035.53	\$116,269.16
180	\$953.44	\$409.89	\$543.56	\$121,579.09	\$115,859.27
181	\$953.44	\$411.80	\$541.64	\$122,120.73	\$115,447.47
182	\$953.44	\$413.73	\$539.72	\$122,660.45	\$115,033.75
183	\$953.44	\$415.66	\$537.78	\$123,198.23	\$114,618.09
184	\$953.44	\$417.60	\$535.84	\$123,734.07	\$114,200.48
185	\$953.44	\$419.56	\$533.89	\$124,267.96	\$113,780.93
186	\$953.44	\$421.52	\$531.93	\$124,799.88	\$113,359.41
187	\$953.44	\$423.49	\$529.96	\$125,329.84	\$112,935.92
188	\$953.44	\$425.47	\$527.98	\$125,857.81	\$112,510.45
189	\$953.44	\$427.46	\$525.99	\$126,383.80	\$112,082.99

Figure 6.27 Amortization table

✓ **Solution**

1. Reading at the top of the table, we see the interest rate is 5.61%.
2. Reading from the top of the table or from the column labeled Payment, we see the payments are \$953.44 per month.
3. In the row for payment 175, we see that the amount that goes to principal is \$400.44.
4. In the row for payment 180, we see that the amount that goes to interest is \$543.56.
5. In the row for payment 170, we see the remaining balance is \$119,873.35.

> **YOUR TURN 6.113**

Below is a portion of an amortization table for a 30-year, \$228,320 mortgage. Use that table to answer the following questions.

Amortization Schedule Calculator					
Loan Amount	\$228,320.00				
Interest Rate (annual)	6.10%				
Term (Years)	30				
Monthly Payment	\$1,383.61				
Month	Payment	Principal	Interest	Total Interest	Balance
213	\$1,383.61	\$655.29	\$728.32	\$209,405.96	\$143,013.61
214	\$1,383.61	\$656.61	\$727.00	\$210,130.96	\$142,359.00
215	\$1,383.61	\$659.95	\$723.66	\$210,854.62	\$141,699.05
216	\$1,383.61	\$663.30	\$720.30	\$211,574.92	\$141,035.74
217	\$1,383.61	\$666.68	\$716.93	\$212,291.85	\$140,369.07
218	\$1,383.61	\$670.06	\$713.54	\$213,005.39	\$139,699.00
219	\$1,383.61	\$673.47	\$710.14	\$213,715.53	\$139,025.53
220	\$1,383.61	\$676.89	\$706.71	\$214,422.24	\$138,348.64
221	\$1,383.61	\$680.34	\$703.27	\$215,125.52	\$137,668.30
222	\$1,383.61	\$683.79	\$699.81	\$215,825.33	\$136,984.51
223	\$1,383.61	\$687.27	\$696.34	\$216,521.67	\$136,297.24
224	\$1,383.61	\$690.76	\$692.84	\$217,214.51	\$135,606.48
225	\$1,383.61	\$694.27	\$689.33	\$217,903.84	\$134,912.20
226	\$1,383.61	\$697.80	\$685.80	\$218,589.65	\$134,214.40
227	\$1,383.61	\$701.35	\$682.26	\$219,271.90	\$133,513.05
228	\$1,383.61	\$704.92	\$678.69	\$219,950.60	\$132,808.13
229	\$1,383.61	\$708.50	\$675.11	\$220,625.70	\$132,099.63
230	\$1,383.61	\$712.10	\$671.51	\$221,297.21	\$131,387.53
231	\$1,383.61	\$715.72	\$667.89	\$221,965.10	\$130,671.81
232	\$1,383.61	\$719.36	\$664.25	\$222,629.35	\$129,952.45
233	\$1,383.61	\$723.02	\$660.59	\$223,289.94	\$129,229.44
234	\$1,383.61	\$726.69	\$656.92	\$223,946.85	\$128,502.75
235	\$1,383.61	\$730.38	\$653.22	\$224,600.08	\$127,772.36
236	\$1,383.61	\$734.10	\$649.51	\$225,249.59	\$127,038.26

Amortization table

1. What is the interest rate?
2. How much are the payments?
3. How much of payment 235 goes to principal?
4. How much of payment 215 goes to interest?
5. What's the remaining balance on the mortgage after payment 227?
6. At what payment does the amount that is applied to mortgage finally exceed the amount applied to interest?

Escrow Payments

The last few examples have looked at mortgage payments, which cover the principal and interest. However, when you take out a mortgage, the payment is sometimes much higher than that. This is because your mortgage company also has you pay into an **escrow account**, which is a savings account maintained by the mortgage company.

Your insurance payments will be set by your insurer and the mortgage company will pay them on time for you from your escrow account. Your property taxes are set by where you live and are typically a percentage of your property's **assessed value**. The assessed value is the estimation of the value of your home and does not necessarily reflect the purchase or resale value of the home. Your property taxes will also be paid on time by the mortgage company from your escrow account.

For example, in Kalamazoo, Michigan, the effective tax rate for property is 1.69% of the assessed value of the home. These escrow payments, which cover bills for the home, can increase the monthly payments for your home well beyond the basic principal and interest payment.

EXAMPLE 6.114

Adding Escrow Payments to Mortgage Payments

Jenna decides to purchase a home, with mortgage of \$108,450 at 6% interest for 30 years. The assessed value of her home is \$75,600. Her property taxes come to 5.7% of her assessed value. Jenna also has to pay her home insurance every 6 months, which is \$744 per six months. How much, including escrow, will Jenna pay per month?

Solution

Using the payment function to find her mortgage payments, $pmt = \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1}$, with $P = \$108,405$, $r =$

0.06, and $t = 30$, her payments are

$$\begin{aligned} pmt &= \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1} \\ &= \frac{\$108,405 \times (0.06/12) \times (1 + 0.06/12)^{12 \times 30}}{(1 + 0.06/12)^{12 \times 30} - 1} \\ &= \$649.95 \end{aligned}$$

Jenna also pays into escrow 1/12 of her property taxes per month. Her property taxes are 5.7% of the assessed value of \$75,600, which comes to $0.057 \times \$75,600 = \$4,309.20$. This is an annual tax, so she pays 1/12 of that each month, or \$359.10. Jenna's home insurance is \$744 per 6 months, so each month she pays \$124.00 for insurance. Adding these together, her monthly payment is $\$649.95 + \$359.10 + \$124.00 = \$1,133.05$. This is quite a bit more than the \$649.95 for the principal and interest.

> YOUR TURN 6.114

1. Destiny decides to purchase a home, with mortgage of \$159,195.50 at 5.75% interest for 30 years. The assessed value of her home is \$100,000. Her property taxes come to 5.42% of her assessed value. Destiny also has to pay her home insurance every 6 months, and that comes to \$843 per 6 months. How much, including escrow, will Destiny pay per month?

Check Your Understanding

78. Does renting or buying have tax advantages?
79. Which has more restrictions, renting or buying?
80. Which has housing cost that does not change?
81. What is the name given to a loan for a home?
82. What are the monthly payments for a 30-year mortgage of \$108,993 with 6.14% interest.
83. What is the cost of financing for a 30-year mortgage of \$108,993 with 6.14% interest if the mortgage is paid off?
84. Consider the following amortization table. What is the amount of payment 141 that goes to principal?

Amortization Schedule Calculator					
Loan Amount	\$274,800.00				
Interest Rate (annual)	5.90%				
Term (Years)	30				
Monthly Payment	\$1,629.94				
Month	Payment	Principal	Interest	Total Interest	Balance
136	\$1,629.94	\$540.64	\$1,089.30	\$167,882.89	\$221,011.17
137	\$1,629.94	\$543.30	\$1,086.64	\$168,969.53	\$220,467.87
138	\$1,629.94	\$545.97	\$1,083.97	\$170,053.50	\$219,921.90
139	\$1,629.94	\$548.66	\$1,081.28	\$171,134.78	\$219,373.24
140	\$1,629.94	\$551.35	\$1,078.59	\$172,213.37	\$218,821.89
141	\$1,629.94	\$554.06	\$1,075.87	\$173,289.24	\$218,267.83
142	\$1,629.94	\$556.79	\$1,073.15	\$174,362.39	\$217,711.04
143	\$1,629.94	\$559.53	\$1,070.41	\$175,432.80	\$217,151.51
144	\$1,629.94	\$562.28	\$1,067.66	\$176,500.47	\$216,589.23
145	\$1,629.94	\$565.04	\$1,064.90	\$177,565.36	\$216,024.19
146	\$1,629.94	\$567.82	\$1,062.12	\$178,627.48	\$215,456.37
147	\$1,629.94	\$570.61	\$1,059.33	\$179,686.81	\$214,885.76
148	\$1,629.94	\$573.42	\$1,056.52	\$180,743.33	\$214,312.34
149	\$1,629.94	\$576.24	\$1,053.70	\$181,797.03	\$213,736.10
150	\$1,629.94	\$579.07	\$1,050.87	\$182,847.90	\$213,157.03
151	\$1,629.94	\$581.92	\$1,048.02	\$183,895.92	\$212,575.12
152	\$1,629.94	\$584.78	\$1,045.16	\$184,941.09	\$211,990.34
153	\$1,629.94	\$587.65	\$1,042.29	\$185,983.37	\$211,402.69
154	\$1,629.94	\$590.54	\$1,039.40	\$187,022.77	\$210,812.14
155	\$1,629.94	\$593.45	\$1,036.49	\$188,059.26	\$210,218.70
156	\$1,629.94	\$596.36	\$1,033.58	\$189,092.84	\$209,622.33
157	\$1,629.94	\$599.30	\$1,030.64	\$190,123.48	\$209,023.04
158	\$1,629.94	\$602.24	\$1,027.70	\$191,151.18	\$208,420.80

85. What is the name of the account that the mortgage company holds your taxes and insurance in?



SECTION 6.12 EXERCISES

In the following exercises, indicate if the advantage listed is for renting or buying a home.

1. Short-term commitment.
2. Tax advantage.
3. Freedom to remodel.
4. Builds equity.
5. Cost is lower.
6. You do not pay for repairs.
7. No pet restrictions
8. More flexibility to move.
9. Housing cost is fixed.
10. May have other amenities.

In the following exercises, find the mortgage payment for the given loan amount, interest rate, and term.

11. Loan amount is \$78,560, interest rate is 5.87%, 30-year mortgage.
12. Loan amount is \$125,800, interest rate is 6.5%, 30-year mortgage.
13. Loan amount is \$96,400, interest rate is 4.9%, 15-year mortgage.
14. Loan amount is \$267,450, interest rate is 5.25%, 20-year mortgage.

In the following exercises, find the total paid on the mortgage if it is fully paid through the term.

15. Loan amount is \$78,560, interest rate is 5.87%, 30-year mortgage.
16. Loan amount is \$125,800, interest rate is 6.5%, 30-year mortgage.
17. Loan amount is \$96,400, interest rate is 4.9%, 15-year mortgage.
18. Loan amount is \$267,450, interest rate is 5.25%, 20-year mortgage.

In the following exercises, find the cost of financing for the mortgages if they are fully paid.

19. Loan amount is \$78,560, interest rate is 5.87%, 30-year mortgage.
20. Loan amount is \$125,800, interest rate is 6.5%, 30-year mortgage.
21. Loan amount is \$96,400, interest rate is 4.9%, 15-year mortgage.
22. Loan amount is \$267,450, interest rate is 5.25%, 20-year mortgage.

In the following exercises, use the amortization table to answer the question.

Amortization Schedule Calculator					
Loan Amount	\$94,176.00				
Interest Rate (annual)	6.30%				
Term (Years)	30				
Monthly Payment	\$582.92				
Month	Payment	Principal	Interest	Total Interest	Balance
147	\$582.92	\$190.09	\$392.83	\$66,149.28	\$74,635.48
148	\$582.92	\$191.09	\$391.84	\$66,541.12	\$74,444.39
149	\$582.92	\$192.09	\$390.83	\$66,931.95	\$74,252.30
150	\$582.92	\$193.10	\$389.82	\$67,321.77	\$74,059.20
151	\$582.92	\$194.11	\$388.81	\$67,710.59	\$73,865.09
152	\$582.92	\$195.13	\$387.79	\$68,098.38	\$73,669.96
153	\$582.92	\$196.16	\$386.77	\$68,485.14	\$73,473.80
154	\$582.92	\$197.19	\$385.74	\$68,870.88	\$73,276.61
155	\$582.92	\$198.22	\$384.70	\$69,255.58	\$73,078.39
156	\$582.92	\$199.26	\$383.66	\$69,639.25	\$72,879.13
157	\$582.92	\$200.31	\$382.62	\$70,021.86	\$72,678.82
158	\$582.92	\$201.36	\$381.56	\$70,403.42	\$72,477.46
159	\$582.92	\$202.42	\$380.51	\$70,783.93	\$72,275.05
160	\$582.92	\$203.48	\$379.44	\$71,163.38	\$72,071.57
161	\$582.92	\$204.55	\$378.38	\$71,541.75	\$71,867.02
162	\$582.92	\$205.62	\$377.30	\$71,919.05	\$71,661.40
163	\$582.92	\$206.70	\$376.22	\$72,295.28	\$71,454.69
164	\$582.92	\$207.79	\$375.14	\$72,670.41	\$71,246.91
165	\$582.92	\$208.88	\$374.05	\$73,044.46	\$71,038.03
166	\$582.92	\$209.97	\$372.95	\$73,417.41	\$70,828.06
167	\$582.92	\$211.08	\$371.85	\$73,789.26	\$70,616.98
168	\$582.92	\$212.18	\$370.74	\$74,159.99	\$70,404.79
169	\$582.92	\$213.30	\$369.63	\$74,529.62	\$70,191.50

23. What is the term of the mortgage?
24. How much of payment 165 applies to interest?

25. What is the remaining balance after payment 155?
26. How much total interest was paid after payment 149?

In the following exercises, use the amortization schedule to answer the question.

Amortization Schedule Calculator					
Loan Amount	\$168,458.00				
Interest Rate (annual)	5.85%				
Term (Years)	15				
Monthly Payment	\$1,407.93				
Month	Payment	Principal	Interest	Total Interest	Balance
93	\$1,407.93	\$917.74	\$490.19	\$62,112.60	\$99,633.22
94	\$1,407.93	\$922.22	\$485.71	\$62,598.32	\$98,711.00
95	\$1,407.93	\$926.71	\$481.22	\$63,079.53	\$97,784.29
96	\$1,407.93	\$931.23	\$476.70	\$63,556.23	\$96,853.06
97	\$1,407.93	\$935.77	\$472.16	\$64,028.39	\$95,917.29
98	\$1,407.93	\$940.33	\$467.60	\$64,495.99	\$94,976.96
99	\$1,407.93	\$944.92	\$463.01	\$64,959.00	\$94,032.04
100	\$1,407.93	\$949.52	\$458.41	\$65,417.40	\$93,082.52
101	\$1,407.93	\$954.15	\$453.78	\$65,871.18	\$92,128.37
102	\$1,407.93	\$958.80	\$449.13	\$66,320.31	\$91,169.56
103	\$1,407.93	\$963.48	\$444.45	\$66,764.76	\$90,206.09
104	\$1,407.93	\$968.17	\$439.75	\$67,204.51	\$89,237.91
105	\$1,407.93	\$972.89	\$435.03	\$67,639.55	\$88,265.02
106	\$1,407.93	\$977.64	\$430.29	\$68,069.84	\$87,287.38
107	\$1,407.93	\$982.40	\$425.53	\$68,495.37	\$86,304.98
108	\$1,407.93	\$987.19	\$420.74	\$68,916.10	\$85,317.79
109	\$1,407.93	\$992.00	\$415.92	\$69,332.03	\$84,325.78
110	\$1,407.93	\$996.84	\$411.09	\$69,743.12	\$83,328.94
111	\$1,407.93	\$1,001.70	\$406.23	\$70,149.34	\$82,327.24
112	\$1,407.93	\$1,006.58	\$401.35	\$70,550.69	\$81,320.66
113	\$1,407.93	\$1,011.49	\$396.44	\$70,947.13	\$80,309.17
114	\$1,407.93	\$1,016.42	\$391.51	\$71,338.63	\$79,292.74
115	\$1,407.93	\$1,021.38	\$386.55	\$71,725.19	\$78,271.37

27. What is the interest rate for the mortgage?
28. How much of payment 110 applies to principal?
29. What is the remaining balance after payment 94?
30. How much total interest was paid after payment 111?

In the following exercises, find the total monthly payment including both the mortgage payment and the escrow payment.

31. Mortgage of \$87,690 at 6.2% interest for 30 years. Assessed value of the home is \$75,600. Property taxes come to 5.65% of assessed value. Home insurance of \$815 paid every 6 months.
32. Mortgage of \$143,900 at 5.05% interest for 30 years. Assessed value of the home is \$90,150. Property taxes come to 5.88% of assessed value. Home insurance of \$924 paid every 6 months.
33. Mortgage of \$65,175 at 6.48% interest for 30 years. Assessed value of the home is \$62,800. Property taxes come to 6.75% of assessed value. Home insurance of \$558 paid every 6 months.
34. Mortgage of \$245,950 at 5.35% interest for 30 years. Assessed value of the home is \$156,500. Property taxes come to 6.41% of assessed value. Home insurance of \$972 paid every 6 months.

For the following exercises, read the following: **Fifteen-year mortgage compared to 30-year mortgage.** Mortgage interest rates are often higher for 30-year mortgages than 15-year mortgages. However, the payments for 15-year mortgages are considerably higher. The following exercises explore the difference between a 15- and 30-year mortgage for a mortgage of \$100,000.

35. The 15-year mortgage interest rate is 5.65%.
- Find the payment.
 - Determine the total that would be paid if the mortgage was completed.
 - Find the cost of financing for this mortgage.
36. The 30-year mortgage rate is 6.4%.
- Find the payment.
 - Determine the total that would be paid if the mortgage was completed.
 - Find the cost of financing for this mortgage.
37. How different are the payments, the total paid, and the cost to finance?
38. Summarize the answer in the previous question.

For the following exercises, read the following: **Fifteen-year mortgage compared to 30-year mortgage.** A 15-year

mortgage comes with advantages, the biggest being the home is paid off much sooner, and equity is built much more quickly. Mortgage interest rates are often higher for 30-year mortgages than 15-year mortgages. However, the payments for 15-year mortgages are considerably higher. The following exercises explore the difference between a 15- and 30-year mortgage for a mortgage of \$200,000.

39. The 15-year mortgage interest rate is 5.6%.
 - a. Find the payment.
 - b. Determine the total that would be paid if the mortgage was completed.
 - c. Find the cost of financing for this mortgage.
40. The 30-year mortgage rate is 6.25%.
 - a. Find the payment.
 - b. Determine the total that would be paid if the mortgage was completed.
 - c. Find the cost of financing for this mortgage.
41. How different are the payments, the total paid, and the cost to finance?
42. Summarize the answer in the previous question.

6.13 Income Tax



Figure 6.28 Federal income tax is a concern for most US citizens. (credit: "1040 US tax form" by Marco Verch Professional Photographer/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Determine gross, adjusted gross, and taxable income.
2. Apply exemptions, deductions, and credits to basic income tax calculations.
3. Compute FICA tax.
4. Solve tax application problems for working students.

Before the start of the American Civil War in 1861, most of the country's revenue came from tariffs on trade and excise taxes. However, this fell far short of the high cost of the war. Because of this, the federal government enacted the nation's first income tax with the Revenue Act of 1861, which created the Internal Revenue Service as we know it today.

No one likes paying income tax, but it is a reality of life. In this section, we will learn about Form 1040, the U.S. Individual Income Tax Return, and ways to prepare for tax time.

The U.S. tax code may change from year to year. Because of this, this section includes examples of how taxes, deductions, and exemptions might be computed. The types of income, deductions, and exemptions that are used in the examples are used in the current tax code.

Gross, Adjusted Gross, and Taxable Income

Your income drives how much you pay in taxes. The more you earn, the more you are likely to pay. But your income alone is not the full story. When you add all the money you earned from your job, freelance work, interest from savings, and other sources, you have your **gross income**. If you are an employee, your income from your job will be reported on a W-2, which is sent to you by your employer. Income from freelance work will be reported on a 1099-MISC form, and is sent by the company that paid you. Income from interest is reported on a 1099-INT form and comes from the entity that paid the interest.

Before you determine how much you owe in taxes, you will make certain adjustments to that gross income. You will deduct, or subtract, some of income from the gross income. That's your **adjusted gross income**, or AGI. That is still not what you are taxed on. Next, you need to apply **exemptions** to your income. These are pieces of income that the government does not tax. After that is done, you reach your **taxable income**. We will look at each of these parts of the taxable income.

? WHO KNEW?

Gifts and Winning

Money given as a gift may be taxed if the gift amount is high enough. If you win \$50,000 in the lottery, that money is taxed as income. If you give a family member a large cash gift, that gift will be subject to a tax provided that the gift exceeds the federally set limits.

You will notice that your paycheck already has taxes taken out of it. Your employer will withhold some of your income, sending it directly to the federal, state, and local governments. It is an estimate of how much you will owe in income tax. In the end, it reduces how much you will pay when your taxes are due. If they withhold too much income, you will receive the extra they withheld in the form of a refund.

EXAMPLE 6.115

Computing Gross Income

Roger is preparing to do his taxes. He worked two jobs, with reported income of \$27,500 and \$13,200. His two CDs yielded \$327 together. He also won a split club raffle for \$2,000. What was Roger's gross earnings?

✓ Solution

This is the sum of his wages, winnings and interest earned. Add these Roger's gross income was \$43,027.

> YOUR TURN 6.115

1. Chloe prepares for her taxes by collecting her W-2 and her 1099-MISC documents. The wages from her regular job were \$41,780. She did some freelance work and earned \$5,500 from that job. What is Chloe's gross income?

Your adjusted gross income (AGI) is computed before your taxes are determined. It begins with the gross income, and then subtracts from that income any deduction. **Deductions** are expenditures on your part that the government won't tax. These deductions include money deposited into tax-deferred investments, and mortgage interest that you paid, charitable contributions if you made any, medical bills over a threshold, medical insurance under certain circumstances, and property taxes. If you add all these up, and they are all legal deductions, the sum is subtracted from your gross income, leaving the AGI.

EXAMPLE 6.116

Compute Adjusted Gross Income

Chandra's gross income is \$58,400. She deposited \$3,000 into her tax-deferred retirement account for the year. Her mortgage interest paid was \$1,250 for the year, and her property taxes came to \$4,200. What is her AGI?

✓ Solution

To get Chandra's AGI, total her deductions and subtract that total from her gross income. The total of her deductions is $\$3,000 + \$1,250 + \$4,200 = \$8,450$. She subtracts that total from her gross income, making her AGI $\$58,400 - \$8,450 = \$57,950$.

> YOUR TURN 6.116

1. Isabelle's gross income is \$93,450. She paid \$7,840 in mortgage interest and \$3,810 in property taxes. She also donated \$1,500 to her favorite charity. What is Isabelle's AGI?

Remember that your AGI is not your **taxable income**. Exemptions need to be subtracted from the AGI to reach your taxable income. Exemptions are income that the government does not tax. Some examples of exempt income are disbursements from health savings accounts for qualified medical expenses, bond interest, some IRA distributions, and gifts given that are under \$16,000. Note that exemptions are different from deductions: exemptions are excused incomes, whereas deductions are excused expenditures.

EXAMPLE 6.117

Taxable Income

Yelizaveta's AGI is \$75,490. However, she gave a \$2,000 gift to her mother. What is her taxable income?

✓ Solution

Taxable income is AGI minus any exemptions. Gifts below \$16,000 are exempted, so her taxable income is $\$75,490 - \$2,000 = \$75,490$.

> YOUR TURN 6.117

1. Ammie has an AGI of \$43,100. However, \$3,400 was a disbursement from her health savings account. What is her taxable income?

Tax Credits

Another piece of the tax puzzle is **tax credits**. This is money subtracted from the tax you owe.

Tax credits are very different from deductions or exemptions. Deductions and exemptions are taken away from your gross income before the tax you owe is calculated. A tax credit, is subtracted, dollar for dollar, from your tax bill. Once the tax you owe is calculated, subtract the any tax credits from that calculated tax.

Some of the tax credits are refundable. This means that if subtracting them from your tax results in a negative number, you receive a tax refund. For more details, see [this article about tax credits \(https://openstax.org/r/taxcredit.asp\)](https://openstax.org/r/taxcredit.asp).

The federal government has placed income limits and restrictions and on those eligible to receive tax credits because their value is so high. Here is a partial list of tax credits that you might qualify for:

- **Earned income credit** is a refundable tax credit for low- to moderate-income workers and ranges from \$560 to \$6,935 depending on dependents and income. This is refundable
- **American opportunity credit** is a credit taken by parents who have children enrolled in college at least half time and pursuing a degree. This credit is worth \$2,500 per student for the first 4 years of undergraduate school, subject to income limits. This is a refundable tax credit.
- **Lifetime learning credit** is a credit is equivalent to 20% of educational expenses, up to \$2,000 per year, subject to income limits. There is no cap to how many years you can apply for this credit.
- **Child tax credit** is worth \$2,000 per child under the age of 17 if that child lives at home at least half the year, subject to income limits. This is a refundable tax credit.
- **Child and dependent care tax credit** was designed to help pay for child care while the parent works. The amount of the credit is dependent on your income. However, the maximum amount that can be received is, in 2022, \$4,000 for one eligible person, or \$8,000 for two or more qualifying people. A dependent qualifies if they are a child under

13 years old, a spouse who is unable to care for themselves, or some other qualifying person. This is a refundable tax credit.

- **Premium tax credit** was created by the Affordable Care Act, and it is one that is received by many people throughout the year. In essence it is a health insurance premium subsidy. The amount of the credit is based on your income and the price of health insurance in your area. This is a refundable tax credit.

EXAMPLE 6.118

Apply a Tax Credit

Kaitlyn has calculated the tax she owes to be \$5,200. However, she receives an earned income tax credit of \$1,715. How much does Kaitlyn owe after applying the earned income tax credit?

Solution

The amount of taxes Kaitlyn owes is her calculated tax of \$5,200 minus the credit she receives. The amount she owes in taxes is $\$5,200 - \$1,715 = \$3,485$.

YOUR TURN 6.118

1. Antonio's taxes owed based on his income are \$3,950. However, he qualifies for the earned income tax credit of \$1,925 and a child tax credit of \$2,000. How much does Antonio owe after applying the tax credits?

EXAMPLE 6.119

Tax Credits and a Refund

Chanajah calculated their tax owed, which came to \$4,300. They have an earned income tax credit of \$2,190, a child tax credit of \$2,000, and a child and dependent care tax credit of \$4,000. How much tax does Chanajah owe, or how much will they get in a refund?

Solution

Adding Chanajah's tax credits together, we find their total to be \$8,190. That is more than the tax they owe, which was \$4,300. Each of those credits is refundable, which means they will receive a refund. Subtracting the credits from the tax owed yields $\$4,300 - \$8,190 = -\$3,890$. This is negative, so represents a refund of \$3,890.

YOUR TURN 6.119

1. Ismail owed \$2,350 in taxes. He has a child tax credit of \$2,000 and an earned income tax credit of \$1,640. After applying the tax credit, how much does Ismail owe or how much does he get in a refund?

VIDEO

[Deductions Versus Credits \(https://openstax.org/r/Versus_Credits\)](https://openstax.org/r/Versus_Credits)

Computing FICA Taxes

FICA stands for the Federal Insurance Contributions Act of 1935. FICA taxes are used solely to fund Social Security and Medicare and are separate from federal income tax. It amounts to 7.65% of your gross pay, which is withheld from your paycheck automatically. Your employer is required to match the 7.65% amount. Of the 7.65%, 6.2% goes to Social Security (SSI), and 1.45% goes to Medicare.

As of 2022, SSI tax only applies to the first \$147,000 of earnings. Any gross income above that is not taxed for social security. This limit changes every year.

Medicare tax, on the other hand, applies to the entirety of your gross income.

EXAMPLE 6.120**Computed FICA Taxes**

McKenzi earned \$2,700 in gross income, before taxes, in a given 2-week period. How much does she owe in FICA taxes, and how much of that is for SSI?

✓ **Solution**

The FICA tax is 7.65% of her gross earnings. 7.65% of her \$2,700 is \$206.55. Also, the SSI is 6.2% of her income, or \$167.40

> YOUR TURN 6.120

1. Arianne earned \$3,200 over a 2-week pay period. How much FICA tax does she pay, and how much of that is for SSI?

EXAMPLE 6.121**Social Security Tax with Higher Income**

Renard earned \$195,000 in wages for the year. How much in SSI taxes does Renard owe for the year?

✓ **Solution**

Since Renard earned more than the taxable limit of \$147,000 dollars, he only pays the 6.2% SSI tax on \$147,000. This comes to \$9,114.

> YOUR TURN 6.121

1. Andy earned \$169,450 in wages this year. How much in SSI taxes does Andy owe?

Calculating Your Income Tax

Your income tax bill and your income tax rate are based on your taxable income. The tax system in the United States is progressive, meaning that the tax rates are marginal so the higher your taxable income the higher the tax rate you will pay. Taxable income is broken into brackets, or ranges of income. Each bracket has a different tax rate. The tax brackets and rates for single filers as of 2022 are given below:

Bracket	Lower Income Limit	Upper Income Limit	Tax Rate
1	0	\$10,275	10%
2	\$10,276	\$41,775	12%
3	\$41,776	\$89,075	22%
4	\$89,076	\$170,050	24%
5	\$170,051	\$215,950	32%

Table 6.3 Federal Income Tax Brackets for Single Filers, 2021–2022
(data source: <https://www.irs.gov/newsroom/irs-provides-tax-inflation-adjustments-for-tax-year-2022>)

Bracket	Lower Income Limit	Upper Income Limit	Tax Rate
6	\$215,951	\$539,900	35%
7	\$539,901		37%

Table 6.3 Federal Income Tax Brackets for Single Filers, 2021–2022
(data source: <https://www.irs.gov/newsroom/irs-provides-tax-inflation-adjustments-for-tax-year-2022>)

So if your taxable income is \$76,500 and you are filing as a single filer, your tax bill will be 22% of that \$76,500, right?

Wrong. Your income is split among those brackets and the money in each bracket is taxed at that bracket's tax rate. Seems confusing. Here is a list of steps to follow to find the tax owed.

Step 1: Find the bracket for the income.

Step 2: For each bracket below the income bracket, the tax from that bracket is:

Step 2a: Find the difference between the upper limit of that bracket and upper limit of the next lower bracket. If this is bracket 1, use 0 as the upper limit of the previous bracket.

Step 2b: The tax from that bracket is the bracket tax rate applied to the difference from Step 2a.

Step 3: For the bracket that the income belongs to, find the income minus the lower limit for the bracket.

Step 4: The tax for the bracket of the income is tax rate for that bracket applied to the difference found in Step 3.

Step 5: Add these various tax values to get the total income tax.

There are various tax brackets, and the rates may change in any given year. The income limits may also change. For all examples going forward, we will use the single filer tax brackets, even if those brackets are not appropriate (e.g., married or head of household filers).

EXAMPLE 6.122

Income Tax on Taxable Income

Faith has a taxable income of \$103,650. How much income tax does Faith owe?

Solution

Step 1: Faith's income belongs to the 4th tax bracket, \$89,076 to \$170,050. So the taxes from the first three brackets follow steps 2a and 2b.

For bracket 1:

Step 2a: The upper limit is \$10,275, the upper limit of the previous bracket was 0, so the difference is \$10,275.

Step 2b: For the first bracket, she owes 10% of \$10,275, or \$1,027.50.

For bracket 2:

Step 2a: The upper limit is \$41,775, the upper limit from the previous bracket was \$10,275, so the difference is \$31,500

Step 2b: For the second bracket, she owes 12% of \$31,500, or \$3,780.

For bracket 3:

Step 2a: The upper limit is \$89,075, the upper limit from the previous bracket is \$41,175, so the difference was \$47,300

Step 2b: For the third bracket, she owes 22% of \$47,300, or \$10,406.

Step 3: For bracket 4, her income is \$103,650, the upper limit of the previous bracket is \$89,075. The difference of those is $\$103,650 - \$89,075 = \$14,575$.

Step 4: The tax he owes for that bracket is 24% of the difference, which is \$3,498.

Step 5: The total in taxes that Emmanuel owes is the sum of the taxes found above, or

$$\$1,027.5 + \$3,780 + \$10,406 + \$3,498 = \$18,711.50$$

> YOUR TURN 6.122

1. Jenna's taxable income is \$73,500. Find how much income tax she owes.

 Remember that your employer will estimate how much tax you will owe and withholds it from paychecks. This means you may have already paid some, if not all and more, of the income tax you owe.

▶ VIDEO

[Tax Brackets \(https://openstax.org/r/Brackets\)](https://openstax.org/r/Brackets)

EXAMPLE 6.123

Finding Income Tax Owed

Emmanuel is preparing his taxes. His W-2 from work shows gross income for the year of \$95,250. He also has a 1099-MISC for some freelance art work he did, amounting to \$7,500. Emmanuel also deposited \$4,500 into a tax-deferred retirement plan. He paid \$7,920 in mortgage interest for the year and \$3,740 in property taxes. He also qualified for \$4,000 in child tax credits. Based on this information, how much tax does Emmanuel owe or how much does he get in a refund?

✓ Solution

We need to know Emmanuel's taxable income, based on gross income, deductions, and exemptions.

His gross income is the amount from his W-2 and his 1099-MISC. Adding these gives a gross income of $\$95,250 + \$7,500 = \$102,750$.

Subtracting his deductions will yield his AGI. His deductions are \$7,920 for mortgage interest, \$3,740 for property taxes, and \$4,500 deposited into his retirement account. Adding these, his total deductions are \$16,160. Subtracting the deductions from the gross income, we find his AGI to be \$86,590.

Emmanuel seems to have no exemptions, so his AGI and his taxable income are the same.

We find the taxes Emmanuel owes using the process outlined above.

Step 1: His income belongs to the third tax bracket, \$41,776 to \$89,0875. So the taxes from the first two brackets follow steps 2a and 2b.

For bracket 1:

Step 2a: The upper limit is \$10,275, the upper limit of the previous bracket was 0, so the difference is \$10,275.

Step 2b: For the first bracket, he owes 10% of \$10,275, or \$1,027.50.

For bracket 2:

Step 2a: The upper limit is \$41,775, the upper limit from the previous bracket was \$10,275, so the difference is \$31,500

Step 2b: For the second bracket, he owes 12% of \$31,500, or \$3,780.

Step 3: For bracket 3, his income is \$86,590, the upper limit of the previous bracket is \$41,775. The difference of those is $\$86,590 - \$41,775 = \$44,815$.

Step 4: The tax he owes for that bracket is 22% of the difference, which is \$9,859.30.

Step 5: The total in taxes that Emmanuel owes is the sum of the taxes found above, or $\$1,027.5 + \$3,780 + \$9,859.30 = \$14,666.80$.

Once his taxes are computed, he subtracts his tax credits. His only tax credit is \$4,000. The total he owes in taxes is $\$14,666.80 - \$4,000 = \$10,666.80$.

 YOUR TURN 6.123

1. Jacob is preparing his taxes. His W-2 from work shows gross income for the year of \$39,885. He also has a 1099-INT for a savings account, amounting to \$378. Jacob also deposited \$2,500 into a tax-deferred retirement plan. He also qualified for \$2,000 in child tax credits and \$4,000 in child and dependent care tax credit. Based on this information, how much tax does Jacob owe or how much does he receive in a refund?

Check Your Understanding

86. Gross income consists of what?
87. Taxable income is found after what is done to gross income?
88. If the gross income is \$50,000 and there are three deductions of \$2,000, \$3,000, and \$8,000, what is the adjusted gross income?
89. When is a tax credit applied?
90. Do you pay 6.2% for SSI tax for all your earnings?
91. For a taxable income of \$41,800, how much income tax is owed?
92. If \$5,600 is owed in taxes but there are \$4,000 in tax credits, how much income tax is owed?



SECTION 6.13 EXERCISES

1. What is gross income?
2. How is adjusted gross income derived from gross income?
3. Is there a difference between AGI and taxable income? If so, what is the difference?
4. How are deductions different from tax credits?
5. What are the two components of the FICA tax?
6. What taxes do employers match?
7. If you are in the 32% tax bracket, is your tax computed on your full income?
8. What does it mean for a tax credit to be refundable?

In the following exercises, compute the gross income.

9. Wages from W-2 are \$32,800, freelance income from a 1099-MISC is \$1,050, and a gift given to a family member of \$2,760.
10. Wages from W-2 are \$59,380, interest income from 1099-INT is \$1,500, freelance income from a 1099-MISC is \$3,500, and a split club raffle winning of \$6,755.
11. Wages from job 1 on W-2 are \$36,200, wages from job 2 on W-2 are \$21,400, interest income from a 1099-INT is \$374.
12. Wages from W-2 are \$121,450, interest from a 1099-INT is \$3,400.

In the following exercises find the adjusted gross income, AGI, based on gross income and deductions.

13. Gross income was \$65,700, \$2,280 deposited in a tax-deferred IRA, mortgage interest was \$4,715, property taxes were \$3,065.
14. Gross income was \$183,200, \$5,000 deposited in a tax-deferred IRA, \$7,300 in charitable contributions, \$8,350 in mortgage interest, and \$6,900 in property taxes.
15. Gross income was \$31,200, \$1,500 deposited in a tax-deferred IRA account.
16. Gross income was \$41,500, \$1,250 deposited in a tax-deferred IRA account, \$4,210 in mortgage interest, \$2,980 in property tax.

In the following exercises, find the taxable income based on AGI and exemptions.

17. AGI of \$34,560, \$2,500 disbursement from a health savings account for a qualifying medical expense.
18. AGI of \$56,750, gift given to a family member of \$8,000, \$550 in bond interest.
19. AGI of \$120,940, gift given to a family member of \$15,000, bond interest of \$4,500.
20. AGI of \$28,450, gift given to a family member of \$2,000.

In the following exercises, find the taxes owed or refund received based on income tax bill and tax credits.

21. Tax bill of \$5,300, child tax credit of \$4,000, earned income tax credit of \$1,630.
22. Tax bill is \$17,300, child tax credit of \$4,000, child and dependent care tax credit \$8,000, lifetime learning credit of \$1,000.
23. Tax bill of \$5,205, child tax credit of \$6,000, earned income tax credit of \$2,450.
24. Tax bill of \$11,300, child tax credit of \$2,000, earned income tax credit of \$650, child and dependent care credit of \$4,000.
25. Tax bill of \$13,750, child tax credit of \$2,000, earned income tax credit of \$780, child and dependent care credit of \$2,000.

In the following exercises, determine the SSI tax and the total FICA tax for the given incomes.

26. Earning for pay period were \$3,500.
27. Earnings for pay period were \$1,400.
28. Earnings for pay period were \$3,150.
29. Earnings for the year were \$135,000.
30. Earnings for the year were \$203,400.

In the following exercises, find the income tax owed using the taxable income and the tax table below, for a person filing single.

Bracket	Lower Income Limit	Upper Income Limit	Tax Rate
1	0	\$10,275	10%
2	\$10,276	\$41,775	12%
3	\$41,776	\$89,075	22%
4	\$89,076	\$170,050	24%
5	\$170,051	\$215,950	32%
6	\$215,951	\$539,900	35%
7	\$539,901		37%

31. Taxable income of \$36,250.
32. Taxable income of \$63,500.
33. Taxable income of \$209,450.
34. Taxable income of \$92,250.
35. Alexandra is preparing her taxes. Her W-2 from work shows gross income for the year of \$51,300. She also has a 1099-INT for interest on a savings account for \$910. Alexandra also deposited \$2,750 into a tax-deferred retirement plan. She paid \$2,150 in mortgage interest for the year and \$2,060 in property taxes. Based on this information, how much tax does Alexandra owe or how much does she get in a refund?
36. Tymoteusz is preparing his taxes. His W-2 from work shows gross income for the year of \$47,680. He also has a 1099-MISC for some freelance consulting he did, amounting to \$1,800. Tymoteusz deposited \$3,000 into a tax-deferred retirement plan. He also qualified for \$2,000 in child tax credits and \$4,000 in child and dependent care tax credits. Based on this information, how much tax does Tymoteusz owe or how much does he get in a refund?

Chapter Summary

Key Terms

6.1 Understanding Percent

- Percent
- Fractional form
- Decimal form
- Total
- Base
- Percent of the total
- Part
- Amount

6.2 Discounts, Markups, and Sales Tax

- Discount
- Cost
- Markup
- Retail price

6.3 Simple Interest

- Interest
- Principal
- Annual percentage rate
- Simple interest
- Term
- Due
- Origination date
- Payoff amount
- Future value
- Partial payment
- Present value

6.4 Compound Interest

- Compound interest
- Effective annual yield

6.5 Making a Personal Budget

- Budget
- Necessary expenses
- Fixed expenses
- Variable expenses
- 50-30-20 budget philosophy

6.6 Methods of Savings

- Savings account
- 1099 form
- Certificate of deposit
- Money market account
- Return on investment
- Ordinary annuity

6.7 Investments

- Bonds
- Maturity date
- Stocks
- Dividend
- Mutual fund

- Prospectus
- Issue price
- Shares
- Stock table
- Individual retirement account
- Roth IRA
- 401(k)

6.8 The Basics of Loans

- Fixed interest rate
- Variable interest rate
- Installment loan
- Loan amortization
- Revolving credit
- Amortization table
- Cost of finance

6.9 Understanding Student Loans

- FAFSA
- College funding gap
- Subsidized loan
- Unsubsidized loan
- Parent loan for undergraduate students
- Private student loan
- School-channel loan
- Direct-to-consumer loan
- Standard repayment plan
- Federal consolidation
- Refinancing
- Private consolidation
- Graduated repayment plan
- Extended repayment plan
- Discretionary income
- Pay as you earn (PAYE) repayment plan
- Revised pay as you earn (REPAYE) repayment plan
- Income-based (IBR) repayment plan
- Income-contingent (ICR) repayment plan
- Delinquent
- Default
- Rehabilitation

6.10 Credit Cards

- Reward program
- Annual fee
- Credit limit
- Bank-issued credit card
- Store-issued credit card
- Travel and entertainment cards
- Charge cards
- Billing period
- Balance
- Minimum payment
- Average daily balance

6.11 Buying or Leasing a Car

- Title and registration fees
- Destination fee
- Documentation fee

- Dealer preparation fee
- Extended warranty
- Down payment
- Acquisition fee
- Security deposit
- Disposition fees
- Liability insurance
- Collision insurance
- Comprehensive insurance
- Uninsured or underinsured motorist insurance
- Medical payment insurance
- Personal injury insurance
- Gap insurance
- Rental reimbursement insurance

6.12 Renting and Homeownership

- Lease
- Landlord
- Mortgage
- Escrow account
- Assessed value

6.13 Income Tax

- Gross income
- Adjusted gross income
- Exemption
- Taxable income
- Deduction
- Tax credit
- Earned income credit
- American opportunity credit
- Lifetime learning credit
- Child tax credit
- Child and dependent care tax credit
- Premium tax credit

Key Concepts

6.1 Understanding Percent

- Know what a percent is as a fraction, a decimal, and as a part of the whole.
- Use the percent equation to find any of the three values that are related by the equation.
- Apply the percent equation in applications.

6.2 Discounts, Markups, and Sales Tax

- Discounts are markdowns from an original price.
- Mark-ups are increases to the price paid by a retailer to cover their costs.
- be able to calculate the markup based on a percentage of the cost
- Sales taxes vary from state to state and often county to county.
- Retail prices, sales prices and percent discounts can be calculated if the other two values are known.
- Original costs, retail prices, and percent markup can be calculated if the other two values are known.
- In calculations, sales tax acts like a markup.

6.3 Simple Interest

- Interest is money that is paid by a borrower for the privilege of borrowing the money.
- Simple interest is computed by substituting the principal, interest rate, and number of years into the formula

$$I = P \times r \times t$$
- The payoff for a loan is the amount of principal remaining on a loan plus the interest that accumulated on the loan since the last payment.

- The future value of an investment yielding simple interest is the original principal plus the interest earned on the investment.
- When making a partial payment, some of the payment pays off all the accumulated interest, while the remainder of the payment is applied to the principal of the loan.
- Finding the present value of an investment is used to determine how much should be invested now in order to achieve a specific goal.

6.4 Compound Interest

- Compound interest means that the interest earned during one period will earn interest in later periods. Essentially, the amount of the principal grows from period to period.
- The important values in computing compound interest are the interest rate, the principal, the length of time the investment, and the number of times the investment is compounded.
- Compound interest has minimal impact early, but later has a very large impact.
- You can determine how much to invest today in order to reach a goal for some time later.
- Compound interest can be translated into an effective annual yield, which allows for comparison between investment options.

6.5 Making a Personal Budget

- A budget is a set of guidelines for how to allocate your income.
- Budgeting helps to plan for many of life's expenses
- Budgets are used to compare income to expenses. When expenses exceed income, changes have to be made.
- Budgets can help evaluate the affordability of life changes.
- One guideline for setting a budget is the 50-30-20 budget philosophy. The guidelines suggest that 50% of income is allocated to necessary expenses, 30% to expenses that wants, and 20% to savings and other debt reduction.

6.6 Methods of Savings

- There are three main types of savings accounts, saving accounts, certificates of deposit (CD), and money market accounts.
- Savings account are very risk free, and so yield low interest rates.
- The differences in the three types of savings accounts relate to their convenience.
- Savings account typically have a lower interest rate than money market accounts, which typically have lower interest rates than CDs.
- Ordinary annuities more accurately reflect how we save, in that money is deposited repeatedly over time.
- Spreadsheet software, such as Google Sheets, have built in functions that can be used to quickly calculate both the future value of an ordinary annuity account, but also the payment necessary to reach a goal using an ordinary annuity.

6.7 Investments

- There are many different investments with different returns and risks.
- Bonds are loans from the purchaser to the entity selling the bond.
- Bonds have some tax benefits, low to no risk, and a low return.
- Stocks represent part ownership in a company. As such, stock holders share in the profits, and losses, of the company.
- Information, including price, P/E, yearly highs and lows, and dividend amount can be found in online stock tables available on many websites.
- Mutual funds represent collections of professionally administered investment vehicles. Have shares in a mutual fund has lower risk than ownership of stocks.
- Retirement accounts employ some of the same strategies as mutual funds, in that they spread the risk and are professionally managed.
- IRAs and Roth IRAs differ on when taxes are paid on the money, and who can use them. Roth IRAs have income limits while traditional IRAs do not.

6.8 The Basics of Loans

- There are many reasons for a loan, but primarily it is taken out for a large expense when cash is not available.
- Each payment for an installment loan consists of an interest portion and a principal portion.
- There is a formula to calculate the payment necessary to pay off a loan in installments.
- Amortization schedules, or tables, show how each payment is applied to principal and interest. It also includes other details such as remaining balance and total interest paid.

- Loans often have other fees associated with them such as origination fees or application fees. The total of the interest paid and the fees is the cost of finance.

6.9 Understanding Student Loans

- The FAFSA must be filled out each year that a student wishes to borrow for.
- A student's funding gap determines how much they need in loans to pay for college.
- Federal subsidized student loans defer payments until after graduation and interest does not accrue on these loans.
- Unsubsidized student loans defer payment until after graduation but interest begins accruing as soon as the loan funds are disbursed.
- There are both yearly and aggregate limits for student loans to prevent over-borrowing, among other reasons.
- Federal direct loans have a low interest rate set by the government, but other student loans have varying rates of interest set by the banks.
- The standard repayment plan lasts 10 years and is made up of monthly payments.
- Consolidating or refinancing student loans merges many student loans into one loan.
- If only federal loans are consolidated, the interest rate is the same as the individual loans, currently set at 4.99%.
- If other loans are refinanced together, the interest rate may be lower with the new loan.
- Other repayment plans are available. Such a plan may have payment that start small and grow as the loan is paid off, or it may have a longer term, or may be based on the discretionary income of the student.
- Being delinquent on a student loan is a precursor to being in default. Making payments in a timely fashion allows the student to avoid this situation.

6.10 Credit Cards

- Credit cards can be a flexible way to pay for almost anything, but can become a financial hazard if used unwisely.
- When deciding which credit card to apply for, evaluate the interest rate, fees (annual and late), reward programs and credit limit. Be sure they meet your criteria.
- Paying off the balance of your credit card every month will control your spending and will never result in paying interest.
- Credit card statements hold all important information about your credit card, including payment, balances, charges and billing cycle dates.
- Although the minimum payment is attractive precisely because it is so small, paying only the minimum results in a long payoff term and higher interest costs.

6.11 Buying or Leasing a Car

- There are many factors to consider when choosing to buy or lease a car.
- The cost of the car is increased by a number of fees and sales tax.
- There are advantages to buying a car and advantages to leasing a car. The decision between the two depends on the preference of the buyer.
- Insurance covers costs associated with accidents. It is made up of various components.
- The costs of owning a car, including insurance and maintenance, should be a part of the budgeting process.
- Budgeting for unexpected repairs can ease the stress of encountering large repair bill.

6.12 Renting and Homeownership

- There are many points of comparison between renting and buying a house.
- Before deciding to buy a house, you should carefully consider all the responsibilities that come with home ownership.
- Renting comes with more restrictions on the renter, but with fewer costs and is easier to move from.
- Owning a house has more costs but has more freedom, plus the owner creates equity.
- Mortgages are loans, and payments are calculated in the same way as any other loan.
- Amortization tables help a homeowner understand the mortgage and how the payments are applied to the principal and interest.
- In addition to paying the amount financed for a mortgage, the monthly payment will include an escrow payment, which covers insurance and taxes.

6.13 Income Tax

- Federal income tax is based on income after certain adjustments.
- Gross income is income from all sources, including gifts and winnings.
- Before taxes are calculated, the taxable income is found by subtracting deductions and exemptions from gross income.

- Income tax is progressive, increasing in rate as income increases.
- Being in the 32% tax bracket means some of your income is taxed at 10%, some at 12%, some at 22%, some at 24%, and the rest at 32%.
- Income in each tax bracket is taxed at that bracket's rate, which means in 2022 the first \$10,275 earned is taxed at 10% only.
- Tax credits are subtracted from the taxes that are owed.
- Some tax credits are refundable, which means they can make the amount you owe negative, which results in a refund.

Videos

6.1 Understanding Percent

- [Finding Percent of a Total \(https://openstax.org/r/solve_percent_problem1\)](https://openstax.org/r/solve_percent_problem1)
- [Finding the Total from the Percent and the Part \(https://openstax.org/r/solve_percent_problem2\)](https://openstax.org/r/solve_percent_problem2)
- [Finding the Percent When the Total and the Part Are Known \(https://openstax.org/r/solve_percent_problem3\)](https://openstax.org/r/solve_percent_problem3)

6.2 Discounts, Markups, and Sales Tax

- [Computing Price Based on a Percent Off Coupon \(https://openstax.org/r/Computing_Price_Based\)](https://openstax.org/r/Computing_Price_Based)
- [Finding Sales Tax Percentage \(https://openstax.org/r/Finding_Sales_Tax\)](https://openstax.org/r/Finding_Sales_Tax)

6.4 Compound Interest

- [Compound Interest \(https://openstax.org/r/compound_interest_beginners\)](https://openstax.org/r/compound_interest_beginners)
- [Compare Simple Interest to Interest Compounded Annually \(https://openstax.org/r/compare_simple_compound_interest1\)](https://openstax.org/r/compare_simple_compound_interest1)
- [Compare Simple Interest and Compound Interest for Different Number of Periods Per Year \(https://openstax.org/r/compare_simple_compound_interest2\)](https://openstax.org/r/compare_simple_compound_interest2)

6.5 Making a Personal Budget

- [Creating a Budget \(https://openstax.org/r/creating_budget\)](https://openstax.org/r/creating_budget)
- [50-30-20 Budget Philosophy \(https://openstax.org/r/50-30-20_budgeting_rule\)](https://openstax.org/r/50-30-20_budgeting_rule)

6.6 Methods of Savings

- [Return on Investment, ROI \(https://openstax.org/r/This_video\)](https://openstax.org/r/This_video)
- [Future Value Using Google Sheets \(https://openstax.org/r/cell_references\)](https://openstax.org/r/cell_references)

6.7 Investments

- [Bonds \(https://openstax.org/r/investing_basics_bonds\)](https://openstax.org/r/investing_basics_bonds)
- [Reading Stock Summary Online \(https://openstax.org/r/Reading_Stock\)](https://openstax.org/r/Reading_Stock)
- [Mutual Funds \(https://openstax.org/r/investing_basics_mutual_funds\)](https://openstax.org/r/investing_basics_mutual_funds)
- [401\(k\) Accounts \(https://openstax.org/r/401\(k\)s\)](https://openstax.org/r/401(k)s)

6.8 The Basics of Loans

- [Credit Scores Explained \(https://openstax.org/r/credit_scores_explained\)](https://openstax.org/r/credit_scores_explained)
- [Reading an Amortization Table \(https://openstax.org/r/Amortization_Table\)](https://openstax.org/r/Amortization_Table)

6.9 Understanding Student Loans

- [Types of Student Loans \(https://openstax.org/r/Student_Loans\)](https://openstax.org/r/Student_Loans)
- [Repayment Plans \(https://openstax.org/r/student_loan_repayment_plans\)](https://openstax.org/r/student_loan_repayment_plans)

6.10 Credit Cards

- [Choosing a Credit Card \(https://openstax.org/r/Credit_Card\)](https://openstax.org/r/Credit_Card)
- [Reading Credit Card Statements \(https://openstax.org/r/Reading_Credit\)](https://openstax.org/r/Reading_Credit)

6.12 Renting and Homeownership

- [Rent or Buy \(https://openstax.org/r/Rent_or_Buy\)](https://openstax.org/r/Rent_or_Buy)
- [Closing Costs \(https://openstax.org/r/closing_costs\)](https://openstax.org/r/closing_costs)

6.13 Income Tax

- [Deductions Versus Credits \(https://openstax.org/r/Versus_Credits\)](https://openstax.org/r/Versus_Credits)
- [Tax Brackets \(https://openstax.org/r/Brackets\)](https://openstax.org/r/Brackets)

Formula Review

6.1 Understanding Percent

part = percent $\times$ total

6.2 Discounts, Markups, and Sales Tax

discount = percent discount $\times$ original price

sale price = original price – discount

sale price = original price – percent discount $\times$ original price = original price $\times$ (1 – percent discount)

markup = percent markup $\times$ cost

retail price = cost + markup

retail price = cost + percent markup $\times$ cost = cost $\times$ (1 + percent markup)

sales tax = purchase price $\times$ tax rate

Total price = purchase price + purchase price $\times$ tax rate = purchase price $\times$ (1 + tax rate)

6.3 Simple Interest

$$I = P \times r \times t$$

$$T = P + I$$

$$T = P + P \times r \times t$$

$$I = P \times \frac{r}{365} \times t$$

$$FV = P + I = P + P \times r \times t$$

$$A = P \times \frac{r \times (1 + r)^t}{(1 + r)^t - 1}$$

$$PV = \frac{FV}{(1 + rt)}$$

6.4 Compound Interest

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$PV = \frac{A}{\left(1 + \frac{r}{n}\right)^{n \times t}}$$

$$Y = \left(1 + \frac{r}{n}\right)^n - 1$$

6.6 Methods of Savings

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$ROI = \frac{FV - P}{P}$$

$$FV = pmt \times \frac{(1 + r/n)^{n \times t} - 1}{r/n}$$

$$pmt = \frac{FV \times (r/n)}{(1 + r/n)^{n \times t} - 1}$$

6.7 Investments

$$\text{annual return} = \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1$$

$$P/E = \frac{\text{Share Price}}{\text{Dividend}}$$

$$\text{Yld}\% = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\%$$

6.8 The Basics of Loans

$$I = P \times \frac{r}{n}$$

$$pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$$

6.9 Understanding Student Loans

funding gap = total cost – all aid

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$$

discretionary income = gross income – 1.5 × poverty guideline

6.10 Credit Cards

$$I = \frac{\text{ADB} \times r \times d}{365}$$

6.11 Buying or Leasing a Car

$$pmt = \frac{P \times (r/n) \times (1 + r/n)^{n \times t}}{(1 + r/n)^{n \times t} - 1}$$

$$\text{MD} = \frac{P - R}{n}$$

$$\text{APR} = 2400 \times \text{MF}$$

$$\text{MF} = \text{APR}/2,400$$

$$\text{PMT} = \frac{(P - R)}{n} + (P + R) \times \text{MF}$$

6.12 Renting and Homeownership

$$pmt = \frac{P \times (r/12) \times (1 + r/12)^{12 \times t}}{(1 + r/12)^{12 \times t} - 1}$$

$$T = pmt \times 12 \times t$$

$$\text{CoF} = T - P$$

Projects

Creating Your Future Budget

In this project, you will create a budget based on a job you are likely to have after you graduate.

1. Go online and research the average starting salary for the profession you are studying for. Use at least two sources. Be sure to record the web address from your search.
2. Approximate your monthly take-home pay. You may use the [SmartAsset \(https://openstax.org/r/paycheck-calculator\)](https://openstax.org/r/paycheck-calculator) website to estimate this.
3. Use the 50-30-20 budget philosophy to determine how much you should budget for needs, wants and savings, or extra debt reduction.
4. Create a list of likely expenses. This list must include rent/mortgage, utilities, food, and school loan repayment. You may also want to include car payments, gasoline, and other items.
5. Categorize each expense as need, want, or savings.
6. Using the amounts found in step 3, decide how much to allocate to each of your expenses. It may help to quickly research how much rent is where you want to live.
7. Discuss the choices you had to make, and why you prioritized some expenses over others.

Interest Rate and Time: What Is the Relationship?

The interplay between interest rate and time for an annuity is not easily seen. How the amount that must be deposited per compounding period, pmt , changes based on the time and interest rate would be useful to understand. In this project, you will explore this relationship. We will use a fixed future value of $FV = \$1,000,000$ and a fixed number of periods per year, 12 (monthly compounding). With those, we'll find various annuity payments that must be made to

reach the goal.

The annual interest rate for the investment is in the top row. The number of years for the investment is in the left column. In each cell (or box), find the monthly payment necessary to reach the goal of \$1,000,000.

		Annual Interest Rate					
		1.5%	2.0%	3.0%	5.0%	7.5%	10.0%
Number of Years	10						
	15						
	20						
	30						
	40						
	45						

Describe how the interest rates and number of years impact the payment necessary to reach the goal of \$1,000,000.

Finding a Home

In this project you will identify a home you like, and then estimate the costs associated with that home.

1. Find a home in your region that you would like to buy using an online search of listings in your area. Zillow is a good place to begin.
2. Find the asking price for this home. Assume you would pay that price.
3. Find an estimation for closing costs in your area. Assume you finance those costs also.
4. Estimate the taxes to be paid on the home per year. It is likely that the online listing of the home has an estimate for the taxes for the house.
5. Use Google to determine the average homeowner's insurance cost in your region.
6. Use the internet to determine the average interest rate for a 30-year mortgage.
7. Find how much you would pay per month, based on the answers to the previous questions, including the escrow payments for taxes and insurance.
8. Assume you will pay \$50 per \$100,000 borrowed in PMI. Add this to the monthly payment.

Chapter Review

Understanding Percent

1. Convert 0.45 to percent form.
2. What is 70% of 200?
3. Rewrite the fraction $\frac{240}{100}$ as a percentage.
4. 20 is what percent of 500?

Discounts, Markups, and Sales Tax

5. An item with a retail price of \$250.00 is on sale for 30% off. What is the sale price of the item? Round to the nearest penny.
6. The sales price of an item is \$38.50 after a 40% discount. What was the retail price of the item? Round to the nearest penny.
7. A retailer buys an item for \$1,000.00 and has a 65% markup. What is the retail price of the item? Round to the nearest penny.
8. The sale price of an item is \$46.00 and the retail price is \$65.00. What is the percent discount? Round to two decimal places.
9. The retail price of an item is \$345.38 and sales tax in the region is 8.25%. How much is the sales tax?

Simple Interest

10. \$3,000 is invested in a 5-year CD earning 2.25% interest. How much is the CD worth in 5 years?
11. A \$4,000.00 simple interest loan is taken out for 3 years at 12.5%. How much is owed when the loan comes due?
12. A \$10,000 loan with annual simple interest of 14.9% is taken out for 90 days. How much is due in 90 days?
13. A simple interest loan for \$20,000 is taken out at 13.9% annual interest rate. A partial payment of \$13,500.00 is made 30 days into the loan period. After this payment, what will the remaining balance be?
14. Find the present value of an investment with future value of \$15,000 if the investment earns 3.55% simple interest for 10 years?

Compound Interest

15. Find the future value after 20 years of \$15,000 deposited in an account bearing 4.26% interest compounded quarterly.
16. What is the effective annual yield for an account bearing 3.21% interest compounded quarterly? Round to two decimal places if necessary.
17. Find the present value of \$100,000 in an account bearing 5.25% interest compounded daily after 30 years.

Making a Personal Budget

18. In budgeting, what is the difference between an expense that is a need and an expense that is a want?
19. Apply the 50-30-20 budget philosophy to a monthly income of \$6,000.00.
20. Create the budget for a person with the following income and bills, and determine how much the income exceeds or falls short of the expenses:
Job = \$5,250, Side job = \$550, rent = \$1,150.00, utilities = \$150.00, internet = \$39.99, student loans = \$375.00, food = \$550.00, bus and subway = \$112.00, credit cards = \$200.00, entertainment = \$400.00, clothing = \$150.00.
21. In question 20, the budget for a person was created. Can the person afford to buy a car if the monthly payments will be \$365.50, monthly car insurance will be \$114.75, and gasoline will be \$200.00?

Methods of Savings

22. Why is a CD less flexible than a money market account?
23. Which is likely to have the highest interest rate, a savings account, CD, or money market account?
24. \$3,000 is deposited in a money market account bearing 3.88% interest compounded monthly. How much is the account worth in 15 years?
25. Refer to question 24. What is the return on investment for the money market account? Round to two decimal

places, if necessary.

26. \$300 is deposited monthly in an ordinary annuity that bears 6.5% interest compounded monthly. How much is in the annuity after 30 years?
27. How much must be deposited quarterly in an account bearing 4.89% interest compounded quarterly for 30 years so the account has a future value of \$500,000?

Investments

28. Why is investing in a stock riskier than in a mutual fund?
29. Which investment offers high rates of return with reduced risk?
30. Which investment offer a fixed rate of return over time?
31. Which is the riskiest investment?
32. How much is earned on a 10-year bond with issue price of \$1,000 that pays 3.9% annually?
33. Henri's employer matches up to 6% of salary for IRA contributions. Henri's annual income is \$52,500. If Henri wants to deposit \$3,000 annually in his IRA, how much in matching funds does the employer deposit in the IRA?
34. How much must be deposited quarterly in a mutual fund that is expected to earn 11.2% interest compounded quarterly if the account is to be worth \$1,000,000 after 38 years?
35. Lorraine purchases stocks for \$5,000. The stocks paid dividends by reinvesting the dividends in stock. She sold the stock for \$13,750 after 6 years. What was Lorraine's annual return on that investment? Round to two decimal places if necessary.
36. What are two ways stocks earn money?

The Basics of Loans

37. What is an installment loan?
38. An installment loan with monthly payments has an outstanding balance of \$3,560. If the annual interest rate is 9%, how much interest will be paid that month?
39. A loan for \$42,000 is taken out for 6 years at 7.5% interest. What are the payments for that loan?
40. The total interest paid on a loan was \$4,500. The loan had an origination fee of \$400.00, a \$125.00 processing fee, and a filing fee of \$250.00. How much was the cost of financing for that loan?
41. Using the amortization table below, what is the remaining balance after payment 25?

Amortization Schedule Calculator					
Loan Amount	\$17,500.00				
Interest Rate (annual)	8.25%				
Term (Years)	5				
Monthly Payment	\$356.93				
Month	Payment	Principal	Interest	Total Interest	Balance
					\$17,500.00
1	\$356.93	\$236.62	\$120.31	\$120.31	\$17,263.38
2	\$356.93	\$238.25	\$118.69	\$239.00	\$17,025.13
3	\$356.93	\$239.89	\$117.05	\$356.05	\$16,785.24
4	\$356.93	\$241.54	\$115.40	\$471.44	\$16,543.71
5	\$356.93	\$243.20	\$113.74	\$585.18	\$16,300.51
6	\$356.93	\$244.87	\$112.07	\$697.25	\$16,055.64
7	\$356.93	\$246.55	\$110.38	\$807.63	\$15,809.09
8	\$356.93	\$248.25	\$108.69	\$916.32	\$15,560.84
9	\$356.93	\$249.95	\$106.98	\$1,023.30	\$15,310.89
10	\$356.93	\$251.67	\$105.26	\$1,128.56	\$15,059.22
11	\$356.93	\$253.40	\$103.53	\$1,232.09	\$14,805.82
12	\$356.93	\$255.14	\$101.79	\$1,333.88	\$14,550.67
13	\$356.93	\$256.90	\$100.04	\$1,433.92	\$14,293.77
14	\$356.93	\$258.66	\$98.27	\$1,532.19	\$14,035.11
15	\$356.93	\$260.44	\$96.49	\$1,628.68	\$13,774.66
16	\$356.93	\$262.23	\$94.70	\$1,723.38	\$13,512.43
17	\$356.93	\$264.04	\$92.90	\$1,816.28	\$13,248.39
18	\$356.93	\$265.85	\$91.08	\$1,907.36	\$12,982.54
19	\$356.93	\$267.68	\$89.25	\$1,996.62	\$12,714.86
20	\$356.93	\$269.52	\$87.41	\$2,084.03	\$12,445.34
21	\$356.93	\$271.37	\$85.56	\$2,169.59	\$12,173.97
22	\$356.93	\$273.24	\$83.70	\$2,253.29	\$11,900.73
23	\$356.93	\$275.12	\$81.82	\$2,335.11	\$11,625.62
24	\$356.93	\$277.01	\$79.93	\$2,415.03	\$11,348.61
25	\$356.93	\$278.91	\$78.02	\$2,493.06	\$11,069.69
26	\$356.93	\$280.83	\$76.10	\$2,569.16	\$10,788.86

Understanding Student Loans

42. If a college program takes 5 years, how long is the student's eligibility period for student loans?
43. What is the funding gap for a student with cost of college equal to \$35,750 and non-loan financial aid of \$24,150?
44. What is the maximum amount of federal subsidized and unsubsidized loans for a dependent student in their third year of an undergraduate data science program?
45. A single person has gross income of \$28,500. Their poverty guideline is \$12,000. What is their discretionary income?

Credit Cards

46. The billing cycle for a credit card goes from March 15 to April 14. The balance at the start of the billing cycle is \$450.00. The list of transactions on the card is below. Find the average daily balance for the billing cycle.

Date	Activity	Amount
15-Mar	Billing Date Balance	\$450.00
21-Mar	Movie	\$50.00
28-Mar	Gasoline	\$65.00
31-Mar	Snacks	\$15.50
1-Apr	Dinner	\$63.60
1-Apr	Payment	\$300.00
1-Apr	Gasoline	\$48.90
9-Apr	Plane Tickets	\$288.50

47. The average daily balance for Greg's last credit card statement was \$1,403.50. The card charges 15.9% interest. If the billing cycle for that statement was 30 days, how much interest is Greg charged?
48. The billing cycle for a credit card goes from March 15 to April 14. The balance at the start of the billing cycle is \$450.00. The list of transactions on the card is below. The interest rate is 15.9%. What is the balance due at the end of the billing cycle?

Date	Activity	Amount
15-Mar	Billing Date Balance	\$450.00
21-Mar	Movie	\$50.00
28-Mar	Gasoline	\$65.00
31-Mar	Snacks	\$15.50
1-Apr	Dinner	\$63.60
1-Apr	Payment	\$300.00
1-Apr	Gasoline	\$48.90
9-Apr	Plane Tickets	\$288.50

Buying or Leasing a Car

49. Name three common fees when leasing a car.
50. Name three common fees when buying a car.
51. If the cost of a car is \$35,500, and the residual value of the car after 3 years is \$27,800, what is the monthly depreciation for the car?
52. What is the annual interest rate if the money factor is 0.000045?
53. What is the payment for a lease on a car if the price of the car is \$29,900, the residual price of the car is \$16,500, the lease is for 24 months, and the APR is 8.9%?

Renting and Homeownership

54. Name two advantages of renting a residence.
55. Name two advantages of buying a home.
56. What is PMI?
57. What are the monthly payments for a 25-year mortgage for \$165,000 at 4.8% interest, not including escrow?
58. What are the monthly payments, including escrow, for a 25-year mortgage for \$165,000 at 4.8% interest, if the property taxes are \$5,200 annually, the homeowner's insurance is \$825 every 6 months, and the PMI is \$45 per month?

Income Tax

59. What is the maximum gift that a person can receive that is an exemption on federal income taxes?
60. Find the gross income if a person earns \$51,500 at their full-time job, they earn \$5,300 for contracted services, and \$5,000 in gambling winnings.
61. Find adjusted gross income for a person with a gross income of \$67,850, if they had a \$2,000 gift, they paid \$1,000 in mortgage interest, \$4,250 in property taxes, and they contributed \$750 to a qualifying charity.
62. How much SSI is paid by a person with an annual income of \$183,500?
63. How much in FICA taxes are paid from a person's paycheck with gross income of \$3,450?
64. How much in federal income tax does a person filing single pay if their taxable income is \$205,240? Use the tax table below.

Bracket	Lower Income Limit	Upper Income Limit	Tax Rate
1	0	\$10,275	10%
2	\$10,276	\$41,775	12%
3	\$41,776	\$89,075	22%
4	\$89,076	\$170,050	24%
5	\$170,051	\$215,950	32%
6	\$215,951	\$539,900	35%
7	\$539,901		37%

Chapter Test

1. 45 is what percent of 180?
2. Find 47% of 31. Round to two decimal places.
3. What is the sale price of a \$90 shirt if there is a 25% discount for the shirt?
4. A small boutique buys purses for \$27.50 per purse. What is the boutique's retail price for the purses if they add a 90% markup?
5. What was the discount on a computer if the retail price was \$1,200 and the sale price is \$950? Round to two decimal places if necessary.
6. A 15% simple interest loan of \$2,000 is taken out for 75 days. How much is owed when the loan is due?
7. What is the future value of a \$15,000 investment made at 4.65% interest compounded yearly for 10 years?
8. Apply the 50-30-20 budget philosophy to a monthly income of \$3,920.

9. \$2,500 is deposited in an account that bears 5.25% interest compounded quarterly for 15 years. What is the return on investment for that investment? Round to two decimal places if necessary.
10. How much per month must be paid into an ordinary annuity so its future value is \$150,000 after 10 years if the annuity earns 3.85% interest compounded monthly?
11. Bethanie's employer matches up to 7% of annual salary for deposits into the company sponsored IRA. Bethanie earns \$62,400 annually. She deposits \$8,000 annually into her IRA. How much does the company deposit in Bethanie's IRA?
12. If \$360 is deposited monthly in a mutual fund that is expected to earn 9.35% compounded monthly, how much will the mutual fund be worth after 23 years?
13. A \$14,250, 4-year loan is taken out at 9.15% interest. What are the monthly payments for the loan?
14. Using the amortization table below, how much total interest has been paid after payment 20?

Amortization Schedule Calculator					
Loan Amount	\$17,500.00				
Interest Rate (annual)	8.25%				
Term (Years)	5				
Monthly Payment	\$356.93				
Month	Payment	Principal	Interest	Total Interest	Balance
					\$17,500.00
1	\$356.93	\$236.62	\$120.31	\$120.31	\$17,263.38
2	\$356.93	\$238.25	\$118.69	\$239.00	\$17,025.13
3	\$356.93	\$239.89	\$117.05	\$356.05	\$16,785.24
4	\$356.93	\$241.54	\$115.40	\$471.44	\$16,543.71
5	\$356.93	\$243.20	\$113.74	\$585.18	\$16,300.51
6	\$356.93	\$244.87	\$112.07	\$697.25	\$16,055.64
7	\$356.93	\$246.55	\$110.38	\$807.63	\$15,809.09
8	\$356.93	\$248.25	\$108.69	\$916.32	\$15,560.84
9	\$356.93	\$249.95	\$106.98	\$1,023.30	\$15,310.89
10	\$356.93	\$251.67	\$105.26	\$1,128.56	\$15,059.22
11	\$356.93	\$253.40	\$103.53	\$1,232.09	\$14,805.82
12	\$356.93	\$255.14	\$101.79	\$1,333.88	\$14,550.67
13	\$356.93	\$256.90	\$100.04	\$1,433.92	\$14,293.77
14	\$356.93	\$258.66	\$98.27	\$1,532.19	\$14,035.11
15	\$356.93	\$260.44	\$96.49	\$1,628.68	\$13,774.66
16	\$356.93	\$262.23	\$94.70	\$1,723.38	\$13,512.43
17	\$356.93	\$264.04	\$92.90	\$1,816.28	\$13,248.39
18	\$356.93	\$265.85	\$91.08	\$1,907.36	\$12,982.54
19	\$356.93	\$267.68	\$89.25	\$1,996.62	\$12,714.86
20	\$356.93	\$269.52	\$87.41	\$2,084.03	\$12,445.34
21	\$356.93	\$271.37	\$85.56	\$2,169.59	\$12,173.97
22	\$356.93	\$273.24	\$83.70	\$2,253.29	\$11,900.73
23	\$356.93	\$275.12	\$81.82	\$2,335.11	\$11,625.62
24	\$356.93	\$277.01	\$79.93	\$2,415.03	\$11,348.61
25	\$356.93	\$278.91	\$78.02	\$2,493.06	\$11,069.69
26	\$356.93	\$280.83	\$76.10	\$2,569.16	\$10,788.86

15. What is the maximum amount of federal subsidized and unsubsidized loans per year for an independent student in a graduate program?

16. Braden takes out a private student loan for \$6,250 in his second year of his undergraduate IT program. He begins paying the loan after 39 months. If Braden's interest rate was 8%, how much is his balance when he begins to pay off the loan?
17. The billing cycle for Therese's credit card is July 7 to August 6 (31 days). The interest rate on the card is 17.9%. The balance on her card at the start of the billing cycle is \$925.00. The table below has her transactions for the billing cycle. What is her balance due at the end of the cycle?

Date	Activity	Amount
7-Jul	Billing Date Balance	\$925.00
10-Jul	Child Care	\$850.00
10-Jul	Gasoline	\$51.00
12-Jul	Food	\$135.50
18-Jul	Payment	\$750.00
18-Jul	Pizza	\$35.00
18-Jul	Gift	\$25.00
21-Jul	Food	\$121.75
30-Jul	Gasoline	\$47.50
1-Aug	Department store	\$173.00
4-Aug	Food	\$98.00

18. The cost of a car is \$45,750. The residual price after 2 years is \$28,822.50. What is the monthly depreciation for the car?
19. Find the payment for a 3-year lease on a car if the price of the car is \$30,000, the residual price of the car is \$20,000, and the APR is 8.5%.
20. What are the monthly payments, excluding escrow, for a 20-year mortgage for \$131,500 at 5.15% interest?
21. What are the monthly payments, including escrow, for a 20-year mortgage for \$131,500 at 5.15% interest, with property taxes of \$3,170 per year and homeowner's insurance of \$715 every 6 months?
22. If a person earns \$2700 gross on a paycheck, how much is taken out in FICA taxes?
23. Find the taxable income for a person who has \$81,200 in earnings, \$1,250 reported on a 1099-INT for a savings account, \$5,400 reported on a 1099-MISC for a contract job they did, paid \$5,250 in mortgage interest, paid \$4,700 in property taxes, and has \$4,200 in bond interest.
24. Find the federal income tax owed by a person filing single who has a taxable income of \$114,750. Use the table below to determine the taxes.

Bracket	Lower Income Limit	Upper Income Limit	Tax Rate
1	0	\$10,275	10%
2	\$10,276	\$41,775	12%

Bracket	Lower Income Limit	Upper Income Limit	Tax Rate
3	\$41,776	\$89,075	22%
4	\$89,076	\$170,050	24%
5	\$170,051	\$215,950	32%
6	\$215,951	\$539,900	35%
7	\$539,901		37%

25. If a person owes \$20,000 in federal income tax, but qualifies for \$4,000 in child tax credit and \$8,000 for child and dependent care credit, how much federal tax does the person actually pay?

7

PROBABILITY

Figure 7.1 Roulette is a game whose outcomes are based entirely on the concept of probability. (credit: modification of work "Roulette wheel" by Håkan Dahlström/Flickr, CC BY 2.0)

Chapter Outline

- 7.1 The Multiplication Rule for Counting
- 7.2 Permutations
- 7.3 Combinations
- 7.4 Tree Diagrams, Tables, and Outcomes
- 7.5 Basic Concepts of Probability
- 7.6 Probability with Permutations and Combinations
- 7.7 What Are the Odds?
- 7.8 The Addition Rule for Probability
- 7.9 Conditional Probability and the Multiplication Rule
- 7.10 The Binomial Distribution
- 7.11 Expected Value



Introduction

Casinos are big business; according to the American Gaming Association, commercial casinos in the United States brought in over \$43 billion in revenue in 2019. Casinos must walk a fine line in order to be profitable. Their customers must lose more money than they win, on average, in order to stay in business. But if the chances of a single customer winning more money than they lose is too small, people will stop coming in the door to play the games.

In this chapter, we'll study the techniques a casino must use to determine how likely it is that a customer will win a particular game, and then how the casino decides how much money a winner will rake in so that the customers are happy, but the casino also turns a profit in the long run. In order to figure out those likelihoods, we have to be able to somehow consider every possible outcome of these games. For example, in a game that involves players receiving 5 cards from a deck of 52, there are 2,598,960 possibilities for each player. We'll start off this chapter by learning how to count those possible outcomes.

7.1 The Multiplication Rule for Counting

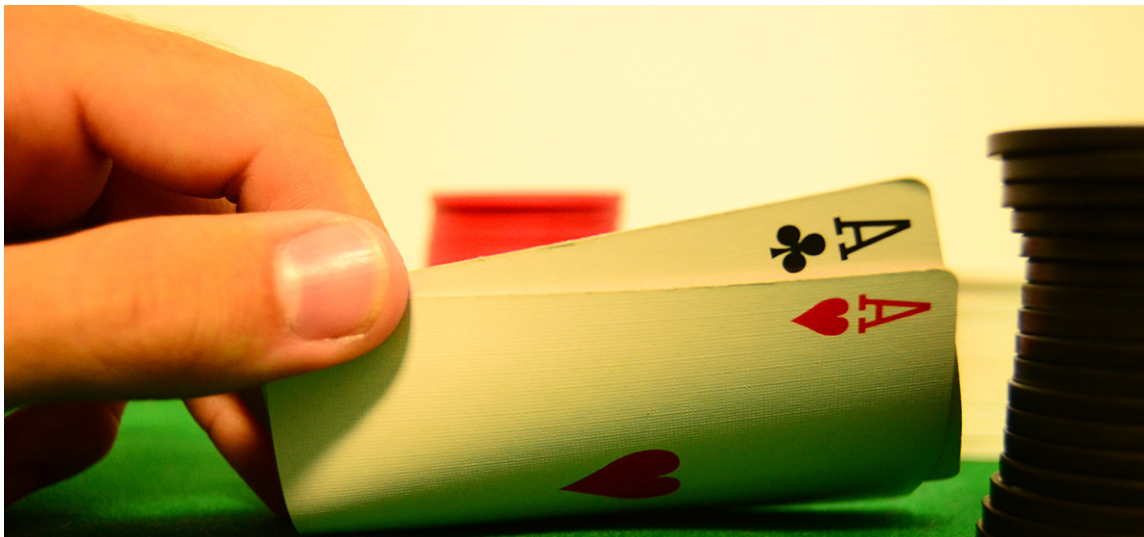


Figure 7.2 The Multiplication Rule for Counting allows us to compute more complicated probabilities, like drawing two aces from a deck. (credit: “Pair of Aces – Poker” by Poker Photos/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Apply the Multiplication Rule for Counting to solve problems.

One of the first bits of mathematical knowledge children learn is how to count objects by pointing to them in turn and saying: “one, two, three, ...” That’s a useful skill, but when the number of things that we need to count grows large, that method becomes onerous (or, for very large numbers, impossible for humans to accomplish in a typical human lifespan). So, mathematicians have developed short cuts to counting big numbers. These techniques fall under the mathematical discipline of **combinatorics**, which is devoted to counting.

Multiplication as a Combinatorial Short Cut

One of the first combinatorial short cuts to counting students learn in school has to do with areas of rectangles. If we have a set of objects to be counted that can be physically arranged into a rectangular shape, then we can use multiplication to do the counting for us. Consider this set of objects ([Figure 7.3](#)):



Figure 7.3

Certainly we can count them by pointing and running through the numbers, but it’s more efficient to group them ([Figure 7.4](#)).

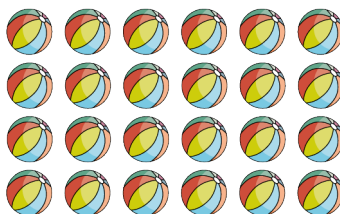


Figure 7.4

If we group the balls by 4s, we see that we have 6 groups (or, we can see this arrangement as 4 groups of 6 balls). Since multiplication is repeated addition (i.e., $6 \times 4 = 4 + 4 + 4 + 4 + 4 + 4$), we can use this grouping to quickly see that there are 24 balls.

Let’s generalize this idea a little bit. Let’s say that we’re visiting a bakery that offers customized cupcakes. For the cake, we have three choices: vanilla, chocolate, and strawberry. Each cupcake can be topped with one of four types of frosting: vanilla, chocolate, lemon, and strawberry. How many different cupcake combinations are possible? We can think of laying out all the possibilities in a grid, with cake choices defining the rows and frosting choices defining the columns

(Figure 7.5).

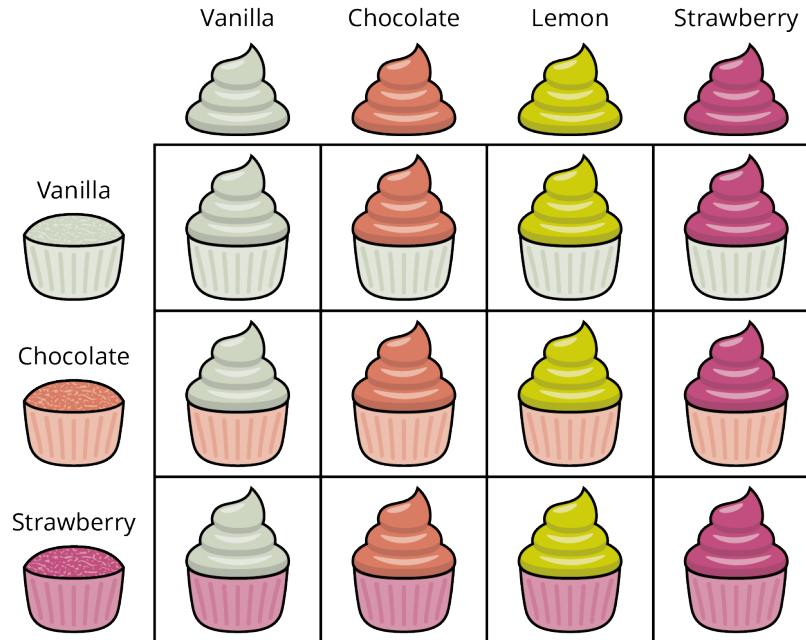


Figure 7.5

Since there are 3 rows (cakes) and 4 columns (frostings), we have $3 \times 4 = 12$ possible combinations. This is the reasoning behind the **Multiplication Rule for Counting**, which is also known as the Fundamental Counting Principle. This rule says that if there are n ways to accomplish one task and m ways to accomplish a second task, then there are $n \times m$ ways to accomplish both tasks. We can tack on additional tasks by multiplying the number of ways to accomplish those tasks to our previous product.

EXAMPLE 7.1

Using the Multiplication Rule for Counting

Every card in a standard deck of cards has two identifying characteristics: a suit (clubs, diamonds, hearts, or spades; these are indicated by these symbols, respectively: ♣, ♦, ♥, ♠) and a rank (ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king; the letters A, J, Q, and K are used to represent the words). Each possible pair of suit and rank appears exactly once in the deck. How many cards are in the standard deck?

✓ Solution

Since there are 4 suits and 13 ranks, the number of cards must be $4 \times 13 = 52$ (Figure 7.6).



Figure 7.6 Standard Deck of Cards, Sorted by Rank and Suit (credit: "Playing Cards, USS Arkansas" by Naval History & Heritage Command/Flickr, CC BY 2.0)

> YOUR TURN 7.1

1. Joe's Pizza Shack offers pizzas with 4 different types of crust and a choice of 15 toppings. How many different one-topping pizzas can be made at Joe's?

EXAMPLE 7.2

Using the Multiplication Rule for Counting for 4 Groups

The University Combinatorics Club has 31 members: 8 seniors, 7 juniors, 5 sophomores, and 11 first-years. How many possible 4-person committees can be formed by selecting 1 member from each class?

✓ Solution

Since we have 8 choices for the senior, 7 choices for the junior, 5 for the sophomore, and 11 for the first-year, there are $8 \times 7 \times 5 \times 11 = 3,080$ different ways to fill out the committee.

> YOUR TURN 7.2

1. The menu for Joe's Pizza Shack offers pizzas with 4 different types of crust and a choice of 15 toppings. Suppose that Joe's also offers a choice of 3 sauces and 2 cheese blends. How many different one-topping pizzas can be made at Joe's now?

EXAMPLE 7.3

Using the Multiplication Rule for Counting for More Groups

The standard license plates for vehicles in a certain state consist of 6 characters: 3 letters followed by 3 digits. There are 26 letters in the alphabet and 10 digits (0 through 9) to choose from. How many license plates can be made using this format?

✓ Solution

Since there are 26 different letters and 10 different digits, the total number of possible license plates is

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000.$$

YOUR TURN 7.3

1. At a certain college, ID cards are issued to all students, faculty, and staff. These cards have unique ID codes for each person: a letter to indicate the person's status (S for students, F for faculty, and E for staff), followed by 5 digits and finally 3 letters (these letters can be anything). How many different ID codes can be created using this scheme?

Check Your Understanding

1. A website that lets you build custom belts has 18 different buckles and 30 different straps. How many different belts can be made using those materials?
2. A chain of chicken restaurants offers a combo that includes your choice of 3 or 5 chicken strips, along with your choice of side dish. If there are 7 side dishes, how many different ways are there to build this combo meal?
3. When you flip a coin, there are 2 possible outcomes: heads and tails. Let's say you flip a coin 10 times, and after each you write down the result of the flip (H for heads, T for tails). How many different results (strings of 10 characters, where each is either an H or a T) are possible?
4. A T-shirt company allows shoppers to customize their shirts in several ways. There are 5 sizes, 8 shirt colors, 4 designs, and 5 design colors. How many different shirts can be made?
5. Josephine is trying to build her class schedule for next semester. Because of her work schedule, she has only 4 class periods that can work for her, and she must take 4 classes. If there are 15 classes that she could take during the first period, 18 during the second, 12 during the third, and 8 during the fourth, how many different schedules could Josephine build?



SECTION 7.1 EXERCISES

An ice-cream parlor sells 26 different flavors of ice cream. A basic sundae has one scoop of any flavor of ice cream, your choice of one of 3 sauces, and any one of 8 different toppings.

1. How many different basic sundaes are possible?
2. The ice-cream parlor also sells a medium sundae. The options are the same except it starts with 2 scoops of ice cream, which can be the same flavor or different flavors. How many different medium sundaes are there?
3. The ice-cream parlor also sells a large sundae. The choice of a large sundae allows you to choose any 3 scoops of ice cream, any 2 sauces (they can be the same, or you can choose 2 different ones), and any 3 toppings (that might be 3 servings of the same topping, or 2 servings of one topping and a single serving of another, or 3 different toppings). How many different large sundaes are possible?
4. A company that builds custom computers offers 4 hard drive sizes, 4 memory sizes, 3 graphics cards, and 3 display options. How many computer configurations do they offer, if customers choose one of each customization?
5. A video game allows users to customize their avatars. There are 12 hair styles that users may choose from, as well as 5 hair colors, 8 skin tones, 24 shirts, 12 pants, and 8 shoes. How many different avatars are possible?
6. A small company has 3 divisions: Sales, Research and Development, and Manufacturing. One person from each division will be chosen to create an advisory board for the management group. If there are 8 people in Sales, 15 in Research and Development, and 48 in Manufacturing, how many different compositions of the advisory board are possible?
7. A multiple-choice quiz has 5 questions, each of which has 4 possible answers. How many different ways are there to respond to this quiz?
8. The teacher decides to make the quiz from above a little harder by offering 5 responses on each of the 5 questions. How many ways are there to respond to this quiz?
9. In the United States, radio and television broadcast stations are assigned unique identifiers known as call signs. Call signs consist of 4 letters. The first is either K or W (generally speaking, stations with a K call sign are west of

the Mississippi River and stations with a W call sign are east of the river, though there are several exceptions to this rule); the remaining 3 letters can be anything. How many possible call signs are there under this system?

10. Little sister has asked big brother to play a new game she's invented. It uses a modified deck of cards with 3 suits and only the numbered cards (those with rank 2 through 10). How many cards are in her deck?
11. The board game *Mastermind* has 2 players. One of them is designated the codemaker who creates a code that consists of a series of 4 colors (indicated in the game with 4 colored pegs), which may contain repeats. The other player, who is the codebreaker, tries to guess the code. If there are 6 colors that the codemaker can use to make the code, how many possible codes can be made?

7.2 Permutations

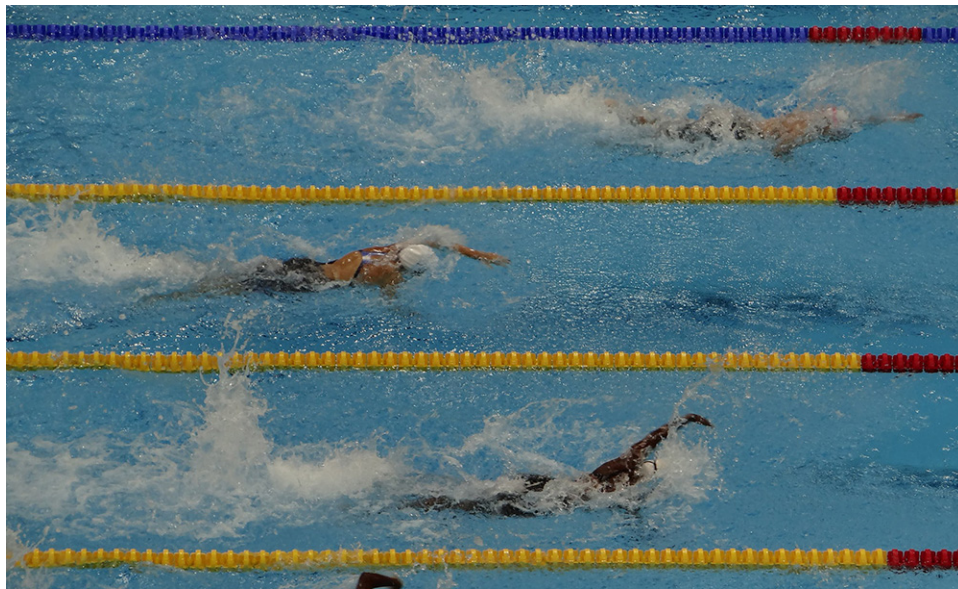


Figure 7.7 We can use permutations to calculate the number of different orders of finish in an Olympic swimming heat. (credit: "London 2012 Olympics Park Stratford London" by Gary Bembridge/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Use the Multiplication Rule for Counting to determine the number of permutations.
2. Compute expressions containing factorials.
3. Compute permutations.
4. Apply permutations to solve problems.

Swimming events are some of the most popular events at the summer Olympic Games. In the finals of each event, 8 swimmers compete at the same time, making for some exciting finishes. How many different orders of finish are possible in these events? In this section, we'll extend the Multiplication Rule for Counting to help answer questions like this one, which relate to **permutations**. A permutation is an ordered list of objects taken from a given population. The length of the list is given, and the list cannot contain any repeated items.

Applying the Multiplication Rule for Counting to Permutations

In the case of the swimming finals, one possible permutation of length 3 would be the list of medal winners (first, second, and third place finishers). A permutation of length 8 would be the full order of finish (first place through eighth place). Let's use the Multiplication Rule for Counting to figure out how many of each of these permutations there are.

EXAMPLE 7.4

Using the Multiplication Rule for Counting to Find the Number of Permutations

The final heat of Olympic swimming events features 8 swimmers (or teams of swimmers).

1. How many different podium placements (first place, second place, and third place) are possible?
2. How many different complete orders of finish (first place through eighth place) are possible?

✓ **Solution**

1. Let's start with the first place finisher. How many options are there? Since 8 swimmers are competing, there are 8 possibilities. Once that first swimmer completes the race, there are 7 swimmers left competing for second place. After the second finisher is decided, there are 6 swimmers remaining who could possibly finish in third place. Thus, there are 8 possibilities for first place, 7 for second place, and 6 for third place. The Multiplication Rule for Counting then tells us there are $8 \times 7 \times 6 = 336$ different ways the winners' podium can be filled out.
2. To look at the complete order of finish, we can continue the pattern we can see in part 1 of this example: There are 5 possibilities for fourth place, 4 for fifth place, 3 for sixth place, 2 for seventh place, and then just 1 swimmer is left to finish in eighth place. Using the Multiplication Rule for Counting, we see that there are $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$ possible orders of finish.

> **YOUR TURN 7.4**

1. You have a hand of 5 cards (that happen to create what's called a royal flush in the game of poker): $10\spadesuit, J\spadesuit, Q\spadesuit, K\spadesuit, \text{ and } A\spadesuit$. Into how many different orders can you put those cards?

Factorials

The pattern we see in [Example 7.4](#) occurs commonly enough that we have a name for it: **factorial**.

For any positive whole number n , we define the factorial of n (denoted $n!$ and read " n factorial") to be the product of every whole number less than or equal to n . We also define $0!$ to be equal to one. We will use factorials in a couple of different contexts, so let's get some practice doing computations with them.

EXAMPLE 7.5

Computing Factorials

Compute the following:

1. $4!$
2. $\frac{8!}{6!}$
3. $\frac{9!}{3!4!}$

✓ **Solution**

1. $4! = 4 \times 3 \times 2 \times 1 = 24$
2. There are two ways to approach this calculation. The first way is to compute the factorials first, then divide:

$$\frac{8!}{6!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{40,320}{720} = 56$$

However, there is an easier way! You may notice in the second step that there are several terms that can be canceled; that's always the case whenever we divide factorials. In this case, notice that we can rewrite the numerator like this:

$$8! = 8 \times 7 \times (6 \times 5 \times 4 \times 3 \times 2 \times 1) = 8 \times 7 \times 6!$$

With that in mind, we can proceed this way by canceling out the $6!$:

$$\frac{8!}{6!} = \frac{8 \times 7 \times \cancel{6!}}{\cancel{6!}} = 8 \times 7 = 56$$

That's much easier!

3. Let's approach this one using our canceling technique. When we see two factorials in either the numerator or denominator, we should focus on the larger one first. So:

$$\frac{9!}{3!4!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4!}}{\cancel{3!4!}} = \frac{9 \times 8 \times 7 \times \cancel{6} \times 5}{\cancel{3} \times \cancel{2} \times \cancel{1}} = 9 \times 8 \times 7 \times 5 = 2,520$$

> YOUR TURN 7.5

Compute the following:

1. $6!$
2. $\frac{12!}{10!}$
3. $\frac{8!}{4!4!}$

Permutations

As we've seen, factorials can pop up when we're computing permutations. In fact, there is a formula that we can use to make that connection explicit. Let's define some notation first. If we have a collection of n objects and we wish to create an ordered list of r of the objects (where $1 \leq r \leq n$), we'll call the number of those permutations ${}_nP_r$ (read "the number of permutations of n objects taken r at a time"). We formalize the formula we'll use to compute permutations below.

FORMULA

$${}_nP_r = \frac{n!}{(n-r)!}$$

If you wondered why we defined $0! = 1$ earlier, it was to make formulas like this one work; if we have n objects and want to order all of them (so, we want the number of permutations of n objects taken n at a time), we get ${}_nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$. Next, we'll get some practice computing these permutations.

EXAMPLE 7.6

Computing Permutations

Find the following numbers:

1. The number of permutations of 12 objects taken 3 at a time
2. The number of permutations of 8 objects taken 5 at a time
3. The number of permutations of 32 objects taken 2 at a time

Solution

1. ${}_{12}P_3 = \frac{12!}{(12-3)!} = \frac{12!}{9!} = \frac{12 \times 11 \times 10 \times \cancel{9!}}{\cancel{9!}} = 12 \times 11 \times 10 = 1,320$
2. ${}_8P_5 = \frac{8!}{(8-5)!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 8 \times 7 \times 6 \times 5 \times 4 = 6,720$
3. ${}_{32}P_2 = \frac{32!}{(32-2)!} = \frac{32 \times 31 \times \cancel{30!}}{\cancel{30!}} = 32 \times 31 = 992$

> YOUR TURN 7.6

Find the following numbers:

1. The number of permutations of 6 objects taken 2 at a time
2. The number of permutations of 14 objects taken 4 at a time
3. The number of permutations of 19 objects taken 3 at a time

EXAMPLE 7.7

Applying Permutations

1. A high school graduating class has 312 students. The top student is declared valedictorian, and the second-best is named salutatorian. How many possible outcomes are there for the valedictorian and salutatorian?

- In the card game blackjack, the dealer's hand of 2 cards is dealt with 1 card faceup and 1 card facedown. If the game is being played with a single deck of (52) cards, how many possible hands could the dealer get?
- The University Combinatorics Club has 3 officers: president, vice president, and treasurer. If there are 18 members of the club, how many ways are there to fill the officer positions?

 Solution

- This is the number of permutations of 312 students taken 2 at a time, and ${}_{312}P_2 = 97,032$.
- We want the number of permutations of 52 cards taken 2 at a time, and ${}_{52}P_2 = 2,652$.
- Here we're looking for the number of permutations of 18 members taken 3 at a time, and ${}_{18}P_3 = 4,896$.

 YOUR TURN 7.7

- One of the big draws at this year's state fair is the pig race. There are 15 entrants, and prizes are given to the top three finishers. How many different combinations of top-three finishes could there be?

 WHO KNEW?

Very Big Permutations

Permutations involving relatively small sets of objects can get very big, very quickly. A standard deck contains 52 cards. So, the number of different ways to shuffle the cards—in other words, the number of permutations of 52 objects taken 52 at a time—is $52! \approx 8 \times 10^{67}$ (written out, that's an 8 followed by 67 zeroes). The estimated age of the universe is *only* about 4×10^{17} seconds. So, if a very bored all-powerful being started shuffling cards at the instant the universe began, it would have to have averaged at least $\frac{8 \times 10^{67}}{4 \times 10^{17}} \approx 2 \times 10^{50}$ shuffles *per second since the beginning of time* to have covered every possible arrangement of a deck of cards. That means the next time you pick up a deck of cards and give it a good shuffle, it's almost certain that the particular arrangement you created has never been created before and likely never will be created again.

Check Your Understanding

- Compute $5!$.
- Compute $\frac{10!}{7!3!}$.
- Compute ${}_{12}P_3$.
- Compute ${}_8P_4$.
- The standard American edition of the board game *Monopoly* has a deck of 15 orange Chance cards. In how many different ways could the first 4 Chance cards drawn in a game appear?



SECTION 7.2 EXERCISES

For the following exercises, give a whole number that's equal to the given expression.

- $3!$
- $9!$
- $\frac{7!}{2!2!3!}$
- $\frac{8!}{5!2!}$
- $\frac{21!}{18!2!}$
- $\frac{28!}{26!2!}$
- $\frac{34!}{30!3!}$
- $\frac{17!}{12!5!}$
- ${}_4P_3$
- ${}_7P_5$

11. ${}_{12}P_{10}$
12. ${}_{14}P_{10}$
13. ${}_{10}P_8$
14. ${}_{15}P_{11}$

The following exercises are about the card game euchre, which uses a partial standard deck of cards: It only has the cards with ranks 9, 10, J, Q, K, and A for a total of 24 cards. Some variations of the game use the 8s or the 7s and 8s, but we'll stick with the 24-card version.

15. A euchre hand contains 5 cards. How many ways are there to receive a 5-card hand (where the order in which the cards are received matters, i.e., $9\heartsuit, J\heartsuit, K\clubsuit, 9\spadesuit, 10\spadesuit$ is different from $9\spadesuit, J\heartsuit, 9\heartsuit, K\clubsuit, 10\spadesuit$)?
16. After all 4 players get their hands, the remaining 4 cards are placed facedown in the center of the table. How many arrangements of 4 cards are there from this deck?
17. Euchre is played with partners. How many ways are there for 2 partners to receive 5-card hands (where the order in which the cards are received matters)?
18. How many different arrangements of the full euchre deck are possible (i.e., how many different shuffles are there)?

The following exercises involve a horse race with 13 entrants.

19. How many possible complete orders of finish are there?
20. An exacta bet is one where the player tries to predict the top two finishers in order. How many possible exacta bets are there for this race?
21. A trifecta bet is one where the player tries to predict the top three finishers in order. How many possible trifecta bets are there for this race?
22. A superfecta bet is one where the player tries to predict the top four finishers in order. How many possible superfecta bets are there for this race?

7.3 Combinations



Figure 7.8 Combinations help us count things like the number of possible card hands, when the order in which the cards were drawn doesn't matter. (credit: "IMG_3177" by Zanaca/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Distinguish between permutation and combination uses.
2. Compute combinations.
3. Apply combinations to solve applications.

In [Permutations](#), we studied permutations, which we use to count the number of ways to generate an ordered list of a given length from a group of objects. An important property of permutations is that the order of the list matters: The results of a race and the selection of club officers are examples of lists where the order is important. In other situations, the order is not important. For example, in most card games where a player receives a hand of cards, the order in which the cards are received is irrelevant; in fact, players often rearrange the cards in a way that helps them keep the cards organized.

Combinations: When Order Doesn't Matter

In situations in which the order of a list of objects doesn't matter, the lists are no longer permutations. Instead, we call

them **combinations**.

EXAMPLE 7.8

Distinguishing Between Permutations and Combinations

For each of the following situations, decide whether the chosen subset is a permutation or a combination.

1. A social club selects 3 members to form a committee. Each of the members has an equal share of responsibility.
2. You are prompted to reset your email password; you select a password consisting of 10 characters without repeats.
3. At a dog show, the judge must choose first-, second-, and third-place finishers from a group of 16 dogs.
4. At a restaurant, the special of the day comes with the customer's choice of 3 sides taken from a list of 6 possibilities.

Solution

1. Since there is no distinction among the responsibilities of the 3 committee members, the order isn't important. So, this is a combination.
2. The order of the characters in a password matter, so this is a permutation.
3. The order of finish matters in a dog show, so this is a permutation.
4. A plate with mashed potatoes, peas, and broccoli is functionally the same as a plate with peas, broccoli, and mashed potatoes, so this is a combination.

YOUR TURN 7.8

Decide whether the following represent permutations or combinations:

1. On Halloween, you give each kid who comes to your door 3 pieces of candy, taken randomly from a candy dish.
2. Your class is going on a field trip, but there are too many people for one vehicle. Your instructor chooses half the class to take the first vehicle.

Counting Combinations

Permutations and combinations are certainly related, because they both involve choosing a subset of a large group. Let's explore that connection, so that we can figure out how to use what we know about permutations to help us count combinations. We'll take a basic example. How many ways can we select 3 letters from the group A, B, C, D, and E? If order matters, that number is ${}_5P_3 = 60$. That's small enough that we can list them all out in the table below.

ABC	ABD	ABE	ACB	ACD	ACE
ADB	ADC	ADE	AEB	AEC	AED
BAC	BAD	BAE	BCA	BCD	BCE
BDA	BDC	BDE	BEA	BEC	BED
CAB	CAD	CAE	CBA	CBD	CBE
CDA	CDB	CDE	CEA	CEB	CED
DCA	DAC	DAE	DBA	DBC	DBE
DCA	DCB	DCE	DEA	DEB	DEC
EAB	EAC	EAD	EBA	EBC	EBD
ECA	ECB	ECD	EDA	EDB	EDC

Now, let's look back at that list and color-code it so that groupings of the same 3 letters get the same color, as shown in [Figure 7.9](#):

ABC	ABD	ABE	ACB	ACD	ACE
ADB	ADC	ADE	AEB	AEC	AED
BAC	BAD	BAE	BCA	BCD	BCE
BDA	BDC	BDE	BEA	BEC	BED
CAB	CAD	CAE	CBA	CBD	CBE
CDA	CDB	CDE	CEA	CEB	CED
DAB	DAC	DAE	DBA	DBC	DBE
DCA	DCB	DCE	DEA	DEB	DEC
EAB	EAC	EAD	EBA	EBC	EBD
ECA	ECB	ECD	EDA	EDB	EDC

Figure 7.9

After color-coding, we see that the 60 cells can be seen as 10 groups (colors) of 6. That's no coincidence! We've already seen how to compute the number of permutations using the [formula](#). To compute the number of combinations, let's count them another way using the Multiplication Rule for Counting. We'll do this in two steps:

Step 1: Choose 3 letters (paying no attention to order).

Step 2: Put those letters in order.

The number of ways to choose 3 letters from this group of 5 (A, B, C, D, E) is the number of combinations we're looking for; let's call that number ${}_5C_3$ (read "the number of combinations of 5 objects taken 3 at a time"). We can see from our chart that this is ten (the number of colors used). We can generalize our findings this way: remember that the number of permutations of n things taken r at a time is ${}_nP_r = \frac{n!}{(n-r)!}$. That number is *also* equal to ${}_nC_r \times r!$, and so it must be the case that $\frac{n!}{(n-r)!} = {}_nC_r \times r!$. Dividing both sides of that equation by $r!$ gives us the formula below.

FORMULA

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

EXAMPLE 7.9

Using the Combination Formula

Compute the following:

- ${}_8C_3$
- ${}_{12}C_5$
- ${}_{15}C_9$

Solution

- ${}_8C_3 = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \times 5!} = 8 \times 7 = 56$
- ${}_{12}C_5 = \frac{12!}{5!(12-5)!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{5 \times 4 \times 3 \times 2 \times 1 \times 7!} = 11 \times 9 \times 8 = 792$
- ${}_{15}C_9 = \frac{15!}{9!(15-9)!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9!}{9! \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{14 \times 13 \times 11 \times 10}{4} = 5,005$

> YOUR TURN 7.9

Compute the following:

1. ${}_6C_4$
2. ${}_{10}C_8$
3. ${}_{14}C_5$

EXAMPLE 7.10**Applying the Combination Formula**

1. In the card game Texas Hold'em (a variation of poker), players are dealt 2 cards from a standard deck to form their hands. How many different hands are possible?
2. The board game *Clue* uses a deck of 21 cards. If 3 people are playing, each person gets 6 cards for their hand. How many different 6-card *Clue* hands are possible?
3. Palmetto Cash 5 is a game offered by the South Carolina Education Lottery. Players choose 5 numbers from the whole numbers between 1 and 38 (inclusive); the player wins the jackpot of \$100,000 if the randomizer selects those numbers in any order. How many different sets of winning numbers are possible?

✓ Solution

1. A standard deck has 52 cards, and a hand has 2 cards. Since the order doesn't matter, we use the formula for counting combinations:

$${}_{52}C_2 = \frac{52!}{2!(52-2)!} = \frac{52 \times 51 \times \mathbf{50!}}{2 \times 1 \times \mathbf{50!}} = \frac{52 \times 51}{2} = 1,326.$$

2. Again, the order doesn't matter, so the number of combinations is:

$${}_{21}C_6 = \frac{21!}{6!(21-6)!} = \frac{21 \times \mathbf{20} \times 19 \times \mathbf{18} \times 17 \times 16 \times \mathbf{15!}}{\mathbf{6} \times \mathbf{5} \times \mathbf{4} \times \mathbf{3} \times 2 \times 1 \times \mathbf{15!}} = \frac{21 \times 19 \times 17 \times 16}{2} = 54,264.$$

3. There are 38 numbers to choose from, and we must pick 5. Since order doesn't matter, the number of combinations is:

$${}_{38}C_5 = \frac{38!}{5!(38-5)!} = \frac{38 \times 37 \times 36 \times 35 \times 34 \times \mathbf{33!}}{5 \times 4 \times 3 \times 2 \times 1 \times \mathbf{33!}} = 501,492.$$

> YOUR TURN 7.10

1. At a charity event with 58 people in attendance, 3 raffle winners are chosen. All receive the same prize, so order doesn't matter. How many different groups of 3 winners can be chosen?
2. A sorority with 42 members needs to choose a committee with 4 members, each with equal responsibility. How many committees are possible?

 The notation and nomenclature used for the number of combinations is not standard across all sources. You'll sometimes see $\binom{n}{r}$ instead of ${}_nC_r$. Sometimes you'll hear that expression read as “*n* choose *r*” as shorthand for “the number of combinations of *n* objects taken *r* at a time.”

**PEOPLE IN MATHEMATICS****Early Eastern Mathematicians**

Although combinations weren't really studied in Europe until around the 13th century, mathematicians of the Middle and Far East had already been working on them for hundreds of years. The Indian mathematician known as Pingala had described them by the second century BCE; Varāhamihira (fl. sixth century) and Halayudha (fl. 10th century) extended Pingala's work. In the ninth century, a Jain mathematician named Mahāvīra gave the formula for

combinations that we use today.

In 10th-century Baghdad, a mathematician named Al-Karaji also knew formulas for combinations; though his work is now lost, it was known to (and repeated by) Persian mathematician Omar Khayyam, whose work survives. Khayyam is probably best remembered as a poet, with his *Rubaiyat* being his most famous work.

Meanwhile, in 11th-century China, Jia Xian also was working with combinations, as was his 13th-century successor Yang Hui.

It is not known whether the discoveries of any of these men were known in the other regions, or if the Indians, Persians, and Chinese all came to their discoveries independently. We do know that mathematical knowledge and sometimes texts did get passed along trade routes, so it can't be ruled out.

EXAMPLE 7.11

Combining Combinations with the Multiplication Rule for Counting

The student government at a university consists of 10 seniors, 8 juniors, 6 sophomores, and 4 first-years.

1. How many ways are there to choose a committee of 8 people from this group?
2. How many ways are to choose a committee of 8 people if the committee must consist of 2 people from each class?

Solution

1. There are 28 people to choose from, and we need 8. So, the number of possible committees is ${}_{28}C_8 = 3,108,105$.
2. Break the selection of the committee members down into a 4-step process: Choose the seniors, then choose the juniors, then the sophomores, and then the first-years, as shown in the table below:

Class	Number of Ways to Choose Committee Representatives
senior	${}_{10}C_2 = 45$
junior	${}_8C_2 = 28$
sophomore	${}_6C_2 = 15$
first-year	${}_4C_2 = 6$

The Multiplication Rule for Counting tells us that we can get the total number of ways to complete this task by multiplying together the number of ways to do each of the four subtasks. So, there are $45 \times 28 \times 15 \times 6 = 113,400$ possible committees with these restrictions.

YOUR TURN 7.11

1. How many ways are there to choose a hand of 6 cards from a standard deck with the constraint that 3 are , 2 are , and 1 is .

Check Your Understanding

11. Suppose you want to count the number of ways that you can arrange the apps on the home screen on your phone. Should you use permutations or combinations?
12. Your little brother is packing up for a family vacation, but there's only room for 3 of his toys. If you want to know how many possible groups of toys he can bring, should you use permutations or combinations?
13. Compute ${}_{12}C_{10}$.
14. Compute ${}_{16}C_3$.

15. You're planning a road trip with some friends. Though you have 6 friends you'd consider bringing along, you only have room for 3 other people in the car. How many different possibilities are there for your road trip squad?
16. You're packing for a trip, for which you need 3 shirts and 3 skirts. If you have 8 shirts and 5 skirts that would work for the trip, how many different ways are there to pack for the trip?



SECTION 7.3 EXERCISES

For the following exercises, decide whether the situation describes a permutation or a combination.

1. You're packing for vacation, and you need to pick 5 shirts.
2. You and your friends are about to play a game, and you need to decide who will have the first turn, second turn, and so on.
3. You are watching your favorite reality show, and you want to know how many possibilities there are for the order of finish for the top three.
4. You are going to be working in groups of 4 with your classmates, and you want to know how many possibilities there are for the composition of your group.

For the following exercises, express your answers as whole numbers.

5. ${}_5C_3$
6. ${}_8C_2$
7. ${}_8C_6$
8. ${}_{12}C_3$
9. ${}_{12}C_5$
10. ${}_{14}C_3$
11. ${}_{14}C_{10}$
12. ${}_{15}C_5$
13. ${}_{15}C_{13}$
14. ${}_{18}C_3$
15. ${}_{18}C_6$
16. ${}_{20}C_4$
17. In most variations of the card game poker, a hand consists of 5 cards, where the order doesn't matter. How many different poker hands are there?
18. A professor starts each class by choosing 3 students to present solutions to homework problems to the class. If there are 41 students in the class, in how many different ways can the professor make those selections?
19. An election for at-large members of a school board has 7 candidates; 3 will be elected. How many different ways can those 3 seats be filled?
20. There are 20 contestants on a reality TV show; at the end of the first episode, 10 are eliminated. How many different groups of eliminated contestants are possible?
21. At a horse race, bettors can place a bet called an exacta box. For this bet, the player chooses 2 horses; if those horses finish first and second (in either order), the player wins. In a race with 12 horses in the field, how many possible exacta box bets are there?

The following exercises are about the card game euchre, which uses a partial standard deck of cards: it only has the cards with ranks 9, 10, J, Q, K, and A (for a total of 24 cards). Some variations of the game use the 8s or the 7s and 8s, but we'll stick with the 24-card version.

22. A euchre hand contains 5 cards. How many ways are there to receive a 5-card hand (where the order in which the cards are received doesn't matter, i.e., $9\heartsuit, J\heartsuit, K\clubsuit, 9\spadesuit, 10\spadesuit$ is the same as $9\spadesuit, J\heartsuit, 9\heartsuit, K\clubsuit, 10\spadesuit$)?
23. After all 4 players get their hands, the remaining 4 cards are placed face down in the center of the table. How many different groups of 4 cards are there from this deck?
24. Euchre is played with partners. How many ways are there for 2 partners to receive 5-card hands (where, as above, the order doesn't matter)? Hint: After the first person gets their cards, there are $52 - 5 = 47$ cards left for the second person.

You and 5 of your friends are at an amusement park, and are about to ride a roller coaster. The cars have room for 6 people arranged in 3 rows of 2, so you and your friends will perfectly fill one car.

25. How many ways are there to choose the 2 people in the front row?
26. Assuming the front row has been selected, how many ways are there to choose the 2 people in the middle row?

27. Assuming the first 2 rows have been selected, how many ways are there to choose the 2 people in the back row?
28. Using the Multiplication Rule for Counting and your answers to the earlier parts of this exercise, how many ways are there for your friends to sort yourselves into rows to board the roller coaster?

The University Combinatorics Club has 18 members. Four of them will be selected to form a committee.

29. How many different committees of 4 are possible, assuming all of the duties are shared equally?
30. Instead of sharing responsibility equally, one person will be chosen to be the committee chair. How many different committees are possible? Count these by selecting a chair first, then selecting the remaining 3 members of the committee from the remaining club members and use the Multiplication Rule for Counting. Show your work.
31. Let's count the number of committees with chairs a different way: First, choose 4 people for the committee (as in the first question), then choose 1 of the 4 to be chair. Show your work. Do you get the same number?

Powerball® is a multistate lottery game, which costs \$2 to play. Players fill out a ticket by choosing 5 numbers between 1 and 69 (these are the white balls) and then a single number between 1 and 26 (this is the Powerball).

32. How many different ways are there to choose the white balls? Players who match these 5 numbers exactly (but not the Powerball) win \$1 million.
33. How many ways are there to choose the Powerball? Players who correctly pick the Powerball win \$4.
34. How many ways are there to play the game altogether? Players who match all 5 white balls and the Powerball win (or share) the grand prize. (The grand prize starts at \$40 million; if no players win the grand prize, the value goes up for the next drawing. The highest value it has ever reached is \$1.586 *billion*!) How many ways are there to fill out a single Powerball ticket?
35. You are in charge of programming for a music festival. The festival has a main stage, a secondary stage, and several smaller stages. There are 40 bands confirmed for the festival. Five of those will play the main stage, and 8 will play the secondary stage. How many ways are there for you to allocate bands to these 2 stages?

7.4 Tree Diagrams, Tables, and Outcomes



Figure 7.10 In genetics, the characteristics of an offspring organism depends on the characteristics of its parents. (credit: "Pea Plant" by Maria Keays/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Determine the sample space of single stage experiment.
2. Use tables to list possible outcomes of a multistage experiment.
3. Use tree diagrams to list possible outcomes of a multistage experiment.

In the 19th century, an Augustinian friar and scientist named Gregor Mendel used his observations of pea plants to set out his theory of genetic propagation. In his work, he looked at the offspring that resulted from breeding plants with different characteristics together. For applications like this, it is often insufficient to only know in how many ways a

process might end; we need to be able to list all of the possibilities. As we've seen, the number of possible outcomes can be very large! Thus, it's important to have a strategy that allows us to systematically list these possibilities to make sure we don't leave any out. In this section, we'll look at two of these strategies.

Single Stage Experiments

When we are talking about combinatorics or probability, the word “**experiment**” has a slightly different meaning than it does in the sciences. Experiments can range from very simple (“flip a coin”) to very complex (“count the number of uranium atoms that undergo nuclear fission in a sample of a given size over the course of an hour”). Experiments have unknown outcomes that generally rely on something random, so that if the experiment is repeated (or **replicated**) the outcome might be different. No matter what the experiment, though, analysis of the experiment typically begins with identifying its sample space.

The **sample space** of an experiment is the set of all of the possible outcomes of the experiment, so it's often expressed as a set (i.e., as a list bound by braces; if the experiment is “randomly select a number between 1 and 4,” the sample space would be written $\{1, 2, 3, 4\}$).

EXAMPLE 7.12

Finding the Sample Space of an Experiment

For each of the following experiments, identify the sample space.

1. Flip a coin (which has 2 faces, typically called “heads” and “tails”) and note which face is up.
2. Flip a coin 10 times and count the number of heads.
3. Roll a 6-sided die and note the number that is on top.
4. Roll two 6-sided dice and note the sum of the numbers on top.

✓ Solution

1. If we use “H” to denote “heads is facing up” and “T” to denote “tails is facing up”, then the sample space is $\{H, T\}$.
2. It's possible (though unlikely) that there will be no heads flipped; the outcome in that case would be “0.” It's also possible (more likely, but still quite unlikely) that only one flip will result in heads. Any other whole number is possible, up to the maximum: We're flipping the coin 10 times, so we can't get any more than 10 heads. So, the sample space is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
3. There are 6 numbers on the die: 1, 2, 3, 4, 5, and 6. So, the sample space for a single roll of the die is $\{1, 2, 3, 4, 5, 6\}$.
4. If we roll 2 dice, the smallest possible sum we could get is $1 + 1 = 2$ and the biggest is $6 + 6 = 12$. Every other whole number between those two is possible. So, the sample space is $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

> YOUR TURN 7.12

Identify the sample space of each experiment.

1. You draw a card from a standard deck and note its suit.
2. You draw a card from a standard deck and note its rank.
3. You roll a 4-sided die and note the number on the bottom. (A 4-sided die is shaped like a pyramid, so when it comes to rest, there's no single side facing up.)
4. You roll three 4-sided dice and note the sum of the numbers on the bottom.

Multistage Experiments

Some experiments have more complicated sample spaces because they occur in stages. These stages can occur in succession (like drawing cards one at a time) or simultaneously (rolling 2 dice). Sample spaces get more complicated as the complexity of the experiment increases, so it's important to choose a systematic method for identifying all of the possible outcomes. The first method we'll discuss is the table.

Using Tables to Find Sample Spaces

Tables are useful for finding the sample space for experiments that meet two criteria: (1) The experiment must have only two stages, and (2) the outcomes of each stage must have no effect on the outcomes of the other. When the stages do not affect each other, we say the stages are **independent**. Otherwise, the stages are **dependent** and so we can't use tables; we'll look at a method for analyzing dependent stages soon.

EXAMPLE 7.13**Determining Independence**

Decide whether the two stages in these experiments are independent or dependent.

1. You flip a coin and note the result, and then flip the coin again and note the result.
2. You draw 2 cards from a standard deck (52 cards), one at a time.

✓ **Solution**

1. No matter what happens on the first flip, the second flip has the same sample space: {H, T} (You'll sometimes hear the phrase "The coin has no memory"). So, these stages are independent.
2. Let's say that the first card you draw is A♠. The sample space for the second draw consists of all the cards *except* A♠ (since that card is no longer in the deck, you can't draw it again). If instead that first card was 2♥, the sample space for the second draw is different: it's every card except 2♥. Since the sample space for the second card changes based on the result of the first draw, these stages are dependent.

> **YOUR TURN 7.13**

Decide whether the two stages in these experiments are dependent or independent.

1. You're getting dressed to go to a party, and you plan to wear a blouse and a skirt. You choose the blouse first, then the skirt (assume that you'd be comfortable wearing any of your skirts with any of your blouses).
2. On further reflection, you realize that some of your skirts clash with some of your blouses. So, you choose the blouse first, and then choose a skirt that goes with your chosen blouse.

If you have a two-stage experiment with independent stages, a table is the most straightforward way to identify the sample space. To build a table, you list the outcomes of one stage of the experiment along the top of the table and the outcomes of the other stage down the side. The cells in the interior of the table are then filled using the outcomes associated with each cell's row and column. Let's look at an example.

EXAMPLE 7.14**Using Tables to Identify Sample Spaces**

Identify the sample spaces of these experiments using tables.

1. You roll two dice: one 4-sided and one 6-sided.
2. You're in an ice-cream shop, and you're going to get a single scoop of ice cream with a topping. The flavors of ice cream you're considering are vanilla, chocolate, and rocky road; the toppings are fudge, whipped cream, and sprinkles.
3. The pea plants you're breeding have two possible pod colors: green and yellow. These colors are decided by a particular gene, which comes in two types: "G" for green, and "g" for yellow (In genetics, capital letters usually denote *dominant* genes, while lower-case letters denote *recessive* genes). Each plant has two genes. If you breed a Gg pea plant with a gg plant, the offspring plant will get one gene from each parent. What are the possible outcomes?

✓ **Solution**

1. **Step 1:** Make the outline of a table, with the results of the 4-sided roll on one side and the results of the 6-sided roll on the other. In practice, it doesn't matter which you choose; for this example, we'll put the 4-sided results on top (labeling the columns of the chart) and the 6-sided results on the side (labeling the rows of the chart) as shown in the following table: