Investigations In Experimental Physics

Tenth Edition

Peter J. Polito, James E. Walsh, and Jeff L. Gagnon

Department of Math, Physics, and Computer Science

Springfield College

This lab manual was typeset using LaTeX software.

Forward

As stated in previous editions, the purpose of the physics laboratory is not only to reinforce the physical principles studied in the classroom but also to stimulate the development of skills and techniques required to carry out well-designed experimental investigations. It is the hope of the authors that the experimental skills and techniques acquired in this laboratory will be carried over by the student to their respective areas of interest. It is, in fact, strongly suggested that the student make an effort early in their undergraduate career to become familiar with the current research problems in their area of interest. The student should always be aware of the applicability of skills and techniques developed in this laboratory to their field of study.

This manual can be used in a one semester or two semester course. At our institution, we use it in a two semester General Physics course designed primarily for health science majors (Physical Therapy, Biology, Physician Assistant, etc.). It it also used in a one semester Physics for Movement Science course designed for movement science majors (Exercise Science, Athletic Training, Physical Education, etc.).

The tenth edition of the manual is a major revision of the ninth edition. Nearly all of the diagrams were updated, pictures of the lab equipment were included for every lab, and more tables were added to record data for each lab. Six new labs on Vector Addition and Subtraction with Applications, Graphical Interpretation, Motion Analysis, Applications of Rotational Equilibrium, Impulse-Momentum Vertical Jump Application, and Basic Exploration of Electric Fields Due to Point Charges were added. In addition, three older labs that we no longer have equipment to perform were removed (Hygrometry, Measurement of the Speed of Sound in Air by Kundt's Method, and Spectrometry). Many of the other labs were also updated and supplemented by adding a simulation component using the Phet online simulations.

The student is required to read and understand the entire laboratory experiment before their assigned laboratory period. Although the theoretical treatment included in the following experiments is in most cases sufficient for the student to develop a complete understanding of the physical principles involved in the experiment, it is strongly recommended that the student refer to their textbook for a more thorough discussion of these principles, if necessary.

Acknowlegements

The authors would like to thank Zenobia Lojewska for her help in designing lab 8 on motion analysis and lab 12 on applications of rotational equilibrium. The authors would also like to thank Lisa Bromberg for carefully reading the entire lab manual and making excellent suggestions for improvement as well as providing expert guidance on the subtleties of LaTeX usage.

Table of Contents

Part I

Measurement

Lab 1

Measurements: Space and Time

An integral part of any laboratory procedure is the ability to make accurate measurements. In this experiment you will be introduced to the measurement of three fundamental quantities - mass, length, and time. You will make all measurements in the metric system.

1.1 Theory

1.1.1 Measurement of Length

1.1.1.1 The Meter Stick

The meter was originally defined to be one ten-millionth of the distance from the North Pole to the equator along a meridian passing through Paris, France. The original standard meter is the distance between two fine lines engraved on gold plugs near the ends of a platinum-iridium alloy bar, maintained at a temperature of 0.00° Celsius at the International Bureau of weights and Measures located at Sevres, France. The meter sticks which you will use in this laboratory are meant to be reproductions of this standard and are accurate to within 1%. The meter is now defined as the distance light travels in free space in a time of $\frac{1}{299792458}$ second. An example of a meter stick is shown in Figure 1.1 below. It has a resolution of 0.1 cm.

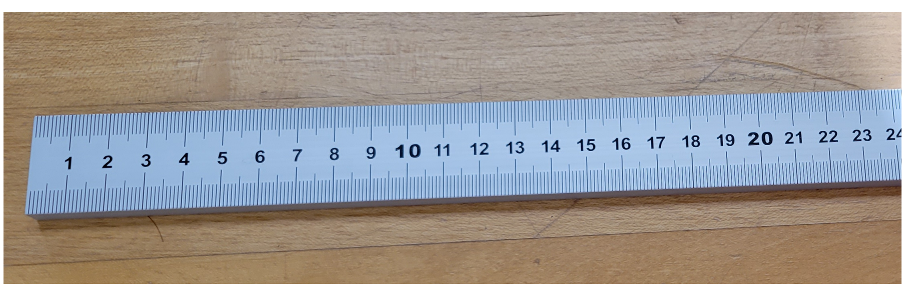


Figure 1.1: Typical Meter Stick

1.1.1.2 The Vernier Caliper

The device known as the vernier caliper is used for very accurate linear measurements. It has a resolution of 0.01 cm. Figure 1.2 below shows an example of a typical student vernier caliper. Note that the vernier scale has ten divisions that occupy the same length as nine divisions on the main scale. Therefore, each division on the vernier scale is $\frac{9}{10}$ of a division on the main scale. The first line on the left of the vernier scale (0 on the vernier scale) is called the index.

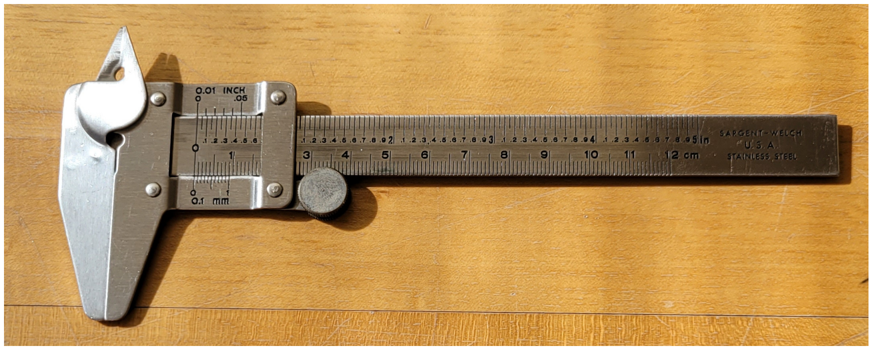
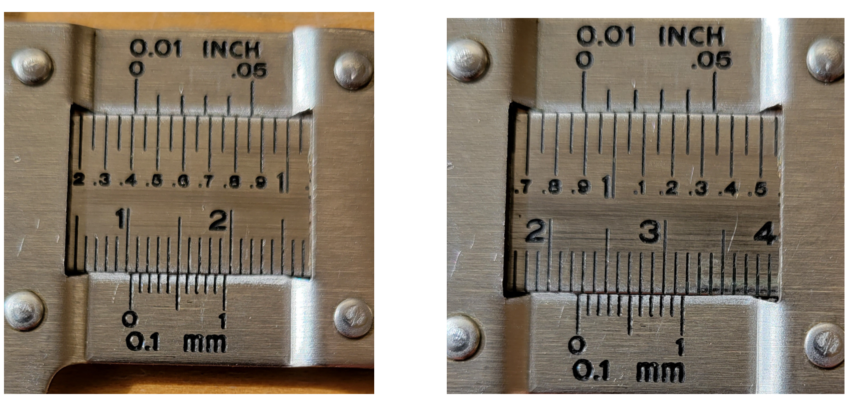


Figure 1.2: Typical Vernier Caliper

Note that the numbers etched on the main scale are in centimeters (cm). Therefore, the main scale is 12 cm long. Also note that the index on the vernier scale should coincide with the zero mark on the main scale when the jaws of the caliper are closed. If this is not the case, bring this to the attention of your instructor, as a correction must be made. Finally, you should notice that the first division on the vernier scale is 0.1 mm from the first division on the main scale. The second division on the vernier scale is 0.2 mm from the second division on the main scale, etc. To measure the length of an object, first place the object between the jaws of the vernier caliper. With the jaws closed snugly against the object, read the location of the index on the main scale. The reading on the main scale that is to the left of the index will give the length to the nearest 0.1 cm. To obtain the second decimal place (nearest 0.01 cm) you must determine which division from the vernier scale lines up best with a division from the main scale. Figure 1.3 below shows some examples of vernier caliper readings.



(a) 1.05 cm (b) 2.27 cm

Figure 1.3: Examples of Vernier Caliper Readings

1.1.1.3 The Micrometer Caliper

Another device used frequently for making measurements of short lengths very accurately is the micrometer caliper (see Figure 1.4 below). The micrometer has a resolution of 0.001 cm. The instrument is equipped with a longitudinal scale of measuring lengths up to 25 mm, and a circular scale of 50 divisions etched on the thimble. The pitch of the screw is 0.5 mm. Therefore, one complete revolution of the thimble advances the screw 0.5 mm, and each division on the circular scale advances the screw by 0.01 mm. To read the instrument, note where the edge of the thimble is on the main longitudinal scale and add to that the appropriate reading on the circular scale. Because one complete revolution on the circular scale advances the screw 0.5 mm, care must be taken to determine which half of the longitudinal scale division the edge of the thimble is on. In other words, if the edge of the thimble is past the half division mark make sure you add an additional 0.5 mm to your measurement. See Figure 1.5 below for some examples of micrometer readings.

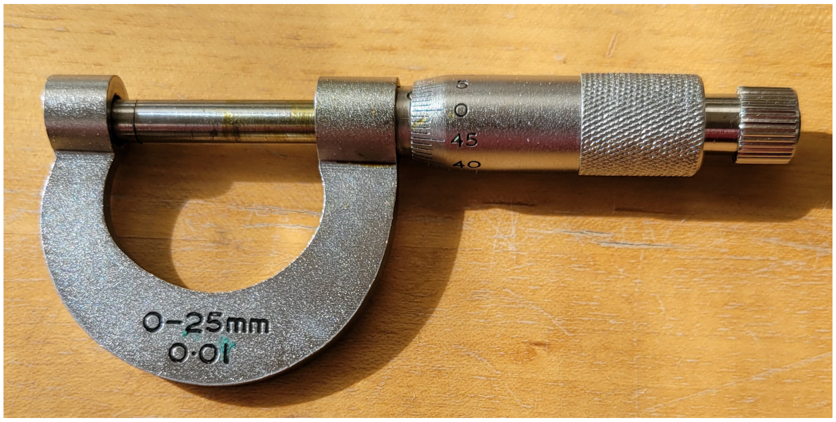
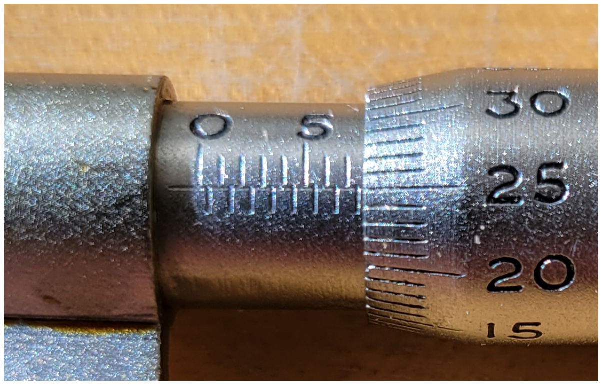
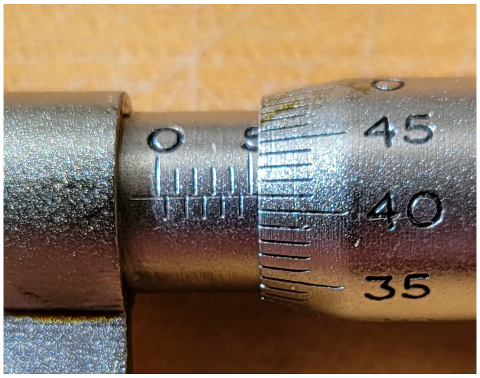


Figure 1.4: Typical Micrometer Caliper



(a) 7.75 mm = 0.775 cm



(b) 5.41 mm = 0.541 cm

Figure 1.5: Examples of Micrometer Caliper Readings

1.1.2 Measurement of Position

Choice of which type of coordinates to use is based in general upon convenience, which is usually displayed by certain symmetries. That is, depending upon the situation, one type of coordinates may be favored over all others. There are many different types of coordinates that one may choose from. However, four types (or sets) of coordinates that are quite popular and useful in physics are Cartesian or rectangular coordinates, plane polar coordinates, spherical polar coordinates, and cylindrical coordinates. If for example a particular problem displays spherical symmetry, then most probably it would be best to use spherical polar coordinates. In this course you will become well acquainted with Cartesian and plane polar coordinates. The interested student should refer to the standard college mathematics textbooks for a discussion concerning spherical polar, cylindrical, and other types of coordinate systems.

1.1.2.1 Cartesian Coordinates

To define the position (or location) of a point P in three-dimensional space using Cartesian coordinates, it is necessary to specify three distances from a common origin usually labeled O (see Figure 1.6). These three distances are measured along three mutually perpendicular directions, called the x-axis, y-axis, and z-axis, respectively. It is customary to then label the point P by the set of numbers (x, y, z) where x is the x coordinate, y is the y coordinate, and z is the z coordinate. For example, the point P in Figure 1.6 has x, y, and z coordinates (2, 3, 4).

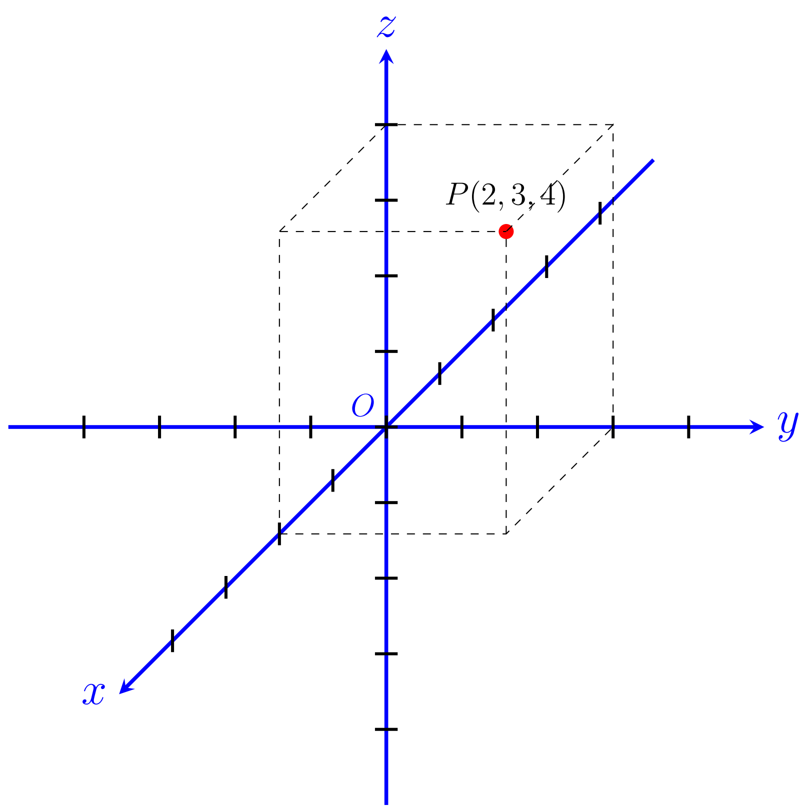


Figure 1.6: Position of Point P in Cartesian Coordinates

1.1.2.2 Plane Polar Coordinates

To locate the position of point P using plane polar coordinates, one must specify both the linear distance r of the point P from the origin O and the polar angle θ (theta), which the length r makes with respect to the positive x-axis (see Figure 1.7).

1.1.3 Measurement of Mass

The mass of an object is a quantitative measure of the inertia of the object. The inertia of an object is that property of the object that tends to resist a change in the state of motion of the object. Common units of mass in the metric system are the gram (g) and the kilogram (kg) where 1 kg =

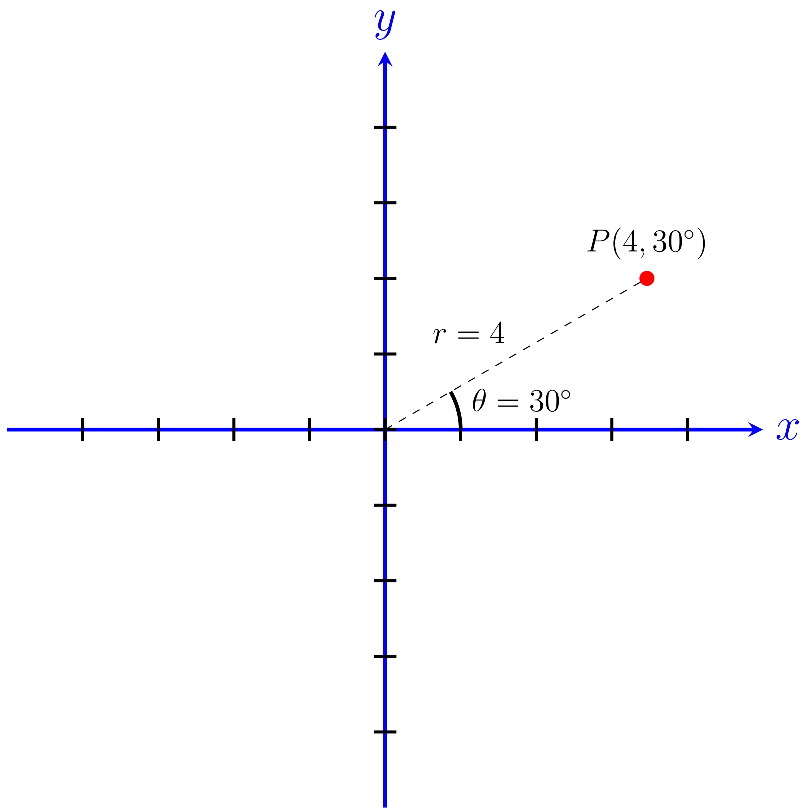


Figure 1.7: Position of point P in Plane Polar Coordinates

1000 g. For many years, the kilogram was defined as the mass of a platinum standard kept at the International Bureau of Weights and Measures at Sevres, France. Recently, however, the kilogram was redefined in terms of the Planck constant which is an invariant of nature. Your laboratory is equipped with an electronic scale for measuring mass (see Figure 1.8). The resolution of the scale is 0.1 g.

Note: The mass of an object is not to be confused with the weight of the object. The nonrelativistic (moving at speeds much less than the speed of light) mass of an object is a constant property that is independent of the motion of the object. However, weight which is the gravitational force that the Earth exerts on an object is not a characteristic property of an object and can change. For example, the weight of an object on the surface of the moon is about $\frac{1}{6}$ the weight of the same object on the surface of the Earth. The mass of the object, however, has the same value on the moon and on the Earth.

1.1.4 Measurement of Time

The flow of time can be measured by counting any regularly recurring event. The interval elasped between successive recurrences of the chosen event is defined as a unit of time. Such an event chosen early in history was the apparent daily motion of the sun. The interval between successive noons was defined as the solar day. However, the sun does not move at a uniform rate, and it is therefore necessary to define the mean solar day as the average length of the solar day taken over the entire year. The system of time based on this unit is known as Mean Solar Time or Civil Time.

The standard unit of time used in the laboratory is the second. The mean solar second was originally defined as $\frac{1}{86400}$ of the mean solar day. Because the revolution of the earth about the sun

Figure 1.8: Typical Scale for Measuring Mass

is not uniform, the length of the year, hence the mean solar day, varies. The Tropical Year is defined as the time elapsed from vernal equinox to vernal equinox (first day of spring - about March 21st). Careful measurements have shown the length of the tropical year has been increasing slightly over the past few decades (for example, 1970 was 41 seconds longer than 1900). In 1960 the standard of time adopted by the 11th General Conference on Weights and Measures was the second defined as $\frac{1}{31556925.9747}$ of the tropical year in 1900; that is, the number of mean solar seconds in 1900. This is called Ephemeris Time.

In an effort to get away from long-term astronomical predictions, scientists have redefined time in terms of the atom. Changes in the energy states of isolated atoms results in the emission of electromagnetic radiation of extremely constant energy, wavelength, and frequency. This remarkably precise periodicity has led to the use of the atom as an extremely accurate clock. In 1967 the second was redefined as 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the Cesium 133 atom.

1.2 Experiment

1.2.1 Measurement of Length

1.2.1.1 The Meter Stick

After you have acquainted yourself with the divisional markings on the meter stick, measure the length and width of your lab bench table top. Perform 5 trials and record your data in Table 1.1 below. Note that the meter stick has a resolution of 0.1 cm.

Question 1: Describe the procedure you used in making the above measurements. Describe how the meter stick was read.

Question 2: Compute the average length and average width of the lab bench. Compute the percent relative average absolute deviation (PRAAD) for the length and the width. If you neeed to review how to compute PRAAD, please refer to Section B.1 in Appendix B. Fill in Table 1.1 with your results.

Trial	Length (cm)	Absolute Deviation (cm)	Width (cm)	Absolute Deviation (cm)
1				
2				
3				
4				
5				
	Average Length (cm)	Average Abs. Dev. (cm)	Average Width (cm)	Average Abs. Dev. (cm)
	PRAAD (%)		PRAAD (%)	

Table 1.1: Measurements of the Laboratory Bench Table Top

Question 3: From your measurements of length and width compute the area of the lab bench table top. Also compute the PRAAD for area. Fill in Table 1.2 with your results.

Area of Lab Bench Table Top (cm^2)	
PRAAD for Area (%)	

Table 1.2: Area of the Laboratory Bench Table Top

1.2.1.2 The Vernier Caliper

Carefully measure the diameter and height of the aluminum cylinder using the vernier caliper. Perform 5 trials and record your data in Table 1.3. Note that the vernier caliper has a resolution of 0.01 cm.

Question 4: Describe the procedure you used in making the above measurements. Describe how the vernier caliper was read.

Question 5: Calculate the average height and the average diameter of the aluminum cylinder and compute the PRAAD for each of these measurements. Include your results in Table 1.3.

Question 6: Calculate the volume of the cylinder using

$$V = \frac{\pi d^2 h}{4}$$

then compute the PRAAD for volume. Include your results in Table 1.4.

Trial	Height (cm)	Absolute Deviation (cm)	Diameter (cm)	Absolute Deviation (cm)
1				
2				
3				
4				
5				
	Average Height (cm)	Average Abs. Dev. (cm)	Average Diameter (cm)	Average Abs. Dev. (cm)
	PRAAD (%)		PRAAD (%)	

Table 1.3: Aluminum Cylinder Measurements (Vernier Caliper)

Volume of Aluminum Cylinder (cm^3)	
PRAAD for Volume (%)	

Table 1.4: Volume of the Aluminum Cylinder (Vernier Caliper)

1.2.1.3 The Micrometer Caliper

Carefully measure the diameter and height of the aluminum cylinder using the micrometer caliper. Perform 5 trials and record your data in Table 1.5. Note that the micrometer caliper has a resolution of 0.001 cm.

Question 7: Calculate the average height and the average diameter of the aluminum cylinder and compute the PRAAD for each of these measurements. Include your results in Table 1.5.

Question 8: Calculate the volume of the cylinder using

$$V = \frac{\pi d^2 h}{4}$$

then compute the PRAAD for volume. Include your results in Table 1.6.

Using the meter stick, perform 5 measurements of the length of the stainless steel rod. Using the micrometer caliper, perform 5 measurements of the diameter of the stainless steel rod.

Question 9: Calculate the average height and the average diameter of the stainless steel rod and compute the PRAAD for each of these measurements. Include your results in Table 1.7.

Question 10: Calculate the volume of the stainless steel rod using

$$V = \frac{\pi d^2 h}{4}$$

then compute the PRAAD for volume. Include your results in Table 1.8.

Trial	Height (cm)	Absolute Deviation (cm)	Diameter (cm)	Absolute Deviation (cm)
1				
2				
3				
4				
5				
	Average Height (cm)	Average Abs. Dev. (cm)	Average Diameter (cm)	Average Abs. Dev. (cm)
	PRAAD (%)		PRAAD (%)	

Table 1.5: Aluminum Cylinder Measurements (Micrometer Caliper)

Volume of Aluminum Cylinder (cm^3)	
PRAAD for Volume (%)	

Table 1.6: Volume of the Aluminum Cylinder (Micrometer Caliper)

Trial	Height (cm)	Absolute Deviation (cm)	Diameter (cm)	Absolute Deviation (cm)
1				
2				
3				
4				
5				
	Average Height (cm)	Average Abs. Dev. (cm)	Average Diameter (cm)	Average Abs. Dev. (cm)
	PRAAD (%)		PRAAD (%)	

Table 1.7: Stainless Steel Rod Measurements (Meter Stick and Micrometer Caliper)

Volume of Stainless Steel Rod (cm^3)	
PRAAD for Volume (%)	

Table 1.8: Volume of the Stainless Steel Rod (Meter Stick and Micrometer Caliper)

1.2.2 Measurement of Mass and Density

Using the electronic balance, determine the mass of the aluminum cylinder. Perform 5 trials and record your data in Table 1.9.

The mass density of an object is defined as

$$\rho = \frac{m}{V} \tag{1.1}$$

where m is the mass and V is the volume of the object.

Question 11: Calculate the average mass and the PRAAD for mass of the aluminum cylinder. Record your values in Table 1.9. Using Equation 1.1 and your experimental values for the average mass and volume of the cylinder, compute the mass density of the cylinder. Also compute the PRAAD for density. Finally, compare your experimental value for the density to the accepted value of 2.70 $\frac{\text{g}}{\text{cm}^3}$ by computing the percent experimental error. Record your values in Table 1.10.

Trial	Mass (g)	Absolute Deviation (g)
1		
2		
3		
4		
5		
	Average Mass (g)	Average Abs. Dev. (g)
	PRAAD (%)	

Table 1.9: Mass of Aluminum Cylinder

Density of Aluminum Cylinder ($\frac{g}{cm^3}$)	
PRAAD for Density (%)	
Percent Experimental Error for Density (%)	

Table 1.10: Density of the Aluminum Cylinder

1.2.3 Measurement of Time

Using the wooden pendulum or the PhET pendulum lab simulation (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)) start by displacing the pendulum bob so it makes and angle of approximately 20° with the vertical. Measure the time for one oscillation (this is called the period of the pendulum). Perform 5 trials and record your data in Table 1.11.

Question 12: Calculate the average period for one swing. Compute the PRAAD for the period. Record your results in Table 1.11.

Again determine the period of the pendulum. In making this determination, measure the time elapsed as the pendulum makes ten complete oscillations. Perform 5 trials and record your data in Table 1.12.

Question 13: Calculate the average time for ten oscillations and the PRAAD for ten oscillations. Record your results in Table 1.12.

Trial	Time for One Oscillation (s)	Absolute Deviation (s)
1		
2		
3		
4		
5		
	Average Period (s)	Average Abs. Dev. (s)
	PRAAD (%)	

Table 1.11: Period of Pendulum (One Oscillation)

Trial	Time for Ten Oscillations (s)	Absolute Deviation (s)
1		
2		
3		
4		
5		
	Average Time for Ten Oscillations (s)	Average Abs. Dev. (s)
	PRAAD (%)	

Table 1.12: Period of Pendulum (Ten Oscillations)

Question 14: Compare the experimental values for the period of the pendulum as determined by measuring the time for one oscillation versus measuring the time for ten oscillations. To compute the period of the pendulum for ten oscillations simply divide the average time elapsed for ten oscillations by 10. Calculate the percent difference between your two values for the period using

$$\text{Percent Difference} = \frac{|\text{Period for one Oscillation} - \text{Period for Ten Oscillations}|}{\text{Period for Ten Oscillations}} \cdot 100\%$$

Record your results in Table 1.13. Which determination of the period do you think is more accurate?

Period for One Oscillation (s)	
Period for Ten Oscillations (s)	
Percent Difference (%)	

Table 1.13: Period of the Pendulum Comparison

Measure ten oscillations of the pendulum for initial angular displacements of 10°, 20°, 30°, 40°, 50°, and 60°. Compute the period for each initial angular displacement. Record your values in Table 1.14.

Initial Angular Displacement (°)	Time for Ten Oscillations (s)	Period (s)
10		
20		
30		
40		
50		
60		

Table 1.14: Period of Pendulum with Various Initial Angular Displacements

Question 15: Plot Period vs. Initial Angular Displacement. Based on the shape of your graph, does period depend on the initial displacement?

Starting with an angular displacement of the pendulum of about 20°, measure ten oscillations of the pendulum for lengths of 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm. Compute the period and the period squared for each length. Record your values in Table 1.15.

Question 16: Plot period vs. length for the pendulum. What is the shape of your graph? Perform a square root (sqrt) fit to the data.

Question 17: Plot period squared vs. length for the pendulum. What is the shape of your graph? Perform a linear fit to the data.

Question 18: Using the results from question 17, write an equation for the curve in question 16 (ie. write an equation that gives the period (T) as a function of the length (l)).

Length (cm)	Time for Ten Oscillations (s)	Period (s)	Period Squared (s^2)
0			
10			
20			
30			
40			
50			

Table 1.15: Period of Pendulum with Various Lengths

Part II

Vectors

Lab 2

An Experimental Investigation of Vector Quantities

In this experiment you will discover that vector quantities behave differently under the mathematical operation of addition than does the set of real numbers. To aid your understanding of the nature of vector addition, you will use the Force Table to experimentally determine the sum of two or more *force* vectors.

2.1 Theory

Physical quantities that possess both magnitude and direction, and under the operation of addition yield a physical quantity that also possesses a magnitude and direction, are called vector quantities. Examples of physical quantities that are vectors are displacement, velocity, acceleration, force, torque (moment of force), the electric field, and the magnetic field. Such physical quantities as force need, in addition to magnitude and direction, a line of action in order to be completely defined. That is, we must know along what *line of action* a force acts in order to realize the effect of the force.

We shall denote a vector symbolically here by a letter with a small arrow drawn above it. For example, the vector named A will be represented as $\vec{A}$. The magnitude of the vector $\vec{A}$ will be denoted simply by A with no arrow. Under the operation of addition, vector quantities enjoy two important properties.

a.) Commutative Law - Given two vectors $\vec{A}$ and $\vec{B}$,

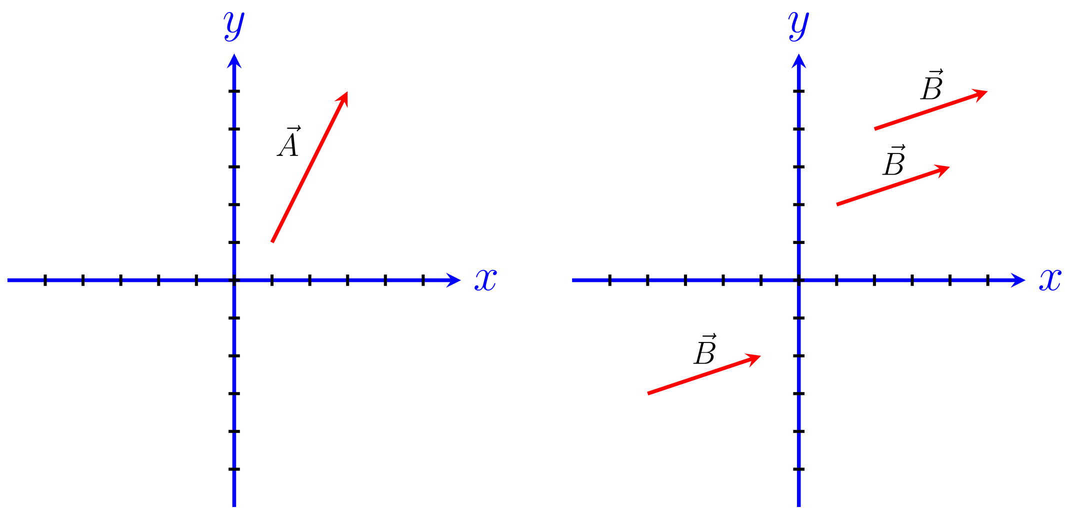
$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

b.) Associative Law - Given three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$,

$$\vec{A} + \left(\vec{B} + \vec{C}\right) = \left(\vec{A} + \vec{B}\right) + \vec{C}$$

A vector quantity is represented graphically by means of an arrow. Needless to say, in order to specify both the magnitude and direction of a vector, a coordinate system must be defined. The simplest, or most familiar, coordinate system is the Cartesian coordinate system that was introduced in Lab 1. In Figure 2.1a we draw the vector $\vec{A}$ in the two-dimensional space of the $xy-$plane. The scale assigned to the coordinate axes fixes the length of vector $\vec{A}$. The direction of vector $\vec{A}$ is given by measuring the angle that $\vec{A}$ makes with the positive x-axis.

In addition to the two properties of vector quantities mentioned above, there is the property of the translational invariance of vectors. Translating a vector quantity along a straight line direction does not change the nature of the vector. For example, as long as vector $\vec{B}$ maintains the same length and direction it can be moved around the coordinate system. Mathematically, it represents the same vector $\vec{B}$ (see Figure 2.1b).



(a) Graphical Representation of Vector $\vec{A}$ (b) Translational Invariance of Vector $\vec{B}$

Figure 2.1: Graphical Representation of a Vector

There are three methods that are customarily employed for adding two or more vectors together graphically. The triangle method and parallelogram method are used when adding two vectors together and the polygon method is used when adding three or more vectors together.

2.1.1 The Triangle Method for Adding Two Vectors

To add the vector $\vec{B}$ to the vector $\vec{A}$ graphically by means of the triangle method, simply draw the vector $\vec{B}$ such that its tail is at the tip of vector $\vec{A}$. The sum (i.e. resultant) is then obtained by drawing a vector $\vec{R}$ with its tail at the tail of $\vec{A}$ and its tip at the tip of $\vec{B}$ (see Figure 2.2).

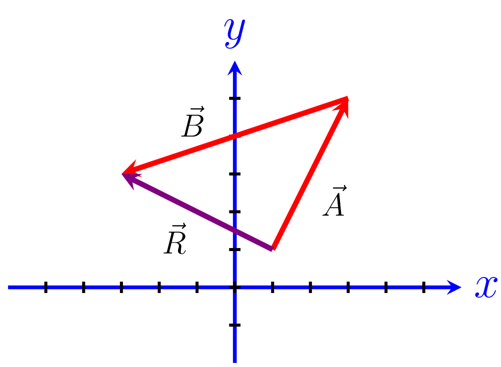


Figure 2.2: Triangle Method for the Addition of Two Vectors

The commutative law of addition $\vec{A}+\vec{B} = \vec{B}+\vec{A}$ can be shown graphically by using the triangle method of vector addition (see Figure 2.3).

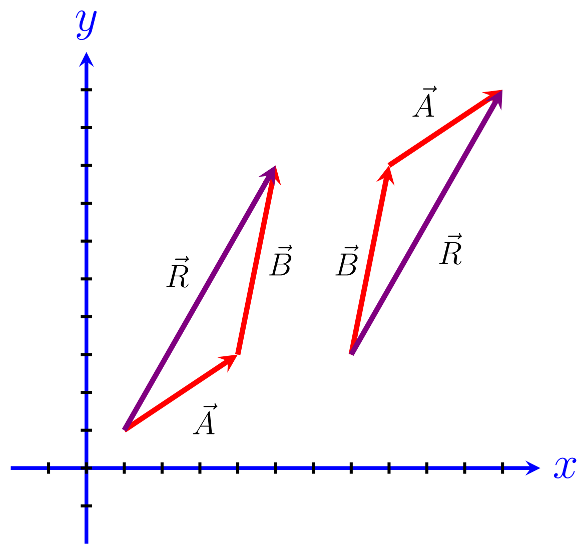


Figure 2.3: Commutative Law of Vector Addition Illustrated by the Triangle Method of Vector Addition

2.1.2 The Parallelogram Method for Adding Vectors

To add two vectors $\vec{A}$ and $\vec{B}$ together by means of the parallelogram method, simply place the tails of the vectors at a common point O. Then complete the parallelogram by drawing the side parallel to vector $\vec{A}$ starting at the tip of vector $\vec{B}$ and the side parallel to vector $\vec{B}$ starting at the tip of vector $\vec{A}$. The sum of the two vectors $\vec{R}$ is then the diagonal of the resulting parallelogram, drawn from point O to the opposite vertex (see Figure 2.4). You should also convince yourself that the parallelogram method also displays the commutative law of vector addition.

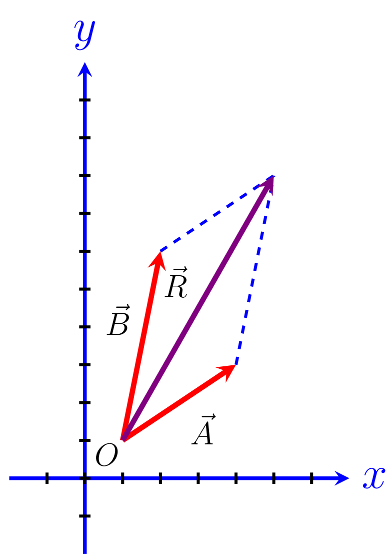


Figure 2.4: Parallelogram Method for the Addition of Two Vectors

2.1.3 The Polygon Method

To add three or more vectors graphically, the easiest way is to employ the polygon method. This method is a result of repetitive application of the triangle method (or parallelogram method) applied to two vectors at a time (it doesn't really matter which two since the commutative law and associative law apply to vector addition). Essentially, you can think of the polygon method as continuing to add vectors tip to tail. The tail of each new vector is placed at the tip of the previous vector. The resultant is then drawn from the tail of the first vector to the tip of the last vector. The polygon method applied to the vector addition of $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ is shown in Figure 2.5.

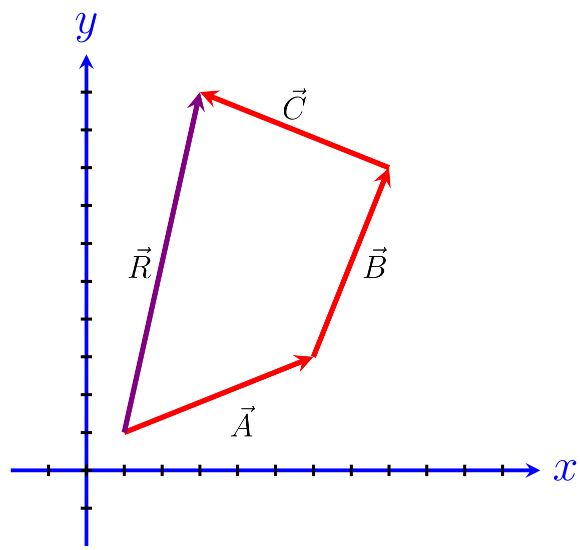


Figure 2.5: Polygon Method of Vector Addition

2.1.4 The Analytical Method

Although the graphical methods of adding vector quantities are instrumental in learning the fundamentals of vector addition, tremendous facility is to be found in the addition of vector quantities by analytical means. The most straightforward and basic of the analytical methods for adding vector quantities makes use of the decomposition of a vector into its components along the x, y, and z directions.

Figure 2.6 shows a vector $\vec{A}$ an the xy-plane. The vector $\vec{A}$, being translationally invariant, may be translated so that its tail is located at the origin of the coordinate system. $\vec{A}$ makes an angle θ with the $+x$ direction. In order to determine the x component of the vector $\vec{A}$, which we will call A_x, and the y component of $\vec{A}$, which we will call A_y, we find the projection of the vector $\vec{A}$ along the x and y directions, respectively. In order to project the vector $\vec{A}$ onto the x direction you draw a perpendicular line from the tip of $\vec{A}$ onto the x axis. In order to project the vector $\vec{A}$ onto the y direction you draw a perpendicular line from the tip of $\vec{A}$ onto the y axis. From Figure 2.6 it should be readily apparent that

$$A_x = A\cos\theta$$

and

$$A_y = A\sin\theta$$

Now consider the addition of the vector $\vec{A}$ and the vector $\vec{B}$. Set the vectors up as if you were graphically adding the vector $\vec{B}$ to the vector $\vec{A}$ by the triangle method, however, in this case you

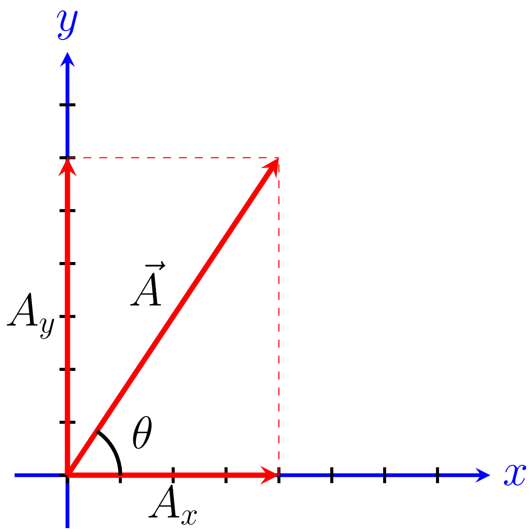


Figure 2.6: Decomposition of a Vector into its x and y Components

don't need to draw the vectors correctly to scale. Then, as is shown in Figure 2.7 below, the vector $\vec{A}$ and the vector $\vec{B}$ may each be decomposed into x and y components.

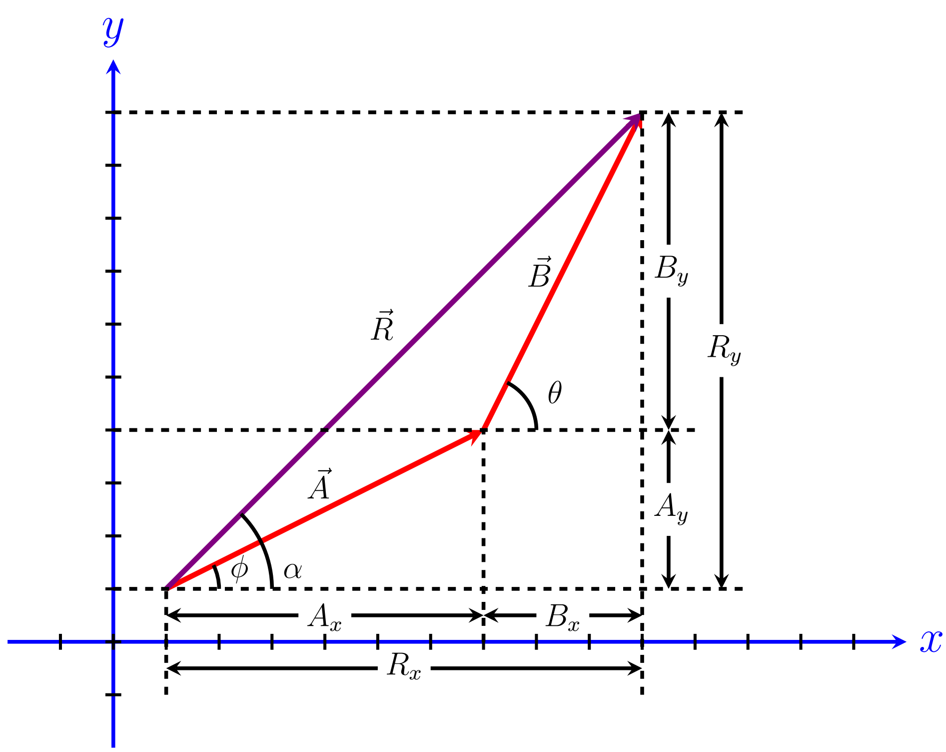


Figure 2.7: Analytical Addition of the Vectors $\vec{A}$ and $\vec{B}$

From Figure 2.7, we have

$$\begin{aligned} A_x &= A\cos\phi \\ A_y &= A\sin\phi \\ B_x &= B\cos\theta \\ B_y &= B\sin\theta \end{aligned}$$

Now notice that if we draw in the resultant vector $\vec{R} = \vec{A} + \vec{B}$, that its x and y components are given simply in terms of the x and y components of the vectors $\vec{A}$ and $\vec{B}$ according to

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y$$

respectively. Notice also that

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

and

$$\tan\alpha = \frac{R_y}{R_x} \quad \text{or} \quad \alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

therefore, the magnitude and direction of vector $\vec{R}$ has been found. To summarize, if the vectors $\vec{A}$ and $\vec{B}$ are completely known, then the vector $\vec{R} = \vec{A} + \vec{B}$ can be found by

(a) finding the x and y components of the vectors $\vec{A}$ and $\vec{B}$.

(b) computing the x and y components of the vector $\vec{R}$ in terms of the x and y components of the vectors $\vec{A}$ and $\vec{B}$.

(c) computing the magnitude of the vector $\vec{R}$ by employing the pythagorean theorem $R = \sqrt{{R_x}^2 + {R_y}^2}$.

(d) computing the angle that the vector $\vec{R}$ makes with the $+x$ direction by determining the tangent of this angle in terms of the ratio $\frac{R_y}{R_x}$.

The analytical method given above for the addition of two vector quantities is readily extended to the addition of three or more vectors. Suppose, for example, that one desires to find

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

where $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ are vectors defined in the xy plane. Then

$$\begin{aligned} R_x &= A_x + B_x + C_x + D_x \\ R_y &= A_y + B_y + C_y + D_y \end{aligned}$$

To find the magnitude of vector $\vec{R}$ we would again use

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

To find the angle θ that $\vec{R}$ makes with the x axis we would use

$$\tan\theta = \frac{R_y}{R_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

There are two types of vector products that must be defined. The first product is known as the scalar or dot product. Given two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

where θ is the angle between the two vectors when drawn with their tails together (see Figure 2.8). You can also see in the figure that $\vec{A}\cdot\vec{B}$ is equal to the projection of vector $\vec{B}$ along the direction of vector $\vec{A}$ times the magnitude of vector $\vec{A}$. You can also show that $\vec{A}\cdot\vec{B}$ is equal to the projection of vector $\vec{A}$ along the direction of vector $\vec{B}$ times the magnitude of vector $\vec{B}$ which is $\vec{B}\cdot\vec{A}$. One area where the dot product is useful is in calculating the work done by a force as an object undergoes a displacement.

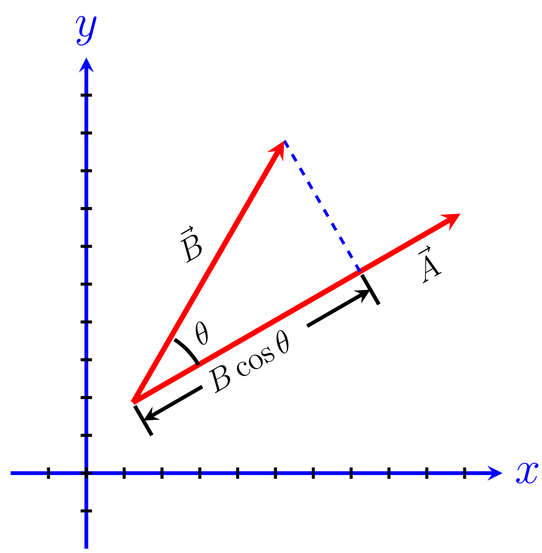


Figure 2.8: Scalar (Dot) Product Between Vectors $\vec{A}$ and $\vec{B}$

The vector cross product is defined by

$$\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$$

where θ is the angle between vector $\vec{A}$ and vector $\vec{B}$ and $\hat{n}$ is a unit vector (magnitude of 1) whose direction is perpendicular to both $\vec{A}$ and $\vec{B}$ (see Figure 2.9). To determine the direction of $\hat{n}$ one may use the "right hand rule" which involves starting by pointing your fingers in the direction of vector $\vec{A}$ then curling them towards vector $\vec{B}$. The direction of $\hat{n}$ is then the direction that your thumb points in. For the example shown, both vectors $\vec{A}$ and $\vec{B}$ lie in the xy plane so the direction of $\hat{n}$ would be perpendicular to the plane which is in the $+z$ direction. The vector cross product has many uses. One example involves computing the torque due to a force about an axis of rotation.

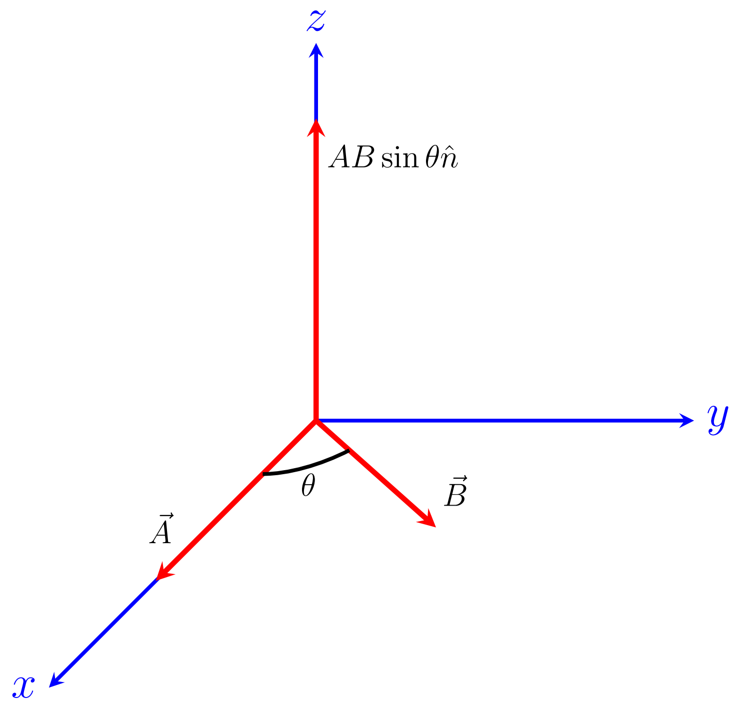


Figure 2.9: Vector (Cross) Product Between Vectors $\vec{A}$ and $\vec{B}$

2.2 Experiment

The force table consists of a large heavy circular disk that is mounted on a strong rigid stand. The disc is etched with marks whose range goes from 0° to 360°. The stand is equipped with adjustment feet for the purpose of leveling the apparatus. Pulleys can be mounted at any desired position on the disc by means of clamp attachments. At the center of the disk you will find a pin and ring. Cord may be tied to the ring and passed over the pulley. Mass hangers are supplied for the purpose of adding loads of known mass. Because the hangers are in equilibrium, the tension in the cord is equal to the weight of the hanger with its added mass. The pulley simply redirects the force so the force of the cord pulling on the center ring is ultimately equal to the weight of the hanger. The location of the pulley gives the direction of the force in degrees. If the sum of the forces (due to the tension in the cords) acting on the ring is zero, then upon removing the pin, the ring will remain stationary (see Figure 2.10).

2.2.1 Determination of the Sum of Two Forces

Place one pulley at the 30° mark and another at the 120° mark. Hang a load of 100 g from the pulley at 30° and a load of 200 g from the pulley at 120°. Now find the position on the disc at which a third pulley must be located and the load that must be suspended from this pulley in order that the ring be in equilibrium upon removal of the pin. The *force* that the load on the third pulley supplies is sometimes called the equilibrant force $\vec{E}$. The resultant $\vec{R}$ of the two forces supplied by the loads on the first two pulleys (call them $\vec{F}_1$ and $\vec{F}_2$) is equal in magnitude but opposite in direction to the equilibrant force $\vec{E}$. This is summarized by the following:

If the ring is in equilibrium, then

$$\vec{F}_1 + \vec{F}_2 + \vec{E} = 0$$

Figure 2.10: Typical Force Platform

however,

$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

therefore

$$\vec{R} + \vec{E} = 0$$
$$\vec{R} = -\vec{E}$$

Question 1: Using graph paper, sketch the vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{R}_{\text{exp}}$. The vectors should all be drawn with their tails at the origin of the coordinate system. Calculate the magnitude of the force for each vector given the mass of the hangers ($F = w = mg$). Calculate the direction of the experimental resultant vector $\vec{R}_{\text{exp}}$ by subtracting 180° from the direction of the equilibrant vector $\vec{E}$. Show your calculations and record your results in Table 2.1 below. Make sure you label both the magnitude and direction of each vector in your sketch.

	Corresponding Force	Mass (g)	Direction From Positive x-axis (°)	Force (N)
Mass 1	$\vec{F}_1$	100	30	
Mass 2	$\vec{F}_2$	200	120	
Mass 3 Resultant	$\vec{R}_{\text{exp}}$			

Table 2.1: Force Platform Data - Adding Two Vectors

Question 2: Using graph paper, now draw vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{R}_{\text{exp}}$ to scale using the scale factor 5 cm = 1 N. The vectors should again be drawn with their tails starting at the origin. Make sure you label both the magnitude and direction of each vector in your drawing.

Question 3: Using graph paper and the same scale factor (5 cm = 1 N) use the triangle method of vector addition (tip to tail) to add vectors $\vec{F}_1$ and $\vec{F}_2$. Then draw the graphical resultant vector $\vec{R}_{\text{graph}}$ from the tail of $\vec{F}_1$ to the tip of $\vec{F}_2$. Measure the length of $\vec{R}_{\text{graph}}$ then use the scale factor to compute the magnitude in Newtons. Finally, measure the direction (angle) of $\vec{R}_{\text{graph}}$ with the protractor. Make sure you label both the magnitude and direction of each vector in your drawing.

Question 4: Repeat the procedure in question 3 using the parallelogram method of vector addition.

Question 5: Compare your graphical results from question 3 to your experimental results from question 1 by computing the percent experimental error for both magnitude and direction. You can use the graphical results from question 3 as the accepted values. Show both calculations and record your results in Table 2.2.

$$\text{Percent Experimental Error for Magnitude} = \frac{|R_{\text{exp}} - R_{\text{graph}}|}{R_{\text{graph}}}$$

$$\text{Percent Experimental Error for Direction} = \frac{|\theta_{\text{exp}} - \theta_{\text{graph}}|}{\theta_{\text{graph}}}$$

Percent Experimental Error for the Magnitude (%)	
Percent Experimental Error for the Direction (Angle) (%)	

Table 2.2: Percent Experimental Errors - Adding Two Vectors

2.2.2 Determination of the Sum of Two Perpendicular Forces

Now place a pulley at the 0° mark and another at the 90° mark. From the pulley at 0°, suspend a load of mass 400 g and from the pulley at 90° suspend a load of mass 300 g. Determine the position at which a third pulley must be placed and the mass load that must be suspended from it in order that the ring be in equilibrium upon removal of the pin. The force supplied by the load on the third pulley is again the equilibrant force.

Question 6: Using graph paper, sketch the vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{R}_{\text{exp}}$. The vectors should all be drawn with their tails at the origin of the coordinate system. Calculate the magnitude of the force for each vector given the mass of the hangers ($F = w = mg$). Calculate the direction of the experimental resultant vector $\vec{R}_{\text{exp}}$ by subtracting 180° from the direction of the equilibrant vector $\vec{E}$. Show your calculations and record your results in Table 2.3 below. Make sure you label both the magnitude and direction of each vector in your sketch.

Question 7: Using graph paper, now draw vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{R}_{\text{exp}}$ to scale using the scale factor 3 cm = 1 N. The vectors should again be drawn with their tails starting at the origin. Make

	Corresponding Force	Mass (g)	Direction From Positive x-axis (°)	Force (N)
Mass 1	$\vec{F}_1$	400	0	
Mass 2	$\vec{F}_2$	300	90	
Mass 3 Resultant	$\vec{R}_{\text{exp}}$			

Table 2.3: Force Platform Data - Adding Two Perpendicular Vectors

sure you label both the magnitude and direction of each vector in your drawing. Now project the experimental resultant vector onto the x and y axes, respectively. What can you conclude from this procedure, taking into account the experimental accuracy of your experiment and knowing that the direction of $\vec{F}_1$ is 0° and the direction of $\vec{F}_2$ is 90°?

2.2.3 Determination of the Sum of Three Forces

Now place one pulley at 20°, another at 120°, and a third at 220°. From the pulley at 20°, suspend a load of mass 100 g, from the pulley at 120°, suspend a load of mass 200 g, and finally from the pulley at 220°, suspend a load of mass 150 g. Determine the position at which a fourth pulley must be placed and the mass load that must be suspended from it in order that the ring be in equilibrium upon removal of the pin. The force supplied by the load on the fourth pulley is again the equilibrant force.

Question 8: Using graph paper, sketch the vectors $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and $\vec{R}_{\text{exp}}$. The vectors should all be drawn with their tails at the origin of the coordinate system. Calculate the magnitude of the force for each vector given the mass of the hangers ($F = w = mg$). Calculate the direction of the experimental resultant vector $\vec{R}_{\text{exp}}$ by subtracting 180° from the direction of the equilibrant vector $\vec{E}$. Show your calculations and record your results in Table 2.4 below. Make sure you label both the magnitude and direction of each vector in your sketch.

	Corresponding Force	Mass (g)	Direction From Positive x-axis (°)	Force (N)
Mass 1	$\vec{F}_1$	100	20	
Mass 2	$\vec{F}_2$	200	120	
Mass 3	$\vec{F}_3$	150	220	
Mass 4 Resultant	$\vec{R}_{\text{exp}}$			

Table 2.4: Force Platform Data - Adding Three Vectors

Question 9: Using graph paper and the scale factor (5 cm = 1 N) use the polygon method of vector addition (extended tip to tail) to add vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Then draw the graphical resultant vector $\vec{R}_{\text{graph}}$ from the tail of $\vec{F}_1$ to the tip of $\vec{F}_3$. Measure the length of $\vec{R}_{\text{graph}}$ then use the scale factor to compute the magnitude in Newtons. Finally, measure the direction (angle) of $\vec{R}_{\text{graph}}$ with the protractor. Make sure you label both the magnitude and direction of each vector in your drawing.

Question 10: Compare your graphical results from question 9 to your experimental results from question 8 by computing the percent experimental error for both magnitude and direction. You can use the graphical results from question 9 as the accepted values. Show both calculations and record your results in Table 2.5.

Percent Experimental Error for the Magnitude (%)	
Percent Experimental Error for the Direction (Angle) (%)	

Table 2.5: Percent Experimental Errors - Adding three Vectors

Question 11: Use the analytical method of vector addition to add vectors $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$. Record the magnitude and direction of your resultant vector $\vec{R}_{\text{analytical}}$ in Table 2.6 below. To get more detail on adding vectors analytically, please refer to Lab 3.

Magnitude of $\vec{R}_{\text{analytical}}$	
Direction of $\vec{R}_{\text{analytical}}$	

Table 2.6: Vector Addition - Analytical Method

Question 12: Compare your analytical results from question 11 to your experimental results from question 8 by computing the percent experimental error for both magnitude and direction. You can use the analytical results from question 11 as the accepted values. Show both calculations and record your results in Table 2.7.

Percent Experimental Error for the Magnitude (%)	
Percent Experimental Error for the Direction (Angle) (%)	

Table 2.7: Percent Experimental Errors - Analytical Method

Lab 3

Vector Addition and Subtraction with Applications

In the last chapter, we briefly introduced the analytical method of vector addition. In this chapter, we will solve some applications that involve vector addition and subtraction.

3.1 Theory

Let's review in detail how to add two force vectors together.

Assume you are given two forces ($\vec{F}_1$ and $\vec{F}_2$) that act on an object and you want to compute the net force ($\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$). As an example, assume $\vec{F}_1$ is 100 N at 30° and $\vec{F}_2$ is 200 N at 135°. To add these vectors together, you should start by finding the x and y components of each vector. Drawing the vectors on a cartesian coordinate system is often very helpful. Please see Figure 3.1.

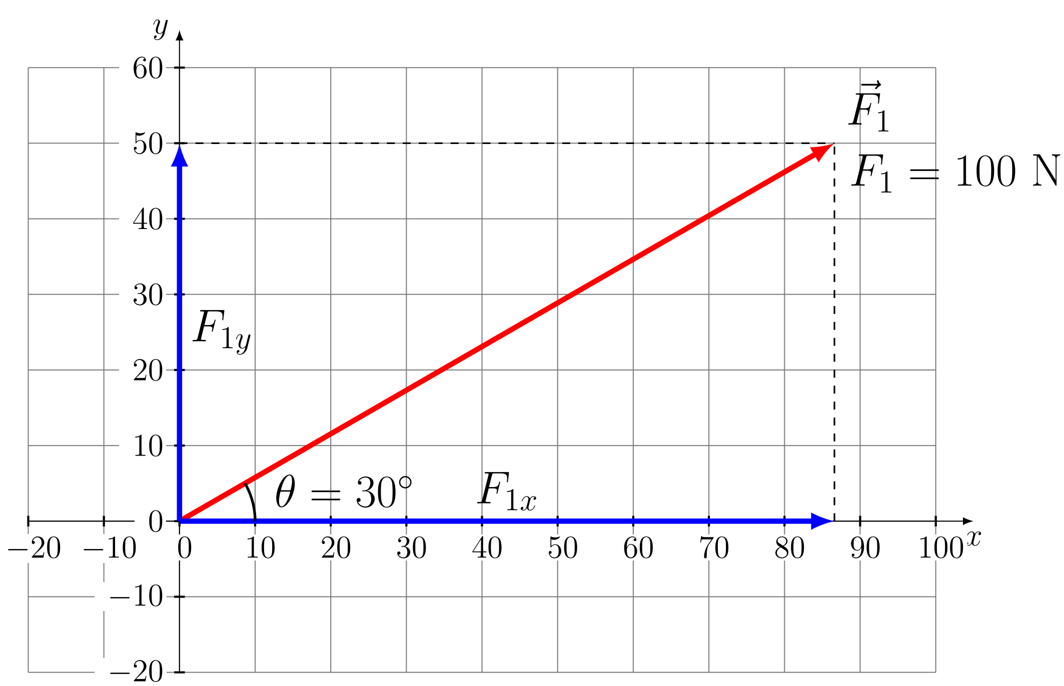


Figure 3.1: $\vec{F}_1$ and its components F_{1x} and F_{1y}

From this diagram, we can compute the following components for $\vec{F}_1$.

$$F_{1x} = F_1 \cos\theta = 100 \cos 30 = 86.6\,\text{N}$$
$$F_{1y} = F_1 \sin\theta = 100 \sin 30 = 50\,\text{N}$$

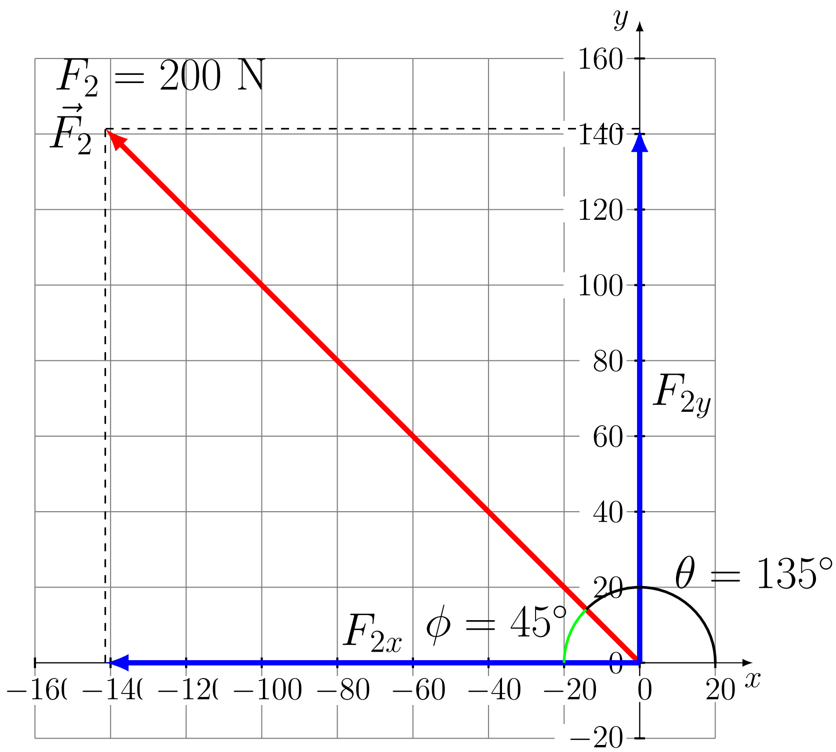


Figure 3.2: $\vec{F}_2$ and its components F_{2x} and F_{2y}

Referring to Figure 3.2, we can compute the following components for $\vec{F}_2$. Note that we have a choice as to which angle to use for the calculation. We could use the angle as measured from the positive x axis directly (θ) or we could use the reference angle (ϕ) and then attach the correct signs (+/-) to the components after. Each has its advantages.

Using θ directly gives

$$F_{2x} = F_2 \cos\theta = 200 \cos 135 = -141.4\,\text{N}$$
$$F_{2y} = F_2 \sin\theta = 200 \sin 135 = 141.4\,\text{N}$$

Using ϕ gives

$$F_{2x} = F_2 \cos\phi = 200 \cos 45 = 141.4\,\text{N}$$
$$F_{2y} = F_2 \sin\phi = 200 \sin 45 = 141.4\,\text{N}$$

Since F_{2x} is directed to the left, it must be negative. F_{2y} is directed upward so it must be positive. Therefore,

$$F_{2x} = -141.4\,\mathrm{N}$$
$$F_{2y} = 141.4\,\mathrm{N}$$

as before. The force vectors may now be written in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. This gives

$$\vec{F}_1 = 86.6\,\mathrm{N}\,\hat{\imath} + 50\,\mathrm{N}\,\hat{\jmath}$$
$$\vec{F}_2 = -141.4\,\mathrm{N}\,\hat{\imath} + 141.4\,\mathrm{N}\,\hat{\jmath}$$

To add $\vec{F}_1$ and $\vec{F}_2$ together, we just add the components together. This could be done using a table but it is particularly easy if the vectors are already written in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

$$\begin{aligned}\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 &= \left(86.6\,\mathrm{N}\,\hat{\imath} + 50\,\mathrm{N}\,\hat{\jmath}\right) + \left(-141.4\,\mathrm{N}\,\hat{\imath} + 141.4\,\mathrm{N}\,\hat{\jmath}\right)\\ &= (86.6\,\mathrm{N} + -141.4\,\mathrm{N})\ \hat{\imath} + (50\,\mathrm{N} + 141.4\,\mathrm{N})\ \hat{\jmath}\\ &= -54.8\,\mathrm{N}\,\hat{\imath} + 191.4\,\mathrm{N}\,\hat{\jmath}\end{aligned}$$

We can now rewrite the vector as a magnitude and a direction. Again, it is helpful to draw the resultant vector $\vec{F}_{\text{net}}$. Please see Figure 3.3.

We can then extract out the right triangle that includes the reference angle as shown in Figure 3.3 on the right. Notice now that since the lengths of the sides of a right triangle are always positive, we can treat the x and y components (the lengths of the two legs) as positive values. To calculate the magnitude of $\vec{F}_{\text{net}}$, we can then use the pythagorean theorem. To calculate the reference angle ϕ, we can use the arctan $\tan^{-1}$ function.

The magnitude is

$$\left|\vec{F}_{\text{net}}\right| = F_{\text{net}} = \sqrt{(F_{\text{net},x})^2 + (F_{\text{net},y})^2}$$
$$F_{\text{net}} = \sqrt{(54.8)^2 + (191.4)^2} = 199.1\,\mathrm{N}$$

The measure of the reference angle is then

$$\phi = \tan^{-1}\left(\frac{|F_{\text{net},y}|}{|F_{\text{net},x}|}\right)$$
$$\phi = \tan^{-1}\left(\frac{191.4}{54.8}\right) = 74.0^\circ$$

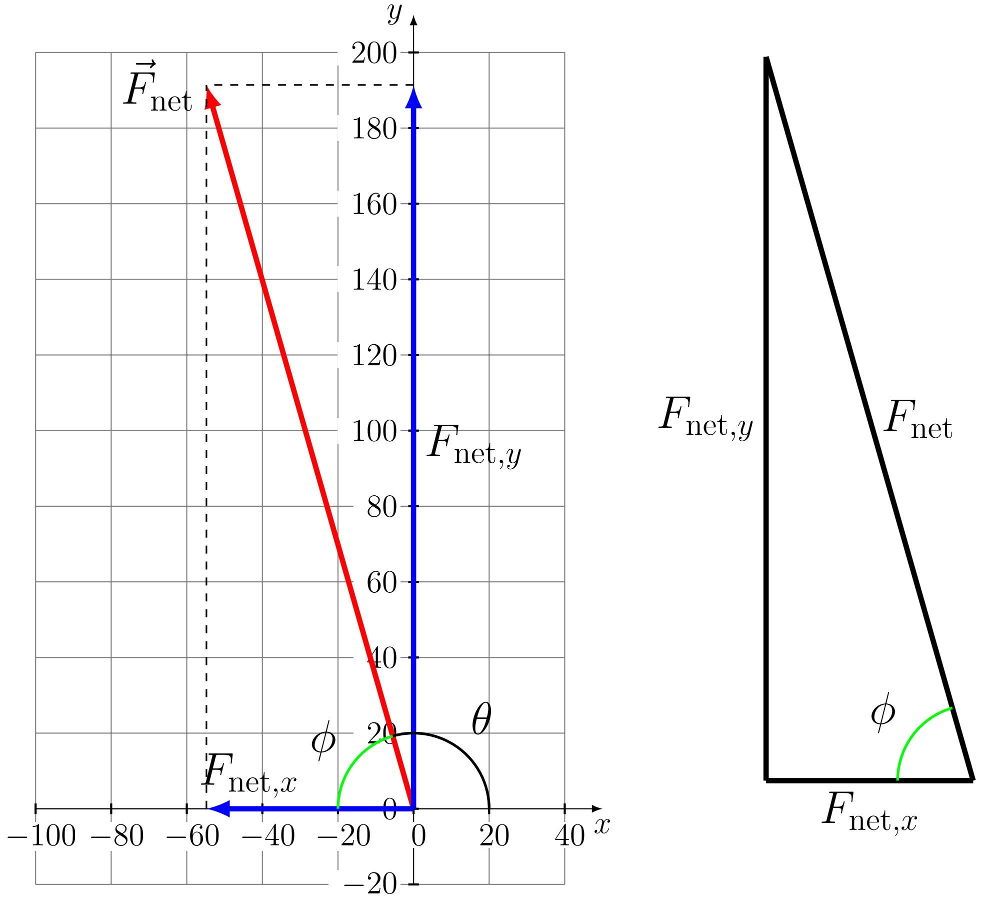


Figure 3.3: $\vec{F}_{\text{net}}$ and its components $F_{\text{net},x}$ and $F_{\text{net},y}$

Finally, we can calculate the angle of the vector θ as measured relative to the positive x-axis and write the vector in terms of magnitude and direction.

$$\theta = 180 - \phi = 180 - 74.0 = 106.0^\circ$$
$$\vec{F}_{\text{net}} = 199.1\,\text{N at } 106.0^\circ \text{ or}$$
$$\vec{F}_{\text{net}} = 199.1\,\text{N at } 74.0^\circ \text{ N of W}$$

3.2 Experiment

This experiment involves a series of applications to solve using your knowledge of vector addition and subtraction.

Question 1: Charge Q_1 and charge Q_2 each exert a force on charge Q_3 (see Figure 3.4). If the magnitude of the force on Q_3 due to Q_1 is $F_{31} = 140$ N and the magnitude of the force on Q_3 due to Q_2 is $F_{32} = 325$ N, find the net force $\vec{F}_{\text{net}}$ acting on Q_3. Represent the net force in terms of a magnitude and a direction.

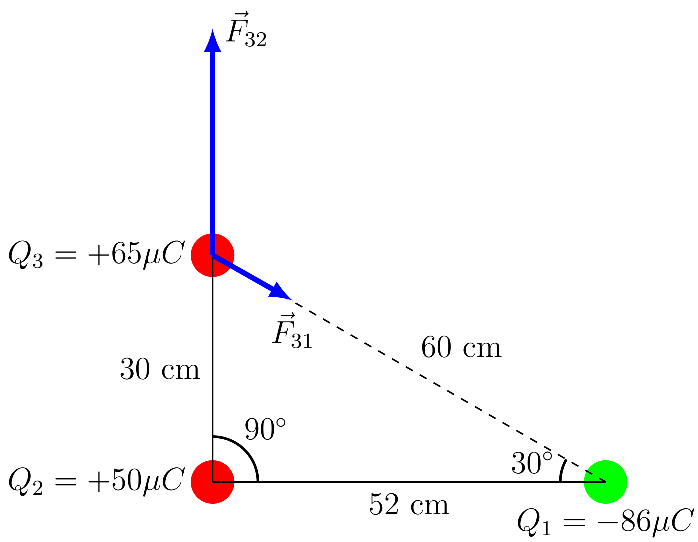


Figure 3.4: Forces on Charges

Question 2: A ball collides with a wall with a velocity $\vec{v}_1$ of 100 $\frac{\text{m}}{\text{s}}$ at an angle of 40° with respect to the wall. The ball undergoes an elastic collision and rebounds at a velocity $\vec{v}_2$ of 100 $\frac{\text{m}}{\text{s}}$ at an angle of 40° with respect to the wall (see Figure 3.5). Find the change in velocity of the ball. Hint: $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ but we can write this as $\Delta\vec{v} = \vec{v}_2 + -\vec{v}_1$ and treat it as vector addition.

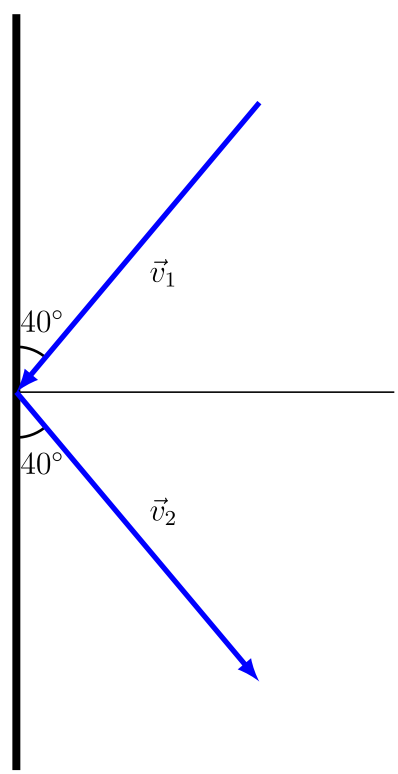


Figure 3.5: Change in Velocity

Question 3: Four forces are acting on a central ring (see Figure 3.6). $\vec{F}_1$ has a magnitude of 100 N. $\vec{F}_2$ has a magnitude of 70 N. $\vec{F}_3$ has a magnitude of 80 N. Finally, $\vec{F}_4$ has a magnitude of 120 N. Using the directions of each vector given in the figure, find the net force $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$ acting on the ring. Give the final answer as a force with a magnitude and a direction.

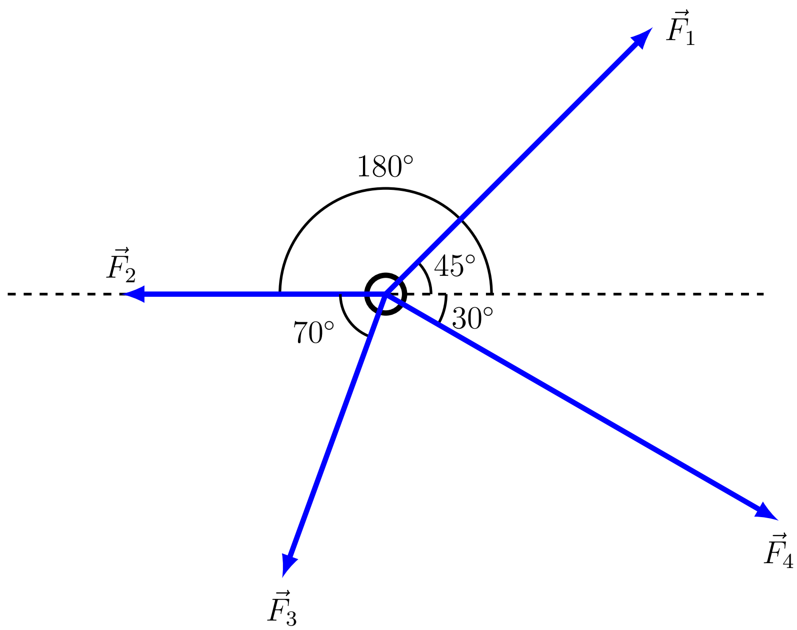


Figure 3.6: Forces on Ring

Question 4: The purpose of this next activity is to utilize your knowledge of geometry, trigonometry, and vectors to solve a problem involving a robotic vacuum cleaner. You have a 15 ft by 12 ft room that needs to be cleaned. A robotic vacuum clearer starts in the southwest side of the room as shown in the diagram. The diameter of the vacuum cleaner is 1.2 ft. A cartesian coordinate system is attached. At the start, the center of the robotic vacuum is at the origin $(0, 0)$ of the coordinate system. The robot starts moving at an average speed of 45.72 $\frac{\text{cm}}{\text{s}}$ at an angle of 20° with respect to the x-axis until it encounters the east wall. It then rotates 114° counterclockwise and then travels at an average speed of 45.72 $\frac{\text{cm}}{\text{s}}$ for 3.5 s until it hits a piece of furniture. The robot then rotates 136° counterclockwise and travels at a average speed of 45.72 $\frac{\text{cm}}{\text{s}}$ until it encounters the south wall. Finally, the robot rotates 150° clockwise and travels at an average speed of 45.72 $\frac{\text{cm}}{\text{s}}$ for 6 s and then comes to a complete stop (see Figure 3.7 below).

a.) We want to solve this problem in US Customary units. Therefore, you want to start by converting the robotic vacuum speed from units of $\frac{\text{cm}}{\text{s}}$ to $\frac{\text{feet}}{\text{s}}$.

b.) In Figure 3.7 below, I have drawn the individual displacements $\vec{d}_1$ and $\vec{d}_2$ as well as the net displacement $\vec{d}_{\text{net}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4$. The displacement refers to the change in position of the center of the robotic vacuum cleaner. Using a protractor and a ruler, sketch $\vec{d}_3$ and $\vec{d}_4$ on the diagram. Make sure when you print the pdf, you choose the option "actual size" from the print menu. Then the scale factor will be exactly 1 cm = 1 foot. Check this by measuring the length of the room, it should be exactly 15 cm.

c.) For each part of the path, find the individual displacements of the robotic vacuum cleaner ($\vec{d}_1$,

$\vec{d}_2$, $\vec{d}_3$, and $\vec{d}_4$). These quantities should be found analytically and presented in component form (for example $\vec{d}_1 = d_{1x}\,\hat{\imath} + d_{1y}\,\hat{\jmath}$).

d.) Find the width w of the piece of furniture.

e.) Find the net displacement of the robotic vacuum cleaner $\left(\vec{d}_{\text{net}}\right)$ in component form.

f.) Find the magnitude and direction of the net displacement vector. Check your analytical answer graphically by measuring the length of the net displacement vector and its angle. Were they equal?

g.) Given that the average speed of the robotic vacuum cleaner was 45.72 $\frac{\text{cm}}{\text{s}}$ for each part of the path and assuming that it took no time for the vacuum to rotate when it changed direction, what was the average velocity ($\vec{v}_{\text{ave}}$) of the robotic vacuum cleaner? Remember that the average velocity has a magnitude and a direction.

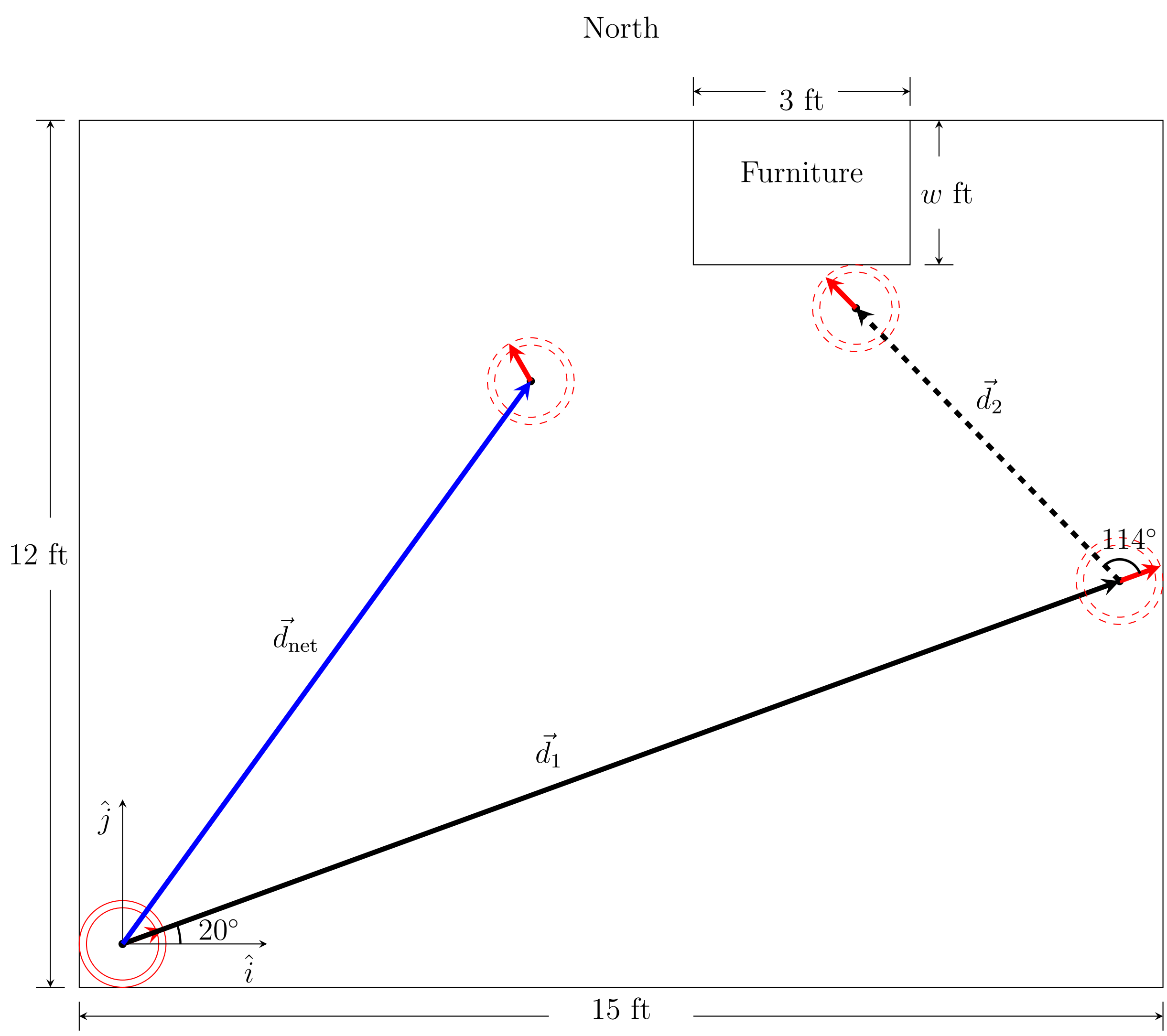


Figure 3.7: Robot Vacuum Cleaner

Part III

Kinematics

Lab 4

Study of Velocity and Acceleration in One Dimension

Two fundamental physical quantities that are descriptive of the motion of an object are velocity and acceleration. In this experiment you will obtain measures of constant velocity, instantaneous velocity, and uniform acceleration.

4.1 Theory

The average velocity of an object is defined by

$$\bar{\vec{v}} = \frac{\vec{x}_{\mathrm{f}} - \vec{x}_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} = \frac{\Delta \vec{x}}{\Delta t}$$

where $\Delta \vec{x}$ is the displacement of the object as it moves from an initial position to a final position and Δt is the corresponding time interval elapsed during this change in position.

The instantaneous velocity of an object is defined by

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

where $d\vec{x}$ is the infinitesimal increment of displacement and dt is the infinitesimal increment of time. The speed of an object is the magnitude of the velocity of that object.

The average acceleration on an object is defined by

$$\bar{\vec{a}} = \frac{\vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} = \frac{\Delta \vec{v}}{\Delta t}$$

where $\Delta \vec{v} = \vec{v}_{\mathrm{f}} - \vec{v}_{\mathrm{i}}$ is the change in the velocity of the object as it moves from the initial position to the final position and where $\Delta t = t_{\mathrm{f}} - t_{\mathrm{i}}$ is the time interval elapsed as the object moves from the initial position to the final position.

Instantaneous acceleration is given by

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

When the acceleration of the object is uniform, then one may write the average velocity in terms of the initial velocity $\vec{v}_{\mathrm{i}}$ at position 1 and the final velocity $\vec{v}_{\mathrm{f}}$ at position 2 as

$$\bar{\vec{v}} = \frac{\vec{v}_{\mathrm{i}} + \vec{v}_{\mathrm{f}}}{2}$$

If the object starts from rest ($\vec{v}_{\rm i} = 0$), then the final velocity is given as

$$\vec{v}_{\rm f} = 2\frac{\Delta \vec{x}}{\Delta t} = 2\vec{\bar{v}}$$

For this special case, the final velocity is twice the average velocity.

4.2 Experiment

This experiment will be performed using the linear air tracks (see Figure 4.1). Time intervals Δt will be measured by the use of a photocell gate. While the flag attached to the cart passes through the gate, the light beam is broken and the timer is activated.

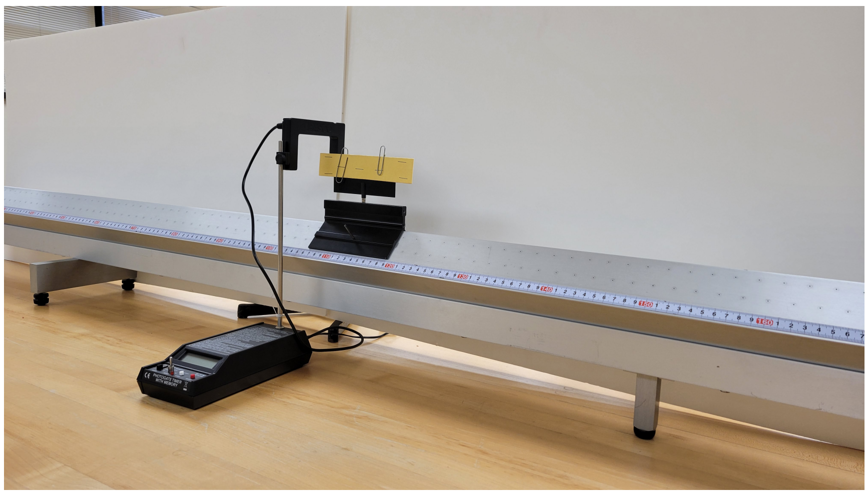


Figure 4.1: Linear Air Track and Photogate Set-up

4.2.1 Measurement of Constant Velocity

Make sure the air track is level on the lab bench. You can check this by placing the cart gently on the air track and releasing it. If the air track is level, the cart should not move. If it does move, adjust the air track feet to correct for this. Now attach the 10 cm flag to the cart (this is the standard plastic flag that comes with the cart). The flag should be adjusted so that it will occlude the photogate beam when the flag passes through the photogate. You may also have to adjust the height of the photogate. Make sure that the beam is broken while the entire length of the flag passes through.

Place the photogate about halfway down the air track. Now launch the cart using the rubber band launcher (see Figure 4.2). When you pull the cart back on the ruuber band launcher, observe the amount that it is displaced. Practice launching the cart until you are repeatedly able to send the cart through the photogate with approximately the same Δt. Taking care always to launch the cart with the same speed, measure 5 values of Δt for flags of length 10 cm, 15 cm, 20 cm, 25 cm, 30 cm. The longer flag lengths can be achieved by attaching paper flags to the standard 10 cm flag using paper clips. Record all of your data in Table 4.1 below.

By using the different flag lengths and performing several trials, we are slowly building up the time and position values of the cart as it travels at a constant velocity down the air track.

Imagine that the origin of the x- axis of the coordinate system is at the photogate. When the flag first occludes the photogate, that represents the initial time $t_i = 0s$. After the 10 cm flag passes through, we have the time for the cart to achieve a position of 10 cm. When the 15 cm flag passes through, we have the time for the cart to achieve a position of 15 cm, and so on.

Figure 4.2: Rubber Band Launcher

Position x (cm)	Time Δt (s)					Average Time $\overline{\Delta t}$ (s)	Average Velocity $\frac{\Delta x}{\Delta t}$ $\left(\frac{\text{cm}}{\text{s}}\right)$
0	0	0	0	0	0	0	
10							
15							
20							
25							
30							

Table 4.1: Data for Constant Velocity Motion of Cart

Question 1: Plot position versus time (plot your position values on the y axis and the average time values on the x axis). Based on the shape of your graph, perform a proper curve fit.

Question 2: What is the shape of your graph from question 1? What does the slope of the graph represent? Record the value of the slope from your graph.

Question 3: Compute the average velocity values using $\bar{v} = \frac{\Delta x}{\Delta t}$ where $\Delta x_1 = 10 - 0 = 10$ cm, $\Delta x_2 = 20 - 0 = 20$ cm, etc. and your Δt is your average time $\overline{\Delta t}$. Plot average velocity versus time (plot your average velocity values on the y axis and the average time values on the x axis). Perform an average fit to the data. What can you conclude from the shape of your graph?

4.2.2 Motion with Constant Acceleration

Elevate the end of the air track by placing a 2 kg mass under the single leg (see Figure 4.3). Attach the 10 cm flag to the cart. Adjust the height of the photogate so that it is occluded as the entire length of the flag passes through. Place the cart at rest in a position such that the flag is just about to trigger the photogate. Release the cart and record the time elapsed as the flag passes through the gate. Make 5 time Δt measurements for each of the flag lengths. We are now building up the time and position values of the cart as it accelerates down the air track. Record all of your data in Table 4.2 below.

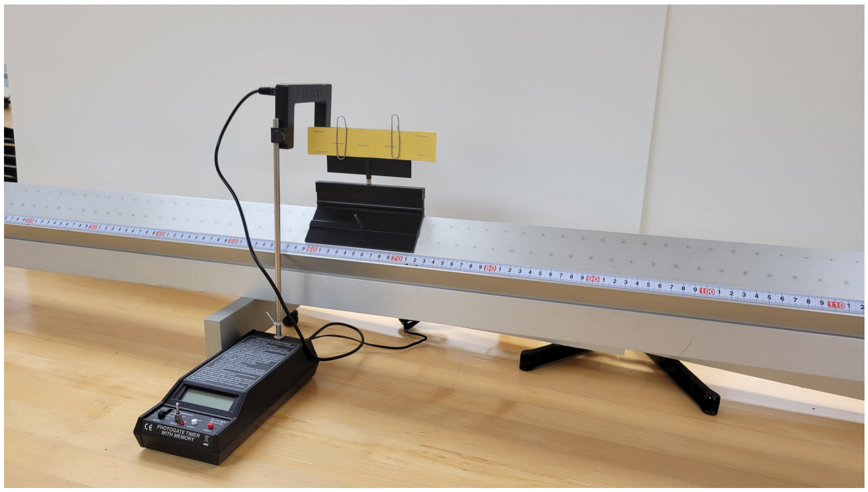


Figure 4.3: Inclined Air Track for Constant Acceleration

Position x (cm)	Time Δt (s)					Average Time $\overline{\Delta t}$ (s)	Final Velocity $v_{\text{f}}\left(\frac{\text{cm}}{\text{s}}\right)$
0	0	0	0	0	0	0	0
10							
15							
20							
25							
30							

Table 4.2: Data for Constant Acceleration Motion of Cart

Question 4: As in Question 1, plot position versus time. Based on the shape of your graph, perform a proper curve fit. How does this curve differ from the plot in question 1? Is the slope constant or is it changing over time? Explain.

Question 5: Using the equation $v_{\text{f}} = 2\frac{\Delta x}{\Delta t}$, compute the values of the final speed as the hind edge of the cart passes through the photogate. Record your values in Table 4.2.

Question 6: Plot velocity versus time (plot your final velocity values v_f on the y axis and the average time values on the x axis). Based on the shape of your graph, perform a proper curve fit.

Question 7: What is the shape of your graph obtained in question 6? What does the slope of this graph represent? Record the value of the slope from your graph.

The theoretical acceleration of the cart as it moves down the inclined air track can be found by applying Newton's second law in the x direction along the air track. See Figure 4.4.

In the x direction we have

$$\begin{aligned} mg\sin\theta &= ma_x \\ a_x &= g\sin\theta \end{aligned}$$

Since the air track has a length of 200 cm, we have

$$\sin\theta = \frac{\Delta h}{200} = \frac{h_2 - h_1}{200}$$

Therefore, the acceleration of the cart down the air track is

$$a_x = g\left(\frac{h_2 - h_1}{200}\right)$$

where h_1 and h_2 are the heights of the ends of the air track measured relative to the table.

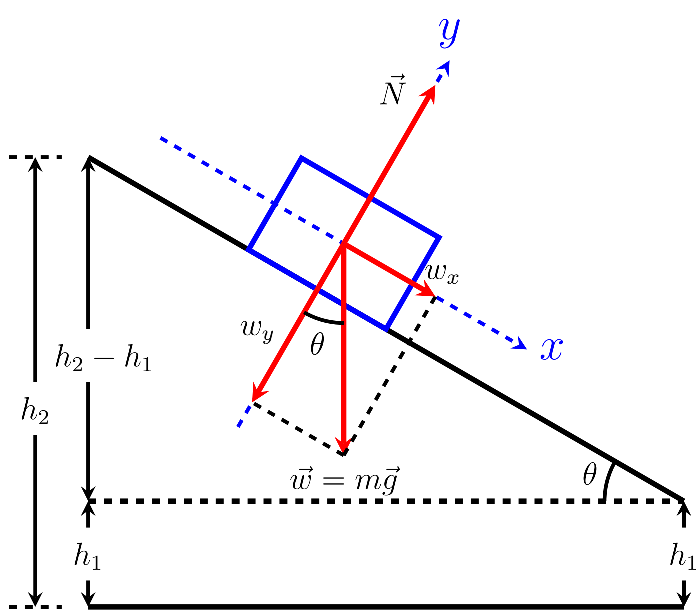


Figure 4.4: Cart Accelerating Down an Inclined Air Track

Question 8: Measure the height at each end of the air track in centimeters. Record your values in Table 4.3.

Height of Air Track at Lower End h_1 (cm)	
Height of Air Track at Higher End h_2 (cm)	

Table 4.3: Heights of the Ends of the Air Track

Question 9: Compute the theoretical value for the acceleration using $a_x = g\left(\frac{h_2 - h_1}{200}\right)$. Compare it to the experimental value of the acceleration (from the slope of the graph in question 6) by computing the percent experimental error. Record your results in Table 4.4.

a_x (theory) (cm/s^2)	a_x (experimental) (cm/s^2)	Percent Experimental Error (%)

Table 4.4: Comparison of Acceleration Values

Lab 5

Alternate Experiment on Velocity and Acceleration

This alternate experiment on velocity and acceleration uses the PASCO Dynamics System along with the PASCO Smart Timer. The PASCO dynamics system contains the PAScar which rides on an aluminum track. The PAScar is equipped with plastic wheels with precision bearings, therefore, the friction can be considered negligible as the car travels down the track. For the theory explaining constant velocity and constant accelerated motion, please refer to Lab 4.

5.1 Experiment

5.1.1 Motion at a Constant Velocity

You will use the PASCO Dynamics System for this lab. Please see Figure 5.1.

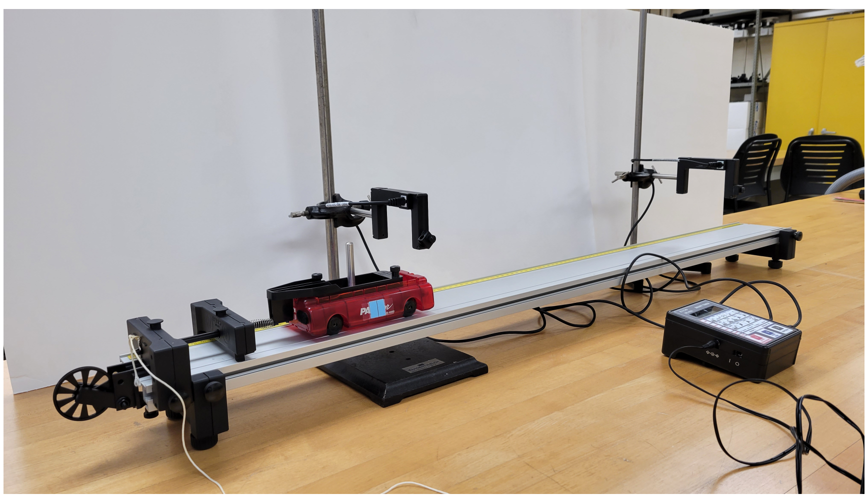


Figure 5.1: PASCO Dynamics System - Constant Velocity

Adjust the feet of the track so that the track is level both along its length and along its width. You may use the level that is provided to check for level along both directions.

In order to release the PAScar with a consistent velocity, you will use the spring cart launcher assembly. This assembly can be attached to the PAScar by simply aligning the built-in thumbscrews over the threaded holes in the PAScar and tightening them in place. You will notice that the

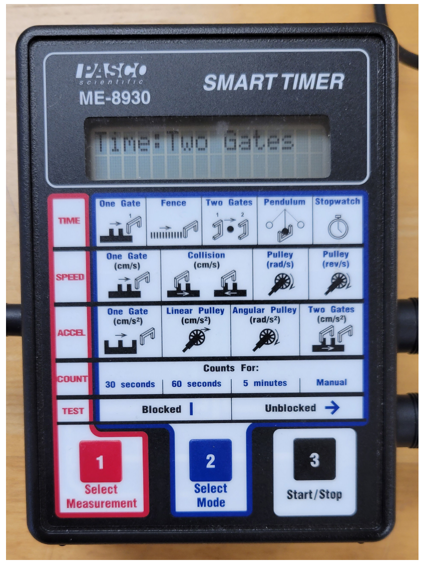


Figure 5.2: Smart Timer in Two Gate Mode

assembly has a shaft affixed to it. The spring that comes with this assembly is to be slipped over the shaft such that the flared end is facing away from the car and the coil at the other end is run through the small hole in the shaft that is next to the car. The spring should then be rotated clockwise so it is locked in place. As can be seen in Figure 5.1, the end of the track has two end stops attached. These should be adjusted so they are approximately 4-5 cm apart.

Compress the spring by pushing the shaft through the holes in each of the end stops until you see the small pinhole near the end of the shaft. Pass the release pin with the attached string through this small pinhole. When you pull up on the string with a smooth jerk, this will release the car. Practice doing this to get your technique down.

You will now take measurements of position and time to verify whether or not the car assumes motion at constant velocity once it has been released. To take time measurements you will use the Pasco Smart Timer. You will put the smart timer in "two gate" mode (see Figure 5.2) and time the motion of the PAScar to travel from the first photogate (consider this to be the origin of the coordinate system) to the second photogate (this will represent the position of the PAScar at the time presented by the smart timer). The attached 0.935 cm flag (the "flag" is the short steel post) will trigger the first photogate to start the timer and trigger the second photogate to stop the timer.

The small marker (shown in blue in Figure 5.1) on the either side of the car contains a vertical line which can be lined up with the built in tape measured attached to the track. Place the car at some initial position (past where the car has left the spring launcher) by lining up the vertical line with the tape measure. While holding the car at that position, move the photogate so that it is just ready to be triggered by the flag. This is the initial position of the car (x_1). Now move the car to a second position (x_2) that is 10 cm away from the first position. Set up the second photogate so that it is just ready to be triggered by the flag at x_2. Record your values of x_1, x_2 and the position of the car relative to x_1 which is $|x_2 - x_1|$ in Table 5.1. Launch the car and measure the time Δt_{12} it takes to travel from x_1 to x_2. Leaving the initial photogate at x_1 move the second photogate to position x_2 which is 20 cm away from x_1. Again launch the car and measure the time interval Δt_{12}. Record your data in Table 5.1. Do this for a total of 5 conditions where the position of the second photogate relative to the first is 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm.

Question 1: Plot position versus time (plot your position values on the y axis and the time

Condition	Time Δt_{12} (s)	Position of Photogate 1 x_1 (cm)	Position of Photogate 2 x_2 (cm)	Position of Car $\lvert x_2 - x_1 \rvert$ (cm)
1				10
2				20
3				30
4				40
5				50

Table 5.1: Position and Time Data for PAScar - Constant Velocity

values on the x axis and include the point (0, 0) in your graph). Based on the shape of your graph, perform a proper curve fit.

Question 2: What is the shape of your graph?

Question 3: Based on this shape, what does this graph tell you about the motion of the car?

Question 4: What does the slope of the graph represent? Record the value of the slope.

Now disconnect photogate 1 from the smart timer and connect photogate 2 to port one on the timer. Place the smart timer in stopwatch mode. You will now measure the time Δt it takes the flag to pass through the photogate at each of the positions x_2. Release the car in the exact same way that you did in the above procedure and record your data in Table 5.2 below.

Condition	Position of Photogate 2 x_2 (cm)	Time Δt (s)	Flag Length Δx (cm)	Velocity (v_2) $\frac{\Delta x}{\Delta t}$ (cm/s)
1			0.935	
2			0.935	
3			0.935	
4			0.935	
5			0.935	

Table 5.2: Constant Velocity Data for PAScar

Question 5: Compute the velocity (v_2) of the car at position x_2 by using $v_2 = \frac{\Delta x}{\Delta t}$. Record your results in Table 5.2.

Question 6: Plot velocity (v_2) versus the time elapsed from position 1 to position 2 (Δt_{12})

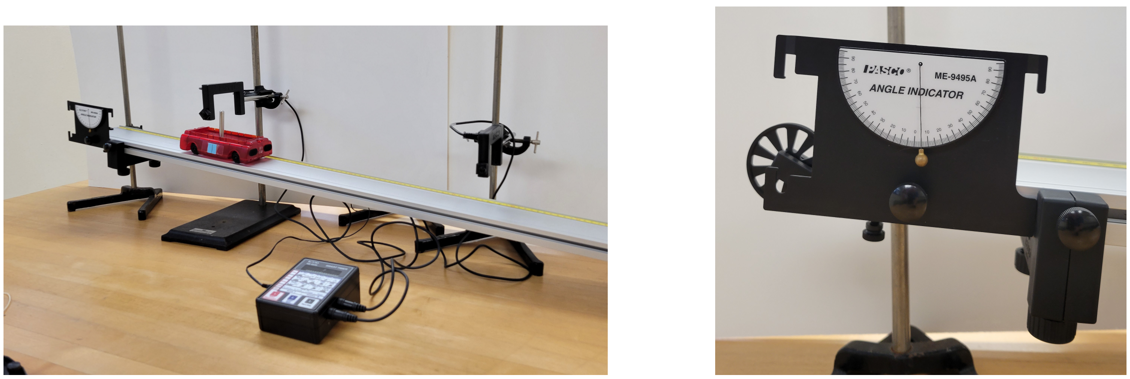
and perform an average fit to the data.

Question 7: Based on the shape of your graph, what can you conclude about the velocity of the car?

Question 8: For objects that move with constant velocity, what can you conclude about how the object's position varies with time?

5.1.2 Motion at a Constant Acceleration

For this activity, you will incline the dynamics track at an angle of about 5°. This can be done with the special clamp and angle indicator that came with the PASCO Dynamics System (see Figure 5.3).



(a) PASCO Dynamics System Inclined (b) Angle Indicator

Figure 5.3: PASCO Dynamics System - Constant Acceleration

Place the car at some initial position by lining up the vertical line with the tape measure. While holding the car at that position, move the first photogate so that it is just ready to be triggered by the flag. This is the initial position of the car (x_1). Now move the car to a second position (x_2) that is 10 cm away from the first position. Set up the second photogate so that it is just ready to be triggered by the flag at x_2. Record your values of x_1, x_2 and the position of the car relative to x_1 which is $|x_2 - x_1|$ in Table 5.3. Release the car from rest and record the time Δt_{12} it takes to travel from x_1 to x_2. Leaving the initial photogate at x_1 move the second photogate to position x_2 which is 20 cm away from x_1. Again release the car and measure the time interval Δt_{12}. Record your data in Table 5.3. Do this for a total of 5 conditions where the position of the second photogate relative to the first is 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm.

Question 9: Plot position versus time (plot your position values on the y axis and the time values on the x axis and include the point (0, 0) in your graph). Based on the shape of your graph, perform a proper curve fit.

Question 10: What is the shape of your graph?

Question 11: Based on this shape, what does this graph tell you about the velocity of the car?

Condition	Time Δt_{12} (s)	Position of Photogate 1 x_1 (cm)	Position of Photogate 2 x_2 (cm)	Position of Car $\lvert x_2 - x_1 \rvert$ (cm)
1				10
2				20
3				30
4				40
5				50

Table 5.3: Position and Time Data for PAScar - Constant Acceleration

Now disconnect photogate 1 from the smart timer and connect photogate 2 to port one on the timer. Place the smart timer in stopwatch mode. You will now measure the time Δt it takes the flag to pass through the photogate at each of the positions x_2. Release the car from rest in the exact same way that you did in the above procedure and record your data in Table 5.4 below.

Condition	Position of Photogate 2 x_2 (cm)	Time Δt (s)	Flag Length Δx (cm)	Velocity (v_2) $\frac{\Delta x}{\Delta t}$ (cm/s)
1			0.935	
2			0.935	
3			0.935	
4			0.935	
5			0.935	

Table 5.4: Velocity Data for PAScar

Question 12: Compute the velocity v_2 of the car at position x_2 by using $v_2 = \frac{\Delta x}{\Delta t}$. Record your results in Table 5.4.

Question 13: From the data in Table 5.3 and Table 5.4 construct Table 5.5 below. Compute five values of the acceleration as the car travels down the inclined track using

$$a = \frac{v_2 - v_1}{\Delta t_{12}}$$

Note that the velocity (v_1) at position 1 (x_1) is 0 because the car was released from rest at x_1.

Question 14: Compute an average value for the acceleration of the car from the 5 values that were recorded in Table 5.5.

Condition	Time Δt_{12}	Velocity at Position 1 (v_1) (m/s)	Velocity at Position 2 (v_2) (m/s)	Acceleration (cm/s^2)
1		0		
2		0		
3		0		
4		0		
5		0		

Table 5.5: Acceleration Data for PAScar

Question 15: Plot velocity (v_2) versus the time elapsed from position 1 to position 2 (Δt_{12}) (plot your velocity values on the y axis and the time values on the x axis and include the point (0, 0) in your graph). Based on the shape of your graph, perform a proper curve fit.

Question 16: What is the shape of your graph? What does the slope of the graph represent? Compare the value of the slope with the average acceleration value calculated in question 14. Are the values similar?

Question 17: Summarize how the position and velocity of the car vary with time as the car accelerates down the track.

Lab 6

Graphical Interpretation

In this lab, we will explore how to interpret idealized graphs of one dimensional motion.

6.1 Theory

The position versus time graph in Figure 6.1 represents the one dimensional position of an object (or particle) as a function of time. You can think of the object as moving along the x-axis. It can either move forward (+ x direction), backward (- x direction), or it is at rest (either for a prolonged period or just for an instant). If a question is asked about the position of the particle at a particular time, you can read the answer directly from the position versus time graph. If a question is asked about the velocity of the particle at a particular time, then you must calculate the slope of the tangent line that touches the graph at that time.

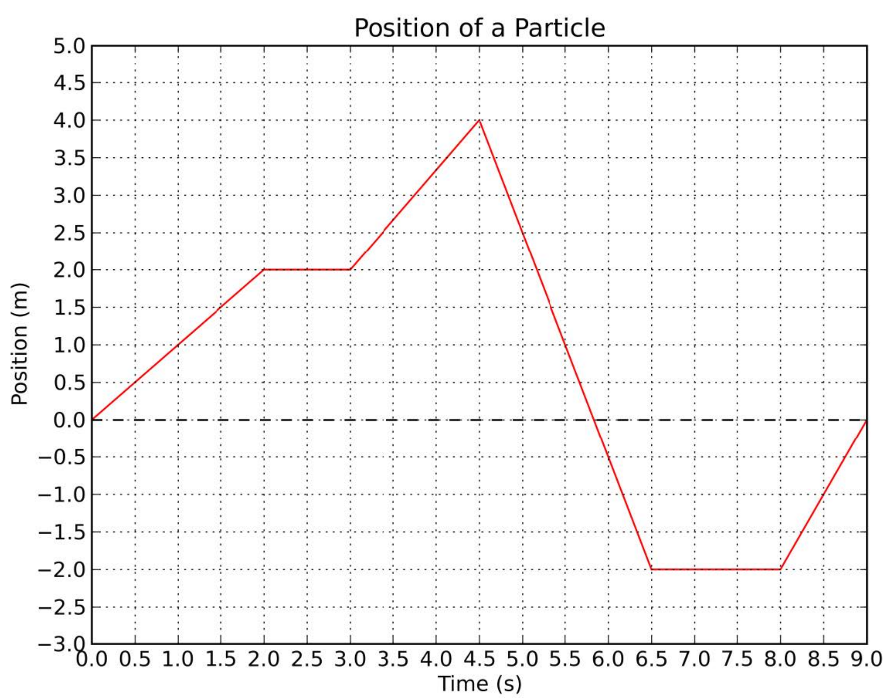


Figure 6.1: Practice Q1 - Position vs. Time

Practice Question 1:

- What is the position of the particle at $t = 1$ s?

  The answer can be read directly from the graph $x(1) = 1$ m.

- What is the velocity of the particle at $t = 5.5$ s?

  To answer this question, we draw a tangent line to the graph at 5.5 s and calculate its slope. Notice, however, that the slope (velocity) is constant from 4.5 s to 6.5 s so we only need to calculate the slope of the line that begins at $(4.5, 4.0)$ and ends at $(6.5, -2.0)$. The slope is

  $$m = v(5.5) = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-2.0 - 4.0}{6.5 - 4.5} = \frac{-6.0}{2.0} = -3.0\frac{\text{m}}{\text{s}}$$

  You may also quickly calculate the slope mentally by computing

  $$m = \frac{\text{rise}}{\text{run}} = \frac{\text{down } 6}{\text{right } 2} = \frac{-6}{2} = -3\frac{\text{m}}{\text{s}}$$

- Is the particle ever moving backwards?

  The velocity will indicate the direction of the particle. The slope of the position versus time graph indicates the velocity. Therefore, if the slope is positive, the particle is moving forwards ($+x$ direction). If the slope is negative, the particle is moving backwards (-x direction). If the slope is 0, the particle is at rest (either for a prolonged period or just for an instant). In this case the particle is moving backwards from 4.5 s to 6.5 s.

- Over what time interval is the speed (magnitude of the velocity) the greatest?

  The greatest speed occurs during the interval with the steepest slope. This occurs again between 4.5 s and 6.5 s. The velocity is -3.0 m/s but the speed is $|-3.0| = 3.0$ m/s.

- Is the particle ever at rest?

  The particle is at rest if the velocity is 0 m/s. The velocity is 0 m/s if the slope of the position versus time graph is 0. Therefore, the particle is at rest between 2.0 s and 3.0 s. It is also at rest between 6.5 s and 8.0 s.

  Note: We are ignoring what is happening where the slope (velocity) jumps on the graph. For example, at 4.5 s there is a "kink" in the graph which represents the velocity changing from a positive value to a negative value in an instant. This is unrealistic in the real world but it is easier to teach these concepts using these idealized graphs. In reality, the graph would show a smooth transition from the positive velocity a little before 4.5 s to the negative velocity a little after 4.5 s. However, a more realistic graph is more difficult to interpret. For now, we will just ignore the "kinks" in the graph.

- In what direction is the particle moving at $t = 4$ s?

  Since the slope of the tangent line to the graph is positive at 4 s, that means the velocity is positive and the particle is moving forward.

Practice Question 2: Sketch the velocity versus time graph that corresponds to the position versus time graph from question 1.

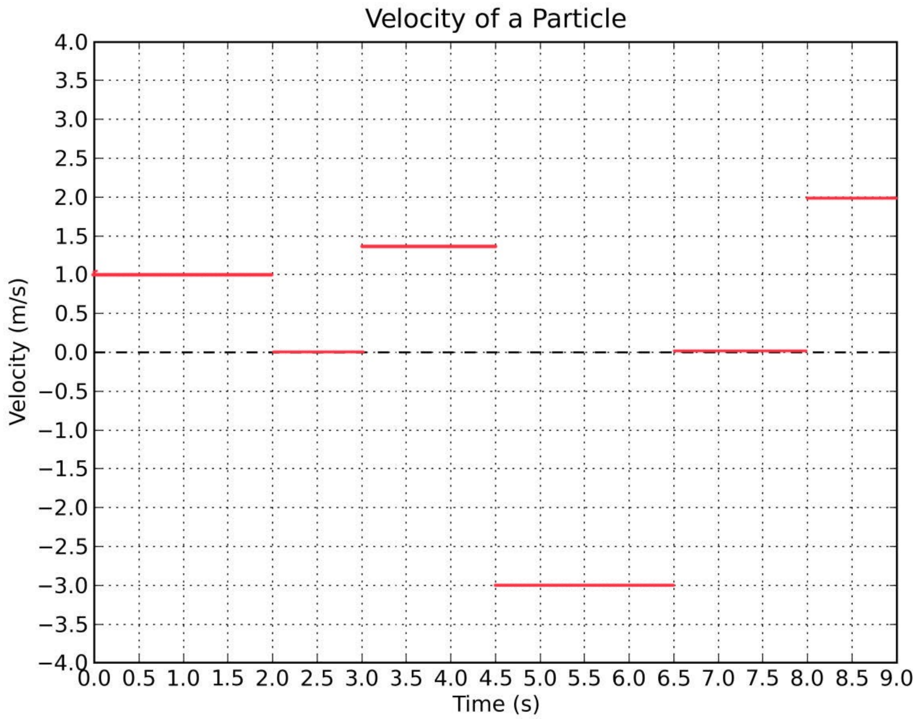


Figure 6.2: Practice Q2 - Velocity vs. Time

To do this, we need to measure the slope of the position versus time graph at each time. Fortunately, the slope (velocity) is constant over the different time intervals. The velocity versus time graph should look similar to that in Figure 6.2.

Practice Question 3:
Another position versus time graph is shown in Figure 6.3 below.

- What is the position of the particle at $t = 5$ s?

  The position of the particle is 16 m.

- Over what time interval, if any, is the particle accelerating?

  The particle will be accelerating if the velocity is changing. To determine where the velocity is changing on a position versus time graph, we locate the positions where the slope is changing. Again, we are going to ignore the "kinks" where the velocity jumps and only look at the intervals where the slope is smoothly changing. Therefore, the particle is accelerating between 4 s and 6 s.

- Is the particle ever at rest?

  The particle is at rest between 0s and 1s, between 8s and 12 s, and just for an instant at 5 s.

- What is the velocity of the particle at 4 s?

  The velocity is constant from 1 s to 4 s. It's slope is

$$m = v = \frac{\text{rise}}{\text{run}} = \frac{12}{3} = 4 \text{ m/s}$$

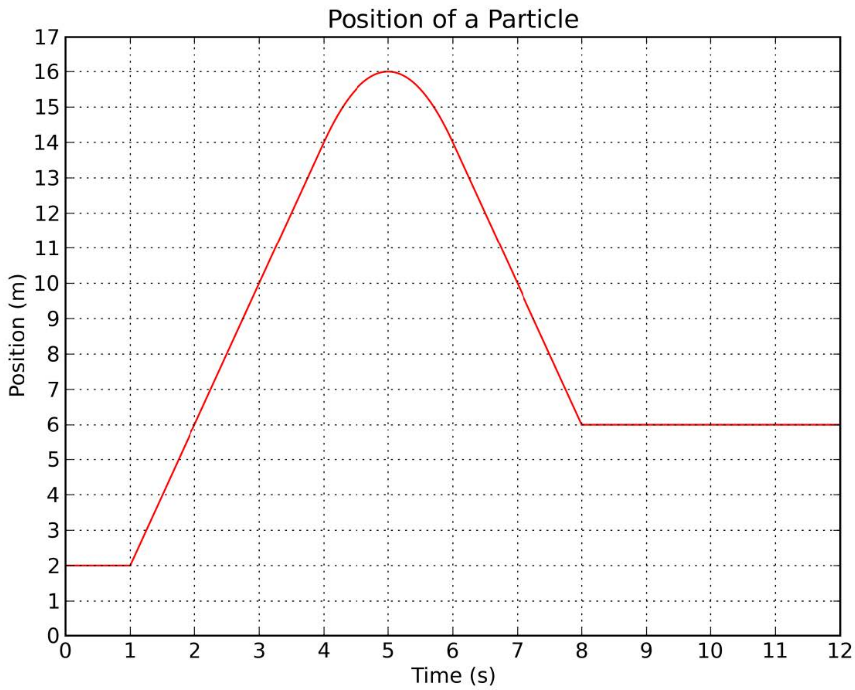


Figure 6.3: Practice Q3 - Position vs. Time

Therefore, $v(4) = 4$ m/s.

- What direction is the particle moving at $t = 5.5$ s?

  The slope of the tangent line to the graph at 5.5 s is negative. Therefore the velocity is negative and the particle is moving backwards.

- Describe in more detail what is happening between 4 s and 6 s.

  At 4 s the particle is moving forward at 4 m/s. Then as time progresses, the particle is slowing down (the slope is still positive but becoming less steep. At 5 s the particle has a velocity of 0 m/s just for an instant. After 5 s, the particle is moving backwards but speeding up (slope is becoming more negative). At 6 s the particle has a velocity of -4 m/s (it is traveling backwards at a speed of 4 m/s).

Practice Question 4: Sketch the velocity versus time graph that corresponds to the position versus time graph from question 3.

To do this, we need to measure the slope of the position versus time graph at each time. Fortunately, the slope (velocity) is constant over most of the different time intervals (the velocity will be constant during these time intervals). However, the velocity is not constant from 4 s to 6 s. The velocity is decresing over that time interval from 4 m/s to -4 m/s. How it is exactly decreasing is not easy to determine from the graph but you may just draw it as linearly decreasing over that time interval (in this particular case, the position graph is quadratic so the velocity is actually linear and the acceleration is constant). The velocity versus time graph should look similar to that in Figure 6.4.

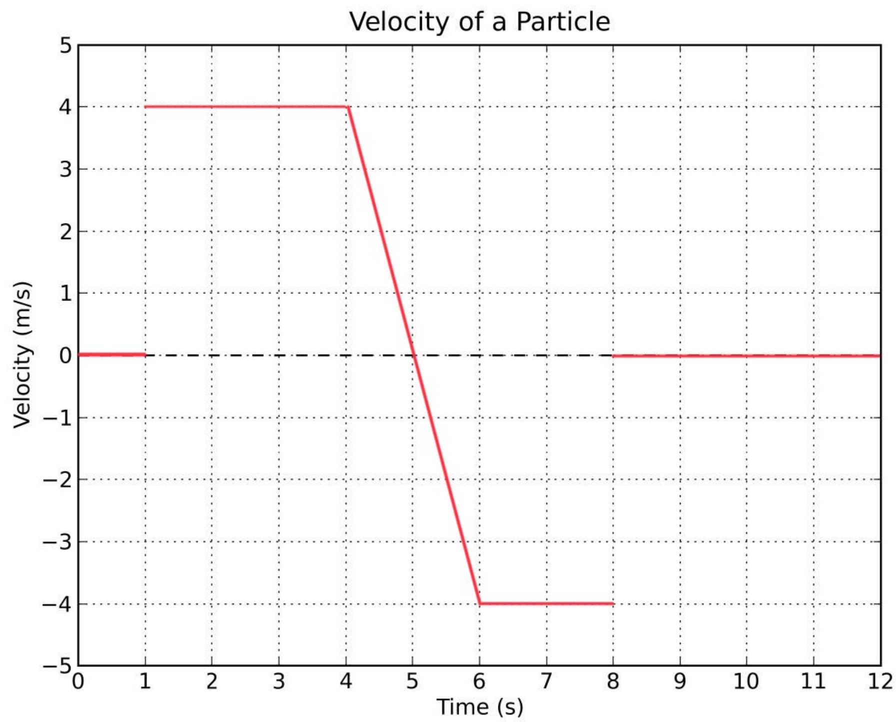


Figure 6.4: Practice Q4 - Velocity vs. Time

The velocity versus time graph in Figure 6.5 represents the one dimensional velocity of an object (or particle) as a function of time. You can again think of the object as moving along the x-axis. It can either move forward (+ x direction), backward (- x direction), or it is at rest (either for a prolonged period or just for an instant). Because this is a velocity versus time graph, if a question is asked about the velocity of the particle you can read it directly from the graph. If a question is asked about the acceleration of the particle at a particular time, then you must calculate the slope of the tangent line that touches the graph at that time. Finally, if a question is asked about the change in position (displacement) of the particle over a particular time interval, you must calculate the area underneath the velocity versus time graph.

Practice Question 5:

- What is the velocity of the particle at $t = 3$ s?

  The value can be read directly from the graph. $v(3) = 4.0$ m/s.

- During which time interval(s) is the velocity of the particle positive?

  Again, since the question is asking about velocity, the answer can be read directly from the graph. In this case, the velocity is positive for the entire 11 s (although the particle is speeding up and slowing down at times, it is moving forward the entire 11 s).

  Note: Be aware of the graph that you are reading. If this were a position versus time graph, the slope would indicate the velocity and hence the direction that the particle travels. A velocity versus time graph on the other hand indicates the velocity directly.

- During which time interval(s) is the velocity of the particle negative?

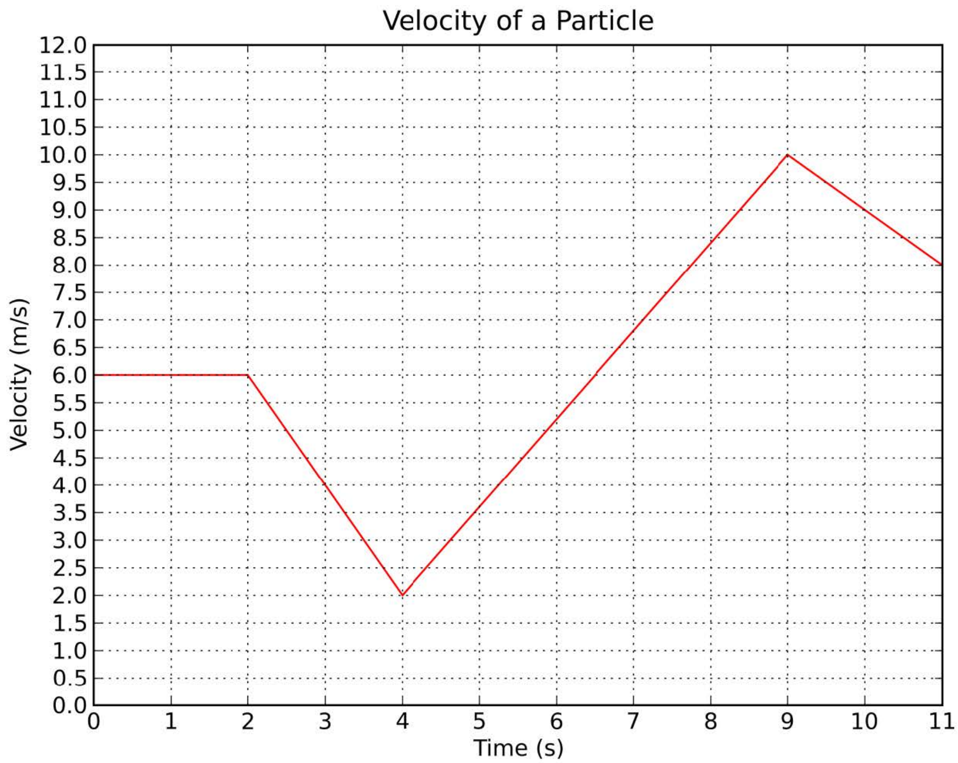


Figure 6.5: Practice Q5 - Velocity vs. Time

In this example, the velocity of the particle is never negative. The graph would have to go below zero to indicate negative velocity.

- Is the particle ever at rest?

  In this example, the particle is never at rest. The velocity graph would either have to go to zero and stay there (indicating a prolonged period of no movement) or pass through zero (indicating that the particle has a velocity of 0 m/s just for an instant).

- What is the acceleration of the particle at $t = 10$ s?

  To answer this question, we draw a tangent line to the graph at 10 s and calculate its slope. In this case it is

$$m = a(10) = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1 \text{ m/s}^2$$

- During which time interval is the magnitude of the acceleration the greatest?

  The greatest magnitude of the acceleration occurs during the interval with the steepest slope. This occurs between 2 s and 4 s. The acceleration is -2.0 m/s^2 but the magnitude of the acceleration is $|-2.0| = 2.0$ m/s^2.

- What is the particle's displacement over the first 4 s?

  To determine the displacement, we need to find the area underneath the velocity versus time graph. Normally, this requires integrating the velocity function. However, in these examples the velocity is piece-wise constant and linear so we can use geometry to find the area.

From 0 s to 2 s, I can form a rectangle and from 2 s to 4 s, I can form a rectangle and a triangle (or one trapezoid). The area of a rectangle is $A = l \cdot w$ and the area of a triangle is $A = \frac{1}{2} b \cdot h$ where l is the length, w is the width, b is the base, and h is the height. These areas are denoted as A_1, A_2, and A_3 in Figure 6.6. Since $A_1 = 6 \cdot 2 = 12$ m, $A_2 = \frac{1}{2} \cdot 2 \cdot 4 = 4$ m, and $A_3 = 2 \cdot 2 = 4$ m then the displacement from 0s to 4 s is $\Delta x = A_1 + A_2 + A_3 = 12 + 4 + 4 = 20$ m.

Note that if the velocity becomes negative, you must calculate the area from the velocity graph up to the time axis. And the resulting displacement is negative. Also, if the velocity graph is more complicated, you may estimate the area by fitting rectangles of appropriate height and small widths to the graph. This will give a good estimate of the displacement and as the widths of the rectangles decrease the estimate becomes better.

- What is the particle's displacement over the entire 11 s?

  Since $A_1 = 6 \cdot 2 = 12$ m, $A_2 = \frac{1}{2} \cdot 2 \cdot 4 = 4$ m, $A_3 = 2 \cdot 2 = 4$ m, $A_4 = \frac{1}{2} \cdot 5 \cdot 8 = 20$ m, $A_5 = 5 \cdot 2 = 10$ m, $A_6 = \frac{1}{2} \cdot 2 \cdot 2 = 2$ m, and $A_7 = 2 \cdot 8 = 16$ m then the displacement from 0s to 11 s is $\Delta x = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 = 12 + 4 + 4 + 20 + 10 + 2 + 16 = 68$ m.

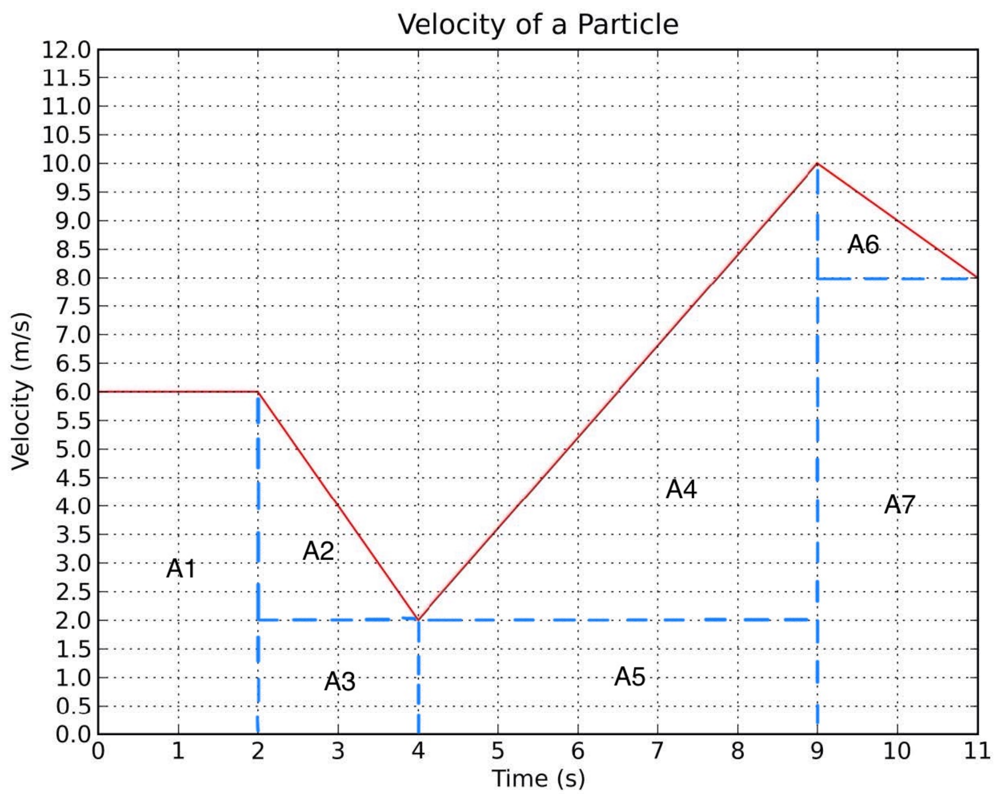


Figure 6.6: Practice Q5 - Velocity vs. Time (Compute Area)

Practice Question 6: Sketch the acceleration versus time graph that corresponds to the velocity versus time graph from question 5.

The acceleration versus time graph should look similar to that in Figure 6.7.

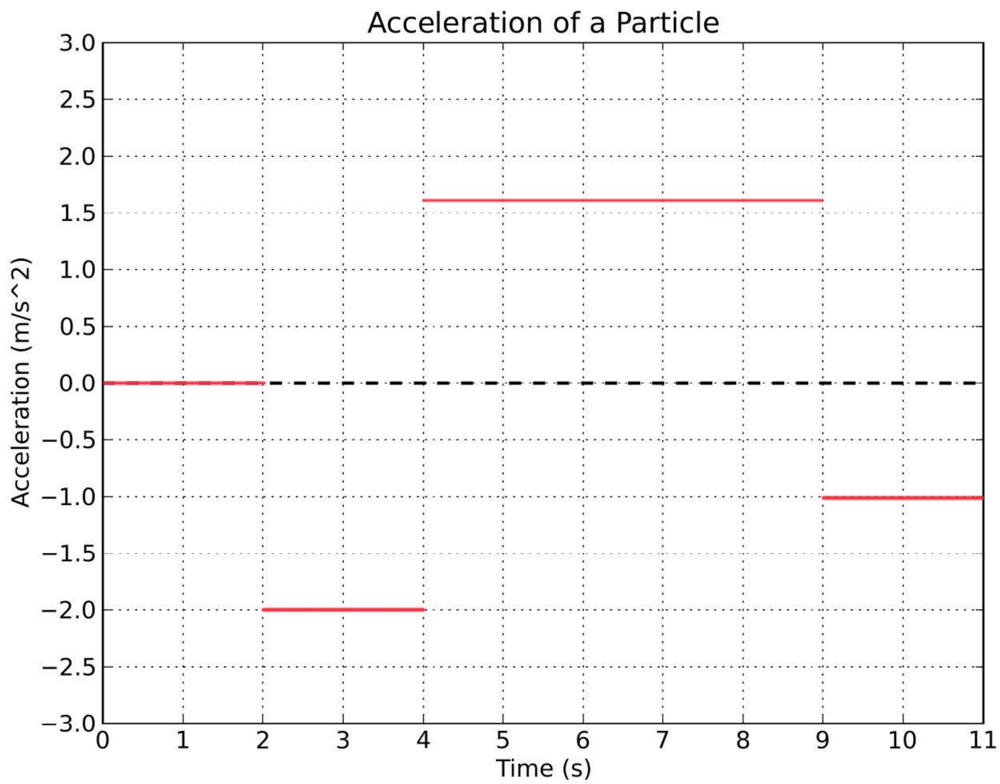


Figure 6.7: Practice Q6 - Acceleration vs. Time

6.2 Experiment

The experiment will consist of interpreting position versus time and velocity versus time graphs, drawing graphs based on other graphs, and drawing graphs based on a description of the motion.

Question 1: Answer the following questions about the position versus time graph in Figure 6.8.

- What is the particle's position at 3 s? The particle's position at 3 s is _____ m.
- What is the particle's velocity at 1 s? The particle's velocity at 1 s is _____ m/s.
- Is the particle ever moving backwards? Yes _____ No _____
  If yes, over what time interval(s) _____
- Where is the speed of the particle the greatest? The speed of the particle is greatest between _____ s.
- Is the particle ever at rest? Yes _____ No _____
  If yes, over what time interval(s) _____
- In what direction is the particle moving at $t = 9$ s? The direction of the particle at 9 s is _____.

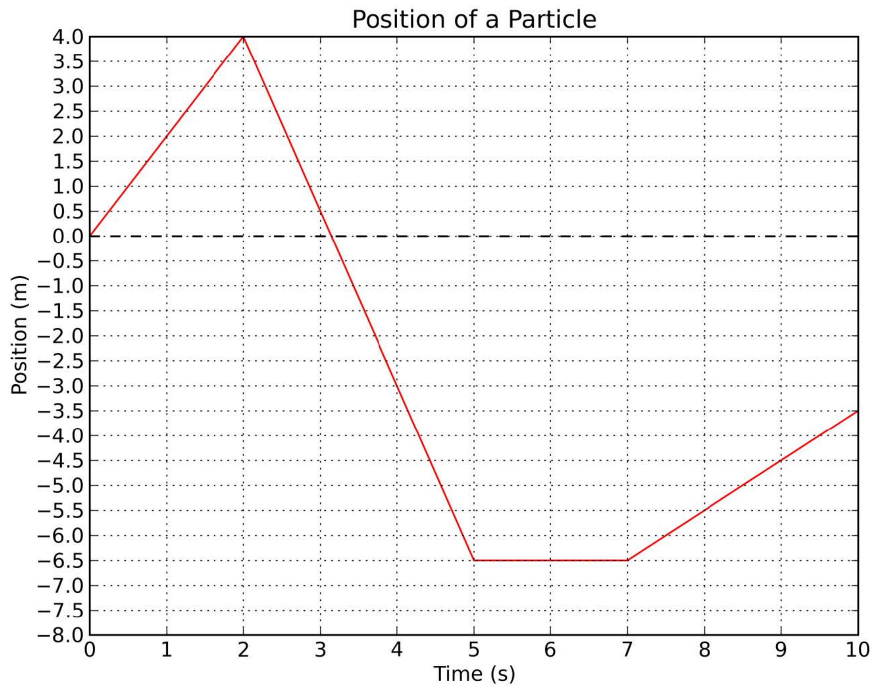


Figure 6.8: Experiment Q1 - Position vs. Time

Question 2: Sketch the velocity versus time graph (Figure 6.9) that corresponds to the position versus time graph in question 1.

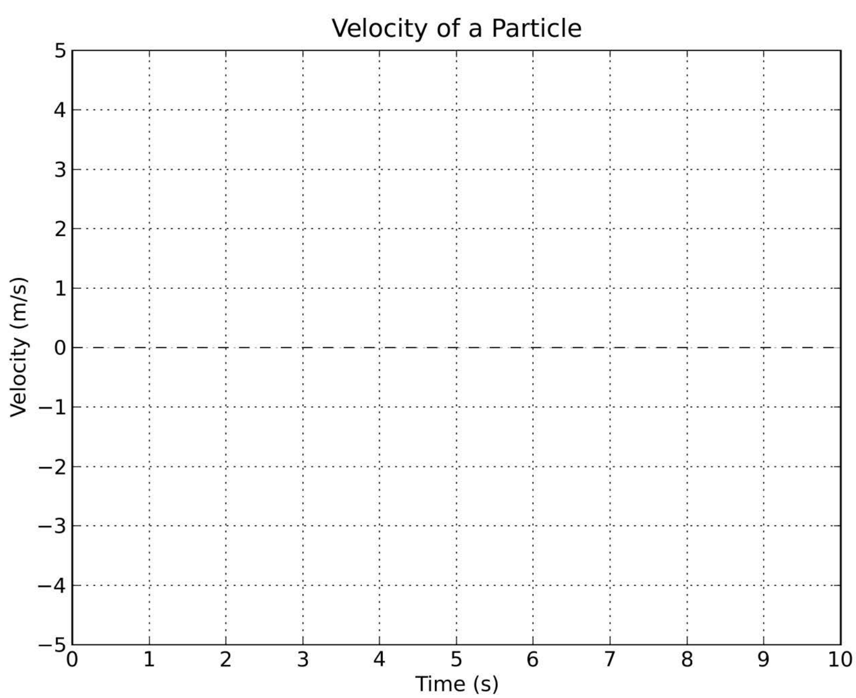


Figure 6.9: Experiment Q2 - Velocity vs. Time

Question 3: Answer the following questions about the position versus time graph in Figure 6.10.

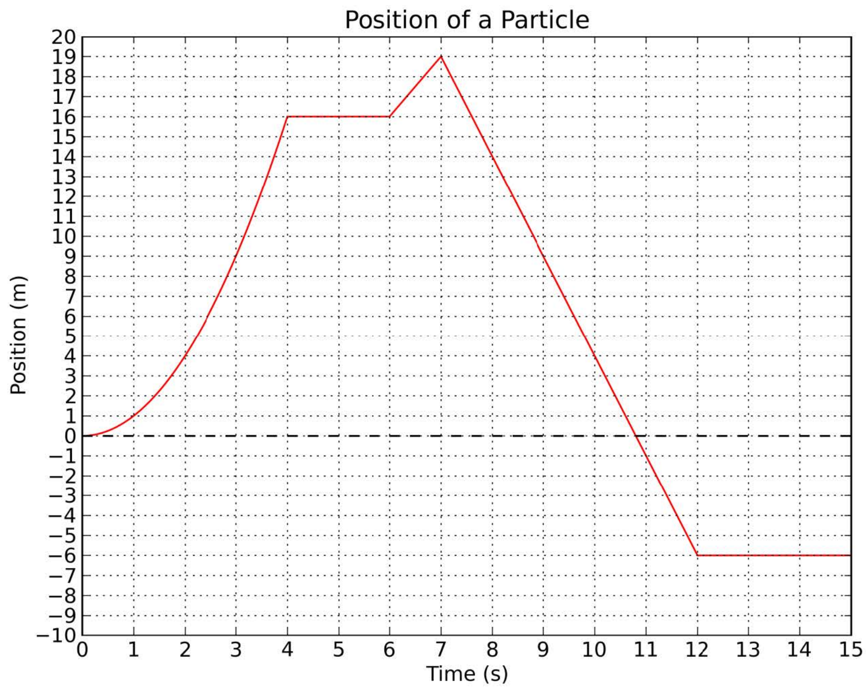


Figure 6.10: Experiment Q3 - Position vs. Time

- What is the speed of the particle at $t = 5$ s? The speed of the particle at 5 s is _____ m/s.
- Over what time interval is the particle accelerating? The particle is accelerating from _____ s.
- Is the particle ever at rest? Yes _____ No _____

  If yes, over what time interval(s) _____
- When is the velocity of the particle negative? The velocity of the particle is negative from _____ s.
- What is the velocity of the particle at time $t = 0$ s? The velocity of the particle at 0 s is _____ m/s.
- What is the velocity of the particle at time $t = 6.5$ s? The velocity of the particle at 6.5 s is _____ m/s.

Question 4: Sketch the velocity versus time graph (Figure 6.11) that corresponds to the position versus time graph in question 3.

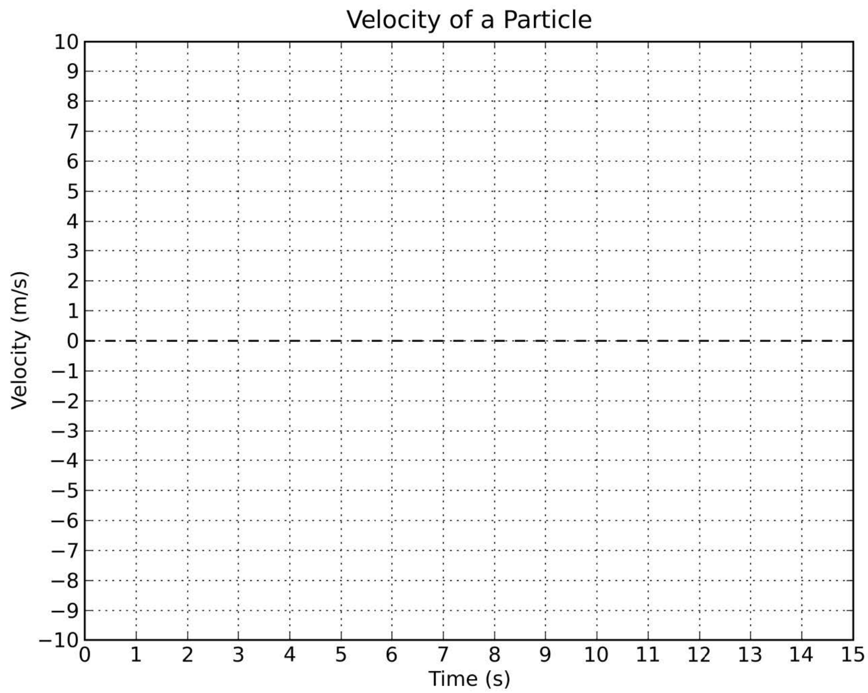


Figure 6.11: Experiment Q4 - Velocity vs. Time

Question 5: Answer the following questions about the velocity versus time graph in Figure 6.12.

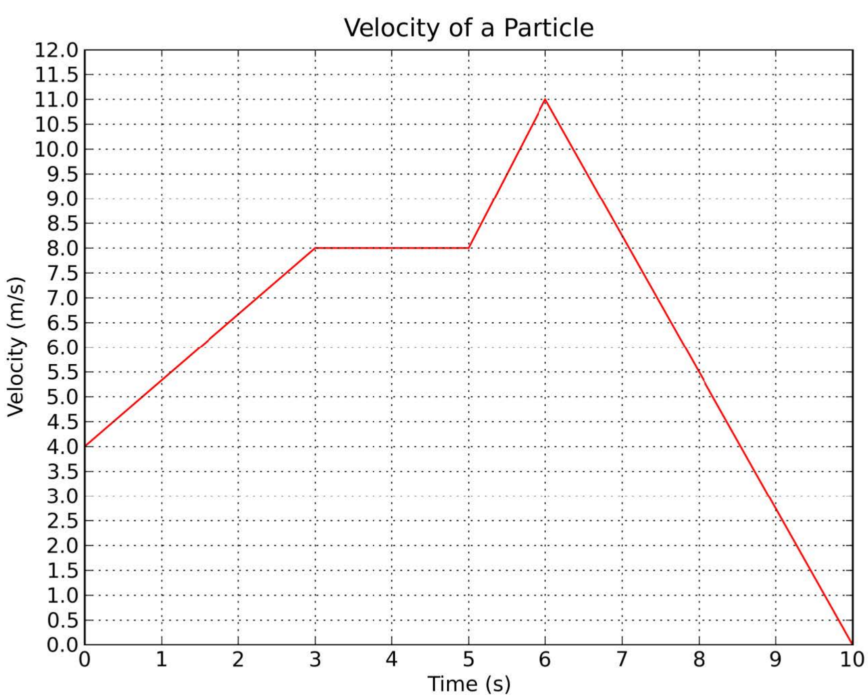


Figure 6.12: Experiment Q5 - Velocity vs. Time

- Is the particle ever moving in the negative direction? Yes ______ No ______
  If yes, over what time interval(s) ______
- Is the particle ever at rest? Yes ______ No ______
  If yes, over what time interval(s) ______
- When is the velocity of the particle the greatest? The velocity of the particle is greatest at ______ s.
- When is the acceleration of the particle the greatest? The acceleration of the particle is greatest from ______ s.
- What is the sign (+/-) of the acceleration at time $t = 8$ s? Is the partcle speeding up or slowing down? The sign (+/-) of the acceleration at 8 s is ______. (circle one) It is slowing down, speeding up.
- What is the acceleration of the particle at time $t = 1$ s? The acceleration of the particle at 1 s is ______ m/s^2.
- What displacement did the particle undergo over the first 5 s? The displacement over the first 5 s is ______ m.
- What displacement did the particle undergo over the entire 10 s? The displacement over the entire 10 s is ______ m.

Question 6: Sketch the acceleration versus time graph (Figure 6.13) that corresponds to the velocity versus time graph in question 5.

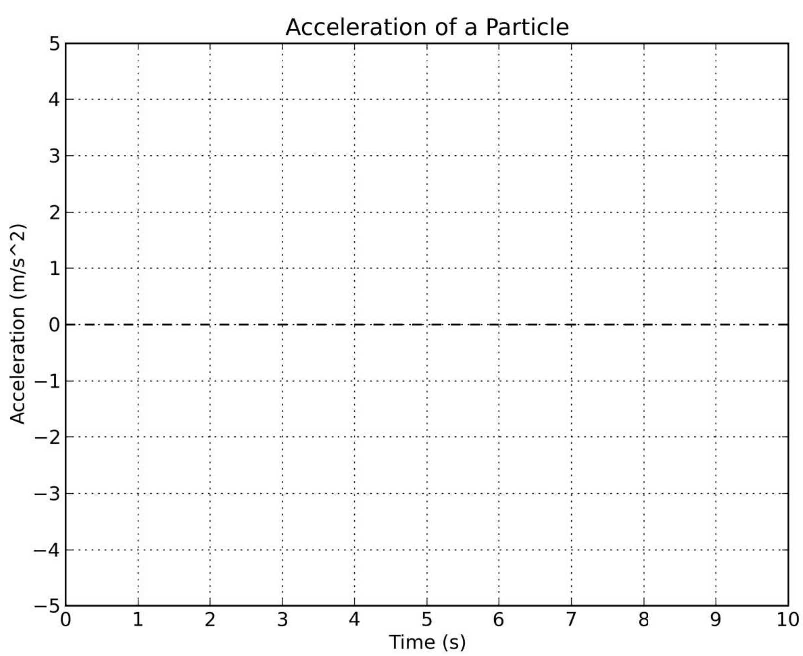


Figure 6.13: Experiment Q6 - Acceleration vs. Time

Question 7: Answer the following questions about the velocity versus time graph in Figure 6.14.

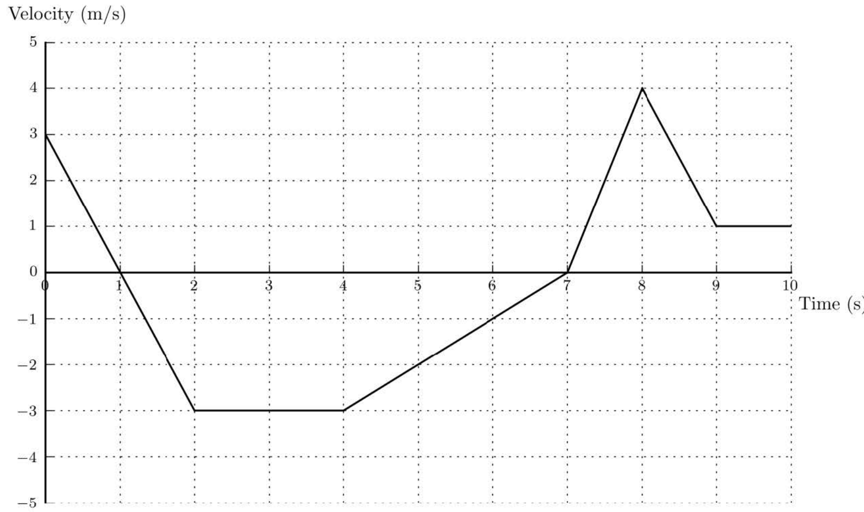


Figure 6.14: Experiment Q7 - Velocity vs. Time

- What is the acceleration of the particle at time $t = 1.5$ s? The acceleration of the particle at 1.5 s is _____ m/s^2.
- Is the particle ever moving in the negative direction? Yes _____ No _____

  If yes, over what time interval(s) _____
- Is the particle ever at rest? Yes _____ No _____

  If yes, when is it at rest _____
- What direction is the particle moving at time $t = 8.5$ s? (circle one) forwards or backwards

  Is it speeding up or slowing down in the time interval around 8.5 s? (circle one) speeding up or slowing down
- When is the velocity of the particle the greatest? The velocity of the particle is the greatest at _____ s.
- What is the displacement of the particle over the entire 10 s? The displacement is _____ m.
- If the particle started at a position $x_\text{i} = 20$ m at time $t_\text{i} = 0$ s, where did it end up at $t_\text{f} = 10$ s? $x_\text{f} =$ _____ m. Note: Use the fact that the displacement is defined as the final position minus the initial position $\Delta x = x_\text{f} - x_\text{i}$, therefore, $x_\text{f} = x_\text{i} + \Delta x$.

Question 8: The below acceleration versus time graph (see Figure 6.15), velocity versus time graph (see Figure 6.16), and position versus time graph (see Figure 6.17) are independent of one another (they don't represent the same motion). Locations on each graph are indicated by letters. The task is to choose all letters where the acceleration is 0 m/s^2. Letters __________

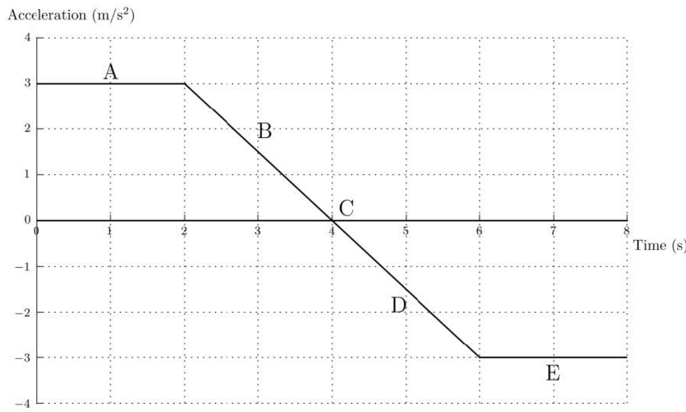


Figure 6.15: Experiment Q8 - Acceleration vs. Time

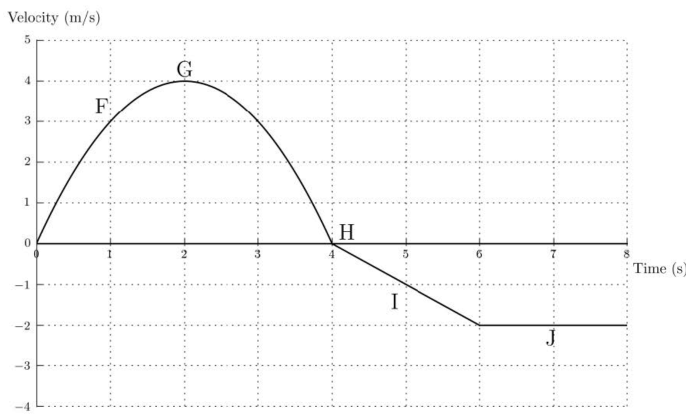


Figure 6.16: Experiment Q8 - Velocity vs. Time

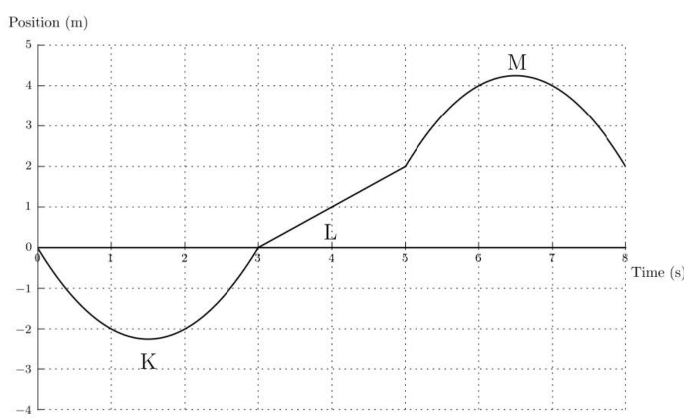


Figure 6.17: Experiment Q8 - Position vs. Time

Question 9: Given the velocity versus time graph below (see Figure 6.18) estimate the displacement by approximating the area under the graph as the sum of the areas of rectangles each of width 1 s. The height of the rectangle should be drawn so as to minimize the error of the area under the graph for that 1 s time interval. For example, between 3 s and 4 s, the rectangle should be drawn with height 9 m/s to have no error. The process is started for you. Don't forget that between 8 s and 12 s, the displacement will be negative.

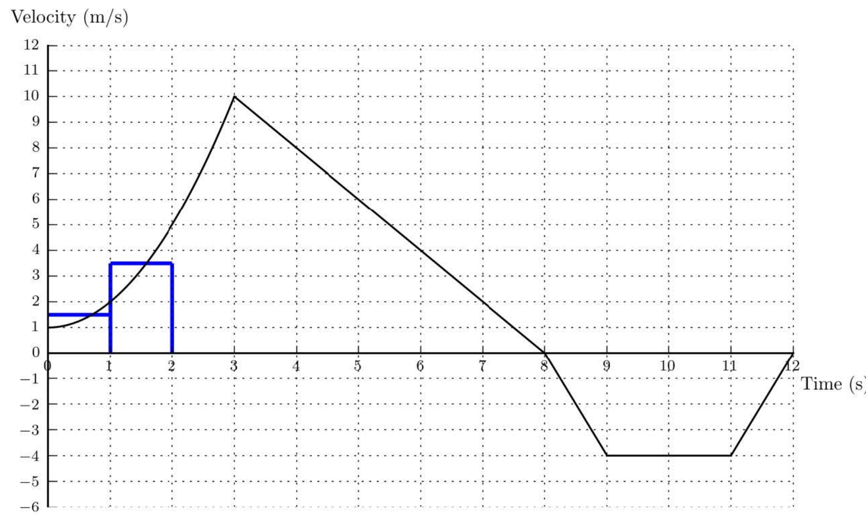


Figure 6.18: Experiment Q9 - Velocity vs. Time

Question 10: Answer the following questions about the velocity versus time graph in Figure 6.19.

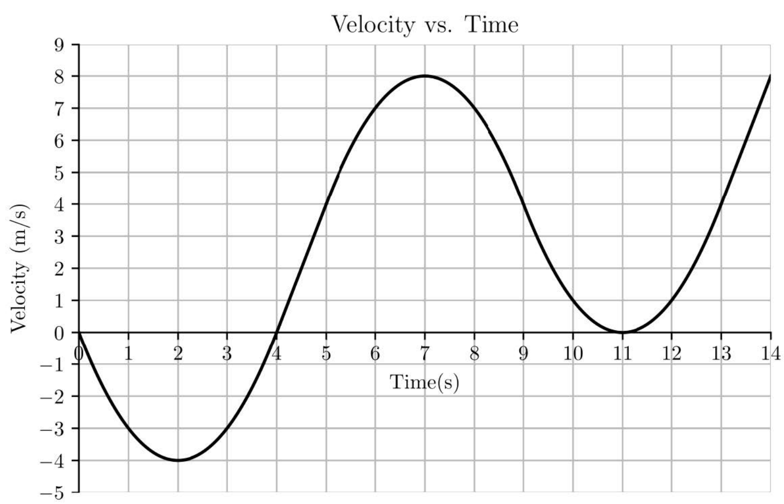


Figure 6.19: Experiment Q10 - Velocity vs. Time

- What is the velocity of the particle at time $t = 6$ s? The velocity at 6 s is ____ m/s.

- What is the velocity of the particle at time $t = 11$ s? The velocity at 11 s is _____ m/s.
- What is the velocity of the particle at time $t = 3$ s? The velocity at 3 s is _____ m/s.
- What is the acceleration of the particle at time $t = 9$ s? The acceleration at 9 s is _____ m/s^2.
- What is the acceleration of the particle at time $t = 11$ s? The acceleration at 11 s is _____ m/s^2.
- What is the acceleration of the particle at time $t = 13.5$ s? The acceleration at 13.5 s is _____ m/s^2.
- Use whatever method you want (ie. fit rectangles of width 1 s) to estimate the displacement of the particle over the first 4 s. Displacement of the particle from 0 s to 4 s is _____ m.
- Estimate the displacement of the particle over the entire 14 s. Displacement of the particle from 0 s to 14 s is _____ m.

Question 11: Sketch the acceleration versus time graph (see Figure 6.20) that corresponds to the velocity versus time graph in question 10. Hint: The acceleration of the particle at 0 s is -4 m/s^2. Also, the velocity is linearly increasing between 4 s and 5 s and between 13 s and 14 s.

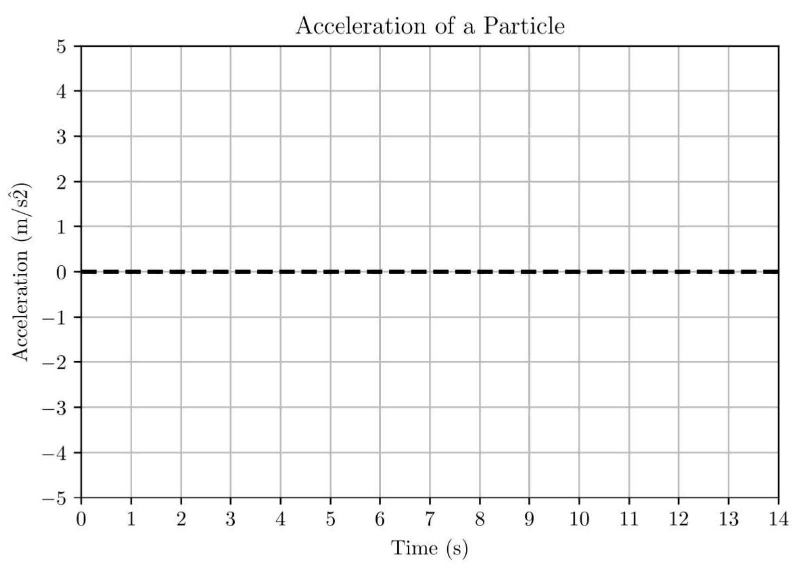


Figure 6.20: Experiment Q11 - Acceleration vs. Time

Question 12: A car is traveling on a straight road (think of it as traveling along the x axis in the positive direction). It starts at rest at a position $x = 0$ m. It accelerates for 10 seconds at a constant rate until a velocity of 30 m/s is reached. It then travels with zero acceleration for 30 seconds. Finally, it decelerates for 10 seconds at a constant rate until it is stopped. Please sketch the correct position vs. time, velocity vs. time, and acceleration vs. time graphs of the car (see Figure 6.21, Figure 6.22, and Figure 6.23 below). Hint: From the information given, you will find it easiest to sketch the velocity vs. time graph first.

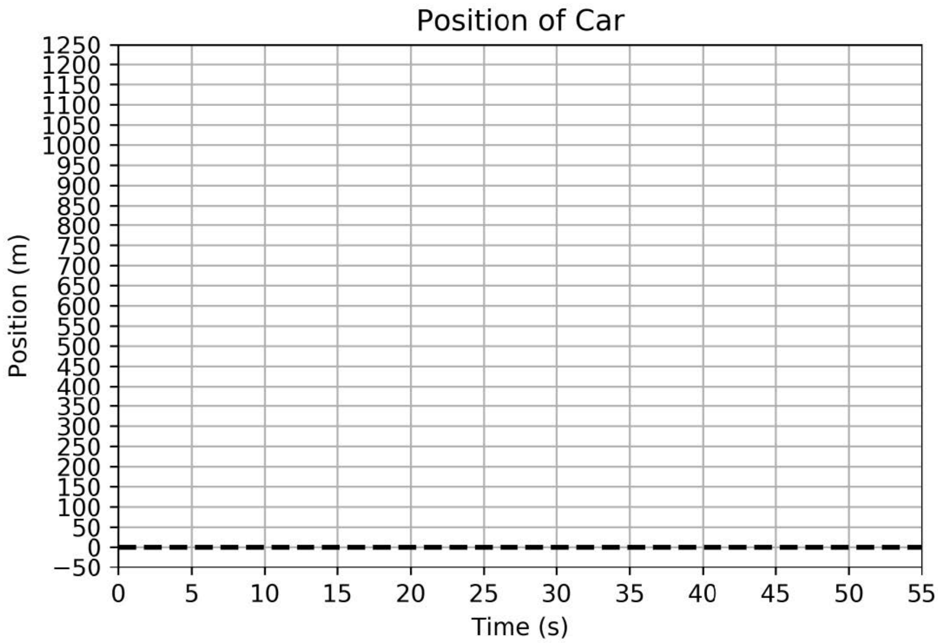


Figure 6.21: Experiment Q12 - Position vs. Time

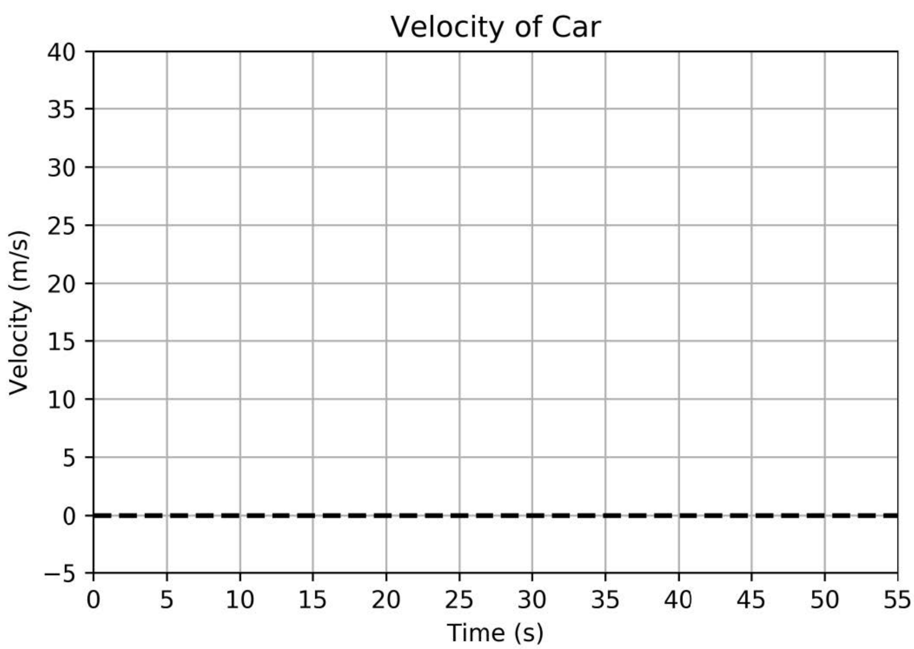


Figure 6.22: Experiment Q12 - Velocity vs. Time

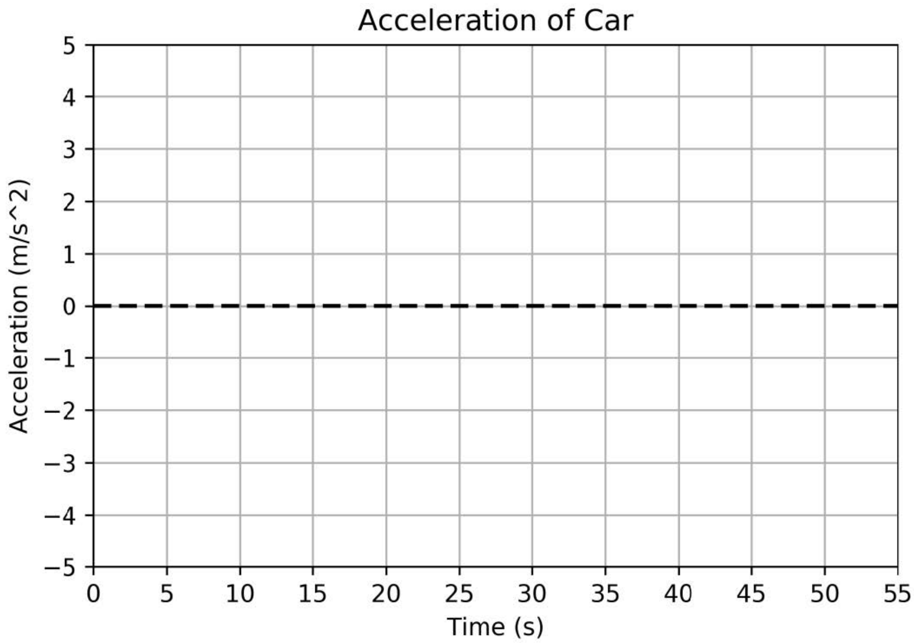


Figure 6.23: Experiment Q12 - Acceleration vs. Time

Lab 7

Determination of the Acceleration Due to Gravity

The acceleration of a freely falling (or rising) object due to the gravitational force of attraction between the earth and the object is called the acceleration due to gravity. In this experiment, you will make three independent determinations of the acceleration due to gravity, denoted by the symbol g, by using (1) the free-fall apparatus, (2) the simple pendulum, and (3) an inclined linear air track.

7.1 Theory

The existence of the acceleration due to gravity, g, physicists believe, has something to do with the observation that bodies within reasonable proximity of one another always attract one another with forces that are very nearly proportional to the inverse of the square of the distance between them, if we treat the bodies as if they were point particles. The mechanism that explains the way in which these forces are created is not fully understood. One of the more popular theories proposes the existence of particlelike things called gravitons, whose exchange between particles is supposed to account for the existence of the forces between the particles. Sufficient experimental data to justify their existence is, to date, still lacking however. Part of the difficulty is due to the fact that gravitational forces are extremely weak compared to other forces in nature. The result is that in order to detect the graviton, experimental measurements of phenomena involving gravitational interaction among particles must be made with painstaking precision and by ultra sophisticated techniques. The field of physics that concerns itself with gravitational phenomena in nature is known as General Relativity.

The value of g is really not a constant. In fact, the classical theory of gravitation proposed by Isaac Newton (1642-1727) demonstrates that g should vary inversely as the square of the distance from the center of the earth. However, for distances that are not too far from the earth's surface it is found that g does not change very much. For example, if we travel a distance of 10 miles from the earth's surface, g changes by only 0.57%. The reason for this small change is that the radius of the earth is relatively large, i.e., about 4,000 miles. Also, there are very slight changes in the value for g with latitude because of the oblate spheroidal shape of the earth. In addition, changes in the value of g occur due to variation between the subsurface strata, a fact that is of extreme interest to the geophysicist. Of course when we start to go hundreds and thousands of miles away from earth the changes in g begin to become dramatic. For purposes of experimentation in this laboratory g will be assumed to be a constant. The theory in each method determining g in this experiment is briefly discussed below.

7.1.1 Free-fall Apparatus

Figure 7.1 shows the free-fall apparatus that will be used in this experiment. It consists of a ball dropper that is used to release a small ball from rest, a set of photogates and a "ball catcher." The instantaneous velocity of the ball at two positions, along with the time elapsed between these positions, can be estimated using the photogate timing system. The acceleration due to gravity is then computed using the definition of average acceleration.

$$g = \bar{a} = \frac{\Delta v}{\Delta t} \tag{7.1}$$

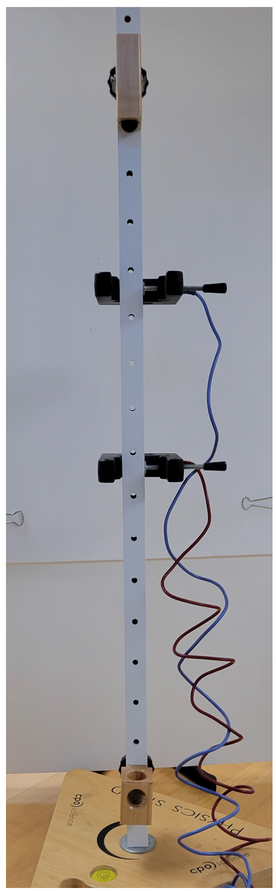
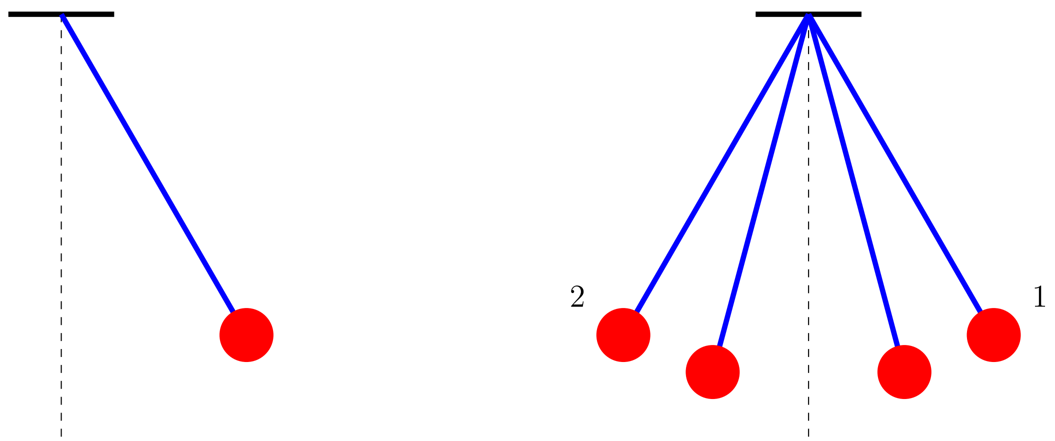


Figure 7.1: Free-fall Apparatus

7.1.2 The Simple Pendulum

If a ball is threaded with a piece of string and the string is suspended from a rigid support, then one has constructed a rather simple device to measure time. If the ball is displaced slightly to the right, as is shown in Figure 7.2a, it will oscillate back and forth (forever, if there is no damping effect due to air friction or damping due to friction between the string and support at the pivot). Under the ideal condition where no friction is present, each oscillation of the ball is just like the one preceding it. Hence, by counting the number of oscillations that the ball makes, one has discovered a novel way of both passing the time of day and also measuring the time of day in units of the number of complete oscillations that the ball makes. Figure 7.2b shows the position of the ball at various times in its swing. Positions 1 and 2 respectively represent the two extremes in the position of the ball. Either of these two positions determine what is referred to as the amplitude of the swing. One complete oscillation of the ball is then, for example, going from position 1 to position 2 and then back to position 1 again. It takes a total wrist watch type time T (in seconds), for the ball to make one complete oscillation. T is called the period.



(a) Diagram of a Simple Pendulum (b) Pendulum Ball at Various Times During Its Swing

Figure 7.2: The Simple Pendulum

When the ball is a point mass m, the device discussed above is called a simple pendulum. Since point masses are only mathematical abstractions, however, we can only approximate a simple pendulum in the laboratory. A workable simple pendulum can be made if one assumes that the mass at the end of the pendulum is small relative to the length of the pendulum. In this lab, a wooden bob weighted with washers connected to a light weight string approximates a simple pendulum (see Figure 7.3).

The length L of the pendulum is from the pivot point (bottom of the dowel) to the center of mass of the bob (which we will consider to be the bottom of the washers as a good approximation). The period T of one complete oscillation for the pendulum is then given in terms of the length L and the acceleration due to gravity, g, by

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{7.2}$$

Since both the period T and the length L of the pendulum can be measured, we can obtain an experimental determination of g. Solving Equation 7.2 for g gives

$$g = \frac{4\pi^2 L}{T^2} \tag{7.3}$$

7.1.3 The Linear Air Track

The linear air track provides a nearly frictionless surface along which a cart can glide. Suppose that the track is inclined at some small angle with respect to a horizontal surface along which a cart can

Figure 7.3: Simple Pendulum

glide (see Figure 7.4). Then, as a detailed analysis of the motion of the cart shows, using Newton's second law of motion, the acceleration of the cart down the air track is given in terms of g by

$$a_x = \frac{w_x}{m} = \frac{mg\sin\theta}{m} = g\sin\theta \tag{7.4}$$

If the cart is released from rest, then the distance that it travels along the track is given as a function of the acceleration, a_x, and the time elapsed, t, by

$$s = \frac{1}{2}a_x t^2 \tag{7.5}$$

where s represents the displacement of the cart which is equal to the flag length. Solving Equation 7.5 for a_x and substituting it into Equation 7.4 we find

$$g = \frac{2s}{t^2\sin\theta} \tag{7.6}$$

Since the length of the air track is 200 cm, we can write $\sin\theta$ in terms of the heights h_1 and h_2 measured relative to the lab bench at the ends of the air track as

$$\sin\theta = \frac{h_2 - h_1}{200} \tag{7.7}$$

Substituting Equation 7.7 into Equation 7.6 gives

$$g = \frac{400s}{t^2\left(h_2 - h_1\right)} \tag{7.8}$$

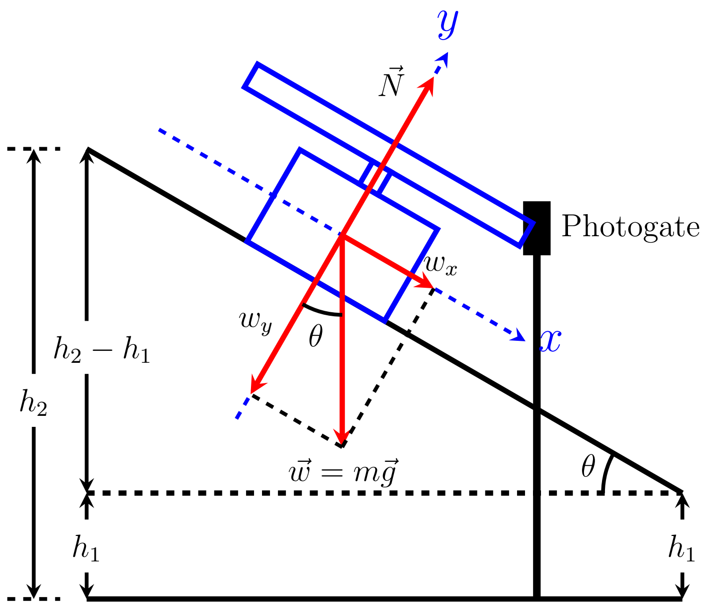


Figure 7.4: Cart Accelerating Down an Inclined Air Track to Measure g

7.2 Experiment

7.2.1 Measuring g Using the Free Fall Apparatus

In order to obtain accurate estimates of instantaneous velocity in the experiment, it will be necessary to measure them beginning 15 cm from the ball dropper. In order to plot instantaneous velocity versus time from release, it will be necessary to determine the time for the ball to achieve a position 15 cm from the dropper. The reason for this will become clearer later.

Place photogate A such that the ball in the ball dropper is just about to intercept the beam in gate A. Position photogate B 15 cm below photogate A. See Figure 7.5.

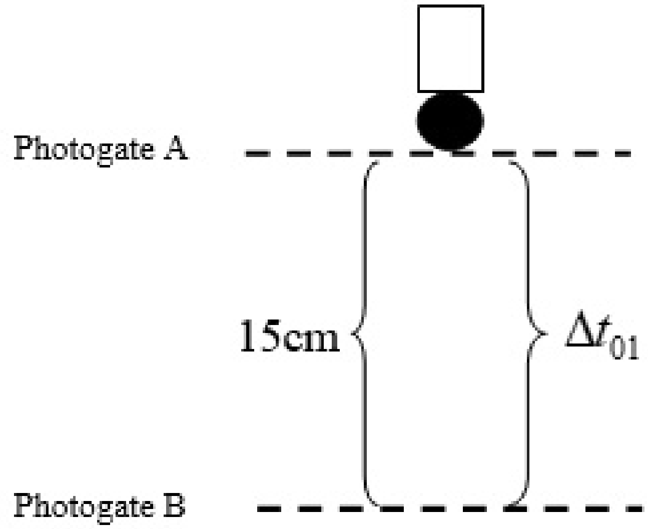
Connect the timer to photogates A and B (the upper photogate should be connected to port A and the lower photogate to port B). Place the timer in interval mode. Perform five trials of dropping the ball and measure the time it takes the ball to travel from photogate A to photogate B (Δt_{01}). Δt_{01} is the time interval displayed when both lights A and B are illuminated on the timer. Record your values in Table 7.1.

Trial	1	2	3	4	5	Average
Δt_{01} (s)						
Absolute Deviations (s)						
PRAAD (%)						

Table 7.1: Time for Ball to Fall 15 cm

Question 1: Compute the average value of Δt_{01} and the PRAAD for Δt_{01}.

(a) Initial Set-up for Apparatus



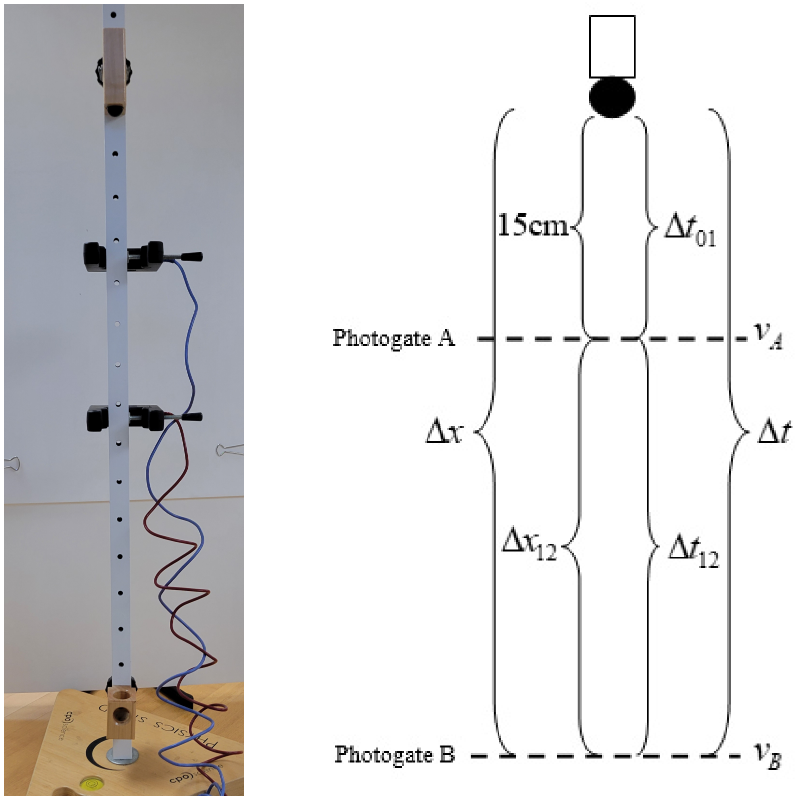
(b) Measurements

Figure 7.5: Measuring Time for the Ball to Fall 15 cm

Remove photogate B. Now position photogate A at position 1 such that it is 15 cm below the position of the ball in the ball dropper. See Figure 7.6. With photogate A at this position, place photogate B at positions 2 below. The positions 2 will be varied so that Δx_{12} is 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm, respectively. Note that these positions 2 of photogate B place it at displacements Δx from the ball dropper of 25 cm (10 cm + 15 cm), 35 cm (20 cm + 15 cm), etc. Let Δt_{12} represent the time for the ball to drop from the position 1 to each position 2. Then, the time it takes the ball to drop from the ball dropper to position 2 is:

$$\Delta t = \Delta t_{01} + \Delta t_{12}$$

Δt_{12} can be measured by reading the timer while in interval mode. For each drop, we can also measure the time that it takes the diameter of the ball to pass through each of the photogates. Let Δt_{A} represent the time for the ball to pass through photogate A. Let Δt_{B} represent the time for the ball to pass through photogate B.



(a) Main Set-up for Apparatus

(b) Measurements

Figure 7.6: Measuring Time for the Ball to Fall Different Distances

Question 2: For each drop described in Table 7.2, measure five trials of the times Δt_A, Δt_B, and Δt_{12}. The time interval Δt_A is displayed when just light A is illuminated on the timer. The time interval Δt_B is displayed when just light B is illuminated on the timer. The time interval Δt_{12} is displayed when both lights A and B are illuminated on the timer. Compute the average values of these times as well as the PRAAD for each time. Record all of your values in the Table 7.2.

	$\Delta x_{12} = 10$ cm			$\Delta x_{12} = 20$ cm			$\Delta x_{12} = 30$ cm		
Trial	Δt_A (s)	Δt_B (s)	Δt_{12} (s)	Δt_A (s)	Δt_B (s)	Δt_{12} (s)	Δt_A (s)	Δt_B (s)	Δt_{12} (s)
1									
2									
3									
4									
5									
Average (s)									
PRAAD (%)									

	$\Delta x_{12} = 40$ cm			$\Delta x_{12} = 50$ cm		
Trial	Δt_A (s)	Δt_B (s)	Δt_{12} (s)	Δt_A (s)	Δt_B (s)	Δt_{12} (s)
1						
2						
3						
4						
5						
Average (s)						
PRAAD (%)						

Table 7.2: Main Data Table for Ball Drop

Question 3: From your average vales of Δt_{12} in Table 7.2 and your average Δt_{01} value from Table 7.1, compute the total time Δt for the ball to fall from the dropper to the second photogate at position 2. Record your vales in Table 7.3 below. Plot Position (Δx) versus Time (Δt). Make sure you also include the point (0, 0). What is the shape of your graph?

Condition	Total Time from Dropper to Position 2 Δt (s)	Total Distance from Dropper to Position 2 Δx (cm)
1		25
2		35
3		45
4		55
5		65

Table 7.3: Total Time from Dropper to Second Photogate

Condition	$\overline{\Delta t_A}$ (s)	v_A (m/s)	$\overline{\Delta t_B}$ (s)	v_B (m/s)	$\overline{\Delta t_{12}}$ (s)	g_{exp} (cm/s^2)	Absolute Deviations (cm/s^2)
1							
2							
3							
4							
5							
Average							
PRAAD (%)							
Percent Experimental Error (%)							

Table 7.4: Measuring g Using Multiple Conditions

Question 4: Measure the diameter of the ball d with the vernier caliper. Copy over your $\overline{\Delta t_A}$, $\overline{\Delta t_B}$, and $\overline{\Delta t_{12}}$ values from Table 7.2 and place them into Table 7.4. Estimate the instantaneous velocity of the ball through photogate A and photogate B for each condition using the following formulas:

$$v_A = \frac{d}{\overline{\Delta t_A}}$$

$$v_B = \frac{d}{\overline{\Delta t_B}}$$

where d is the diameter of the ball, $\overline{\Delta t_A}$ is the average time through photogate A and $\overline{\Delta t_B}$ is the average time through photogate B. Compute the acceleration of the ball g_{exp} using

$$g_{\text{exp}} = \frac{v_{\text{B}} - v_{\text{A}}}{\overline{\Delta t_{12}}}$$

where $\overline{\Delta t_{12}}$ is the average time interval for the ball to travel from photogate 1 to photogate 2. Finally, compute the average g_{exp}, the PRAAD for g_{exp}, and the percent experimental error comparing your g_{exp} to the accepted value of 980 cm/s^2. Record your values in Table 7.4.

Question 5: For each condition, your value of v_{B} occurs at the time $\Delta t = \Delta t_{01} + \Delta t_{12}$. Copy over the Δt values from Table 7.3 and the v_{B} values from Table 7.4 and place them into Table 7.5. Plot v_{B} versus Δt and perform a linear fit to the data. Record the slope of your line. This slope represents the acceleration of the ball g. Record your acceleration value in Table 7.6. Finally, compare your value of g with the accepted value of 980 cm/s^2 by computing the percent experimental error.

	Condition 1	Condition 2	Condition 3	Condition 4	Condition 5
Δt					
v_{B}					

Table 7.5: Velocity and Time Data for a Ball Drop

Experimental g from Graph (cm/s^2)	Accepted g (cm/s^2)	Percent Experimental Error (%)
	980	

Table 7.6: Comparison of Acceleration g Values

7.2.2 Measuring g Using the Simple Pendulum

Set up the wooden pendulum. The length of the pendulum is defined to be from the bottom of the dowel (pivot point) to the bottom of the washers. Displace the pendulum by approximately 10° to 20° and measure the time for 10 complete oscillations. Do this for pendulum lengths of 20 cm, 30 cm, 40 cm, 50 cm, and 60 cm. Record your values in Table 7.7. An alternative to using the wooden pendulum is to use the PhET pendulum lab simulation (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Either option is fine for collecting your data.

Question 6: Compute the period T for each of the pendulum lengths. Compute the acceleration due to gravity g_{exp} using Equation 7.3. Round the g_{exp} values to the nearest whole number.

Question 7: Using the 5 values of g_{exp} that you have calculated, compute an average value for g_{exp}. Also compute the PRAAD for g_{exp}.

Length of Pendulum (L) (cm)	Time for 10 Oscillations (s)	Period (T) (s)	Experimental Value of g g_{exp} (cm/s^2)	Absolute Deviations (cm/s^2)
20				
30				
40				
50				
60				
Average				
PRAAD (%)				
Percent Experimental Error (%)				

Table 7.7: Acceleration g Values Using the Simple Pendulum

Question 8: Compare your experimental average value for g_{exp} with the accepted value of 980 cm/s^2 by computing the percent experimental error.

7.2.3 Measuring g Using the Linear Air Track

Condition	Trial 1 Time (s)	Trial 2 Time (s)	Trial 3 Time (s)	Trial 4 Time (s)	Trial 5 Time (s)	Average Time (s)	g_{exp} (cm/s^2)	Absolute Deviations (cm/s^2)
$s = 10$ cm								
$s = 20$ cm								
$s = 30$ cm								
Average								
PRAAD (%)								
Percent Experimental Error (%)								

Table 7.8: Measuring g Using Inclined Air Track

Incline the air track slightly by placing the end with the single leg on a 2 kg mass. Measure the height relative to the table at the two ends of the 200 cm length air track. h_1 is the height to the lower end of the track and h_2 is the height to the higher end. Now place the 10 cm flag on the cart and start the cart at rest right before the flag is about to trigger the photogate. Release the cart and measure the time it takes for the flag to pass through the photogate (this is the time for the cart to undergo a displacement of 10 cm). Perform five trials each with the 10 cm, 20 cm, and 30 cm flags. Record your data in Table 7.8.

Question 9: Calculate the average time of the five trials for each condition. Using Equation 7.8, calculate the acceleration due to gravity g_{exp}.

Question 10: Compute the average value for g_{exp} and compute the PRAAD for g_{exp}.

Question 11: Compare your average value for g_{exp} to the accepted value of 980 cm/s^2 by computing the percent experimental error.

Lab 8

Motion Analysis

In this lab we will digitize several movies of various objects in motion (a dropped ball, a ball rolling of the lab bench, a student performing a vertical jump, and a student shooting a free throw in basketball) and then analyze the graphs of those motions.

8.1 Theory

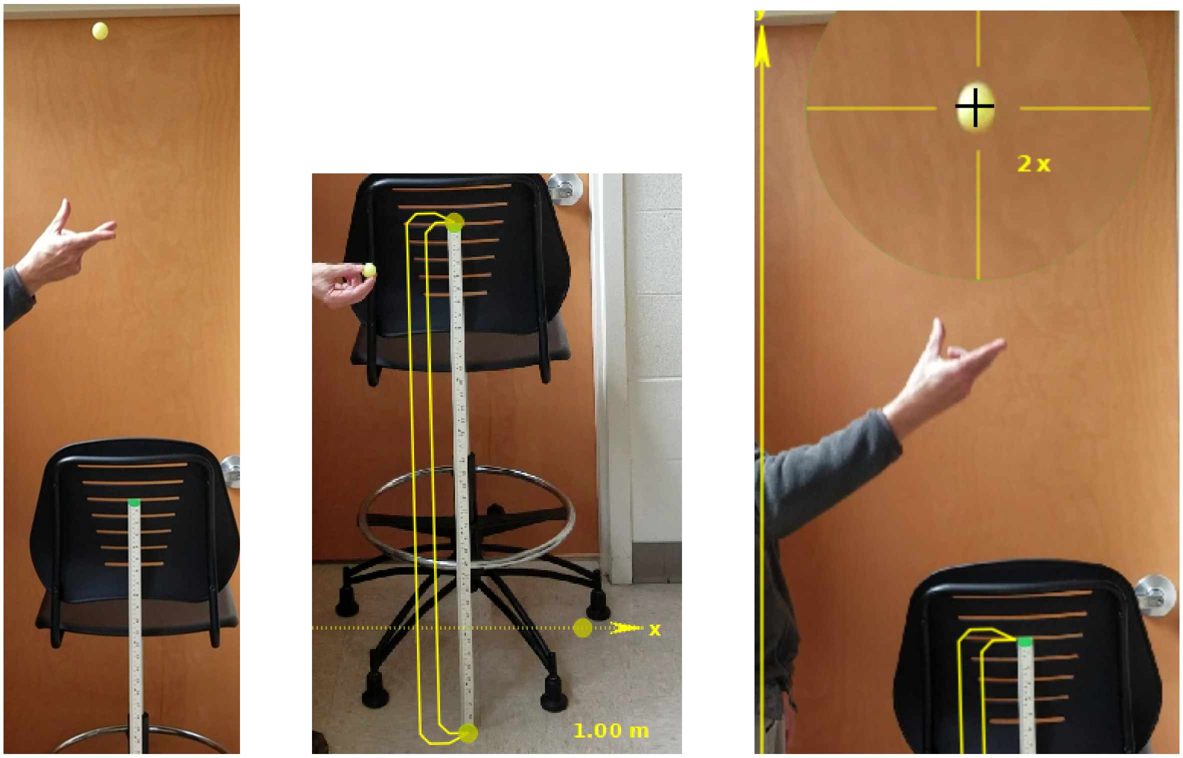
In all of these motions we are only interested in the time interval when the object is in the air and acted on only by the gravitational force. In other words, we aren't interested in the forces that generated the initial velocity of the object and we will neglect air resistance while the object is in the air. For each motion, we will generate position versus time, velocity versus time, and acceleration versus time graphs. To do this, we will record a movie of the motion and then digitize that motion. The digitization process involves using a crosshair cursor to locate the position of the object in each frame of the movie. To ensure that the position data is in real world units of meters, you will also need to calibrate the video. This is done by digitizing the two ends of an object of known size in one of the video frames (typically, this is a meter stick that is placed in the field of view). Once the position of the object is determined, one can also compute the velocity and acceleration of the object. This is done by numerically differentiating the position data with respect to time and the velocity data with respect to time, respectively (this is all done automatically by the software). Finally, one can plot graphs of the vertical or horizontal position of the object versus time, the vertical or horizontal velocity of the object versus time, and the vertical or horizontal acceleration of the object versus time.

8.1.1 Demonstration - Ball Thrown Vertically Upward

To demonstrate this process we will analyze the motion of a small ball that is thrown up vertically (see Figure 8.1a). We will be using the video analysis option in Pasco Capstone to digitize our movie. If you don't have access to Pasco Capstone, there are other options for digitizing two dimensional motions such as the free Tracker software (https://physlets.org/tracker/).

8.1.1.1 Digitizing the Video and Generating the Graphs

To begin, open the Pasco Capstone software, then choose Video Analysis followed by Open Movie. At this point, you can choose the movie that contains the motion you want to analyze, so we will choose the movie containing the vertical ball throw. The first thing that should be done with



(a) Vertical Throw (b) Calibration (c) Tracking Tool

Figure 8.1: Ball Thrown Vertically Upward

any motion is the calibration. Make sure that an object of known length (usually a meter stick) is in the current frame (Note: Make sure the entire meter stick is in the field of view when you record the movie). Then bring the top of the calibration tool to the top of the meter stick and the bottom of the calibration tool to the bottom of the meter stick (see Figure 8.1b). Now indicate the known length on the calibration tool (the default is 1.00 meters so in this case you can leave the default value). Essentially, you are indicating to the computer that the known length (meter stick) represents a certain number of pixels. In the future, all positions will be automatically converted from pixels to meters.

Now we are ready to locate (track) the position of the ball in each frame of the movie. Make sure that you are only analyzing the frames in which the ball is in the air (from having just left the individual's hand to just before hitting the floor). You can either choose these frames manually or clip the original movie so that these are the only recorded frames. To locate the position of the ball, we will choose the "magnify video in region around cursor" option (see Figure 8.1c). For each frame, we will place the cross hairs of the magnifying glass icon over the center of the ball and click the left mouse button. That position will be recorded and the movie will automatically increment to the next frame. Continue by locating the center of the ball in each frame for the entire motion while the ball is in the air.

Now we will create three graphs that represent the motion of the ball. Since the motion occurs in the vertical direction, we will create a vertical position versus time, a vertical velocity versus time, and a vertical acceleration versus time graph. To create the vertical position graph, select the "graph" icon. On the vertical axis of the graph choose Object #1 and then "y". You should select the "scale axes to show all data" icon at the top left of the graph. This will zoom in to your data. You should now remove the default line connections that connect each data point. You can do this by hovering over a data point, right click, then deselect "show connecting lines". Depending on how noisy your data is, you may want to select the "apply smoothing to active data" icon to remove some of the noise in the data that results from the digitizing process (if you use this feature, I would recommend choosing no more than 12 or 15 as the amount of smoothing). Next, select the "apply selected curve fits to active data" icon and choose a quadratic fit. Finally, make sure you give the plot a descriptive title which can be entered at the bottom left of the graph.

To create the vertical velocity versus time graph, select the "graph" icon again and on the vertical axis of the graph now choose Object #1 and then "vy". Follow similar steps that were used to generate the vertical position versus time graph from above. For the fit, you should choose "linear". Finally, to create the acceleration versus time graph, select the "graph" icon, and choose Object #1 and then "ay". For the acceleration graph I would rescale the y-axis so that the graph looks horizontal. To do this, you can click on the lowest value on the y-axis and change it to a new value (maybe -40 m/s^2) then click on the largest value on the y-axis and change it to a new value as well (maybe $+20$ m/s^2). For the fit, you want to choose a constant fit. Unfortunately, "constant" is not an option in the fit menu. However, you can get a constant fit by choosing the "display selected statistics for active data" icon. By default it will create a constant (horizontal) line fit and display the average value of the fitted line. Once all three graphs are created, you can copy and paste them into a google doc or word document to be further analyzed. To do this, highlight the border of the graph, right click, then select "copy display". This will copy the graph to the clipboard and allow you to paste the graph into other applications.

You should now have three graphs (vertical position versus time, vertical velocity versus time, and vertical acceleration versus time) that are similar to those in Figures 8.2, 8.3, and 8.4.

8.1.1.2 Interpreting the Graphs

Now we are ready to interpret the graphs. First, let's examine the vertical position versus time graph. Notice that the graph represents one-dimensional motion (motion in just the vertical direction) as a function of time. From the graph, we see that the ball left the individual's hand at about 0.8 m (relative to the reference frame which was attached just above the floor), it then rises to a peak height of about 1.37 m at a time of 0.36 s. The ball then falls from the peak height until it's just about to strike the floor. Notice that the height of the ball just before it hits the floor is -0.2 m. This is because the reference frame happened to be a little above the floor.

It is also useful to look at the rate of change of the vertical position versus time graph. The rate of change or slope of the vertical position graph represents the vertical velocity of the ball. In this case, we see that the slope starts out with a positive value, decreases until about 0.36 s where the slope is 0 m/s, then continues to decrease after 0.36 s becoming more and more negative.

Because the slope of the vertical position versus time graph represents the velocity, the values of those slopes should be represented on the vertical velocity versus time graph. This is exactly what we observe. The object initially has a velocity of about 3.0 m/s as the ball leaves the individual's hand. The velocity of the ball then decreases until about 0.36 s at which time it's value is instantaneously 0 m/s. After 0.36 s, the velocity continues to decrease (it is becoming more and more negative). Since velocity is a vector that has both magnitude and direction, let's examine the motion by looking at each quantity separately. When the ball is rising, the direction of the vertical velocity of the ball is positive and the magnitude of the velocity (speed) of the ball is decreasing. At the peak the velocity of the ball is instantaneously 0 m/s. When the ball is falling, the direction of the vertical velocity of the ball is negative and the magnitude of the velocity of the ball is increasing (it is falling faster and faster). The last thing to notice is that the shape of the vertical velocity versus time graph is linear. It's rate of change (or slope), which represents the vertical acceleration of the ball, is constant. In fact, it is negative and constant.

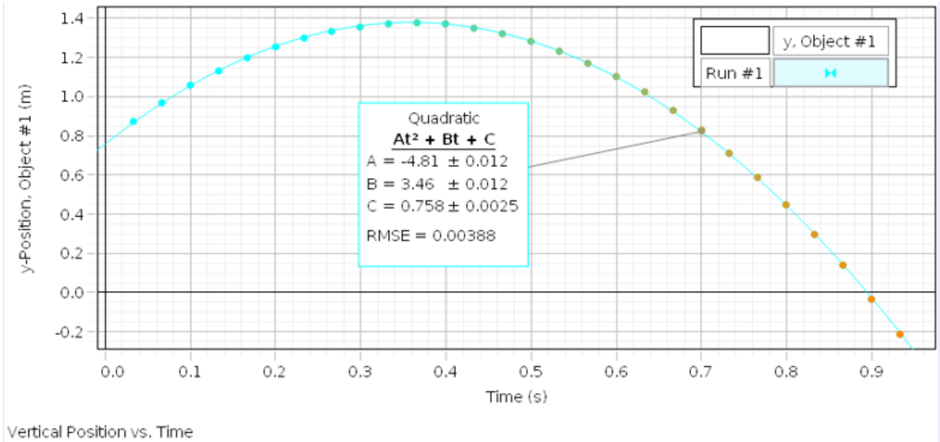


Figure 8.2: Vertical Position vs. Time of Ball

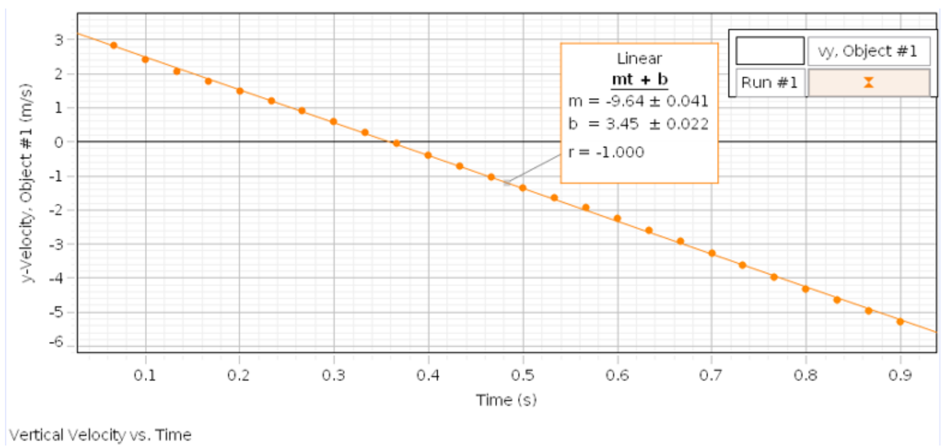


Figure 8.3: Vertical Velocity vs. Time of Ball

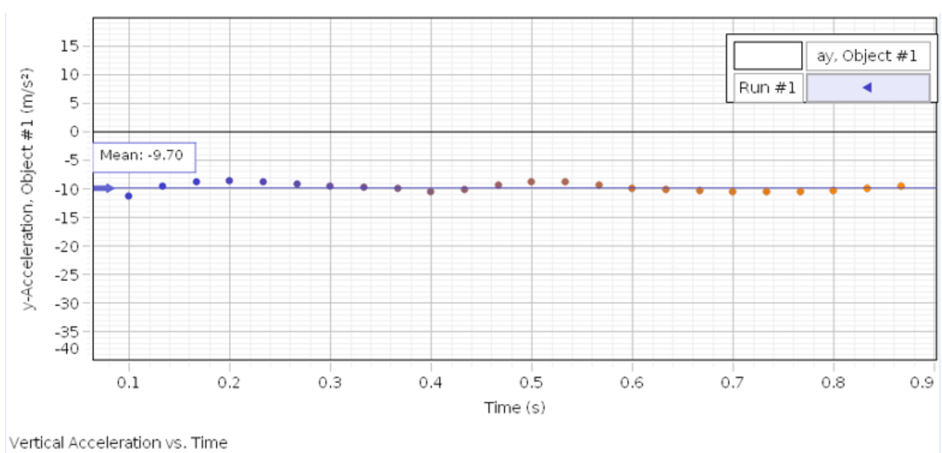


Figure 8.4: Vertical Acceleration vs. Time of Ball

When we examine the vertical acceleration versus time graph, its values should reflect the slopes of the vertical velocity versus time graph. Ignoring the little bit of noise that is present in the data, the values are fairly constant. In fact, the mean of the values is about $-9.70\ \text{m/s}^2$. From kinematics, we know that when an object is in free rise / free fall motion (only under the influence of the gravitational force) then its acceleration in the vertical direction can be determined from

$$-mg\hat{j} = ma_y\hat{j}$$

Dividing both sides by the mass m gives

$$a_y = -g$$

Using $g = 9.8\ \text{m/s}^2$ gives $a_y = -9.8\ \text{m/s}^2$ for the acceleration of any object near the surface of the Earth in free rise / free fall motion (note that we are ignoring the presence of any other forces like air resistance).

By digitizing the motion of the ball being throw upward, we computed a value of the vertical acceleration of $a_y = -9.70\ \text{m/s}^2$ which is very close to the expected value of $a_y = -9.80\ \text{m/s}^2$. Had we dropped a ball, or launched it at some angle, or performed a vertical jump and analyzed the position of the center of mass of the human body we should compute the same value ($a_y = -9.8\ \text{m/s}^2$) for the vertical acceleration while the object is in the air.

8.2 Experiment

In this experiment, we will analyze four motions by creating movies of each motion (or using movies that have already been created) and then digitizing them. Those motions will be dropping a ball from rest, performing a maximal vertical jump, rolling a ball horizontally off of a lab bench, and performing a free throw shot in basketball.

8.2.1 Dropping a Ball From Rest

In this experiment we will analyze the motion of a ball being dropped from rest (see Figure 8.5). You may create your own movie of the motion with your smart phone or download a provided movie to analyze. If you create your movie don't forget to place a meter stick in the field of view in the same plane of motion as the ball being dropped. Once you have created (or downloaded) your movie, you are now ready to analyze it. Using the directions given in section 8.1.1.1 above, digitize the movie and create graphs of vertical position versus time, vertical velocity versus time, and vertical acceleration versus time. For the vertical position versus time graph, perform a quadratic fit. For the vertical velocity versus time graph, perform a linear fit. For the vertical acceleration versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so that the graph looks horizontal. Make sure you are only digitizing the frames when the ball is in the air (from right when the ball leaves the hand to right before hitting the floor). Copy the graphs into your lab report for further analysis.

Question 1: Since the ball is dropped, we know that the y-position of the ball is decreasing over time. We can also see this from the vertical position versus time graph. However, is the vertical position of the ball decreasing at a constant rate (ie. is the slope of the vertical position versus time graph constant)? Explain.

Question 2: What quantity does the slope of the vertical position versus time graph represent? Does your vertical velocity versus time graph support your answer?

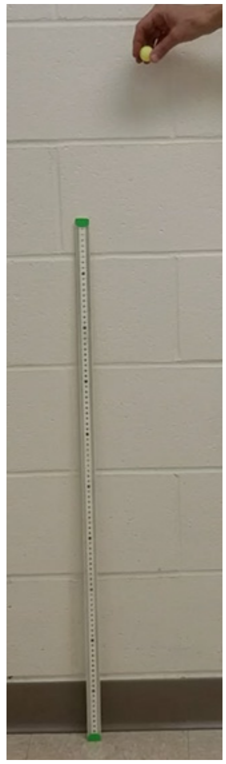


Figure 8.5: Ball Dropped From Rest

Question 3: How is the magnitude of the vertical velocity (speed) changing as time progresses?

Question 4: What quantity does the slope of the vertical velocity versus time graph represent? Does your vertical acceleration versus time graph support your answer?

Question 5: Theoretically, if we ignore air resistance, what should the average value of the vertical acceleration be? What was your experimental average vertical acceleration from your graph? Was the value close to what you expected?

8.2.2 Maximal Vertical Jump

In this experiment we will analyze the motion of an individual performing a maximal vertical jump (see Figure 8.6). Again, you may create your own movie with your smart phone or download a provided movie to analyze. If you record your own movie, make sure that you place a marker on the individual's hip. This marker will approximate the location of the center of mass of the body which is the point that we will track. Also make sure a meter stick is placed in the field of view in the same plane as the hip marker. Using the directions given in section 8.1.1.1 above, digitize the movie and create graphs of vertical position versus time, vertical velocity versus time, and vertical acceleration versus time. For the vertical position versus time graph, perform a quadratic fit. For the vertical velocity versus time graph, perform a linear fit. For the vertical acceleration versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. Make sure you are only digitizing the frames when the jumper is in the air (from right when their feet leave the ground to right before the feet again make contact with the ground). Copy the graphs into your lab report for further analysis.

Figure 8.6: Vertical Jump

Question 6: Is the vertical position of the jumper changing at a constant rate (ie. is the slope of the vertical position versus time graph constant)? Explain.

Question 7: As the jumper is rising to the peak height, what direction is the vertical velocity (positive or negative)? As the jumper is falling from the peak height, what direction is the vertical velocity (positive or negative)? What is the vertical velocity at the peak height?

Question 8: As the jumper is rising to the peak height, is the vertical speed (magnitude of vertical velocity) increasing or decreasing? As the student is falling from the peak height, is the vertical speed increasing or decreasing?

Question 9: Theoretically, what should the vertical acceleration be while the jumper is in the air throughout the jump? What was your average vertical acceleration value from your graph?

Question 10: When the jumper reaches the peak of the jump, what was their vertical acceleration?

8.2.3 Projectile Launched Horizontally

In this experiment we will analyze the motion of a ball which is rolled horizontally off of a lab bench (see Figure 8.7). Again, you may create your own movie with your smart phone or download a provided movie to analyze. If you record your own movie, make sure a meter stick is placed in the field of view in the same plane as the motion of the ball. Using the directions given in section 8.1.1.1 above, digitize the movie and create graphs for both the horizontal and vertical directions (horizontal position versus time, horizontal velocity versus time, horizontal acceleration versus time, vertical position versus time, vertical velocity versus time, and vertical acceleration versus time). For the horizontal position versus time graph, perform a linear fit. For the horizontal velocity versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. For the horizontal acceleration versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. For the vertical position versus time graph, perform a quadratic fit. For the vertical velocity versus time graph, perform a linear fit. For the vertical acceleration versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. Make sure you are only digitizing the frames when the ball is in the air (from right after the ball leaves the lab bench to right before the ball makes contact with the ground). Copy the graphs into your lab report for further analysis.

Figure 8.7: Ball Rolling Off Lab Bench

Question 11: Since the ball is moving forward, the horizontal position of the ball is increasing over time as can be seen in the graph. Is the horizontal position of the ball increasing at a constant rate (ie. is the slope of the horizontal position versus time graph constant)? Explain.

Question 12: What quantity does the slope of the horizontal position versus time graph represent? Does your horizontal velocity versus time graph support your answer?

Question 13: What quantity does the slope of the horizontal velocity versus time graph represent? Does your horizontal acceleration versus time graph support your answer? Theoretically, what should the value of the horizontal acceleration value be?

Question 14: Compare your vertical position versus time, vertical velocity versus time, and vertical acceleration versus time graphs to those generated from the motion of the dropped ball. Comment on the general shapes of each graph. What can you conclude about the motion in the vertical direction for a dropped ball and one rolled off the bench?

8.2.4 Free Throw Shot

In this experiment we will analyze the motion of an individual shooting a free throw in basketball. Again, you may create your own movie with your smart phone or download a provided movie to analyze. If you record your own movie, make sure a meter stick is placed in the field of view in the same plane as the motion of the ball (see Figure 8.8). Using the directions given in section 8.1.1.1 above, digitize the movie and create graphs for both the horizontal and vertical directions (horizontal position versus time, horizontal velocity versus time, horizontal acceleration versus time, vertical position versus time, vertical velocity versus time, and vertical acceleration versus time). For the horizontal position versus time graph, perform a linear fit. For the horizontal velocity versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. For the horizontal acceleration versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. For the vertical position versus time graph, perform a quadratic fit. For the vertical velocity versus time graph, perform a linear fit. For the vertical acceleration versus time graph, perform an average fit using the statistics summary icon and scale the vertical axis so the graph looks horizontal. Make sure you track the center of the basketball (or some other consistent location such as the bottom of the basketball) and are only digitizing the frames when the ball is in the air (from right after the ball leaves the individual's hand to right before the ball makes contact with the rim). Copy the graphs into your lab report for further analysis.

Figure 8.8: Basketball Free Throw

Question 15: Compare your horizontal position versus time, horizontal velocity versus time, and horizontal acceleration versus time graphs to those generated from the motion of the ball rolled off of the lab bench. Comment on the general shapes of each graph. What can you conclude about the motion of a ball in the horizontal direction for a free throw shot and one rolled off the bench?

Question 16: Compare your vertical position versus time, vertical velocity versus time, and vertical acceleration versus time graphs to those generated from the motion of the vertical jump (or the ball thrown upward as in the theory section). Comment on the general shapes of each graph. What can you conclude about the motion of a ball in the vertical direction for a free throw shot and the motion of an individual performing a vertical jump (or the motion of a ball thrown upward)?

Lab 9

The Motion of a Projectile

Galileo (1564–1642) was perhaps the first to give any scientific thought to the motion of a projectile. The form of the trajectory of a projectile had been incorrectly understood by pre-Galilean thinkers. Some of these men had even gone so far as to try to explain the motion of a projectile by the encouragement of demons and mysterious swooshes of air. Figure 9.1 below, illustrates a popular pre-Galilean conception of projectile motion.

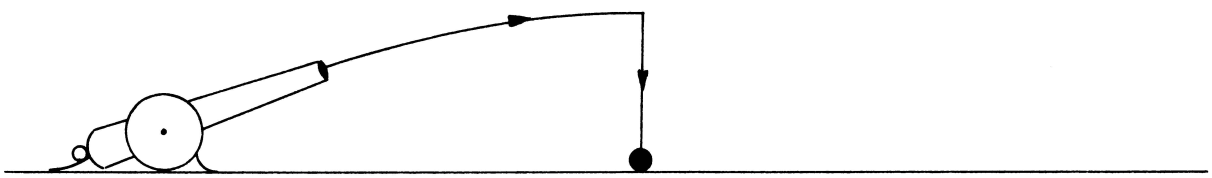


Figure 9.1: Pre-Galilean Conception of the Trajectory of a Projectile

Galileo argued that, in the absence of complications such as air resistance, the trajectory of a projectile should be symmetrical like the trajectory shown in Figure 9.2 below. His argument rested on his keen observation and reasoning that, in the absence of complications such as air resistance, all bodies accelerate toward the center of the earth at the same rate. The motion of a projectile wasn't rigorously analyzed, however, until Isaac Newton formulated the laws of motion.

In this experiment you will both observe and analyze the motion of a projectile. Your projectile will be a ball.

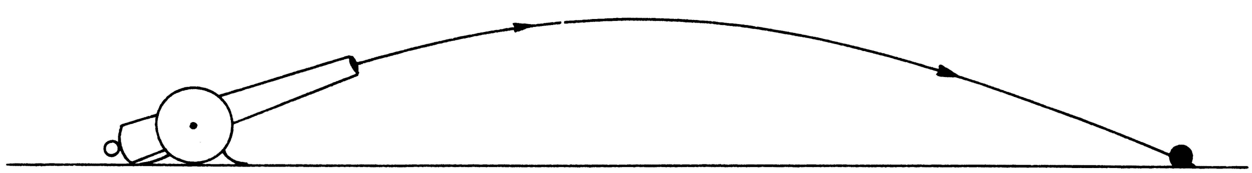


Figure 9.2: Galileo's Conception of the Trajectory of a Projectile

9.1 Theory

9.1.1 General Analysis

The Galilean conception (see Figure 9.2) of projectile motion is the correct one. Below we briefly outline the theoretical analysis of projectile motion and sketch the derivations of interesting results. Before you enter the laboratory, you should thoroughly understand the analysis and also be able to work out the derivations.

In the absence of air resistance, there is one external force that acts on a projectile, the force due to gravity.

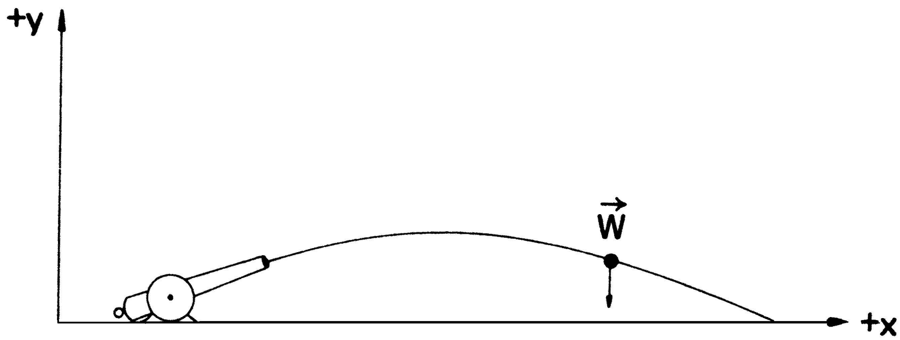


Figure 9.3: The only external force acting on the projectile is the force due to gravity $\vec{w}$

Now, according to Newton's second law of motion, the relationship between the net force $\vec{F}_{\rm net}$ and the acceleration $\vec{a}$ is

$$\vec{F}_{\rm net} = m\vec{a} \tag{9.1}$$

when the object experiencing the net force suffers no change in mass m. For a projectile of mass m, $\vec{F}_{\rm net}$ is the force due to gravity $\vec{w}$, the weight of the projectile. $\vec{w}$ is directed toward the center of the earth in the $-y$ direction as shown in Figure 9.3.

Experiments show that $\vec{a} = \vec{g}$ when $\vec{F}_{\rm net} = \vec{w}$ where the magnitude of $\vec{g}$ is $g = 9.8\ \text{m/s}^2 = 980\ \text{cm/s}^2 = 32\ \text{ft/s}^2$.

It is easy to see (with the choice of coordinates shown in Figure 9.3 that $F_{{\rm net},x} = w_x = 0$ and $F_{{\rm net},y} = w_y = -mg$. Therefore, since $F_{{\rm net},x} = 0 = ma_x$ we have

$$a_x = 0 \tag{9.2}$$

And since $F_{{\rm net},y} = -mg = ma_y$, this implies that

$$a_y = -g \tag{9.3}$$

Equation 9.2 implies that the x-component of the velocity vector of the projectile remains constant throughout the flight of the projectile. If the projectile experiences an initial velocity $\vec{v}_0$ and is released at an angle of θ with respect to the x-axis (see Figure 9.4), then

$$v_x = v_{0x} = v_0 \cos\theta \tag{9.4}$$

The projectile accelerates in the y direction as Equation 9.3 indicates. Now

$$v_{0y} = v_0 \sin\theta \tag{9.5}$$

and

$$a_y = \frac{v_y - v_{0y}}{t} \tag{9.6}$$

where v_y is the y-component of the velocity at time t (note: we have chosen $t_0 = 0$). Equations 9.3 and 9.6 clearly imply that

$$v_y = v_{0y} - gt \tag{9.7}$$

Equation 9.4 shows that the x-component of the velocity of the projectile remains constant during the flight of the projectile, and Equation 9.7 shows that the y-component of $\vec{v}$ changes with the time of flight of the projectile. Notice that Equation 9.7 has the same form as the corresponding equation for a freely falling object.

The x-component of the displacement vector $\vec{s}$ of the projectile at time t is given by

$$s_x = x - x_0 = \overline{v}_x t = v_{0x} t \tag{9.8}$$

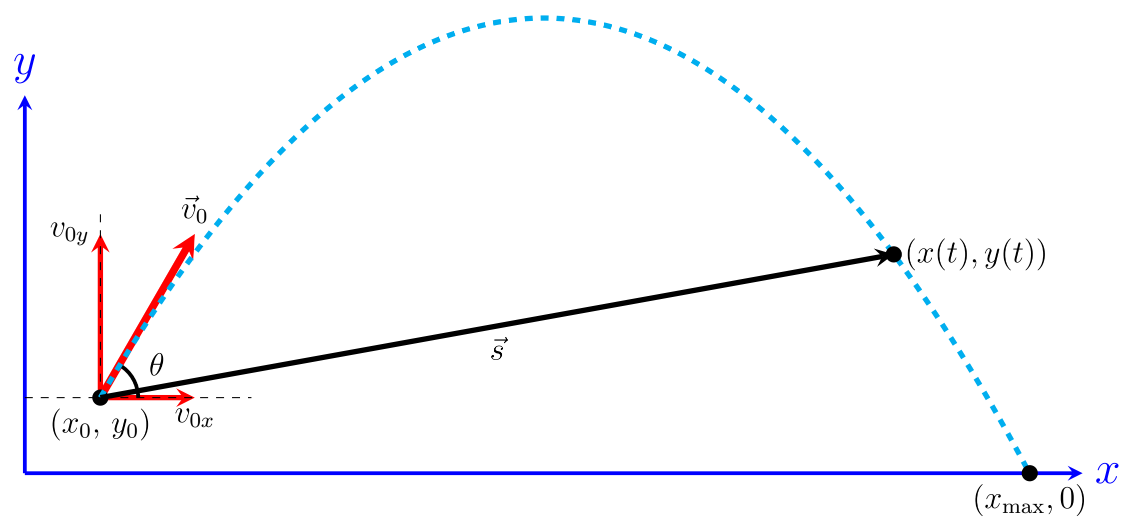


Figure 9.4: Trajectory of a Projectile: $\vec{s}$ is the displacement vector that locates the projectile at time t.

The y-component of the displacement vector $\vec{s}$ of the projectile at time t is given by

$$s_y = y - y_0 = \overline{v}_y t = \frac{v_{0y} + v_y}{2} t = \frac{v_{0y} + v_{0y} - gt}{2} t = v_{0y} t - \frac{1}{2} g t^2 \tag{9.9}$$

Equivalently, we may write that the x-coordinate of the position of the projectile at time t is given by

$$x = x_0 + v_{0x} t \tag{9.10}$$

and the y coordinate of the position of the projectile at time t is given by

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \tag{9.11}$$

Let T be the amount of time that the projectile is in flight. Then, the horizontal distance that the projectile has travelled in time T is

$$X = x - x_0 = v_{0x} T \tag{9.12}$$

At the end of time T, the projectile has struck the ground (see Figure 9.4) so that $y = 0$. Substituting $t = T$ and $y = 0$ into Equation 9.11 gives

$$0 = y_0 + v_{0y}T - \frac{1}{2}gT^2 \tag{9.13}$$

Equation 9.13 may be solved for T giving the physical solution

$$T = \frac{v_{0y}}{g} + \sqrt{\frac{v_{0y}^2}{g^2} + \frac{2y_0}{g}} \tag{9.14}$$

Equations 9.4, 9.5, 9.12, and 9.13 may be combined to yield the general result for the initial speed v_0 of the projectile

$$v_0 = X\left(\frac{2y_0}{g}\cos^2\theta + \frac{\sin(2\theta)}{g}X\right)^{-\frac{1}{2}} \tag{9.15}$$

For the simple case when the projectile is fired horizontally, $\theta = 0°$ and Equation 9.15 simplifies to

$$v_0 = X\sqrt{\frac{g}{2y_0}} \tag{9.16}$$

9.1.2 The Range of a Projectile

The range of a projectile is defined as the horizontal distance from the starting position of the projectile to the position at which the projectile returns to its original elevation. Figure 9.5 illustrates what is commonly referred to as the range of a projectile. Let t_{R} be the time it takes the projectile to return to its initial elevation y_0. Then Equation 9.11 yields

$$y_0 = y_0 + v_{0y}t_{\mathrm{R}} - \frac{1}{2}gt_{\mathrm{R}}^2 \tag{9.17}$$

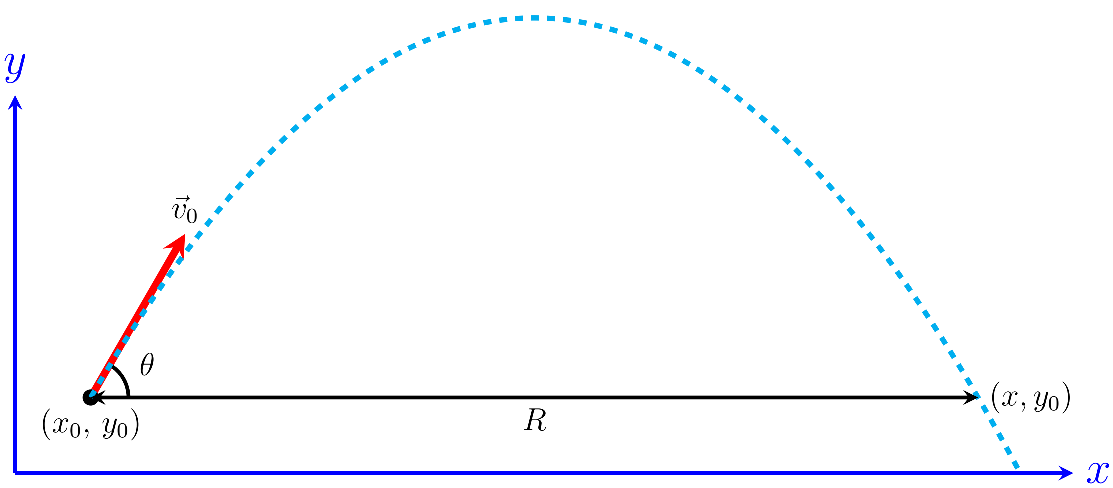


Figure 9.5: The Range R of a Projectile

Solving Equation 9.17 for t_{R}, we find the nontrivial solution

$$t_{\mathrm{R}} = \frac{2v_{0y}}{g} \tag{9.18}$$

The horizontal distance traveled in the time t_{R} is just what we have called the range R.

$$R = v_{0x} t_{\mathrm{R}} \tag{9.19}$$

substituting Equation 9.18 into Equation 9.19 we find that

$$R = \frac{2 v_{0x} v_{0y}}{g} \tag{9.20}$$

Using Equations 9.4 and 9.5 we can write Equation 9.20 as

$$R = \frac{2 v_0 \cos\theta v_0 \sin\theta}{g} \tag{9.21}$$

Finally, using the well known trigonometry identity $\sin(2\theta) = 2\cos\theta\sin\theta$ we can write Equation 9.21 as

$$R = \frac{{v_0}^2 \sin(2\theta)}{g} \tag{9.22}$$

which is often referred to as the range formula.

9.2 Experiment

9.2.1 Part 1 - Observing a Projectile Launched Horizontally

Using the blue projectile launcher given, shoot the solid steel ball horizontally off the top of your laboratory table. Note its motion.

Question 1: Sketch a picture of the motion you have just observed.

Question 2: Why does the ball immediately fall instead of going straight?

Question 3: Are there any external forces acting on the ball to account for the motion that you have observed (only consider the motion of the ball after it leaves the launcher and neglect air resistance)? If so, what are they? Give the direction of the force(s).

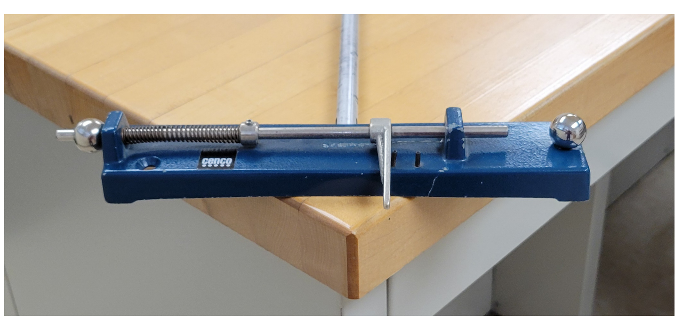


Figure 9.6: Solid Ball Launched Horizontally, Slotted Ball Dropped

9.2.2 Part 2 - Comparing a Projectile Launched Horizontally to One Dropped

Using the blue launcher, pull back the firing pin and place one solid metal ball on the shelf at the edge of the launcher. Place the slotted ball on the axle on the other side of the launcher (see Figure 9.6). When the launcher is fired, the solid ball will initially leave the launcher horizontally and the slotted ball will fall vertically. Repeat this procedure with a plastic ball and the slotted metal ball.

Question 4: What happened? In each case, which ball hit first? Was this unexpected? Explain.

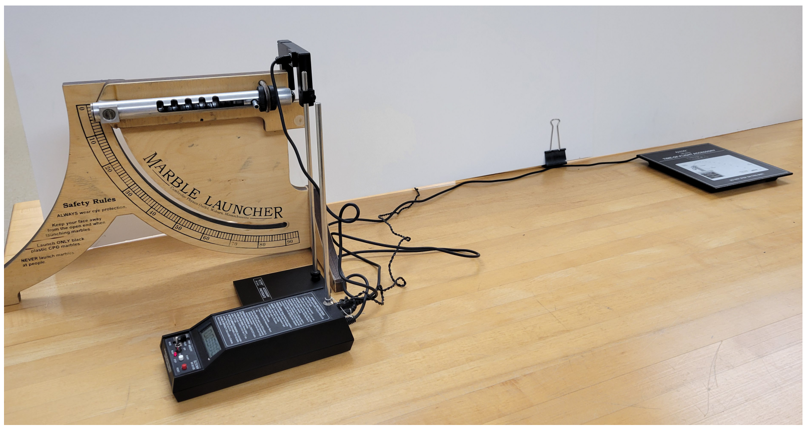


Figure 9.7: Projectile Launched Horizontally Using Wooden Launcher

9.2.3 Part 3 - Measuring Initial Velocity and Horizontal Displacement of a Projectile Launched Horizontally

In this part of the experiment you will measure the initial velocity of a plastic marble projectile using two different techniques as it is launched horizontally from the wooden launcher (see Figure 9.7). You will also measure the horizontal displacement of the marble from the end of the launcher to the landing position on the landing platform using two different techniques. First, measure the height of the bottom of the launching tube from the top of the time of flight landing platform (you can get this value by calculating the difference between the height of the bottom of the launching tube above the table and the height of the landing platform above the table). Make sure the photogate is set up in front of the barrel of the launcher so that it is triggered as soon as the marble leaves the launcher. When the marble hits the landing platform, the timer will be triggered to turn off, giving the time of flight of the marble. Use a piece of carbon paper on top of a sheet of white paper on the landing platform so that the marble leaves a mark when it strikes the platform. Fire the marble from the launcher using the lowest power setting. Measure the horizontal distance that the marble travels (from the end of the launch tube to the mark on the paper). Also record the time of flight of the marble (the timer should be set in pulse mode). Repeat each measurement five times and then calculate the averages. Record all of your results in Table 9.1.

Question 5: Draw a set of coordinate axes and draw a picture of the motion of the marble. Label the coordinates of the initial and final positions of the marble. Write down the two equations

Trial	Height of Launcher (h) (cm)	Horizontal Displacement (Δx_{exp}) (cm)	Time of Flight (T) (s)
1			
2			
3			
4			
5			
Average			

Table 9.1: Data for Projectile Launched Horizontally

that correctly give the x and y coordinates of the marble as a function of time. Draw and label the velocity of projection of the marble on your drawing.

Question 6: Determine the time of flight of the marble using the following equation derived from kinematics:

$$t_{\mathrm{f}} = \sqrt{\frac{2h_{\mathrm{ave}}}{g}}$$

Question 7: Determine the magnitude of the predicted initial velocity of the marble using the average measured horizontal displacement and the time of flight calculated in question 6. Use the following equation

$$v_{0,\mathrm{pred}} = \frac{\Delta x_{\mathrm{exp,ave}}}{t_{\mathrm{f}}}$$

Record all values in Table 9.2.

Time of Flight (t_{f}) (s)	
Initial Velocity ($v_{0,\mathrm{pred}}$) (cm/s)	

Table 9.2: Initial Velocity Predicted

Question 8: Determine the magnitude of the experimental initial velocity of the marble by using the CPO photogate and timer system (see Figure 9.8). Measure the diameter d of the marble and the time t_{pg} it takes the marble to pass through the photogate as it's being launched. The initial velocity can then be calculated as $v_{0,\mathrm{exp}} = \frac{d}{t_{\mathrm{pg}}}$. Finally, compare this experimental velocity to the predicted velocity calculated in Question 7. Compute the percent experimental error. Record all values in Table 9.3 and Table 9.4.

Question 9: Determine the horizontal displacement Δx_{pred} of the marble using the velocity $v_{0,\mathrm{exp}}$ of the ball measured in Question 8 and the average time T as recorded by the time of flight

Figure 9.8: Experimental Initial Velocity of Marble Measured Using CPO Timer

Angle (θ) (°)	0
Diameter (d) (cm)	
Time through Photogate (t_{pg}) (s)	
Initial Velocity ($v_{0,exp}$) (cm/s)	

Table 9.3: Initial Velocity Experimental

Predicted Initial Velocity ($v_{0,pred}$) (cm/s^2)	Experimental Initial Velocity ($v_{0,exp}$) (cm/s^2)	Percent Experimental Error (%)

Table 9.4: Comparison of Initial Velocity Values

apparatus (in Table 9.1). Use the following equation:

$$\Delta x_{pred} = v_{0,exp} T$$

Compare this predicted displacement with the average experimental displacement $\Delta x_{exp,ave}$ that was measured directly (in Table 9.1) by computing the percent experimental error. Record your results in Table 9.5.

Predicted Displacement Δx_{pred} (cm)	Experimental Displacement $\Delta x_{exp,ave}$ (cm)	Percent Experimental Error (%)

Table 9.5: Comparison of Horizontal Displacement Values

9.2.4 Part 4 - Measuring the Range of a Projectile with the Wooden Launcher

Using the wooden launcher on the lowest power setting, launch the marble at different angles varying from 15° to 75°. Make sure that the height of the yellow landing platform is adjusted so that it is level with the bottom of the barrel of the launcher (see Figure 9.9). Also make sure you have a piece of carbon paper and a sheet of white paper taped to the landing platform to record the landing position of the marble. Measure the range of the marble from the end of the launching tube of the launcher to the landing position using a tape measure. Perform three launches at each given angle then calculate the average range for each angle. Record your results in Table 9.6.

Figure 9.9: Measuring the Range of the Projectile Using a Landing Platform

Range (R) (cm)				
Angle (θ) (°)	Trial 1	Trial 2	Trial 3	Average Range
0	0	0	0	0
15				
25				
35				
45				
55				
65				
75				
90	0	0	0	0

Table 9.6: Range Values at Different Launch Angles

Question 10: Plot the range versus the launch angle (plot average range on the y axis and angle on the x axis). Perform a sine function fit to the data. Using the fitted curve, determine the angle at which the range is maximum and determine the experimental maximum range (this can be done by using the coordinate tool). At what launch angle does theory predict that the range will be maximum? Compare the experimental angle corresponding to the experimental maximum range to the theoretical angle at which the maximum range will occur by calculating a percent experimental error. Record your results in Table 9.7 and Table 9.8.

Optimal Angle (Theory) (°)	Optimal Angle (Experimental) (°)	Percent Experimental Error (%)

Table 9.7: Comparison of Optimal Angle Values

Question 11: Compute the theoretical maximum range of the marble using

$$R = \frac{v_{0\text{exp}}^2 \sin(2\theta)}{g} = \frac{v_{0\text{exp}}^2 \sin(2 \cdot 45)}{g} = \frac{v_{0\text{exp}}^2}{g}$$

Compare this theoretical value with the value you found from the experiment in question 10 by computing the percent experimental error. Record your results in Table 9.8.

Maximum Range (Theory) (cm)	Maximum Range (Experimental) (cm)	Percent Experimental Error (%)

Table 9.8: Comparison of Maximum Range Values

9.2.5 Part 5 - Measuring the Range of a Projectile Using the PhET Simulation

For this part of the lab, we will use the projectile motion simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation. https://phet.colorado.edu/en/simulation/projectile-motion

To begin the simulation, press play and select the intro icon. Set the initial height of the cannon so that it is at 0 m. Set the initial speed to be 18 m/s (see Figure 9.10). Select the cannonball as your projectile object and fire it at angles of 25, 30, 35, 40, 45, 50, 55, 60, and 65°. Use the tape measure to measure the range for each of those angles and fill in Table 9.9. Note that the simulation uses SI units whereas the experiment with the wooden launcher was done in cgs units.

Question 12: Plot the range versus the launch angle (plot range on the y axis and angle on the x axis). Perform a sine function fit to the data. Using the fitted curve, determine the angle at which the range is maximum and determine the experimental maximum range (this can be done by using the coordinate tool). At what launch angle does theory predict that the range will be maximum? Compare the experimental angle corresponding to the experimental maximum range to

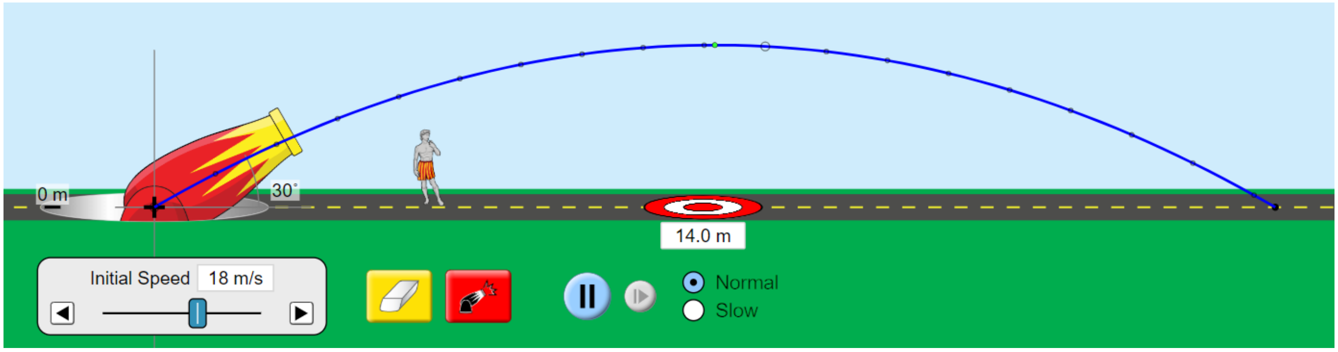


Figure 9.10: PhET Projectile Motion Simulation

Angle (θ) (°)	Range (R) (m)
0	0
25	
35	
40	
45	
50	
55	
60	
65	
90	0

Table 9.9: Range Values for PhET Simulation

Optimal Angle (Theory) (°)	Optimal Angle (Experimental) (°)	Percent Experimental Error (%)

Table 9.10: Comparison of Optimal Angle Values for the PhET Simulation

the theoretical angle at which the maximum range will occur by calculating a percent experimental error. Record your results in Table 9.10 and Table 9.11.

Question 13: Compute the theoretical maximum range of the cannonball using

$$R = \frac{v_0^2 \sin(2\theta)}{g} = \frac{v_0^2 \sin(2 \cdot 45)}{g} = \frac{v_0^2}{g}$$

Compare this theoretical value with the value you found from the experiment in question 12 by computing the percent experimental error. Record your results in Table 9.11.

Maximum Range (Theory) (m)	Maximum Range (Experimental) (m)	Percent Experimental Error (%)

Table 9.11: Comparison of Maximum Range Values for the PhET Simulation

Question 14: Using the simulation, now set the bullseye target to be 20 m from the base of the cannon. Keep the height of the cannon at 0 m. With the angle at 25°, what speed should the cannonball be fired at to hit the center of the bullseye? Now change the angle to 45°, and determine the correct speed to hit the bullseye. This can be done by trial and error. Record your results in Table 9.12.

Angle (θ) (°)	Initial Speed Needed to Hit Bullseye at a Range of 20 Meters (m/s)
25	
45	

Table 9.12: Initial Speed Values Needed to Hit Target at 20 m

Question 15: Using the range equation $R = \frac{v_0^2 \sin(2\theta)}{g}$, calculate the theoretical speeds needed to hit the target at 20 m when the projectile was launched at an angle of 25° and 45°. Were your values the same as those found using the simulation?

9.2.6 Part 6 - Computer Analysis of the Shot Put Event and Long Jump Event

The shot put event and long jump event are two interesting applications of projectile motion. In this laboratory exercise, you will analyze each of these events with the aid of computer programs.

9.2.6.1 Theory

Both analyses result by application of the equations of motion of a projectile. These equations have already been derived in lecture and are summarized in Table 9.13 below.

Equations of Motion of a Projectile	
x-Motion	**y-Motion**
$a_x = 0$	$a_y = -g$
$v_x = v_{0x} = v_0 \cos\theta_0$	$v_y = v_{0y} - gt$
$\Delta x = v_{0x}t$	$\Delta y = v_{0y}t - \frac{1}{2}gt^2$
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$
$R = \frac{v_0^2 \sin(2\theta_0)}{g}$	

Table 9.13: Equations of Motion

9.2.6.2 Analysis of the Shot Put Event

In the shot put event, the shot is customarily released at a height of about seven feet above the ground (see Figure 9.11).

h = initial height of shot above the ground at the time of release
$\vec{v}_0$ = velocity of projection
θ_0 = angle of projection
X = horizontal distance covered by shot

Consider the motion of the shot from the athlete's hand to the ground. The time that it takes the shot to go from the athlete's hand to the ground can be found by applying the equation

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Substituting in the following values

$$y_0 = h$$
$$y = 0$$
$$v_{0y} = v_0 \sin\theta_0$$

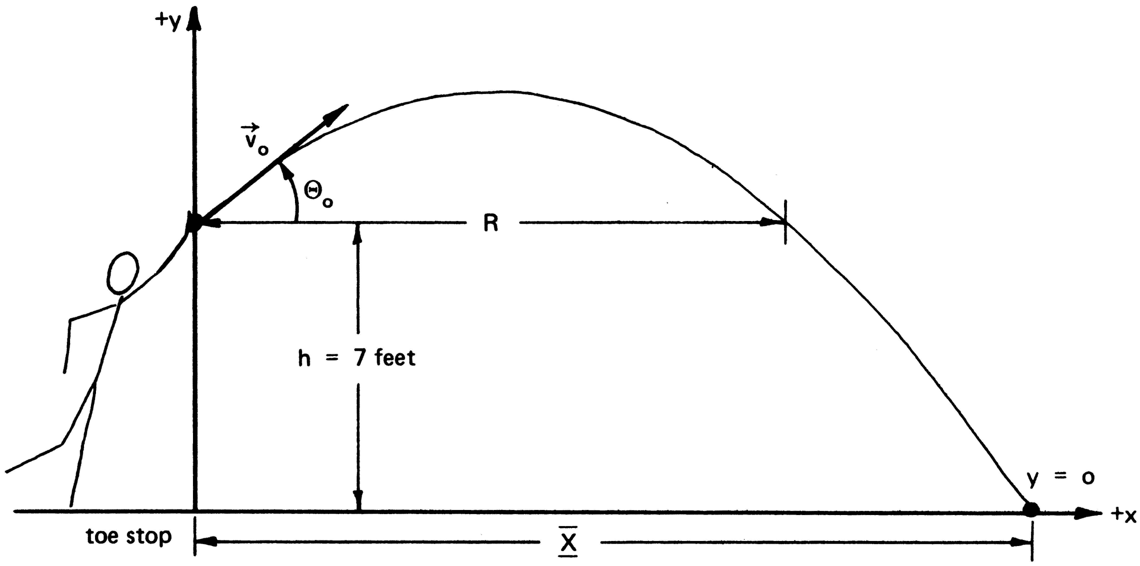


Figure 9.11: Shot Put Event

gives

$$0 = h + (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2$$

This equation may be rewritten as

$$t^2 - \frac{2v_0 \sin\theta_0}{g} t - \frac{2h}{g} = 0 \tag{9.23}$$

This is a quadratic equation of the form

$$at^2 + bt + c = 0$$

where

$$\begin{aligned} a &= 1 \\ b &= -\frac{2v_0 \sin\theta_0}{g} \\ c &= -\frac{2h}{g} \end{aligned}$$

The solutions of the equations are

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ t &= \frac{\frac{2v_0 \sin\theta_0}{g} \pm \sqrt{\frac{4v_0^2 \sin^2\theta_0}{g^2} + \frac{8h}{g}}}{2} \\ t &= \frac{v_0 \sin\theta_0}{g} \pm \sqrt{\frac{v_0^2 \sin^2\theta_0}{g^2} + \frac{2h}{g}} \end{aligned}$$

The solution one gets using the minus sign is nonphysical since it yields a negative time. The physical solution for the time for the motion is given by

$$t = \frac{v_0 \sin\theta_0}{g} + \sqrt{\frac{v_0^2 \sin^2\theta_0}{g^2} + \frac{2h}{g}} \tag{9.24}$$

Now that we know the time for the motion, we can find the horizontal distance x by using the equation

$$x = x_0 + v_{0x} t$$

For this motion

$$\begin{aligned} x_0 &= 0 \\ x &= X \\ v_{0x} &= v_0 \cos\theta_0 \end{aligned}$$

giving

$$\begin{aligned} X &= v_0 \cos\theta_0 \left(\frac{v_0 \sin\theta_0}{g} + \sqrt{\frac{v_0^2 \sin^2\theta_0}{g^2} + \frac{2h}{g}} \right) \\ X &= \frac{v_0^2 \sin\theta_0 \cos\theta_0}{g} + v_0 \cos\theta_0 \sqrt{\frac{v_0^2 \sin^2\theta_0}{g^2} + \frac{2h}{g}} \\ X &= \frac{v_0^2 \sin\theta_0 \cos\theta_0}{g} + \sqrt{\frac{v_0^4 \sin^2\theta_0 \cos^2\theta_0}{g^2} + \frac{2h}{g} v_0^2 \cos^2\theta_0} \end{aligned}$$

This result can be simplified by using the trigonometric identity

$$\sin(2\theta_0) = 2 \sin\theta_0 \cos\theta_0$$

giving

$$X = \frac{v_0^2 \sin(2\theta_0)}{2g} + \sqrt{\left(\frac{v_0^2 \sin(2\theta_0)}{2g}\right)^2 + \frac{2h}{g} v_0^2 \cos^2\theta_0} \tag{9.25}$$

This result gives the horizontal distance covered by the shot as a function of the speed of projection v_0, the angle of projection θ_0, and the initial elevation of the shot h.

9.2.6.3 Analysis of the Long Jump Event

Another common track and field event is the long jump. A diagram of the long jump indicating the variables of interest is given in Figure 9.12. The variables are defined as the following:

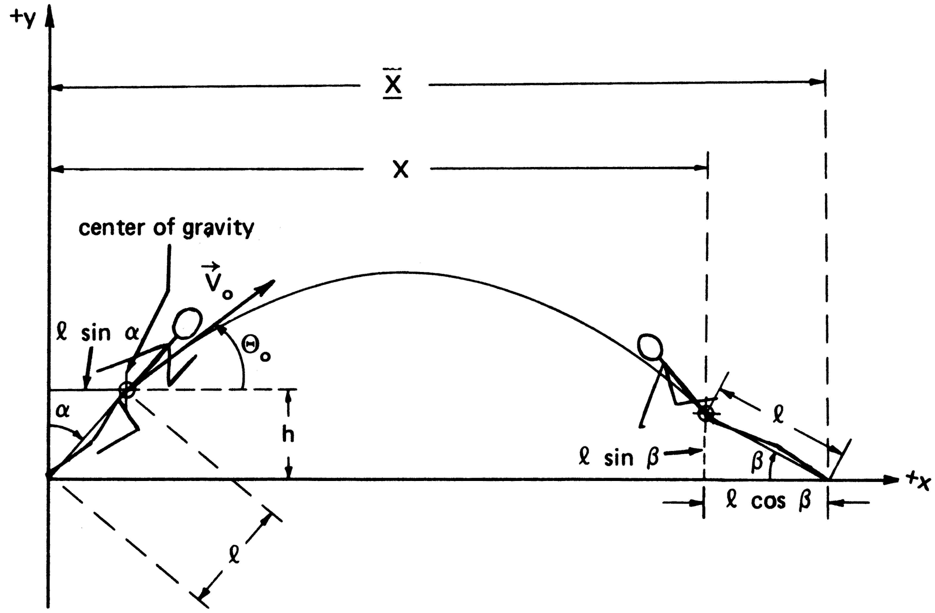


Figure 9.12: Long Jump Event

$l =$ distance from take-off foot to center of gravity
$\alpha =$ angle of lean at take-off
$\beta =$ angle of legs at the moment of landing
$h =$ initial height of center of gravity at take-off
$X = x$ coordinate of center of gravity at landing
$\underline{\overline{X}} =$ total horizontal distance travelled for the jump
$\vec{v}_0 =$ velocity of projection of center of gravity
$\theta_0 =$ angle of projection of center of gravity

In this analysis, we will take

$l = 3$ ft
$h = 2.6$ ft
$\alpha = 30°$
$\beta = 30°$

Consider the motion of the center of gravity from take-off to landing. The center of gravity undergoes projectile motion. Referring to Figure 9.12, we have

$$x_0 = l \sin\alpha$$
$$v_{0x} = v_0 \cos\theta_0$$
$$y_0 = h$$
$$v_{0y} = v_0 \sin\theta_0$$
$$y = l \sin\beta$$

The time for this motion of the center of gravity is determined from

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$l\sin\beta = h + v_0\sin\theta_0 t - \frac{1}{2}gt^2$$

This equation may be rewritten in the form

$$t^2 - \frac{2v_0\sin\theta_0}{g}t - \frac{2(h - l\sin\beta)}{g} = 0 \tag{9.26}$$

Equation 9.26 is similar to Equation 9.23. Note that

$$\frac{2(h - l\sin\beta)}{g} \quad \text{replaces} \quad \frac{2h}{g}$$

Therefore, the solution is obtained by simply replacing

$$\frac{2h}{g} \quad \text{in Equation 9.24 by} \quad \frac{2(h - l\sin\beta)}{g}$$

This gives

$$t = \frac{v_0\sin\theta_0}{g} + \sqrt{\frac{v_0^2\sin^2\theta_0}{g^2} + \frac{2(h - l\sin\beta)}{g}} \tag{9.27}$$

The x coordinate X of the position of the center of gravity at the time of landing can be found by applying

$$x = x_0 + v_{0x}t$$

giving

$$X = l\sin\alpha + v_0\cos\theta_0\left(\frac{v_0\sin\theta_0}{g} + \sqrt{\frac{v_0^2\sin^2\theta_0}{g^2} + \frac{2(h - l\sin\beta)}{g}}\right)$$

This result can be simplified using the trigonometric identity

$$\sin(2\theta_0) = 2\sin\theta_0\cos\theta_0$$

giving

$$X = l\sin\alpha + \frac{v_0^2\sin(2\theta_0)}{2g} + \sqrt{\frac{2(h - l\sin\beta)}{g}v_0^2\cos^2\theta_0 + \left(\frac{v_0^2\sin(2\theta_0)}{2g}\right)^2}$$

The total horizontal distance $\underline{\overline{X}}$ covered by the long jumper in executing the jump is

$$\underline{\overline{X}} = X + l\cos\beta$$

therefore

$$\underline{\overline{X}} = l\sin\alpha + l\cos\beta + \frac{v_0^2\sin(2\theta_0)}{2g} + \sqrt{\frac{2(h - l\sin\beta)}{g}v_0^2\cos^2\theta_0 + \left(\frac{v_0^2\sin(2\theta_0)}{2g}\right)^2}$$
$$\underline{\overline{X}} = l(\sin\alpha + \cos\beta) + \frac{v_0^2\sin(2\theta_0)}{2g} + \sqrt{\frac{2(h - l\sin\beta)}{g}v_0^2\cos^2\theta_0 + \left(\frac{v_0^2\sin(2\theta_0)}{2g}\right)^2} \tag{9.28}$$

9.2.6.4 Computer Analysis of Shot Put Event

A computer program was written to evaluate X as a function of v_0, θ_0, and h. For each of the following speeds of projection 10 ft/s, 20 ft/s, 30 ft/s, 40 ft/s, 50 ft/s, 60 ft/s, 70 ft/s, and 80 ft/s, the horizontal distance X for angles of projection of 0°, 5°, 10°, 15°, 20°, 30°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48°, 49°, and 50° were computed and are displayed in Table 9.14 and Table 9.15. The entries in the table are the values of X.

Angle (°)	Speed of Projection (ft/s)			
	10	20	30	40
0	6.6	13.19	19.79	26.39
5	6.85	14.27	22.29	30.96
10	7.05	15.29	24.85	35.85
15	7.2	16.23	27.35	40.79
20	7.28	17.02	29.65	45.49
30	7.22	18.02	33.1	52.94
34	7.1	18.13	33.88	54.84
35	7.06	18.13	34.02	55.2
36	7.02	18.11	34.12	55.51
37	6.97	18.09	34.2	55.77
38	6.92	18.05	34.25	55.98
39	6.87	18	34.27	56.14
40	6.81	17.94	34.26	56.24
41	6.75	17.86	34.22	56.3
42	6.69	17.77	34.16	56.29
43	6.62	17.67	34.06	56.24
44	6.55	17.56	33.93	56.13
45	6.47	17.43	33.77	55.96
46	6.39	17.29	33.59	55.73
47	6.31	17.13	33.37	55.45
48	6.22	16.96	33.12	55.12
49	6.13	16.78	32.84	54.73
50	6.04	16.58	32.53	54.28

Table 9.14: Horizontal Distance of Shot for Different Launch Speeds and Launch Angles

Question 16: For each speed of projection, circle the largest value of X in the table. This value will indicate the optimum angle of projection for that speed of projection.

Question 17: Plot the optimum angle of projection $\tilde{\theta}$ versus the speed of projection v_0. Discuss the graph. That is, what can you conclude about the variation of the optimum angle of projection $\tilde{\theta}$ with v_0?

Question 18: Discuss the variation of X with respect to θ_0, keeping v_0 fixed, versus the variation of X with respect to v_0 keeping θ_0 fixed. Which is the more significant variation?

Angle (°)	Speed of Projection (ft/s)			
	50	60	70	80
0	32.98	39.58	46.18	52.78
5	40.29	50.33	61.09	72.61
10	48.39	62.56	78.46	96.14
15	56.74	75.35	96.73	120.96
20	64.78	87.7	114.37	144.86
30	77.79	107.81	143.09	183.66
34	81.26	113.26	150.94	194.31
35	81.94	114.35	152.51	196.46
36	82.54	115.32	153.93	198.39
37	83.06	116.17	155.18	200.11
38	83.5	116.9	156.27	201.61
39	83.85	117.51	157.18	202.89
40	84.12	118	157.93	203.94
41	84.31	118.36	158.5	204.75
42	84.41	118.59	158.89	205.34
43	84.42	118.69	159.11	205.7
44	84.34	118.67	159.16	205.82
45	84.17	118.52	159.02	205.71
46	83.92	118.23	158.71	205.37
47	83.58	117.82	158.21	204.78
48	83.15	117.27	157.54	203.97
49	82.62	116.6	156.69	202.91
50	82.01	115.8	155.66	201.63

Table 9.15: Horizontal Distance of Shot for Different Launch Speeds and Launch Angles

Question 19: Based on your analysis of the shot put event, which factors would you emphasize in order to effect the most successful event?

9.2.6.5 Computer Analysis of Long Jump Event

A computer program was written to evaluate $\underline{X}$ as a function of v_0, θ_0, h, l, α, and β. For each of the following speeds of projection 5 ft/s, 10 ft/s, 15 ft/s, 20 ft/s, 25 ft/s, 30 ft/s, 35 ft/s, and 40 ft/s, the horizontal distance $\underline{X}$ for angles of projection of 0°, 5°, 10°, 15°, 20°, 30°, 34°, 35°, 36°, 37°, 38°, 39°, 40°, 41°, 42°, 43°, 44°, 45°, 46°, 47°, 48°, 49°, 50° were computed and are displayed in Table 9.16 and Table 9.17. The entries in the table are the values of $\underline{X}$.

Question 20: For each speed of projection, circle the largest value of $\underline{X}$ in the table. This value will indicate the optimum angle of projection for that speed of projection.

Question 21: Plot the optimum angle of projection $\tilde{\theta}$ versus the speed of projection v_0. Discuss the graph. That is, what can you conclude about the variation of the optimum angle of projection $\tilde{\theta}$ with v_0?

Angle (°)	Speed of Projection (ft/s)			
	5	10	15	20
0	5.41	6.71	8.02	9.33
5	5.47	6.99	8.66	10.5
10	5.53	7.26	9.34	11.8
15	5.57	7.52	10.02	13.14
20	5.6	7.75	10.66	14.43
30	5.62	8.08	11.68	16.52
34	5.6	8.14	11.93	17.08
35	5.59	8.15	11.98	17.18
36	5.59	8.16	12.02	17.28
37	5.58	8.16	12.05	17.37
38	5.57	8.16	12.08	17.44
39	5.56	8.16	12.1	17.49
40	5.55	8.15	12.11	17.54
41	5.54	8.14	12.12	17.57
42	5.53	8.13	12.11	17.58
43	5.52	8.11	12.1	17.59
44	5.5	8.09	12.08	17.57
45	5.49	8.07	12.06	17.55
46	5.47	8.04	12.02	17.51
47	5.46	8.01	11.98	17.45
48	5.44	7.98	11.93	17.39
49	5.42	7.94	11.88	17.3
50	5.4	7.9	11.81	17.21

Table 9.16: Horizontal Distance of the Long Jump for Different Launch Speeds and Launch Angles

Question 22: Discuss the variation of $\overline{X}$ with respect to θ_0, keeping v_0 fixed, versus the variation of $\overline{X}$ with respect to v_0 keeping θ_0 fixed. Which is the more significant variation?

Question 23: Based on your analysis of the long jump event, which factors would you emphasize in order to effect the most successful event?