

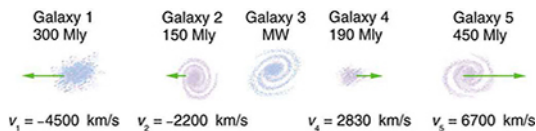
- 39.** Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.
- 40.** An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.
- 41.** An owl is carrying a mouse to the chicks in its nest. Its position at that time is 4.00 m west and 12.0 m above the center of the 30.0 cm diameter nest. The owl is flying east at 3.50 m/s at an angle  $30.0^\circ$  below the horizontal when it accidentally drops the mouse. Is the owl lucky enough to have the mouse hit the nest? To answer this question, calculate the horizontal position of the mouse when it has fallen 12.0 m.
- 42.** Suppose a soccer player kicks the ball from a distance 30 m toward the goal. Find the initial speed of the ball if it just passes over the goal, 2.4 m above the ground, given the initial direction to be  $40^\circ$  above the horizontal.
- 43.** Can a goalkeeper at her/his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.
- 44.** The free throw line in basketball is 4.57 m (15 ft) from the basket, which is 3.05 m (10 ft) above the floor. A player standing on the free throw line throws the ball with an initial speed of 8.15 m/s, releasing it at a height of 2.44 m (8 ft) above the floor. At what angle above the horizontal must the ball be thrown to exactly hit the basket? Note that most players will use a large initial angle rather than a flat shot because it allows for a larger margin of error. Explicitly show how you follow the steps involved in solving projectile motion problems.
- 45.** In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of  $38.0^\circ$  above the horizontal? (Although the maximum distance for a projectile on level ground is achieved at  $45^\circ$  when air resistance is neglected, the actual angle to achieve maximum range is smaller; thus,  $38^\circ$  will give a longer range than  $45^\circ$  in the shot put.)
- 46.** A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?
- 47.** A football player punts the ball at a  $45.0^\circ$  angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?
- 48.** Prove that the trajectory of a projectile is parabolic, having the form  $y = ax + bx^2$ . To obtain this expression, solve the equation  $x = v_{0x}t$  for  $t$  and substitute it into the expression for  $y = v_{0y}t - (1/2)gt^2$  (These equations describe the  $x$  and  $y$  positions of a projectile that starts at the origin.) You should obtain an equation of the form  $y = ax + bx^2$  where  $a$  and  $b$  are constants.
- 49.** Derive  $R = \frac{v_0^2 \sin 2\theta_0}{g}$  for the range of a projectile on level ground by finding the time  $t$  at which  $y$  becomes zero and substituting this value of  $t$  into the expression for  $x - x_0$ , noting that  $R = x - x_0$ .
- 50.** Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

- 51. Construct Your Own Problem** Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

### 3.5 Addition of Velocities

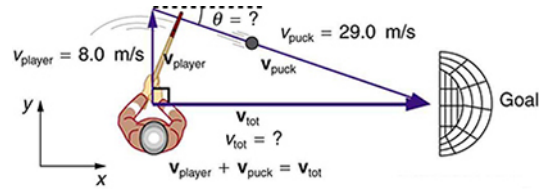
- 52.** Bryan Allen pedaled a human-powered aircraft across the English Channel from the cliffs of Dover to Cap Gris-Nez on June 12, 1979. (a) He flew for 169 min at an average velocity of 3.53 m/s in a direction  $45^\circ$  south of east. What was his total displacement? (b) Allen encountered a headwind averaging 2.00 m/s almost precisely in the opposite direction of his motion relative to the Earth. What was his average velocity relative to the air? (c) What was his total displacement relative to the air mass?
- 53.** A seagull flies at a velocity of 9.00 m/s straight into the wind. (a) If it takes the bird 20.0 min to travel 6.00 km relative to the Earth, what is the velocity of the wind? (b) If the bird turns around and flies with the wind, how long will he take to return 6.00 km? (c) Discuss how the wind affects the total round-trip time compared to what it would be with no wind.
- 54.** Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?
- 55.** Verify that the coin dropped by the airline passenger in the [Example 3.8](#) travels 144 m horizontally while falling 1.50 m in the frame of reference of the Earth.
- 56.** A football quarterback is moving straight backward at a speed of 2.00 m/s when he throws a pass to a player 18.0 m straight downfield. The ball is thrown at an angle of  $25.0^\circ$  relative to the ground and is caught at the same height as it is released. What is the initial velocity of the ball *relative to the quarterback*?
- 57.** A ship sets sail from Rotterdam, The Netherlands, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction  $40.0^\circ$  north of east. What is the velocity of the ship relative to the Earth?
- 58.** (a) A jet airplane flying from Darwin, Australia, has an air speed of 260 m/s in a direction  $5.0^\circ$  south of west. It is in the jet stream, which is blowing at 35.0 m/s in a direction  $15^\circ$  south of east. What is the velocity of the airplane relative to the Earth? (b) Discuss whether your answers are consistent with your expectations for the effect of the wind on the plane's path.
- 59.** (a) In what direction would the ship in [Exercise 3.57](#) have to travel in order to have a velocity straight north relative to the Earth, assuming its speed relative to the water remains 7.00 m/s? (b) What would its speed be relative to the Earth?
- 60.** (a) Another airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction  $20^\circ$  south of east (as in [Exercise 3.58](#)). Its direction of motion relative to the Earth is  $45.0^\circ$  south of west, while its direction of travel relative to the air is  $5.00^\circ$  south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?
- 61.** A sandal is dropped from the top of a 15.0-m-high mast on a ship moving at 1.75 m/s due south. Calculate the velocity of the sandal when it hits the deck of the ship: (a) relative to the ship and (b) relative to a stationary observer on shore. (c) Discuss how the answers give a consistent result for the position at which the sandal hits the deck.
- 62.** The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction  $30.0^\circ$  east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of  $50.0^\circ$  south of west relative to the Earth. What is the velocity of the wind relative to the water?

- 63.** The great astronomer Edwin Hubble discovered that all distant galaxies are receding from our Milky Way Galaxy with velocities proportional to their distances. It appears to an observer on the Earth that we are at the center of an expanding universe. [Figure 3.59](#) illustrates this for five galaxies lying along a straight line, with the Milky Way Galaxy at the center. Using the data from the figure, calculate the velocities: (a) relative to galaxy 2 and (b) relative to galaxy 5. The results mean that observers on all galaxies will see themselves at the center of the expanding universe, and they would likely be aware of relative velocities, concluding that it is not possible to locate the center of expansion with the given information.



**FIGURE 3.59** Five galaxies on a straight line, showing their distances and velocities relative to the Milky Way (MW) Galaxy. The distances are in millions of light years (Mly), where a light year is the distance light travels in one year. The velocities are nearly proportional to the distances. The sizes of the galaxies are greatly exaggerated; an average galaxy is about 0.1 Mly across.

- 64.** (a) Use the distance and velocity data in [Figure 3.59](#) to find the rate of expansion as a function of distance.  
(b) If you extrapolate back in time, how long ago would all of the galaxies have been at approximately the same position? The two parts of this problem give you some idea of how the Hubble constant for universal expansion and the time back to the Big Bang are determined, respectively.
- 65.** An athlete crosses a 25-m-wide river by swimming perpendicular to the water current at a speed of 0.5 m/s relative to the water. They reach the opposite side at a distance 40 m downstream from the starting point. How fast is the water in the river flowing with respect to the ground? What is the speed of the swimmer with respect to a friend at rest on the ground?
- 66.** A ship sailing in the Gulf Stream is heading  $25.0^\circ$  west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is 4.80 m/s  $5.00^\circ$  west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)
- 67.** An ice hockey player is moving at 8.00 m/s when they hit the puck toward the goal. The speed of the puck relative to the player is 29.0 m/s. The line between the center of the goal and the player makes a  $90.0^\circ$  angle relative to their path as shown in [Figure 3.60](#). What angle must the puck's velocity make relative to the player (in their frame of reference) to hit the center of the goal?



**FIGURE 3.60** An ice hockey player moving across the rink must shoot backward to give the puck a velocity toward the goal.

- 68. Unreasonable Results** Suppose you wish to shoot supplies straight up to astronauts in an orbit 36,000 km above the surface of the Earth. (a) At what velocity must the supplies be launched? (b) What is unreasonable about this velocity? (c) Is there a problem with the relative velocity between the supplies and the astronauts when the supplies reach their maximum height? (d) Is the premise unreasonable or is the available equation inapplicable? Explain your answer.
- 69. Unreasonable Results** A commercial airplane has an air speed of 280 m/s due east and flies with a strong tailwind. It travels 3000 km in a direction  $5^\circ$  south of east in 1.50 h. (a) What was the velocity of the plane relative to the ground? (b) Calculate the magnitude and direction of the tailwind's velocity. (c) What is unreasonable about both of these velocities? (d) Which premise is unreasonable?
- 70. Construct Your Own Problem** Consider an airplane headed for a runway in a cross wind. Construct a problem in which you calculate the angle the airplane must fly relative to the air mass in order to have a velocity parallel to the runway. Among the things to consider are the direction of the runway, the wind speed and direction (its velocity) and the speed of the plane relative to the air mass. Also calculate the speed of the airplane relative to the ground. Discuss any last minute maneuvers the pilot might have to perform in order for the plane to land with its wheels pointing straight down the runway.

- 71. Critical Thinking** Two identical spring-loaded launchers are fixed to the end of a table. Launcher A is inclined  $40^\circ$  above the horizontal and launcher B is inclined  $50^\circ$  above the horizontal. (a) Which of these launchers will fire the ball the greatest vertical height above the launcher before it crashes into the ground? (b) Based on experimental data, an equation that fits the data is suggested for the vertical height above the launcher,  $y$ , which may not be correct:  $y = KV_0^2 \sin \theta$ , where  $K$  is a constant with appropriate units,  $V_0$  is the initial velocity of the launcher, and  $\theta$  is the angle relative to the  $x$ -axis of the launcher. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Graph the magnitude of the velocity vs. vertical height for both launcher A and B.

## CHAPTER 4

# Dynamics: Force and Newton's Laws of Motion



**FIGURE 4.1** Newton's laws of motion describe the motion of the dolphin's path. (credit: Jin Jang)

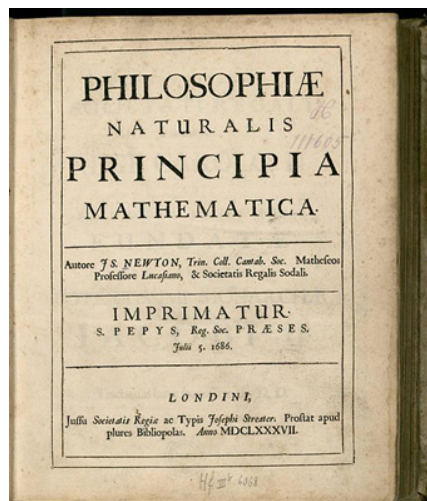
### CHAPTER OUTLINE

- 4.1 Development of Force Concept**
- 4.2 Newton's First Law of Motion: Inertia**
- 4.3 Newton's Second Law of Motion: Concept of a System**
- 4.4 Newton's Third Law of Motion: Symmetry in Forces**
- 4.5 Normal, Tension, and Other Examples of Forces**
- 4.6 Problem-Solving Strategies**
- 4.7 Further Applications of Newton's Laws of Motion**
- 4.8 Extended Topic: The Four Basic Forces—An Introduction**

**INTRODUCTION TO DYNAMICS: NEWTON'S LAWS OF MOTION** Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Isaac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the

many great thinkers who contributed to this change were Newton and Galileo.



**FIGURE 4.2** Isaac Newton’s monumental work, *Philosophiæ Naturalis Principia Mathematica*, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l’Université de Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than “logical” argument. Galileo’s use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton’s first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton’s laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about  $10^{-9}$  m in diameter). These constraints define the realm of classical mechanics, as discussed in [Introduction to the Nature of Science and Physics](#). At the beginning of the 20<sup>th</sup> century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in [Special Relativity](#), are in the realm of classical physics.

### Making Connections: Past and Present Philosophy

*The importance of observation* and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

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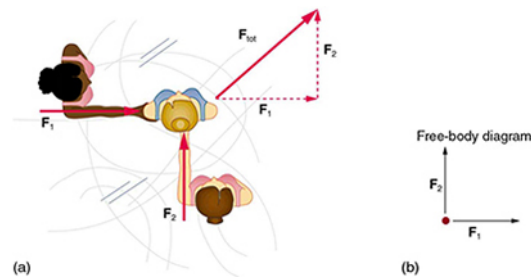
## 4.1 Development of Force Concept

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand the definition of force.

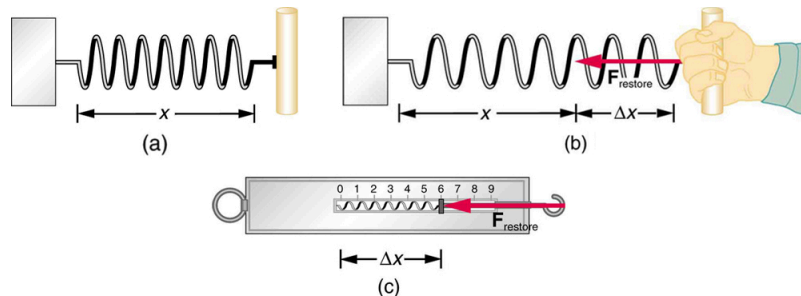
**Dynamics** is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in [Figure 4.3](#), we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors, as illustrated in [Figure 4.3\(a\)](#) for two ice skaters. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or by trigonometric methods. These ideas were developed in [Two-Dimensional Kinematics](#).



**FIGURE 4.3** Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

[Figure 4.3\(b\)](#) is our first example of a **free-body diagram**, which is a technique used to illustrate all the **external forces** acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting *on* the body from the outside (external forces) are shown. (These forces are the only ones shown, because only external forces acting on the body affect its motion. We can ignore any internal forces within the body.) Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in [Figure 4.4](#), and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. (One that we will encounter in [Magnetism](#) is the magnetic force between two wires carrying electric current.) Some alternative definitions of force will be given later in this chapter.



**FIGURE 4.4** The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length  $x$  when undistorted. (b) When stretched a distance  $\Delta x$ , the spring exerts a restoring force,  $\mathbf{F}_{\text{restore}}$ , which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force  $\mathbf{F}_{\text{restore}}$  is exerted on whatever is attached to the hook. Here  $\mathbf{F}_{\text{restore}}$  has a magnitude of 6 units in the force standard being employed.

### Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

## 4.2 Newton's First Law of Motion: Inertia

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

### Newton's First Law of Motion

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton's first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, “What is the cause?” Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as “Why does a tiger have stripes?” would have been answered in Aristotelian fashion, “That is the nature of the beast.” True perhaps, but not a useful insight.

## Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of “stuff” (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

### CHECK YOUR UNDERSTANDING

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

#### Solution

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

## 4.3 Newton's Second Law of Motion: Concept of a System

### LEARNING OBJECTIVES

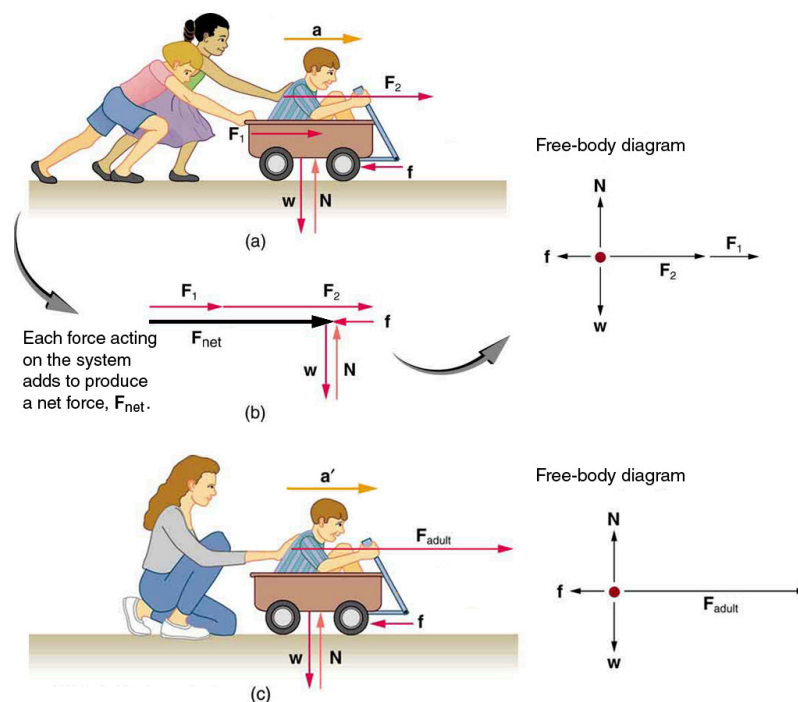
By the end of this section, you will be able to:

- Define net force, external force, and system.
- Understand Newton's second law of motion.
- Apply Newton's second law to determine the weight of an object.

**Newton's second law of motion** is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in [Figure 4.5\(a\)](#) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at [Figure 4.5\(a\)](#), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external.* Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.



**FIGURE 4.5** Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight  $w$  of the system and the support of the ground  $N$  are also shown for completeness and are assumed to cancel. The vector  $f$  represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force,  $F_{net}$ . The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration ( $a' > a$ ) when an adult pushes the child.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in [Figure 4.5](#). In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight  $w$  and the support of the ground  $N$ , and the horizontal force  $f$  represents the force of friction. These will be discussed in more detail in later sections. For now, we will define **friction** as a force that opposes the motion past each other of objects that are touching. [Figure 4.5](#)(b) shows how vectors representing the external forces add together to produce a net force,  $F_{net}$ .

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

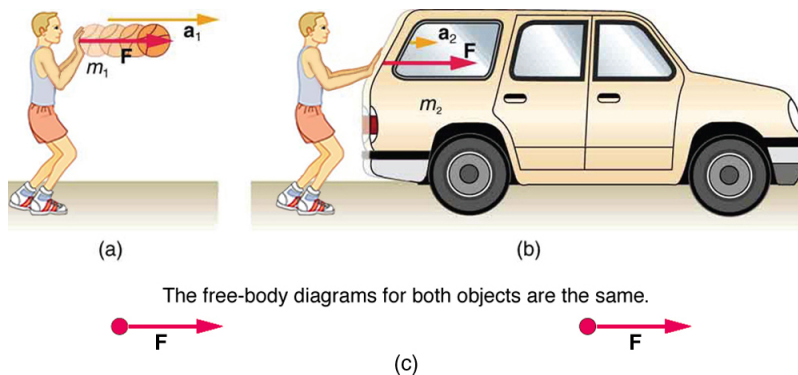
$$\mathbf{a} \propto \mathbf{F}_{net}, \quad 4.1$$

where the symbol  $\propto$  means “proportional to,” and  $F_{net}$  is the **net external force**. (The net external force is the vector sum of all external forces and can be determined graphically, using the head-to-tail method, or analytically, using components. The techniques are the same as for the addition of other vectors, and are covered in [Two-Dimensional Kinematics](#).) This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification.

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in [Figure 4.6](#), the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{1}{m} \quad 4.2$$

where  $m$  is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.



**FIGURE 4.6** The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

### Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \quad 4.3$$

This is often written in the more familiar form

$$\mathbf{F}_{\text{net}} = m\mathbf{a}. \quad 4.4$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\text{net}} = ma. \quad 4.5$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

### Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$  is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton** (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of  $1\text{ m/s}^2$ . That is, since  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ ,

$$1\text{ N} = 1\text{ kg} \cdot \text{m/s}^2. \quad 4.6$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where  $1\text{ N} = 0.225\text{ lb}$ .

### Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the

gravitational force, commonly called its **weight  $w$** . Weight can be denoted as a vector  $\mathbf{w}$  because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as  $w$ . Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration  $g$ . Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass  $m$  falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude  $w$ . Newton's second law states that the magnitude of the net external force on an object is  $F_{\text{net}} = ma$ .

Since the object experiences only the downward force of gravity,  $F_{\text{net}} = w$ . We know that the acceleration of an object due to gravity is  $g$ , or  $a = g$ . Substituting these into Newton's second law gives

### Weight

This is the equation for *weight*—the gravitational force on a mass  $m$ :

$$w = mg. \quad 4.7$$

Since  $g = 9.80 \text{ m/s}^2$  on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = mg = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}. \quad 4.8$$

Recall that  $g$  can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity  $g$  varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only  $1.625 \text{ m/s}^2$ . A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

### Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object ( $m$ ) multiplied by the acceleration due to gravity ( $g$ ). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However,

because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is  $1.625 \text{ m/s}^2$  (which is much less than the acceleration due to gravity on Earth,  $9.80 \text{ m/s}^2$ ). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you “weigh” much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are “losing weight,” they really mean that they are losing “mass” (which in turn causes them to weigh less).

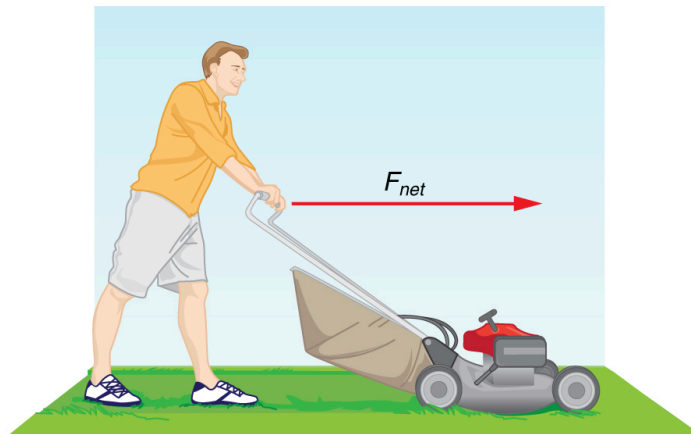
### Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same “mass” on Earth as on the Moon?

### EXAMPLE 4.1

#### What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is  $51 \text{ N}$  (about  $11 \text{ lb}$ ) parallel to the ground. The mass of the mower is  $24 \text{ kg}$ . What is its acceleration?



**FIGURE 4.7** The net force on a lawn mower is  $51 \text{ N}$  to the right. At what rate does the lawn mower accelerate to the right?

#### Strategy

Since  $\mathbf{F}_{\text{net}}$  and  $m$  are given, the acceleration can be calculated directly from Newton's second law as stated in  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ .

#### Solution

The magnitude of the acceleration  $a$  is  $a = \frac{F_{\text{net}}}{m}$ . Entering known values gives

$$a = \frac{51 \text{ N}}{24 \text{ kg}} \quad 4.9$$

Substituting the units  $\text{kg} \cdot \text{m/s}^2$  for  $\text{N}$  yields

$$a = \frac{51 \text{ kg} \cdot \text{m/s}^2}{24 \text{ kg}} = 2.1 \text{ m/s}^2. \quad 4.10$$

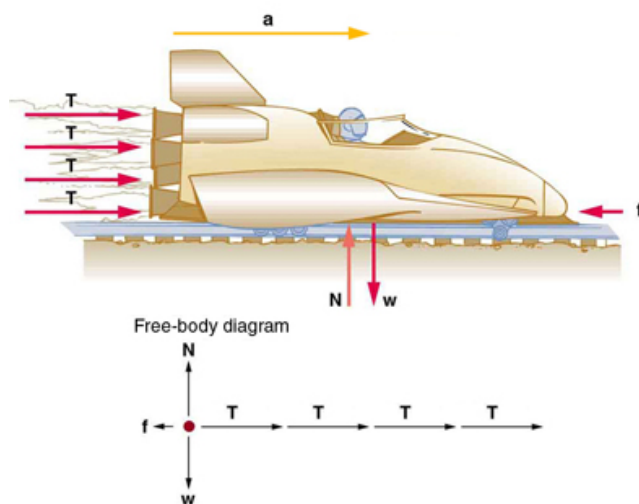
### Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

## EXAMPLE 4.2

### What Rocket Thrust Accelerates This Sled?

Prior to space flights carrying astronauts, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust  $\mathbf{T}$ , for the four-rocket propulsion system shown in [Figure 4.8](#). The sled's initial acceleration is  $49 \text{ m/s}^2$ , the mass of the system is  $2100 \text{ kg}$ , and the force of friction opposing the motion is known to be  $650 \text{ N}$ .



**FIGURE 4.8** A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust  $\mathbf{T}$ . As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force  $\mathbf{N}$  on the system that is equal in magnitude and opposite in direction to its weight,  $\mathbf{w}$ . The system here is the sled, its rockets, and rider, so none of the forces *between* these objects are considered. The arrow representing friction ( $\mathbf{f}$ ) is drawn larger than scale.

### Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

### Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{\text{net}} = ma, \quad 4.11$$

where  $F_{\text{net}}$  is the net force along the horizontal direction. We can see from [Figure 4.8](#) that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{\text{net}} = 4T - f. \quad 4.12$$

Substituting this into Newton's second law gives

$$F_{\text{net}} = ma = 4T - f. \quad 4.13$$

Using a little algebra, we solve for the total thrust  $4T$ :

$$4T = ma + f. \quad 4.14$$

Substituting known values yields

$$4T = ma + f = (2100 \text{ kg})(49 \text{ m/s}^2) + 650 \text{ N}. \quad 4.15$$

So the total thrust is

$$4T = 1.0 \times 10^5 \text{ N}, \quad 4.16$$

and the individual thrusts are

$$T = \frac{1.0 \times 10^5 \text{ N}}{4} = 2.6 \times 10^4 \text{ N}. \quad 4.17$$

### Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45  $g$ 's. (Recall that  $g$ , the acceleration due to gravity, is  $9.80 \text{ m/s}^2$ . When we say that an acceleration is 45  $g$ 's, it is  $45 \times 9.80 \text{ m/s}^2$ , which is approximately  $440 \text{ m/s}^2$ .) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

## 4.4 Newton's Third Law of Motion: Symmetry in Forces

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

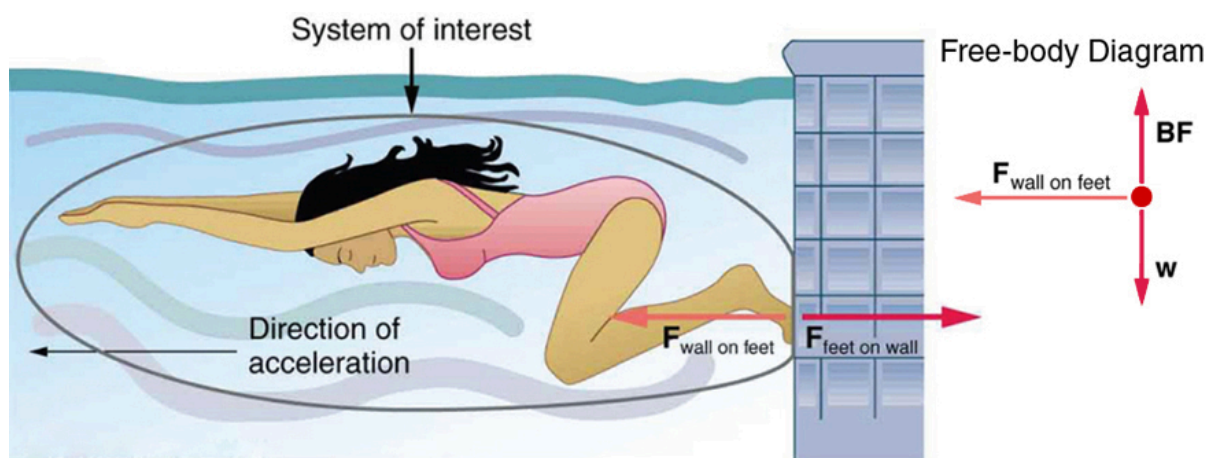
Baseball relief pitcher Mariano Rivera was so highly regarded that during his retirement year, opposing teams conducted farewell presentations when he played at their stadiums. The Minnesota Twins offered a unique gift: A chair made of broken bats. Any pitch can break a bat, but with Rivera's signature pitch—known as a cutter—the ball and the bat frequently came together at a point that shattered the hardwood. Typically, we think of a baseball or softball hitter exerting a force on the incoming ball, and baseball analysts focus on the resulting "exit velocity" as a key statistic. But the force of the ball can do its own damage. This is exactly what happens whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or pushing off the floor during a jump, confirm this. It is precisely stated in **Newton's third law of motion**.

### Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as “action-reaction,” where the force exerted is the action and the force experienced as a consequence is the reaction. Newton’s third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton’s third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in [Figure 4.9](#). She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then  $\mathbf{F}_{\text{wall on feet}}$  is an external force on this system and affects its motion. The swimmer moves in the direction of  $\mathbf{F}_{\text{wall on feet}}$ . In contrast, the force  $\mathbf{F}_{\text{feet on wall}}$  acts on the wall and not on our system of interest. Thus  $\mathbf{F}_{\text{feet on wall}}$  does not directly affect the motion of the system and does not cancel  $\mathbf{F}_{\text{wall on feet}}$ . Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.



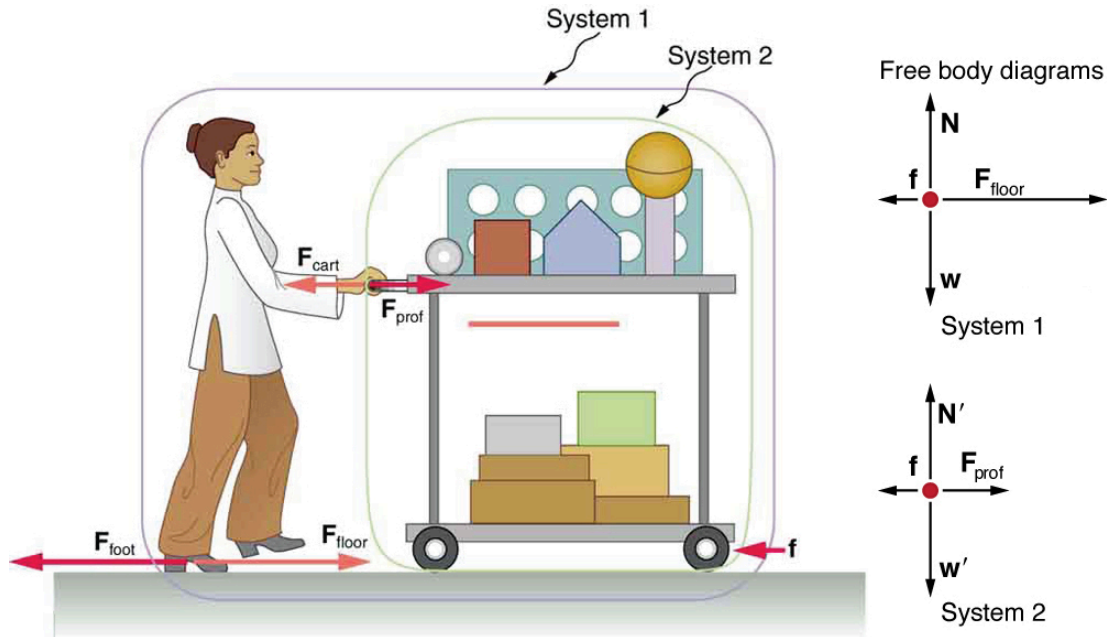
**FIGURE 4.9** When the swimmer exerts a force  $\mathbf{F}_{\text{feet on wall}}$  on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to  $\mathbf{F}_{\text{feet on wall}}$ . This opposition occurs because, in accordance with Newton’s third law of motion, the wall exerts a force  $\mathbf{F}_{\text{wall on feet}}$  on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that  $\mathbf{F}_{\text{feet on wall}}$  does not act on this system (the swimmer) and, thus, does not cancel  $\mathbf{F}_{\text{wall on feet}}$ . Thus the free-body diagram shows only  $\mathbf{F}_{\text{wall on feet}}$ ,  $\mathbf{w}$ , the gravitational force, and  $\mathbf{BF}$ , the buoyant force of the water supporting the swimmer’s weight. The vertical forces  $\mathbf{w}$  and  $\mathbf{BF}$  cancel since there is no vertical motion.

Other examples of Newton’s third law are easy to find. As a professor walks in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. Boxers and other martial arts fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent’s body.

### EXAMPLE 4.3

#### Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in [Figure 4.10](#). Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.



**FIGURE 4.10** A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for  $f$ , since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for this example, since it asks for the acceleration of the entire group of objects. Only  $F_{\text{floor}}$  and  $f$  are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for [Example 4.4](#) so that  $F_{\text{prof}}$  will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

#### Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in [Figure 4.10](#). The professor pushes backward with a force  $F_{\text{foot}}$  of 150 N. According to Newton's third law, the floor exerts a forward reaction force  $F_{\text{floor}}$  of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted,  $f$  opposes the motion and is thus in the opposite direction of  $F_{\text{floor}}$ . Note that we do not include the forces  $F_{\text{prof}}$  or  $F_{\text{cart}}$  because these are internal forces, and we do not include  $F_{\text{foot}}$  because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

#### Solution

Newton's second law is given by

$$a = \frac{F_{\text{net}}}{m}. \quad 4.18$$

The net external force on System 1 is deduced from [Figure 4.10](#) and the discussion above to be

$$F_{\text{net}} = F_{\text{floor}} - f = 150 \text{ N} - 24.0 \text{ N} = 126 \text{ N}. \quad 4.19$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0) \text{ kg} = 84 \text{ kg}. \quad 4.20$$

These values of  $F_{\text{net}}$  and  $m$  produce an acceleration of

$$a = \frac{F_{\text{net}}}{m},$$

$$a = \frac{126 \text{ N}}{84 \text{ kg}} = 1.5 \text{ m/s}^2. \quad 4.21$$

### Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

## EXAMPLE 4.4

### Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in [Figure 4.10](#) using data from the previous example if needed.

#### Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in [Figure 4.10](#)), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart,  $\mathbf{F}_{\text{prof}}$ , is an external force acting on System 2.  $\mathbf{F}_{\text{prof}}$  was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

#### Solution

Newton's second law can be used to find  $\mathbf{F}_{\text{prof}}$ . Starting with

$$a = \frac{F_{\text{net}}}{m} \quad 4.22$$

and noting that the magnitude of the net external force on System 2 is

$$F_{\text{net}} = F_{\text{prof}} - f, \quad 4.23$$

we solve for  $F_{\text{prof}}$ , the desired quantity:

$$F_{\text{prof}} = F_{\text{net}} + f. \quad 4.24$$

The value of  $f$  is given, so we must calculate net  $F_{\text{net}}$ . That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{\text{net}} = ma, \quad 4.25$$

where the mass of System 2 is 19.0 kg ( $m = 12.0 \text{ kg} + 7.0 \text{ kg}$ ) and its acceleration was found to be  $a = 1.5 \text{ m/s}^2$  in the previous example. Thus,

$$F_{\text{net}} = ma, \quad 4.26$$

$$F_{\text{net}} = (19.0 \text{ kg})(1.5 \text{ m/s}^2) = 29 \text{ N}. \quad 4.27$$

Now we can find the desired force:

$$F_{\text{prof}} = F_{\text{net}} + f, \quad 4.28$$

$$F_{\text{prof}} = 29 \text{ N} + 24.0 \text{ N} = 53 \text{ N}. \quad 4.29$$

### Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

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## PHET EXPLORATIONS

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### Gravity Force Lab

Visualize the gravitational force that two objects exert on each other. Change properties of the objects in order to see how it changes the gravity force.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/4-4-newtons-third-law-of-motion-symmetry-in-forces\)](https://openstax.org/books/college-physics-2e/pages/4-4-newtons-third-law-of-motion-symmetry-in-forces)



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## 4.5 Normal, Tension, and Other Examples of Forces

### LEARNING OBJECTIVES

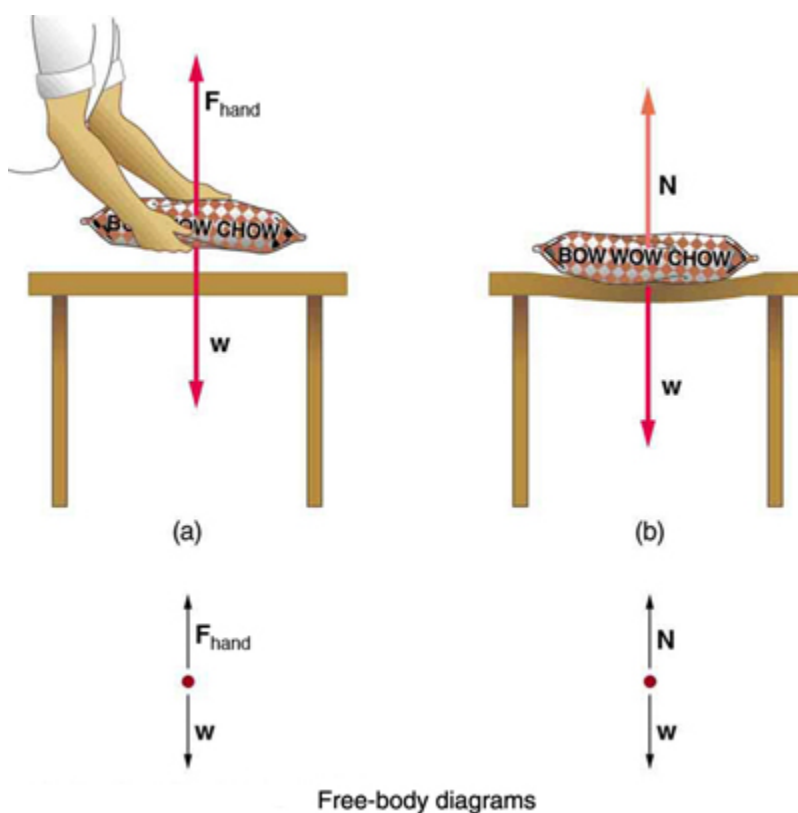
By the end of this section, you will be able to:

- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.
- Use trigonometric identities to resolve weight into components.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

### Normal Force

**Weight** (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in [Figure 4.11\(a\)](#). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in [Figure 4.11\(b\)](#)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.



**FIGURE 4.11** (a) The person holding the bag of dog food must supply an upward force  $F_{\text{hand}}$  equal in magnitude and opposite in direction to the weight of the food  $w$ . (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force  $N$  equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol  $N$ . (This is not the unit for force  $N$ .) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

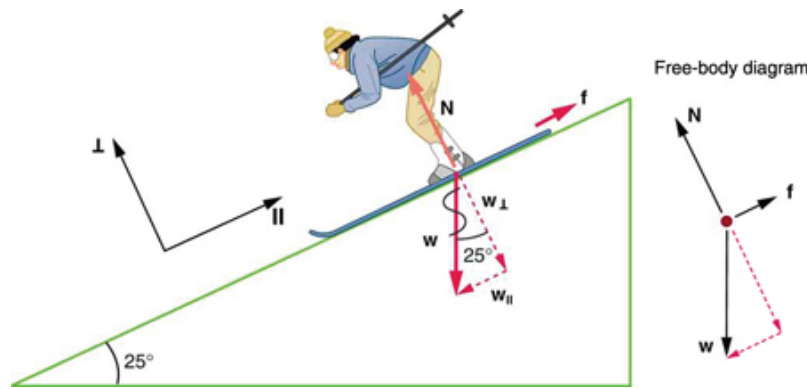
### Common Misconception: Normal Force ( $N$ ) vs. Newton ( $N$ )

In this section we have introduced the quantity normal force, which is represented by the variable  $N$ . This should not be confused with the symbol for the newton, which is also represented by the letter  $N$ . These symbols are particularly important to distinguish because the units of a normal force ( $N$ ) happen to be newtons ( $N$ ). For example, the normal force  $N$  that the floor exerts on a chair might be  $N = 100 \text{ N}$ . One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work ( $W$ ) and the unit watts ( $W$ ).

## EXAMPLE 4.5

### Weight on an Incline, a Two-Dimensional Problem

Consider the skier on a slope shown in [Figure 4.12](#). Her mass including equipment is  $60.0 \text{ kg}$ . (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is known to be  $45.0 \text{ N}$ ?



**FIGURE 4.12** Since motion and friction are parallel to the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).  $\mathbf{N}$  is perpendicular to the slope and  $\mathbf{f}$  is parallel to the slope, but  $\mathbf{w}$  has components along both axes, namely  $w_{\perp}$  and  $w_{\parallel}$ .  $\mathbf{N}$  is equal in magnitude to  $w_{\perp}$ , so that there is no motion perpendicular to the slope, but  $f$  is less than  $w_{\parallel}$ , so that there is a downslope acceleration (along the parallel axis).

### Strategy

This is a two-dimensional problem, since the forces on the skier (the system of interest) are not parallel. The approach we have used in two-dimensional kinematics also works very well here. Choose a convenient coordinate system and project the vectors onto its axes, creating *two* connected *one*-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Remember that motions along mutually perpendicular axes are independent.) We use the symbols  $\perp$  and  $\parallel$  to represent perpendicular and parallel, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and because friction is always parallel to the surface between two objects. The only external forces acting on the system are the skier's weight, friction, and the support of the slope, respectively labeled  $\mathbf{w}$ ,  $\mathbf{f}$ , and  $\mathbf{N}$  in Figure 4.12.  $\mathbf{N}$  is always perpendicular to the slope, and  $\mathbf{f}$  is parallel to it. But  $\mathbf{w}$  is not in the direction of either axis, and so the first step we take is to project it into components along the chosen axes, defining  $w_{\parallel}$  to be the component of weight parallel to the slope and  $w_{\perp}$  the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

### Solution

The magnitude of the component of the weight parallel to the slope is  $w_{\parallel} = w \sin(25^\circ) = mg \sin(25^\circ)$ , and the magnitude of the component of the weight perpendicular to the slope is  $w_{\perp} = w \cos(25^\circ) = mg \cos(25^\circ)$ .

(a) Neglecting friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the amount of the skier's weight parallel to the slope  $w_{\parallel}$  and friction  $f$ . Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} \quad 4.30$$

where  $F_{\text{net}\parallel} = w_{\parallel} = mg \sin(25^\circ)$ , assuming no friction for this part, so that

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{mg \sin(25^\circ)}{m} = g \sin(25^\circ) \quad 4.31$$

$$(9.80 \text{ m/s}^2)(0.4226) = 4.14 \text{ m/s}^2 \quad 4.32$$

is the acceleration.

(b) Including friction. We now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$F_{\text{net}\parallel} = w_{\parallel} - f, \quad 4.33$$

and substituting this into Newton's second law,  $a_{\parallel} = \frac{F_{\text{net}\parallel}}{m}$ , gives

$$a_{\parallel} = \frac{F_{\text{net}\parallel}}{m} = \frac{w_{\parallel} - f}{m} = \frac{mg \sin(25^\circ) - f}{m}. \quad 4.34$$

We substitute known values to obtain

$$a_{\parallel} = \frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(0.4226) - 45.0 \text{ N}}{60.0 \text{ kg}}, \quad 4.35$$

which yields

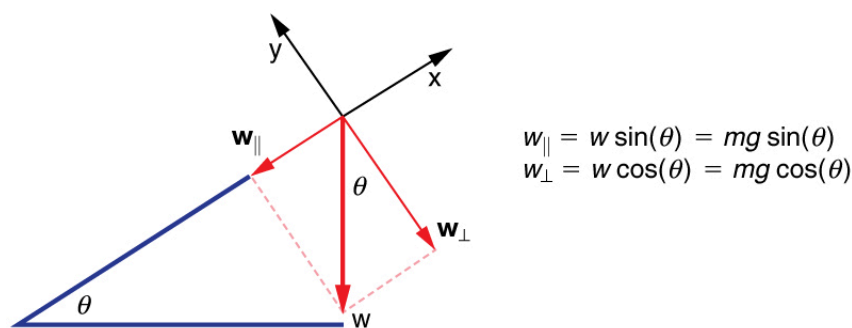
$$a_{\parallel} = 3.39 \text{ m/s}^2, \quad 4.36$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

### Discussion

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. In fact, it is a general result that if friction on an incline is negligible, then the acceleration down the incline is  $a = g \sin\theta$ , *regardless of mass*. This is related to the previously discussed fact that all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

### Resolving Weight into Components



**FIGURE 4.13** An object rests on an incline that makes an angle  $\theta$  with the horizontal.

When an object rests on an incline that makes an angle  $\theta$  with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane,  $w_{\perp}$ , and a force acting parallel to the plane,  $w_{\parallel}$ . The perpendicular force of weight,  $w_{\perp}$ , is typically equal in magnitude and opposite in direction to the normal force,  $\mathbf{N}$ . The force acting parallel to the plane,  $w_{\parallel}$ , causes the object to accelerate down the incline. The force of friction,  $\mathbf{f}$ , opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle  $\theta$  to the horizontal, then the magnitudes of the weight components are

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta) \quad 4.37$$

and

$$w_{\perp} = w \cos(\theta) = mg \cos(\theta). \quad 4.38$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle  $\theta$  of the incline is the same as the angle formed between  $\mathbf{w}$  and  $\mathbf{w}_{\perp}$ . Knowing this property, you can use trigonometry to determine the magnitude of the weight components:

$$\cos(\theta) = \frac{w_{\perp}}{w} \quad 4.39$$

$$w_{\perp} = w \cos(\theta) = mg \cos(\theta)$$

$$\sin(\theta) = \frac{w_{\parallel}}{w} \quad 4.40$$

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta)$$

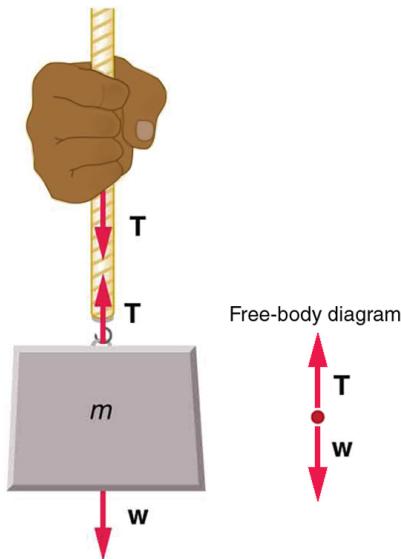
### Take-Home Experiment: Force Parallel

To investigate how a force parallel to an inclined plane changes, find a rubber band, some objects to hang from the end of the rubber band, and a board you can position at different angles. How much does the rubber band stretch when you hang the object from the end of the board? Now place the board at an angle so that the object slides off when placed on the board. How much does the rubber band extend if it is lined up parallel to the board and used to hold the object stationary on the board? Try two more angles. What does this show?

### Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: “You can’t push a rope.” The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in [Figure 4.14](#).



**FIGURE 4.14** When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force  $\mathbf{T}$ , that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton’s third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton’s second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus  $\mathbf{F}_{\text{net}} = 0$ . The only external forces acting on the mass are its weight  $\mathbf{w}$  and the tension  $\mathbf{T}$  supplied by the rope. Thus,

$$F_{\text{net}} = T - w = 0, \quad 4.41$$

where  $T$  and  $w$  are the magnitudes of the tension and weight and their signs indicate direction, with up being

positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

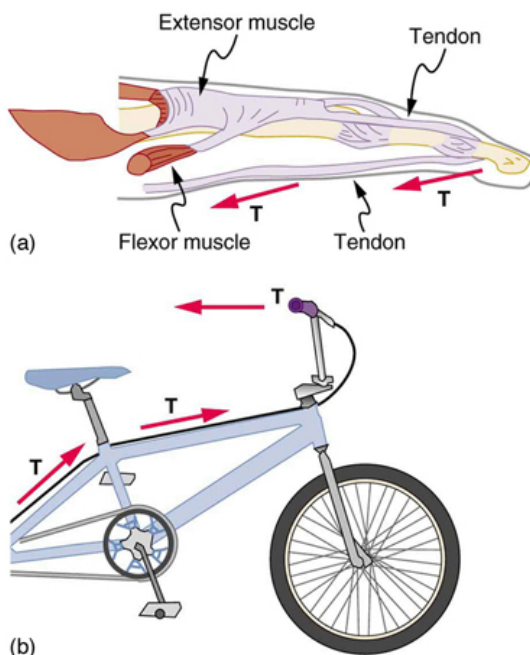
$$T = w = mg. \quad 4.42$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T = mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}. \quad 4.43$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in [Figure 4.15](#) (a) and (b).

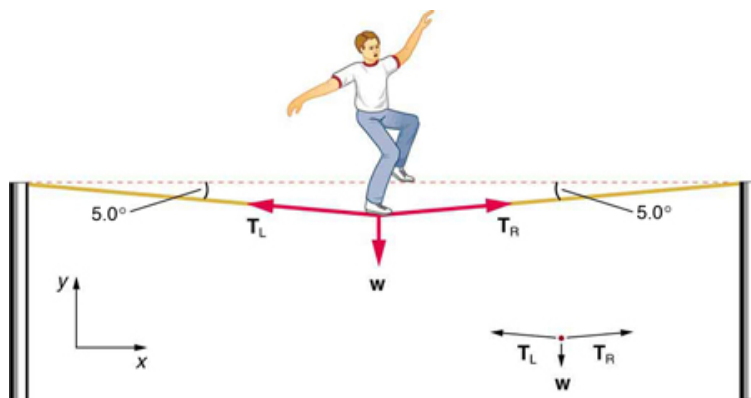


**FIGURE 4.15** (a) Tendons in the finger carry force  $\mathbf{T}$  from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension  $\mathbf{T}$  from the handlebars to the brake mechanism. Again, the direction but not the magnitude of  $\mathbf{T}$  is changed.

### EXAMPLE 4.6

#### What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in [Figure 4.16](#).



**FIGURE 4.16** The weight of a tightrope walker causes a wire to sag by 5.0 degrees. The system of interest here is the point in the wire at which the tightrope walker is standing.

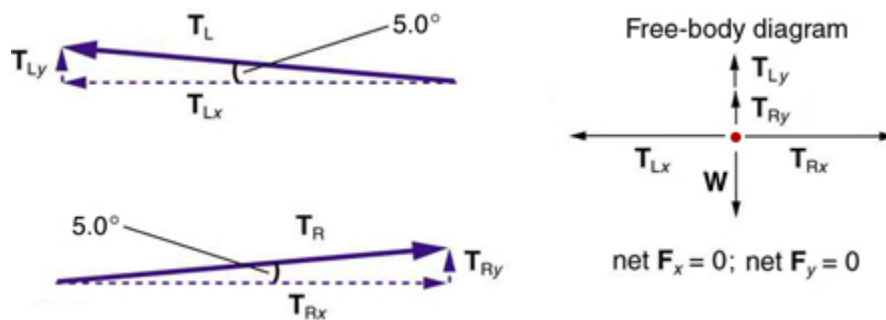
### Strategy

As you can see in the figure, the wire is not perfectly horizontal (it cannot be!), but is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows having the same directions as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight  $\mathbf{w}$  and the two tensions  $\mathbf{T}_L$  (left tension) and  $\mathbf{T}_R$  (right tension), as illustrated. It is reasonable to neglect the weight of the wire itself. The net external force is zero since the system is stationary. A little trigonometry can now be used to find the tensions. One conclusion is possible at the outset—we can see from part (b) of the figure that the magnitudes of the tensions  $T_L$  and  $T_R$  must be equal. This is because there is no horizontal acceleration in the rope, and the only forces acting to the left and right are  $T_L$  and  $T_R$ . Thus, the magnitude of those forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case the best coordinate system has one axis horizontal and the other vertical. We call the horizontal the  $x$ -axis and the vertical the  $y$ -axis.

### Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to draw a new free-body diagram showing all of the horizontal and vertical components of each force acting on the system.



**FIGURE 4.17** When the vectors are projected onto vertical and horizontal axes, their components along those axes must add to zero, since the tightrope walker is stationary. The small angle results in  $T$  being much greater than  $w$ .

Consider the horizontal components of the forces (denoted with a subscript  $x$ ):

$$F_{\text{net}x} = T_{Lx} - T_{Rx}. \quad 4.44$$

The net external horizontal force  $F_{\text{net}x} = 0$ , since the person is stationary. Thus,

$$\begin{aligned} F_{\text{net}x} = 0 &= T_{Lx} - T_{Rx} \\ T_{Lx} &= T_{Rx}. \end{aligned} \quad 4.45$$

Now, observe [Figure 4.17](#). You can use trigonometry to determine the magnitude of  $T_L$  and  $T_R$ . Notice that:

$$\begin{aligned} \cos(5.0^\circ) &= \frac{T_{Lx}}{T_L} \\ T_{Lx} &= T_L \cos(5.0^\circ) \\ \cos(5.0^\circ) &= \frac{T_{Rx}}{T_R} \\ T_{Rx} &= T_R \cos(5.0^\circ). \end{aligned} \quad 4.46$$

Equating  $T_{Lx}$  and  $T_{Rx}$ :

$$T_L \cos(5.0^\circ) = T_R \cos(5.0^\circ). \quad 4.47$$

Thus,

$$T_L = T_R = T, \quad 4.48$$

as predicted. Now, considering the vertical components (denoted by a subscript  $y$ ), we can solve for  $T$ . Again, since the person is stationary, Newton's second law implies that net  $F_y = 0$ . Thus, as illustrated in the free-body diagram in [Figure 4.17](#),

$$F_{\text{net}y} = T_{Ly} + T_{Ry} - w = 0. \quad 4.49$$

Observing [Figure 4.17](#), we can use trigonometry to determine the relationship between  $T_{Ly}$ ,  $T_{Ry}$ , and  $T$ . As we determined from the analysis in the horizontal direction,  $T_L = T_R = T$ :

$$\begin{aligned} \sin(5.0^\circ) &= \frac{T_{Ly}}{T_L} \\ T_{Ly} = T_L \sin(5.0^\circ) &= T \sin(5.0^\circ) \\ \sin(5.0^\circ) &= \frac{T_{Ry}}{T_R} \\ T_{Ry} = T_R \sin(5.0^\circ) &= T \sin(5.0^\circ). \end{aligned} \quad 4.50$$

Now, we can substitute the values for  $T_{Ly}$  and  $T_{Ry}$ , into the net force equation in the vertical direction:

$$\begin{aligned} F_{\text{net}y} &= T_{Ly} + T_{Ry} - w = 0 \\ F_{\text{net}y} &= T \sin(5.0^\circ) + T \sin(5.0^\circ) - w = 0 \\ 2T \sin(5.0^\circ) - w &= 0 \\ 2T \sin(5.0^\circ) &= w \end{aligned} \quad 4.51$$

and

$$T = \frac{w}{2 \sin(5.0^\circ)} = \frac{mg}{2 \sin(5.0^\circ)}, \quad 4.52$$

so that

$$T = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)}{2(0.0872)}, \quad 4.53$$

and the tension is

$$T = 3900 \text{ N}. \quad 4.54$$

### Discussion

Note that the vertical tension in the wire acts as a normal force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a small fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, and so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to *create* a very large tension, all we have to do is exert a force perpendicular to a flexible connector, as illustrated in [Figure 4.18](#). As we saw in the last example, the weight of the tightrope walker acted as a force perpendicular to the rope. We saw that the tension in the roped related to the weight of the tightrope walker in the following way:

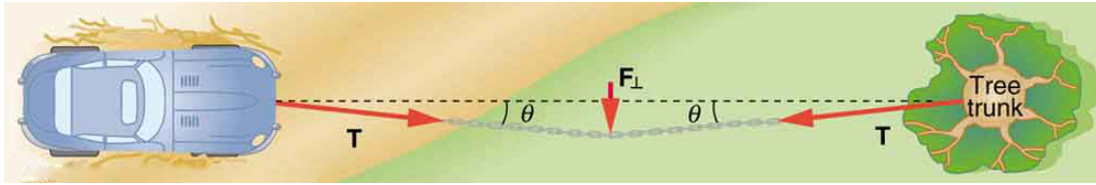
$$T = \frac{w}{2 \sin(\theta)}. \quad 4.55$$

We can extend this expression to describe the tension  $T$  created when a perpendicular force ( $F_\perp$ ) is exerted at the middle of a flexible connector:

$$T = \frac{F_\perp}{2 \sin(\theta)}. \quad 4.56$$

Note that  $\theta$  is the angle between the horizontal and the bent connector. In this case,  $T$  becomes very large as  $\theta$  approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite

tension would result if it were horizontal (i.e.,  $\theta = 0$  and  $\sin \theta = 0$ ). (See [Figure 4.18](#).)



**FIGURE 4.18** We can create a very large tension in the chain by pushing on it perpendicular to its length, as shown. Suppose we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as nearly straight as possible. The tension in the chain is given by  $T = \frac{F_{\perp}}{2 \sin(\theta)}$ ; since  $\theta$  is small,  $T$  is very large. This situation is analogous to the tightrope walker shown in [Figure 4.16](#), except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where  $F_{\perp}$  is applied.



**FIGURE 4.19** Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

### Extended Topic: Real Forces and Inertial Frames

There is another distinction among forces in addition to the types already mentioned. Some forces are real, whereas others are not. *Real forces* are those that have some physical origin, such as the gravitational pull. Contrastingly, *fictitious forces* are those that arise simply because an observer is in an accelerating frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's northern hemisphere, then to an observer on Earth it will appear to experience a force to the west that has no physical origin. Of course, what is happening here is that Earth is rotating toward the east and moves east under the satellite. In Earth's frame this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). An **inertial frame of reference** is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On the large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are considered in inertial frames.

All the forces discussed in this section are real forces, but there are a number of other real forces, such as lift and thrust, that are not discussed in this section. They are more specialized, and it is not necessary to discuss every type of force. It is natural, however, to ask where the basic simplicity we seek to find in physics is in the long list of forces. Are some more basic than others? Are some different manifestations of the same underlying force? The answer to

both questions is yes, as will be seen in the next (extended) section and in the treatment of modern physics later in the text.



## PHET EXPLORATIONS

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### Forces in 1 Dimension

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

[Click to view content \(https://openstax.org/l/21forcesatwork\)](https://openstax.org/l/21forcesatwork).



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## 4.6 Problem-Solving Strategies

### LEARNING OBJECTIVES

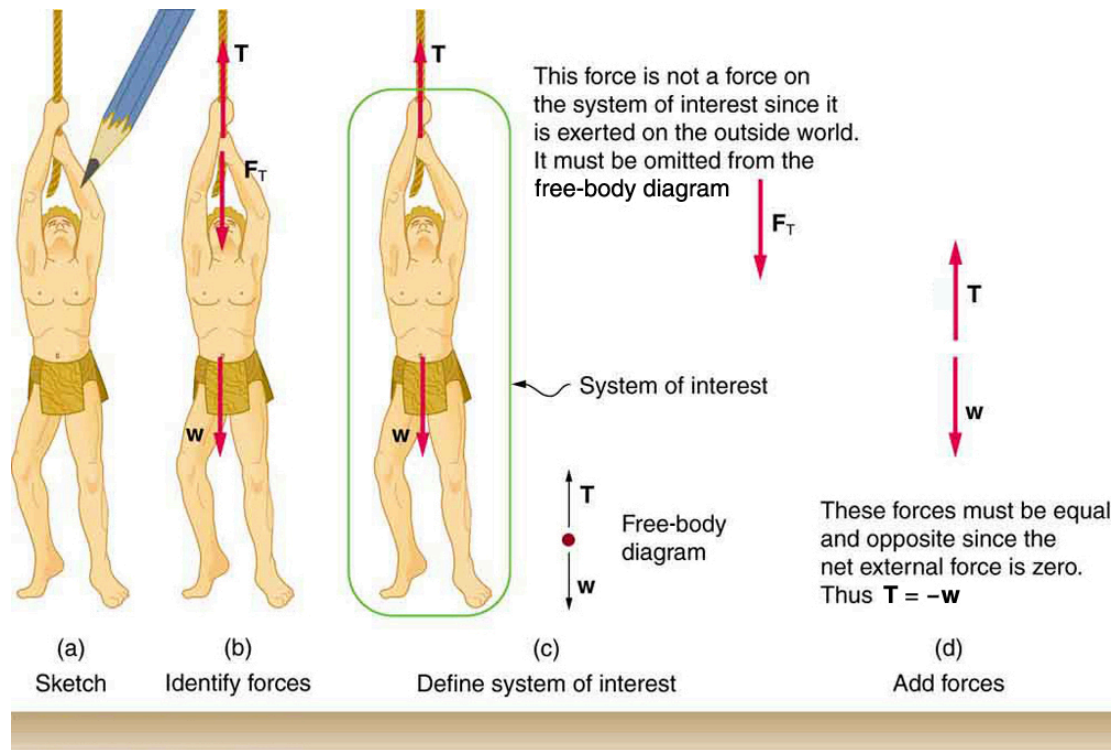
By the end of this section, you will be able to:

- Understand and apply a problem-solving procedure to solve problems using Newton's laws of motion.

Success in problem solving is obviously necessary to understand and apply physical principles, not to mention the more immediate need of passing exams. The basics of problem solving, presented earlier in this text, are followed here, but specific strategies useful in applying Newton's laws of motion are emphasized. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, and so the following techniques should reinforce skills you have already begun to develop.

### Problem-Solving Strategy for Newton's Laws of Motion

Step 1. As usual, it is first necessary to identify the physical principles involved. *Once it is determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation.* Such a sketch is shown in [Figure 4.20\(a\)](#). Then, as in [Figure 4.20\(b\)](#), use arrows to represent all forces, label them carefully, and make their lengths and directions correspond to the forces they represent (whenever sufficient information exists).



**FIGURE 4.20** (a) A sketch of Tarzan hanging from a vine. (b) Arrows are used to represent all forces.  $\mathbf{T}$  is the tension in the vine above Tarzan,  $\mathbf{F}_T$  is the force he exerts on the vine, and  $\mathbf{w}$  is his weight. All other forces, such as the nudge of a breeze, are assumed negligible. (c) Suppose we are given the ape man's mass and asked to find the tension in the vine. We then define the system of interest as shown and draw a free-body diagram.  $\mathbf{F}_T$  is no longer shown, because it is not a force acting on the system of interest; rather,  $\mathbf{F}_T$  acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that  $\mathbf{T} = -\mathbf{w}$ , if Tarzan is stationary.

Step 2. Identify what needs to be determined and what is known or can be inferred from the problem as stated. That is, make a list of knowns and unknowns. *Then carefully determine the system of interest.* This decision is a crucial step, since Newton's second law involves only external forces. Once the system of interest has been identified, it becomes possible to determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See [Figure 4.20\(c\)](#).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated earlier in this chapter, the system of interest depends on what question we need to answer. This choice becomes easier with practice, eventually developing into an almost unconscious process. Skill in clearly defining systems will be beneficial in later chapters as well.

A diagram showing the system of interest and all of the external forces is called a **free-body diagram**. Only forces are shown on free-body diagrams, not acceleration or velocity. We have drawn several of these in worked examples. [Figure 4.20\(c\)](#) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Step 3. Once a free-body diagram is drawn, *Newton's second law can be applied to solve the problem.* This is done in [Figure 4.20\(d\)](#) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then they add like scalars. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. This is done by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known.

### Applying Newton's Second Law

Before you write net force equations, it is critical to determine whether the system is accelerating in a particular direction. If the acceleration is zero in a particular direction, then the net force is zero in that direction. Similarly, if the acceleration is nonzero in a particular direction, then the net force is described by the equation:

$$F_{\text{net}} = ma.$$

For example, if the system is accelerating in the horizontal direction, but it is not accelerating in the vertical direction, then you will have the following conclusions:

$$F_{\text{net } x} = ma, \quad 4.57$$

$$F_{\text{net } y} = 0. \quad 4.58$$

You will need this information in order to determine unknown forces acting in a system.

Step 4. As always, *check the solution to see whether it is reasonable*. In some cases, this is obvious. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving, and with experience it becomes progressively easier to judge whether an answer is reasonable. Another way to check your solution is to check the units. If you are solving for force and end up with units of m/s, then you have made a mistake.

## 4.7 Further Applications of Newton's Laws of Motion

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

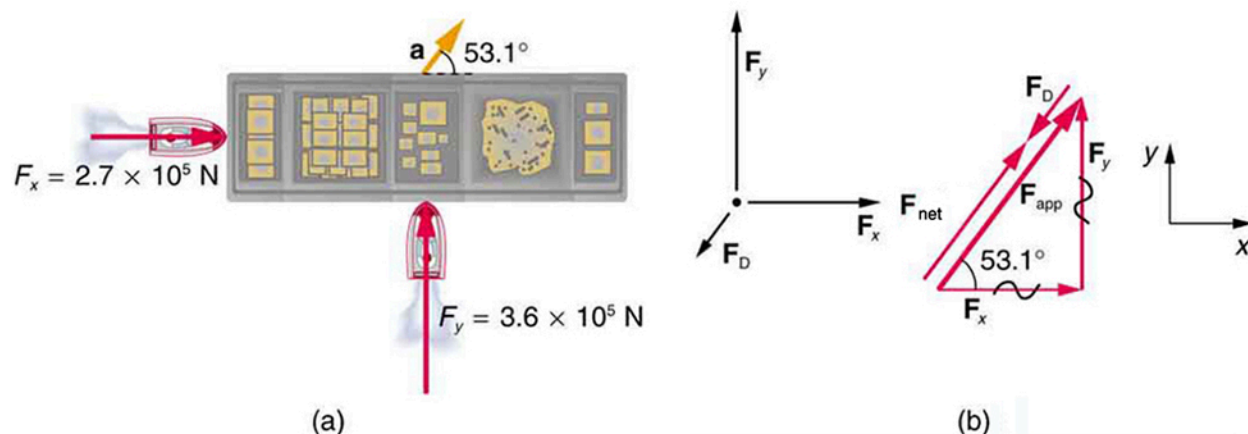
- Apply problem-solving techniques to solve for quantities in more complex systems of forces.
- Integrate concepts from kinematics to solve problems using Newton's laws of motion.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills.

### EXAMPLE 4.7

#### Drag Force on a Barge

Suppose two tugboats push on a barge at different angles, as shown in [Figure 4.21](#). The first tugboat exerts a force of  $2.7 \times 10^5$  N in the  $x$ -direction, and the second tugboat exerts a force of  $3.6 \times 10^5$  N in the  $y$ -direction.



**FIGURE 4.21** (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Since the applied forces are perpendicular, the  $x$ - and  $y$ -axes are in the same direction as  $F_x$  and  $F_y$ . The problem quickly becomes a one-dimensional problem along the direction of  $F_{\text{app}}$ , since friction is in the direction opposite to  $F_{\text{app}}$ .

If the mass of the barge is  $5.0 \times 10^6$  kg and its acceleration is observed to be  $7.5 \times 10^{-2}$  m/s<sup>2</sup> in the direction

shown, what is the drag force of the water on the barge resisting the motion? (Note: drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object.)

### Strategy

The directions and magnitudes of acceleration and the applied forces are given in [Figure 4.21\(a\)](#). We will define the total force of the tugboats on the barge as  $\mathbf{F}_{\text{app}}$  so that:

$$\mathbf{F}_{\text{app}} = \mathbf{F}_x + \mathbf{F}_y \quad 4.59$$

Since the barge is flat bottomed, the drag of the water  $\mathbf{F}_D$  will be in the direction opposite to  $\mathbf{F}_{\text{app}}$ , as shown in the free-body diagram in [Figure 4.21\(b\)](#). The system of interest here is the barge, since the forces on it are given as well as its acceleration. Our strategy is to find the magnitude and direction of the net applied force  $\mathbf{F}_{\text{app}}$ , and then apply Newton's second law to solve for the drag force  $\mathbf{F}_D$ .

### Solution

Since  $\mathbf{F}_x$  and  $\mathbf{F}_y$  are perpendicular, the magnitude and direction of  $\mathbf{F}_{\text{app}}$  are easily found. First, the resultant magnitude is given by the Pythagorean theorem:

$$F_{\text{app}} = \sqrt{F_x^2 + F_y^2} \quad 4.60$$

$$F_{\text{app}} = \sqrt{(2.7 \times 10^5 \text{ N})^2 + (3.6 \times 10^5 \text{ N})^2} = 4.5 \times 10^5 \text{ N}.$$

The angle is given by

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \quad 4.61$$

$$\theta = \tan^{-1}\left(\frac{3.6 \times 10^5 \text{ N}}{2.7 \times 10^5 \text{ N}}\right) = 53^\circ,$$

which we know, because of Newton's first law, is the same direction as the acceleration.  $\mathbf{F}_D$  is in the opposite direction of  $\mathbf{F}_{\text{app}}$ , since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as  $\mathbf{F}_{\text{app}}$ , but its magnitude is slightly less than  $\mathbf{F}_{\text{app}}$ . The problem is now one-dimensional. From [Figure 4.21\(b\)](#), we can see that

$$F_{\text{net}} = F_{\text{app}} - F_D. \quad 4.62$$

But Newton's second law states that

$$F_{\text{net}} = ma. \quad 4.63$$

Thus,

$$F_{\text{app}} - F_D = ma. \quad 4.64$$

This can be solved for the magnitude of the drag force of the water  $F_D$  in terms of known quantities:

$$F_D = F_{\text{app}} - ma. \quad 4.65$$

Substituting known values gives

$$F_D = (4.5 \times 10^5 \text{ N}) - (5.0 \times 10^6 \text{ kg})(7.5 \times 10^{-2} \text{ m/s}^2) = 7.5 \times 10^4 \text{ N}. \quad 4.66$$

The direction of  $\mathbf{F}_D$  has already been determined to be in the direction opposite to  $\mathbf{F}_{\text{app}}$ , or at an angle of  $53^\circ$  south of west.

### Discussion

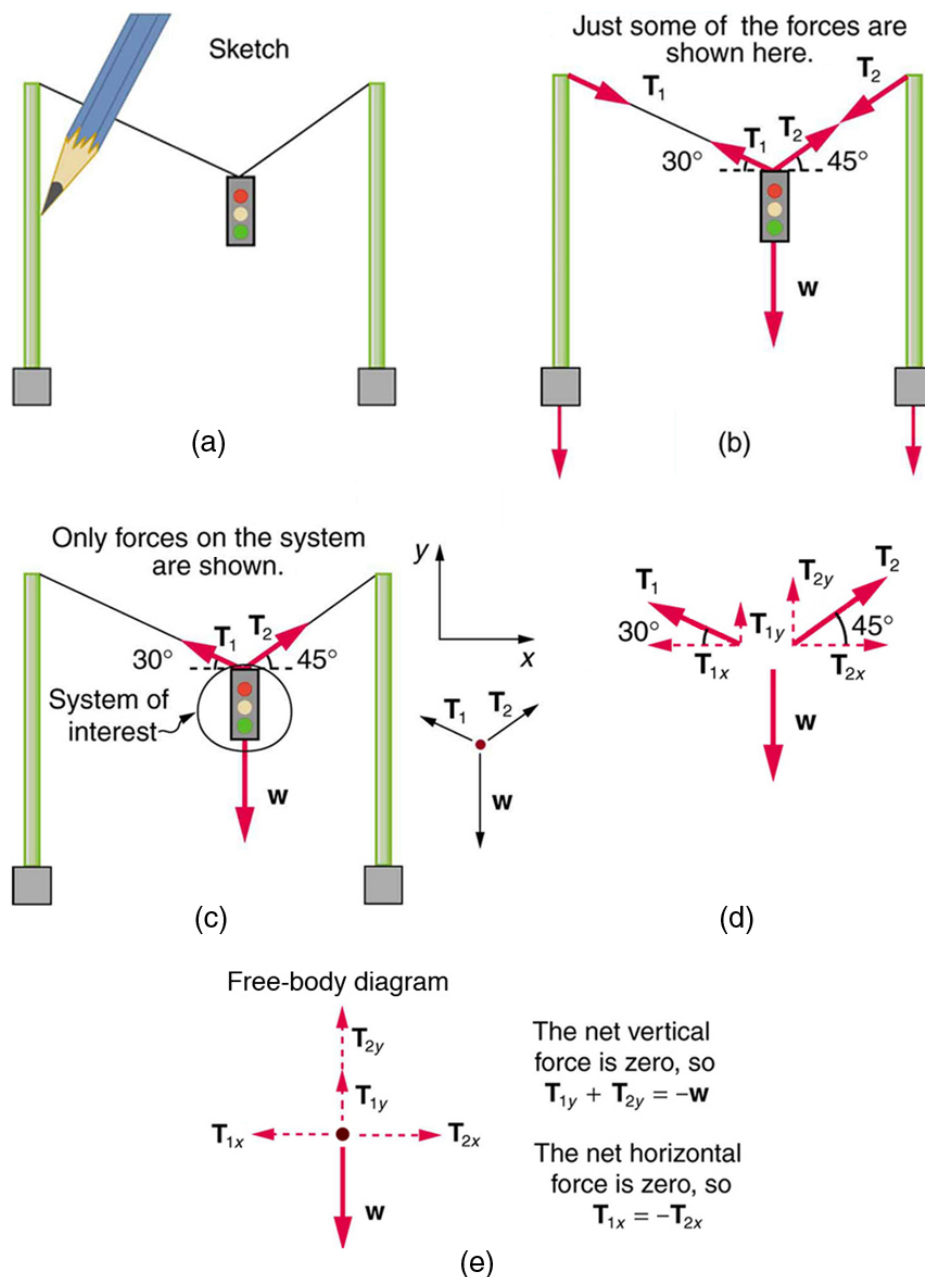
The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where  $F_D$  is less than 1/600th of the weight of the ship.

In the earlier example of a tightrope walker we noted that the tensions in wires supporting a mass were equal only because the angles on either side were equal. Consider the following example, where the angles are not equal; slightly more trigonometry is involved.

### EXAMPLE 4.8

#### Different Tensions at Different Angles

Consider the traffic light (mass 15.0 kg) suspended from two wires as shown in [Figure 4.22](#). Find the tension in each wire, neglecting the masses of the wires.



**FIGURE 4.22** A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical ( $y$ ) and horizontal ( $x$ ) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

**Strategy**

The system of interest is the traffic light, and its free-body diagram is shown in [Figure 4.22\(c\)](#). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in part (d) of the figure. There are two unknowns in this problem ( $T_1$  and  $T_2$ ), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

**Solution**

First consider the horizontal or  $x$ -axis:

$$F_{\text{net}x} = T_{2x} - T_{1x} = 0. \quad 4.67$$

Thus, as you might expect,

$$T_{1x} = T_{2x}. \quad 4.68$$

This gives us the following relationship between  $T_1$  and  $T_2$ :

$$T_1 \cos(30^\circ) = T_2 \cos(45^\circ). \quad 4.69$$

Thus,

$$T_2 = (1.225)T_1. \quad 4.70$$

Note that  $T_1$  and  $T_2$  are not equal in this case, because the angles on either side are not equal. It is reasonable that  $T_2$  ends up being greater than  $T_1$ , because it is exerted more vertically than  $T_1$ .

Now consider the force components along the vertical or  $y$ -axis:

$$F_{\text{net}y} = T_{1y} + T_{2y} - w = 0. \quad 4.71$$

This implies

$$T_{1y} + T_{2y} = w. \quad 4.72$$

Substituting the expressions for the vertical components gives

$$T_1 \sin(30^\circ) + T_2 \sin(45^\circ) = w. \quad 4.73$$

There are two unknowns in this equation, but substituting the expression for  $T_2$  in terms of  $T_1$  reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg, \quad 4.74$$

which yields

$$(1.366)T_1 = (15.0 \text{ kg})(9.80 \text{ m/s}^2). \quad 4.75$$

Solving this last equation gives the magnitude of  $T_1$  to be

$$T_1 = 108 \text{ N}. \quad 4.76$$

Finally, the magnitude of  $T_2$  is determined using the relationship between them,  $T_2 = 1.225 T_1$ , found above. Thus we obtain

$$T_2 = 132 \text{ N}. \quad 4.77$$

**Discussion**

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker).

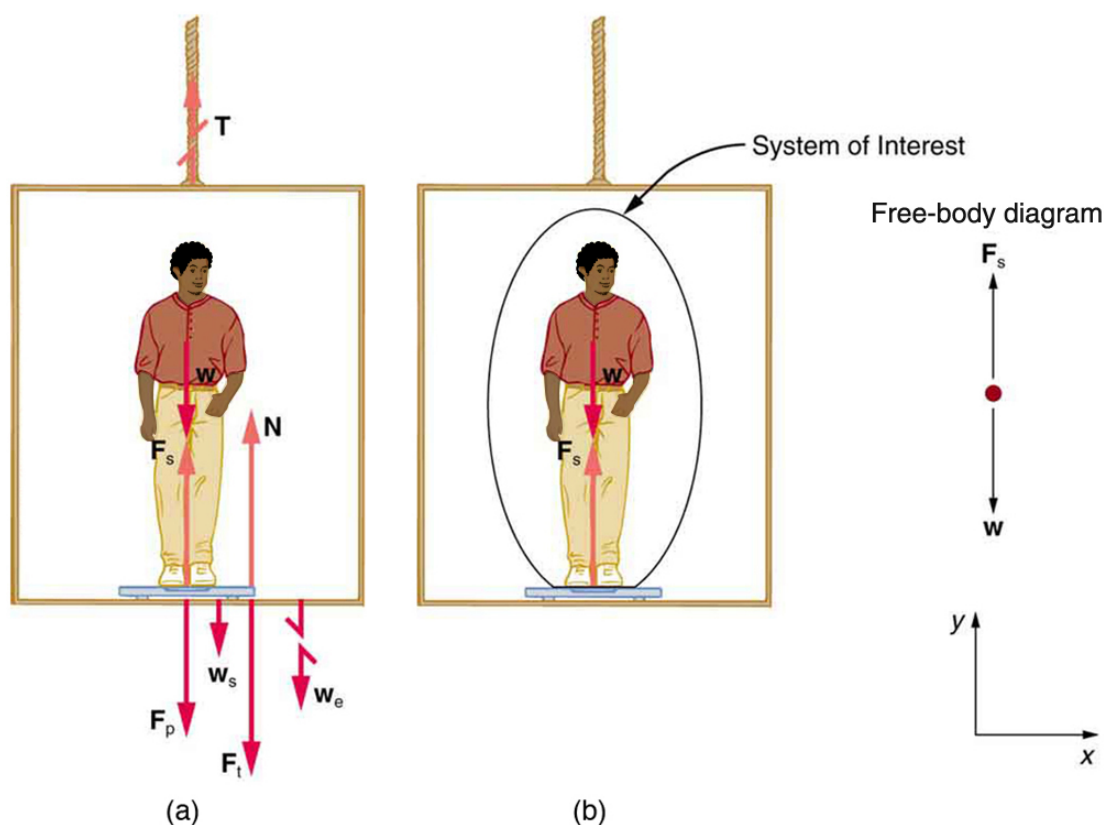
The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the

dial of a bathroom scale if you stood on it during an elevator ride? Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed: will the scale still read more than your weight at rest? Consider the following example.

### EXAMPLE 4.9

#### What Does the Bathroom Scale Read in an Elevator?

Figure 4.23 shows a 75.0-kg man (weight of about 165 lb) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of  $1.20 \text{ m/s}^2$ , and (b) if the elevator moves upward at a constant speed of  $1 \text{ m/s}$ .



**FIGURE 4.23** (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale.  $\mathbf{T}$  is the weight of the scale,  $\mathbf{w}_e$  is the weight of the elevator,  $\mathbf{F}_s$  is the force of the scale on the person,  $\mathbf{F}_p$  is the force of the person on the scale,  $\mathbf{F}_i$  is the force of the scale on the floor of the elevator, and  $\mathbf{N}$  is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting on the designated system of interest—the person.

#### Strategy

If the scale is accurate, its reading will equal  $F_p$ , the magnitude of the force the person exerts downward on it. Figure 4.23(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn as in Figure 4.23(b). Analysis of the free-body diagram using Newton's laws can produce answers to both parts (a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight  $\mathbf{w}$  and the upward force of the scale  $\mathbf{F}_s$ . According to Newton's third law  $\mathbf{F}_p$  and  $\mathbf{F}_s$  are equal in magnitude and opposite in direction, so that we need to find  $F_s$  in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

$$F_{\text{net}} = ma. \quad 4.78$$

From the free-body diagram we see that  $F_{\text{net}} = F_s - w$ , so that

$$F_s - w = ma. \quad 4.79$$

Solving for  $F_s$  gives an equation with only one unknown:

$$F_s = ma + w, \quad 4.80$$

or, because  $w = mg$ , simply

$$F_s = ma + mg. \quad 4.81$$

No assumptions were made about the acceleration, and so this solution should be valid for a variety of accelerations in addition to the ones in this exercise.

### Solution for (a)

In this part of the problem,  $a = 1.20 \text{ m/s}^2$ , so that

$$F_s = (75.0 \text{ kg})(1.20 \text{ m/s}^2) + (75.0 \text{ kg})(9.80 \text{ m/s}^2), \quad 4.82$$

yielding

$$F_s = 825 \text{ N}. \quad 4.83$$

### Discussion for (a)

This is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$\begin{aligned} F_{\text{net}} &= ma = 0 = F_s - w \\ F_s &= w = mg \\ F_s &= (75.0 \text{ kg})(9.80 \text{ m/s}^2) \\ F_s &= 735 \text{ N}. \end{aligned} \quad 4.84$$

So, the scale reading in the elevator is greater than his 735-N (165 lb) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators.

### Solution for (b)

Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because  $a = \frac{\Delta v}{\Delta t}$ , and  $\Delta v = 0$ .

Thus,

$$F_s = ma + mg = 0 + mg. \quad 4.85$$

Now

$$F_s = (75.0 \text{ kg})(9.80 \text{ m/s}^2), \quad 4.86$$

which gives

$$F_s = 735 \text{ N}. \quad 4.87$$

### Discussion for (b)

The scale reading is 735 N, which equals the person's weight. This will be the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward,  $a$  is negative, and the scale reading is *less* than the weight of the person, until a constant downward velocity is reached, at which time the scale reading again becomes equal to the person's weight. If the elevator is in free-fall and accelerating downward at  $g$ , then the scale reading will be zero and the person will *appear* to be weightless.

## Integrating Concepts: Newton's Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton's laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters. When approaching problems that involve various types of forces, acceleration, velocity, and/or position, use the following steps to approach the problem:

### Problem-Solving Strategy

Step 1. *Identify which physical principles are involved.* Listing the givens and the quantities to be calculated will allow you to identify the principles involved.

Step 2. *Solve the problem using strategies outlined in the text.* If these are available for the specific topic, you should refer to them. You should also refer to the sections of the text that deal with a particular topic. The following worked example illustrates how these strategies are applied to an integrated concept problem.



### EXAMPLE 4.10

#### What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts from rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What was his average acceleration? (b) What average force did he exert backward on the ground to achieve this acceleration? The player's mass is 70.0 kg, and air resistance is negligible.

#### Strategy

- To solve an *integrated concept problem*, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example considers *acceleration* along a straight line. This is a topic of *kinematics*. Part (b) deals with *force*, a topic of *dynamics* found in this chapter.
- The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so forth.

#### Solution for (a)

We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is  $\Delta v = 8.00$  m/s. We are given the elapsed time, and so  $\Delta t = 2.50$  s. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}. \quad 4.88$$

Substituting the known values yields

$$\begin{aligned} a &= \frac{8.00 \text{ m/s}}{2.50 \text{ s}} \\ &= 3.20 \text{ m/s}^2. \end{aligned} \quad 4.89$$

#### Discussion for (a)

This is an attainable acceleration for an athlete in good condition.

#### Solution for (b)

Here we are asked to find the average force the player exerts backward to achieve this forward acceleration. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes his acceleration. Since we now know the player's acceleration and are given his mass, we can use Newton's second law to find the force exerted. That is,

$$F_{\text{net}} = ma. \quad 4.90$$

Substituting the known values of  $m$  and  $a$  gives

$$\begin{aligned} F_{\text{net}} &= (70.0 \text{ kg})(3.20 \text{ m/s}^2) \\ &= 224 \text{ N.} \end{aligned} \quad 4.91$$

**Discussion for (b)**

This is about 50 pounds, a reasonable average force.

This worked example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These strategies are found throughout the text, and many worked examples show how to use them for single topics. You will find these techniques for integrated concept problems useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

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## 4.8 Extended Topic: The Four Basic Forces—An Introduction

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand the four basic forces that underlie the processes in nature.

One of the most remarkable simplifications in physics is that only four distinct forces account for all known phenomena. In fact, nearly all of the forces we experience directly are due to only one basic force, called the electromagnetic force. (The gravitational force is the only force we experience directly that is not electromagnetic.) This is a tremendous simplification of the myriad of *apparently* different forces we can list, only a few of which were discussed in the previous section. As we will see, the basic forces are all thought to act through the exchange of microscopic carrier particles, and the characteristics of the basic forces are determined by the types of particles exchanged. Action at a distance, such as the gravitational force of Earth on the Moon, is explained by the existence of a **force field** rather than by “physical contact.”

The *four basic forces* are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force. Their properties are summarized in [Table 4.1](#). Since the weak and strong nuclear forces act over an extremely short range, the size of a nucleus or less, we do not experience them directly, although they are crucial to the very structure of matter. These forces determine which nuclei are stable and which decay, and they are the basis of the release of energy in certain nuclear reactions. Nuclear forces determine not only the stability of nuclei, but also the relative abundance of elements in nature. The properties of the nucleus of an atom determine the number of electrons it has and, thus, indirectly determine the chemistry of the atom. More will be said of all of these topics in later chapters.

### Concept Connections: The Four Basic Forces

The four basic forces will be encountered in more detail as you progress through the text. The gravitational force is defined in [Uniform Circular Motion and Gravitation](#), electric force in [Electric Charge and Electric Field](#), magnetic force in [Magnetism](#), and nuclear forces in [Radioactivity and Nuclear Physics](#). On a macroscopic scale, electromagnetism and gravity are the basis for all forces. The nuclear forces are vital to the substructure of matter, but they are not directly experienced on the macroscopic scale.

Force	Approximate Relative Strengths	Range	Attraction/Repulsion	Carrier Particle
Gravitational	$10^{-38}$	$\infty$	attractive only	Graviton
Electromagnetic	$10^{-2}$	$\infty$	attractive and repulsive	Photon
Weak nuclear	$10^{-6}$	$< 10^{-18}$ m	attractive and repulsive	$W^+$ , $W^-$ , $Z^0$
Strong nuclear	1	$\infty$	attractive and repulsive	gluons

**TABLE 4.1** Properties of the Four Basic Forces<sup>1</sup>

The gravitational force is surprisingly weak—it is only because gravity is always attractive that we notice it at all. Our weight is the gravitational force due to the *entire* Earth acting on us. On the very large scale, as in astronomical systems, the gravitational force is the dominant force determining the motions of moons, planets, stars, and galaxies. The gravitational force also affects the nature of space and time. As we shall see later in the study of general relativity, space is curved in the vicinity of very massive bodies, such as the Sun, and time actually slows down near massive bodies.

Electromagnetic forces can be either attractive or repulsive. They are long-range forces, which act over extremely large distances, and they nearly cancel for macroscopic objects. (Remember that it is the *net* external force that is important.) If they did not cancel, electromagnetic forces would completely overwhelm the gravitational force. The electromagnetic force is a combination of electrical forces (such as those that cause static electricity) and magnetic forces (such as those that affect a compass needle). These two forces were thought to be quite distinct until early in the 19th century, when scientists began to discover that they are different manifestations of the same force. This discovery is a classical case of the *unification of forces*. Similarly, friction, tension, and all of the other classes of forces we experience directly (except gravity, of course) are due to electromagnetic interactions of atoms and molecules. It is still convenient to consider these forces separately in specific applications, however, because of the ways they manifest themselves.

### Concept Connections: Unifying Forces

Attempts to unify the four basic forces are discussed in relation to elementary particles later in this text. By “unify” we mean finding connections between the forces that show that they are different manifestations of a single force. Even if such unification is achieved, the forces will retain their separate characteristics on the macroscopic scale and may be identical only under extreme conditions such as those existing in the early universe.

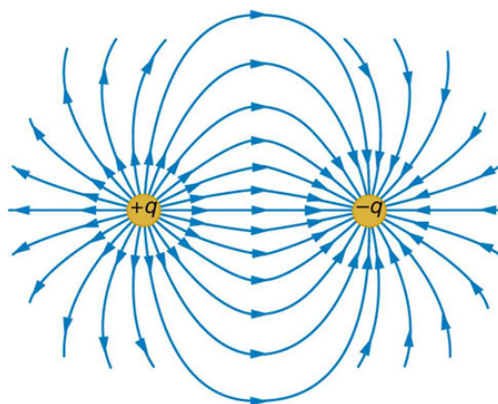
Physicists are now exploring whether the four basic forces are in some way related. Attempts to unify all forces into one come under the rubric of Grand Unified Theories (GUTs), with which there has been some success in recent years. It is now known that under conditions of extremely high density and temperature, such as existed in the early universe, the electromagnetic and weak nuclear forces are indistinguishable. They can now be considered to be different manifestations of one force, called the *electroweak* force. So the list of four has been reduced in a sense to only three. Further progress in unifying all forces is proving difficult—especially the inclusion of the gravitational force, which has the special characteristics of affecting the space and time in which the other forces exist.

While the unification of forces will not affect how we discuss forces in this text, it is fascinating that such underlying simplicity exists in the face of the overt complexity of the universe. There is no reason that nature must be simple—it simply is.

<sup>1</sup> The graviton is a proposed particle, though it has not yet been observed by scientists. See the discussion of gravitational waves later in this section. The particles  $W^+$ ,  $W^-$ , and  $Z^0$  are called vector bosons; these were predicted by theory and first observed in 1983. There are eight types of gluons proposed by scientists, and their existence is indicated by meson exchange in the nuclei of atoms.

## Action at a Distance: Concept of a Field

All forces act at a distance. This is obvious for the gravitational force. Earth and the Moon, for example, interact without coming into contact. It is also true for all other forces. Friction, for example, is an electromagnetic force between atoms that may not actually touch. What is it that carries forces between objects? One way to answer this question is to imagine that a **force field** surrounds whatever object creates the force. A second object (often called a *test object*) placed in this field will experience a force that is a function of location and other variables. The field itself is the “thing” that carries the force from one object to another. The field is defined so as to be a characteristic of the object creating it; the field does not depend on the test object placed in it. Earth’s gravitational field, for example, is a function of the mass of Earth and the distance from its center, independent of the presence of other masses. The concept of a field is useful because equations can be written for force fields surrounding objects (for gravity, this yields  $w = mg$  at Earth’s surface), and motions can be calculated from these equations. (See [Figure 4.24](#).)

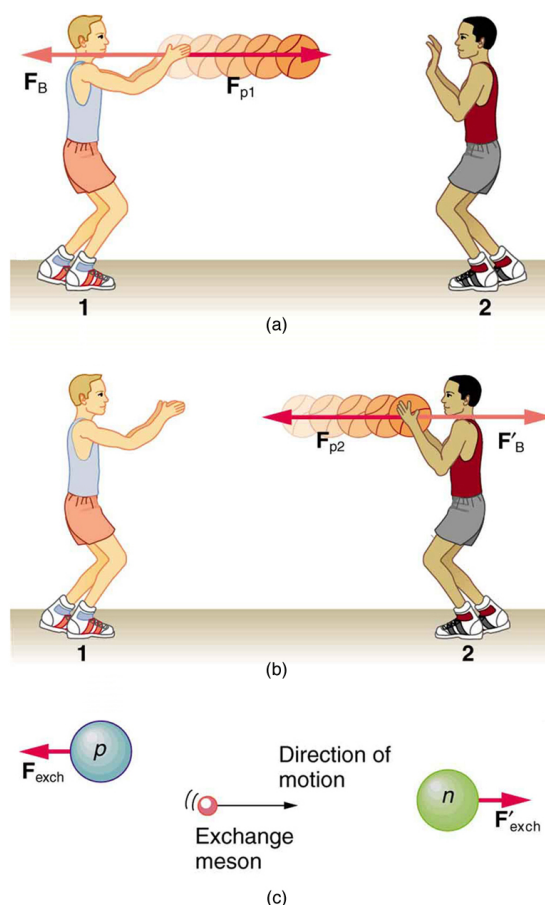


**FIGURE 4.24** The electric force field between a positively charged particle and a negatively charged particle. When a positive test charge is placed in the field, the charge will experience a force in the direction of the force field lines.

### Concept Connections: Force Fields

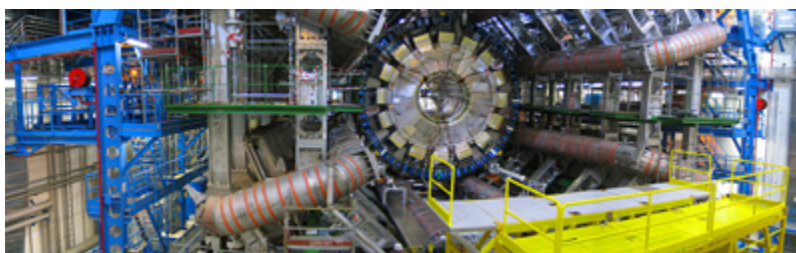
The concept of a *force field* is also used in connection with electric charge and is presented in [Electric Charge and Electric Field](#). It is also a useful idea for all the basic forces, as will be seen in [Particle Physics](#). Fields help us to visualize forces and how they are transmitted, as well as to describe them with precision and to link forces with subatomic carrier particles.

The field concept has been applied very successfully; we can calculate motions and describe nature to high precision using field equations. As useful as the field concept is, however, it leaves unanswered the question of what carries the force. It has been proposed in recent decades, starting in 1935 with Hideki Yukawa’s (1907–1981) work on the strong nuclear force, that all forces are transmitted by the exchange of elementary particles. We can visualize particle exchange as analogous to macroscopic phenomena such as two people passing a basketball back and forth, thereby exerting a repulsive force without touching one another. (See [Figure 4.25](#).)



**FIGURE 4.25** The exchange of masses resulting in repulsive forces. (a) The person throwing the basketball exerts a force  $\mathbf{F}_{p1}$  on it toward the other person and feels a reaction force  $\mathbf{F}_B$  away from the second person. (b) The person catching the basketball exerts a force  $\mathbf{F}_{p2}$  on it to stop the ball and feels a reaction force  $\mathbf{F}'_B$  away from the first person. (c) The analogous exchange of a meson between a proton and a neutron carries the strong nuclear forces  $\mathbf{F}_{\text{exch}}$  and  $\mathbf{F}'_{\text{exch}}$  between them. An attractive force can also be exerted by the exchange of a mass—if person 2 pulled the basketball away from the first person as he tried to retain it, then the force between them would be attractive.

This idea of particle exchange deepens rather than contradicts field concepts. It is more satisfying philosophically to think of something physical actually moving between objects acting at a distance. [Table 4.1](#) lists the exchange or **carrier particles**, both observed and proposed, that carry the four forces. But the real fruit of the particle-exchange proposal is that searches for Yukawa's proposed particle found it *and* a number of others that were completely unexpected, stimulating yet more research. All of this research eventually led to the proposal of quarks as the underlying substructure of matter, which is a basic tenet of GUTs. If successful, these theories will explain not only forces, but also the structure of matter itself. Yet physics is an experimental science, so the test of these theories must lie in the domain of the real world. As of this writing, scientists at the CERN laboratory in Switzerland are starting to test these theories using the world's largest particle accelerator: the Large Hadron Collider. This accelerator (27 km in circumference) allows two high-energy proton beams, traveling in opposite directions, to collide. An energy of 14 trillion electron volts will be available. It is anticipated that some new particles, possibly force carrier particles, will be found. (See [Figure 4.26](#).) One of the force carriers of high interest that researchers hope to detect is the Higgs boson. The observation of its properties might tell us why different particles have different masses.

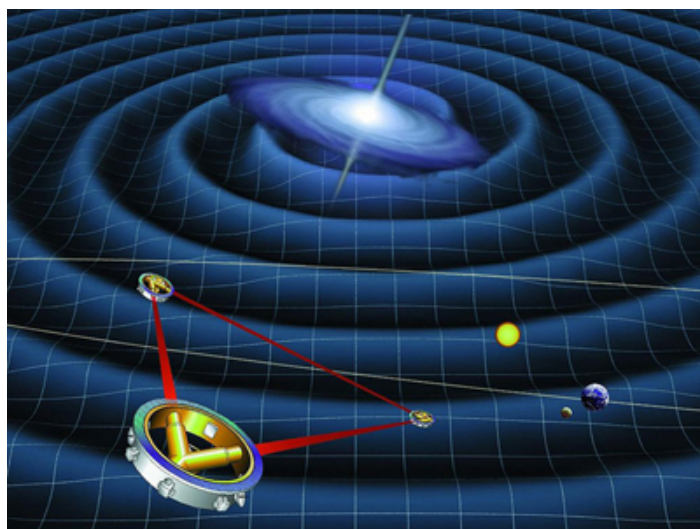


**FIGURE 4.26** The world's largest particle accelerator spans the border between Switzerland and France. Two beams, traveling in opposite directions close to the speed of light, collide in a tube similar to the central tube shown here. External magnets determine the beam's path. Special detectors will analyze particles created in these collisions. Questions as broad as what is the origin of mass and what was matter like the first few seconds of our universe will be explored. This accelerator began preliminary operation in 2008. (credit: Frank Hommes)

Tiny particles also have wave-like behavior, something we will explore more in a later chapter. To better understand force-carrier particles from another perspective, let us consider gravity. The search for gravitational waves has been going on for a number of years. Over 100 years ago, Einstein predicted the existence of these waves as part of his general theory of relativity. Gravitational waves are created during the collision of massive stars, in black holes, or in supernova explosions—like shock waves. These gravitational waves will travel through space from such sites much like a pebble dropped into a pond sends out ripples—except these waves move at the speed of light. A detector apparatus has been built in the U.S., consisting of two large installations nearly 3000 km apart—one in Washington state and one in Louisiana! The facility is called the Laser Interferometer Gravitational-Wave Observatory (LIGO). Each installation is designed to use optical lasers to examine any slight shift in the relative positions of two masses due to the effect of gravity waves. The two sites allow simultaneous measurements of these small effects to be separated from other natural phenomena, such as earthquakes. Initial operation of the detectors began in 2002, and work is proceeding on increasing their sensitivity. Similar installations have been built in Italy (VIRGO), Germany (GEO600), and Japan (TAMA300) to provide a worldwide network of gravitational wave detectors.

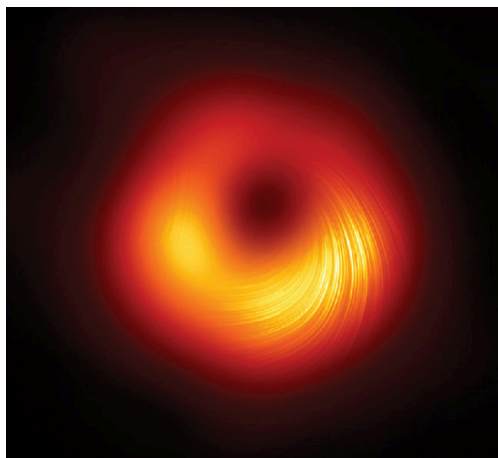
In September, 2015, LIGO fulfilled its promise and helped prove Einstein's predictions. The system detected the first gravitational waves arising from the merger of two black holes—one 29 times the mass of our Sun and the other 36 times the mass of our Sun—that occurred 1.3 billion years ago. About 3 times the mass of the Sun was converted into gravitational waves in a fraction of a second—with a peak power output about 50 times that of the whole visible universe. Due to the 7 millisecond delay in detection, researchers established that the merger occurred on the southern hemisphere side of Earth. Since then, LIGO and VIRGO have combined to detect about a dozen similar events, with better and more precise measurements. Waves from neutron star mergers and different-sized black holes have deepened our understanding of these objects and their impact on the universe.

International collaboration in this area is moving into space with the joint EU/US project LISA (Laser Interferometer Space Antenna). Earthquakes and other Earthly noises will be no problem for these monitoring spacecraft. LISA will complement LIGO by looking at much more massive black holes through the observation of gravitational-wave sources emitting much larger wavelengths. Three satellites will be placed in space above Earth in an equilateral triangle (with 5,000,000-km sides) (Figure 4.27). The system will measure the relative positions of each satellite to detect passing gravitational waves. Accuracy to within 10% of the size of an atom will be needed to detect any waves. The launch of this project will likely be in the 2030s.



**FIGURE 4.27** Space-based future experiments for the measurement of gravitational waves. Shown here is a drawing of LISA's orbit. Each satellite of LISA will consist of a laser source and a mass. The lasers will transmit a signal to measure the distance between each satellite's test mass. The relative motion of these masses will provide information about passing gravitational waves. (credit: NASA)

As you can see above, some of the most groundbreaking developments in physics are made with a relatively long gap from theoretical prediction to experimental detection. This pattern continues the process of science from its earliest days, where early thinkers and researchers made discoveries that only led to more questions. Einstein was unique in many ways, but he was not unique in that later scientists, building on his and each other's work, would prove his theories. Evidence for black holes became more and more concrete as scientists developed new and better ways to look for them. Some of the most prominent have been Roger Penrose, who developed new mathematical models related to black holes, as well as Reinhard Genzel and Andrea Ghez, who independently used telescope observations to identify a region of our galaxy where a massive unseen gravity source (4 million times the size of our Sun) was pulling on stars. And soon after, collaborators on the Event Horizon Telescope project produced the first actual image of a black hole.



**FIGURE 4.28** This updated image of the supermassive black hole at the center of the M87 galaxy shows the polarization created by the powerful magnetic field that surrounds the object. The image was created by the Event Horizon Telescope, which combines data from telescopes around the world. The project involves over 200 researchers and produced the first image of a black hole in 2019. (credit: EHT Collaboration/NASA APOD)

## Glossary

**acceleration** the rate at which an object's velocity changes over a period of time

**carrier particle** a fundamental particle of nature that is surrounded by a characteristic force field; photons are carrier particles of the electromagnetic force

**dynamics** the study of how forces affect the motion of objects and systems

**external force** a force acting on an object or system that originates outside of the object or system

**force** a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

**force field** a region in which a test particle will experience a force

**free-body diagram** a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

**free-fall** a situation in which the only force acting on an object is the force due to gravity

**friction** a force past each other of objects that are touching; examples include rough surfaces and air resistance

**inertia** the tendency of an object to remain at rest or remain in motion

**inertial frame of reference** a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

**law of inertia** see Newton's first law of motion

**mass** the quantity of matter in a substance; measured in kilograms

**net external force** the vector sum of all external forces acting on an object or system; causes a mass

to accelerate

**Newton's first law of motion** a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

**Newton's second law of motion** the net external force  $\mathbf{F}_{\text{net}}$  on an object with mass  $m$  is proportional to and in the same direction as the acceleration of the object,  $\mathbf{a}$ , and inversely proportional to the mass; defined mathematically as  $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$

**Newton's third law of motion** whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

**normal force** the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

**system** defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

**tension** the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

**thrust** a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

**weight** the force  $\mathbf{w}$  due to gravity acting on an object of mass  $m$ ; defined mathematically as:  $\mathbf{w} = m\mathbf{g}$ , where  $\mathbf{g}$  is the magnitude and direction of the acceleration due to gravity

## Section Summary

### 4.1 Development of Force Concept

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

### 4.2 Newton's First Law of Motion: Inertia

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

### 4.3 Newton's Second Law of Motion: Concept of a System

- Acceleration,  $\mathbf{a}$ , is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is  $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ .
- This is often written in the more familiar form:  $\mathbf{F}_{\text{net}} = m\mathbf{a}$ .
- The weight  $\mathbf{w}$  of an object is defined as the force of gravity acting on an object of mass  $m$ . The object experiences an acceleration due to gravity  $\mathbf{g}$ :  $\mathbf{w} = m\mathbf{g}$ .
- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

### 4.4 Newton's Third Law of Motion: Symmetry in Forces

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

### 4.5 Normal, Tension, and Other Examples of Forces

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force,  $\mathbf{N}$ .
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:  $N = mg$ .
- When objects rest on an inclined plane that makes an angle  $\theta$  with the horizontal surface, the weight

of the object can be resolved into components that act perpendicular ( $\mathbf{w}_{\perp}$ ) and parallel ( $\mathbf{w}_{\parallel}$ ) to the surface of the plane. These components can be calculated using:

$$w_{\parallel} = w \sin(\theta) = mg \sin(\theta)$$

$$w_{\perp} = w \cos(\theta) = mg \cos(\theta).$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension,  $\mathbf{T}$ . When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:  $T = mg$ .
- In any inertial frame of reference (one that is not accelerated or rotated), Newton's laws have the simple forms given in this chapter and all forces are real forces having a physical origin.

### 4.6 Problem-Solving Strategies

- To solve problems involving Newton's laws of motion, follow the procedure described:
  1. Draw a sketch of the problem.
  2. Identify known and unknown quantities, and identify the system of interest. Draw a free-body diagram, which is a sketch showing all of the forces acting on an object. The object is represented by a dot, and the forces are represented by vectors extending in different directions from the dot. If vectors act in directions that are not horizontal or vertical, resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
  3. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the  $x$ -direction) then  $F_{\text{net } x} = 0$ . If the object does accelerate in that direction,  $F_{\text{net } x} = ma$ .
  4. Check your answer. Is the answer reasonable? Are the units correct?

### 4.7 Further Applications of Newton's Laws of Motion

- Newton's laws of motion can be applied in numerous situations to solve problems of motion.
- Some problems will contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether  $F_{\text{net}} = ma$  or  $F_{\text{net}} = 0$ .
- The normal force on an object is not always equal

in magnitude to the weight of the object. If an object is accelerating, the normal force will be less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force will always be less than the full weight of the object.

- Some problems will contain various physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics in order to solve these problems of motion.

#### 4.8 Extended Topic: The Four Basic Forces—An Introduction

- The various types of forces that are categorized for

### Conceptual Questions

#### 4.1 Development of Force Concept

1. Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.
2. What properties do forces have that allow us to classify them as vectors?

#### 4.2 Newton's First Law of Motion: Inertia

3. How are inertia and mass related?
4. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

#### 4.3 Newton's Second Law of Motion: Concept of a System

5. Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.
6. Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?
7. Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.
8. Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.
9. A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.
10. A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

use in many applications are all manifestations of the *four basic forces* in nature.

- The properties of these forces are summarized in [Table 4.1](#).
- Everything we experience directly without sensitive instruments is due to either electromagnetic forces or gravitational forces. The nuclear forces are responsible for the submicroscopic structure of matter, but they are not directly sensed because of their short ranges. Attempts are being made to show all four forces are different manifestations of a single unified force.
- A force field surrounds an object creating a force and is the carrier of that force.

11. (a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.
12. If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.
13. If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?
14. The gravitational force on the basketball in [Figure 4.6](#) is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

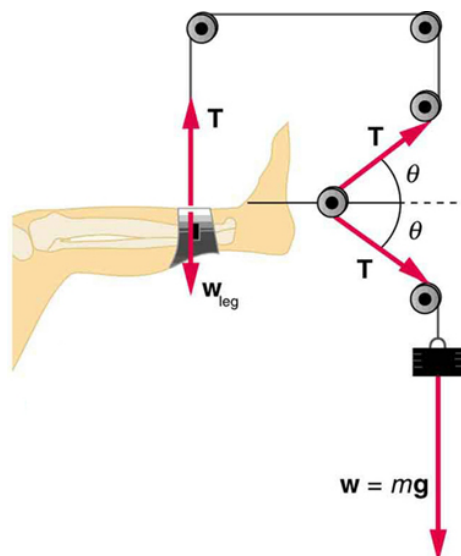
#### 4.4 Newton's Third Law of Motion: Symmetry in Forces

15. When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)
16. A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

17. Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?
18. Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?
19. An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.
20. Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

#### 4.5 Normal, Tension, and Other Examples of Forces

21. If a leg is suspended by a traction setup as shown in [Figure 4.29](#), what is the tension in the rope?



**FIGURE 4.29** A leg is suspended by a traction system in which wires are used to transmit forces. Frictionless pulleys change the direction of the force  $T$  without changing its magnitude.

22. In a traction setup for a broken bone, with pulleys and rope available, how might we be able to increase the force along the tibia using the same weight? (See [Figure 4.29](#).) (Note that the tibia is the shin bone shown in this image.)

#### 4.7 Further Applications of Newton's Laws of Motion

23. To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at  $g$ . Why will they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?
24. A cartoon shows the hat coming off the head of an elevator passenger when the elevator rapidly stops during an upward ride. Can this really happen without the person being tied to the floor of the elevator? Explain your answer.

#### 4.8 Extended Topic: The Four Basic Forces—An Introduction

25. Explain, in terms of the properties of the four basic forces, why people notice the gravitational force acting on their bodies if it is such a comparatively weak force.
26. What is the dominant force between astronomical objects? Why are the other three basic forces less significant over these very large distances?
27. Give a detailed example of how the exchange of a particle can result in an *attractive* force. (For example, consider one child pulling a toy out of the hands of another.)

## Problems & Exercises

### 4.3 Newton's Second Law of Motion: Concept of a System

You may assume data taken from illustrations is accurate to three digits.

1. A 63.0-kg sprinter starts a race with an acceleration of  $4.20 \text{ m/s}^2$ . What is the net external force on him?
2. If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?
3. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.
4. Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be  $0.893 \text{ m/s}^2$ . (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.
5. In [Figure 4.7](#), the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force  $F$  (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force  $F$  is removed. How far will the mower go before stopping?
6. The same rocket sled drawn in [Figure 4.30](#) is decelerated at a rate of  $196 \text{ m/s}^2$ . What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

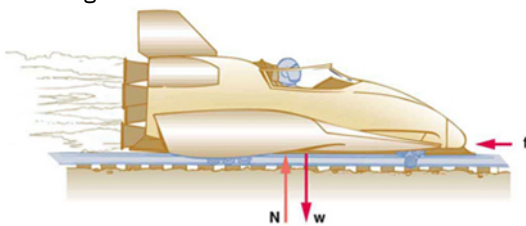


FIGURE 4.30

7. (a) If the rocket sled shown in [Figure 4.31](#) starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust  $T$  is  $2.4 \times 10^4 \text{ N}$ , and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

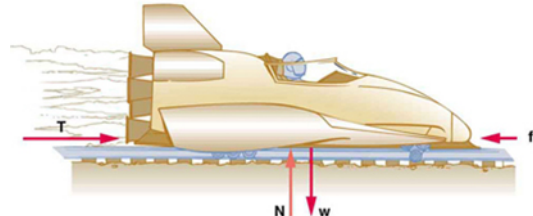


FIGURE 4.31

8. What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)
9. Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg.
  - a. What is the system of interest if the acceleration of the child in the wagon is to be calculated?
  - b. Draw a free-body diagram, including all forces acting on the system.
  - c. Calculate the acceleration.
  - d. What would the acceleration be if friction were 15.0 N?
10. A powerful motorcycle can produce an acceleration of  $3.50 \text{ m/s}^2$  while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?
11. The rocket sled shown in [Figure 4.32](#) accelerates at a rate of  $49.0 \text{ m/s}^2$ . Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

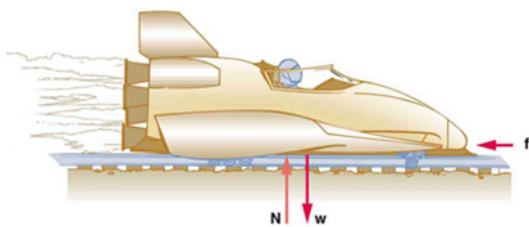


FIGURE 4.32

12. Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of  $201 \text{ m/s}^2$ . In this problem, the forces are exerted by the seat and restraining belts.
13. The weight of an astronaut plus their space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?
14. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

#### 4.4 Newton's Third Law of Motion: Symmetry in Forces

15. What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at  $2.40 \times 10^4 \text{ m/s}^2$ ? What is the magnitude of the force exerted on the ship by the artillery shell?
16. A rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at  $1.20 \text{ m/s}^2$  backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

#### 4.5 Normal, Tension, and Other Examples of Forces

17. Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?
18. What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at  $7.50 \text{ m/s}^2$ ? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.
19. (a) Calculate the tension in a vertical strand of spider web if a spider of mass  $8.00 \times 10^{-5} \text{ kg}$  hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in [Figure 4.16](#). The strand sags at an angle of  $12^\circ$  below the horizontal. Compare this with the tension in the vertical strand (find their ratio).
20. Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if they climb at a constant speed? (b) What is the tension in the rope if they accelerate upward at a rate of  $1.50 \text{ m/s}^2$ ?
21. Show that, as stated in the text, a force  $F_\perp$  exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in [Figure 4.16](#)) gives rise to a tension of magnitude  $T = \frac{F_\perp}{2 \sin(\theta)}$ .
22. Consider the baby being weighed in [Figure 4.33](#). (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension  $T_1$  in the cord attaching the baby to the scale? (c) What is the tension  $T_2$  in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

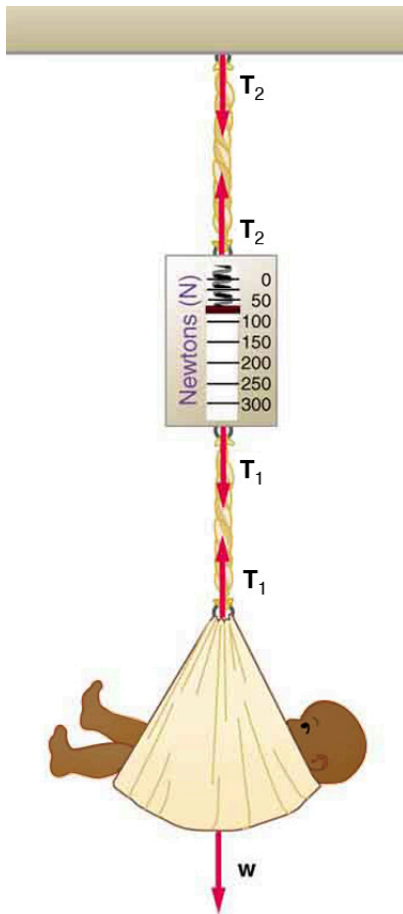


FIGURE 4.33 A baby is weighed using a spring scale.

#### 4.6 Problem-Solving Strategies

- 23.** A  $5.00 \times 10^5$ -kg rocket is accelerating straight up. Its engines produce  $1.250 \times 10^7$  N of thrust, and air resistance is  $4.50 \times 10^6$  N. What is the rocket's acceleration? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.
- 24.** The wheels of a midsize car exert a force of 2100 N backward on the road to accelerate the car in the forward direction. If the force of friction including air resistance is 250 N and the acceleration of the car is  $1.80 \text{ m/s}^2$ , what is the mass of the car plus its occupants? Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. For this situation, draw a free-body diagram and write the net force equation.
- 25.** Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.
- 26.** When landing after a spectacular somersault, a 40.0-kg gymnast decelerates by pushing straight down on the mat. Calculate the force she must exert if her deceleration is 7.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.
- 27.** A freight train consists of two  $8.00 \times 10^4$ -kg engines and 45 cars with average masses of  $5.50 \times 10^4$  kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of  $5.00 \times 10^{-2} \text{ m/s}^2$  if the force of friction is  $7.50 \times 10^5$  N, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?
- 28.** Commercial airplanes are sometimes pushed out of the passenger loading area by a tractor. (a) An 1800-kg tractor exerts a force of  $1.75 \times 10^4$  N backward on the pavement, and the system experiences forces resisting motion that total 2400 N. If the acceleration is  $0.150 \text{ m/s}^2$ , what is the mass of the airplane? (b) Calculate the force exerted by the tractor on the airplane, assuming 2200 N of the friction is experienced by the airplane. (c) Draw two sketches showing the systems of interest used to solve each part, including the free-body diagrams for each.
- 29.** A 1100-kg car pulls a boat on a trailer. (a) What total force resists the motion of the car, boat, and trailer, if the car exerts a 1900-N force on the road and produces an acceleration of  $0.550 \text{ m/s}^2$ ? The mass of the boat plus trailer is 700 kg. (b) What is the force in the hitch between the car and the trailer if 80% of the resisting forces are experienced by the boat and trailer?
- 30.** (a) Find the magnitudes of the forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  that add to give the total force  $\mathbf{F}_{\text{tot}}$  shown in Figure 4.34. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ . (c) Find the direction and magnitude of some other pair of vectors that add to give  $\mathbf{F}_{\text{tot}}$ . Draw these to scale on the same drawing used in part (b) or a similar picture.

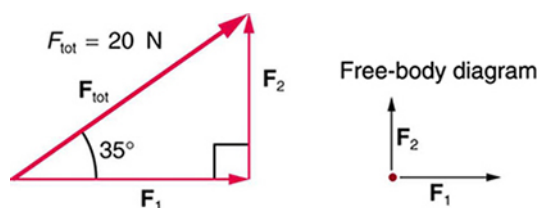


FIGURE 4.34

31. Two children pull a third child on a snow saucer sled exerting forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  as shown from above in [Figure 4.35](#). Find the acceleration of the 49.00-kg sled and child system. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of  $\mathbf{F}_1$  and  $\mathbf{F}_2$ .

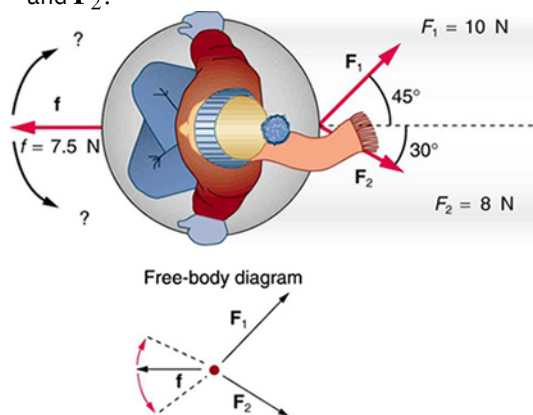


FIGURE 4.35 An overhead view of the horizontal forces acting on a child's snow saucer sled.

32. Suppose your car was mired deeply in the mud and you wanted to use the method illustrated in [Figure 4.36](#) to pull it out. (a) What force would you have to exert perpendicular to the center of the rope to produce a force of 12,000 N on the car if the angle is  $2.00^\circ$ ? In this part, explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion. (b) Real ropes stretch under such forces. What force would be exerted on the car if the angle increases to  $7.00^\circ$  and you still apply the force found in part (a) to its center?

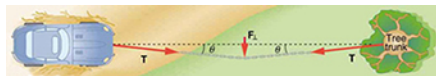
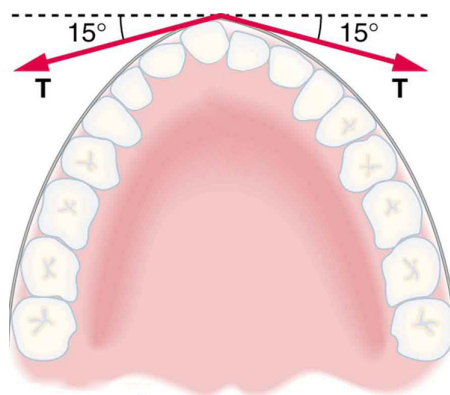


FIGURE 4.36

33. What force is exerted on the tooth in [Figure 4.37](#) if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.

FIGURE 4.37 Braces are used to apply forces to teeth to realign them. Shown in this figure are the tensions applied by the wire to the protruding tooth. The total force applied to the tooth by the wire,  $\mathbf{F}_{\text{app}}$ , points straight toward the back of the mouth.

34. [Figure 4.38](#) shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.



FIGURE 4.38 Superhero and Trusty Sidekick hang motionless on a rope as they try to figure out what to do next. Will the tension be the same everywhere in the rope?

- 35.** A nurse pushes a cart by exerting a force on the handle at a downward angle  $35.0^\circ$  below the horizontal. The loaded cart has a mass of  $28.0\text{ kg}$ , and the force of friction is  $60.0\text{ N}$ . (a) Draw a free-body diagram for the system of interest. (b) What force must the nurse exert to move at a constant velocity?
- 36.** Construct Your Own Problem Consider the tension in an elevator cable during the time the elevator starts from rest and accelerates its load upward to some cruising velocity. Taking the elevator and its load to be the system of interest, draw a free-body diagram. Then calculate the tension in the cable. Among the things to consider are the mass of the elevator and its load, the final velocity, and the time taken to reach that velocity.
- 37.** Construct Your Own Problem Consider two people pushing a toboggan with four children on it up a snow-covered slope. Construct a problem in which you calculate the acceleration of the toboggan and its load. Include a free-body diagram of the appropriate system of interest as the basis for your analysis. Show vector forces and their components and explain the choice of coordinates. Among the things to be considered are the forces exerted by those pushing, the angle of the slope, and the masses of the toboggan and children.
- 38.** Unreasonable Results (a) Repeat [Exercise 4.29](#), but assume an acceleration of  $1.20\text{ m/s}^2$  is produced. (b) What is unreasonable about the result? (c) Which premise is unreasonable, and why is it unreasonable?
- 39.** Unreasonable Results (a) What is the initial acceleration of a rocket that has a mass of  $1.50 \times 10^6\text{ kg}$  at takeoff, the engines of which produce a thrust of  $2.00 \times 10^6\text{ N}$ ? Do not neglect gravity. (b) What is unreasonable about the result? (This result has been unintentionally achieved by several real rockets.) (c) Which premise is unreasonable, or which premises are inconsistent? (You may find it useful to compare this problem to the rocket problem earlier in this section.)

#### 4.7 Further Applications of Newton's Laws of Motion

- 40.** A flea jumps by exerting a force of  $1.20 \times 10^{-5}\text{ N}$  straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of  $0.500 \times 10^{-6}\text{ N}$  on the flea. Find the direction and magnitude of the acceleration of the flea if its mass is  $6.00 \times 10^{-7}\text{ kg}$ . Do not neglect the gravitational force.
- 41.** Two muscles in the back of the leg pull upward on the Achilles tendon, as shown in [Figure 4.39](#). (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?

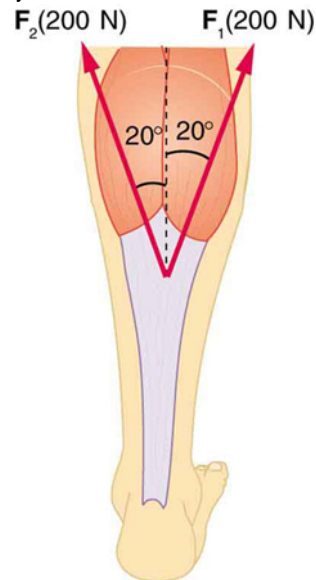
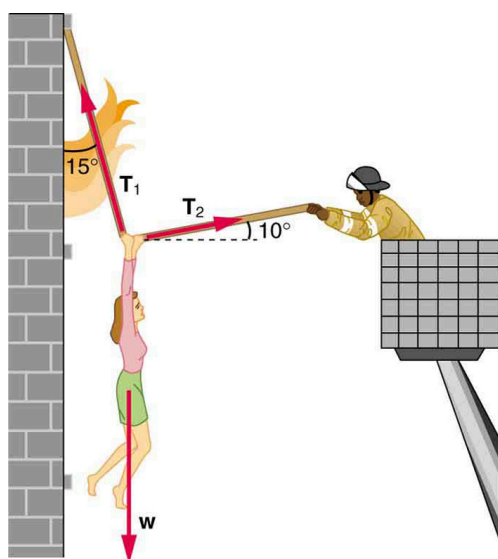


FIGURE 4.39 Achilles tendon

- 42.** A  $76.0\text{-kg}$  person is being pulled away from a burning building as shown in [Figure 4.40](#). Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.



**FIGURE 4.40** The force  $T_2$  needed to hold steady the person being rescued from the fire is less than her weight and less than the force  $T_1$  in the other rope, since the more vertical rope supports a greater part of her weight (a vertical force).

- 43. Integrated Concepts** A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow him if he was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)
- 44. Integrated Concepts** When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?
- 45. Integrated Concepts** A large rocket has a mass of  $2.00 \times 10^6$  kg at takeoff, and its engines produce a thrust of  $3.50 \times 10^7$  N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust? (c) In reality, the mass of a rocket decreases significantly as its fuel is consumed. Describe qualitatively how this affects the acceleration and time for this motion.
- 46. Integrated Concepts** A basketball player jumps straight up for a ball. To do this, they lower their body 0.300 m and then accelerate through this distance by forcefully straightening their legs. This player leaves the floor with a vertical velocity sufficient to carry the player 0.900 m above the floor. (a) Calculate the player's velocity when they leave the floor. (b) Calculate their acceleration while they are straightening their legs. They go from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force they exert on the floor to do this, given that their mass is 110 kg.
- 47. Integrated Concepts** A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell's velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.
- 48. Integrated Concepts** Repeat [Exercise 4.47](#) for a shell fired at an angle  $10.0^\circ$  from the vertical.
- 49. Integrated Concepts** An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of  $1.20 \text{ m/s}^2$  for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of  $0.600 \text{ m/s}^2$  for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?
- 50. Unreasonable Results** (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of  $0.400 \text{ m/s}^2$  for 50.0 s? (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
- 51. Unreasonable Results** A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

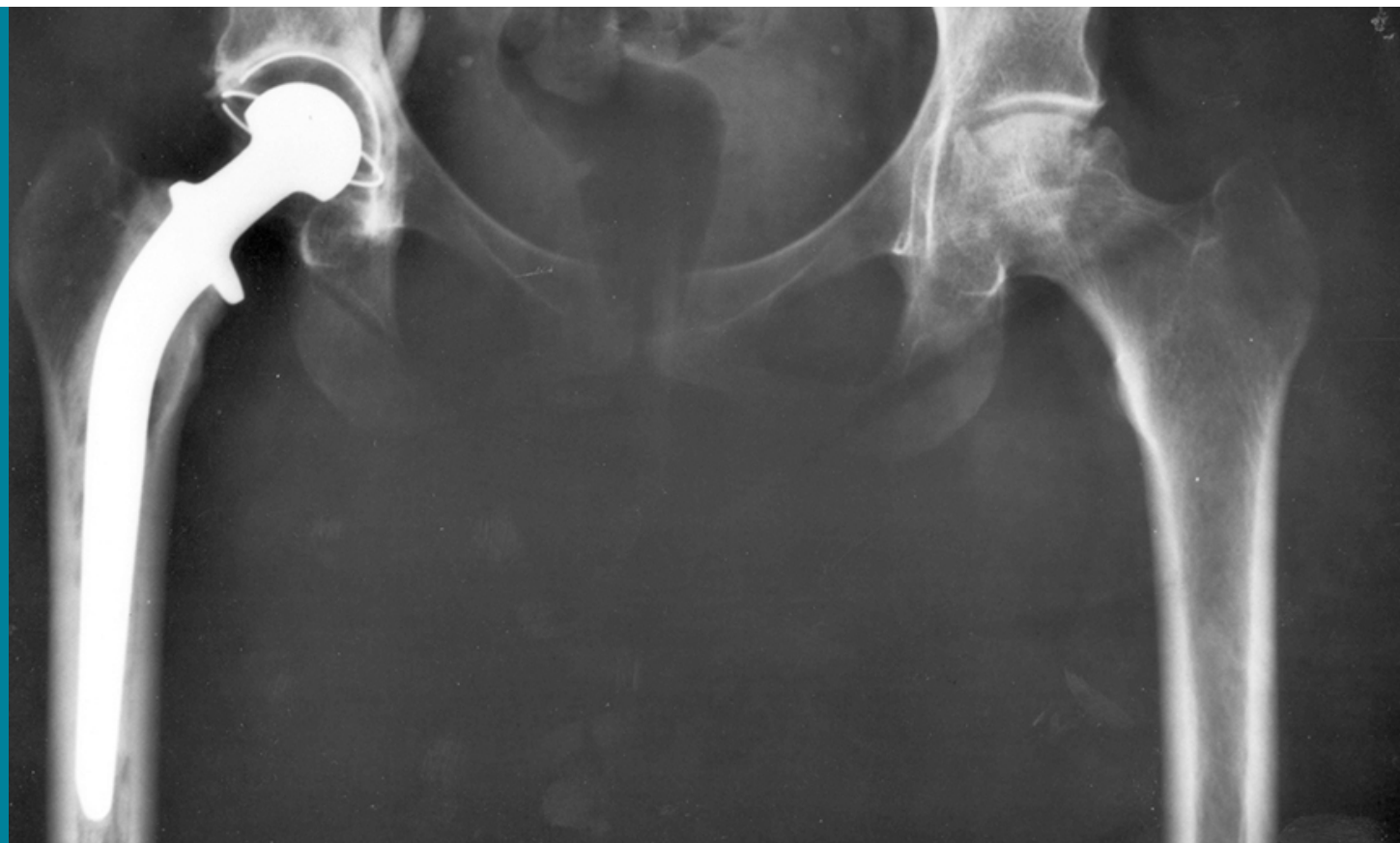
#### 4.8 Extended Topic: The Four Basic Forces—An Introduction

- 52.** (a) What is the strength of the weak nuclear force relative to the strong nuclear force? (b) What is the strength of the weak nuclear force relative to the electromagnetic force? Since the weak nuclear force acts at only very short distances, such as inside nuclei, where the strong and electromagnetic forces also act, it might seem surprising that we have any knowledge of it at all. We have such knowledge because the weak nuclear force is responsible for beta decay, a type of nuclear decay not explained by other forces.
- 53.** (a) What is the ratio of the strength of the gravitational force to that of the strong nuclear force? (b) What is the ratio of the strength of the gravitational force to that of the weak nuclear force? (c) What is the ratio of the strength of the gravitational force to that of the electromagnetic force? What do your answers imply about the influence of the gravitational force on atomic nuclei?
- 54.** What is the ratio of the strength of the strong nuclear force to that of the electromagnetic force? Based on this ratio, you might expect that the strong force dominates the nucleus, which is true for small nuclei. Large nuclei, however, have sizes greater than the range of the strong nuclear force. At these sizes, the electromagnetic force begins to affect nuclear stability. These facts will be used to explain nuclear fusion and fission later in this text.
- 55.** Critical Thinking Two boxes of equal mass slide start from rest and slide with different horizontal applied forces across frictionless surfaces to reach the same distance away. Box A has a horizontal force of three times that of box B. (a) Which box is traveling faster at this distance away from the starting point? Briefly explain your reasoning. (b) Based on experimental data, an equation that fits the data is suggested for the final speed,  $V$ , which may not be correct:  $V = KF$ , where  $K$  is a constant with appropriate units and  $F$  is the horizontal applied force. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Now the applied force can be any value and the distance from the starting point is measured when the speed gets to a consistent value. Graph the distance the box gets to at a given speed as a function of the applied force.



## CHAPTER 5

# Further Applications of Newton's Laws: Friction, Drag, and Elasticity



**FIGURE 5.1** Total hip replacement surgery has become a common procedure. The head (or ball) of the patient's femur fits into a cup that has a hard plastic-like inner lining. (credit: National Institutes of Health, via Wikimedia Commons)

### CHAPTER OUTLINE

#### 5.1 Friction

#### 5.2 Drag Forces

#### 5.3 Elasticity: Stress and Strain

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**INTRODUCTION: FURTHER APPLICATIONS OF NEWTON'S LAWS** Describe the forces on the hip joint. What means are taken to ensure that this will be a good movable joint? From the photograph (for an adult) in [Figure 5.1](#), estimate the dimensions of the artificial device.

It is difficult to categorize forces into various types (aside from the four basic forces discussed in previous chapter). We know that a net force affects the motion, position, and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton's laws of motion. We have in mind the forces of friction, air or liquid drag, and deformation.

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## 5.1 Friction

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

**Friction** is a force that is around us all the time that opposes relative motion between surfaces in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

### Friction

Friction is a force that opposes relative motion between surfaces in contact.

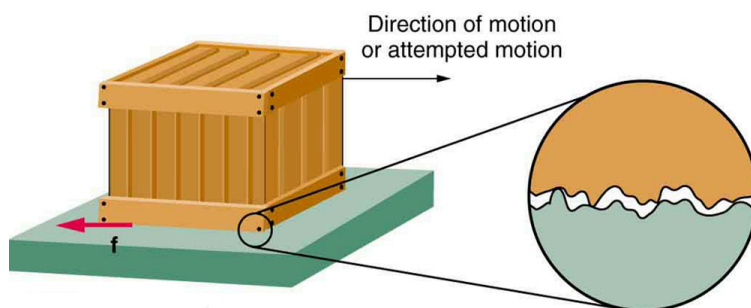
One of the simpler characteristics of friction is that it is parallel to the contact surface between surfaces and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two surfaces are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the surfaces.

### Kinetic Friction

If two surfaces are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

[Figure 5.2](#) is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



**FIGURE 5.2** Frictional forces, such as  $f$ , always oppose motion or attempted motion between surfaces in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction  $f_s$**  is

$$f_s \leq \mu_s N, \quad 5.1$$

where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force (the force perpendicular to the surface).

### Magnitude of Static Friction

Magnitude of static friction  $f_s$  is

$$f_s \leq \mu_s N, \quad 5.2$$

where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force.

The symbol  $\leq$  means *less than or equal to*, implying that static friction can have a minimum and a maximum value of  $\mu_s N$ . Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds  $f_{s(\max)}$ , the object will move. Thus

$$f_{s(\max)} = \mu_s N. \quad 5.3$$

Once an object is moving, the **magnitude of kinetic friction  $f_k$**  is given by

$$f_k = \mu_k N, \quad 5.4$$

where  $\mu_k$  is the coefficient of kinetic friction. A system in which  $f_k = \mu_k N$  is described as a system in which *friction behaves simply*.

### Magnitude of Kinetic Friction

The magnitude of kinetic friction  $f_k$  is given by

$$f_k = \mu_k N, \quad 5.5$$

where  $\mu_k$  is the coefficient of kinetic friction.

As seen in [Table 5.1](#), the coefficients of kinetic friction are less than their static counterparts. That values of  $\mu$  in [Table 5.1](#) are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	Static friction $\mu_s$	Kinetic friction $\mu_k$
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.04	0.02

**TABLE 5.1** Coefficients of Static and Kinetic Friction

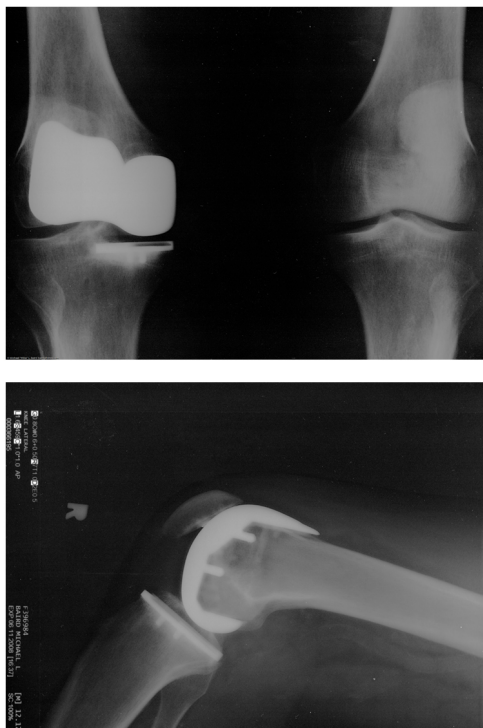
The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight,  $W = mg = (100 \text{ kg})(9.80 \text{ m/s}^2) = 980 \text{ N}$ , perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than  $f_{s(\text{max})} = \mu_s N = (0.45)(980 \text{ N}) = 440 \text{ N}$  to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ( $f_k = \mu_k N = (0.30)(980 \text{ N}) = 290 \text{ N}$ ) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unitless quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

### Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia)

and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (Figure 5.3). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.



**FIGURE 5.3** Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op X-rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to move freely over the skin.

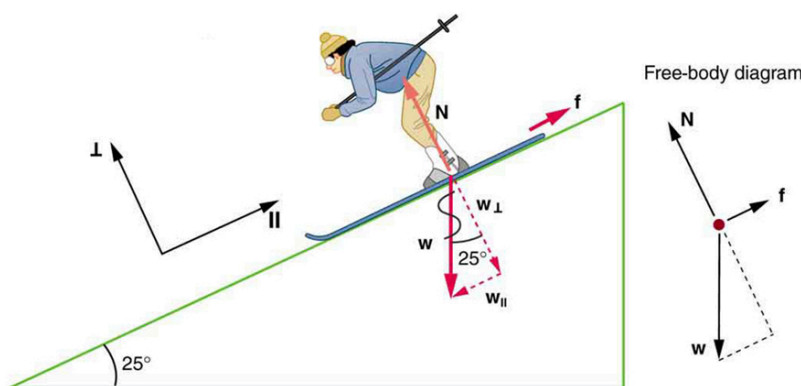
### **EXAMPLE 5.1**

#### **Skiing Exercise**

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

#### **Strategy**

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force  $N$  as  $f_k = \mu_k N$ ; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in Figure 5.4.)



**FIGURE 5.4** The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier).  $\mathbf{N}$  (the normal force) is perpendicular to the slope, and  $\mathbf{f}$  (the friction) is parallel to the slope, but  $\mathbf{w}$  (the skier's weight) has components along both axes, namely  $w_{\perp}$  and  $w_{\parallel}$ .  $\mathbf{N}$  is equal in magnitude to  $w_{\perp}$ , so there is no motion perpendicular to the slope. However,  $\mathbf{f}$  is less than  $w_{\parallel}$  in magnitude, so there is acceleration down the slope (along the  $x$ -axis).

That is,

$$N = w_{\perp} = w \cos 25^{\circ} = mg \cos 25^{\circ}. \quad 5.6$$

Substituting this into our expression for kinetic friction, we get

$$f_k = \mu_k mg \cos 25^{\circ}, \quad 5.7$$

which can now be solved for the coefficient of kinetic friction  $\mu_k$ .

### Solution

Solving for  $\mu_k$  gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w \cos 25^{\circ}} = \frac{f_k}{mg \cos 25^{\circ}}. \quad 5.8$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0 \text{ N}}{(62 \text{ kg})(9.80 \text{ m/s}^2)(0.906)} = 0.082. \quad 5.9$$

### Discussion

This result is a little smaller than the coefficient listed in [Table 5.1](#) for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass  $m$  slides down a slope that makes an angle  $\theta$  with the horizontal, friction is given by  $f_k = \mu_k mg \cos \theta$ . All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

### Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in [Example 5.1](#), the kinetic friction on a slope  $f_k = \mu_k mg \cos \theta$ . The component of the weight down the slope is equal to  $mg \sin \theta$  (see the free-body diagram in [Figure 5.4](#)). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_k = mg_x \quad 5.10$$

$$\mu_k mg \cos \theta = mg \sin \theta. \quad 5.11$$

Solving for  $\mu_k$ , we find that

$$\mu_k = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta. \quad 5.12$$

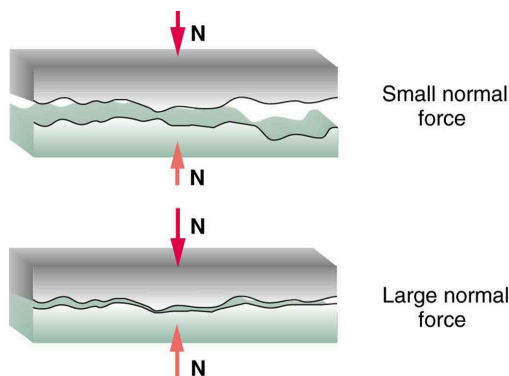
Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find  $\mu_k$ . Note that the coin will not start to slide at all until an angle greater than  $\theta$  is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for  $\mu_k$  and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

### Making Connections: Submicroscopic Explanations of Friction

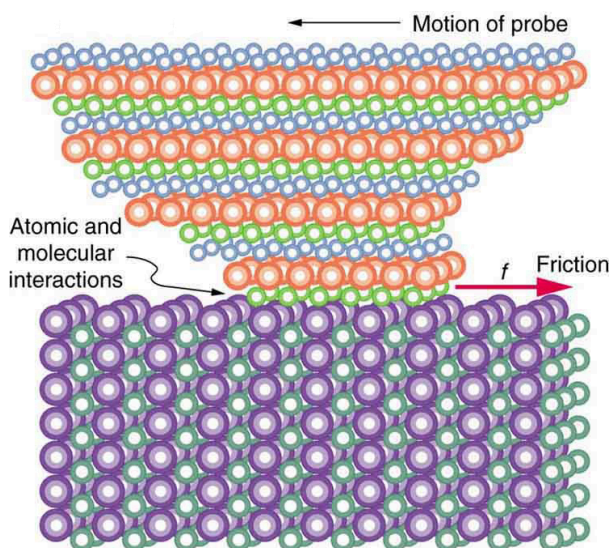
The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

[Figure 5.5](#) illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



**FIGURE 5.5** Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. [Figure 5.6](#) shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of  $10^{12}$ ) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



**FIGURE 5.6** The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.



## PHET EXPLORATIONS

### Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/5-1-friction\)](https://openstax.org/books/college-physics-2e/pages/5-1-friction)



## 5.2 Drag Forces

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Express mathematically the drag force.
- Discuss the applications of drag force.
- Define terminal velocity.
- Determine the terminal velocity given mass.

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force  $F_D$  is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as  $F_D \propto v^2$ . When taking into account other factors, this relationship becomes

$$F_D = \frac{1}{2}C\rho Av^2, \quad 5.13$$

where  $C$  is the drag coefficient,  $A$  is the area of the object facing the fluid, and  $\rho$  is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as  $F_D = bv^2$ , where  $b$  is a constant equivalent to  $0.5C\rho A$ . We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the

speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

### Drag Force

Drag force  $F_D$  is found to be proportional to the square of the speed of the object. Mathematically

$$F_D \propto v^2 \quad 5.14$$

$$F_D = \frac{1}{2} C \rho A v^2, \quad 5.15$$

where  $C$  is the drag coefficient,  $A$  is the area of the object facing the fluid, and  $\rho$  is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See [Figure 5.7](#)). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.



**FIGURE 5.7** From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient,  $C$ , is determined empirically, usually with the use of a wind tunnel. (See [Figure 5.8](#)).



**FIGURE 5.8** NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. [Table 5.2](#) lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Object	$C$
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

**TABLE 5.2** Drag Coefficient Values Typical values of drag coefficient  $C$ .

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See [Figure 5.9](#)). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.



**FIGURE 5.9** Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother “skin” and more compression forces on a swimmer’s body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* ( $v_t$ ). Since  $F_D$  is proportional to the speed, a heavier skydiver must go faster for  $F_D$  to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{\text{net}} = mg - F_D = ma = 0. \quad 5.16$$

Thus,

$$mg = F_D. \quad 5.17$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}\rho CA v^2. \quad 5.18$$

Solving for the velocity, we obtain

$$v = \sqrt{\frac{2mg}{\rho CA}}. \quad 5.19$$

Assume the density of air is  $\rho = 1.21 \text{ kg/m}^3$ . A 75-kg skydiver descending head first will have an area approximately  $A = 0.18 \text{ m}^2$  and a drag coefficient of approximately  $C = 0.70$ . We find that

$$\begin{aligned} v &= \sqrt{\frac{2(75 \text{ kg})(9.80 \text{ m/s}^2)}{(1.21 \text{ kg/m}^3)(0.70)(0.18 \text{ m}^2)}} \\ &= 98 \text{ m/s} \\ &= 350 \text{ km/h}. \end{aligned} \quad 5.20$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a headfirst position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

### Take-Home Experiment

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity  $v$  versus mass. Also plot  $v^2$  versus mass. Which of these relationships is more linear? What can you conclude from these graphs?



### EXAMPLE 5.2

#### A Terminal Velocity

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

#### Strategy

At terminal velocity,  $F_{\text{net}} = 0$ . Thus the drag force on the skydiver must equal the force of gravity (the person's

weight). Using the equation of drag force, we find  $mg = \frac{1}{2}\rho CAv^2$ .

Thus the terminal velocity  $v_t$  can be written as

$$v_t = \sqrt{\frac{2mg}{\rho CA}}. \quad 5.21$$

### Solution

All quantities are known except the person's projected area. This is an adult (85 kg) falling spread eagle. We can estimate the frontal area as

$$A = (2 \text{ m})(0.35 \text{ m}) = 0.70 \text{ m}^2. \quad 5.22$$

Using our equation for  $v_t$ , we find that

$$\begin{aligned} v_t &= \sqrt{\frac{2(85 \text{ kg})(9.80 \text{ m/s}^2)}{(1.21 \text{ kg/m}^3)(1.0)(0.70 \text{ m}^2)}} \\ &= 44 \text{ m/s}. \end{aligned} \quad 5.23$$

### Discussion

This result is consistent with the value for  $v_t$  mentioned earlier. The 75-kg skydiver going feet first had a  $v = 98 \text{ m/s}$ . He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled "On Being the Right Size."

*To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.*

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by **Stokes' law**, which states that

$$F_s = 6\pi r\eta v, \quad 5.24$$

where  $r$  is the radius of the object,  $\eta$  is the viscosity of the fluid, and  $v$  is the object's velocity.

#### Stokes' Law

$$F_s = 6\pi r\eta v, \quad 5.25$$

where  $r$  is the radius of the object,  $\eta$  is the viscosity of the fluid, and  $v$  is the object's velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about  $1 \mu\text{m}$ ) can be about  $2 \mu\text{m/s}$ . To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about  $5 \mu\text{m/s}$ ), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see [Figure 5.10](#)). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.



**FIGURE 5.10** Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

### Galileo's Experiment

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren't readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

## 5.3 Elasticity: Stress and Strain

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State Hooke's law.
- Explain Hooke's law using graphical representation between deformation and applied force.
- Discuss the three types of deformations such as changes in length, sideways shear and changes in volume.
- Describe with examples the young's modulus, shear modulus and bulk modulus.
- Determine the change in length given mass, length and radius.

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, **Hooke's law** is given by

$$F = k\Delta L, \quad 5.26$$

where  $\Delta L$  is the amount of deformation (the change in length, for example) produced by the force  $F$ , and  $k$  is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation  $\Delta L$ —it is not constant as a kinetic friction force is. Rearranging this to

$$\Delta L = \frac{F}{k} \quad 5.27$$

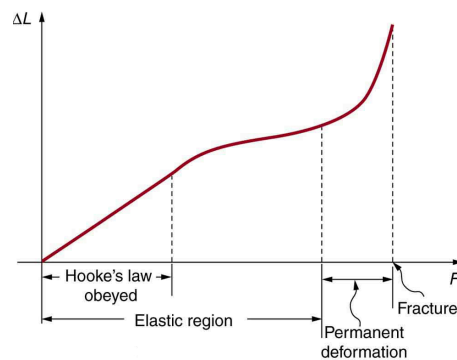
makes it clear that the deformation is proportional to the applied force. [Figure 5.11](#) shows the Hooke's law relationship between the extension  $\Delta L$  of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

### Hooke's Law

$$F = k\Delta L, \quad 5.28$$

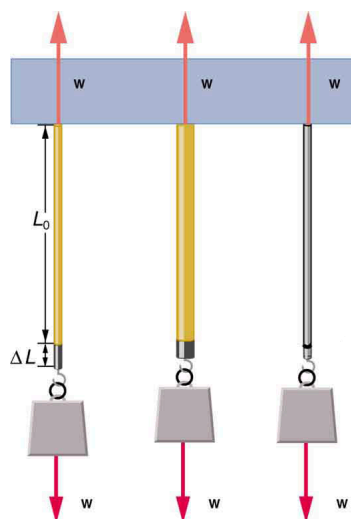
where  $\Delta L$  is the amount of deformation (the change in length, for example) produced by the force  $F$ , and  $k$  is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\Delta L = \frac{F}{k} \quad 5.29$$



**FIGURE 5.11** A graph of deformation  $\Delta L$  versus applied force  $F$ . The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is  $\frac{1}{k}$ . For larger forces, the graph is curved but the deformation is still elastic— $\Delta L$  will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force  $F$  is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in  $F$  is producing a large increase in  $L$  near the fracture.

The proportionality constant  $k$  depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation  $\Delta L$  is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger  $k$  (see [Figure 5.12](#)). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in  $10^3$ .



**FIGURE 5.12** The same force, in this case a weight ( $w$ ), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on

the right is steel.

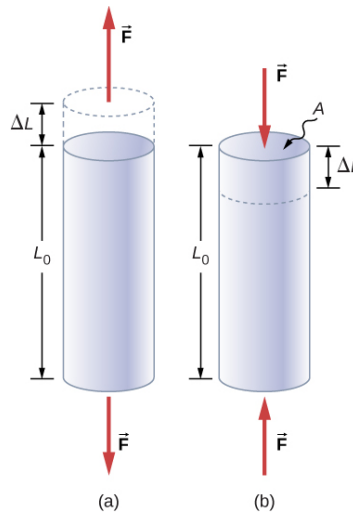
### Stretch Yourself a Little

How would you go about measuring the proportionality constant  $k$  of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

We now consider three specific types of deformations: changes in length (tension and compression), sideways shear (stress), and changes in volume. All deformations are assumed to be small unless otherwise stated.

### Changes in Length—Tension and Compression: Elastic Modulus

A change in length  $\Delta L$  is produced when a force is applied to a wire or rod parallel to its length  $L_0$ , either stretching it (a tension) or compressing it. (See [Figure 5.13](#).)



**FIGURE 5.13** (a) Tension. The rod is stretched a length  $\Delta L$  when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials,  $\Delta L$  is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Experiments have shown that the change in length ( $\Delta L$ ) depends on only a few variables. As already noted,  $\Delta L$  is proportional to the force  $F$  and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length  $L_0$  and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for  $\Delta L$ :

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_0, \quad 5.30$$

where  $\Delta L$  is the change in length,  $F$  the applied force,  $Y$  is a factor, called the elastic modulus or Young's modulus, that depends on the substance,  $A$  is the cross-sectional area, and  $L_0$  is the original length. [Table 5.3](#) lists values of  $Y$  for several materials—those with a large  $Y$  are said to have a large tensile stiffness because they deform less for a given tension or compression.

Material	Young's modulus (tension–compression) $Y$ ( $10^9$ N/m <sup>2</sup> )	Shear modulus $S$ ( $10^9$ N/m <sup>2</sup> )	Bulk modulus $B$ ( $10^9$ N/m <sup>2</sup> )
Aluminum	70	25	75
Bone – tension	16	80	8
Bone – compression	9		
Brass	90	35	75
Brick	15		
Concrete	20		
Glass	70	20	30
Granite	45	20	45
Hair (human)	10		
Hardwood	15	10	
Iron, cast	100	40	90
Lead	16	5	50
Marble	60	20	70
Nylon	5		
Polystyrene	3		
Silk	6		
Spider thread	3		
Steel	210	80	130
Tendon	1		
Acetone			0.7
Ethanol			0.9
Glycerin			4.5

**TABLE 5.3** Elastic Moduli<sup>1</sup>

<sup>1</sup> Approximate and average values. Young's moduli  $Y$  for tension and compression sometimes differ but are averaged here. Bone has significantly different Young's moduli for tension and compression.

Material	Young's modulus (tension–compression) $Y$ ( $10^9$ N/m <sup>2</sup> )	Shear modulus $S$ ( $10^9$ N/m <sup>2</sup> )	Bulk modulus $B$ ( $10^9$ N/m <sup>2</sup> )
Mercury			25
Water			2.2

**TABLE 5.3** Elastic Moduli<sup>1</sup>

Young's moduli are not listed for liquids and gases in [Table 5.3](#) because they cannot be stretched or compressed in only one direction. Note that there is an assumption that the object does not accelerate, so that there are actually two applied forces of magnitude  $F$  acting in opposite directions. For example, the strings in [Figure 5.13](#) are being pulled down by a force of magnitude  $w$  and held up by the ceiling, which also exerts a force of magnitude  $w$ .

### EXAMPLE 5.3

#### The Stretch of a Long Cable

Suspension cables are used to carry gondolas at ski resorts. (See [Figure 5.14](#)) Consider a suspension cable that includes an unsupported span of 3020 m. Calculate the amount of stretch in the steel cable. Assume that the cable has a diameter of 5.6 cm and the maximum tension it can withstand is  $3.0 \times 10^6$  N.



**FIGURE 5.14** Gondolas travel along suspension cables at the Gala Yuzawa ski resort in Japan. (credit: Rudy Herman, Flickr)

#### Strategy

The force is equal to the maximum tension, or  $F = 3.0 \times 10^6$  N. The cross-sectional area is  $\pi r^2 = 2.46 \times 10^{-3}$  m<sup>2</sup>. The equation  $\Delta L = \frac{1}{Y} \frac{F}{A} L_0$  can be used to find the change in length.

#### Solution

All quantities are known. Thus,

$$\begin{aligned} \Delta L &= \left( \frac{1}{210 \times 10^9 \text{ N/m}^2} \right) \left( \frac{3.0 \times 10^6 \text{ N}}{2.46 \times 10^{-3} \text{ m}^2} \right) (3020 \text{ m}) \\ &= 18 \text{ m.} \end{aligned} \quad \mathbf{5.31}$$

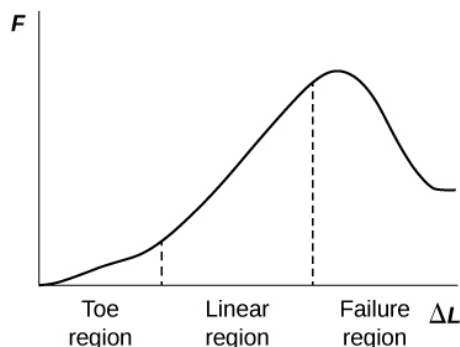
#### Discussion

This is quite a stretch, but only about 0.6% of the unsupported length. Effects of temperature upon length might be important in these environments.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is

arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Another biological example of Hooke's law occurs in tendons. Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. Figure 5.15 shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.



**FIGURE 5.15** Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling exactly this—the elastic behavior of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out. Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

### EXAMPLE 5.4

#### Calculating Deformation: How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

#### Strategy

The force is equal to the weight supported, or

$$F = mg = (62.0 \text{ kg})(9.80 \text{ m/s}^2) = 607.6 \text{ N}, \quad 5.32$$

and the cross-sectional area is  $\pi r^2 = 1.257 \times 10^{-3} \text{ m}^2$ . The equation  $\Delta L = \frac{1}{Y} \frac{F}{A} L_0$  can be used to find the change in length.

**Solution**

All quantities except  $\Delta L$  are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\begin{aligned}\Delta L &= \left( \frac{1}{9 \times 10^9 \text{ N/m}^2} \right) \left( \frac{607.6 \text{ N}}{1.257 \times 10^{-3} \text{ m}^2} \right) (0.400 \text{ m}) \\ &= 2 \times 10^{-5} \text{ m.}\end{aligned}\tag{5.33}$$

**Discussion**

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in [Table 5.3](#) have larger values of Young's modulus  $Y$ . In other words, they are more rigid.

The equation for change in length is traditionally rearranged and written in the following form:

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}.\tag{5.34}$$

The ratio of force to area,  $\frac{F}{A}$ , is defined as **stress** (measured in  $\text{N/m}^2$ ), and the ratio of the change in length to length,  $\frac{\Delta L}{L_0}$ , is defined as **strain** (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.\tag{5.35}$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form

$$F = YA \frac{\Delta L}{L_0},\tag{5.36}$$

we see that it is the same as Hooke's law with a proportionality constant

$$k = \frac{YA}{L_0}.\tag{5.37}$$

This general idea—that force and the deformation it causes are proportional for small deformations—applies to changes in length, sideways bending, and changes in volume.

**Stress**

The ratio of force to area,  $\frac{F}{A}$ , is defined as stress measured in  $\text{N/m}^2$ .

**Strain**

The ratio of the change in length to length,  $\frac{\Delta L}{L_0}$ , is defined as strain (a unitless quantity). In other words,

$$\text{stress} = Y \times \text{strain}.\tag{5.38}$$

**Sideways Stress: Shear Modulus**

[Figure 5.16](#) illustrates what is meant by a sideways stress or a *shearing force*. Here the deformation is called  $\Delta x$  and it is perpendicular to  $L_0$ , rather than parallel as with tension and compression. Shear deformation behaves similarly to tension and compression and can be described with similar equations. The expression for **shear deformation** is

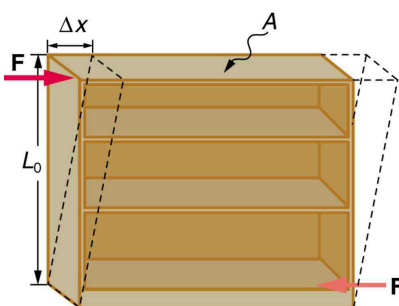
$$\Delta x = \frac{1}{S} \frac{F}{A} L_0,\tag{5.39}$$

where  $S$  is the shear modulus (see [Table 5.3](#)) and  $F$  is the force applied perpendicular to  $L_0$  and parallel to the cross-sectional area  $A$ . Again, to keep the object from accelerating, there are actually two equal and opposite forces  $F$  applied across opposite faces, as illustrated in [Figure 5.16](#). The equation is logical—for example, it is easier to bend a long thin pencil (small  $A$ ) than a short thick one, and both are more easily bent than similar steel rods (large  $S$ ).

### Shear Deformation

$$\Delta x = \frac{1}{S} \frac{F}{A} L_0, \quad 5.40$$

where  $S$  is the shear modulus and  $F$  is the force applied perpendicular to  $L_0$  and parallel to the cross-sectional area  $A$ .



**FIGURE 5.16** Shearing forces are applied perpendicular to the length  $L_0$  and parallel to the area  $A$ , producing a deformation  $\Delta x$ . Vertical forces are not shown, but it should be kept in mind that in addition to the two shearing forces,  $F$ , there must be supporting forces to keep the object from rotating. The distorting effects of these supporting forces are ignored in this treatment. The weight of the object also is not shown, since it is usually negligible compared with forces large enough to cause significant deformations.

Examination of the shear moduli in [Table 5.3](#) reveals some telling patterns. For example, shear moduli are less than Young's moduli for most materials. Bone is a remarkable exception. Its shear modulus is not only greater than its Young's modulus, but it is as large as that of steel. This is why bones are so rigid.

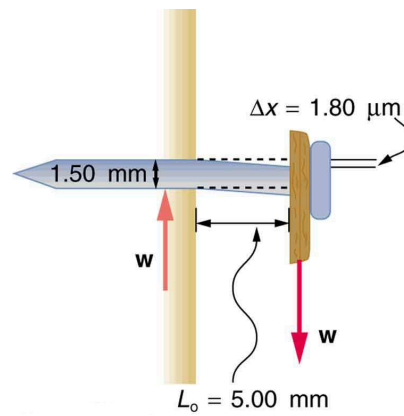
The spinal column (consisting of 26 vertebral segments separated by discs) provides the main support for the head and upper part of the body. The spinal column has normal curvature for stability, but this curvature can be increased, leading to increased shearing forces on the lower vertebrae. Discs are better at withstanding compressional forces than shear forces. Because the spine is not vertical, the weight of the upper body exerts some of both. Pregnant women and people that are overweight (with large abdomens) need to move their shoulders back to maintain balance, thereby increasing the curvature in their spine and so increasing the shear component of the stress. An increased angle due to more curvature increases the shear forces along the plane. These higher shear forces increase the risk of back injury through ruptured discs. The lumbosacral disc (the wedge shaped disc below the last vertebrae) is particularly at risk because of its location.

The shear moduli for concrete and brick are very small; they are too highly variable to be listed. Concrete used in buildings can withstand compression, as in pillars and arches, but is very poor against shear, as might be encountered in heavily loaded floors or during earthquakes. Modern structures were made possible by the use of steel and steel-reinforced concrete. Almost by definition, liquids and gases have shear moduli near zero, because they flow in response to shearing forces.

### EXAMPLE 5.5

#### Calculating Force Required to Deform: That Nail Does Not Bend Much Under a Load

Find the mass of the picture hanging from a steel nail as shown in [Figure 5.17](#), given that the nail bends only  $1.80 \mu\text{m}$ . (Assume the shear modulus is known to two significant figures.)



**FIGURE 5.17** Side view of a nail with a picture hung from it. The nail flexes very slightly (shown much larger than actual) because of the shearing effect of the supported weight. Also shown is the upward force of the wall on the nail, illustrating that there are equal and opposite forces applied across opposite cross sections of the nail. See [Example 5.5](#) for a calculation of the mass of the picture.

### Strategy

The force  $F$  on the nail (neglecting the nail's own weight) is the weight of the picture  $w$ . If we can find  $w$ , then the mass of the picture is just  $\frac{w}{g}$ . The equation  $\Delta x = \frac{1}{S} \frac{F}{A} L_0$  can be solved for  $F$ .

### Solution

Solving the equation  $\Delta x = \frac{1}{S} \frac{F}{A} L_0$  for  $F$ , we see that all other quantities can be found:

$$F = \frac{SA}{L_0} \Delta x. \quad 5.41$$

$S$  is found in [Table 5.3](#) and is  $S = 80 \times 10^9 \text{ N/m}^2$ . The radius  $r$  is 0.750 mm (as seen in the figure), so the cross-sectional area is

$$A = \pi r^2 = 1.77 \times 10^{-6} \text{ m}^2. \quad 5.42$$

The value for  $L_0$  is also shown in the figure. Thus,

$$F = \frac{(80 \times 10^9 \text{ N/m}^2)(1.77 \times 10^{-6} \text{ m}^2)}{(5.00 \times 10^{-3} \text{ m})} (1.80 \times 10^{-6} \text{ m}) = 51 \text{ N}. \quad 5.43$$

This 51 N force is the weight  $w$  of the picture, so the picture's mass is

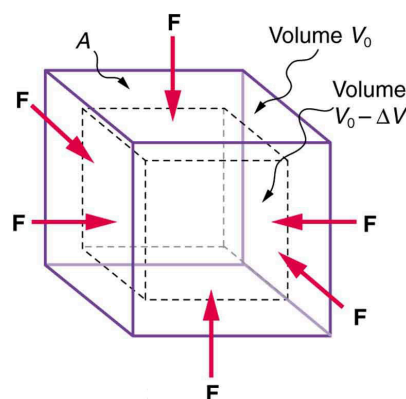
$$m = \frac{w}{g} = \frac{F}{g} = 5.2 \text{ kg}. \quad 5.44$$

### Discussion

This is a fairly massive picture, and it is impressive that the nail flexes only  $1.80 \mu\text{m}$ —an amount undetectable to the unaided eye.

## Changes in Volume: Bulk Modulus

An object will be compressed in all directions if inward forces are applied evenly on all its surfaces as in [Figure 5.18](#). It is relatively easy to compress gases and extremely difficult to compress liquids and solids. For example, air in a wine bottle is compressed when it is corked. But if you try corking a brim-full bottle, you cannot compress the wine—some must be removed if the cork is to be inserted. The reason for these different compressibilities is that atoms and molecules are separated by large empty spaces in gases but packed close together in liquids and solids. To compress a gas, you must force its atoms and molecules closer together. To compress liquids and solids, you must actually compress their atoms and molecules, and very strong electromagnetic forces in them oppose this compression.



**FIGURE 5.18** An inward force on all surfaces compresses this cube. Its change in volume is proportional to the force per unit area and its original volume, and is related to the compressibility of the substance.

We can describe the compression or volume deformation of an object with an equation. First, we note that a force “applied evenly” is defined to have the same stress, or ratio of force to area  $\frac{F}{A}$  on all surfaces. The deformation produced is a change in volume  $\Delta V$ , which is found to behave very similarly to the shear, tension, and compression previously discussed. (This is not surprising, since a compression of the entire object is equivalent to compressing each of its three dimensions.) The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0, \quad 5.45$$

where  $B$  is the bulk modulus (see [Table 5.3](#)),  $V_0$  is the original volume, and  $\frac{F}{A}$  is the force per unit area applied uniformly inward on all surfaces. Note that no bulk moduli are given for gases.

What are some examples of bulk compression of solids and liquids? One practical example is the manufacture of industrial-grade diamonds by compressing carbon with an extremely large force per unit area. The carbon atoms rearrange their crystalline structure into the more tightly packed pattern of diamonds. In nature, a similar process occurs deep underground, where extremely large forces result from the weight of overlying material. Another natural source of large compressive forces is the pressure created by the weight of water, especially in deep parts of the oceans. Water exerts an inward force on all surfaces of a submerged object, and even on the water itself. At great depths, water is measurably compressed, as the following example illustrates.

### EXAMPLE 5.6

#### Calculating Change in Volume with Deformation: How Much Is Water Compressed at Great Ocean Depths?

Calculate the fractional decrease in volume ( $\frac{\Delta V}{V_0}$ ) for seawater at 5.00 km depth, where the force per unit area is  $5.00 \times 10^7 \text{ N/m}^2$ .

#### Strategy

Equation  $\Delta V = \frac{1}{B} \frac{F}{A} V_0$  is the correct physical relationship. All quantities in the equation except  $\frac{\Delta V}{V_0}$  are known.

#### Solution

Solving for the unknown  $\frac{\Delta V}{V_0}$  gives

$$\frac{\Delta V}{V_0} = \frac{1}{B} \frac{F}{A}. \quad 5.46$$

Substituting known values with the value for the bulk modulus  $B$  from [Table 5.3](#),

$$\begin{aligned}\frac{\Delta V}{V_0} &= \frac{5.00 \times 10^7 \text{ N/m}^2}{2.2 \times 10^9 \text{ N/m}^2} \\ &= 0.023 = 2.3\%.\end{aligned}$$
5.47

### Discussion

Although measurable, this is not a significant decrease in volume considering that the force per unit area is about 500 atmospheres (1 million pounds per square foot). Liquids and solids are extraordinarily difficult to compress.

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Conversely, very large forces are created by liquids and solids when they try to expand but are constrained from doing so—which is equivalent to compressing them to less than their normal volume. This often occurs when a contained material warms up, since most materials expand when their temperature increases. If the materials are tightly constrained, they deform or break their container. Another very common example occurs when water freezes. Water, unlike most materials, expands when it freezes, and it can easily fracture a boulder, rupture a biological cell, or crack an engine block that gets in its way.

Other types of deformations, such as torsion or twisting, behave analogously to the tension, shear, and bulk deformations considered here.



## PHET EXPLORATIONS

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### Masses & Springs

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/5-3-elasticity-stress-and-strain\)](https://openstax.org/books/college-physics-2e/pages/5-3-elasticity-stress-and-strain)



## Glossary

**deformation** change in shape due to the application of force

**drag force**  $F_D$ , found to be proportional to the square of the speed of the object; mathematically  $F_D \propto v^2$

$$F_D = \frac{1}{2} C \rho A v^2,$$

where  $C$  is the drag coefficient,  $A$  is the area of the object facing the fluid, and  $\rho$  is the density of the fluid

**friction** a force that opposes relative motion or attempts at motion between systems in contact

**Hooke's law** proportional relationship between the force  $F$  on a material and the deformation  $\Delta L$  it causes,  $F = k\Delta L$

**kinetic friction** a force that opposes the motion of two systems that are in contact and moving relative

to one another

**magnitude of kinetic friction**  $f_k = \mu_k N$ , where  $\mu_k$  is the coefficient of kinetic friction

**magnitude of static friction**  $f_s \leq \mu_s N$ , where  $\mu_s$  is the coefficient of static friction and  $N$  is the magnitude of the normal force

**shear deformation** deformation perpendicular to the original length of an object

**static friction** a force that opposes the motion of two systems that are in contact and are not moving relative to one another

**Stokes' law**  $F_s = 6\pi r\eta v$ , where  $r$  is the radius of the object,  $\eta$  is the viscosity of the fluid, and  $v$  is the object's velocity

**strain** ratio of change in length to original length

**stress** ratio of force to area

**tensile strength** the breaking stress that will cause permanent deformation or fracture of a material

## Section Summary

### 5.1 Friction

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force  $N$  pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction  $f_s$  between systems stationary relative to one another is given by

$$f_s \leq \mu_s N,$$

where  $\mu_s$  is the coefficient of static friction, which depends on both of the materials.

- The kinetic friction force  $f_k$  between systems moving relative to one another is given by

$$f_k = \mu_k N,$$

where  $\mu_k$  is the coefficient of kinetic friction, which also depends on both materials.

### 5.2 Drag Forces

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity  $v$  in air, the drag force is given by

$$F_D = \frac{1}{2} C \rho A v^2,$$

where  $C$  is the drag coefficient (typical values are given in [Table 5.2](#)),  $A$  is the area of the object facing the fluid, and  $\rho$  is the fluid density.

- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is

given by Stokes' law,

$$F_s = 6\pi r\eta v,$$

where  $r$  is the radius of the object,  $\eta$  is the fluid viscosity, and  $v$  is the object's velocity.

### 5.3 Elasticity: Stress and Strain

- Hooke's law is given by

$$F = k\Delta L,$$

where  $\Delta L$  is the amount of deformation (the change in length),  $F$  is the applied force, and  $k$  is a proportionality constant that depends on the shape and composition of the object and the direction of the force. The relationship between the deformation and the applied force can also be written as

$$\Delta L = \frac{1}{Y} \frac{F}{A} L_0,$$

where  $Y$  is *Young's modulus*, which depends on the substance,  $A$  is the cross-sectional area, and  $L_0$  is the original length.

- The ratio of force to area,  $\frac{F}{A}$ , is defined as *stress*, measured in  $\text{N/m}^2$ .
- The ratio of the change in length to length,  $\frac{\Delta L}{L_0}$ , is defined as *strain* (a unitless quantity). In other words,  $\text{stress} = Y \times \text{strain}$ .
- The expression for shear deformation is  $\Delta x = \frac{1}{S} \frac{F}{A} L_0$ , where  $S$  is the shear modulus and  $F$  is the force

applied perpendicular to  $L_0$  and parallel to the cross-sectional area  $A$ .

- The relationship of the change in volume to other physical quantities is given by

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0,$$

where  $B$  is the bulk modulus,  $V_0$  is the original volume, and  $\frac{F}{A}$  is the force per unit area applied uniformly inward on all surfaces.

## Conceptual Questions

### 5.1 Friction

1. Define normal force. What is its relationship to friction when friction behaves simply?
2. The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.
3. When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.
4. When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

### 5.2 Drag Forces

5. Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.
6. Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?
7. As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

## Problems & Exercises

### 5.1 Friction

1. A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

8. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

### 5.3 Elasticity: Stress and Strain

9. The elastic properties of the arteries are essential for blood flow. Explain the importance of this in terms of the characteristics of the flow of blood (pulsating or continuous).
10. What are you feeling when you feel your pulse? Measure your pulse rate for 10 s and for 1 min. Is there a factor of 6 difference?
11. Examine different types of shoes, including sports shoes and thongs. In terms of physics, why are the bottom surfaces designed as they are? What differences will dry and wet conditions make for these surfaces?
12. Would you expect your height to be different depending upon the time of day? Why or why not?
13. Would you expect a large or small stress to be required to deform a spider web? Why is this elasticity an important feature for a spider web?
14. Explain why pregnant women often suffer from back strain late in their pregnancy.
15. An old carpenter's trick to keep nails from bending when they are pounded into hard materials is to grip the center of the nail firmly with pliers. Why does this help?
16. When a glass bottle full of vinegar warms up, both the vinegar and the glass expand, but vinegar expands significantly more with temperature than glass. The bottle will break if it was filled to its tightly capped lid. Explain why, and also explain how a pocket of air above the vinegar would prevent the break. (This is the function of the air above liquids in glass containers.)

2. (a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

- (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.
- Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?
- (a) If half of the weight of a small  $1.00 \times 10^3$  kg utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.
- A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

- Consider the 65.0-kg ice skater being pushed by two others shown in [Figure 5.19](#). (a) Find the direction and magnitude of  $\mathbf{F}_{\text{tot}}$ , the total force exerted on her by the others, given that the magnitudes  $F_1$  and  $F_2$  are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of  $\mathbf{F}_{\text{tot}}$ ? (c) What is her acceleration assuming she is already moving in the direction of  $\mathbf{F}_{\text{tot}}$ ? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

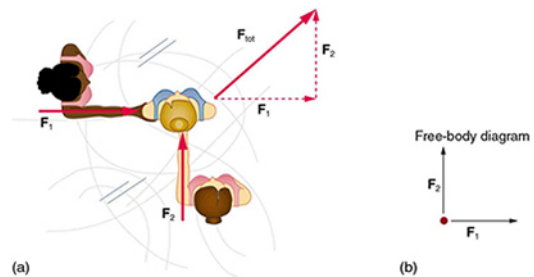
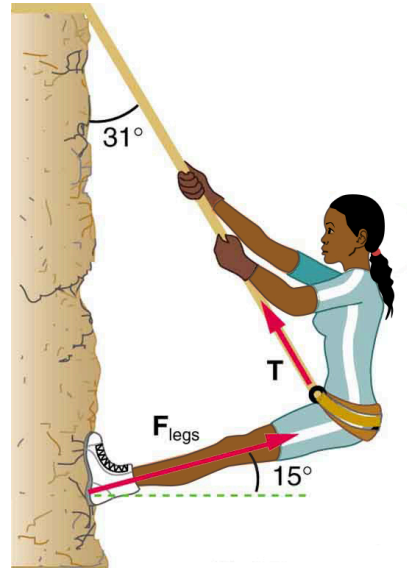


FIGURE 5.19

- Show that the acceleration of any object down a frictionless incline that makes an angle  $\theta$  with the horizontal is  $a = g \sin \theta$ . (Note that this acceleration is independent of mass.)
- Show that the acceleration of any object down an incline where friction behaves simply (that is, where  $f_k = \mu_k N$ ) is  $a = g(\sin \theta - \mu_k \cos \theta)$ . Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small ( $\mu_k = 0$ ).
- Calculate the deceleration of a snow boarder going up a  $5.0^\circ$  slope assuming the coefficient of friction for waxed wood on wet snow. The result of [Exercise 5.9](#) may be useful, but be careful to consider the fact that the snow boarder is going uphill. Explicitly show how you follow the steps in [Problem-Solving Strategies](#).
- (a) Calculate the acceleration of a skier heading down a  $10.0^\circ$  slope, assuming the coefficient of friction for waxed wood on wet snow. (b) Find the angle of the slope down which this skier could coast at a constant velocity. You can neglect air resistance in both parts, and you will find the result of [Exercise 5.9](#) to be useful. Explicitly show how you follow the steps in the [Problem-Solving Strategies](#).

12. If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is  $\theta = \tan^{-1} \mu_s$ . You may use the result of the previous problem. Assume that  $a = 0$  and that static friction has reached its maximum value.
13. Calculate the maximum deceleration of a car that is heading down a  $6^\circ$  slope (one that makes an angle of  $6^\circ$  with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that  $\mu_s = 0.100$ , the same as for shoes on ice.
14. Calculate the maximum acceleration of a car that is heading up a  $4^\circ$  slope (one that makes an angle of  $4^\circ$  with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that  $\mu_s = 0.100$ , the same as for shoes on ice.
15. Repeat [Exercise 5.14](#) for a car with four-wheel drive.
16. A freight train consists of two  $8.00 \times 10^5$ -kg engines and 45 cars with average masses of  $5.50 \times 10^5$  kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of  $5.00 \times 10^{-2} \text{ m/s}^2$  if the force of friction is  $7.50 \times 10^5 \text{ N}$ , assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

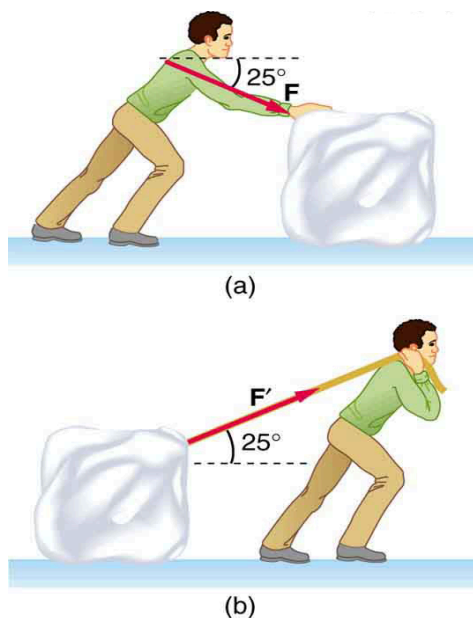
17. Consider the 52.0-kg mountain climber in [Figure 5.20](#). (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



**FIGURE 5.20** Part of the climber's weight is supported by her rope and part by friction between her feet and the rock face.

18. A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown in [Figure 5.21\(a\)](#). (a) Calculate the minimum force  $F$  he must exert to get the block moving. (b) What is the magnitude of its acceleration once it starts to move, if that force is maintained?

19. Repeat [Exercise 5.18](#) with the contestant pulling the block of ice with a rope over his shoulder at the same angle above the horizontal as shown in [Figure 5.21](#)(b).



**FIGURE 5.21** Which method of sliding a block of ice requires less force—(a) pushing or (b) pulling at the same angle above the horizontal?

## 5.2 Drag Forces

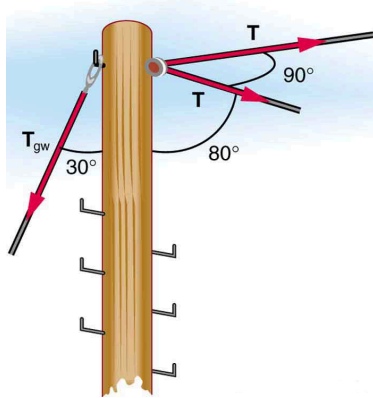
20. The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a headfirst position with a cross-section area facing the fluid of  $0.140 \text{ m}^2$ .
21. A 60-kg and a 90-kg skydiver jump from an airplane at an altitude of 6000 m, both falling in a headfirst position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.
22. A 560-g squirrel with a cross-section area facing the fluid of  $144 \text{ cm}^2$  falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?
23. To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the magnitudes of drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is  $0.70 \text{ m}^2$ ) (b) What is the magnitude of drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is  $2.44 \text{ m}^2$ ) Assume all values are accurate to three significant digits.
24. By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?
25. Calculate the speed a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be  $1.00 \times 10^3 \text{ kg/m}^3$ , and the cross-section area facing the fluid to be  $\pi r^2$ .
26. Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.
27. Find the terminal velocity of a spherical bacterium (diameter  $2.00 \text{ }\mu\text{m}$ ) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be  $1.10 \times 10^3 \text{ kg/m}^3$ .
28. Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density  $7.8 \times 10^3 \text{ kg/m}^3$ , diameter 3.0 mm) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

## 5.3 Elasticity: Stress and Strain

29. During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.
30. During a wrestling match, a 150 kg wrestler briefly stands on one hand during a maneuver designed to perplex her adversary. By how much does the upper arm bone shorten in length? The bone can be represented by a uniform rod 38.0 cm in length and 2.10 cm in radius.

- 31.** (a) The “lead” in pencils is a graphite composition with a Young’s modulus of about  $1 \times 10^9 \text{ N/m}^2$ . Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of 4.0 N. The lead is 0.50 mm in diameter and 60 mm long. (b) Is the answer reasonable? That is, does it seem to be consistent with what you have observed when using pencils?
- 32.** TV broadcast antennas are the tallest artificial structures on Earth. In 1987, a 72.0-kg physicist placed himself and 400 kg of equipment at the top of one 610-m high antenna to perform gravity experiments. By how much was the antenna compressed, if we consider it to be equivalent to a steel cylinder 0.150 m in radius?
- 33.** (a) By how much does a 65.0-kg mountain climber stretch her 0.800-cm diameter nylon rope when they hang 35.0 m below a rock outcropping? (b) Does the answer seem to be consistent with what you have observed for nylon ropes? Would it make sense if the rope were actually a bungee cord?
- 34.** A 20.0-m tall hollow aluminum flagpole is equivalent in stiffness to a solid cylinder 4.00 cm in diameter. A strong wind bends the pole much as a horizontal force of 900 N exerted at the top would. How far to the side does the top of the pole flex?
- 35.** As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in stiffness to a solid cylinder 5.00 cm in diameter.
- 36.** Calculate the force a piano tuner applies to stretch a steel piano wire 8.00 mm, if the wire is originally 0.850 mm in diameter and 1.35 m long.
- 37.** A vertebra is subjected to a shearing force of 500 N. Find the shear deformation, taking the vertebra to be a cylinder 3.00 cm high and 4.00 cm in diameter.
- 38.** A disk between vertebrae in the spine is subjected to a shearing force of 600 N. Find its shear deformation, taking it to have the shear modulus of  $1 \times 10^9 \text{ N/m}^2$ . The disk is equivalent to a solid cylinder 0.700 cm high and 4.00 cm in diameter.
- 39.** When using a pencil eraser, you exert a vertical force of 6.00 N at a distance of 2.00 cm from the hardwood-eraser joint. The pencil is 6.00 mm in diameter and is held at an angle of  $20.0^\circ$  to the horizontal. (a) By how much does the wood flex perpendicular to its length? (b) How much is it compressed lengthwise?
- 40.** To consider the effect of wires hung on poles, we take data from [Example 4.8](#), in which tensions in wires supporting a traffic light were calculated. The left wire made an angle  $30.0^\circ$  below the horizontal with the top of its pole and carried a tension of 108 N. The 12.0 m tall hollow aluminum pole is equivalent in stiffness to a 4.50 cm diameter solid cylinder. (a) How far is it bent to the side? (b) By how much is it compressed?
- 41.** A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is,  $\Delta V/V_0 = 2 \times 10^{-3}$ ) relative to the space available. Calculate the magnitude of the normal force exerted by the juice per square centimeter if its bulk modulus is  $1.8 \times 10^9 \text{ N/m}^2$ , assuming the bottle does not break. In view of your answer, do you think the bottle will survive?
- 42.** (a) When water freezes, its volume increases by 9.05% (that is,  $\Delta V/V_0 = 9.05 \times 10^{-2}$ ). What force per unit area is water capable of exerting on a container when it freezes? (It is acceptable to use the bulk modulus of water in this problem.) (b) Is it surprising that such forces can fracture engine blocks, boulders, and the like?
- 43.** This problem returns to the tightrope walker studied in [Example 4.6](#), who created a tension of  $3.94 \times 10^3 \text{ N}$  in a wire making an angle  $5.0^\circ$  below the horizontal with each supporting pole. Calculate how much this tension stretches the steel wire if it was originally 15 m long and 0.50 cm in diameter.

44. The pole in [Figure 5.22](#) is at a  $90.0^\circ$  bend in a power line and is therefore subjected to more shear force than poles in straight parts of the line. The tension in each line is  $4.00 \times 10^4$  N, at the angles shown. The pole is 15.0 m tall, has an 18.0 cm diameter, and can be considered to have half the stiffness of hardwood. (a) Calculate the compression of the pole. (b) Find how much it bends and in what direction. (c) Find the tension in a guy wire used to keep the pole straight if it is attached to the top of the pole at an angle of  $30.0^\circ$  with the vertical. (Clearly, the guy wire must be in the opposite direction of the bend.)



**FIGURE 5.22** This telephone pole is at a  $90^\circ$  bend in a power line. A guy wire is attached to the top of the pole at an angle of  $30^\circ$  with the vertical.

45. **Critical Thinking** Two spherical beads of the same mass fall at a constant speed due to a drag force. Bead A takes twice as long to fall the same distance as bead B. (a) Which of these beads has the larger radius? (b) Based on experimental data, an equation that fits the data is suggested for radius of a bead,  $r$ , which may not be correct:  $r = Kt - \frac{1}{2}$ , where  $K$  is a constant with appropriate units and  $t$  is the time it takes for a bead to reach a determined distance. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Beads of different radii are selected and the time it takes to fall a set distance is recorded. Graph the radius of these beads as a function of the time it takes to fall a determined distance.

## CHAPTER 6

# Uniform Circular Motion and Gravitation



**FIGURE 6.1** This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly—the latter completing many revolutions, the former only part of one (a circular arc). The same physical principles are involved in each. (credit: Richard Munckton)

### CHAPTER OUTLINE

#### 6.1 Rotation Angle and Angular Velocity

#### 6.2 Centripetal Acceleration

#### 6.3 Centripetal Force

#### 6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

#### 6.5 Newton's Universal Law of Gravitation

#### 6.6 Satellites and Kepler's Laws: An Argument for Simplicity

**INTRODUCTION TO UNIFORM CIRCULAR MOTION AND GRAVITATION** Many motions, such as the arc of a bird's flight or Earth's path around the Sun, are curved. Recall that Newton's first law tells us that motion is along a straight line at constant speed unless there is a net external force. We will therefore study not only motion along curves, but also the forces that cause it, including gravitational forces. In some ways, this chapter is a continuation of [Dynamics: Newton's Laws of Motion](#) as we study more applications of Newton's laws of motion.

This chapter deals with the simplest form of curved motion, **uniform circular motion**, motion in a circular path at constant speed. Studying this topic illustrates most concepts associated with rotational motion and leads to the study of many new topics we group under the name *rotation*. Pure *rotational motion* occurs when points in an object move in circular paths centered on one point. Pure *translational motion* is motion with no rotation. Some motion combines both types, such as a rotating hockey puck moving along ice.

## 6.1 Rotation Angle and Angular Velocity

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define arc length, rotation angle, radius of curvature and angular velocity.
- Calculate the angular velocity of a car wheel spin.

In [Kinematics](#), we studied motion along a straight line and introduced such concepts as displacement, velocity, and acceleration. [Two-Dimensional Kinematics](#) dealt with motion in two dimensions. Projectile motion is a special case of two-dimensional kinematics in which the object is projected into the air, while being subject to the gravitational force, and lands a distance away. In this chapter, we consider situations where the object does not land but moves in a curve. We begin the study of uniform circular motion by defining two angular quantities needed to describe rotational motion.

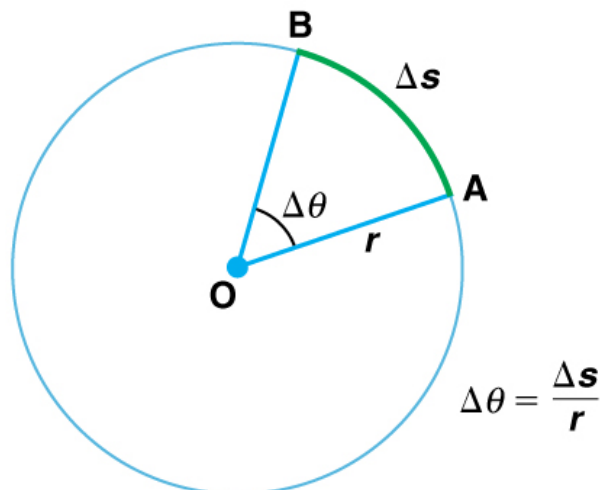
### Rotation Angle

When objects rotate about some axis—for example, when the CD (compact disc) in [Figure 6.2](#) rotates about its center—each point in the object follows a circular arc. Consider a line from the center of the CD to its edge. Each **pit** used to record sound along this line moves through the same angle in the same amount of time. The rotation angle is the amount of rotation and is analogous to linear distance. We define the **rotation angle**  $\Delta\theta$  to be the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r}. \quad 6.1$$



**FIGURE 6.2** All points on a CD travel in circular arcs. The pits along a line from the center to the edge all move through the same angle  $\Delta\theta$  in a time  $\Delta t$ .



**FIGURE 6.3** The radius of a circle is rotated through an angle  $\Delta\theta$ . The arc length  $\Delta s$  is described on the circumference.

The **arc length**  $\Delta s$  is the distance traveled along a circular path as shown in [Figure 6.3](#). Note that  $r$  is the **radius of curvature** of the circular path.

We know that for one complete revolution, the arc length is the circumference of a circle of radius  $r$ . The circumference of a circle is  $2\pi r$ . Thus for one complete revolution the rotation angle is

$$\Delta\theta = \frac{2\pi r}{r} = 2\pi. \quad 6.2$$

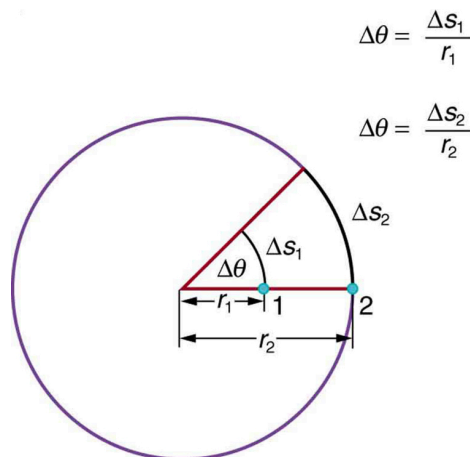
This result is the basis for defining the units used to measure rotation angles,  $\Delta\theta$  to be **radians** (rad), defined so that

$$2\pi \text{ rad} = 1 \text{ revolution}. \quad 6.3$$

A comparison of some useful angles expressed in both degrees and radians is shown in [Table 6.1](#).

Degree Measures	Radian Measure
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	$\pi$

**TABLE 6.1** Comparison of Angular Units



**FIGURE 6.4** Points 1 and 2 rotate through the same angle ( $\Delta\theta$ ), but point 2 moves through a greater arc length ( $\Delta s$ ) because it is at a greater distance from the center of rotation ( $r$ ).

If  $\Delta\theta = 2\pi$  rad, then the CD has made one complete revolution, and every point on the CD is back at its original position. Because there are  $360^\circ$  in a circle or one revolution, the relationship between radians and degrees is thus

$$2\pi \text{ rad} = 360^\circ \quad 6.4$$

so that

$$1 \text{ rad} = \frac{360^\circ}{2\pi} \approx 57.3^\circ. \quad 6.5$$

## Angular Velocity

How fast is an object rotating? We define **angular velocity**  $\omega$  as the rate of change of an angle. In symbols, this is

$$\omega = \frac{\Delta\theta}{\Delta t}, \quad 6.6$$

where an angular rotation  $\Delta\theta$  takes place in a time  $\Delta t$ . The greater the rotation angle in a given amount of time, the greater the angular velocity. The units for angular velocity are radians per second (rad/s).

Angular velocity  $\omega$  is analogous to linear velocity  $v$ . To get the precise relationship between angular and linear velocity, we again consider a pit on the rotating CD. This pit moves an arc length  $\Delta s$  in a time  $\Delta t$ , and so it has a linear velocity

$$v = \frac{\Delta s}{\Delta t}. \quad 6.7$$

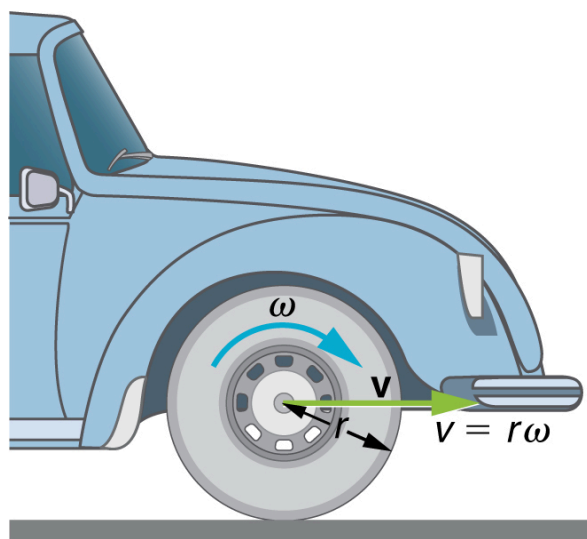
From  $\Delta\theta = \frac{\Delta s}{r}$  we see that  $\Delta s = r\Delta\theta$ . Substituting this into the expression for  $v$  gives

$$v = \frac{r\Delta\theta}{\Delta t} = r\omega. \quad 6.8$$

We write this relationship in two different ways and gain two different insights:

$$v = r\omega \text{ or } \omega = \frac{v}{r}. \quad 6.9$$

The first relationship in  $v = r\omega$  or  $\omega = \frac{v}{r}$  states that the linear velocity  $v$  is proportional to the distance from the center of rotation, thus, it is largest for a point on the rim (largest  $r$ ), as you might expect. We can also call this linear speed  $v$  of a point on the rim the *tangential speed*. The second relationship in  $v = r\omega$  or  $\omega = \frac{v}{r}$  can be illustrated by considering the tire of a moving car. Note that the speed of a point on the rim of the tire is the same as the speed  $v$  of the car. See [Figure 6.5](#). So the faster the car moves, the faster the tire spins—large  $v$  means a large  $\omega$ , because  $v = r\omega$ . Similarly, a larger-radius tire rotating at the same angular velocity ( $\omega$ ) will produce a greater linear speed ( $v$ ) for the car.



**FIGURE 6.5** A car moving at a velocity  $v$  to the right has a tire rotating with an angular velocity  $\omega$ . The speed of the tread of the tire relative to the axle is  $v$ , the same as if the car were jacked up. Thus the car moves forward at linear velocity  $v = r\omega$ , where  $r$  is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

### EXAMPLE 6.1

#### How Fast Does a Car Tire Spin?

Calculate the angular velocity of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See

Figure 6.5.

**Strategy**

Because the linear speed of the tire rim is the same as the speed of the car, we have  $v = 15.0$  m/s. The radius of the tire is given to be  $r = 0.300$  m. Knowing  $v$  and  $r$ , we can use the second relationship in  $v = r\omega$ ,  $\omega = \frac{v}{r}$  to calculate the angular velocity.

**Solution**

To calculate the angular velocity, we will use the following relationship:

$$\omega = \frac{v}{r}. \quad 6.10$$

Substituting the knowns,

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}} = 50.0 \text{ rad/s}. \quad 6.11$$

**Discussion**

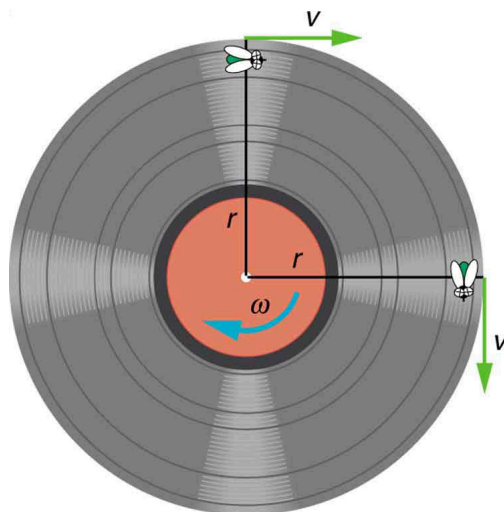
When we cancel units in the above calculation, we get 50.0/s. But the angular velocity must have units of rad/s. Because radians are actually unitless (radians are defined as a ratio of distance), we can simply insert them into the answer for the angular velocity. Also note that if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular velocity

$$\omega = (15.0 \text{ m/s}) / (1.20 \text{ m}) = 12.5 \text{ rad/s}. \quad 6.12$$

Both  $\omega$  and  $v$  have directions (hence they are angular and linear *velocities*, respectively). Angular velocity has only two directions with respect to the axis of rotation—it is either clockwise or counterclockwise. Linear velocity is tangent to the path, as illustrated in [Figure 6.6](#).

**Take-Home Experiment**

Tie an object to the end of a string and swing it around in a horizontal circle above your head (swing at your wrist). Maintain uniform speed as the object swings and measure the angular velocity of the motion. What is the approximate speed of the object? Identify a point close to your hand and take appropriate measurements to calculate the linear speed at this point. Identify other circular motions and measure their angular velocities.



**FIGURE 6.6** As an object moves in a circle, here a fly on the edge of an old-fashioned vinyl record, its instantaneous velocity is always tangent to the circle. The direction of the angular velocity is clockwise in this case.

### Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

[Click to view content \(https://openstax.org/l/28ladybugrevolutionrotation\).](https://openstax.org/l/28ladybugrevolutionrotation)

## 6.2 Centripetal Acceleration

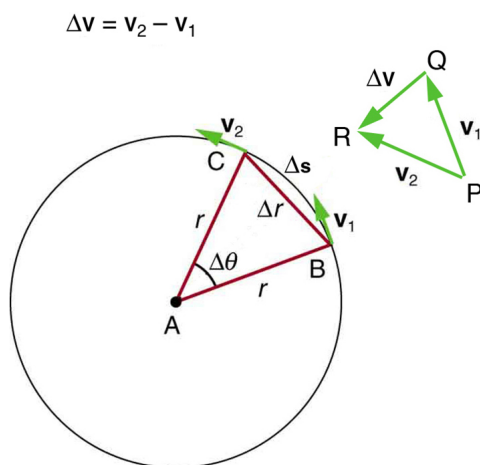
### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Establish the expression for centripetal acceleration.
- Explain the centrifuge.

We know from kinematics that acceleration is a change in velocity, either in its magnitude or in its direction, or both. In uniform circular motion, the direction of the velocity changes constantly, so there is always an associated acceleration, even though the magnitude of the velocity might be constant. You experience this acceleration yourself when you turn a corner in your car. (If you hold the wheel steady during a turn and move at constant speed, you are in uniform circular motion.) What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we examine the direction and magnitude of that acceleration.

[Figure 6.7](#) shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration** ( $a_c$ ); centripetal means “toward the center” or “center seeking.”



**FIGURE 6.7** The directions of the velocity of an object at two different points are shown, and the change in velocity  $\Delta v$  is seen to point directly toward the center of curvature. (See small inset.) Because  $a_c = \Delta v / \Delta t$ , the acceleration is also toward the center;  $a_c$  is called centripetal acceleration. (Because  $\Delta\theta$  is very small, the arc length  $\Delta s$  is equal to the chord length  $\Delta r$  for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? Note that the triangle formed by the velocity vectors and the one formed by the radii  $r$  and  $\Delta s$  are similar. Both the triangles ABC and PQR are isosceles triangles (two equal sides). The two equal sides of the velocity vector triangle are the speeds  $v_1 = v_2 = v$ . Using the properties of two similar triangles, we obtain

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}. \quad 6.13$$

Acceleration is  $\Delta v / \Delta t$ , and so we first solve this expression for  $\Delta v$ :

$$\Delta v = \frac{v}{r} \Delta s. \quad 6.14$$

Then we divide this by  $\Delta t$ , yielding

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t}. \quad 6.15$$

Finally, noting that  $\Delta v/\Delta t = a_c$  and that  $\Delta s/\Delta t = v$ , the linear or tangential speed, we see that the magnitude of the centripetal acceleration is

$$a_c = \frac{v^2}{r}, \quad 6.16$$

which is the acceleration of an object in a circle of radius  $r$  at a speed  $v$ . So, centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that  $a_c$  is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that  $a_c$  is greater for tighter turns, as you have probably noticed.

It is also useful to express  $a_c$  in terms of angular velocity. Substituting  $v = r\omega$  into the above expression, we find  $a_c = (r\omega)^2/r = r\omega^2$ . We can express the magnitude of centripetal acceleration using either of two equations:

$$a_c = \frac{v^2}{r}; \quad a_c = r\omega^2. \quad 6.17$$

Recall that the direction of  $a_c$  is toward the center. You may use whichever expression is more convenient, as illustrated in examples below.

A **centrifuge** (see [Figure 6.8b](#)) is a rotating device used to separate specimens of different densities. High centripetal acceleration significantly decreases the time it takes for separation to occur, and makes separation possible with small samples. Centrifuges are used in a variety of applications in science and medicine, including the separation of single cell suspensions such as bacteria, viruses, and blood cells from a liquid medium and the separation of macromolecules, such as DNA and protein, from a solution. Centrifuges are often rated in terms of their centripetal acceleration relative to acceleration due to gravity ( $g$ ); maximum centripetal acceleration of several hundred thousand  $g$  is possible in a vacuum. Human centrifuges, extremely large centrifuges, have been used to test the tolerance of astronauts to the effects of accelerations larger than that of Earth's gravity.



## EXAMPLE 6.2

### How Does the Centripetal Acceleration of a Car Around a Curve Compare with That Due to Gravity?

What is the magnitude of the centripetal acceleration of a car following a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h)? Compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed. See [Figure 6.8\(a\)](#).

#### Strategy

Because  $v$  and  $r$  are given, the first expression in  $a_c = \frac{v^2}{r}$ ;  $a_c = r\omega^2$  is the most convenient to use.

#### Solution

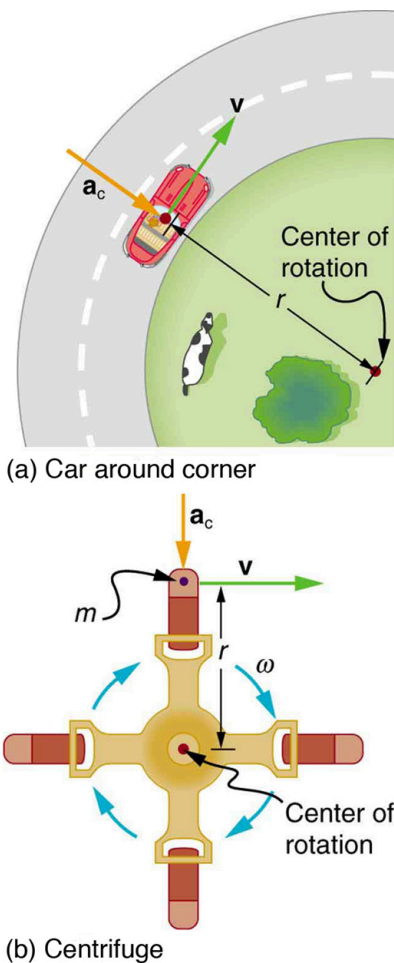
Entering the given values of  $v = 25.0$  m/s and  $r = 500$  m into the first expression for  $a_c$  gives

$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}} = 1.25 \text{ m/s}^2. \quad 6.18$$

#### Discussion

To compare this with the acceleration due to gravity ( $g = 9.80 \text{ m/s}^2$ ), we take the ratio of

$a_c/g = (1.25 \text{ m/s}^2)/(9.80 \text{ m/s}^2) = 0.128$ . Thus,  $a_c = 0.128 g$  and is noticeable especially if you were not wearing a seat belt.



**FIGURE 6.8** (a) The car following a circular path at constant speed is accelerated perpendicular to its velocity, as shown. The magnitude of this centripetal acceleration is found in [Example 6.2](#). (b) A particle of mass  $m$  in a centrifuge is rotating at constant angular velocity  $\omega$ . It must be accelerated perpendicular to its velocity or it would continue in a straight line. The magnitude of the necessary acceleration is found in [Example 6.3](#).

### EXAMPLE 6.3

#### How Big Is the Centripetal Acceleration in an Ultracentrifuge?

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an **ultracentrifuge** spinning at  $7.5 \times 10^4$  rev/min. Determine the ratio of this acceleration to that due to gravity. See [Figure 6.8\(b\)](#).

#### Strategy

The term rev/min stands for revolutions per minute. By converting this to radians per second, we obtain the angular velocity  $\omega$ . Because  $r$  is given, we can use the second expression in the equation  $a_c = \frac{v^2}{r}$ ;  $a_c = r\omega^2$  to calculate the centripetal acceleration.

#### Solution

To convert  $7.5 \times 10^4$  rev/min to radians per second, we use the facts that one revolution is  $2\pi$  rad and one minute is 60.0 s. Thus,

$$\omega = 7.50 \times 10^4 \frac{\text{rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60.0 \text{ s}} = 7854 \text{ rad/s.} \quad 6.19$$

Now the centripetal acceleration is given by the second expression in  $a_c = \frac{v^2}{r}$ ;  $a_c = r\omega^2$  as

$$a_c = r\omega^2. \quad 6.20$$

Converting 7.50 cm to meters and substituting known values gives

$$a_c = (0.0750 \text{ m})(7854 \text{ rad/s})^2 = 4.63 \times 10^6 \text{ m/s}^2. \quad 6.21$$

Note that the unitless radians are discarded in order to get the correct units for centripetal acceleration. Taking the ratio of  $a_c$  to  $g$  yields

$$\frac{a_c}{g} = \frac{4.63 \times 10^6}{9.80} = 4.72 \times 10^5. \quad 6.22$$

### Discussion

This last result means that the centripetal acceleration is 472,000 times as strong as  $g$ . It is no wonder that such high  $\omega$  centrifuges are called ultracentrifuges. The extremely large accelerations involved greatly decrease the time needed to cause the sedimentation of blood cells or other materials.

Of course, a net external force is needed to cause any acceleration, just as Newton proposed in his second law of motion. So a net external force is needed to cause a centripetal acceleration. In [Centripetal Force](#), we will consider the forces involved in circular motion.



## PHET EXPLORATIONS

### Ladybug Motion 2D

Learn about position, velocity and acceleration vectors. Move the ladybug by setting the position, velocity or acceleration, and see how the vectors change. Choose linear, circular or elliptical motion, and record and playback the motion to analyze the behavior.

[Click to view content \(https://openstax.org/l/28ladybugmotion\)](https://openstax.org/l/28ladybugmotion).



## 6.3 Centripetal Force

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate coefficient of friction on a car tire.
- Calculate ideal speed and angle of a car on a turn.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net  $F = ma$ . For uniform circular motion, the acceleration is the centripetal acceleration— $a = a_c$ . Thus, the magnitude of centripetal force  $F_c$  is

$$F_c = ma_c. \quad 6.23$$

By using the expressions for centripetal acceleration  $a_c$  from  $a_c = \frac{v^2}{r}$ ;  $a_c = r\omega^2$ , we get two expressions for the centripetal force  $F_c$  in terms of mass, velocity, angular velocity, and radius of curvature:

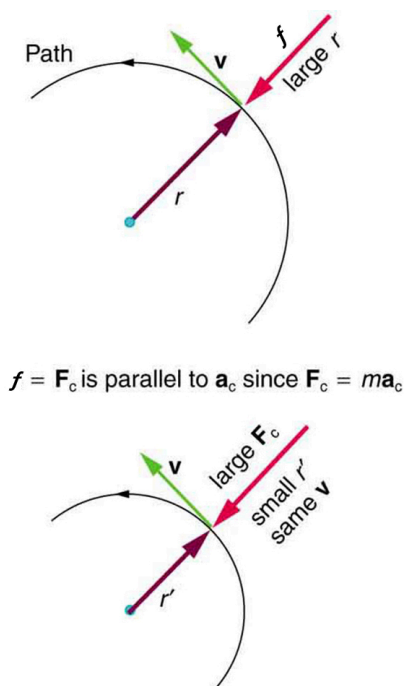
$$F_c = m\frac{v^2}{r}; F_c = mr\omega^2. \quad 6.24$$

You may use whichever expression for centripetal force is more convenient. Centripetal force  $F_c$  is always perpendicular to the path and pointing to the center of curvature, because  $\mathbf{a}_c$  is perpendicular to the velocity and pointing to the center of curvature.

Note that if you solve the first expression for  $r$ , you get

$$r = \frac{mv^2}{F_c}. \quad 6.25$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.



**FIGURE 6.9** The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the  $F_c$ , the smaller the radius of curvature  $r$  and the sharper the curve. The second curve has the same  $v$ , but a larger  $F_c$  produces a smaller  $r'$ .

### EXAMPLE 6.4

#### What Coefficient of Friction Do Car Tires Need on a Flat Curve?

- (a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.
- (b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see [Figure 6.10](#)).

#### Strategy and Solution for (a)

We know that  $F_c = \frac{mv^2}{r}$ . Thus,

$$F_c = \frac{mv^2}{r} = \frac{(900 \text{ kg})(25.0 \text{ m/s})^2}{(500 \text{ m})} = 1125 \text{ N}. \quad 6.26$$

#### Strategy for (b)

[Figure 6.10](#) shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping

the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is  $\mu_s N$ , where  $\mu_s$  is the static coefficient of friction and  $N$  is the normal force. The normal force equals the car's weight on level ground, so that  $N = mg$ . Thus the centripetal force in this situation is

$$F_c = f = \mu_s N = \mu_s mg. \quad 6.27$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the first expression for  $F_c$  from the equation

$$\left. \begin{aligned} F_c &= m \frac{v^2}{r} \\ F_c &= mr\omega^2 \end{aligned} \right\} \quad 6.28$$

$$m \frac{v^2}{r} = \mu_s mg. \quad 6.29$$

We solve this for  $\mu_s$ , noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}. \quad 6.30$$

### Solution for (b)

Substituting the knowns,

$$\mu_s = \frac{(25.0 \text{ m/s})^2}{(500 \text{ m})(9.80 \text{ m/s}^2)} = 0.13. \quad 6.31$$

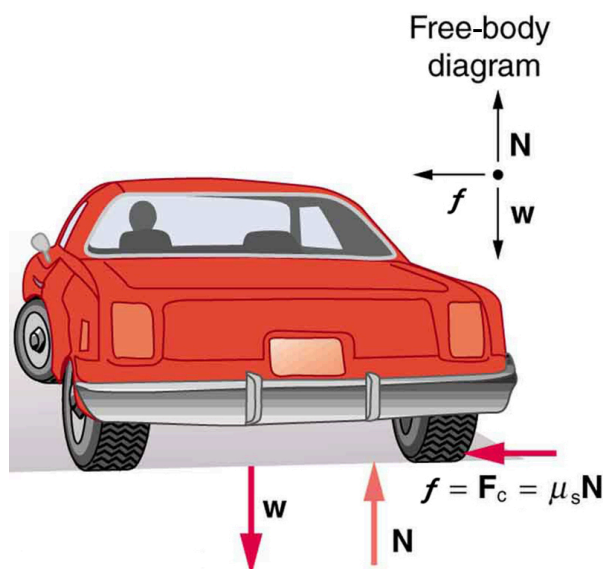
(Because coefficients of friction are approximate, the answer is given to only two digits.)

### Discussion

We could also solve part (a) using the first expression in  $\left. \begin{aligned} F_c &= m \frac{v^2}{r} \\ F_c &= mr\omega^2 \end{aligned} \right\}$ , because  $m$ ,  $v$ , and  $r$  are given. The

coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than  $\mu_s N$ . A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

---



**FIGURE 6.10** This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve. See [Figure 6.11](#). The greater the angle  $\theta$ , the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an “ideally banked curve,” the angle  $\theta$  is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for  $\theta$  for an ideally banked curve and consider an example related to it.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force  $N$  in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

[Figure 6.11](#) shows a free body diagram for a car on a frictionless banked curve. If the angle  $\theta$  is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight  $w$  and the normal force of the road  $N$ . (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude  $mv^2/r$ . Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, and so this must equal the centripetal force—that is,

$$N \sin \theta = \frac{mv^2}{r}. \quad 6.32$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From the figure, we see that the vertical component of the normal force is  $N \cos \theta$ , and the only other vertical force is the car’s weight. These must be equal in magnitude; thus,

$$N \cos \theta = mg. \quad 6.33$$

Now we can combine the last two equations to eliminate  $N$  and get an expression for  $\theta$ , as desired. Solving the second equation for  $N = mg/(\cos \theta)$ , and substituting this into the first yields

$$mg \frac{\sin \theta}{\cos \theta} = \frac{mv^2}{r} \quad 6.34$$

$$mg \tan(\theta) = \frac{mv^2}{r}$$

$$\tan \theta = \frac{v^2}{rg} \quad 6.35$$

Taking the inverse tangent gives

$$\theta = \tan^{-1} \left( \frac{v^2}{rg} \right) \text{ (ideally banked curve, no friction).} \quad 6.36$$

This expression can be understood by considering how  $\theta$  depends on  $v$  and  $r$ . A large  $\theta$  will be obtained for a large  $v$  and a small  $r$ . That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve is frictionless. Note that  $\theta$  does not depend on the mass of the vehicle.

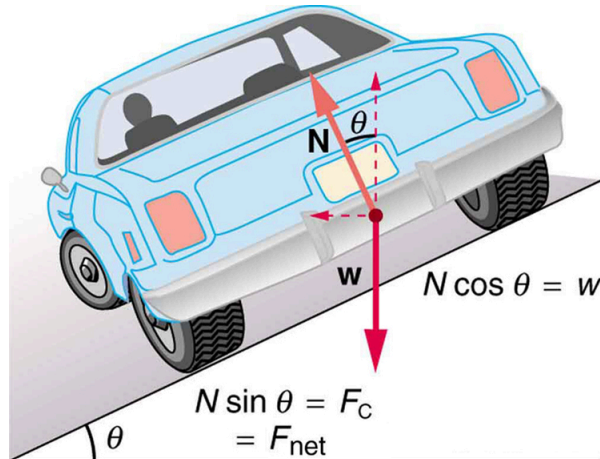


FIGURE 6.11 The car on this banked curve is moving away and turning to the left.



### EXAMPLE 6.5

#### What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as the Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at  $65.0^\circ$  should be driven if the road is frictionless.

#### Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

#### Solution

Starting with

$$\tan \theta = \frac{v^2}{rg} \quad 6.37$$

we get

$$v = (rg \tan \theta)^{1/2}. \quad 6.38$$

Noting that  $\tan 65.0^\circ = 2.14$ , we obtain

$$v = [(100 \text{ m})(9.80 \text{ m/s}^2)(2.14)]^{1/2} \quad 6.39$$

$$= 45.8 \text{ m/s.}$$

### Discussion

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Calculations similar to those in the preceding examples can be performed for a host of interesting situations in which centripetal force is involved—a number of these are presented in this chapter's Problems and Exercises.

### Take-Home Experiment

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.



## PHET EXPLORATIONS

### Gravity and Orbits

Move the sun, earth, moon and space station to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it!

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/6-3-centripetal-force\)](https://openstax.org/books/college-physics-2e/pages/6-3-centripetal-force)



## 6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

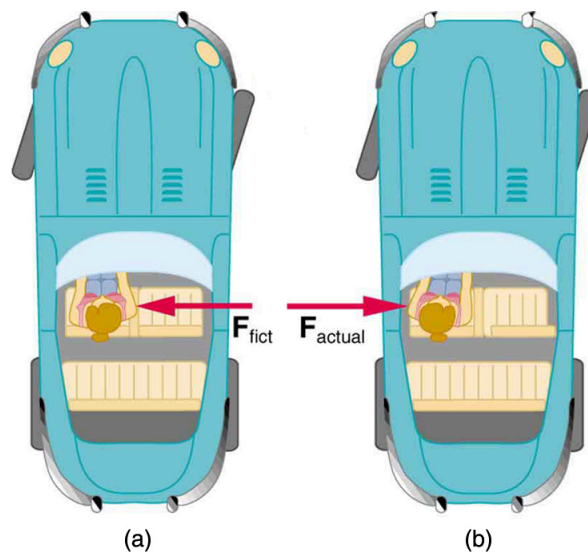
### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the inertial frame of reference.
- Discuss the non-inertial frame of reference.
- Describe the effects of the Coriolis force.

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits fictitious forces—unreal forces that arise from motion and may seem real, because the observer's frame of reference is accelerating or rotating.

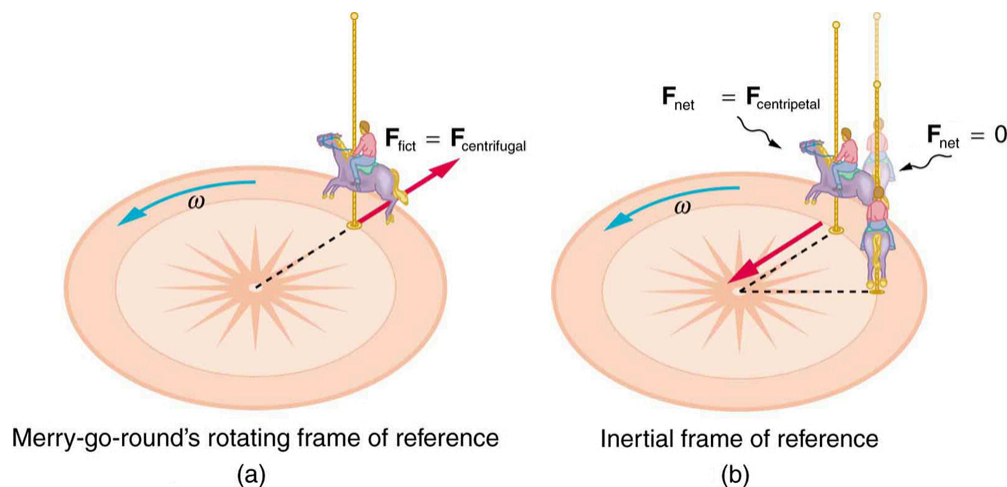
When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you, and there is no real force backward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right. You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line but the *car* moves to the right, and there is no real force on you to the left. Recall Newton's first law.



**FIGURE 6.12** (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is a fictitious force arising from the use of the car as a frame of reference. (b) In the Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no real force to the left on the driver relative to Earth. There is a real force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, while a physicist uses Earth. The physicist chooses Earth because it is very nearly an inertial frame of reference—one in which all forces are real (that is, in which all forces have an identifiable physical origin). In such a frame of reference, Newton's laws of motion take the form given in [Dynamics: Newton's Laws of Motion](#). The car is a **non-inertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is a **fictitious force** having no physical origin. There is nothing real pushing them left—the car, as well as the driver, is actually accelerating to the right.

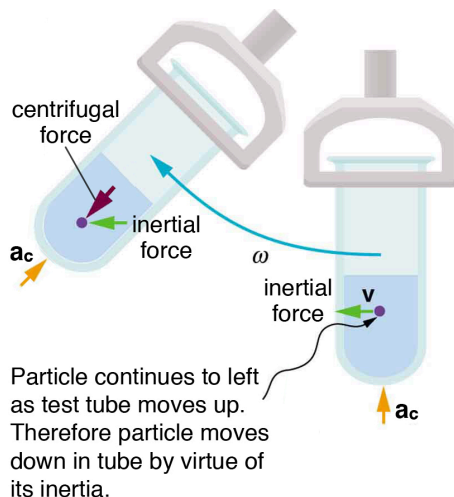
Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round. You take the merry-go-round to be your frame of reference because you rotate together. In that non-inertial frame, you feel a fictitious force, named **centrifugal force** (not to be confused with centripetal force), trying to throw you off. You must hang on tightly to counteract the centrifugal force. In Earth's frame of reference, there is no force trying to throw you off. Rather you must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round.



**FIGURE 6.13** (a) A rider on a merry-go-round feels as if he is being thrown off. This fictitious force is called the centrifugal force—it explains the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off and not a real force (the unshaded rider has  $F_{\text{net}} = 0$  and heads in a straight line). A real force,  $F_{\text{centripetal}}$ , is needed to cause a circular path.

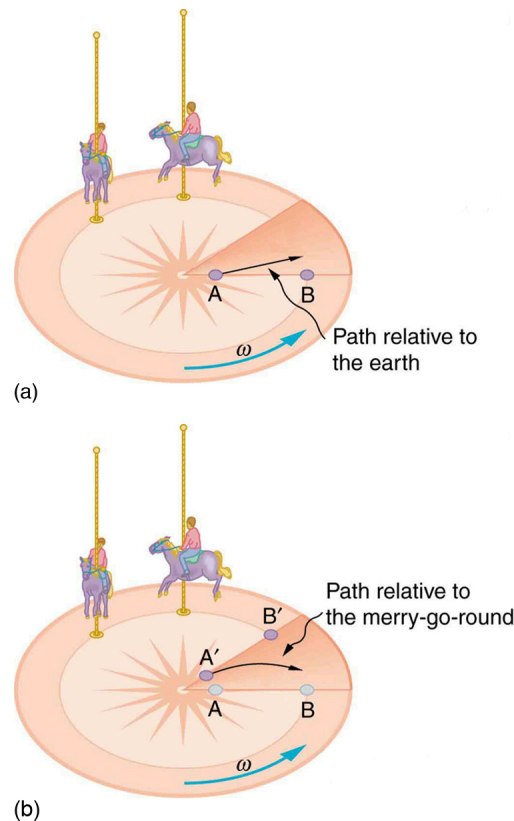
This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (see [Figure 6.14](#)). A centrifuge spins a sample very rapidly, as mentioned

earlier in this chapter. Viewed from the rotating frame of reference, the fictitious centrifugal force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.



**FIGURE 6.14** Centrifuges use inertia to perform their task. Particles in the fluid sediment come out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles will come into contact with the test tube walls, which will then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a frame of reference that rotates. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in [Figure 6.15](#)? The ball follows a straight path relative to Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using a fictitious force, called the **Coriolis force**, that causes the ball to curve to the right. The fictitious Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's Laws in non-inertial frames of reference.

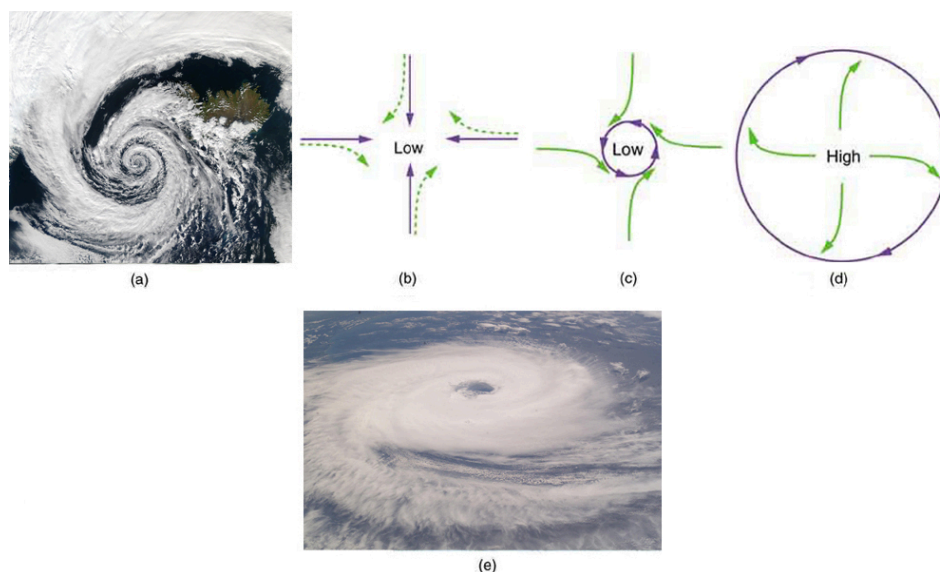


**FIGURE 6.15** Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point B, starting at point A. Both points rotate to the shaded positions (A' and B') shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in [Figure 6.15](#). As on the merry-go-round, any motion in Earth's northern hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the southern hemisphere; there, the force is to the left. Because Earth's angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the northern hemisphere to rotate in the counterclockwise direction, while the tropical cyclones (what hurricanes are called below the equator) in the southern hemisphere rotate in the clockwise direction. The terms hurricane, typhoon, and tropical storm are regionally-specific names for tropical cyclones, storm systems characterized by low pressure centers, strong winds, and heavy rains. [Figure 6.16](#) helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the northern hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the northern hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When non-inertial frames are used, fictitious forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these fictitious forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest and truest, in the sense that all forces have real origins and explanations.



**FIGURE 6.16** (a) The counterclockwise rotation of this northern hemisphere hurricane is a major consequence of the Coriolis force. (credit: NASA) (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the southern hemisphere, leading to tropical cyclones. (credit: NASA)

## 6.5 Newton's Universal Law of Gravitation

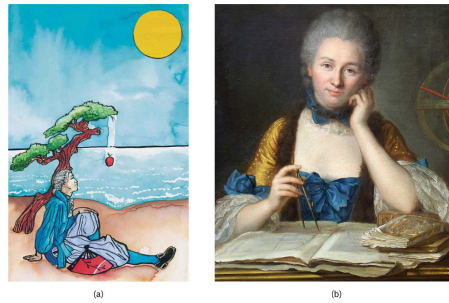
### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain Earth's gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.
- Examine the Cavendish experiment

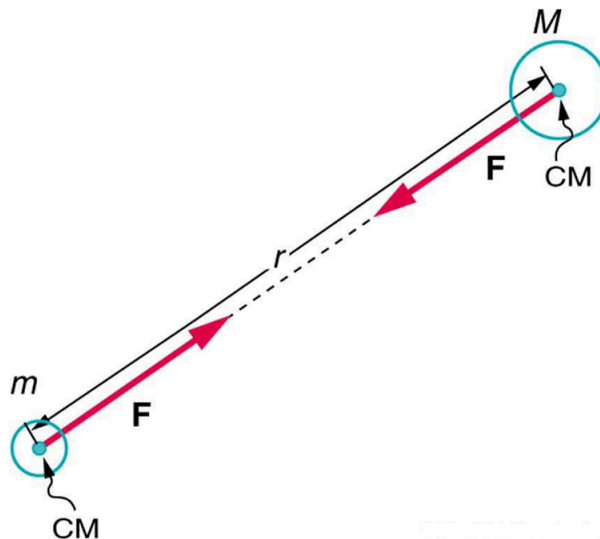
What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth's gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth's surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See [Figure 6.17](#). But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others. Other prominent scientists and mathematicians of the time, particularly those outside of England, were reluctant to accept Newton's principles. It took the work of another prominent philosopher, writer, and scientist, Émilie du Châtelet, to establish the Newtonian gravitation as the accurate and overarching law. Du Châtelet, who had earlier laid the foundation for the understanding of conservation of energy as well as the principle that light had no mass, translated and augmented Newton's key work. She also utilized calculus to explain gravity, which helped lead to its acceptance.



**FIGURE 6.17** According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Like many revolutionary discoveries, it was not immediately accepted. Prominent French scientist and philosopher Émilie du Châtelet helped establish Newton's theory in France and mainland Europe.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



**FIGURE 6.18** Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

### Misconception Alert

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM), which will be further explored in [Linear Momentum and Collisions](#). For two bodies having masses  $m$  and  $M$  with a distance  $r$  between their centers of mass, the equation for Newton's universal law of gravitation is

$$F = G \frac{mM}{r^2}, \quad 6.40$$

where  $F$  is the magnitude of the gravitational force and  $G$  is a proportionality factor called the **gravitational constant**.  $G$  is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G = 6.674 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \quad 6.41$$

in SI units. Note that the units of  $G$  are such that a force in newtons is obtained from  $F = G \frac{mM}{r^2}$ , when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of  $6.674 \times 10^{-11}$  N. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of  $6 \times 10^{24}$  kg.

Recall that the acceleration due to gravity  $g$  is about  $9.80 \text{ m/s}^2$  on Earth. We can now determine why this is so. The weight of an object  $mg$  is the gravitational force between it and Earth. Substituting  $mg$  for  $F$  in Newton's universal law of gravitation gives

$$mg = G \frac{mM}{r^2}, \quad 6.42$$

where  $m$  is the mass of the object,  $M$  is the mass of Earth, and  $r$  is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [Figure 6.19](#). The mass  $m$  of the object cancels, leaving an equation for  $g$ :

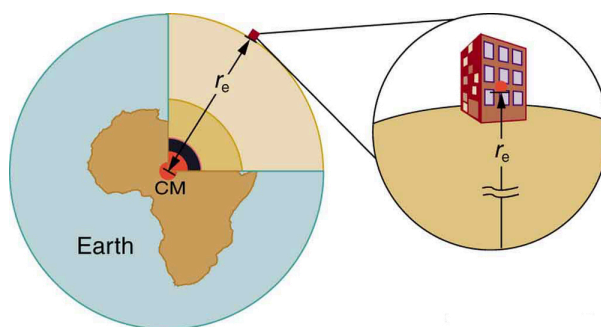
$$g = G \frac{M}{r^2}. \quad 6.43$$

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = \left( 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \right) \times \frac{5.98 \times 10^{24} \text{ kg}}{(6.38 \times 10^6 \text{ m})^2}, \quad 6.44$$

and we obtain a value for the acceleration of a falling body:

$$g = 9.80 \text{ m/s}^2. \quad 6.45$$



**FIGURE 6.19** The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

### Take-Home Experiment

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

### Making Connections

Attempts are still being made to understand the gravitational force. As we shall see in [Particle Physics](#), modern physics is exploring the connections of gravity to other forces, space, and time. General relativity alters our view of gravitation, leading us to think of gravitation as bending space and time.

In the following example, we make a comparison similar to one made by Newton himself. He noted that if the gravitational force caused the Moon to orbit Earth, then the acceleration due to gravity should equal the centripetal acceleration of the Moon in its orbit. Newton found that the two accelerations agreed “pretty nearly.”



### EXAMPLE 6.6

#### Earth's Gravitational Force Is the Centripetal Force Making the Moon Move in a Curved Path

(a) Find the acceleration due to Earth's gravity at the distance of the Moon.

(b) Calculate the centripetal acceleration needed to keep the Moon in its orbit (assuming a circular orbit about a fixed Earth), and compare it with the value of the acceleration due to Earth's gravity that you have just found.

#### Strategy for (a)

This calculation is the same as the one finding the acceleration due to gravity at Earth's surface, except that  $r$  is the distance from the center of Earth to the center of the Moon. The radius of the Moon's nearly circular orbit is  $3.84 \times 10^8$  m.

#### Solution for (a)

Substituting known values into the expression for  $g$  found above, remembering that  $M$  is the mass of Earth not the Moon, yields

$$\begin{aligned} g &= G \frac{M}{r^2} = \left( 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \times \frac{5.98 \times 10^{24} \text{ kg}}{(3.84 \times 10^8 \text{ m})^2} \\ &= 2.70 \times 10^{-3} \text{ m/s}^2 \end{aligned} \quad 6.46$$

#### Strategy for (b)

Centripetal acceleration can be calculated using either form of

$$\left. \begin{aligned} a_c &= \frac{v^2}{r} \\ a_c &= r\omega^2 \end{aligned} \right\} \quad 6.47$$

We choose to use the second form:

$$a_c = r\omega^2, \quad 6.48$$

where  $\omega$  is the angular velocity of the Moon about Earth.

#### Solution for (b)

Given that the period (the time it takes to make one complete rotation) of the Moon's orbit is 27.3 days, (d) and using

$$1 \text{ d} \times 24 \frac{\text{hr}}{\text{d}} \times 60 \frac{\text{min}}{\text{hr}} \times 60 \frac{\text{s}}{\text{min}} = 86,400 \text{ s} \quad 6.49$$

we see that

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{(27.3 \text{ d})(86,400 \text{ s/d})} = 2.66 \times 10^{-6} \frac{\text{rad}}{\text{s}}. \quad 6.50$$

The centripetal acceleration is

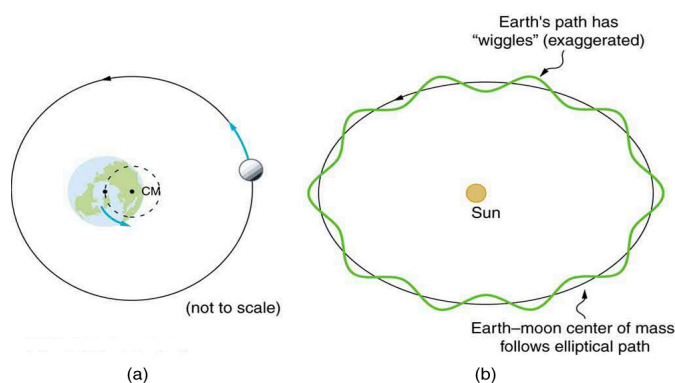
$$\begin{aligned} a_c &= r\omega^2 = (3.84 \times 10^8 \text{ m})(2.66 \times 10^{-6} \text{ rad/s})^2 \\ &= 2.72 \times 10^{-3} \text{ m/s}^2 \end{aligned} \quad 6.51$$

The direction of the acceleration is toward the center of the Earth.

### Discussion

The centripetal acceleration of the Moon found in (b) differs by less than 1% from the acceleration due to Earth's gravity found in (a). This agreement is approximate because the Moon's orbit is slightly elliptical, and Earth is not stationary (rather the Earth-Moon system rotates about its center of mass, which is located some 1700 km below Earth's surface). The clear implication is that Earth's gravitational force causes the Moon to orbit Earth.

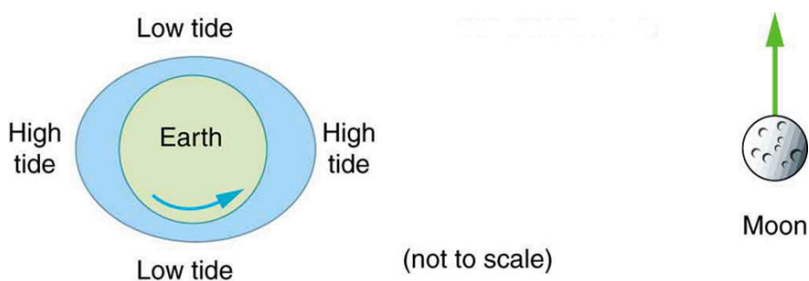
Why does Earth not remain stationary as the Moon orbits it? This is because, as expected from Newton's third law, if Earth exerts a force on the Moon, then the Moon should exert an equal and opposite force on Earth (see [Figure 6.20](#)). We do not sense the Moon's effect on Earth's motion, because the Moon's gravity moves our bodies right along with Earth but there are other signs on Earth that clearly show the effect of the Moon's gravitational force as discussed in [Satellites and Kepler's Laws: An Argument for Simplicity](#).



**FIGURE 6.20** (a) Earth and the Moon rotate approximately once a month around their common center of mass. (b) Their center of mass orbits the Sun in an elliptical orbit, but Earth's path around the Sun has "wiggles" in it. Similar wiggles in the paths of stars have been observed and are considered direct evidence of planets orbiting those stars. This is important because the planets' reflected light is often too dim to be observed.

### Tides

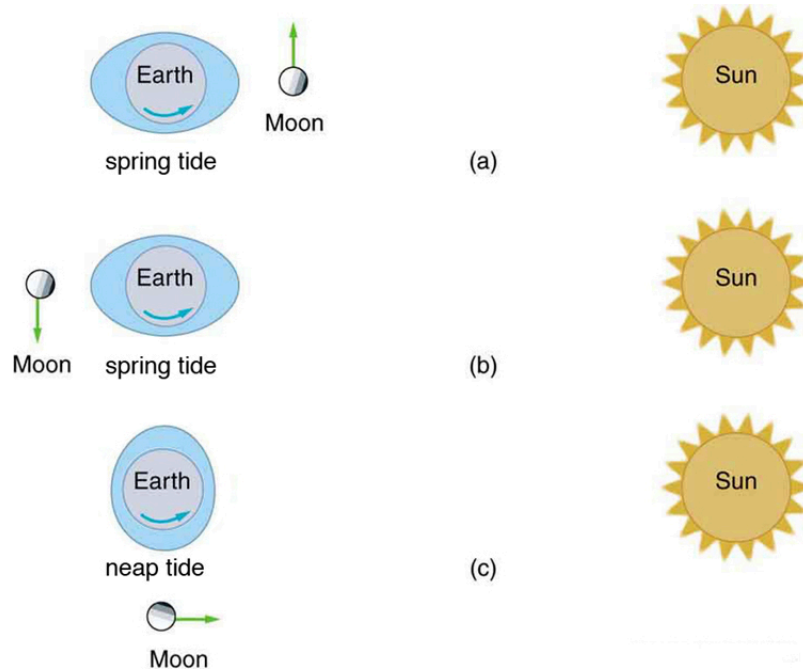
Ocean tides are one very observable result of the Moon's gravity acting on Earth. [Figure 6.21](#) is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).



**FIGURE 6.21** The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the

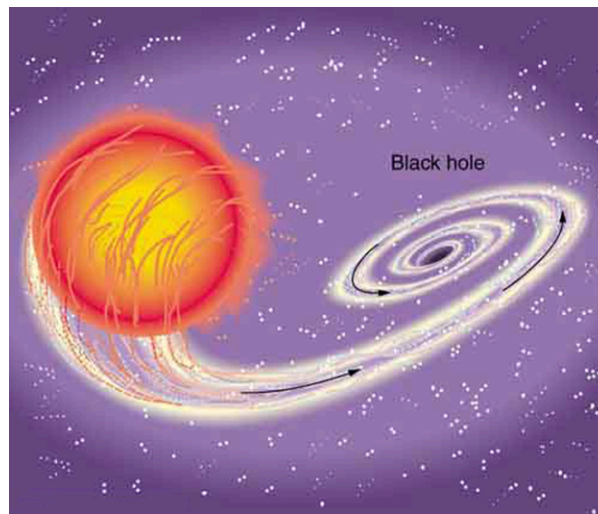
water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a  $90^\circ$  angle to the Earth-Moon alignment.



**FIGURE 6.22** (a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at  $90^\circ$  to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see [Figure 6.23](#)). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.



**FIGURE 6.23** A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

### "Weightlessness" and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by

astronauts orbiting Earth. What is the effect of “weightlessness” upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn’t mean that an astronaut is not being acted upon by the gravitational force. There is no “zero gravity” in an astronaut’s orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.



**FIGURE 6.24** Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

**Microgravity** refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

### The Cavendish Experiment: Then and Now

As previously noted, the universal gravitational constant  $G$  is determined experimentally. This definition was first done accurately by Henry Cavendish (1731–1810), an English scientist, in 1798, more than 100 years after Newton published his universal law of gravitation. The measurement of  $G$  is very basic and important because it determines the strength of one of the four forces in nature. Cavendish’s experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most), using apparatus like that in [Figure 6.25](#). Remarkably, his value for  $G$  differs by less than 1% from the best modern value.

One important consequence of knowing  $G$  was that an accurate value for Earth's mass could finally be obtained. This was done by measuring the acceleration due to gravity as accurately as possible and then calculating the mass of Earth  $M$  from the relationship Newton's universal law of gravitation gives

$$mg = G \frac{mM}{r^2}, \quad 6.52$$

where  $m$  is the mass of the object,  $M$  is the mass of Earth, and  $r$  is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [Figure 6.18](#). The mass  $m$  of the object cancels, leaving an equation for  $g$ :

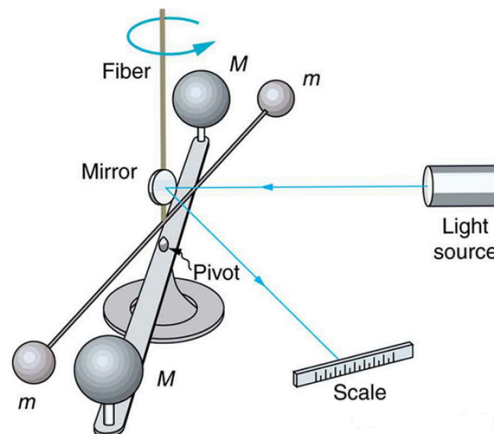
$$g = G \frac{M}{r^2}. \quad 6.53$$

Rearranging to solve for  $M$  yields

$$M = \frac{gr^2}{G}. \quad 6.54$$

So  $M$  can be calculated because all quantities on the right, including the radius of Earth  $r$ , are known from direct measurements. We shall see in [Satellites and Kepler's Laws: An Argument for Simplicity](#) that knowing  $G$  also allows for the determination of astronomical masses. Interestingly, of all the fundamental constants in physics,  $G$  is by far the least well determined.

The Cavendish experiment is also used to explore other aspects of gravity. One of the most interesting questions is whether the gravitational force depends on substance as well as mass—for example, whether one kilogram of lead exerts the same gravitational pull as one kilogram of water. A Hungarian scientist named Roland von Eötvös pioneered this inquiry early in the 20th century. He found, with an accuracy of five parts per billion, that the gravitational force does not depend on the substance. Such experiments continue today, and have improved upon Eötvös' measurements. Cavendish-type experiments such as those of Eric Adelberger and others at the University of Washington, have also put severe limits on the possibility of a fifth force and have verified a major prediction of general relativity—that gravitational energy contributes to rest mass. Ongoing measurements there use a torsion balance and a parallel plate (not spheres, as Cavendish used) to examine how Newton's law of gravitation works over sub-millimeter distances. On this small-scale, do gravitational effects depart from the inverse square law? So far, no deviation has been observed.



**FIGURE 6.25** Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres ( $m$ ) and the two on the stand ( $M$ ) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

## 6.6 Satellites and Kepler's Laws: An Argument for Simplicity

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State Kepler's laws of planetary motion.
- Derive the third Kepler's law for circular orbits.
- Discuss the Ptolemaic model of the universe.

Examples of gravitational orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The Moon's orbit about Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets about the Sun are no less interesting. If we look further, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force, and it is possible to describe them to various degrees of precision. Precise descriptions of complex systems must be made with large computers. However, we can describe an important class of orbits without the use of computers, and we shall find it instructive to study them. These orbits have the following characteristics:

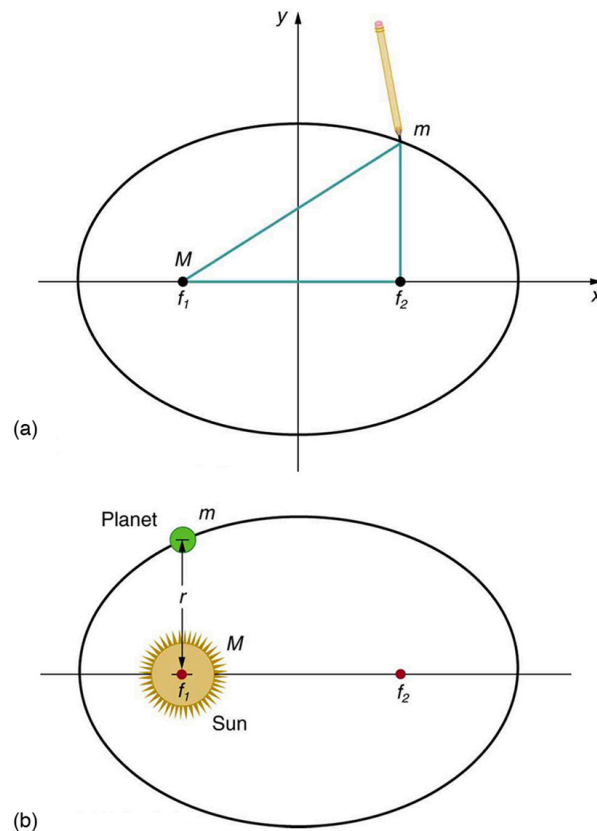
1. *A small mass  $m$  orbits a much larger mass  $M$ .* This allows us to view the motion as if  $M$  were stationary—in fact, as if from an inertial frame of reference placed on  $M$ —without significant error. Mass  $m$  is the satellite of  $M$ , if the orbit is gravitationally bound.
2. *The system is isolated from other masses.* This allows us to neglect any small effects due to outside masses.

The conditions are satisfied, to good approximation, by Earth's satellites (including the Moon), by objects orbiting the Sun, and by the satellites of other planets. Historically, planets were studied first, and there is a classical set of three laws, called Kepler's laws of planetary motion, that describe the orbits of all bodies satisfying the two previous conditions (not just planets in our solar system). These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630), who devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed.

### Kepler's Laws of Planetary Motion

#### Kepler's First Law

The orbit of each planet about the Sun is an ellipse with the Sun at one focus.



**FIGURE 6.26** (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci ( $f_1$  and  $f_2$ ) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit,  $m$  follows an elliptical path with  $M$  at one focus. Kepler's first law states this fact for planets orbiting the Sun.

### Kepler's Second Law

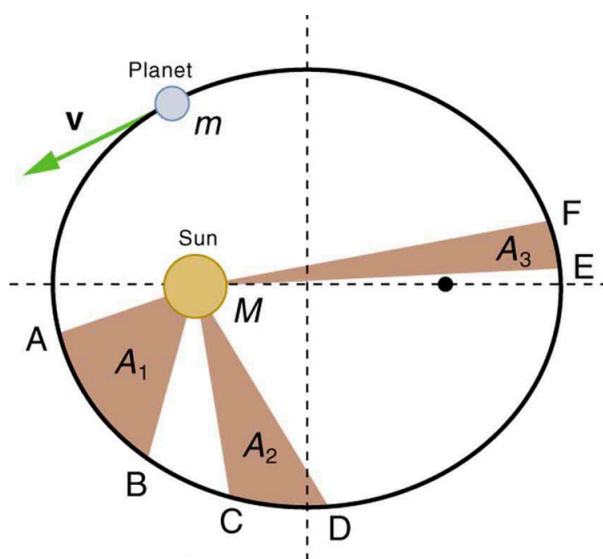
Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times (see [Figure 6.27](#)).

### Kepler's Third Law

The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun. In equation form, this is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}, \quad 6.55$$

where  $T$  is the period (time for one orbit) and  $r$  is the average radius. This equation is valid only for comparing two small masses orbiting the same large one. Most importantly, this is a descriptive equation only, giving no information as to the cause of the equality.



**FIGURE 6.27** The shaded regions have equal areas. It takes equal times for  $m$  to go from A to B, from C to D, and from E to F. The mass  $m$  moves fastest when it is closest to  $M$ . Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Note again that while, for historical reasons, Kepler's laws are stated for planets orbiting the Sun, they are actually valid for all bodies satisfying the two previously stated conditions.

### EXAMPLE 6.7

#### Find the Time for One Orbit of an Earth Satellite

Given that the Moon orbits Earth each 27.3 d and that it is an average distance of  $3.84 \times 10^8$  m from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1500 km above Earth's surface.

#### Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form in  $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$ . Let us use the subscript 1 for the Moon and the subscript 2 for the satellite. We are asked to find

$T_2$ . The given information tells us that the orbital radius of the Moon is  $r_1 = 3.84 \times 10^8$  m, and that the period of the Moon is  $T_1 = 27.3$  d. The height of the artificial satellite above Earth's surface is given, and so we must add the radius of Earth (6380 km) to get  $r_2 = (1500 + 6380)$  km = 7880 km. Now all quantities are known, and so  $T_2$  can be found.

#### Solution

Kepler's third law is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \quad 6.56$$

To solve for  $T_2$ , we cross-multiply and take the square root, yielding

$$T_2^2 = T_1^2 \left( \frac{r_2}{r_1} \right)^3 \quad 6.57$$

$$T_2 = T_1 \left( \frac{r_2}{r_1} \right)^{3/2} \quad 6.58$$

Substituting known values yields

$$\begin{aligned}
 T_2 &= 27.3 \text{ d} \times \frac{24.0 \text{ h}}{\text{d}} \times \left( \frac{7880 \text{ km}}{3.84 \times 10^5 \text{ km}} \right)^{3/2} \\
 &= 1.93 \text{ h.}
 \end{aligned}
 \tag{6.59}$$

### Discussion

This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will orbit in the same amount of time. This fact is related to the condition that the satellite's mass is small compared with that of Earth.

People immediately search for deeper meaning when broadly applicable laws, like Kepler's, are discovered. It was Newton who took the next giant step when he proposed the law of universal gravitation. While Kepler was able to discover *what* was happening, Newton discovered that gravitational force was the cause.

### Derivation of Kepler's Third Law for Circular Orbits

We shall derive Kepler's third law, starting with Newton's laws of motion and his universal law of gravitation. The point is to demonstrate that the force of gravity is the cause for Kepler's laws (although we will only derive the third one).

Let us consider a circular orbit of a small mass  $m$  around a large mass  $M$ , satisfying the two conditions stated at the beginning of this section. Gravity supplies the centripetal force to mass  $m$ . Starting with Newton's second law applied to circular motion,

$$F_{\text{net}} = ma_c = m \frac{v^2}{r}. \tag{6.60}$$

The net external force on mass  $m$  is gravity, and so we substitute the force of gravity for  $F_{\text{net}}$ :

$$G \frac{mM}{r^2} = m \frac{v^2}{r}. \tag{6.61}$$

The mass  $m$  cancels, yielding

$$G \frac{M}{r} = v^2. \tag{6.62}$$

The fact that  $m$  cancels out is another aspect of the oft-noted fact that at a given location all masses fall with the same acceleration. Here we see that at a given orbital radius  $r$ , all masses orbit at the same speed. (This was implied by the result of the preceding worked example.) Now, to get at Kepler's third law, we must get the period  $T$  into the equation. By definition, period  $T$  is the time for one complete orbit. Now the average speed  $v$  is the circumference divided by the period—that is,

$$v = \frac{2\pi r}{T}. \tag{6.63}$$

Substituting this into the previous equation gives

$$G \frac{M}{r} = \frac{4\pi^2 r^2}{T^2}. \tag{6.64}$$

Solving for  $T^2$  yields

$$T^2 = \frac{4\pi^2}{GM} r^3. \tag{6.65}$$

Using subscripts 1 and 2 to denote two different satellites, and taking the ratio of the last equation for satellite 1 to satellite 2 yields

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}. \tag{6.66}$$

This is Kepler's third law. Note that Kepler's third law is valid only for comparing satellites of the same parent body, because only then does the mass of the parent body  $M$  cancel.

Now consider what we get if we solve  $T^2 = \frac{4\pi^2}{GM}r^3$  for the ratio  $r^3/T^2$ . We obtain a relationship that can be used to determine the mass  $M$  of a parent body from the orbits of its satellites:

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2}M. \quad 6.67$$

If  $r$  and  $T$  are known for a satellite, then the mass  $M$  of the parent can be calculated. This principle has been used extensively to find the masses of heavenly bodies that have satellites. Furthermore, the ratio  $r^3/T^2$  should be a constant for all satellites of the same parent body (because  $r^3/T^2 = GM/4\pi^2$ ). (See [Table 6.2](#)).

It is clear from [Table 6.2](#) that the ratio of  $r^3/T^2$  is constant, at least to the third digit, for all listed satellites of the Sun, and for those of Jupiter. Small variations in that ratio have two causes—uncertainties in the  $r$  and  $T$  data, and perturbations of the orbits due to other bodies. Interestingly, those perturbations can be—and have been—used to predict the location of new planets and moons. This is another verification of Newton's universal law of gravitation.

### Making Connections

Newton's universal law of gravitation is modified by Einstein's general theory of relativity, as we shall see in [Particle Physics](#). Newton's gravity is not seriously in error—it was and still is an extremely good approximation for most situations. Einstein's modification is most noticeable in extremely large gravitational fields, such as near black holes. However, general relativity also explains such phenomena as small but long-known deviations of the orbit of the planet Mercury from classical predictions.

### The Case for Simplicity

The development of the universal law of gravitation by Newton played a pivotal role in the history of ideas. While it is beyond the scope of this text to cover that history in any detail, we note some important points. The definition of planet set in 2006 by the International Astronomical Union (IAU) states that in the solar system, a planet is a celestial body that:

1. is in orbit around the Sun,
2. has sufficient mass to assume hydrostatic equilibrium and
3. has cleared the neighborhood around its orbit.

A non-satellite body fulfilling only the first two of the above criteria is classified as “dwarf planet.”

In 2006, Pluto was demoted to a ‘dwarf planet’ after scientists revised their definition of what constitutes a “true” planet.

Parent	Satellite	Average orbital radius $r(\text{km})$	Period $T(\text{y})$	$r^3 / T^2$ ( $\text{km}^3 / \text{y}^2$ )
Earth	Moon	$3.84 \times 10^5$	0.07481	$1.01 \times 10^{19}$
Sun	Mercury	$5.79 \times 10^7$	0.2409	$3.34 \times 10^{24}$
	Venus	$1.082 \times 10^8$	0.6150	$3.35 \times 10^{24}$
	Earth	$1.496 \times 10^8$	1.000	$3.35 \times 10^{24}$
	Mars	$2.279 \times 10^8$	1.881	$3.35 \times 10^{24}$

**TABLE 6.2** Orbital Data and Kepler's Third Law

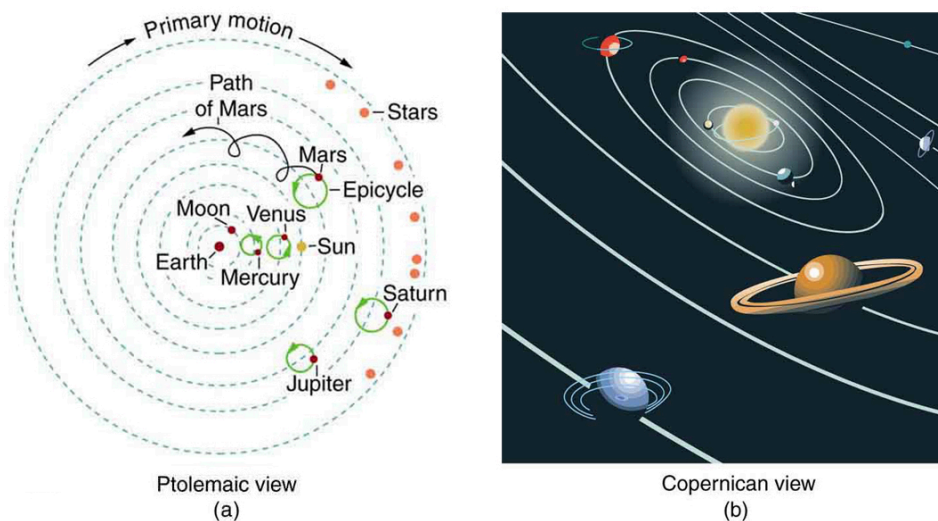
Parent	Satellite	Average orbital radius $r(\text{km})$	Period $T(\text{y})$	$r^3 / T^2 (\text{km}^3 / \text{y}^2)$
	Jupiter	$7.783 \times 10^8$	11.86	$3.35 \times 10^{24}$
	Saturn	$1.427 \times 10^9$	29.46	$3.35 \times 10^{24}$
	Neptune	$4.497 \times 10^9$	164.8	$3.35 \times 10^{24}$
	Pluto	$5.90 \times 10^9$	248.3	$3.33 \times 10^{24}$
Jupiter	Io	$4.22 \times 10^5$	0.00485 (1.77 d)	$3.19 \times 10^{21}$
	Europa	$6.71 \times 10^5$	0.00972 (3.55 d)	$3.20 \times 10^{21}$
	Ganymede	$1.07 \times 10^6$	0.0196 (7.16 d)	$3.19 \times 10^{21}$
	Callisto	$1.88 \times 10^6$	0.0457 (16.19 d)	$3.20 \times 10^{21}$

**TABLE 6.2** Orbital Data and Kepler's Third Law

The universal law of gravitation is a good example of a physical principle that is very broadly applicable. That single equation for the gravitational force describes all situations in which gravity acts. It gives a cause for a vast number of effects, such as the orbits of the planets and moons in the solar system. It epitomizes the underlying unity and simplicity of physics.

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in [Figure 6.28\(a\)](#). This is called the Ptolemaic view, for the Greek philosopher who lived in the second century AD. This model is characterized by a list of facts for the motions of planets with no cause and effect explanation. There tended to be a different rule for each heavenly body and a general lack of simplicity.

[Figure 6.28\(b\)](#) represents the modern or Copernican model. In this model, a small set of rules and a single underlying force explain not only all motions in the solar system, but all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling. As our knowledge of nature has grown, the basic simplicity of its laws has become ever more evident.



**FIGURE 6.28** (a) The Ptolemaic model of the universe has Earth at the center with the Moon, the planets, the Sun, and the stars revolving about it in complex superpositions of circular paths. This geocentric model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints as to what are the causes of these motions. (b) The Copernican model has the Sun at the center of the solar system. It is fully explained by a small number of laws of physics, including Newton's universal law of gravitation.

## Glossary

**angular velocity**  $\omega$ , the rate of change of the angle with which an object moves on a circular path

**arc length**  $\Delta s$ , the distance traveled by an object along a circular path

**banked curve** the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

**center of mass** the point where the entire mass of an object can be thought to be concentrated

**centrifugal force** a fictitious force that tends to throw an object off when the object is rotating in a non-inertial frame of reference

**centripetal acceleration** the acceleration of an object moving in a circle, directed toward the center

**centripetal force** any net force causing uniform circular motion

**Coriolis force** the fictitious force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

**fictitious force** a force having no physical origin

**gravitational constant,  $G$**  a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

**ideal angle** the angle at which a car can turn safely on a steep curve, which is in proportion to the ideal speed

**ideal banking** the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the

aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

**ideal speed** the maximum safe speed at which a vehicle can turn on a curve without the aid of friction between the tire and the road

**microgravity** an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

**Newton's universal law of gravitation** every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

**non-inertial frame of reference** an accelerated frame of reference

**pit** a tiny indentation on the spiral track moulded into the top of the polycarbonate layer of CD

**radians** a unit of angle measurement

**radius of curvature** radius of a circular path

**rotation angle** the ratio of the arc length to the radius of curvature on a circular path:  $\Delta\theta = \frac{\Delta s}{r}$

**ultracentrifuge** a centrifuge optimized for spinning a rotor at very high speeds

**uniform circular motion** the motion of an object in a circular path at constant speed

## Section Summary

### 6.1 Rotation Angle and Angular Velocity

- Uniform circular motion is motion in a circle at constant speed. The rotation angle  $\Delta\theta$  is defined as the ratio of the arc length to the radius of curvature:

$$\Delta\theta = \frac{\Delta s}{r},$$

where arc length  $\Delta s$  is distance traveled along a circular path and  $r$  is the radius of curvature of the circular path. The quantity  $\Delta\theta$  is measured in units of radians (rad), for which

$$2\pi \text{ rad} = 360^\circ = 1 \text{ revolution.}$$

- The conversion between radians and degrees is  $1 \text{ rad} = 57.3^\circ$ .
- Angular velocity  $\omega$  is the rate of change of an angle,
 
$$\omega = \frac{\Delta\theta}{\Delta t},$$
 where a rotation  $\Delta\theta$  takes place in a time  $\Delta t$ . The units of angular velocity are radians per second

(rad/s). Linear velocity  $v$  and angular velocity  $\omega$  are related by

$$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

### 6.2 Centripetal Acceleration

- Centripetal acceleration  $a_c$  is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity  $v$  and has the magnitude

$$a_c = \frac{v^2}{r}; a_c = r\omega^2.$$

- The unit of centripetal acceleration is  $\text{m/s}^2$ .

### 6.3 Centripetal Force

- Centripetal force  $F_c$  is any force causing uniform circular motion. It is a “center-seeking” force that always points toward the center of rotation. It is perpendicular to linear velocity  $v$  and has

magnitude

$$F_c = ma_c,$$

which can also be expressed as

$$\left. \begin{array}{l} F_c = m \frac{v^2}{r} \\ \text{or} \\ F_c = m r \omega^2 \end{array} \right\}$$

### 6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

- Rotating and accelerated frames of reference are non-inertial.
- Fictitious forces, such as the Coriolis force, are needed to explain motion in such frames.

### 6.5 Newton's Universal Law of Gravitation

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

$$F = G \frac{mM}{r^2},$$

where  $F$  is the magnitude of the gravitational force.

$G$  is the gravitational constant, given by

$$G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2.$$

- Newton's law of gravitation applies universally.

### 6.6 Satellites and Kepler's Laws: An Argument for Simplicity

- Kepler's laws are stated for a small mass  $m$

## Conceptual Questions

### 6.1 Rotation Angle and Angular Velocity

1. There is an analogy between rotational and linear physical quantities. What rotational quantities are analogous to distance and velocity?

### 6.2 Centripetal Acceleration

2. Can centripetal acceleration change the speed of circular motion? Explain.

### 6.3 Centripetal Force

3. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

orbiting a larger mass  $M$  in near-isolation.

Kepler's laws of planetary motion are then as follows:

Kepler's first law

The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

Kepler's second law

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

Kepler's third law

The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

where  $T$  is the period (time for one orbit) and  $r$  is the average radius of the orbit.

- The period and radius of a satellite's orbit about a larger body  $M$  are related by

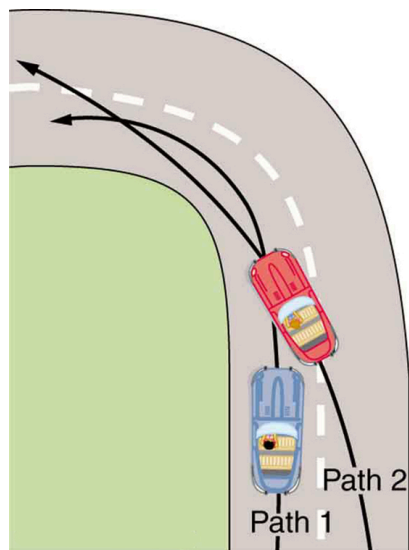
$$T^2 = \frac{4\pi^2}{GM} r^3$$

or

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M.$$

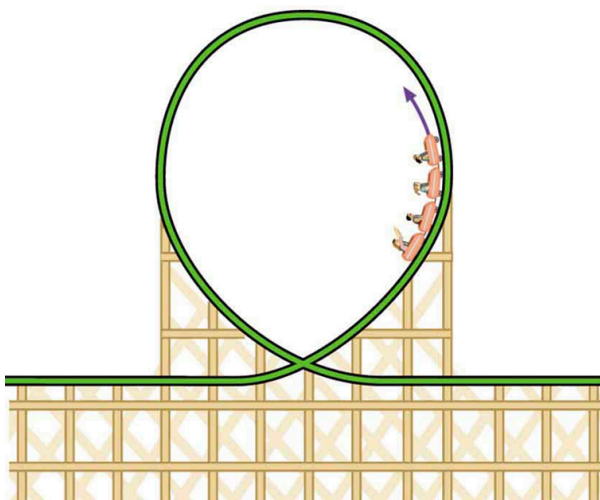
4. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?
5. If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

6. Race car drivers routinely cut corners as shown in [Figure 6.29](#). Explain how this allows the curve to be taken at the greatest speed.



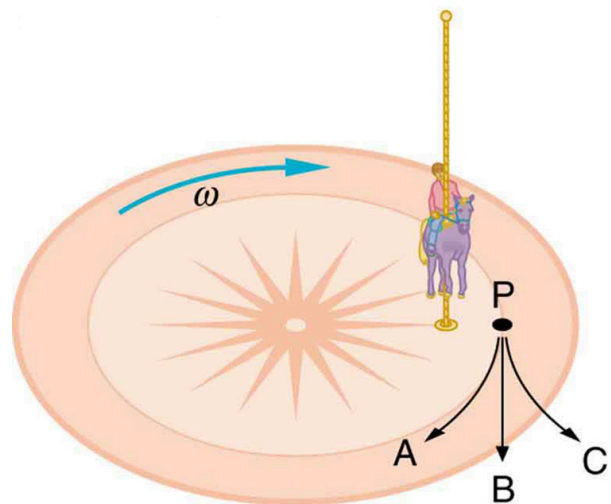
**FIGURE 6.29** Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

7. A number of amusement parks have rides that make vertical loops like the one shown in [Figure 6.30](#). For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:
- The car goes over the top at faster than this speed?
  - The car goes over the top at slower than this speed?



**FIGURE 6.30** Amusement rides with a vertical loop are an example of a form of curved motion.

8. What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in [Figure 6.30](#) under the following circumstances:
- The car goes over the top at such a speed that the gravitational force is the only force acting?
  - The car goes over the top faster than this speed?
  - The car goes over the top slower than this speed?
9. As a skater forms a circle, what force is responsible for making her turn? Use a free body diagram in your answer.
10. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in [Figure 6.31](#) will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.

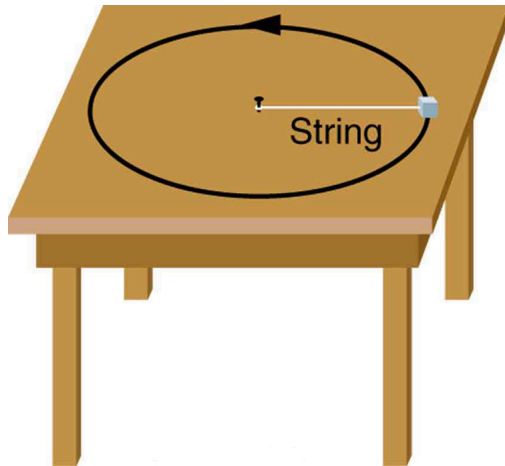


### Merry-go-round's rotating frame of reference

**FIGURE 6.31** A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?

11. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

- 12.** Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



**FIGURE 6.32** A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

### 6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

- 13.** When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the northern hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?
- 14.** Is there a real force that throws water from clothes during the spin cycle of a washing machine? Explain how the water is removed.
- 15.** In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is a fictitious force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all of the real forces acting on them.
- 16.** Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?
- 17.** Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not  $9.80 \text{ m/s}^2$ . Who do you agree with and why?
- 18.** A non-rotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

### 6.5 Newton's Universal Law of Gravitation

- 19.** Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?
- 20.** Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not  $9.80 \text{ m/s}^2$ . Who do you agree with and why?
- 21.** Draw a free body diagram for a satellite in an elliptical orbit showing why its speed increases as it approaches its parent body and decreases as it moves away.
- 22.** Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

## 6.6 Satellites and Kepler's Laws: An Argument for Simplicity

- 23.** In what frame(s) of reference are Kepler's laws valid? Are Kepler's laws purely descriptive, or do they contain causal information?

## Problems & Exercises

### 6.1 Rotation Angle and Angular Velocity

- Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?
- Microwave ovens rotate at a rate of about 6 rev/min. What is this in revolutions per second? What is the angular velocity in radians per second?
- An automobile with 0.260 m radius tires travels 80,000 km before wearing them out. How many revolutions do the tires make, neglecting any backing up and any change in radius due to wear?
- (a) What is the period of rotation of Earth in seconds? (b) What is the angular velocity of Earth? (c) Given that Earth has a radius of  $6.4 \times 10^6$  m at its equator, what is the linear velocity at Earth's surface?
- A baseball pitcher brings his arm forward during a pitch, rotating the forearm about the elbow. If the velocity of the ball in the pitcher's hand is 35.0 m/s and the ball is 0.300 m from the elbow joint, what is the angular velocity of the forearm?
- In lacrosse, a ball is thrown from a net on the end of a stick by rotating the stick and forearm about the elbow. If the angular velocity of the ball about the elbow joint is 30.0 rad/s and the ball is 1.30 m from the elbow joint, what is the velocity of the ball?
- A truck with 0.420-m-radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?
- Integrated Concepts When kicking a football, the kicker rotates his leg about the hip joint.
  - If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip joint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity?
  - The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s?
  - Find the maximum range of the football, neglecting air resistance.

### 9. Construct Your Own Problem

Consider an amusement park ride in which participants are rotated about a vertical axis in a cylinder with vertical walls. Once the angular velocity reaches its full value, the floor drops away and friction between the walls and the riders prevents them from sliding down. Construct a problem in which you calculate the necessary angular velocity that assures the riders will not slide down the wall. Include a free body diagram of a single rider. Among the variables to consider are the radius of the cylinder and the coefficients of friction between the riders' clothing and the wall.

### 6.2 Centripetal Acceleration

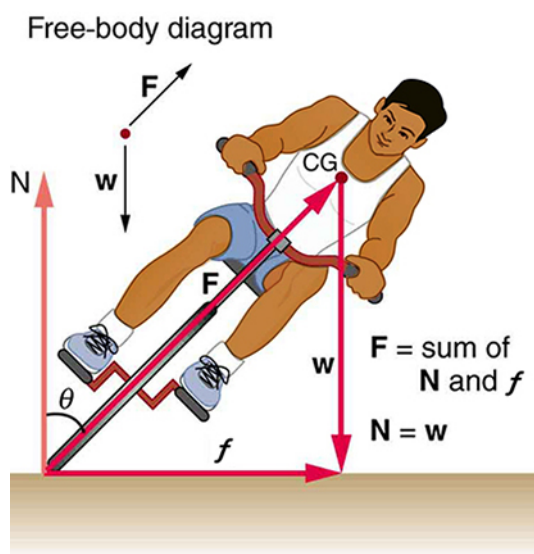
- A fairground ride spins its occupants inside a flying saucer-shaped container. If the horizontal circular path the riders follow has an 8.00 m radius, at how many revolutions per minute will the riders be subjected to a centripetal acceleration whose magnitude is 1.50 times that due to gravity?
- A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If the runner completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of their centripetal acceleration as they run the curved portion of the track?
- Taking the age of Earth to be about  $4 \times 10^9$  years and assuming its orbital radius of  $1.5 \times 10^{11}$  m has not changed and is circular, calculate the approximate total distance Earth has traveled since its birth (in a frame of reference stationary with respect to the Sun).
- The propeller of a World War II fighter plane is 2.30 m in diameter.
  - What is its angular velocity in radians per second if it spins at 1200 rev/min?
  - What is the linear speed of its tip at this angular velocity if the plane is stationary on the tarmac?
  - What is the centripetal acceleration of the propeller tip under these conditions? Calculate it in meters per second squared and convert to multiples of  $g$ .

- 14.** An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.  
 (a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of  $g$ .  
 (b) What is the linear speed of a point on its edge?
- 15.** Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.  
 (a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.  
 (b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).
- 16.** Olympic ice skaters are able to spin at about 5 rev/s.  
 (a) What is their angular velocity in radians per second?  
 (b) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?  
 (c) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?  
 (d) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.
- 17.** What percentage of the acceleration at Earth's surface is the acceleration due to gravity at the position of a satellite located 300 km above Earth?
- 18.** Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating:  
 (a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min.  
 (b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).
- 19.** A rotating space station is said to create “artificial gravity”—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what angular velocity would produce an “artificial gravity” of  $9.80 \text{ m/s}^2$  at the rim?
- 20.** At takeoff, a commercial jet has a 60.0 m/s speed. Its tires have a diameter of 0.850 m.  
 (a) At how many rev/min are the tires rotating?  
 (b) What is the centripetal acceleration at the edge of the tire?  
 (c) With what force must a determined  $1.00 \times 10^{-15} \text{ kg}$  bacterium cling to the rim?  
 (d) Take the ratio of this force to the bacterium's weight.
- 21. Integrated Concepts**  
 Riders in an amusement park ride shaped like a Viking ship hung from a large pivot are rotated back and forth like a rigid pendulum. Sometime near the middle of the ride, the ship is momentarily motionless at the top of its circular arc. The ship then swings down under the influence of gravity. The speed at the bottom of the arc is 23.4 m/s.  
 (a) What is the centripetal acceleration at the bottom of the arc?  
 (b) Draw a free body diagram of the forces acting on a rider at the bottom of the arc.  
 (c) Find the force exerted by the ride on a 60.0 kg rider and compare it to her weight.  
 (d) Discuss whether the answer seems reasonable.
- 22. Unreasonable Results**  
 A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.  
 (a) What is the magnitude of the centripetal acceleration of the child at the low point?  
 (b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?  
 (c) What is unreasonable about these results?  
 (d) Which premises are unreasonable or inconsistent?

### 6.3 Centripetal Force

23. (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?  
 (b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?  
 (c) Compare each force with her weight.
24. Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.
25. What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?
26. What is the ideal speed to take a 100 m radius curve banked at a  $20.0^\circ$  angle?
27. (a) What is the radius of a bobsled turn banked at  $75.0^\circ$  and taken at 30.0 m/s, assuming it is ideally banked?  
 (b) Calculate the centripetal acceleration.  
 (c) Does this acceleration seem large to you?

28. Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen in [Figure 6.33](#). To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force), and the vertical normal force (which must equal the system's weight).  
 (a) Show that  $\theta$  (as defined in the figure) is related to the speed  $v$  and radius of curvature  $r$  of the turn in the same way as for an ideally banked roadway—that is,  $\theta = \tan^{-1} v^2 / rg$   
 (b) Calculate  $\theta$  for a 12.0 m/s turn of radius 30.0 m (as in a race).

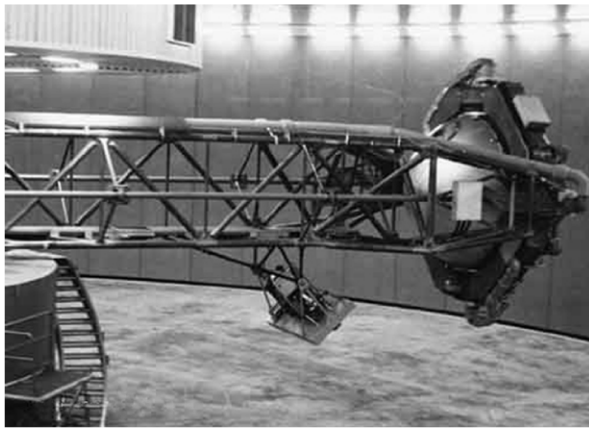


**FIGURE 6.33** A bicyclist negotiating a turn on level ground must lean at the correct angle—the ability to do this becomes instinctive. The force of the ground on the wheel needs to be on a line through the center of gravity. The net external force on the system is the centripetal force. The vertical component of the force on the wheel cancels the weight of the system while its horizontal component must supply the centripetal force. This process produces a relationship among the angle  $\theta$ , the speed  $v$ , and the radius of curvature  $r$  of the turn similar to that for the ideal banking of roadways.

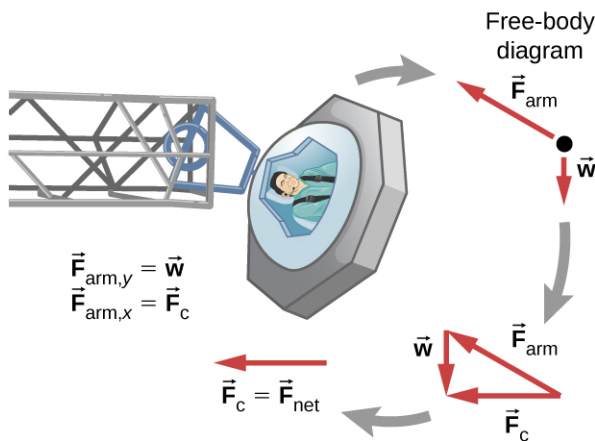
29. A large centrifuge, like the one shown in [Figure 6.34\(a\)](#), is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries.

(a) At what angular velocity is the centripetal acceleration  $10g$  if the rider is  $15.0\text{ m}$  from the center of rotation?

(b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in [Figure 6.34\(b\)](#). At what angle  $\theta$  below the horizontal will the cage hang when the centripetal acceleration is  $10g$ ? (Hint: The arm supplies centripetal force and supports the weight of the cage. Draw a free-body diagram of the forces to see what the angle  $\theta$  should be.)



(a)



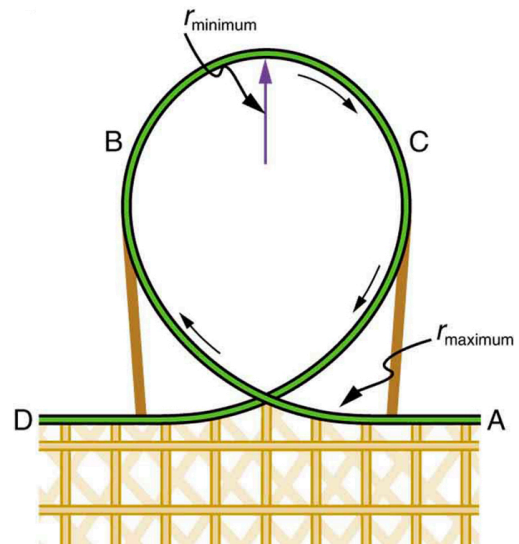
(b)

**FIGURE 6.34** (a) NASA centrifuge used to subject trainees to accelerations similar to those experienced in rocket launches and reentries. (credit: NASA) (b) Rider in cage showing how the cage pivots outward during rotation. This allows the total force exerted on the rider by the cage to be along its axis at all times.

30. Integrated Concepts

If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a real problem on icy mountain roads). (a) Calculate the ideal speed to take a  $100\text{ m}$  radius curve banked at  $15.0^\circ$ . (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at  $20.0\text{ km/h}$ ?

31. Modern roller coasters have vertical loops like the one shown in [Figure 6.35](#). The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is  $15.0\text{ m}$  and the downward acceleration of the car is  $1.50g$ ?



**FIGURE 6.35** Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than  $g$  so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place.

32. Unreasonable Results

(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked  $50.0\text{ m}$  radius curve at  $30.0\text{ m/s}$ .

(b) What is unreasonable about the result?

(c) Which premises are unreasonable or inconsistent?

## 6.5 Newton's Universal Law of Gravitation

- 33.** (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is  $9.830 \text{ m/s}^2$  and the radius of the Earth is 6371 km from center to pole.  
(b) Compare this with the accepted value of  $5.979 \times 10^{24} \text{ kg}$ .
- 34.** (a) Calculate the magnitude of the acceleration due to gravity on the surface of Earth due to the Moon.  
(b) Calculate the magnitude of the acceleration due to gravity at Earth due to the Sun.  
(c) Take the ratio of the Moon's acceleration to the Sun's and comment on why the tides are predominantly due to the Moon in spite of this number.
- 35.** (a) What is the acceleration due to gravity on the surface of the Moon?  
(b) On the surface of Mars? The mass of Mars is  $6.418 \times 10^{23} \text{ kg}$  and its radius is  $3.38 \times 10^6 \text{ m}$ .
- 36.** (a) Calculate the acceleration due to gravity on the surface of the Sun.  
(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)
- 37.** The Moon and Earth rotate about their common center of mass, which is located about 4700 km from the center of Earth. (This is 1690 km below the surface.)  
(a) Calculate the magnitude of the acceleration due to the Moon's gravity at that point.  
(b) Calculate the magnitude of the centripetal acceleration of the center of Earth as it rotates about that point once each lunar month (about 27.3 d) and compare it with the acceleration found in part (a). Comment on whether or not they are equal and why they should or should not be.
- 38.** Solve part (b) of [Example 6.6](#) using  $a_c = v^2/r$ .
- 39.** (a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child).  
(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some  $6.29 \times 10^{11} \text{ m}$  away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)
- 40.** The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But it now appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared with the closest planet to Neptune:  
(a) Calculate the acceleration due to gravity at Neptune due to Pluto when they are  $4.50 \times 10^{12} \text{ m}$  apart, as they are at present. The mass of Pluto is  $1.4 \times 10^{22} \text{ kg}$ .  
(b) Calculate the acceleration due to gravity at Neptune due to Uranus, presently about  $2.50 \times 10^{12} \text{ m}$  apart, and compare it with that due to Pluto. The mass of Uranus is  $8.62 \times 10^{25} \text{ kg}$ .
- 41.** (a) The Sun orbits the Milky Way galaxy once each  $2.60 \times 10^8 \text{ y}$ , with a roughly circular orbit averaging  $3.00 \times 10^4$  light years in radius. (A light year is the distance traveled by light in 1 y.) Calculate the centripetal acceleration of the Sun in its galactic orbit. Does your result support the contention that a nearly inertial frame of reference can be located at the Sun?  
(b) Calculate the average speed of the Sun in its galactic orbit. Does the answer surprise you?
- 42. Unreasonable Result**  
A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.  
(a) Calculate the mass of the mountain.  
(b) Compare the mountain's mass with that of Earth.  
(c) What is unreasonable about these results?  
(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

## 6.6 Satellites and Kepler's Laws: An Argument for Simplicity

- 43.** A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit based on the data for the moon in [Table 6.2](#).

- 44.** Calculate the mass of the Sun based on data for Earth's orbit and compare the value obtained with the Sun's actual mass.
- 45.** Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.
- 46.** Find the ratio of the mass of Jupiter to that of Earth based on data in [Table 6.2](#).
- 47.** Astronomical observations of our Milky Way galaxy indicate that it has a mass of about  $8.0 \times 10^{11}$  solar masses. A star orbiting on the galaxy's periphery is about  $6.0 \times 10^4$  light years from its center. (a) What should the orbital period of that star be? (b) If its period is  $6.0 \times 10^7$  years instead, what is the mass of the galaxy? Such calculations are used to imply the existence of "dark matter" in the universe and have indicated, for example, the existence of very massive black holes at the centers of some galaxies.
- 48. Integrated Concepts**  
Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same radius that intersects the satellite's orbit at an angle of  $90^\circ$  relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)
- 49. Unreasonable Results**  
(a) Based on Kepler's laws and information on the orbital characteristics of the Moon, calculate the orbital radius for an Earth satellite having a period of 1.00 h. (b) What is unreasonable about this result? (c) What is unreasonable or inconsistent about the premise of a 1.00 h orbit?
- 50. Construct Your Own Problem**  
On February 14, 2000, the NEAR spacecraft was successfully inserted into orbit around Eros, becoming the first artificial satellite of an asteroid. Construct a problem in which you determine the orbital speed for a satellite near Eros. You will need to find the mass of the asteroid and consider such things as a safe distance for the orbit. Although Eros is not spherical, calculate the acceleration due to gravity on its surface at a point an average distance from its center of mass. Your instructor may also wish to have you calculate the escape velocity from this point on Eros.
- 51. Critical Thinking** A car travels around a loop with negligible friction at a constant speed and never loses contact with the loop. The top of the loop is labeled A and the bottom of the loop is labeled B. (a) At what point would the normal force be greatest? Briefly explain your reasoning. (b) Based on experimental data, an equation that fits the data is suggested for the normal force,  $F_N$ , which may not be correct:  $F_N = Kr^{1/2}$ , where  $K$  is a constant with appropriate units and  $r$  is the radius of the loop. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) Now the car travels around loops of various radii and the speed at which the car barely makes it around the loop is measured. Graph that speed vs. the radius of the loop.



## CHAPTER 7

# Work, Energy, and Energy Resources



**FIGURE 7.1** How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

### CHAPTER OUTLINE

#### 7.1 Work: The Scientific Definition

#### 7.2 Kinetic Energy and the Work-Energy Theorem

#### 7.3 Gravitational Potential Energy

#### 7.4 Conservative Forces and Potential Energy

#### 7.5 Nonconservative Forces

#### 7.6 Conservation of Energy

#### 7.7 Power

#### 7.8 Work, Energy, and Power in Humans

#### 7.9 World Energy Use

**INTRODUCTION TO WORK, ENERGY, AND ENERGY RESOURCES** Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

**Conservation of energy** (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. For example, scientists Willem 's Gravesande and Émilie du Châtelet undertook (separate) experiments where they dropped heavy lead balls into beds of clay. Du Châtelet showed that the balls that hit the clay with twice the velocity penetrated four times as deep into the clay; those with three times the velocity reached a depth nine times greater. This led her to develop a more accurate concept of energy conservation, expressed as  $E = \frac{1}{2}mv^2$ . Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation  $E = mc^2$ ).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are

key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

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## 7.1 Work: The Scientific Definition

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

### What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = |\mathbf{F}|(\cos \theta)|\mathbf{d}|, \quad 7.1$$

where  $W$  is work,  $\mathbf{d}$  is the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ , as in [Figure 7.2](#). We can also write this as

$$W = Fd \cos \theta. \quad 7.2$$

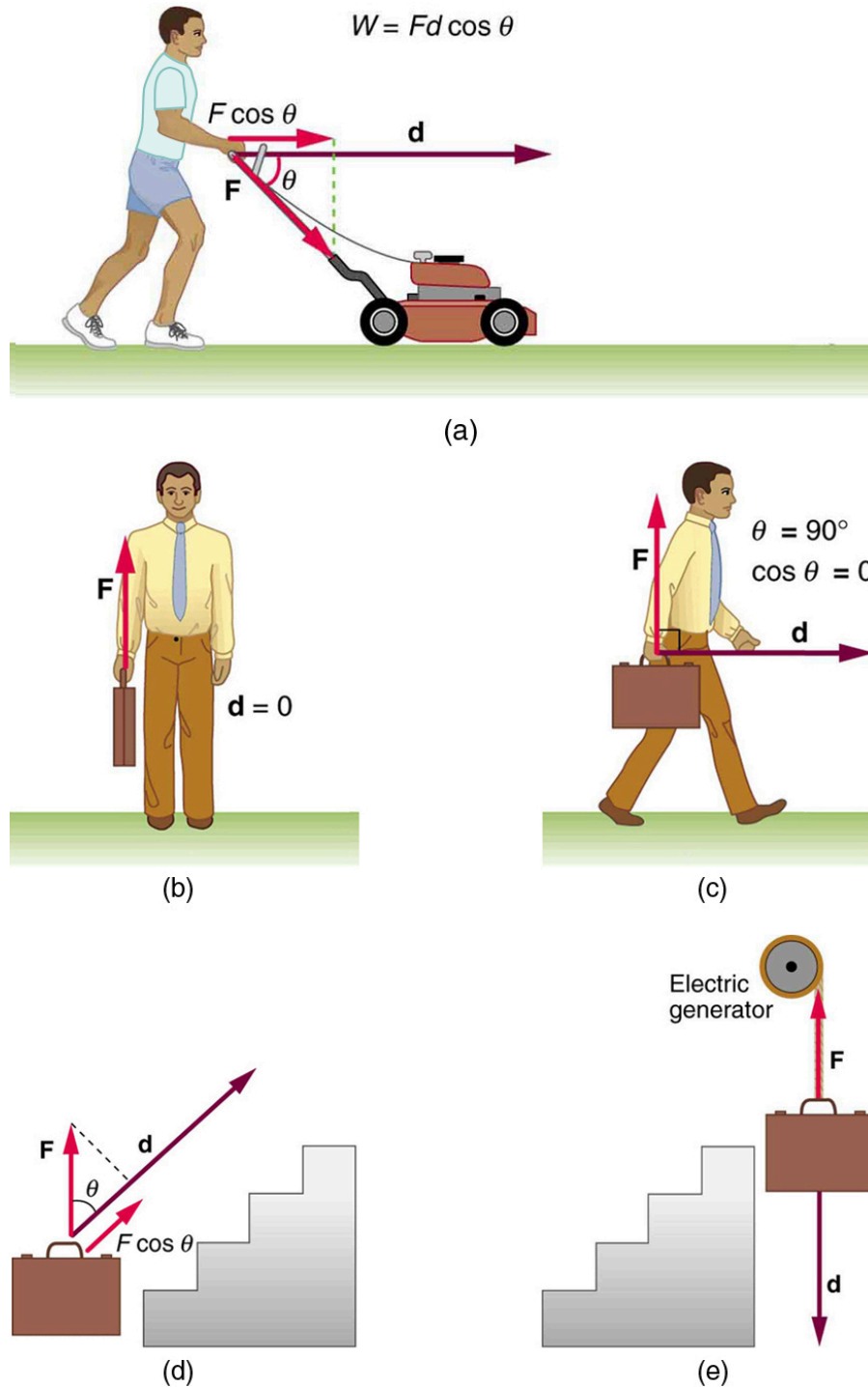
To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

### What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd \cos \theta, \quad 7.3$$

where  $W$  is work,  $F$  is the magnitude of the force on the system,  $d$  is the magnitude of the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ .



**FIGURE 7.2** Examples of work. (a) The work done by the force  $\mathbf{F}$  on this lawn mower is  $Fd \cos \theta$ . Note that  $F \cos \theta$  is the component of the force in the direction of the motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work is done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force  $\mathbf{F}$  in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because  $\mathbf{F}$  and  $\mathbf{d}$  are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in [Figure 7.2](#). The person holding the briefcase in [Figure 7.2\(b\)](#) does no work, for example. Here  $d = 0$ , so  $W = 0$ . Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase–Earth system”—see [Gravitational Potential Energy](#) for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the

motion. For example, the person carrying the briefcase on level ground in [Figure 7.2\(c\)](#) does no work on it, because the force is perpendicular to the motion. That is,  $\cos 90^\circ = 0$ , and so  $W = 0$ .

In contrast, when a force exerted on the system has a component in the direction of motion, such as in [Figure 7.2\(d\)](#), work *is* done—energy is transferred to the briefcase. Finally, in [Figure 7.2\(e\)](#), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase's weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes  $\theta = 180^\circ$ , and  $\cos 180^\circ = -1$ ; therefore,  $W$  is negative.

## Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and  $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$ . One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.



### EXAMPLE 7.1

#### Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in [Figure 7.2\(a\)](#) if he exerts a constant force of 75.0 N at an angle  $35^\circ$  below the horizontal and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of 10,000 kJ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by  $1^\circ\text{C}$ , and is equivalent to 4.186 J, while one *food calorie* (1 kcal) is equivalent to 4186 J.

#### Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation  $W = Fd \cos \theta$ . The force, angle, and displacement are given, so that only the work  $W$  is unknown.

#### Solution

The equation for the work is

$$W = Fd \cos \theta. \quad 7.4$$

Substituting the known values gives

$$\begin{aligned} W &= (75.0 \text{ N})(25.0 \text{ m}) \cos (35.0^\circ) \\ &= 1536 \text{ J} = 1.54 \times 10^3 \text{ J}. \end{aligned} \quad 7.5$$

Converting the work in joules to kilocalories yields  $W = (1536 \text{ J})(1 \text{ kcal}/4186 \text{ J}) = 0.367 \text{ kcal}$ . The ratio of the work done to the daily consumption is

$$\frac{W}{2400 \text{ kcal}} = 1.53 \times 10^{-4}. \quad 7.6$$

#### Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

## 7.2 Kinetic Energy and the Work-Energy Theorem

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

### Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if the lawn mower in [Figure 7.2\(a\)](#) is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on the briefcase by the person carrying it up stairs in [Figure 7.2\(d\)](#) is stored in the briefcase-Earth system and can be recovered at any time, as shown in [Figure 7.2\(e\)](#). In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

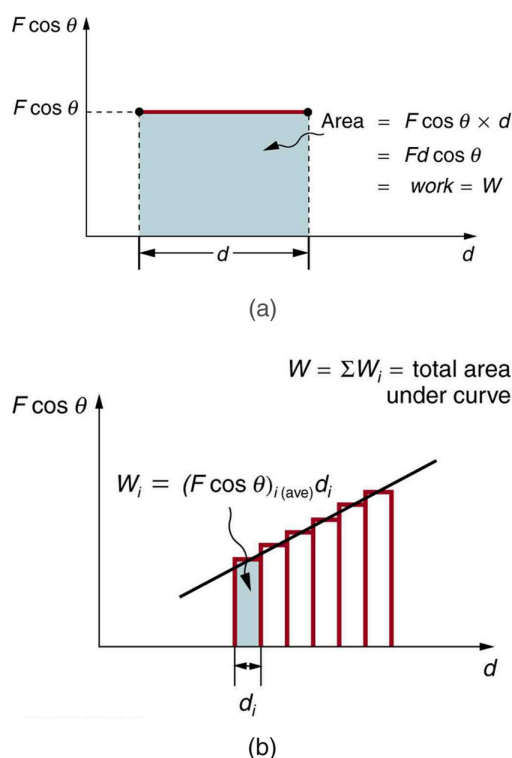
In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

### Net Work and the Work-Energy Theorem

We know from the study of Newton's laws in [Dynamics: Force and Newton's Laws of Motion](#) that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

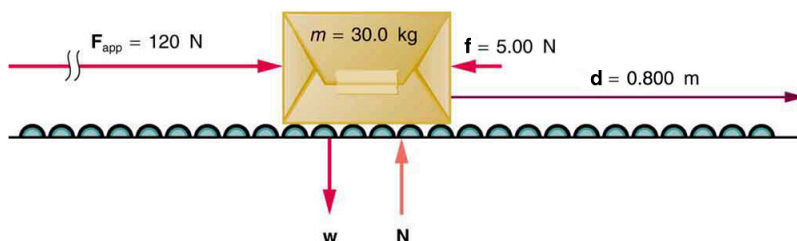
Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work on an object. The net work can be written in terms of the net force on an object,  $\mathbf{F}_{\text{net}}$ . In equation form, this is  $W_{\text{net}} = F_{\text{net}}d \cos \theta$  where  $\theta$  is the angle between the force vector and the displacement vector.

[Figure 7.3\(a\)](#) shows a graph of force versus displacement for the component of the force in the direction of the displacement—that is, an  $F \cos \theta$  vs.  $d$  graph. In this case,  $F \cos \theta$  is constant. You can see that the area under the graph is  $Fd \cos \theta$ , or the work done. [Figure 7.3\(b\)](#) shows a more general process where the force varies. The area under the curve is divided into strips, each having an average force  $(F \cos \theta)_{i(\text{ave})}$ . The work done is  $(F \cos \theta)_{i(\text{ave})}d_i$  for each strip, and the total work done is the sum of the  $W_i$ . Thus the total work done is the total area under the curve, a useful property to which we shall refer later.



**FIGURE 7.3** (a) A graph of  $F \cos \theta$  vs.  $d$ , when  $F \cos \theta$  is constant. The area under the curve represents the work done by the force. (b) A graph of  $F \cos \theta$  vs.  $d$  in which the force varies. The work done for each interval is the area of each strip; thus, the total area under the curve equals the total work done.

Net work will be simpler to examine if we consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in [Figure 7.4](#).



**FIGURE 7.4** A package on a roller belt is pushed horizontally through a distance  $d$ .

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force  $F_{\text{app}}$  and the horizontal friction force  $f$ . Thus, as expected, the net force is parallel to the displacement, so that  $\theta = 0^\circ$  and  $\cos \theta = 1$ , and the net work is given by

$$W_{\text{net}} = F_{\text{net}}d. \quad 7.7$$

The effect of the net force  $F_{\text{net}}$  is to accelerate the package from  $v_0$  to  $v$ . The kinetic energy of the package increases, indicating that the net work done on the system is positive. (See [Example 7.2](#).) By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting  $F_{\text{net}} = ma$  from Newton's second law gives

$$W_{\text{net}} = mad. \quad 7.8$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take  $d = x - x_0$  and use the equation studied in [Motion Equations for Constant Acceleration in One Dimension](#) for the change in speed over a distance  $d$  if the acceleration has the constant value  $a$ ; namely,  $v^2 = v_0^2 + 2ad$  (note that  $a$  appears in the expression for the net work). Solving for acceleration gives  $a = \frac{v^2 - v_0^2}{2d}$ . When  $a$  is substituted into

the preceding expression for  $W_{\text{net}}$ , we obtain

$$W_{\text{net}} = m \left( \frac{v^2 - v_0^2}{2d} \right) d. \quad 7.9$$

The  $d$  cancels, and we rearrange this to obtain

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \quad 7.10$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ . This quantity is our first example of a form of energy.

### The Work-Energy Theorem

The net work on a system equals the change in the quantity  $\frac{1}{2}mv^2$ .

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \quad 7.11$$

The quantity  $\frac{1}{2}mv^2$  in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass  $m$  moving at a speed  $v$ . (*Translational* kinetic energy is distinct from *rotational* kinetic energy, which is considered later.) In equation form, the translational kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2, \quad 7.12$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

We are aware that it takes energy to get an object, like a car or the package in [Figure 7.4](#), up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 100 km/h has four times the kinetic energy it has at 50 km/h, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.



### EXAMPLE 7.2

#### Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in [Figure 7.4](#) is moving at 0.500 m/s. What is its kinetic energy?

#### Strategy

Because the mass  $m$  and speed  $v$  are given, the kinetic energy can be calculated from its definition as given in the equation  $\text{KE} = \frac{1}{2}mv^2$ .

#### Solution

The kinetic energy is given by

$$\text{KE} = \frac{1}{2}mv^2. \quad 7.13$$

Entering known values gives

$$\text{KE} = 0.5(30.0 \text{ kg})(0.500 \text{ m/s})^2, \quad 7.14$$

which yields

$$KE = 3.75 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 3.75 \text{ J.} \quad 7.15$$

### Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

## EXAMPLE 7.3

### Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in [Figure 7.4](#) with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

#### Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See [Figure 7.4](#).) As expected, the net work is the net force times distance.

#### Solution for (a)

The net force is the push force minus friction, or  $F_{\text{net}} = 120 \text{ N} - 5.00 \text{ N} = 115 \text{ N}$ . Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115 \text{ N})(0.800 \text{ m}) \\ &= 92.0 \text{ N} \cdot \text{m} = 92.0 \text{ J.} \end{aligned} \quad 7.16$$

#### Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

#### Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

#### Solution for (b)

The applied force does work.

$$\begin{aligned} W_{\text{app}} &= F_{\text{app}}d \cos(0^\circ) = F_{\text{app}}d \\ &= (120 \text{ N})(0.800 \text{ m}) \\ &= 96.0 \text{ J} \end{aligned} \quad 7.17$$

The friction force and displacement are in opposite directions, so that  $\theta = 180^\circ$ , and the work done by friction is

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}}d \cos(180^\circ) = -F_{\text{fr}}d \\ &= -(5.00 \text{ N})(0.800 \text{ m}) \\ &= -4.00 \text{ J.} \end{aligned} \quad 7.18$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_{\text{N}} &= 0, \\ W_{\text{app}} &= 96.0 \text{ J}, \\ W_{\text{fr}} &= -4.00 \text{ J}. \end{aligned} \quad 7.19$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0 \text{ J}. \quad 7.20$$

### Discussion for (b)

The calculated total work  $W_{\text{total}}$  as the sum of the work by each force agrees, as expected, with the work  $W_{\text{net}}$  done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

## EXAMPLE 7.4

### Determining Speed from Work and Energy

Find the speed of the package in [Figure 7.4](#) at the end of the push, using work and energy concepts.

#### Strategy

Here the work-energy theorem can be used, because we have just calculated the net work,  $W_{\text{net}}$ , and the initial kinetic energy,  $\frac{1}{2}mv_0^2$ . These calculations allow us to find the final kinetic energy,  $\frac{1}{2}mv^2$ , and thus the final speed  $v$ .

#### Solution

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \quad 7.21$$

Solving for  $\frac{1}{2}mv^2$  gives

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2. \quad 7.22$$

Thus,

$$\frac{1}{2}mv^2 = 92.0 \text{ J} + 3.75 \text{ J} = 95.75 \text{ J}. \quad 7.23$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75 \text{ J})}{m}} = \sqrt{\frac{191.5 \text{ kg}\cdot\text{m}^2/\text{s}^2}{30.0 \text{ kg}}} \\ &= 2.53 \text{ m/s}. \end{aligned} \quad 7.24$$

#### Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

## EXAMPLE 7.5

### Work and Energy Can Reveal Distance, Too

How far does the package in [Figure 7.4](#) coast after the push, assuming friction remains constant? Use work and energy considerations.

#### Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction is the force of friction times the distance traveled times the cosine of the angle between the friction force and displacement; hence, this gives us a way of finding the distance traveled after the person stops pushing.

#### Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement, so  $\theta = 180^\circ$ . To reduce the kinetic energy of the package to zero, the work  $W_{\text{fr}}$  by friction must be minus the kinetic energy that the package started with plus what the package accumulated due to the pushing. Thus  $W_{\text{fr}} = -95.75 \text{ J}$ . Furthermore,  $W_{\text{fr}} = f d' \cos \theta = -f d'$ , where  $d'$  is the distance it takes to stop. Thus,

$$d' = -\frac{W_{\text{fr}}}{f} = -\frac{-95.75 \text{ J}}{5.00 \text{ N}}, \quad 7.25$$

and so

$$d' = 19.2 \text{ m}. \quad 7.26$$

#### Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

## 7.3 Gravitational Potential Energy

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass  $m$  at height  $h$  on Earth is given by  $PE_g = mgh$ .
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

### Work Done Against Gravity

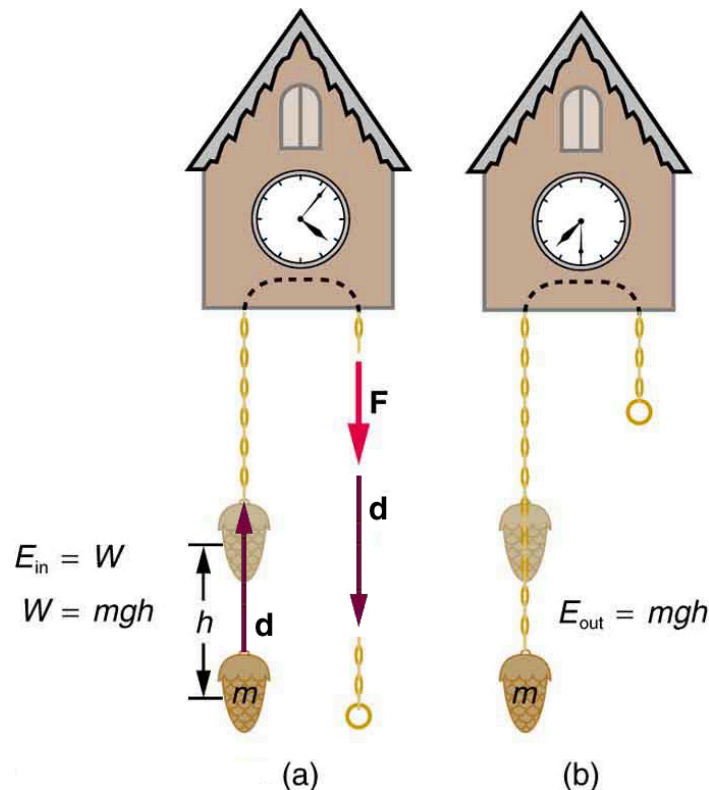
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass  $m$  through a height  $h$ , such as in [Figure 7.5](#). If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight  $mg$ . The work done on the mass is then  $W = Fd = mgh$ . We define this to be the **gravitational potential energy** ( $PE_g$ ) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the  $PE_g$  gained by the object, recognizing

that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

### Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to  $mgh$  on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of  $PE_g$  to KE without explicitly considering the intermediate step of work. (See [Example 7.7](#).) This shortcut makes it easier to solve problems using energy (if possible) rather than explicitly using forces.



**FIGURE 7.5** (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy  $\Delta PE_g$  to be

$$\Delta PE_g = mgh, \quad 7.27$$

where, for simplicity, we denote the change in height by  $h$  rather than the usual  $\Delta h$ . Note that  $h$  is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

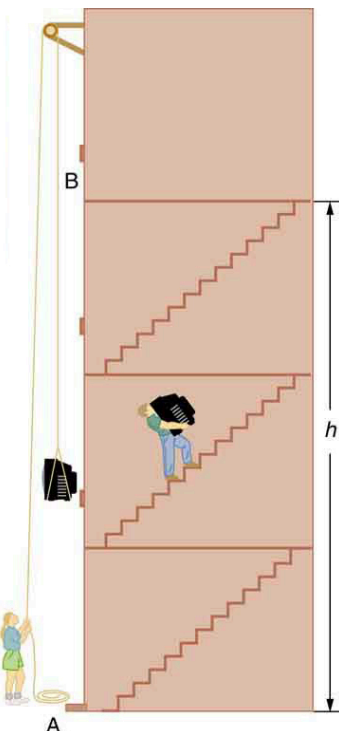
$$\begin{aligned} mgh &= (0.500 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \text{ m}) \\ &= 4.90 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 4.90 \text{ J}. \end{aligned} \quad 7.28$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of

gravitational potential energy, *without directly considering the force of gravity that does the work.*

### Using Potential Energy to Simplify Calculations

The equation  $\Delta PE_g = mgh$  applies for any path that has a change in height of  $h$ , not just when the mass is lifted straight up. (See [Figure 7.6.](#)) It is much easier to calculate  $mgh$  (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position  $h$  of a mass  $m$  is accompanied by a change in gravitational potential energy  $mgh$ , and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



**FIGURE 7.6** The change in gravitational potential energy ( $\Delta PE_g$ ) between points A and B is independent of the path.  $\Delta PE_g = mgh$  for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

### EXAMPLE 7.6

#### The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

#### Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial  $PE_g$  is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

#### Solution

The work done on the person by the floor as he stops is given by

$$W = Fd \cos \theta = -Fd, \quad 7.29$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions ( $\cos \theta = \cos 180^\circ = -1$ ). The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through

height  $h$ :

$$\text{KE} = -\Delta\text{PE}_g = -mgh, \quad 7.30$$

The distance  $d$  that the person's knees bend is much smaller than the height  $h$  of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work  $W$  done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\text{KE} = mgh. \quad 7.31$$

Combining this equation with the expression for  $W$  gives

$$-Fd = mgh. \quad 7.32$$

Recalling that  $h$  is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mgh}{d} = -\frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(-3.00 \text{ m})}{5.00 \times 10^{-3} \text{ m}} = 3.53 \times 10^5 \text{ N}. \quad 7.33$$

### Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump. (See [Figure 7.7](#).)



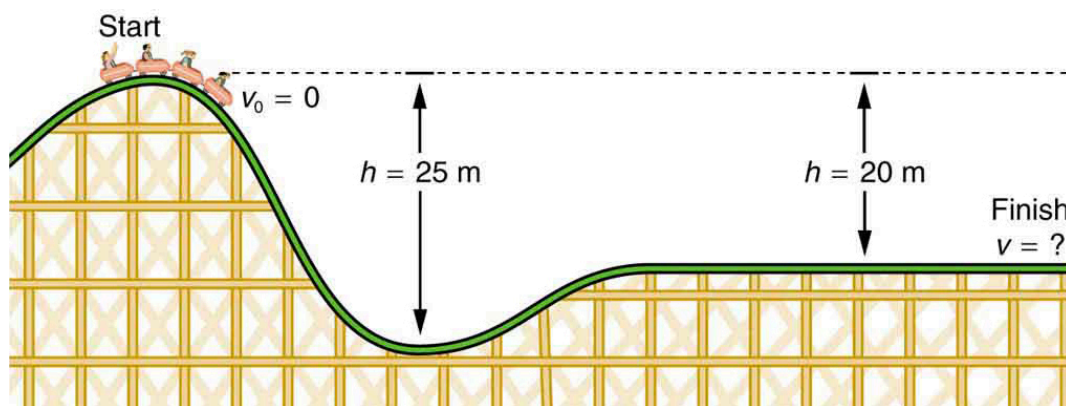
**FIGURE 7.7** The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)



### EXAMPLE 7.7

#### Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in [Figure 7.8](#) if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



**FIGURE 7.8** The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all  $\Delta PE_g$  is converted to KE.

### Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance  $h$  equals the *gain* in kinetic energy. This can be written in equation form as  $-\Delta PE_g = \Delta KE$ . Using the equations for  $PE_g$  and KE, we can solve for the final speed  $v$ , which is the desired quantity.

### Solution for (a)

Here the initial kinetic energy is zero, so that  $\Delta KE = \frac{1}{2}mv^2$ . The equation for change in potential energy states that  $\Delta PE_g = mgh$ . Since  $h$  is negative in this case, we will rewrite this as  $\Delta PE_g = -mg|h|$  to show the minus sign clearly. Thus,

$$-\Delta PE_g = \Delta KE \quad 7.34$$

becomes

$$mg|h| = \frac{1}{2}mv^2. \quad 7.35$$

Solving for  $v$ , we find that mass cancels and that

$$v = \sqrt{2g|h|}. \quad 7.36$$

Substituting known values,

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m})} \\ &= 19.8 \text{ m/s.} \end{aligned} \quad 7.37$$

### Solution for (b)

Again  $-\Delta PE_g = \Delta KE$ . In this case there is initial kinetic energy, so  $\Delta KE = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ . Thus,

$$mg|h| = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \quad 7.38$$

Rearranging gives

$$\frac{1}{2}mv^2 = mg|h| + \frac{1}{2}mv_0^2. \quad 7.39$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v = \sqrt{2g|h| + v_0^2}. \quad 7.40$$

This equation is very similar to the kinematics equation  $v = \sqrt{v_0^2 + 2ad}$ , but it is more general—the kinematics equation is valid only for constant acceleration, whereas our equation above is valid for any path regardless of whether the object moves with a constant acceleration. Now, substituting known values gives

$$\begin{aligned} v &= \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m}) + (5.00 \text{ m/s})^2} \\ &= 20.4 \text{ m/s}. \end{aligned} \quad 7.41$$

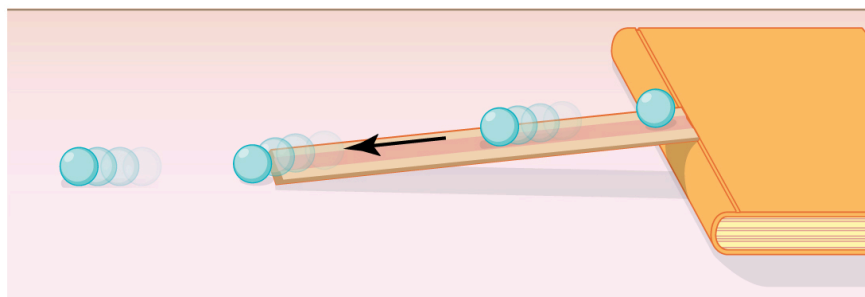
### Discussion and Implications

First, note that mass cancels. This is quite consistent with observations made in [Falling Objects](#) that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of  $h$  at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

### Making Connections: Take-Home Investigation—Converting Potential to Kinetic Energy

One can study the conversion of gravitational potential energy into kinetic energy in this experiment. On a smooth, level surface, use a ruler of the kind that has a groove running along its length and a book to make an incline (see [Figure 7.9](#)). Place a marble at the 10-cm position on the ruler and let it roll down the ruler. When it hits the level surface, measure the time it takes to roll one meter. Now place the marble at the 20-cm and the 30-cm positions and again measure the times it takes to roll 1 m on the level surface. Find the velocity of the marble on the level surface for all three positions. Plot velocity squared versus the distance traveled by the marble. What is the shape of each plot? If the shape is a straight line, the plot shows that the marble's kinetic energy at the bottom is proportional to its potential energy at the release point.



**FIGURE 7.9** A marble rolls down a ruler, and its speed on the level surface is measured.

## 7.4 Conservative Forces and Potential Energy

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define conservative force, potential energy, and mechanical energy.
- Explain the potential energy of a spring in terms of its compression when Hooke's law applies.
- Use the work-energy theorem to show how having only conservative forces implies conservation of mechanical energy.

### Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy (PE)** for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

#### Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

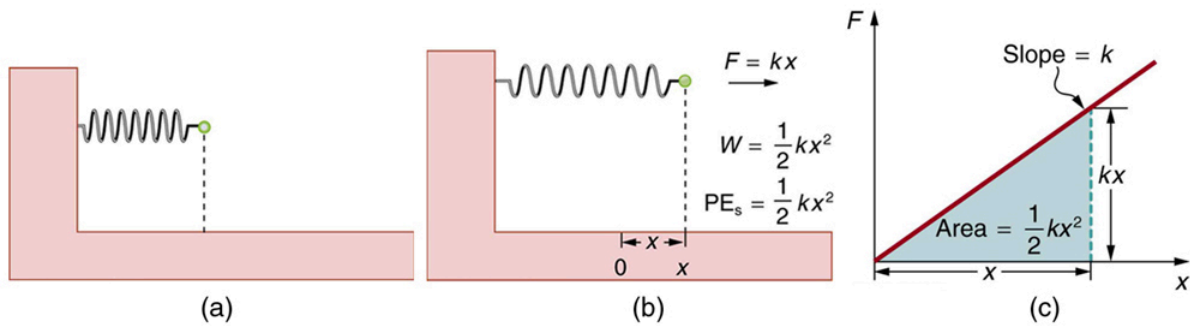
We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

### Potential Energy of a Spring

First, let us obtain an expression for the potential energy stored in a spring ( $PE_s$ ). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in [Elasticity: Stress and Strain](#), and states that the magnitude of force  $F$  on the spring and the resulting deformation  $\Delta L$  are proportional,  $F = k\Delta L$ .) (See [Figure 7.10](#).) For our spring, we will replace  $\Delta L$  (the amount of deformation produced by a force  $F$ ) by the distance  $x$  that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude  $F = kx$ , where  $k$  is the spring's force constant. The force increases linearly from 0 at the start to  $kx$  in the fully stretched position. The average force is  $kx/2$ . Thus the work done in stretching or compressing the spring is  $W_s = Fd = \left(\frac{kx}{2}\right)x = \frac{1}{2}kx^2$ . Alternatively, we noted in [Kinetic Energy and the Work-Energy Theorem](#) that the area under a graph of  $F$  vs.  $x$  is the work done by the force. In [Figure 7.10\(c\)](#) we see that this area is also  $\frac{1}{2}kx^2$ . We therefore define the **potential energy of a spring**,  $PE_s$ , to be

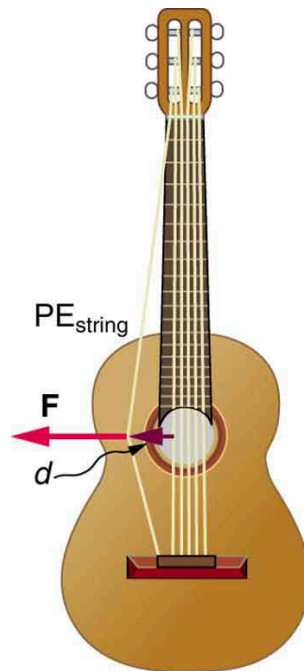
$$PE_s = \frac{1}{2}kx^2, \quad 7.42$$

where  $k$  is the spring's force constant and  $x$  is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance  $x$ . The potential energy of the spring  $PE_s$  does not depend on the path taken; it depends only on the stretch or squeeze  $x$  in the final configuration.



**FIGURE 7.10** (a) An undeformed spring has no  $PE_s$  stored in it. (b) The force needed to stretch (or compress) the spring a distance  $x$  has a magnitude  $F = kx$ , and the work done to stretch (or compress) it is  $\frac{1}{2}kx^2$ . Because the force is conservative, this work is stored as potential energy ( $PE_s$ ) in the spring, and it can be fully recovered. (c) A graph of  $F$  vs.  $x$  has a slope of  $k$ , and the area under the graph is  $\frac{1}{2}kx^2$ . Thus the work done or potential energy stored is  $\frac{1}{2}kx^2$ .

The equation  $PE_s = \frac{1}{2}kx^2$  has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is  $PE_s = \frac{1}{2}kx^2$ , where  $k$  is the force constant of the particular system and  $x$  is its deformation. Another example is seen in [Figure 7.11](#) for a guitar string.



**FIGURE 7.11** Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

### Conservation of Mechanical Energy

Let us now consider what form the work-energy theorem takes when only conservative forces are involved. This will lead us to the conservation of energy principle. The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy. In equation form, this is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta KE. \quad 7.43$$

If only conservative forces act, then

$$W_{\text{net}} = W_c, \quad 7.44$$

where  $W_c$  is the total work done by all conservative forces. Thus,

$$W_c = \Delta KE. \quad 7.45$$

Now, if the conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy. That is,  $W_c = -\Delta PE$ . Therefore,

$$-\Delta PE = \Delta KE \quad 7.46$$

or

$$\Delta KE + \Delta PE = 0. \quad 7.47$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

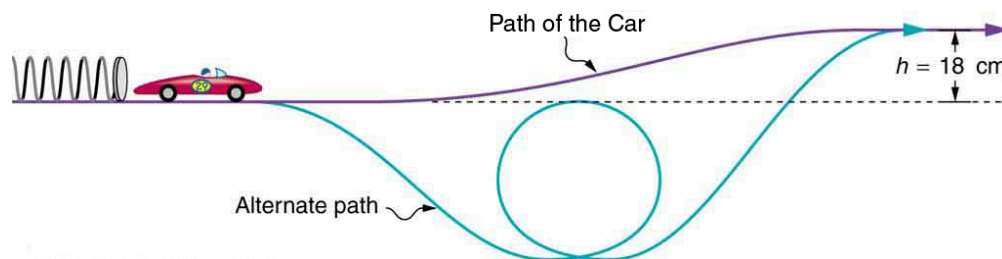
$$\left. \begin{array}{l} KE + PE = \text{constant} \\ \text{or} \\ KE_i + PE_i = KE_f + PE_f \end{array} \right\} \text{(conservative forces only),} \quad 7.48$$

where  $i$  and  $f$  denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**,  $(KE + PE)$ . In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

## EXAMPLE 7.8

### Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in [Figure 7.12](#). The car follows a track that rises 0.180 m above the starting point. The spring is compressed 4.00 cm and has a force constant of 250.0 N/m. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.



**FIGURE 7.12** A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

#### Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$KE_i + PE_i = KE_f + PE_f \quad 7.49$$

or

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2, \quad 7.50$$

where  $h$  is the height (vertical position) and  $x$  is the compression of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial

and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

### Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both  $h_i$  and  $h_f$  are zero. Furthermore, the initial speed  $v_i$  is zero and the final compression of the spring  $x_f$  is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2. \quad 7.51$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$\begin{aligned} v_f &= \sqrt{\frac{k}{m}x_i} \\ &= \sqrt{\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}(0.0400 \text{ m})} \\ &= 2.00 \text{ m/s}. \end{aligned} \quad 7.52$$

### Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f. \quad 7.53$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for  $v_f$  and substituting known values gives

$$\begin{aligned} v_f &= \sqrt{\frac{kx_i^2}{m} - 2gh_f} \\ &= \sqrt{\left(\frac{250.0 \text{ N/m}}{0.100 \text{ kg}}\right)(0.0400 \text{ m})^2 - 2(9.80 \text{ m/s}^2)(0.180 \text{ m})} \\ &= 0.687 \text{ m/s}. \end{aligned} \quad 7.54$$

### Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in [Example 7.8](#). Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.



## PHET EXPLORATIONS

### Energy Skate Park

Learn about conservation of energy with a skater dude! Build tracks, ramps and jumps for the skater and view the kinetic energy, potential energy and friction as he moves. You can also take the skater to different planets or even space!

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/7-4-conservative-forces-and-potential-energy\)](https://openstax.org/books/college-physics-2e/pages/7-4-conservative-forces-and-potential-energy)



## 7.5 Nonconservative Forces

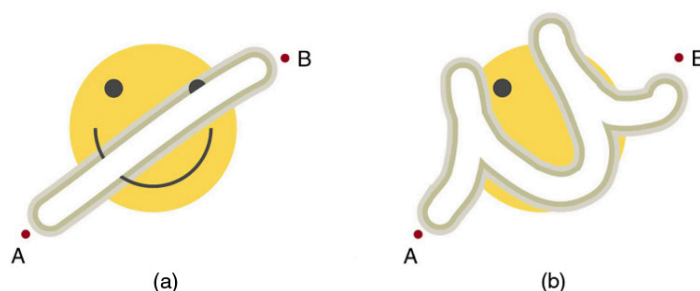
### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define nonconservative forces and explain how they affect mechanical energy.
- Show how the principle of conservation of energy can be applied by treating the conservative forces in terms of their potential energies and any nonconservative forces in terms of the work they do.

### Nonconservative Forces and Friction

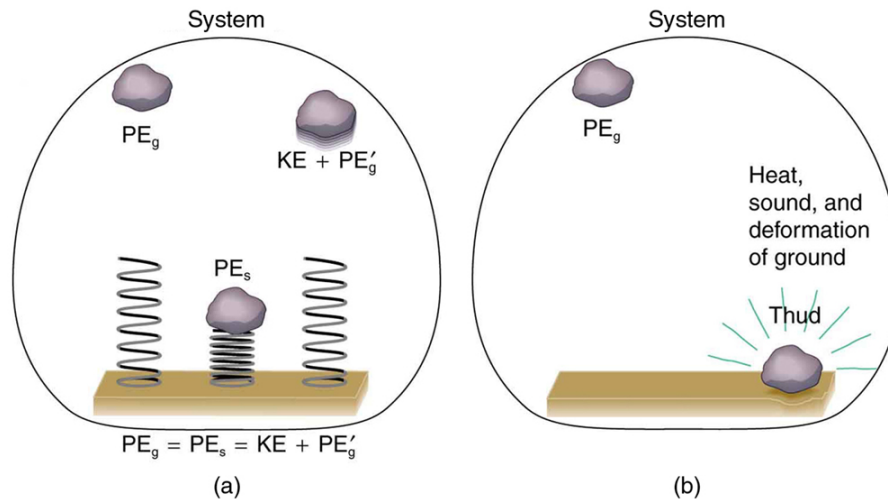
Forces are either conservative or nonconservative. Conservative forces were discussed in [Conservative Forces and Potential Energy](#). A **nonconservative force** is one for which work depends on the path taken. Friction is a good example of a nonconservative force. As illustrated in [Figure 7.13](#), work done against friction depends on the length of the path between the starting and ending points. Because of this dependence on path, there is no potential energy associated with nonconservative forces. An important characteristic is that the work done by a nonconservative force *adds or removes mechanical energy from a system*. **Friction**, for example, creates **thermal energy** that dissipates, removing energy from the system. Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.



**FIGURE 7.13** The amount of the happy face erased depends on the path taken by the eraser between points A and B, as does the work done against friction. Less work is done and less of the face is erased for the path in (a) than for the path in (b). The force here is friction, and most of the work goes into thermal energy that subsequently leaves the system (the happy face plus the eraser). The energy expended cannot be fully recovered.

### How Nonconservative Forces Affect Mechanical Energy

*Mechanical energy may not be conserved* when nonconservative forces act. For example, when a car is brought to a stop by friction on level ground, it loses kinetic energy, which is dissipated as thermal energy, reducing its mechanical energy. [Figure 7.14](#) compares the effects of conservative and nonconservative forces. We often choose to understand simpler systems such as that described in [Figure 7.14\(a\)](#) first before studying more complicated systems as in [Figure 7.14\(b\)](#).



**FIGURE 7.14** Comparison of the effects of conservative and nonconservative forces on the mechanical energy of a system. (a) A system with only conservative forces. When a rock is dropped onto a spring, its mechanical energy remains constant (neglecting air resistance) because the force in the spring is conservative. The spring can propel the rock back to its original height, where it once again has only potential energy due to gravity. (b) A system with nonconservative forces. When the same rock is dropped onto the ground, it is stopped by nonconservative forces that dissipate its mechanical energy as thermal energy, sound, and surface distortion. The rock has lost mechanical energy.

### How the Work-Energy Theorem Applies

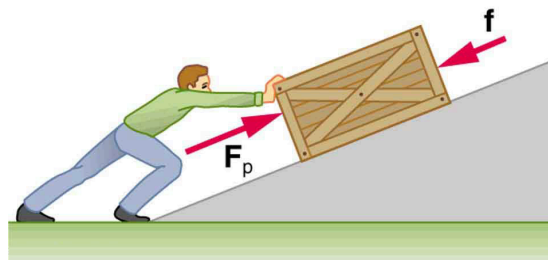
Now let us consider what form the work-energy theorem takes when both conservative and nonconservative forces act. We will see that the work done by nonconservative forces equals the change in the mechanical energy of a system. As noted in [Kinetic Energy and the Work-Energy Theorem](#), the work-energy theorem states that the net work on a system equals the change in its kinetic energy, or  $W_{\text{net}} = \Delta KE$ . The net work is the sum of the work by nonconservative forces plus the work by conservative forces. That is,

$$W_{\text{net}} = W_{\text{nc}} + W_{\text{c}}, \quad 7.55$$

so that

$$W_{\text{nc}} + W_{\text{c}} = \Delta KE, \quad 7.56$$

where  $W_{\text{nc}}$  is the total work done by all nonconservative forces and  $W_{\text{c}}$  is the total work done by all conservative forces.



**FIGURE 7.15** A person pushes a crate up a ramp, doing work on the crate. Friction and gravitational force (not shown) also do work on the crate; both forces oppose the person's push. As the crate is pushed up the ramp, it gains mechanical energy, implying that the work done by the person is greater than the work done by friction.

Consider [Figure 7.15](#), in which a person pushes a crate up a ramp and is opposed by friction. As in the previous section, we note that work done by a conservative force comes from a loss of gravitational potential energy, so that  $W_{\text{c}} = -\Delta PE$ . Substituting this equation into the previous one and solving for  $W_{\text{nc}}$  gives

$$W_{\text{nc}} = \Delta KE + \Delta PE. \quad 7.57$$

This equation means that the total mechanical energy ( $KE + PE$ ) changes by exactly the amount of work done by nonconservative forces. In [Figure 7.15](#), this is the work done by the person minus the work done by friction. So even if energy is not conserved for the system of interest (such as the crate), we know that an equal amount of work was done to cause the change in total mechanical energy.

We rearrange  $W_{nc} = \Delta KE + \Delta PE$  to obtain

$$KE_i + PE_i + W_{nc} = KE_f + PE_f. \quad 7.58$$

This means that the amount of work done by nonconservative forces adds to the mechanical energy of a system. If  $W_{nc}$  is positive, then mechanical energy is increased, such as when the person pushes the crate up the ramp in [Figure 7.15](#). If  $W_{nc}$  is negative, then mechanical energy is decreased, such as when the rock hits the ground in [Figure 7.14\(b\)](#). If  $W_{nc}$  is zero, then mechanical energy is conserved, and nonconservative forces are balanced. For example, when you push a lawn mower at constant speed on level ground, your work done is removed by the work of friction, and the mower has a constant energy.

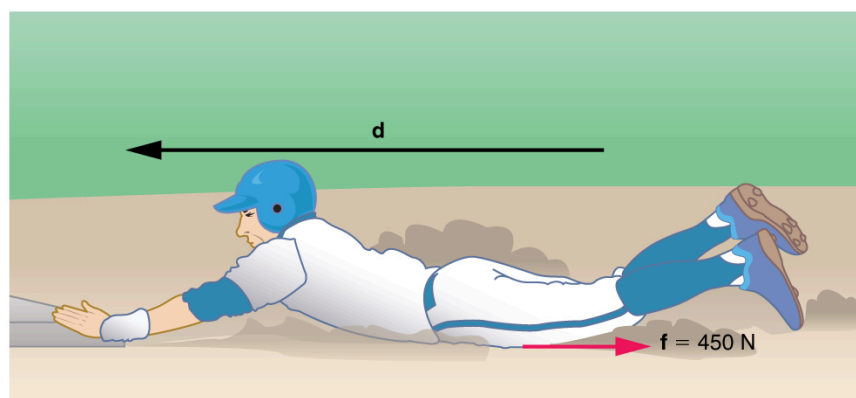
### Applying Energy Conservation with Nonconservative Forces

When no change in potential energy occurs, applying  $KE_i + PE_i + W_{nc} = KE_f + PE_f$  amounts to applying the work-energy theorem by setting the change in kinetic energy to be equal to the net work done on the system, which in the most general case includes both conservative and nonconservative forces. But when seeking instead to find a change in total mechanical energy in situations that involve changes in both potential and kinetic energy, the previous equation  $KE_i + PE_i + W_{nc} = KE_f + PE_f$  says that you can start by finding the change in mechanical energy that would have resulted from just the conservative forces, including the potential energy changes, and add to it the work done, with the proper sign, by any nonconservative forces involved.

## EXAMPLE 7.9

### Calculating Distance Traveled: How Far a Baseball Player Slides

Consider the situation shown in [Figure 7.16](#), where a baseball player slides to a stop on level ground. Using energy considerations, calculate the distance the 65.0-kg baseball player slides, given that his initial speed is 6.00 m/s and the force of friction against him is a constant 450 N.



**FIGURE 7.16** The baseball player slides to a stop in a distance  $d$ . In the process, friction removes the player's kinetic energy by doing an amount of work  $fd$  equal to the initial kinetic energy.

#### Strategy

Friction stops the player by converting his kinetic energy into other forms, including thermal energy. In terms of the work-energy theorem, the work done by friction, which is negative, is added to the initial kinetic energy to reduce it to zero. The work done by friction is negative, because  $\mathbf{f}$  is in the opposite direction of the motion (that is,  $\theta = 180^\circ$ , and so  $\cos \theta = -1$ ). Thus  $W_{nc} = -fd$ . The equation simplifies to

$$\frac{1}{2}mv_i^2 - fd = 0 \quad 7.59$$

or

$$fd = \frac{1}{2}mv_i^2. \quad 7.60$$

This equation can now be solved for the distance  $d$ .

**Solution**

Solving the previous equation for  $d$  and substituting known values yields

$$\begin{aligned} d &= \frac{mv_i^2}{2f} \\ &= \frac{(65.0 \text{ kg})(6.00 \text{ m/s})^2}{(2)(450 \text{ N})} \\ &= 2.60 \text{ m.} \end{aligned} \quad 7.61$$

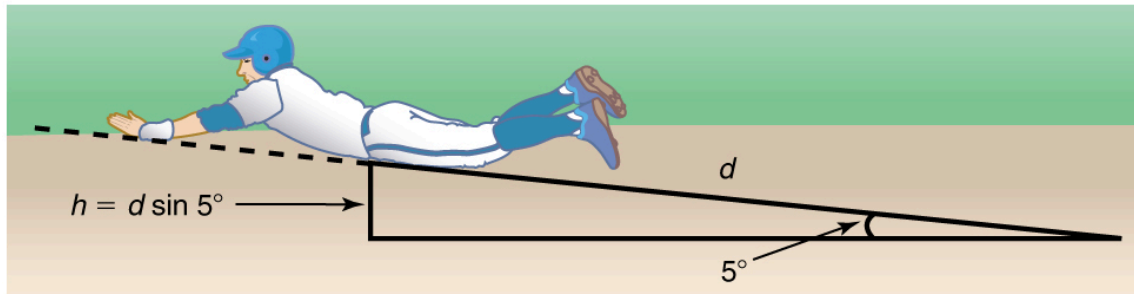
**Discussion**

The most important point of this example is that the amount of nonconservative work equals the change in mechanical energy. For example, you must work harder to stop a truck, with its large mechanical energy, than to stop a mosquito.

### EXAMPLE 7.10

**Calculating Distance Traveled: Sliding Up an Incline**

Suppose that the player from [Example 7.9](#) is running up a hill having a  $5.00^\circ$  incline upward with a surface similar to that in the baseball stadium. The player slides with the same initial speed, and the frictional force is still 450 N. Determine how far he slides.



**FIGURE 7.17** The same baseball player slides to a stop on a  $5.00^\circ$  slope.

**Strategy**

In this case, the work done by the nonconservative friction force on the player reduces the mechanical energy he has from his kinetic energy at zero height, to the final mechanical energy he has by moving through distance  $d$  to reach height  $h$  along the hill, with  $h = d \sin 5.00^\circ$ . This is expressed by the equation

$$KE_i + PE_i + W_{nc} = KE_f + PE_f. \quad 7.62$$

**Solution**

The work done by friction is again  $W_{nc} = -fd$ ; initially the potential energy is  $PE_i = mg \cdot 0 = 0$  and the kinetic energy is  $KE_i = \frac{1}{2}mv_i^2$ ; the final energy contributions are  $KE_f = 0$  for the kinetic energy and  $PE_f = mgh = mgd \sin \theta$  for the potential energy.

Substituting these values gives

$$\frac{1}{2}mv_i^2 + 0 + (-fd) = 0 + mgd \sin \theta. \quad 7.63$$

Solve this for  $d$  to obtain

$$\begin{aligned}
 d &= \frac{\left(\frac{1}{2}\right)mv_1^2}{f+mg \sin \theta} \\
 &= \frac{(0.5)(65.0 \text{ kg})(6.00 \text{ m/s})^2}{450 \text{ N}+(65.0 \text{ kg})(9.80 \text{ m/s}^2) \sin (5.00^\circ)} \\
 &= 2.31 \text{ m.}
 \end{aligned}$$
7.64

### Discussion

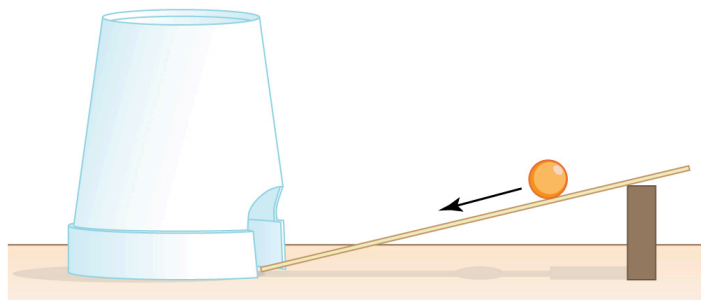
As might have been expected, the player slides a shorter distance by sliding uphill. Note that the problem could also have been solved in terms of the forces directly and the work energy theorem, instead of using the potential energy. This method would have required combining the normal force and force of gravity vectors, which no longer cancel each other because they point in different directions, and friction, to find the net force. You could then use the net force and the net work to find the distance  $d$  that reduces the kinetic energy to zero. By applying conservation of energy and using the potential energy instead, we need only consider the gravitational potential energy  $mgh$ , without combining and resolving force vectors. This simplifies the solution considerably.

### Making Connections: Take-Home Investigation—Determining Friction from the Stopping Distance

This experiment involves the conversion of gravitational potential energy into thermal energy. Use the ruler, book, and marble from [Take-Home Investigation—Converting Potential to Kinetic Energy](#). In addition, you will need a foam cup with a small hole in the side, as shown in [Figure 7.18](#). From the 10-cm position on the ruler, let the marble roll into the cup positioned at the bottom of the ruler. Measure the distance  $d$  the cup moves before stopping. What forces caused it to stop? What happened to the kinetic energy of the marble at the bottom of the ruler? Next, place the marble at the 20-cm and the 30-cm positions and again measure the distance the cup moves after the marble enters it. Plot the distance the cup moves versus the initial marble position on the ruler. Is this relationship linear?

With some simple assumptions, you can use these data to find the coefficient of kinetic friction  $\mu_k$  of the cup on the table. The force of friction  $f$  on the cup is  $\mu_k N$ , where the normal force  $N$  is just the weight of the cup plus the marble. The normal force and force of gravity do no work because they are perpendicular to the displacement of the cup, which moves horizontally. The work done by friction is  $f d$ . You will need the mass of the marble as well to calculate its initial kinetic energy.

It is interesting to do the above experiment also with a steel marble (or ball bearing). Releasing it from the same positions on the ruler as you did with the glass marble, is the velocity of this steel marble the same as the velocity of the marble at the bottom of the ruler? Is the distance the cup moves proportional to the mass of the steel and glass marbles?



**FIGURE 7.18** Rolling a marble down a ruler into a foam cup.



## PHET EXPLORATIONS

### The Ramp

Explore forces, energy and work as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces acting on the file cabinet. Graphs show forces, energy and work.

[Click to view content \(https://openstax.org/l/21ramp\)](https://openstax.org/l/21ramp).



## 7.6 Conservation of Energy

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.
- Define efficiency of an energy conversion process as the fraction left as useful energy or work, rather than being transformed, for example, into thermal energy.

### Law of Conservation of Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

*Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.*

We have explored some forms of energy and some ways it can be transferred from one system to another. This exploration led to the definition of two major types of energy—mechanical energy ( $KE + PE$ ) and energy transferred via work done by nonconservative forces ( $W_{nc}$ ). But energy takes *many* other forms, manifesting itself in *many* different ways, and we need to be able to deal with all of these before we can write an equation for the above general statement of the conservation of energy.

### Other Forms of Energy than Mechanical Energy

At this point, we deal with all other forms of energy by lumping them into a single group called other energy (OE). Then we can state the conservation of energy in equation form as

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f. \quad 7.65$$

All types of energy and work can be included in this very general statement of conservation of energy. Kinetic energy is  $KE$ , work done by a conservative force is represented by  $PE$ , work done by nonconservative forces is  $W_{nc}$ , and all other energies are included as  $OE$ . This equation applies to all previous examples; in those situations  $OE$  was constant, and so it subtracted out and was not directly considered.

#### Making Connections: Usefulness of the Energy Conservation Principle

The fact that energy is conserved and has many forms makes it very important. You will find that energy is discussed in many contexts, because it is involved in all processes. It will also become apparent that many situations are best understood in terms of energy and that problems are often most easily conceptualized and solved by considering energy.

When does  $OE$  play a role? One example occurs when a person eats. Food is oxidized with the release of carbon dioxide, water, and energy. Some of this chemical energy is converted to kinetic energy when the person moves, to potential energy when the person changes altitude, and to thermal energy (another form of  $OE$ ).

## Some of the Many Forms of Energy

What are some other forms of energy? You can probably name a number of forms of energy not yet discussed. Many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

[Table 7.1](#) gives the amount of energy stored, used, or released from various objects and in various phenomena. The range of energies and the variety of types and situations is impressive.

### Problem-Solving Strategies for Energy

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

**Step 1.** Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

**Step 2.** Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

**Step 3.** If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. The equation expressing conservation of energy is

$$KE_i + PE_i = KE_f + PE_f. \quad 7.66$$

**Step 4.** If you know the potential energy for only some of the forces, possibly because some of them are nonconservative and do not have a potential energy, or if there are other energies that are not easily treated in terms of force and work, then the conservation of energy law in its most general form must be used.

$$KE_i + PE_i + W_{nc} + OE_i = KE_f + PE_f + OE_f. \quad 7.67$$

In most problems, one or more of the terms is zero, simplifying its solution. Do not calculate  $W_c$ , the work done by conservative forces; it is already incorporated in the PE terms.

**Step 5.** You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose  $h = 0$  at either the initial or final point, so that  $PE_g$  is zero there. Then solve for the unknown in the customary manner.

**Step 6.** *Check the answer to see if it is reasonable.* Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

## Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is

converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see [Figure 7.19](#)) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.



**FIGURE 7.19** Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Object/phenomenon	Energy in joules
Big Bang	$10^{68}$
Energy released in a supernova	$10^{44}$
Fusion of all the hydrogen in Earth's oceans	$10^{34}$
Annual world energy use	$4 \times 10^{20}$
Large fusion bomb (9 megaton)	$3.8 \times 10^{16}$
1 kg hydrogen (fusion to helium)	$6.4 \times 10^{14}$
1 kg uranium (nuclear fission)	$8.0 \times 10^{13}$
Hiroshima-size fission bomb (10 kiloton)	$4.2 \times 10^{13}$
90,000-metric ton aircraft carrier at 30 knots	$1.1 \times 10^{10}$
1 barrel crude oil	$5.9 \times 10^9$
1 ton TNT	$4.2 \times 10^9$

**TABLE 7.1** Energy of Various Objects and Phenomena

Object/phenomenon	Energy in joules
1 gallon of gasoline	$1.2 \times 10^8$
Daily home electricity use (developed countries)	$7 \times 10^7$
Daily adult food intake (recommended)	$1.2 \times 10^7$
1000-kg car at 90 km/h	$3.1 \times 10^5$
1 g fat (9.3 kcal)	$3.9 \times 10^4$
ATP hydrolysis reaction	$3.2 \times 10^4$
1 g carbohydrate (4.1 kcal)	$1.7 \times 10^4$
1 g protein (4.1 kcal)	$1.7 \times 10^4$
Tennis ball at 100 km/h	22
Mosquito ( $10^{-2}$ g at 0.5 m/s)	$1.3 \times 10^{-6}$
Single electron in a TV tube beam	$4.0 \times 10^{-15}$
Energy to break one DNA strand	$10^{-19}$

**TABLE 7.1** Energy of Various Objects and Phenomena

## Efficiency

Even though energy is conserved in an energy conversion process, the output of *useful energy* or work will be less than the energy input. The **efficiency**  $Eff$  of an energy conversion process is defined as

$$\text{Efficiency}(Eff) = \frac{\text{useful energy or work output}}{\text{total energy input}} = \frac{W_{\text{out}}}{E_{\text{in}}}. \quad 7.68$$

[Table 7.2](#) lists some efficiencies of mechanical devices and human activities. In a coal-fired power plant, for example, about 40% of the chemical energy in the coal becomes useful electrical energy. The other 60% transforms into other (perhaps less useful) energy forms, such as thermal energy, which is then released to the environment through combustion gases and cooling towers.

Activity/device	Efficiency (%) <sup>1</sup>
Cycling and climbing	20
Swimming, surface	2

**TABLE 7.2** Efficiency of the Human Body and Mechanical Devices

<sup>1</sup> Representative values

Activity/device	Efficiency (%) <sup>1</sup>
Swimming, submerged	4
Shoveling	3
Weightlifting	9
Steam engine	17
Gasoline engine	30
Diesel engine	35
Nuclear power plant	35
Coal power plant	42
Electric motor	98
Compact fluorescent light	20
Gas heater (residential)	90
Solar cell	10

**TABLE 7.2** Efficiency of the Human Body and Mechanical Devices



## PHET EXPLORATIONS

### Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energies for each spring.

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## 7.7 Power

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

### What is Power?

*Power*—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in [Figure 7.20](#).



**FIGURE 7.20** This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** ( $P$ ) as the rate at which work is done.

### Power

Power is the rate at which work is done.

$$P = \frac{W}{t} \quad 7.69$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second ( $1 \text{ W} = 1 \text{ J/s}$ ).

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

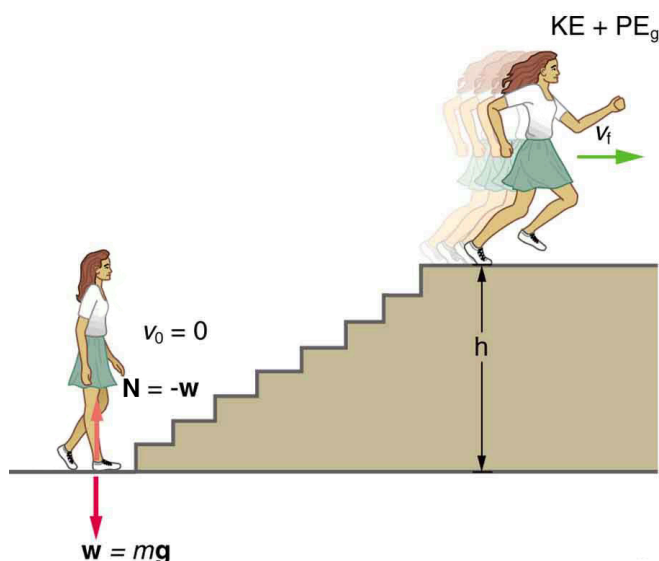
### Calculating Power from Energy



#### EXAMPLE 7.11

#### Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See [Figure 7.21](#).)



**FIGURE 7.21** When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

### Strategy and Concept

The work going into mechanical energy is  $W = KE + PE$ . At the bottom of the stairs, we take both  $KE$  and  $PE_g$  as initially zero; thus,  $W = KE_f + PE_g = \frac{1}{2}mv_f^2 + mgh$ , where  $h$  is the vertical height of the stairs. Because all terms are given, we can calculate  $W$  and then divide it by time to get power.

### Solution

Substituting the expression for  $W$  into the definition of power given in the previous equation,  $P = W/t$  yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 + mgh}{t} \quad 7.70$$

Entering known values yields

$$\begin{aligned} P &= \frac{0.5(60.0 \text{ kg})(2.00 \text{ m/s})^2 + (60.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})}{3.50 \text{ s}} \\ &= \frac{120 \text{ J} + 1764 \text{ J}}{3.50 \text{ s}} \\ &= 538 \text{ W}. \end{aligned} \quad 7.71$$

### Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** ( $1 \text{ hp} = 746 \text{ W}$ )! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

### Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect

that your output will be more than about 0.5 hp.

## Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See [Table 7.3](#) for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter ( $\text{kW/m}^2$ ). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is  $10^6$  W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See [Figure 7.22](#).)



**FIGURE 7.22** Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	$5 \times 10^{37}$
Milky Way galaxy	$10^{37}$
Crab Nebula pulsar	$10^{28}$
The Sun	$4 \times 10^{26}$
Volcanic eruption (maximum)	$4 \times 10^{15}$
Lightning bolt	$2 \times 10^{12}$

**TABLE 7.3** Power Output or Consumption

Object or Phenomenon	Power in Watts
Nuclear power plant (total electric and heat transfer)	$3 \times 10^9$
Aircraft carrier (total useful and heat transfer)	$10^8$
Dragster (total useful and heat transfer)	$2 \times 10^6$
Car (total useful and heat transfer)	$8 \times 10^4$
Football player (total useful and heat transfer)	$5 \times 10^3$
Clothes dryer	$4 \times 10^3$
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	$10^{-3}$

**TABLE 7.3** Power Output or Consumption

### Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is  $P = W/t = E/t$ , where  $E$  is the energy supplied by the electricity company. So the energy consumed over a time  $t$  is

$$E = Pt. \quad 7.72$$

Electricity bills state the energy used in units of **kilowatt-hours** ( $\text{kW} \cdot \text{h}$ ), which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

### EXAMPLE 7.12

#### Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is \$0.120 per  $\text{kW} \cdot \text{h}$ ?

#### Strategy

Cost is based on energy consumed; thus, we must find  $E$  from  $E = Pt$  and then calculate the cost. Because electrical energy is expressed in  $\text{kW} \cdot \text{h}$ , at the start of a problem such as this it is convenient to convert the units into  $\text{kW}$  and hours.

**Solution**

The energy consumed in kW · h is

$$\begin{aligned} E &= Pt = (0.200 \text{ kW})(6.00 \text{ h/d})(30.0 \text{ d}) \\ &= 36.0 \text{ kW} \cdot \text{h}, \end{aligned} \quad 7.73$$

and the cost is simply given by

$$\text{cost} = (36.0 \text{ kW} \cdot \text{h})(\$0.120 \text{ per kW} \cdot \text{h}) = \$4.32 \text{ per month.} \quad 7.74$$

**Discussion**

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

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The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in [Thermodynamics](#), the potential for energy to produce useful work has been “degraded” in the energy transformation.

## 7.8 Work, Energy, and Power in Humans

### LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the human body’s consumption of energy when at rest vs. when engaged in activities that do useful work.
- Calculate the conversion of chemical energy in food into useful work.

### Energy Conversion in Humans

Our own bodies, like all living organisms, are energy conversion machines. Conservation of energy implies that the chemical energy stored in food is converted into work, thermal energy, and/or stored as chemical energy in fatty tissue. (See [Figure 7.23](#).) The fraction going into each form depends both on how much we eat and on our level of physical activity. If we eat more than is needed to do work and stay warm, the remainder goes into body fat.