Advanced Waterworks Mathematics

ZERO TEXTBOOK COST

WATER-131

ADVANCED WATERWORKS MATHEMATICS

Learning and Understanding Mathematical Concepts in Water Distribution and Water Treatment

An Open Educational Resources Publication by College of the Canyons

Written by Michael Alvord, Regina Blasberg, Stephanie Anagnoson, and Ernesto Velazquez
Edited by Regina Blasberg

Version 1.6

2021

ZERO TEXTBOOK COST

Acknowledgements

College of the Canyons would like to extend appreciation to the following people and organizations for allowing this textbook to be created:

California Community Colleges Chancellor's Office
Chancellor Dianne G. Van Hook
Santa Clarita Community College District
College of the Canyons Online Education

Table of Contents

Unit 1: Unit Dimensional Analysis (UDA) 6

1.1 Converting Units 6

1.2 Applying the Math of the UDA 16

Unit 2: Geometric Shapes 22

2.1 Area 22

2.2 Volumes 42

2.3 Area and Volume Word Problems 51

2.4 Flow Rate 67

Unit 3 74

3.1 Density and Specific Gravity 74

3.2 Parts-per Notation 80

3.3 Mixing and Diluting Solutions 89

Unit 4 96

4.1 Chemical Dosage Analysis 96

Unit 5 116

5.1 Weir Overflow Rate 116

Unit 6 127

6.1 Detention Time 127

6.2 Filtration Rates 142

Unit 7 152

7.1 CT Calculations 152

Unit 8 173

8.1 Pressure 173

8.2 Head Loss 183

Unit 9 191

9.1 Well Yield, Specific Capacity, and Drawdown 191

Unit 10 201
10.1 Horsepower and Efficiency 201
10.2 Head Loss and Horsepower 209
10.3 Calculating Power Costs 215
Unit 11 231
11.1 Per Capita Water Use and Water Use Efficiency 231
Unit 12 243
12.1 Blending and Diluting 243
Unit 13 253
13.1 SCADA and the use of MA SCADA 253
Unit 14 266
14.1 Water Utility Management 266
Practice Problem Solutions 285
CT Tables 367
State Water Resources Control Board Exam Formula Sheets 378

UNIT 1: UNIT DIMENSIONAL ANALYSIS (UDA)

1.1 CONVERTING UNITS

Converting units is the most important concept in waterworks mathematics. You may work with million gallons, cubic feet, acre feet, feet per second, gallons per minute, milligrams per liter, pounds per gallon, yards, ounces, and parts per billion. Units are the "driver" for problem solving in many waterworks math questions because operators measure processes and need units to describe these processes. For example, many water meters measure usage in hundred cubic feet. However, when utility managers report total usage by a class of customers, it is typically expressed as acre-feet or million gallons. So, a conversion is necessary to report out this water usage. Many times, flow rates are measured as gallons per minute, but dosage computations require million gallons per day. An operator must convert correctly to administer the right dose.

In an introductory water mathematics course, unit conversion is typically explained in a "unit by unit" breakdown. **Unit dimensional analysis** or UDA is the process through which we convert units by writing all units out and looking for opportunities to convert and cancel units.

Example: Convert seconds to days using UDA.

$$\frac{\text{sec}}{1}\times\frac{\text{min}}{\text{sec}}\times\frac{\text{hour}}{\text{min}}\times\frac{\text{day}}{\text{hour}}$$

If we break the problem up into parts, you can see how the canceling of units occurs.

$$\frac{\cancel{\text{sec}}}{1}\times\frac{\text{min}}{\cancel{\text{sec}}}=\text{min}$$

In the above example, if we just do the first part you can see that canceling seconds will yield minutes as the result. By continuing the same process, you can convert to the appropriate units.

$$\frac{\cancel{\text{sec}}}{1}\times\frac{\cancel{\text{min}}}{\cancel{\text{sec}}}\times\frac{\cancel{\text{hour}}}{\cancel{\text{min}}}\times\frac{\text{day}}{\cancel{\text{hour}}}=\text{day}$$

The reason seconds is set up over a "1" is because it is the units you begin with. Remember anything over 1 is that same value.

Multiple units may need to be converted. In the text for Water 130, you were instructed to convert one unit at a time. In this text, we will look at common unit conversions that include multiple units, which is the preferred method to minimize mistakes and avoid costly errors. As you become more familiar with water-related math problems, you will notice these recurring numbers and conversions.

A common waterworks mathematics conversion is from cubic feet per second (cfs) to gallons per minute (gpm). It takes two steps to convert cfs to gpm: one step to convert the seconds to minutes and the other step to convert cubic feet to gallons.

Example: Convert from cubic feet per second (cfs) to gallons per minute (gpm).

$$\frac{1 \cancel{\text{cf}}}{\cancel{\text{sec}}} \times \frac{60 \cancel{\text{sec}}}{1 \text{ min}} \times \frac{7.48 \text{ gal}}{1 \cancel{\text{cf}}} = \frac{448.8 \text{ gal}}{\text{min}}$$

Units in the numerators and denominators cancel out.

Cubic feet are converted to gallons and seconds are converted to minutes. This results in a single, very useful, conversion factor that can be used any time you need to convert from cfs to gpm or from gpm to cfs.

$$\frac{1 \text{ cfs}}{448.8 \text{ gpm}} \quad \text{or} \quad \frac{448.8 \text{ gpm}}{1 \text{ cfs}}$$

Example: Convert 5 cubic feet per second (cfs) to gallons per minute (gpm) using the combined conversion factor.

$$\frac{5 \text{ cfs}}{1} \times \frac{448.8 \text{ gpm}}{1 \text{ cfs}} = 2{,}244 \text{ gpm}$$

Instead of breaking up cubic feet per second into cubic feet and seconds, cubic feet per seconds are kept together as cfs.

There are additional combined conversion factors. Let’s look at converting millions of gallons per day (MGD) to cfs.

Example: Show how 1 MGD is equivalent to 1.55 cfs.

$$\frac{1{,}000{,}000\ \cancel{\text{gal}}}{\cancel{\text{day}}} \times \frac{1\ \cancel{\text{day}}}{24\ \cancel{\text{hr}}} \times \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{sec}} \times \frac{1\ \text{cf}}{7.48\ \cancel{\text{gal}}} = 1.55\ \text{cfs}$$

In converting MGD to cfs, there are multiple units to convert including gallons to cubic feet, days to hours, hours to minutes, and minutes to seconds. Now, you can use the combined conversion factor when you need to convert from MGD to cfs.

$$\frac{1\ \text{MGD}}{1.55\ \text{cfs}} \quad \text{or} \quad \frac{1.55\ \text{cfs}}{1\ \text{MGD}}$$

Practice Problems 1.1a

Show how each of the following “combined” conversion factors are calculated.

1. 1 MGD = 694 gpm

2. 86,400 seconds = 1 day

3. 1 MGD = 3.069 AF/day

Exercise 1.1a

Show how each of the following combined conversion factors are calculated.

1. 1 day = 1,440 minutes

2. 1 cfs = 0.646 MGD

3. 1 cf = 62.4 lbs

Word problems may involve unit conversion. When solving word problems, there is always a specific question that must be answered. Identifying that question or problem is critical in solving the problem. Then you need to be able to identify which information provided is necessary to solve for the answer and which is not necessary. Most students skip checking their answer to make sure it makes sense. This is a critical step.

Use these steps to solve word problems.

Step 1: Read and understand the problem to determine what is being asked. Know what you are trying to solve for.

Step 2: Identify the information you need. You may want to underline key parts or circle units.

Step 3: Make a sketch to clarify the information. This step works well in geometric problems when you are trying to solve for missing dimensions, but you may find other times when it is helpful to make a quick sketch.

Step 4: Put the information into an equation or formula. Using a reference sheet for formulas is often key in this step. Make sure you copied the formula correctly!

Step 5: Solve and double check your answer. Does the solution make sense? If your solution does not make sense, read the problem again.

Practice Problems 1.1b

Solve the following conversion problems using combined conversion factors when possible.

1. Convert 3,837,000 lbs to AF

2. Convert 12.75 cfs to MGD

3. A well is pumping water at a rate of 8.25 gpm. If the pump runs 12 hours per day, how many acre feet are pumped out of the well in one year?

4. A rectangular basin contains 4.45 AF of water. How many gallons are in the basin?

5. A fire hydrant is leaking at a rate of 10 ounces per minute. How many gallons will be lost in one week? (There are 128 ounces in one gallon.)

6. A pipe flows at a rate of 6.3 cfs. How many MG will flow through the pipe in 3 days?

7. A water utility operator needs to report the total water drained from two separate basins. The 18” pipe in Basin A drained water at a rate of 12.4 cfs for four hours each day. The 12” pipe in Basin B drained water at a rate of 3.1 cfs for 16 hours each day. What is the total amount of water drained in million gallons in 30 days?

8. A water utility operator while on duty drove a total of 22,841 miles in one year. What were the average miles driven per day? Assume that the vehicle operated 6 days per week.

9. Water travels 52 miles per day through an aqueduct. What is the velocity of the water in feet per second?

10. How many days will it take to fill an Olympic size swimming pool with 660,000 gallons of water if the flow rate is 150 gpm?

Exercise 1.1b

Solve the following conversion problems using combined conversion factors when possible.

1. Convert 3 cfs to gpm

2. Convert 10.55 MGD to cfs

3. A pipe is flowing at a rate of 5.65 cfs. If this pipe flowed 10 hours per day for 365 days, how many AFY would this produce?

4. A water tank has 3,200,000 gallons in it. How much does the water weigh in lbs?

5. A faucet is dripping at a rate of 3 drops per 15 seconds. How many gallons will leak out of this faucet in 30 days? (Assume 18 drops equals 1 ounce)

6. A well flows at a rate of 1,250 gpm. How many MGD can this well produce?

7. A water utility operator needs to report the total annual production from 3 wells. The report needs to be expressed in AFY. Well 1 pumps at a rate of 750 gpm and runs for 7 hours per day. Well 2 pumps at a rate of 3,400 gpm and runs for 10 hours per day. Well 3 pumps at a rate of 1,340 gpm and runs for 5 hours per day.

8. A water utility has a fleet of 10 vehicles. Group A vehicles drove 13,330 miles last year, Group B vehicles drove 12,200 miles last year, and Group C vehicles drove 9,540 miles last year. What were the average miles per day that each vehicle grouping drove? (Assume that the vehicles were only operated during the week.)

9. Water flows through an aqueduct at a velocity of 0.55 fps. How many miles will the water travel in one day?

10. How many gallons will flow into a tank in 5 hours if the rate is 500 gpm?

1.2 APPLYING THE MATH OF THE UDA

Converting units is an everyday task in the water industry. Flow rates, hours of operation, and water sources are only part of the big picture in terms of water supply. To plan sustainable, long term groundwater and surface water supplies, it is better to have different or diverse sources of water. Pumping the lowest cost source, but typically groundwater, isn't always a sustainable solution over the long term.

Example: A water utility manager has been asked to prepare an end of year report for the utility's board of directors. The utility has two groundwater wells and one connection to a surface water treatment plant. Complete the table below.

Source of Supply	Flow Rate (gpm)	Daily Operation (Hrs)	Total Flow (MGD)	Annual Flow (AFY)
Well 1	800	10		
Well 2	1,000	8		
SW Pump	1,750	7		

In order to solve this problem, you will need to work with each well and the pump separately to calculate the total flow and then the annual flow. Let's start with Well 1.

Well 1: Total Flow in MGD

$$\frac{800 \text{ gal}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{10 \text{ hrs}}{1 \text{ day}} = 480{,}000 \frac{\text{gal}}{\text{day}} \times \frac{1 \text{ MG}}{1{,}000{,}000 \text{ gal}} = 0.48 \text{ MGD}$$

Well 1: Annual Flow in AFY

$$\frac{480{,}000 \text{ gal}}{\text{day}} \times \frac{1 \text{ AF}}{325{,}851 \text{ gal}} \times \frac{365 \text{ day}}{1 \text{ year}} = 537.7 \text{ AFY}$$

These calculations are repeated for the remaining wells and pumps.

Well 2: Total Flow in MGD

$$\frac{1{,}000 \text{ gal}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{8 \text{ hrs}}{1 \text{ day}} = 480{,}000 \frac{\text{gal}}{\text{day}} \times \frac{1 \text{ MG}}{1{,}000{,}000 \text{ gal}} = 0.48 \text{ MGD}$$

Well 2: Annual Flow in AFY

$$\frac{480{,}000 \text{ gal}}{\text{day}} \times \frac{1 \text{ AF}}{325{,}851 \text{ gal}} \times \frac{365 \text{ day}}{1 \text{ year}} = 537.7 \text{ AFY}$$

SW Pump: Total Flow in MGD

$$\frac{1{,}750 \text{ gal}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{7 \text{ hrs}}{1 \text{ day}} = 735{,}000 \frac{\text{gal}}{\text{day}} \times \frac{1 \text{ MG}}{1{,}000{,}000 \text{ gal}} = 0.735 \text{ MGD}$$

Well 1: Annual Flow in AFY

$$\frac{735{,}000 \text{ gal}}{\text{day}} \times \frac{1 \text{ AF}}{325{,}851 \text{ gal}} \times \frac{365 \text{ day}}{1 \text{ year}} = 823.3 \text{ AFY}$$

Example: Using the information from the above example problem, fill in the table below.

Source of Supply	Annual Production (AFY)	Cost per AF ($/AF)	Total Annual Cost ($)
Well 1	537.7	60	$32,262
Well 2	537.7	60	$32,262
SW Pump	823.3	450	$370,485
Total Annual Cost			$435,009

To calculate the total annual cost for Well 1, multiply the AFY times the cost per acre foot.

$$\frac{537.7 \text{ AF}}{1 \text{ year}} \times \frac{\$ 60}{1 \text{ AF}} = \$ 32{,}262 \text{ per year}$$

Repeat this calculation for the remaining wells and pump.

$$\frac{537.7 \text{ AF}}{1 \text{ year}} \times \frac{\$ 60}{1 \text{ AF}} = \$ 32{,}262 \text{ per year}$$

$$\frac{823.3 \text{ AF}}{1 \text{ year}} \times \frac{\$ 450}{1 \text{ AF}} = \$ 370{,}485 \text{ per year}$$

The total annual cost for the water utility is determined by adding the annual cost of each well and pump.

$32,262 + $32,262 + $370,485 = $ 435,009 per year

Key Terms

- **unit dimensional analysis (UDA)** – the process through which we convert units by writing all units out and looking for opportunities to convert and cancel units

Practice Problems 1.2

1. A water utility manager has been asked to prepare an end of year report for the utility's board of directors. The utility has four groundwater wells and two connections to a surface water treatment plant. Complete the table below.

Source of Supply	Flow Rate (cfs)	Daily Operation (Hr)	Total Flow (MGD)	Annual Flow (AFY)
Well 1	3.2	5		
Well 2	5	10		
Well 3	1.4	12		
Well 4	2.7	18		
SW Pump 1	4.2	4		
SW Pump 2	0.5	20		

2. Using the information from the above problem, fill in the table below.

Source of Supply	Annual Production (AFY)	Cost per AF ($/AF)	Total Annual Cost ($)
Well 1		55	
Well 2		64	
Well 3		35	
Well 4		70	
SW Pump 1		325	
SW Pump 2		275	
Total Annual Cost			$

Exercise 1.2

1. A utility manager has been asked to prepare an end of year report for the utility's board of directors. The utility has four groundwater wells and two connections to a surface water treatment plant. Complete the table below.

Source of Supply	Flow Rate (gpm)	Daily Operation (Hrs)	Total Flow (MGD)	Annual Flow (AFY)
Well 1	625	7.5		
Well 2	1,122	9		
Well 3	495	15		
Well 4	2,325	8		
SW Pump 1	1,347	6		
SW Pump 2	1,400	10		

2. Using the information from the above problem, fill in the table below.

Source of Supply	Annual Production (AFY)	Cost per AF ($/AF)	Total Annual Cost ($)
Well 1		60	
Well 2		60	
Well 3		95	
Well 4		95	
SW Pump 1		450	
SW Pump 2		450	
Total Annual Cost			

3. A water utility has 12,300 service connections. 80% of the connections are residential, 15% commercial, and 5% industrial. Complete the following table. (Assume an average month has 30 days)

Connection Type	Number of Connections	Average usage per day per connection (gallons)	Average Monthly Usage per Connection Type (CCF)
Residential		835	
Commercial		1,370	
Industrial		2,200	

4. Based on the total combined monthly usage and a unit cost of water equaling \$1.15/CCF, how much money will the utility generate in one year?

UNIT 2: GEOMETRIC SHAPES

2.1 AREA

In order to transport water from the source to treatment, to the distribution system, and eventually to the customer, water flows through geometric shapes. In cross section, the aqueduct is a trapezoid. Canals are also trapezoids in cross section. The canal below transports irrigation water from the Central Valley Project to farmers within Madera County.

Figure 2.1[1]

Reservoirs and tanks store water before it enters the treatment process. The surface of a reservoir could approximate a rectangle.

[1] Photo used with permission of Stephanie Anagnoson

The shape of a tank is often a cylinder. The tank below is in Santa Clarita and placed on top of a hill in order to use elevation to create pressure.

Figure 2.2[2]

Simple residential tanks are also cylinders and sometimes have a partial sphere on the top for extra storage.

Figure 2.3[3]

[2] Photo used with permission of SCV Water
[3] Photo used with permission of Stephanie Anagnoson

Water flows through pipes in the treatment plant and through the distribution system. Pipes are cylindrical and used throughout water distribution systems.

Figure 2.4[4]

You can see why geometric shapes are important in the water industry both in terms of area and in terms of volume.

To solve these problems correctly, you will need to apply what you already know about the Order of Operations:

Step 1: Complete anything in parentheses.

Step 2: Complete all exponents.

Step 3: Complete all multiplication and division from left to right.

Step 4: Complete all addition and subtraction from left to right.

Calculating areas is the first step in working with geometric shapes. Areas are used to determine how much paint to buy, how much water can flow through a pipe, and many other things. A circle, a rectangle, and a trapezoid are probably the most common shapes you will encounter in the water industry.

[4] Photo used with permission of SCV Water

Circles

A **circle** is a round figure whose boundary is an equal distance from the center. The distance around the circle is the **circumference.** The **diameter** is the distance across the circle through the center. The **radius** is the distance from the center to the edge of the circle.

To calculate the area of a circle, square the diameter and then multiply by 0.785. This means multiply the diameter times the diameter and then multiply the product by 0.785. This follows the Order of Operations in which exponents are worked out before multiplication and division. If you recall from the Water 130 course, we use 0.785 in the "area" formula rather than the more typical formula you probably learned in high school.

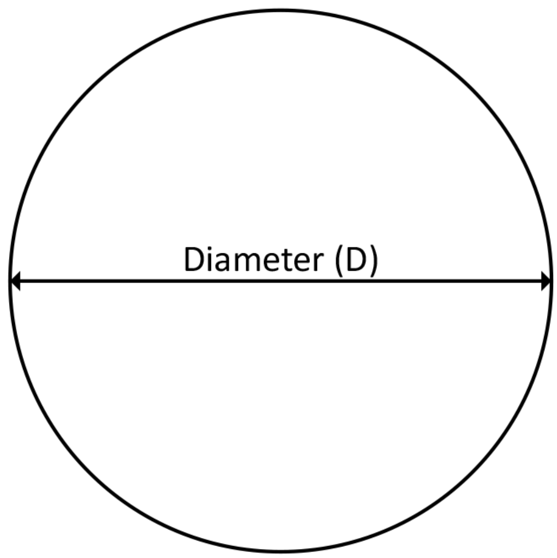


Figure 2.5[5]

$\text{Area} = \pi r^2$ or $\text{Area} = 0.785 \times D^2$

If we compare the two formulas, you can see that 0.785 replaces Pi and the diameter replaces radius. The diameter squared is four times greater than radius squared and 0.785 is one fourth of Pi.

Let's look at why these formulas and equations are both correct.

$\text{Area} = \pi r^2$ and $r = \frac{D}{2}$ where r is the radius and D is the diameter.

Now substitute D/2 into the Area formula for r.

[5] Image by Marilyn Hightower is licensed under CC BY 4.0

$$\text{Area} = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$$

To simplify the equation, divide the number pi by 4.

$$\text{Area} = \frac{\pi D^2}{4} = \frac{\pi}{4} \times D^2 = 0.785 \times D^2$$

Pin It! Misconception Alert
Take special note of the units for the diameter. Many times, especially when talking about pipes, the diameter will be given on some other unit besides feet (e.g. inches). Converting the diameter to feet as your first step will avoid ending up with squared units other than square feet. Sometimes the diameter of a pipe might be given in metric units. This is common when working with the California Department of Transportation.

Example: What is the area of a 24" diameter pipe?

When the diameter is provided in inches, it is easiest to convert the inches to feet first and then solve for area.

$$\frac{24\ \cancel{\text{in}}}{1} \times \frac{1\ \text{ft}}{12\ \cancel{\text{in}}} = 2\ \text{ft}$$

Now calculate the area.

$$\text{Area} = 0.785 \times D^2 = 0.785 \times (2\ \text{ft})^2 = 0.785 \times 4\ \text{ft}^2 = 3.14\ \text{ft}^2$$

Example: What is the area of a 130" diameter pipe?

When the diameter is provided in inches, it is easiest to convert the inches to feet first and then solve for area.

$$\frac{130\ \text{in}}{1} \times \frac{1\ \text{ft}}{12\ \text{in}} = 10.8\ \text{ft}$$

Now calculate the area.

$$0.785 \times (10.8\ \text{ft})^2 = 0.785 \times 116.64\ \text{ft}^2 = 91.56\ \text{ft}^2$$

Example: What is the area of an 813 mm diameter pipe?

Measurements may be provided in either standard or metric units. In either case, convert to feet and then solve for area. Note that there are 304.8 mm in 1 foot.

$$\frac{813 \text{ mm}}{1} \times \frac{1 \text{ ft}}{304.8 \text{ mm}} = 2.67 \text{ ft}$$

Now calculate the area.

$$0.785 \times (2.67 \text{ ft})^2 = 0.785 \times 7.1289 \text{ ft}^2 = 5.60 \text{ ft}^2$$

Rectangles

A **rectangle** is a four-sided shape with four right (90 degree) angles. A **square** is a type of rectangle with four sides that are the same length. Calculating the area of a rectangle or a square simply involves multiplying the length by the width. If you are painting the walls, ceiling or floors of a room the perspective changes slightly. For example, the dimensions of a wall might look like a width and height when you are standing looking at it. A floor might look like a width and a length. Regardless of the perspective, the area formula is the same.

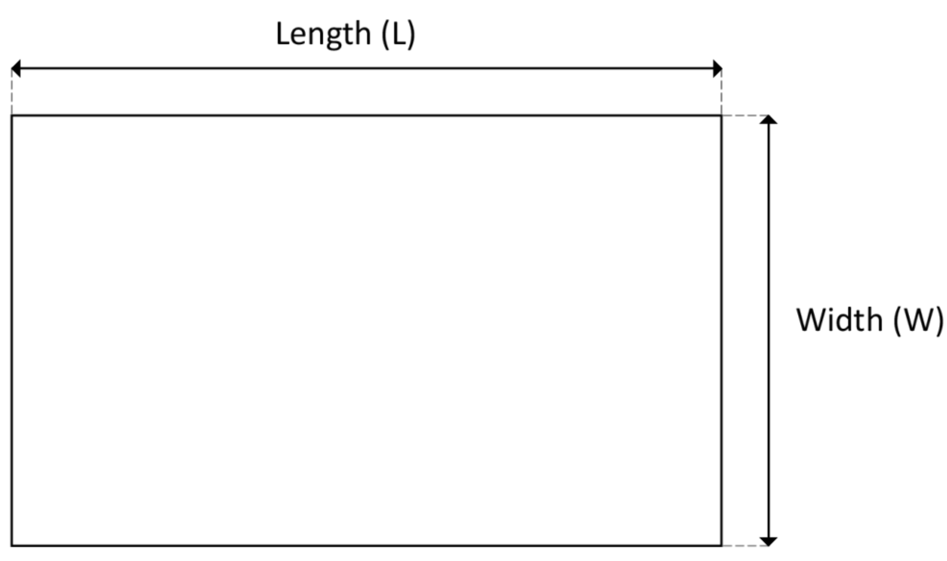


Figure 2.6[6]

$$\text{Area of a Rectangle} = L \times W$$

[6] Image by Marilyn Hightower is licensed under CC BY 4.0

Example: What is the area of a rectangle that is 30' long and 10' wide?

Since both measurements are provided in units of feet, you can calculate the area directly.

$$30 \text{ ft} \times 10 \text{ ft} = 300 \text{ ft}^2$$

Example: What is the area of a rectangle that has a height of 15' and a width of 7"?

Since the height is provided in feet and the width is provided in inches, you need to convert inches to feet before you calculate the area.

$$\frac{7 \cancel{\text{in}}}{1} \times \frac{1 \text{ ft}}{12 \cancel{\text{in}}} = 0.5833 \text{ ft}$$

$$15 \text{ ft} \times 0.5833 \text{ ft} = 8.75 \text{ ft}^2$$

Trapezoids

Trapezoids are four-sided shapes with one set of parallel sides. Aqueducts are commonly a trapezoid in cross section with a parallel top and bottom called bases. Aqueducts have narrow flat bottoms and wider flat tops at the water level. Aqueducts are typically miles and miles of trapezoidal shaped concrete channels. They have flat narrow bottoms that slope up to wider distances at the top.

Figure 2.7: Public Domain from U.S.G.S. of California Aqueduct through the Central Valley transporting water from Northern California to Southern California[7]

In order to calculate the varying distances across a trapezoid, add the distance (width, b_2) across the bottom to the distance (width, b_1) across the top and divide by 2. This gives the average width. Then multiply the average width by the height or depth of the trapezoid to calculate the area.

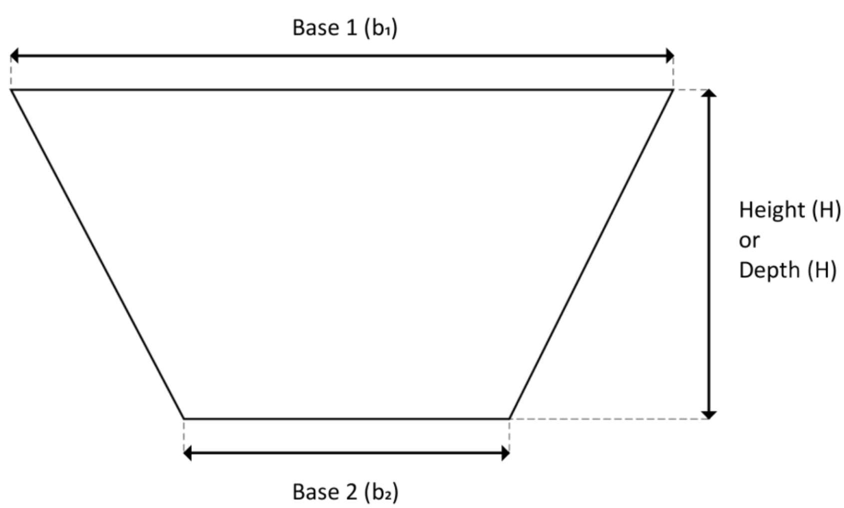


Figure 2.8[8]

$$\text{Area of a Trapezoid} = \frac{b_1 + b_2}{2} \times H$$

[7] Image by the USGS is in the public domain
[8] Image by Marilyn Hightower is licensed under CC BY 4.0

Example: What is the area of a cross section of an aqueduct that is 5 feet across the bottom, 7 feet across the top and 6 feet deep?

$$\text{Area of a Trapezoid} = \frac{b_1 + b_2}{2} \times H$$

$$\text{Area of a Trapezoid} = \frac{5\text{ ft} + 7\text{ ft}}{2} \times 6\text{ ft} =$$

$$\left(\frac{12\text{ ft}}{2}\right)(6\text{ ft}) = (6\text{ ft})(6\text{ ft}) = 36\text{ ft}^2$$

Example: What is the area of a cross section of an aqueduct that is 8 feet across the bottom, 12 feet across the top and 10 feet deep?

$$\text{Area of a Trapezoid} = \frac{b_1 + b_2}{2} \times H$$

$$\text{Area of a Trapezoid} = \frac{8\text{ ft} + 12\text{ ft}}{2} \times 10\text{ ft} =$$

$$\left(\frac{20\text{ ft}}{2}\right)(10\text{ ft}) = (10\text{ ft})(10\text{ ft}) = 100\text{ ft}^2$$

Surface Area

Circles, rectangles, and trapezoids are the most common shapes in the water industry. However, large standpipes shaped like a cylinder with a sphere on top or an elevated storage tank shaped like a sphere can be very common in flat areas. Half circles and rectangles can also be found as reservoirs or sedimentation basins. Therefore, understanding how to calculate the area for these types of structures is also important.

Cylinder

The circumference is the distance around a circle. The circumference is importance in waterworks mathematics as it is used to calculate the surface area of the side of a cylinder or tank. This is especially helpful when painting or coating a tank. The picture below shows a cylinder and the surface area of a cylinder.

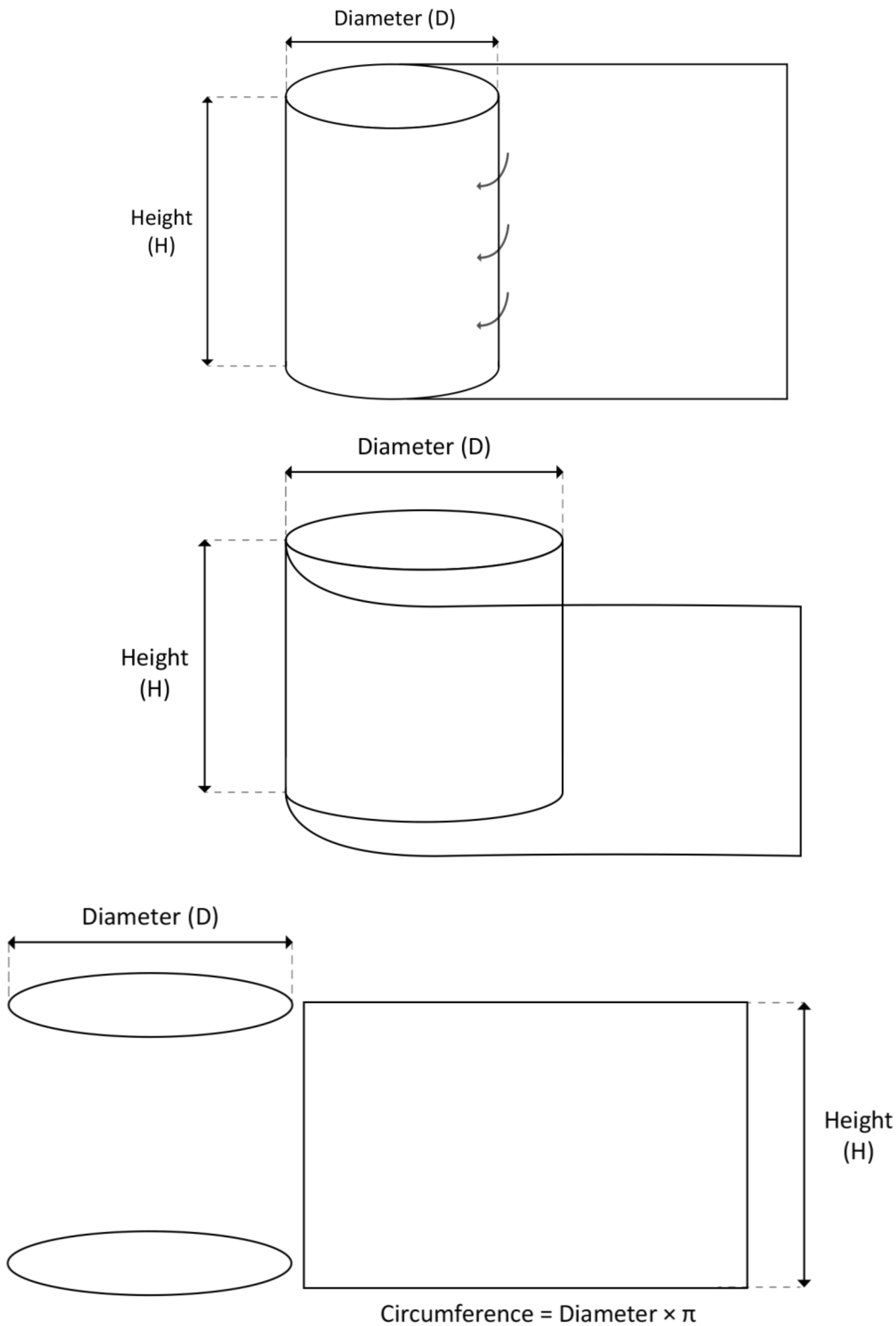


Figure 2.9[9]

Circumference of a Circle = π × D

[9] Image by Marilyn Hightower is licensed under CC BY 4.0

The circumference can also be looked at as the "length" around a cylinder. To calculate the circumference requires the unitless constant, Pi, which is typically represented as the number 3.14.

If you "slice" open a cylinder and unwrap it, it becomes a rectangle as shown in the above image. The length of this rectangle is the circumference of the cylinder. In order to calculate the surface area of a cylinder, multiply the height or depth of the cylinder by the circumference.

$$\text{Surface Area of a Cylinder} = H \times \pi \times D$$

Height or depth may be used interchangeably in the surface area calculation and are dependent on your perspective.

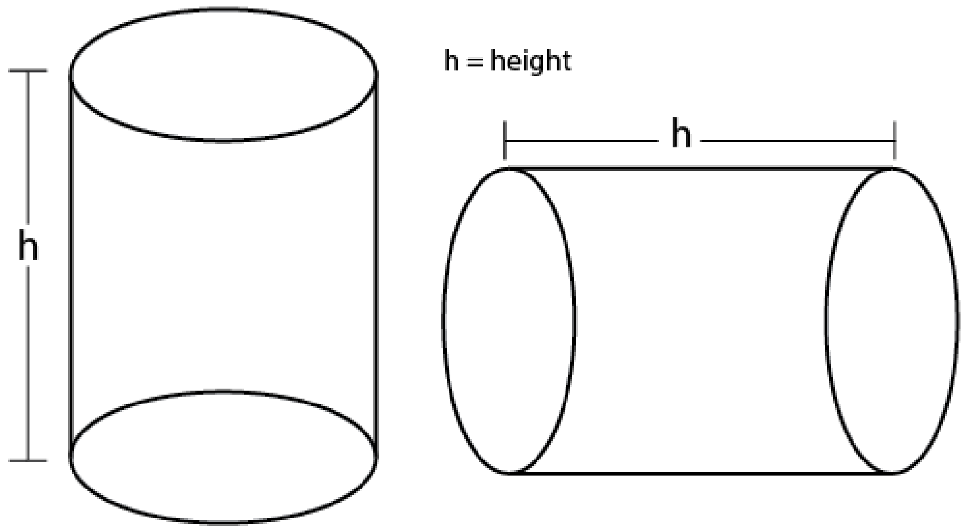


Figure 2.10[10]

Example: What is the surface area of the wall of a 20 ft tall tank with a 100 ft diameter?

$$\text{Circumference of a Circle} = \pi \times D =$$
$$3.14 \times 20 \text{ ft} = 62.8 \text{ ft}$$
$$\text{Surface Area} = 62.8 \text{ ft} \times 100 \text{ ft} = 6{,}280 \text{ ft}^2$$

Sphere

A **sphere** is a three-dimensional solid whose surface is made up of points all the same distance from a center. It is commonly thought to be shaped like a tennis ball. The surface area of a sphere is the entire surface area of a "ball." Spheres can be commonplace in flat areas, such as California's Central Valley, as elevated storage structures.

[10] Image by Marilyn Hightower is licensed under CC BY 4.0

The formula for the area of a sphere is:

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times D^2$$ Where D is the diameter of the sphere.

Example: What is the surface area of a sphere with a 50 ft diameter?

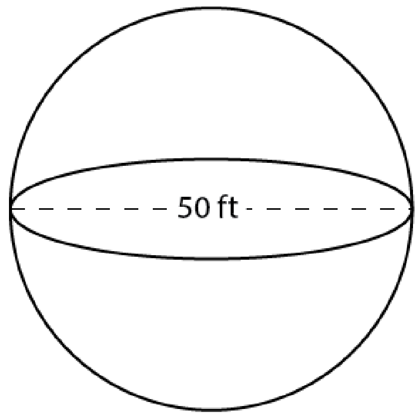


Figure 2.11[11]

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times D^2$$

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times (50\text{ ft})^2 = 7{,}850\text{ ft}^2$$

Example: What is the surface area of a sphere with a diameter of 35 ft?

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times D^2$$

$$4 \times 0.785 \times (35\text{ ft})^2 = 3{,}846.5\text{ ft}^2 = 3{,}847\text{ ft}^2$$

Some of these shapes may be combined in practical applications. You may see a cylinder with a sphere on top for example or a myriad of other combinations.

[11] Image by Marilyn Hightower is licensed under CC BY 4.0

Figure 2.12[12]

Figure 2.13[13]

[12] Image by the USGS is in the public domain
[13] Image by the USGS is in the public domain

Example: What is the entire interior surface area of a 350-foot-long, 20-inch diameter pipe that is capped with half of a sphere. The sphere is not included in the length of the pipe.

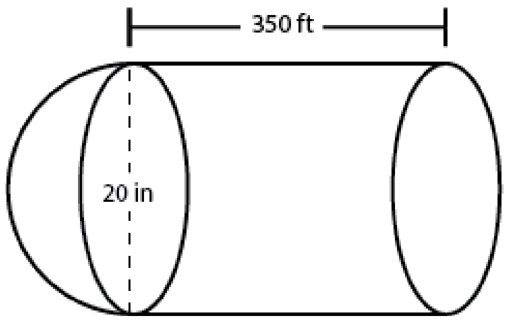


Figure 2.14[14]

First convert inches to feet:

$$\frac{20\ \cancel{in}}{1} \times \frac{1\ ft}{12\ \cancel{in}} = 1.67\ ft$$

Now we can calculate the surface area of the interior of the pipe. Remember that when you unroll a cylinder, you end up with a rectangle. The length of one side of the rectangle is the circumference of the pipe or cylinder.

$$\text{Circumference} = \pi \times D$$

$$\text{Circumference} = 3.14 \times 1.67\ ft = 5.2438\ ft$$

Using the circumference, you can calculate the surface area of the interior of the pipe.

$$\text{Surface Area} = L \times W$$

$$\text{Surface Area} = 350\ ft \times 5.2438\ ft = 1{,}835.33\ ft^2$$

Now we will calculate the surface area of the interior of half of the sphere.

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times D^2$$

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times (1.67\ ft)^2$$

$$\text{Surface Area of a Sphere} = 4 \times 0.785 \times 2.7889\ ft^2 = 8.757146\ ft^2$$

[14] Image by Marilyn Hightower is licensed under CC BY 4.0

Since only half a sphere is capping the pipe, we need to divide the total surface area by two to get the actual area for this problem.

$$\text{Half the Surface Area} = \frac{8.757146 \text{ ft}^2}{2} = 4.378573 \text{ ft}^2 = 4.38 \text{ ft}^2$$

The entire interior surface area is the sum of the pipe surface area and the surface area of the half sphere.

$$\text{Total Area} = 1{,}835.33 \text{ ft}^2 + 4.38 \text{ ft}^2 = 1{,}839.71 \text{ ft}^2$$

Practice Problems 2.1

1. What is the area of the opening of a 21” diameter pipe?

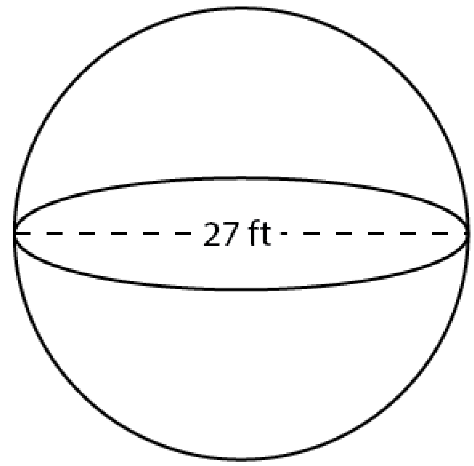
2. What is the cross-sectional area of a rectangular channel that has a width of 5 feet 8 inches and a height of 8 feet 5 inches?

3. A trapezoidal channel is 12 feet wide at the bottom and 22 feet wide at the water line when the water is 7 feet deep. What is the cross-sectional area of the channel?

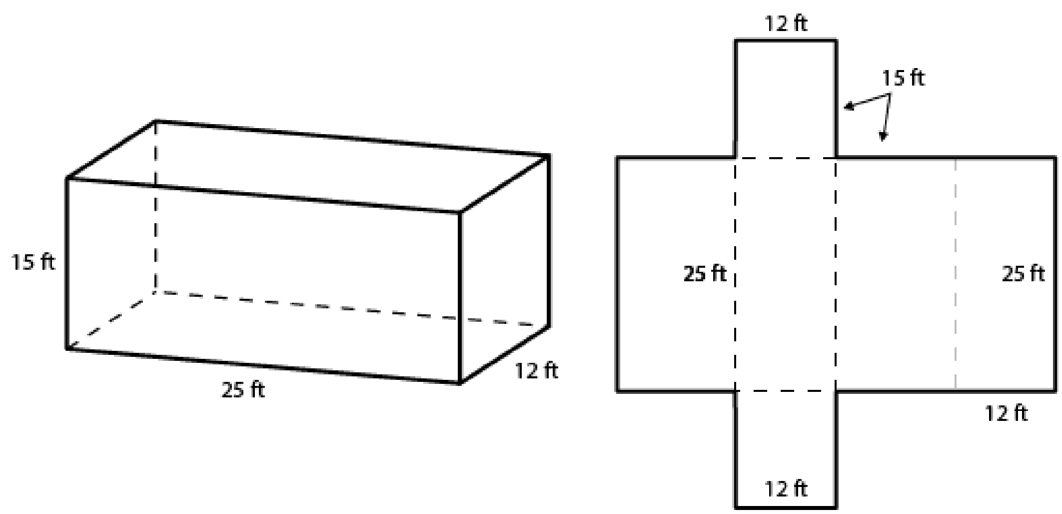
4. A 35-foot diameter spherical tank needs to be painted. If one gallon of paint will cover 400 sf, how many gallons of paint will be required to put two coats of paint on the exterior of the tank?

5. What is the surface area of a 45-foot tall standpipe with a diameter of 20 feet?

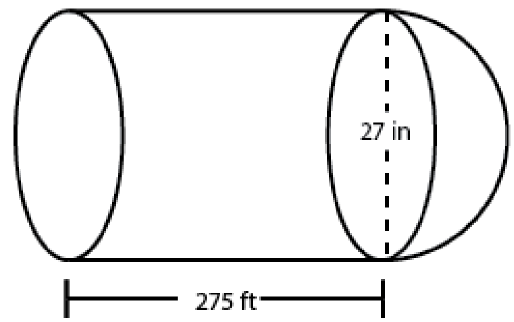
6. What is the surface area of a 27-foot diameter sphere?



7. The inside of a rectangular structure measuring 15 feet tall by 25 feet long by 12 feet wide needs painting. What is the total surface area? Include all six interior surfaces.



8. What is the entire interior surface area of a 275 foot long, 27 inch diameter pipe that is capped with half of a sphere? The sphere is not included in the length of the pipe.



Exercise 2.1

1. What is the area of the opening of a 10” diameter pipe?

2. A rectangular channel flows millions of gallons of water through it and dumps into a storage reservoir. The channel is 2 miles long 3 feet wide and 2 feet deep. What is the area of the channel opening?

3. A trapezoidal-shaped channel is 3 feet wide at the bottom and 5 feet wide at the top and the water is 4 feet deep when the channel is full. What is the area of a cross section of the channel?

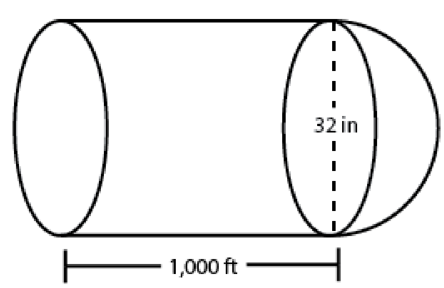
4. An elevated storage tank is shaped like a sphere and needs to be recoated. If the diameter of the tank is 65 feet, what is the surface area?

5. A standpipe needs to be painted. It has a diameter of 30 feet and is 80 feet tall. What is the surface area of the entire standpipe?

6. What is the surface area of a spherical structure that has a 42-foot diameter?

7. A box structure needs to be painted. It is 20 feet wide, 30 feet long and 10 feet tall. What is the total area of all six surfaces (inside only)?

8. What is the inside surface area of a 32” diameter pipe that is 1,000 feet long and is capped with half a sphere at the end? (Assume the sphere diameter is not included in the length)



2.2 VOLUMES

To calculate the volume inside a structure, a third dimension needs to be included in the "area" formula. For example, if a circle (a two-dimensional object) becomes part of a cylinder (a three-dimensional object), a volume can be calculated using the height.

Cylinder

A **cylinder** is a three-dimensional solid with circular bases and parallel sides. The formula for volume of a cylinder uses the area formula for a circle with the addition of a third dimension. Notice that a cylinder can have either a length or a height depending on your perspective. In fact, all three-dimensional structures can have a depth or a height.

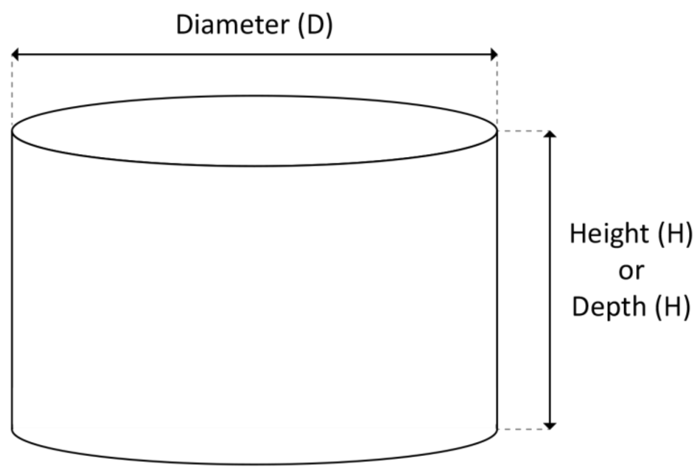


Figure 2.15

$$\text{Volume of a Cylinder} = 0.785 \times D^2 \times H$$

Example: What is the volume of a cylindrical tank that has an 80" diameter and is 30 feet tall?

$$\text{Volume of a Cylinder} = 0.785 \times D^2 \times H$$

Notice that the diameter of the tank is in inches. First, we need to convert the units for the diameter from inches to feet.

$$\frac{80\ \cancel{\text{in}}}{1} \times \frac{1\ \text{ft}}{12\ \cancel{\text{in}}} = 6.67\ \text{ft}$$

Then, calculate the volume of the tank.

$$\text{Volume of a Cylinder} = 0.785 \times (6.67\text{ ft})^2 \times 30\text{ ft} = 1{,}047.713595\text{ ft}^3 = 1{,}047.7\text{ ft}^3$$

Rectangular Prism

A **prism** is a three-dimensional solid with two bases that are the same size and shape polygons. A **rectangular prism** has bases that are rectangles.

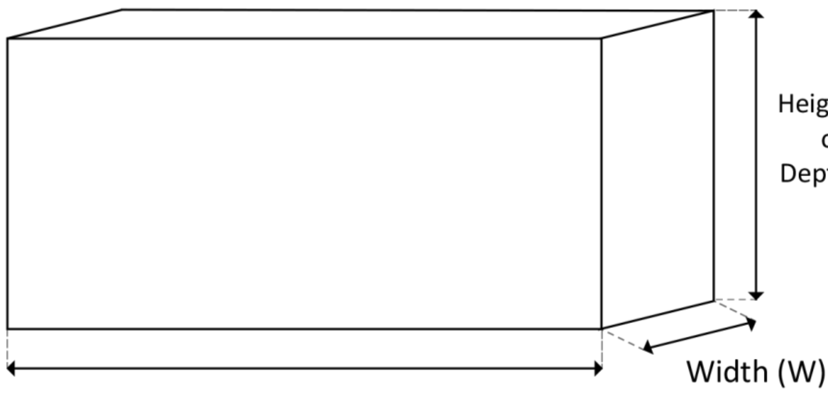


Figure 2.16[15]

$$\text{Volume of a Rectangular Prism} = L \times W \times H$$

Example: A rectangular basin is 225 feet long, 37 feet wide, and has a depth of 45 feet. How much water can the basin hold?

$$\text{Volume of a Rectangular Prism} = L \times W \times H$$

$$\text{Volume of a Rectangular Prism} = 225\text{ ft} \times 37\text{ ft} \times 45\text{ ft} = 374{,}625\text{ ft}^3$$

Note that the question is asking how much water will fit in the basin. You need to convert cubic feet to gallons in order to answer the question.

$$\frac{374{,}625\text{ ft}^3}{1} \times \frac{7.48\text{ gal}}{\text{ft}^3} = 2{,}802{,}195\text{ gal}$$

$$2{,}802{,}195\text{ gal} \times \frac{1\text{ MG}}{1{,}000{,}000\text{ gal}} = 2.8\text{ MG}$$

Note that 2.802195 MG rounds to 2.8 MG.

---

[15] Image by Marilyn Hightower is licensed under CC BY 4.0

Trapezoidal Prism

A **trapezoidal prism** is a prism with bases that are trapezoids.

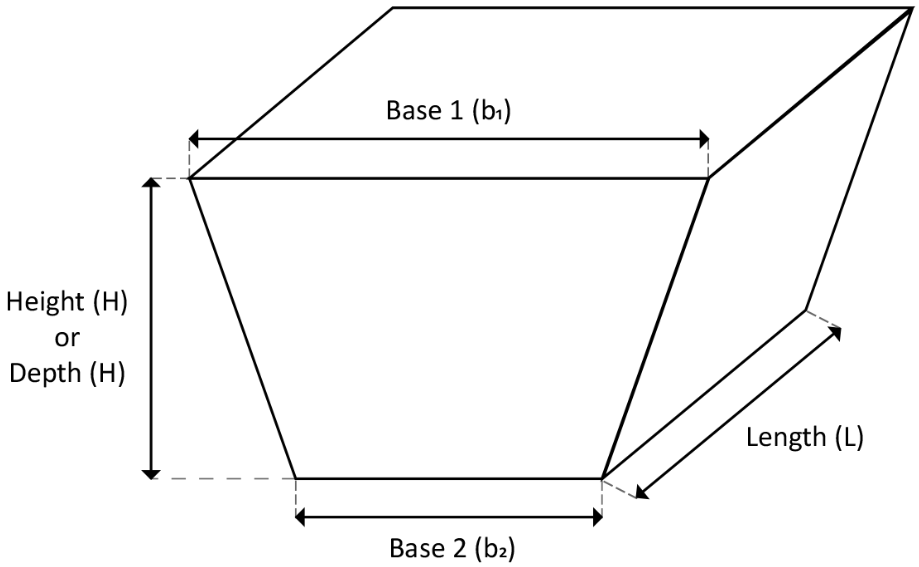


Figure 2.17[16]

$$\text{Volume of a Trapezoidal Prism} = \frac{b_1 + b_2}{2} \times H \times L$$

Example: The width across the bottom of an aqueduct measures 8 feet 6 inches. The width across the water level measures 14 feet 8 inches and the water is 10 feet deep. The aqueduct extends for 8,475 feet. How much water is in the aqueduct?

$$\text{Volume of a Trapezoidal Prism} = \frac{b_1 + b_2}{2} \times H \times L$$

First, we need to convert all of the measurements to feet.

$$8 \text{ ft } 6 \text{ in} = 8 \text{ ft} + \left(6 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}}\right) = 8 \text{ ft} + 0.5 \text{ ft} = 8.5 \text{ ft}$$

$$14 \text{ ft } 8 \text{ in} = 14 \text{ ft} + \left(8 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}}\right) = 14 \text{ ft} + 0.667 \text{ ft} = 14.667 \text{ ft}$$

Now we can calculate the volume of the aqueduct.

---

[16] Image by Marilyn Hightower is licensed under CC BY 4.0

$$\text{Trapezoidal Prism} = \frac{b_1 + b_2}{2} \times H \times L$$

$$\text{Trapezoidal Prism} = \frac{8.5 \text{ ft} + 14.67 \text{ ft}}{2} \times 10 \text{ ft} \times 8{,}475 \text{ ft} = 981{,}828.75 \text{ ft}^3$$

Note that the question is asking how much water is in the aqueduct. You need to convert cubic feet to gallons in order to answer the question.

$$\frac{981{,}828.75 \text{ ft}^3}{1} \times \frac{7.48 \text{ gal}}{\text{ft}^3} = 7{,}344{,}079.05 \text{ gal} = 7.3 \text{ MG}$$

Sphere

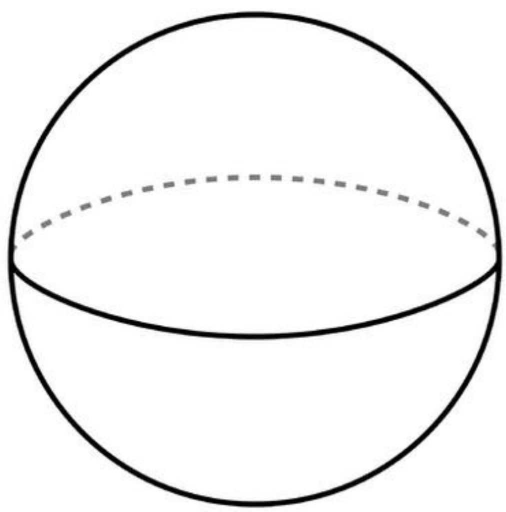


Figure 2.18[17]

$$\text{Volume of a Sphere} = \frac{\pi D^3}{6}$$ Where D is the diameter of the sphere.

Example: How many gallons of water can a 20-foot diameter spherical tank hold?

$$\text{Volume of a Sphere} = \frac{\pi D^3}{6}$$

$$\text{Volume} = \frac{\pi (20 \text{ ft})^3}{6} = \frac{3.14(8{,}000 \text{ ft}^3)}{6} =$$

[17] Image by Marilyn Hightower is licensed under CC BY 4.0

$$\text{Volume} = \frac{25{,}120 \text{ ft}^3}{6} = 4{,}187 \text{ ft}^3$$

You have found the volume of the sphere. However, the question is asking how many gallons of water can the sphere hold? To calculate the volume in gallons, convert cubic feet to gallons. Both units are units of volume.

$$\text{Volume} = 4{,}187 \text{ ft}^3 \times \frac{7.48 \text{ gal}}{1 \text{ cf}} = 31{,}319 \text{ gal}$$

Practice Problems 2.2

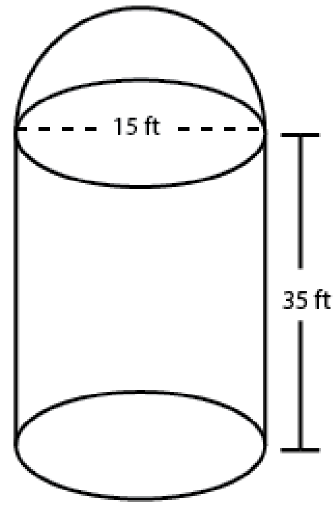
1. What is the volume of a 52-foot diameter sphere?

2. What is the volume of a 36” diameter pipe that is 1,500 feet long?

3. A half-full aqueduct is 10 miles long. It is 15 feet wide at the bottom, 24 feet wide at the top, and 25 feet tall. How many acre feet of water are in the aqueduct?

4. A sedimentation basin is 110 feet long, 40 feet wide, and 30 feet deep. How much water can it hold in million gallons?

5. A 35-foot tall cylinder with a 15-foot diameter is topped with a half sphere. How many gallons will it hold?



Exercise 2.2

1. What is the volume of a 30-foot diameter sphere?

2. What is the volume in 2,000 feet of 18-inch diameter pipe?

3. A 5-mile long aqueduct is 5 feet wide at the bottom and 8 feet wide at the water line. If the water depth is 6 ½ feet, how many acre-feet of water are in the aqueduct?

4. A sedimentation basin is 100 feet long, 30 feet wide, and 20 feet deep. How many gallons can it hold?

5. A cylinder is 80 feet tall and has a 25-foot diameter. The cylinder is topped with a half sphere. How many gallons will it hold?

2.3 AREA AND VOLUME WORD PROBLEMS

Water structures and components are often made up of multiple geometric shapes. An operator may need to determine the volume of water within a tank that sits on top of a tall pipe. An operator might calculate the volume of water in a storage structure or pipeline to determine how much chlorine is needed to disinfect the structure. A contractor might calculate the internal surface area of a tank to determine the amount of coating that is required. You might be asked to paint the interior walls of a room.

When solving word problems, there is always a specific question that must be answered. Identifying that question or problem is critical in solving the problem. Then you need to be able to pull out the information in the question needed to solve for the answer. Remember that the question may have extra information that is not needed to solve for what is being asked.

Use these steps to solve word problems.

> **Step 1:** Read and understand the problem to determine what is being asked. Know what you are trying to solve for.
>
> **Step 2:** Identify the information you need. You may want to underline key parts or circle units.
>
> **Step 3:** Make a sketch to clarify the information. This step works well in geometric problems when you are trying to solve for missing dimensions, but you may find other times when it is helpful to make a quick sketch.
>
> **Step 4:** Put the information into an equation or formula. Using a reference sheet for formulas is often key in this step. Make sure you copied the formula correctly!
>
> **Step 5:** Solve and double check your answer. Does the solution make sense? If your solution does not make sense, read the problem again.

Figure 2.19[18]

Example: A construction crew will be installing 1,250 feet of 10-inch diameter pipe. The width of the trench will be 30 inches and the depth 36 inches. After the pipe has been installed, how many cubic yards of dirt will be needed to backfill the trench (as seen above)? (Assume the trench will be backfilled up to 6 inches from the ground surface.)

Step 1: What is being asked in this problem?
How many cubic yards of dirt are needed to backfill the trench?

Step 2: Identify the information you need.
You need to know the size/volume of the trench and the size/volume of the pipe being placed in the trench in order to compute the total amount of dirt required to backfill the trench once the pipe has been installed.

Step 3: Make a sketch to clarify the information.

[18] Photo used with permission of SCV Water

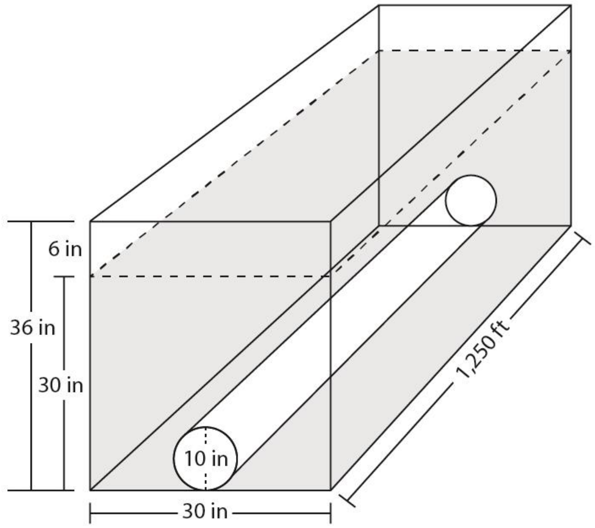


Figure 2.20[19]

Step 4: Put the information into an equation or formula. Solve.

Calculate the volume of the trench to the backfill height in cubic feet.

The height of the trench is 36 inches and the backfill is required up to 6 inches from the top of the trench. Therefore, the backfill will be at 30 inches (36" – 6" = 30"). Note that the width of the trench is also 30 inches.

$$\frac{30\ \text{in}}{1} \times \frac{1\ \text{ft}}{12\ \text{in}} = 2.5\ \text{ft}$$

$$\text{Volume of the trench at backfill height} = 2.5\ \text{ft} \times 2.5\ \text{ft} \times 1{,}250\ \text{ft} = 7{,}812.5\ \text{ft}^3$$

Calculate the volume of the pipe in cubic feet.

$$\frac{10\ \text{in}}{1} \times \frac{1\ \text{ft}}{12\ \text{in}} = 0.8333\ \text{ft}$$

$$\text{Volume of the pipe} = 0.785 \times (0.8333\ \text{ft})^2 \times 1{,}250\ \text{ft} = 681.42\ \text{ft}^3$$

The total volume of back fill required in cubic feet is the difference between the backfill trench volume and the pipe volume.

$$7{,}812.5\ \text{ft}^3 - 681.42\ \text{ft}^3 = 7{,}131.08\ \text{ft}^3$$

[19] Image by Marilyn Hightower is licensed under CC BY 4.0

Now convert the cubic feet to cubic yards to determine how many cubic yards of dirt are required to backfill the trench.

$$7{,}131.08 \text{ ft}^3 \times \frac{1 \text{ yd}^3}{27 \text{ ft}^3} = 264.1 \text{ yd}^3$$

Step 5: Solve and double check your answer. Does the solution make sense?

You can make sure that the answer is in cubic yards as the question was asking for an answer in cubic yards.

Note that when converting from cubic feet to cubic yards, there should be fewer cubic yards than cubic feet. This is because a cubic yard is larger than a cubic foot.

Currently, you may not have any idea if approximately 264 cubic yards makes sense as a solution to this problem or not. However, as you gain experience as an operator, you can work on developing a better sense of scale related to the various units frequently used.

Example: In the above example, the construction crew also needs to place 6 inches of aggregate base on top of the fill. How many cubic feet of base is needed?

Step 1: What is being asked in this problem?
The problem is asking how many cubic feet of aggregate base are needed to fill the trench on top of the dirt backfill.

Step 2: Identify the information you need.
You need to know the size/volume of the trench that still needs to be filled with aggregate base.

Step 3: Make a sketch to clarify the information.

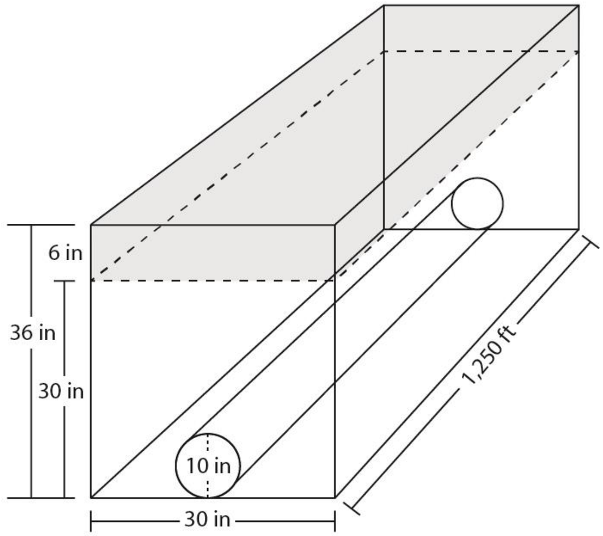


Figure 2.21[20]

Step 4/5: Put the information into an equation or formula. Solve.

Option #1: One way to solve this problem is to determine the difference between the entire volume of the trench and the volume of the trench at the dirt backfill height since the original problem stated that the backfill height is 6 in from the top of the trench.

The volume of the trench to the backfill height in cubic feet was previously calculated.

$$\text{Volume of the trench at backfill height} = 2.5\text{ ft} \times 2.5\text{ ft} \times 1{,}250\text{ ft} = 7{,}812.5\text{ ft}^3$$

Calculate the volume of the entire trench in cubic feet.

$$\frac{36\not{\text{in}}}{1} \times \frac{1\text{ ft}}{12\not{\text{in}}} = 3\text{ ft}$$

$$\text{Volume of the trench at backfill height} = 2.5\text{ ft} \times 3\text{ ft} \times 1{,}250\text{ ft} = 9{,}375\text{ ft}^3$$

The difference between these is the total cubic feet of aggregate base required to fill the trench in cubic feet.

$$9{,}375\text{ ft}^3 - 7{,}812.5\text{ ft}^3 = 1{,}562.5\text{ ft}^3$$

[20] Image by Marilyn Hightower is licensed under CC BY 4.0

Option #2: The other way to approach the solution to this problem is to directly calculate the amount of aggregate base. The problem statement indicates that 6 inches of aggregate base are needed.

The volume of aggregate base is then calculated as follows.

$$\frac{6\ \cancel{in}}{1} \times \frac{1\ ft}{12\ \cancel{in}} = 0.5\ ft$$

$$\text{Volume of the aggregate base} = 0.5\ ft \times 2.5\ ft \times 1{,}250\ ft = 1{,}562.5\ ft^3$$

Step 5: Solve and double check your answer. Does the solution make sense?

First check to make sure your solution is in the correct units, which are the units being requested in the problem. In this case, our answer is in cubic feet and the problem statement is asking for cubic feet.

Both ways of approaching the problem provide the correct answer. Choose the approach that seems most logical to you.

Sometimes you many need to solve multiple smaller problems to find what is being asked. Multiple shapes may be combined to create other system fixtures. In this image, you can see a large tank built on top of a pipe.

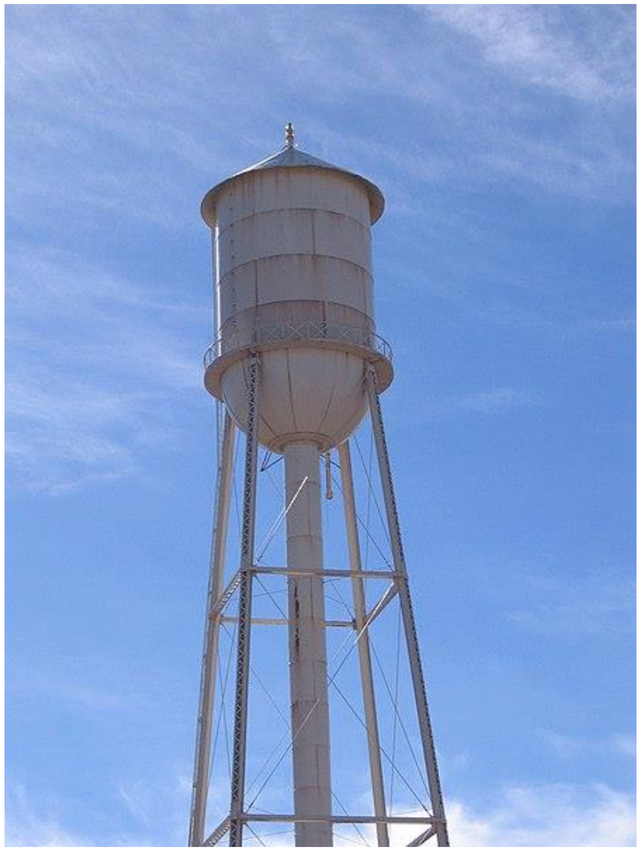


Figure 2.22[21]

When looking at a problem that contains an unfamiliar shape, the best approach is to divide the shape into smaller, more common shapes like rectangles, circles, or semi-circles. You can compute areas, volumes, etc. for these more common shapes and then add it all together to get the solution for the entire shape.

Example: A water storage tank is shaped like a "pill" with a cylinder and half of a sphere on each end. Each end has a 22-foot diameter and the center section is 15 feet long. How much water in gallons, can be stored in this tank?

Step 1: What is being asked in this problem?
The volume of water in gallons that can be stored in the tank.

Step 2: Identify the information you need.
You need to know the volume of the tank.

[21] Image by J. Nguyen~commonswiki is licensed under CC BY-SA 3.0

Step 3: Make a sketch to clarify the information.

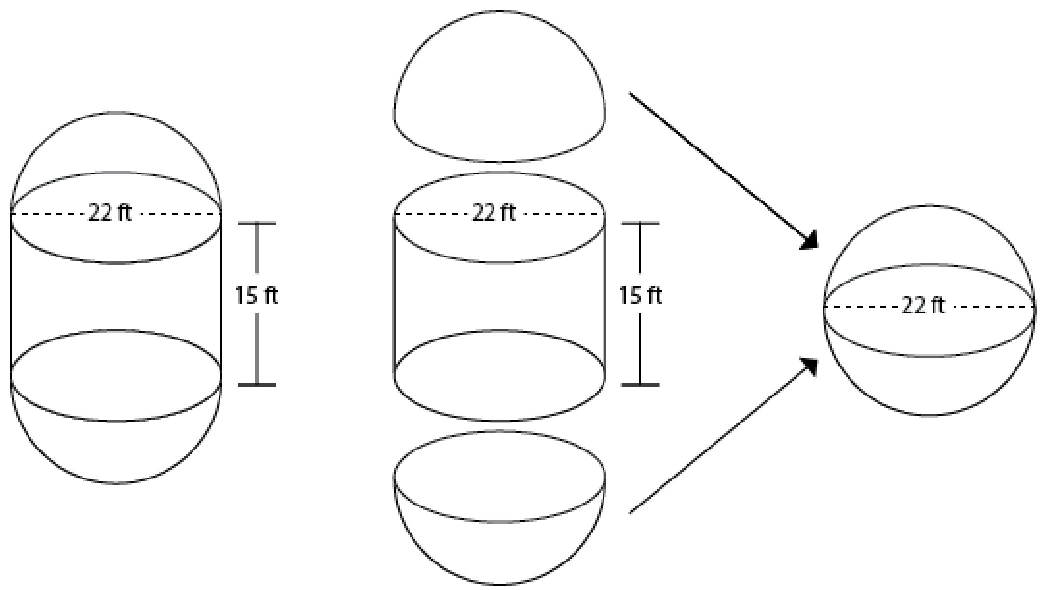


Figure 2.23[22]

Step 4: Put the information into an equation or formula. Solve.

To calculate the volume of the tank, you need to break the tank down into more common shapes. Each end of the tank is an identical half sphere with a 22-foot diameter. Put together, they combine to form a whole sphere with a 22-foot diameter. The center part of the tank is a cylinder with a diameter of 22 feet and a height or length of 15 feet.

Let's calculate the volume of the sphere first. (You can calculate either volume first).

$$\text{Volume of a Sphere} = \frac{\pi D^3}{6}$$

$$\text{Volume} = \frac{\pi(22\text{ ft})^3}{6} = \frac{3.14(10{,}648\text{ ft}^3)}{6} =$$

$$\text{Volume} = \frac{33{,}434.72\text{ ft}^3}{6} = 5{,}572.45\text{ ft}^3$$

[22] Image by Marilyn Hightower is licensed under CC BY 4.0

Calculate the volume of the cylinder in cubic feet.

$$\text{Volume of the cylinder} = 0.785 \times (22\text{ ft})^2 \times 15\text{ ft} = 5{,}699.1\text{ ft}^3$$

To calculate the total volume in the tank in cubic feet add the volume of the sphere to the volume of the cylinder.

$$5{,}572.45\text{ ft}^3 + 5{,}699.1\text{ ft}^3 = 11{,}271.55\text{ ft}^3$$

Now convert the cubic feet to gallons to determine how many gallons of water the tank can hold.

$$11{,}271.55\text{ ft}^3 \times \frac{7.48\text{ gal}}{1\text{ ft}^3} = 84{,}311.2\text{ gal}$$

Step 5: Solve and double check your answer. Does the solution make sense?

You can make sure that the answer is in gallons as the question was asking for an answer in gallons.

Note that when converting from cubic feet to gallons, there should be significantly more gallons than cubic feet. This is because there are more than 7 gallons in every cubic foot.

Practice Problems 2.3

1. A water utility operator needs to determine the cost of painting an above ground storage tank. The tank is 50 feet tall and has a diameter of 28 feet. One gallon of paint can cover 200 sf and costs $36.27 per gallon. What is the total cost to paint the storage tank?

2. A local amusement park requires a 0.5 MG storage tank. If the diameter of the tank is 55 feet, how tall will the tank need to be in order to store the 4.2 MG?

3. How many gallons of water are in a 32-foot diameter storage tank that sits on a 15-foot diameter, 45 foot tall pipe?

4. A water tank truck delivered 30 loads of water to a construction site. The water tank on the truck is shaped like a pill. Each end has a 10-foot diameter and the center section is 15 feet long. If the water costs $352 an AF, how much did the construction site pay for the water?

5. A maintenance crew is replacing an 18” meter at a pump. The specifications state that there needs to be 6.5 times the pipe diameter in feet of straight pipe before the meter and 4 times the pipe diameter in feet of straight pipe after the meter. How many feet of 18” pipe are needed?

6. A 1,200-foot section of a trapezoidal shaped aqueduct needs to be drained for maintenance. The aqueduct contains 5 AF of water, is 8 feet wide at the bottom, and is 14 feet wide at the water line. What is the water depth?

7. A water utility has installed 900 feet of 28” diameter pipe. They want to wrap a corrosion resistant sleeve around the pipe and fill the pipe to pressure test it. How many gallons of water will the pipe hold and how many square feet of corrosion resistant sleeve are required to cover the whole pipe?

8. Which of the following tanks will provide storage for 50,000 gallons of water?
    a. A spherical tank with a 20-foot diameter.
    b. A rectangular tank that is 20 feet by 30 feet by 12 feet

Exercise 2.3

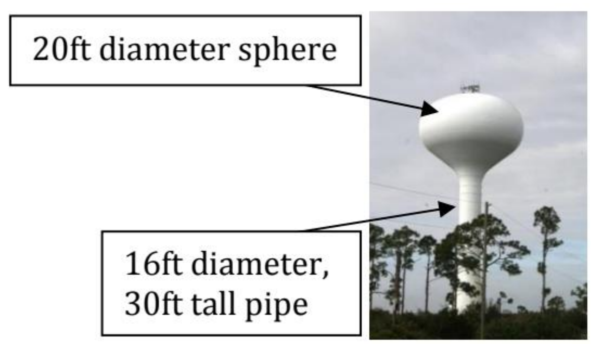
1. A water utility operator needs to determine how many gallons of paint are needed to paint the outside of an above ground storage tank and the cost of the paint. The tank has a 120-foot diameter and is 32 feet tall. (Assume that one gallon of paint can cover 125 ft^2 and costs $25.75 per gallon.)

2. A utility manager needs to find a site for a 3.1 MG storage tank. The tank cannot be taller than 33 feet. What diameter should this tank have?

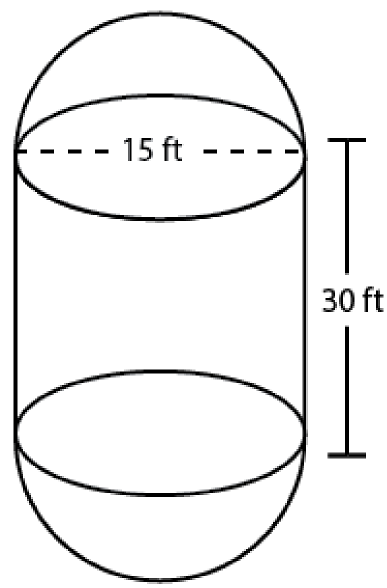
3. A construction crew will be installing 2,500 feet of 18-inch diameter pipe. The width of the trench will be 40 inches and the depth 45 inches. After the pipe has been installed, how many cubic yards of dirt will be needed to backfill the trench? (Assume the trench will be backfilled up to 8 inches from the ground surface.)

4. In the above problem, the construction crew also needs to place an aggregate base on top of the dirt fill in order to completely fill the trench. How many cubic feet of base is needed?

5. A water quality technician needs to disinfect an elevated storage tank, but first she needs to calculate the volume of water in the tank. The 20-foot diameter storage tank sits on a 16-foot diameter, 30 foot tall pipe. How many gallons are in the structure?



6. A private contractor needs water for a grading project. In a similar sized job, he used 155 tank loads from a water tower. The tower is shaped like a "pill." Each end has a 15-foot diameter and the center section is 30 feet long. If the water costs $425 an acre-foot, how much does the contractor need to budget for water?



7. A maintenance crew is replacing a 12" meter at a well. The specifications state that there needs to be 3 times the pipe diameter in feet of straight pipe before the meter and 5 times the pipe diameter in feet of straight pipe after the meter. How many feet of 12" pipe are needed?

8. A 1.25-mile section of trapezoidal shaped aqueduct needs to be drained. The aqueduct is 5 feet wide at the base and 10 feet wide at the water line. If there is 9-acre feet of water in the aqueduct, what is the depth?

9. A contractor just installed 350 feet of 8" diameter pipe. They want to wrap a corrosion resistant sleeve around the pipe and fill the pipe to pressure test it. How many gallons of water will the pipe hold and how many square feet of corrosion resistant sleeve are required to cover the whole pipe?

10. A water utility manager is determining what shaped storage tank should be used to store water for a small mobile home park. The mobile home park needs 110,000 gallons of storage. There is room for a 25-foot diameter and 30 foot tall cylinder shaped tank or a 30 foot diameter sphere shaped tank. Which tank will provide the adequate storage?

2.4 FLOW RATE

Flow rate is the measurement of a volume of liquid that passes through a given cross-sectional area (i.e., pipe) per unit in time.

$$\text{Flow Rate} = \frac{\text{Volume}}{\text{Time}}$$

In the waterworks industry, flow rates are expressed in several different units. The most common ones are shown below.

$$\text{cfs} = \frac{\text{cubic feet}}{\text{sec}} \qquad \text{gpm} = \frac{\text{gallons}}{\text{min}} \qquad \text{MGD} = \frac{\text{million gallons}}{\text{day}}$$

Depending on the application, flow rates are expressed in these or potentially other units. For example, the flow rate from a well or booster pump is commonly expressed as gpm, whereas annual production might be expressed as acre-feet per year (AFY). However, when solving a problem for flow rate the common unit of expression is cfs. The reason for this is in part due to the measurement of unit area of the structure that the water is passing through (i.e., pipe, culvert, aqueduct.) The areas for these structures are typically expressed as square feet (ft^2.) In addition, the speed (distance over time) at which the water is flowing is commonly expressed as feet per second. The flow rate formula and how the units are expressed are shown in the example below.

$$\text{Flow Rate (cfs)} = \text{Area } (ft^2) \times \text{Velocity (ft/sec)}$$

The letters Q, A, and V are used to represent Flow Rate, Area, and Velocity respectively in the equation.

$$Q\text{ (cfs)} = A\ (ft^2) \times V\text{ (ft/sec)}$$

Understanding flow rates and velocities can help with the design of pipe sizes for wells, pump stations, and treatment plants. Typically, velocities are in the range of 2 – 7 feet per second and can be used with a known flow rate to calculate the pipe diameter. For example, if a new well is being drilled and the pump test data determines that the well can produce a specific flow of 1,500 gpm, and you do not want the velocity to exceed 6.5 fps, then you can calculate the required pipe diameter.

Example: Given a flow rate of 1,500 gpm and a velocity of 6.5 fps, what is the required pipe diameter?

The problem statement is asking for a pipe diameter which means that you need to solve for Area in the flow rate equation.

$$Q\ (cfs) = A\ (ft^2) \times V\ (ft/sec)$$

$$A\ (ft^2) = \frac{Q\ (cfs)}{V\ (ft/sec)}$$

Before you can substitute the values provided in the problem statement into the formula, you must make sure that all of the units align. The flow rate is provided in gpm and the velocity is provided as fps. Convert gpm to cfs.

$$\frac{1{,}500\ gpm}{1} \times \frac{1\ cfs}{448.8\ gpm} = 3.3422\ cfs = 3.34\ cfs$$

Now you can substitute the values into the equation and solve for Area.

$$A\ (ft^2) = \frac{Q\ (cfs)}{V\ (ft/sec)} = \frac{3.34\ cfs}{6.5\ fps} = 0.513846\ ft^2 = 0.51\ ft^2$$

To determine the diameter of the pipe, use the formula for the area of a circle and solve for diameter.

$$Area = 0.785 \times D^2$$

$$D^2 = \frac{Area}{0.785} = \frac{0.51\ ft^2}{0.785} = 0.6496815\ ft^2$$

Note that this is what D squared equals, not D. In order to solve for D, you need to take the square root of both sides of the equation.

$$\sqrt{D^2} = \sqrt{0.6496815\ ft^2}$$

$$D = 0.806028\ ft \times \frac{12\ in}{1\ ft} = 9.6723\ in = 9.7\ in$$

Length in small diameter pipes is expressed in inches, so you would need a 10-inch pipe.

Key Terms

- **circumference** – the distance around the circle
- **circle** – a round figure whose boundary is an equal distance from the center
- **cylinder** – a three-dimensional solid with two bases that are circles (or ovals)
- **diameter –** the distance across the circle through the diameter
- **flow rate** – the measurement of a volume of liquid that flows over a cross-sectional area over time
- **prism** – a three-dimensional solid with two bases that are the same size and shape polygon
- **radius** – the distance from the center to the edge of the circle
- **rectangle** – a four-sided figure with four right (90 degree) angles
- **rectangular prism** – a prism with bases that are trapezoids
- **sphere** – a three-dimensional solid is entirely made of points an equal distance from the center; a ball
- **square** – a type of rectangle with four sides that are the same length
- **trapezoid** – four-sided shape with one set of parallel sides called bases; aqueducts are commonly trapezoids in cross-section
- **trapezoidal prism** – a prism with bases that are trapezoids

Practice Problems 2.4

1. What is the flow rate in MGD of a 30" diameter pipe with a velocity of 5.5 fps?

2. What is the velocity through a box culvert that is 8 feet wide and 5 feet deep if the daily flow is 44 AF?

3. What is the area of a pipe that flows 3.1 MGD and has a velocity of 9 fps?

4. What is the diameter of a pipe that flows 1,425 gpm with a velocity of approximately 2.7 fps?

5. A 20-mile aqueduct flows 22,200 AFY at an average velocity of 0.32 fps. If the distance across the top is 20 feet and the depth is 6 feet, what is the distance across the bottom?

Exercise 2.4

Solve the following problems.

1. What is the flow rate in MGD of a 24" diameter pipe with a velocity of 3 fps?

2. What is the velocity through a box culvert that is 3 feet wide and 2 feet deep if the daily flow is 27 AF?

3. What is the area of a pipe that flows 1.5 MGD and has a velocity of 5 fps?

4. What is the diameter of a pipe that flows 2,500 gpm with a velocity of approximately 7 fps?

5. A 15-mile aqueduct flows 30,000 AFY at an average velocity of 0.45 fps. If the distance across the top is 13 feet and the depth is 8 feet, what is the distance across the bottom?

UNIT 3

3.1 DENSITY AND SPECIFIC GRAVITY

How much does water actually weigh? There are a few variables, such as temperature, that determine the weight of water, but for all practical purposes in waterworks mathematics, water weighs 8.34 pounds per gallon.

The **density** or mass per unit volume of water is 1 gram per cubic centimeter. The ratio of a solution's density compared to the density of water is known as **specific gravity.** Water has a specific gravity of 1.00.

$$\frac{\text{Density of a Solution}}{\text{Density of Water}} = \text{Specific Gravity} = \frac{\text{lbs/gal of a Solution}}{\text{lbs/gal of Water}}$$

Tip: The weight ratio of a solution compared to that of water can also be used to calculate specific gravity.

In order to understand specific gravity, we need to have a better understanding of density and its relationship to mass. Mass is the amount of matter (atoms) in a given substance. Take saltwater for example. We know that one gallon of water weighs 8.34 gallons. However, if one pound (453 grams) of salt is added to create a saltwater solution, then that same gallon of water would weigh 9.34 pounds per gallon instead. It is the same volume (one gallon) but has a different weight due to the additional mass that was added from the salt. Hence the definition of density, mass per unit volume. The amount of "stuff" in a given volume.

As discussed, the specific gravity of water is used as the reference point for comparisons. If something has a specific gravity less than water (<1), then the substance will float on water. Conversely, if a substance has a specific gravity greater than 1, it will sink in water. The table below lists common specific gravities and weights of substances used in the waterworks industry. On any State exam, you will be given the specific gravity or corresponding weight of the substance in the question.

Substance	Specific Gravity	Weight
Crude Oil	0.815	6.80 lbs/gal
Water	1.00	8.34 lbs/gal
8% Sodium Hypochlorite	1.12	9.34 lbs/gal
12.5% Sodium Hypochlorite	1.20	10.0 lbs/gal
Alum	1.16 – 1.40	9.67 – 11.68 lbs/gal
Ferric chloride	1.43	11.93 lbs/gal

Substance	Specific Gravity	Weight
Calcium hypochlorite	2.35	19.60 lbs/gal
Chlorine (g)	2.49	20.77 lbs/gal

The specific gravities in the table are approximate. Professionally, you will consult the Safety Data Sheets for the value.

Note that everything listed in the table after water is heavier, and before water is lighter. Also, it's important to consider that sodium hypochlorite has a different specific gravity depending on the concentration of the solution. This is true of all chemical solutions.

Since water is the reference, then a specific gravity (SG) of 1 and a weight of 8.34 lbs/gal are the reference numbers needed to calculate the SG and weight of other substances. We can use this information as a conversion factor to determine the SG or weight of other substances. The conversion can be expressed as follows:

$$\frac{8.34 \text{ lbs/gal}}{1 \text{ SG}} \quad \text{or} \quad \frac{1 \text{ SG}}{8.34 \text{ lbs/gal}}$$

Let's take a look at how to apply this information.

Example: What is the weight of ferric chloride in lbs/gal if it has a SG of 1.43?

The problem statement indicates a SG of 1.43 for ferric chloride. Based on the SG of water being 1 and weighing 8.34 lbs/gal, we can determine the weight of the ferric chloride.

$$\frac{8.34 \text{ lbs/gal}}{1 \text{ SG}} \times \frac{1.43 \text{ SG}}{1} = 11.9262 = 11.93 \frac{\text{lbs}}{\text{gal}}$$

Remember, anything that has a SG >1 will weigh more than 8.34 lbs/gal.

Example: What is the SG of ferric chloride that weighs 11.93 lbs/gal?

In this problem, the weight of ferric chloride is provided, and you are asked to calculate the specific gravity (SG). Using the conversion above, you can calculate the SG for ferric chloride.

$$\frac{1 \text{ SG}}{8.34 \text{ lbs/gal}} \times \frac{11.93 \text{ lbs/gal}}{1} = 1.43 \text{ SG}$$

Remember, anything with a weight <8.34 lbs/gal will have a SG <1.

Practice Problems 3.1

1. Liquid sodium hypochlorite has a specific gravity of 1.69. What is the corresponding weight in pounds per gallon?

2. Chlorine gas is cooled and pressurized into a liquid state. It weighs 17.31 lbs/gal. What is the specific gravity?

3. What is the weight difference between 111 gallons of water and 61 gallons of sodium hypochlorite with a specific gravity of 1.37?

4. A treatment operator has 75 gallons of 14.5% sodium hypochlorite. How many pounds of the 75 gallons are available chlorine?

5. The specific gravity of 25% Alum is 1.24. How much does 83 gallons of 25% Alum weigh?

6. Ferric chloride weighs 19.44 lbs/gal. What is the specific gravity?

7. How many pounds of ferric chloride are in 92 gallons of 33% strength? (Assume the specific gravity is 1.52.)

8. What is the weight in lbs/cf of a substance that has a specific gravity of 1.47?

9. A shipment of crude oil has a specific gravity of 0.674. What is the weight in lbs/cf?

Exercise 3.1

Solve the following density-related problems.

1. Liquid sodium hypochlorite has a specific gravity of 1.47. What is the corresponding weight in pounds per gallon?

2. Chlorine gas has formed into a liquid state. It weighs 19.75 lbs/gal. What is the specific gravity?

3. What is the weight difference between 75 gallons of water and 42 gallons of sodium hypochlorite with a specific gravity of 1.42?

4. A treatment operator has 50 gallons of 12.5% sodium hypochlorite. How many pounds of the 50 gallons are available chlorine?

5. The specific gravity of 25% Alum is 1.35. How much does 45 gallons of 25% Alum weigh?

6. Ferric chloride weighs 14.25 lbs/gal. What is the specific gravity?

7. How many pounds of ferric chloride are in 250 gallons of 22% strength? (Assume the specific gravity is 1.41)

8. What is the weight in lbs/cf of a substance that has a specific gravity of 2.05?

9. A shipment of crude oil has a specific gravity of 0.825. What is the weight in lbs/cf?

3.2 PARTS-PER NOTATION

Parts-per notation is used to describe very small quantities of chemical concentrations. Most of the time, chemical concentrations are expressed in percentages. However, it is important to understand the relationship between the concentration expressed as a percentage and the concentration expressed in parts-per notation.

Commonly used parts-per notation is below:

Parts-per Notation	Parts-per Acronym	Value	Value in Scientific Notation
Parts per Hundred	pph	0.01	10^{-2}
Parts per Million	ppm	0.000001	10^{-6}
Parts per Billion	ppb	0.000000001	10^{-9}
Parts per Trillion	ppt	0.000000000001	10^{-12}

Because the amounts are small, examples will help.

One part per million is the equivalent of a drop of ink in 55 gallons of water.

One part per billion is the equivalent of one drop of ink in 55 barrels of water.
One part per billion is the equivalent of the width of a human hair within 68 miles.

One part per trillion is the equivalent of one drop in 500,000 barrels of water.
One part per trillion is the equivalent to six inches in 93 million miles, the distance to the sun.[23]

In chemical dosage-related problems, concentrations are typically expressed in parts per million, ppm. Therefore, we are most interested in converting a percentage to ppm and ppm to a percentage. If you divide 1,000,000 (1 ppm) by 100 (100%), you get the following:

$$\frac{1 \text{ ppm}}{100\%} = \frac{1{,}000{,}000}{100} = 10{,}000$$

Therefore, a solution with a 1% chemical concentration can also be expressed as a solution with a 10,000-ppm chemical concentration.

$$1\% = 10{,}000 \text{ ppm}$$

[23] https://www.watereducation.org/aquapedia-background/parts-notation and

https://www.secnav.navy.mil/eie/Pages/DrinkingWaterConcentrations.aspx

To convert between a percentage and ppm, multiply the percent solution by 10,000. Note that you do not convert the percentage to a decimal before multiplying. It has already been accounted for in the conversion.

Percent Concentration	Ppm
1%	10,000 ppm
2%	20,000 ppm
3%	30,000 ppm
10%	100,000 ppm

Example: What is the ppm of a 25% solution?

$$25 \times 10{,}000 = 25{,}000 \text{ ppm}$$

Example: A water utility uses a 0.5% sodium hypochlorite solution to disinfect a well. What is the ppm concentration of the solution?

To solve this problem, you need to take the percent of the solution given in the problem and multiply by 10,000 in order to have an answer in parts per million.

$$0.5 \times 10{,}000 = 5{,}000 \text{ ppm}$$

Now let's look at the differences between parts per million (ppm), parts per billion (ppb), and parts per trillion (ppt.) As water quality regulations become more stringent and laboratory analysis techniques get better and better, contaminants are being identified at smaller and smaller concentrations. Most water quality standards are expressed in ppm or milligrams per liter (mg/L), but many are expressed in ppb or micrograms per liter (ug/L), and a few are expressed in ppt or nanograms per liter (ng/L). Another way to express the amount of contaminant in water supplies is as follows.

$$1{,}000{,}000 = 1 \text{ million}$$

$$1{,}000{,}000{,}000 = 1 \text{ billion}$$

$$1{,}000{,}000{,}000{,}000 = 1 \text{ trillion}$$

Based on this, one billion is equal to one thousand million and one trillion is equal to one thousand billion.

$$1 \text{ billion} = 1{,}000 \text{ million}$$

$$1 \text{ trillion} = 1{,}000 \text{ billion}$$

So how does this relate to our very small concentrations of ppm, ppb, and ppt?

$$1 \text{ ppm} = 1{,}000 \text{ ppb} = 1{,}000{,}000 \text{ ppt}$$

The expression above says that 1 part of a small number (ppm) equals 1,000 parts of a smaller number (ppb) which equals 1,000,000 parts of an even smaller number (ppt). You can further simplify the difference between ppb and ppt as follows.

1 ppb = 1,000 ppt

Example: Complete the following table with the corresponding unit for the various water quality Maximum Contaminant Levels (MCL).

Constituent	ppm	ppb	ppt
Arsenic		14	
Chromium	0.19		
Nitrate (NO_3)	57		
Perchlorate			7,400
Vinyl chloride		0.8	

To complete the chart, we will start with the first constituent, Arsenic, and work our way down completing one row at a time.

To convert ppb to ppm, you divide ppb by 1,000.

$$\text{Arsenic: } 14 \text{ ppb} \times \frac{1 \text{ ppm}}{1{,}000 \text{ ppb}} = 0.014 \text{ ppm}$$

To convert ppb to ppt, you multiply ppb by 1,000.

$$\text{Arsenic: } 14 \text{ ppb} \times \frac{1{,}000{,}000 \text{ ppt}}{1{,}000 \text{ ppb}} = 14 \text{ ppb} \times \frac{1{,}000 \text{ ppt}}{1 \text{ ppb}} = 14{,}000 \text{ ppt}$$

Now, we are going to move to the row for Chromium. To convert ppm to ppb, you multiply ppm by 1,000.

$$\text{Chromium: } 0.19 \text{ ppm} \times \frac{1{,}000 \text{ ppb}}{1 \text{ ppm}} = 190 \text{ ppb}$$

To convert ppb to ppt, you multiply ppb by 1,000.

$$\text{Chromium: } 190 \text{ ppb} \times \frac{1{,}000{,}000 \text{ ppt}}{1{,}000 \text{ ppb}} = 190 \text{ ppb} \times \frac{1{,}000 \text{ ppt}}{1 \text{ ppb}} = 190{,}000 \text{ ppt}$$

And now we are moving to the row for Nitrate. To convert ppm to ppb, you multiply ppm by 1,000.

$$\text{Nitrate } (NO_3) : 57 \text{ ppm} \times \frac{1{,}000 \text{ ppb}}{1 \text{ ppm}} = 57{,}000 \text{ ppb}$$

To convert ppb to ppt, you multiply ppb by 1,000.

$$\text{Nitrate } (NO_3) : 57{,}000 \text{ ppb} \times \frac{1{,}000 \text{ ppt}}{1 \text{ ppb}} = 57{,}000{,}000 \text{ ppt}$$

Now, we are moving to the row for Perchlorate. To convert ppt to ppb, you divide ppt by 1,000.

$$\text{Perchlorate} : 7{,}400 \text{ ppt} \times \frac{1 \text{ ppb}}{1{,}000 \text{ ppt}} = 7.4 \text{ ppb}$$

To convert ppb to ppm, you divide ppb by 1,000.

$$\text{Perchlorate} : 7.4 \text{ ppb} \times \frac{1 \text{ ppm}}{1{,}000 \text{ ppb}} = 0.0074 \text{ ppm}$$

And finally, we are moving to the row for Vinyl Chloride. To convert ppb to ppm, you divide ppb by 1,000.

$$\text{Vinyl chloride: } 0.8 \text{ ppb} \times \frac{1 \text{ ppm}}{1{,}000 \text{ ppb}} = 0.0008 \text{ ppm}$$

To convert ppb to ppt, you multiply ppb by 1,000.

$$\text{Vinyl chloride: } 0.8 \text{ ppb} \times \frac{1{,}000 \text{ ppt}}{1 \text{ ppb}} = 800 \text{ ppt}$$

Here is the completed table with the numbers calculated in bold.

Constituent	ppm	ppb	ppt
Arsenic	**0.014**	14	**14,000**
Chromium	0.19	**190**	**190,000**
Nitrate (NO_3)	57	**57,000**	**57,000,000**
Perchlorate	**0.0074**	**7.4**	7,400
Vinyl chloride	**0.0008**	0.8	**800**

Practice Problems 3.2

1. An 87.5% chlorine solution has a ppm concentration of?

2. What is the percent concentration of a 471-ppm solution?

3. A water utility uses a 12.7% sodium hypochlorite solution to disinfect a well. What is the ppm concentration of the solution?

4. A container of liquid chlorine has a concentration of 390 ppm. What is the percent concentration of the solution?

5. Complete the following table with the corresponding unit for the various water quality Maximum Contaminant Levels (MCL).

Constituent	ppm	ppb	ppt
Arsenic		51	
Chromium	1.74		
Nitrate (NO3)	112		
Perchlorate			90,832
Vinyl chloride		0.75	

Exercise 3.2

Solve the following problems. Think of the "%" symbol as "pph" (parts per hundred).

1. A 12.5% chlorine solution has a ppm concentration of?

2. What is the percent concentration of a 100-ppm solution?

3. A water utility uses a 0.8% sodium hypochlorite solution to disinfect a well. What is the ppm concentration of the solution?

4. A container of liquid chlorine has a concentration of 1,250 ppm. What is the percent concentration of the solution?

5. Complete the following table with the corresponding unit for the various water quality Maximum Contaminant Levels (MCL).

Constituent	Ppm	ppb	ppt
Arsenic		10	
Chromium	0.05		
Nitrate (NO3)	45		
Perchlorate			6,000
Vinyl chloride		0.5	

3.3 MIXING AND DILUTING SOLUTIONS

The water industry uses a variety of different chemicals for various processes in water treatment. These chemicals can come in different forms including gas, solid, or liquid (typically a solution). A **solution** is a liquid often made up of water and one or more chemicals. The concentration of a solution is dependent on the amount of chemical that is diluted within the water and can be represented as a percentage.

The percentage strength of a disinfectant solution represents the amount of active chemical available to inactivate pathogenic bacteria. For example, if you were to separate the two components of a 10% bleach solution, you would see that it is made up of 90% water and 10% sodium hypochlorite (active chemical).

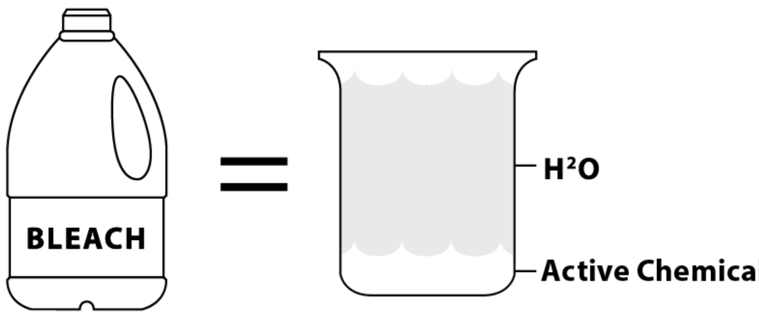


Figure3.1 [24]

Chemicals sometimes need to be diluted or mixed so that operators can safely work with them or to match the needs of a treatment process. The following formula can be used to calculate either the new concentration of a mixed solution, or the required volume needed to achieve a desired concentration.

$$C_1V_1 = C_2V_2$$

C = Concentration represented as a percentage

V = Volume of the solution in mL, gallons, etc.

Example: If 700 mL of water is added to 250 mL of a 65% solution, what is the resulting solution's diluted concentration strength?

First you need to identify which information is being provided and what is being asked. We know that one of the solutions is 250 mL at a 65% concentration.

[24] Image by Marilyn Hightower is licensed under CC BY 4.0

$C_1 = 65\%$

$V_1 = 250$ mL

Then you add 700 mL of water to the existing solution and want to know what the resulting concentration is of the new solution. Since we are adding water, the solution is being diluted.

Total Volume of the new solution is:

$$250 \text{ mL} + 700 \text{ mL} = 950 \text{ mL}$$

C_2 = ? %
V_2 = 950 mL

Now you can substitute all the information into the equation and solve for the unknown quantity.

$$C_1V_1 = C_2V_2$$

$$(65\%)(250 \text{ mL}) = C_2(950 \text{ mL})$$

Rearrange the terms to isolate the variable and convert the percent to a decimal in order to perform the calculation. To convert from a percent to a decimal, move the decimal two places to the left.

$$C_2 = \frac{(0.65)(250 \text{ mL})}{(950 \text{ mL})}$$

$$C_2 = \frac{162.5}{950} = 0.17105$$

Multiply the answer by 100 to convert from a decimal to a percent or move the decimal two places to the right.

$$C_2 = 0.17105 \times 100 = 17.1\%$$

This says if 250 mL of a 65% solution is diluted with 700 mL of water, the diluted solution's concentration will be 17%.

Key Terms

- **density** – mass per unit of volume
- **parts-per notation** – a way of describing small quantities of chemical concentrates, often expressed as percentages
- **solution** – a liquid often made up of water and one or more chemicals
- **specific gravity** – the ratio of a solution's density compared to the density of water

Practice Problems 3.3

1. How many gallons are needed to dilute 30-gallons of 18.75% sodium hypochlorite solution to a 10% solution?

2. If a 500-gallon container is 1/2 full of a 14% solution and is then completely filled with fresh water, what would the resulting ppm of the water be?

3. A chlorine storage tank that is 6 ft high with a 3ft diameter contains 227 gallons of 30% chlorine solution. If the tank is filled up with water, what will the new diluted concentration be?

4. 45 gallons of a 223,000ppm solution are mixed with 100 gallons of water. What is the concentration of the diluted solution? (Express the answer as a percentage.)

Exercise 3.3

1. How many gallons are needed to dilute 15-gallons of 12.5% sodium hypochlorite solution to a 6% solution?

2. If a 100-gallon container is ¾ full of a 5.25% solution and is then completely filled with fresh water, what would the resulting ppm of the water be?

3. A chlorine storage tank that is 10 ft high with a 5ft diameter contains 400 gallons of 20% chlorine solution. If the tank is filled up with water, what will the new diluted concentration be?

4. 10 gallons of a 50,000ppm solution are mixed with 15 gallons of water. What is the concentration of the diluted solution? (Express the answer as a percentage)

UNIT 4

4.1 CHEMICAL DOSAGE ANALYSIS

One of the most common and useful formulas in waterworks mathematics is used to calculate the amount of chemical needed to add to water. It is commonly known as the **Pound Formula** because it is a calculation for the weight of a chemical that is being added to water. Typically, this chemical is chlorine or a chlorine-related compound. However, it can be also used to calculate alum, ferric chloride, or any other type of chemical dosage.

Recall that the pound formula can be used in two forms.

The first is used when you are calculating chemical dosage in a volume of water, for example a tank, reservoir, or pipeline.

Pound Formula: $$\frac{\text{MG}}{1} \times \frac{8.34 \text{ lbs}}{\text{gal}} \times \frac{\text{parts}}{\text{million parts}} = \frac{\text{lbs}}{1}$$

The second is used when you are calculating chemical dosage on a flow rate, for example through a pipeline, in a channel, or through a treatment facility.

Pound Formula: $$\frac{\text{MG}}{\text{D}} \times \frac{8.34 \text{ lbs}}{\text{gal}} \times \frac{\text{parts}}{\text{million parts}} = \frac{\text{lbs}}{\text{day}}$$

Note that the only difference between the two formulas is the unit for time. It is important to remember that MGD means Million Gallons per Day and this expression can be written as the following.

MGD or $\frac{\text{MG}}{\text{D}}$

Specific units for volume and flow must be used in the Pound Formulas. Volume must be in million gallons and flow must be in million gallons per day before you solve.

You may want to use the Pie Wheel to solve pound formula problems. In the pie wheel, the horizontal line across the middle of the pie wheel represents a division sign. Anything above the line should be divided by anything below the line. Everything below the line, which are variables next to one another, are multiplied together. Anything in the bottom half of the pie wheel is multiplied together to get the answer in the top half of the wheel.

The following charts or Pie Wheels are helpful in illustrating how to apply the Pound Formula.

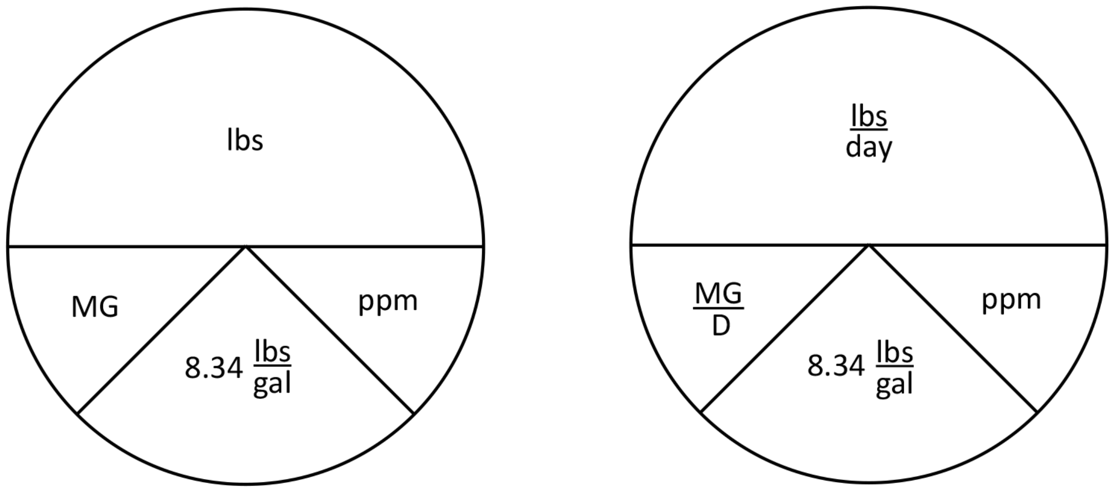


Figure 4.1[25]

Example: How many pounds of chlorine are needed to dose 2 MG of water to a dosage of 3.25 ppm?

Here is the Pound Formula.

$$\text{MG} \times \frac{8.34\ \text{lbs}}{\text{gal}} \times \text{ppm} = \text{lbs}$$

To calculate the lbs of chlorine, plug the values into the formula and solve.

$$2\ \text{MG} \times \frac{8.34\ \text{lbs}}{\text{gal}} \times 3.25\ \text{ppm} = 54.21\ \text{lbs}$$

It's pretty easy to use the Pound Formula to calculate how many pounds of chlorine are needed to provide a certain dosage if we are using 100% concentration of any chemical. However, most chemicals used are not in pure 100% form.

In many treatment plants and at treatment sites within distribution systems, the use of chlorine gas is in decline, unless the plant is of considerable size. In gas form, 100 lbs of gas chlorine is 100 lbs of available chlorine. The reduction in chlorine gas usage is primarily due to safety concerns and other forms of chlorine being less expensive. For example, groundwater wells are commonly disinfected with solid (calcium hypochlorite) or liquid (sodium hypochlorite) chlorine. In addition, other chemicals such as alum, ferric chloride, sodium hydroxide are used in varying concentration strengths at treatment plants in addition to chlorine.

When solving dosage problems with chemicals of different concentrations, you will still use the Pound Formula, but you need to adjust for the concentration strength of the chemical in the calculation.

$$\text{Pound Formula:} \quad \frac{\text{MG} \times \frac{8.34\text{ lbs}}{\text{gal}} \times \text{ppm}}{\%\text{ concentration}} = \text{lbs}$$

$$\text{Pound Formula:} \quad \frac{\frac{\text{MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}} \times \text{ppm}}{\%\text{ concentration}} = \frac{\text{lbs}}{\text{day}}$$

If you are calculating the number of pounds needed, you divide by the decimal equivalent of the percent concentration. Since it is not at 100% concentration, you need more of the chemical. When you divide by a number less than one, you get a larger number, in this case, the lbs of chemical needed at the reduced concentration level.

Example: How many pounds of 10% Alum are needed to dose a treatment flow of 5 MGD to a dosage of 10 ppm?

$$\text{Pound Formula} \rightarrow \frac{\frac{\text{MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}} \times \text{ppm}}{\%\text{ concentration}} = \frac{\text{lbs}}{\text{day}}$$

First, solve the pound formula in the numerator by taking information from the problem and substituting it into the formula. In this case, you know 5 MGD and 10 ppm

$$\frac{5\text{ MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}} \times 10\text{ ppm} = \frac{417\text{ lbs}}{\text{day}}$$

It takes 417 lbs per day of 100% Alum to dose 5 MGD to 10 ppm. However, the Alum being used is only a 10% concentration. To adjust for the percent concentration, divide lbs per day by the percent concentration. Remember that to change a percent to a decimal, you need to move the decimal point over two places to the left.

$$\frac{\frac{417\text{ lbs}}{\text{day}}}{\%\text{ concentration}} = \frac{\frac{417\text{ lbs}}{\text{day}}}{10\%} = \frac{\frac{417\text{ lbs}}{\text{day}}}{0.10} = \frac{4{,}170\text{ lbs}}{\text{day}}$$

At only a 10% concentration, you will need 4,170 lbs of 10% Alum, significantly more, to get the equivalent of 417 lbs at full concentration. Therefore, to determine the amount of 10% Alum needed, you divide the total amount by 10% (or 0.1).

If you are solving for pounds, you divide by the percent concentration.

If the number of pounds is known, multiplying by the decimal equivalent of the percent concentration will calculate how much of that chemical is available in the total pounds of the substance. Multiplying by a number less than one yields a smaller number.

Example: An operator added 382 pounds of 15% ferric chloride to a treatment flow of 12.9 MGD. What was the corresponding dosage?

$$\text{Pound Formula} \rightarrow \frac{\frac{\text{MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}} \times \text{ppm}}{\%\text{ concentration}} = \frac{\text{lbs}}{\text{day}}$$

First, substitute all of the known information into the formula.

$$\text{Pound Formula} \rightarrow \frac{\frac{12.9\text{ MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}} \times \text{ppm}}{15\%} = \frac{382\text{ lbs}}{\text{day}}$$

To solve for the dosage, you need to rearrange the terms in the equation. In this step, you can see that you are multiplying the lbs per day by the decimal equivalent of the percent concentration (15% is equal to 0.15)

$$\frac{12.9\text{ MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}} \times \text{ppm} = \frac{382\text{ lbs}}{\text{day}} \times 0.15$$

Now, isolate the variable, ppm.

$$\text{ppm} = \frac{\frac{382\text{ lbs}}{\text{day}} \times 0.15}{\frac{12.9\text{ MG}}{\text{D}} \times \frac{8.34\text{ lbs}}{\text{gal}}}$$

$$\text{ppm} = \frac{\frac{57.3\text{ lbs}}{\text{day}}}{\frac{107.586\text{ lbs}}{\text{day}}} = 0.533$$

The dosage applied is 0.533 ppm.

In problems where pounds are given, you multiply by the percent concentration.

Once you understand the concept behind the problem, it makes solving them easier. Think of it this way: it takes much more 10% ferric chloride in the coagulation process than 75% ferric chloride. The same is true if you are using calcium hypochlorite as opposed to gas chlorine, because gas is at a greater strength (100%) than calcium hypochlorite.

Example: You have 100 pounds of 65% calcium hypochlorite solution. How many pounds are calcium hypochlorite?

$$100 \text{ lbs} \times 65\% =$$

$$100 \text{ lbs} \times 0.65 = 65 \text{ lbs of calcium hypochlorite}$$

Therefore, if you have 100 pounds of 65% calcium hypochlorite solution only 65 pounds of the substance is actually calcium hypochlorite, the portion actively working as the disinfectant.

The last chemical dosage concept we need to look at is when the chemical being used is in the form of a liquid. Since the Pound Formula is measuring chemicals in pounds, the liquid chemical needs to be expressed as pounds. In Unit 3, we learned about specific gravity and how it affects the weight of a substance. You will need to use that information when presented with a pound formula question where the chemical used is a liquid.

Example: The specific gravity of 12.5% sodium hypochlorite is 1.89. If 220 lbs per day are used, how many gallons of sodium hypochlorite are needed per hour?

From Unit 3, you can use the specific gravity to calculate the lbs per gallon.

$$\frac{8.34 \text{ lbs/gal}}{1 \text{ SG}} \times \frac{1.89 \text{ SG}}{1} = 15.7626 = 15.76 \frac{\text{lbs}}{\text{gal}}$$

You know that 220 lbs per day are being used and that it weighs 15.76 lbs per gallon. This allows you to calculate the gallons per day being used. Make sure to write your fractions in a way that allows for the lbs to cancel and the units in your answer to be gallons per day.

$$\frac{220 \text{ lbs}}{\text{day}} \times \frac{\text{gal}}{15.76 \text{ lbs}} = \frac{13.96 \text{ gal}}{\text{day}}$$

Now the question is asking how many gallons are needed per hour. Assuming a 24-hour day, you can calculate the gallons needed per hour.

$$\frac{13.96 \text{ gal}}{\text{day}} \times \frac{1 \text{ day}}{24 \text{ hours}} = \frac{0.58 \text{ gal}}{\text{hour}}$$

Therefore, slightly more than a half-gallon is required every hour.

Example: How many gallons of 15% strength sodium hypochlorite are needed to dose a well flowing 1,500 gpm to a dosage of 1.75 ppm? (Assume the sodium hypochlorite has a specific gravity of 1.42).

In order to use the Pound Formula equation, you first need to convert the flow rate from gpm to MGD.

$$\frac{1{,}500 \text{ gal}}{\text{min}} \times \frac{1{,}440 \text{ min}}{1 \text{ day}} \times \frac{1 \text{ MG}}{1{,}000{,}000 \text{ gal}} = 2.16 \text{ MGD}$$

Now you can substitute the information into the Pound Formula.

$$\text{Pound Formula} \rightarrow \frac{\frac{\text{MG}}{\text{D}} \times \frac{8.34 \text{ lbs}}{\text{gal}} \times \text{ppm}}{\% \text{ concentration}} = \frac{\text{lbs}}{\text{day}}$$

First, solve the pound formula in the numerator.

$$\frac{2.16 \text{ MG}}{\text{D}} \times \frac{8.34 \text{ lbs}}{\text{gal}} \times 1.75 \text{ ppm} = \frac{31.5252 \text{ lbs}}{\text{day}}$$

To adjust for the percent concentration, divide lbs per day by the percent concentration. You know that 15% is equivalent to 0.15.

$$\frac{\frac{31.5252 \text{ lbs}}{\text{day}}}{\% \text{ concentration}} = \frac{\frac{31.5252 \text{ lbs}}{\text{day}}}{15\%} = \frac{\frac{31.5252 \text{ lbs}}{\text{day}}}{0.15} = \frac{210.17 \text{ lbs}}{\text{day}}$$

210 lbs of 15% sodium hypochlorite are needed to dose 1,500 gpm to 1.75 ppm. Now the 210 lbs needs to be converted to gallons.

$$\frac{8.34 \text{ lbs/gal}}{1 \text{ SG}} \times \frac{1.42 \text{ SG}}{1} = 11.8428 = \frac{11.84 \text{ lbs}}{\text{gal}}$$

$$\frac{210.17\text{ lbs}}{\text{day}} \times \frac{\text{gal}}{11.84\text{ lbs}} = \frac{17.75\text{ gal}}{\text{day}}$$

17.75 gallons per day of 15% strength sodium hypochlorite are needed to dose a well flowing 1,500 gpm to a dosage of 1.75 ppm.

The reason drinking water is disinfected is to prevent pathogenic organisms from causing illness in the population drinking the water. These organisms are controlled by the amount of disinfectant, typically chlorine, added to the water. The amount of chlorine required to inactivate the various pathogens is called the **chlorine demand**.

Once the demand is satisfied, the remaining chlorine in the water supply is termed the **residual**. In order to keep the water supply safe for consumption, there must be a residual after the **dosage** has been applied. Water operators may be required to measure the original dosage or the residual in a water supply.

dosage = residual + demand

Example: An operator is disinfecting a line. He doses the line to 50 ppm. An hour later, the dosage is measured as 40 ppm. What is the chlorine demand in the line?

First identify the information provided. The line was dosed to 50 ppm and later it was measured as 40 ppm. Therefore, the dosage was 50 ppm and the residual was 40 ppm.

Substituting this into the equation will allow you to solve for the demand.

dosage = residual + demand
50 ppm = 40 ppm + demand

Now isolate the demand in the equation and solve.

demand = 50 ppm - 40 ppm
demand = 10 ppm

Therefore, the chlorine demand in the line is 10 ppm.

Key Terms

- **chlorine demand** – the amount of chlorine required to inactivate the various pathogens
- **dosage** – the amount added to satisfy demand
- **Pound Formula** – a calculation for the weight of a chemical, such as chlorine or chlorine-related, that is added to water; can be used to calculate alum, ferric chloride or other chemical doses
- **residual** – remaining chlorine in the water supply once the demand is satisfied

Practice Problems 4.1

1. How many gallons of water can be treated with 325 pounds of 80% High Test Hypochlorite (HTH) to a dosage of 1.25 mg/L?

2. An operator added 85 pounds of 10% ferric chloride to a treatment flow of 4.1 MGD. What was the corresponding dosage?

3. How many pounds of 24.5% sodium hypochlorite are needed to dose a well with a flow rate of 2,150 gpm to a dosage of 3.45 ppm? (Assume the well runs 10 hours a day).

4. In the above problem, how many gallons of chemical are needed per hour? (Assume the SG is 1.9).

5. A treatment operator has set a chemical pump to dose 145 gallons of NaOH (sodium hydroxide) per day for a flow rate of 6.35 MGD. What is the corresponding dosage? (Assume the SG is 2.41).

6. 3 miles of 24” diameter main line needs to be dosed to 100 ppm. Answer the following questions.

   a. How many gallons of 15% (SG = 1.60) sodium hypochlorite are needed?

   b. How many pounds of 45% HTH are needed?

   c. Assuming the following costs, which one is least expensive?

      i. Sodium hypochlorite = $2.75 per gallon

      ii. HTH = $1.35 per pound

7. A water treatment operator adjusted the amount of 15% Alum dosage from 90 mg/L to 65 mg/L. Based on a treatment flow of 8 MGD, what is the cost savings if 15% Alum costs $1.20 per pound?

8. A water utility produced 6,000 AF of water last year. The entire amount was dosed at an average rate of 0.6 ppm. If the chemical of choice was 35% HTH at a per pound cost of $2.43, what was the annual budget?

9. Ferric chloride is used as the coagulant of choice at a 10.1 MGD rated capacity treatment plant. If the plant operated at the rated capacity for 60% of the year and operated at 30% of rated capacity for 40% of the year, how many pounds of the coagulant was needed to maintain a dosage of 65 mg/L?

10. A water softening treatment process uses 30% NaOH during 40% of the year and 40% NaOH for 60% of the year. Assuming a constant flow rate of 500 gpm and a dosage of 55 mg/L, what is the annual budget if the 30% NaOH (SG = 1.55) costs $1.20 per gallon and the 40% NaOH (SG = 1.87) costs $2.10 per gallon?

11. An operator added 422 gallons of 15% sodium hypochlorite (SG=1.57) in to 5,340 ft of 3 feet diameter pipe. After 36 hours, the residual was measured at 122.65 ppm. What was the demand?

Exercise 4.1

Solve the following chemical dosage problems. Be sure to account for the differences in chemical percent concentrations.

1. How many gallons of water can be treated with 100 pounds of 65% High Test Hypochlorite (HTH) to a dosage of 2.55 mg/L?

2. An operator added 165 pounds of 25% ferric chloride to a treatment flow of 10.5 MGD. What was the corresponding dosage?

3. How many pounds of 12.5% sodium hypochlorite are needed to dose a well with a flow rate of 1,000 gpm to a dosage of 1.75 ppm? (Assume the well runs 17 hours a day).

4. In the above problem, how many gallons of chemical are needed per hour? (Assume the SG is 1.4).

5. A treatment operator has set a chemical pump to dose 75 gallons of NaOH (sodium hydroxide) per day for a flow rate of 2.25 MGD. What is the corresponding dosage? (Assume the SG is 1.65).

6. 11,250 feet of 18” diameter main line needs to be dosed to 50 ppm. Answer the following questions.

   a. How many gallons of 12.5% (SG = 1.44) sodium hypochlorite are needed?

   b. How many pounds of 65% HTH are needed?

   c. Assuming the following costs, which one is least expensive?

      i. Sodium hypochlorite = $2.45 per gallon

      ii. HTH = $1.65 per pound

7. A water treatment operator adjusted the amount of 20% Alum dosage from 85 mg/L to 70 mg/L. Based on a treatment flow of 10 MGD, what is the cost savings if 20% Alum costs $2.50 per pound?

8. A water utility produced 11,275 AF of water last year. The entire amount was dosed at an average rate of 1.5 ppm. If the chemical of choice was 65% HTH at a per pound cost of $1.85, what was the annual budget?

9. Ferric chloride is used as the coagulant of choice at a 5.75 MGD rated capacity treatment plant. If the plant operated at the rated capacity for 75% of the year and operated at 60% of rated capacity for 25% of the year, how many pounds of the coagulant was needed to maintain a dosage of 45 mg/L?

10. A water softening treatment process uses 25% NaOH during 20% of the year and 50% NaOH for 80% of the year. Assuming a constant flow rate of 1,100 gpm and a dosage of 70 mg/L, what is the annual budget if the 25% NaOH (SG = 1.18) costs $0.95 per gallon and the 50% NaOH (SG = 1.53) costs $1.70 per gallon?

11. An operator added 275 gallons of 12.5% sodium hypochlorite (SG=1.32) into 2,550 ft of 12-foot diameter pipe. After 24 hours, the residual was measured at 10.25 ppm. What was the demand?

UNIT 5

5.1 WEIR OVERFLOW RATE

A **weir** is an overflow structure that is used to alter flow characteristics. Weirs are used in many different circumstances, including water treatment facilities and irrigation canals.

The weir raises the water level and causes the water to flow over the weir structure at a constant flow rate known as the **Weir Overflow Rate (WOR)**. WORs are expressed as the flow of water by the length of the weir, typically as MGD per foot (MGD/ft) or gpm per foot (gpm/ft).

Figure 5.1[26]

[26] Image by Titico is in the public domain

Weirs can either be sharp-crested or broad-crested. Broad-crested weirs are flat-crested structures and are commonly used in dam spillways. Sharp-crested weirs (most common are "V" notch) allow the water to fall cleanly away from the weir and are typically found in water treatment plants. The same WOR formula can be used no matter which style of weir is used.

Figure 5.2: Sharp crested weir[27]

Figure 5.3: Broad crested weir[28]

Weir Overflow Rate (WOR) Formula:

$$\text{Weir Overflow Rate (gpm/ft)} = \frac{\text{Flow (gpm)}}{\text{Weir Length (ft)}}$$

[27] Image by Regina Blasberg is used with permission

[28] Image by Ernesto Velazquez is used with permisison

Calculating the length of the weir is required in order to calculate the WOR. Sometimes the weir can be a circular structure requiring the circumference to be calculated in order to find the actual length. Other times it is a linear structure, in which case the length would be known.

Example: What is the weir overflow rate through a 12.85 MGD treatment plant if the weir is 90 feet long? (Express your answer in MGD/ft and gpm/ft).

The values provided in the problem statement are in MGD and feet. Therefore, to calculate the weir overflow rate in MGD/ft, substitute the values for flow and weir length into the equation.

$$\text{Weir Overflow Rate (MGD/ft)} = \frac{12.85\text{ MGD}}{90\text{ ft}} = 0.142778\text{ MGD/ft}$$

You can approach calculating the solution in gpm/ft in two ways.

Option #1: One way is to convert the MGD/ft solution previously calculated to gpm/ft.

$$\frac{\dfrac{0.142778\text{ MG}}{\text{D}} \times \dfrac{1{,}000{,}000\text{ gal}}{1\text{ MG}} \times \dfrac{1\text{ day}}{24\text{ hour}} \times \dfrac{1\text{ hour}}{60\text{ min}}}{\text{ft}} = \frac{99.15\text{ gpm}}{\text{ft}}$$

Option #2: You can convert the treatment plant flow rate provided in the problem statement from MGD to gpm and then calculate the weir overflow rate.

$$\frac{12.85\text{ MG}}{\text{D}} \times \frac{1{,}000{,}000\text{ gal}}{1\text{ MG}} \times \frac{1\text{ day}}{24\text{ hour}} \times \frac{1\text{ hour}}{60\text{ min}} = 8{,}923.611\text{ gpm}$$

$$\text{Weir Overflow Rate (gpm/ft)} = \frac{8{,}923.611\text{ gpm}}{90\text{ ft}} = \frac{99.15\text{ gpm}}{\text{ft}}$$

Note that the final answer is the same using either option.

Example: A drainage channel has a 32-foot weir and a weir overflow rate of 14.5 gpm/ft. What is the daily flow expressed in MGD?

Substitute the values provided in the question into the Weir Overflow Rate formula and solve for the unknown flow rate.

$$\text{Weir Overflow Rate (gpm/ft)} = \frac{\text{? gpm}}{\text{32 ft}} = \text{14.5 gpm/ft}$$

Rearrange the terms and solve for the flow rate in gpm.

$$\text{? gpm} = \frac{\text{14.5 gpm}}{\text{ft}} \times \text{32 ft = 464 gpm}$$

Now you can convert the gpm to MGD.

$$\frac{\text{464 gal}}{\text{min}} \times \frac{\text{1 MG}}{\text{1,000,000 gal}} \times \frac{\text{24 hour}}{\text{1 day}} \times \frac{\text{60 min}}{\text{1 hour}} = \text{0.66816 MGD = 0.67 MGD}$$

Example: What is the length of a weir if the daily flow is 4.3 MG and the weir overflow rate is 52 gpm/ft?

The units of the values provided in the problem statement, do not match. You have MGD for the daily flow and gpm/ft for the weir overflow rate. Therefore, in order to solve for length, either MGD must be converted to gpm or gpm/ft must be converted to MGD/ft. Both will provide you with the same final answer.

Converting 4.3 MGD to gpm.

$$\frac{\text{4.3 MG}}{\text{D}} \times \frac{\text{1,000,000 gal}}{\text{1 MG}} \times \frac{\text{1 day}}{\text{24 hour}} \times \frac{\text{1 hour}}{\text{60 min}} = \text{2,986.1 gpm}$$

Now you can substitute the values into the Weir Overflow Rate formula and solve for the weir length.

$$\text{Weir Overflow Rate (gpm/ft)} = \frac{\text{2,986.1 gpm}}{\text{? ft}} = \text{52 gpm/ft}$$

Rearrange the terms and solve for the weir length in feet.

$$\text{Weir Length (ft)} = \frac{\text{2,986.1 gpm}}{\text{52 gpm/ft}} = \text{57.425 ft = 57.43 ft}$$

Converting 52 gpm/ft to MGD/ft.

$$\frac{\frac{\text{52 gal}}{\text{min}} \times \frac{\text{1 MG}}{\text{1,000,000 gal}} \times \frac{\text{24 hour}}{\text{1 day}} \times \frac{\text{60 min}}{\text{1 hour}}}{\text{ft}} = \frac{\text{0.07488 MGD}}{\text{ft}}$$

Now you can substitute the values into the Weir Overflow Rate formula and solve for the weir length.

$$\text{Weir Overflow Rate (MGD/ft)} = \frac{4.3 \text{ MGD}}{? \text{ ft}} = 0.07488 \text{ MGD/ft}$$

Rearrange the terms and solve for the weir length in feet.

$$\text{Weir Length (ft)} = \frac{4.3 \text{ MGD}}{0.07488 \text{ MGD/ft}} = 57.425 \text{ ft} = 57.43 \text{ ft}$$

It doesn't matter whether you calculate the weir length using MGD or gpm. In either case, the weir length is 57.43 ft. However, it is important that all of the units in the problem are the same – MGD to MGD/ft or gpm to gpm/ft.

Example: A treatment plant processes 12.5 MGD. The weir overflow rate through a circular clarifier is 33.8 gpm/ft. What is the diameter of the clarifier?

Since the plant flow rate is provided in MGD and the WOR is provided in gpm per foot, you have the option of converting gpm to MGD or MGD to gpm in order to solve. Here is the conversion for gpm to MGD.

$$\frac{\frac{33.8 \text{ gal}}{\text{min}} \times \frac{1 \text{ MG}}{1{,}000{,}000 \text{ gal}} \times \frac{24 \text{ hour}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hour}}}{\text{ft}} = \frac{0.048672 \text{ MGD}}{\text{ft}}$$

$$\text{Weir Overflow Rate (MGD/ft)} = \frac{12.5 \text{ MGD}}{? \text{ ft}} = 0.048672 \text{ MGD/ft}$$

$$\text{Weir Length (ft)} = \frac{12.5 \text{ MGD}}{0.048672 \text{ MGD/ft}} = 256.8212 \text{ ft}$$

It is a circular weir. Therefore, the length calculated is actually the circumference of a circle. To calculate the diameter, use the circumference formula and solve for D.

$$\text{Circumference} = \pi \times D = 256.8212 \text{ ft}$$

$$D = \frac{256.8212 \text{ ft}}{3.14} = 81.79 \text{ ft} = 81.8 \text{ ft}$$

Key Terms

- **weir** – an overflow structure that is used to alter flow characteristics
- **weir overflow rate (WOR)** – the rate at which water flows over the weir structure; the flow of water by the length of the weir

Practice Problems 5.1

1. What is the weir overflow rate through a 3.2 MGD treatment plant if the weir is 18 feet long? (Express your answer in MGD/ft and gpm/ft).

2. A drainage channel has a 210-foot weir and a weir overflow rate of 28 gpm/ft. What is the daily flow expressed in MGD?

3. What is the length of a weir if the daily flow is 6.9 MG and the weir overflow rate is 41 gpm/ft?

4. A 37 ft diameter circular clarifier has a weir overflow rate of 25 gpm/ft. What is the daily flow in MGD?

5. A treatment plant processes 8.4 MGD. The weir overflow rate through a circular clarifier is 17.6 gpm/ft. What is the diameter of the clarifier?

6. An aqueduct that flowed 44,500 acre-feet of water last year has a weir overflow structure to control the flow. If the weir is 315 feet long, what was the average weir overflow rate in gpm/ft?

7. An aqueduct is being reconstructed to widen the width across the top. The width across the bottom is 25 feet and the average water depth is 40 feet. The aqueduct must maintain a constant weir overflow rate of 15 gpm per foot with a daily flow of 0.88 MGD. What is the length of the weir?

8. An engineering report determined that a minimum weir overflow rate of 25 gpm per foot and a maximum weir overflow rate of 30 gpm per foot were needed to meet the water quality objectives of a certain treatment plant. The existing weir is 120 feet long. What is the daily treatment flow range of the plant?

Exercise 5.1

1. What is the weir overflow rate through a 7 MGD treatment plant if the weir is 30 feet long? (Express your answer in MGD/ft and gpm/ft).

2. A drainage channel has a 10-foot weir and a weir overflow rate of 7 gpm/ft. What is the daily flow expressed in MGD?

3. What is the length of a weir if the daily flow is 8.45 MG and the weir overflow rate is 28 gpm/ft?

4. A 60 ft diameter circular clarifier has a weir overflow rate of 15 gpm/ft. What is the daily flow in MGD?

5. A treatment plant processes 15 MGD. The weir overflow rate through a circular clarifier is 29.5 gpm/ft. What is the diameter of the clarifier?

6. An aqueduct that flowed 36,000 acre-feet of water last year has a weir overflow structure to control the flow. If the weir is 250 feet long, what was the average weir overflow rate in gpm/ft?

7. A 75-mile aqueduct is being reconstructed to widen the width across the top. The width across the bottom is 10 feet and the average water depth is 15 feet. The aqueduct must maintain a constant weir overflow rate of 25 gpm per foot with a daily flow of 0.63 MGD. What is the length of the weir?

8. An engineering report determined that a minimum weir overflow rate of 15 gpm per foot and a maximum weir overflow rate of 20 gpm per foot were needed to meet the water quality objectives of a certain treatment plant. The existing weir is 80 feet long. What is the daily treatment flow range of the plant?

9. An aqueduct has a weir that is 5 feet narrower than the distance across the aqueduct. Assuming a constant weir overflow rate of 28.75 gpm/ft, an average depth of 12 feet, a distance across the bottom of 8 feet, a length of 22 miles, and a daily flow of 0.85 MG, what is the capacity of the aqueduct in AF?

UNIT 6

6.1 DETENTION TIME

Detention Time is an important process that allows large particles to "settle out" from the flow of water through gravity, prior to filtration. It is the time it takes a particle to travel from one end of a sedimentation basin to the other end. Conventional filtration plants require large areas of land to construct sedimentation basins and employ the detention time process. Not all treatment plants have the available land and may decide that direct filtration is suitable. Therefore, in direct filtration plants the sedimentation process is eliminated. However, in direct filtration plants, the filters have shorter run times and require more frequent backwashing cycles to clean the filters.

A term used that is interchangeable with detention time is **contact time**. Note that this should not be confused with CT, Concentration Time which will be discussed in Unit 7. Contact times represent how long a chemical (typically chlorine) is in contact with the water supply prior to delivery to customers. For example, contact time can be measured from the time a well is chlorinated until it reaches the first customer within a community or, it could be how long the water mixes in a storage tank before it reaches a customer.

Calculating the Detention Time and Contact Time requires two elements, the volume of the structure holding the water (sedimentation basin, pipeline, and storage tank) and the flow rate of the water (gallons per minute, million gallons per day.) Since detention times and contact times are typically expressed in hours, it is important that the correct units are used. When solving D_t problems be sure to convert to the requested unit of time.

As with all water math-related problems, there are other parameters that can be calculated within the problem. For example, if the detention time and volume are known, then the flow rate can be calculated. Or, if the flow rate and detention time are known, the volume can be calculated. Sometimes the flow rate and the desired detention time is known and the size of the vessel holding the water needs to be designed. In this example, the area or dimensions of the structure can be calculated.

The pie wheel (below) shows a simple way of calculating the variables. If the variables are next to each other (D_t and Flow Rate) then multiply. If they are over each other (Volume and D_t or Volume and Flow Rate) then divide.

The following formula is used for calculating detention times.

$$D_t = \frac{\text{Volume}}{\text{Flow}}$$

You may also use the Pie Wheel to solve for Detention Time as shown below.

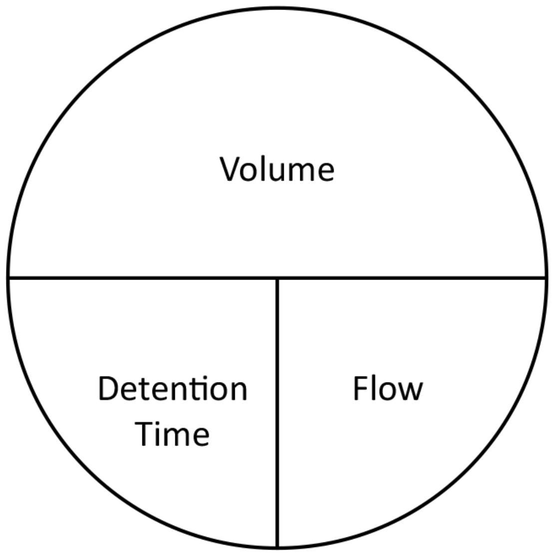


Figure 6.1[29]

Units are extremely important when using this formula. There are three variables in this formula: Detention Time, Flow, and Volume. Each variable can be provided using a variety of units. You cannot calculate the solution unless all of the units align. If the units are similar (matching), then dividing volume by flow will yield a time (Detention Time). However, simply dividing a volume by a flow will not result in a time. For example, if you divide gallons by cubic feet per second there is no resulting answer. This is because "gallons" and "cubic feet" will not cancel each other.

Making sure the units are correct is important before solving this equation. Take a look at the examples below.

Example: Calculate the detention time.

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{\text{gallons}}{\text{gallons/minute}} = \text{minute}$$

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{\text{cubic feet}}{\text{cubic feet/second}} = \text{second}$$

[29] Image by Marilyn Hightower is licensed under CC BY 4.0

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{\text{gallons}}{\text{million gallons/day}}$$

In the first two examples, the terms can be divided. However, in the third example they cannot. D_t should be expressed as a unit of time (i.e., sec, min, hours). If you divide the first two examples (gal/gpm and cf/cfs), you will end up with minutes and seconds respectively. However, in the third example, gallons and million gallons cannot cancel each other out. Therefore, if you had 100,000 gallons as the volume and 1 MGD as the flow rate:

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{\text{100,000 gallons}}{\text{1 MGD}}$$

Then you would need to convert 1 MGD to 1,000,000 gallons per day in order to cancel the unit gallons. The gallons then cancel leaving "day" as the remaining unit.

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{\text{100,000 gallons}}{\frac{\text{1,000,000 gallons}}{\text{1 day}}} = \text{0.1 day}$$

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{\text{0.1 MG}}{\text{1 MGD}} = \text{0.1 day or 2.4hrs}$$

Converting the days to hours is easy since there are 24 hours in one day.

$$\text{0.1 day} \times \frac{\text{24 hours}}{\text{1 day}} = \text{2.4 hours}$$

Sometimes this can be the simplest way to solve detention time problems. However, people can be confused when they get an answer such as 0.1 days. There are other ways to solve these problems. One way is to convert MGD to gpm. Using the above example, convert 1 MGD to gpm.

$$\frac{\text{1,000,000 gallons}}{\text{1 day}} \times \frac{\text{1 day}}{\text{1,440 minutes}} = \text{694.4 gpm}$$

Now solve for the Detention Time.

$$\frac{\text{100,000 gallons}}{\frac{\text{694.4 gallons}}{\text{min}}} = \text{144 minutes} \times \frac{\text{1 hour}}{\text{60 minutes}} = \text{2.4 hours}$$

If the question is asking for hours there still needs to be a conversion. However, 144 minutes is more understandable than 0.1 days.

Example: What is the detention time in a circular clarifier with a depth of 50 ft and a 30 ft diameter if the daily flow is 2.2 MG. (Express your answer in hours:minutes.)

First you need to calculate the volume of the clarifier.

$$\text{Clarifier Volume} = 0.785 \times D^2 \times H = 0.785 \times (30\text{ ft})^2 \times 50\text{ ft} = 35{,}325\text{ ft}^3$$

In order to calculate the detention time, the units for Volume and the units for Flow must align. Since the flow rate is provided in MG, convert the cubic foot volume to MG.

$$35{,}325\text{ ft}^3 \times \frac{7.48\text{ gal}}{1\text{ cf}} \times \frac{1\text{ MG}}{1{,}000{,}000\text{ gal}} = 0.264231\text{ MG}$$

Now you can calculate the detention time by substituting the volume calculated and the daily flow provided into the detention time formula.

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{0.264231\text{ MG}}{2.2\text{ MGD}} = 0.120105\text{ days} \times \frac{24\text{ hour}}{1\text{ day}} = 2.88252\text{ hours}$$

The problem asks for the detention time to be expressed in hours and minutes. Based on the calculation above, you know that you have 2 full hours and a portion of a third hour. To calculate the exact number of minutes, take the decimal amount and multiply by 60 min per hour.

$$0.88252\text{ hours} \times \frac{60\text{ min}}{1\text{ hour}} = 52.95\text{ min} = 53\text{ min}$$

$$D_t = 2\text{ hours } 53\text{ minutes} = 2{:}53$$

Example: A water utility is designing a transmission pipeline collection system in order to achieve a chlorine contact time of 2 hours 10 mins once a 1,125 gpm well is chlorinated. How many feet of 30” diameter pipe are needed?

The problem statement provides both the detention time and the flow rate. However, in order to use the detention time provided, you need to convert it to minutes.

$$\left(2\text{ hours} \times \frac{60\text{ min}}{1\text{ hour}}\right) + 10\text{ min} = 130\text{ min}$$

Rearranging the terms in the detention time equation to solve for volume results in volume equals detention time times flow. Substitute the minutes calculated and the flow rate into this equation in order to solve for the total volume in gallons.

$$\text{Volume} = D_t \times \text{Flow} = 130 \text{ min} \times \frac{1{,}125 \text{ gal}}{\text{min}} = 146{,}250 \text{ gal}$$

Since the problem statement is asking for how many feet of pipe, convert the gallons to cubic feet.

$$\text{Volume} = 146{,}250 \text{ gal} \times \frac{1 \text{ cf}}{7.48 \text{ gal}} = 19{,}552.1390374 \text{ cf} = 19{,}552.1 \text{ cf}$$

Now that you know the total volume of the pipe in cubic feet, you can use the formula for volume of a pipe to determine the pipe length. Substitute the pipe diameter and the pipe volume into the equation and rearrange the terms to solve for length.

$$\text{Pipe Volume} = 0.785 \times D^2 \times H =$$

$$0.785 \times \left(30 \text{ in x } \frac{1 \text{ ft}}{12 \text{ in}}\right)^2 \text{ x Length} = 19{,}552.1 \text{ ft}^3$$

$$\text{Length} = \frac{19{,}552.1 \text{ ft}^3}{0.785 \times (2.5 \text{ ft})^2} = \frac{19{,}552.1 \text{ ft}^3}{4.90625 \text{ ft}^2} = 3{,}985.1414 \text{ ft} = 3{,}985 \text{ ft}$$

Practice Problems 6.1

1. What is the detention time in hours of a 300 ft by 50 ft by 25 ft sedimentation basin with a flow of 8.1 MGD?

2. What is the detention time in a circular clarifier with a depth of 65 ft and a 95 ft diameter if the daily flow is 7.3 MG. (Express your answer in hours:minutes.)

3. A water utility engineer is designing a sedimentation basin to treat 12 MGD and maintain a minimum detention time of 3 hours 30 minutes. The basin cannot be longer than 100 feet and wider than 65 feet. Under this scenario, how deep must the basin be?

4. A water utility is designing a transmission pipeline collection system in order to achieve a chlorine contact time of 75 minutes once a 3,400 gpm well is chlorinated. How many feet of 36” diameter pipe are needed?

5. The chlorine residual decay rate is 0.7 mg/L per 3/4 hour in a 4 MG water storage tank. If the storage tank needs to maintain a minimum chlorine residual of 6.5 mg/L what is the required dosage if the tank is filling at a rate of 900 gpm until the tank is full?

6. A 44-foot-tall water storage tank is disinfected with chloramines through an onsite disinfection system. The average constant effluent from the tank is 680 gpm through a 20-inch diameter pipe. If the first customer that receives water from the tank is 4,972 feet from the tank, would the required 30-minute contact time be achieved?

7. A 70-foot diameter, 35-foot-deep clarifier maintains a constant weir overflow rate of 22.6 gpm/ft. What is the detention time in hours:min?

8. A circular clarifier processes 9.5 MGD with a detention time of 3.7 hours. If the clarifier is 45 feet deep, what is the diameter?

Exercise 6.1

Solve the following problems. Be sure you provide the answer in the correct units.

1. What is the detention time in hours of a 100 ft by 20 ft by 10 ft sedimentation basin with a flow of 5 MGD?

2. What is the detention time in a circular clarifier with a depth of 30 ft and a 70 ft diameter if the daily flow is 4.5 MG. (Express your answer in hours:minutes.)

3. A water utility engineer is designing a sedimentation basin to treat 10 MGD and maintain a minimum detention time of 2 hours 15 minutes. The basin cannot be longer than 80 feet and wider than 40 feet. Under this scenario, how deep must the basin be?

4. Chlorine is injected into an 18" diameter pipe at a well site. The pipeline is 2,000 ft long before it reaches the first customer. Assuming a well flow rate of 1,700 gpm, what is the detention time (contact time) in minutes?

5. A water utility is designing a transmission pipeline collection system in order to achieve a chlorine contact time of 40 minutes once a 2,250 gpm well is chlorinated. How many feet of 24" diameter pipe are needed?

6. A fluoride tracer study is being conducted at a 15.5 MGD capacity water treatment plant. The contact time through the coagulation and flocculation process is 2.45 hours. If the sedimentation basin has a capacity of 500,000 gallons, what is the total detention time through the 3 processes?

7. The chlorine residual decay rate is 0.2 mg/L per ½ hour in a 5 MG water storage tank. If the storage tank needs to maintain a minimum chlorine residual of 10.0 mg/L what is the required dosage if the tank is filling at a rate of 1,500 gpm until the tank is full?

8. A drinking water well serves a community of 2,000 people. The customer closest to the well is 1,250 feet away. The above ground portion of the well piping is 12" diameter and 25 feet long. The below ground portion is 750 feet of 10" diameter and 475 feet of 8" diameter piping. What is the chlorine contact time in minutes from the well head to the first customer? Assume a constant flow rate of 3.90 cfs.

9. A 32-foot-tall water storage tank is disinfected with chloramines through an onsite disinfection system. The average constant effluent from the tank is 550 gpm through a 16-inch diameter pipe. If the first customer that receives water from the tank is 3,220 feet from the tank, would the required 45-minute contact time be achieved?

10. A 90-foot diameter, 20-foot-deep clarifier maintains a constant weir overflow rate of 15.25 gpm/ft. What is the detention time in hours:min?

11. A circular clarifier processes 12.5 MGD with a detention time of 2.35 hours. If the clarifier is 50 feet deep, what is the diameter?

12. A water treatment plant is in the process of redesigning their sedimentation basin. The plant treats 4.5 MGD with an average detention time of 1.85 hours. Portable storage tanks will be used when the basin is under construction. The portable storage tanks are 25 ft tall and 20 ft in diameter. How many tanks will be needed?

6.2 FILTRATION RATES

One of the most important processes in a Water Treatment Plant is **filtration**. It is the last barrier between the treatment process and the customer. Filters trap or remove particles from the water further reducing the cloudiness or turbidity. There are different shapes, sizes, and types of filters containing one bed or a combination of beds of sand, anthracite coal, or some other form of granular material.

Slow sand filters are the oldest type of municipal water filtration and have filtration rates varying from 0.015 to 0.15 gallons per minute per square foot of filter bed area, depending on the gradation of filter medium and raw water quality. Rapid sand filters on the other hand can have filtration rates ranging from 2.0 to 10 gallons per minute per square foot of filter bed area. Typically, rapid sand filters will require more frequent backwash cycles to remove the trapped debris from the filters.

Backwashing is the reversal of flow through the filters at a higher rate to remove clogged particles from the filters. Backwash run times can be anywhere from 5 – 20 minutes with rates ranging from 8 to 25 gallons per minute per square foot of filter bed area, depending on the quality of the pre-filtered water.

Filtration and backwash rates are calculated by dividing the flow rate through the filter by the surface area of the filter bed. Typically, these rates are measured in gallons per minute per square foot of filter bed area.

Filtration Rate Formula:

$$\text{Filtration Rate (gpm/ft}^2\text{)} = \frac{\text{Flow (gpm)}}{\text{Surface Area (ft}^2\text{)}}$$

You may also use the Pie Wheel to solve Filtration Rate problems.

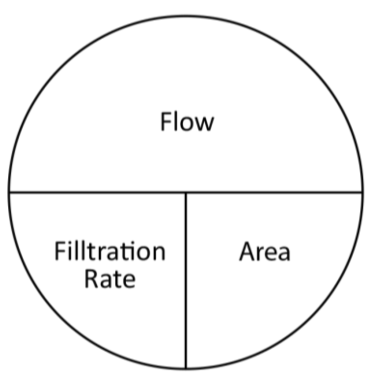


Figure 6.2[30]

[30] Image by Marilyn Hightower is licensed under CC BY 4.0

Although filtration rates are commonly expressed as gpm/ft^2 they are also expressed as the distance of fall (in inches) within the filter per unit of time (in minutes). This "fall" references the fact that filtration rates are reduced over time as particles are lodged into the filtration media during operation. Backwashing, a process of cleaning the media by reversing the flow through the filter, is then employed in order to try and recover some of the filtering capacity and prolong the operational life of the filter. During backwashing, the formula is expressed with the same units as "fall" but is described as "rise" in the filter instead. This lets you know how much filtering capacity is recovered through your backwashing cycle. Use the formulas below.

$$\text{Filtration Rate} = \frac{\text{Fall (inches)}}{\text{Time (min)}}$$

$$\text{Backwash Rate} = \frac{\text{Rise (inches)}}{\text{Time (min)}}$$

The conversion from gallons per minute g per square foot to inches per min requires converting gallons to cubic feet, then feet to inches.

Example: Express 2.5 gpm/ft^2 as in/min.

First, convert gpm to cfm. This is the first step toward converting the gallons to inches.

$$\frac{2.5\text{ gpm}}{\text{ft}^2} \times \frac{1\text{ ft}^3}{7.48\text{ gal}} = \frac{0.33\text{ cfm}}{\text{ft}^2}$$

In the above conversion, the gallons canceled, and you were left with cubic feet per min divided by square feet. This can cancel further.

$$\frac{0.33\text{ ft}^3/\text{m}}{\text{ft}^2} = 0.33\text{ ft/min} = \frac{0.33\text{ ft}}{\text{min}}$$

When you divide cubic feet by square feet, you are left with feet. The result in this example is feet per minute.

Feet per minute can easily be converted to inches per minute by multiplying by the conversion 12 inches equals 1 foot.

$$\frac{0.33\text{ ft}}{\text{min}} \times \frac{12\text{ in}}{1\text{ ft}} = \frac{4\text{ in}}{\text{min}}$$

However, this can be simplified by using the following unit conversion.

$$\frac{1.6 \text{ in}}{\text{min}} = \frac{1 \text{ gpm}}{\text{sqft}}$$

$$\frac{1.6 \text{ in/min}}{1 \text{ gpm/sqft}} \quad \text{or} \quad \frac{1 \text{ gpm/sqft}}{1.6 \text{ in/min}}$$

Example: Express 2.5 gpm/ft^2 as in/min.

Use the unit conversion to solve this problem.

$$2.5 \text{ gpm/ft}^2 \times \frac{1.6 \text{ in/min}}{1 \text{ gpm/sqft}} = 4 \text{ in/min}$$

Example: What is the filtration rate through a 20' by 20' filter if the average flow through the treatment process is 2.5 MG? Express the filtration rate as in/min.

First, convert 2.5 MGD to gpm. To do this divide 2.5 MGD by 1,440.

$$\frac{2{,}500{,}000 \text{ gal}}{\text{day}} \times \frac{1 \text{ day}}{1{,}440 \text{ min}} = 1{,}736 \text{ gpm}$$

Next, calculate the surface area of the filter in square feet.

$$20 \text{ ft} \times 20 \text{ ft} = 400 \text{ ft}^2$$

To calculate the filtration rate, substitute the values into the formula.

$$\text{Filtration Rate (gpm/ft}^2\text{)} = \frac{\text{Flow (gpm)}}{\text{Surface Area (ft}^2\text{)}}$$

$$\text{Filtration Rate (gpm/ft}^2\text{)} = \frac{1{,}736 \text{ gpm}}{400 \text{ ft}^2} = 4.32 \text{ gpm/ft}^2$$

Now use the unit conversion to calculate the filtration rate as inches per minute.

$$\text{Filtration Rate} = 4.32 \text{ gpm/ft}^2 \times \frac{1.6 \text{ in/min}}{1 \text{ gpm/sqft}} = 6.912 \text{ in/min}$$

Key Terms

- **contact time** – detention time; do not confuse contact time with concentration time
- **detention time** – time that allows large particles to settle out from the flow of water through gravity
- **filtration** –the last barrier between the treatment process and the customer; filters trap or remove particles from the water further reducing the cloudiness or turbidity; conventional filtrate plants require large areas of land to construct sedimentation basins and employ detention time process; the direct filtration process eliminates sedimentation to have a shorter run time, use less land, and requires more frequent backwashing to clean the filters.

Practice Problems 6.2

1. A water treatment plant processes 15.2 MGD. What is the required filter bed area needed to maintain a filtration rate of 2.80 gpm/ft^2?

2. A 30 ft by 35 ft filter needs to be back washed at a rate of 25 gpm/ft^2 for a minimum of 22 minutes. How many gallons are used during the backwashing process?

3. In order to properly back wash a certain filter a back wash rate of 15 inches per minute rise is needed. If the filter is 40 ft by 30 ft, what is the backwash flow rate in gpm?

4. A water treatment plant processes a maximum of 7.50 MGD. The plant has 3 filters measuring 28 ft by 33 ft each. Assuming that each filter receives an equal amount of flow what is the filtration rate in gpm/ft^2?

5. A water treatment plant processes 10.3 MGD through a 50 ft by 50 ft filter. What is the corresponding inches per minute through the filter?

6. A filter is backwashed at a rate of 27.0 inches per minute for 17 minutes. If the filter is 225 ft^2, how many gallons were used?

7. An Engineer is designing a circular filter to handle 2.14 MGD and maintain a filtration rate of 1.25 inches per minute. What will the diameter be?

8. A filter needs to be backwashed when the fall rate exceeds 6.3 inches per minute. It was determined that this rate is reached after 4.7 MG flows through a 27 ft by 28 ft filter. How often does the filter need backwashing? Give your answer in the most logical time unit.

Exercise 6.2

1. A water treatment plant processes 10.5 MGD. What is the required filter bed area needed to maintain a filtration rate of 1.75 gpm/ft^2?

2. A 15 ft by 17 ft filter needs to be back washed at a rate of 17 gpm/ft^2 for a minimum of 20 minutes. How many gallons are used during the backwashing process?

3. In order to properly back wash a certain filter a back wash rate of 20 inches per minute rise is needed. If the filter is 20 ft by 25 ft, what is the backwash flow rate in gpm?

4. A water treatment plant processes a maximum of 18.65 MGD. The plant has 6 filters measuring 20 ft by 22 ft each. Assuming that each filter receives an equal amount of flow what is the filtration rate in gpm/ft^2?

5. A water treatment plant processes 4.55 MGD through a 35 ft by 35 ft filter. What is the corresponding inches per minute through the filter?

6. A filter is backwashed at a rate of 15.5 inches per minute for 25 minutes. If the filter is 150 ft^2, how many gallons were used?

7. An Engineer is designing a circular filter to handle 5.75 MGD and maintain a filtration rate of 1.75 inches per minute. What will the diameter be?

8. A filter needs to be backwashed when the fall rate exceeds 3.1 inches per minute. It was determined that this rate is reached after 2.3 MG flows through a 17 ft by 17 ft filter. How often does the filter need backwashing? Give your answer in the most logical time unit.

UNIT 7

7.1 CT CALCULATIONS

Concentration and Time are critical variables in water treatment. **CT** stands for Concentration and Time. As soon as a disinfectant is added to water, it begins the disinfection process. What is the concentration of the disinfectant and how long does it need to be in contact with the water? Well, it takes time to complete the disinfection process once chemical is added to water. In addition, there are other variables that can delay the disinfection process such as, pH, water temperature, turbidity, and the amount of pathogens in the water, among other things. Therefore, knowing the concentration of the disinfectant and the time the disinfectant has to do its "work" is very important in ensuring water is properly disinfected and safe for human consumption.

There are many different types of bacteria in natural water sources that can cause sickness if not properly treated. The disinfection process kills and/or inactivates pathogenic (disease causing) bacteria to make it safe for human consumption. In order to follow the Surface Water Treatment Rule (SWTR), drinking water treatment plants must meet the following inactivation requirements:

Cryptosporidium parvum – 2.0 Log or 99% Inactivation
Giardia lamblia – 3.0 Log or 99.9% Inactivation
Viruses – 4.0 Log or 99.99% Inactivation

The table below compares the Log and Percent Inactivation values.

Table 7.1 - Log Percent and Inactivation Values

Log Inactivation	Expressed as Log	Log Value	Percent Inactivation
1.0	$10^{1.0}$	10	90.00
2.0	$10^{2.0}$	100	99.00
3.0	$10^{3.0}$	1,000	99.90
4.0	$10^{4.0}$	10,000	99.99
5.0	$10^{5.0}$	100,000	99.999
6.0	$10^{6.0}$	1,000,000	99.9999
7.0	$10^{7.0}$	10,000,000	99.99999

See Appendix for further information regarding Logs

Pin It! Misconception Alert
Sometimes logs (logarithms) can seem scary! Think of the Log as the power to which a number has to be raised to get another number.

Cryptosporidium will not be discussed in this class due to the complexities in the 2003 update to the SWTR known as the Long Term 2 Enhanced Surface Water Treatment Rule. Instead, we will focus on Giardia and viruses.

Table 7.2 (below) depicts the log requirements that are required for both Giardia and viruses. Notice in the second column that Giardia must be disinfected to 3 Log (99.9%) and viruses to 4 Log (99.99%). Various treatment processes (shown in the first column) account for some of the inactivation or removal of pathogens from the raw water. Therefore, the SWTR provides "credits" toward the inactivation of *Giardia* and viruses (shown in the third column). Credits are subtracted from the inactivation requirements to determine the level of disinfection still required after water goes through treatment.

Table 7.2 - Treatment Credits and Log Inactivation Requirements

Treatment	**Log Inactivation Requirements**		**Removal Credit Logs**		**Required Log Inactivation from Disinfection**	
	Giardia	Viruses	*Giardia*	Viruses	*Giardia*	Viruses
Conventional	3	4	2.5	2	0.5	2
Direct Filtration, DE, or Slow Sand	3	4	2	1	1	3

Direct Filtration, DE, or Slow Sand	3	4	2	1	1	3

4 Log (Required) - 1 Log (Credit) = 3 Log (Remaining)

For example, the CT requirement for viruses is 4 Log. This requirement can be satisfied through disinfection, treatment, or a combination of the two.

If raw water is treated using direct filtration, it would receive 1 Log credit. After water leaves the treatment plant, the disinfection requirement remaining for viruses would be 3 Log, as shown in the fourth column of Table 7.2 (complete calculation illustrated below the table). The remaining 3 Logs will need to be inactivated by disinfecting the water using the appropriate concentration and time. Disinfectant can either be added before or after water goes through the treatment process to make sure the pathogens are thoroughly inactivated and meet the requirements.

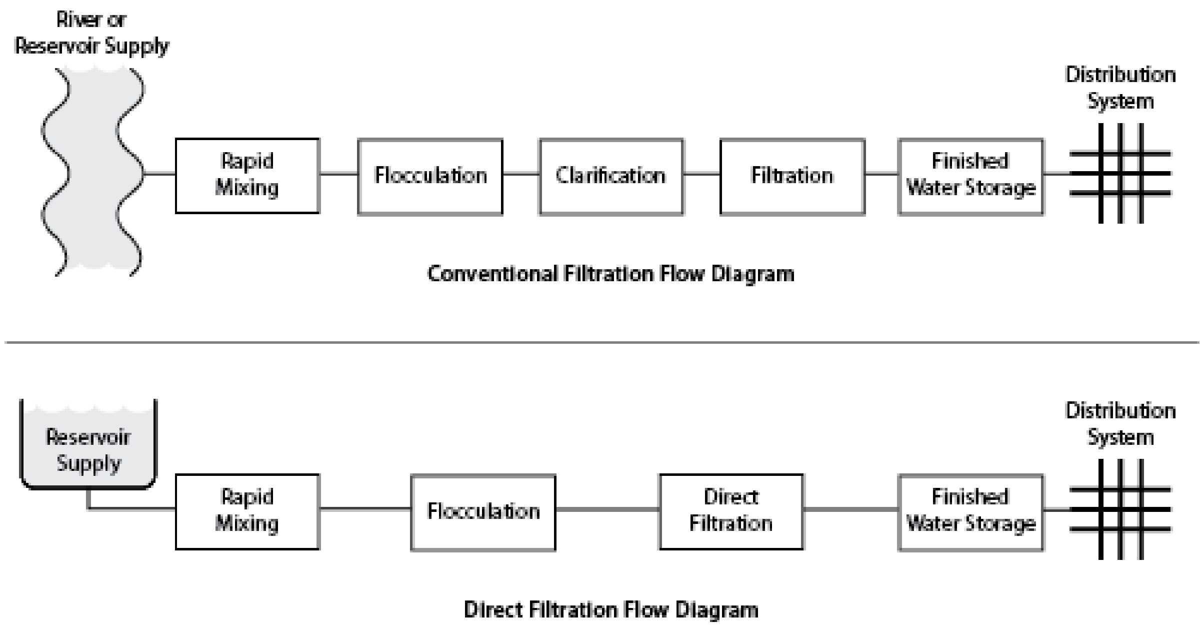


Figure 7.1[31]

Now let's discuss CT units. When dealing with CT calculations, concentration will be expressed in mg/L and Time will be expressed in minutes. Therefore, CT is expressed as mg/L · min (all one unit). When solving CT problems, the concentration of the disinfectant is typically provided in the question. However, there may be times when the "Pound Formula" is needed to calculate the chemical concentration. In order to calculate the contact time of an applied chemical, the detention time (D_t) formula from Unit 6 will be needed.

Time is defined as the moment the disinfectant is in contact with the water to the point where the Concentration is measured. These times are easily calculated through pipelines and reservoirs of known volumes but can be difficult to calculate through various treatment plant processes. To solve this issue, Tracer Studies (aka T_{10}) are sometimes conducted.

A tracer study can be accomplished by adding a unique tracer chemical to the raw water before it goes through the treatment plant and measuring how long before it is detected in the effluent of the plant. More specifically, T_{10} represents the time for 10% of an applied tracer mass to be detected through a treatment process or, the time that 90% of the water and pathogens are exposed to the disinfectant within a given treatment process. Some problems will require the calculation of the contact time using the D_t formula while others will provide T_{10} values.

[31] Image by Marilyn Hightower is licensed under [CC BY 4.0]

Next, there are two crucial terms that are required in order to calculate whether water has been adequately disinfected, "actual CT" and "required CT." Actual CT is the actual concentration of chemical through the treatment process and the actual time the disinfectant is in contact with the water. Required CT is found in the CT tables using the information provided in the problem. Once the concentration of chemical and the contact time are calculated, they can be multiplied together to determine the Actual CT.

Example: Determine the Actual CT given 2 mg/L concentration in a pipeline and 10 minutes of contact time (either calculated using D_t formula or provided as a T_{10})

Remember that the units for the Actual CT are mg/L · min.

$$2 \text{ mg/L} \times 10 \text{ min} = 20 \text{ mg/L min}$$

Once the actual CT values have been calculated, the final step in the CT calculation process involves CT Tables. The U.S. Environmental Protection Agency (USEPA) as part of the SWTR, created a series of tables that list the type of disinfectant, the pH of the water, the concentration of the disinfectant, the contact time, and the pathogen in question. Using all this information, the required CT (mg/L · min) values can be found. For your reference, the CT Tables are provided at the end of this text. They can be confusing at first, but once you understand what information you need to look for, the CT values can be easily found. See the example below.

Example: What is the required 1.0 log inactivation from disinfection (value after credits are applied) for the Inactivation for Giardia in 10 degrees Celsius water with a pH of 7.5 using a free chlorine dosage of 1 mg/L?

To answer this question, you need to look at the CT Values table for Inactivation of Giardia Cysts by Free Chlorine at 10°C.

EPA Guidance Manual Disinfection Profiling and Benchmarking

C-4

Table C-3. CT Values for Inactivation of Giardia Cysts by Free Chlorine at 10°C

CHLORINE CONCENTRATION (mg/L)	pH<=6 Log Inactivation						pH=6.5 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	12	24	37	49	61	73	15	29	44	59	73	88
0.6	13	25	38	50	63	75	15	30	45	60	75	90
0.8	13	26	39	52	65	78	15	31	46	61	77	92
1	13	26	40	53	66	79	16	31	47	63	78	94
1.2	13	27	40	53	67	80	16	32	48	63	79	95
1.4	14	27	41	55	68	82	16	33	49	65	82	98
1.6	14	28	42	55	69	83	17	33	50	66	83	99
1.8	14	29	43	57	72	86	17	34	51	67	84	101
2	15	29	44	58	73	87	17	35	52	69	87	104
2.2	15	30	45	59	74	89	18	35	53	70	88	105
2.4	15	30	45	60	75	90	18	36	54	71	89	107
2.6	15	31	46	61	77	92	18	37	55	73	92	110
2.8	16	31	47	62	78	93	19	37	56	74	93	111
3	16	32	48	63	79	95	19	38	57	75	94	113

CHLORINE CONCENTRATION (mg/L)	pH=7.0 Log Inactivation						pH=7.5 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	17	35	52	69	87	104	21	42	63	83	104	125
0.6	18	36	54	71	89	107	21	43	64	85	107	128
0.8	18	37	55	73	92	110	22	44	66	87	109	131
1	19	37	56	75	93	112	22	45	67	89	112	134
1.2	19	38	57	76	95	114	23	46	69	91	114	137
1.4	19	39	58	77	97	116	23	47	70	93	117	140
1.6	20	40	60	79	99	119	24	48	72	96	120	144
1.8	20	41	61	81	102	122	25	49	74	98	123	147
2	21	41	62	83	103	124	25	50	75	100	125	150
2.2	21	42	64	85	106	127	26	51	77	102	128	153
2.4	22	43	65	86	108	129	26	52	79	105	131	157
2.6	22	44	66	87	109	131	27	53	80	107	133	160
2.8	22	45	67	89	112	134	27	54	82	109	136	163
3	23	46	69	91	114	137	28	55	83	111	138	166

Figure 7.2[32]

[32] Image by the EPA is in the public domain

The information provided in the question statement will help determine which CT table to use. In this case, it is Table C-3 which is specifically for inactivation of Giardia by Free Chlorine at 10°C

The problem states that the water has a pH of 7.5 so you look in the column that says pH = 7.5.

The supporting information (outlined in the boxes above) will help determine which exact column and row you need to use to find the answer. For this example, the chlorine concentration is 1 mg/L and the Log Inactivation is 1.0.

Therefore, the answer is 45 mg/L · min which is circled in red. It is the intersection of the 1 mg/L row and the 1.0 Log inactivation column.

The CT Tables provide the required CT needed to inactivate either *Giardia* or viruses . The ratio of the actual CT (calculated portion of the problem) and the required CT (found in the CT tables) is then calculated to determine if the water has been properly disinfected. If the actual CT is equal to or greater than the required CT then the ratio is equal to or greater than 1.0 and CT is met. If the actual CT is less than the required CT then the ratio would be less than 1.0 and CT would not be met.

Example: If the Actual CT is 28 mg/L · min and the Required CT is 22 mg/L · min, has the water been properly disinfected?

$$\frac{\text{Actual CT}}{\text{Required CT}} = \frac{28\text{ mg/L min}}{22\text{ mg/L min}} = 1.27$$

Since the ratio is greater than 1.0, the water has been properly treated and the CT is met.

Example: If the Actual CT is 16 mg/L · min and the Required CT is 32 mg/L · min, has the water been properly disinfected?

$$\frac{\text{Actual CT}}{\text{Required CT}} = \frac{16\text{ mg/L min}}{32\text{ mg/L min}} = 0.5$$

Since the ratio is less than 1.0, the water has NOT been properly treated and the CT is NOT met.

Finding the Correct CT Table

Typical CT problems will provide the pH, the temperature, the pathogen of interest, the type of disinfectant, the dosage or a way to calculate the dosage, and the type of treatment in the problem statement. You can use this information to identify which CT table to use.

Example: Given the following information, which CT table would you use?

pH – 7.5
Temperature – 10°C
Disinfectant – Free chlorine
Dosage – 0.2 mg/L
Pathogen – Giardia
Treatment – Direct Filtration

EPA Guidance Manual
Disinfection Profiling and Benchmarking

APPENDIX C. CT VALUES FOR INACTIVATIONS ACHIEVED BY VARIOUS DISINFECTANTS

Table C-3. CT Values for Inactivation of Giardia Cysts by Free Chlorine at 10°C

CHLORINE CONCENTRATION (mg/L)	pH<=6 Log Inactivation						pH=6.5 Log Inactivation						pH=7.0 Log Inactivation						pH=7.5 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	12	24	37	49	61	73	15	29	44	59	73	88	17	35	52	69	87	104	21	42	63	83	104	125
0.6	13	25	38	50	63	75	15	30	45	60	75	90	18	36	54	71	89	107	21	43	64	85	107	128
0.8	13	26	39	52	65	78	15	31	46	61	77	92	18	37	55	73	92	110	22	44	66	87	109	131
1	13	26	40	53	66	79	16	31	47	63	78	94	19	37	56	75	93	112	22	45	67	89	112	134
1.2	13	27	40	53	67	80	16	32	48	63	79	95	19	38	57	76	95	114	23	46	69	91	114	137
1.4	14	27	41	55	68	82	16	33	49	65	82	98	19	39	58	77	97	116	23	47	70	93	117	140
1.6	14	28	42	55	69	83	17	33	50	66	83	99	20	40	60	79	99	119	24	48	72	96	120	144
1.8	14	29	43	57	72	86	17	34	51	67	84	101	20	41	61	81	102	122	25	49	74	98	123	147
2	15	29	44	58	73	87	17	35	52	69	87	104	21	41	62	83	103	124	25	50	75	100	125	150
2.2	15	30	45	59	74	89	18	35	53	70	88	105	21	42	64	85	106	127	26	51	77	102	128	153
2.4	15	30	45	60	75	90	18	36	54	71	89	107	22	43	65	86	108	129	26	52	79	105	131	157
2.6	15	31	46	61	77	92	18	37	55	73	92	110	22	44	66	87	109	131	27	53	80	107	133	160
2.8	16	31	47	62	78	93	19	37	56	74	93	111	22	45	67	89	112	134	27	54	82	109	136	163
3	16	32	48	63	79	95	19	38	57	75	94	113	23	46	69	91	114	137	28	55	83	111	138	166

CHLORINE CONCENTRATION (mg/L)	pH=8.0 Log Inactivation						pH=8.5 Log Inactivation						pH=9.0 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	25	50	75	99	124	149	30	59	89	118	148	177	35	70	105	139	174	209
0.6	26	51	77	102	128	153	31	61	92	122	153	183	36	73	109	145	182	218
0.8	26	53	79	105	132	158	32	63	95	126	158	189	38	75	113	151	188	226
1	27	54	81	108	135	162	33	65	98	130	163	195	39	78	117	156	195	234
1.2	28	55	83	111	138	166	33	67	100	133	167	200	40	80	120	160	200	240
1.4	28	57	85	113	142	170	34	69	103	137	172	206	41	82	124	165	206	247
1.6	29	58	87	116	145	174	35	70	106	141	176	211	42	84	127	169	211	253
1.8	30	60	90	119	149	179	36	72	108	143	179	215	43	86	130	173	216	259
2	30	61	91	121	152	182	37	74	111	147	184	221	44	88	133	177	221	265
2.2	31	62	93	124	155	186	38	75	113	150	188	225	45	90	136	181	226	271
2.4	32	63	95	127	158	190	38	77	115	153	192	230	46	92	138	184	230	276
2.6	32	65	97	129	162	194	39	78	117	156	195	234	47	94	141	187	234	281
2.8	33	66	99	131	164	197	40	80	120	159	199	239	48	96	144	191	239	287
3	34	67	101	134	168	201	41	81	122	162	203	243	49	97	146	195	243	292

Source: AWWA, 1991.

C-4

August 1999

Figure 7.3[33]

Therefore, Table C-3 is the correct table to use for this data set. The title of the table tells you which CT value the table will provide. Table C-3 above is for *Giardia*, with free chlorine as the disinfectant, at a temperature of 10°C.

[33] Image by the EPA is in the public domain

Finding Required CT

Now you need the other information in the problem statement. Specifically, the pH and the dosage concentration. There are seven (7) boxes in the table each with different pH values. On the far left of the table, you can find the varying disinfectant concentrations, starting with less than or equal to 0.4 mg/L going up to 3 mg/L.

Example: Given the following information, what is the Required CT?

pH – 7.5
Temperature – 10°C
Disinfectant – Free chlorine
Dosage – 0.2 mg/L
Pathogen – Giardia
Treatment – Direct Filtration

EPA Guidance Manual
Disinfection Profiling and Benchmarking

APPENDIX C. CT VALUES FOR INACTIVATIONS ACHIEVED BY VARIOUS DISINFECTANTS

Table C-3. CT Values for Inactivation of Giardia Cysts by Free Chlorine at 10°C

CHLORINE CONCENTRATION (mg/L)	pH<=6 Log Inactivation						pH=6.5 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	12	24	37	49	61	73	15	29	44	59	73	88
0.6	13	25	38	50	63	75	15	30	45	60	75	90
0.8	13	26	39	52	65	78	15	31	46	61	77	92
1	13	26	40	53	66	79	16	31	47	63	78	94
1.2	13	27	40	53	67	80	16	32	48	63	79	95
1.4	14	27	41	55	68	82	16	33	49	65	82	98
1.6	14	28	42	55	69	83	17	33	50	66	83	99
1.8	14	29	43	57	72	86	17	34	51	67	84	101
2	15	29	44	58	73	87	17	35	52	69	87	104
2.2	15	30	45	59	74	89	18	35	53	70	88	105
2.4	15	30	45	60	75	90	18	36	54	71	89	107
2.6	15	31	46	61	77	92	18	37	55	73	92	110
2.8	16	31	47	62	78	93	19	37	56	74	93	111
3	16	32	48	63	79	95	19	38	57	75	94	113

CHLORINE CONCENTRATION (mg/L)	pH=7.0 Log Inactivation						pH=7.5 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	17	35	52	69	87	104	21	42	63	83	104	125
0.6	18	36	54	71	89	107	21	43	64	85	107	128
0.8	18	37	55	73	92	110	22	44	66	87	109	131
1	19	37	56	75	93	112	22	45	67	89	112	134
1.2	19	38	57	76	95	114	23	46	69	91	114	137
1.4	19	39	58	77	97	116	23	47	70	93	117	140
1.6	20	40	60	79	99	119	24	48	72	96	120	144
1.8	20	41	61	81	102	122	25	49	74	98	123	147
2	21	41	62	83	103	124	25	50	75	100	125	150
2.2	21	42	64	85	106	127	26	51	77	102	128	153
2.4	22	43	65	86	108	129	26	52	79	105	131	157
2.6	22	44	66	87	109	131	27	53	80	107	133	160
2.8	22	45	67	89	112	134	27	54	82	109	136	163
3	23	46	69	91	114	137	28	55	83	111	138	166

CHLORINE CONCENTRATION (mg/L)	pH=8.0 Log Inactivation						pH=8.5 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	25	50	75	99	124	149	30	59	89	118	148	177
0.6	26	51	77	102	128	153	31	61	92	122	153	183
0.8	26	53	79	105	132	158	32	63	95	126	158	189
1	27	54	81	108	135	162	33	65	98	130	163	195
1.2	28	55	83	111	138	166	33	67	100	133	167	200
1.4	28	57	85	113	142	170	34	69	103	137	172	206
1.6	29	58	87	116	145	174	35	70	106	141	176	211
1.8	30	60	90	119	149	179	36	72	108	143	179	215
2	30	61	91	121	152	182	37	74	111	147	184	221
2.2	31	62	93	124	155	186	38	75	113	150	188	225
2.4	32	63	95	127	158	190	38	77	115	153	192	230
2.6	32	65	97	129	162	194	39	78	117	156	195	234
2.8	33	66	99	131	164	197	40	80	120	159	199	239
3	34	67	101	134	168	201	41	81	122	162	203	243

CHLORINE CONCENTRATION (mg/L)	pH=9.0 Log Inactivation					
	0.5	1.0	1.5	2.0	2.5	3.0
<=0.4	35	70	105	139	174	209
0.6	36	73	109	145	182	218
0.8	38	75	113	151	188	226
1	39	78	117	156	195	234
1.2	40	80	120	160	200	240
1.4	41	82	124	165	206	247
1.6	42	84	127	169	211	253
1.8	43	86	130	173	216	259
2	44	88	133	177	221	265
2.2	45	90	136	181	226	271
2.4	46	92	138	184	230	276
2.6	47	94	141	187	234	281
2.8	48	96	144	191	239	287
3	49	97	146	195	243	292

Source: AWWA, 1991.

C-4

August 1999

Figure 7.4[34]

To find the required CT on the table, look for the box in the table that says pH = 7.5. Now look at the far left of the table and find the dosage of 0.2 mg/L as indicated in the problem statement. You'll need to use the first row of the table.

You now need the last bit of information, the treatment process. In this instance, it is Direct Filtration. Remember, *Giardia* has an inactivation requirement of 3 Log. Referring back to Table 7.1 you can identify the Log credit

[34] Image by the EPA is in the public domain

and resulting disinfection inactivation requirements. You should come up with a required inactivation from disinfection of 1 Log.

3 Log required – 2 Log credit for direct filtration = 1 Log remaining
Using the first row for disinfectant concentration and the second column from the 7.5 pH portion of the table, you should come up with a required CT of 42 mg/L · min (circled above).

Calculating Actual CT

There can be multiple locations where a disinfectant is added to the water during the treatment process. Sometimes the water is pre-chlorinated in the raw water pipeline leaving a storage reservoir prior to entering the treatment facility. Sometimes the water is disinfected before the coagulation flocculation process and many times the water is disinfected after filtration prior to delivery to customers. Every time chlorine is added to the water supply it counts towards the inactivation of pathogens. Each step of the way CT will need to be calculated. The example information below will help illustrate this concept.

Example: Free chlorine is added at a concentration of 0.2 mg/L in a 12” diameter 5,000-foot-long pipeline leaving a storage reservoir prior to entering the treatment plant. The flow through the pipeline is 2 MGD. What is the time 0.2 mg/L of free chlorine is in contact with the water?

First you need to calculate the detention time.

$$D_t = \frac{\text{Volume}}{\text{Flow}}$$

$$\text{Pipe Volume} = 0.785 \times D^2 \times L$$

$$0.785 \times (1\text{ ft})^2 \text{ x } 5{,}000\text{ ft} = 3{,}925\ \text{ft}^3$$

Convert the volume to gallons.

$$3{,}925\ \text{ft}^3 \times \frac{7.48\text{ gal}}{\text{ft}^3} = 29{,}359\text{ gal}$$

Convert the flow rate to gallons per minute.

$$\frac{2{,}000{,}000\text{ gallons}}{1\text{ day}} \times \frac{1\text{ day}}{1{,}440\text{ minutes}} = 1{,}388.88\text{ gpm} = 1{,}389\text{ gpm}$$

Now you can calculate the detention time.

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{29{,}359 \text{ gal}}{1{,}389 \text{ gpm}} = 21.136789 \text{ mins} = 21 \text{ mins}$$

Multiply the detention time by the concentration and you get CT.

$$0.2 \text{ mg/L} \times 21 \text{ min} = 4.2 \text{ mg/L min}$$

Therefore, the actual CT through the pipeline is 4.2 mg/L min.

Example: Tracer studies (T_{10}) have determined that a free chlorine concentration of 1.2 mg/L through the treatment plant is 20 minutes. What is the CT through the plant?

$$1.2 \text{ mg/L} \times 20 \text{ min} = 24 \text{ mg/L min}$$

Example: Using the information from the previous examples, is CT met for this treatment plant?

Since both sections are disinfected with the same chemical, the two CT values can be added together.

$$4.2 \text{ mg/L} + 24 \text{ mg/L min} = 28.2 \text{ mg/L min}$$

To answer the question, it is helpful to organize the data in a table and to calculate the CT Ratio.

Location and Type of Disinfection	Actual CT	Required CT	CT Ratio
Pipeline + Plant (free chlorine)	28.2 mg/L · min	42 mg/L · min	0.67

Since the ratio of actual to required CT is less than 1.0, then CT is **not** met. If a treatment plant does not meet CT it can either increase the detention time through the pipeline or plant or it can increase the dosage.

In a situation where two different disinfection chemicals are used, the required CT values would be different, and you would not add the different disinfecting locations together. The next example illustrates this scenario.

Example: A conventional water treatment plant receives water with a 0.4 mg/L free chlorine residual from 9,000 feet of 3-foot diameter pipe at a constant flow rate of 10 MGD. The water has a pH of 7.5 and a temperature of 10°C. Tracer studies have shown a contact time (T_{10}) for the treatment plant to be 30 minutes. The plant maintains a chloraminated residual of 1.2 mg/L. Does the plant meet CT compliance for *Giardia*?

The first step in solving this problem is identifying the CT Tables to use to find the required CT values. This particular problem uses CT Tables C-3 and C-10. Remember to subtract out the 2.5 Log credit for conventional treatment.

The next step is to organize the data in a table.

Location and Type of Disinfection	Actual CT	Required CT	CT Ratio
Pipeline (free chlorine)		21 mg/L · min	
Plant (chloramines)		310 mg/L · min	

Now calculate the actual CT using the formula for detention time.

$$D_t = \frac{\text{Volume}}{\text{Flow}}$$

First determine the volume in the pipe in gallons.

$$\text{Pipe Volume} = 0.785 \times D^2 \times L$$

$$0.785 \times (3\text{ ft})^2 \text{ x } 9{,}000\text{ ft} = 63{,}585\ \text{ft}^3$$

Convert the volume to gallons.

$$63{,}585\ \text{ft}^3 \times \frac{7.48\text{ gal}}{\text{ft}^3} = 475{,}615.8\text{ gal} = 475{,}616\text{ gal}$$

Convert the flow rate to gallons per minute.

$$\frac{10{,}000{,}000\text{ gallons}}{1\text{ day}} \times \frac{1\text{ day}}{1{,}440\text{ minutes}} = 6{,}944.44\text{ gpm} = 6{,}944\text{ gpm}$$

Now you can calculate the detention time.

$$D_t = \frac{\text{Volume}}{\text{Flow}} = \frac{475{,}616\text{ gal}}{6{,}944\text{ gpm}} = 68.4930\text{ mins} = 68.5\text{ mins}$$

Now you multiply the detention time by the concentration, and you get CT through the pipeline.

$$0.4 \text{ mg/L} \times 68.5 \text{ min} = 27.4 \text{ mg/L min}$$

Therefore, the actual CT through the pipeline is 27.4 mg/L min.

Next determine the actual CT through the plant.

$$1.2 \text{ mg/L} \times 30 \text{ min} = 36 \text{ mg/L min}$$

Now you can finish populating the table and calculating the CT Ratios.

Location and Type of Disinfection	Actual CT	Required CT	CT Ratio
Pipeline (free chlorine)	27.4 mg/L · min	21 mg/L · min	1.3
Plant (chloramines)	36 mg/L · min	310 mg/L · min	0.12

The sum of the CT ratios equals 1.42 mg/L · min. Therefore, CT is met. You may have noticed that CT was achieved through the pipeline only and the chloramination through the plant is not needed. This is true. So, when solving one of these problems, once you meet the ratio of 1.0 or greater, CT is met, and you can stop solving the problem.

Key Terms

- **CT** – concentration and time for a disinfectant

Practice Problems 7.1

1. What is the required CT inactivation in a conventional filtration plant for Giardia by free chlorine at 20°C with a pH of 8.0 and a chlorine concentration of 2.0 mg/L? (Look up value in the CT tables and remember to apply any credits.)

2. What is the required CT inactivation for viruses with chlorine dioxide, a pH of 9.0, and a temperature of 10°C?

3. What is the required CT inactivation in a direct filtration plant for Giardia by free chlorine at 15°C with a pH of 7.0 and a chlorine concentration of 2.8 mg/L?

4. A conventional water treatment plant is fed from a reservoir 1.5 miles away through a 7-foot diameter pipe. Disinfection is provided from the supply reservoir to the plant influent at a free chlorine residual of 0.6 mg/L. The daily flow is a constant 40 MGD. And the water is 10°C and has a pH of 8.5. The treatment plant maintains a chloramines residual of 2.0 mg/L. Tracer studies have shown the contact time (T_{10}) for the treatment plant at the rated capacity of 40 MGD to be 22 minutes. Does this plant meet compliance for CT inactivation for *Giardia*?

5. A conventional water treatment plant is fed from a reservoir 4 miles away through a 4-foot diameter pipe. Disinfection is provided from the supply reservoir to the plant influent at a free chlorine residual of 0.1 mg/L. The daily flow is a constant 25 MGD. The water is 10°C and has a pH of 7.0. The treatment plant maintains a chloramines residual of 1.5 mg/L. Tracer studies have shown the contact time (T_{10}) for the treatment plant at the rated capacity of 25 MGD to be 55 minutes. Does this plant meet compliance for CT inactivation for viruses?

6. A direct filtration water treatment plant is fed from a reservoir 0.5 miles away through a 3-foot diameter pipe. Disinfection is provided from the supply reservoir to the plant influent at a free chlorine residual of 0.6 mg/L. The daily flow is a constant 15 MGD. The water is 15°C and has a pH of 7.0. The treatment plant maintains a chloramines residual of 0.4 mg/L. Tracer studies have shown the contact time (T_{10}) for the treatment plant at the rated capacity of 15 MGD to be 30 minutes. Does this plant meet compliance for CT inactivation for *Giardia*?

7. A direct filtration plant is operated at a designed flow of 20 MGD with a contact time of 35 minutes. A free chlorine dose of 1.2 mg/L is maintained through the plant. Upon leaving the plant, the effluent is chloraminated (and maintained to the distribution system) to a dose of 0.4 mg/L through a pipeline with a contact time of 12 minutes into a 650,000-gallon reservoir. The pH of the water is 8.5 and has a temperature of 15°C. Does this treatment process meet compliance for CT inactivation for viruses?

8. Does a water utility meet CT for viruses by disinfection if only the free chlorine concentration is 0.5 ppm through 200 ft of 24" diameter pipe at a flow rate of 730 gpm? Assume the water is 15°C and has a pH of 8.0.

Exercise 7.1

1. What is the required CT inactivation in a conventional filtration plant for Giardia by free chlorine at 20°C with a pH of 9.0 and a chlorine concentration of 1.0 mg/L? (Look up value in the CT tables and remember to apply any credits.)

2. What is the required CT inactivation for viruses with chlorine dioxide, a pH of 8.5, and a temperature of 7°C?

3. What is the required CT inactivation in a direct filtration plant for Giardia by free chlorine at 5°C with a pH of 8.0 and a chlorine concentration of 1.6 mg/L?

4. A conventional water treatment plant is fed from a reservoir 3 miles away through a 5-foot diameter pipe. Disinfection is provided from the supply reservoir to the plant influent at a free chlorine residual of 0.3 mg/L. The daily flow is a constant 50 MGD. And the water is 10°C and has a pH of 8.0. The treatment plant maintains a chloramines residual of 1.0 mg/L. Tracer studies have shown the contact time (T_{10}) for the treatment plant at the rated capacity of 50 MGD to be 30 minutes. Does this plant meet compliance for CT inactivation for *Giardia*?

5. A conventional water treatment plant is fed from a reservoir 2 miles away through a 6-foot diameter pipe. Disinfection is provided from the supply reservoir to the plant influent at a free chlorine residual of 0.2 mg/L. The daily flow is a constant 55 MGD. The water is 10°C and has a pH of 7.5. The treatment plant maintains a chloramines residual of 1.0 mg/L. Tracer studies have shown the contact time (T_{10}) for the treatment plant at the rated capacity of 55 MGD to be 40 minutes. Does this plant meet compliance for CT inactivation for viruses?

6. A direct filtration water treatment plant is fed from a reservoir 2.5 miles away through a 4-foot diameter pipe. Disinfection is provided from the supply reservoir to the plant influent at a free chlorine residual of 0.4 mg/L. The daily flow is a constant 30 MGD. The water is 15°C and has a pH of 8.5. The treatment plant maintains a chloramines residual of 0.75 mg/L. Tracer studies have shown the contact time (T_{10}) for the treatment plant at the rated capacity of 30 MGD to be 20 minutes. Does this plant meet compliance for CT inactivation for *Giardia*?

7. A direct filtration plant is operated at a designed flow of 10 MGD with a contact time of 15 minutes. A free chlorine dose of 0.5 mg/L is maintained through the plant. Upon leaving the plant, the effluent is chloraminated (and maintained to the distribution system) to a dose of 1.0 mg/L through a pipeline with a contact time of 10 minutes into a 500,000-gallon reservoir. The pH of the water is 8.0 and has a temperature of 20°C. Does this treatment process meet compliance for CT inactivation for viruses?

8. Does a water utility meet CT for viruses by disinfection if only the free chlorine concentration is 2.60 ppm through 150 ft of 48" diameter pipe at a flow rate of 1,200 gpm? Assume the water is 15°C and has a pH of 7.0.

UNIT 8

8.1 PRESSURE

Pressure is the amount of force that is "pushing" on a specific unit area. What does this mean? When you turn on your water faucet or shower you feel the water flowing out, but why is it flowing out? Water flows through pipes and out of faucets because it is under pressure. It could be that a pump is turned on in which case the pump and motor are providing the pressure. More commonly, the pressure is being provided by water being stored at a higher elevation. This is why you see water tanks on top of hills.

Figure 8.1[35]

Pressures are usually expressed as pounds per square inch (psi), but they can be expressed as pounds per square foot or pounds per square yard as well. The key is that the force is expressed per unit area.

Typically, water operators will measure pressures with gauges and express the unit answer as psig. The "g" is this case represents gauge. However, it is also common to express pressure in feet. Feet represent the height of the water in relation to the location that the pressure is being measured.

There are two commonly used factors to convert from feet to psi and vice versa. For every foot in elevation change there is a 0.433 change in psi. Conversely, for every psi change there is a 2.31 foot in elevation change.

[35] Photo used with permission of SCV Water

1 foot = 0.433 psi

2.31 feet = 1 psi

Written as conversion factors, these relationships can be expressed as follows.

$$\frac{1 \text{ psi}}{2.31 \text{ ft}} \quad \text{or} \quad \frac{1 \text{ ft}}{0.433 \text{ psi}}$$

Remember, as with all conversion factors, these conversion factors can also be written as the inverse.

$$\frac{2.31 \text{ ft}}{1 \text{ psi}} \quad \text{or} \quad \frac{0.433 \text{ psi}}{1 \text{ ft}}$$

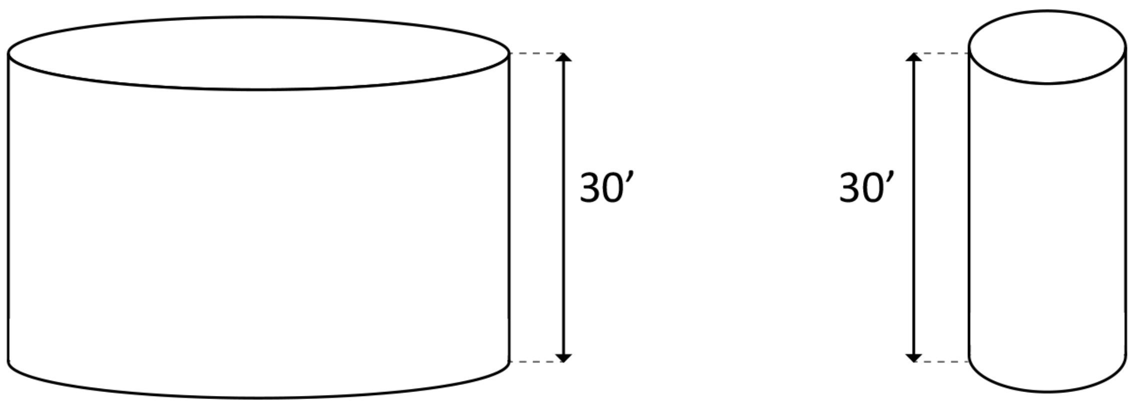


Figure 8.2[36]

Example: If both tanks above are filled with water, which one has a greater pressure at the base of it?

This is a trick question. The answer is neither! The pressure at the base of each tank is the same because the height of the water is the same.

What is the pressure at the base of each cylinder?

$$\frac{30 \text{ ft}}{1} \times \frac{1 \text{ psi}}{2.31 \text{ ft}} = 12.99 \text{ psi} \quad \text{or} \quad \frac{30 \text{ ft}}{1} \times \frac{0.433 \text{ psi}}{1 \text{ ft}} = 12.99 \text{ psi}$$

As you can see, it doesn’t matter which conversion factor you use. The answer remains the same and can be rounded to 13 psi.

[36] Image by Marilyn Hightower is licensed under CC BY 4.0

From this example, it is clear that the pressure exerted on the bottom of the tank is the same for either tank, but what about the force? Is the force exerted on the bottom of both tanks the same too?

Remember from Unit 3 that the density or weight of water is approximately 8.34 pounds per gallon. Using this conversion factor, the actual force exerted by the water can be calculated.

Example: Looking at these tanks again, assume that the diameter of the narrower tank is 10 feet and the diameter of the wider tank is 40 feet. Assume both tanks are filled with water. What is the force exerted on the bottom of each tank?

There are a couple ways to calculate the force. First, let's look at it in terms of tank volume. We'll start with the smaller 10-foot diameter tank.

$$\text{Volume of 10' diam tank} = 0.785 \times (10\text{ ft})^2 \times 30\text{ ft} = 2{,}355\text{ ft}^3$$

Now convert the volume to gallons.

$$2{,}355\text{ cf} \times \frac{7.48\text{ gal}}{1\text{ cf}} = 17{,}615.4\text{ gal}$$

$$\text{Volume of 40' diam tank} = 0.785 \times (40\text{ ft})^2 \times 30\text{ ft} = 37{,}680\text{ ft}^3$$

Now convert the volume to gallons.

$$37{,}680\text{ cf} \times \frac{7.48\text{ gal}}{1\text{ cf}} = 281{,}846.4\text{ gal}$$

Force is always expressed in pounds. If you multiply the volume in gallons by 8.34 lbs per gallon, you will be left with the pounds of force being exerted on the bottom of each tank.

For the 10 ft diameter tank the calculation is as follows:

$$17{,}615.4\text{ gal} \times \frac{8.34\text{ lbs}}{\text{gal}} = 146{,}912.436\text{ lbs} = 146{,}912\text{ lbs}$$

For the 40 ft diameter tank the calculation is as follows:

$$281{,}846.4\text{ gal} \times \frac{8.34\text{ lbs}}{\text{gal}} = 2{,}350{,}598.976\text{ lbs} = 2{,}350{,}599\text{ lbs}$$

Clearly, the force exerted on the bottom of the larger, 40-foot diameter tank, is significantly greater than the force exerted on the 10-foot diameter tank.
Let's look at this problem a different way. The formula for force is:

$$\text{Force} = \text{Pressure} \times \text{Area}$$

Typically, pressure would be calculated in psi, but this requires that the area be expressed in square inches in order to calculate force. Since the tank diameters are given in feet, you can convert pressure from psi, pounds per square inch, to pounds per square foot.

From the previous example, you know that the pressure on the bottom of both tanks is 13 psi. Convert pounds per square inch to pounds per square foot.

$$\frac{13 \text{ lbs}}{\text{in}^2} \times \frac{144 \text{ in}^2}{1 \text{ ft}^2} = \frac{1{,}872 \text{ lbs}}{\text{ft}^2}$$

The bottom of each tank is a circle and the area can be calculated as follows:

$$\text{Area of a Circle} = 0.785 \times D^2$$

$$\text{Area of 10 ft diam tank} = 0.785 \times (10 \text{ ft})^2 = 78.5 \text{ ft}^2$$

$$\text{Area of 40 ft diam tank} = 0.785 \times (40 \text{ ft})^2 = 1{,}256 \text{ ft}^2$$

Now you can substitute these values into the Force equation. For the 10-foot diameter tank the force is the following.

$$\text{Force} = \frac{1{,}872 \text{ lbs}}{\text{ft}^2} \times 78.5 \text{ ft}^2 = 146{,}952 \text{ lbs}$$

For the 40-foot diameter tank the force is the following.

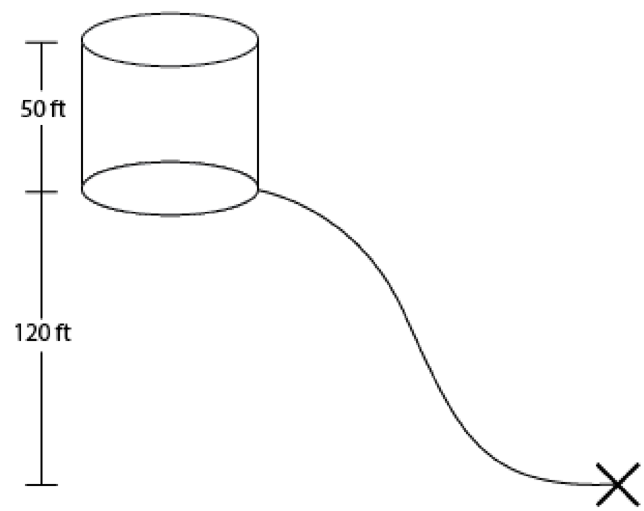
$$\text{Force} = \frac{1{,}872 \text{ lbs}}{\text{ft}^2} \times 1{,}256 \text{ ft}^2 = 2{,}351{,}232 \text{ lbs}$$

As you can see, the outcome is the same. The force being exerted on the bottom of the larger tank is significantly greater. The calculated value of the force is slightly different from the previous calculation due to rounding in the standard conversion factors.

Practice Problems 8.1

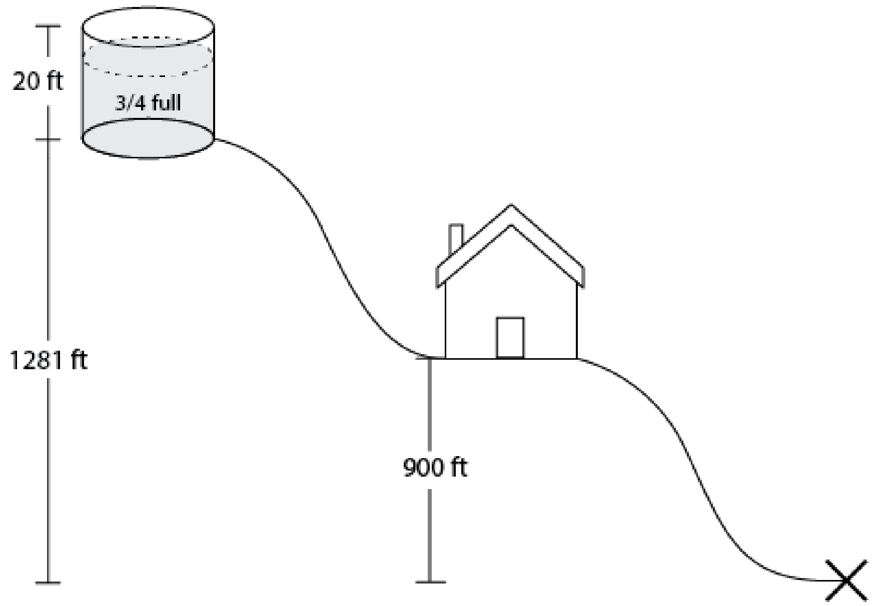
1. What is the pressure at the bottom of a 65-ft tank if the tank is half full?

2. A 50-foot-tall tank sits on a 120 foot tall hill. Assuming the tank is full, what is the pressure at the bottom of the hill?

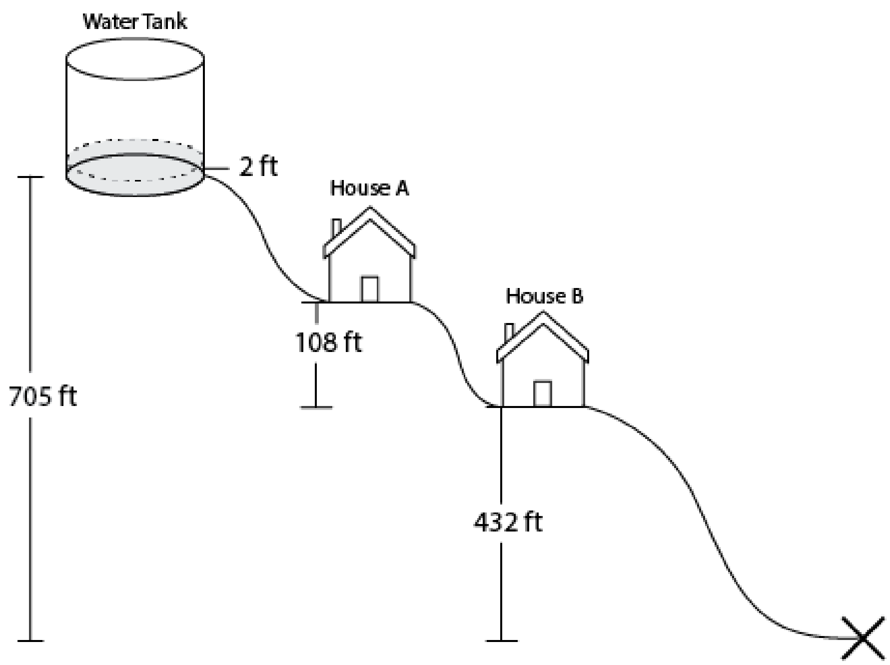


3. The opening of a 3.7” fire hydrant nozzle has a pressure of 212 psi. What is the corresponding force in pounds?

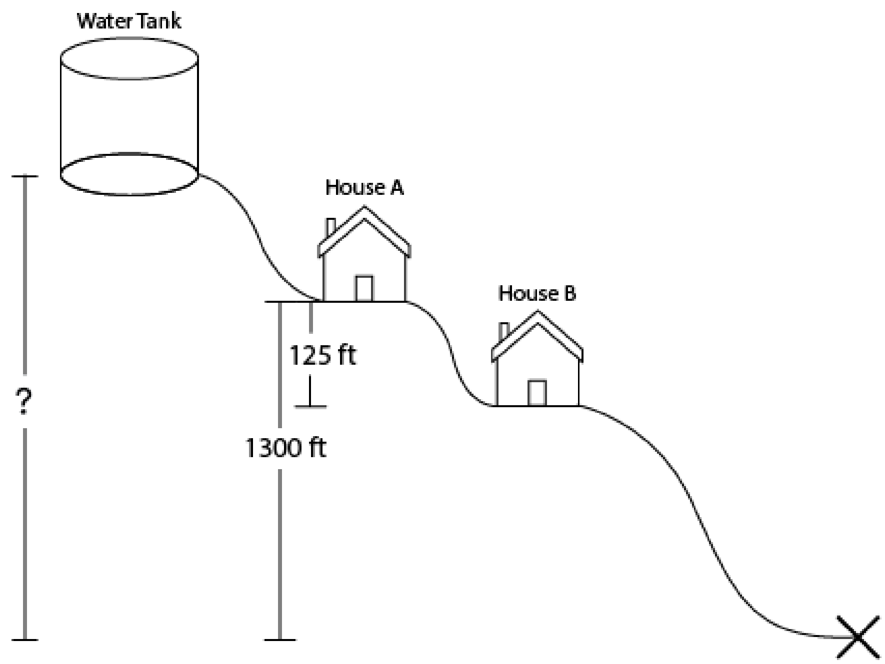
4. A home sits at an elevation of 900 ft above sea level. The base of a water tank that serves the home sits at an elevation of 1,281 ft above sea level. The tank is 20 feet tall and ¾ full. What is the pressure in psi at the home?



5. Two houses are served by a nearby water storage tank. House A is 108 ft above House B which sits at 432 ft above sea level. The base of the tank sits at 705 ft above sea level. The low water level in the tank is at 2.0 ft. At the low level, will House A meet the minimum pressure requirements of 60 psi?



6. House A sits at an elevation of 1,300 ft. Another house (B) needs to be built 125 ft below House A. At what elevation should the tank be built in order to give House B the maximum pressure of 210 psi?



7. A flowing pipeline has a pressure of 65 psi and a corresponding force of 2,398 pounds. What is the diameter of the pipe?

Exercise 8.1

Solve the following pressure- and force-related problems.

1. What is the pressure at the bottom of a 30-ft tank if the tank is half full?

2. A 28-foot-tall tank sits on a 75foot tall hill. Assuming the tank is full, what is the pressure at the bottom of the hill?

3. The opening of a 2 1/2” fire hydrant nozzle has a pressure of 135 psi. What is the corresponding force in pounds?

4. A home sits at an elevation of 1,301 ft above sea level. The base of a water tank that serves the home sits at an elevation of 1,475 ft above sea level. The tank is 35 feet tall and ¾ full. What is the pressure in psi at the home?

5. Two houses are served by a nearby water storage tank. House A is 55 ft above House B which sits at 725 ft above sea level. The base of the tank sits at 855 ft above sea level. The low water level in the tank is at 7.5 ft. At the low level, will House A meet the minimum pressure requirements of 35 psi?

6. House A sits at an elevation of 975 ft. Another house (B) needs to be built 75 ft below House A. At what elevation should the tank be built in order to give House B the maximum pressure of 130 psi?

7. A flowing pipeline has a pressure of 98 psi and a corresponding force of 4,924 pounds. What is the diameter of the pipe?

8.2 HEAD LOSS

As water travels through objects including pipes, valves, and angle points, or goes up hill, there are losses in pressure (or head) due to the friction. These losses are called “friction” or **head loss**. There are standard tables listing head loss factors (also termed C factor) for pipes of differing age and material, different types of valves and angle points. However, in this text we will focus on the theory more than the actual values.

Example: If water is traveling through 10,000 feet of pipe that has head loss of 3 feet, passes through 4 valves that have head loss of 1 foot for each valve, and passes through 2 angle points that have head loss of 0.5 feet each, calculate the total head loss.

To calculate the total head loss, you find the sum all the individual head losses.

$$3 \text{ ft} + 1 \text{ ft} + 1 \text{ ft} + 1 \text{ ft} + 1 \text{ ft} + 0.5 \text{ ft} + 0.5 \text{ ft} = 8 \text{ ft of head loss}$$

In distribution systems, water is pumped from lower elevations to higher elevations in order to supply customers with water in different areas called zones. Water is also pumped out of the ground using groundwater wells and from treatment plants that typically treat water from surface water sources. The water is then sent throughout the distribution system to supply customers in different pressure zones. As water makes its way through the distribution system head loss is realized (as mentioned in the previous paragraph) and pumps must overcome the head loss from elevation changes, interior pipe conditions, valves, and sudden angles.

Knowing the head losses will help determine what size pump is needed in a specific pressure zone. However, there are other forces acting on the pump that either help or hinder its ability to pump water. The diagrams below help illustrate the differences between suction lift and suction head. Suction lift requires more work by the pump to move the water from point A to point B because it has to lift water from a lower elevation. Suction head provides some help (head pressure) to the pump in order to get water from point A to point B.

Suction Lift and Suction Head

Let's take a look at how to identify suction head versus suction lift.

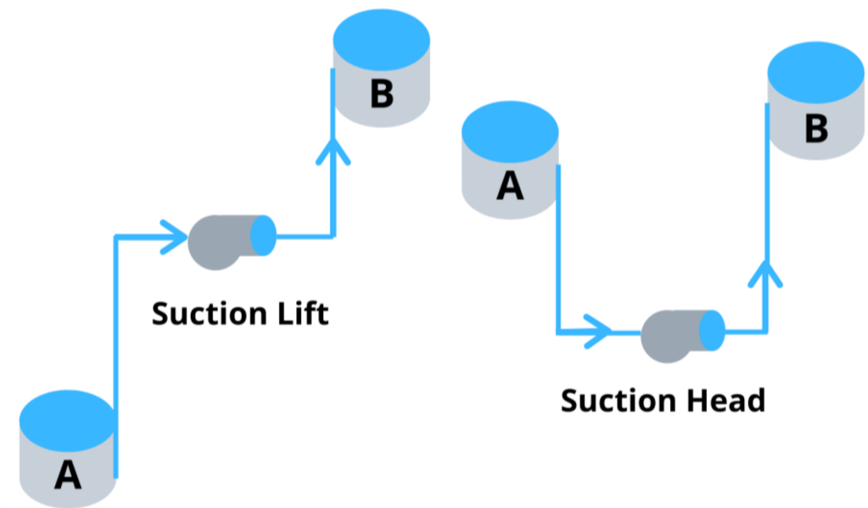


Figure 8.3[37]

Suction Lift

(Tank A Elevation + Pump Elevation) + (Pump Elevation + Tank B Elevation) = Total Head

Suction Head

(Tank B Elevation - Pump Elevation) - (Tank A Elevation – Pump Elevation) = Total Head

Friction can either be added or subtracted depending on where the measurement is being taken. See the two measuring points below (in red).

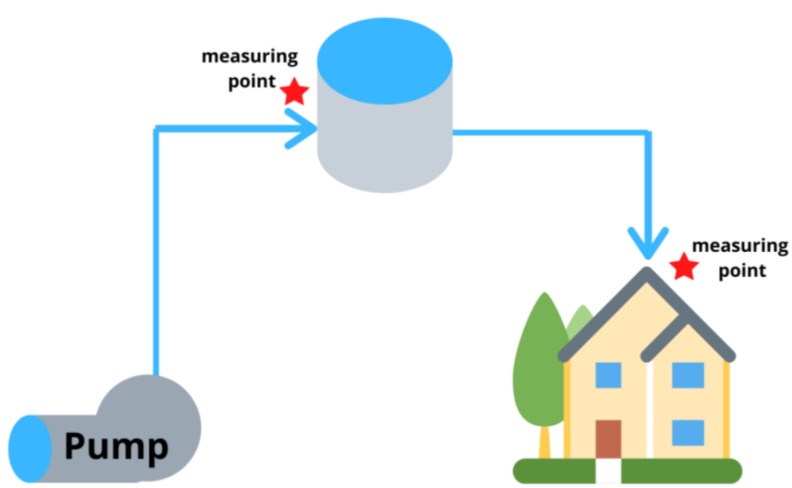


Figure 8.4[38]

[37] Image by College of the Canyons OER Team is licensed under CC BY 4.0

[38] Image by College of the Canyons OER Team is licensed under CC BY 4.0

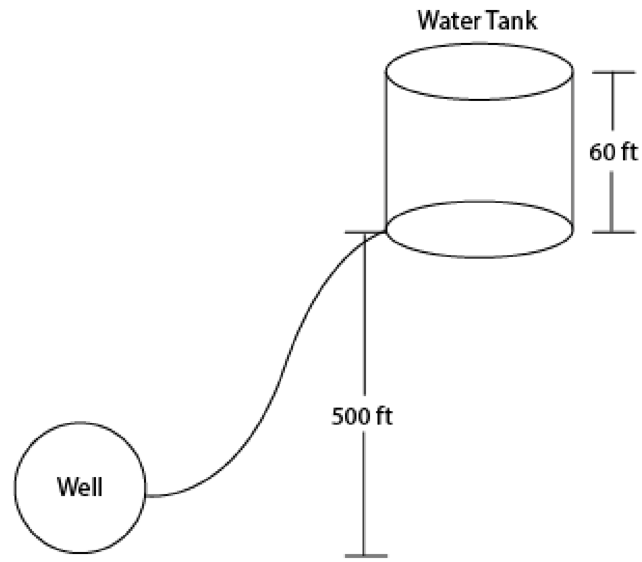
Friction will need to be added at the first measuring point because the pump will need to work harder to push water up to the tank. However, if water is traveling to a lower elevation like a home, then friction will need to be subtracted because the home will experience less pressure than it would have if there was no friction in the supply pipe.

Key Terms

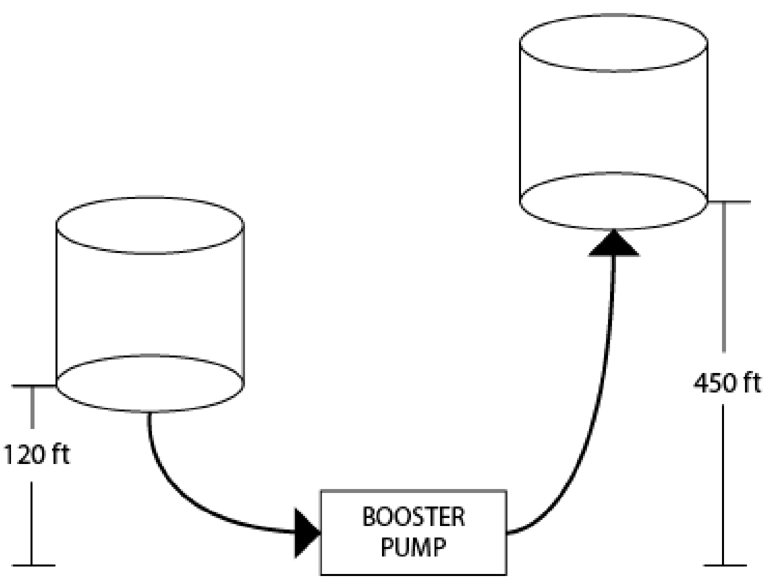
- **head loss** – loss in pressure or head in a water system
- **pressure** – the amount of force “pushing” on a specific unit area
- **suction lift** – (Tank A Elevation + Pump Elevation) + (Pump Elevation + Tank B Elevation) = Total Head
- **suction head** – (Tank B Elevation - Pump Elevation) - (Tank A Elevation – Pump Elevation) = Total Head

Practice Problems 8.2

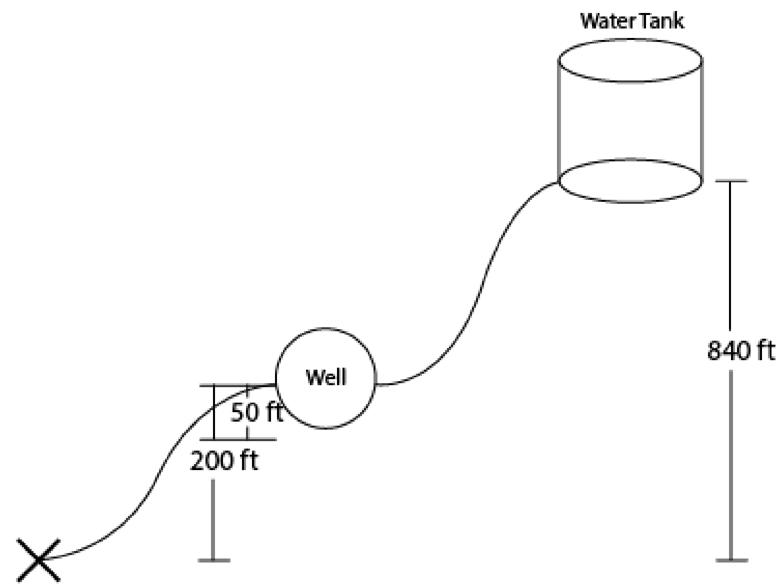
1. A well pumps directly to a 60-foot tall water tank that sits 500 feet above the elevation of the well. If the total head loss in the piping up to the tank is 14 feet, what is the total pressure in psi on the discharge side of the well?



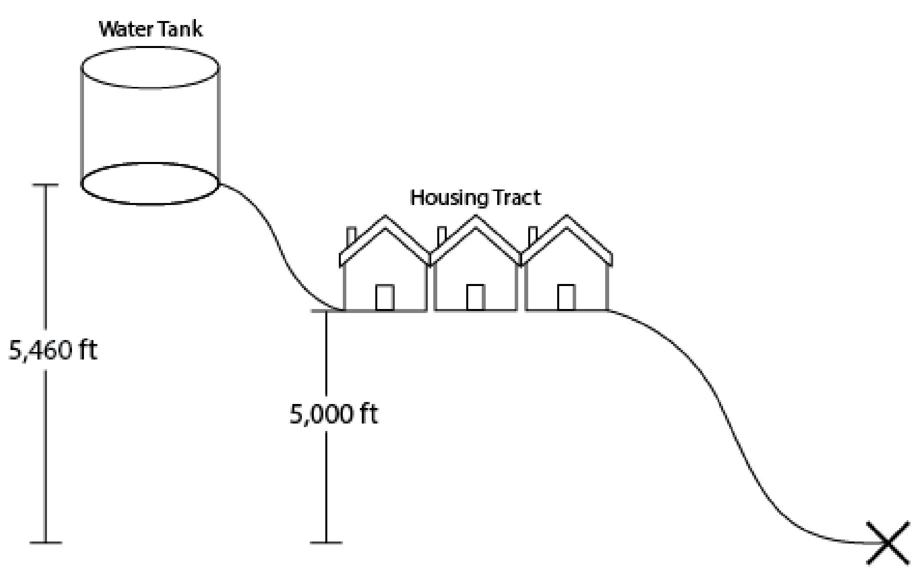
2. A booster pump receives water from a tank that is 120 feet above the pump and discharges to a tank that is 450 feet above the pump. What is the total head (TH)?



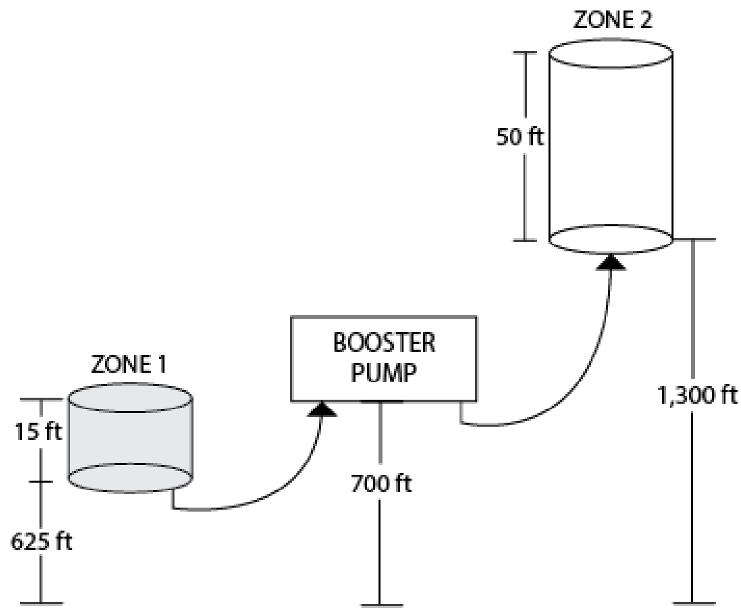
3. A well located at 200 feet above sea level has a below ground surface water depth of 50 ft and pumps to a water tank at an elevation of 840 ft above sea level. The water main from the well to the tank has a total head loss of 6 psi. What is the TH in feet?



4. A housing tract is located at an approximate average elevation of 5,000 ft above sea level and is served from a storage tank that is at 5,460 ft. The average head loss from the tank to the housing tract is 20.3 psi. What is the minimum water level in the tank to maintain a minimum pressure 30 psi?



5. A water utility has two different pressure zones (1 and 2.) The zone 1 Tank is 15 ft tall and sits at an elevation of 625 ft and the zone 2 Tank is 50 feet tall and sits at 1,300 ft. The booster pump from zone 1 to 2 sits at an elevation of 700 ft. The head loss is 11 psi. Tank 1 is full, and Tank 2 needs to be 1/2 full. What is the TH?



Exercise 8.2

Solve the following problems.

1. A well pumps directly to a 25-foot tall water tank that sits 200 feet above the elevation of the well. If the total head loss in the piping up to the tank is 5 feet, what is the total pressure in psi on the discharge side of the well?

2. A booster pump receives water from a tank that is 40 feet above the pump and discharges to a tank that is 275 feet above the pump. What is the total head (TH)?

3. A well located at 750 feet above sea level has a below ground surface water depth of 38 ft and pumps to a water tank at an elevation of 1,030 ft above sea level. The water main from the well to the tank has a total head loss of 11 psi. What is the TH in feet?

4. A housing tract is located at an approximate average elevation of 2,225 ft above sea level and is served from a storage tank that is at 2,330 ft. The average head loss from the tank to the housing tract is 15.5 psi. What is the minimum water level in the tank to maintain a minimum pressure 40 psi?

5. A water utility has two different pressure zones (1 and 2.) The zone 1 Tank is 30 ft tall and sits at an elevation of 850 ft and the zone 2 Tank is 40 feet tall and sits at 1,061 ft. The booster pump from zone 1 to 2 sits at an elevation of 925 ft. The head loss is 19 psi. Tank 1 is half full and Tank 2 needs to be ¾ full. What is the TH?