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2e.

College Physics 2e

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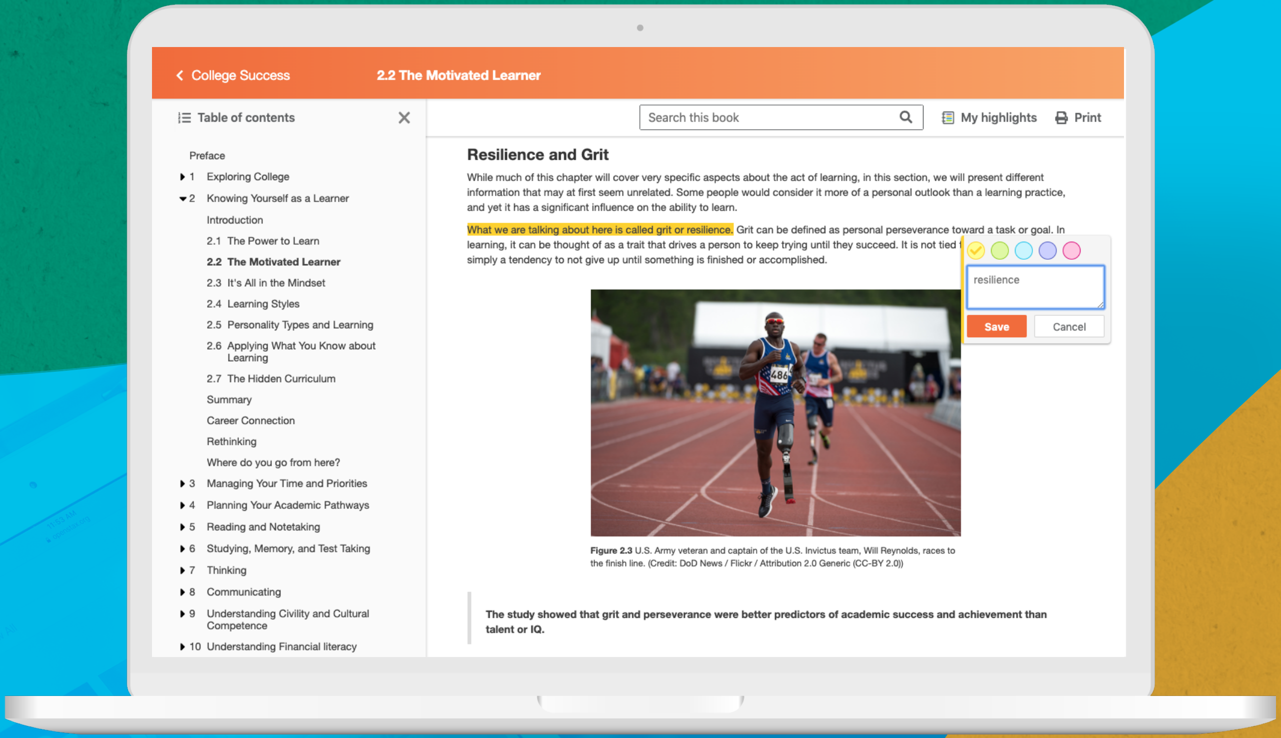

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Resilience and Grit

While much of this chapter will cover very specific aspects about the act of learning, in this section, we will present different information that may at first seem unrelated. Some people would consider it more of a personal outlook than a learning practice, and yet it has a significant influence on the ability to learn.

What we are talking about here is called grit or resilience. Grit can be defined as personal perseverance toward a task or goal. In learning, it can be thought of as a trait that drives a person to keep trying until they succeed. It is not tied t simply a tendency to not give up until something is finished or accomplished.

Figure 2.3 U.S. Army veteran and captain of the U.S. Invictus team, Will Reynolds, races to the finish line. (Credit: DoD News / Flickr / Attribution 2.0 Generic (CC-BY 2.0))

> **The study showed that grit and perseverance were better predictors of academic success and achievement than talent or IQ.**

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PREFACE

About OpenStax

OpenStax is part of Rice University, which is a 501(c)(3) nonprofit charitable corporation. As an educational initiative, it's our mission to transform learning so that education works for every student. Through our partnerships with philanthropic organizations and our alliance with other educational resource companies, we're breaking down the most common barriers to learning. Because we believe that everyone should and can have access to knowledge.

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Format

You can access this textbook for free in web view or PDF through OpenStax.org, and for a low cost in print.

About *College Physics 2e*

College Physics 2e provides a comprehensive and welcoming introduction to the principles, concepts, and applications typically covered in introductory physics courses. The book progresses through a learning model intended to support students at various levels, and offers faculty a great deal of flexibility in the approach and organization of their course. The text is grounded in real-world examples to help students understand the importance of physics in their lives and especially their future careers. It requires knowledge of algebra and some trigonometry, but not calculus; quantitative explanations and solutions are extremely detailed in order to build a thorough and useful understanding among students. *College Physics 2e* orients its coverage around clear and widely accepted learning outcomes. It includes links to simulations and other multimedia, and each section contains ample practice opportunities in a wide array of question types.

Coverage and Scope

College Physics 2e introduces topics conceptually and progresses through clear explanations in the context of career-oriented, practical applications. Consistency among the various definitions, foundational concepts, worked examples, and features provides a memorable pathway for student learning and helps maximize the impacts of study and practice.

The text aligns to the scope and sequence of most introductory physics courses and uses algebra as a basis for calculations. Extensive faculty feedback informed the sequence presented in the standard table of contents, but the open nature of the book—both in license and available formats—allows for significant rearrangement by faculty. Mechanics and electricity & magnetism anchor each half of the text, and optics, waves, modern physics, and other topics are arranged in units of their own for flexibility in course placement.

Changes to the Second Edition

College Physics 2e builds on the first edition's guiding principle that physics is a discipline undertaken by and for people. Throughout the text, the human impact of physics understanding, phenomena, discoveries, and applications is made clear through widespread examples, scenarios, and explanations. The narrative of physics and scientific discovery has been even further expanded to focus on including more diverse contributors to the field. From Ibn al-Haytham's 11th century foundation of the scientific method to Gladys West's complex models enabling GPS, the second edition broadens the discussion of pioneering and current researchers in an effort to tell a more accurate and inclusive scientific and societal story.

Relevance and Responsiveness

The impact of physics on engineering, urban development, the environment, medicine, energy production, and other aspects of everyday life have been updated and expanded to reflect more student experiences and interests. Techniques and developments in related disciplines are covered in context—not only in opening vignettes—so that students encounter the deep impact of evolving knowledge relevant to their potential fields of study.

Since many introductory physics students are focused on medicine, sections and examples related to biology have been significantly expanded. The section on electric forces in biology (18.6), for example, has been deepened to include Ernest Everett Just's work on electronegativity in ova, as well as the emerging practice of electrical stimulation in wound healing. Additional biological application narratives include Yalow and Berson's development of radioimmunoassay, and Strickland and Mourou's invention of chirped lasers used in vision correction.

Currency and Accuracy

We have updated sections related to ongoing research, frontiers of physics, and emerging information. In particular, section 4.8 on the four basic forces has been revised with information about recent discoveries and ongoing research, as well as with additional context about the ongoing process of discovery—for example, the progression from Einstein's black hole predictions to the first black hole images produced in 2019. The section on world energy use (7.9), the section on ozone depletion (24.3), and several sections discussing space telescopes have been similarly updated to reflect current research and data.

Over ten years of widespread usage, OpenStax *College Physics* has benefitted from suggestions, corrections, and clarifications submitted by hundreds of faculty and also from students. We have made the requisite corrections and improvements over time, but the second edition unifies those edits for more consistency and ease of use.

Improving Problem-Solving and Deepening Understanding

College Physics 2e employs the best practices of physics teaching, informed by education research and extensive adopter feedback. In order to unify conceptual, analytical, and calculation skills within the learning process, the authors have integrated a wide array of strategies and supports throughout the text.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Module Summaries

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Engaging Students

Concept Trailers

Click to view content (https://openstax.org/r/concepttrailers)

Concept Trailers are twenty-four videos designed to engage and introduce students to key chapter concepts. These professionally produced videos are like a movie trailer and are approximately 90 seconds in length. These can be used independently by students or in lecture. Concept Trailers are available on YouTube.

Simulations

Where applicable, students are directed to the interactive PhET physics simulations developed by the University of Colorado Boulder. There they can further explore the physics concepts they have learned about in the module.

Flexibility and Progressions in Practice and Assessment

College Physics 2e provides a rich array of question types that promote faculty choice and ample opportunity for student practice and advancement.

- **Conceptual Questions** challenge students' ability to explain what they have learned conceptually, independent of the mathematical details.
- **Problems & Exercises** challenge students to apply both concepts and skills to solve mathematical physics problems.
- **Integrated Concept Problems** ask students to apply what they have learned about two or more concepts to arrive at a solution to a problem.
- **Create Your Own Problems** require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem's solution, and discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer. Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.
- **Unreasonable Results Problems** drive students to both solve a problem and analyze the answer's likelihood and realism. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.
- **Critical Thinking Questions** are new additions to the text. These challenging, multi-part problems typically integrate conceptual, quantitative, and graphical response elements in order to deeply investigate student understanding. Most chapters provide one Critical Thinking Question, and we have reserved additional questions and solutions only for instructor use.

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Additional Resources

Student and Instructor Resources

We've compiled additional resources for both students and instructors, including Getting Started Guides, an instructor's manual, a test bank, and image slides. Instructor resources require a verified instructor account, which you can apply for when you log in or create your account on OpenStax.org. Take advantage of these resources to supplement your OpenStax book.

Instructor's solutions manual. The instructor solutions manual contains the instructor-facing answers to the problems and exercises within the textbook. Since many instructors use these questions in graded assignments, we ask that you not post these questions and the answers on any publicly available websites.

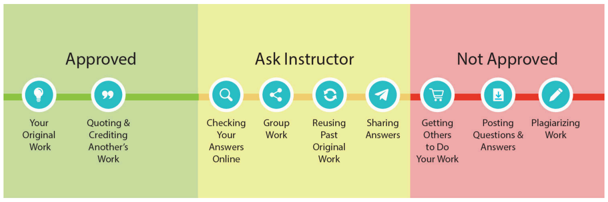
PowerPoint lecture slides. The PowerPoint slides provide images and descriptions as a starting place for instructors to build their lectures.

Concept Trailer instructor notes. These teaching notes support implementation of the OpenStax Physics Concept Trailers. The notes contain tips for usage, clarifications of coverage, and guidance on how to use the trailers in different educational situations.

Academic Integrity

Academic integrity builds trust, understanding, equity, and genuine learning. While students may encounter significant challenges in their courses and their lives, doing their own work and maintaining a high degree of authenticity will result in meaningful outcomes that will extend far beyond their college career. Faculty, administrators, resource providers, and students can work together to maintain a fair and positive experience.

We realize that students benefit when academic integrity ground rules are established early in the course. To that end, OpenStax has created an interactive to aid with academic integrity discussions in your course.



Visit our academic integrity slider (https://view.genial.ly/61e08a7af6db870d591078c1/interactive-image-defining-academic-integrity-interactive-slider). Click and drag icons along the continuum to align these practices with your institution and course policies. You may then include the graphic on your syllabus, present it in your first course meeting, or create a handout for students.

At OpenStax we are also developing resources supporting authentic learning experiences and assessment. Please visit this book's page for updates. For an in-depth review of academic integrity strategies, we highly recommend visiting the International Center of Academic Integrity (ICAI) website at https://academicintegrity.org/ (https://academicintegrity.org/).

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CHAPTER 1

Introduction: The Nature of Science and Physics

FIGURE 1.1 The laws of physics describe the smallest and largest forces and structures, such as the Veil Nebula, a supernova remnant from a star that was 20 times larger than our Sun. Stretching over 110 light years across, the spectacular image is created by the stellar explosion's blast wave impacting clouds of gas. Scientists use successive observations of the still-moving wave to learn about nebula formation in ways they can apply to other systems. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature’s apparent complexity. (credit: NASA, ESA, Hubble Heritage, and the Digitized Sky Survey 2)

CHAPTER OUTLINE

INTRODUCTION TO SCIENCE AND THE REALM OF PHYSICS, PHYSICAL QUANTITIES, AND UNITS What is your first reaction when you hear the word “physics”? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

Consider the Veil Nebula, a cloud of heated dust and gas located about 2,400 light years from Earth (a light year is the distance light travels in one year, or approximately 9.5 trillion kilometers). The unique structure is the ongoing result of a supernova that occurred 8,000 years ago. The shock wave from the explosion is colliding with a cloud of gas and dust, creating the rope-like filaments and sheet-like appearance. Scientists compare newer images, such as the one above, with detailed images taken by the Hubble Space Telescope in 1997 in order to understand nebula expansion and other properties of astronomical objects. The forces that cause the supernova remnant to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply heating a new home. Although the scale is much different, the interaction of gasses in the Veil Nebula resembles those on Earth; and the vibrant color combinations are created by the familiar glows of hydrogen, sulfur, oxygen, and similar elements that make up everything we know. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Humans have created and manufactured millions of different objects over the history of our species. Successive technological periods (often referred to as the Stone Age, the Bronze Age, the Iron Age, and so on) were marked by our knowledge of the physical properties of certain materials and our ability to manipulate them. This knowledge all stems from physics, whether it's the way a rock would flake when constructing a spear point, the effect of integrating carbon with iron in South Indian and Sri Lankan furnaces to create the earliest high-quality steel, or the proper way to combine perfectly ground and polished pieces of glass to create optical instruments. Our current technological age, the Information Age, builds on all that knowledge and can be traced to critical innovations made by people from all backgrounds working together. Mohamed M. Atalla and Dawon Kahng, for example, invented the MOSFET (metal-oxide-semiconductor field-effect transistor). Although unknown to most people, this tiny device, created in 1959 by an Egyptian-born scientist and Korean-born scientist working in a lab in New Jersey, is the basis for modern electronics. More MOSFETs have been produced than any other object in human history. They are used in computers, smart phones, microwave ovens, automotive controls, medical instruments, and nearly every other electronic device.

Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient thinkers from the Middle East and the Mediterranean, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

1.1 Physics: An Introduction

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

FIGURE 1.2 The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone (Figure 1.3). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. GPS relies on precise calculations that account for variations in the Earth's landscapes, the exact distance between orbiting satellites, and even the effect of a complex occurrence of time dilation. Most of these calculations are founded on algorithms developed by Gladys West, a mathematician and computer scientist who programmed the first computers capable of highly accurate remote sensing and positioning. When you use a GPS device, it utilizes these algorithms to recognize where you are and how your position relates to other objects on Earth.

FIGURE 1.3 The Apple iPhone is a common smart phone with a GPS function, sophisticated camera, haptic (vibration) capabilities, and many other functions. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: Tinh tế Photo/Flickr)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See Figure 1.4 and Figure 1.5.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (Figure 1.6 and Figure 1.7). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

FIGURE 1.4 The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

FIGURE 1.5 These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)

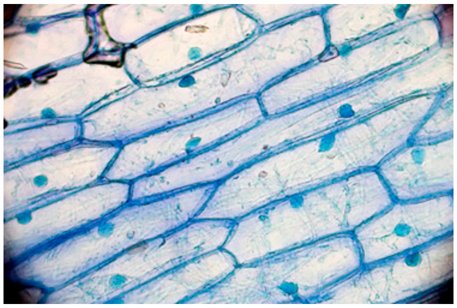

FIGURE 1.6 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

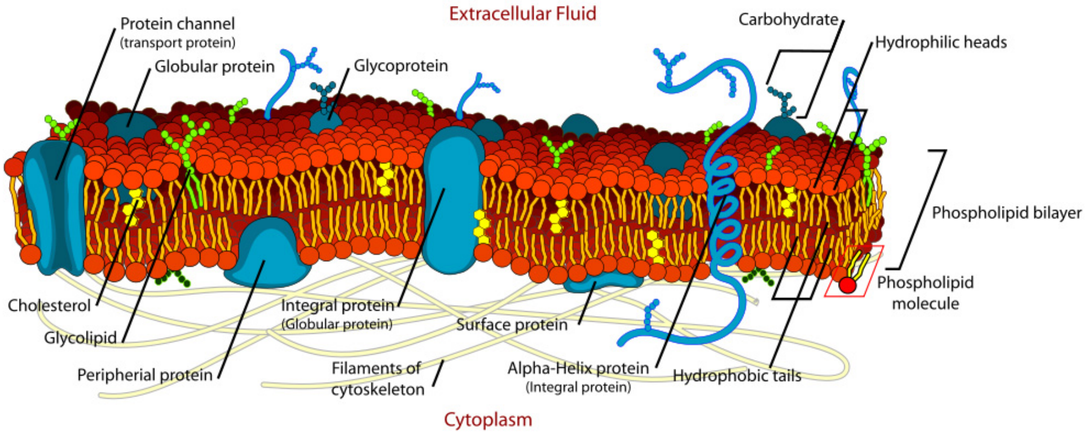

FIGURE 1.7 An artist's rendition of the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See Figure 1.8 and Figure 1.9.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

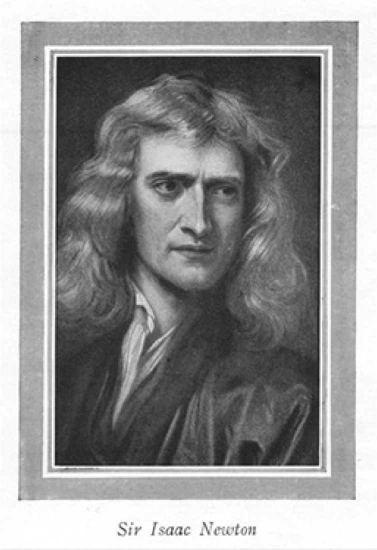

FIGURE 1.8 **Isaac Newton** (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

FIGURE 1.9 **Marie Curie** (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See Figure 1.10.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

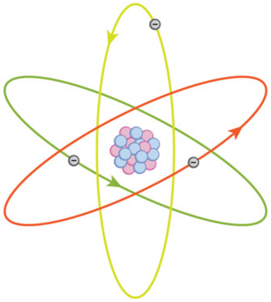

FIGURE 1.10 What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

Ibn al-Haytham (sometimes referred to as Alhazen), a 10th-11th century scientist working in Cairo, significantly advanced the understanding of optics and vision. But his contributions go much further. In demonstrating that previous approaches were incorrect, he emphasized that scientists must be ready to reject existing knowledge and become "the enemy" of everything they read; he expressed that scientists must trust only objective evidence. Al-Haytham emphasized repeated experimentation and validation, and acknowledged that senses and predisposition could lead to poor conclusions. His work was a precursor to the scientific method that we use today.

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See Figure 1.11, Figure 1.12, and Figure 1.13.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

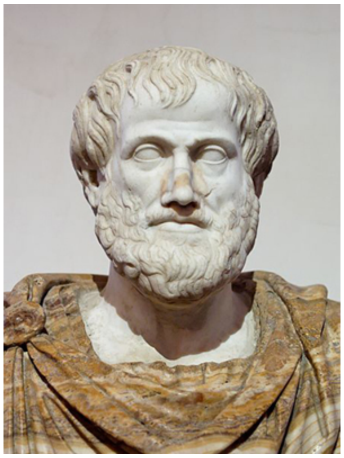

FIGURE 1.11 Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection)

FIGURE 1.12 Ibn al-Haytham (965–1040) and Galileo Galilei (1564–1642) were so critical to the advancement of scientific practice and to the fields of mathematics, optics, physics, and astronomy that a later prominent scientist featured them on the cover of his major work. (credit: Johannes Hevelius (author), Adolph Boÿ (artist), J. Falck (engraver); Houghton Library, Harvard University/Wikimedia Commons)

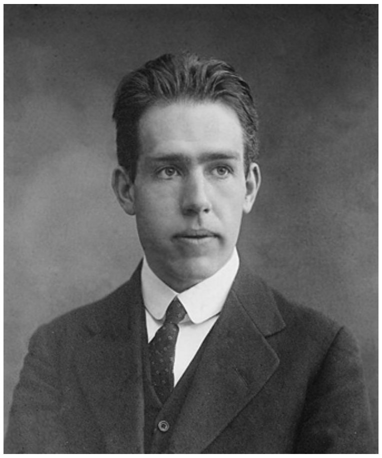

FIGURE 1.13 **Niels Bohr** (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom’s properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually “picture” the atom.

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

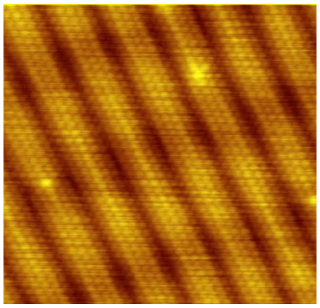

FIGURE 1.14 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results. We will find, however, that we can do a great deal of modern physics with the algebra and trigonometry used in this text.

CHECK YOUR UNDERSTANDING

A friend tells you they have learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you they had learned about a scientific theory rather than a law?

Solution

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

PHET EXPLORATIONS

Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

Click to view content (https://openstax.org/books/college-physics-2e/pages/1-1-physics-an-introduction)

1.2 Physical Quantities and Units

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

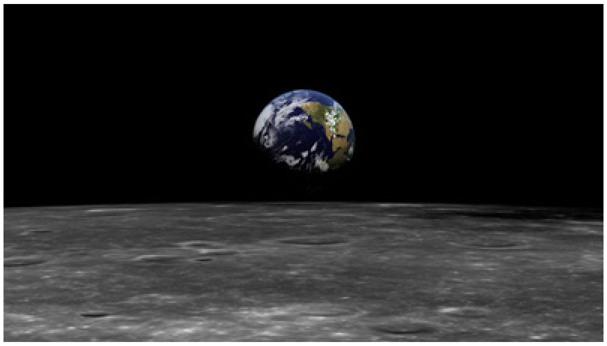

FIGURE 1.15 The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—most physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See Figure 1.16.)

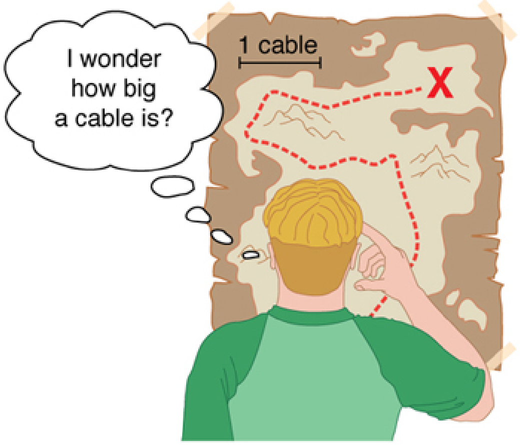

FIGURE 1.16 Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1.1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

TABLE 1.1 Fundamental SI Units

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second** (abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See Figure 1.17.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

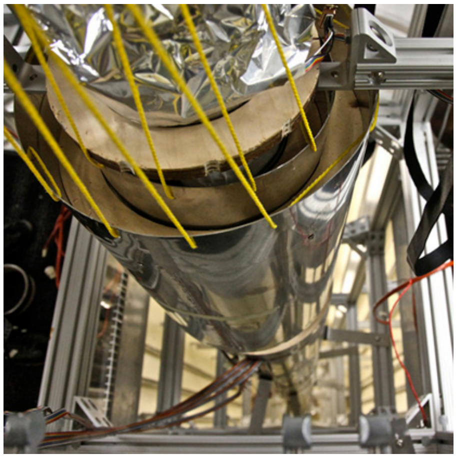

FIGURE 1.17 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See Figure 1.18.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

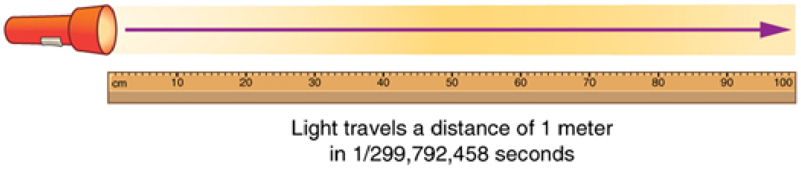

FIGURE 1.18 The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it was previously defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the previously defined kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses could be ultimately traced to a comparison with the standard mass. Even though the platinum-iridium cylinder was resistant to corrosion, airborne contaminants were able to adhere to its surface, slightly changing its mass over time. In May 2019, the scientific community adopted a more stable definition of the kilogram. The kilogram is now defined in terms of the second, the meter, and Planck's constant, *h* (a quantum mechanical value that relates a photon's energy to its frequency).

Electric current and its accompanying unit, the ampere, will be introduced in Introduction to Electric Current, Resistance, and Ohm's Law when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. Table 1.2 gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-10}\ \text{m}$, while the diameter of the Sun is on the order of $10^9\ \text{m}$.

The Quest for Microscopic Standards for Basic Units

The fundamental units described in this chapter are those that produce the greatest accuracy and precision in measurement. There is a sense among physicists that, because there is an underlying microscopic substructure to matter, it would be most satisfying to base our standards of measurement on microscopic objects and fundamental physical phenomena such as the speed of light. A microscopic standard has been accomplished for the standard of time, which is based on the oscillations of the cesium atom.

The standard for length was once based on the wavelength of light (a small-scale length) emitted by a certain type of atom, but it has been supplanted by the more precise measurement of the speed of light. If it becomes possible to measure the mass of atoms or a particular arrangement of atoms such as a silicon sphere to greater precision than the kilogram standard, it may become possible to base mass measurements on the small scale. There are also possibilities that electrical phenomena on the small scale may someday allow us to base a unit of charge on the charge of electrons and protons, but at present current and charge are related to large-scale currents and forces between wires.

Prefix	Symbol	Value	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	$10^{18}\ \text{m}$	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	$10^{15}\ \text{s}$	30 million years
tera	T	10^{12}	terawatt	TW	$10^{12}\ \text{W}$	powerful laser output

TABLE 1.2 Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value	Example (some are approximate)			
giga	G	10^9	gigahertz	GHz	10^9 Hz	a microwave frequency
mega	M	10^6	megacurie	MCi	10^6 Ci	high radioactivity
kilo	k	10^3	kilometer	km	10^3 m	about 6/10 mile
hecto	h	10^2	hectoliter	hL	10^2 L	26 gallons
deka	da	10^1	dekagram	dag	10^1 g	teaspoon of butter
—	—	10^0 (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

TABLE 1.2 Metric Prefixes for Powers of 10 and their Symbols

Known Ranges of Length, Mass, and Time

The vastness of the universe and the breadth over which physics applies are illustrated by the wide range of examples of known lengths, masses, and times in Table 1.3. Examination of this table will give you some feeling for the range of possible topics and numerical values. (See Figure 1.19 and Figure 1.20.)

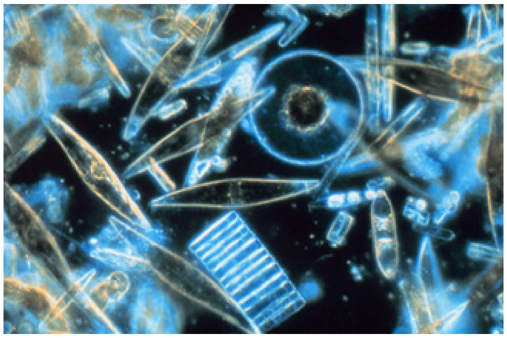

FIGURE 1.19 Tiny phytoplankton swims among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (credit: Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

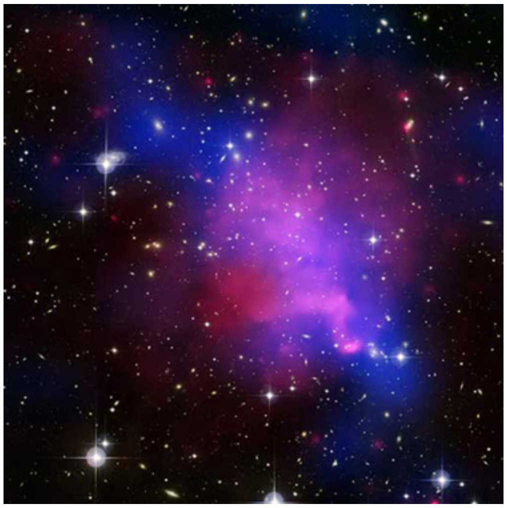

FIGURE 1.20 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (credit: NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\,\cancel{\text{m}}\times\frac{1\text{ km}}{1000\,\cancel{\text{m}}}=0.080\text{ km}. \tag{1.1}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Click Appendix C for a more complete list of conversion factors.

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $(9.11\times10^{-31}\text{ kg})$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $(1.67\times10^{-27}\text{ kg})$	10^{-22}	Mean life of an extremely unstable nucleus

TABLE 1.3 Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $\left(8.64 \times 10^{4}\ \text{s}\right)$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $\left(3.16 \times 10^{7}\ \text{s}\right)$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $\left(7.35 \times 10^{22}\ \text{kg}\right)$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $\left(5.97 \times 10^{24}\ \text{kg}\right)$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $\left(1.99 \times 10^{30}\ \text{kg}\right)$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

TABLE 1.3 Approximate Values of Length, Mass, and Time

EXAMPLE 1.1

Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your school to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}. \tag{1.2}$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\ \text{km}}{20.0\ \text{min}} = 0.500 \frac{\text{km}}{\text{min}}. \tag{1.3}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is $60\ \text{min/hr}$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{1\ \text{hr}}{60\ \text{min}} = \frac{1}{60} \frac{\text{km} \cdot \text{hr}}{\text{min}^2}, \tag{1.4}$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3{,}600\text{ s}} \times \frac{1{,}000\text{ m}}{1\text{ km}}, \tag{1.5}$$

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}. \tag{1.6}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces? The module Accuracy, Precision, and Significant Figures will help you answer these questions.

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

CHECK YOUR UNDERSTANDING

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

CHECK YOUR UNDERSTANDING

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

1.3 Accuracy, Precision, and Significant Figures

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Determine the appropriate number of significant figures in both addition and subtraction, as well as multiplication and division calculations.
- Calculate the percent uncertainty of a measurement.

FIGURE 1.21 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The “known masses” are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (credit: Serge Melki)

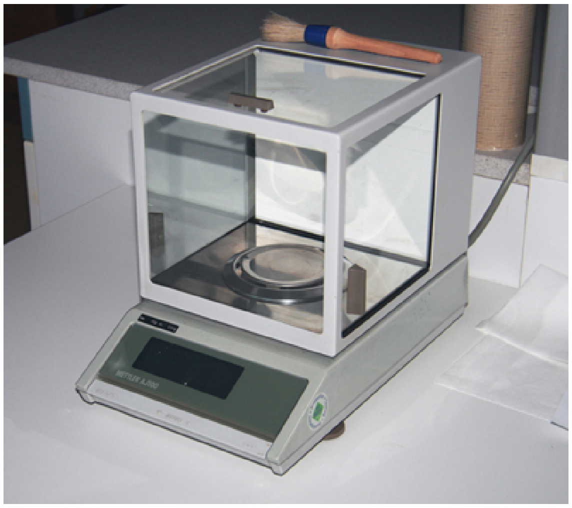

FIGURE 1.22 Many mechanical balances, such as double-pan balances, have been replaced by digital scales, which can typically measure the mass of an object more precisely. Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, many digital scales can measure the mass of an object up to the nearest thousandth of a gram. (credit: Karel Jakubec)

Accuracy and Precision of a Measurement

Science is based on observation and experiment—that is, on measurements. **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard computer paper. The packaging in which you purchased the paper states that it is 11.0 inches long. You measure the length of the paper three times and obtain the following measurements: 11.1 in., 11.2 in., and 10.9 in. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate.

The **precision** of a measurement system refers to how close the agreement is between repeated measurements (which are repeated under the same conditions). Consider the example of the paper measurements. The precision of the measurements refers to the spread of the measured values. One way to analyze the precision of the measurements would be to determine the range, or difference, between the lowest and the highest measured values. In that case, the lowest value was 10.9 in. and the highest value was 11.2 in. Thus, the measured values deviated from each other by at most 0.3 in. These measurements were relatively precise because they did not vary too much in value. However, if the measured values had been 10.9, 11.1, and 11.9, then the measurements would not be very precise because there would be significant variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider an example of a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull’s-eye target, and think of each GPS attempt to locate the restaurant as a black dot. In Figure 1.23, you can see that the GPS measurements are spread out far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 1.24, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system.

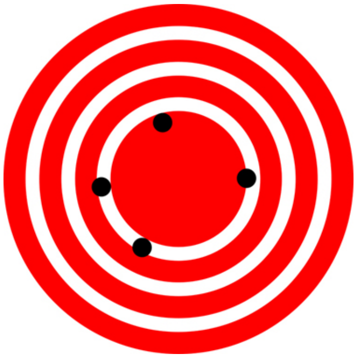

FIGURE 1.23 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (credit: Dark Evil)

FIGURE 1.24 In this figure, the dots are concentrated rather closely to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (credit: Dark Evil)

Accuracy, Precision, and Uncertainty

The degree of accuracy and precision of a measuring system are related to the **uncertainty** in the measurements. Uncertainty is a quantitative measure of how much your measured values deviate from a standard or expected value. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 in., plus or minus 0.2 in. The uncertainty in a measurement, A, is often denoted as δA ("delta A"), so the measurement result would be recorded as $A \pm \delta A$. In our paper example, the length of the paper could be expressed as $11 \text{ in.} \pm 0.2$.

The factors contributing to uncertainty in a measurement include:

1. Limitations of the measuring device,
2. The skill of the person making the measurement,
3. Irregularities in the object being measured,
4. Any other factors that affect the outcome (highly dependent on the situation).

In our example, such factors contributing to the uncertainty could be the following: the smallest division on the ruler is 0.1 in., the person using the ruler has bad eyesight, or one side of the paper is slightly longer than the other. At any rate, the uncertainty in a measurement must be based on a careful consideration of all the factors that might contribute and their possible effects.

Making Connections: Real-World Connections – Fevers or Chills?

Uncertainty is a critical piece of information, both in physics and in many other real-world applications. Imagine you are caring for a sick child. You suspect the child has a fever, so you check their temperature with a thermometer. What if the uncertainty of the thermometer were 3.0°C? If the child's temperature reading was 37.0°C (which is normal body temperature), the "true" temperature could be anywhere from a hypothermic 34.0°C to a dangerously high 40.0°C. A thermometer with an uncertainty of 3.0°C would be useless.

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement A is expressed with uncertainty, δA, the **percent uncertainty** (%unc) is defined to be

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%. \tag{1.7}$$

EXAMPLE 1.2

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.8\ \text{lb}$
- Week 2 weight: $5.3\ \text{lb}$
- Week 3 weight: $4.9\ \text{lb}$
- Week 4 weight: $5.4\ \text{lb}$

You determine that the weight of the 5-lb bag has an uncertainty of $\pm 0.4\ \text{lb}$. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight:

$$\%\ \text{unc} = \frac{\delta A}{A} \times 100\%. \tag{1.8}$$

Solution

Plug the known values into the equation:

$$\%\ \text{unc} = \frac{0.4\ \text{lb}}{5\ \text{lb}} \times 100\% = 8\%. \tag{1.9}$$

Discussion

We can conclude that the weight of the apple bag is $5\ \text{lb} \pm 8\%$. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100%. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainties in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements going into the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used for multiplication or division. This method says that *the percent uncertainty in a quantity calculated by multiplication* *or division is the sum of the percent uncertainties in the items used to make the calculation*. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2% and 1%, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3%. (Expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter.)

CHECK YOUR UNDERSTANDING

A high school track coach has just purchased a new stopwatch. The stopwatch manual states that the stopwatch has an uncertainty of ±0.05 s. The team's top sprinter clocked a 100 meter sprint at 12.04 seconds last week and at 11.96 seconds this week. Can we conclude that this week's time was faster?

Solution

No, the uncertainty in the stopwatch is too great to effectively differentiate between the sprint times.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements involves the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, a standard ruler can measure length to the nearest millimeter, while a caliper can measure length to the nearest 0.01 millimeter. The caliper is a more precise measuring tool because it can measure extremely small differences in length. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool. For example, if you use a standard ruler to measure the length of a stick, you may measure it to be 36.7 cm. You could not express this value as 36.71 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36.6 cm and 36.7 cm, and they must estimate the value of the last digit. Using the method of **significant figures**, the rule is that *the last digit written down in a measurement is the first digit with some uncertainty*. In order to determine the number of significant digits in a value, start with the first measured value at the left and count the number of digits through the last digit written on the right. For example, the measured value 36.7 cm has three digits, or significant figures. Significant figures indicate the precision of a measuring tool that was used to measure a value.

Zeros

Special consideration is given to zeros when counting significant figures. The zeros in 0.053 are not significant, because they are only placekeepers that locate the decimal point. There are two significant figures in 0.053. The zeros in 10.053 are not placekeepers but are significant—this number has five significant figures. The zeros in 1300 may or may not be significant depending on the style of writing numbers. They could mean the number is known to the last digit, or they could be placekeepers. So 1300 could have two, three, or four significant figures. (To avoid this ambiguity, write 1300 in scientific notation.) *Zeros are significant except when they serve only as placekeepers*.

CHECK YOUR UNDERSTANDING

Determine the number of significant figures in the following measurements:

a. 0.0009
b. 15,450.0
c. 6×10^3
d. 87.990
e. 30.42

Solution

(a) 1; the zeros in this number are placekeepers that indicate the decimal point

(b) 6; here, the zeros indicate that a measurement was made to the 0.1 decimal point, so the zeros are significant

(c) 1; the value 10^3 signifies the decimal place, not the number of measured values

(d) 5; the final zero indicates that a measurement was made to the 0.001 decimal point, so it is significant

(e) 4; any zeros located in between significant figures in a number are also significant

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Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, *the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value*. There are two different rules, one for multiplication and division and the other for addition and subtraction, as discussed below.

1. For multiplication and division: *The result should have the same number of significant figures as the quantity having the least significant figures entering into the calculation*. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area has if the radius has only two—say, $r = 1.2\ \text{m}$. Then,

$$A = \pi r^2 = (3.1415927...)\times(1.2\ \text{m})^2 = 4.5238934\ \text{m}^2 \tag{1.10}$$

is what you would get using a calculator that has an eight-digit output. But because the radius has only two significant figures, it limits the calculated quantity to two significant figures or

$$A = 4.5\ \text{m}^2, \tag{1.11}$$

even though π is good to at least eight digits.

2. For addition and subtraction: *The answer can contain no more decimal places than the least precise measurement.* Suppose that you buy 7.56-kg of potatoes in a grocery store as measured with a scale with precision 0.01 kg. Then you drop off 6.052-kg of potatoes at your laboratory as measured by a scale with precision 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with precision 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{r} 7.56\ \ \ \text{kg} \\ -\ 6.052\ \text{kg} \\ \dfrac{+13.7\ \ \ \ \text{kg}}{15.208\ \text{kg}} = 15.2\ \text{kg}. \end{array} \tag{1.12}$$

Next, we identify the least precise measurement: 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer is rounded to the tenths place, giving us 15.2 kg.

Significant Figures in this Text

In this text, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits, for example. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, particularly in optics, more accurate numbers are needed and more than three significant figures will be used. Finally, if a number is *exact*, such as the two in the formula for the circumference of a circle, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

CHECK YOUR UNDERSTANDING

Perform the following calculations and express your answer using the correct number of significant digits.
(a) A woman has two bags weighing 13.5 pounds and one bag with a weight of 10.2 pounds. What is the total weight of the bags?
(b) The force F on an object is equal to its mass m multiplied by its acceleration a. If a wagon with mass 55 kg accelerates at a rate of $0.0255\ \text{m/s}^2$, what is the force on the wagon? (The unit of force is called the newton, and it is expressed with the symbol N.)

Solution

(a) 37.2 pounds; Because the number of bags is an exact value, it is not considered in the significant figures.

(b) 1.4 N; Because the value 55 kg has only two significant figures, the final value must also contain two significant figures.

1.4 Approximation

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Make reasonable approximations based on given data.

On many occasions, physicists, other scientists, and engineers need to make **approximations** or “guesstimates” for a particular quantity. What is the distance to a certain destination? What is the approximate density of a given item? About how large a current will there be in a circuit? Many approximate numbers are based on formulae in which the input quantities are known only to a limited accuracy. As you develop problem-solving skills (that can be applied to a variety of fields through a study of physics), you will also develop skills at approximating. You will develop these skills through thinking more quantitatively, and by being willing to take risks. As with any endeavor, experience helps, as well as familiarity with units. These approximations allow us to rule out certain scenarios or unrealistic numbers. Approximations also allow us to challenge others and guide us in our approaches to our scientific world. Let us do two examples to illustrate this concept.

EXAMPLE 1.3

Approximate the Height of a Building

Can you approximate the height of one of the buildings on your campus, or in your neighborhood? Let us make an approximation based upon the height of a person. In this example, we will calculate the height of a 39-story building.

Strategy

Think about the average height of an adult male. We can approximate the height of the building by scaling up from the height of a person.

Solution

Based on information in the example, we know there are 39 stories in the building. If we use the fact that the height of one story is approximately equal to about the length of two adult humans (each human is about 2-m tall), then we can estimate the total height of the building to be

$$\frac{2\text{ m}}{1\text{ person}} \times \frac{2\text{ person}}{1\text{ story}} \times 39\text{ stories} = 156\text{ m}. \tag{1.13}$$

Discussion

You can use known quantities to determine an approximate measurement of unknown quantities. If your hand measures 10 cm across, how many hand lengths equal the width of your desk? What other measurements can you approximate besides length?

EXAMPLE 1.4

Approximating Vast Numbers: a Trillion Dollars

FIGURE 1.25 A bank stack contains one-hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (credit: Andrew Magill)

The U.S. federal debt in 2021 was a little more than $28 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

(1) Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is:

$$\begin{aligned}\text{volume of stack} &= \text{length} \times \text{width} \times \text{height},\\ \text{volume of stack} &= 6 \text{ in.} \times 3 \text{ in.} \times 0.5 \text{ in.},\\ \text{volume of stack} &= 9 \text{ in.}^3.\end{aligned} \tag{1.14}$$

(2) Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $\$100$ bills is equal to $\$10{,}000$, or $\$1 \times 10^4$. The number of stacks you will have is:

$$\$1 \times 10^{12} (\text{a trillion dollars})/\ \$1 \times 10^4 \text{ per stack} = 1 \times 10^8 \text{ stacks.} \tag{1.15}$$

(3) Calculate the area of a football field in square inches. The area of a football field is $100 \text{ yd} \times 50 \text{ yd}$, which gives $5{,}000 \text{ yd}^2$. Because we are working in inches, we need to convert square yards to square inches:

$$\begin{aligned}\text{Area} = 5{,}000 \text{ yd}^2 \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{12 \text{ in.}}{1 \text{ ft}} &= 6{,}480{,}000 \text{ in.}^2,\\ \text{Area} \approx 6 \times 10^6 \text{ in.}^2.\end{aligned} \tag{1.16}$$

This conversion gives us $6 \times 10^6 \text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

(4) Calculate the total volume of the bills. The volume of all the $\$100$-bill stacks is

$9 \text{ in.}^3/\text{stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{ in.}^3.$

(5) Calculate the height. To determine the height of the bills, use the equation:

$$\begin{aligned} \text{volume of bills} &= \text{area of field} \times \text{height of money:} \\ \text{Height of money} &= \frac{\text{volume of bills}}{\text{area of field}}, \\ \text{Height of money} &= \frac{9 \times 10^8 \text{in.}^3}{6 \times 10^6 \text{in.}^2} = 1.33 \times 10^2 \text{in.}, \\ \text{Height of money} &\approx 1 \times 10^2 \text{in.} = 100 \text{ in.} \end{aligned} \tag{1.17}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft}. \tag{1.18}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough “guesstimates” versus carefully calculated approximations?

CHECK YOUR UNDERSTANDING

Using mental math and your understanding of fundamental units, approximate the area of a regulation basketball court. Describe the process you used to arrive at your final approximation.

Solution

An average male is about two meters tall. It would take approximately 15 males laid out end to end to cover the length, and about 7 to cover the width. That gives an approximate area of 420 m^2.

Glossary

accuracy the degree to which a measured value agrees with correct value for that measurement

approximation an estimated value based on prior experience and reasoning

classical physics physics that was developed from the Renaissance to the end of the 19th century

conversion factor a ratio expressing how many of one unit are equal to another unit

derived units units that can be calculated using algebraic combinations of the fundamental units

English units system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units units that can only be expressed relative to the procedure used to measure them

kilogram the SI unit for mass, abbreviated (kg)

law a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

meter the SI unit for length, abbreviated (m)

method of adding percents the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation

metric system a system in which values can be calculated in factors of 10

model representation of something that is often too difficult (or impossible) to display directly

modern physics the study of relativity, quantum mechanics, or both

order of magnitude refers to the size of a quantity as it relates to a power of 10

percent uncertainty the ratio of the uncertainty of a measurement to the measured value, expressed as a percentage

physical quantity a characteristic or property of an object that can be measured or calculated from other measurements

physics the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

precision the degree to which repeated measurements agree with each other

quantum mechanics the study of objects smaller than can be seen with a microscope

relativity the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

scientific method a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

second the SI unit for time, abbreviated (s)

SI units the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

significant figures express the precision of a measuring tool used to measure a value

theory an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

uncertainty a quantitative measure of how much your measured values deviate from a standard or expected value

units a standard used for expressing and comparing measurements

Section Summary

1.1 Physics: An Introduction

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

1.2 Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and

ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

1.3 Accuracy, Precision, and Significant Figures

- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- The precision of a *measuring tool* is related to the size of its measurement increments. The smaller the measurement increment, the more precise the tool.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

1.4 Approximation

Scientists often approximate the values of quantities to perform calculations and analyze systems.

Conceptual Questions

1.1 Physics: An Introduction

1. Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?
2. How does a model differ from a theory?
3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?
4. What determines the validity of a theory?
5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?
6. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?
7. Classical physics is a good approximation to modern physics under certain circumstances. What are they?
8. When is it *necessary* to use relativistic quantum mechanics?
9. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

1.2 Physical Quantities and Units

10. Identify some advantages of metric units.

1.3 Accuracy, Precision, and Significant Figures

11. What is the relationship between the accuracy and uncertainty of a measurement?
12. Prescriptions for vision correction are given in units called *diopters* (D). Determine the meaning of that unit. Obtain information (perhaps by calling an optometrist or performing an internet search) on the minimum uncertainty with which corrections in diopters are determined and the accuracy with which corrective lenses can be produced. Discuss the sources of uncertainties in both the prescription and accuracy in the manufacture of lenses.

Problems & Exercises

1.2 Physical Quantities and Units

1. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?
2. A car is traveling at a speed of $33\ \mathrm{m/s}$. (a) What is its speed in kilometers per hour? (b) Is it exceeding the $90\ \mathrm{km/h}$ speed limit?
3. Show that $1.0\ \mathrm{m/s} = 3.6\ \mathrm{km/h}$. Hint: Show the explicit steps involved in converting $1.0\ \mathrm{m/s} = 3.6\ \mathrm{km/h}$.
4. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

5. Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

6. What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

7. Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

8. The speed of sound is measured to be $342\ \text{m/s}$ on a certain day. What is this in km/h?

9. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

10. (a) Refer to Table 1.3 to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?

1.3 Accuracy, Precision, and Significant Figures

Express your answers to problems in this section to the correct number of significant figures and proper units.

11. Suppose that your bathroom scale reads your mass as 65 kg with a 3% uncertainty. What is the uncertainty in your mass (in kilograms)?

12. A good-quality measuring tape can be off by 0.50 cm over a distance of 20 m. What is its percent uncertainty?

13. (a) A car speedometer has a 5.0% uncertainty. What is the range of possible speeds when it reads $90\ \text{km/h}$? (b) Convert this range to miles per hour. $(1\ \text{km} = 0.6214\ \text{mi})$

14. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?

15. (a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats do they have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?

16. A can contains 375 mL of soda. How much is left after 308 mL is removed?

17. State how many significant figures are proper in the results of the following calculations: (a) $(106.7)(98.2)/(46.210)(1.01)$ (b) $(18.7)^2$ (c) $\left(1.60 \times 10^{-19}\right)(3712)$.

18. (a) How many significant figures are in the numbers 99 and 100? (b) If the uncertainty in each number is 1, what is the percent uncertainty in each? (c) Which is a more meaningful way to express the accuracy of these two numbers, significant figures or percent uncertainties?

19. (a) If your speedometer has an uncertainty of $2.0\ \text{km/h}$ at a speed of $90\ \text{km/h}$, what is the percent uncertainty? (b) If it has the same percent uncertainty when it reads $60\ \text{km/h}$, what is the range of speeds you could be going?

20. (a) A person's blood pressure is measured to be $120 \pm 2\ \text{mm Hg}$. What is its percent uncertainty? (b) Assuming the same percent uncertainty, what is the uncertainty in a blood pressure measurement of $80\ \text{mm Hg}$?

21. A person measures their heart rate by counting the number of beats in $30\ \text{s}$. If 40 ± 1 beats are counted in $30.0 \pm 0.5\ \text{s}$, what is the heart rate and its uncertainty in beats per minute?

22. What is the area of a circle $3.102\ \text{cm}$ in diameter?

23. If a marathon runner averages 9.5 mi/h, how long does it take him or her to run a 26.22-mi marathon?

24. A marathon runner completes a 42.188-km course in $2\ \text{h}$, 30 min, and $12\ \text{s}$. There is an uncertainty of $25\ \text{m}$ in the distance traveled and an uncertainty of 1 s in the elapsed time. (a) Calculate the percent uncertainty in the distance. (b) Calculate the uncertainty in the elapsed time. (c) What is the average speed in meters per second? (d) What is the uncertainty in the average speed?

25. The sides of a small rectangular box are measured to be $1.80 \pm 0.01\ \text{cm}$, $2.05 \pm 0.02\ \text{cm}$, and $3.1 \pm 0.1\ \text{cm}$ long. Calculate its volume and uncertainty in cubic centimeters.

26. When non-metric units were used in the United Kingdom, a unit of mass called the *pound-mass* (lbm) was employed, where $1\ \text{lbm} = 0.4539\ \text{kg}$. (a) If there is an uncertainty of $0.0001\ \text{kg}$ in the pound-mass unit, what is its percent uncertainty? (b) Based on that percent uncertainty, what mass in pound-mass has an uncertainty of 1 kg when converted to kilograms?

27. The length and width of a rectangular room are measured to be $3.955 \pm 0.005\ \text{m}$ and $3.050 \pm 0.005\ \text{m}$. Calculate the area of the room and its uncertainty in square meters.

28. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002\ \text{cm}$ diameter a distance of $3.250 \pm 0.001\ \text{cm}$ to compress the gas in the cylinder. (a) By what amount is the gas decreased in volume in cubic centimeters? (b) Find the uncertainty in this volume.

1.4 Approximation

29. How many heartbeats are there in a lifetime?

30. A generation is about one-third of a lifetime. Approximately how many generations have passed since the year 0 AD?

31. How many times longer than the mean life of an extremely unstable atomic nucleus is the lifetime of a human? (Hint: The lifetime of an unstable atomic nucleus is on the order of $10^{-22}\ \text{s}$.)

32. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint: The mass of a hydrogen atom is on the order of $10^{-27}\ \text{kg}$ and the mass of a bacterium is on the order of $10^{-15}\ \text{kg}$.)

FIGURE 1.26 This color-enhanced photo shows *Salmonella typhimurium* (red) attacking human cells. These bacteria are commonly known for causing foodborne illness. Can you estimate the number of atoms in each bacterium? (credit: Rocky Mountain Laboratories, NIAID, NIH)

33. Approximately how many atoms thick is a cell membrane, assuming all atoms there average about twice the size of a hydrogen atom?

34. (a) What fraction of Earth's diameter is the greatest ocean depth? (b) The greatest mountain height?

35. (a) Calculate the number of cells in a hummingbird assuming the mass of an average cell is ten times the mass of a bacterium. (b) Making the same assumption, how many cells are there in a human?

36. Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

CHAPTER 2
Kinematics

FIGURE 2.1 The motion of an American kestrel through the air can be described by the bird’s displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

CHAPTER OUTLINE

INTRODUCTION TO ONE-DIMENSIONAL KINEMATICS Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars? Where will a football land if it is thrown at a certain angle?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). In one-dimensional kinematics and Two-Dimensional Kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion. In Two-Dimensional Kinematics, we apply concepts developed here to study motion along curved paths (two- and three-dimensional motion); for example, that of a car rounding a curve.

Click to view content (https://openstax.org/books/college-physics-2e/pages/2-introduction-to-onedimensional-kinematics)

2.1 Displacement

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

FIGURE 2.2 These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor’s position could be described in terms of where she is in relation to the nearby white board. (See Figure 2.3.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See Figure 2.4.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object’s position changes. This change in position is known as **displacement**. The word “displacement” implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_{\mathrm{f}} - x_0, \tag{2.1}$$

where Δx is displacement, x_{f} is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means “change in” whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_f.

Note that the SI unit for displacement is the meter (m) (see Physical Quantities and Units), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

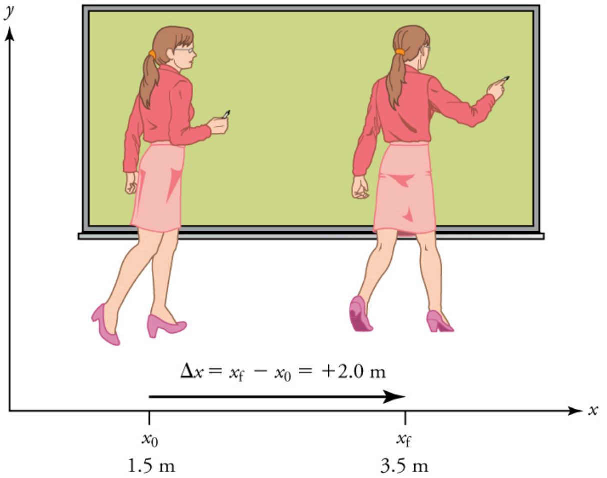


FIGURE 2.3 A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0$ m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

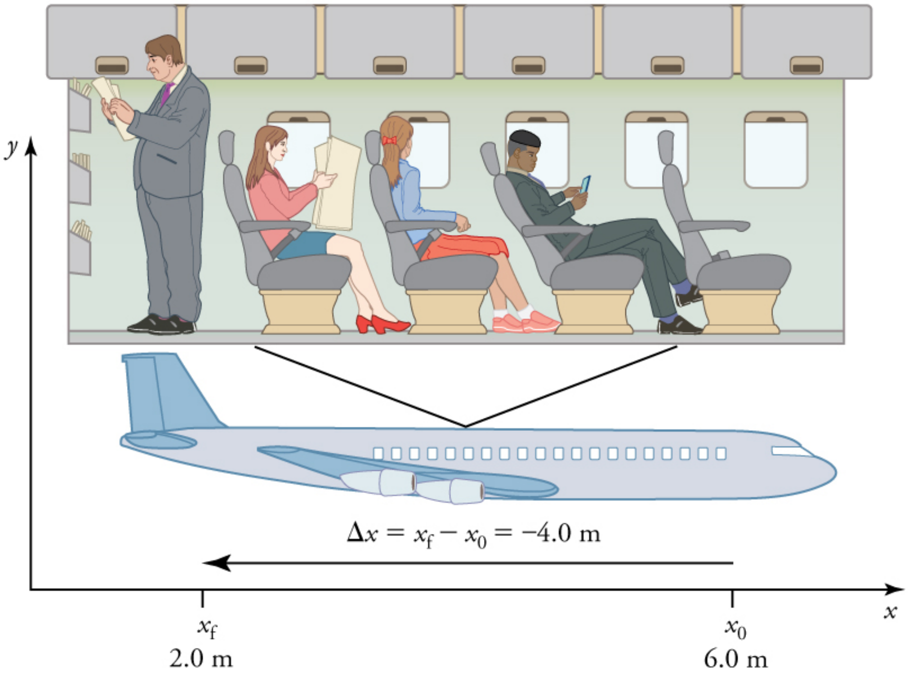


FIGURE 2.4 A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in Figure 2.3.

Note that displacement has a direction as well as a magnitude. The professor’s displacement is 2.0 m to the right, and the airline passenger’s displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor’s initial position is $x_0 = 1.5\text{ m}$ and her final position is $x_f = 3.5\text{ m}$. Thus her displacement is

$$\Delta x = x_f - x_0 = 3.5\text{ m} - 1.5\text{ m} = +2.0\text{ m}. \tag{2.2}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0\text{ m}$ and his final position is $x_\text{f} = 2.0\text{ m}$, so his displacement is

$$\Delta x = x_\text{f} - x_0 = 2.0\text{ m} - 6.0\text{ m} = -4.0\text{ m}. \tag{2.3}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

CHECK YOUR UNDERSTANDING

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is their displacement? (b) What distance do they ride? (c) What is the magnitude of their displacement?

Solution

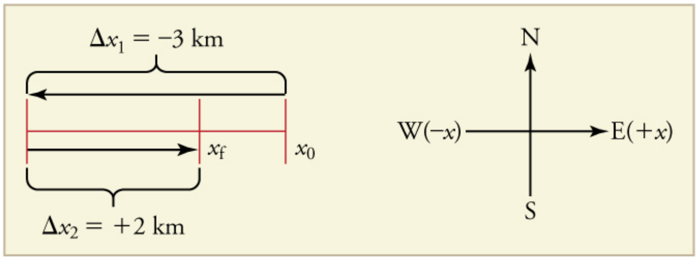


FIGURE 2.5

(a) The rider's displacement is $\Delta x = x_\text{f} - x_0 = -1\text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\text{ km} + 2\text{ km} = 5\text{ km}$.

(c) The magnitude of the displacement is 1 km.

2.2 Vectors, Scalars, and Coordinate Systems

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

FIGURE 2.6 The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure 2.6, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

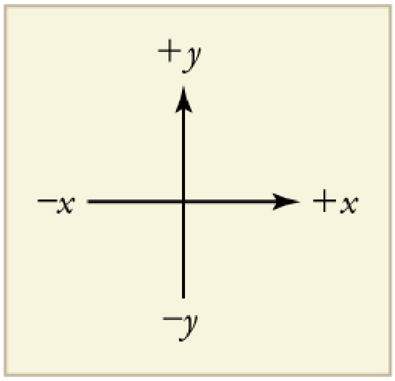


FIGURE 2.7 It is usually convenient to consider motion upward or to the right as positive (+) and motion downward or to the left as negative (−).

CHECK YOUR UNDERSTANDING

A person’s speed can stay the same as they round a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

2.3 Time, Velocity, and Speed

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

FIGURE 2.8 The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

As discussed in Physical Quantities and Units, the most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

$$\Delta t = t_{\mathrm{f}} - t_0, \tag{2.4}$$

where Δt is the change in time or elapsed time, t_{f} is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Life is simpler if the beginning time t_0 is taken to be zero, as when we use a stopwatch. If we were using a stopwatch, it would simply read zero at the start of the lecture and 50 min at the end. If $t_0 = 0$, then $\Delta t = t_{\mathrm{f}} \equiv t$.

In this text, for simplicity's sake,

- motion starts at time equal to zero $(t_0 = 0)$
- the symbol *t* is used for elapsed time unless otherwise specified $(\Delta t = t_{\mathrm{f}} \equiv t)$

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel,*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\mathrm{f}} - x_0}{t_{\mathrm{f}} - t_0}, \tag{2.5}$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_{f} and x_0 are the final and beginning positions at times t_{f} and t_0, respectively. If the starting time t_0 is taken to be zero, then the average velocity is simply

$$\bar{v} = \frac{\Delta x}{t}. \tag{2.6}$$

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{t} = \frac{-4\ \mathrm{m}}{5\ \mathrm{s}} = -0.8\ \mathrm{m/s}. \tag{2.7}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

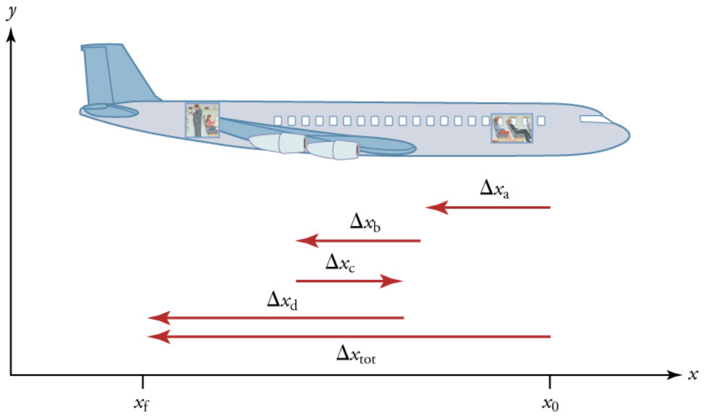


FIGURE 2.9 A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than the magnitude of displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

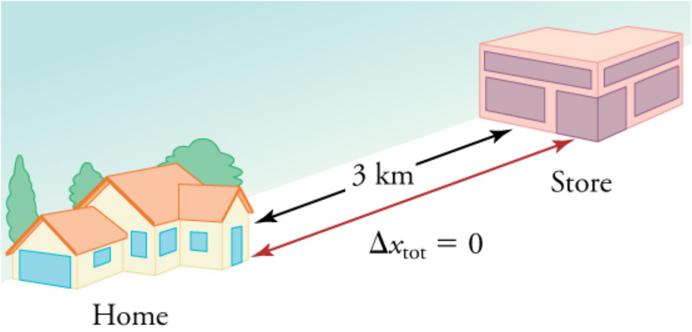


FIGURE 2.10 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in Figure 2.11. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

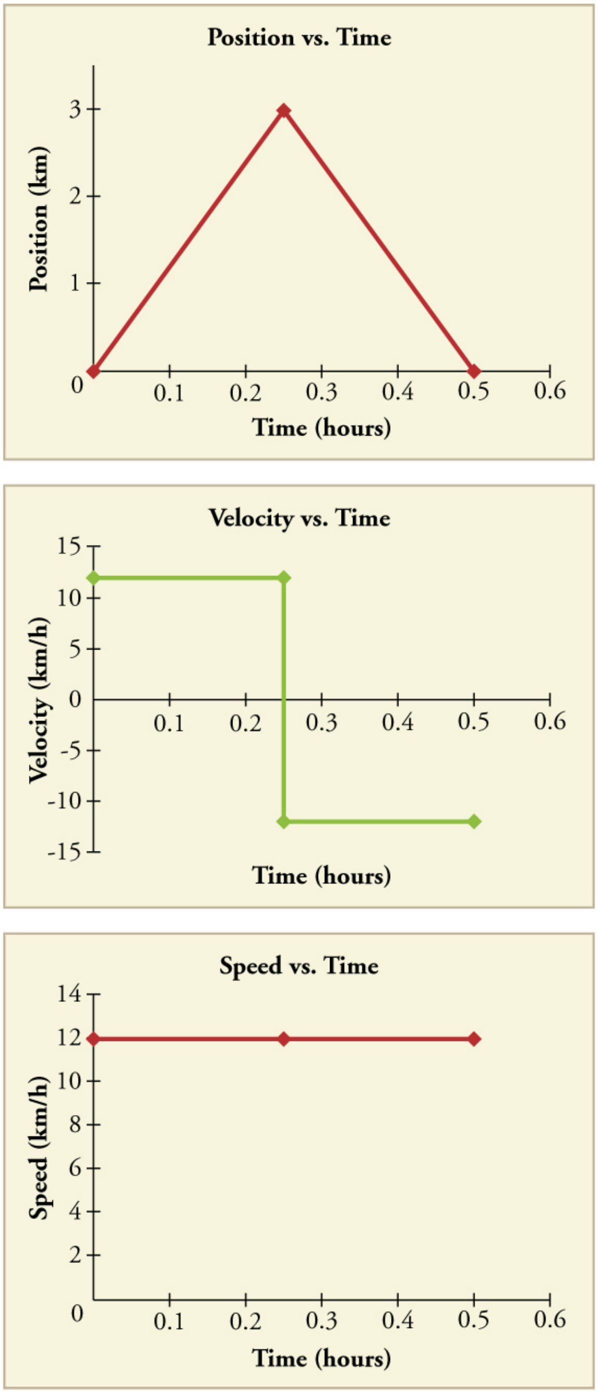


FIGURE 2.11 Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h

- determine the speed of an ant, snail, or falling leaf

CHECK YOUR UNDERSTANDING

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution

(a) The average velocity of the train is zero because $x_\mathrm{f} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80\text{ miles}}{105\text{ minutes}} \tag{2.8}$$

$$\frac{80\text{ miles}}{105\text{ minutes}} \times \frac{5280\text{ feet}}{1\text{ mile}} \times \frac{1\text{ meter}}{3.28\text{ feet}} \times \frac{1\text{ minute}}{60\text{ seconds}} = 20\text{ m/s} \tag{2.9}$$

2.4 Acceleration

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define and distinguish between instantaneous acceleration, average acceleration, and deceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

FIGURE 2.12 A plane decelerates, or slows down, as it comes in for landing in St. Maarten. Its acceleration is opposite in direction to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes,*

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\mathrm{f} - v_0}{t_\mathrm{f} - t_0}, \tag{2.10}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the *a* means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are $\mathrm{m/s^2}$, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. When an object slows down, its acceleration is opposite to the direction of its motion. This is known as **deceleration**.

FIGURE 2.13 A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Misconception Alert: Deceleration vs. Negative Acceleration

Deceleration always refers to acceleration in the direction opposite to the direction of the velocity. Deceleration always reduces speed. Negative acceleration, however, is acceleration *in the negative direction in the chosen coordinate system*. Negative acceleration may or may not be deceleration, and deceleration may or may not be considered negative acceleration. For example, consider Figure 2.14.

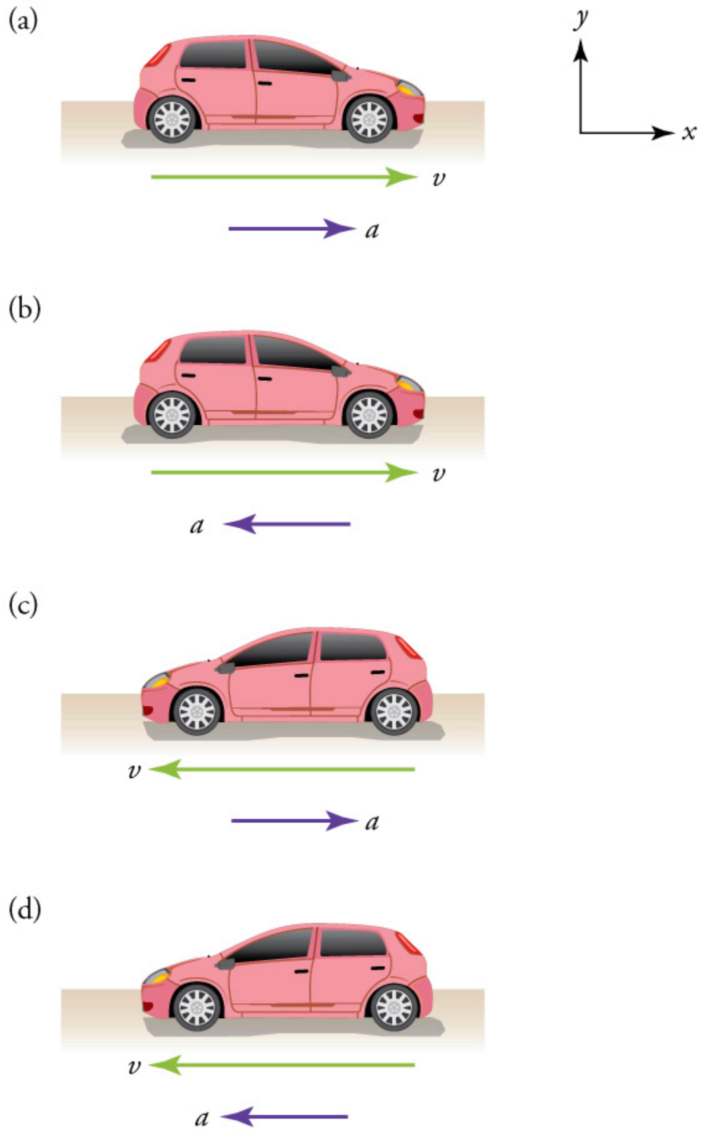


FIGURE 2.14 (a) This car is speeding up as it moves toward the right. It therefore has positive acceleration in our coordinate system. (b) This car is slowing down as it moves toward the right. Therefore, it has negative acceleration in our coordinate system, because its acceleration is toward the left. The car is also decelerating: the direction of its acceleration is opposite to its direction of motion. (c) This car is moving toward the left, but slowing down over time. Therefore, its acceleration is positive in our coordinate system because it is toward the right. However, the car is decelerating because its acceleration is opposite to its motion. (d) This car is speeding up as it moves toward the left. It has negative acceleration because it is accelerating toward the left. However, because its acceleration is in the same direction as its motion, it is speeding up (*not* decelerating).

EXAMPLE 2.1

Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

FIGURE 2.15 (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

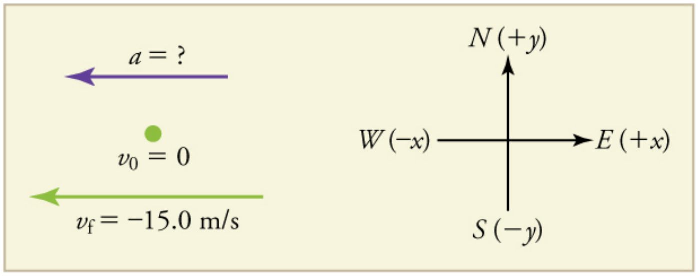


FIGURE 2.16

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2. \tag{2.11}$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity in Time, Velocity, and Speed—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. Figure 2.17 shows graphs of instantaneous acceleration versus time for two very different motions. In Figure 2.17(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about $1.8\ \text{m/s}^2$). In Figure 2.17(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\ \text{m/s}^2$ and $–2.0\ \text{m/s}^2$, respectively.

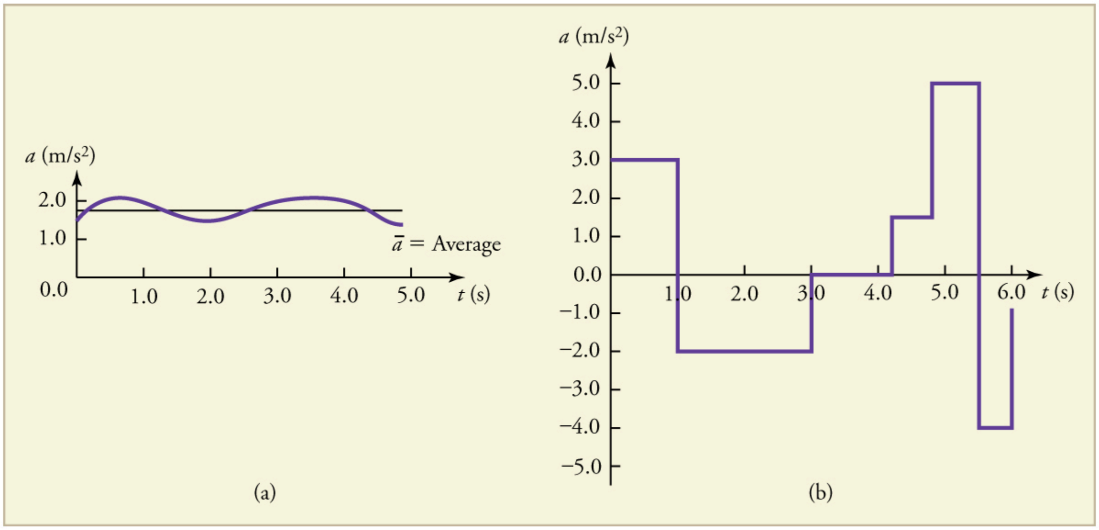


FIGURE 2.17 Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in Figure 2.18. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

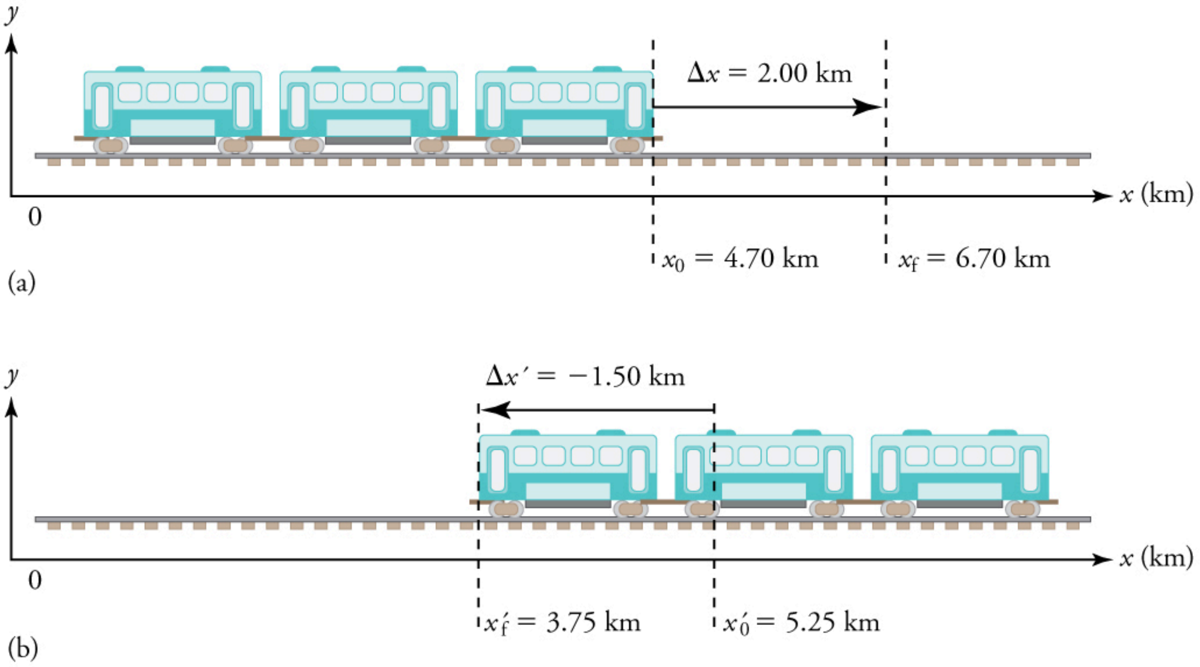


FIGURE 2.18 One-dimensional motion of a subway train considered in Example 2.2, Example 2.3, Example 2.4, Example 2.5, Example 2.6, and Example 2.7. Here we have chosen the x-axis so that + means to the right and − means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_f. Its displacement $\Delta x'$ is -1.5 km. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

EXAMPLE 2.2

Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of Figure 2.18?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_f - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_f = 6.70$ km and $x_0 = 4.70$ km for part (a), and $x'_f = 3.75$ km and $x'_0 = 5.25$ km for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_f - x_0 = 6.70 \text{ km} - 4.70 \text{ km} = +2.00 \text{ km} \tag{2.12}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_f - x'_0 = 3.75 \text{ km} - 5.25 \text{ km} = -1.50 \text{ km} \tag{2.13}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

EXAMPLE 2.3

Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in Figure 2.18?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in Example 2.2. Distance traveled is the total length of the path traveled between the two positions. (See Displacement.) In the case of the subway train shown in Figure 2.18, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was -1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

EXAMPLE 2.4

Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in Figure 2.18(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

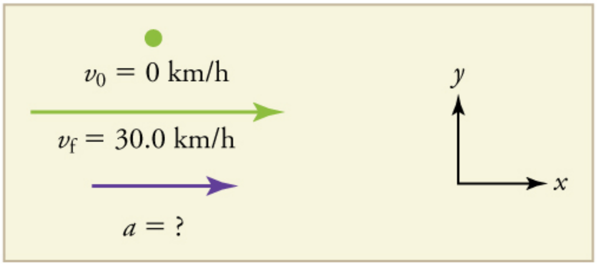


FIGURE 2.19

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_\text{f} = 30.0\text{ km/h}$, and $\Delta t = 20.0\text{ s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0 \text{ km/h}}{20.0 \text{ s}} \tag{2.14}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds. (See Physical Quantities and Units for more guidance.)

$$\bar{a} = \left(\frac{+30 \text{ km/h}}{20.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 0.417 \text{ m/s}^2 \tag{2.15}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

EXAMPLE 2.5

Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in Figure 2.18(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

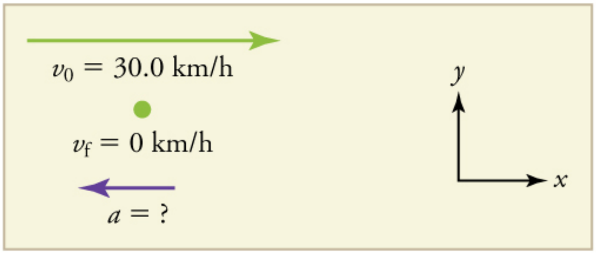


FIGURE 2.20

In this case, the train is decelerating and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0 \text{ km/h}$, $v_f = 0 \text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00 \text{ s}$.

2. Solve for the change in velocity, Δv.

$$\Delta v = v_f - v_0 = 0 - 30.0 \text{ km/h} = -30.0 \text{ km/h} \tag{2.16}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0 \text{ km/h}}{8.00 \text{ s}} \tag{2.17}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0 \text{ km/h}}{8.00 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = -1.04 \text{ m/s}^2. \tag{2.18}$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion. Again, acceleration is in the same direction as the *change* in velocity, which is negative here. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

The graphs of position, velocity, and acceleration vs. time for the trains in Example 2.4 and Example 2.5 are displayed in Figure 2.21. (We have taken the velocity to remain constant from 20 to 40 s, after which the train decelerates.)

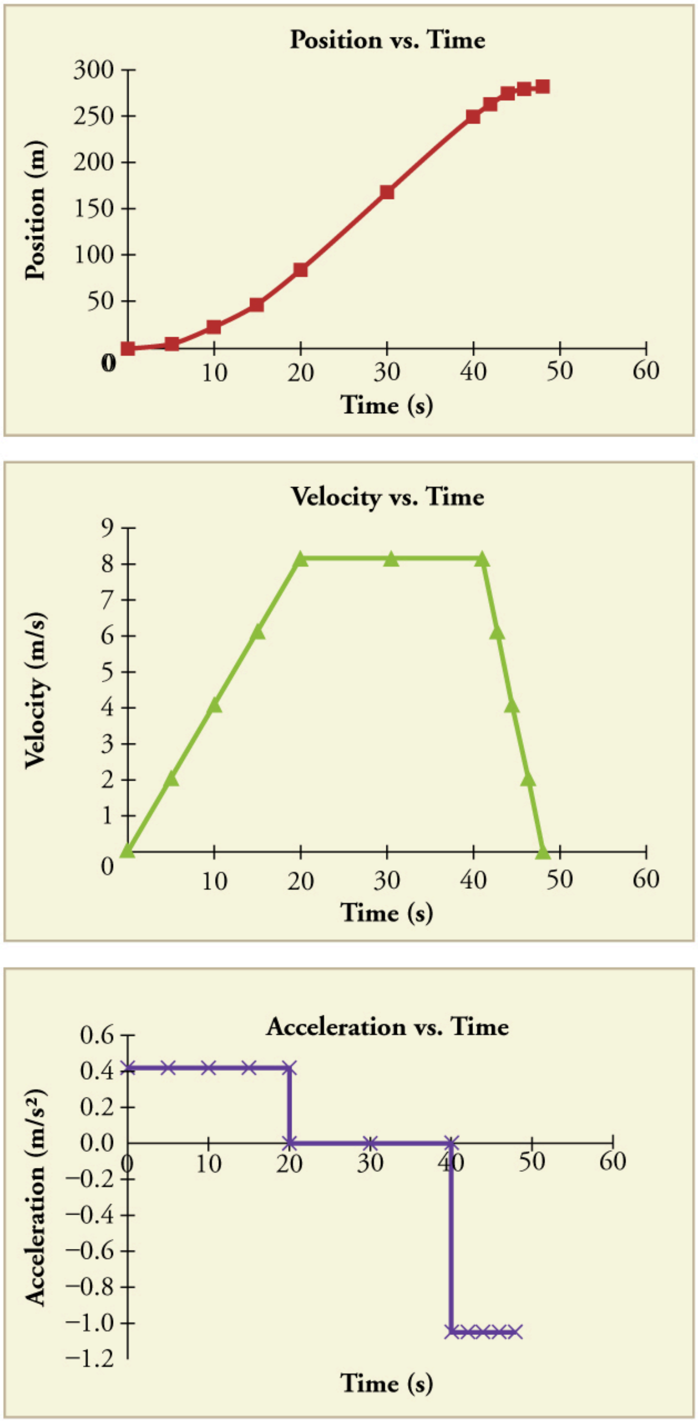


FIGURE 2.21 (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train decelerates at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

EXAMPLE 2.6

Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of Example 2.2, and shown again below, if it takes 5.00 min to make its trip?

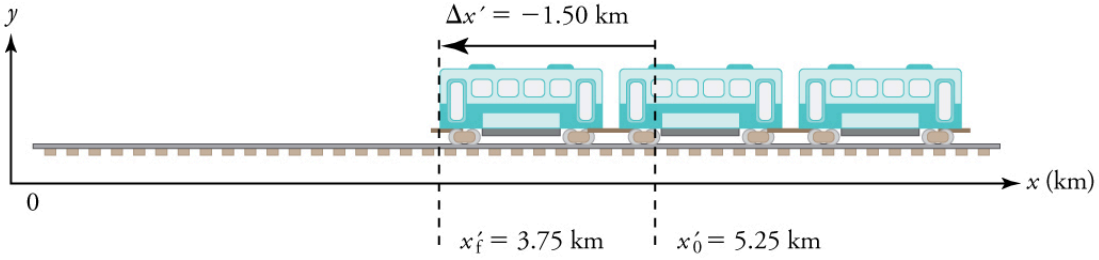


FIGURE 2.22

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_f = 3.75\text{ km}$, $x'_0 = 5.25\text{ km}$, $\Delta t = 5.00\text{ min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be -1.5 km in Example 2.2.

3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50\text{ km}}{5.00\text{ min}} \qquad 2.19$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50\text{ km}}{5.00\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = -18.0\text{ km/h} \qquad 2.20$$

Discussion

The negative velocity indicates motion to the left.

EXAMPLE 2.7

Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 2.22 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

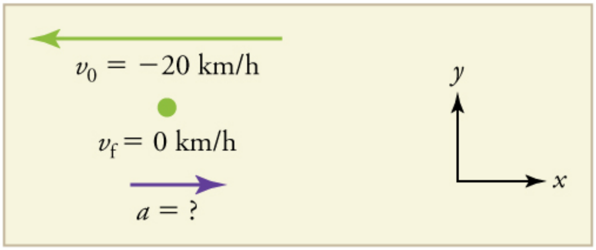


FIGURE 2.23

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\text{ km/h}$, $v_\text{f} = 0\text{ km/h}$, $\Delta t = 10.0\text{ s}$.
2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_\text{f} - v_0 = 0 - (-20\text{ km/h}) = +20\text{ km/h}. \tag{2.21}$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\text{ km/h}}{10.0\text{ s}} \tag{2.22}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0\text{ km/h}}{10.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = +0.556\text{ m/s}^2 \tag{2.23}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right). Again, acceleration is in the same direction as the *change* in velocity, which is positive here. As in Example 2.5, this acceleration can be called a deceleration since it is in the direction opposite to the velocity.

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in Example 2.7, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in Figure 2.22 is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the velocity, the object is speeding up. If acceleration has the opposite sign as the velocity, the object is slowing down.

✓ CHECK YOUR UNDERSTANDING

An airplane lands on a runway traveling east. Describe its acceleration.

Solution

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also decelerating: its acceleration is opposite in direction to its velocity.

PHET EXPLORATIONS

Moving Man Simulation

Learn about position, velocity, and acceleration graphs. Move the little man back and forth with the mouse and plot his motion. Set the position, velocity, or acceleration and let the simulation move the man for you.

Click to view content (https://openstax.org/l/02moving_man).

2.5 Motion Equations for Constant Acceleration in One Dimension

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

FIGURE 2.24 Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_{\mathrm{f}} - t_0$, taking $t_0 = 0$ means that $\Delta t = t_{\mathrm{f}}$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{array}{lcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}\right\} \qquad 2.24$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant}, \tag{2.25}$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2.26}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{2.27}$$

Solving for x yields

$$x = x_0 + \bar{v}t, \tag{2.28}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{)}. \tag{2.29}$$

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\text{ km/h} + 60\text{ km/h}}{2} = 45\text{ km/h}, \tag{2.30}$$

which seems logical.

EXAMPLE 2.8

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

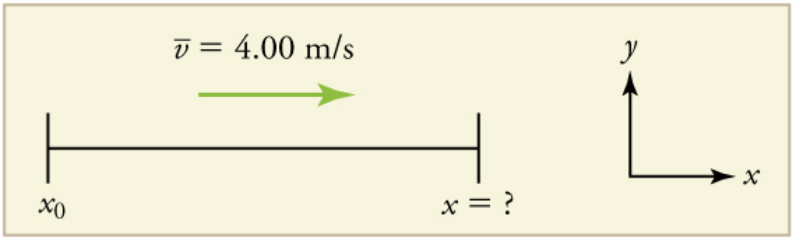


FIGURE 2.25

The final position x is given by the equation

$$x = x_0 + \bar{v}t. \tag{2.31}$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00 \text{ m/s}$, $\Delta t = 2.00 \text{ min}$, and $x_0 = 0 \text{ m}$.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00 \text{ m/s})(120 \text{ s}) = 480 \text{ m} \tag{2.32}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

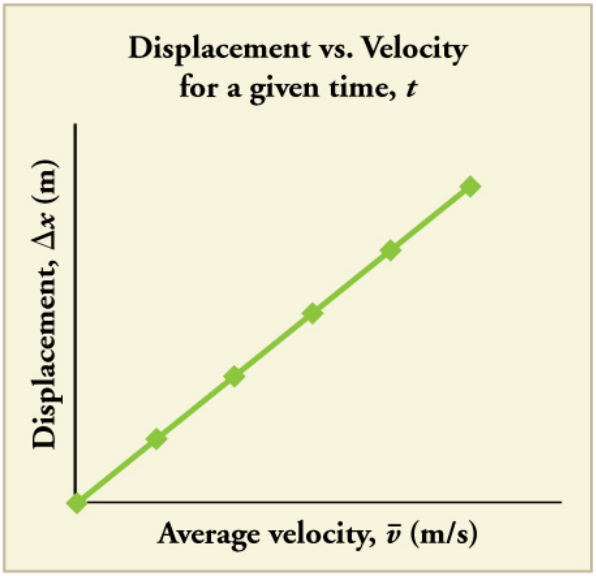


FIGURE 2.26 There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t} \tag{2.33}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{).} \tag{2.34}$$

Solving for v yields

$$v = v_0 + at \text{ (constant } a\text{).} \tag{2.35}$$

EXAMPLE 2.9

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then decelerates at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is decelerating.

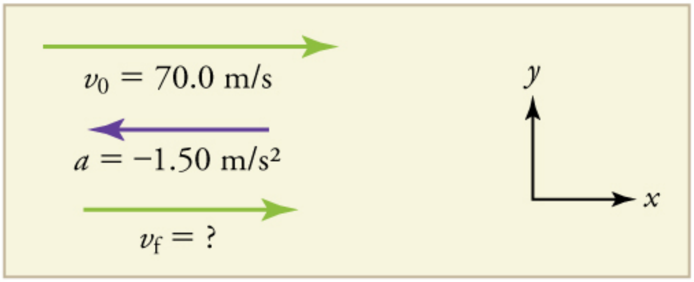


FIGURE 2.27

Solution

1. Identify the knowns. $v_0 = 70.0 \text{ m/s}$, $a = -1.50 \text{ m/s}^2$, $t = 40.0 \text{ s}$.

2. Identify the unknown. In this case, it is final velocity, v_{f}.

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0 \text{ m/s} + \left(-1.50 \text{ m/s}^2\right)(40.0 \text{ s}) = 10.0 \text{ m/s} \tag{2.36}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

FIGURE 2.28 The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal.

Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection

FIGURE 2.29 The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant $(a \neq 0)$

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at. \tag{2.37}$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at. \tag{2.38}$$

Since $\frac{v_0+v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at. \tag{2.39}$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}. \tag{2.40}$$

EXAMPLE 2.10

Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of $26.0\ \text{m/s}^2$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

FIGURE 2.30 U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.

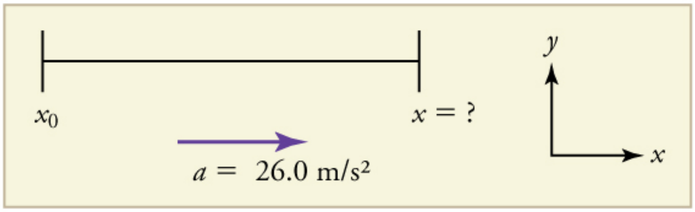


FIGURE 2.31

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as $26.0\ \text{m/s}^2$ and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + \frac{1}{2}at^2. \tag{2.41}$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}at^2. \tag{2.42}$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}\left(26.0\ \text{m/s}^2\right)(5.56\ \text{s})^2, \tag{2.43}$$

yielding

$$x = 402\ \text{m}. \tag{2.44}$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In Example 2.10, the dragster covers only one fourth of the total distance in the first half of the elapsed time
- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t, we get

$$t = \frac{v - v_0}{a}. \qquad 2.45$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0) \text{ (constant } a\text{).} \qquad 2.46$$

EXAMPLE 2.11

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in Example 2.10 without using information about time.

Strategy

Draw a sketch.

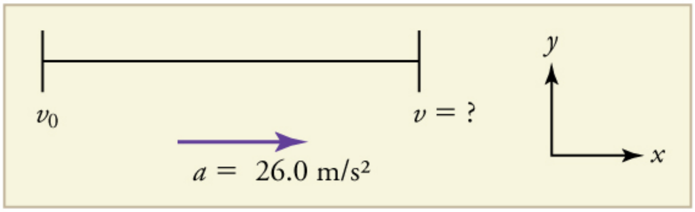


FIGURE 2.32

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402 \text{ m}$ (this was the answer in Example 2.10). Finally, the average acceleration was given to be $a = 26.0 \text{ m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0 \text{ m/s}^2\right)(402 \text{ m}). \qquad 2.47$$

Thus

$$v^2 = 2.09 \times 10^4 \text{ m}^2/\text{s}^2. \tag{2.48}$$

To get v, we take the square root:

$$v = \sqrt{2.09 \times 10^4 \text{ m}^2/\text{s}^2} = 145 \text{ m/s}. \tag{2.49}$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed deceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{2.50}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{2.51}$$

$$v = v_0 + at \tag{2.52}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2.53}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{2.54}$$

EXAMPLE 2.12

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can decelerate at a rate of 7.00 m/s^2, whereas on wet concrete it can decelerate at only 5.00 m/s^2. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

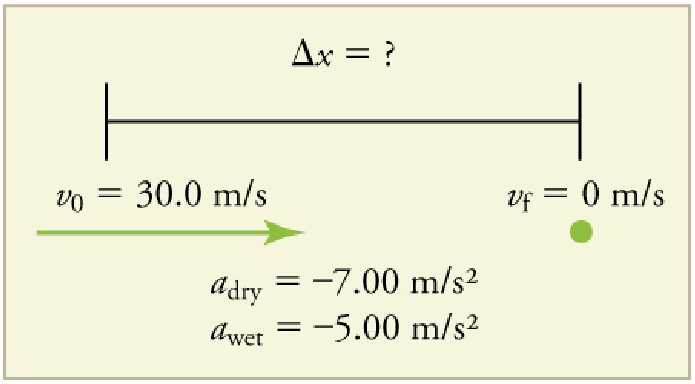


FIGURE 2.33

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0 \text{ m/s}$; $v = 0$; $a = -7.00 \text{ m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2.55}$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x.

$$x - x_0 = \frac{v^2 - v_0^2}{2a} \tag{2.56}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0 \text{ m/s})^2}{2\left(-7.00 \text{ m/s}^2\right)} \tag{2.57}$$

Thus,

$$x = 64.3 \text{ m on dry concrete.} \tag{2.58}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the deceleration is -5.00 m/s^2. The result is

$$x_{\text{wet}} = 90.0 \text{ m on wet concrete.} \tag{2.59}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0 \text{ m/s}$; $t_{\text{reaction}} = 0.500 \text{ s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m}. \tag{2.60}$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}} \tag{2.61}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet

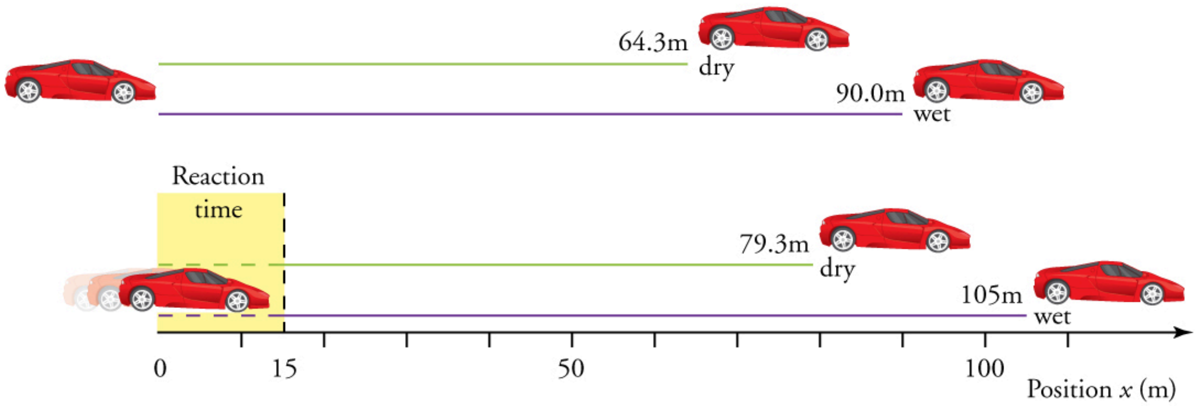


FIGURE 2.34 The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

EXAMPLE 2.13

Calculating Time: A Car Merges into Traffic

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)

Strategy

Draw a sketch.

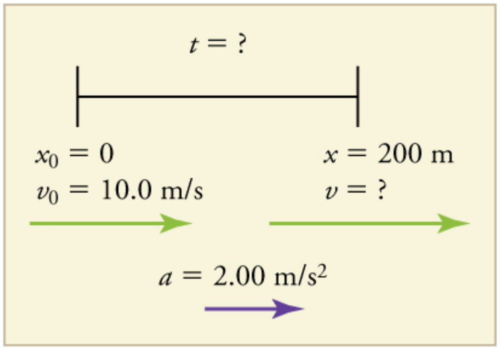


FIGURE 2.35

We are asked to solve for the time t. As before, we identify the known quantities in order to choose a convenient physical relationship (that is, an equation with one unknown, t).

Solution

1. Identify the knowns and what we want to solve for. We know that $v_0 = 10\text{ m/s}$; $a = 2.00\text{ m/s}^2$; and $x = 200\text{ m}$.

2. We need to solve for t. Choose the best equation. $x = x_0 + v_0 t + \frac{1}{2}at^2$ works best because the only unknown in the equation is the variable t for which we need to solve.

3. We will need to rearrange the equation to solve for t. In this case, it will be easier to plug in the knowns first.

$$200\text{ m} = 0\text{ m} + (10.0\text{ m/s})t + \frac{1}{2}\left(2.00\text{ m/s}^2\right)t^2 \tag{2.62}$$

4. Simplify the equation. The units of meters (m) cancel because they are in each term. We can get the units of seconds (s) to cancel by taking $t = t\text{ s}$, where t is the magnitude of time and s is the unit. Doing so leaves

$$200 = 10t + t^2. \tag{2.63}$$

5. Use the quadratic formula to solve for t.

(a) Rearrange the equation to get 0 on one side of the equation.

$$t^2 + 10t - 200 = 0 \tag{2.64}$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0, \tag{2.65}$$

where the constants are $a = 1.00$, $b = 10.0$, and $c = -200$.

(b) Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{2.66}$$

This yields two solutions for t, which are

$$t = 10.0 \text{ and } -20.0. \tag{2.67}$$

In this case, then, the time is $t = t$ in seconds, or

$$t = 10.0\text{ s and } -20.0\text{ s}. \tag{2.68}$$

A negative value for time is unreasonable, since it would mean that the event happened 20 s before the motion began. We can discard that solution. Thus,

$$t = 10.0\text{ s}. \tag{2.69}$$

Discussion

Whenever an equation contains an unknown squared, there will be two solutions. In some problems both solutions are meaningful, but in others, such as the above, only one solution is reasonable. The 10.0 s answer seems reasonable for a typical freeway on-ramp.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships. Problem-Solving Basics discusses problem-solving basics and outlines an approach that will help you succeed in this invaluable task.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration or deceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking deceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the deceleration in miles per hour per second. Convert this to meters per second squared and compare with other decelerations mentioned in this chapter. Calculate the distance traveled in braking.

CHECK YOUR UNDERSTANDING

A rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket to reach a velocity of 400 m/s?

Solution

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.

$$v = v_0 + at \tag{2.70}$$

Rearrange to solve for t.

$$t = \frac{v - v_0}{a} = \frac{400 \text{ m/s} - 0 \text{ m/s}}{20 \text{ m/s}^2} = 20 \text{ s} \tag{2.71}$$

2.6 Problem-Solving Basics for One-Dimensional Kinematics

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Apply problem-solving steps and strategies to solve problems of one-dimensional kinematics.
- Apply strategies to determine whether or not the result of a problem is reasonable, and if not, determine the cause.

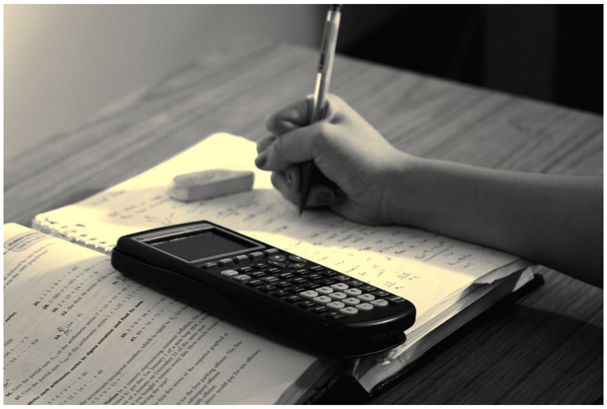


FIGURE 2.36 Problem-solving skills are essential to your success in Physics. (credit: scui3asteveo, Flickr)

Problem-solving skills are obviously essential to success in a quantitative course in physics. More importantly, the ability to apply broad physical principles, usually represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Such analytical skills are useful both for solving problems in this text and for applying physics in everyday and professional life.

Problem-Solving Steps

While there is no simple step-by-step method that works for every problem, the following general procedures facilitate problem solving and make it more meaningful. A certain amount of creativity and insight is required as well.

Step 1

Examine the situation to determine which physical principles are involved. It often helps to *draw a simple sketch* at the outset. You will also need to decide which direction is positive and note that on your sketch. Once you have identified the physical principles, it is much easier to find and apply the equations representing those principles. Although finding the correct equation is essential, keep in mind that equations represent physical principles, laws of nature, and relationships among physical quantities. Without a conceptual understanding of a problem, a numerical solution is meaningless.

Step 2

Make a list of what is given or can be inferred from the problem as stated (identify the knowns). Many problems are stated very succinctly and require some inspection to determine what is known. A sketch can also be very useful at this point. Formally identifying the knowns is of particular importance in applying physics to real-world situations. Remember, “stopped” means velocity is zero, and we often can take initial time and position as zero.

Step 3

Identify exactly what needs to be determined in the problem (identify the unknowns). In complex problems, especially, it is not always obvious what needs to be found or in what sequence. Making a list can help.

Step 4

Find an equation or set of equations that can help you solve the problem. Your list of knowns and unknowns can help here. It is easiest if you can find equations that contain only one unknown—that is, all of the other variables are known, so you can easily solve for the unknown. If the equation contains more than one unknown, then an additional equation is needed to solve the problem. In some problems, several unknowns must be determined to get at the one needed most. In such problems it is especially important to keep physical principles in mind to avoid going astray in a sea of equations. You may have to use two (or more) different equations to get the final answer.

Step 5

Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units. This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made. However, be warned that correct units do not guarantee that the numerical part of the answer is also correct.

Step 6

Check the answer to see if it is reasonable: Does it make sense? This final step is extremely important—the goal of physics is to accurately describe nature. To see if the answer is reasonable, check both its magnitude and its sign, in addition to its units. Your judgment will improve as you solve more and more physics problems, and it will become possible for you to make finer and finer judgments regarding whether nature is adequately described by the answer to a problem. This step brings the problem back to its conceptual meaning. If you can judge whether the answer is reasonable, you have a deeper understanding of physics than just being able to mechanically solve a problem.

When solving problems, we often perform these steps in different order, and we also tend to do several steps simultaneously. There is no rigid procedure that will work every time. Creativity and insight grow with experience, and the basics of problem solving become almost automatic. One way to get practice is to work out the text’s examples for yourself as you read. Another is to work as many end-of-section problems as possible, starting with the easiest to build confidence and progressing to the more difficult. Once you become involved in physics, you will see it all around you, and you can begin to apply it to situations you encounter outside the classroom, just as is done in many of the applications in this text.

Unreasonable Results

Physics must describe nature accurately. Some problems have results that are unreasonable because one premise is unreasonable or because certain premises are inconsistent with one another. The physical principle applied correctly then produces an unreasonable result. For example, if a person starting a foot race accelerates at 0.40 m/s^2 for 100 s, his final speed will be 40 m/s (about 150 km/h)—clearly unreasonable because the time of 100 s is an unreasonable premise. The physics is correct in a sense, but there is more to describing nature than just manipulating equations correctly. Checking the result of a problem to see if it is reasonable does more than help uncover errors in problem solving—it also builds intuition in judging whether nature is being accurately described.

Use the following strategies to determine whether an answer is reasonable and, if it is not, to determine what is the cause.

Step 1

Solve the problem using strategies as outlined and in the format followed in the worked examples in the text. In the example given in the preceding paragraph, you would identify the givens as the acceleration and time and use the equation below to find the unknown final velocity. That is,

$$v = v_0 + at = 0 + \left(0.40\text{ m/s}^2\right)(100\text{ s}) = 40\text{ m/s}. \tag{2.72}$$

Step 2

Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, ...? In this case, you may need to convert meters per second into a more familiar unit, such as miles per hour.

$$\left(\frac{40\text{ m}}{\text{s}}\right)\left(\frac{3.28\text{ ft}}{\text{m}}\right)\left(\frac{1\text{ mi}}{5280\text{ ft}}\right)\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = 89\text{ mph} \tag{2.73}$$

This velocity is about four times greater than a person can run—so it is too large.

Step 3

If the answer is unreasonable, look for what specifically could cause the identified difficulty. In the example of the runner, there are only two assumptions that are suspect. The acceleration could be too great or the time too long. First look at the acceleration and think about what the number means. If someone accelerates at 0.40 m/s^2, their velocity is increasing by 0.4 m/s each second. Does this seem reasonable? If so, the time must be too long. It is not possible for someone to accelerate at a constant rate of 0.40 m/s^2 for 100 s (almost two minutes).

2.7 Falling Objects

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

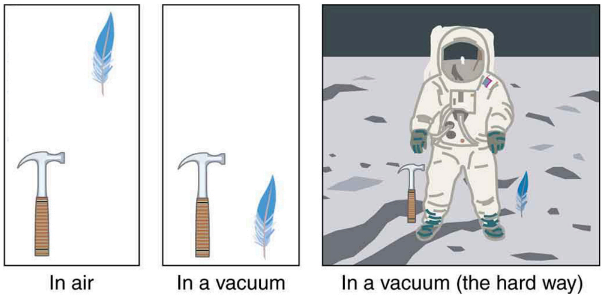


FIGURE 2.37 A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only $1.67\ \text{m/s}^2$.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g. It is constant at any given location on Earth and has the average value

$$g = 9.80\ \text{m/s}^2. \qquad 2.74$$

Although g varies from $9.78\ \text{m/s}^2$ to $9.83\ \text{m/s}^2$, depending on latitude, altitude, underlying geological formations, and local topography, the average value of $9.80\ \text{m/s}^2$ will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80\ \text{m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80\ \text{m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration = -*g*

$$v = v_0 - gt \qquad 2.75$$

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \qquad 2.76$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad 2.77$$

EXAMPLE 2.14

Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

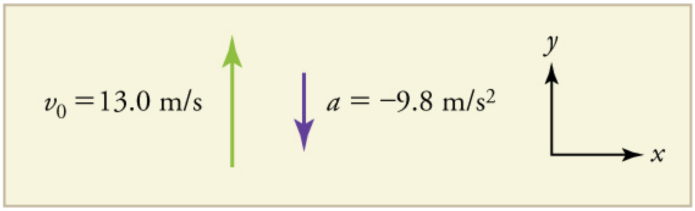


FIGURE 2.38

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position

y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{ m/s}$; $a = -g = -9.80 \text{ m/s}^2$; and $t = 1.00 \text{ s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.

3. Plug in the known values and solve for y_1.

$$y_1 = 0 + (13.0 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)(1.00 \text{ s})^2 = 8.10 \text{ m} \tag{2.78}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity

v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{ m/s}$; $a = -g = -9.80 \text{ m/s}^2$; and $t = 1.00 \text{ s}$. We also know from the solution above that $y_1 = 8.10 \text{ m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0 \text{ m/s} - \left(9.80 \text{ m/s}^2\right)(1.00 \text{ s}) = 3.20 \text{ m/s} \tag{2.79}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\text{ s}$. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\text{ s}$ and 3.00 s are the same as those above. The results are summarized in Table 2.1 and illustrated in Figure 2.39.

Time, *t*	Position, *y*	Velocity, *v*	Acceleration, *a*
1.00 s	8.10 m	3.20 m/s	-9.80 m/s^2
2.00 s	6.40 m	−6.60 m/s	-9.80 m/s^2
3.00 s	−5.10 m	−16.4 m/s	-9.80 m/s^2

TABLE 2.1 Results

Graphing the data helps us understand it more clearly.

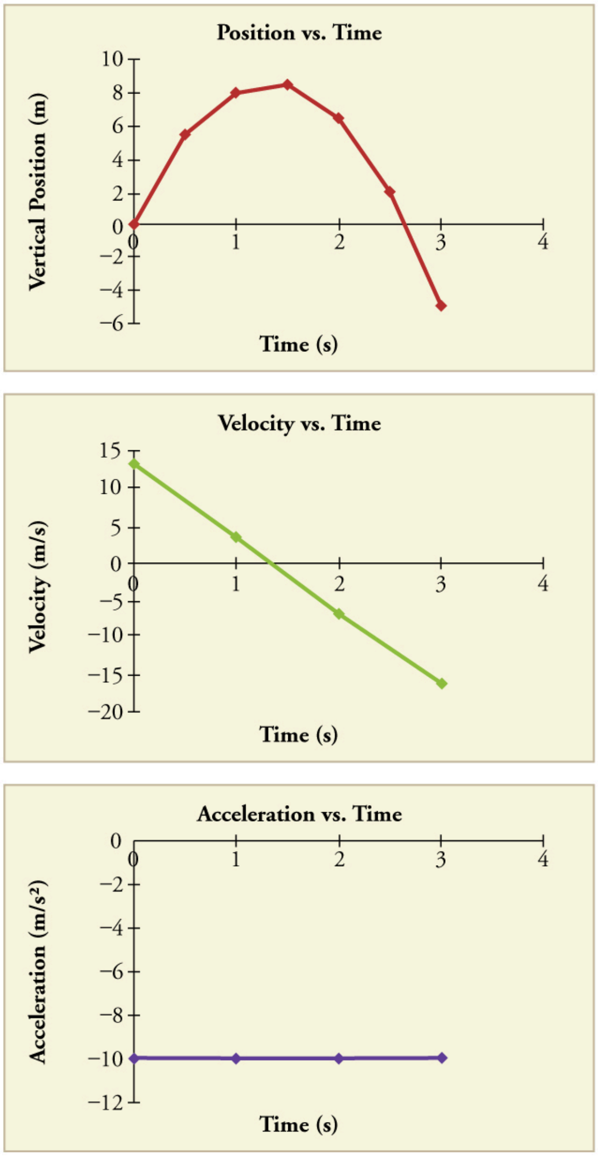


FIGURE 2.39 Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

EXAMPLE 2.15

Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

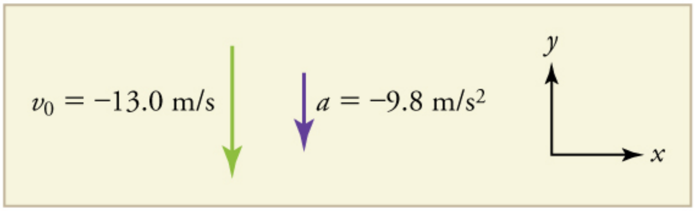


FIGURE 2.40

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2, \quad 2.80$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4\text{ m/s}. \quad 2.81$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4\text{ m/s}. \quad 2.82$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See Example 2.14 and Figure 2.41(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from Example 2.14) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20 \text{ m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

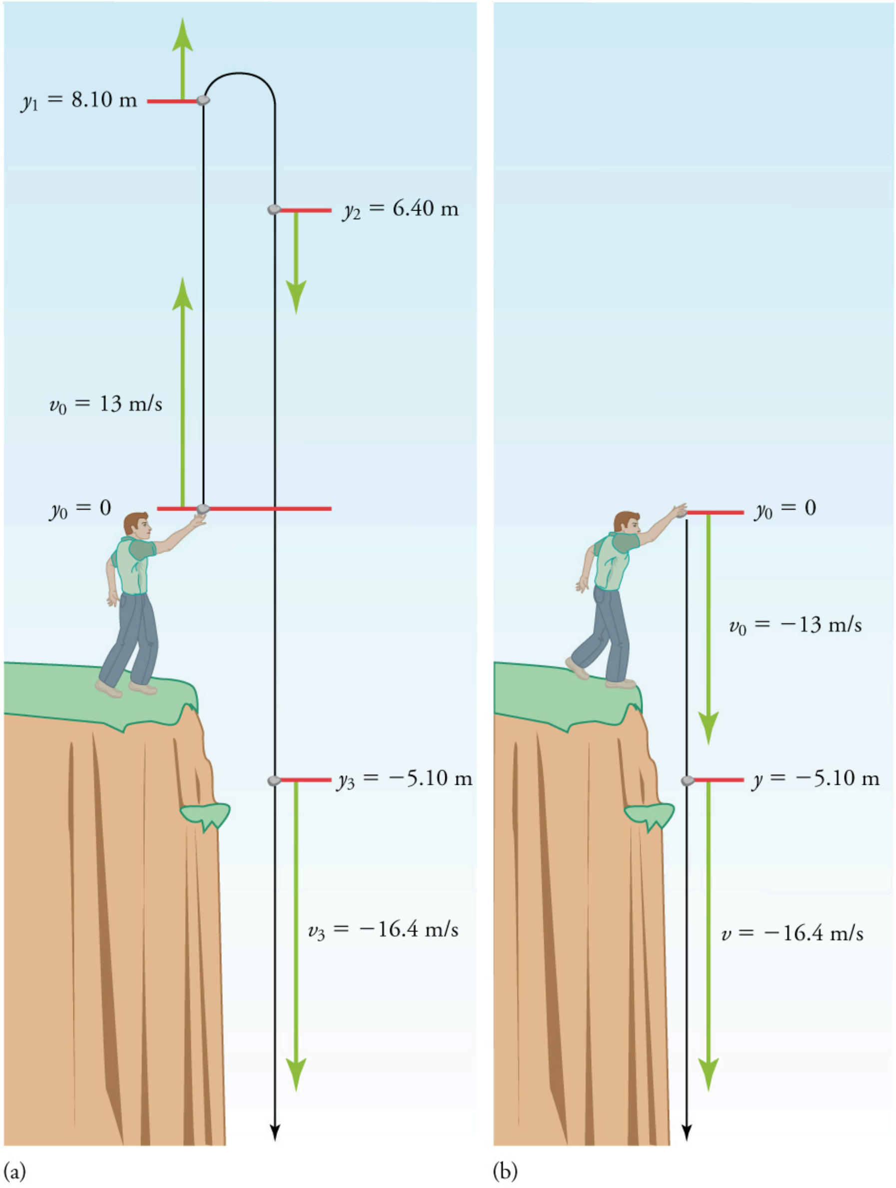


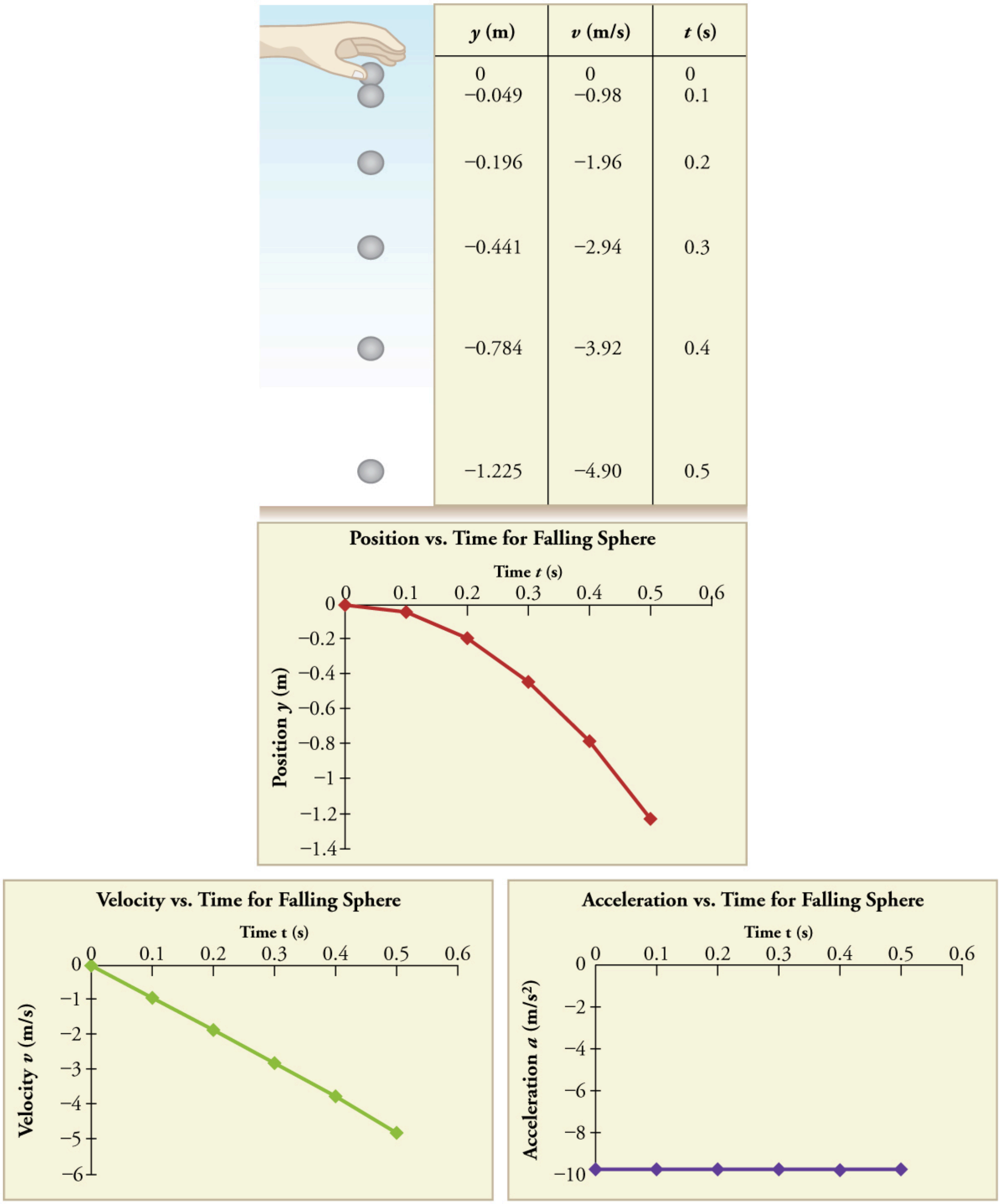
FIGURE 2.41 (a) A person throws a rock straight up, as explored in Example 2.14. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in Example 2.15. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In Example 2.14, the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is -13.0 m/s. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10 \text{ m}$ to be the same whether we have thrown it upwards at $+13.0 \text{ m/s}$ or thrown it downwards at -13.0 m/s. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

EXAMPLE 2.16

Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, Figure 2.42. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

FIGURE 2.42 Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

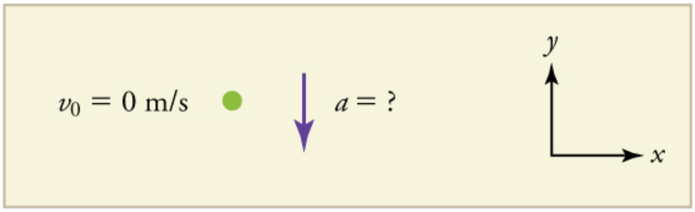


FIGURE 2.43

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = -1.0000\text{ m}$; $t = 0.45173$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \tag{2.83}$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2. \tag{2.84}$$

Solving for a gives

$$a = \frac{2(y - y_0)}{t^2}. \tag{2.85}$$

4. Substitute known values yields

$$a = \frac{2(-1.0000\text{ m} - 0)}{(0.45173\text{ s})^2} = -9.8010\text{ m/s}^2, \tag{2.86}$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010\text{ m/s}^2. \tag{2.87}$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80 m/s^2, so 9.8010 m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80 m/s^2; it represents the local value for the acceleration due to gravity.

CHECK YOUR UNDERSTANDING

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution

We know that initial position $y_0 = 0$, final position $y = -30.0\text{ m}$, and $a = -g = -9.80\text{ m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$$\begin{aligned} y &= 0 + 0 - \tfrac{1}{2}gt^2 \\ t^2 &= \frac{2y}{-g} \\ t &= \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0\text{ m})}{-9.80\text{ m/s}^2}} = \pm\sqrt{6.12\text{ s}^2} = 2.47\text{ s} \approx 2.5\text{ s} \end{aligned} \tag{2.88}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

PHET EXPLORATIONS

Equation Grapher

Learn about graphing polynomials. The shape of the curve changes as the constants are adjusted. View the curves for the individual terms (e.g. $y = bx$) to see how they add to generate the polynomial curve.

Click to view content (https://openstax.org/books/college-physics-2e/pages/2-7-falling-objects)

2.8 Graphical Analysis of One-Dimensional Motion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe a straight-line graph in terms of its slope and *y*-intercept.
- Determine average velocity or instantaneous velocity from a graph of position vs. time.
- Determine average or instantaneous acceleration from a graph of velocity vs. time.
- Derive a graph of velocity vs. time from a graph of position vs. time.
- Derive a graph of acceleration vs. time from a graph of velocity vs. time.

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information; they also reveal relationships between physical quantities. This section uses graphs of position, velocity, and acceleration versus time to illustrate one-dimensional kinematics.

Slopes and General Relationships

First note that graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against one another in such a graph, the horizontal axis is usually considered to be an **independent variable** and the vertical axis a **dependent variable**. If we call the horizontal axis the x-axis and the vertical axis the y-axis, as in Figure 2.44, a straight-line graph has the general form

$$y = mx + b. \tag{2.89}$$

Here m is the **slope**, defined to be the rise divided by the run (as seen in the figure) of the straight line. The letter b is used for the ***y*-intercept**, which is the point at which the line crosses the vertical axis.

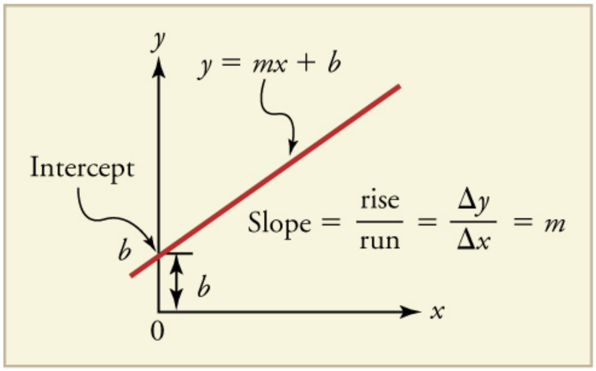


FIGURE 2.44 A straight-line graph. The equation for a straight line is $y = mx + b$.

Graph of Position vs. Time ($a = 0$, so v is constant)

Time is usually an independent variable that other quantities, such as position, depend upon. A graph of position versus time would, thus, have x on the vertical axis and t on the horizontal axis. Figure 2.45 is just such a straight-line graph. It shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.

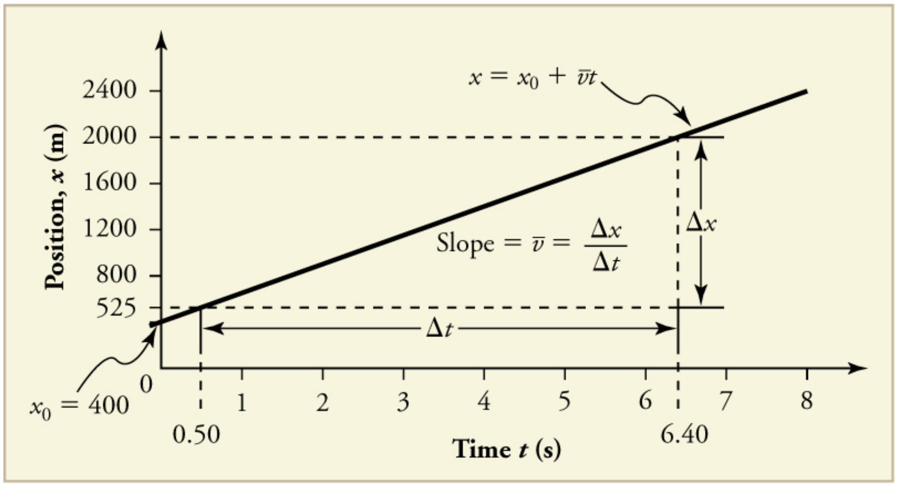


FIGURE 2.45 Graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph above is average velocity $\bar{v}$ and the intercept is position at time zero—that is, x_0. Substituting these symbols into $y = mx + b$ gives

$$x = \bar{v}t + x_0 \tag{2.90}$$

or

$$x = x_0 + \bar{v}t. \tag{2.91}$$

Thus a graph of position versus time gives a general relationship among displacement(change in position), velocity, and time, as well as giving detailed numerical information about a specific situation.

The Slope of x vs. t

The slope of the graph of position x vs. time t is velocity v.

$$\text{slope} = \frac{\Delta x}{\Delta t} = v \tag{2.92}$$

Notice that this equation is the same as that derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

From the figure we can see that the car has a position of 525 m at 0.50 s and 2000 m at 6.40 s. Its position at other times can be read from the graph; furthermore, information about its velocity and acceleration can also be obtained from the graph.

EXAMPLE 2.17

Determining Average Velocity from a Graph of Position versus Time: Jet Car

Find the average velocity of the car whose position is graphed in Figure 2.45.

Strategy

The slope of a graph of x vs. t is average velocity, since slope equals rise over run. In this case, rise = change in position and run = change in time, so that

$$\text{slope} = \frac{\Delta x}{\Delta t} = \bar{v}. \tag{2.93}$$

Since the slope is constant here, any two points on the graph can be used to find the slope. (Generally speaking, it is most accurate to use two widely separated points on the straight line. This is because any error in reading data from the graph is proportionally smaller if the interval is larger.)

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)

2. Substitute the x and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2000\text{ m} - 525\text{ m}}{6.4\text{ s} - 0.50\text{ s}}, \tag{2.94}$$

yielding

$$\bar{v} = 250\text{ m/s}. \tag{2.95}$$

Discussion

This is an impressively large land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 60 mi/h (27 m/s or 96 km/h), but considerably shy of the record of 343 m/s (1234 km/h or 766 mi/h) set in 1997.

Graphs of Motion when a is constant but $a \neq 0$

The graphs in Figure 2.46 below represent the motion of the jet-powered car as it accelerates toward its top speed, but only during the time when its acceleration is constant. Time starts at zero for this motion (as if measured with a stopwatch), and the position and velocity are initially 200 m and 15 m/s, respectively.

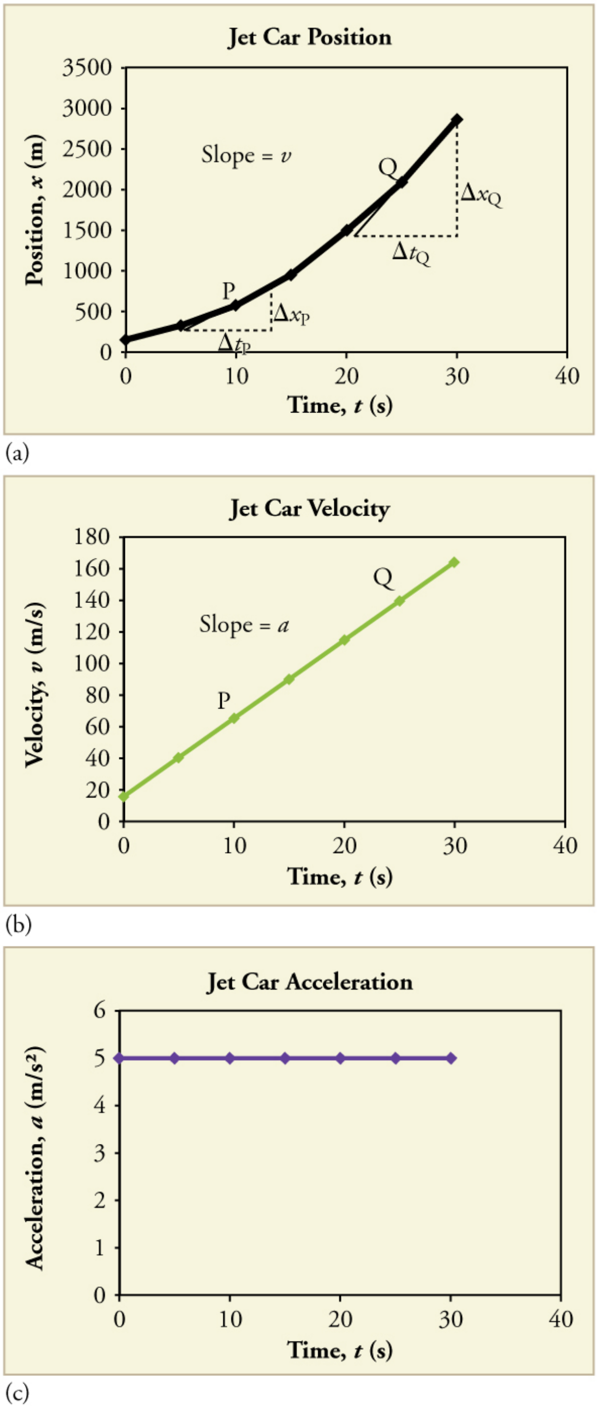


FIGURE 2.46 Graphs of motion of a jet-powered car during the time span when its acceleration is constant. (a) The slope of an x vs. t graph is velocity. This is shown at two points, and the instantaneous velocities obtained are plotted in the next graph. Instantaneous velocity at any point is the slope of the tangent at that point. (b) The slope of the v vs. t graph is constant for this part of the motion, indicating constant acceleration. (c) Acceleration has the constant value of 5.0 m/s^2 over the time interval plotted.

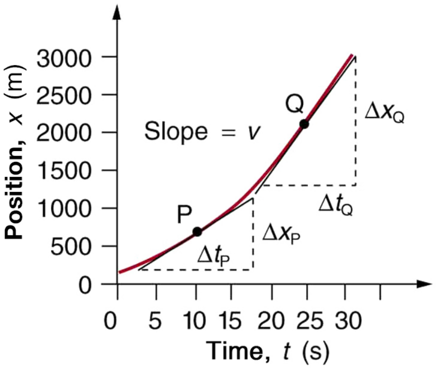
FIGURE 2.47 A U.S. Air Force jet car speeds down a track. (credit: Matt Trostle, Flickr)

The graph of position versus time in Figure 2.46(a) is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line tangent to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.46(a). If this is done at every point on the curve and the values are plotted against time, then the graph of velocity versus time shown in Figure 2.46(b) is obtained. Furthermore, the slope of the graph of velocity versus time is acceleration, which is shown in Figure 2.46(c).

EXAMPLE 2.18

Determining Instantaneous Velocity from the Slope at a Point: Jet Car

Calculate the velocity of the jet car at a time of 25 s by finding the slope of the x vs. t graph in the graph below.



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

FIGURE 2.48 The slope of an x vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point. This principle is illustrated in Figure 2.48, where Q is the point at $t = 25\text{ s}$.

Solution

1. Find the tangent line to the curve at $t = 25\text{ s}$.

2. Determine the endpoints of the tangent. These correspond to a position of 1300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, v.

$$\text{slope} = v_{\mathrm{Q}} = \frac{\Delta x_{\mathrm{Q}}}{\Delta t_{\mathrm{Q}}} = \frac{(3120\text{ m} - 1300\text{ m})}{(32\text{ s} - 19\text{ s})} \quad \mathbf{2.96}$$

Thus,

$$v_{\mathrm{Q}} = \frac{1820\text{ m}}{13\text{ s}} = 140\text{ m/s}. \quad \mathbf{2.97}$$

Discussion

This is the value given in this figure's table for v at $t = 25$ s. The value of 140 m/s for v_{Q} is plotted in Figure 2.48. The entire graph of v vs. t can be obtained in this fashion.

Carrying this one step further, we note that the slope of a velocity versus time graph is acceleration. Slope is rise divided by run; on a v vs. t graph, rise = change in velocity Δv and run = change in time Δt.

The Slope of *v* vs. *t*

The slope of a graph of velocity v vs. time t is acceleration a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = a \quad \mathbf{2.98}$$

Since the velocity versus time graph in Figure 2.46(b) is a straight line, its slope is the same everywhere, implying that acceleration is constant. Acceleration versus time is graphed in Figure 2.46(c).

Additional general information can be obtained from Figure 2.48 and the expression for a straight line, $y = mx + b$.

In this case, the vertical axis y is V, the intercept b is v_0, the slope m is a, and the horizontal axis x is t. Substituting these symbols yields

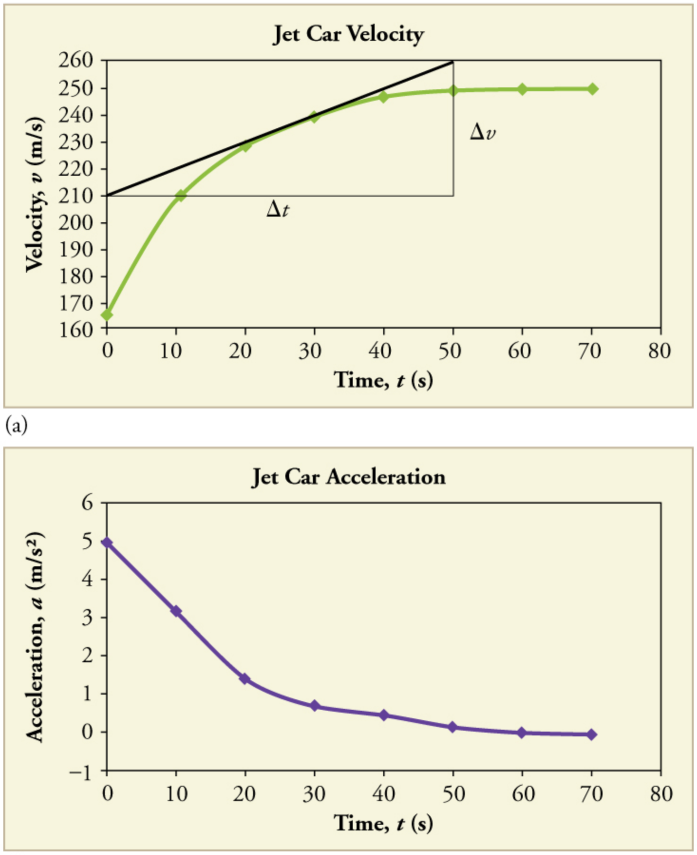
$$v = v_0 + at. \quad \mathbf{2.99}$$

A general relationship for velocity, acceleration, and time has again been obtained from a graph. Notice that this equation was also derived algebraically from other motion equations in Motion Equations for Constant Acceleration in One Dimension.

It is not accidental that the same equations are obtained by graphical analysis as by algebraic techniques. In fact, an important way to *discover* physical relationships is to measure various physical quantities and then make graphs of one quantity against another to see if they are correlated in any way. Correlations imply physical relationships and might be shown by smooth graphs such as those above. From such graphs, mathematical relationships can sometimes be postulated. Further experiments are then performed to determine the validity of the hypothesized relationships.

Graphs of Motion Where Acceleration is Not Constant

Now consider the motion of the jet car as it goes from 165 m/s to its top velocity of 250 m/s, graphed in Figure 2.49. Time again starts at zero, and the initial velocity is 165 m/s. (This was the final velocity of the car in the motion graphed in Figure 2.46.) Acceleration gradually decreases from 5.0 m/s^2 to zero when the car hits 250 m/s. The velocity increases until 55 s and then becomes constant, since acceleration decreases to zero at 55 s and remains zero afterward.



(a)

(b)

FIGURE 2.49 Graphs of motion of a jet-powered car as it reaches its top velocity. This motion begins where the motion in Figure 2.46 ends. (a) The velocity gradually approaches its top value. The slope of this graph is acceleration. It is plotted in the final graph. (b) Acceleration gradually declines to zero when velocity becomes constant. Notice in each of the three graphs that the acceleration drops down to zero and the velocity levels out. This results in a position-time graph that is almost linear. A close-up of the position time graph would show a slight curvature, as indicated in the velocity graph.

EXAMPLE 2.19

Calculating Acceleration from a Graph of Velocity versus Time

Calculate the acceleration of the jet car at a time of 25 s by finding the slope of the v vs. t graph in Figure 2.49(a).

Strategy

The slope of the curve at $t = 25\text{ s}$ is equal to the slope of the line tangent at that point, as illustrated in Figure 2.49(a).

Solution

Determine endpoints of the tangent line from the figure, and then plug them into the equation to solve for slope, a.

$$\text{slope} = \frac{\Delta v}{\Delta t} = \frac{(260\text{ m/s} - 210\text{ m/s})}{(51\text{ s} - 1.0\text{ s})} \qquad \textbf{2.100}$$

$$a = \frac{50\text{ m/s}}{50\text{ s}} = 1.0\text{ m/s}^2. \qquad \textbf{2.101}$$

Discussion

Note that this value for a is consistent with the value plotted in Figure 2.49(b) at $t = 25\text{ s}$.

A graph of position versus time can be used to generate a graph of velocity versus time, and a graph of velocity versus time can be used to generate a graph of acceleration versus time. We do this by finding the slope of the graphs at every point. If the graph is linear (i.e., a line with a constant slope), it is easy to find the slope at any point and you have the slope for every point. Graphical analysis of motion can be used to describe both specific and general characteristics of kinematics. Graphs can also be used for other topics in physics. An important aspect of exploring physical relationships is to graph them and look for underlying relationships.

CHECK YOUR UNDERSTANDING

A graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b)What would a graph of the ship's acceleration look like?

FIGURE 2.50

Solution

(a) The ship moves at constant velocity and then begins to decelerate at a constant rate. At some point, its deceleration rate decreases. It maintains this lower deceleration rate until it stops moving.

(b) A graph of acceleration vs. time would show zero acceleration in the first leg, large and constant negative acceleration in the second leg, and constant negative acceleration.

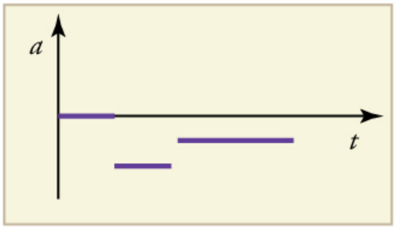


FIGURE 2.51

Glossary

acceleration the rate of change in velocity; the change in velocity over time

acceleration due to gravity acceleration of an object as a result of gravity

average acceleration the change in velocity divided by the time over which it changes

average speed distance traveled divided by time during which motion occurs

average velocity displacement divided by time over which displacement occurs

deceleration acceleration in the direction opposite to velocity; acceleration that results in a decrease in velocity

dependent variable the variable that is being measured; usually plotted along the y-axis

displacement the change in position of an object

distance the magnitude of displacement between two positions

distance traveled the total length of the path traveled between two positions

elapsed time the difference between the ending time and beginning time

free-fall the state of movement that results from gravitational force only

independent variable the variable that the dependent variable is measured with respect to; usually plotted along the x-axis

instantaneous acceleration acceleration at a specific point in time

instantaneous speed magnitude of the instantaneous velocity

instantaneous velocity velocity at a specific instant, or the average velocity over an infinitesimal time interval

kinematics the study of motion without considering its causes

model simplified description that contains only those elements necessary to describe the physics of a physical situation

position the location of an object at a particular time

scalar a quantity that is described by magnitude, but not direction

slope the difference in y-value (the rise) divided by the difference in x-value (the run) of two points on a straight line

time change, or the interval over which change occurs

vector a quantity that is described by both magnitude and direction

y-intercept the y-value when x= 0, or when the graph crosses the y-axis

Section Summary

2.1 Displacement

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.
- In symbols, displacement Δx is defined to be
  $\Delta x = x_{\mathrm{f}} - x_0,$
  where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means “change in” whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

2.2 Vectors, Scalars, and Coordinate Systems

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3 Time, Velocity, and Speed

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is
  $\Delta t = t_{\mathrm{f}} - t_0,$
  where t_{f} is the final time and t_0 is the initial time. The initial time is often taken to be zero, as if measured with a stopwatch; the elapsed time is then just t.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4 Acceleration

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$
- The SI unit for acceleration is m/s^2.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- Deceleration is an acceleration with a direction opposite to that of the velocity.

2.5 Motion Equations for Constant Acceleration in One Dimension

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,
$$\left.\begin{array}{lcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}\right\}$$
- The following kinematic equations for motion with constant a are useful:
$$x = x_0 + \bar{v}t$$
$$\bar{v} = \frac{v_0 + v}{2}$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
- In vertical motion, y is substituted for x.

2.6 Problem-Solving Basics for One-Dimensional Kinematics

- *The six basic problem solving steps for physics are:*
Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

  Step 6. Check the answer to see if it is reasonable: Does it make sense?

2.7 Falling Objects

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages $g = 9.80\ \text{m/s}^2$.
- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\ \text{m/s}^2$ is negative. In the opposite case, $a = +g = 9.80\ \text{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

2.8 Graphical Analysis of One-Dimensional Motion

- Graphs of motion can be used to analyze motion.
- Graphical solutions yield identical solutions to mathematical methods for deriving motion equations.
- The slope of a graph of displacement x vs. time t is velocity v.
- The slope of a graph of velocity v vs. time t graph is acceleration a.
- Average velocity, instantaneous velocity, and acceleration can all be obtained by analyzing

graphs.

Conceptual Questions

2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}\ \left(50 \times 10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in Exercise 2.4?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be -5°C the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure distance traveled or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6 Problem-Solving Basics for One-Dimensional Kinematics

18. What information do you need in order to choose which equation or equations to use to solve a problem? Explain.

19. What is the last thing you should do when solving a problem? Explain.

2.7 Falling Objects

20. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

21. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

22. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

23. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

24. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

25. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

2.8 Graphical Analysis of One-Dimensional Motion

26. (a) Explain how you can use the graph of position versus time in Figure 2.52 to describe the change in velocity over time. Identify (b) the time (t_a, t_b, t_c, t_d, or t_e) at which the instantaneous velocity is greatest, (c) the time at which it is zero, and (d) the time at which it is negative.

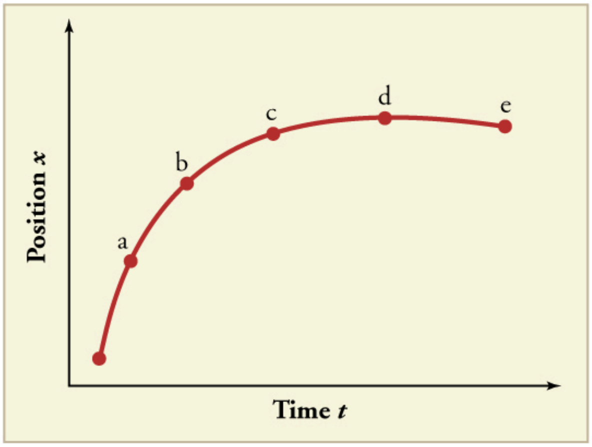


FIGURE 2.52

27. (a) Sketch a graph of velocity versus time corresponding to the graph of position versus time given in Figure 2.53. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the instantaneous velocity is greatest. (c) At which times is it zero? (d) At which times is it negative?

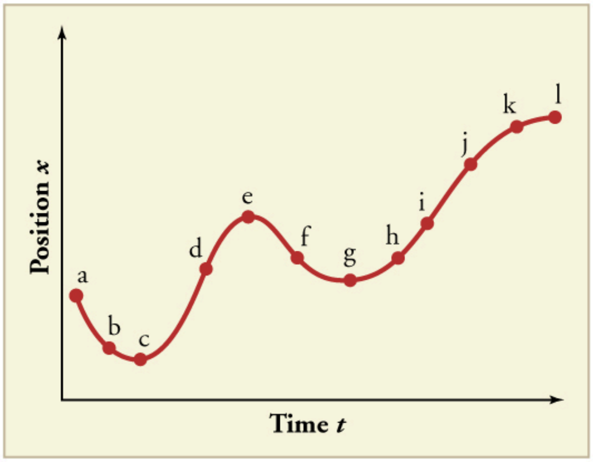


FIGURE 2.53

28. (a) Explain how you can determine the acceleration over time from a velocity versus time graph such as the one in Figure 2.54. (b) Based on the graph, how does acceleration change over time?

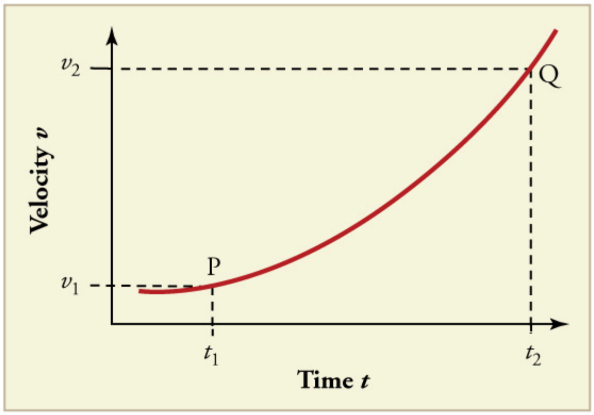


FIGURE 2.54

29. (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in Figure 2.55. (b) Identify the time or times (t_a, t_b, t_c, etc.) at which the acceleration is greatest. (c) At which times is it zero? (d) At which times is it negative?

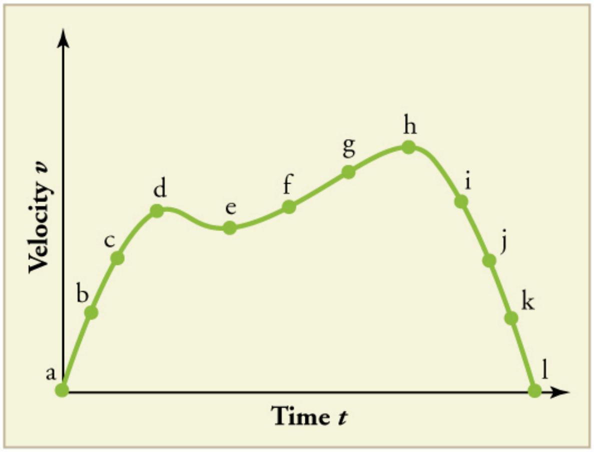


FIGURE 2.55

30. Consider the velocity vs. time graph of a person in an elevator shown in Figure 2.56. Suppose the elevator is initially at rest. It then accelerates for 3 seconds, maintains that velocity for 15 seconds, then decelerates for 5 seconds until it stops. The acceleration for the entire trip is not constant so we cannot use the equations of motion from Motion Equations for Constant Acceleration in One Dimension for the complete trip. (We could, however, use them in the three individual sections where acceleration is a constant.) Sketch graphs of (a) position vs. time and (b) acceleration vs. time for this trip.

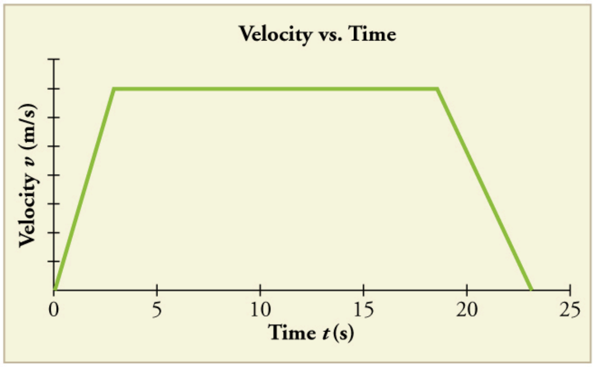


FIGURE 2.56

31. A cylinder is given a push and then rolls up an inclined plane. If the origin is the starting point, sketch the position, velocity, and acceleration of the cylinder vs. time as it goes up and then down the plane.

Problems & Exercises

2.1 Displacement

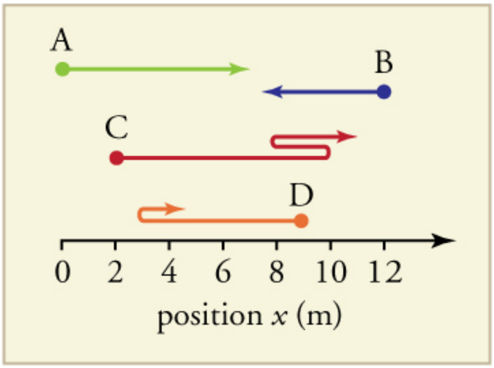


FIGURE 2.57

1. Find the following for path A in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2. Find the following for path B in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3. Find the following for path C in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4. Find the following for path D in Figure 2.57: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.3 Time, Velocity, and Speed

5. (a) Calculate Earth’s average speed relative to the Sun. (b) What is its average velocity over a period of one year?

6. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

7. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

8. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

9. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

10. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84 \times 10^6\ \text{m}$ (1%)?

11. A student drove to the university from their home and noted that the odometer reading of their car increased by 12.0 km. The trip took 18.0 min. (a) What was their average speed? (b) If the straight-line distance from their home to the university is 10.3 km in a direction 25.0° south of east, what was their average velocity? (c) If they returned home by the same path 7 h 30 min after they left, what were their average speed and velocity for the entire trip?

12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00 \times 10^8\ \text{m/s})$.

14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06 \times 10^{-10}\ \text{m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20 \times 10^6\ \text{m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity per revolution?

2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

17. Professional Application
Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of $g\ (9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

18. A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her deceleration?

19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in $\mathrm{m/s^2}$ and in multiples of $g\ (9.80\ \mathrm{m/s^2})$?

2.5 Motion Equations for Constant Acceleration in One Dimension

20. An Olympic-class sprinter starts a race with an acceleration of $4.50\ \mathrm{m/s^2}$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

21. A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is $2.10\times10^4\ \mathrm{m/s^2}$, and 1.85 ms $(1\ \mathrm{ms}=10^{-3}\ \mathrm{s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

22. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20\times10^5\ \mathrm{m/s^2}$ for $8.10\times10^{-4}\ \mathrm{s}$. What is its muzzle velocity (that is, its final velocity)?

23. (a) A light-rail commuter train accelerates at a rate of $1.35\ \mathrm{m/s^2}$. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of $1.65\ \mathrm{m/s^2}$. How long does it take to come to a stop from its top speed? (c) In emergencies the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency deceleration in $\mathrm{m/s^2}$?

24. While entering a freeway, a car accelerates from rest at a rate of $2.40\ \mathrm{m/s^2}$ for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in part (c), showing all steps explicitly.

25. At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of $2.00\ \mathrm{m/s^2}$. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense?

26. Professional Application:
Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

27. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33\times10^{-2}\ \mathrm{s}$, calculate the distance over which the puck accelerates.

28. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

29. Freight trains can produce only relatively small accelerations and decelerations. (a) What is the final velocity of a freight train that accelerates at a rate of $0.0500\ \mathrm{m/s^2}$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \mathrm{m/s^2}$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

30. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) How long did the acceleration last? (b) Calculate the acceleration.

31. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \mathrm{m/s^2}$, how far will it travel before becoming airborne? (b) How long does this take?

32. **Professional Application:**
A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in $\mathrm{m/s^2}$ and in multiples of g $\left(g=9.80\ \mathrm{m/s^2}\right)$. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of g?

33. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his deceleration? (b) How long does the collision last?

34. In World War II, there were several reported cases of airmen who jumped from their flaming airplanes with no parachute to escape certain death. Some fell about 20,000 feet (6000 m), and some of them survived, with few life-threatening injuries. For these lucky pilots, the tree branches and snow drifts on the ground allowed their deceleration to be relatively small. If we assume that a pilot's speed upon impact was 123 mph (54 m/s), then what was his deceleration? Assume that the trees and snow stopped him over a distance of 3.0 m.

35. Consider a grey squirrel falling out of a tree to the ground. (a) If we ignore air resistance in this case (only for the sake of this problem), determine a squirrel's velocity just before hitting the ground, assuming it fell from a height of 3.0 m. (b) If the squirrel stops in a distance of 2.0 cm through bending its limbs, compare its deceleration with that of the airman in the previous problem.

36. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of 0.150 m/s^2 as it goes through. The station is 210 m long. (a) How long did the nose of the train stay in the station? (b) How fast is it going when the nose leaves the station? (c) If the train is 130 m long, when does the end of the train leave the station? (d) What is the velocity of the end of the train as it leaves?

37. Dragsters can actually reach a top speed of 145 m/s in only 4.45 s—considerably less time than given in Example 2.10 and Example 2.11. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? *Hint*: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.

38. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of 0.500 m/s^2 for 7.00 s. (a) What is his final velocity? (b) The racer continues at this velocity to the finish line. If he was 300 m from the finish line when he started to accelerate, how much time did he save? (c) One other racer was 5.00 m ahead when the winner started to accelerate, but he was unable to accelerate, and traveled at 11.8 m/s until the finish line. How far ahead of him (in meters and in seconds) did the winner finish?

39. In 1967, New Zealander Burt Munro set the world record for an Indian motorcycle, on the Bonneville Salt Flats in Utah, with a maximum speed of 183.58 mi/h. The one-way course was 5.00 mi long. Acceleration rates are often described by the time it takes to reach 60.0 mi/h from rest. If this time was 4.00 s, and Burt accelerated at this rate until he reached his maximum speed, how long did it take Burt to complete the course?

40. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race?

2.7 Falling Objects

Assume air resistance is negligible unless otherwise stated.

41. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

42. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

43. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

44. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

45. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

46. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s, and her takeoff point is 1.80 m above the pool. (a) How long are her feet in the air? (b) What is her highest point above the board? (c) What is her velocity when her feet hit the water?

47. (a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?

48. A very strong, but inept, shot putter puts the shot straight up vertically with an initial velocity of 11.0 m/s. How long does he have to get out of the way if the shot was released at a height of 2.20 m, and he is 1.80 m tall?

49. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

50. A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

51. Standing at the base of one of the cliffs of Mt. Arapiles in Victoria, Australia, a hiker hears a rock break loose from a height of 105 m. He can’t see the rock right away but then does, 1.50 s later. (a) How far above the hiker is the rock when he can see it? (b) How much time does he have to move before the rock hits his head?

52. An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

53. There is a 250-m-high cliff at Half Dome in Yosemite National Park in California. Suppose a boulder breaks loose from the top of this cliff. (a) How fast will it be going when it strikes the ground? (b) Assuming a reaction time of 0.300 s, how long will a tourist at the bottom have to get out of the way after hearing the sound of the rock breaking loose (neglecting the height of the tourist, which would become negligible anyway if hit)? The speed of sound is 335 m/s on this day.

54. A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 0.312 s to go past the window. What was the ball’s initial velocity? Hint: First consider only the distance along the window, and solve for the ball's velocity at the bottom of the window. Next, consider only the distance from the ground to the bottom of the window, and solve for the initial velocity using the velocity at the bottom of the window as the final velocity.

55. Suppose you drop a rock into a dark well and, using precision equipment, you measure the time for the sound of a splash to return. (a) Neglecting the time required for sound to travel up the well, calculate the distance to the water if the sound returns in 2.0000 s. (b) Now calculate the distance taking into account the time for sound to travel up the well. The speed of sound is 332.00 m/s in this well.

56. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms $(8.00 \times 10^{-5}\ \text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

57. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

58. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms $(3.50 \times 10^{-3}\ \text{s})$. (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid?

2.8 Graphical Analysis of One-Dimensional Motion

Note: There is always uncertainty in numbers taken from graphs. If your answers differ from expected values, examine them to see if they are within data extraction uncertainties estimated by you.

59. (a) By taking the slope of the curve in Figure 2.58, verify that the velocity of the jet car is 115 m/s at $t = 20\ \text{s}$. (b) By taking the slope of the curve at any point in Figure 2.59, verify that the jet car's acceleration is $5.0\ \text{m/s}^2$.

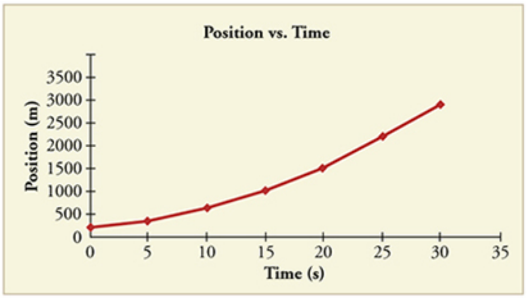


FIGURE 2.58

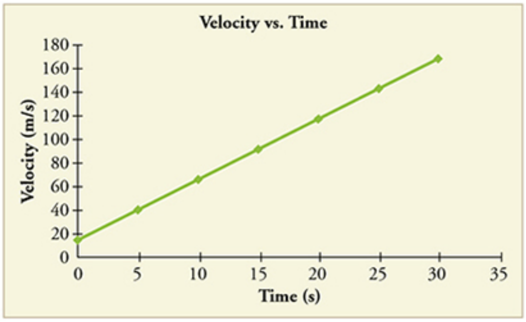


FIGURE 2.59

60. Using approximate values, calculate the slope of the curve in Figure 2.60 to verify that the velocity at $t = 30.0\ \text{s}$ is approximately 0.24 m/s. Assume all values are known to 2 significant figures.

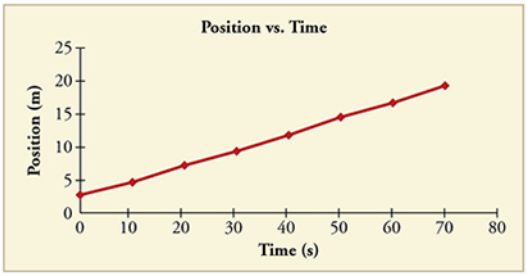


FIGURE 2.60

61. Using approximate values, calculate the slope of the curve in Figure 2.60 to verify that the velocity at $t = 30.0\ \text{s}$ is approximately 0.24 m/s.

62. By taking the slope of the curve in Figure 2.61, verify that the acceleration is $3.2\ \text{m/s}^2$ at $t = 10\ \text{s}$.

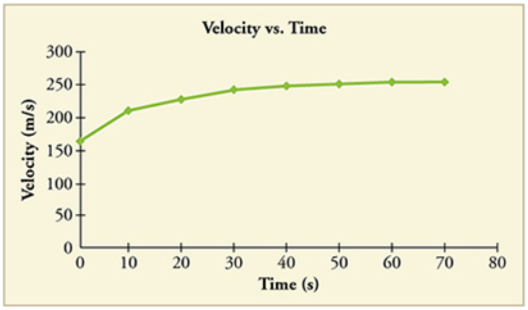


FIGURE 2.61

63. Construct the position graph for the subway shuttle train as shown in Figure 2.18(a). Your graph should show the position of the train, in kilometers, from t = 0 to 20 s. You will need to use the information on acceleration and velocity given in the examples for this figure.

64. (a) Take the slope of the curve in Figure 2.62 to find the jogger's velocity at $t = 2.5\text{ s}$. (b) Repeat at 7.5 s. These values must be consistent with the graph in Figure 2.63.

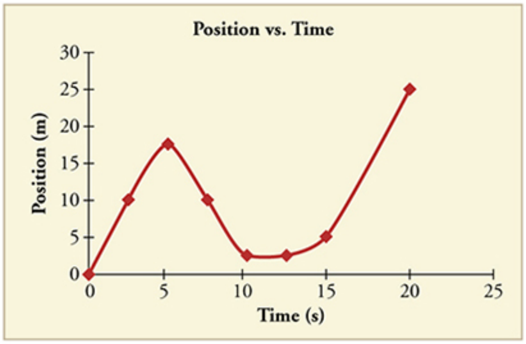


FIGURE 2.62

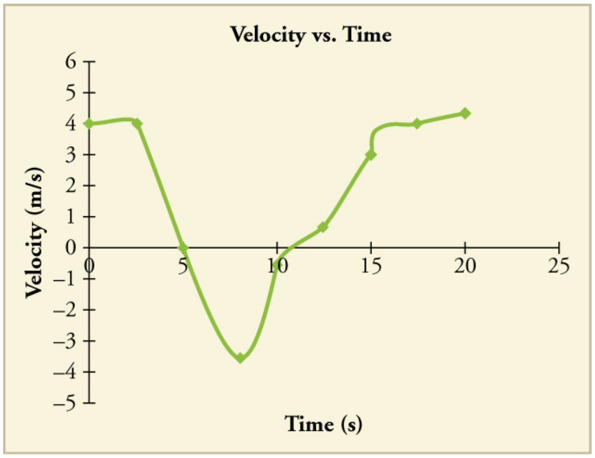


FIGURE 2.63

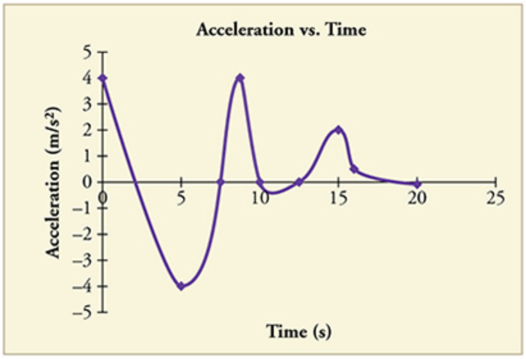


FIGURE 2.64

65. A graph of $v(t)$ is shown for a world-class track sprinter in a 100-m race. (See Figure 2.65). (a) What is his average velocity for the first 4 s? (b) What is his instantaneous velocity at $t = 5\text{ s}$? (c) What is his average acceleration between 0 and 4 s? (d) What is his time for the race?

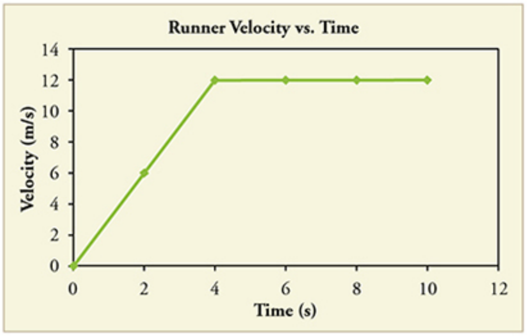


FIGURE 2.65

66. Figure 2.66 shows the position graph for a particle for 6 s. (a) Draw the corresponding Velocity vs. Time graph. (b) What is the acceleration between 0 s and 2 s? (c) What happens to the acceleration at exactly 2 s?

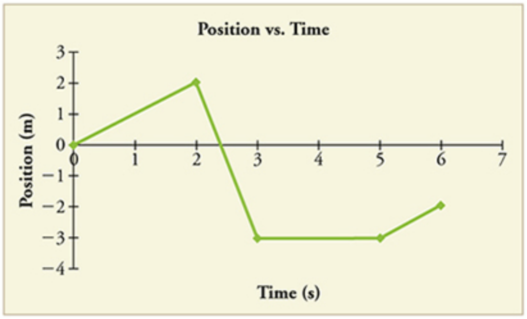


FIGURE 2.66

67. Critical Thinking Two cars are side by side on a path where friction is negligible. Car A starts from rest and has a positive, constant acceleration while car B travels at a constant speed. The cars reach the same check point down the road at the same time. (a) Which car is traveling faster at the check point? Briefly explain your reasoning. (b) Based on experimental data, an equation that fits the data is suggested for the speed of car A at the check point, *V*, which may not be correct: $V = KV_0$, where *K* is a constant with appropriate units and V_0 is the initial velocity of car B. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Graph the velocity of car A and the velocity of car B vs. distance along the path, starting when the cars are side by side.

CHAPTER 3

Two-Dimensional Kinematics

FIGURE 3.1 Hitting a tennis ball is difficult enough when it is traveling at high speeds in a relatively straight line. However, most motion is in curved, rather than straight-line, paths. Tennis players must account for the arc of the ball and the impact of its spin, while they also predict how it will bounce and plan the location of their shot. Motion along a curved path is two- or three-dimensional motion, and can be described in a similar fashion to one-dimensional motion. (credit: Edwin Martinez/Flickr)

CHAPTER OUTLINE

INTRODUCTION TO TWO-DIMENSIONAL KINEMATICS The arc of a basketball, the orbit of a satellite, a bicycle rounding a curve, a swimmer diving into a pool, blood gushing out of a wound, and a puppy chasing its tail are but a few examples of motions along curved paths. In fact, most motions in nature follow curved paths rather than straight lines. Motion along a curved path on a flat surface or a plane (such as that of a ball on a pool table or a skater on an ice rink) is two-dimensional, and thus described by two-dimensional kinematics. Motion not confined to a plane, such as a car following a winding mountain road, is described by three-dimensional kinematics. Both two- and three-dimensional kinematics are simple extensions of the one-dimensional kinematics developed for straight-line motion in the previous chapter. This simple extension will allow us to apply physics to many more situations, and it will also yield unexpected insights about nature.

Click to view content (https://openstax.org/books/college-physics-2e/pages/3-introduction-to-twodimensional-kinematics)

3.1 Kinematics in Two Dimensions: An Introduction

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Observe that motion in two dimensions consists of horizontal and vertical components.
- Understand the independence of horizontal and vertical vectors in two-dimensional motion.

FIGURE 3.2 Walkers and drivers in a city like New York are rarely able to travel in straight lines to reach their destinations. Instead, they must follow roads and sidewalks, making two-dimensional, zigzagged paths. (credit: Margaret W. Carruthers)

Two-Dimensional Motion: Walking in a City

Suppose you want to walk from one point to another in a city with uniform square blocks, as pictured in Figure 3.3.

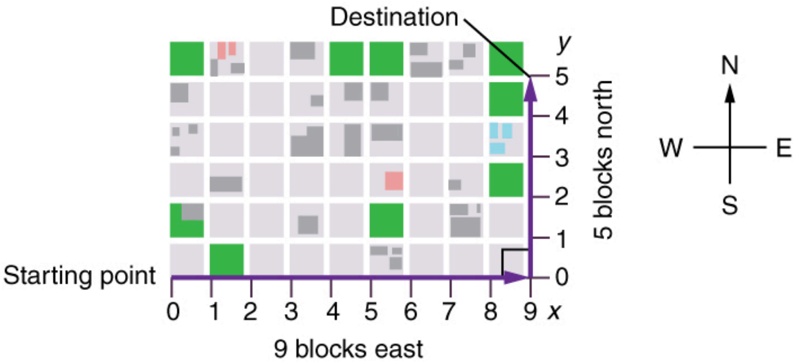


FIGURE 3.3 A pedestrian walks a two-dimensional path between two points in a city. In this scene, all blocks are square and are the same size.

The straight-line path that a helicopter might fly is blocked to you as a pedestrian, and so you are forced to take a two-dimensional path, such as the one shown. You walk 14 blocks in all, 9 east followed by 5 north. What is the straight-line distance?

An old adage states that the shortest distance between two points is a straight line. The two legs of the trip and the straight-line path form a right triangle, and so the Pythagorean theorem, $a^2 + b^2 = c^2$, can be used to find the straight-line distance.

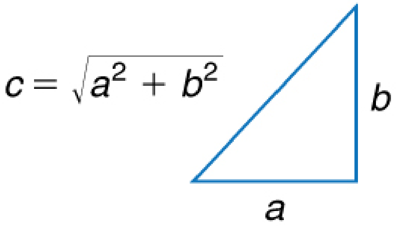


FIGURE 3.4 The Pythagorean theorem relates the length of the legs of a right triangle, labeled a and b, with the hypotenuse, labeled c. The relationship is given by: $a^2 + b^2 = c^2$. This can be rewritten, solving for $c: c = \sqrt{a^2 + b^2}$.

The hypotenuse of the triangle is the straight-line path, and so in this case its length in units of city blocks is $\sqrt{(9\text{ blocks})^2 + (5\text{ blocks})^2} = 10.3\text{ blocks}$, considerably shorter than the 14 blocks you walked. (Note that we are using three significant figures in the answer. Although it appears that “9” and “5” have only one significant digit, they are discrete numbers. In this case “9 blocks” is the same as “9.0 or 9.00 blocks.” We have decided to use three significant figures in the answer in order to show the result more precisely.)

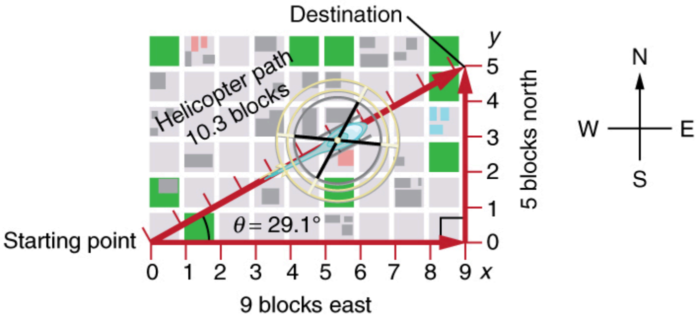


FIGURE 3.5 The straight-line path followed by a helicopter between the two points is shorter than the 14 blocks walked by the pedestrian. All blocks are square and the same size.

The fact that the straight-line distance (10.3 blocks) in Figure 3.5 is less than the total distance walked (14 blocks) is one example of a general characteristic of vectors. (Recall that **vectors** are quantities that have both magnitude and direction.)

As for one-dimensional kinematics, we use arrows to represent vectors. The length of the arrow is proportional to the vector’s magnitude. The arrow’s length is indicated by hash marks in Figure 3.3 and Figure 3.5. The arrow points in the same direction as the vector. For two-dimensional motion, the path of an object can be represented with three vectors: one vector shows the straight-line path between the initial and final points of the motion, one vector shows the horizontal component of the motion, and one vector shows the vertical component of the motion. The horizontal and vertical components of the motion add together to give the straight-line path. For example, observe the three vectors in Figure 3.5. The first represents a 9-block displacement east. The second represents a 5-block displacement north. These vectors are added to give the third vector, with a 10.3-block total displacement. The third vector is the straight-line path between the two points. Note that in this example, the vectors that we are adding are perpendicular to each other and thus form a right triangle. This means that we can use the Pythagorean theorem to calculate the magnitude of the total displacement. (Note that we cannot use the Pythagorean theorem to add vectors that are not perpendicular. We will develop techniques for adding vectors having any direction, not just those perpendicular to one another, in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods.)

The Independence of Perpendicular Motions

The person taking the path shown in Figure 3.5 walks east and then north (two perpendicular directions). How far they walk east is only affected by their motion eastward. Similarly, how far they walk north is only affected by their motion northward.

Independence of Motion

The horizontal and vertical components of two-dimensional motion are independent of each other. Any motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

This is true in a simple scenario like that of walking in one direction first, followed by another. It is also true of more complicated motion involving movement in two directions at once. For example, let’s compare the motions of two baseballs. One baseball is dropped from rest. At the same instant, another is thrown horizontally from the same height and follows a curved path. A stroboscope has captured the positions of the balls at fixed time intervals as they fall.

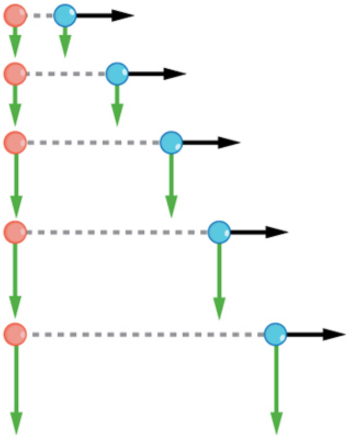


FIGURE 3.6 This shows the motions of two identical balls—one falls from rest, the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity, while the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls. This shows that the vertical and horizontal motions are independent.

It is remarkable that for each flash of the strobe, the vertical positions of the two balls are the same. This similarity implies that the vertical motion is independent of whether or not the ball is moving horizontally. (Assuming no air resistance, the vertical motion of a falling object is influenced by gravity only, and not by any horizontal forces.) Careful examination of the ball thrown horizontally shows that it travels the same horizontal distance between flashes. This is due to the fact that there are no additional forces on the ball in the horizontal direction after it is thrown. This result means that the horizontal velocity is constant, and affected neither by vertical motion nor by gravity (which is vertical). Note that this case is true only for ideal conditions. In the real world, air resistance will affect the speed of the balls in both directions.

The two-dimensional curved path of the horizontally thrown ball is composed of two independent one-dimensional motions (horizontal and vertical). The key to analyzing such motion, called *projectile motion*, is to *resolve* (break) it into motions along perpendicular directions. Resolving two-dimensional motion into perpendicular components is possible because the components are independent. We shall see how to resolve vectors in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods. We will find such techniques to be useful in many areas of physics.

PHET EXPLORATIONS

Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

Click to view content (https://openstax.org/l/28ladybugmotion).

3.2 Vector Addition and Subtraction: Graphical Methods

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand the rules of vector addition, subtraction, and multiplication.
- Apply graphical methods of vector addition and subtraction to determine the displacement of moving objects.

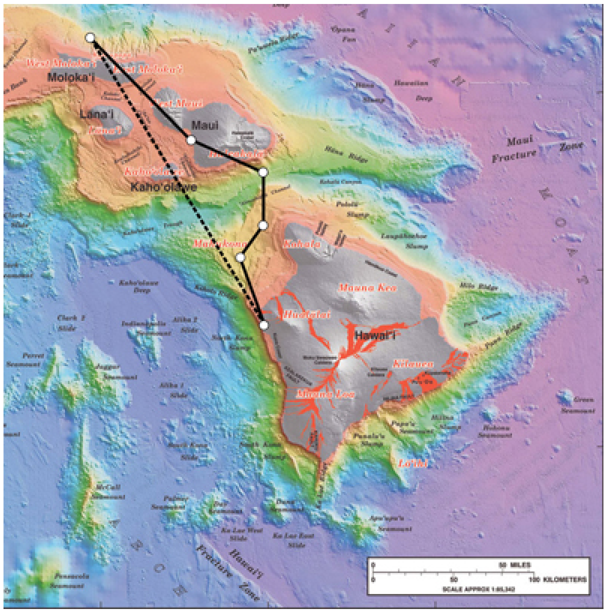


FIGURE 3.7 Displacement can be determined graphically using a scale map, such as this one of the Hawaiian Islands. A journey from Hawai'i to Moloka'i has a number of legs, or journey segments. These segments can be added graphically with a ruler to determine the total two-dimensional displacement of the journey. (credit: US Geological Survey)

Vectors in Two Dimensions

A **vector** is a quantity that has magnitude and direction. Displacement, velocity, acceleration, and force, for example, are all vectors. In one-dimensional, or straight-line, motion, the direction of a vector can be given simply by a plus or minus sign. In two dimensions (2-d), however, we specify the direction of a vector relative to some reference frame (i.e., coordinate system), using an arrow having length proportional to the vector's magnitude and pointing in the direction of the vector.

Figure 3.8 shows such a *graphical representation of a vector*, using as an example the total displacement for the person walking in a city considered in Kinematics in Two Dimensions: An Introduction. We shall use the notation that a boldface symbol, such as $\mathbf{D}$, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction by θ.

Vectors in this Text

In this text, we will represent a vector with a boldface variable. For example, we will represent the quantity force with the vector $\mathbf{F}$, which has both magnitude and direction. The magnitude of the vector will be represented by a variable in italics, such as F, and the direction of the variable will be given by an angle θ.

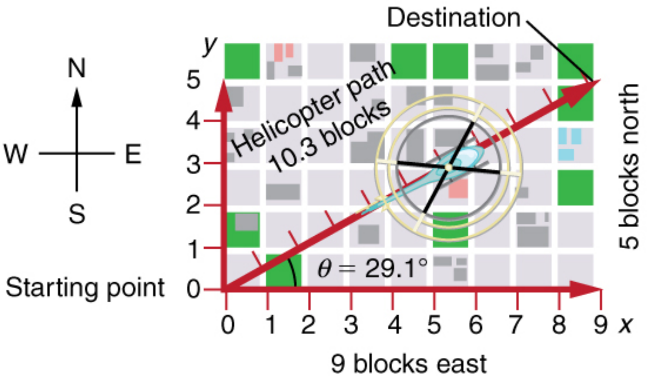


FIGURE 3.8 A person walks 9 blocks east and 5 blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

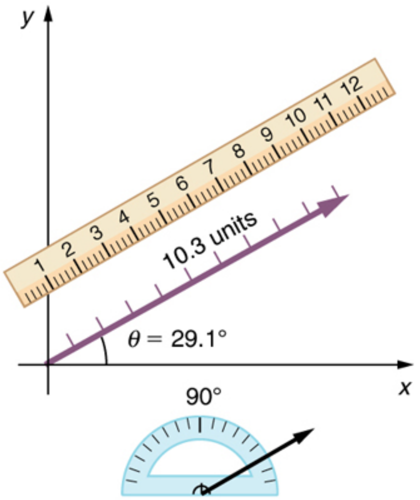


FIGURE 3.9 To describe the resultant vector for the person walking in a city considered in Figure 3.8 graphically, draw an arrow to represent the total displacement vector $\mathbf{D}$. Using a protractor, draw a line at an angle θ relative to the east-west axis. The length D of the arrow is proportional to the vector's magnitude and is measured along the line with a ruler. In this example, the magnitude D of the vector is 10.3 units, and the direction θ is 29.1° north of east.

Vector Addition: Head-to-Tail Method

The **head-to-tail method** is a graphical way to add vectors, described in Figure 3.10 below and in the steps following. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the final, pointed end of the arrow.

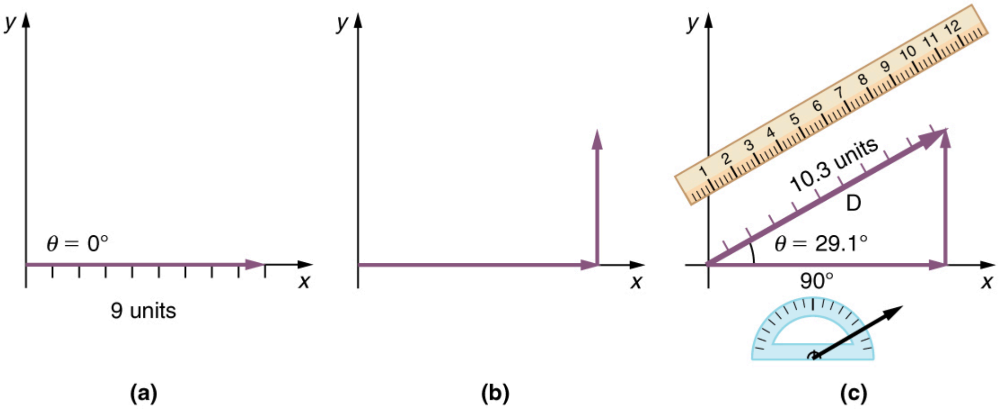


FIGURE 3.10 **Head-to-Tail Method:** The head-to-tail method of graphically adding vectors is illustrated for the two displacements of the person walking in a city considered in Figure 3.8. (a) Draw a vector representing the displacement to the east. (b) Draw a vector representing the displacement to the north. The tail of this vector should originate from the head of the first, east-pointing vector. (c) Draw a line from the tail of the east-pointing vector to the head of the north-pointing vector to form the sum or **resultant vector** $\mathbf{D}$. The length of the arrow $\mathbf{D}$ is proportional to the vector's magnitude and is measured to be 10.3 units . Its direction, described as the angle with respect to the east (or horizontal axis) θ is measured with a protractor to be 29.1°.

Step 1. *Draw an arrow to represent the first vector (9 blocks to the east) using a ruler and protractor.*

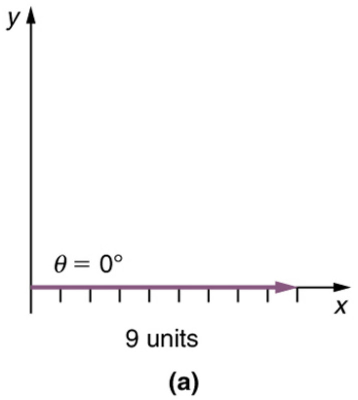


FIGURE 3.11

Step 2. Now draw an arrow to represent the second vector (5 blocks to the north). *Place the tail of the second vector at the head of the first vector.*

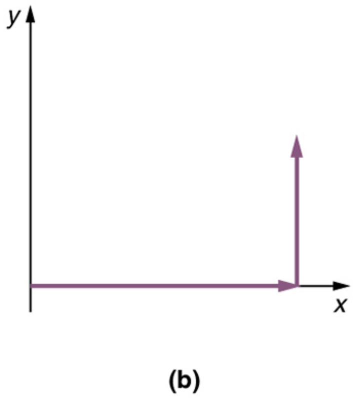


FIGURE 3.12

Step 3. *If there are more than two vectors, continue this process for each vector to be added. Note that in our example, we have only two vectors, so we have finished placing arrows tip to tail.*

Step 4. *Draw an arrow from the tail of the first vector to the head of the last vector*. This is the **resultant**, or the sum, of the other vectors.

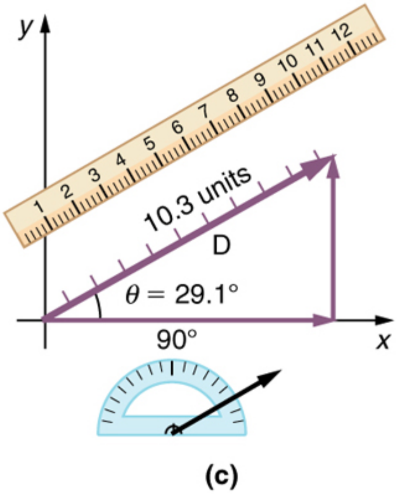


FIGURE 3.13

Step 5. To get the **magnitude** of the resultant, *measure its length with a ruler. (Note that in most calculations, we will use the Pythagorean theorem to determine this length.)*

Step 6. To get the **direction** of the resultant, *measure the angle it makes with the reference frame using a protractor. (Note that in most calculations, we will use trigonometric relationships to determine this angle.)*

The graphical addition of vectors is limited in accuracy only by the precision with which the drawings can be made and the precision of the measuring tools. It is valid for any number of vectors.

EXAMPLE 3.1

Adding Vectors Graphically Using the Head-to-Tail Method: A Woman Takes a Walk

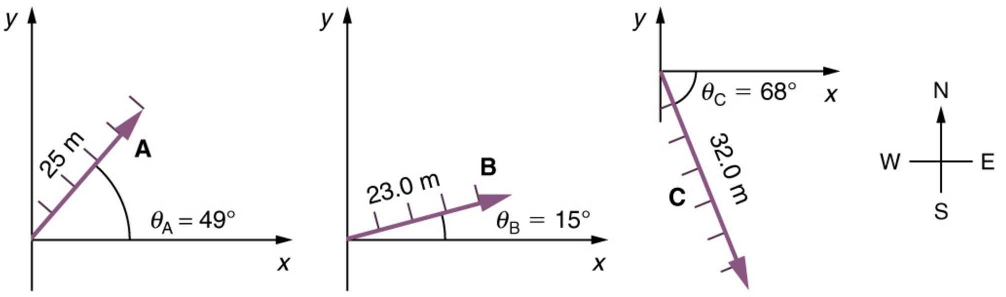
Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, she walks 25.0 m in a direction 49.0° north of east. Then, she walks 23.0 m heading 15.0° north of east. Finally, she turns and walks 32.0 m in a direction 68.0° south of east.

Strategy

Represent each displacement vector graphically with an arrow, labeling the first $\mathbf{A}$, the second $\mathbf{B}$, and the third $\mathbf{C}$, making the lengths proportional to the distance and the directions as specified relative to an east-west line. The head-to-tail method outlined above will give a way to determine the magnitude and direction of the resultant displacement, denoted $\mathbf{R}$.

Solution

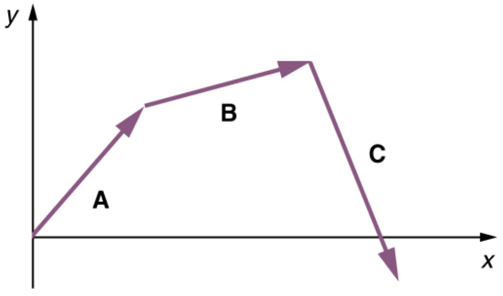
(1) Draw the three displacement vectors.



(a)

FIGURE 3.14

(2) Place the vectors head to tail retaining both their initial magnitude and direction.



(a)

FIGURE 3.15

(3) Draw the resultant vector, $\mathbf{R}$.

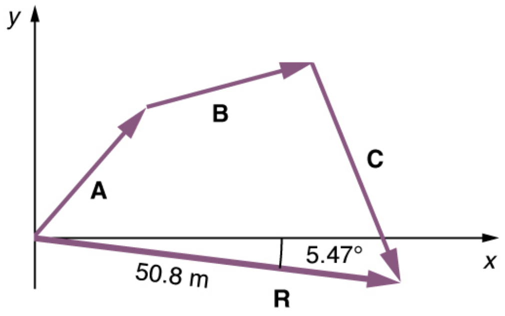


FIGURE 3.16

(4) Use a ruler to measure the magnitude of $\mathbf{R}$, and a protractor to measure the direction of $\mathbf{R}$. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since the resultant vector is south of the eastward pointing axis, we flip the protractor upside down and measure the angle between the eastward axis and the vector.

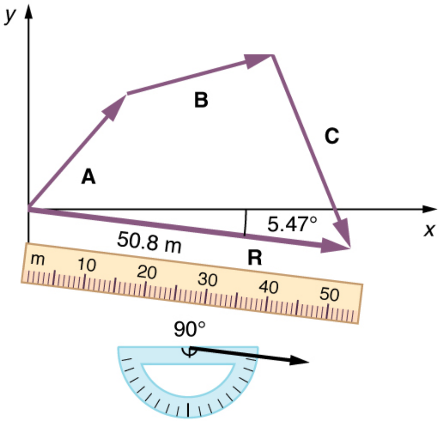


FIGURE 3.17

In this case, the total displacement $\mathbf{R}$ is seen to have a magnitude of 50.8 m and to lie in a direction 5.47° south of east. By using its magnitude and direction, this vector can be expressed as $R = 50.8\text{ m}$ and $\theta = 5.47^{\circ}$ south of east.

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that the resultant is independent of the order in which the vectors are added. Therefore, we could add the vectors in any order as illustrated in Figure 3.18 and we will still get the same solution.

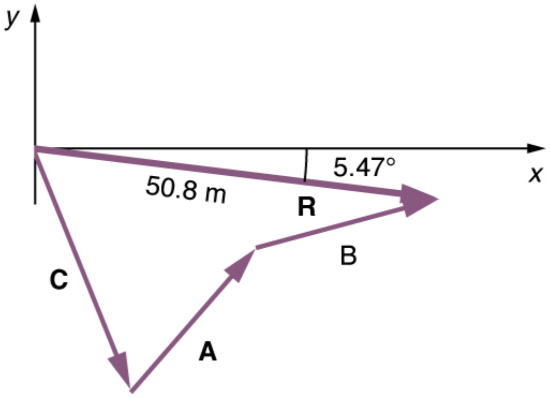


FIGURE 3.18

Here, we see that when the same vectors are added in a different order, the result is the same. This characteristic is true in every case and is an important characteristic of vectors. Vector addition is **commutative**. Vectors can be added in any order.

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}. \tag{3.1}$$

(This is true for the addition of ordinary numbers as well—you get the same result whether you add $2 + 3$ or $3 + 2$, for example).

Vector Subtraction

Vector subtraction is a straightforward extension of vector addition. To define subtraction (say we want to subtract $\mathbf{B}$ from $\mathbf{A}$, written $\mathbf{A} – \mathbf{B}$, we must first define what we mean by subtraction. The *negative* of a vector $\mathbf{B}$ is defined to be $–\mathbf{B}$; that is, graphically *the negative of any vector has the same magnitude but the opposite direction*, as shown in Figure 3.19. In other words, $\mathbf{B}$ has the same length as $–\mathbf{B}$, but points in the opposite direction. Essentially, we just flip the vector so it points in the opposite direction.

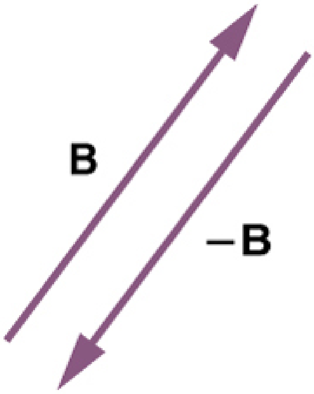


FIGURE 3.19 The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So $\mathbf{B}$ is the negative of $–\mathbf{B}$; it has the same length but opposite direction.

The *subtraction* of vector $\mathbf{B}$ from vector $\mathbf{A}$ is then simply defined to be the addition of $–\mathbf{B}$ to $\mathbf{A}$. Note that vector subtraction is the addition of a negative vector. The order of subtraction does not affect the results.

$$\mathbf{A} – \mathbf{B} = \mathbf{A} + (–\mathbf{B}). \tag{3.2}$$

This is analogous to the subtraction of scalars (where, for example, $5 – 2 = 5 + (–2)$). Again, the result is independent of the order in which the subtraction is made. When vectors are subtracted graphically, the techniques outlined above are used, as the following example illustrates.

EXAMPLE 3.2

Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? Compare this location with the location of the dock.

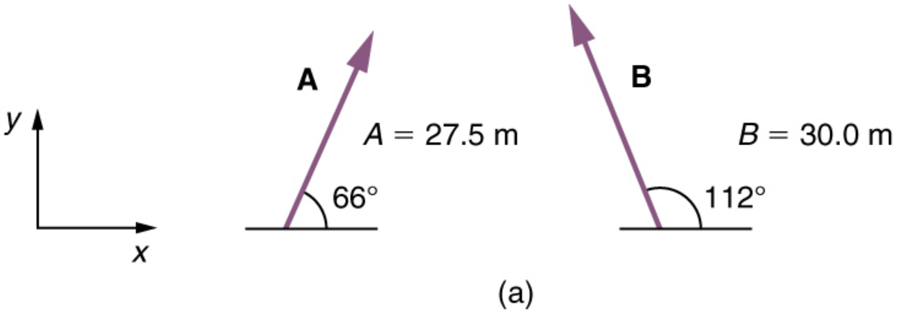


FIGURE 3.20

Strategy

We can represent the first leg of the trip with a vector $\mathbf{A}$, and the second leg of the trip with a vector $\mathbf{B}$. The dock is located at a location $\mathbf{A}+\mathbf{B}$. If the woman mistakenly travels in the *opposite* direction for the second leg of the journey, she will travel a distance B (30.0 m) in the direction $180^{\circ}-112^{\circ}=68^{\circ}$ south of east. We represent this as $-\mathbf{B}$, as shown below. The vector $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction. Thus, she will end up at a location $\mathbf{A}+(-\mathbf{B})$, or $\mathbf{A}-\mathbf{B}$.

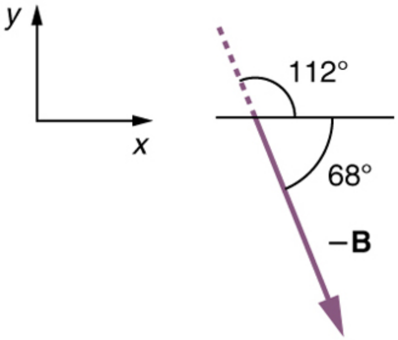


FIGURE 3.21

We will perform vector addition to compare the location of the dock, $\mathbf{A}+\mathbf{B}$, with the location at which the woman mistakenly arrives, $\mathbf{A}+(-\mathbf{B})$.

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors $\mathbf{A}$ and $-\mathbf{B}$.

(2) Place the vectors head to tail.

(3) Draw the resultant vector $\mathbf{R}$.

(4) Use a ruler and protractor to measure the magnitude and direction of $\mathbf{R}$.

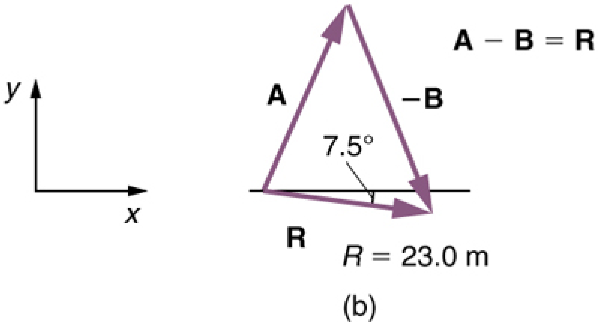


FIGURE 3.22

In this case, $R=23.0\text{ m}$ and $\theta=7.5^{\circ}$ south of east.

(5) To determine the location of the dock, we repeat this method to add vectors $\mathbf{A}$ and $\mathbf{B}$. We obtain the resultant vector $\mathbf{R}$':

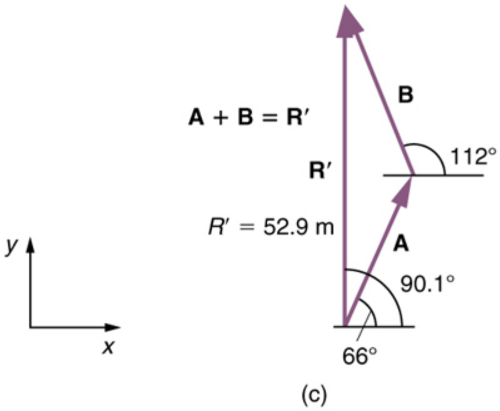


FIGURE 3.23

In this case $R = 52.9\text{ m}$ and $\theta = 90.1^{\text{o}}$ north of east.

We can see that the woman will end up a significant distance from the dock if she travels in the opposite direction for the second leg of the trip.

Discussion

Because subtraction of a vector is the same as addition of a vector with the opposite direction, the graphical method of subtracting vectors works the same as for addition.

Multiplication of Vectors and Scalars

If we decided to walk three times as far on the first leg of the trip considered in the preceding example, then we would walk $3 \times 27.5\text{ m}$, or 82.5 m, in a direction 66.0^{o} north of east. This is an example of multiplying a vector by a positive **scalar**. Notice that the magnitude changes, but the direction stays the same.

If the scalar is negative, then multiplying a vector by it changes the vector's magnitude and gives the new vector the *opposite* direction. For example, if you multiply by –2, the magnitude doubles but the direction changes. We can summarize these rules in the following way: When vector $\mathbf{A}$ is multiplied by a scalar c,

- the magnitude of the vector becomes the absolute value of cA,
- if c is positive, the direction of the vector does not change,
- if c is negative, the direction is reversed.

In our case, $c = 3$ and $A = 27.5\text{ m}$. Vectors are multiplied by scalars in many situations. Note that division is the inverse of multiplication. For example, dividing by 2 is the same as multiplying by the value (1/2). The rules for multiplication of vectors by scalars are the same for division; simply treat the divisor as a scalar between 0 and 1.

Resolving a Vector into Components

In the examples above, we have been adding vectors to determine the resultant vector. In many cases, however, we will need to do the opposite. We will need to take a single vector and find what other vectors added together produce it. In most cases, this involves determining the perpendicular **components** of a single vector, for example the *x- and y*-components, or the north-south and east-west components.

For example, we may know that the total displacement of a person walking in a city is 10.3 blocks in a direction 29.0^{o} north of east and want to find out how many blocks east and north had to be walked. This method is called *finding the components (or parts)* of the displacement in the east and north directions, and it is the inverse of the process followed to find the total displacement. It is one example of finding the components of a vector. There are many applications in physics where this is a useful thing to do. We will see this soon in Projectile Motion, and much more when we cover **forces** in Dynamics: Newton's Laws of Motion. Most of these involve finding components along perpendicular axes (such as north and east), so that right triangles are involved. The analytical techniques presented in Vector Addition and Subtraction: Analytical Methods are ideal for finding vector components.

PHET EXPLORATIONS

Maze Game

Learn about position, velocity, and acceleration in the "Arena of Pain". Use the green arrow to move the ball. Add more walls to the arena to make the game more difficult. Try to make a goal as fast as you can.

Click to view content (https://openstax.org/l/28mazegame).

3.3 Vector Addition and Subtraction: Analytical Methods

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand the rules of vector addition and subtraction using analytical methods.
- Apply analytical methods to determine vertical and horizontal component vectors.
- Apply analytical methods to determine the magnitude and direction of a resultant vector.

Analytical methods of vector addition and subtraction employ geometry and simple trigonometry rather than the ruler and protractor of graphical methods. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization. However, analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Analytical methods are limited only by the accuracy and precision with which physical quantities are known.

Resolving a Vector into Perpendicular Components

Analytical techniques and right triangles go hand-in-hand in physics because (among other things) motions along perpendicular directions are independent. We very often need to separate a vector into perpendicular components. For example, given a vector like $\mathbf{A}$ in Figure 3.24, we may wish to find which two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it.

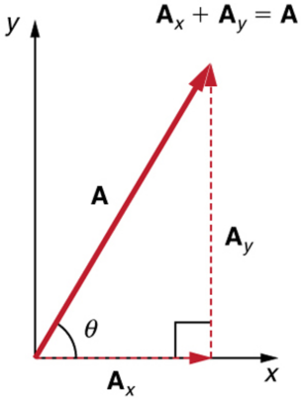


FIGURE 3.24 The vector $\mathbf{A}$, with its tail at the origin of an *x*, *y*-coordinate system, is shown together with its *x*- and *y*-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the *x*- and *y*-axes. The three vectors $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$ form a right triangle:

$$\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}. \tag{3.3}$$

Note that this relationship between vector components and the resultant vector holds only for vector quantities (which include both magnitude and direction). The relationship does not apply for the magnitudes alone. For example, if $\mathbf{A}_x = 3\text{ m}$ east, $\mathbf{A}_y = 4\text{ m}$ north, and $\mathbf{A} = 5\text{ m}$ north-east, then it is true that the vectors $\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$. However, it is *not* true that the sum of the magnitudes of the vectors is also equal. That is,

$$3\text{ m}+4\text{ m}\neq 5\text{ m} \tag{3.4}$$

Thus,

$$A_x+A_y\neq A \tag{3.5}$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its *x*- and *y*-components, we use the following relationships for a right triangle.

$$A_x=A\cos\theta \tag{3.6}$$

and

$$A_y=A\sin\theta. \tag{3.7}$$

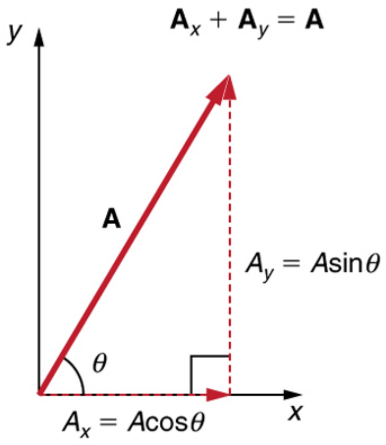


FIGURE 3.25 The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x=A\cos\theta$ and $A_y=A\sin\theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city considered in Kinematics in Two Dimensions: An Introduction and Vector Addition and Subtraction: Graphical Methods.

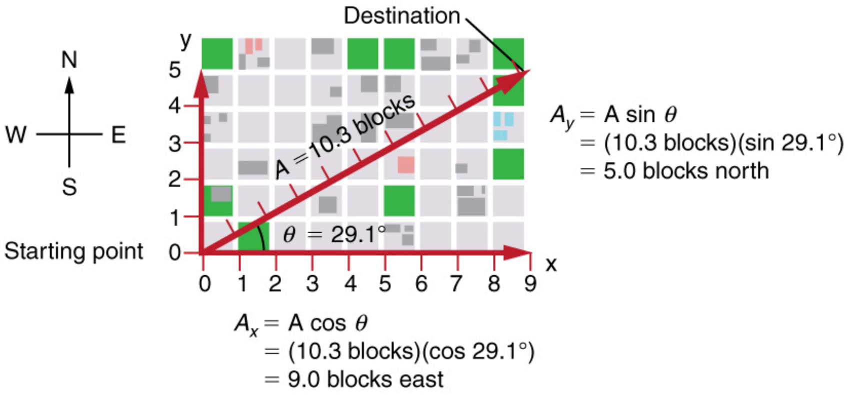


FIGURE 3.26 We can use the relationships $A_x=A\cos\theta$ and $A_y=A\sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then $A=10.3$ blocks and $\theta=29.1^\circ$, so that

$$A_x=A\cos\theta=(10.3\text{ blocks})(\cos 29.1^\circ)=9.0\text{ blocks} \tag{3.8}$$

$$A_y=A\sin\theta=\left(10.3\text{ blocks}\right)\left(\sin 29.1^\circ\right)=5.0\text{ blocks}. \tag{3.9}$$

Calculating a Resultant Vector

If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then $\mathbf{A}$ can also be found analytically. To find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, relative to the *x*-axis, we use the following relationships:

$$A = \sqrt{{A_x}^2 + {A_y}^2} \qquad \mathbf{3.10}$$

$$\theta = \tan^{-1}(A_y/A_x). \qquad \mathbf{3.11}$$

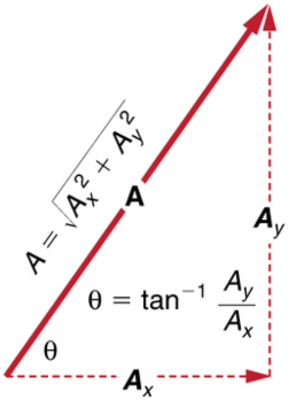


FIGURE 3.27 The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components A_x and A_y have been determined.

Note that the equation $A = \sqrt{A_x^2 + A_y^2}$ is just the Pythagorean theorem relating the legs of a right triangle to the length of the hypotenuse. For example, if A_x and A_y are 9 and 5 blocks, respectively, then $A = \sqrt{9^2{+}5^2}{=}10.3$ blocks, again consistent with the example of the person walking in a city. Finally, the direction is $\theta = \tan^{-1}(5/9){=}29.1^{\mathrm{o}}$, as before.

Determining Vectors and Vector Components with Analytical Methods

Equations $A_x = A \cos\theta$ and $A_y = A \sin\theta$ are used to find the perpendicular components of a vector—that is, to go from A and θ to A_x and A_y. Equations $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ are used to find a vector from its perpendicular components—that is, to go from A_x and A_y to A and θ. Both processes are crucial to analytical methods of vector addition and subtraction.

Adding Vectors Using Analytical Methods

To see how to add vectors using perpendicular components, consider Figure 3.28, in which the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$.

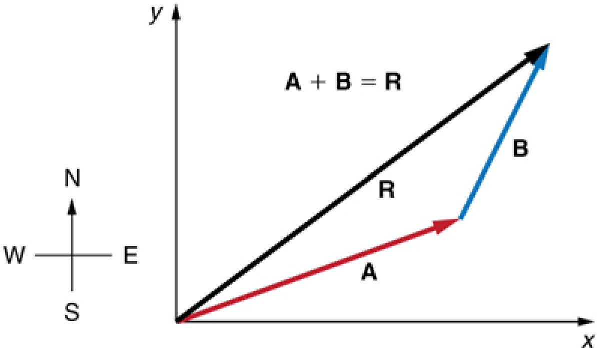


FIGURE 3.28 Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. In particular, the person could have walked first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $A = \sqrt{{A_x}^2 + {A_y}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$. When you use the analytical method of vector addition, you can determine the components or the magnitude and direction of a vector.

Step 1. *Identify the x- and y-axes that will be used in the problem. Then, find the components of each vector to be added along the chosen perpendicular axes.* Use the equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to find the components. In Figure 3.29, these components are A_x, A_y, B_x, and B_y. The angles that vectors $\mathbf{A}$ and $\mathbf{B}$ make with the *x*-axis are θ_{A} and θ_{B}, respectively.

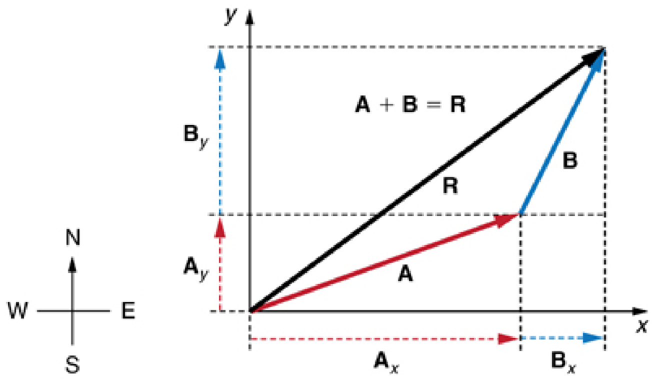


FIGURE 3.29 To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.

Step 2. *Find the components of the resultant along each axis by adding the components of the individual vectors along that axis.* That is, as shown in Figure 3.30,

$$R_x = A_x + B_x \tag{3.12}$$

and

$$R_y = A_y + B_y. \tag{3.13}$$

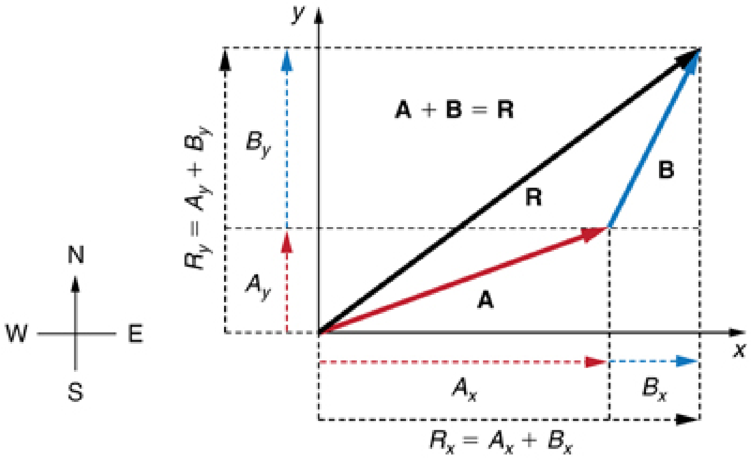


FIGURE 3.30 The magnitude of the vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude R_x of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude R_y of the resultant vector in the vertical direction.

Components along the same axis, say the *x*-axis, are vectors along the same line and, thus, can be added to one another like ordinary numbers. The same is true for components along the *y*-axis. (For example, a 9-block eastward walk could be taken in two legs, the first 3 blocks east and the second 6 blocks east, for a total of 9, because they are along the same direction.) So resolving vectors into components along common axes makes it easier to add them. Now that the components of $\mathbf{R}$ are known, its magnitude and direction can be found.

Step 3. *To get the magnitude R of the resultant, use the Pythagorean theorem:*

$$R = \sqrt{R_x^2 + R_y^2}. \tag{3.14}$$

Step 4. *To get the direction of the resultant relative to the x-axis:*

$$\theta = \tan^{-1}(R_y/R_x). \tag{3.15}$$

The following example illustrates this technique for adding vectors using perpendicular components.

EXAMPLE 3.3

Adding Vectors Using Analytical Methods

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in Figure 3.31, using perpendicular components along the *x*- and *y*-axes. The *x*- and *y*-axes are along the east–west and north–south directions, respectively. Vector $\mathbf{A}$ represents the first leg of a walk in which a person walks 53.0 m in a direction 20.0° north of east. Vector $\mathbf{B}$ represents the second leg, a displacement of 34.0 m in a direction 63.0° north of east.

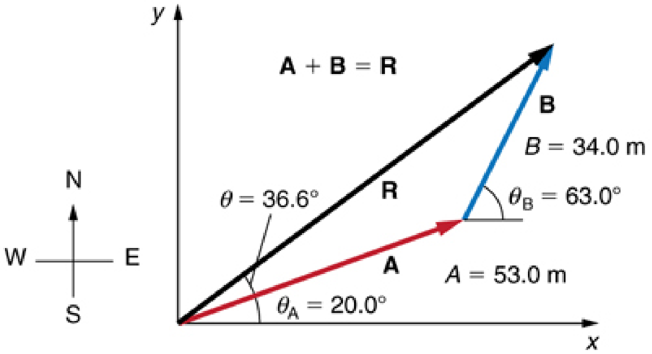


FIGURE 3.31 Vector $\mathbf{A}$ has magnitude 53.0 m and direction 20.0° north of the *x*-axis. Vector $\mathbf{B}$ has magnitude 34.0 m and direction 63.0° north of the *x*-axis. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. Once found, they are combined to produce the resultant.

Solution

Following the method outlined above, we first find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes. Note that $A = 53.0\text{ m}$, $\theta_\text{A} = 20.0^\circ$, $B = 34.0\text{ m}$, and $\theta_\text{B} = 63.0^\circ$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$\begin{aligned} A_x &= A\cos\theta_\text{A} = (53.0\text{ m})(\cos 20.0^\circ) \\ &= (53.0\text{ m})(0.940) = 49.8\text{ m} \end{aligned} \tag{3.16}$$

and

$$\begin{aligned} B_x &= B\cos\theta_\text{B} = (34.0\text{ m})(\cos 63.0^\circ) \\ &= (34.0\text{ m})(0.454) = 15.4\text{ m}. \end{aligned} \tag{3.17}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_\text{A}$:

$$\begin{aligned} A_y &= A\sin\theta_\text{A} = (53.0\text{ m})(\sin 20.0^\circ) \\ &= (53.0\text{ m})(0.342) = 18.1\text{ m} \end{aligned} \tag{3.18}$$

and

$$\begin{aligned} B_y &= B\sin\theta_\text{B} = (34.0\text{ m})(\sin 63.0^\circ) \\ &= (34.0\text{ m})(0.891) = 30.3\text{ m}. \end{aligned} \tag{3.19}$$

The *x*- and *y*-components of the resultant are thus

$$R_x = A_x + B_x = 49.8\text{ m} + 15.4\text{ m} = 65.2\text{ m} \tag{3.20}$$

and

$$R_y = A_y + B_y = 18.1\text{ m} + 30.3\text{ m} = 48.4\text{ m}. \tag{3.21}$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2}\text{ m} \tag{3.22}$$

so that

$$R = 81.2\text{ m}. \tag{3.23}$$

Finally, we find the direction of the resultant:

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2). \tag{3.24}$$

Thus,

$$\theta = \tan^{-1}(0.742) = 36.6^\circ. \tag{3.25}$$

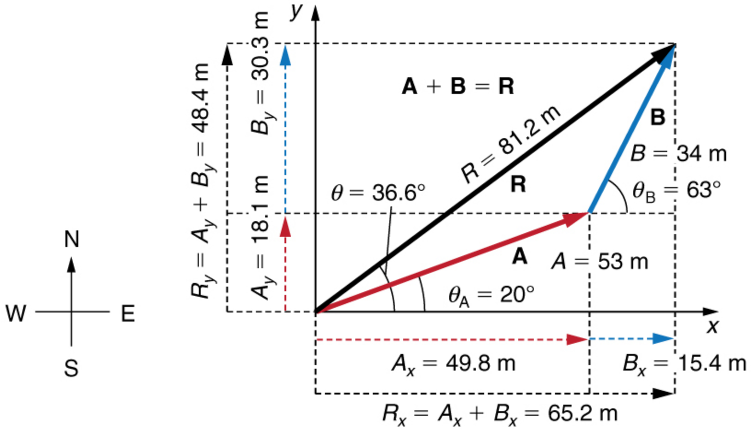


FIGURE 3.32 Using analytical methods, we see that the magnitude of $\mathbf{R}$ is 81.2 m and its direction is 36.6° north of east.

Discussion

This example illustrates the addition of vectors using perpendicular components. Vector subtraction using perpendicular components is very similar—it is just the addition of a negative vector.

Subtraction of vectors is accomplished by the addition of a negative vector. That is, $\mathbf{A} - \mathbf{B} \equiv \mathbf{A} + (-\mathbf{B})$. Thus, *the method for the subtraction of vectors using perpendicular components is identical to that for addition*. The components of $-\mathbf{B}$ are the negatives of the components of $\mathbf{B}$. The *x*- and *y*-components of the resultant $\mathbf{A} - \mathbf{B} = \mathbf{R}$ are thus

$$R_x = A_x + (-B_x) \tag{3.26}$$

and

$$R_y = A_y + \left(-B_y\right) \tag{3.27}$$

and the rest of the method outlined above is identical to that for addition. (See Figure 3.33.)

Analyzing vectors using perpendicular components is very useful in many areas of physics, because perpendicular quantities are often independent of one another. The next module, Projectile Motion, is one of many in which using perpendicular components helps make the picture clear and simplifies the physics.

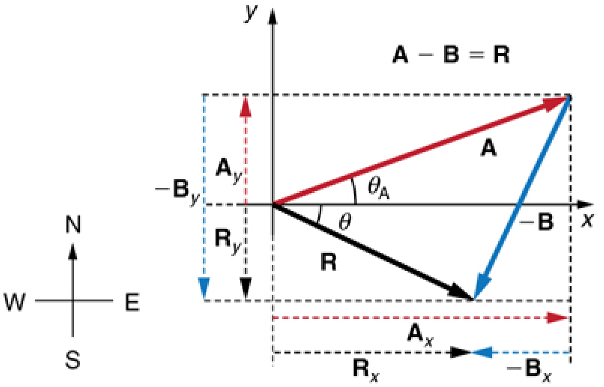


FIGURE 3.33 The subtraction of the two vectors shown in Figure 3.28. The components of $\mathbf{-B}$ are the negatives of the components of $\mathbf{B}$. The method of subtraction is the same as that for addition.

PHET EXPLORATIONS

Vector Addition

Learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

Click to view content (https://openstax.org/books/college-physics-2e/pages/3-3-vector-addition-and-subtraction-analytical-methods)

3.4 Projectile Motion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory.
- Determine the location and velocity of a projectile at different points in its trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects, as covered in Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to remember here is that *motions along perpendicular axes are independent* and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. Figure 3.34 illustrates the notation for displacement, where $\mathbf{s}$ is defined to be the total displacement and $\mathbf{x}$ and $\mathbf{y}$ are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are *s*, *x*, and *y*. (Note that in the last section we used the notation $\mathbf{A}$ to represent a vector with components $\mathbf{A}_x$ and $\mathbf{A}_y$. If we continued this format, we would call displacement $\mathbf{s}$ with components $\mathbf{s}_x$ and $\mathbf{s}_y$. However, to simplify the notation, we will simply represent the component vectors as $\mathbf{x}$ and $\mathbf{y}$.)

Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the *x*- and *y*-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: $a_y = -g = -9.80 \text{ m/s}^2$. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, $a_x = 0$. Both accelerations are constant, so the kinematic equations can be used.

Review of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{3.28}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{3.29}$$

$$v = v_0 + at \tag{3.30}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{3.31}$$

$$v^2 = v_0^2 + 2a(x - x_0). \tag{3.32}$$

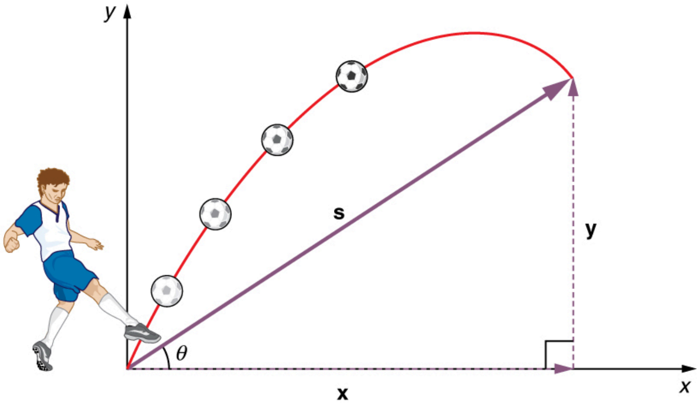


FIGURE 3.34 The total displacement $\mathbf{s}$ of a soccer ball at a point along its path. The vector $\mathbf{s}$ has components $\mathbf{x}$ and $\mathbf{y}$ along the horizontal and vertical axes. Its magnitude is s, and it makes an angle θ with the horizontal.

Given these assumptions, the following steps are then used to analyze projectile motion:

Step 1. *Resolve or break the motion into horizontal and vertical components along the x- and y-axes.* These axes are perpendicular, so $A_x = A \cos\theta$ and $A_y = A \sin\theta$ are used. The magnitude of the components of displacement $\mathbf{s}$ along these axes are x and y. The magnitudes of the components of the velocity $\mathbf{v}$ are $v_x = v \cos\theta$ and $v_y = v \sin\theta$, where v is the magnitude of the velocity and θ is its direction, as shown in Figure 3.35. Initial values are denoted with a subscript 0, as usual.

Step 2. *Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical.* The kinematic equations for horizontal and vertical motion take the following forms:

$$\text{Horizontal Motion}(a_x = 0) \tag{3.33}$$

$$x = x_0 + v_x t \tag{3.34}$$

$$v_x = v_{0x} = v_x = \text{velocity is a constant.} \tag{3.35}$$

$$\begin{gathered}\text{Vertical Motion}\\ \text{(assuming positive is up}\\ a_y = -g = -9.80\text{m/s}^2)\end{gathered} \tag{3.36}$$

$$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t \tag{3.37}$$

$$v_y = v_{0y} - gt \tag{3.38}$$

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \tag{3.39}$$

$$v_y^2 = v_{0y}^2 - 2g(y - y_0). \tag{3.40}$$

Step 3. *Solve for the unknowns in the two separate motions—one horizontal and one vertical.* Note that the only common variable between the motions is time t. The problem solving procedures here are the same as for one-dimensional **kinematics** and are illustrated in the solved examples below.

Step 4. *Recombine the two motions to find the total displacement* $\mathbf{s}$ *and velocity* $\mathbf{v}$. Because the x - and y -motions are perpendicular, we determine these vectors by using the techniques outlined in the Vector Addition and Subtraction: Analytical Methods and employing $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ in the following form, where θ is the direction of the displacement $\mathbf{s}$ and θ_v is the direction of the velocity $\mathbf{v}$:

Total displacement and velocity

$$s = \sqrt{x^2 + y^2} \tag{3.41}$$

$$\theta = \tan^{-1}(y/x) \tag{3.42}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.43}$$

$$\theta_v = \tan^{-1}(v_y/v_x). \tag{3.44}$$

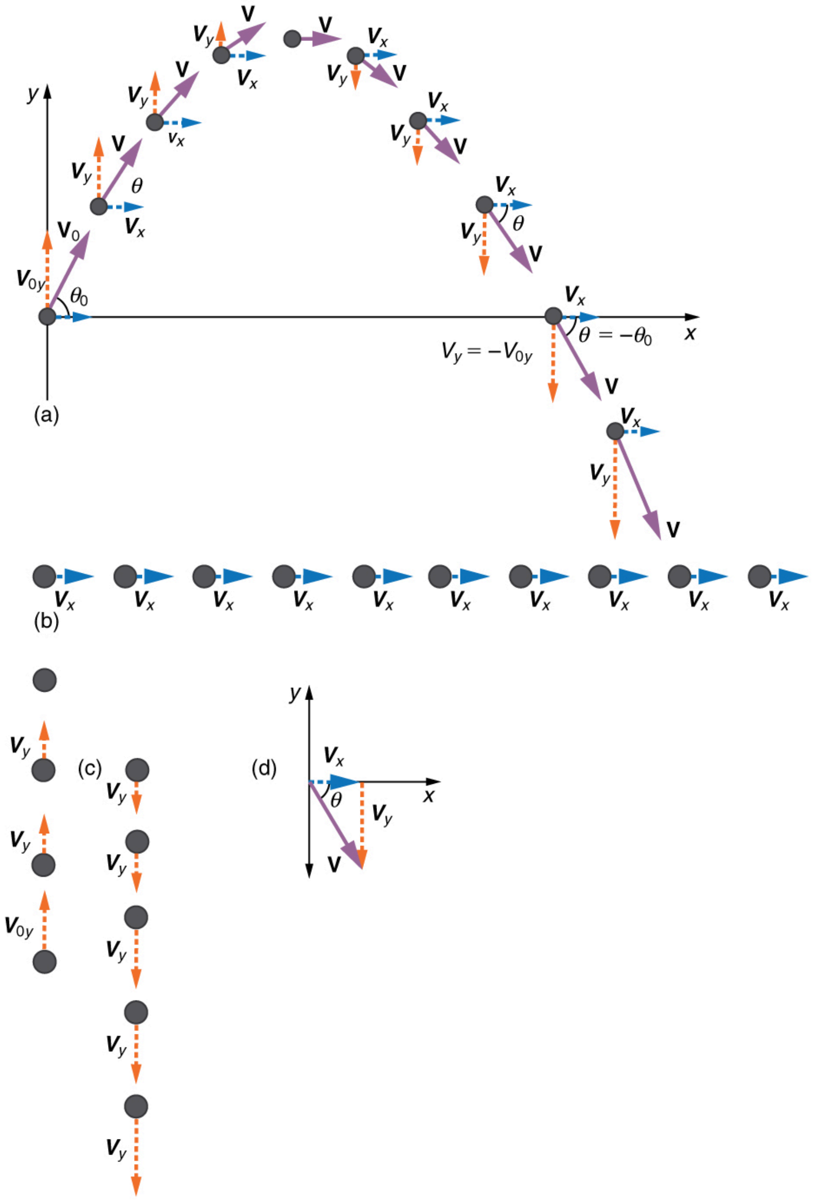


FIGURE 3.35 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The *x* - and *y* -motions are recombined to give the total velocity at any given point on the trajectory.

EXAMPLE 3.4

A Fireworks Projectile Explodes High and Away

During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal, as illustrated in Figure 3.36. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

Strategy

Because air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which $a_x = 0$ and $a_y = -g$. We can then define x_0 and y_0 to be zero and solve for the desired quantities.

Solution for (a)

By “height” we mean the altitude or vertical position y above the starting point. The highest point in any trajectory, called the apex, is reached when $v_y = 0$. Since we know the initial and final velocities as well as the initial position, we use the following equation to find y:

$$v_y^2 = v_{0y}^2 - 2g(y - y_0). \tag{3.45}$$

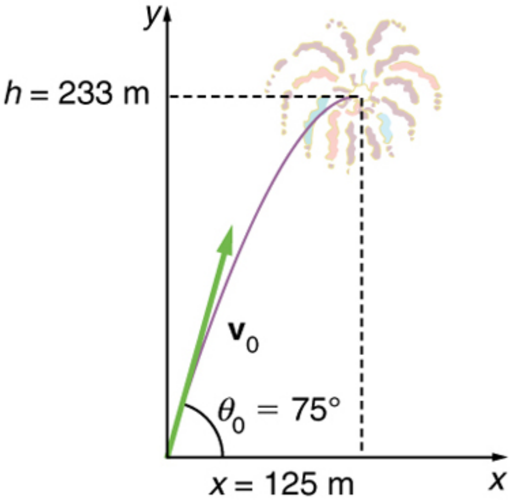


FIGURE 3.36 The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.

Because y_0 and v_y are both zero, the equation simplifies to

$$0 = v_{0y}^2 - 2gy. \tag{3.46}$$

Solving for y gives

$$y = \frac{v_{0y}^2}{2g}. \tag{3.47}$$

Now we must find v_{0y}, the component of the initial velocity in the y-direction. It is given by $v_{0y} = v_0 \sin \theta$, where v_{0y} is the initial velocity of 70.0 m/s, and $\theta_0 = 75.0°$ is the initial angle. Thus,

$$v_{0y} = v_0 \sin \theta_0 = (70.0 \text{ m/s})(\sin 75°) = 67.6 \text{ m/s}. \tag{3.48}$$

and y is

$$y = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)}, \tag{3.49}$$

so that

$$y = 233\text{m}. \tag{3.50}$$

Discussion for (a)

Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.

Solution for (b)

As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$. Because y_0 is zero, this equation reduces to simply

$$y = \frac{1}{2}(v_{0y} + v_y)t. \tag{3.51}$$

Note that the final vertical velocity, v_y, at the highest point is zero. Thus,

$$\begin{aligned} t &= \frac{2y}{(v_{0\text{y}}+v_y)} = \frac{2(233\ \text{m})}{(67.6\ \text{m/s})} \\ &= 6.90\ \text{s}. \end{aligned} \tag{3.52}$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, and solving the quadratic equation for t.)

Solution for (c)

Because air resistance is negligible, $a_x = 0$ and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by $x = x_0 + v_x t$, where x_0 is equal to zero:

$$x = v_x t, \tag{3.53}$$

where v_x is the x-component of the velocity, which is given by $v_x = v_0 \cos \theta_0$. Now,

$$v_x = v_0 \cos \theta_0 = (70.0\ \text{m/s})(\cos 75.0^\text{o}) = 18.1\ \text{m/s}. \tag{3.54}$$

The time t for both motions is the same, and so x is

$$x = (18.1\ \text{m/s})(6.90\ \text{s}) = 125\ \text{m}. \tag{3.55}$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below.

In solving part (a) of the preceding example, the expression we found for y is valid for any projectile motion where air resistance is negligible. Call the maximum height $y = h$; then,

$$h = \frac{v_{0y}^2}{2g}. \tag{3.56}$$

This equation defines the *maximum height of a projectile* and depends only on the vertical component of the initial velocity.

Defining a Coordinate System

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the x and y positions. Often, it is convenient to choose the initial position of the object as the origin such that $x_0 = 0$ and $y_0 = 0$. It is also important to define the positive and negative directions in the x and y directions. Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion. When this is the case, the vertical acceleration, g, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, g takes a positive value.

EXAMPLE 3.5

Calculating Projectile Motion: Hot Rock Projectile

Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0° above the horizontal, as shown in Figure 3.37. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact?

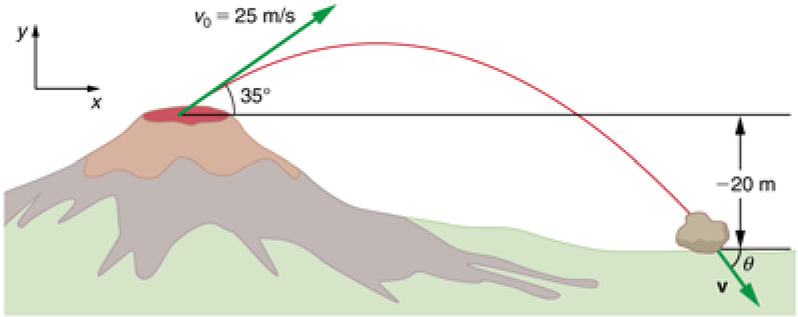


FIGURE 3.37 The trajectory of a rock ejected from the Kilauea volcano.

Strategy

Again, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities. The time a projectile is in the air is governed by its vertical motion alone. We will solve for t first. While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain v and θ_v at the final time t determined in the first part of the example.

Solution for (a)

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2. \tag{3.57}$$

If we take the initial position y_0 to be zero, then the final position is $y = -20.0 \text{ m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from $v_{0y} = v_0 \sin \theta_0 = (25.0 \text{ m/s})(\sin 35.0^{\circ}) = 14.3 \text{ m/s}$. Substituting known values yields

$$-20.0 \text{ m} = (14.3 \text{ m/s})t - \left(4.90 \text{ m/s}^2\right)t^2. \tag{3.58}$$

Rearranging terms gives a quadratic equation in t:

$$\left(4.90 \text{ m/s}^2\right)t^2 - (14.3 \text{ m/s})t - (20.0 \text{ m}) = 0. \tag{3.59}$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c = -20.0$. Its solutions are given by the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{3.60}$$

This equation yields two solutions: $t = 3.96$ and $t = -1.03$. (It is left as an exercise for the reader to verify these solutions.) The time is $t = 3.96\text{ s}$ or -1.03 s. The negative value of time implies an event before the start of motion, and so we discard it. Thus,

$$t = 3.96\text{ s}. \tag{3.61}$$

Discussion for (a)

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Solution for (b)

From the information now in hand, we can find the final horizontal and vertical velocities v_x and v_y and combine them to find the total velocity v and the angle θ_0 it makes with the horizontal. Of course, v_x is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:

$$v_x = v_0 \cos \theta_0 = (25.0\text{ m/s})(\cos 35^\circ) = 20.5\text{ m/s}. \tag{3.62}$$

The final vertical velocity is given by the following equation:

$$v_y = v_{0y} - gt, \tag{3.63}$$

where v_{0y} was found in part (a) to be 14.3 m/s. Thus,

$$v_y = 14.3\text{ m/s} - (9.80\text{ m/s}^2)(3.96\text{ s}) \tag{3.64}$$

so that

$$v_y = -24.5\text{ m/s}. \tag{3.65}$$

To find the magnitude of the final velocity v we combine its perpendicular components, using the following equation:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20.5\text{ m/s})^2 + (-24.5\text{ m/s})^2}, \tag{3.66}$$

which gives

$$v = 31.9\text{ m/s}. \tag{3.67}$$

The direction θ_v is found from the equation:

$$\theta_v = \tan^{-1}(v_y/v_x) \tag{3.68}$$

so that

$$\theta_v = \tan^{-1}(-24.5/20.5) = \tan^{-1}(-1.19). \tag{3.69}$$

Thus,

$$\theta_v = -50.1^\circ. \tag{3.70}$$

Discussion for (b)

The negative angle means that the velocity is 50.1° below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure 3.37.)

One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define **range** to be the horizontal distance R traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

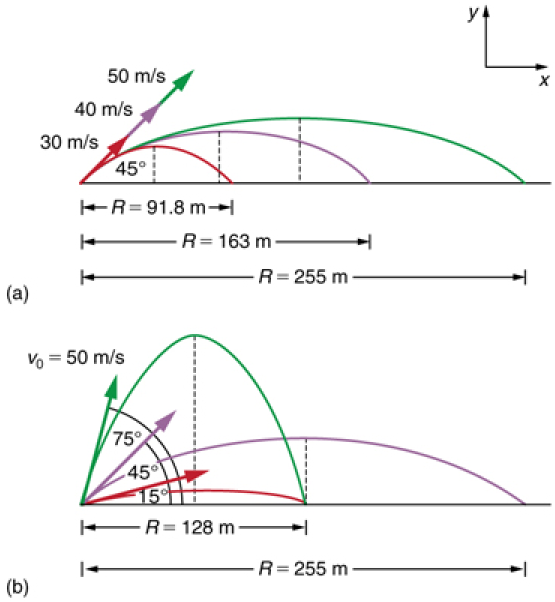


FIGURE 3.38 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for $15°$ and $75°$, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in Figure 3.38(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in Figure 3.38(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45°$. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately $38°$. Interestingly, for every initial angle except $45°$, there are two angles that give the same range—the sum of those angles is $90°$. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}, \tag{3.71}$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure 3.39.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

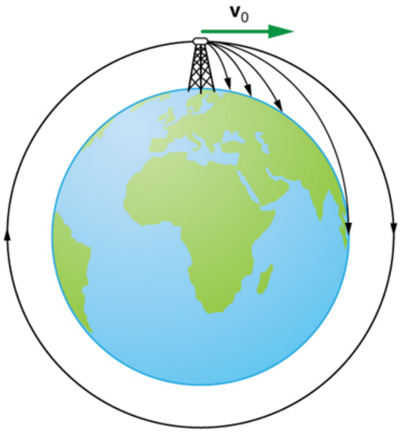


FIGURE 3.39 Hypothetical projectile to satellite. From this theoretical tower, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

PHET EXPLORATIONS

Projectile Motion

Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target.

Click to view content (https://openstax.org/books/college-physics-2e/pages/3-4-projectile-motion)

3.5 Addition of Velocities

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Apply principles of vector addition to determine relative velocity.
- Explain the significance of the observer in the measurement of velocity.

Relative Velocity

If a person rows a boat across a rapidly flowing river and tries to head directly for the other shore, the boat instead moves *diagonally* relative to the shore, as in Figure 3.40. The boat does not move in the direction in which it is pointed. The reason, of course, is that the river carries the boat downstream. Similarly, if a small airplane flies overhead in a strong crosswind, you can sometimes see that the plane is not moving in the direction in which it is pointed, as illustrated in Figure 3.41. The plane is moving straight ahead relative to the air, but the movement of the air mass relative to the ground carries it sideways.

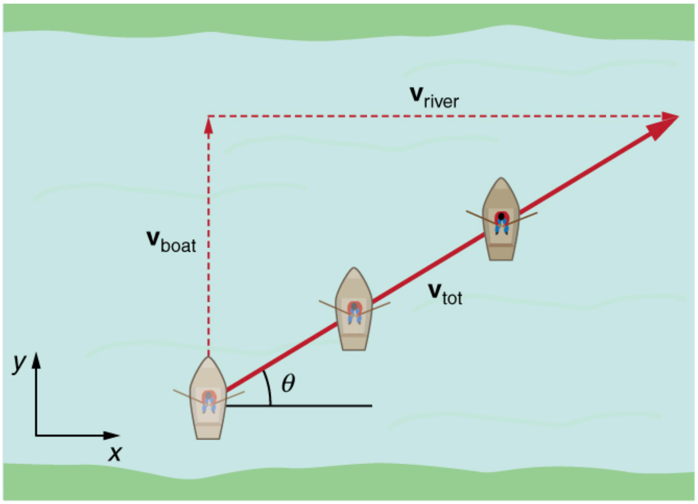


FIGURE 3.40 A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.

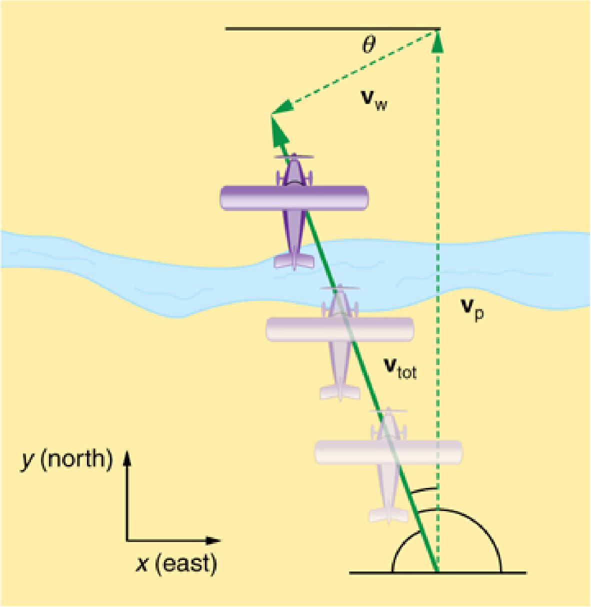


FIGURE 3.41 An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).

In each of these situations, an object has a **velocity** relative to a medium (such as a river) and that medium has a velocity relative to an observer on solid ground. The velocity of the object *relative to the observer* is the sum of these velocity vectors, as indicated in Figure 3.40 and Figure 3.41. These situations are only two of many in which it is useful to add velocities. In this module, we first re-examine how to add velocities and then consider certain aspects of what relative velocity means.

How do we add velocities? Velocity is a vector (it has both magnitude and direction); the rules of **vector addition** discussed in Vector Addition and Subtraction: Graphical Methods and Vector Addition and Subtraction: Analytical Methods apply to the addition of velocities, just as they do for any other vectors. In one-dimensional motion, the addition of velocities is simple—they add like ordinary numbers. For example, if a field hockey player is moving at 5 m/s straight toward the goal and drives the ball in the same direction with a velocity of 30 m/s relative to her body, then the velocity of the ball is 35 m/s relative to the stationary, profusely sweating goalkeeper standing in front of the goal.

In two-dimensional motion, either graphical or analytical techniques can be used to add velocities. We will concentrate on analytical techniques. The following equations give the relationships between the magnitude and direction of velocity (v and θ) and its components (v_x and v_y) along the *x*- and *y*-axes of an appropriately chosen coordinate system:

$$v_x = v \cos \theta \tag{3.72}$$

$$v_y = v \sin \theta \tag{3.73}$$

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.74}$$

$$\theta = \tan^{-1}(v_y/v_x). \tag{3.75}$$

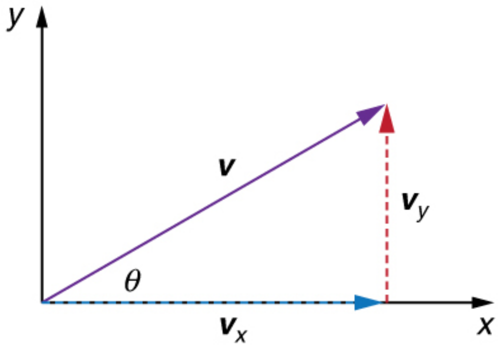


FIGURE 3.42 The velocity, v, of an object traveling at an angle θ to the horizontal axis is the sum of component vectors $\mathbf{v}_x$ and $\mathbf{v}_y$.

These equations are valid for any vectors and are adapted specifically for velocity. The first two equations are used to find the components of a velocity when its magnitude and direction are known. The last two are used to find the magnitude and direction of velocity when its components are known.

Take-Home Experiment: Relative Velocity of a Boat

Fill a bathtub half-full of water. Take a toy boat or some other object that floats in water. Unplug the drain so water starts to drain. Try pushing the boat from one side of the tub to the other and perpendicular to the flow of water. Which way do you need to push the boat so that it ends up immediately opposite? Compare the directions of the flow of water, heading of the boat, and actual velocity of the boat.

EXAMPLE 3.6

Adding Velocities: A Boat on a River

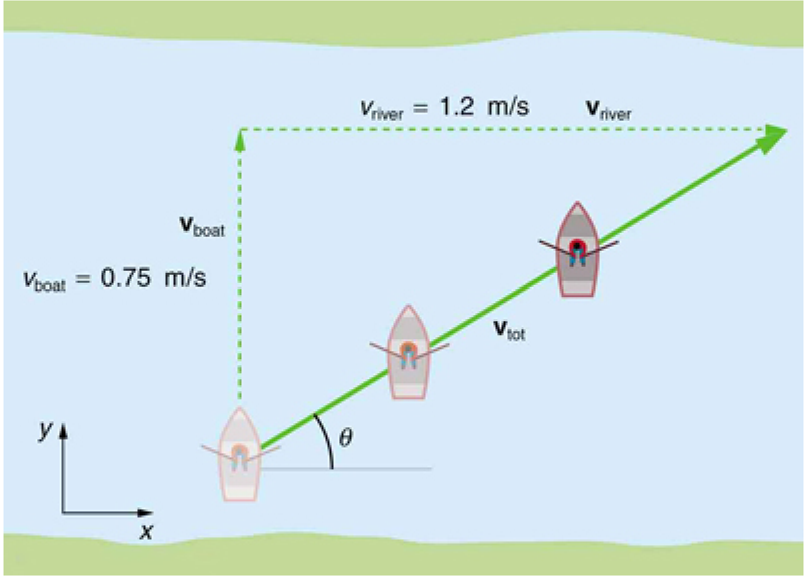


FIGURE 3.43 A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right.

Refer to Figure 3.43, which shows a boat trying to go straight across the river. Let us calculate the magnitude and direction of the boat's velocity relative to an observer on the shore, $\mathbf{v}_{\text{tot}}$. The velocity of the boat, $\mathbf{v}_{\text{boat}}$, is 0.75 m/s in the y-direction relative to the river and the velocity of the river, $\mathbf{v}_{\text{river}}$, is 1.20 m/s to the right.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river, as shown in Figure 3.43. Because the boat is directed straight toward the other shore, its velocity relative to the water is parallel to the y-axis and perpendicular to the velocity of the river. Thus, we can add the two velocities by using the equations $v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}$ and $\theta = \tan^{-1}(v_y/v_x)$ directly.

Solution

The magnitude of the total velocity is

$$v_{\text{tot}} = \sqrt{v_x^2 + v_y^2}, \tag{3.76}$$

where

$$v_x = v_{\text{river}} = 1.20 \text{ m/s} \tag{3.77}$$

and

$$v_y = v_{\text{boat}} = 0.750 \text{ m/s}. \tag{3.78}$$

Thus,

$$v_{\text{tot}} = \sqrt{(1.20 \text{ m/s})^2 + (0.750 \text{ m/s})^2} \tag{3.79}$$

yielding

$$v_{\text{tot}} = 1.42 \text{ m/s}. \tag{3.80}$$

The direction of the total velocity θ is given by:

$$\theta = \tan^{-1}(v_y / v_x) = \tan^{-1}(0.750/1.20). \tag{3.81}$$

This equation gives

$$\theta = 32.0^\circ. \tag{3.82}$$

Discussion

Both the magnitude v and the direction θ of the total velocity are consistent with Figure 3.43. Note that because the velocity of the river is large compared with the velocity of the boat, it is swept rapidly downstream. This result is evidenced by the small angle (only 32.0°) the total velocity has relative to the riverbank.

EXAMPLE 3.7

Calculating Velocity: Wind Velocity Causes an Airplane to Drift

Calculate the wind velocity for the situation shown in Figure 3.44. The plane is known to be moving at 45.0 m/s due north relative to the air mass, while its velocity relative to the ground (its total velocity) is 38.0 m/s in a direction 20.0° west of north.

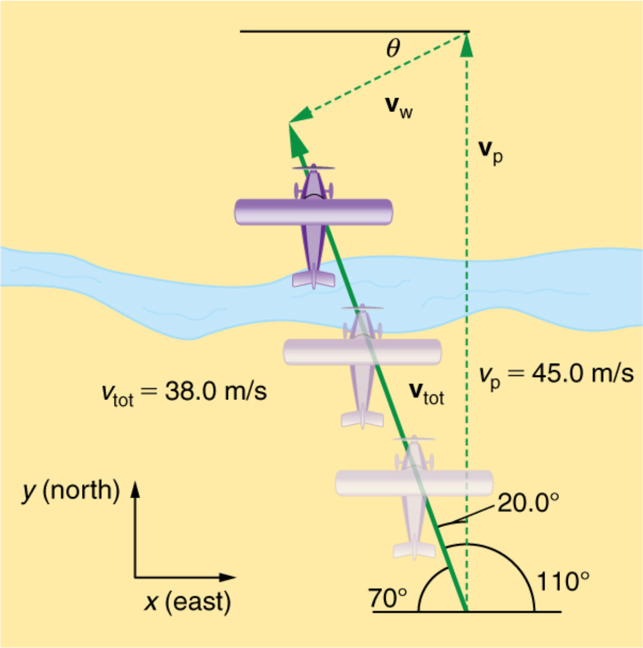


FIGURE 3.44 An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?

Strategy

In this problem, somewhat different from the previous example, we know the total velocity $\mathbf{v}_{tot}$ and that it is the sum of two other velocities, $\mathbf{v}_w$ (the wind) and $\mathbf{v}_p$ (the plane relative to the air mass). The quantity $\mathbf{v}_p$ is known, and we are asked to find $\mathbf{v}_w$. None of the velocities are perpendicular, but it is possible to find their components along a common set of perpendicular axes. If we can find the components of $\mathbf{v}_w$, then we can combine them to solve for its magnitude and direction. As shown in Figure 3.44, we choose a coordinate system with its *x*-axis due east and its *y*-axis due north (parallel to $\mathbf{v}_p$). (You may wish to look back at the discussion of the addition of vectors using perpendicular components in Vector Addition and Subtraction: Analytical Methods.)

Solution

Because $\mathbf{v}_{tot}$ is the vector sum of the $\mathbf{v}_w$ and $\mathbf{v}_p$, its *x*- and *y*-components are the sums of the *x*- and *y*-components of the wind and plane velocities. Note that the plane only has vertical component of velocity so $v_{px} = 0$ and

$v_{py} = v_p$. That is,

$$v_{totx} = v_{wx} \tag{3.83}$$

and

$$v_{toty} = v_{wy} + v_p. \tag{3.84}$$

We can use the first of these two equations to find v_{wx}:

$$v_{wx} = v_{totx} = v_{tot}\cos 110°. \tag{3.85}$$

Because $v_{tot} = 38.0\text{ m/s}$ and $\cos 110° = -0.342$ we have

$$v_{wx} = (38.0\text{ m/s})(-0.342) = -13\text{ m/s}. \tag{3.86}$$

The minus sign indicates motion west which is consistent with the diagram.

Now, to find v_{wy} we note that

$$v_{toty} = v_{wy} + v_p \tag{3.87}$$

Here $v_{toty} = v_{tot}\sin 110°$; thus,

$$v_{wy} = (38.0\text{ m/s})(0.940) - 45.0\text{ m/s} = -9.29\text{ m/s}. \tag{3.88}$$

This minus sign indicates motion south which is consistent with the diagram.

Now that the perpendicular components of the wind velocity v_{wx} and v_{wy} are known, we can find the magnitude and direction of $\mathbf{v}_w$. First, the magnitude is

$$\begin{aligned} v_w &= \sqrt{v_{wx}^2 + v_{wy}^2} \\ &= \sqrt{(-13.0\text{ m/s})^2 + (-9.29\text{ m/s})^2} \end{aligned} \tag{3.89}$$

so that

$$v_w = 16.0\text{ m/s}. \tag{3.90}$$

The direction is:

$$\theta = \tan^{-1}(v_{wy}/v_{wx}) = \tan^{-1}(-9.29/-13.0) \tag{3.91}$$

giving

$$\theta = 35.6°. \tag{3.92}$$

Discussion

The wind’s speed and direction are consistent with the significant effect the wind has on the total velocity of the plane, as seen in Figure 3.44. Because the plane is fighting a strong combination of crosswind and head-wind, it ends up with a total velocity significantly less than its velocity relative to the air mass as well as heading in a different direction.

Note that in both of the last two examples, we were able to make the mathematics easier by choosing a coordinate system with one axis parallel to one of the velocities. We will repeatedly find that choosing an appropriate coordinate system makes problem solving easier. For example, in projectile motion we always use a coordinate system with one axis parallel to gravity.

Relative Velocities and Classical Relativity

When adding velocities, we have been careful to specify that the *velocity is relative to some reference frame*. These velocities are called **relative velocities**. For example, the velocity of an airplane relative to an air mass is different from its velocity relative to the ground. Both are quite different from the velocity of an airplane relative to its passengers (which should be close to zero). Relative velocities are one aspect of **relativity**, which is defined to be the study of how different observers moving relative to each other measure the same phenomenon.

Nearly everyone has heard of relativity and immediately associates it with Albert Einstein (1879–1955), the greatest physicist of the 20th century. Einstein revolutionized our view of nature with his *modern* theory of relativity, which we shall study in later chapters. The relative velocities in this section are actually aspects of classical relativity, first discussed correctly by Galileo and Isaac Newton. **Classical relativity** is limited to situations where speeds are less than about 1% of the speed of light—that is, less than $3{,}000\ \text{km/s}$. Most things we encounter in daily life move slower than this speed.

Let us consider an example of what two different observers see in a situation analyzed long ago by Galileo. Suppose a sailor at the top of a mast on a moving ship drops their binoculars. Where will it hit the deck? Will it hit at the base of the mast, or will it hit behind the mast because the ship is moving forward? The answer is that if air resistance is negligible, the binoculars will hit at the base of the mast at a point directly below its point of release. Now let us consider what two different observers see when the binoculars drop. One observer is on the ship and the other on shore. The binoculars have no horizontal velocity relative to the observer on the ship, and so he sees them fall straight down the mast. (See Figure 3.45.) To the observer on shore, the binoculars and the ship have the *same* horizontal velocity, so both move the same distance forward while the binoculars are falling. This observer sees the curved path shown in Figure 3.45. Although the paths look different to the different observers, each sees the same result—the binoculars hit at the base of the mast and not behind it. To get the correct description, it is crucial to correctly specify the velocities relative to the observer.

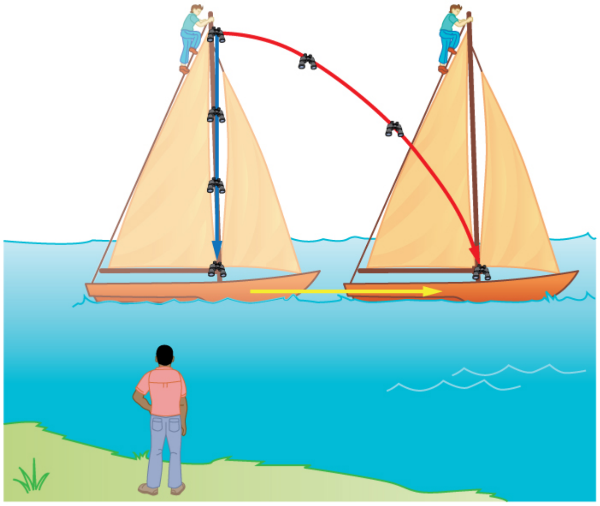


FIGURE 3.45 Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers. (The ship is shown moving rather fast to emphasize the effect.)

EXAMPLE 3.8

Calculating Relative Velocity: An Airline Passenger Drops a Coin

An airline passenger drops a coin while the plane is moving at 260 m/s. What is the velocity of the coin when it strikes the floor 1.50 m below its point of release: (a) Measured relative to the plane? (b) Measured relative to the Earth?

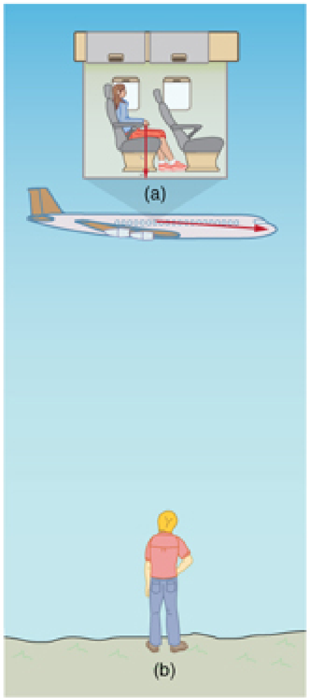


FIGURE 3.46 The motion of a coin dropped inside an airplane as viewed by two different observers. (a) An observer in the plane sees the coin fall straight down. (b) An observer on the ground sees the coin move almost horizontally.

Strategy

Both problems can be solved with the techniques for falling objects and projectiles. In part (a), the initial velocity of the coin is zero relative to the plane, so the motion is that of a falling object (one-dimensional). In part (b), the initial velocity is 260 m/s horizontal relative to the Earth and gravity is vertical, so this motion is a projectile motion. In both parts, it is best to use a coordinate system with vertical and horizontal axes.

Solution for (a)

Using the given information, we note that the initial velocity and position are zero, and the final position is 1.50 m. The final velocity can be found using the equation:

$$v_y{}^2 = v_{0,y}{}^2 - 2g(y - y_0). \tag{3.93}$$

Substituting known values into the equation, we get

$$v_y{}^2 = 0^2 - 2(9.80 \text{ m/s}^2)(-1.50 \text{ m} - 0 \text{ m}) = 29.4 \text{ m}^2/\text{s}^2 \tag{3.94}$$

yielding

$$v_y = -5.42 \text{ m/s}. \tag{3.95}$$

We know that the square root of 29.4 has two roots: 5.42 and -5.42. We choose the negative root because we know that the velocity is directed downwards, and we have defined the positive direction to be upwards. There is no initial horizontal velocity relative to the plane and no horizontal acceleration, and so the motion is straight down relative to the plane.

Solution for (b)

Because the initial vertical velocity is zero relative to the ground and vertical motion is independent of horizontal motion, the final vertical velocity for the coin relative to the ground is $v_y = -5.42 \text{ m/s}$, the same as found in part (a). In contrast to part (a), there now is a horizontal component of the velocity. However, since there is no horizontal acceleration, the initial and final horizontal velocities are the same and $v_x = 260 \text{ m/s}$. The *x*- and *y*-components of velocity can be combined to find the magnitude of the final velocity:

$$v = \sqrt{{v_x}^2 + {v_y}^2}. \tag{3.96}$$

Thus,

$$v = \sqrt{(260\text{ m/s})^2 + (-5.42\text{ m/s})^2} \tag{3.97}$$

yielding

$$v = 260.06\text{ m/s}. \tag{3.98}$$

The direction is given by:

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(-5.42/260) \tag{3.99}$$

so that

$$\theta = \tan^{-1}(-0.0208) = -1.19^\circ. \tag{3.100}$$

Discussion

In part (a), the final velocity relative to the plane is the same as it would be if the coin were dropped from rest on the Earth and fell 1.50 m. This result fits our experience; objects in a plane fall the same way when the plane is flying horizontally as when it is at rest on the ground. This result is also true in moving cars. In part (b), an observer on the ground sees a much different motion for the coin. The plane is moving so fast horizontally to begin with that its final velocity is barely greater than the initial velocity. Once again, we see that in two dimensions, vectors do not add like ordinary numbers—the final velocity v in part (b) is *not* $(260 - 5.42)\text{ m/s}$; rather, it is 260.06 m/s. The velocity's magnitude had to be calculated to five digits to see any difference from that of the airplane. The motions as seen by different observers (one in the plane and one on the ground) in this example are analogous to those discussed for the binoculars dropped from the mast of a moving ship, except that the velocity of the plane is much larger, so that the two observers see *very* different paths. (See Figure 3.46.) In addition, both observers see the coin fall 1.50 m vertically, but the one on the ground also sees it move forward 144 m (this calculation is left for the reader). Thus, one observer sees a vertical path, the other a nearly horizontal path.

Making Connections: Relativity and Einstein

Because Einstein was able to clearly define how measurements are made (some involve light) and because the speed of light is the same for all observers, the outcomes are spectacularly unexpected. Time varies with observer, energy is stored as increased mass, and more surprises await.

PHET EXPLORATIONS

Motion in 2D

Try the "Motion in 2D" simulation to Learn about position, velocity, and acceleration vectors. Move the ball with the mouse or let the simulation move the ball in four types of motion (2 types of linear, simple harmonic, circle).

Click to view content (https://openstax.org/l/motion-2d).

Glossary

air resistance a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

analytical method the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

classical relativity the study of relative velocities in situations where speeds are less than about 1% of the speed of light—that is, less than 3000 km/s

commutative refers to the interchangeability of order in a function; vector addition is commutative because the order in which vectors are added together does not affect the final sum

component (of a 2-d vector) a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

direction (of a vector) the orientation of a vector in space

head (of a vector) the end point of a vector; the location of the tip of the vector’s arrowhead; also referred to as the “tip”

head-to-tail method a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

kinematics the study of motion without regard to mass or force

magnitude (of a vector) the length or size of a vector; magnitude is a scalar quantity

motion displacement of an object as a function of time

projectile an object that travels through the air and experiences only acceleration due to gravity

projectile motion the motion of an object that is subject only to the acceleration of gravity

range the maximum horizontal distance that a projectile travels

relative velocity the velocity of an object as observed from a particular reference frame

relativity the study of how different observers moving relative to each other measure the same phenomenon

resultant the sum of two or more vectors

resultant vector the vector sum of two or more vectors

scalar a quantity with magnitude but no direction

tail the start point of a vector; opposite to the head or tip of the arrow

trajectory the path of a projectile through the air

vector a quantity that has both magnitude and direction; an arrow used to represent quantities with both magnitude and direction

vector addition the rules that apply to adding vectors together

velocity speed in a given direction

Section Summary

3.1 Kinematics in Two Dimensions: An Introduction

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

3.2 Vector Addition and Subtraction: Graphical Methods

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A}+\mathbf{B}=\mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})=\mathbf{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is **commutative** such that $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$.
- The **head-to-tail method** of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by cA. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

3.3 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:
  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations
  $$\begin{aligned} A_x &= A\cos\theta \\ B_x &= B\cos\theta \end{aligned}$$
  and
  $$\begin{aligned} A_y &= A\sin\theta \\ B_y &= B\sin\theta. \end{aligned}$$
  Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, $\mathbf{R}$:
  $$R_x = A_x + B_x$$
  and
  $$R_y = A_y + B_{y.}$$
  Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector $\mathbf{R}$:
  $$R = \sqrt{R_x^2 + R_y^2}.$$
  Step 4: Use a trigonometric identity to determine the direction, θ, of $\mathbf{R}$:
  $$\theta = \tan^{-1}(R_y/R_x).$$

3.4 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:
  1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $\mathbf{s}$ are given by the quantities x and y, and the components of the velocity $\mathbf{v}$ are given by $v_x = v\cos\theta$ and $v_y = v\sin\theta$, where v is the magnitude of the velocity and θ is its direction.
  2. Analyze the motion of the projectile in the horizontal direction using the following equations:
     $$\text{Horizontal motion}(a_x = 0)$$
     $$x = x_0 + v_x t$$
     $$v_x = v_{0x} = \mathbf{v}_\text{x} = \text{velocity is a constant.}$$
  3. Analyze the motion of the projectile in the vertical direction using the following equations:
     $$\text{Vertical Motion}$$
     $$(\text{assuming positive is up } a_y = -g = -9.80\text{m/s}^2)$$
     $$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$$
     $$v_y = v_{0y} - gt$$
     $$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
     $$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$
  4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:
     $$s = \sqrt{x^2 + y^2}$$
     $$\theta = \tan^{-1}(y/x)$$
     $$v = \sqrt{v_x^2 + v_y^2}$$
     $$\theta_\text{v} = \tan^{-1}(v_y/v_x).$$
- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by
  $$h = \frac{v_{0y}^2}{2g}.$$
- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by
  $$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

3.5 Addition of Velocities

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as
  $$v_x = v\cos\theta$$
  $$v_y = v\sin\theta$$
  $$v = \sqrt{v_x^2 + v_y^2}$$
  $$\theta = \tan^{-1}(v_y/v_x).$$
- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).

Conceptual Questions

3.2 Vector Addition and Subtraction: Graphical Methods

1. Which of the following is a vector: a person's height, the altitude on Mt. Everest, the age of the Earth, the boiling point of water, the cost of this book, the Earth's population, the acceleration of gravity?
2. Give a specific example of a vector, stating its magnitude, units, and direction.
3. What do vectors and scalars have in common? How do they differ?
4. Two campers in a national park hike from their cabin to the same spot on a lake, each taking a different path, as illustrated below. The total distance traveled along Path 1 is 7.5 km, and that along Path 2 is 8.2 km. What is the final displacement of each camper?

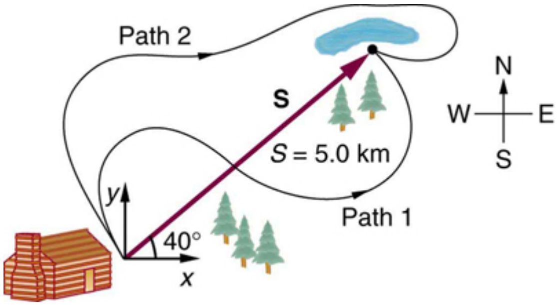


FIGURE 3.47

5. If an airplane pilot is told to fly 123 km in a straight line to get from San Francisco to Sacramento, explain why he could end up anywhere on the circle shown in Figure 3.48. What other information would he need to get to Sacramento?

FIGURE 3.48

6. Suppose you take two steps $\mathbf{A}$ and $\mathbf{B}$ (that is, two nonzero displacements). Under what circumstances can you end up at your starting point? More generally, under what circumstances can two nonzero vectors add to give zero? Is the maximum distance you can end up from the starting point $\mathbf{A} + \mathbf{B}$ the sum of the lengths of the two steps?
7. Explain why it is not possible to add a scalar to a vector.
8. If you take two steps of different sizes, can you end up at your starting point? More generally, can two vectors with different magnitudes ever add to zero? Can three or more?

3.3 Vector Addition and Subtraction: Analytical Methods

9. Suppose you add two vectors $\mathbf{A}$ and $\mathbf{B}$. What relative direction between them produces the resultant with the greatest magnitude? What is the maximum magnitude? What relative direction between them produces the resultant with the smallest magnitude? What is the minimum magnitude?
10. Give an example of a nonzero vector that has a component of zero.

11. Explain why a vector cannot have a component greater than its own magnitude.
12. If the vectors $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, what is the component of $\mathbf{A}$ along the direction of $\mathbf{B}$? What is the component of $\mathbf{B}$ along the direction of $\mathbf{A}$?

3.4 Projectile Motion

13. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?
14. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0° nor 90°): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?
15. For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?
16. During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

3.5 Addition of Velocities

17. What frame or frames of reference do you instinctively use when driving a car? When flying in a commercial jet airplane?
18. A basketball player dribbling down the court usually keeps his eyes fixed on the players around him. He is moving fast. Why doesn't he need to keep his eyes on the ball?
19. If someone is riding in the back of a pickup truck and throws a softball straight backward, is it possible for the ball to fall straight down as viewed by a person standing at the side of the road? Under what condition would this occur? How would the motion of the ball appear to the person who threw it?
20. The hat of a jogger running at constant velocity falls off the back of his head. Draw a sketch showing the path of the hat in the jogger's frame of reference. Draw its path as viewed by a stationary observer.
21. A clod of dirt falls from the bed of a moving truck. It strikes the ground directly below the end of the truck. What is the direction of its velocity relative to the truck just before it hits? Is this the same as the direction of its velocity relative to ground just before it hits? Explain your answers.

Problems & Exercises

3.2 Vector Addition and Subtraction: Graphical Methods

Use graphical methods to solve these problems. You may assume data taken from graphs is accurate to three digits.

1. Find the following for path A in Figure 3.49: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.

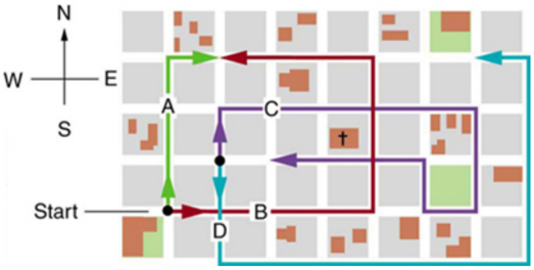


FIGURE 3.49 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

2. Find the following for path B in Figure 3.49: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.
3. Find the north and east components of the displacement for the hikers shown in Figure 3.47.
4. Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.50, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

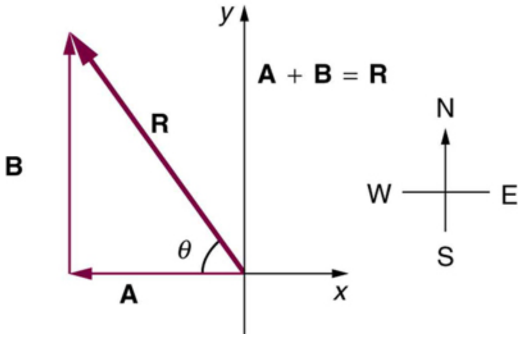


FIGURE 3.50 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

5. Suppose you first walk 12.0 m in a direction 20° west of north and then 20.0 m in a direction 40.0° south of west. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.51, then this problem finds their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

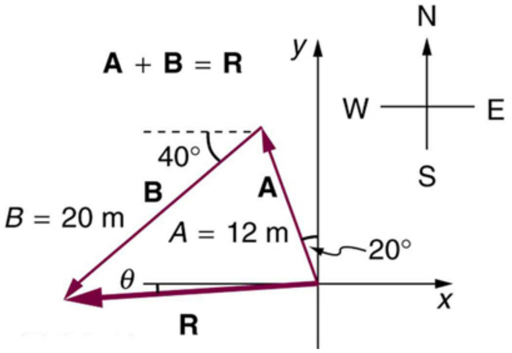


FIGURE 3.51

6. Repeat the problem above, but reverse the order of the two legs of the walk; show that you get the same final result. That is, you first walk leg $\mathbf{B}$, which is 20.0 m in a direction exactly 40° south of west, and then leg $\mathbf{A}$, which is 12.0 m in a direction exactly 20° west of north. (This problem shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.)
7. (a) Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction 40.0°). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction 40.0° east of south (which is equivalent to subtracting $\mathbf{A}$ from $\mathbf{B}$ —that is, to finding $\mathbf{R}'' = \mathbf{B} - \mathbf{A} = -\mathbf{R}'$). Show that this is the case.
8. Show that the *order* of addition of three vectors does not affect their sum. Show this property by choosing any three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, all having different lengths and directions. Find the sum $\mathbf{A} + \mathbf{B} + \mathbf{C}$ then find their sum when added in a different order and show the result is the same. (There are five other orders in which $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ can be added; choose only one.)
9. Show that the sum of the vectors discussed in Example 3.2 gives the result shown in Figure 3.23.
10. Find the magnitudes of velocities v_A and v_B in Figure 3.52

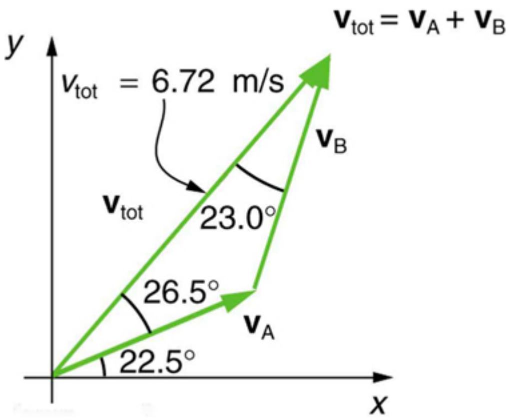


FIGURE 3.52 The two velocities $\mathbf{v}_\mathrm{A}$ and $\mathbf{v}_\mathrm{B}$ add to give a total $\mathbf{v}_\mathrm{tot}$.

11. Find the components of v_tot along the *x*- and *y*-axes in Figure 3.52.
12. Find the components of v_tot along a set of perpendicular axes rotated 30° counterclockwise relative to those in Figure 3.52.

3.3 Vector Addition and Subtraction: Analytical Methods

13. Find the following for path C in Figure 3.53: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

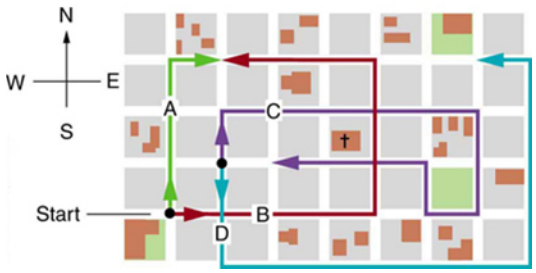


FIGURE 3.53 The various lines represent paths taken by different people walking in a city. All blocks are 120 m on a side.

14. Find the following for path D in Figure 3.53: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.

15. Find the north and east components of the displacement from San Francisco to Sacramento shown in Figure 3.54.

FIGURE 3.54

16. Solve the following problem using analytical techniques: Suppose you walk 18.0 m straight west and then 25.0 m straight north. How far are you from your starting point, and what is the compass direction of a line connecting your starting point to your final position? (If you represent the two legs of the walk as vector displacements $\mathbf{A}$ and $\mathbf{B}$, as in Figure 3.55, then this problem asks you to find their sum $\mathbf{R} = \mathbf{A} + \mathbf{B}$.)

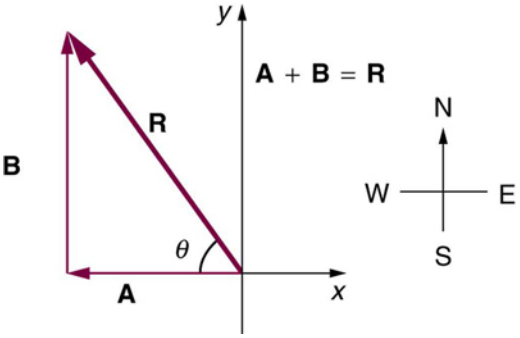


FIGURE 3.55 The two displacements $\mathbf{A}$ and $\mathbf{B}$ add to give a total displacement $\mathbf{R}$ having magnitude R and direction θ.

Note that you can also solve this graphically. Discuss why the analytical technique for solving this problem is potentially more accurate than the graphical technique.

17. Repeat Exercise 3.16 using analytical techniques, but reverse the order of the two legs of the walk and show that you get the same final result. (This problem shows that adding them in reverse order gives the same result—that is, $\mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B}$.) Discuss how taking another path to reach the same point might help to overcome an obstacle blocking your other path.

18. You drive 7.50 km in a straight line in a direction 15° east of north. (a) Find the distances you would have to drive straight east and then straight north to arrive at the same point. (This determination is equivalent to find the components of the displacement along the east and north directions.) (b) Show that you still arrive at the same point if the east and north legs are reversed in order.

19. Do Exercise 3.16 again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$ —that is, finding $\mathbf{R}' = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$ —that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)

20. A new landowner has a triangular piece of flat land she wishes to fence. Starting at the west corner, she measures the first side to be 80.0 m long and the next to be 105 m. These sides are represented as displacement vectors $\mathbf{A}$ from $\mathbf{B}$ in Figure 3.56. She then correctly calculates the length and orientation of the third side $\mathbf{C}$. What is her result?

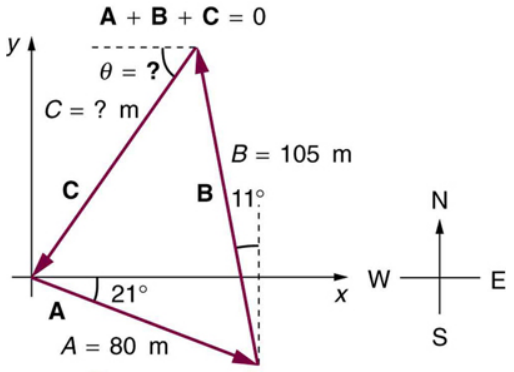


FIGURE 3.56

21. You fly 32.0 km in a straight line in still air in the direction 35.0° south of west. (a) Find the distances you would have to fly straight south and then straight west to arrive at the same point. (This determination is equivalent to finding the components of the displacement along the south and west directions.) (b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes—one rotated 45°.

22. A farmer wants to fence off a four-sided plot of flat land. They measure the first three sides, shown as $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in Figure 3.57, and then correctly calculate the length and orientation of the fourth side $\mathbf{D}$. What is their result?

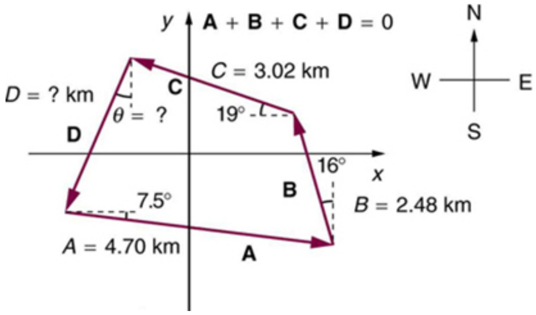


FIGURE 3.57

23. In an attempt to escape his island, Gilligan builds a raft and sets to sea. The wind shifts a great deal during the day, and he is blown along the following straight lines: 2.50 km 45.0° north of west; then 4.70 km 60.0° south of east; then 1.30 km 25.0° south of west; then 5.10 km straight east; then 1.70 km 5.00° east of north; then 7.20 km 55.0° south of west; and finally 2.80 km 10.0° north of east. What is his final position relative to the island?

24. Suppose a pilot flies 40.0 km in a direction 60° north of east and then flies 30.0 km in a direction 15° north of east as shown in Figure 3.58. Find her total distance R from the starting point and the direction θ of the straight-line path to the final position. Discuss qualitatively how this flight would be altered by a wind from the north and how the effect of the wind would depend on both wind speed and the speed of the plane relative to the air mass.

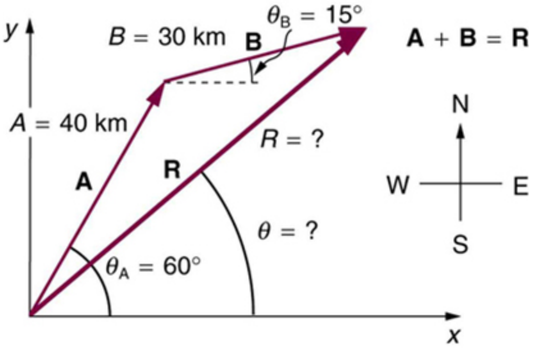


FIGURE 3.58

3.4 Projectile Motion

25. A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target above the ground 3.00 seconds later. What are the x and y distances from where the projectile was launched to where it lands?

26. A ball is kicked with an initial velocity of 16 m/s in the horizontal direction and 12 m/s in the vertical direction. (a) At what speed does the ball hit the ground? (b) For how long does the ball remain in the air? (c)What maximum height is attained by the ball?

27. A ball is thrown horizontally from the top of a 60.0-m building and lands 100.0 m from the base of the building. Ignore air resistance. (a) How long is the ball in the air? (b) What must have been the initial horizontal component of the velocity? (c) What is the vertical component of the velocity just before the ball hits the ground? (d) What is the velocity (including both the horizontal and vertical components) of the ball just before it hits the ground?

28. (a) A daredevil is attempting to jump his motorcycle over a line of buses parked end to end by driving up a 32^{o} ramp at a speed of $40.0\ \mathrm{m/s}\ (144\ \mathrm{km/h})$. How many buses can he clear if the top of the takeoff ramp is at the same height as the bus tops and the buses are 20.0 m long? (b) Discuss what your answer implies about the margin of error in this act—that is, consider how much greater the range is than the horizontal distance he must travel to miss the end of the last bus. (Neglect air resistance.)

29. An archer shoots an arrow at a 75.0 m distant target; the bull's-eye of the target is at same height as the release height of the arrow. (a) At what angle must the arrow be released to hit the bull's-eye if its initial speed is 35.0 m/s? In this part of the problem, explicitly show how you follow the steps involved in solving projectile motion problems. (b) There is a large tree halfway between the archer and the target with an overhanging horizontal branch 3.50 m above the release height of the arrow. Will the arrow go over or under the branch?

30. A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?

31. Verify the ranges for the projectiles in Figure 3.38(a) for $\theta = 45^{\mathrm{o}}$ and the given initial velocities.

32. Verify the ranges shown for the projectiles in Figure 3.38(b) for an initial velocity of 50 m/s at the given initial angles.

33. The cannon on a battleship can fire a shell a maximum distance of 32.0 km. (a) Calculate the initial velocity of the shell. (b) What maximum height does it reach? (At its highest, the shell is above 60% of the atmosphere—but air resistance is not really negligible as assumed to make this problem easier.) (c) The ocean is not flat, because the Earth is curved. Assume that the radius of the Earth is $6.37 \times 10^{3}\ \mathrm{km}$. How many meters lower will its surface be 32.0 km from the ship along a horizontal line parallel to the surface at the ship? Does your answer imply that error introduced by the assumption of a flat Earth in projectile motion is significant here?

34. An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60^{o} above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (c) What is the arrow's impact speed just before hitting the cliff?

35. In the standing broad jump, one squats and then pushes off with the legs to see how far one can jump. Suppose the extension of the legs from the crouch position is 0.600 m and the acceleration achieved from this position is 1.25 times the acceleration due to gravity, g. How far can they jump? State your assumptions. (Increased range can be achieved by swinging the arms in the direction of the jump.)

36. The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

37. Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle θ below the horizontal. The base line is 11.9 m from the net, which is 0.91 m high. What is the angle θ such that the ball just crosses the net? Will the ball land in the service box, whose service line is 6.40 m from the net?

38. A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. (a) If the ball is thrown at an angle of 25^{o} relative to the ground and is caught at the same height as it is released, what is its initial speed relative to the ground? (b) How long does it take to get to the receiver? (c) What is its maximum height above its point of release?