

in solid-state physics and hydrodynamics. Mathematical physicists are so used to the appearance of Maxwell-like equations in other domains that they seldom pay it much attention. The real tests for any model of electrodynamics, quantum or classical, are the deviations that the model predicts from electrodynamics, especially at high energies.

CURIOSITIES AND FUN CHALLENGES ABOUT QED

Challenge 157 e Can you confirm that the strand model of quantum electrodynamics does not violate charge conjugation C nor parity P at any energy?

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Challenge 158 e Can you confirm that the strand model of quantum electrodynamics conserves colour and weak charge at all energies?

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Challenge 159 e Can you determine whether the U(1) gauge group deduced here is that of electrodynamics or that of weak hypercharge?

* *

Challenge 160 d Can you find a measurable deviation of the strand model from QED?

SUMMARY ON QED AND EXPERIMENTAL PREDICTIONS

In the strand model, photons are single, helically twisted strands, randomly exchanged between charges; charges are chiral tangles, and therefore they effectively emit and absorb real and virtual photons. This is the complete description of QED using strands.

Page 161 In particular, we have shown that Reidemeister I moves – or twists – of tangle cores lead to U(1) gauge invariance, Coulomb's inverse square relation, Maxwell's equations of electrodynamics and to Feynman diagrams. In short, we have deduced all experimental properties of quantum electrodynamics, except one: the strength of the coupling. Despite this open point, we have settled one line of the millennium list of open issues: we know the origin of the electromagnetic interaction and of its properties.

Is there a difference between the strand model and quantum electrodynamics? The precise answer is: there are *no measurable* differences between the strand model and QED. For example, the g -factor of the electron or the muon predicted by QED is not changed by the strand model. The U(1) gauge symmetry and the whole of QED remain valid at all energies. There are no magnetic charges. There are no other gauge groups. QED remains exact in all cases – as long as gravity plays no role.

The strand model prediction of a lack of larger gauge symmetries is disconcerting. There is thus *no* grand unification in nature; there is no general gauge group in nature, be it SU(5), SO(10), E6, E7, E8, SO(32) or any other. This result indirectly also rules out supersymmetry and supergravity. This unpopular result contrasts with many cherished habits of thought.

In the strand model, the equivalence of Feynman diagrams and strand diagrams implies that deviations of the strand model from QED are expected *only* when gravity starts to play a role. The strand model predicts that this will only happen just near the Planck energy $\sqrt{\hbar c^5/4G}$. At lower energies, QED is predicted to remain valid.

The strand model also confirms that the combination of gravity and quantum theory turns all Planck units into *limit* values, because there is a maximum density of strand crossings in nature, due to the fundamental principle. In particular, the strand model confirms the maximum electric field value $E_{\max} = c^4/4Ge \approx 1.9 \cdot 10^{62}$ V/m and a maximum magnetic field value $B_{\max} = c^3/4Ge \approx 6.3 \cdot 10^{53}$ T. So far, these predictions are not in contrast with observations.

Thus the strand model predicts that approaching the electric or magnetic field limit values – given by quantum gravity – is the only option to observe deviations from QED. But measurements are not possible in those domains. Therefore we can state that there are no measurable differences between the strand model and QED.

Our exploration of QED has left open only two points: the calculation of the electromagnetic coupling constant and the determination of the spectrum of possible tangles for the elementary particles. Before we clarify these points, we look at the next Reidemeister move.

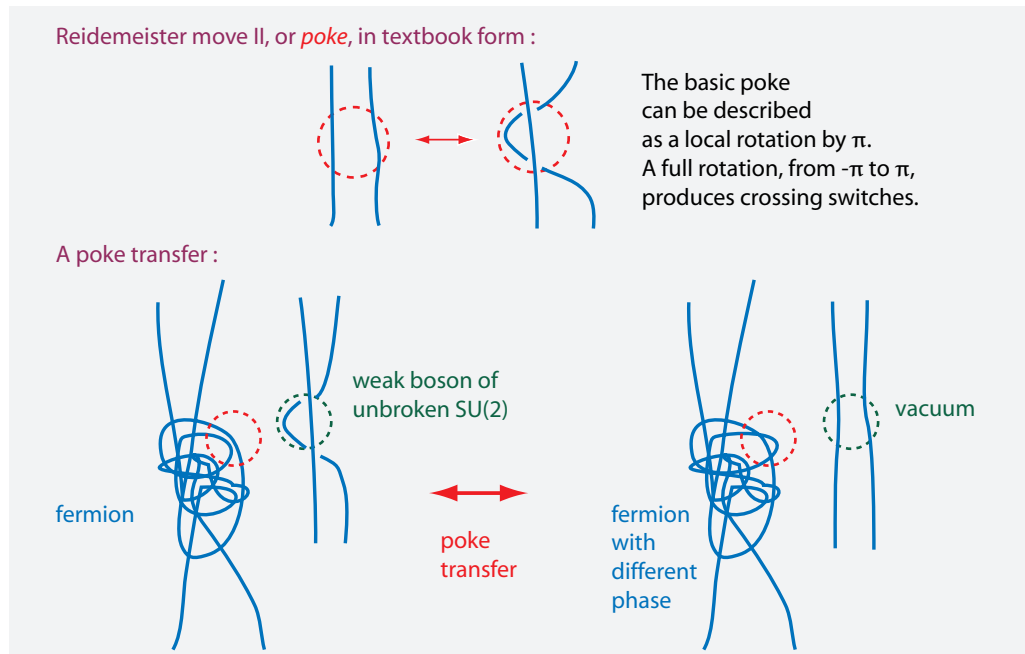


FIGURE 60 *Poke transfer* is the basis of the weak interaction in the strand model. No strand is cut or reglued; the transfer occurs only through the excluded volume due to the impenetrability of strands.

THE WEAK NUCLEAR INTERACTION AND THE SECOND REIDEMEISTER MOVE

In nature, the weak interaction is the result of the absorption and the emission of massive spin-1 bosons that form a broken weak triplet. The W and the Z bosons are emitted or absorbed by particles with weak charge; these are the left-handed fermions and right-handed antifermions. In other words, the weak interaction breaks parity P maximally. The W boson has unit electric charge, the Z boson has vanishing electric charge. The emission or absorption of W bosons changes the particle type of the involved fermion. The weak bosons also interact among themselves. All weakly charged particles are massive and move slower than light. The Lagrangian of matter coupled to the weak field has a broken SU(2) gauge symmetry. There are fundamental Feynman diagrams with triple and with quartic vertices. The weak coupling constant is determined by the electromagnetic coupling constant and the weak boson masses; its energy dependence is fixed by renormalization. The Higgs boson ensures full consistency of the quantum field theory of the weak interaction.

The previous paragraph summarizes the main observations about the weak interaction. More precisely, all observations related to the weak interaction are described by its Lagrangian. Therefore, we need to check whether the weak interaction Lagrangian follows from the strand model.

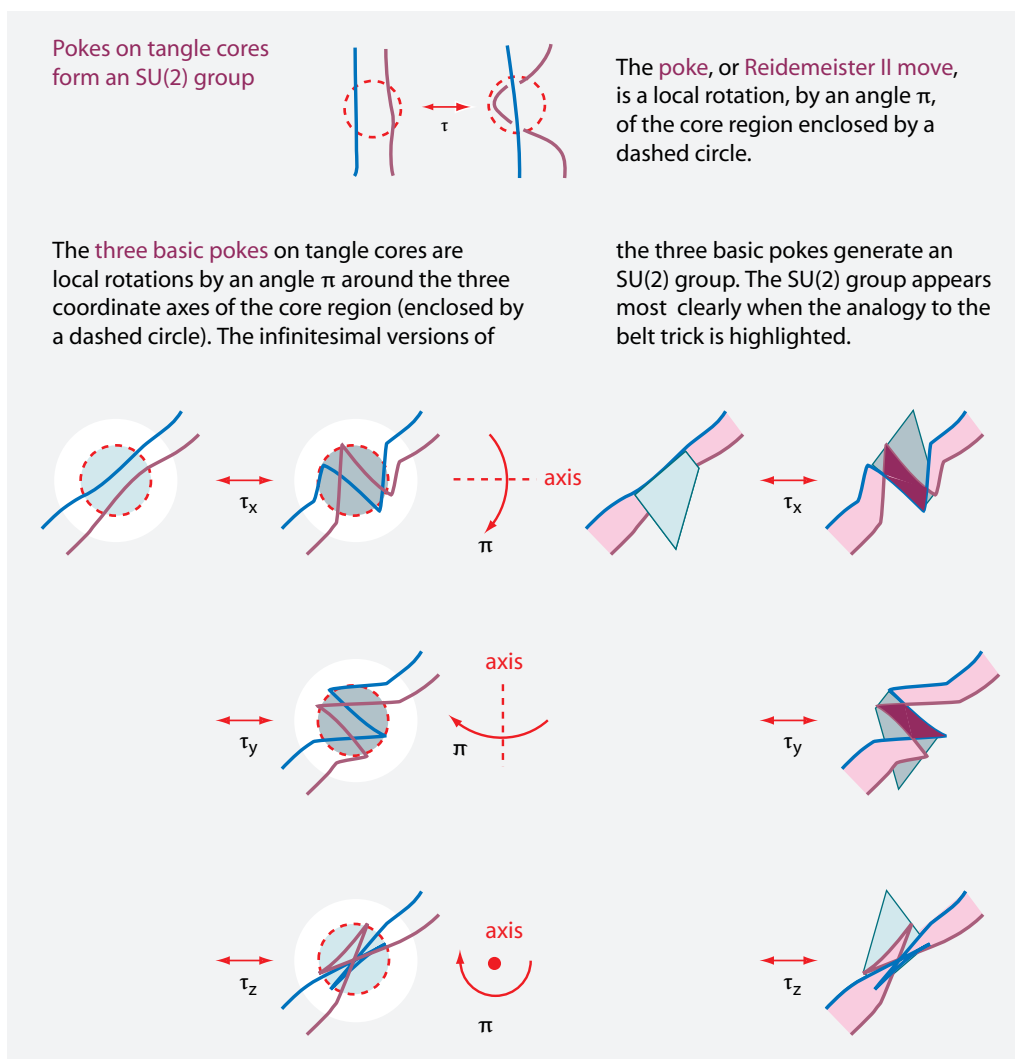


FIGURE 61 How the set of all pokes – the set of all deformations induced on tangle cores by the weak interaction – forms an $SU(2)$ gauge group: the three pokes lead to the belt trick, illustrated here with a pointed buckle and two belts. For clarity, deformations of two strands are shown, instead of the deformation of a single strand.

STRANDS, POKES AND $SU(2)$

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As explained above, any gauge interaction involving a fermion is a deformation of the tangle core that changes the phase and rotation of the fermion tangle. We start directly with the main definition.

- ▷ The *weak interaction* is the transfer of a poke, i.e., the transfer of a Reidemeister II move, between two particles. An illustration is given in [Figure 60](#). Strands are not cut in this process; they simply transfer the deformation as a result of their impenetrability.

Strands describe the weak interaction as exchange of pokes. In tangle cores, the *basic* pokes induce local rotations by an angle π , as shown in Figure 61: each basic poke rotates the region enclosed by the dotted circle. There are *three*, linearly independent, basic pokes, in three mutually orthogonal directions. The three basic pokes τ_x , τ_y and τ_z act on the local region in the same way as the three possible mutually orthogonal rotations act on a belt buckle. For completeness, we note that the following arguments do not depend on whether the two strands involved in a poke are parallel, orthogonal, or at a general angle. The following arguments also do not depend on whether the pokes are represented by deforming *two* strands or only *one* strand. Both cases lead to crossing switches, for each possible poke type.

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Figure 61 illustrates that the product of two different basic pokes gives the third basic poke, together with a sign $-$ which depends on whether the sequence is cyclic or not $-$ and a factor of i . Using the definition of -1 as a local rotation of the buckle region by 2π , we also find that the square of each basic poke is -1 . In detail, we can read off the following multiplication table for the three basic pokes:

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$$\begin{array}{c|ccc}
 \cdot & \tau_x & \tau_y & \tau_z \\
 \hline
 \tau_x & -1 & i\tau_z & -i\tau_y \\
 \tau_y & -i\tau_z & -1 & i\tau_x \\
 \tau_z & i\tau_y & -i\tau_x & -1
 \end{array} \tag{157}$$

In other terms, the three basic pokes $-$ and in particular also their infinitesimal versions $-$ behave like the generators of an $SU(2)$ group. Because pokes can be seen as local rotations of a buckle region, they can be generalized to arbitrary angles. Such arbitrary pokes can be concatenated. We thus find that arbitrary pokes form a full $SU(2)$ group. This is the reason for their equivalence with the belt trick.

The different gauge choices for a particle are not illustrated in Figure 61. The gauge choices arise from the different ways in which the basic pokes τ_x , τ_y and τ_z can be assigned to the set of deformations that describe the belt trick.

In summary, we can state that in any definition of the phase of a tangled fermion core, there is an $SU(2)$ gauge freedom; in addition, there exists an interaction with $SU(2)$ gauge symmetry. In other words, the strand model implies, through the second Reidemeister move, *the existence of the unbroken weak interaction with a gauge group $SU(2)$.*

WEAK CHARGE AND PARITY VIOLATION

A particle has weak charge if, when subject to many random pokes, a non-zero average phase change occurs. Surrounded by a bath of strands that continuously induce random pokes, not all tangles will change their phase on a long-time average: only tangles that lack symmetry will. One symmetry that must be lacking is spherical symmetry. Therefore, only tangles whose cores lack *spherical symmetry* have the chance to be influenced by random pokes. Since all tangles with knotted or braided cores lack spherical symmetry, all such tangles, i.e., all massive particles, are candidates to be influenced, and thus are candidates for weakly charged particles. We therefore explore them in detail.

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If a tangle is made of *two or more* knotted or braided strands, it represents a massive spin-1/2 particle (except for a simple twist, which represents the graviton). All such fer-

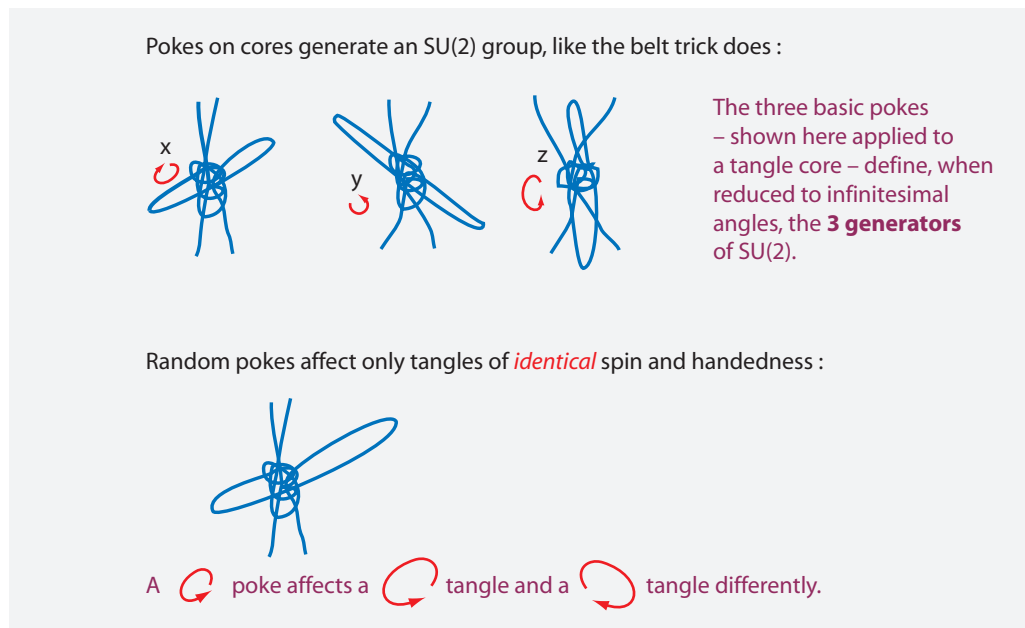


FIGURE 62 The three basic pokes and weak charge in the strand model.

mion cores lack spherical and cylindrical symmetry. When a fermion spins, two things happen: the core rotates and the belt trick occurs, which untangles the tails. Compared to the direction of motion, the rotation and the untangling can be either left-handed or right-handed.

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Every poke is a shape transformation of the core with a preferred handedness. The chirality is of importance in the following.

A particle has weak charge if random pokes lead to a long-time phase change. In order to feel any average effect when large numbers of random pokes are applied, a core must undergo different effects for a poke and its reverse. As already mentioned, this requires a lack of core symmetry. Whenever the core has no symmetry, non-compensating phase effects will occur: if the core rotation with its tail untangling and the poke are of the same handedness, the phase will increase, whereas for opposite handedness, the phase will decrease a bit less.

- ▷ Non-vanishing *weak charge* for fermions appears only for tangle cores whose handedness leads to average poke effects.

In other words, the strand model predicts that random pokes will only affect a core if the core handedness and the randomly applied belt trick are of the *same* handedness. In physical terms, random pokes will only affect left-handed particles or right-handed anti-particles. Thus, the strand model predicts that *the weak interaction violates parity maximally*, This is exactly as observed. In other terms, weak charge and the parity violation of the weak interaction are consequences of the belt trick. This relation is summarized in [Figure 62](#).

If an elementary particle is described by a *two tangled* strands, we expect it to be influenced by average pokes. Such tangle cores are spin-1 bosons; their cores lack spherical and cylindrical symmetry. The core rotation will induce a left-right asymmetry that will lead to a higher effect of a poke than of its reverse. Two-stranded particles are thus predicted to carry weak charge. We therefore expect that quarks – to be explored below – and the weak bosons themselves interact weakly.

Because the weak bosons interact weakly, the strand model implies that the weak interaction is a *non-Abelian* gauge theory, as is observed.*

If a tangle is made of a *single unknotted* strand, it is not affected by random pokes. The strand model thus predicts that the photon has no weak charge, as is observed. The same also holds for gluons.

The strand definition of weak charge leads to two conclusions that can be checked by experiment. First, all electrically charged particles – having cores that are chiral and thus lack cylindrical symmetry – are predicted to be weakly charged. Secondly, in the strand model, only massive particles interact weakly; in fact, *all* massive particles interact weakly, because their cores lack cylindrical symmetry. In other words, all weakly charged particles move more slowly than light and vice versa. Both conclusions agree with observation.

In summary, all properties of weak charge found in nature are reproduced by the tangle model.

WEAK BOSONS

Gauge bosons are those particles that are exchanged between interacting fermions: gauge bosons induce phase changes of fermions. This implies that the (unbroken) weak bosons are the particles** that induce the three poke moves:

- ▷ *Weak intermediate bosons* are described by double strands. An illustration is given in [Figure 63](#).

Single strands that induce phase changes in fermions interacting weakly are shown on the left side of [Figure 63](#). They correspond to the three basic pokes τ_x , τ_y and τ_z .

We note two additional points. First of all, the (unbroken) spin-1 bosons could also be described by the motion of a single strand in a strand group. This makes them spin 1 particles.

Furthermore, unknotted tangles are *massless*. In the strand model, tangles that induce pokes *differ* from the massive weak intermediate bosons, shown on the right of [Figure 63](#). This difference is due to the *breaking* of the SU(2) gauge symmetry, as we will find out soon.

* Non-Abelian gauge theory was introduced by Wolfgang Pauli. In the 1950s, he explained the theory in series of talks. Two physicists, Yang Chen Ning and Robert Mills, then wrote down his ideas. Yang later received the Nobel Prize in Physics with Lee Tsung Dao for a different topic, namely for the violation of parity of the weak interaction.

** This reworked strand model of the W and Z bosons arose in 2015.

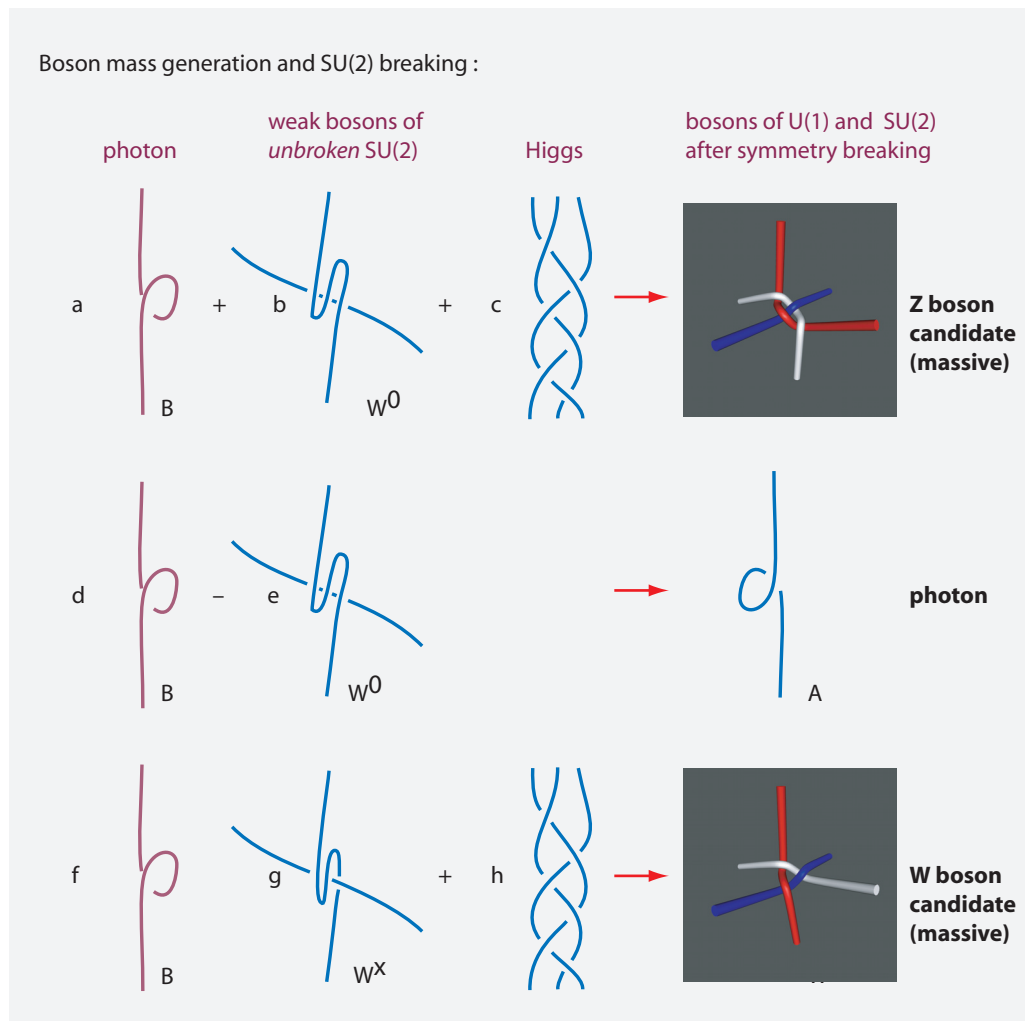


FIGURE 63 Poke-inducing strand motions (left) become massive weak vector tangles (right) through symmetry breaking and tail braiding. Tail braiding is related to the Higgs boson, whose tangle model will be clarified later on.

THE LAGRANGIAN OF THE UNBROKEN SU(2) GAUGE INTERACTION

The energy of the weak field is given by the density of weak gauge boson strands. As long as the SU(2) symmetry is not broken, the energy of the weak field and the energy of fermions are both SU(2) invariant. As a consequence, we are now able to deduce a large part of the Lagrangian of the weak interaction, namely the Lagrangian for the case that the SU(2) symmetry is unbroken.

As long as SU(2) is unbroken, the vector bosons are described as unknotted tangles that induce pokes, as shown on the left of [Figure 63](#). There are three such bosons. Since they can be described by a single strand that moves, they have spin 1; since they are unknotted, they have zero mass and electric charge.

Energy is the number of crossing switches per time. As long as SU(2) is unbroken

and the weak bosons are massless, the energy of the weak boson field and thus their Lagrangian density is given by the same expression as the energy of the photon field. In particular, the strand model implies that energy density is quadratic in the field intensities. We only have to add the energies of all three bosons together to get:

$$\mathcal{L} = -\frac{1}{4} \sum_{a=1}^3 W_a^{\mu\nu} W_a^{\mu\nu}, \quad (158)$$

This expression is SU(2) gauge invariant. Indeed, SU(2) gauge transformations have no effect on the number of crossing switches due to weak bosons or to the motion of pokes. Thus, gauge transformations leave weak field intensities and thus also the energy of the weak fields invariant, as observed.

We can now write down the Lagrangian for weakly charged fermions interacting with the weak vector bosons. Starting from the idea that tangle core deformations lead to phase redefinitions, we have found that pokes imply that the *unbroken* weak Lagrangian density for matter and radiation fields is SU(2) gauge invariant. In parallel to electrodynamics we thus get the Lagrangian

$$\mathcal{L}_{\text{unbroken weak}} = \sum_f \bar{\Psi}_f (i\hbar c \mathcal{D} - m_f c^2) \Psi_f - \frac{1}{4} \sum_{a=1}^3 W_a^{\mu\nu} W_a^{\mu\nu}, \quad (159)$$

where \mathcal{D} is now the SU(2) gauge covariant derivative and the first sum is taken over all fermions. In this Lagrangian, only the left-handed fermions and the right-handed antifermions carry weak charge. This Lagrangian, however, does *not* describe nature: the observed SU(2) breaking is missing.

SU(2) BREAKING

In nature, the weak interaction does *not* have an SU(2) gauge symmetry. The symmetry is only approximate; is said to be *broken*. The main effect of SU(2) symmetry breaking are the non-vanishing – and different – masses for the W and Z bosons, and thus the weakness and the short range of the weak interaction. In addition, the symmetry breaking implies a *mixing* of the weak and the electromagnetic interaction: it yields the so-called *electroweak* interaction. This mixing is often called electroweak ‘unification’.

The strand model suggests the following description:

- ▷ **Mass generation** for bosons and the related SU(2) **symmetry breaking** are due to *tail braiding* at the border of space. [Figure 63](#) illustrates the idea.

In this description, tail braiding* is assumed to occur at a distance outside the domain of observation; in that region – which can be also the border of physical space – tail braiding is *not* forbidden and *can* occur. The probability of tail braiding is low, because

* In the original strand model of the weak bosons, from the year 2008, the role of tail braiding was taken by strand overcrossing.

the crossings have first to fluctuate to that distance and then fluctuate back. Nevertheless, the process of tail braiding can take place.

Tail braiding appears *only* in the weak interaction. It does not appear in the other two gauge interactions, as the other Reidemeister moves are not affected by processes at the border of space. In the strand model, this is the reason that only SU(2) is broken in nature. In short, SU(2) breaking is a natural consequence of the second Reidemeister move.

Page 353 Tail braiding transforms the unbraided, and thus massless, photon strands into the braided, and thus massive W and Z strands. Tail braiding leads to particle cores: therefore is a mass-generating process. The precise mass values that it generates will be determined below. The strand model thus confirms that mass generation is related to the breaking of the weak interaction.

Page 247 Tail braiding mixes the W^0 with the ‘original’ photon. This is shown in Figure 63. The mixing is due to the topological similarities of the strand models of the two particles. The resulting Z boson is achiral, and thus electrically neutral, as observed. We note that the existence of a neutral, massive Z boson implies that elastic neutrino scattering in matter occurs in nature, as was observed for the first time in 1974. Since any electrically charged particle also has weak charge, the existence of a Z boson implies that any two electrically charged particles can interact both by exchange of photons and by exchange of Z bosons. In other words, SU(2) breaking implies electroweak mixing, or, as is it usually called, electroweak ‘unification’.

Page 254 Tail braiding takes place in several weak interaction processes, as shown in Figure 66. Page 253 Tail braiding thus can change particle topology, and thus particle type. The strand model thus predicts that the weak interaction *changes* particle flavours (types), as is observed. In fact, the strand model also predicts that *only* the weak interaction has this property. This is also observed.

Page 317 On the other hand, strands are never cut or glued back together in the strand model, not even in the weak interaction. As a result, the strand model predicts that the weak interaction conserves electric charge, spin and, as we will see below, colour charge, baryon number and lepton number. All this is observed.

Page 353 Tail braiding also implies that the tangles for the Z boson and for the W boson shown above are only the *simplest* tangles associated with each boson; more complicated tangles are higher order propagating states of the same basic open knots. This will be of great importance later on, for the proof that all gauge bosons of nature are already known today.

Page 355 In summary, the second Reidemeister move leads to *tail braiding*; tail braiding leads to the observed properties of SU(2) symmetry breaking. (Equivalently, the strand model implies that the simplest tangles of the weak interaction bosons show SU(2) symmetry, whereas the more complicated, knotted tangles break this symmetry.) The value of the mixing angle and the particle masses have still to be determined. This will be done below.

OPEN ISSUE: ARE THE W AND Z TANGLES CORRECT?

In 2014, Sergei Fadeev raised an issue: A *tangle* version of the W and Z that does *not* contain any knot and does not require an actual strand overcrossing process at spatial

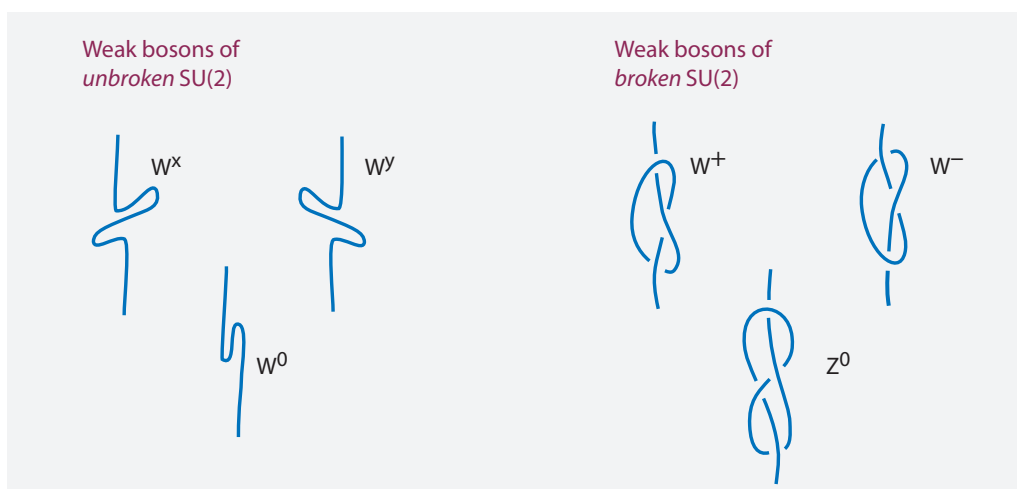


FIGURE 64 The supposed models for the weak gauge bosons from 2008, now seen to be incorrect.

infinity, the strand model would gain in simplicity and elegance. Thinking about the issue, it became clear that such a tangle could occur when vacuum strands were included, as shown above.

In contrast, in 2008, in the first version of the strand model, the W boson after symmetry breaking was thought to be an open overhand knot, and the Z boson an open figure-eight knot.

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It might well be that the new, 2015/2016 strand models for the two intermediate vector bosons, shown in Figure 63 are still not correct. The possibility remains intriguing and a definitive issue still needs to be found.

THE ELECTROWEAK LAGRANGIAN

We can now use the results on $SU(2)$ symmetry breaking to deduce the *electroweak* Lagrangian density. We have seen that symmetry breaking leaves the photon massless but introduces masses to the weak vector bosons, as shown in Figure 63. The non-vanishing boson masses M_W and M_Z add kinetic terms for the corresponding fields in the Lagrangian.

Due to the symmetry breaking induced by tail braiding, the Z boson results from the mixing with the (unbroken) photon. The strand model predicts that the mixing can be described by an angle, the so-called weak mixing angle θ_w . In particular, the strand model implies that $\cos \theta_w = M_W/M_Z$.

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As soon as symmetry breaking is described by a mixing angle due to tail braiding, we get the known electroweak Lagrangian, though at first without the terms due to the Higgs boson. (We will come back to the Higgs boson later on.) We do not write down the Lagrangian of the weak interaction predicted by the strand model, but the terms are the same as those found in the standard model of elementary particles. There is one important difference: the Lagrangian so derived does not yet contain quark and lepton mixing. Indeed, experiments show that the weak fermion eigenstates are not the same as the strong or electromagnetic eigenstates: quarks mix, and so do neutrinos. The reason

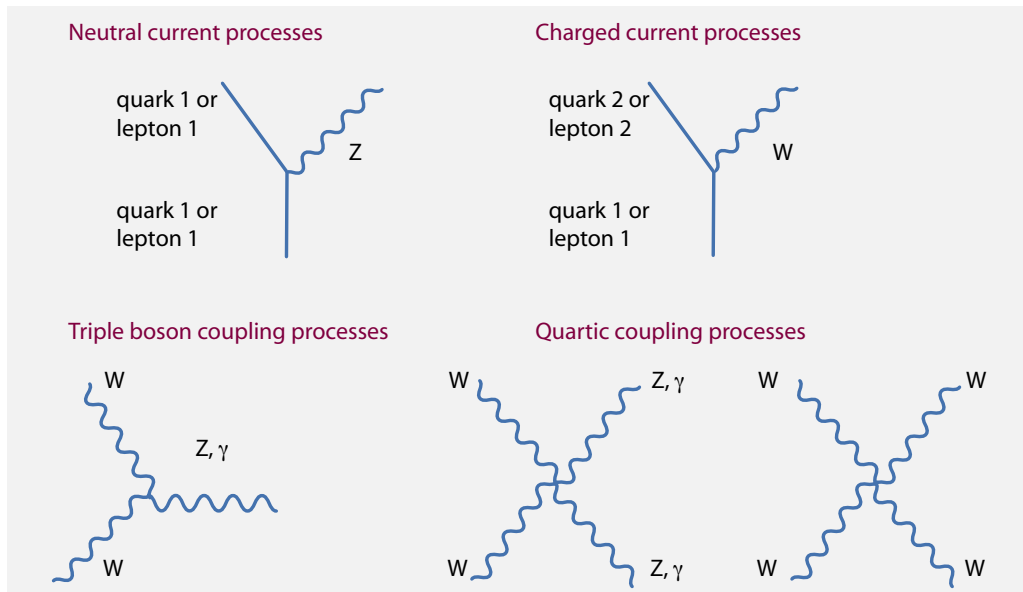


FIGURE 65 The fundamental Feynman diagrams of the weak interaction that do not involve the Higgs boson.

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for this observation, and the effect that mixing has on the weak Lagrangian, will become clear once we have determined the tangles for each fermion.

In summary, the strand model implies the largest part of the Lagrangian of the weak interaction. The issue of the Higgs boson is still open, and the electroweak Lagrangian contains a number of constants that are not yet clarified. These unexplained constants are the number of the involved elementary particles, their masses, couplings, mixing angles and CP violation phases, as well as the value of the weak mixing angle.

THE WEAK FEYNMAN DIAGRAMS

In nature, the weak interaction is described by a small number of fundamental Feynman diagrams. Those not containing the Higgs boson are shown in Figure 65. These Feynman diagrams encode the corresponding Lagrangian of the weak interaction.

In the strand model, pokes lead naturally to strand versions of the fundamental Feynman diagrams. This happens as shown in Figure 66. We see again that the strand model reproduces the weak interaction: each Feynman diagram is due to a strand diagram for which only crossing switches are considered, and for which Planck size is approximated as zero size. In particular, the strand model does not allow any *other* fundamental diagrams for the weak interaction.

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The finite and small number of possible strand diagrams and thus of Feynman diagrams implies that the weak interaction is *renormalizable*. For example, the change or ‘running’ of the weak coupling with energy is reproduced by the strand model, because the running can be determined through the appropriate Feynman diagrams.

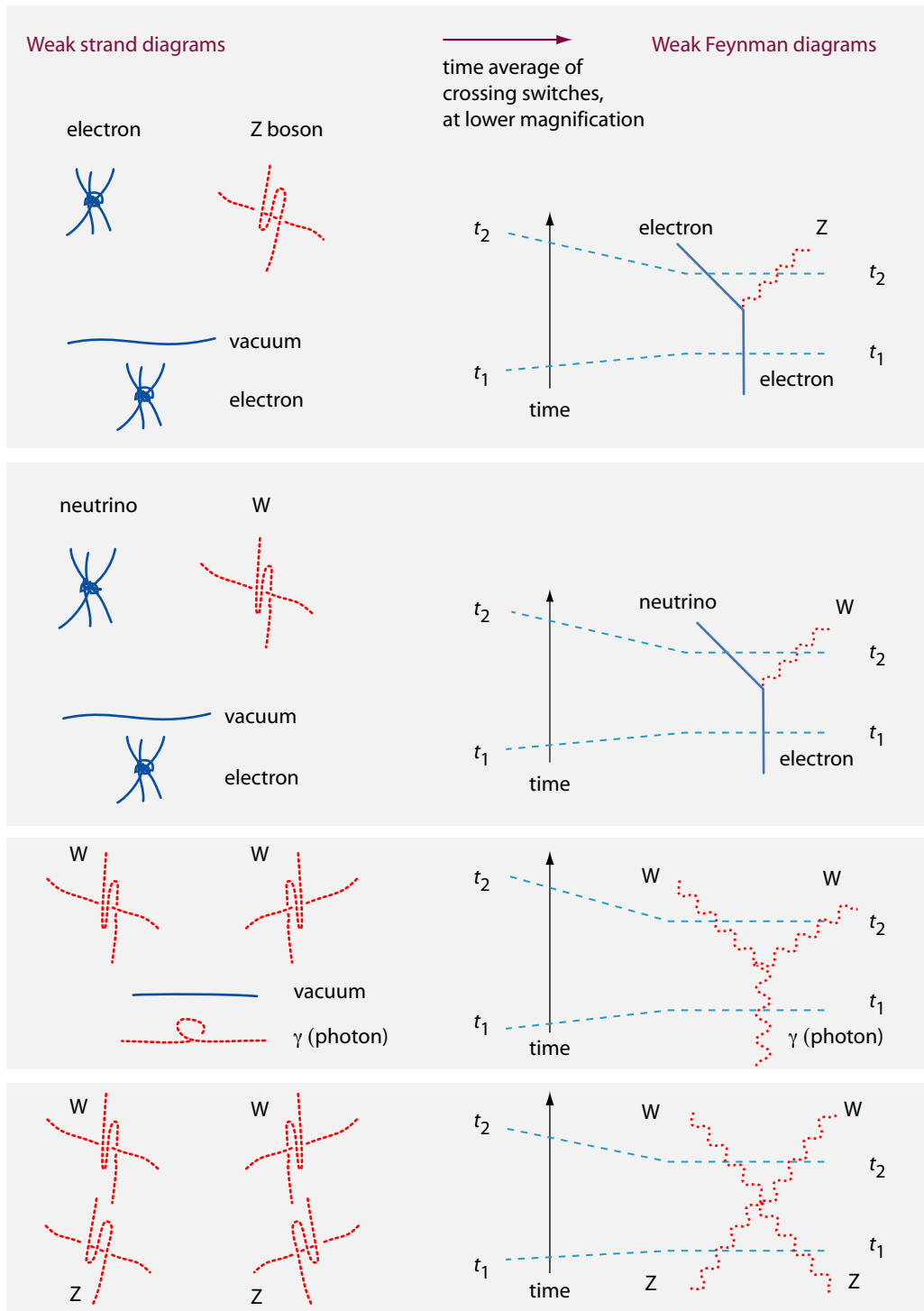


FIGURE 66 The strand model for the fundamental Feynman diagrams of the weak interaction. The tangles for the fermions are introduced later on.

FUN CHALLENGES AND CURIOSITIES ABOUT THE WEAK INTERACTION

Challenge 165 e The W boson and its antiparticle are observed to annihilate through the electromagnetic interaction, yielding two or more photons. The tangle model of the weak bosons has a lot of advantages compared to the knot model: The annihilation is much easier to understand.

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The strand model, like the standard model of particle physics, predicts that everything about the weak interaction is already known. Nevertheless, the most important weak process, the *decay of the neutron*, is being explored by many precision experiments. The strand model predicts that none of these experiments will yield any surprise.

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Ref. 182 The strand model makes clear that the weak interaction and the electromagnetic interaction *mix*, but do not unify. There is only electroweak mixing, and *no* electroweak unification, despite claims to the contrary by the Nobel Prize committee and many other physicists. In fact, Sheldon Glashow, who received the Nobel Prize in Physics for this alleged ‘unification’, agrees with this assessment. So do Richard Feynman and, above all, Martin Veltman, who was also involved in the result; he even makes this very point in his Nobel Prize lecture. The incorrect habit to call electroweak mixing a ‘unification’ was one of the main reason for the failure of past unification attempts: it directed the attention of researchers in the wrong direction.

In the strand model, the mixing of the electromagnetic and the weak interaction can be seen as a consequence of knot geometry: the poke generators of the weak interaction also contain twists, i.e., also contain generators of the electromagnetic interaction. In contrast, generators of other Reidemeister moves do not mix among them or with pokes; and indeed, no other type of interaction mixing is observed in nature.

SUMMARY ON THE WEAK INTERACTION AND EXPERIMENTAL PREDICTIONS

We have deduced the main properties of the weak Lagrangian from the strand model. We have shown that Reidemeister II moves – or pokes – in tangle cores lead to a broken SU(2) gauge group and to massive weak bosons. We found that the deviation from tangle core sphericity plus chirality is weak charge, and that the weak interaction is non-Abelian. We have also shown that the weak interaction naturally breaks parity maximally and mixes with the electromagnetic interaction. In short, we have deduced the main experimental properties of the weak interaction.

Page 327 Is there a difference between the strand model and the electroweak Lagrangian of the standard model of particle physics? Before we can fully answer the question on deviations between the strand model and the standard model, we must settle the issue of the Higgs boson. This is done later on.

In any case, the strand model predicts that the broken SU(2) gauge symmetry remains valid at all energies. No other gauge groups appear in nature. The strand model thus predicts again that there is no grand unification, and thus no larger gauge group, be it SU(5), SO(10), E6, E7, E8, SO(32) or any other group. Also this result indirectly rules out supersymmetry and supergravity.

The strand model also predicts that the combination of gravity and quantum theory turns all Planck units into *limit* values, because there is a maximum density of strand crossings in nature, due to the fundamental principle. Therefore, the strand model predicts a *maximum weak field* value given by the Planck force divided by the smallest weak charge. All physical systems – including all astrophysical objects, such as neutron stars, quark stars, gamma-ray bursters or quasars – are predicted to conform to this limit. So far, no observed field value is near this limit, so that the prediction does not contradict observation.

So far, our exploration of the weak interaction has left us with a few open issues: we need to calculate the weak coupling constant and determine the tangle for each particle of the standard model, including the Higgs boson. But we also need to explain weak fermion mixing, CP violation and the masses of all particles. Despite these open points, we have settled another line of the millennium list: we know the origin of the weak interaction and of its main properties. Before we clarify the open issues, we explore the third Reidemeister move.

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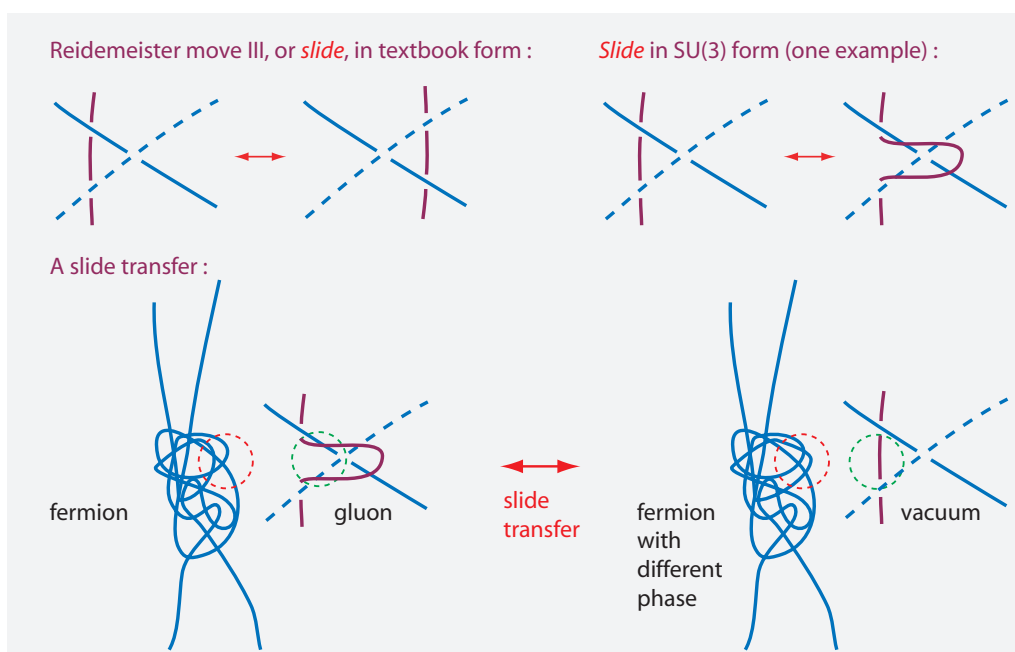


FIGURE 67 A gluon changes the phase of a tangle: *slide transfer* is the basis of the strong interaction in the strand model. During the interaction, no strand is cut or reglued; the transfer occurs purely through the excluded volume that results from the impenetrability of strands.

THE STRONG NUCLEAR INTERACTION AND THE THIRD REIDEMEISTER MOVE

In nature, the strong interaction is the result of the absorption and the emission of massless, electrically uncharged, spin-1 gauge bosons that are called *gluons*. Gluons interact with quarks, the only fermions with *colour* charge. Fermions can have three different colour charges, antifermions three different anticolours. Gluons form an octet, are themselves colour charged and therefore also interact among themselves. The Lagrangian of quarks coupled to the gluon field has an unbroken SU(3) gauge symmetry. There are three fundamental Feynman diagrams: one for quark-gluon interaction and two for gluon-gluon interactions: a triple and a quartic gluon vertex. The strong coupling constant is about 0.5 at low energy; its energy dependence is determined by renormalization.

The previous paragraph summarizes the main observations about the strong interaction. All known observations related to the strong interaction, without any known exception, are contained in its Lagrangian. Therefore, we need to show that the strong interaction Lagrangian follows from the strand model.

STRANDS AND THE SLIDE, THE THIRD REIDEMEISTER MOVE

Page 221 As explained above, interactions of fermions are deformations of the tangle core that change its phase. We start directly by presenting the strand model for the strong interaction.

- ▷ The **strong interaction** is the transfer of *slides*, i.e., the transfer of third Reidemeister moves, between a gluon and a particle. As shown in Figure 67, strands are not cut in this process; they simply transfer deformations as a result of their impenetrability.

Such a slide transfer will influence the phase of the affected particle tangle. Therefore, slide transfers are indeed a type of interaction.

AN INTRODUCTION TO SU(3)

Before we show that slides are responsible for the strong nuclear interaction, we summarize the mathematical properties of the Lie group SU(3). This Lie group is the structure generated by the unitary 3×3 matrices with determinant +1. It is a *group*, because matrices can be properly multiplied, because the identity matrix is included, and inverse matrices exist. SU(3) is also a *manifold*; a quick check shows that it has eight dimensions. In short, SU(3) is a *Lie group*: its elements behave like points on a manifold that can be multiplied. The Lie bracket is the commutator. A general element E of SU(3) can be written as an exponential in the well-known way

$$E = e^{\sum_{n=1}^8 \alpha_n i \lambda_n / 2} \tag{160}$$

where the eight real parameters α_n can be thought of as the eight coordinates of the group elements on the group manifold. Since SU(3) is compact and simple, these coordinates are best visualized as angles. Of course, i is the imaginary unit. The generators λ_n are complex, traceless and hermitian 3×3 matrices; they are used to define a basis for the group elements. The eight generators are *not* group elements themselves. They describe the structure of the group manifold near the identity matrix; for a Lie group, this local structure defines the full group manifold. Like for any basis, also set of eight generators λ_n is not unique. Of the many possible choices for the generators, the *Gell-Mann matrices* λ_1 to λ_8 are the most commonly used in physics.

The Gell-Mann matrices λ_n , the corresponding group elements D_n for general angles, and the group elements E_n for the finite angle π are given by:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, D_1(\alpha) = e^{\alpha i \lambda_1 / 2} = \begin{pmatrix} \cos \alpha / 2 & i \sin \alpha / 2 & 0 \\ i \sin \alpha / 2 & \cos \alpha / 2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_1 = e^{\pi i \lambda_1 / 2} = \begin{pmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, D_2(\alpha) = e^{\alpha i \lambda_2 / 2} = \begin{pmatrix} \cos \alpha / 2 & \sin \alpha / 2 & 0 \\ -\sin \alpha / 2 & \cos \alpha / 2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$E_2 = e^{\pi i \lambda_2 / 2} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned}
\lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, D_3(\alpha) = e^{\alpha i \lambda_3/2} = \begin{pmatrix} \cos \alpha/2 + i \sin \alpha/2 & 0 & 0 \\ 0 & \cos \alpha/2 - i \sin \alpha/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
E_3 &= e^{\pi i \lambda_3/2} = \begin{pmatrix} i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, D_4(\alpha) = e^{\alpha i \lambda_4/2} = \begin{pmatrix} \cos \alpha/2 & 0 & i \sin \alpha/2 \\ 0 & 1 & 0 \\ i \sin \alpha/2 & 0 & \cos \alpha/2 \end{pmatrix}, \\
E_4 &= e^{\pi i \lambda_4/2} = \begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ i & 0 & 0 \end{pmatrix} \\
\lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, D_5(\alpha) = e^{\alpha i \lambda_5/2} = \begin{pmatrix} \cos \alpha/2 & 0 & \sin \alpha/2 \\ 0 & 1 & 0 \\ -\sin \alpha/2 & 0 & \cos \alpha/2 \end{pmatrix}, \\
E_5 &= e^{\pi i \lambda_5/2} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \\
\lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, D_6(\alpha) = e^{\alpha i \lambda_6/2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha/2 & i \sin \alpha/2 \\ 0 & i \sin \alpha/2 & \cos \alpha/2 \end{pmatrix}, \\
E_6 &= e^{\pi i \lambda_6/2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{pmatrix} \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, D_7(\alpha) = e^{\alpha i \lambda_7/2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha/2 & \sin \alpha/2 \\ 0 & -\sin \alpha/2 & \cos \alpha/2 \end{pmatrix}, \\
E_7 &= e^{\pi i \lambda_7/2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \\
\lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \\
D_8(\alpha) &= e^{\sqrt{3} \alpha i \lambda_8/2} = \begin{pmatrix} \cos \alpha/2 + i \sin \alpha/2 & 0 & 0 \\ 0 & \cos \alpha/2 + i \sin \alpha/2 & 0 \\ 0 & 0 & \cos \alpha - i \sin \alpha \end{pmatrix}, \\
E_8 &= D_8(\pi) = \begin{pmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{161}
\end{aligned}$$

The eight Gell-Mann matrices λ_n are hermitean, traceless and trace-orthogonal. The corresponding group elements D_n and E_n can be thought as the unnormalized and normalized

basis vectors of the group manifold. We note that the definition of E_8 differs from that of the other group elements E_n : it contains an extra factor $\sqrt{3}$. The fourfold concatenation of each matrix $i\lambda_n$ is the identity matrix – except for the case $i\lambda_8$. Instead, the generator λ_8 commutes with λ_1, λ_2 and λ_3 – though not with the other generators.

There is *no* ninth or tenth Gell-Mann matrix. Such a matrix would not be linearly independent from the first eight ones. Indeed, the two matrices deduced from λ_3 using symmetry considerations, namely

$$\lambda_9 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, D_9(\alpha) = e^{i\alpha\lambda_9/2} = \begin{pmatrix} \cos \alpha/2 - i \sin \alpha/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos \alpha/2 + i \sin \alpha/2 \end{pmatrix},$$

$$E_9 = D_9(\pi) = \begin{pmatrix} -i & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}$$

$$\lambda_{10} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, D_{10}(\alpha) = e^{i\alpha\lambda_{10}/2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha/2 + i \sin \alpha/2 & 0 \\ 0 & 0 & \cos \alpha/2 - i \sin \alpha/2 \end{pmatrix},$$

$$E_{10} = D_{10}(\pi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix} \tag{162}$$

are linear combinations of λ_3 and λ_8 ; in particular, we have $\lambda_3 + \lambda_9 + \lambda_{10} = 0$ and $\sqrt{3}\lambda_8 + \lambda_9 = \lambda_{10}$. Therefore, λ_9 and λ_{10} are *not* Gell-Mann matrices. (Also two further matrices corresponding to λ_8 in the other two triplets can be defined. The sum of these three matrices is 0 as well.)

The multiplication properties of the Gell-Mann generators λ_1 to λ_8 are listed in Table 10. To make the threefold symmetry more evident, the table also lists the products containing the linearly dependent matrices λ_9 and λ_{10} . Writing the table with the commutators would directly show that the generators form a Lie algebra.

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The *centre* of SU(3), the subgroup that commutes with all other elements of the group, is Z_3 ; its threefold symmetry is useful in understanding the behaviour of the group elements and of the generators in more detail.

The group elements E_1 to E_8 listed above share the property that their fourth powers $(E_n)^4$ are the identity matrix. The first matrix triplet E_1, E_2, E_3 , the second triplet E_4, E_5, E_9 and the third triplet E_6, E_7, E_{10} each form a SU(2) subgroup. Reflecting the threefold symmetry of its centre, SU(3) contains three linearly independent SU(2) subgroups. The group element E_8 commutes with the first triplet E_1, E_2, E_3 ; therefore, these four elements generate a U(2) subgroup of SU(3). This U(2) subgroup, often sloppily labeled as SU(2)xU(1), is given by those 3 by 3 matrices that contain a unitary 2 by 2 matrix in the upper left, contain zeroes in the remaining four off-diagonal elements, and contain the inverse value of the determinant of the 2 by 2 matrix in the remaining, lower right diagonal element. In short, SU(3) contains three linearly independent U(2) subgroups.

SU(3) is characterized by the way that the SU(2) triplets are connected. In particular, the product $E_3E_9E_{10}$ is the identity, reflecting the linear dependence of the three corresponding generators λ_n . We also have $E_8E_9 = E_{10}$. Also the product of E_8 with its

TABLE 10 The multiplication table for the generators λ_1 to λ_8 of $SU(3)$, and for the additional, *linearly dependent* matrices $\lambda_9 = -\lambda_3/2 - \lambda_8\sqrt{3}/2$ and $\lambda_{10} = -\lambda_3/2 + \lambda_8\sqrt{3}/2$ that are *not* generators. Note that, despite the appearance, $\lambda_4^2 = \lambda_5^2 = \lambda_9^2$ and $\lambda_6^2 = \lambda_7^2 = \lambda_{10}^2$.

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_9	λ_6	λ_7	λ_{10}	λ_8
λ_1	$2/3$ $+ \lambda_8/\sqrt{3}$	$i\lambda_3$	$-i\lambda_2$	$\lambda_6/2$ $+i\lambda_7/2$	$-i\lambda_6/2$ $+ \lambda_7/2$	$-\lambda_1/2$ $+i\lambda_2/2$	$\lambda_4/2$ $+i\lambda_5/2$	$-i\lambda_4/2$ $+ \lambda_5/2$	$\lambda_1/2$ $+i\lambda_2/2$	$\lambda_1/\sqrt{3}$
λ_2	$-i\lambda_3$ $+ \lambda_8/\sqrt{3}$	$2/3$	$i\lambda_1$	$i\lambda_6/2$ $- \lambda_7/2$	$\lambda_6/2$ $+i\lambda_7/2$	$-i\lambda_1/2$ $- \lambda_2/2$	$-i\lambda_4/2$ $+ \lambda_5/2$	$- \lambda_4/2$ $-i\lambda_5/2$	$-i\lambda_1/2$ $+ \lambda_2/2$	$\lambda_2/\sqrt{3}$
λ_3	$i\lambda_2$	$-i\lambda_1$	$2/3$ $+ \lambda_8/\sqrt{3}$	$\lambda_4/2$ $+i\lambda_5/2$	$-i\lambda_4/2$ $+ \lambda_5/2$	$-1/3 - \lambda_3/3$ $+ \lambda_9/3$	$- \lambda_6/2$ $-i\lambda_7/2$	$i\lambda_6/2$ $- \lambda_7/2$	$-1/3 + \lambda_3/3$ $+ \lambda_{10}/3$	$\lambda_3/\sqrt{3}$
λ_4	$\lambda_6/2$ $-i\lambda_7/2$	$-i\lambda_6/2$ $- \lambda_7/2$	$\lambda_4/2$ $-i\lambda_5/2$	$2/3 + \lambda_3/2$ $- \lambda_8/2\sqrt{3}$	$-i\lambda_9$	$i\lambda_5$	$\lambda_1/2$ $+i\lambda_2/2$	$i\lambda_1/2$ $- \lambda_2/2$	$- \lambda_4/2$ $-i\lambda_5/2$	$- \lambda_4/2\sqrt{3}$ $-i\sqrt{3}\lambda_5/2$
λ_5	$i\lambda_6/2$ $+ \lambda_7/2$	$\lambda_6/2$ $-i\lambda_7/2$	$i\lambda_4/2$ $+ \lambda_5/2$	$i\lambda_9$	$2/3 + \lambda_3/2$ $- \lambda_8/2\sqrt{3}$	$-i\lambda_4$	$-i\lambda_1/2$ $+ \lambda_2/2$	$\lambda_1/2$ $+i\lambda_2/2$	$i\lambda_4/2$ $- \lambda_5/2$	$i\sqrt{3}\lambda_4/2$ $- \lambda_5/2\sqrt{3}$
λ_9	$-\lambda_1/2$ $-i\lambda_2/2$	$i\lambda_1/2$ $- \lambda_2/2$	$-1/3 - \lambda_3/3$ $+ \lambda_9/3$	$-i\lambda_5$	$i\lambda_4$	$2/3 + 2\lambda_3/3$ $+ \lambda_9/3$	$\lambda_6/2$ $-i\lambda_7/2$	$i\lambda_6/2$ $+ \lambda_7/2$	$-1/3 - \lambda_9/3$ $+ \lambda_{10}/3$	-1 $+ \lambda_{10}$
λ_6	$+ \lambda_4/2$ $-i\lambda_5/2$	$i\lambda_4/2$ $+ \lambda_5/2$	$- \lambda_6/2$ $+i\lambda_7/2$	$\lambda_1/2$ $-i\lambda_2/2$	$i\lambda_1/2$ $+ \lambda_2/2$	$\lambda_6/2$ $+i\lambda_7/2$	$2/3 - \lambda_3/2$ $- \lambda_8/2\sqrt{3}$	$i\lambda_{10}$	$-i\lambda_7$	$- \lambda_6/2\sqrt{3}$ $-i\sqrt{3}\lambda_7/2$
λ_7	$i\lambda_4/2$ $+ \lambda_5/2$	$- \lambda_4/2$ $+i\lambda_5/2$	$-i\lambda_6/2$ $- \lambda_7/2$	$-i\lambda_1/2$ $- \lambda_2/2$	$\lambda_1/2$ $-i\lambda_2/2$	$-i\lambda_6/2$ $+ \lambda_7/2$	$-i\lambda_{10}$	$2/3 - \lambda_3/2$ $- \lambda_8/2\sqrt{3}$	$i\lambda_6$	$i\sqrt{3}\lambda_6/2$ $- \lambda_7/2\sqrt{3}$
λ_{10}	$-\lambda_1/2$ $+i\lambda_2/2$	$-i\lambda_1/2$ $- \lambda_2/2$	$-1/3 + \lambda_3/3$ $- \lambda_{10}/3$	$- \lambda_4/2$ $+i\lambda_5/2$	$-i\lambda_4/2$ $- \lambda_5/2$	$-1/3 - \lambda_9/3$ $+ \lambda_{10}/3$	$i\lambda_7$	$-i\lambda_6$	$2/3 - \lambda_3/3$ $+ \lambda_9/3$	1 $+ \lambda_9$
λ_8	$\lambda_1/\sqrt{3}$	$\lambda_2/\sqrt{3}$	$\lambda_3/\sqrt{3}$	$- \lambda_4/2\sqrt{3}$ $+i\sqrt{3}\lambda_5/2$	$-i\sqrt{3}\lambda_4/2$ $- \lambda_5/2\sqrt{3}$	-1 $+ \lambda_{10}$	$- \lambda_6/2\sqrt{3}$ $+i\sqrt{3}\lambda_7/2$	$-i\sqrt{3}\lambda_6/2$ $- \lambda_7/2\sqrt{3}$	1 $+ \lambda_9$	$2/3$ $- \lambda_8/\sqrt{3}$

companions from the other two triplets is the identity.

Finally, the product $(E_k E_l)^3$ for any k taken from the set $\{1, 2, 4, 5, 6, 7\}$ and any l from the same set, but from a *different* triplet, is also the identity matrix. This property of the third powers – taken together with the threefold symmetry of its centre – can be seen as the essential property that distinguishes $SU(3)$ from other Lie groups. We now return to the strand model and show that slides indeed define an $SU(3)$ group.

FROM SLIDES TO $SU(3)$

The *slide*, or *third Reidemeister move*, involves *three* pieces of strands. The textbook version of the third Reidemeister move – which is called E_0 here and is illustrated in [Figure 68](#) – moves or ‘slides’ one strand, taken to be the horizontal blue one in the figure,

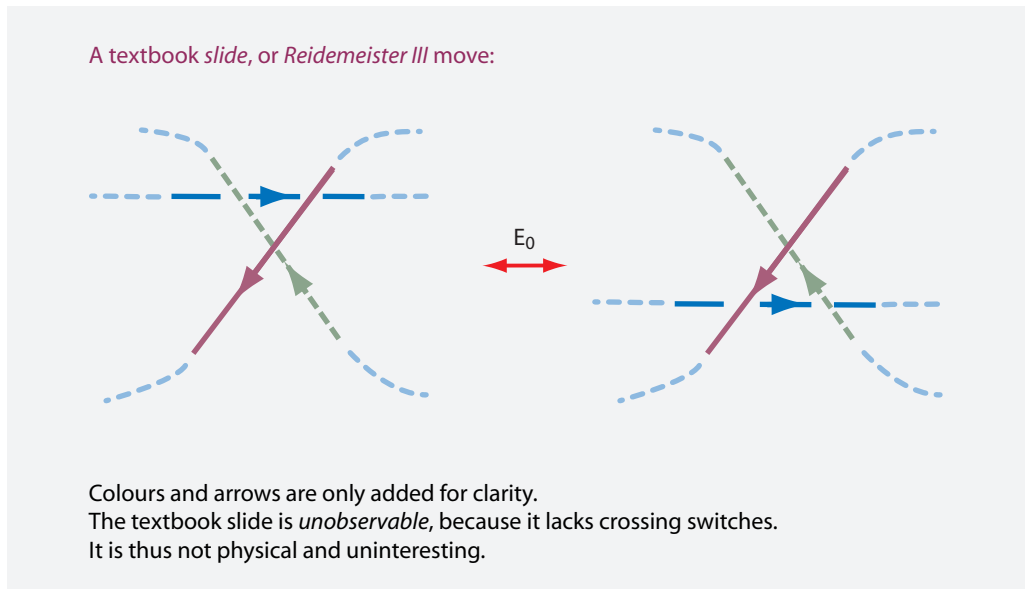


FIGURE 68 The textbook version E_0 of the slide move, or third Reidemeister move, is unobservable, because it does not involve crossing switches.

against a crossing of the other two. Equivalently, we can say that a slide pushes two strands against the blue strand that is kept in place. This textbook slide – we also call it a *pure slide* here – does not contain any crossing switch; following the fundamental principle of the strand model, it is therefore unobservable, or, simply said, of no physical relevance. However, related strand moves that do involve crossing switches do exist.

We introduce eight *generalized slides*, or slide-rotations, for a three-strand configuration; they are shown in Figure 69. We directly call these generalized slides E_1 to E_8 , because they will turn out to correspond to the $SU(3)$ group elements with the same name that were introduced above. In other words, we will show that the *generalized slides* E_n are elements of a Lie group $SU(3)$; in particular, they obey all the properties expected from the correspondence with the $SU(3)$ generators λ_n in Gell-Mann’s choice:

$$E_n = e^{\pi i \lambda_n / 2} . \tag{163}$$

In the strand model, the generators λ_n describe the difference between an infinitesimal generalized slide – thus a slide-rotation with a rotation by an infinitesimal angle – and the identity. For slides, concatenation is equivalent to group multiplication, as expected. Slides form a group. We will now show that the slide generators obey the multiplication table already given in Table 10.

To see how the $SU(3)$ multiplication table follows from Figure 69, we first note that the starting strand configuration of the Reidemeister III move contains, if all spatial configurations are considered, the same threefold symmetry as the centre of $SU(3)$. In particular, like the generators and the basis vectors of $SU(3)$, also the slides of the figure can be grouped into three triplets.

We now focus on the first triplet, the one formed by the three slides E_1 , E_2 and E_3 .

The generalized slides, or Reidemeister III moves, acting on three strands, form an SU(3) group.

The 8 generalized slides are shown below, with grey background. They are local slides and rotations by an angle π of an imaginary buckle formed by (usually) two strands. The strands lie (mostly) in a paper plane.

For each SU(2) triplet inside SU(3), the rotation axes of the finite group elements E_n are arranged at right angles to each other, as are those of the generators λ_n shown on the right. The rotation axes for E_3, E_9 and E_{10} are parallel; they are perpendicular to the paper plane. The three imaginary belt buckles for the three SU(2) subgroups are also shown.

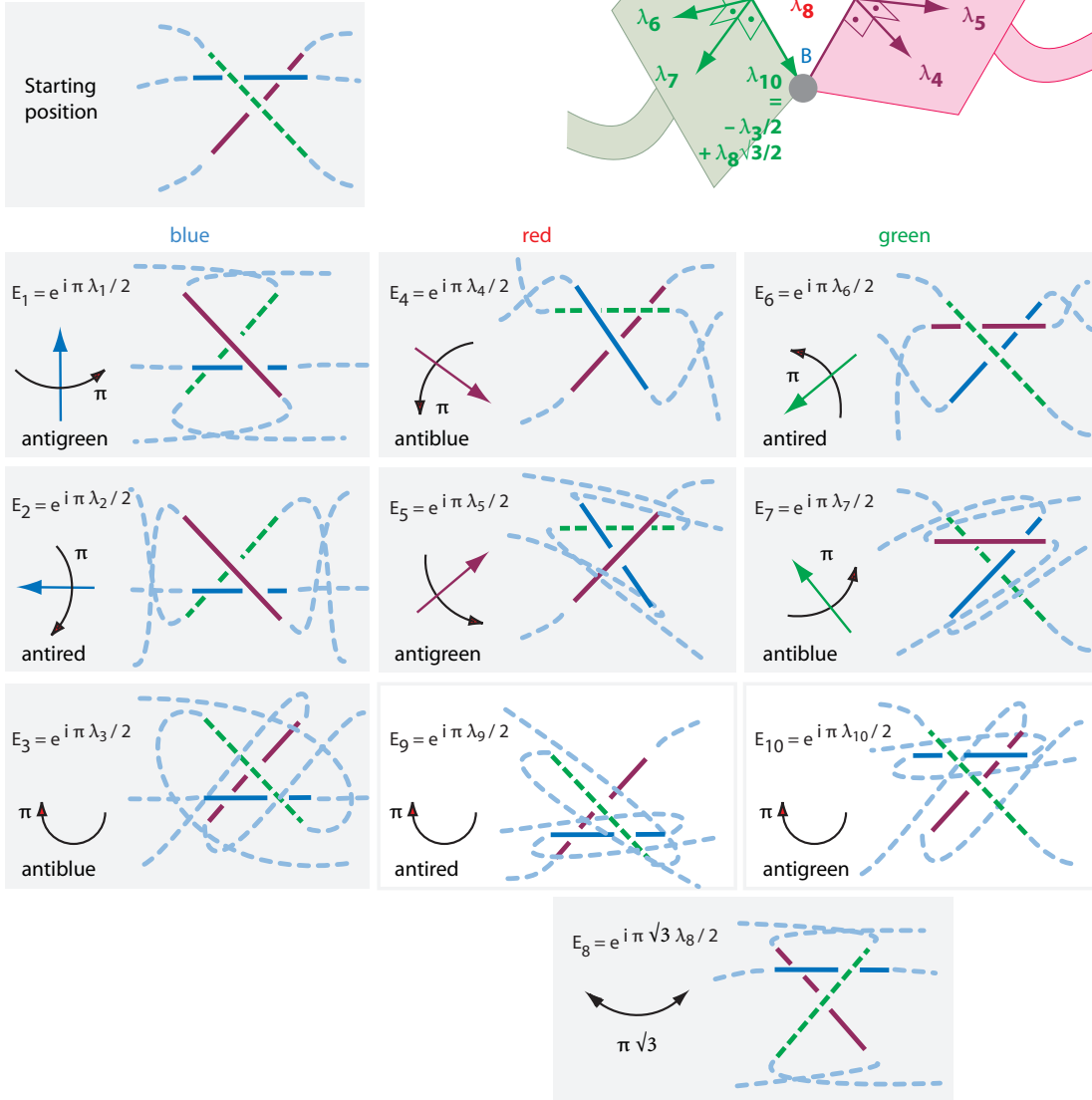


FIGURE 69 The strand deformations for the generalized slide moves E_n . The corresponding generators λ_n lead to an SU(3) structure, as shown in the text. Note that the rotation vectors for the generators λ_n and for the generalized slide moves E_n differ from each other. For clarity, the figure shows, instead of the deformation of the strand under discussion, the complementary deformations of the other two strands.

To make things clear, these moves can be pictured as combined deformations and slides of the red and green strands against the horizontal blue strand. We can imagine these moves like those of the belt trick, but acting on an *imaginary buckle* formed only by the red and green strands. These generalized slides do contain crossing changes; therefore they are observable and are of physical relevance.

We note that ‘slide’ is not a perfect term for the generalized deformations E_1 to E_8 ; in fact, we might prefer to call them *slide-rotations*, because they are slide-rotations by an angle π that are applied to an imaginary belt buckle. Despite the involved construction, these generalized, observable moves remain modelled on the textbook slide E_0 ; in particular, they require *three* strand segments. The generalized, observable moves just defined *differ* from the twists and pokes discussed above, in the sections on the electromagnetic and weak interactions; thus they differ from Reidemeister I and II moves. As a result, we will usually continue to call the generalized, observable moves simply *slides*.

For simplicity, we assume – similarly to what we did in the discussion about the weak interaction – that the three strand segments are (roughly) in a plane. This is an idealized situation; in fact, the arguments given in the following apply also to all other three-dimensional configurations of three strands. In particular, the same results appear if all three strand segments are assumed perpendicular to each other, instead of lying in a plane.

We note that the rotation axes of the generalized slides E_1 and E_2 are neither aligned nor orthogonal to the paper plane. More precisely, the rotation axes of E_1 , E_4 and E_6 are perpendicular to the sides of a cube. E_2 , E_5 and E_7 are perpendicular to them. For the first triplet, the rotation axes E_1 , E_2 and E_3 form an orthonormal basis; the same is valid for the other two triplets. We now show that the slides of the first triplet define an $SU(2)$ group.

The observable, generalized slides in the triplet E_1 , E_2 and E_3 can be concatenated. We distinguish two cases. The first case is the concatenation of any such slide with itself. The result corresponds to a rotation by 2π of the chosen strand pair and its imaginary belt buckle, and thus induces a corresponding amount of tail twisting. In fact, when any slide of the triplet is concatenated *four* times with itself, the result is the identity operation. Comparing a twofold and a fourfold concatenation, we see that they differ only by an entangling, or algebraically, by a minus sign for the imaginary buckle. This already realizes half of the belt trick that visualizes $SU(2)$.

The other case to be checked is the concatenation of two different slides of the triplet. The result is always the third slide of the triplet (up to a sign that depends on whether the combination is cyclical or not). This behaviour realizes the other half of the belt trick. In short, we have shown that the triplet containing the first three generalized slides defines an $SU(2)$ group. More precisely, the infinitesimal slide-rotations λ_1 , λ_2 and λ_3 corresponding to the finite $SU(3)$ elements E_1 , E_2 and E_3 generate the $SU(2)$ Lie algebra of an $SU(2)$ Lie group. The $SU(2)$ subgroup just found is just one of the three linearly independent $SU(2)$ subgroups of $SU(3)$. The generators of the first slide triplet thus reproduce the nine results in the upper left of Table 10. We thus retain that we can indeed visualize the first three generalized slides with the help of the three orthogonal rotations by π of an imaginary belt buckle formed by the red and green strands.

For the visualization of $SU(3)$ it is essential to recall that the direction in three-dimensional space of the vectors visualizing λ_n and those visualizing E_n differ from each

other. This already the case for $U(1)$.

The remaining generalized slides that are possible in the three-strand configuration are easily constructed using the threefold symmetry of the strand configuration; they are illustrated in [Figure 69](#). For each of the three strand segments there is a triplet of observable slides; this yields a total of *nine* possible generalized slides for the observer defined by the paper plane. In the second triplet, the slides corresponding to E_1 and E_2 are called E_4 and E_5 , and in the third triplet they are called E_6 and E_7 . For the three slides corresponding to E_3 – we call the other two E_9 and E_{10} – *only two* generators are linearly independent. Indeed, the figure shows that $E_3E_9E_{10}$ – whose axes are all three parallel – is the identity matrix; this expected from an $SU(3)$ structure. The three operations E_3 , E_9 and E_{10} also commute with all other operations; thus they form the centre of the group defined by all E . The second linearly independent, generalized slide of common use, E_8 , is also shown in the figure; it is a linear combination of E_9 and E_{10} . We note that the strand model also visualizes the factor $\sqrt{3}$ in the definition of E_8 . In total, we get *eight* linearly independent generalized slides. All slides, except for E_8 , act on an imaginary belt buckle that is formed by two strands.

We saw that the generators corresponding to the slides E_1 , E_2 and E_3 generate an $SU(2)$ subgroup. The same holds for the corresponding triplet E_4 , E_5 and the linear combination $E_9 = -E_3/2 - E_8\sqrt{3}/2$ (corresponding to E_3), and for the triplet E_6 , E_7 and $E_{10} = -E_3/2 + E_8\sqrt{3}/2$. For each of these slides, a fourfold concatenation yields the identity; and inside each triplet, the concatenation of two different slides yields a multiple of the third slide. In short, for each triplet, the corresponding infinitesimal slides generate an $SU(2)$ group. These three $SU(2)$ groups are linearly independent. We have thus reproduced an important part of the structure of $SU(3)$. In addition, we have found a visualization of $SU(3)$; since each $SU(2)$ group can be represented by a separate imaginary buckle, the group $SU(3)$ can be visualized – in many, but not all in aspects – with the help of three imaginary buckles. The top right of [Figure 69](#) illustrates this visualization.

The correspondence of the slides and the multiplication table increases further if we change slightly the definition of the first triplet. In this first triplet we can take as imaginary buckle the set of *all three* central segments. Moving all three strands together simplifies the visualization, because for the first triplet, the blue strand is trapped between the other two strands. In this way, generalized slide still consists of a rotation followed by a slide. And we still have a $SU(2)$ subgroup for the first triplet.

The slide E_8 differs from the other slides, as expected from $SU(3)$. It describes a motion that rotates the red and green strands in opposite directions; this is illustrated in [Figure 69](#). E_8 is thus *not* well described with an imaginary belt buckle. It is straightforward to check that the slide E_8 commutes with E_1 , E_2 , E_3 and obviously with itself, but not with the other generalized slides. Together, E_8 and the first triplet thus form a $U(2)$ Lie group, as expected. In addition, we find that E_8 commutes with E_9 and E_{10} , and that $E_8E_9 = E_{10}$, as expected from $SU(3)$. The strand model also implies that the product of E_8 with its two counterparts from the other triplets is the identity matrix, as expected from $SU(3)$.

The last step to show the equivalence of slides and $SU(3)$ requires us to confirm the multiplication properties – between slides E_n or between generators λ_n – from *different* triplets. In fact, because of the three-fold symmetry of the centre, we only need to check two multiplication results between slides from different triplets: one that either involves

λ_3 or λ_8 , and one that does not.

We begin with products involving λ_3 and one of the first two elements of another triplet. Such products yield a weighted sum of generators of the triplet. It is easier to check these product properties by using the exemplary relation between finite group elements $E_5E_3E_4 = E_3$. Note that only this specific permutation of 5, 3 and 4 yields this result. Playing with the strand model confirms the relation. Similar comments apply to $E_6E_3E_7 = E_3$ – and to the corresponding products involving E_9 , such as $E_1E_9E_2 = E_9$, or E_{10} , such as $E_1E_{10}E_2 = E_{10}$ – as well as $E_4E_8E_5 = E_8$ and $E_6E_8E_7 = E_8$. The strand model allows anybody to check that these relations are satisfied.

We continue with the exemplary product $\lambda_5\lambda_7$, respectively E_5E_7 . We note a basic difference between a product like $\lambda_5\lambda_7$ and any product of two generators from the same triplet. The product $\lambda_5\lambda_7$ – like the other concatenations of generators from different triplets – does not yield a single generator, but yields a combination, i.e., a *sum* of generators. The combination is not easy to visualize with strands; an easier way is to check the SU(3) algebra using the properties of the product E_5E_7 .

As mentioned above, in SU(3), for products involving the first two members from different triplets, *the threefold concatenation $(E_iE_j)^3$ is the identity*. And indeed, **Figure 69** confirms that $(E_2E_4)^3$ or $(E_5E_7)^3$ is the identity. Similarly, also the other products can be tested with the help of three strands.

Using the visualization with three strands, we have thus confirmed all products of generators from two different triplets that appear in **Table 10**. We note that **Figure 69** also illustrates that the three slides E_2 , E_5 and E_7 generate an SO(3) group, the rotation group in three dimensions. In order to see this, we observe that the infinitesimal versions of the three slides generate all possible rotations in three dimensions of the central triangle. An SO(3) group also appears for the slides 1, 4 and 7, for the slides 1, 5 and 6, and for the slides 2, 4 and 6. These are the four basic SO(3) subgroups of SU(3). The remaining combinations of three operations from three different triplets – such as 1, 4 and 6, or the combination 1, 5 and 7, or the combination 2, 4 and 7, or the combination 2, 5 and 6 – do *not* generate any subgroup. This can be confirmed by exploring the corresponding strand moves.

We can conclude: in a region with three strands crossing each other, the eight linearly independent, generalized slides that can be applied to that region define the group SU(3). In other words, *the group SU(3) follows from the third Reidemeister move*.

In the same way as for the other gauge groups, we find that particles whose strand models contain configurations with three strand segments can be subject to an SU(3) gauge interaction. In experiments, this interaction is called the *strong nuclear interaction*. The strong interaction is due to the Reidemeister III move. Like for the other interactions, a particle will only interact strongly if its tangle is not too symmetric, because in the symmetric case, averaged over time, there will be no net interaction. We will clarify the details below, when we discuss the specific tangles and colour charges of the different elementary matter particles.

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THE STRAND MODEL FOR GLUONS

Physically, the eight slides corresponding to the Gell-Mann matrices represent the effects of the eight *gluons*, the intermediate vector bosons of the strong interaction, that can act

on a particle.

- ▷ Given that the eight slides E_1 to E_8 represent the effects of the eight gluons, they also represent the gluons themselves.

Interactions are transfers of a tangle process to another tangle. Therefore

- ▷ The absorption of a gluon is a slide that is transferred to another particle.
- ▷ The emission of a gluon is a slide that is transferred to three vacuum strands.

To visualize the concept of gluon even further, we can say that every gluon can be described as a strand structure that continuously performs an $SU(3)$ operation, i.e., a generalized slide continuously repeating itself. We found a similar correspondence for the other gauge interactions. In case of the electromagnetic interaction, the intermediate vector boson, the photon, can be described as a strand that continuously performs a $U(1)$ operation, i.e., a rotation. In case of the weak interaction, a weak intermediate vector boson can be described as a strand that continuously performs an $SU(2)$ operation, i.e., an operation from the belt trick. This is most evident in the unbroken form of the weak bosons.

Challenge 167 e

Every gluon can also be seen as the deformation of a single strand that drags its surrounding with it. This single strand description of gluons implies that gluons have vanishing mass and vanishing charge. This single strand description of gluons also implies that they have spin 1, as is observed. The strand model of the gluon also implies that free gluons would have a huge energy.

The $SU(3)$ multiplication table confirms that the eight gluons transform according to the adjoint (and faithful) representation of $SU(3)$. Therefore, each row or column in a Gell-Mann matrix thus corresponds to one of the three *colours* of the strong interaction. The exploration of slide concatenation also showed that two general slides do not commute and do not anticommute. The group $SU(3)$ is *non-Abelian*. This implies that gluons interact among themselves. Both the multiplication table and the strand model for gluons imply that two interacting gluons can yield either one or two new gluons, but not more. This is illustrated in [Figure 70](#). The strand model, through its generation of $SU(3)$, thus implies that gluons interact among themselves, but only in triple and quartic gluon vertices.

Slides – i.e., gluon emission or absorption – never change the topology of tangles, and in particular, of matter tangles. Therefore, the strand model predicts that the strong interactions conserve electric charge, baryon number, weak isospin, flavour, spin and all parities. This is indeed observed. In particular, there is a natural lack of C, P and CP violation by slides. This is precisely what is observed for the strong interaction.

Because gluons do not change the topology of the particle tangles they act upon, but only change their shape, gluons are predicted to be massless in the strand model, despite interacting among themselves. And because gluons interact among themselves, free gluons are predicted not to appear in nature. And of course, all these conclusions agree with experiments.

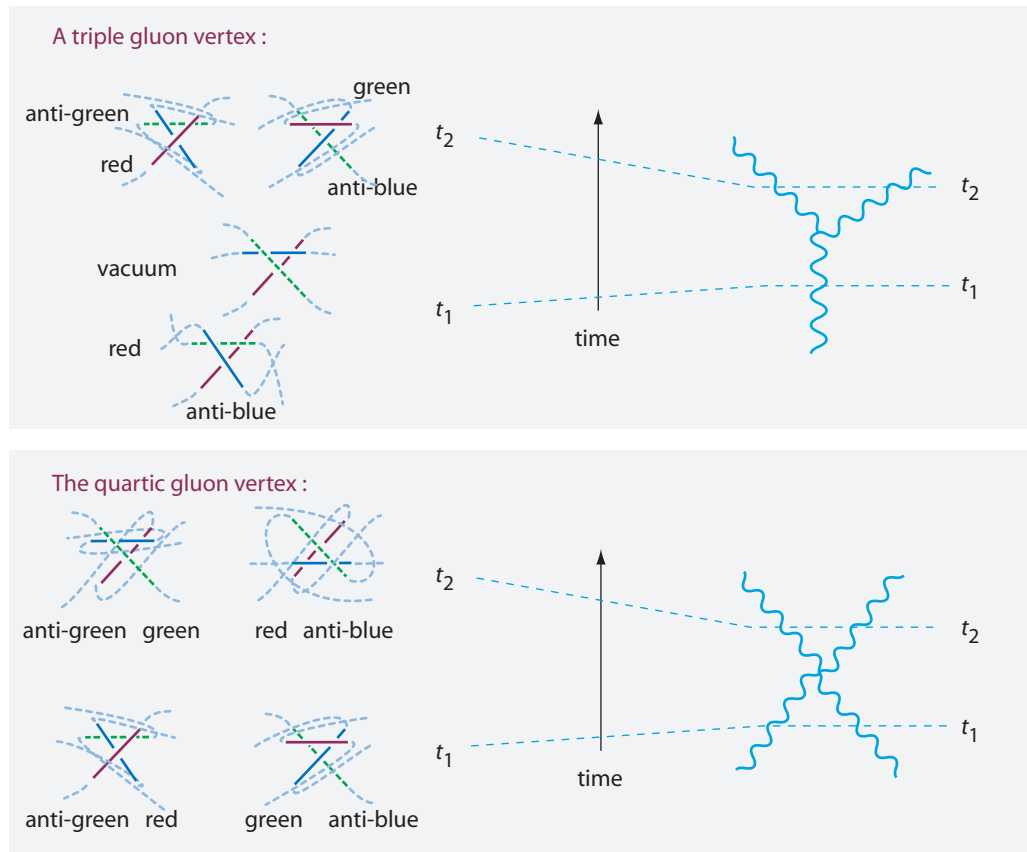


FIGURE 70 The two types of self-interaction of gluons in the strand model.

In summary, we have shown that in the strand model, the strong nuclear interaction and all its properties appear automatically from slides, i.e., from Reidemeister III moves. In particular, the strand model implies that the Lagrangian of strongly interacting fermions has a SU(3) gauge invariance that is due to generalized slide deformations.

THE GLUON LAGRANGIAN

Gluons are massless particles with spin 1. As a result, the field intensities and the Lagrangian are determined in the same way as for photons: energy density is the square of crossing density, i.e., the ‘square’ of field intensity. Since there are 8 gluons, the Lagrangian density becomes

$$\mathcal{L}_{\text{gluons}} = -\frac{1}{4} \sum_{a=1}^8 G_{\mu\nu}^a G_a^{\mu\nu} \tag{164}$$

where the gluon field intensities, with *two* greek indices, are given naturally as

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - gf^{abc} G_\mu^b G_\nu^c, \tag{165}$$

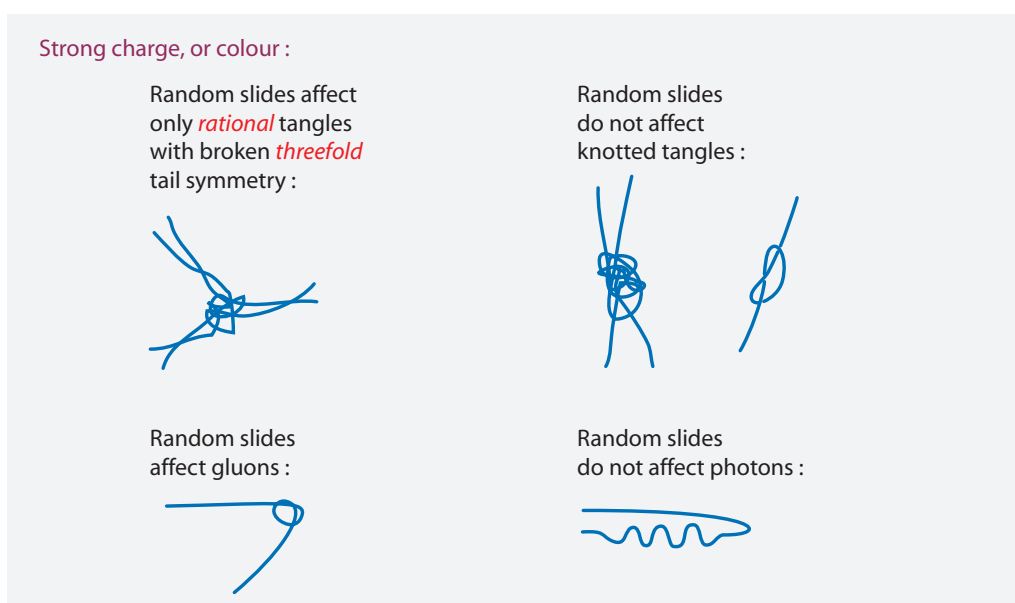


FIGURE 71 Tangles with and without colour charge. (This figure needs to be updated.)

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and f_{abc} are the structure constants of $SU(3)$ that can be deduced from the multiplication table given above. The quantities G_{μ}^a , with *one* greek index, are the gluon vector potentials. The last term in the definition of the field intensities corresponds to the triple and quartic vertices in the Feynman diagrams of gluon interactions. They are shown in Figure 70. The Lagrangian is simply the natural generalization from the $U(1)$ case of photons to the $SU(3)$ case of gluons. In short, we obtain the usual free gluon Lagrangian from the strand model.

COLOUR CHARGE

Surrounded by a bath of gluons that randomly induce slides of all kinds, not all fermion cores will change their rotation state. Generally speaking, particles have colour if a bath of random gluons changes their phase. Only tangles which lack some symmetry will therefore possess colour charge. Tangle that are symmetric will be neutral, or 'white'. Which symmetry is important here?

We see directly that the *photon* tangle is not sensitive to a gluon bath. The same is valid for W and Z bosons. These tangles are too simple. The strand model predicts that these particles are colour-neutral, i.e., that they are 'white', as is observed.

On the other hand, the multiplication properties given above shows that *gluons* interact among themselves and thus that they have colour charge. In fact, group theory shows that their properties are best described by saying that gluons have a colour and an anticoulour; this is the simplest way to describe the representation to which they belong. In short, the strand model of gluons automatically implies that they carry both a colour and an anti-colour.

Fermions behave differently. In the strand model, a fermion has colour charge if the corresponding triple belt model is affected by large numbers of random gluons. The first

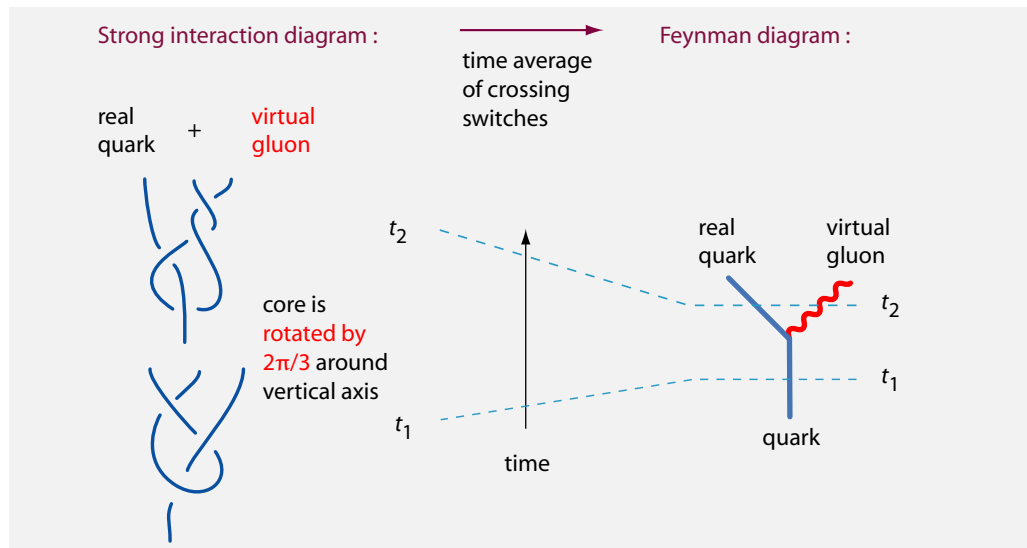


FIGURE 72 The Feynman diagram of the strong interaction for a quark. The upper triplet of tails correspond to the three belts.

tangles that come to mind are tangles made of three strands, such as the simple tangles shown in Figure 69. But a short investigation shows that such tangles are colour-neutral, or ‘white’. We will see below that this implies that *leptons* are colour-neutral, or ‘white’. In contrast, a rational fermion tangle made of *two* strands does not suffer this fate. In a bath of gluon strands that induce slides, i.e., third Reidemeister moves, a general *rational tangle* made of two strands is expected to be influenced, and thus to be colour-charged.

Rational tangles made of two strands are the simplest possible tangles with colour. A tangle is called *rational* if it can be untangled just by moving the tails around. An example of a rational tangle is shown in Figure 72. Such tangles break the three-fold symmetry of the three-belt structure, and are thus colour-charged. We will show below how these tangles are related to *quarks*. We can thus say:

- ▷ A fermion tangle has *colour charge* if its three-belt model is not symmetric for rotations by $\pm 2\pi/3$.

Coloured rational tangles automatically have *three* possible colours:

- ▷ The *three colour charges* are the three possibilities to map a tangle to the three belt model.* *Each colour is thus a particular orientation in ordinary space.*

Page 317 If we explore more complicated types of tangles of two strands, such as *prime* tangles or *locally knotted* tangles, we find that their colour depends on their structure. The strand model thus predicts that rational tangles made of two strands are the basic colour states. And indeed, in nature, quarks are the only fermions with colour charge.

* Can you define a geometric or even a topological knot invariant that reproduces colour charge?

We can summarize that colour charge is related to orientation in space. The three possible colours and anticolours are consequences of the possible orientations along the three dimensions of space.

PROPERTIES OF THE STRONG INTERACTION

In the strand model, all interactions are *deformations* of the tangle core. Specifically, the strong interaction is due to exchange of *slides*. Particles have strong charge, or colour, if their tangles lack the three-belt symmetry just specified. In the case of coloured fermions, *colour change* is a change of the mapping to the three-belt model, i.e., a change of orientation of the tangle in space.

If we use the strand definition of the strong interaction, visual inspection shows us that slide exchanges, and thus gluon exchanges, are deformations that conserve topology; therefore gluon exchange *conserves colour*. Since the strong interaction conserves the topology of all involved tangles and knots, the strong interaction also *conserves electric charge, parity*, and, as we shall see below, *all other quantum numbers* – except colour itself, of course. All these results correspond to observation.

THE LAGRANGIAN OF QCD

We started from the idea that tangle core deformations lead to phase redefinitions. We then found that slides imply that the strong interaction Lagrangian for matter and for radiation fields is SU(3) gauge invariant. If we include these two gauge invariances into the fermion Lagrangian density from the Dirac equation, we get

$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{\Psi}_q (i\hbar c \mathcal{D} - m_q c^2 \delta_{qq'}) \Psi_{q'} - \frac{1}{4} \sum_{a=1}^8 G_{\mu\nu}^a G_a^{\mu\nu}, \quad (166)$$

where the index q counts the coloured fermion, i.e., the quark. In this Lagrangian density, \mathcal{D} is now the SU(3) gauge covariant derivative

$$\mathcal{D} = \partial - g \gamma^\mu G_\mu^a \lambda_a, \quad (167)$$

where g is the gauge coupling, λ_a are the generators of SU(3), i.e., the Gell-Mann matrices given above, and the G_μ^a are, as before, the gluon vector potentials. The last term in the covariant derivative corresponds to the Feynman diagram and the strand diagram of [Figure 72](#). This is the Lagrangian density of QCD.

In summary: the strand model reproduces QCD. However, we have not yet deduced the number and masses m_q of the quarks, nor the strong gauge coupling g .

RENORMALIZATION OF THE STRONG INTERACTION

The slide move description of the strong interaction implies that only three Feynman diagrams are possible: one QCD Feynman diagram is possible for quarks, and only the triple and the quartic vertices are possible among gluons. This limited range of options allowed us to deduce the QCD Lagrangian. The limited range of options is also essential

for the *renormalization* of QCD. The strand model thus automatically ensures that the strong interaction is renormalizable.

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In short, the strand model provides a new underlying picture for the Feynman diagrams of the strong interaction, but does not change the physical results at any energy scale accessible in the laboratory. In particular, the measured running of the strong coupling constant is reproduced. Indeed, in the strand model, a flux-tube-like bond between the quarks appears automatically, as we will see when exploring hadrons. At high kinetic energies, the bond has little effect, so that quarks behave more like free particles. In short, we find that the strand model reproduces *asymptotic freedom* and also provides an argument for quark confinement. We will return to the issue in more detail below.

CURIOSITIES AND FUN CHALLENGES ABOUT SU(3)

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Ref. 183

Deducing the Lie group SU(3) from a three-dimensional model is a new result. In particular, deducing the gauge group SU(3) as a *deformation gauge group* is new. Frank Wilczek, Alfred Shapere, Alden Mead, Jerry Marsden and several others have confirmed that before this discovery, only the geometric Lie group SO(3) and its subgroups had been found in deformations. The fundamental principle of the strand model shows its power by overcoming this limitation. (Apparently, nobody had even realized that the belt trick already implies the possibility of an SU(2) gauge group for deformations.)

* *

Challenge 169 ny

We have discussed the *shape deformations* that lead to the SU(3) group. But what are the precise *phase choices* for a crossing that lead to SU(3) invariance?

* *

Challenge 170 ny

Do the two linear independent gluons with lined-up tails have the same properties as the other six gluons?

* *

Challenge 171 s

Three strands can cross each other also in another way, such that the three strands are interlocked. Why can we disregard the situation in this section?

* *

Deducing the Lie groups U(1), SU(2) and SU(3) directly from a basic principle contradicts another old dream. Many scholars hoped that the three gauge groups have something to do with the sequence complex numbers, quaternions and octonions. The strand model quashes this hope – or at least changes it in an almost unrecognizable way.

* *

Challenge 172 e

The tangles for the W and Z bosons have no colour charge. Can you confirm this?

* *

Challenge 173 ny

The Lie group SU(3) is also the symmetry group of the three-dimensional harmonic oscillator. What is the geometric relation to the Lie group SU(3) induced by slides?

* *

Challenge 174 e Confirm that the strand model does not contradict the Coleman–Mandula theorem on the possible conserved quantities in quantum field theory.

* *

Challenge 175 e Confirm that the strand model does not contradict the Weinberg–Witten theorem on the possible massless particles in quantum field theory.

* *

Challenge 176 d Are the *Wightman axioms* of quantum field theory fulfilled by the strand model with interactions? The *Haag–Kastler axioms*? Is Haag’s theorem circumvented?

* *

Ref. 184 Challenge 177 ny Show that the BCFW recursion relation for tree level gluon scattering follows from the strand model.

SUMMARY ON THE STRONG INTERACTION AND EXPERIMENTAL PREDICTIONS

We have deduced the Lagrangian density of QCD from the strand model with the help of slides. Is there a difference between the strand model and QCD? No, not as long as gravity plays no role. The strand model predicts that gravitation only comes into play near the Planck energy $\sqrt{\hbar c^5/4G}$. And indeed, accelerator experiments have not yet found any effect that contradicts QCD, and therefore no effect that contradicts the strand model of the strong interaction.

Page 350 The strand model also predicts that the strong interaction is naturally CP-invariant. This means that axions – particles invented to explain the invariance – are unnecessary: as shown below, the strand model even predicts that they do not to exist. Both predictions agree with experiment.

The strand model of the strong interaction implies that the SU(3) gauge symmetry is valid at all energies. No other gauge group plays a role in the strong interaction. The strand model thus predicts again that there is no grand unification in nature, and thus no larger gauge group. Often discussed groups such as SU(5), SO(10), E6, E7, E8 or SO(32) are predicted not to apply to nature. Also this prediction is not contradicted by experiment.

The strand model further predicts that the combination of gravity and quantum theory turns all Planck units into *limit* values. The strand model thus predicts a maximum strong field value given by the Planck force divided by the strong charge of the quark. All physical systems – including all astrophysical objects, such as neutron stars, quark stars, gamma-ray bursters or quasars – are predicted to conform to this field limit. So far, this prediction is validated by experiment.

Page 161 In summary, we have shown that Reidemeister III moves – or slides – in tangle cores lead to an SU(3) gauge invariance and a Lagrangian that reproduces the strong interaction. Colour charge is related to the topology of certain rational tangles. In this way, we have deduced the origin and most observed properties of the strong interaction. We have thus settled another issue of the millennium list. However, we still need to deduce the tangles and the number of quarks, their masses and the strength of the strong coupling.

SUMMARY AND PREDICTIONS ABOUT GAUGE INTERACTIONS

Page 150 At this point of our adventure, we have deduced gauge theory and the three known gauge interactions from strands. Using only the fundamental principle, we explained the dimensions of space-time, the Planck units, the principle of least action, the appearance of the gauge groups $U(1)$, broken $SU(2)$ and $SU(3)$, of renormalization, of Lorentz symmetry and of permutation symmetry. Thus we have deduced all the concepts and all the mathematical structures that are necessary to *formulate* the standard model of elementary particles.

In particular, the strand model provides a description and explanation of the three gauge interactions at Planck scales that is based on *deformations* of strands. The deduction of the three gauge interactions given in this text, with the help of the Reidemeister moves, is the first and, at present, the *only* explanation of the three gauge forces. No other explanation or deduction has ever been given.

Page 147 We have shown that quantum field theory is an *approximation* of the strand model. The approximation appears when the strand diameter is neglected; quantum field theory is thus valid for all energies below the Planck scale. In other words, in contrast to many other attempts at unification, the strand model is *not a generalization* of quantum field theory. The strand model for the three gauge interactions is also unmodifiable. These properties are in agreement with our list of requirements for a final theory.

Page 161 We have not yet deduced the complete standard model: we still need to show which types of particles exist, which properties they have and what couplings they produce. However, we have found that the strand model explains all the mathematical structures from the millennium list that occur in quantum field theory and in the standard model of particle physics. In fact, the strand explanation for the origin of the gauge interactions allows us to make several definite predictions.

PREDICTING THE NUMBER OF INTERACTIONS IN NATURE

Ref. 181 Already in 1926, Kurt Reidemeister proved an important theorem about possible deformations of knots or tangles that lead to changes of crossings. When tangles are described with two-dimensional diagrams, all possible deformations can be reduced to *exactly three* moves, nowadays called after him. In the strand model, the two-dimensional tangle diagram describes what an observer *sees* about a physical system. Together with the equivalence of interactions as crossing-changing deformations, Reidemeister's theorem thus proves that there are *only three gauge interactions* in nature. In particular, there is no fifth force. Searches for additional gauge interactions are predicted to fail. And indeed, they have all failed up to now.

UNIFICATION OF INTERACTIONS

Ref. 142 We can also state that there is only *one* Reidemeister move. This becomes especially clear if we explore the three-dimensional shape of knots instead of their two-dimensional diagrams: all three Reidemeister moves can be deduced from the *same* deformation of a single strand. Only the projection on a two-dimensional diagram creates the distinction between the three moves. In the terms of the strand model, this means that all gauge interactions are in fact aspects of only one basic process, a fluctuation of strand shape,

and that the three gauge interactions are only distinguished by their projections. In this way, the three gauge interactions are thus *unified* by the strand model.

The plane of projection used in a strand diagram defines a mapping from strand fluctuations to Reidemeister moves. The projection plane is defined by the observer, i.e., by the frame of reference. Depending on the projection plane, a general deformation is mapped into different Reidemeister moves. At first sight, the nature of an interaction – whether electromagnetic, strong or weak – seems to depend on the observer. In nature, however, this is not the case. But this contradiction is only apparent. In the strand model, the nature of interaction of a particle results from the type of asymmetry of its tangle core. Certain strand deformations do not lead to interactions, because their effects are suppressed by the averaging of short-time fluctuations underlying every observation. In other words, the averaging process at the basis of observations also ensures that interactions are effectively observer-independent at low energy.

Page 378 In short, the strand model provides a natural *unification* of the interactions. And this unification of the interactions differs completely from any past proposal. The final test, of course, can only be provided by experiment.

NO DIVERGENCES

The strand model implies that there are *no* divergences in the quantum description of nature. This lack of divergence occurs because physical values result after strand effects have been averaged out. As mentioned above, strand effects on space-time disappear through ‘shivering’ and strand effects on particles disappear through wavefunctions.

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In summary, in the strand model, no interaction implies or contains divergences: neither gravity nor the gauge interactions. There are neither ultraviolet nor infrared divergences. The strand model avoids divergences, infinities and singularities of any kind from its very start.

GRAND UNIFICATION, SUPERSYMMETRY AND OTHER DIMENSIONS

The three gauge interactions are due to the three Reidemeister moves. Therefore, the strand model asserts that there is *no* single gauge group for all interactions. In short, the strand model asserts that there is *no* so-called *grand unification*. The absence of grand unification implies the absence of large proton decay rates, the absence of additional, still undiscovered gauge bosons, the absence of neutron–antineutron oscillations, and the absence of sizeable electric dipole moments in elementary particles. All these searches are ongoing at present; the strand model predicts that they yield *null results*.

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Supersymmetry and approaches based on it assume gauge group unification. However, as just explained, the strand model predicts that there is no supersymmetry and therefore no supergravity. The strand model also predicts the absence of all conjectured ‘superparticles’. In 2016 and again in 2017, the numerous experiments at CERN confirmed the prediction: there is no sign of supersymmetry in nature.

Reidemeister moves are confined to three spatial dimensions. Indeed, the strand model is based on exactly three spatial dimensions. It predicts that there are no other, undetected dimensions of space. The strand model also predicts the absence of non-commutative space-time, even though, with some imagination, strands can be seen as remotely related to that approach. Finally, the strand model predicts the lack of different

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vacua: the vacuum is unique.

In short, the strand model differs both experimentally and theoretically from the unification proposals made in the twentieth century. In particular, the strand model predicts the *absence* of additional symmetries and of additional space-time properties at high energy. The strand model predicts that unification is not achieved by searching for higher symmetries, nor for higher dimensions, nor for concepts that contain both. This lack of complex mathematical or symmetry concepts in nature is disappointing; the hopes and search activities in the last fifty years are predicted to have been misguided. In other words, the predictions of the strand model are unpopular. However, these predictions agree with our list of requirements for a final theory; and so far, all these predictions agree with experiment.

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NO NEW OBSERVABLE GRAVITY EFFECTS IN PARTICLE PHYSICS

Page 8 In the ‘cube’ structure of physics shown in [Figure 1](#), the transition from the final, unified description to quantum field theory occurs by neglecting gravity, i.e., by assuming flat space-time. The same transition occurs in the strand model, where neglecting gravity in addition requires neglecting the strand diameter. In this way, the gravitational constant G disappears completely from the description of nature.

We can summarize our findings on quantum field theory also in the following way:

- ▷ The strand model predicts that particle masses are the only observable effect of gravity in quantum physics and in particle physics.

Page 309 This result will be complemented below by a second, equally restrictive result that limits the observable quantum effects in the study of gravity. In short, the strand model keeps particle physics and general relativity almost completely separated from each other. This is a consequence of the different effects produced by tail deformations and by core deformations. And again, the prediction of a lack of additional gravitational effects in particle physics agrees with all experiments so far.

THE STATUS OF OUR QUEST

In this chapter, we have deduced that strands predict exactly three interactions. Interactions are deformations of tangle cores and just three classes of such core deformations exist. The three classes of deformations are given by the three Reidemeister moves. Because of the properties of the Reidemeister moves, the three interactions are described by a $U(1)$, a broken $SU(2)$ and a $SU(3)$ gauge symmetry, respectively.

Strands also show that the three interactions are renormalizable, relativistically invariant, and that they follow the least action principle. Strands thus imply the three interaction Lagrangians of the standard model of particle physics. In addition, strands predict the absence of other interactions, symmetries and space-time structures.

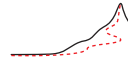
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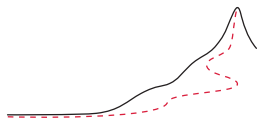
If we look at the millennium list of open issues in fundamental physics, we have now solved all issues concerning the mathematical structures that appear in quantum field theory and in the standard model of particle physics.

- ▷ All mathematical structures found in quantum physics result from the fun-

damental principle of the strand model.

This is an intriguing result that induces us to continue our exploration. Only two groups of issues are still unexplained: the theory of general relativity and the spectrum of elementary particles. We proceed in this order.





GENERAL RELATIVITY DEDUCED FROM STRANDS

General relativity describes the deformations of the vacuum. In everyday life, gravitation is the only such effect that we observe. But on astronomical scale, gravity shows more phenomena: vacuum can deflect light, producing gravitational lenses, can wobble, giving gravitational waves, and can accelerate, yielding the darkness of the sky and the fascinating black holes. All these observations require general relativity for their description. Therefore, general relativity must be part of any unified description of nature.

In the following, we explain the existence of gravity as a consequence of strands. Then we deduce the field equations of general relativity, the entropy of black holes and relativistic cosmology from the strand model. We also predict the outcome of many quantum gravity experiments. Finally, we deduce the consequences of strands for cosmology, including a new experimental prediction. Of all Planck-scale models of space or space-time, strands seem to be the simplest one that provides these deductions.

FLAT SPACE, SPECIAL RELATIVITY AND ITS LIMITATIONS

Page 205 We have seen above that any observer automatically introduces a 3+1-dimensional *background* space-time. We have also seen that in the case of quantum theory, *physical* space-time, the space-time that is formed by the fluctuations of the vacuum strands, is naturally 3+1-dimensional and flat. In the absence of gravity, physical space and background space coincide.

Page 208 Using strands, we have deduced the invariant limit c for all energy speeds and shown that it is realized only for free massless particles, such as photons. Strands also showed us that massive particles move more slowly than light. In short, strands reproduce special relativity.

The strand model thus predicts that *pure* special relativity is correct for all situations and all energies in which gravity and quantum theory play no role. The strand model also predicts that when gravity or quantum effects do play a role, general relativity or quantum theory *must* be taken into account. This means that there is no domain of nature in which intermediate descriptions are valid.

Ref. 85 It is sometimes suggested that the invariant Planck energy limit for elementary particles might lead to a ‘doubly special relativity’ that deviates from special relativity at high particle energy. However, this suggestion is based on two assumptions: that at Planck energy *point masses* are a viable approximation to particles, and that at Planck energy *vacuum and matter differ*. In nature, and in the strand model, both assumptions

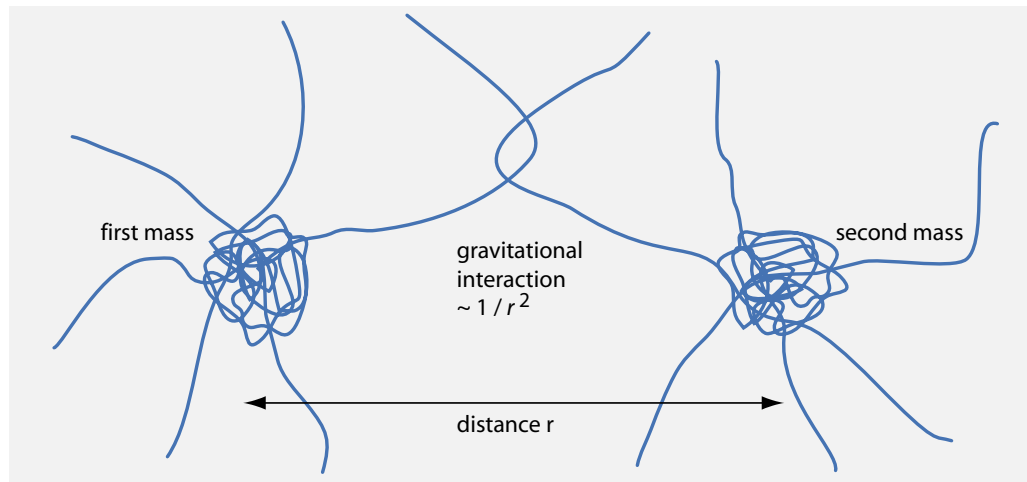


FIGURE 73 Gravitational attraction as result of strands.

are incorrect. Nature, as general relativity shows, does not allow the existence of point masses: the densest objects in nature are black holes, and these are not point-like for any mass value. In addition, quantum theory implies the fuzziness of matter and space. As a result, near Planck energy, matter and vacuum *cannot* be distinguished. Put simply, no system near Planck energy can be described without general relativity or without quantum gravity. In short, the strand model predicts that the approach of ‘doubly special relativity’ cannot be correct. Also Figure 1 makes this point: there is no description of nature besides the usual ones.

Page 64

Ref. 185

Page 8

To sum up, special relativity (incorrectly) assumes that a (small, light and localizable) mass can exist in flat space. In nature this is not the case – especially near Planck energy – and the strand model reproduces this impossibility.

CLASSICAL GRAVITATION

In nature, at low speeds and in the flat space limit, gravitation is observed to lead to an acceleration a of test masses that changes as the inverse square distance from the gravitating mass;

$$a = G \frac{M}{R^2}. \quad (168)$$

This acceleration is called *universal gravitation* or *classical gravitation*. It is an excellent approximation for the solar system and for many star systems throughout the universe.

In the strand model, every space-time effect, including gravitation, is due to the behaviour of tangle tails. In the strand model, every mass, i.e., every system of tangles, is connected to the border of space by tails. The nearer a mass is to a second mass, the more frequently the tails of the two masses cross and get tangled. Figure 73 illustrates the situation. The strand model states:

- ▷ **Gravitation** is due to the fluctuations of tail crossings.

Challenge 178 e

The tail crossings fluctuate; averaged of time, the fluctuations lead to a crossing switch density. The resulting energy density – where energy is the number of crossing switches per time – changes like the inverse distance from the central mass. This is the reason for the $1/r$ -dependence of the gravitational potential and the $1/r^2$ -dependence of gravitational acceleration. (This applies to all those cases where curvature is negligible.) In simple words, in the strand model, the inverse square dependence of gravitational acceleration is due to the three-dimensionality of space combined with the one-dimensionality of strands.

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The strand model also shows that masses and energies are always positive: every tangle has tails. The model also shows qualitatively that larger masses produce stronger attraction, as they generally have more tails. We will show below that the effective number density of tails is indeed proportional to the mass.

In the strand model, crossing switches are not only related to energy; they are also related to entropy. A slightly different – but equivalent – view on gravitation therefore appears when we put the stress on the entropic aspect.

DEDUCING UNIVERSAL GRAVITATION FROM BLACK HOLE PROPERTIES

Black holes have entropy; this implies universal gravitation. There are at least two ways to explain this connection.

Ref. 186

An especially concise explanation was recently given by Erik Verlinde. In this view, *gravity appears because any mass M generates an effective vacuum temperature around it.* A gravitating mass M attracts test masses because during the *fall* of a test mass, the total entropy *decreases*. It is not hard to describe these ideas quantitatively.

Given a spherical surface A enclosing a gravitating mass M at its centre, the acceleration a of a test mass located somewhere on the surface is given by the local vacuum temperature T :

$$a = T \frac{2\pi kc}{\hbar}, \quad (169)$$

where k is the Boltzmann constant. This relation is called the *Fulling–Davies–Unruh effect* and relates vacuum temperature and local acceleration. Thus, an inertial or a freely falling mass (or observer) measures a vanishing vacuum temperature.

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In the strand model, the vacuum temperature at the surface of the enclosing sphere is given by the crossing switches induced by the tails starting at the mass. We can determine the vacuum temperature by dividing the energy E contained inside the sphere by *twice* the *maximum* possible entropy S for that sphere. This maximum value is the entropy that the sphere would have if it were a black hole horizon; it can be calculated by the strand model, as we will see shortly.

The temperature T is thus given by the expression

$$T = \frac{E}{2S} = \frac{M}{A} \frac{2G\hbar}{kc}. \quad (170)$$

The factor 2 needs explanation: it might be due to the combination of the effects of space and matter.

Neglecting spatial curvature, we can set $A = 4\pi R^2$; this gives a temperature at the

enclosing sphere given by

$$T = \frac{M}{R^2} \frac{G\hbar}{2\pi kc} . \quad (171)$$

Page 279 Inserting this expression into the expression (169) for the Fulling–Davies–Unruh acceleration a , we get

$$a = G \frac{M}{R^2} . \quad (172)$$

This is universal gravitation, as discovered by Robert Hooke and popularized by Isaac Newton. Since spatial curvature was neglected, and the central mass was assumed at rest, this expression is only valid for large distances and small speeds. We have thus deduced universal gravity from the effects of gravitating masses on vacuum temperature. Below, we show that in the relativistic case this sequence of arguments – which was given fifteen years earlier by Jacobson – leads to the field equations of general relativity.

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An alternative deduction of universal gravitation from black hole entropy is the following. The gravitational force F on a test mass m is given by the vacuum temperature T created by the central mass M and by the change of entropy S per length that is induced by the motion of the test mass:

$$F = T \frac{dS}{dx} . \quad (173)$$

The change of entropy dS/dx when a test mass m moves by a distance x can be determined from the strand model in a simple manner. When the test mass m moves by a (reduced) Compton wavelength, in the strand model, the mass has rotated by a full turn: the entropy change is thus $2\pi k$ per (reduced) Compton wavelength. Thus we have

$$\frac{dS}{dx} = m \frac{2\pi kc}{\hbar} . \quad (174)$$

Using the temperature T found in expression (171), we get an expression for the gravitational force given by

$$F = G \frac{Mm}{R^2} . \quad (175)$$

This is universal gravitation again. This time we have thus deduced universal gravitation from the entropy and temperature generated by gravitating masses.

Page 35 We note that the temperature and entropy of black holes are limit values. We can thus state that universal gravitation is a consequence of nature's limit values.

SUMMARY ON UNIVERSAL GRAVITATION FROM STRANDS

Page 285 Universal gravitation is due to the temperature and entropy of black holes and the vacuum. In the strand model, these temperature and entropy values are a consequence of the underlying strand crossing switches; we will show this shortly. Universal gravitation thus (again) appears as an effect of the crossing switches induced by masses.

We thus have several explanations of universal gravitation using strands. We have de-

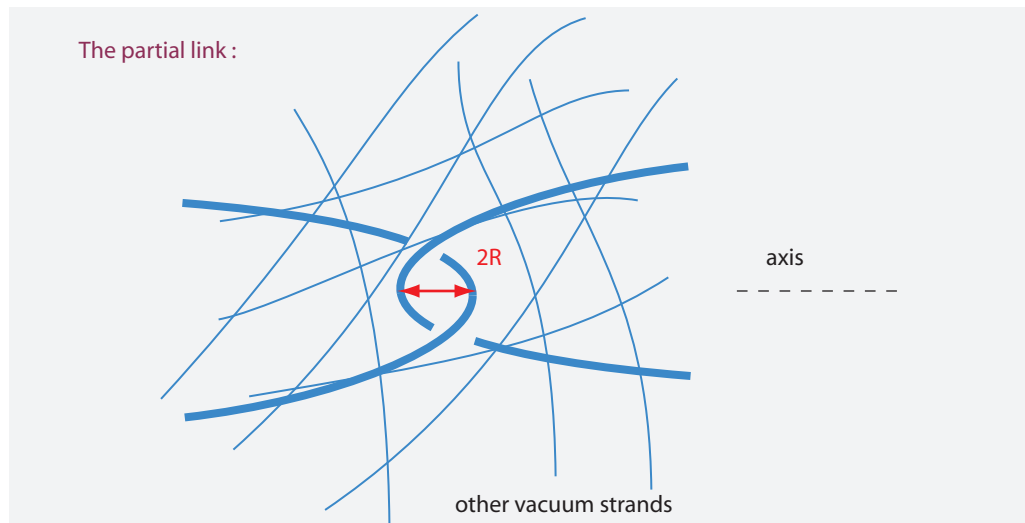


FIGURE 74 A schematic model of the fundamental defect, and thus the fundamental type of curvature: the *partial link*.

duced universal gravitation from the energy of strands, from the temperature of strands and from the entropy of strands. And we have deduced universal gravitation from the maximum force, which strands fulfil as well. In short, strands explain the origin of universal gravitation.

Ref. 187

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Incidentally, modelling mass as a source for strand crossing switches is remotely reminiscent of Georges-Louis Lesage's eighteenth-century model of gravitation. Lesage proposed that gravity appears because many tiny, usually unnoticed corpuscles push masses together. In fact, as we will see shortly, there is a certain similarity between these assumed tiny corpuscles and virtual gravitons. And interestingly, all criticisms of Lesage's model then cease to hold. First, there is no deceleration of free masses in inertial motion, thanks to the built-in special-relativistic invariance. Secondly, there is no heating of masses, because the entangled tails represent virtual gravitons that scatter elastically. Thirdly, and most of all, by replacing the *corpuscules ultra-mondains* of Lesage by virtual gravitons – and finally by strands – we can predict an additional effect of gravity that is not described by the inverse square dependence: space-time curvature.

CURVED SPACE

In nature, observation shows that physical space is not flat around masses, i.e., in the presence of gravity. Near mass and energy, physical space is *curved*. Observations also show that curved space-time remains 3+1-dimensional. The observation of this type of curvature was predicted long before it was measured, because curvature follows unambiguously when the observer-invariance of the speed of light c and the observer-invariance of the gravitational constant G are combined.

We continue directly with the strand model of spatial curvature and show that all observations are reproduced.

- ▷ **Curvature** (of physical space-time) is due to simple, unknotted and weakly localized defects in the tangle of strands that make up the vacuum. An example is shown in [Figure 74](#).
- ▷ In the case of curvature, *physical* space-time, which is due to averaged strand crossing switches, *differs* from flat *background* space-time, which usually corresponds to the tangent or to the asymptotic space-time. In [Figure 74](#), the grey background colour can be taken as visualization of the background space.
- ▷ **Mass** is a localized defect in space and is due to knotted or tangled strands. Thus mass curves space around it.
- ▷ **Energy** in a volume is the number of crossing switches per unit time. As a result, mass is equivalent to energy. As a second result, energy also curves space.
- ▷ **Gravitation** is the space-time curvature originating from compact regions with mass or energy.

These natural definitions show that curvature is due to strand configurations. In particular, curvature is built of unknotted – i.e., massless – *defects*. The massless defects leading to curvature are usually dynamic: they evolve and change. Such curvature defects – virtual gravitons – originate at regions containing matter or energy. In fact, the curvature of space around masses is a natural result of fluctuations of the strands that make up matter tangles.

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We note that curved space, being a time average, is *continuous* and *unique*. Vacuum or curved space, more precisely, curved physical space, thus differs from background space, which is flat (and drawn in grey in the figures).

Incidentally, the distinction between physical and background space also avoids Einstein's hole argument; in fact, the distinction allows discussing it clearly, as only physical space describes nature.

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THE STRUCTURE OF HORIZONS AND BLACK HOLES

In general relativity, another concept plays a fundamental role. In the strand model we have:

- ▷ A **horizon** is a tight, one-sided weave of strands.

Therefore, there are no strands behind the horizon. This implies that behind a horizon, there is no matter, no light, no space and no time – just *nothing*. Indeed, this is the experience of any observer about a horizon. A horizon is thus a structure that limits physical space. It does *not* limit background space.

One particular type of horizon is well-known:

- ▷ A **black hole** is a tight, one-sided and *closed* weave of strands.

In principle, closed horizons can have any shape. The simplest case is the spherical, non-

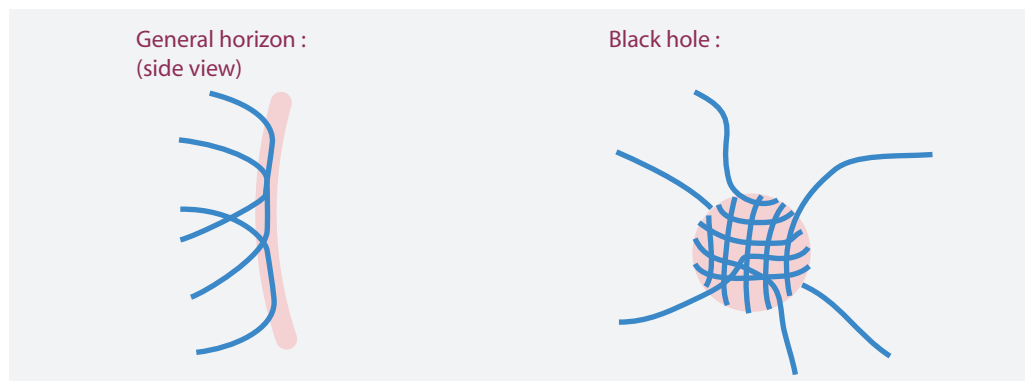


FIGURE 75 A schematic model of a general and a spherical horizon as tight weaves, as pictured by a distant observer. In the strand model there is *nothing*, no strands and thus no space, behind a horizon.

rotating horizon, which defines the *Schwarzschild black hole*. It is illustrated on the right-hand side of [Figure 75](#).

If an observer is located outside a spherical horizon, the strand model states that there is nothing *inside* the horizon: no matter, no light and no vacuum. The strand model thus provides a simple and drastic view of black hole horizons. [Figure 75](#) also illustrates that the concept of radius (or size) of a black hole has to be approached with the (well-known) care. In general, the size of a structure made of strands is the number of crossings encountered when travelling through it. However, an observer cannot travel *through* a black hole: there are no strands inside, thus there is no vacuum there! The size of a black hole must therefore be defined indirectly. The simplest way is to take the square root of the area, divided by 4π , as the radius. Thus the strand model, like general relativity, requires that the size of a compact horizon be defined by travelling *around* it.

We note that the strand model also provides an intuitive explanation for the differences between a rotating and a non-rotating black hole.

IS THERE SOMETHING BEHIND A HORIZON?

A drawing of a horizon weave, such as the one of [Figure 75](#), clearly points out the difference between the background space and the physical space. The *background space* is the space we need for thinking, and is the space in which the drawing is set. The *physical space* is the one that appears as a consequence of the averaging of the strand crossings. Physical, curved space exists only on the observer side – usually outside – of the horizon. The physical space around a black hole is curved; it agrees with the background space only at infinite distance from the horizon. Inside the horizon, there is background space, but no physical space. In short, the strand model implies that – for an observer at spatial infinity – there is *nothing*, not even a singularity, inside a black hole horizon.

- ▷ There is no physical space, no matter and no singularity inside a horizon.

Horizons are observer-dependent. Both the existence and the shape of a horizon depends on the observer. As we will see, this happens in precisely the same way as in usual

general relativity. In the strand model, there is no contradiction between the one observer at spatial infinity who says that there is *nothing* behind a horizon, not even physical space, and another, falling observer, who does not observe a horizon and thus states that there is *something* there. In the strand model, the two statements naturally transform into each other under change of viewpoint. Indeed, the transformation between the two viewpoints contains a deformation of the involved strands.

We note that the equivalence of viewpoints and the statement that there is nothing behind a horizon is based on the combination of general relativity and quantum theory. If we would continue thinking that space and time is a manifold of points – thus disregarding quantum theory – these statements would *not* follow.

In summary, one-sided tight weaves are a *natural* definition of horizons.

ENERGY OF BLACK HOLE HORIZONS

The strand model allows us to calculate the energy content of a closed horizon. Energy is action per unit time. In the strand model, the energy of a non-rotating spherical horizon is thus given by the number N_{cs} of crossing switches per time unit. In a tight weave, crossing switches cannot happen in parallel, but have to happen sequentially. As a result, a crossing switch ‘propagates’ to the neighbouring Planck area on the surface. Since the horizon weave is tight and the propagation speed is one crossing per crossing switch time, this happens at the speed of light. In the time T that light takes to circumnavigate the spherical horizon, all crossings switch. We thus have:

Challenge 179 e

$$E = \frac{N_{cs}}{T} = \frac{4\pi R^2}{2\pi R} \frac{c^4}{4G} = R \frac{c^4}{2G}. \quad (176)$$

Strands thus imply the well-known relation between energy (or mass) and radius of Schwarzschild black holes.

Challenge 180 e

How do the crossing switches occur at a horizon of a black hole? This interesting puzzle is left to the reader.

The tight-weave model of horizons also illustrates and confirms both the *hoop conjecture* and *Penrose conjecture*. For a given mass, because of the minimum size of crossings, a spherical horizon has the smallest possible diameter, compared to other possible shapes. This implies that, for a given mass, spherical black holes indeed are the densest objects in nature.

THE NATURE OF BLACK HOLES

The strand model naturally implies the *no-hair theorem*. Since all strands are the same, independently of the type of matter that formed or fell into the horizon, a black hole has no characteristics other than mass, angular momentum and charge. Here we used a result from the next chapter, when it will become clear that all elementary particles are indeed made of the same featureless strands. Taking that result as given, we deduce that flavour quantum numbers and particle number do not make sense for black holes. We also deduce that weak and strong charge are not defined for black holes. Strands explain naturally why neutral black holes made of antimatter and neutral black holes made of matter do not differ, if their masses and angular momenta are the same. In short, the

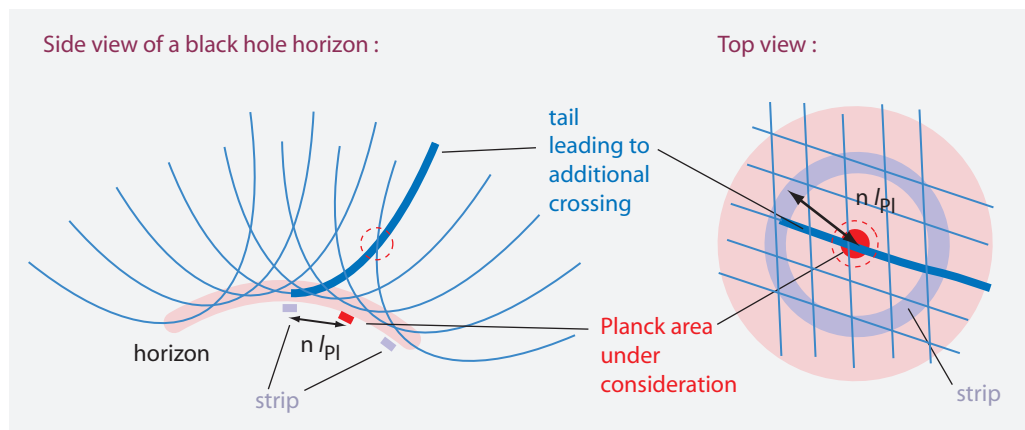


FIGURE 76 The entropy of black holes results from the number of possible crossing switches.

strand model of nature implies the no-hair theorem: *strands, not hairs*.

Horizons and black holes are borderline systems between space and matter. This borderline property must be fulfilled by every final theory. The strand model fulfils this requirement: in the strand model, black holes can either be described as curved space or as particles in free fall.

ENTROPY OF VACUUM AND MATTER

Both vacuum and matter are made of fluctuating strands. We note directly:

- ▷ The flat and infinite vacuum has *vanishing* entropy, because the number of crossing switches is zero on average.

At the same time,

- ▷ Curved space and horizons have *non-vanishing* entropy.

The entropy of vacuum and of horizons differs from that of matter. In the *absence of gravity*, the number of microstates of matter is determined – as in usual thermodynamics (thermostatics) – by the behaviour of *tangle cores*.

In *strong gravity*, when the distinction between matter and vacuum is not so clear-cut, the number of microstates is determined by the possible crossing switches of the strands. In strong gravity, only *tails* play a role. This becomes clear when we calculate the entropy of black holes.

ENTROPY OF BLACK HOLES DEDUCED FROM THE STRAND MODEL

Despite the tight weaving, the strands making up a horizon are fluctuating and moving: the weave shape fluctuates and crossing switch all the time. This fluctuating motion is the reason why horizons – in particular those of black holes – have entropy.

The weave model of a horizon, illustrated in detail in [Figure 76](#), allows us to calculate

the corresponding entropy. Since the horizon is a tight weave, there is a crossing on each Planck area. To a first approximation, on each (corrected) Planck area of the horizon, the strands can cross in *two* different ways. The fundamental principle of the strand model thus yields two microstates per Planck area. The number N of Planck areas is given by $N^2 = Ac^3/4G\hbar$. The resulting number of black hole microstates is 2^{N^2} . The entropy is given by the natural logarithm of the number of the possible microstates times k . This gives an entropy of a horizon of

$$S = A \frac{kc^3}{4G\hbar} \ln 2. \quad (177)$$

Ref. 188

This is the well-known first approximation of black hole entropy: one bit per corrected Planck area. In the strand model, the proportionality of entropy and area is thus a direct consequence of the *extension* of the strands. This proportionality is also well known from studies of quantum gravity and of strings. In those approaches however, the relation between the area proportionality and extension is less obvious.

For Schwarzschild black holes, the entropy value of expression (177) is *not* correct. In the strand model, this incorrect value is explained as a consequence of neglecting the effects of the strand *tails*. Indeed, additional contributions to the entropy appear at a *finite distance* from the horizon, due to the crossing of the tails on their way to the border of space, as shown in Figure 76. The actual entropy will thus be larger than the first approximation, but still be proportional to the area A .

The correct proportionality factor between the area and the entropy of a black hole results when the strand tails are taken into account. (The correction factor is called the *Barbero–Immirzi parameter* in the research literature on quantum gravity.) The calculation is simplest for Schwarzschild black holes. By construction, a black hole with macroscopic radius R , being a tight weave, has R/l_{pl} tails. For each given Planck area, there are, apart from the basic, or lowest crossing, additional crossings ‘above it’, along the radial direction, as shown in Figure 76. These additional crossings are due to the tails from neighbouring and distant Planck areas.

Taking into effect all strand tails allows us to calculate the average number of crossings *above* a given Planck area. The main point is to perform this calculation for all those tails that start in a circular strip of Planck width centred around the Planck area under consideration. We then add the probabilities for all possible circular strips. One such circular strip is drawn in Figure 76.

The definition of horizons as tight weaves implies that a horizon with N^2 Planck areas is made of N strands. This means that for each circular strip of radius nl_{pl} , there is only *one* strand that starts there and reaches spatial infinity as a tail.

For this tail, the average probability p that it crosses above the central Planck area under consideration is

$$p = \frac{1}{n!}. \quad (178)$$

Summing over all strips, i.e., over all values n , we get a total of $\sum_{n=0}^{\infty} 1/n! = e = 2.71828\dots$ microstates *on* and *above* the central Planck area under consideration. Thus the number e replaces the number 2 of the first approximation: the number of horizon microstates

Challenge 181 e

of a Schwarzschild black hole is not 2^{N^2} , but e^{N^2} . As a consequence, the entropy of a macroscopic Schwarzschild horizon becomes

$$S = A \frac{kc^3}{4G\hbar} . \quad (179)$$

This is the Bekenstein–Hawking expression for the entropy of Schwarzschild black holes. The strand model thus reproduces this well-known result. With this explanation of the difference between 2 and $e = 2.71828\dots$, the strand model confirms an old idea:

- ▷ The entropy of a black hole is located *at and near* the horizon.

The above calculation, however, counts some states more than once. Topologically identical spherical horizons can differ in the direction of their north pole and in their state of rotation around the north–south axis. If a spherical horizon is made of N strands, it has N^2 possible physical orientations for the north pole and N possible angular orientations around the north–south axis. The actual number of microstates is thus e^{N^2}/N^3 . Using the relation between N^2 and the surface area A , namely $A = N^2 4G\hbar/c^3$, we get the final result

$$S = A \frac{kc^3}{4G\hbar} - \frac{3k}{2} \ln \frac{A c^3}{4G\hbar} . \quad (180)$$

The strand model thus makes a specific prediction for the logarithmic correction of the entropy of a Schwarzschild black hole. This final prediction of the strand model agrees with many (but not all) calculations using superstrings or other quantum gravity approaches.

Ref. 189

In summary, the entropy value (179), respectively (180), of black holes is due to the *extension* of the fundamental entities in the strand model and to the *three dimensions* of space. If either of these properties were not fulfilled, the entropy of black holes would not result. This is not a surprise; also our deduction of quantum theory was based on the same two properties. In short: like every quantum effect, also the entropy of black holes is a result of extension and three-dimensionality. Only a three-dimensional description of nature agrees with observation.

TEMPERATURE, RADIATION AND EVAPORATION OF BLACK HOLES

The strands that make up a horizon fluctuate in shape. Since every horizon contains energy, the shape fluctuations imply energy fluctuations. In other words, horizons are predicted to have a *temperature*. The value of the temperature can be deduced from the strand model by noting that the characteristic size of the fluctuations for a spherical horizon is the radius R of the horizon. Therefore we have

$$kT = \frac{\hbar c}{2\pi R} . \quad (181)$$

Using the definition of *surface gravity* as $a = c^2/R$, we get

$$T = \frac{\hbar a}{2\pi k c} . \quad (182)$$

Ref. 57, Ref. 58

The strand model predicts that horizons have a temperature proportional to their surface gravity. This result has been known since 1973.

All hot bodies radiate. The strand model thus predicts that Schwarzschild black holes *radiate* thermal radiation of the horizon temperature, with power and wavelength

$$P = 2\pi\hbar c^2/R^2 , \quad \lambda \approx R . \quad (183)$$

This confirms a well-known consequence of the temperature of black holes.

Like all thermal systems, horizons follow thermodynamics. In the strand model, black hole radiation and evaporation occur by reduction of the number of strands that make up the horizon. The strand model thus predicts that black holes *evaporate completely*, until only elementary particles are left over. In particular, the strand model implies that in black hole radiation, there is *no* information loss.

In short, strand model reproduce all aspects of black hole evaporation. The strand model also shows that there is no information loss in this process.

BLACK HOLE LIMITS

In many ways, black holes are *extreme* physical systems. Not only are black holes the limit systems of general relativity; black holes also realize various other limits. As such, black holes resemble light, which realizes the speed limit. We now explore some of these limits.

For a general physical system, not necessarily bound by a horizon, the definitions of energy and entropy with strands allow some interesting conclusions. The entropy of a system is the result of the number of crossing possibilities. The energy of a system is the number of crossing changes per unit time. A large entropy is thus only possible if a system shows many crossing changes per time. Since the typical system time is given by the circumference of the system, the entropy of a physical system is therefore limited:

$$S \leq ER 2\pi k/\hbar c . \quad (184)$$

This relation is known as *Bekenstein's entropy bound*; it thus also follows from the strand model. The equality is realized only for black holes.

In the strand model, horizons are tight, one-sided weaves. For example, this implies that any tangle that encounters a horizon is essentially flat. Because of tangle flatness and the extension of the tails, at most one Planck mass can cross a horizon during a Planck time. This yields the mass rate limit

$$dm/dt \leq c^3/4G \quad (185)$$

that is valid in general relativity and in nature.

Black holes can rotate. The strand model states that there is a highest angular frequency possible; it appears when the equator of the black hole rotates with the speed of light. As a result, the angular momentum J of a black hole is limited by

$$J < 2GM^2/c . \quad (186)$$

Ref. 190 This limit is well known from general relativity.

The electric charge of a black hole is also limited. The force limit in nature implies that the electrical forces between two charged black holes must be lower than their gravitational interaction. This means that

$$\frac{Q^2}{4\pi\epsilon_0 r^2} \leq \frac{GM^2}{r^2} , \quad (187)$$

or

$$Q^2 \leq 4\pi\epsilon_0 GM^2 . \quad (188)$$

This is the well-known charge limit for (static) black holes given by the Reissner-Nordström metric. The maximum charge of a black hole is proportional to its radius. It follows directly from the maximum force principle.

To explain the charge limit, we deduce that the *extremal* charge surface density Q/A of a black hole is proportional to $1/R$. The higher the horizon curvature, the more charge per Planck area is possible. In the strand model, a horizon is a tight weave of strands. We are thus led to conjecture that at Planck scale, electric charge is related to and limited by strand curvature. We will explore this connection in more detail below.

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The strand model limits energy density to the Planck energy per Planck volume, or to the value $c^7/(16G^2\hbar)$. This limit implies a lower size limit for black holes; therefore, the strand model does not allow singularities, be they dressed or naked. And indeed, no singularity has ever been observed.

In summary, the strand model reproduces the known limit properties of horizons. And all these results are independent of the precise fluctuation details of the strands.

CURVATURE AROUND BLACK HOLES

The tails of a black hole extend up to the border of space; the density of tails is highest at the horizon. A black hole is therefore surrounded by partial links at any *finite* distance from the horizon. In other words, the space around a black hole is *curved*. The value of the space-time curvature increases as one approaches the horizon, because of the way in which the partial links hinder each other in their motion. The nearer they are to the horizon, the more they hinder each other. The curvature that appears is proportional to the density of partial links and to their average strand curvature.

At the horizon, the curvature radius is the horizon radius R . By construction, the number of tails departing from a non-rotating black hole is proportional to R . The spatial curvature is given by the average crossing density gradient. Hence at a radial distance r

from a static black hole, the spatial curvature K is

$$K \sim \frac{R}{r^3}. \quad (189)$$

So at the horizon itself, the curvature K is (of the order of) the inverse square of the horizon radius; further away, it decreases rapidly, with the third power of the distance. This result is a well-known property of the Schwarzschild solution and is due to the extension of the strands. The rapid decay with radius is the reason why in everyday situations there is no noticeable curvature of space-time. In short, strands allow us to deduce the correct curvature of space-time around black holes and spherical masses.

THE SHAPE OF NON-ROTATING BLACK HOLES

The strand model also explains and visualizes the importance of spherical horizons in nature. First of all, strands illustrate the non-existence of (uncharged) one-dimensional or toroidal horizons in $3 + 1$ space-time dimensions. Such configurations are unstable, in particular against transverse shear and rearrangement of the strands.

The strand model also implies that non-rotating, closed horizons are spherical. Obviously, spheres are the bodies with the smallest surface for a given volume. The minimum horizon surface appears because the strands, through their fluctuations, effectively ‘pull’ on each Planck area of the horizon. As a result, all non-rotating macroscopic horizons will evolve to the spherical situation in a few Planck times. (Deviations from the spherical shape will mainly occur near Planck scales.) With the definition of gravity waves given below, it also becomes clear that strongly deformed, macroscopic and non-spherical horizons are unstable against emission of gravity waves or of other particles. In short,

- ▷ All non-rotating horizons of non-spherical shape are unstable.

The strand model thus confirms that spherical horizons are favoured and that the most compact bodies with a given mass. The reasoning can be extended to rotating horizons, yielding the well-known shapes.

In summary, strands reproduce all known qualitative and quantitative properties of horizons and of black holes, and thus of general systems with strong gravitational fields. All predictions from strands agree with observations and with other approaches to quantum gravity. These hints already suggest that strands imply the field equations.

THE FIELD EQUATIONS OF GENERAL RELATIVITY

The field equations can be deduced from the fundamental principle in two different, but related ways. Essentially, both derivations repeat the reasoning for universal gravitation given above, but for the relativistic case. The first deduction of the field equations is based on an old argument on the thermodynamics of space-time. Strands show that horizons have three thermodynamic properties:

- an area–entropy relation of $S = A kc^3/4G\hbar$,
- a curvature–temperature relation of $T = a \hbar/2\pi kc$,

– a relation between heat and entropy of $\delta Q = T\delta S$.

Using these three properties, and using the relation

$$\delta Q = \delta E, \quad (190)$$

that is valid *only* in case of horizons, we get the first principle of horizon mechanics

$$\delta E = \frac{c^2}{8\pi G} a \delta A. \quad (191)$$

Page 32 From this relation, using the Raychaudhuri equation, we obtain the field equations of general relativity. This deduction was given above.*

In other words, the field equations result from *the thermodynamics of strands*. It is worth noting that the result is independent of the details of the fluctuations or of the microscopic model of space, as long as the three thermodynamic properties just given are valid. In fact, these properties must be fulfilled by any model of space-time; and indeed, several competing models of space claim to fulfil them.

Page 185 We can use the relation between fluctuations and strands to settle an issue mentioned above, in the section on quantum theory. Strand fluctuations *must* obey the thermodynamic properties to allow us to define space-time. If they obey these properties, then space-time exists and curves according to general relativity.

Ref. 19 Page 32 A second derivation of the field equations of general relativity follows the spirit of the strand model most closely. It is even shorter. Strands imply that all physical quantities are limited by the corresponding Planck limit. These limits are due to the limit to the fundamental principle, in other words, they are due to the packing limit of strands. In particular, the fundamental principle limits force by $F \leq c^4/4G$ and power by $P \leq c^5/4G$. We have already shown above that this limit implies the field equation.

In other words,

* Here is the argument in a few lines. The first principle of horizon mechanics can be rewritten, using the energy-momentum tensor T_{ab} , as

$$\int T_{ab} k^a d\Sigma^b = \frac{c^2}{8\pi G} a \delta A$$

where $d\Sigma^b$ is the general surface element and k is the Killing vector that generates the horizon. The Raychaudhuri equation allows us to rewrite the right-hand side as

$$\int T_{ab} k^a d\Sigma^b = \frac{c^4}{8\pi G} \int R_{ab} k^a d\Sigma^b$$

where R_{ab} is the Ricci tensor describing space-time curvature. This equality implies that

$$T_{ab} = \frac{c^4}{8\pi G} (R_{ab} - (R/2 + \Lambda)g_{ab})$$

where Λ is an undetermined constant of integration. These are Einstein's field equations of general relativity. The field equations are valid everywhere and for all times, because a suitable coordinate transformation can put a horizon at any point and at any time. To achieve this, just change to a suitable accelerating frame, as explained in the volume on relativity.

- ▷ Given that black holes and thus horizons are thermodynamic systems, so is curved space.

The reason: both can be transformed into each other. Therefore:

- ▷ Since black holes have thermodynamic aspects, so has gravity.

And since black holes are built from microscopic degrees of freedom, so is curved space. Or, in simple words:

- ▷ Space is made of many small entities.

And finally we can state:

- ▷ Space is made of strands, because strands are the simplest entities that yield black hole entropy.

Strands are the simplest way to incorporate quantum effects into gravitation. If we take into consideration that strands are the only way known so far to incorporate gauge interactions, we can even conclude that strands are the only way known so far to incorporate all quantum effects into gravitation.

In summary, the strand model asserts that the field equations appear as consequences of fluctuations of impenetrable, featureless strands. In particular, the strand model implies and confirms that a horizon and a particle gas at Planck energy do not differ. However, the value of the cosmological constant is *not* predicted from strand thermodynamics.

EQUATIONS FROM NO EQUATION

The strand model asserts that the field equations of general relativity are not the result of another, more basic evolution equation, but result directly from the fundamental principle. To say it bluntly, the field equations are deduced from a drawing – the fundamental principle shown in [Figure 10](#). This strong, almost unbelievable statement is due to a specific property of the field equations and to two properties of the strand model.

First of all, the field equations are, above all, consequences of the thermodynamics of space-time. In the strand model, the thermodynamic properties are deduced as a consequence of the strand fluctuations. This deduction does not require underlying evolution equations; the field equations follow from the statistical behaviour of strands.

The second, essential property of the strand model is its independence from the underlying motion of the strands. In the strand model we obtain the evolution equations of the vacuum – the field equations of general relativity – without deducing them from another equation. We do not need an evolution equation for the strand shape; the deduction of the field equations works for *any* underlying behaviour of strand shapes, as long as the thermodynamic properties of the strand fluctuations are reproduced.

The third and last essential property that allows us to deduce the field equations directly from a graph, and not from another equation, is the relation between the graph and

natural physical units. The relation with natural units, in particular with the quantum of action \hbar and the Boltzmann constant k , is fundamental for the success of the strand model.

In summary, the fundamental principle of the strand model contains all the essential properties necessary for deducing the field equations of general relativity. In fact, the discussion so far makes another important point: unique, underlying, more basic evolution equations for the tangle shape *cannot* exist. There are two reasons. First, an underlying equation would itself require a deduction, thus would not be a satisfying solution to unification. Secondly, and more importantly, evolution equations are differential equations; they assume well-behaved, smooth space-time. At Planck scales, this is impossible.

- ▷ Any principle that allows deducing the field equations cannot itself be an evolution equation.

THE HILBERT ACTION OF GENERAL RELATIVITY

Page 207 We have just shown that the strand model implies the field equations of general relativity. We have also shown above that, in the strand model, the least action principle is a natural property of all motion of strands. Combining these two results, we find that a natural way to describe the motion of space-time is the (extended) *Hilbert action* given by

$$W = \frac{c^4}{16\pi G} \int (R - 2\Lambda) dV, \quad (192)$$

where R is the Ricci scalar, $dV = \sqrt{\det g} d^4x$ is the invariant 4-volume element of the metric g , and Λ is the cosmological constant, whose value we have not determined yet. As is well known, the description of evolution with the help of an action does not add anything to the field equations; both descriptions are equivalent.

Page 281 For a curved three-dimensional space, the Ricci scalar R is the average amount, at a given point in space, by which the curvature deviates from the zero value of flat space. In the strand model, this leads to a simple statement, already implied by [Figure 74](#):

- ▷ The *Ricci scalar* R is the ratio of additional or missing crossings per spatial volume, compared to flat space.

As usual, the averaging is performed over all spatial orientations. A similar statement can be made for the cosmological constant Λ . In short, we can say: the Hilbert action follows directly from the fundamental principle of the strand model.

SPACE-TIME FOAM

Quantum physics implies that at scales near the Planck length and the Planck time, space-time fluctuates heavily. John Wheeler called the situation *space-time foam*; the term *quantum foam* is also used. In a sense, *quantum gravity* can be defined, if at all, as the description of space-time foam. This reduced view arises because no separate theory of quantum gravity is possible in nature.

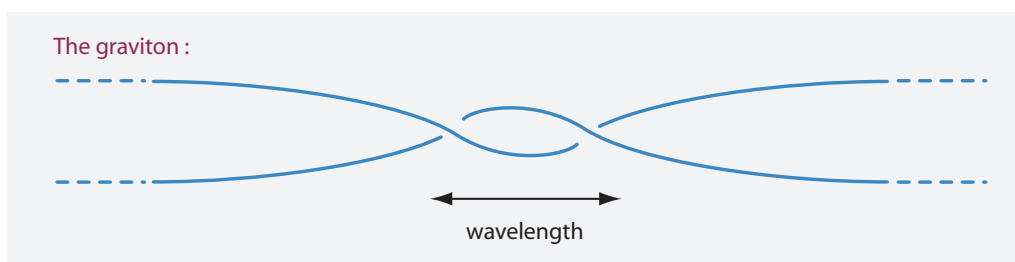


FIGURE 77 The graviton in the strand model.

Historically, there have been many speculations on the details of space-time foam. Apart from its fluctuations, researchers speculated about the appearance of topology changes – such as microscopic wormholes – about the appearance of additional dimensions of space – between six and twenty-two – or about the appearance of other unusual properties – such as microscopic regions of negative energy, networks or loop structures.

The strand model makes a simple prediction that contradicts most previous speculations:

- ▷ Space-time foam is made of fluctuating strands.

At everyday scales, the foam is not noticed, because background space and physical space are indistinguishable. At Planck scales, space-time is not fundamentally different from everyday space-time. No unusual topology, no additional dimensions, and no new or unusual properties appear at Planck scales. Above all, the strand model predicts that there are *no* observable effects of space-time foam; for example, ‘space-time noise’ or ‘particle diffusion’ do not exist. The strand model of space-time foam is both simple and unspectacular.

Ref. 191

GRAVITONS, GRAVITATIONAL WAVES AND THEIR DETECTION

In the strand model, gravitons can be seen as a special kind of partial links. An example is shown in Figure 77. As a twisted pair of parallel strands, the graviton returns to itself after rotation by π ; it thus behaves like a spin-2 boson, as required.

Can single gravitons be observed? The strand model implies that the absorption of a single graviton by an elementary particle changes its spin or position. However, such a change *cannot* be distinguished from a quantum fluctuation, because the graviton is predicted to be massless. Furthermore, the strand model predicts that gravitons do not interact with photons, because they have no electric charge. In summary, the strand model predicts:

- ▷ Single gravitons *cannot* be detected.

The situation changes for gravitational waves. Such waves are coherent superpositions of large numbers of gravitons and are observable classically. In such a case, the argument against the detection of single gravitons does not apply. In short, the strand model predicts that gravitational waves *can* be observed. (This prediction, made by many since

Challenge 182 e

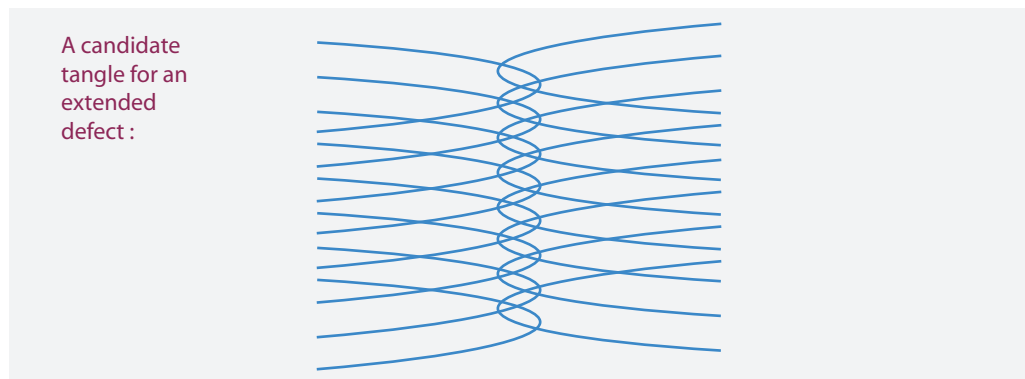


FIGURE 78 A speculative, highly schematic model for a cosmic string, a one-dimensional defect in space-time

Ref. 192 1915 and repeated in this text on the basis of the strand model in 2008, came true in February 2016. The observations also produced the extremely low mass limit of at most $1.2 \times 10^{-22} \text{ eV}/c^2$ for any possible mass of the photon.)

OPEN CHALLENGE: IMPROVE THE ARGUMENT FOR THE GRAVITON TANGLE

Challenge 183 ny

The argument that leads to the graviton tangle is too much hand-waving. Can you make the argument more compelling? Could the four tails form a cross and thus span a plane?

OTHER DEFECTS IN VACUUM

The strand model provides a quantum description of gravitation. The strand model does so by explaining physical space as the average of the crossing switches induced by strand fluctuations among untangled strands. Matter, radiation and horizons are defects in the 'sea' of untangled strands.

So far, we have been concerned with *particles*, i.e., localized, zero-dimensional defects, and with *horizons*, i.e., two-dimensional defects. Now, modelling of the vacuum as a set of untangled strands also suggests the possible existence of *one-dimensional* – equivalent to dislocations and disclinations in solids – of *additional* two-dimensional defects, or of *three-dimensional* defects. Such defects could model cosmic strings, domain walls, wormholes, toroidal black holes, time-like loops and regions of negative energy.

An example of such a possible new defect is illustrated in [Figure 78](#). The illustration can be seen as the image of a one-dimensional defect or as the cross section of a two-dimensional defect. Are such defects stable against fluctuations? The strand model suggests that they are not. These defects are expected to decay into a mixture of gravitons, black holes, matter and radiation particles. However, this issue is still a topic of research, and will not be covered here.

Exploring the stability of wormholes, time-like loops and toroidal black holes leads to similar results. It seems that the strand model should not allow time-like loops of macroscopic size, since any configuration that cannot be embedded locally into three flat spatial dimensions is either a particle or a black hole. Alternatively, macroscopic time-like loops would collapse or decay because of the fluctuations of the strands. In

TABLE 11 Correspondences between physical systems and mathematical tangles.

PHYSICAL SYSTEM	STRANDS	TANGLE TYPE
Vacuum	many infinite unknotted strands	unlinked
Dark energy	many fluctuating infinite strands	unlinked
Elementary vector boson	one infinite strand	a curve
Quark	two infinite strands	rational tangle
Lepton	three infinite strands	braided tangle
Meson, baryon	three or more infinite strands	composed of rational tangles
Higher-order propagating fermion	two or more infinite strands	general rational tangle
Virtual particles	open or unlinked strands	trivial tangles
Composed systems	many strands	separable tangles
Graviton	two infinite twisted strands	specific rational tangle
Gravity wave	many infinite twisted strands	many graviton tangles
Horizon	many tightly woven infinite strands	web-like rational tangle
Young universe	closed strand(s)	knot (link)

the same way, wormholes or black holes with non-trivial topology should be unstable against more usual strand structures, such as particles or black holes.

We also note the strand model does not allow volume defects (black holes being surface-like defects). The most discussed types of volume defects are macroscopic regions of negative energy. Energy being action per unit time, and action being connected to crossing changes, the model does not allow the construction of negative-energy regions. However, the strand model does allow the construction of regions with lower energy than their environment, as in the Casimir effect, by placing restrictions on the wavelengths of photons.

The strand model thus predicts the absence of additional defects and tangle types. The final and general connection between tangle types and defects is shown (again) in Table 11. The next chapter will give details of the tangles corresponding to each particle.

In summary, the strand model reproduces the results of modern quantum gravity and predicts that the more spectacular defects conjectured in the past – linear defects such as cosmic strings, surface defects such as wormholes, volume defects such as negative-energy regions – do *not* appear in nature.

THE GRAVITY OF SUPERPOSITIONS

What is the gravitational field of a quantum system in a macroscopic superposition? The issue has been raised by many scholars as an important step towards the understanding of how to combine gravitation and quantum theory.

The strand model deflates the importance of the issue. The model shows – or predicts, if one prefers – that the gravitational field of a superposition is the temporal and spatial average of the evolving quantum system, possibly under inclusion of decoherence.

Page 199 What is the gravitational field of a single quantum particle in a double-slit experiment? As Figure 38 shows, the gravitational field almost always appears in both slits, and only very rarely in just one slit.

In summary, in the strand model, the combination of gravitation and quantum theory is much simpler than was expected by most researchers. For many decades it was suggested that the combination was an almost unattainable goal. In fact, in the strand model we can almost say that the two descriptions combine naturally.

TORSION, CURIOSITIES AND CHALLENGES ABOUT QUANTUM GRAVITY

Ref. 193 On the one hand, the strand model denies the existence of any specific effects of *torsion* on gravitation. On the other hand, the strand model of matter describes spin with the belt trick. The belt trick is thus the strand phenomenon that is closest to the idea of torsion. Therefore, exaggerating a bit in the other direction, it could also be argued that in the strand model, torsion effects are quantum field theory effects.

* *

The strand model describes three-dimensional space as made of tangled strands. Several similar models have been proposed in the past.

Ref. 194 The model of space as a *nematic world crystal* stands out as the most similar. This model was proposed by Hagen Kleinert in the 1980s. He took his inspiration from the famous analogy by Ekkehart Kröner between the equations of solid-state elasticity around line defects and the equations of general relativity.

Ref. 195 Also in the 1980s, the mentioned posets have been proposed as the fundamental structure of space. Various models of quantum gravity from the 1990s, inspired by spin networks, spin foams and by similar systems, describe empty space as made of extended constituents. These extended constituents tangle, or bifurcate, or are connected, or sometimes all of this at the same time. Depending on the model, the constituents are lines, circles or ribbons. In some models their shapes fluctuate, in others they don't.

Ref. 157 Around the year 2000, another type of Planck-scale crystal model of the vacuum has been proposed by David Finkelstein. In 2008, a specific model of space, a crystal-like network of connected bifurcating lines, has been proposed by Gerard 't Hooft.

Ref. 197 All these models describe space as made of some kind of extended constituents in a three-dimensional background. All these models derive general relativity from these constituents by some averaging procedure. The lesson is clear: it is *not* difficult to derive general relativity from a Planck-scale model of space. It is *not* difficult to unify gravity and quantum theory. As Luca Bombelli said already in the early 1990s, the challenge for a Planck-scale model of nature is not to derive gravity or general relativity; the challenge is to derive the other interactions. So far, the strand model seems to be the only model that has provided such a derivation.

* *

Challenge 184 e The Planck force is the force value necessary to produce a change \hbar in a Planck time over a Planck length. The Planck force thus appears almost exclusively at horizons.

* *

Ref. 198 Already in the 1990s, Leonard Susskind speculated that black holes could be formed by a single wound-up string. Strands differ from strings; they differ in the number of dimensions, in their intrinsic properties, in their symmetry properties, in the fields they carry and in the ways they generate entropy. Nevertheless, the similarity with the strand model of black holes is intriguing.

* *

Page 160 In September 2010, two years after the strand model appeared, independent research confirmed its description of physical space, as already mentioned above. In an extended article exploring the small scale structure of space from several different research perspectives in general relativity, Steven Carlip comes to the conclusion that all these perspectives suggest the common idea that ‘space at a fixed time is thus threaded by rapidly fluctuating lines’.

Ref. 154

Ref. 199

In 2011, also independently, Marcelo Botta Cantcheff modelled space as a statistic ensemble of one-dimensional ‘strings’. He explained the main properties of space, including the thermodynamic properties of black holes.

* *

Challenge 185 e The first version of the strand model assumed that space is not defined at the cosmic horizon, and that therefore, strand impenetrability does not hold there. The same was thought to occur at black hole horizons. The newest version of the strand model does not seem to need this exception to impenetrability. Can you explain black hole entropy without it?

* *

Page 35 The strand model also allows us to answer the question whether quantum particles are black holes: no, they are not. Quantum particles are tangles, like black holes are, but particles do *not* have horizons. As a side result, the mass of all particles is lower than a Planck mass, or more precisely, lower than a Planck mass black hole.

Ref. 200

Strands imply that gravity is weaker than the three gauge interactions. This consequence, like the low particle mass just mentioned, is due to the different origins of gravity and gauge interactions. Gravity is due to the strand tails, whereas gauge interactions are due to the tangle cores. Thus gravity is the weakest interaction in everyday life. The observation of the weakness of gravity at everyday and other energy scales is sometimes called the weak gravity conjecture. It is naturally valid in the strand model.

* *

For an observer at spatial infinity, a black hole horizon is an averaged-out tight web of strands. What does a falling observer experience? The question will still capture the imagination in many years. Such an observer will also see strands; above all, a falling observer will never hit any singularity. The details of the fall are so involved that they are not discussed here, because the fall affects both the black hole appearance and the observer.

* *

Can black hole radiation be seen as the result of trying to tear vacuum apart? Yes and no.

The answer is no, because physical vacuum cannot be torn apart, due to the maximum force principle. But the answer is also yes in a certain sense, because the maximum force is the closest attempt to this idea that can be realized or imagined.

* *

Ref. 201 The strand model makes the point that *entanglement* and the vacuum – and thus quantum gravity – have the same nature: both are due to crossing strands. This idea has been explored independently by Mark van Raamsdonk.

* *

As we have seen, the strand model predicts no *observable* violation of Lorentz-invariance – even though it predicts its violation at Planck scale. Strands predict the lack of dispersion, birefringence and opacity of the vacuum. Strands predict that the vacuum has three dimensions whenever it is observed and that it is unique, without phase transitions. We already mentioned the impossibility of detecting single gravitons.

All these negative predictions are examples of the ‘*no avail*’ conjecture:

- ▷ Quantum gravity effects cannot be distinguished from ordinary quantum fluctuations.

Despite many attempts to disprove it, all experiments so far confirm the conjecture. Because both quantum gravity effects and quantum effects are due to tail fluctuations, the strand model seems to imply the conjecture.

* *

Ref. 202 The strand model of black holes also confirms a result by Zurek and Thorne from the 1980s: the entropy of a black hole is the logarithm of the number of ways in which it could have been made.

* *

Challenge 186 s Argue that because of the strand model, no black hole can have a mass below the (corrected) Planck mass, about 11 μg , and thus that *microscopic black holes* do not exist. Can you find a higher lower limit for the mass?

* *

Do atoms or the elementary fermions moving inside matter emit gravitational radiation, and why? The question was already raised by Albert Einstein in 1916. The strand model answers the issue in the same way as textbook physics. Elementary particles in atoms – in the ground state – do not emit gravitational waves for the same reason that they do not emit electromagnetic waves: for atoms in the ground state, there is no lower state into which they could decay. Excited atomic states do not emit gravitational waves because of the extremely low emission probability; it is due to the extremely low mass quadrupole values.

* *

Ref. 203 In 2009 Mikhail Shaposhnikov and Christof Wetterich argued that if gravitation is ‘asymptotically safe’, there is no physics beyond the standard model and the Higgs mass

must be around 126 GeV – exactly the value that was found experimentally a few years afterwards. A quantum field theory is called *asymptotically safe* if it has a fixed point at extremely high energies. Does the strand model imply that gravity is – maybe only effectively – asymptotically safe?

Challenge 187 ny

* *

Page 58 It is often stated that general relativity does not allow the description of fermions if the topology of space is kept fixed. This is wrong: the strand model shows that fermions can be included in the case that space is seen as an average of extended fundamental entities.

* *

Following the fundamental principle of the strand model, G is the fundamental constant that describes gravitation. The strand model predicts that gravity is the same for all energy scales; in other words, the constant G is not expected to change with energy. This agrees with recent results from quantum gravity.

Ref. 204

PREDICTIONS OF THE STRAND MODEL ABOUT GRAVITY

As just presented, the strand model makes several verifiable predictions about general relativity and quantum gravity.

- The maximum energy speed in nature is c , at all energy scales, in all directions, at all times, at all positions, for every physical observer. This agrees with observations.
- No deviations from special relativity appear for any measurable energy scale, as long as gravity plays no role. No ‘double’ or ‘deformed special relativity’ holds in nature, even though a maximum energy-momentum for elementary particles does exist in nature. Whenever special relativity is not valid, general relativity, or quantum field theory, or both together need to be used. This agrees with observations.
- There is a maximum power or luminosity $c^5/4G$, a maximum force or momentum flow $c^4/4G$, and a maximum mass change rate $c^3/4G$ in nature. This agrees with observations, though only few experimental observations are close to these limit values.
- There is a minimum distance and a minimum time interval in nature. There is a maximum curvature and a maximum mass density in nature. There are no singularities in nature. All this agrees with observations, including the newly discovered black hole mergers.
- The usual black hole entropy expression given by Bekenstein and Hawking holds. The value has never been measured, but is consistently found in all calculations performed so far. In fact, black hole entropy is related to the Fulling–Davies–Unruh effect, which itself is related to the Sokolov–Ternov effect; and this latter effect has already been observed in several accelerators, for the first time in 1971.
- There are no deviations from general relativity, as described by the Hilbert action, for any measurable scale. The only deviations appear in situations with a few strands, i.e., in situations where quantum theory is necessary. This agrees with observations, but experimental data are far from sufficient; undetected deviations could still exist.
- There is no modified Newtonian dynamics, or MOND, with evolution equations that differ from general relativity. The rotation curves of stars in galaxies are due to dark

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Ref. 205

- matter, to other conventional explanations, or both.
- There is no effect of torsion that modifies general relativity. This agrees with observations.
 - There is no effect of higher derivatives of the metric on the motion of bodies. This agrees with observations, but experimental data are far from sufficient.
 - Observations are independent of the precise strand fluctuations. Mathematical consistency checks of this prediction are possible.
 - No wormholes, no negative energy regions and no time-like loops exist. This agrees with observations, but experimental data are far from covering every possible loop-hole.
 - The Penrose conjecture and the hoop conjecture hold. Here, a mathematical consistency check is possible.
 - There are no cosmic strings and no domain walls. This agrees with observations, but experimental data are far from exhaustive.
 - Gravitons have spin 2; they return to their original state after a rotation by π and are bosons. This agrees with expectations.
 - Gravitons cannot be detected, due to the indistinguishability with ordinary quantum fluctuations of the detector. This agrees with data so far.
 - Atoms do not emit gravitational waves or gravitons.
 - Gravitational waves exist and can be detected. This agrees with experiment; the final confirmation occurred in late 2015.
 - The gravitational constant G does not run with energy – as long as the strand diameter can be neglected. In this domain, G is not renormalized. This prediction agrees with expectations and with data, though the available data is sparse.

All listed predictions are unspectacular; they are made also by other approaches that contain general relativity as limiting cases. In particular, the strand model, like many other approaches, predicts:

- Page 305 ▷ With the exception of the cosmological constant and Sokolov-Ternov-like effects, *no quantum gravity effects will be observed.*

Gravity will not yield new measurable quantum effects. So far, this prediction agrees with experiment – and with almost all proposed models of quantum foam in the research literature. In other words, we have found *no unexpected* experimental predictions from the strand model in the domain of quantum gravity. This is the so-called ‘*no avail*’ conjecture; and it is not a surprise.

Ref. 97

Page 8

In fact, the Bronshtein cube of Figure 1 also implies:

- ▷ There is *no* separate theory of quantum gravity that includes relativity but does not include the other interactions.

There is no room for a theory of relativistic quantum gravity in nature.

In short, strands lead us to expect deviations from general relativity only in two domains: in cosmology (such as changes of the cosmological constant) and in particle physics. The rest of this chapter deals with cosmology. The subsequent chapters focus on

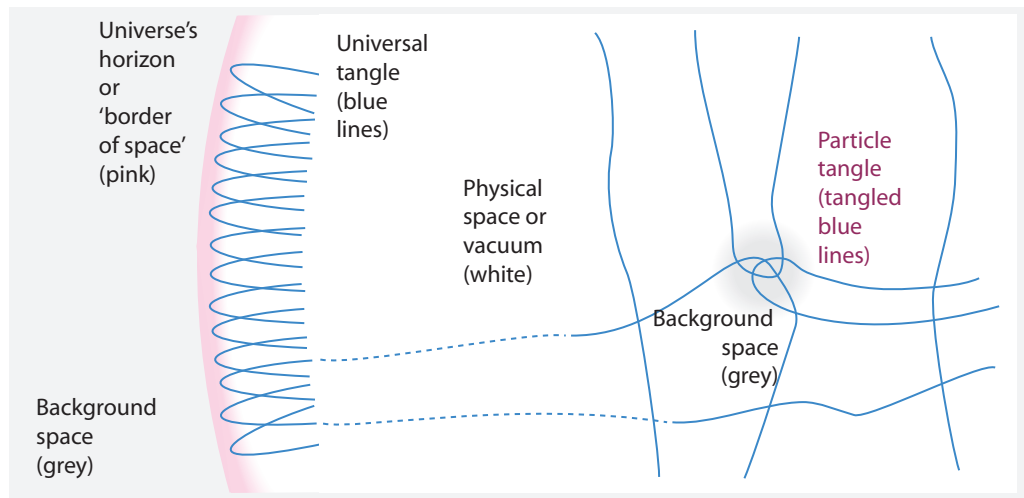


FIGURE 79 In the strand model, the universe is limited by a horizon, as schematically illustrated here. Physical space (white) matches background space (grey) only inside the horizon. Physical space thus only exists inside the cosmic horizon.

particle physics.

COSMOLOGY

Cosmology is an active field of research, and new data are collected all the time. We start with a short summary.

The sky is dark at night. This and other observations about the red shift show that the universe is surrounded by a horizon and is of finite size and age. Precise measurements show that cosmic age is around 13 800 million years. The universe expands; the expansion is described by the field equations of general relativity. The universe's expansion accelerates; the acceleration is described by the *cosmological constant* Λ – also called *dark energy* – that has a small positive value. The universe is observed to be flat, and, averaged over large scales, homogeneous and isotropic. At present, the observed average matter density in the universe is about 18 times smaller than the energy density due to the cosmological constant. In addition, there is a large amount of matter around galaxies that does not radiate; the nature of this *dark matter* is unclear. Galaxy formation started from early density fluctuations; the typical size and amplitude of the fluctuations are known. The topology of space is observed to be simple.

The strand model, like any unified description of nature, must reproduce and explain these measurement results. Otherwise, the strand model is wrong.

THE FINITENESS OF THE UNIVERSE

In the strand model, cosmology is based on the following idea:

- ▷ The *universe* is made of *one* fluctuating strand. Fluctuations increase the

complexity of the strand tangledness over time.

The existence of finite size and of finite age then follows automatically:

- ▷ The *universe's horizon* appears at the age or distance at which the strand crossings cannot be embedded any more into a common three-dimensional background space. The horizon expands over time.

The strand model thus has a simple explanation for the finiteness of the universe and the cosmic horizon that bounds it. A schematic illustration of the cosmic horizon is given in [Figure 79](#).

[Ref. 206](#)
[Ref. 207](#) The strand model predicts that the cosmic horizon is an *event horizon*, like that of a black hole. Until 1998, this possibility seemed questionable; but in 1998, it was discovered that the expansion of the universe is accelerating. This discovery implies that the cosmic horizon is indeed an event horizon, as predicted by the strand model.

[Page 305](#)
[Page 283](#) In fact, the strand model predicts that *all* horizons in nature are of the *same* type. This also means that the universe is predicted to saturate Bekenstein's entropy bound. More precisely, the strand model predicts that the universe is a kind of *inverted back hole*. Like for any situation that involves a horizon, the strand model thus does not allow us to make statements about properties 'before' the big bang or 'outside' the horizon. As explained above, there is nothing behind a horizon.

In particular, the strand model implies that the matter that appears at the cosmic horizon during the evolution of the universe appears through Bekenstein–Hawking radiation. This contrasts with the 'classical' explanation from general relativity that new matter appears simply because it existed behind the horizon beforehand and then crosses the horizon into the 'visible part' of the universe.

[Page 101](#) We note that modelling the universe as a single strand implies that it contains tangles. In other words, the strand model makes the prediction that the universe cannot be empty, but that it must contain particles. Strand cosmology also confirms that the question of initial conditions for the universe does not really make sense: particles appear at the horizon.

We also note that describing the universe as made of a single strand is a natural, but somewhat unusual way to incorporate what particle physicists and cosmologists like to call *holography*. Holography is the idea that all observables of a physical system are defined on a boundary enclosing the system. In other words, if we would know, at Planck scale, everything that happens on the walls of a room, we could know everything that is and goes on inside the room. Instead of holography, we could also call it the *NSA dream*. Holography is a consequence of the extension of the fundamental constituents of nature and is a natural consequence of the strand model. As a consequence, strand cosmology naturally reproduces holographic cosmology – though not fully, as is easy to check.

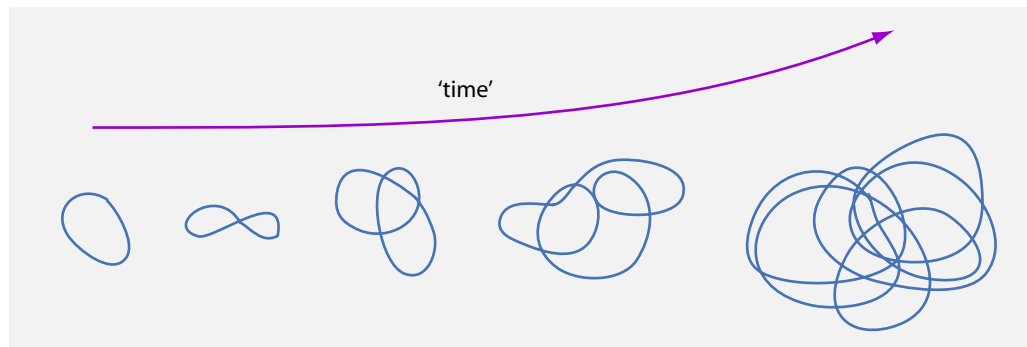


FIGURE 80 An extremely simplified view of how the universe evolved near the big bang. In this situation, physical space is not 'yet' defined.

“ Or cette liaison ou cet accommodement de toutes les choses créées à chacune, et de chacune à toutes les autres, fait que chaque substance simple a des rapports qui expriment toutes les autres, et qu'elle est par conséquent un miroir vivant perpétuel de l'univers.* ”

Gottfried Wilhelm Leibniz, *Monadologie*, 56.

THE BIG BANG – WITHOUT INFLATION

Ref. 208
Page 289

Any expanding, homogeneous and isotropic matter distribution had earlier stages of smaller size and higher density. Also the universe has been hotter and denser in the past. But the strand model also states that singularities do not appear in nature, because there is a highest possible energy density. As a result, the big bang might be imagined as illustrated in [Figure 80](#). Obviously, physical space and time are not well defined near that situation, so that the figure has to be taken with a grain of salt. Nevertheless, it shows how the evolution of the universe can be seen as resulting from the increase in tangledness of the strand that makes up nature.

The strand model leads to the conjecture that the evolution of the universal strand just after the big bang automatically yields both a homogeneous and isotropic matter distribution and a flat space. Also the scale invariance of early density fluctuations seems natural in the strand model. In short, the strand model looks like a promising alternative to *inflation*: the hypothesis of inflation becomes unnecessary in the strand model, because strand cosmology directly makes the predictions that seem so puzzling in classical cosmology. This issue is still subject of research.

THE COSMOLOGICAL CONSTANT

The cosmological constant is due to strands. When three mutually orthogonal strands come together at a point, they cannot be completely straight; they are slightly bent. Equivalently, because the strands of the vacuum touch each other, due to the statistics of the

* 'Now this connexion or adaptation of all created things to each and of each to all, means that each simple substance has relations which express all the others, and, consequently, that it is a perpetual living mirror of the universe.'

fluctuations, there is a slight effective repulsion between them. This is the strand model for the cosmological constant.

Page 209 In short, in the strand model, *vacuum energy*, or *dark energy*, is due to the cosmological constant, which itself is due to strand fluctuations. As we saw above, the strand model predicts that the cosmological constant Λ for infinitely extended flat space vanishes, because the vacuum energy density vanishes in that case. But the strand model also predicts that for *finite* extension, the cosmological constant does *not* vanish. Indeed, in the strand model, a finite size limits the fluctuations of the strands. Fluctuations with sizes larger than the size of space are frozen out; this leads to an effective repulsion of strands that in turn leads to a cosmological constant given by (the square of) the extension of space:

$$\Lambda = \frac{1}{R_{\max}^2}. \quad (193)$$

In particular, the strand model predicts a small *positive* cosmological constant, i.e., a constant that leads to a small repulsion of masses.

Ref. 209 The relation between the cosmological constant and the radius of the universe can be found also with another, more precise argument, based on holography, and given by Balázs and Szapudi. Bekenstein's holographic entropy bound states that for all systems of size R and energy E we have

$$S \leq ER \frac{2\pi k}{\hbar c}. \quad (194)$$

For a spherical system, this yields

$$S \leq A \frac{kc^3}{4G\hbar}. \quad (195)$$

Ref. 210 The application of this inequality to the universe is the Fischler–Susskind holographic conjecture. Using the energy–entropy relation $E = TS$ valid for any holographic system, and introducing the *energy* density ρ_E , we get the limit given by

$$\rho_E \leq \frac{T}{R} \frac{3kc^3}{4\hbar G}. \quad (196)$$

Page 287 Using the formula for temperature $T = \hbar c/2\pi kR$ for a horizon – deduced above from the strand model – we get

$$\rho_E \leq \frac{1}{A} \frac{3c^4}{2G} = \frac{1}{4\pi R^2} \frac{3c^4}{2G}. \quad (197)$$

The strand model predicts that the universe *saturates* the entropy bound. In other words, assuming that R is c times the age of the universe t_0 , the strand model predicts that the total energy density of the universe is equal to the so-called *critical* energy density.

Ref. 211 The equality of the measured total energy density and the critical density is well known. These measurements show that the present total energy density of the universe

is about

$$\rho_{\text{E vac}} \approx 8.5 \cdot 10^{-10} \text{ J/m}^3 \quad \text{or} \quad \rho_{\text{m vac}} = 0.94(9) \cdot 10^{-26} \text{ kg/m}^3 . \quad (198)$$

In other words, the strand model, like the holographic argument, predicts that the cosmological constant is limited by

$$\Lambda \leq \frac{3}{c^2 t_0^2} . \quad (199)$$

Ref. 211 The result confirms the result of expression (193). Modern measurements yield 74 % of the maximum possible value.

Ref. 212 The argument for the value of the cosmological constant can be made for any age of the universe. Therefore, the strand model predicts that the cosmological constant Λ *decreases* with increasing radius of the universe. In particular, there is no need for a scalar field that makes the cosmological constant decrease; the decrease is a natural result of the strand model. The strand model states that the cosmological constant appears in the field equations as a quantum effect due to the finite size of the universe. The strand model thus implies that there is no separate equation of motion for the cosmological constant, but that the constant appears as a large-scale average of quantum effects, as long as the size of the universe is limited.

Ref. 213 In summary, the strand model predicts that not only the field equations of general relativity, but also the amount of dark energy, the expansion of the universe and its acceleration result from strand fluctuations. The cosmological constant changes roughly with the inverse square of time. In particular, the strand model implies that the effect proposed by Wiltshire – that the cosmological constant is an artefact of the inhomogeneity of matter distribution – is *not* fundamental, but may at most influence the value somewhat. (Could the difference between the maximum possible and the measured value of the cosmological constant be due to this effect?)

Challenge 189 s

THE VALUE OF THE MATTER DENSITY

The strand model predicts that horizons emit particles. As a consequence, the strand model predicts an upper limit for the number N_{b} of baryons that could have been emitted by the cosmic horizon during its expansion. For a horizon shining throughout the age of the universe t_0 while emitting the maximum power $c^5/4G$, we get

$$N_{\text{b0}} \leq \frac{t_0 c^5/4G}{m_{\text{b}} c^2} = 2.6 \cdot 10^{79} . \quad (200)$$

Ref. 211 Equality would hold only if the contributions of photons, electrons, neutrinos and dark matter could be neglected. In short, using the age $t_0 = 13.8$ Ga, the strand model predicts that at most $2.6 \cdot 10^{79}$ baryons exist in the universe at present. Modern measurements indeed give values around this limit.

Ref. 211 In other terms, the strand model states that the sum of all particle energies in the universe is at most $t_0 c^5/4G$, or 50 % of the critical density; this includes observable matter as well as dark matter. The experimental value for the total matter density is about 26 % of the critical density. In observations, 4 % of the matter density is observed, and 22 % is

Page 351 dark. We will discuss the nature of dark matter later on.

The strand model also makes a clear statement on the change of matter density with time. As just explained, the number of baryons is predicted to increase with time t , due to their appearance at the horizon. Also the radius will increase (roughly) with time; as a result,

- ▷ The strand model predicts that matter density decreases roughly as $1/t^2$.

This unexpected prediction contrasts with the usually assumed $1/t^3$ dependence in a matter-dominated universe. The prediction has yet to be tested with observations. We note that the strand arguments imply that the ratio between matter density and vacuum energy density is a quantity related to the details of the radius increase during the history of the universe.

OPEN CHALLENGE: WHAT IS DARK MATTER?

Challenge 190 ny

In the arguments above, is there a factor of order 2 missing that induces incorrect conclusions about dark matter density? Might the prediction of dark matter increase, decrease or even disappear after correction of this missing numerical factor?

Conventionally, it is argued that *cold* dark matter exists for two reasons: First, it is necessary to grow the density fluctuations of the cosmic microwave background rapidly enough to achieve the present-day high values. Secondly, it is needed to yield the observed amplitudes for the acoustic peaks in the cosmic background oscillations. Can the strand model change these arguments?

Challenge 191 ny

How does the dark matter prediction of the strand model explain the galaxy rotation curves? A related issue is the following: Could tangle effects at the scale of a full galaxy be related to dark matter?

THE TOPOLOGY OF THE UNIVERSE

In the strand model, physical space-time, whenever it is defined, *cannot* be multiply connected. Also all quantum gravity approaches make this prediction, and the strand model confirms it: because physical space-time is a result of averaging strand crossing switches, non-trivial topologies (except black holes) do not occur as solutions. For example, the strand model predicts that wormholes do not exist. In regions where space-time is undefined – at and beyond horizons – it does not make sense to speak of space-time topology. In these regions, the fluctuations of the universal strand determine observations. In short, the strand model predicts that all searches for non-trivial *macroscopic* (and microscopic) topologies of the universe, at both high and low energies, will yield negative results. So far, this prediction agrees with all observations.

PREDICTIONS OF THE STRAND MODEL ABOUT COSMOLOGY

In the domain of cosmology, the strand model makes the following testable predictions.

- The universe is not empty. (Agrees with observation.)
- Its integrated luminosity saturates the power limit $c^5/4G$. (Agrees with observation.)

- The energy density of the universe saturates the entropy bound. (Agrees with observation.)
- There are no singularities in nature. (Agrees with observation.)
- Dark energy exists and results from vacuum/strand fluctuations. (Agrees with observation.)
- *Dark energy, or vacuum energy, is completely described by a cosmological constant Λ that is positive and changes with the radius R of the universe as $1/R^2$.* (This prediction differs from the usual cosmological models, which assume that Λ is constant or changes with time in other ways. The strand prediction might be checked in the near future by testing whether the minimum acceleration around galaxies changes with distance – if this minimum is related to Λ .)
- The number of baryons in nature is limited by the maximum luminosity times the age of the universe. The present upper limit is $2.6 \cdot 10^{79}$ baryons. (Agrees with observation.)
- The matter density of the universe decreases with age, roughly as $1/t^2$. (Checks are under way. This prediction differs from the usual cosmological models.)
- There is nothing behind the cosmic horizon. Matter, energy and space appear at the horizon. (Agrees with observations and requirements of logic.)
- Early density fluctuations are scale-invariant. (Agrees with observation.)
- The universe is flat and homogeneous. (Agrees with observation.)
- Apart from the cosmological constant Λ , there are all other fundamental constants of nature are constant over time and space. (Agrees with observation, despite regular claims of the contrary.)
- Inflation is unnecessary.
- The universe's topology is trivial. (Agrees with observation.)
- The above statements are independent of the precise fluctuation details. (Can be tested with mathematical investigations.)

All these predictions can and will be tested in the coming years, either by observation or by computer calculations.

SUMMARY ON MILLENNIUM ISSUES ABOUT RELATIVITY AND COSMOLOGY

We have deduced special relativity, general relativity and cosmology from the strand model. The fundamental principle of the strand model implies the invariant Planck units, the Lagrangian and action of general relativity, the finiteness of the universe and, above all, it explains in simple terms the entropy of black holes.

Space-time foam is replaced by the strand model of the vacuum: empty space is the time-average of untangled strands. More precisely, space is the thermodynamic average of crossing switches that are due to shape fluctuations of untangled strands.

The strand model – and in particular, the strand model of the vacuum – explains the number of space-time dimensions, the vacuum energy density, the matter density and the finiteness of the universe. The cosmological constant is a consequence of the finite size of the universe. The issue of the initial conditions of the universe has been defused.

The macroscopic and microscopic topology of the universe has been clarified. And dark matter is predicted to be, as shown in the next chapter, a combination of conventional matter and black holes.

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The most important predictions of the strand model are the decrease of the cosmological constant with time and the absence of inflation. Various experiments will test these predictions with increased precision in the coming years. So far, measurements do not contradict these predictions.

The strand model confirms that the speed of light c and the corrected Planck force $c^4/4G$ are *limit* values. The strand model also predicts that no variation in space and time of c , G , \hbar and k can be detected, because they define all measurement units.

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The strand model predicts that the cosmological constant and the masses of the elementary particles are the *only* quantum effects that will be observed in the study of gravitation. Strands strongly suggest that additional effects of quantum gravity cannot be measured. In particular, no effects of space-time foam will be observed.

The strand model is, at present, the simplest – but not the only – known model of quantum gravity that allows deducing all these results. In particular, the strand explanation of black hole entropy is by far the simplest explanation known.

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General relativity is an approximation of the strand model. The approximation appears when the quantum of action and, in particular, the strand diameter are neglected. Therefore, general relativity is predicted to be valid for all energies below the Planck energy. In other words, the strand model is not a generalization of general relativity, in contrast to other attempts at unification. This conforms to the list of requirements for the final theory.

Page 8

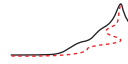
In the ‘cube’ structure of physics shown in [Figure 1](#), the transition from the final, unified theory to general relativity occurs by neglecting quantum effects, i.e., by approximating \hbar as 0. In the strand model, the transition is from the description with strands to a description with continuous variables: neglecting the strands in the strand model – more precisely, averaging over crossing switches of strands with zero diameter – leads to general relativity and cosmology.

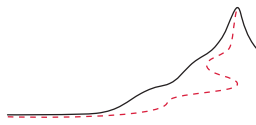
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If we look at the millennium list of open issues in physics, we see that – except for the issue of dark matter – all issues about general relativity and cosmology have been settled. The strand model explains the mathematical description of curved space-time and of general relativity. The strand model also provides a simple model – maybe the simplest known one – of quantum gravity. Above, we had already shown that the strand model explains all mathematical structures that appear in quantum theory and in particle physics. Together with the results from this chapter we can now say: *the strand model explains all mathematical structures that appear in physical theories*. In particular, strands explain the metric, curvature, wave functions, field intensities – and the probabilistic behaviour of all of them. They all result from averaging crossing switches.

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In summary, starting from the fundamental principle of the strand model, we have understood that strands are the origin of gravitation, general relativity, quantum gravity and cosmology. We have also understood the mathematical description of gravitation – and, before, that of quantum physics – found in all textbooks. These results encourage us to continue our quest. Indeed, we are not done yet: we still need to deduce the possible elementary particles and to explain their properties.





THE PARTICLE SPECTRUM DEDUCED FROM STRANDS

Ref. 214

“No problem can withstand the assault of sustained thinking.”
Voltaire**

Page 161

Strands describe quantum theory, gauge interactions and general relativity. But do strands also settle all issues left open by twentieth-century physics? Do they settle the origin of all the elementary particles, their quantum numbers, their masses and their mixing angles? How does the infinite number of possible tangles lead to a finite number of elementary particles? And finally, do strands explain the coupling constants? In the millennium list of open issues in fundamental physics, these are the issues that remain. The strand model is correct only if these issues are resolved.

In this chapter, we show that the strand model indeed explains the known spectrum of elementary particles, including the three generations of quarks and leptons. The strand model is the first approach of modern physics that can provide such an explanation.

It should be stressed that from this point onwards, the ideas are particularly speculative. In the chapters so far, the agreement of the strand model with quantum field theory and general relativity has been remarkable. The following chapters assign specific tangles to specific particles. Such assignments are, by nature, not completely certain. The speculative nature of the ideas now becomes particularly apparent.

PARTICLES AND QUANTUM NUMBERS FROM TANGLES

In nature, we observe three entities: vacuum, horizons and particles. Of these, (quantum) *particles* are *localized* entities with specific *intrinsic* properties, i.e., properties that do not depend on their motion.

In nature, all the intrinsic properties of every particle, every object and every image are completely described by three types of *basic* properties: (1) the elementary particles they contain, (2) their behaviour under space-time transformations, (3) their interactions. The full list of these basic intrinsic properties of particles is given in [Table 12](#). Given the basic properties for each elementary particle, physicists can deduce *all* those intrinsic particle properties that are *not* listed; examples are the half life, decay modes, branching ratios, electric dipole moment, T-parity, gyromagnetic ratio or electric polarizability. Of course, the basic properties also allow physicists to deduce *every* property of every object or image, such as size, shape, colour, density, elasticity, brittleness, magnet-

** Voltaire (b. 1694 Paris, d. 1778 Paris) was an influential philosopher, politician and often satirical writer.

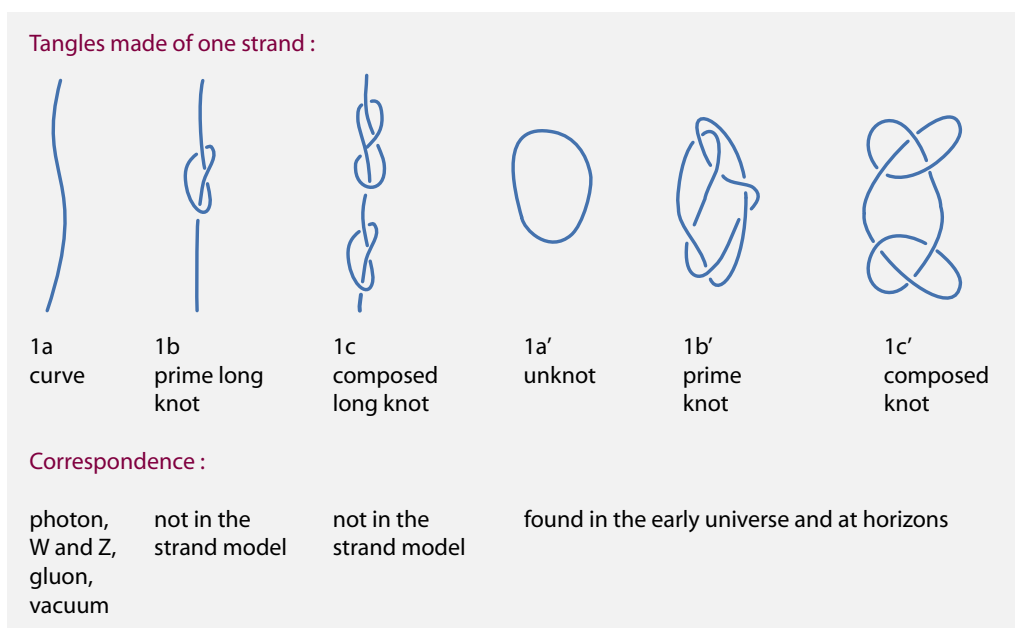


FIGURE 81 Examples for each class of tangles made of one strand.

ism or conductance.

In short, understanding *all* properties of matter and images thus only requires understanding the *basic* properties of quantum particles; and understanding the *basic* properties of quantum particles only requires understanding the *basic* properties of the *elementary* particles.

The strand model states that all elementary (and all composed) particles are tangles of strands. This leads us to ask: Which tangle is associated to each elementary particle? What kinds of elementary particles are possible? Do these tangles reproduce, for each elementary particle, the observed values of the basic properties listed in Table 12?

It turns out that the strand model only allows a *limited number of elementary* particles. In addition, the tangles of these elementary particle have intrinsic properties that *match* the observed properties. To prove these strong statements, we first recall that all massive elementary particles are represented by an *infinite sequence* of tangles. We now explore tangles according to the number of strands they are made of.

PARTICLES MADE OF ONE STRAND

In the strand model, all particles made of *one* strand have spin 1, are elementary, and are bosons. Conversely, all massless elementary spin-1 bosons can only have two tails, and thus must be made of a single strand. Such one-stranded tangles return to the original strand after a core rotation by 2π . Massive elementary spin-1 bosons can have one or more strands. Tangles of more than one strand can only have spin 1 if they represent massive elementary or composed particles. In short, classifying one-stranded tangles allows classifying all elementary gauge bosons.

TABLE 12 The full list of *basic* intrinsic properties of quantum particles, from which all other observed intrinsic properties of particles, objects and images can be deduced.

PROPERTY	POSSIBLE VALUE	DETERMINES
Quantum numbers due to space-time symmetries:		
Spin S or J	integer or half-integer multiple of \hbar	statistics, rotation behaviour, conservation
P parity	even (+1) or odd (-1)	behaviour under reflection, conservation
C parity	even (+1) or odd (-1)	behaviour under charge conjugation, conservation
Interaction properties:		
Mass M	between 0 and the Planck mass	gravitation, inertia
Electric charge Q	integer multiples of one third of electron or proton charge	Lorentz force, coupling to photons, conservation
Weak charge	rational multiple of weak coupling constant	weak scattering and decays, coupling to W and Z, partial conservation
Mixing angles	between 0 and $\pi/2$	mixing of quarks and neutrinos, flavour change
CP-violating phases	between 0 and $\pi/2$	degree of CP violation in quarks and neutrinos
Strong charge, i.e., colour	rational multiple of strong coupling constant	confinement, coupling to gluons, conservation
Flavour quantum numbers, describing elementary particle content:		
Lepton number(s) L'	integer(s)	conservation in strong and e.m. interactions
Baryon number B	integer times $1/3$	conservation in all three gauge interactions
Isospin I_z or I_3	$+1/2$ or $-1/2$	up and down quark content, conservation in strong and e.m. interactions
Strangeness S'	integer	strange quark content, conservation in strong and e.m. interactions
Charmness C'	integer	charm quark content, conservation in strong and e.m. interactions
Bottomness B'	integer	bottom quark content, conservation in strong and e.m. interactions
Topness T'	integer	top quark content, conservation in strong and e.m. interactions

Mathematicians have already classified one-stranded tangles; they are usually called *open knots* or *long knots*. To get an overview, we list an example for each class of one-stranded tangles on the left-hand side of [Figure 81](#). For completeness, closed curves are shown on the right-hand side of the figure. We now explore each of these classes of curves.

UNKNOTTED CURVES

The simplest type of tangle made of one strand is an *unknotted curve*, shown as example 1a in [Figure 81](#). The study of gauge interactions has shown that unknotted strands are, depending on their precise average shape, either vacuum strands or gauge bosons.

The time-average of a vacuum strand is straight. A single strand represents a particle if the time-averaged strand shape is not a straight line.

In the strand model, vacuum strands in flat space are, on average, *straight*. In this property, vacuum strands differ from gauge bosons, which, on average, have *curved* strands, and thus carry energy.

GAUGE BOSONS – AND REIDEMEISTER MOVES

Gauge bosons are the carrier particles of the interactions. In the strand model, the gauge interactions are due to the three Reidemeister moves. The electromagnetic, the weak and the strong interaction correspond to respectively the first, second and third Reidemeister move. As we have seen above, when the three Reidemeister moves deform fermion tangle cores they generate U(1), SU(2) and SU(3) gauge symmetries. The detailed exploration of the correspondence between tangle deformation and gauge theory led us to the gauge boson tangles shown in [Figure 82](#). All gauge bosons – before symmetry breaking when applicable – are single curved strands.

A single strand represents a particle if the time-averaged strand shape is not a straight line. A particle strand can thus be a strand with a bulge or a strand whose tails are not aligned along a straight line.

As explained above, the *first* Reidemeister move, the twist, leads to the modelling of photons as helical strands. Therefore, photons have vanishing mass and two possible polarizations. Photons do not have knotted family members; they are massless. Their specific unknotted and twisted strand shapes also imply that photons generate an Abelian gauge theory and that photons do not interact among themselves. Automatically, photons have no weak and no strong charge. The strand model further implies that photons have negative P-parity and C-parity, as is observed.

The study of the *second* Reidemeister move, the *poke*, showed that deformations induced by pokes can also involve braiding of tangle tails; this leads to the symmetry breaking of the weak interaction. As a result, the observed W and the Z boson strands become massive. The tangle of the W is chiral, and thus it is electrically charged; the tangle of the Z is achiral and thus electrically neutral. Being knotted, the W and the Z also carry weak charge and thus interact among themselves, generating a *non-Abelian* gauge theory. The strand model also implies that the W and the Z have no P-parity, no C-parity and no colour charge, as is observed.

The study of the *third* Reidemeister move, the slide, led us to the existence of eight gluons. The eight gluons are unknotted, thus they carry no mass, no electric charge and

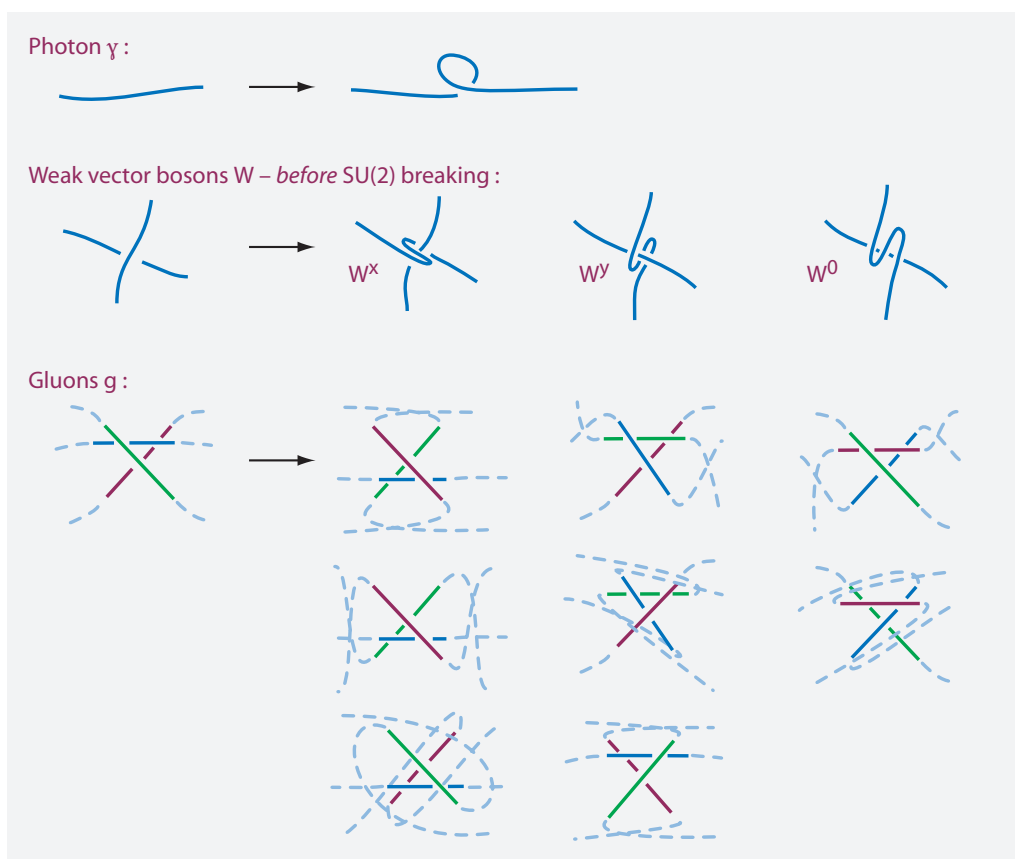


FIGURE 82 The gauge bosons in the strand model. All differ from vacuum by one curved strand – though, for clarity, the gluons are shown here using their complementary two-strand moves.

no weak charge. Each gluon tangle has two possible polarizations. The strand model of gluons also implies that they have negative P-parity and no C-parity, as is observed. Gluons tangles carry colour and interact among themselves, thus they generate a non-Abelian gauge theory. In contrast to the other two interactions, free, single gluons are short-lived, because their structure induces rapid hadronization: when gluons act on the vacuum, quark–antiquark pairs are produced. Gluons do not have knotted family members; they are massless in the high energy limit, when their tails are aligned.

For completeness we mention that by assignment, all gauge bosons differ from vacuum by a single curved strand, have vanishing lepton and baryon numbers, and thus also lack all flavour quantum numbers. All this is as observed.

The strand model explains the lack of *classical* $SU(2)$ field waves as a consequence of the breaking of the $SU(2)$ symmetry and the consequent mass of the weak bosons. Strands explain the lack of *classical* $SU(3)$ waves, also called *gluonic waves*, as a consequence of the topological impossibility to produce such waves, which is related to the infinite mass of single free gluons.

In somewhat sloppy language we can say that the shape and the effects of photons are one-dimensional, those of the unbroken weak bosons are two-dimensional, and those

of the gluons are three-dimensional. This is the essential reason that they reproduce the $U(1)$, $SU(2)$ and $SU(3)$ groups, and that no higher gauge groups exist in nature.

In summary, Reidemeister's theorem implies that the list of known gauge bosons with spin 1 is complete. But the list of possible tangles made of a single strand is much more extensive; we are not done yet.

OPEN OR LONG KNOTS

Page 250 Strands could also contain knotted regions. We have explained earlier on that all such possibilities – mathematically speaking, all so-called *open knots* or *long knots* – have no relation to particles. In the strand model, they cannot appear and thus play no role. The original strand model from 2008 did include such configurations as particles (for example as W and Z bosons), but it now – i.e., after 2014 – seems that this inclusion is an unnecessary complication.

CLOSED TANGLES: KNOTS

Figure 81 shows, on the right-hand side, examples for all classes of *closed* tangles of one strand, i.e., of tangles *without tails*. They are usually just called *knots* in mathematics. In the strand model knots may appear only in the early universe, and maybe near horizons. They do not seem to have physical relevance and we do not explore them here.

SUMMARY ON TANGLES MADE OF ONE STRAND

In summary, a single strand represents a particle if the strand shape is, on average, not a straight line. This distinguishes a vacuum strand from a particle strand. A particle strand can thus be a strand with a bulge or a strand whose tails are not aligned along a straight line. All tangles made of *one strand* represent *elementary* particles of spin 1, thus elementary vector bosons.

Massless elementary spin-1 particles are made of one strand also because other tangles cannot reproduce both zero mass and the spin-1 behaviour under rotations: only one-stranded tangles return to the original strand after a core rotation by 2π and allow vanishing mass at the same time.

In the strand model, *all* tangles made of one curved strand are assigned to the *known* gauge bosons. The strand model correctly reproduces and thus explains the gauge boson spectrum and the quantum numbers for each gauge boson. In short, there is *no* room for additional elementary gauge bosons.

Page 273 In other words, the strand model predicts that all gauge bosons and thus all interactions are already known. We have thus a second argument – after the non-existence of other gauge groups – stating that no other gauge interaction exists in nature. (Both arguments against the existence of other gauge interactions are related; in particular, both are due to the three-dimensionality of space.) In particular, we find again that grand unification and supersymmetry are not allowed in nature.

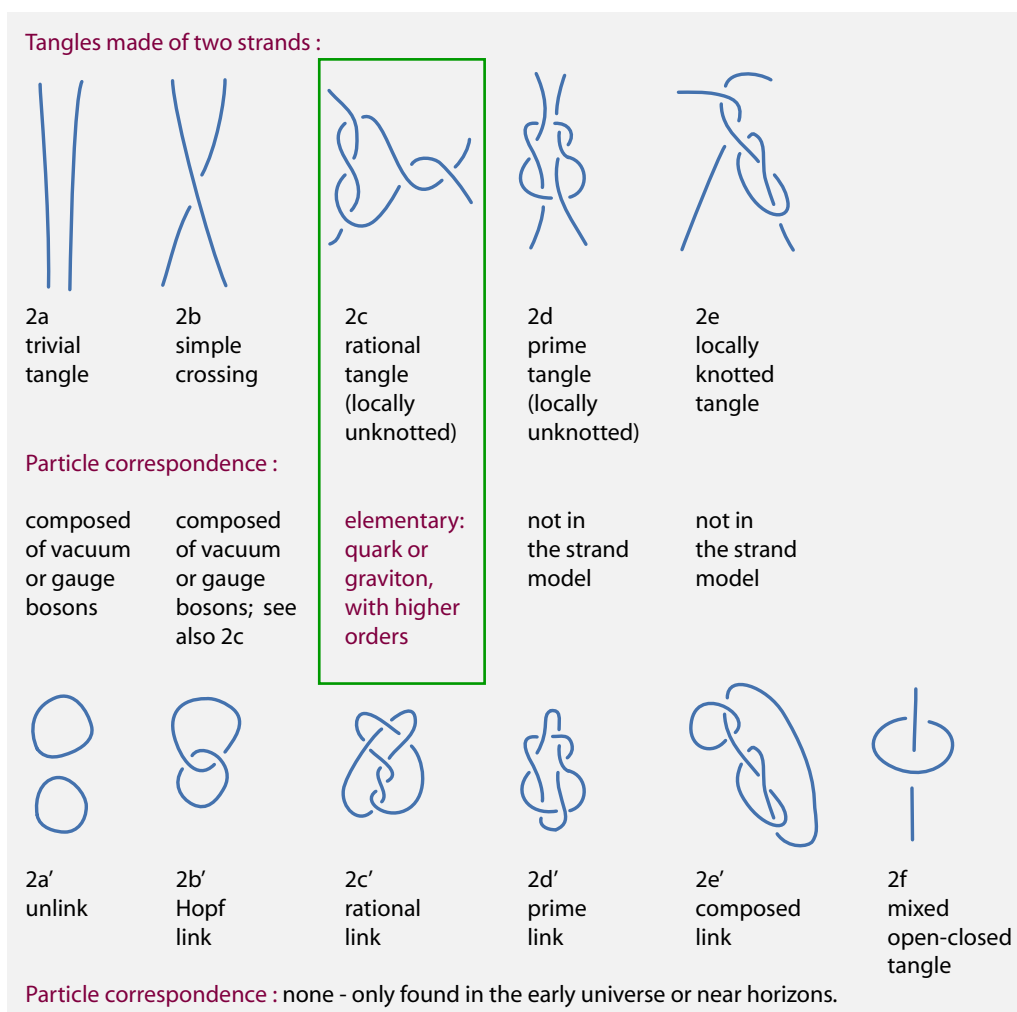


FIGURE 83 Possible tangles made of two strands.

PARTICLES MADE OF TWO STRANDS

In the strand model, particle tangles can also be made of *two* strands. Examples for all the classes of two-stranded tangles are given in Figure 83. Each class has a physical particle assignment.

- The simplest tangle made of two strands is the *trivial tangle*, shown as example 2a in Figure 83. In the strand model, the trivial tangle, like all *separable* tangles, is a *composite* system. Each of the two strands can represent either the vacuum or a gauge boson. Simply stated, the trivial tangle of two strands is not an elementary particle.
- The simplest non-trivial tangle made of two strands is the *crossing*, shown as 2b in Figure 83. In the strand model, the crossing appears as part of the W and Z bosons;

- in addition, for certain tail configurations, it can represent a graviton or the simplest state of a down quark, as we will see below.
- A new class of tangles are the *rational tangles*, represented by example 2c in the figure. A rational tangle is a tangle that can be untangled by moving its tails *around*. (Also example 2b is a rational tangle.) Rational tangles are distinct from prime and from locally knotted tangles, shown as examples 2d and 2e, which require pulling the tail *through* the tangle to untangle it. Rational tangles are thus *weakly* tangled. As we will see, rational tangles represent the *graviton* and the *quarks*; we will discuss them in detail in the next two sections. More complicated rational tangles are higher-order propagating states of the simpler ones.
 - Another class of tangles are *prime tangles*, for which the tangle 2d is an example. Like complicated one-stranded tangles, we conclude that prime tangles are not part of the strand model.
 - Still another class of tangles are *locally knotted tangles*, shown as example 2e. Also this class is not part of the strand model.
 - Finally, *closed tangles, links* and *mixed tangles*, shown in the lower row of [Figure 83](#), have no role in the strand model – except maybe during the very early universe or near horizons.

In short, the only two-stranded tangles of interest in the strand model are the rational tangles. We now explore them in more detail.

QUARKS

[Page 268](#) The exploration of the strand model and of the strong interaction showed: the tangle of a coloured fermion, thus of a quark, must be rational, must reproduce the three possible colour options, and must break the three-belt symmetry.

The simplest tangles that realize these requirements are shown in [Figure 84](#): quark tangles are *rational tangles* made of *two* strands. Higher quark generations have larger crossing numbers. The four tails form the skeleton of a tetrahedron. A particle with two strands tangled in this way automatically has spin $1/2$. The electric charges of the quarks are $1/3$ and $-2/3$, an assignment that is especially obvious for up and down quarks and that will become clearer later on, in the study of hadrons. Parity is naturally assigned as done in [Figure 84](#). Baryon number and the other flavour quantum numbers – isospin, strangeness, charm, bottomness, topness – are naturally assigned as usual. The flavour quantum numbers simply ‘count’ the number of corresponding quark tangles. Like all localized tangles, quarks have weak charge. We will explore weak charge in more detail below. Antiquarks are mirror tangles and have opposite quantum numbers. We will see below that these assignments reproduce the observed quantum numbers of all mesons and baryons, as well as all their other properties.

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[Page 331, page 339](#)

We note that the simplest version of the down quark is a simple crossing; nevertheless, it differs from its antiparticle, because the simple crossing mixes with the braid with seven crossings, 13 crossings, etc.; this mixing is due to the leather trick, as shown below. And for every quark type, these more complicated braids differ from those of their antiparticles.

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For each quark, the four tails form the skeleton of a tetrahedron. In [Figure 84](#) and

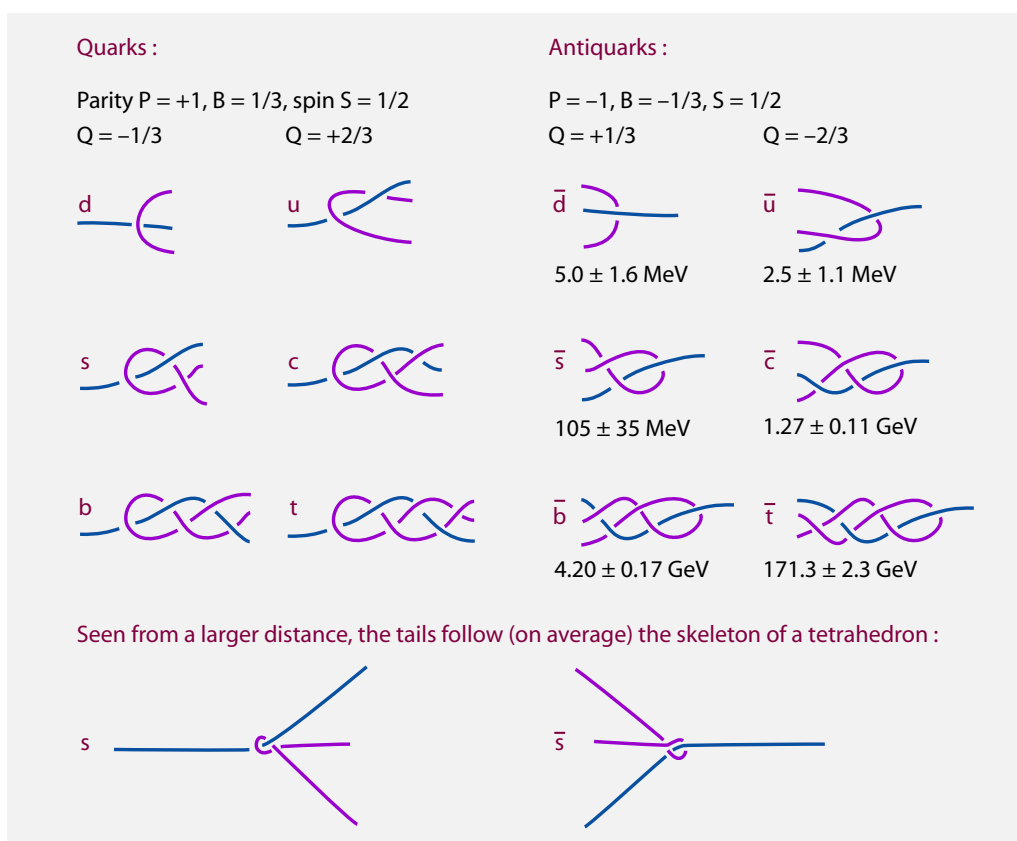


FIGURE 84 The simplest tangles assigned to the quarks and antiquarks. For reference, the experimental mass values are also given.

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Figure 85, the tetrahedral skeletons are drawn with one tail in the paper plane; of the other three tails, the middle one is assumed to be *above* the paper plane, and the outer two tails to be *below* the paper plane. This is important for the drawing of quark compounds later on. The three tails allow us to reproduce the strong interaction and the colour charge of the quarks: each colour is one of three possible orientations in space; more precisely, the three colours result from the three possible ways to map a quark tangle to the three belt structure. Each colour corresponds to a different choice for the tail that lies above the paper plane, as shown in Figure 85. The colour interaction of quarks will be clarified in the section on mesons.

In the strand model, the quark tangles thus carry *colour*. In nature, no free coloured particle has been observed. The strand model reproduces this observation in several ways. First of all, all leptons and baryons are colour-neutral, as we will see shortly. Secondly, only free quark tangles, as shown in Figure 84, have a definite colour state, because they have a fixed orientation in space. Thirdly, free quark states, thus quark states in the tetrahedral configuration of Figure 84, do not fit into vacuum even at large distances from the core; thus free quarks carry infinitely high energy. In practice, this means that free quark states do not occur in nature. Indeed, a free, coloured quark tangle can reduce its energy by interacting with one or several other quarks. The result is a strong

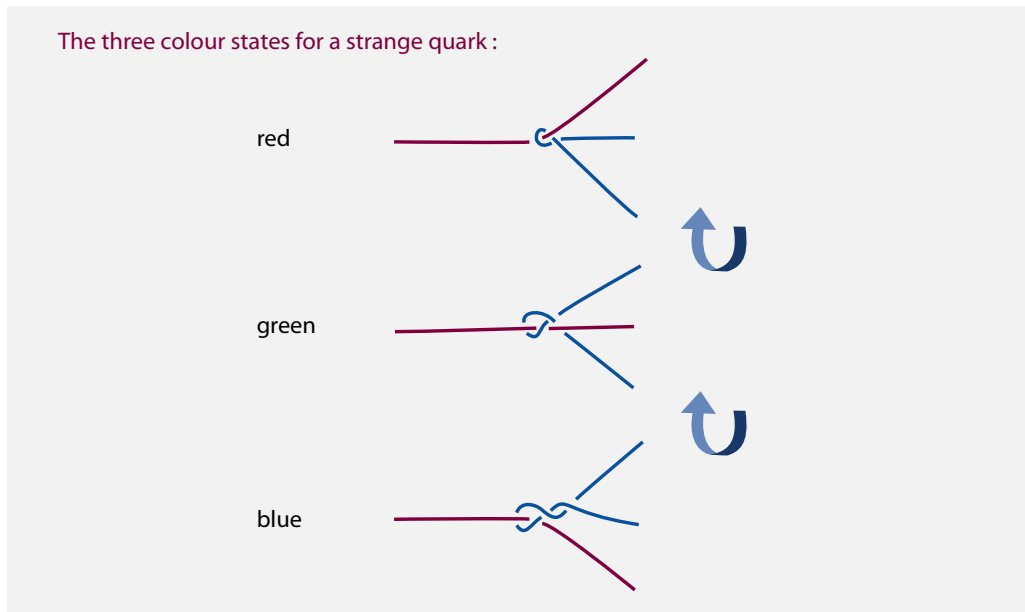


FIGURE 85 The three colour charges correspond to the three possible spatial orientations; the centre tail on the right is always above the paper plane, the other two tails on the right are below the paper plane.

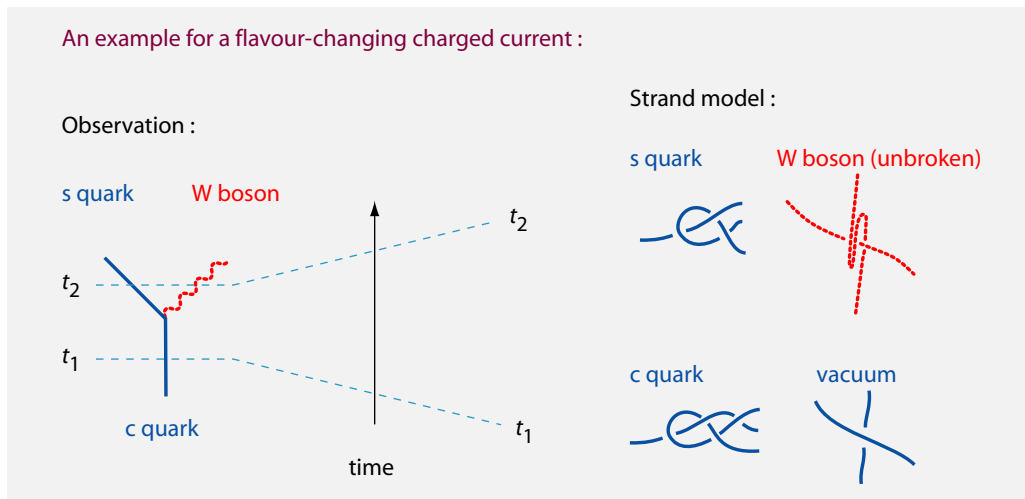


FIGURE 86 Absorption or emission of a W boson changes quark flavour.

colour attraction between quarks that leads to colourless composites.

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Page 336

In short, also in the strand model, only colourless composites of quarks exist as stable free particles. We will explore quark composites and the issue of confinement of quarks in more detail shortly.

In nature, quarks are weakly charged and interact with W bosons. In the strand model, the absorption or the emission of a W boson is the operation that takes a quark tangle and adds or subtracts a braiding step. This process is illustrated in Figure 86, which

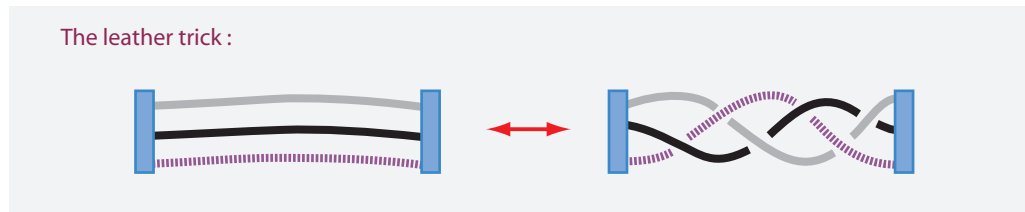


FIGURE 87 The leather trick is the deformation process that changes these two structures into each other. The leather trick limits structures made of three-stranded braids to six basic types.

shows that a braiding (unbraiding) operation corresponds to the emission (absorption) of an W boson before symmetry breaking. It is straightforward to check that this operation fulfils all conservation laws and properties that are observed for these so-called *flavour-changing charged currents*. The absorption or emission of an (unbroken) Z boson has no braiding effect. The strand model thus reproduces the result that only the charged weak bosons can change quark flavours, as is observed.

Page 324 For completeness, we mention that quarks, being tangles of *two* strands, have vanishing lepton number. Indeed, as we will see below, lepton tangles are made of *three* strands.

In summary, all quantum numbers of quarks are reproduced by the strand model, as long as quarks are modelled as braids of two strands with ends directed along the corners of a tetrahedron.

QUARK GENERATIONS

We stress that the quark tangles shown [Figure 84](#) represent only the *simplest* tangle for each quark. First of all, longer braids are mapped to each of the six quarks. This might seem related to the *leather trick* shown in [Figure 87](#). This trick is well-known to all people in the leather trade: if a braid of three strands has $n \geq 6$ crossings, it can be deformed into a braid with $n - 6$ crossings. We might conjecture that, due to the leather trick, there is no way to introduce more than 6 quarks in the strand model.

Page 360 In fact, the leather trick argument assumes that the braid end – and thus the ends of the strands – can be moved *through* the braids. In the strand model, this can only happen at the horizon, the only region where space (and time) are not well-defined, and where such manipulations become possible. The low probability of such a process will be important in the determination of quark masses.

Page 329 Instead of resting on the leather trick, it is simpler to assume that braids with large numbers of crossings are mapped modulo 6 to the braids with the smallest number of crossings. This is consistent, because in the strand model, a braid with six additional crossings is mapped to a particle together with a virtual Higgs boson. The modulo 6 rule thus represents the Yukawa mass generation mechanism in the strand model.

In summary, in the strand model, each quark is not only represented by the tangles shown in [Figure 84](#), but also by tangles with 6 additional crossings, with 12 additional crossings, etc.

As a mathematical check, we can also ask whether *all* other rational tangles are mapped to quarks. Rational tangles of higher complexity arise by repeatedly twisting any pair of tails of a quark tangle. This process produces an infinite number of complex

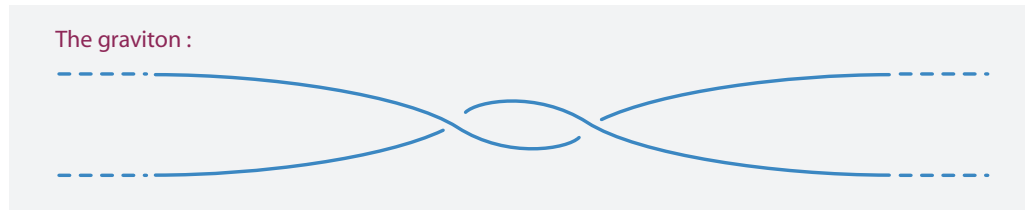


FIGURE 88 The graviton in the strand model.

two-stranded tangles. In the strand model, these tangles are quarks surrounded by virtual particles. Equivalently, we can say that all the more complex rational tangles that do not appear in Figure 84 are higher-order propagators of quarks.

Challenge 192 e

THE GRAVITON

Page 294 One rational tangle made of two strands is special. This special tangle is shown (again) in Figure 88. It differs from a quark tangle in one property: the tails are parallel (and near) to each other, and thus lie in a plane. Its tangle core returns to its original state after rotation by π , and therefore models a spin-2 particle. The tangle is not knotted and not localized; thus it has no mass, no electric and no weak charge. It also has no colour charge, as expected from the graviton. Similar tangles with higher winding numbers represent higher orders in the perturbation theory of gravitation.

Page 277 The chapter on gravitation has already shown how gravitons lead to curvature, horizons and the field equations of general relativity.

GLUEBALLS

Ref. 215, Ref. 216 There is no observational evidence for glueballs yet, even though simulations of QCD on the lattice predict the existence of several such states in the $1.5 \text{ GeV}/c^2$ mass range. The lack of experimental confirmation is usually explained by the strong background noise in the reaction that produces glueballs, and by the expected strong mixing with mesons of similar quantum numbers. The experimental search for glueballs is still ongoing.

The lowest-mass glueball is usually expected to be made of two gluons. In the strand model, a glueball made of two gluons would be made of two curved strands. However, the strand model of gluons does not seem to allow such a tangle.

Could a situation in which two gluons are linked in such a way that the four tails are perpendicular and span a plane lead, through averaging, to a zero spin value? The issue of glueballs needs a more precise investigation.

Challenge 193 ny

Ref. 217 Whatever the situation for glueballs might be, the strand model of gluons seems in contrast with the models of glueballs as knots that were proposed by Buniy and Kephart or by Niemi. These models are based on *closed* knots, not on tangles with tails. The strand model does not seem to allow real particles of *zero spin* that are composed of gluons. On the other hand, if closed knots were somehow possible in the strand model, they would imply the existence of glueballs.

Ref. 218

In summary, the issue of glueballs is not settled; a definitive solution might even lead to additional checks of the strand model.

THE MASS GAP PROBLEM AND THE CLAY MATHEMATICS INSTITUTE

Ref. 219 The Clay Mathematics Institute offers a large prize to anybody who proves the following statement: *For any compact simple non-Abelian gauge group, quantum gauge theory exists in continuous, four-dimensional space-time and produces a mass gap.* This is one of their so-called *millennium problems*.

The strand model does not allow arbitrary gauge groups in quantum field theory. According to the strand model, the only compact simple non-Abelian gauge group of interest is $SU(3)$, the gauge group of the strong nuclear interaction. And since the strand model does not seem to allow for glueballs, for $SU(3)$ an effective mass gap of the order of the Planck mass is predicted. (If glueballs would exist in the strand model, the mass gap would still exist but be smaller.) Indeed, the strand model explains the short range of the strong interaction as a consequence of the details of Reidemeister III moves and the quark tangle topology.

The strand model further states that space-time and gauge groups are low-energy approximations, because neither points nor fields exist at a fundamental level; points and fields are approximations to strands. According to the strand model, the *quantum* properties of nature result from the extension of strands. As a consequence, the strand model denies the existence of *any quantum* gauge theory as a separate, exact theory on *continuous* space-time.

In summary, the strand model does predict a mass gap for $SU(3)$; but the strand model also denies the existence of quantum gauge theory for any other compact simple non-Abelian gauge group. And even in the case of $SU(3)$ it denies – like for any other gauge groups – the existence of a quantum gauge theory on continuous space-time. As deduced above, the strand model allows only the three known gauge groups, and allows their existence only in the non-continuous strand model of space-time. In short, it is *impossible* to realize the wish of the Clay Mathematics Institute.

A PUZZLE

Challenge 194 s The topic of two-stranded tangles also requires to solve the puzzle of [Figure 89](#). To which physical states do the three pictured tangles correspond?

SUMMARY ON TWO-STRANDED TANGLES

In summary, the strand model predicts that apart from the six quarks and the graviton, no other two-stranded elementary particle exists in nature. Concerning composite particles, the two-stranded glueball issue is not completely settled, but points towards non-existence.

Quarks and the graviton, the elementary particles made of two strands, are *rational* tangles. Their strand models are thus not tangled in a complicated way, but tangled in the *least complicated* way possible. This connection will be of importance in our search for elementary particles that are still undiscovered.

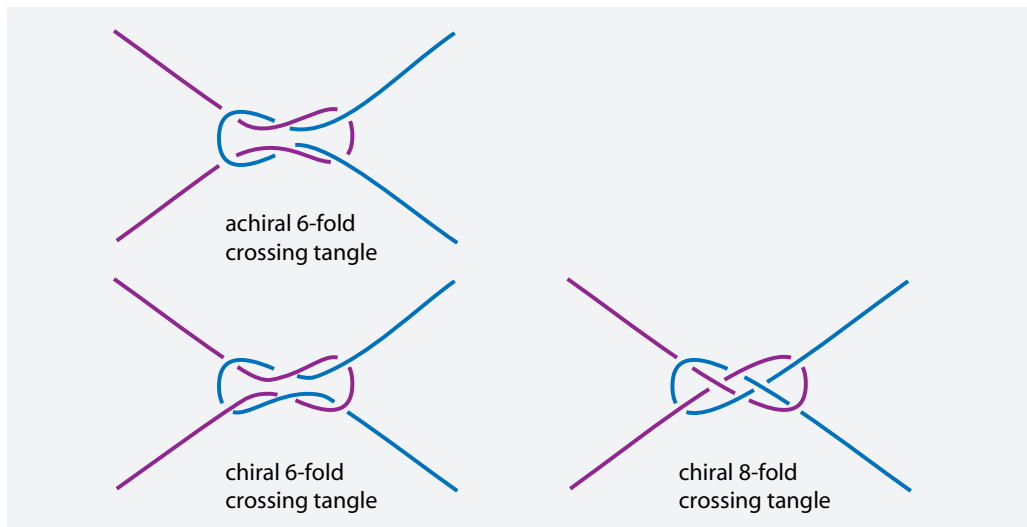


FIGURE 89 Which particle states are described by these tangles?

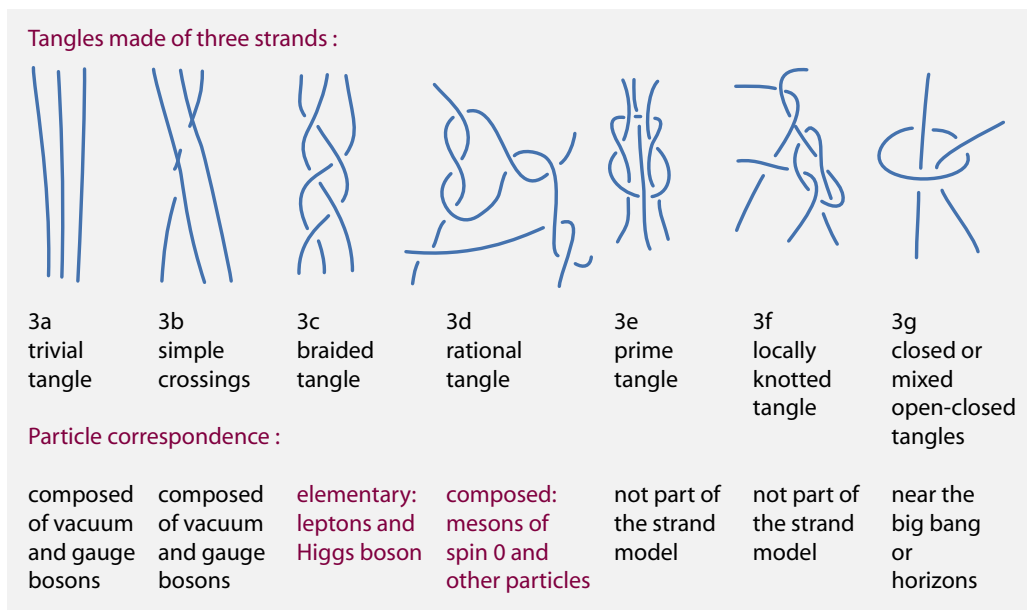


FIGURE 90 Examples for all the classes of tangles made of three strands.

PARTICLES MADE OF THREE STRANDS

In the strand model, the next group are particles made of *three* strands. Examples for all classes of three-stranded tangles are given in Figure 90. Several classes of three-stranded tangles turn out to be composites of two-stranded particles. However, a number of tangles are new and represent elementary particles.

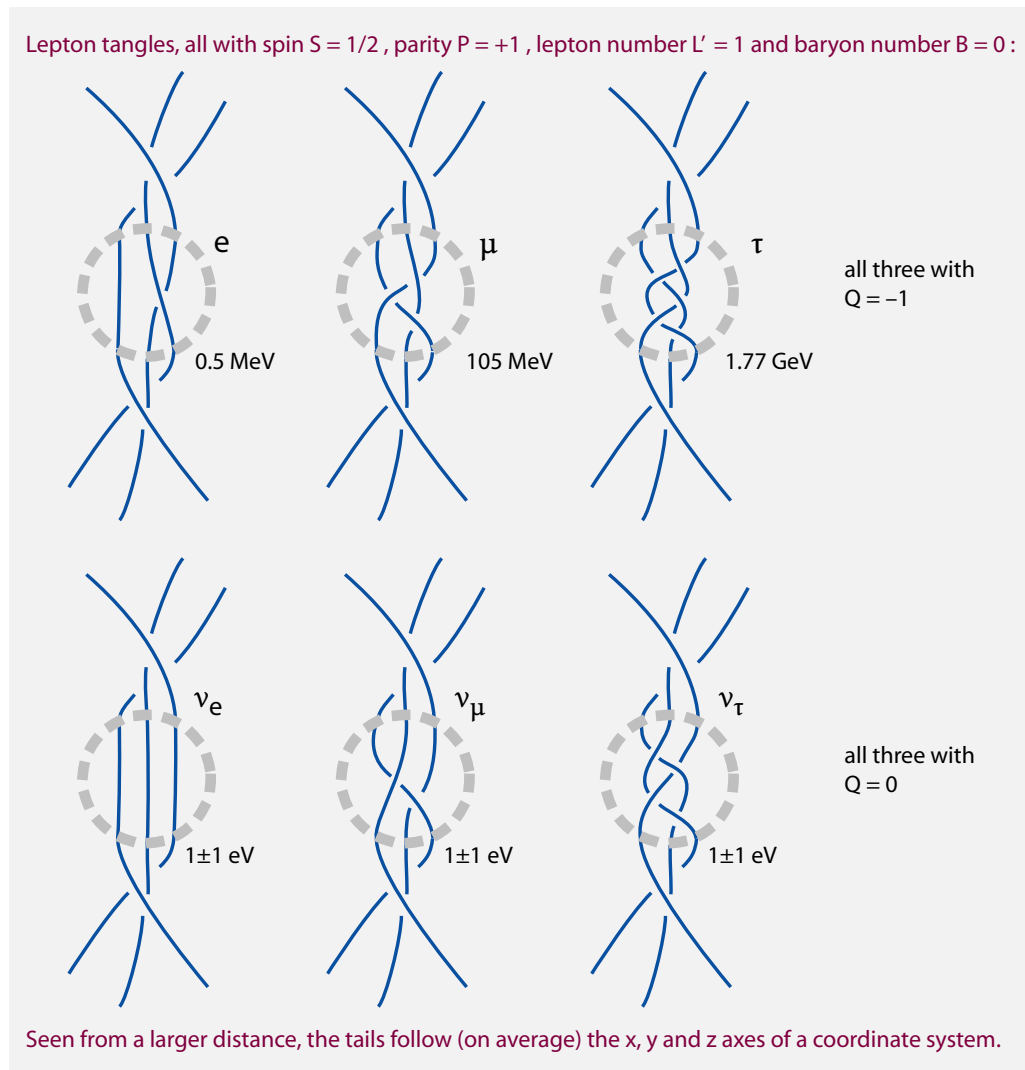


FIGURE 91 The simplest tangles of the leptons, with the experimental mass values. Antileptons are mirror tangles.

LEPTONS

The candidate tangles for the leptons shown in [Figure 91](#) are the simplest possible non-trivial tangles with three strands. These lepton tangles are simple braids with tails reaching the border of space. The six tails probably point along the coordinate axes. These braided tangles have the following properties.

- Each lepton is localized. Each lepton has mass and has spin $1/2$, two possible chiralities (spin orientations), and weak charge. Each lepton thus follows the Dirac equation.
- Leptons and antileptons differ. In particular, neutrinos and antineutrinos differ, and – again – both are predicted to show both chiralities.
- Three of the tangles are topologically chiral, thus electrically charged, and three other

- tangles are topologically achiral, thus uncharged.
- The spatial parity P of the charged lepton tangles is opposite to that of their anti-particles.
 - Being made of three strands, lepton tangles have vanishing colour charge and vanishing baryon number.
 - In contrast to quarks, lepton tangles can be inserted in the vacuum using a localized, i.e., finite amount of energy and are thus predicted to exist as free particles.
 - The three types of lepton (flavour) numbers can be assigned as usual; the lepton numbers are conserved in reactions, apart for neutrino mixing effects, as we will see below.
 - The strand model predicts that the electron, the charged tangle with the lowest mass, is stable, as there is no way for it to decay and conserve charge and spin. The other two generations are predicted to be unstable, due to weak decays that simplify their topology.
 - The number of generations is reproduced by the strand model, as every more complicated braid can be seen as equivalent to one of the first six braids, with the same leather trick/Higgs argument that limits the number of quarks.
 - There is a natural mapping between the six quarks and the six leptons that appears if the final bend of the ‘longer’ quark strand is extended to the border of space, thus transforming a two-stranded quark braid into a three-stranded lepton braid. Thus we get three common generations for quarks and leptons.
 - The neutrino strands differ by tail braiding; the strand model thus predicts that the weak interaction mixes neutrinos.
 - All lepton tangles differ from each other. Thus the mass values are different for each lepton.
 - Due to the small amount of tangling, the strand model predicts that the masses of the leptons are much smaller than those of the W and Z boson. This is indeed observed. (This also suggests a relation between the mass and the total curvature of a tight tangle.)
 - The simplest tangle for the electron neutrino also suggests that the mass values for the electron neutrino is naturally small, as its tangle is almost not tangled.
 - The strand model predicts that lepton masses increase with the generation number. Since the neutrino masses are not precisely known, this prediction cannot yet be checked.

In summary, tangles of three strands have precisely the quantum numbers and properties of leptons. In particular, the strand model predicts exactly three generations of leptons, and neutrinos are predicted to be Dirac particles. This implies that searches for the neutrino-less double beta decay should yield negative results, that the magnetic moments of the neutrinos should have the exceedingly small values predicted by the standard model of particle physics, and that rare muon and other decays should occur at the small rates predicted by the standard model.

OPEN CHALLENGE: FIND BETTER ARGUMENTS FOR THE LEPTON TANGLES

The argument that leads to the lepton tangles is vague. The tangle assignments might need corrections. (Improved tangles might solve the issue about the different charge

Ref. 220

Ref. 221

Page 384

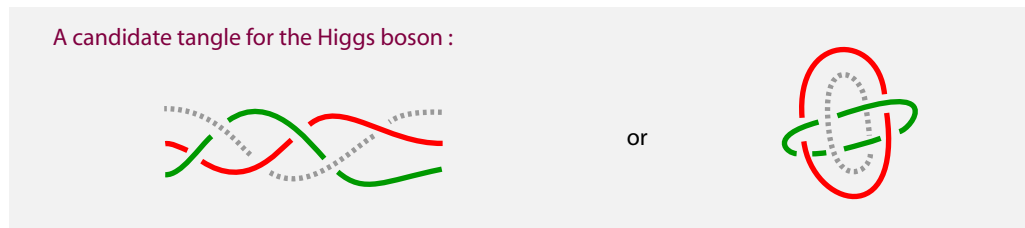


FIGURE 92 A candidate tangle for the Higgs boson in the strand model: the open version (left) and the corresponding closed version (right). For the left version, the tails approach the six coordinate axes at infinity.

Challenge 195 ny definition for leptons that is mentioned below.) Can you improve the situation?

THE HIGGS BOSON – THE MISTAKEN SECTION FROM 2009

The existence of the Higgs boson is predicted from the standard model of elementary particle physics using two arguments. First of all, the Higgs boson prevents unitarity violation in longitudinal W - W and Z - Z boson scattering. Secondly, the Higgs boson confirms the symmetry breaking mechanism of $SU(2)$ and the related mass generation mechanism of fermions. Quantum field theory predicts that the Higgs boson has spin 0, has no electric or strong charge, and has positive C and P parity. In other words, the Higgs boson is predicted to have, apart from its weak charge, the same quantum numbers as the vacuum.

In the strand model, there seems to be only one possible candidate tangle for the Higgs boson, shown on the left of [Figure 92](#). The tangle has positive C and P parity, and has vanishing electric and strong charge. The tangle also corresponds to the tangle added by the leather trick; it thus could be seen to visualize how the Higgs boson gives mass to the quarks and leptons. However, there are two issues with this candidate. First, the tangle is a deformed, higher-order version of the electron neutrino tangle. Secondly, the spin value is not 0. In fact, there is no way at all to construct a spin-0 tangle in the strand model. These issues lead us to reconsider the arguments for the existence of the Higgs boson altogether.

[Page 249](#) We have seen that the strand model proposes a clear mechanism for mass generation:

▷ *Mass* is due to strand braiding.

[Page 355](#) This mechanism, due to the weak interaction, explains the W and Z boson mass ratio, as we will see below. The leather trick that explains fermion masses can be seen as the addition of a sixfold tail braiding. In particular, the rarity of the braiding process explains why particle masses are so much smaller than the Planck mass. In short, the strand model explains mass *without* a Higgs boson.

[Page 206](#) If the Higgs boson does not exist, how is the unitarity of longitudinal W and Z boson scattering maintained? The strand model states that interactions of tangles in particle collisions are described by deformations of tangles. Tangle deformations in turn are described by unitary operators. Therefore, the strand model predicts that unitarity is never violated in nature. In particular, the strand model automatically predicts that the scat-

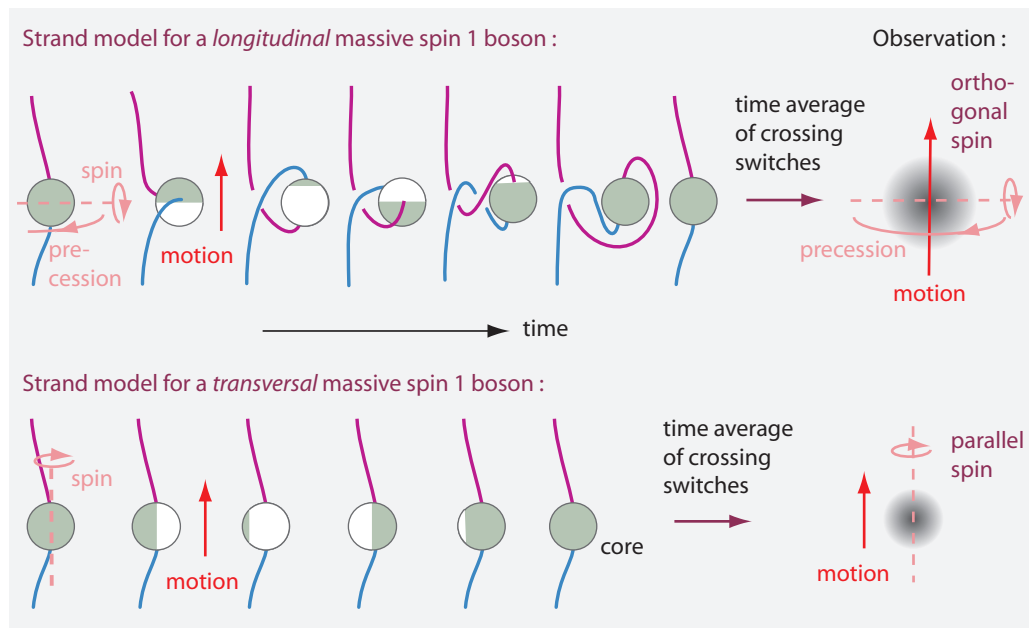


FIGURE 93 In the strand model, transverse and longitudinal W and Z bosons differ. (Note added in 2012: this statement is mistaken.)

tering of longitudinal W or Z bosons does *not* violate unitarity.

Ref. 222 In other terms, the strand model predicts that the conventional argument about unitarity violation, which requires a Higgs boson, must be wrong. How can this be? There are at least two loopholes available in the research literature, and the strand model realizes them both.

Ref. 223 The first known loophole is the appearance of non-perturbative effects. It is known for a long time that non-perturbative effects can mimic the existence of a Higgs boson in usual, perturbative approximations. In this case, the standard model could remain valid at high energy without the Higgs sector. This type of electroweak symmetry breaking would lead to longitudinal W and Z scattering that does not violate unitarity.

Ref. 224 The other loophole in the unitarity argument appears when we explore the details of the longitudinal scattering process. In the strand model, longitudinal and transversal W or Z bosons are modelled as shown in Figure 93. For longitudinal bosons, spin and its precession leads to a different situation than transversal bosons: longitudinal bosons are *more delocalized* than transversal bosons. This is not the case for fermions, where the belt trick leads to the *same* delocalization for longitudinal and transversal polarization. Interestingly, it is also known for a long time that different delocalization for longitudinal and transversal bosons *maintains* scattering unitarity, and that in the case of delocalization the conventional argument for the necessity of the Higgs boson is wrong. These are well-known consequences of the so-called *non-local regularization* in quantum field theory. The strand model thus provides a specific model for this non-locality, and at the same time explains why it *only* appears for longitudinal W and Z bosons.

The issue of different scattering behaviour for longitudinal and transversal weak bosons also raises the question whether the mass of the longitudinal and the transversal

bosons are precisely equal. The possibility, triggered by Figure 93, might seem appealing at first sight in order to solve the unitarity problem. However, the strand model forbids such a mass difference. In the strand model, mass is due to tangle fluctuations, but does not depend on spin direction.

In other words, the strand model predicts that the scattering of longitudinal W and Z bosons is the first system that will show effects specific to the strand model. Such precision scattering experiments might be possible at the Large Hadron Collider in Geneva. These experiments will allow checking the *non-perturbative effects* and the *regularization effects* predicted by the strand model. For example, the strand model predicts that the wave function of a longitudinal and a transversally polarized W or Z boson of the same energy differ in cross section.

In summary, the strand model predicts well-behaved scattering amplitudes for longitudinal W and Z boson scattering in the TeV region, together with the absence of the Higgs boson.* The strand model explains mass generation and lack of unitarity violations in longitudinal W or Z boson scattering as consequences of tail braiding, i.e., as non-perturbative and non-local effects, and not as consequences of an elementary spin-0 Higgs boson. The forthcoming experiments at the Large Hadron Collider in Geneva will test this prediction.

THE HIGGS BOSON – THE CORRECTED SECTION OF 2012

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In July 2012, CERN researchers from two different experiments announced the observation of a new neutral boson with a mass of 125 GeV. Additional data analysis showed that the boson has spin 0 and positive parity. All experimental checks confirm that the boson behaves like the Higgs boson predicted in 1963 by Peter Higgs and a number of other researchers.

The results lead to question several statements made in 2009 in the previous section.

- Is the tangle on the left-hand side of Figure 92 really a higher order version of the electron neutrino? It seems that this statement is wrong: in contrast to the tangle of the neutrino, the tangle of Figure 92 is achiral.
- Does the tangle of Figure 94 have spin 1/2 or spin 0? As mentioned already in 2009, an effective spin 0 might be possible, in a similar way that it is possible for spin-0 mesons. Spin 0 behaviour appears because the tangle can be oriented in different directions; the time average of these orientations has spherical symmetry.
- Does the tangle of Figure 94 have the correct, positive, C and P values expected for a Higgs boson? It seems so.
- Is the mentioned non-locality effect for W and Z bosons real? If the effect were real, it should also appear for other spin-1 particles. In the strand model, mass values should not depend on spin orientation, but only on tangle core topology. The statements made in 2009 on delocalization and longitudinal scattering seem wrong in retrospect.

* If the arguments against the Higgs boson turn out to be wrong, then the strand model might be saved with a dirty trick: we could argue that the tangle on the left-hand side of Figure 92 might effectively have spin 0. In this case, the ropelength of the Borromean rings, 29.03, together with the ropelengths of the weak bosons, lead to a Higgs mass prediction, to first order, in the range from $(29.03/10.1)^{1/3} \cdot 80.4 \text{ GeV} = 114 \text{ GeV}$ to $(29.03/13.7)^{1/3} \cdot 91.2 \text{ GeV} = 117 \text{ GeV}$, plus or minus a few per cent.

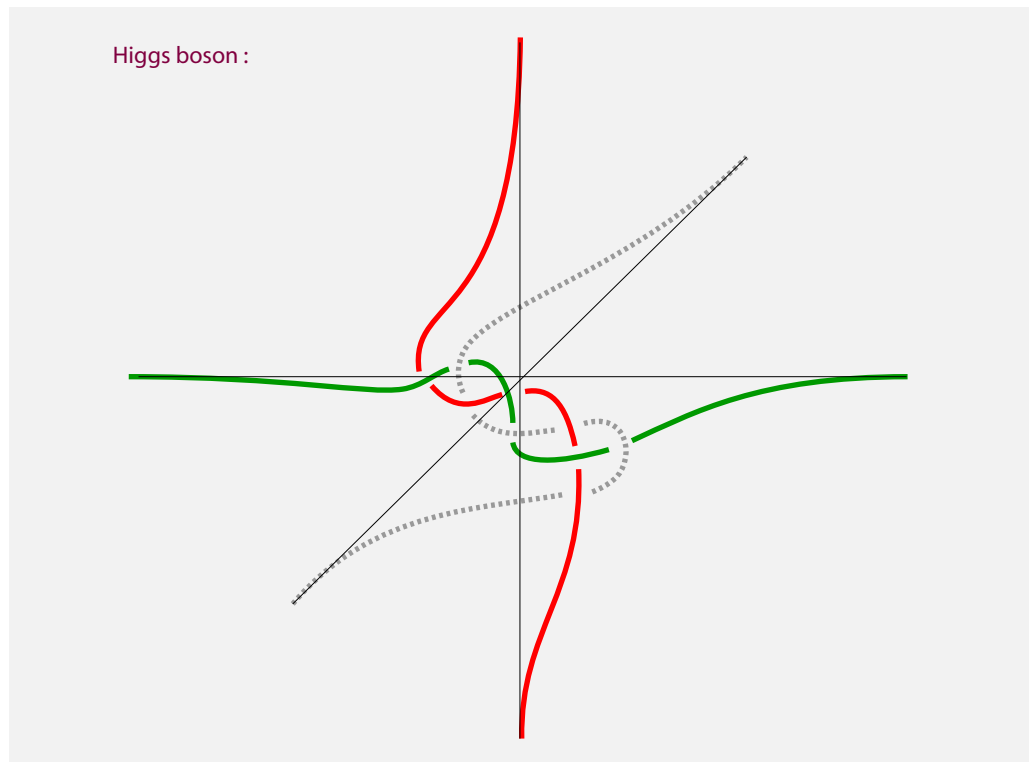


FIGURE 94 The tangle of the Higgs boson in the strand model. Spin 0 appears because the braid can be oriented in different directions, so that the time average has spherical symmetry. The tangle has 9 crossings: 3 crossings appear already in the vacuum configuration of three strands, and the additional 6 crossings (see [Figure 92](#)) are due to the Higgs boson.

- Would the Higgs boson tangle assignment of [Figure 94](#) be testable? Yes; any tangle assignment must yield the observed mass value and the observed branching ratios and decay rates. This is a subject of research. But already at the qualitative level, the proposed tangle structure of the Higgs boson suggests decays into leptons that are similar to those observed at CERN.
- Is the tangle of [Figure 94](#) elementary? Yes.
- Are there other possible Higgs boson tangles? This issue is open. The braid structure seems the most appealing structure, as it embodies the effect of tail braiding, an effect that is important for the appearance of mass.
- Are knots and links, i.e., closed tangles, really forbidden? The discussion about the Higgs boson concerns the open tangle shown in [Figure 94](#), not the Borromean link shown on the right-hand side of [Figure 92](#). So far, there is no evidence for closed tangles in the strand model. Such evidence would mean a departure from the idea that nature is a single strand.
- Does the Higgs boson issue put into question the strand model as a whole? First of all, $SU(2)$ breaking is unaffected. Secondly, a mistaken tangle–particle assignment can be accommodated in the strand model; new forces or symmetries cannot. Therefore the strand model is not put into question.

- Could several, possibly charged, Higgs bosons exist? No such tangles seem possible – as long as a tangle with *two* [Figure 94](#) Higgs cores in sequence is not a separate particle.
- Has some other strand model effect been overlooked? Could other elementary or composed particles exist? For example, the structure of the Higgs boson might be seen to suggest that lepton families reappear (roughly) every 125 GeV. Is that the case? The issue is not completely settled. It seems more probable that those higher tangles simply yield corrections to the Higgs mass.

In short, the existence of the standard model Higgs boson seems compatible with the strand model. The 2009 mistake about the Higgs also shows that the exploration of the strand model is not yet complete. In any case, the strand model has not been falsified by the discovery of the Higgs boson.

[Page 330](#) Assuming that the Higgs tangle shown in [Figure 94](#) is correct, we have an intuitive proposal for the mechanism that produces mass, namely *tail braiding*. The proposed Higgs tangle also allows a number of experimental predictions.

2012 PREDICTIONS ABOUT THE HIGGS

- The Higgs tangle implies a Higgs boson with vanishing charge, positive parity, being elementary – as is observed.
- The Higgs tangle allows us to estimate the Higgs/Z mass ratio. Using the original knotted model for the W and Z bosons, the estimates are in the region of the observed values. Determining the estimates is still subject of research.
- The Higgs tangle and the strand model imply that the standard model is correct up to Planck energy, and that the Higgs mass value should reflect this. The observed Higgs mass of 125 GeV complies also with this expectation.
- Therefore, the strand model suggests that no deviations between the standard model and data should ever be observed in any experiment.
- The strand model again and consistently predicts the lack of supersymmetry.
- In case that several Higgs bosons exist or that the Higgs tangle does not apply, the strand model is in trouble.
- In case that effects, particles or interactions beyond the standard model are observed, the strand model is in trouble.

QUARK-ANTIQUARK MESONS

In the strand model, all three-stranded tangles apart from the leptons, as well as all four-stranded tangles represent *composite* particles. The first example are *mesons*.

In the strand model, rational tangles of three strands are quark-antiquark mesons with spin 0. The quark tangles yield a simple model of these *pseudoscalar* mesons, shown on the left-hand sides of [Figure 95](#), [Figure 97](#) and [Figure 98](#). The right-hand sides of the figures show *vector* mesons, thus with spin 1, that consist of *four* strands. All tangles are rational. Inside mesons, quarks and antiquarks ‘bond’ at three spots that form a triangle oriented perpendicularly to the bond direction and to the paper plane. To increase clarity, the ‘bonds’ are drawn as circles in the figures; however, they consist of two crossed (linked) tails of the involved strands that reach the border of space, as shown in [Figure 96](#).

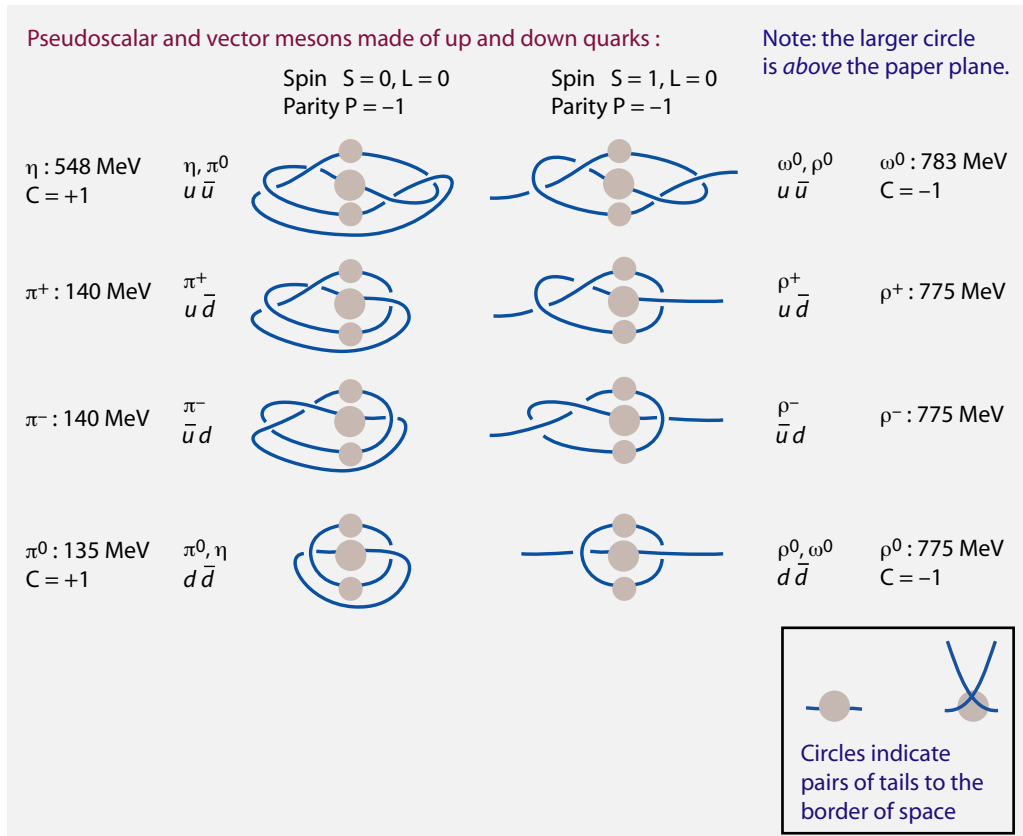


FIGURE 95 The simplest strand models for the light pseudoscalar and vector mesons (circles indicate crossed tail pairs to the border of space), with the observed mass values.

Ref. 225 With this construction, mesons made of two quarks are only possible for the type $\bar{q}q$.
 Ref. 226 Other combinations, such as qq or $\bar{q}\bar{q}$, turn out to be unlinked. We note directly that
 Ref. 227 this model of mesons resembles the original string model of hadrons from 1973, but also the Lund string model and the recent QCD string model.

To compare the meson structures with experimental data, we explore the resulting quantum numbers. As in quantum field theory, also in the strand model the parity of a particle is the product of the intrinsic parities and of wave function parity. The states with orbital angular momentum $L = 0$ are the lowest states. Experimentally, the lightest mesons have quantum numbers $J^{PC} = 0^{-+}$, and thus are pseudoscalars, or have $J^{PC} = 1^{--}$, and thus are vector mesons. The strand model reproduces these observed quantum numbers. (We note that the spin of any composite particle, such as a meson, is low-energy quantity; to determine it from the composite tangle, the tails producing the bonds – drawn as circles in the figures – must be neglected. As a result, the low-energy spin of mesons and of baryons is correctly reproduced by the strand model.)

In the strand model, the meson states are colour-neutral, or ‘white’, by construction, because the quark and the antiquark, in all orientations, always have opposite colours that add up to white.

In the strand model, the electric charge is an integer for all mesons. Chiral tangles are

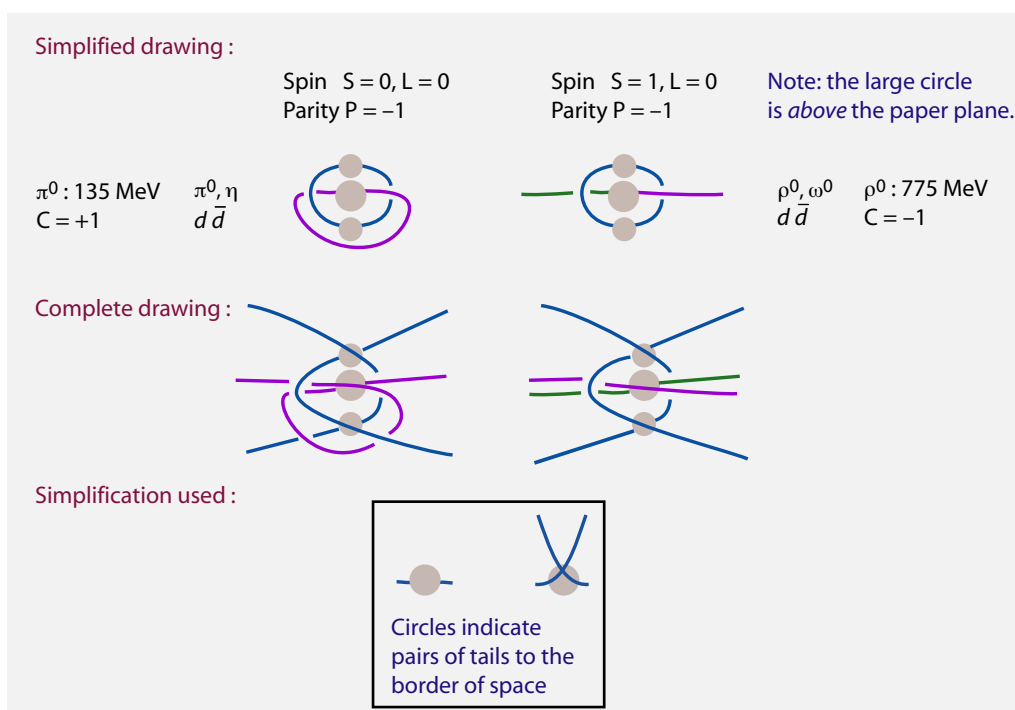


FIGURE 96 The meaning of the circles used in the tangle graphs of mesons and baryons.

charged, achiral tangles uncharged. The charge values deduced from the strand model thus reproduce the observed ones.

In experiments, no mesons with quantum numbers 0^{--} , 0^{+-} , or 1^{+-} are observed. Also this observation is reproduced by the quark tangles, as is easily checked by direct inspection. The strand model thus reproduces the very argument that once was central to the acceptance of the quark model itself.

It is important to realize that in the strand model, each meson is represented by a *tangle family* consisting of *several* tangle structures. This has three reasons. First, the ‘circles’ can be combined in different ways. For example, both the $u\bar{u}$ and the $d\bar{d}$ have as alternate structure a line plus a ring. This common structure is seen as the underlying reason that these two quark structures *mix*, as is indeed observed. (The same structure is also possible for $s\bar{s}$, and indeed, a full description of these mesons must include mixing with this state as well.) The second reason that mesons have several structures are the mentioned, more complicated braid structures possible for each quark, namely with 6, 12, etc. additional braid crossings. The third reason for additional tangle structures is the occurrence of higher-order Feynman diagrams of the weak interaction, which add yet another group of more complicated topologies that also belong to each meson.

In short, the mesons structures of Figure 95, Figure 97 and Figure 98 are only the *simplest* tangles for each meson. Nevertheless, all tangles, both the simplest and the more complicated meson tangles, reproduce spin values, parities, and all the other quantum numbers of mesons. Indeed, in the strand model, the more complicated tangles automatically share the quantum numbers of the simplest one.

Pseudoscalar and vector mesons containing strange and charm quarks :		Note: the large circle is above the paper plane.	
K^- $s\bar{u}$ 494 MeV			K^{*-} $s\bar{u}$ 892 MeV
K^+ $\bar{s}u$ 494 MeV			K^{*+} $\bar{s}u$ 892 MeV
\bar{K}^0 $s\bar{d}$ 498 MeV			\bar{K}^{*0} $s\bar{d}$ 899 MeV
K^0 $\bar{s}d$ 498 MeV			K^{*0} $\bar{s}d$ 899 MeV
η' $s\bar{s}$ 958 MeV			ϕ' $s\bar{s}$ 1020 MeV
D^0 $c\bar{u}$ 1864 MeV			D^{*0} $c\bar{u}$ 2007 MeV
\bar{D}^0 $\bar{c}u$ 1864 MeV			\bar{D}^{*0} $\bar{c}u$ 2007 MeV
D^+ $c\bar{d}$ 1870 MeV			D^{*+} $c\bar{d}$ 2010 MeV
D^- $\bar{c}d$ 1870 MeV			D^{*-} $\bar{c}d$ 2010 MeV
D_s^+ $c\bar{s}$ 1970 MeV			D_s^{*+} $c\bar{s}$ 2112 MeV
D_s^- $\bar{c}s$ 1968 MeV			D_s^{*-} $\bar{c}s$ 2112 MeV
η_c $c\bar{c}$ 2981 MeV			J/ψ $c\bar{c}$ 3097 MeV

FIGURE 97 The simplest strand models for strange and charmed mesons with vanishing orbital angular momentum. Mesons on the left side have spin 0 and negative parity; mesons on the right side have spin 1 and also negative parity. Circles indicate crossed tail pairs to the border of space; grey boxes indicate tangles that mix with their antiparticles and which are thus predicted to show CP violation.

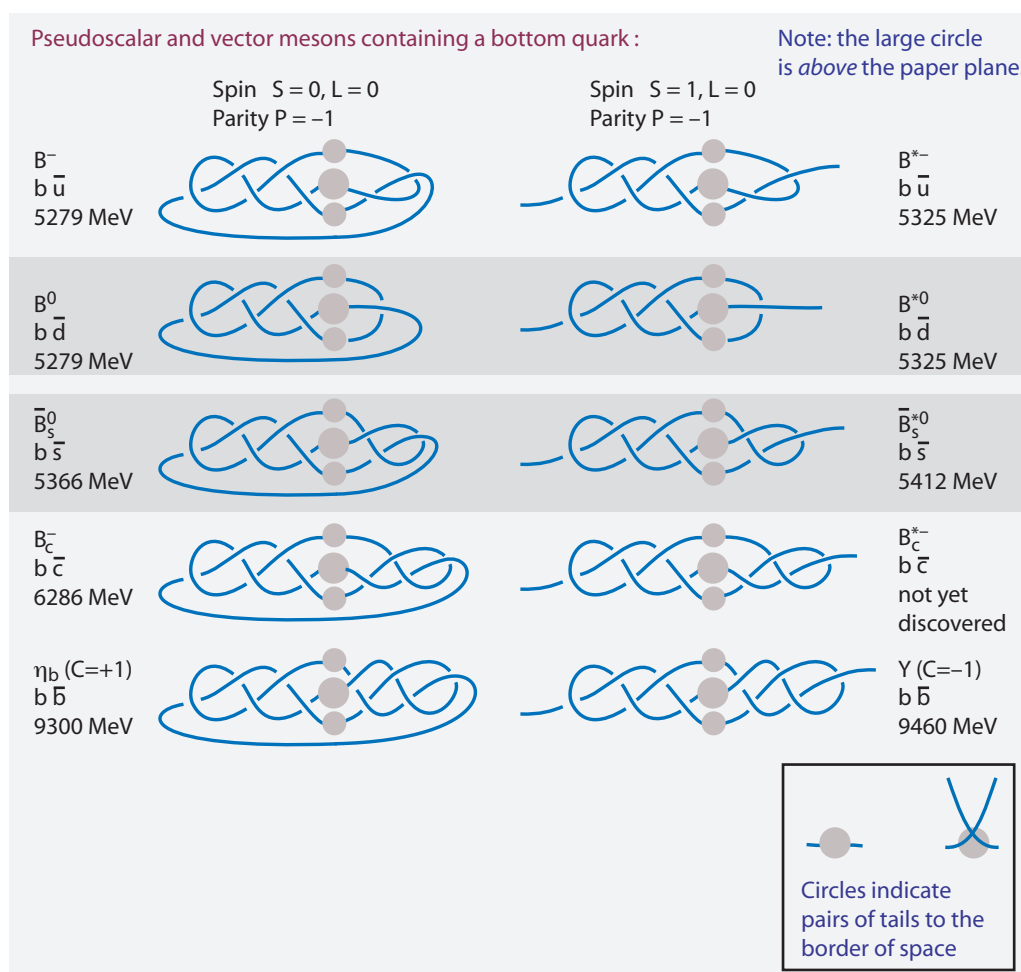


FIGURE 98 The simplest strand models for some heavy pseudoscalar and vector mesons, together with their experimental mass values. Antiparticles are not drawn; their tangles are mirrors of the particle tangles. Circles indicate crossed tail pairs to the border of space; grey boxes indicate tangles that mix with their antiparticles and which are thus predicted to show CP violation.

MESON FORM FACTORS

Ref. 228 The strand model also predicts directly that all mesons from [Figure 95](#), [Figure 97](#) and [Figure 98](#), in fact all mesons with vanishing orbital momentum, are *prolate*. This (un-surprising) result is agreement with observations. Mesons with non-vanishing orbital momentum are also predicted to be prolate. This latter prediction about meson shapes is made also by all other meson models, but has not yet been checked by experiment.

There is another way to put what we have found so far. The strand model makes the following prediction: When the meson tangles are averaged over time, the crossing densities reproduce the measured spatial, quark flavour, spin and colour part of the meson wave functions. This prediction can be checked against measured form factors and against lattice QCD calculations.

MESON MASSES, EXCITED MESONS AND QUARK CONFINEMENT

The strand model also allows us to understand meson masses. We recall that a *topologically complicated* tangle implies a *large* mass. With this relation, Figure 95 predicts that the π^0 , η and $\pi^{+/-}$ have different masses and follow the observed meson mass sequence $m(\pi^0) < m(\pi^{+/-}) < m(\eta)$. The other mass sequences can be checked with the help of Figure 95, Figure 97 and Figure 98; there are no contradictions with observations. However, there is one limit case: the strand model predicts different masses for the ρ^0 , ω , and $\rho^{+/-}$. So far, observations only partly confirm the prediction. Recent precision experiments seem to suggest that ρ^0 and $\rho^{+/-}$ have different mass; this result has not been confirmed yet.

Ref. 229

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More precise mass determinations will be possible with numerical calculations. This will be explored in more detail later on. In any case, the strand model for mesons suggests that the quark masses are not so important for the determination of meson masses, whereas the details of the quark-antiquark bond are. Indeed, the light meson and baryon masses are much higher than the masses of the constituent quarks.

The relative unimportance of quark masses for many meson masses is also confirmed for the case of *excited* mesons, i.e., for mesons with orbital angular momentum L . It is well known that mesons of non-vanishing orbital angular momentum can be grouped into sets which have the same quark content, but different total angular momentum $J = L + S$. These families are observed to follow a well-known relation between total angular momentum J and mass m , called *Regge trajectories*:

$$J = \alpha_0 + \alpha_1 m^2 \quad (201)$$

Ref. 230

with an (almost) constant factor α_1 for all mesons, about 0.9 GeV/fm. These relations, the famous *Regge trajectories*, are explained in quantum chromodynamics as deriving from the linear increase with distance of the effective potential between quarks, thus from the properties of the relativistic harmonic oscillator. The linear potential itself is usually seen as a consequence of a fluxtube-like bond between quarks.

In the strand model, the fluxtube-like bond between the quarks is built-in automatically, as shown in Figure 99. All mesons have three connecting 'bonds' and these three bonds can be seen as forming one common string tube. In the simplified drawings, the bond or string tube is the region containing the circles. In orbitally excited mesons, the three bonds are expected to lengthen and thus to produce additional crossing changes, thus additional effective mass. The strand model also suggests a *linear* relation. Since the mechanism is expected to be similar for all mesons, which all have three bonding circles, the strand model predicts the *same* slope for all meson (and baryon) Regge trajectories. This is indeed observed.

In summary, the strand model reproduces meson mass sequences and quark confinement in its general properties.

CP VIOLATION IN MESONS

Ref. 231

In the weak interaction, the product CP of C and P parity is usually conserved. However, rare exceptions are observed for the decay of the K^0 meson and in various processes

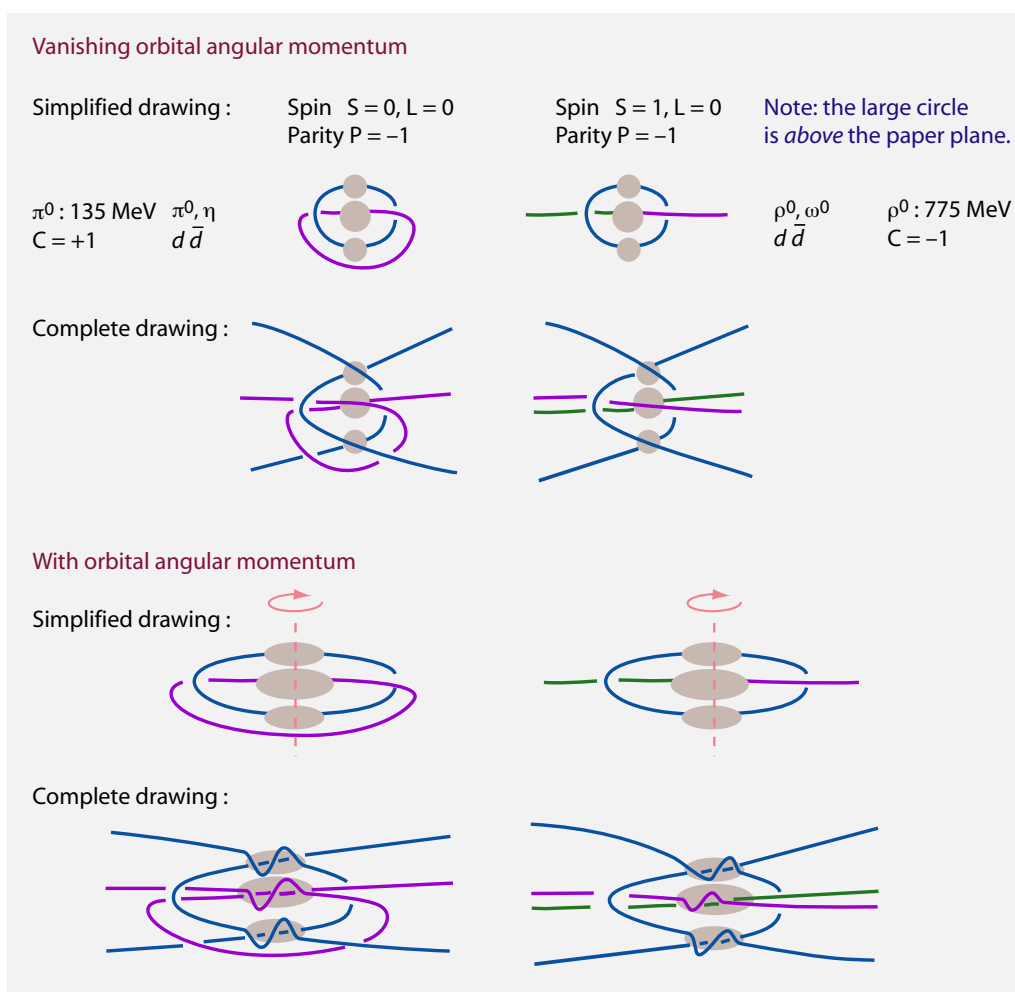


FIGURE 99 The strand model for mesons without (top) and with (bottom) orbital angular momentum.

Ref. 229 that involve the B^0 and B_s^0 mesons. In each of these exceptions, the meson is found to mix with its own antiparticle. CP violation is essential to explain the matter–antimatter asymmetry of the universe.

The strand model allows us to deduce whether the mixing of a meson with its own antiparticle is possible or not. As expected, only neutral mesons are candidates for such mixing, because of charge conservation. In the strand model, particle–antiparticle mixing is possible whenever the transition from a neutral meson to its antiparticle is possible in *two* ways: by taking the mirror of the meson tangle or by shifting the position of the binding strands. All mesons for which this is possible are shown in grey boxes in Figure 95, Figure 97 and Figure 98. The strand model also makes it clear that such mixing requires shifting of the bonds; this is a low-probability process that is due to the weak interaction. The strand model thus predicts that the weak interaction violates CP invariance in mesons that mix with their antiparticles.

Since the spin 1 mesons decay strongly and thus do not live long enough, the small

effect of CP violation is de facto only observed in pseudoscalar, spin-0 mesons. The strand model thus predicts observable mixings and CP violation for the mesons pairs $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B^0 - \bar{B}^0$, $B_s^0 - \bar{B}_s^0$. The prediction by the strand model corresponds precisely to those systems for which CP violation is actually observed. (CP violation in D mesons was finally discovered at CERN in 2011, after it was predicted both by the standard model and the strand model, in earlier editions of this volume.)

Ref. 229

In the strand model, meson–antimeson mixing is possible because the various quarks are braided strands. Because of this braid structure, the existence of meson–antimeson mixing is a consequence of the existence of three quark generations. The meson structures also make it clear that such mixings would not be possible if there were no third quark generation. The strand model thus reproduces the usual explanation of CP violation as the result of three quark generations.

For the strong and the electromagnetic interaction, the strand model predicts that there is no mixing and no CP violation, because gluons and photons do not change particle topology. Therefore, the strand model suggests the absence of axions. The lack of a suitable tangle for axions, shown later on, then turns this suggestions into a prediction.

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In summary, the existence of CP violation in the weak interactions and the lack of CP violation in the strong interaction are natural consequences of the strand model.

OTHER THREE-STRANDED TANGLES AND GLUEBALLS

In the strand model, the aditted complicated tangles made of three strands are either higher-order propagating versions of the tangles just presented or composites of one-stranded or two-stranded particles.

The often conjectured glueball could also be made of three gluons. In the strand model, such a structure would be a simple tangle made of three strands. However, the masslessness of gluons does not seem to allow such a tangle. The argument is not watertight, however, and the issue is, as mentioned above, still subject of research.

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SPIN AND THREE-STRANDED PARTICLES

Why do three strands sometimes form a spin 0 particle, such as the elementary Higgs boson, sometimes a spin 1/2 particle, such as the elementary electron, and sometimes a spin 1 particle, such as a composed meson? The answer depends on how the strands are free to move against each other.

The Higgs tangle appears through tangling of vacuum strands, and inherits the zero spin of vacuum. The W and Z tangles have a special property: two strands can rotate around the third; this makes them bosons as well, but of spin 1. Fermion tangles have neither property; their core can only rotate through the belt trick; thus they are fermions.

SUMMARY ON THREE-STRANDED TANGLES

Compared to two-stranded tangles, one *new class* of *elementary* particles appears for three strands; the new class is somewhat less tangled than general rational tangles but still more tangled than the trivial vacuum tangle: the *braided tangles*. Braided tangles represent the Higgs boson and the leptons; the tangles reproduce all their observed quantum numbers. The braided tangles also imply that neutrinos and anti-neutrinos differ, are

massive, and are Dirac particles.

Page 329 The strand model (corrected in 2012) also predicts that, apart from the six leptons and one Higgs boson, no other elementary particle made of three strands exist in nature.

In the case of *composite* particles made of three strands, the strand model proposes tangles for all pseudoscalar mesons; the resulting quantum numbers and mass sequences match the observed values. In the spectrum of composite particles, the glueball issue is not completely settled.

TANGLES OF FOUR AND MORE STRANDS

If we add one or more strand to a three-strand tangle, no additional class of tangles appears. The tangle classes remain the same as in the three-strand case. In other words, *no additional elementary particles* arise in the strand model. To show this, we start our exploration with the *rational* tangles.

We saw above that the rational tangles made of four strands represent the vector mesons. We have already explored them together with the scalar mesons. But certain more complicated rational tangles are also important in nature, as we consist of them.

BARYONS

In the strand model, rational tangles made of five or six strands are baryons. The quark tangles of the strand model yield the tangles for baryons in a natural way, as Figure 100 shows. Again, not all quark combinations are possible. First of all, quark tangles do not allow mixed $q q \bar{q}$ or $q \bar{q} \bar{q}$ structures, but only $q q q$ or $\bar{q} \bar{q} \bar{q}$ structures. In addition, the tangles do not allow (fully symmetric) spin 1/2 states for $u u u$ or $d d d$, but only spin 3/2 states. The model also naturally predicts that there are only two spin 1/2 baryons made of u and d quarks. All this corresponds to observation. The tangles for the simplest baryons are shown in Figure 100.

Page 383 The electric charges of the baryons are reproduced. In particular, the tangle topologies imply that the proton has the same charge as the positron. Neutral baryons have topologically achiral structures; nevertheless, the neutron differs from its antiparticle, as can be deduced from Figure 100, through its three-dimensional shape. The Δ baryons have different electric charges, depending on their writhe.

Ref. 228 Baryons are naturally colour-neutral, as observed. The model also shows that the baryon wave function usually cannot be factorized into a spin and quark part: the nucleons need *two* graphs to describe them, and tangle shapes play a role. Baryon parities are reproduced; the neutron and the antineutron differ. All this corresponds to known baryon behaviour. Also the observed baryon shapes (in other words, the baryon quadrupole moments) are reproduced by the tangle model.

The particle masses of proton and neutron differ, because their topologies differ. However, the topological difference is ‘small’, as seen in Figure 100, so the mass difference is small. The topological difference between the various Δ baryons is even smaller, and indeed, their mass difference is barely discernible in experiments.

The strand model naturally yields the baryon octet and decuplet, as shown in Figure 101 and Figure 102. In general, complicated baryon tangles have higher mass than

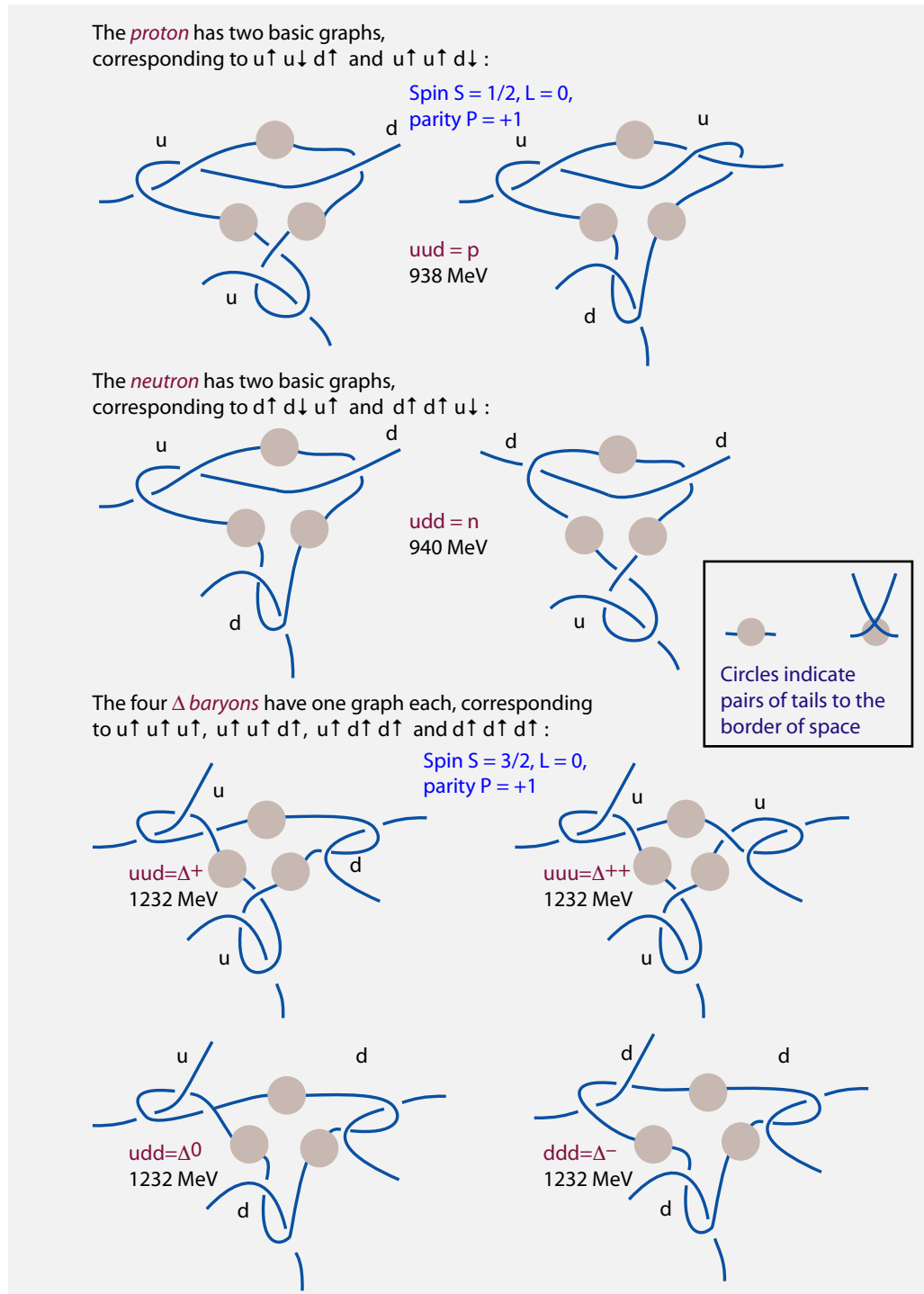


FIGURE 100 The simplest strand models for the lightest baryons made of up and down quarks (circles indicate linked tail pairs to the border of space), together with the measured mass values.

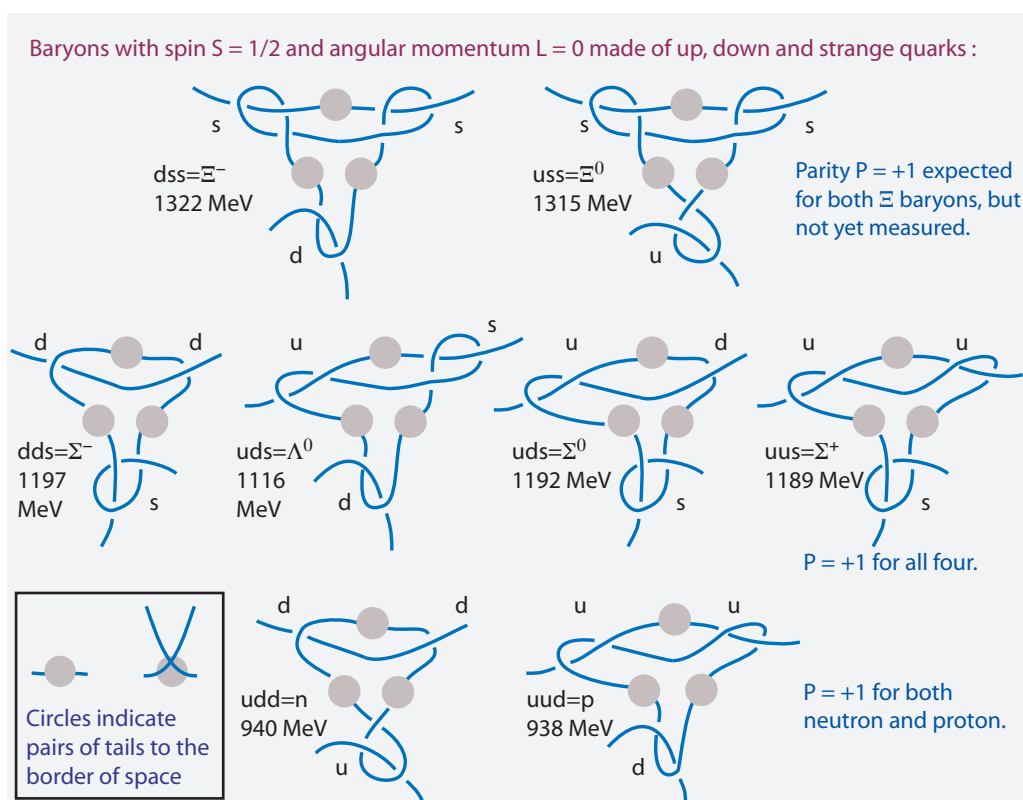


FIGURE 101 One tangle (only) for each baryon in the lowest $J=L+S=1/2$ baryon octet (circles indicate linked tail pairs to the border of space), together with the measured mass values.

simpler ones, as shown in the figures; this is also the case for the baryons, not illustrated here, that include other quarks. And like for mesons, baryon Regge trajectories are due to 'stretching' and tangling of the binding strands. Since the bonds to each quark are again (at most) three, the model qualitatively reproduces the observation that the Regge slope for all baryons is the same and is equal to that for mesons. We note that this also implies that the quark masses play only a minor role in the generation of hadron masses; this old result from QCD is thus reproduced by the strand model.

The arguments presented so far only reproduce mass sequences, not mass values. Actual hadron mass calculations are possible with the strand model: it is necessary to compute the number of crossing changes each tangle produces. There is a chance, but no certainty, that such calculations might be simpler to implement than those of lattice QCD.

TETRAQUARKS AND EXOTIC MESONS

Ref. 216 Among the exotic mesons, tetraquarks are the most explored cases. It is now widely be-
 Ref. 232 lieved that the low-mass scalar mesons are tetraquarks. In the strand model, tetraquarks are possible; an example is given in Figure 103. This is a six-stranded rational tangle. Spin, parities and mass sequences from the strand model seem to agree with observations. If the arrangement of Figure 103 would turn out to be typical, the tetraquark looks more

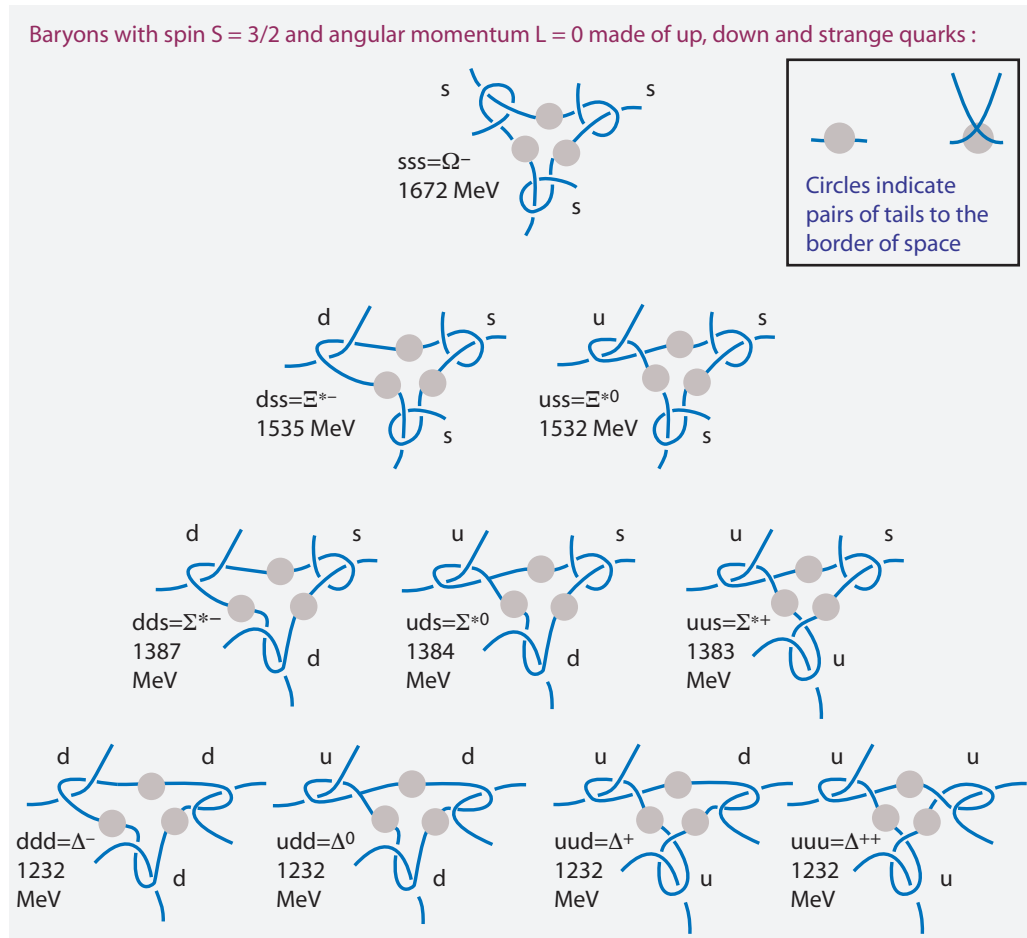


FIGURE 102 One tangle for each baryon in the lowest $J=3/2$ baryon decuplet (circles indicate linked tail pairs to the border of space), together with the measured mass values.

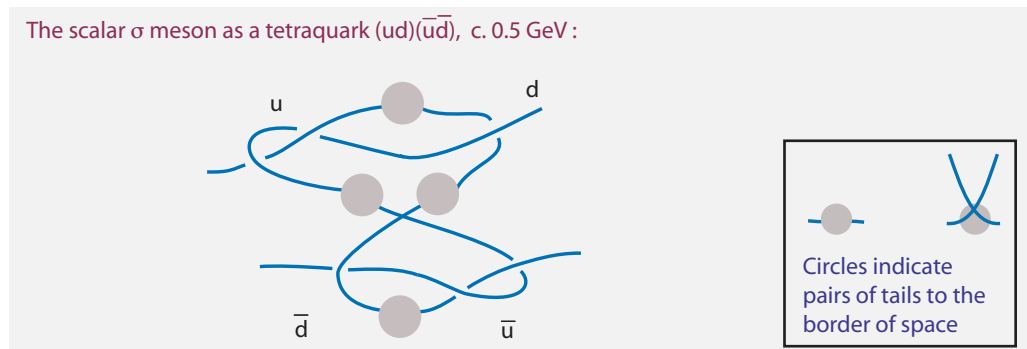


FIGURE 103 The strand model for a specific tetraquark (circles indicate linked tail pairs to the border of space).

like a bound pair of two mesons and not like a state in which all four quarks are bound in

equal way to each other. On the other hand, a tetrahedral arrangement of quarks might also be possible. The details of this topic are left for future exploration.

Ref. 218 The strand model makes an additional statement: knotted (hadronic) strings in quark–antiquark states are impossible. Such states have been proposed by Niemi. In the strand model, such states would not be separate mesons, but usual mesons with one or several added virtual weak vector bosons. This type of exotic mesons is therefore predicted not to exist.

Page 322, page 338 The situation for glueballs, which are another type of exotic mesons, has already been discussed above.

OTHER TANGLES MADE OF FOUR OR MORE STRANDS

We do not need to explore other *prime* tangles or *locally knotted* tangles made of four or more strands. They are either not allowed or are higher-order versions of rational tangles, as explained already in the case of two and three strands. We also do not need to explore *separable* tangles. Separable tangles are composite of tangles with fewer strands.

One class of tangles remains to be discussed: *braided* tangles of four or more strands. Now, a higher-order perturbation of the weak interaction can always lead to the topological entanglement of some vacuum strand with a tangle of fewer strands. Braided tangles of four or more strands are thus higher-order propagating states of three-stranded leptons or hadrons.

We can also state this in another way. There are no tangles of four or more strands that are more tangled than the trivial tangle but less tangled than the lepton tangles. Therefore, no additional elementary particles are possible. In short, *the tangle model does not allow elementary particles with four or more strands*.

SUMMARY ON TANGLES MADE OF FOUR OR MORE STRANDS

By exploring all possible tangle classes in detail, we have shown that *every* localized structure made of strands has an interpretation in the strand model. In particular, the strand model makes a simple statement on any tangle made of four or more strands: such a tangle is *composite* of the elementary tangles made of one, two or three strands. In other terms, there are *no* elementary particles made of four or more strands in nature.

The strand model states that each possible tangle represents a physical particle system: an overview is given in Table 13. The mapping between tangles and particles is only possible because (infinitely) many tangles are assigned to each massive elementary particle.

The result of this exploration is that the strand model limits the number of elementary particles to those contained in the standard model of particle physics.

FUN CHALLENGES AND CURIOSITIES ABOUT PARTICLE TANGLES

Challenge 196 s In the strand model, mass appears due to tail braiding. But mass is also due to tangle rotation and fluctuation. How do the two definitions come together?

TABLE 13 The match between tangles and particles in the strand model.

STRANDS	TANGLE	PARTICLE	TYPE
1	unknotted	elementary	vacuum, (unbroken) gauge boson
1	knotted	–	not in the strand model
2	unknotted	composed	composed of simpler tangles
2	rational	elementary	quark or graviton
2	prime, knotted	–	not in the strand model
3	unknotted	composed	composed of simpler tangles
3	braided	elementary	lepton
3	rational	elementary or composed	leptons
3	prime, knotted	–	not in the strand model
4 & more	like for 3 strands	all composed	composed of simpler tangles

* *

The following statement seems absurd, but is correct:

- ▷ The tangle model implies that all elementary particles are point-like, *without* internal structure.

Indeed, if at all, the strand model implies deviations from point-like behaviour only at Planck scale; particles are point-like for all practical purposes.

* *

In the strand model, only crossing switches are observable. How then can the specific tangle structure of a particle have any observable effects? In particular, how can quantum numbers be related to tangle structure, if the only observables are due to crossing changes?

Challenge 197 e

* *

No neutral weak currents that change strangeness or other flavours are observed. In the strand model this observation is a consequence of the tangle shape of the Z boson.

* *

- Ref. 233 In 2014, Marek Karliner predicted the existence of six-quark states. Can the strand model reproduce them? Can it settle whether they are molecules of three mesons or genuine six-quark states?

Challenge 198 r

* *

Challenge 199 e Can you use the strand model to show that pentaquarks do not exist?

* *

Ref. 234 What is the relation of the model shown here to the ideas of Viro and Viro on skew lines?

* *

Ref. 235 The most prominent proponent of the idea that particles might be knots was, in 1868, William Thomson–Kelvin. He proposed the idea that different atoms might be differently ‘knotted vortices’ in the ‘ether’. The proposal was ignored – and rightly so – because it did not explain anything: neither the properties nor the interactions of atoms were explained. The proposal simply had no relation to reality. In retrospect, the main reason for this failure was that elementary particles and quantum theory were unknown at the time.

* *

Purely topological models for elementary particles have been proposed and explored by various scholars in the past. But only a few researchers ever proposed specific topological structures for each elementary particle. Such proposals are easily criticized, so that it is easy to make a fool of oneself; any such proposal thus needs a certain amount of courage.

- Ref. 236 — Herbert Jehle modelled elementary particles as closed knots already in the 1970s. However, his model did not reproduce quantum theory, nor does it reproduce all particles known today.
- Ref. 143 — Ng Sze Kui has modelled mesons as knots. There is however, no model for quarks, leptons or bosons, nor a description for the gauge interactions.
- Ref. 237 — Tom Mongan has modelled elementary particles as made of three strands that each carry electric charge. However, there is no connection with quantum field theory or general relativity.
- Ref. 139 — Jack Avrin has modelled hadrons and leptons as Moebius bands, and interactions as cut-and-glue processes. The model however, does not explain the masses of the particles or the coupling constants.
- Ref. 141 — Robert Finkelstein has modelled fermions as knots. This approach, however, does not explain the gauge properties of the interactions, nor most properties of elementary particles.
- Ref. 140 — Sundance Bilson-Thompson, later together with his coworkers, modelled elementary fermions and bosons as structures of triple ribbons. The leather trick is used, like in the strand model, to explain the three generations of quarks and leptons. This is by far the most complete model from this list. However, the origin of particle mass, of particle mixing and, most of all, of the gauge interactions is not explained.

* *

Strands are *not* superstrings. In contrast to superstrings, strands have a fundamental principle. (This is the biggest conceptual difference.) The fundamental principle for strands is not fulfilled by superstrings. In contrast to superstrings, strands have no tension, no supersymmetry and no own Lagrangian. (This is the biggest physical difference.) Because strands have no tension, they cannot oscillate. Because strands have no supersymmetry, general relativity follows directly. Because strands have no own Lagrangian, particles are tangles, not oscillating superstrings, and quantum theory follows directly. In fact, the definitions of particles, wave functions, fields, vacuum, mass and horizons

differ completely in the two approaches.

In contrast to superstrings, strands describe the number of gauge interactions and of particle generations. In contrast to superstrings, strands describe quarks, hadrons, confinement, Regge behaviour, asymptotic freedom, particle masses, particle mixing and coupling constants. In the strand model, in contrast to ‘open superstrings’, no important configuration has ends. In contrast to open or closed superstrings, strands move in three spatial dimensions, not in nine or ten; strands resolve the anomaly issue without higher dimensions or supersymmetry, because unitarity is automatically maintained, by construction; strands are not related to membranes or supermembranes. In the strand model, no strand is ‘bosonic’ or ‘heterotic’, there is no $E(8)$ or $SO(32)$ gauge group, there are no general ‘pants diagrams’ for all gauge interactions, there is no AdS/CFT duality, there is no ‘landscape’ with numerous vacuum states, and there is no ‘multiverse’. In contrast to superstrings, strands are based on Planck units. And in contrast to superstrings, strands yield the standard model of elementary particles without any alternative. In fact, not a single statement about superstrings is applicable to strands.

Ref. 144

* *

Strands do not require higher dimensions. On the other hand, it can be argued that strands do produce an additional non-commutative structure at each point in space. In a sense, when strands are averaged over time, a non-commutative inner space is created at each point in space. As a result, when we focus at a specific spatial position over somewhat longer times scales than the Planck time, we can argue that, at that point of space, nature is described by a product of three-dimensional space with an internal, non-commutative space. Since many years, Alain Connes and his colleagues have explored such product spaces in detail. They have discovered that with an appropriately chosen non-commutative inner space, it is possible to reproduce many, but not all, aspects of the standard model of particle physics. Among others, choosing a suitable non-commutative space, they can reproduce the three gauge interactions; on the other hand, they cannot reproduce the three particle generations.

Ref. 238

Connes’ approach and the strand model do not agree completely. One way to describe the differences is to focus on the relation of the inner spaces at different points of space. Connes’ approach assumes that each point has its own inner space, and that these spaces are not related. The strand model, instead, implies that the inner spaces of neighbouring points are related; they are related by the specific topology and entanglement of the involved strands. For this very reason the strand model does allow to understand the origin of the three particle generations and the details of the particle spectrum.

There are further differences between the two approaches. Connes’ approach assumes that quantum theory and general relativity, in particular, the Hilbert space and the spatial manifold, are given from the outset. The strand model, instead, deduces these structures from the fundamental principle. And, as just mentioned, Connes’ approach is not unique or complete, whereas the strand model seems to be. Of the two, only the strand model seems to be unmodifiable, or ‘hard to vary’.

* *

The strand model implies that there is *nothing new* at small distances. At small distances, or high energies, nature consists only of strands. Thus there are no new phenomena

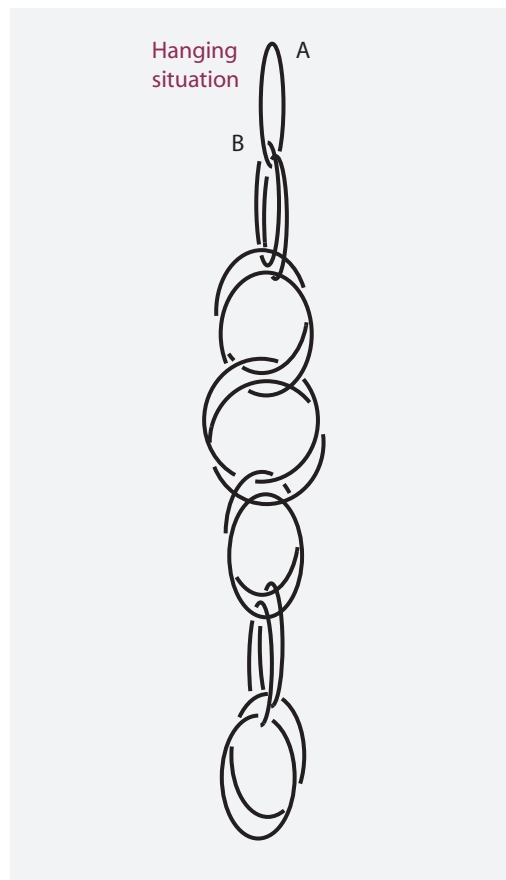


FIGURE 104 A ring chain gives an impression of motion along the chain, when holding ring B while dropping ring A.

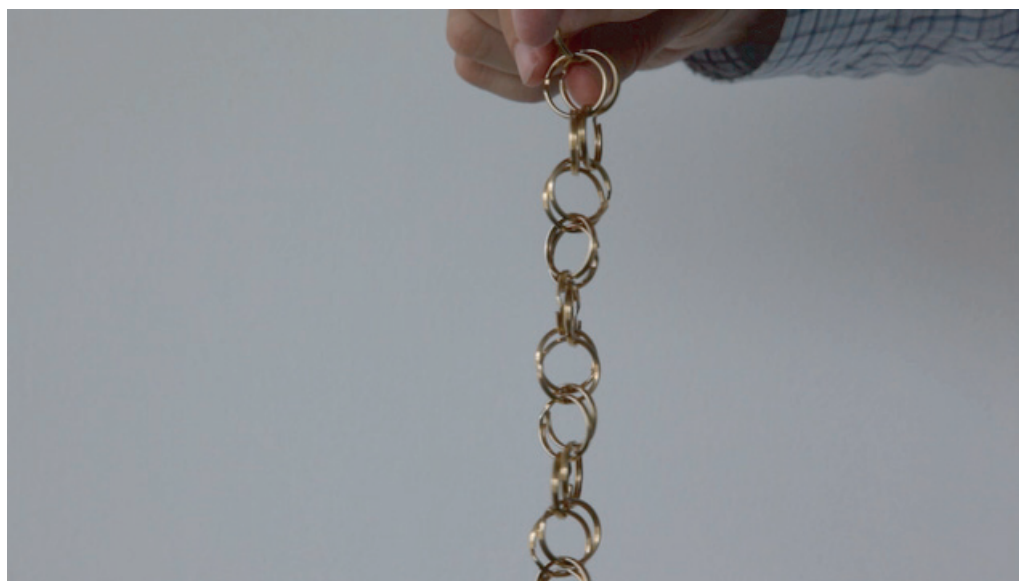
there. Quantum theory states that at small scales, nothing new appears: at small scales, there are *no* new degrees of freedom. For example, quantum theory states that there is no kingdom *Lilliput* in nature. The strand model thus confirms the essence of quantum theory. And indeed, the strand model predicts that between the energy scale of the heaviest elementary particle, the top quark, 173 GeV, and the Planck energy, 10^{19} GeV, nothing is to be found. There is a so-called *energy desert* – empty of interesting features, particles or phenomena – in nature.

* *

Most ropes used in sailing, climbing or other domains of everyday life are produced by braiding. Searching for ‘braiding machine’ on the internet yields a large amount of videos. Searching for ‘LEGO braiding machine’ shows the most simple and beautiful examples and allows you to see how they work.

* *

Not all tangle assignments are self-evident at first sight. [Figure 107](#) shows a tangle whose status in the strand model is not clear. Can you explain what the tangle represents?



Challenge 200 e **FIGURE 105** The ring chain trick produces an illusion of motion (mp4 film © Franz Aichinger). Can more rings be added in horizontal directions?

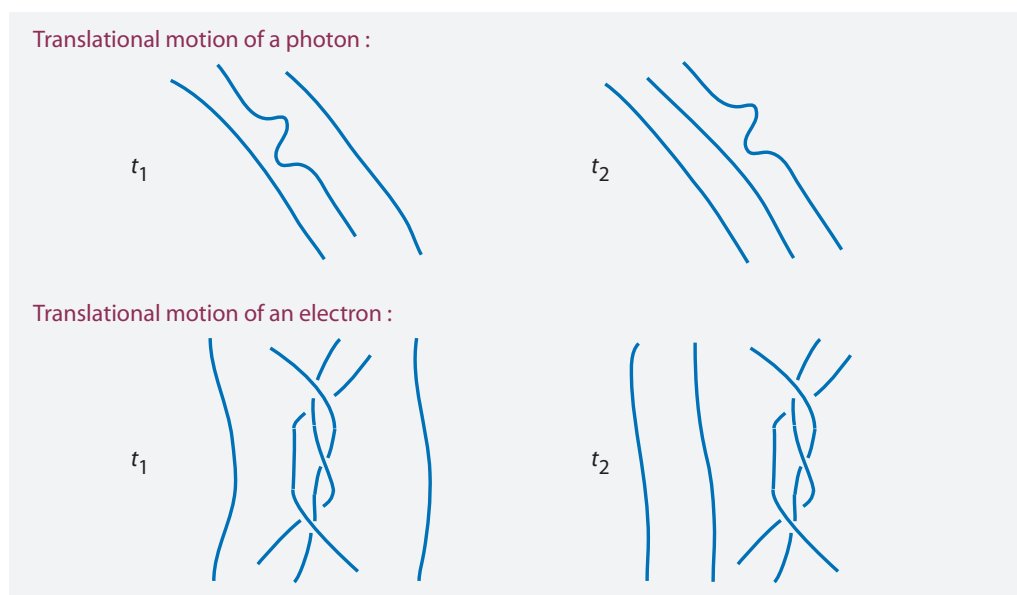


FIGURE 106 Motion of photons and electrons through strand hopping.

* *

Page 150 **Challenge 202 ny** What is the effect of shivering on braiding, and thus on weak particle mixing, on particle tangle families and on the number of generations?

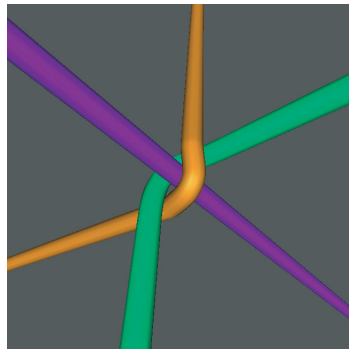


FIGURE 107 A discarded candidate tangle for the W boson.

MOTION THROUGH THE VACUUM – AND THE SPEED OF LIGHT

Up to now, one problem was left open: How can a particle, being a tangle of infinite extension, move through the web of strands that makes up the vacuum? An old trick, known already in France in the nineteenth century, can help preparing for the idea of particle motion in space. [Figure 104](#) shows a special chain that is most easily made with a few dozen key rings. If the ring B is grabbed and the ring A released, this latter ring seems to fall down along the whole chain in a helical path, as shown in the film of [Figure 200](#). If you have never seen the trick, try it yourself; the effect is astonishing. In reality, this is an optical illusion. No ring is actually falling, but the sequence of rings moves in a way that creates the impression of ring motion. And this old trick helps us to solve a number of issues about particle motion that we swept under the carpet so far.

The main idea on particle motion in the strand model is the following:

- ▷ *Translational particle motion* is also due to strand substitution, or ‘strand hopping’.

A schematic illustration of translational motion is given in [Figure 106](#). In the strand model, contrary to the impression given so far, a tangle does not always need to move as a whole along the strand. This is seen most easily in the case of a photon. It is easy to picture that the tangle structure corresponding to a photon can also hop from strand to strand. At any stage, the structure is a photon; but the involved strand is never the same.

The idea of motion through strand hopping also works for massive particles. The motion of a massive particle, such as an electron, is shown schematically in [Figure 106](#). The figure shows that through a tail unbraiding, the structure that describes an electron can get rid of one strand and grab a new one. This process has a low probability, of course. In the strand model, this is one reason that massive particles move more slowly than light, even if the first approximation yields a zero mass value.

We note that this explanation of motion is important also for the mapping from strand diagrams to Feynman diagrams. For many such diagrams, for example for the annihilation of particles and antiparticles in QED, strand hopping and tail unbraiding play a role. Without them, the mapping from strands to quantum field theory would not be possible.

In summary, tangles of massive particles *can* move through the vacuum using hopping – via tail unbraiding – and this naturally happens more slowly than the motion of

photons, which do not need any process at the border of space to hop. The speed of photons is thus a limit speed for massive particles; special relativity is thus recovered.

SUMMARY ON MILLENNIUM ISSUES ABOUT PARTICLES AND THE VACUUM

We have discovered that the strand model makes a strong statement: elementary particles can only be made of one, two or three strands. Each particle is represented by an infinite *family* of tangles of fixed strand number. The family members are related through various degrees of tangling, such as tail braiding or the leather trick.

For *one-stranded* particles, the strand model shows that the photon, the W, the Z and the gluons form the full list of spin-1 bosons. For *two-stranded* particles, the strand model shows that there are precisely three generations of quarks. For *three-stranded* elementary particles, the strand model shows that there is a Higgs boson and three generations of leptons. Neutrinos and antineutrinos differ and are massive Dirac particles. The strand model thus predicts that the neutrino-less double-beta decay will *not* be observed. Glueballs probably do not exist.

Page 153 The strand model explains the origin of all quantum numbers of the observed elementary particles. Also all predicted quantum numbers for composed particles agree with observations. Therefore, we have also completed the proof that all observables in nature are due to crossing switches.

The strand model reproduces the quark model, including all the allowed and all the forbidden hadron states. For mesons and baryons, the strand model predicts the correct mass sequences and quantum numbers. Tetraquarks are predicted to exist. A way to calculate hadron form factors is proposed.

Page 329 In the strand model, all tangles are mapped to known particles. The strand model predicts that *no* elementary particles outside the standard model exist, because no tangles are left over. For example, there are no axions, no leptoquarks and no supersymmetric particles in nature. The strand model also predicts the lack of other gauge bosons and other interactions. In particular, the strand model – corrected in 2012 – reproduces the existence of the Higgs boson. In fact, any new elementary particle found in the future would contradict and invalidate the strand model.

Page 161 In simple words, the strand model explains why the known elementary particles exist and why others do not. We have thus settled two further items from the millennium list of open issues. In fact, the deduction of the elementary particle spectrum given here is, the first and, at present, also the *only* such deduction in the research literature.

THE OMNIPRESENT NUMBER 3

The strand model shows that the number 3 that appears so regularly in the standard model of particle physics – 3 generations, 3 interactions, charge values $e/3$ and $2e/3$ of quarks (as shown below), 3 colours and SU(3) – is, in each case, a consequence of the three-dimensionality of space. In fact, the strand model adds a further, but related number 3 to this list, namely the maximum number of strands that make up elementary particles.

Page 205 The three-dimensionality of space is, as we saw already above, a result of the existence of strands: only three dimensions allow tangles of strands. In short, all numbers 3 that appear in fundamental physics are explained by strands.

PREDICTIONS ABOUT DARK MATTER, THE LHC AND THE VACUUM

Astrophysical observations show that galaxies and galaxy clusters are surrounded by large amounts of matter that does not radiate. This unknown type of matter is called *dark matter*.

In the strand model, the known elementary particles are the only possible ones. Therefore, the galactic clouds made of dark matter must consist of those particles mentioned up to now, or of black holes.

- ▷ The strand model thus predicts that dark matter is a mixture of particles of the standard model and black holes.

Page 161 This statement settles a further item from the millennium list of open issues.

The prediction from 2008 of a lack of new elementary particles in dark matter is at odds with the most favoured present measurement interpretations, but cannot yet be ruled out. The detection of black hole mergers in 2015 can even be seen as a partial confirmation. However, the issue is obviously not yet settled. In fact, the prediction provides another hard test of the model: if dark matter is found to be made of yet unknown particles, the strand model is in trouble.

We can condense all the results on particle physics found so far in the following statement:

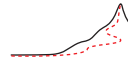
- ▷ There is nothing to be discovered about nature outside general relativity and the standard model of particle physics.

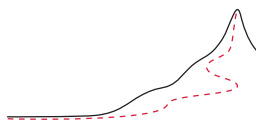
Strands predict that there is no hidden aspect of nature left. In particular, the strand model predicts a so-called high-energy *desert*: it predicts the lack of any additional elementary particle. Equivalently, the strand model predicts that apart from the Planck scale, there is no further energy scale in particle physics. Researchers blinded by beliefs sometimes call this the *nightmare scenario*.

In other words, there is no room for discoveries beyond the Higgs boson at the Large Hadron Collider in Geneva, nor at the various dark matter searches across the world. If any new elementary particle is discovered, the strand model is wrong. More precisely, if any new elementary particle *that contradicts the strand model* is discovered, the strand model is wrong. That some unknown elementary particle has been missed in the present exploration of tangle classes is still a logical possibility.

Because the strand model confirms the standard model and general relativity, a further prediction can be made: *the vacuum is unique and stable*. There is no room for other options. For example, there are no domain walls between different vacuum states and the universe will not decay or change in any drastic manner.

In summary, the strand model predicts a lack of any kind of science fiction in modern physics.





CHAPTER 12

PARTICLE PROPERTIES DEDUCED FROM STRANDS

« Tutto quel che vedete, lo devo agli spaghetti.** »
Sophia Loren

The Planck units, via strands and the fundamental principle, explain almost all that is known about motion: strands explain *what* moves and *how* it moves. But the strand model is only correct if it also explains every measured property of every elementary particle. So far, we only deduced the quantum numbers of the elementary particles. Three kinds of particle properties from the millennium list remain open: the *masses*, the *mixing angles* and the *couplings*. These measured particle properties are important, because they determine the amount of change – or physical action – induced by the motion of each elementary particle.

So far, the strand model has answered all open questions on motion. In particular, we know why quantum field theory, the interactions, the particle spectrum, general relativity and cosmology are what they are. But as long as we do not understand the measured properties of elementary particles, we do not understand motion completely.

In short, the next step is to find a way to *calculate* these particle properties – and obviously, to show that the calculations agree with the measurements. So far, no other unified model has ever achieved such calculations – not even calculations that disagree with measurements.

Because the strand model makes no experimental predictions that go beyond general relativity and the standard model, explaining the properties of elementary particles is the *only way* to confirm the strand model. Many ways to refute the strand model are possible; but only a calculation of the measured particle properties can confirm it.

THE MASSES OF THE ELEMENTARY PARTICLES

The mass describes the inertial and gravitational effects of a body. The strand model must reproduce all mass values observed in nature; if it doesn't, it is wrong.

To reproduce the masses of *all* bodies, it is sufficient that the strand model reproduces the measured masses, mixing angles and coupling strengths of the *elementary* particles. We start with their masses.

** 'Everything you see, I owe it to spaghetti.' Sofia Villani Scicolone is an Italian actress and Hollywood star.

In nature, the *gravitational mass* of a particle results from the space curvature that it induces around it. In the strand model, this curvature is due to the modified fluctuations that result from the presence of the tangle core, in particular to the modified fluctuations of the particle tails and to the modified vacuum strand fluctuations just around the particle position. The modified fluctuations produce a crossing switch distribution around the tangle core; the crossing switch distribution leads to spatial curvature; at sufficiently large distances, this curvature distribution is detected as a gravitational mass.

In contrast, *inertial mass* appears in the Dirac equation. In the strand model, inertial mass is determined by the frequency and wavelength of the rotating phase vector. These quantities in turn are influenced by the type of tangledness, by the fluctuations induced by the particle charges, by the topology changes induced by the weak interaction, and, in the case of fermions, by the average frequency and size of the belt and leather tricks. Also these processes are all due to strand fluctuations.

In short, both gravitational and inertial particle mass are due to strand fluctuations and, more specifically, mainly to the fluctuations of the tails of a particle tangle. *The strand model thus suggests that gravitational and inertial mass are automatically equal.* In particular, the strand model suggests that every mass is surrounded by fluctuating crossing switches whose density decreases with distance and is proportional to the mass itself. As discussed above, this idea leads to universal gravity.

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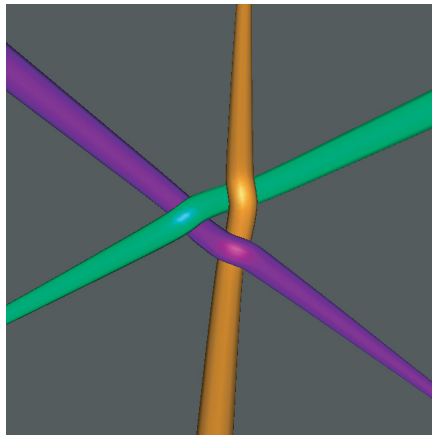
GENERAL PROPERTIES OF PARTICLE MASS VALUES

So far, our adventure allows us to deduce several results on the mass values of elementary particles:

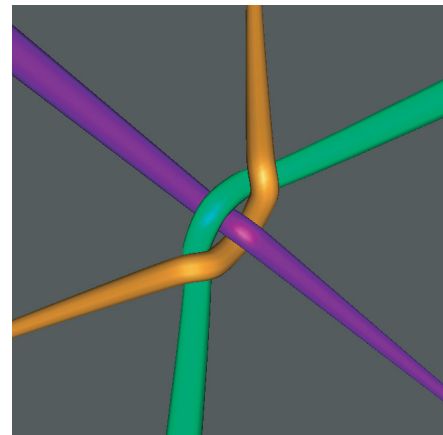
- The strand model implies that the masses of elementary particles are *not free parameters*, but that they are determined by the topology, or tangledness, of the underlying tangles and their tangle families. Particle masses are thus *fixed* and *discrete* in the strand model – as is observed. Of course, we have to take into the many members in each tangle family.
- The strand model implies that masses are always *positive* numbers.
- The strand model implies that the *more complex* a tangle is, the *higher* its mass value is. This follows from the behaviour of tangle tail fluctuations around the tangle core.
- Because particle masses are due to strand fluctuations, the strand model also implies that all elementary particle masses are *much smaller than the Planck mass*, as is observed. Also this result follows from the behaviour of tangle tail fluctuations around the tangle core.
- Because particle masses are due to strand fluctuations, particle and antiparticle masses – their tangles are mirrors of each other – are always *equal*, as is observed.
- Because particle masses are due to strand fluctuations, particle masses do *not* depend on the age of the universe, nor on their position in the universe, nor on any other state variable: The strand model predicts that particle masses are constant and invariant, as is observed.
- Because particle masses are due to strand fluctuations, and the fluctuations differ somewhat for tight and loose tangles of the same shape, the strand model predicts that particle masses change – or *run* – with energy, as is observed.

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Tight W boson tangle candidate



Tight Z boson tangle candidate



Tight Higgs tangle candidate

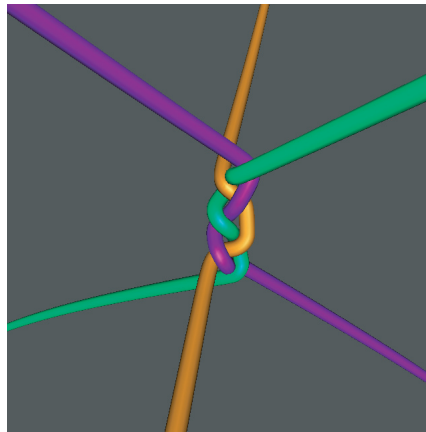


FIGURE 108 Tight tangle candidates (of 2015/2016) for the simplest tangles of the W, the Z and the Higgs bosons.

The general properties of particle masses are thus reproduced by the strand model. Therefore, continuing our exploration makes sense. We start by looking for ways to determine the mass values from the tangle structures. We discuss each particle class separately, and we first look at mass ratios, then at absolute mass values.

BOSON MASSES

Three elementary particles of integer spin have non-vanishing mass: the W boson, the Z boson and the Higgs boson. Mass calculations are especially simple for bosons, because in the strand model, they are *clean* systems: each boson is described by a relatively simple tangle family; furthermore, bosons do not need the belt trick to rotate continuously.

We expect that the induced curvature, and thus the gravitational mass, of an element-

ary boson is due to the disturbance it introduces into the vacuum. At Planck energy, this disturbance will be, to a large extent, a function of the *ropelength* introduced by the corresponding *tight* tangle. Let us clarify these concepts.

Tight or *ideal* tangles or knots are those tangles or knots that appear if we imagine strands as being made of a rope of *constant* diameter that is *infinitely flexible*, *infinitely slippery* and pulled as tight as possible. Examples of tight tangles are shown in [Figure 108](#). With physical ropes from everyday life, tight knots and tangles can only be approximated, because they are not infinitely flexible and slippery; tight tangles are mathematical idealizations. But tight tangles are of special interest: if we recall that each strand has an effective diameter of one Planck length, tight tangles realize the Planck limit of the strand model.

- The *ropelength* of a tight *closed knot* is the length of a perfectly flexible and slippery rope of constant diameter required to tie the tight knot. In other words, the ropelength is the smallest amount of idealized rope needed to tie a knot.
- The *ropelength* of a tight *open knot*, such as the simplest tangle of each weak boson, is the length by which a rope tied into a tight knot is shortened.
- With a bit of care, the concept of ropelength can be also be defined for tangles of several strands.

In the following, the ropelength is assumed to be measured in units of the rope *diameter*. Measuring ropelength in units of the rope radius is less common.

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In the strand model, the ropelength measures, to a large extent, the amount by which a tight knot or tangle disturbs the vacuum around it. The ropelength fulfils all the properties of particle mass mentioned above: the ropelength is discrete, positive, increases with tangle complexity, is equal for particles and antiparticles, and is a constant and invariant quantity. The ropelength will thus play an important role in any estimate of a particle mass.

Ref. 240

It is known from quantum field theory that the masses of W and Z bosons do not change much between Planck energy and everyday energy, whatever renormalization scheme is used. This allows us, with a good approximation, to approximate the weak boson masses at low, everyday energy with their mass values at Planck energy. Thus we can use tight tangles to estimate boson masses.

In the strand model, the *gravitational mass* of a spin 1 boson is proportional to the radius of the disturbance that it induces in the vacuum. For a boson, this radius, and thus the mass, scales as the third root of the ropelength of the corresponding tight tangle.

W/Z BOSON MASS RATIO AND MIXING ANGLE (IN THE 2016 KNOT MODEL)

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The simplest tangles of the W boson and of the Z boson are shown in [Figure 108](#). The corresponding ropelength values for tight tangles, determined numerically, are $L_W = 4.28$ and $L_Z = 7.25$ rope diameters. The strand model estimates the W/Z mass ratio by the cube root of the ropelength ratio:

Ref. 242

$$\frac{m_W}{m_Z} \approx \left(\frac{L_W}{L_Z} \right)^{1/3} = 0.84 . \quad (202)$$

Ref. 229 This value has to be compared with the experimental ratio of $80.4 \text{ GeV}/91.2 \text{ GeV}=0.88$. The agreement between experiment and strand model is acceptable, for two reasons.

Page 249 On the one hand, the strand model reproduces the higher value of the neutral Z boson's mass: a tangle with spatial symmetry is more complex than one without. On the other hand, it is also clear why the calculated mass ratio does not match the experimental result. First of all, the simple tangles represent and approximate W and Z bosons only to the first order. As mentioned above, in the strand model, every massive particle is represented by an infinite family of tangles. The strand model thus also predicts that the match between the calculated and the measured ratio m_W/m_Z should improve when higher-order Feynman diagrams, and thus more complicated tangle topologies, are taken into account. Improving the calculation is still a subject of research. Secondly, approximating the tight knot effects with an effective radius, thus just using the ropelength to determine the mass, implies neglecting the actual shape, and effectively approximating their shape by a sphere. Thirdly, as already mentioned, this calculation assumes that the low energy mass ratio and the mass ratio at Planck energy are equal.

Despite the used approximations, the tight tangle estimate for the W/Z mass ratio gives an acceptable agreement with experiment. The main reason is that we expect the strand fluctuations from the various family members to be similar for particles with the *same* number of strands. For these mass ratios, the tail braiding processes cancel out. Also the other two approximations are expected to be roughly similar for the two weak bosons. This similarity explains why determining the W/Z boson mass *ratio* is possible with acceptable accuracy.

The W/Z mass ratio also determines the weak mixing angle θ_w of the weak interaction Lagrangian, through the relation $\cos \theta_w = m_W/m_Z$. The strand model thus predicts the value of the weak mixing angle to the same accuracy as it predicts the W/Z mass ratio.

Challenge 203 ny This argument leads to a puzzle: Can you deduce from the strand model how the W/Z mass ratio changes with energy?

Page 193, page 211 Also the *inertial masses* of the W and Z bosons can be compared. In quantum theory, the inertial mass relates the wavelength and the frequency of the wave function. In the strand model, a quantum particle that moves through vacuum is a tangle core that rotates while advancing. The frequency and the wavelength of the helix thus generated determine the inertial mass. The process is analogous to the motion of a body moving at constant speed in a viscous fluid at small Reynolds numbers. Despite the appearance of friction, the analogy is possible. If a small body of general shape is pulled through a viscous fluid by a constant force, such as gravity, it follows a *helical* path. This analogy implies that, for spin 1 particles, the frequency and the wavelength are above all determined by the effective radius of the small body. The strand model thus suggests that the inertial mass – inversely proportional to the path frequency and the path wavelength squared – of the W or the Z boson is *approximately* proportional to its tight knot radius. This yields again a cube root of the ropelength and thus gives the same result as for the gravitational mass.

Ref. 162

Also the inertial mass is not exactly proportional to the average tight knot radius; the precise shape of the tight knot and the tangle family members play a role. The strand model thus predicts that a more accurate mass calculation has to take into account these effects.

In summary, the strand model predicts a W/Z mass ratio and thus a weak mixing

angle close to the observed ratio, and explains the deviation of the approximation from the measured value – provided that the tangle assignments are correct.

THE G-FACTOR OF THE W BOSON

Ref. 229 Experiments show that the W boson has a g-factor with the value $g_W = 2.2(2)$. This result – which does not allow to detect any anomalous magnetic moment yet – can be compared to the prediction of the strand model. In particular, the observation might be used to eliminate certain tangle candidates for the W boson.

In fact, the strand model makes a simple prediction for charged elementary particles: because mass rotation and charge rotation are both due to the rotation of the particle core, the g-factor of such particles is always 2 – in the approximation that neglects Feynman diagrams of higher order. In particular, the g-factor of the W boson is 2 in this approximation. Also this strand model prediction thus agrees both with experiment and with the theory of the standard model.

Ref. 241

THE HIGGS/Z BOSON MASS RATIO

The observed mass value of the Higgs boson is 125(1) GeV. The observed mass value for the Z boson is 91.2(1) GeV. Like for the other bosons, the strand model suggests using the ropelength to estimate the mass of the Higgs boson tangle. The candidate tangle for the Higgs boson was given in Figure 94, and the tight version in Figure 108.

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Ref. 242 The ropelength of the tight Higgs tangle turns out to be 17.1, again determined by numerical approximation. This value yields a naive mass estimate for the Higgs boson of $(17.1/7.25)^{1/3} \cdot 91.2$ GeV, i.e.,

$$m_{\text{Higgs}} \approx 121 \text{ GeV} . \quad (203)$$

Starting with the W boson yields an estimate for the Higgs mass of 128 GeV. Both estimates are acceptable, given that the non-sphericity of the W, Z and Higgs boson tangles have not been taken into account. (The strand model suggests that for a strongly non-spherical shape – such as the shape of the W, Z and Higgs tangle – the effective mass is higher than the value deduced from ropelength alone.) Deducing better mass ratio estimates for the W, Z and Higgs tangles is still a subject of research.

In summary, the strand model predicts a Higgs/Z, a Higgs/W and a W/Z mass ratio close to the observed values; and the model suggests explanations for the deviations of the approximation from the observed value – provided that the tangle assignments are correct.

A FIRST APPROXIMATION FOR ABSOLUTE BOSON MASS VALUES

The tangles for the W, Z and Higgs bosons also provide a first approximation for their *absolute* mass values. We notice that each tangle is made of strands that can be pulled straight. This implies, for each strand, no extra strand length and no net core rotation. As a result, in the first approximation, the gravitational mass and the inertial mass of the elementary bosons both *vanish*.

A better approximation requires to determine, for each boson, the probability of crossing switches in and around its tangle core. This probability depends on the probab-

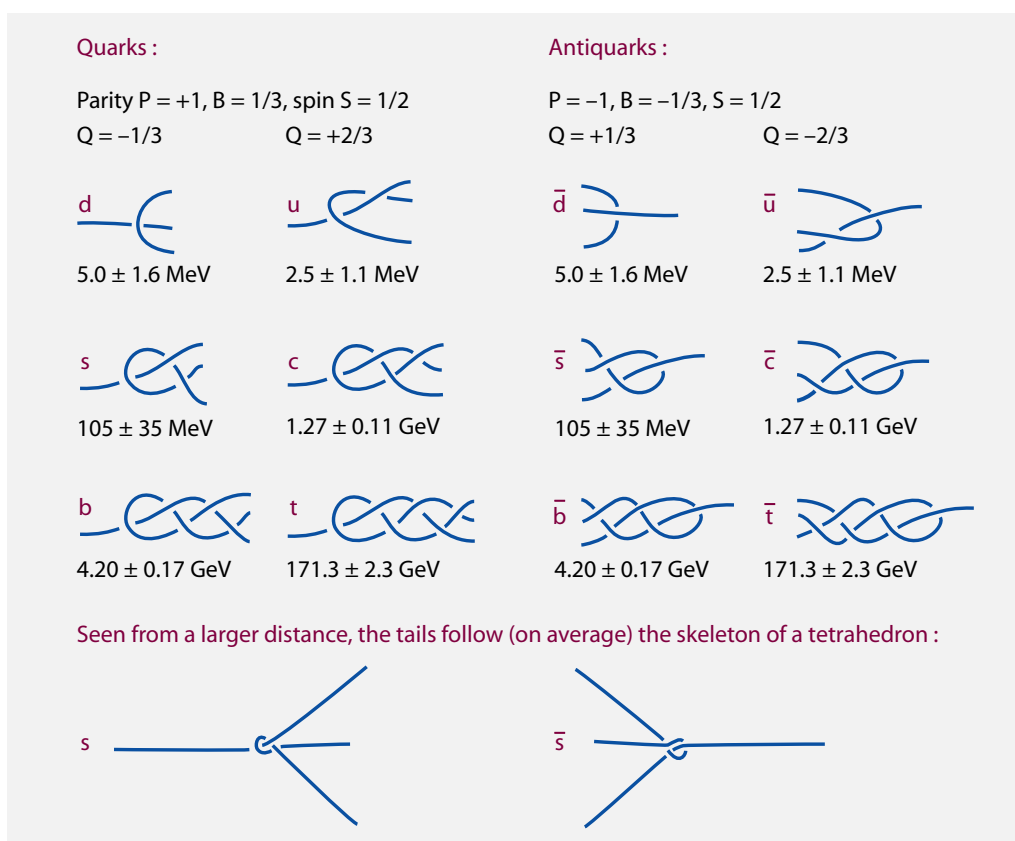


FIGURE 109 The simplest tangles assigned to the quarks and antiquarks. The experimental mass values are also given.

ilities for tail braiding and for core rotation. These probabilities are low, because, sloppily speaking, the corresponding strand fluctuations are rare. The rarity is a due to the specific tangle type: tangles whose strands can be pulled straight have low crossing switch probabilities at their core or at their tails when they propagate.

The strand model thus predicts that elementary boson masses, like all other elementary particle masses, are much smaller than the Planck mass, though not exactly zero. This prediction agrees with observation: experimentally, the three elementary boson mass values are of the order of 10^{-17} Planck masses. We will search for more precise mass estimates below.

QUARK MASS RATIOS

Quarks are fermions. In the strand model, mass estimates for fermions are more difficult than for bosons, because their propagation involves the belt trick. Still, using [Figure 109](#), the strand model allows several predictions about the relations between quark masses.

- The quark masses are predicted to be the same for every possible colour charge. This is observed.

- Furthermore, the progression in ropelength of the tight basic tangles for the six quarks suggests a progression in their masses. This is observed, though with the exception of the up quark mass. For this exceptional case, effects due to tail braiding and to quark mixing are expected to play a role, as argued below.

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Let us try to extract numerical values for the quark mass ratios. We start by exploring the tight quark tangles, thus Planck-scale mass values. For each quark number q , the quark mass will be the weighted average over the mass of its family tangles with q , $q + 6$, $q + 12$, ... crossings, where the period 6 is due to the leather trick. Each tight tangle has a certain ropelength. The mass of each tangle will be determined by the frequency of crossing changes at the core, including those due to the belt trick. The quark mass then is the average over all family tangles; it will be determined by the frequency of tail braiding and all other processes that generate crossing switches.

For determining mass ratios, the frequency of the crossing switches in the core are the most important. Given that the particles are fermions, not bosons, this frequency is expected to be an exponential of the ropelength L . Among quarks, we thus expect a general mass dependence of the type

$$m \sim e^{aL} \quad (204)$$

where a is an unknown number of order 1. We note directly that such a relation promises general agreement with the observed quark mass ratios.

Actual ropelength calculations by Eric Rawdon and Maria Fisher show that the ropelength of quark tangles increases roughly linearly with q , as expected from general knot theoretic arguments. Their results are given in Table 14. Comparing these calculated ropelength differences with the known low-energy quark masses confirms that the number a has an effective value in the range between 0.4 and 0.9, and thus indeed is of order one.

Ref. 243

The results of Table 14 suggest that the top quark should be particularly heavy – as is observed. The results of Table 14 also suggest that something special is going on for the d quark, which is out of sequence with the other quarks. Indeed, the strand model predicts a very small mass, – i.e., Planck energy bare mass – for the down quark. However, in nature, the down mass is observed to be *larger* than the up mass. (We note that despite this issue, meson mass sequences are predicted correctly.) It could well be that the mirror symmetry of the simplest down quark tangle is the reason that the braiding, i.e., the mixing with more massive the family members with six and more additional crossings, is *higher* than that for the up quark.

Ref. 244

The experimental values for the quark masses are given in Table 15; the table also includes the values extrapolated to Planck energy for the pure standard model. The calculation of the strand model *does not agree* with the data. The only encouraging aspect is that the ropelength approximation provides an approximation for older speculations on approximately *fixed mass ratios* between the up-type quarks u , c , t and *fixed mass ratios* between down-type quarks d , s , b . The attempted strand model estimate shows that ropelength alone is *not sufficient* to understand quark mass ratios. Research has yet to show which effect has to be included to improve the correspondence with experiment.

Ref. 245

In fact, the strand model predicts that everyday quark masses result from a combin-

TABLE 14 Calculated ropelengths, in units of the rope *diameter*, of tight quark tangles of Figure 84 (Page 319) with tails oriented along the skeleton of a tetrahedron.

TANGLE	LENGTH	ROPELENGTH	DIFFERENCE
skeleton (vacuum)	138.564065	base value	
simplest d	139.919533	1.355468	1.355468
simplest u	142.627837	4.063773	2.708305
simplest s	146.175507	7.611443	3.547670
simplest c	149.695643	11.131578	3.520136
simplest b	153.250364	14.686299	3.554721
simplest t	157.163826	18.599761	3.913462

TABLE 15 Calculated quark masses at Planck energy, assuming that the standard model is correct up to that value.

QUARK	LOW ENERGY MASS	PLANCK ENERGY MASS
u ($q = 2/3e$)	2.5(1.1) MeV	0.45(0.16) MeV
d ($q = -1/3e$)	5.0(1.6) MeV	0.97(0.10) MeV
s ($q = -1/3e$)	105(35) MeV	19.4(1.2) MeV
c ($q = 2/3e$)	1270(110) MeV	213(8) MeV
b ($q = -1/3e$)	4200(170) MeV	883(10) MeV
t ($q = 2/3e$)	171300(2300) MeV	66993(880) MeV

ation of three effects: the effect of ropelength and of tangle core shape on rotation and the belt trick, the effect of sixfold tail braiding, and the effect of the energy dependence of mass between Planck energy and everyday energy, due to core loosening.

Even though an analytic calculation for quark masses seems difficult, better approximations are possible. With sufficient computer power, it is possible to calculate the effects of the core shape rotations, including the belt trick, and of the energy dependence of the quark masses. The most difficult point remains the calculation of the probabilities for tail braiding. More research is needed on all these points.

LEPTON MASS RATIOS

Mass calculations for leptons are as involved as for quarks. Each lepton, being a fermion, has a large family of associated tangles: there is a simplest tangle and there are the tangles that appear through repeated application of tail braiding. Despite this large tangle family, some results can be deduced from the simplest lepton tangles alone, disregarding the higher-order family members.

Both for neutrinos and for charged leptons, the progression in ropelength of the tight versions of the basic tangles predicts a progression in their masses. This is indeed observed.

For each lepton tangle with l crossings, knot theory predicts a ropelength L that in-

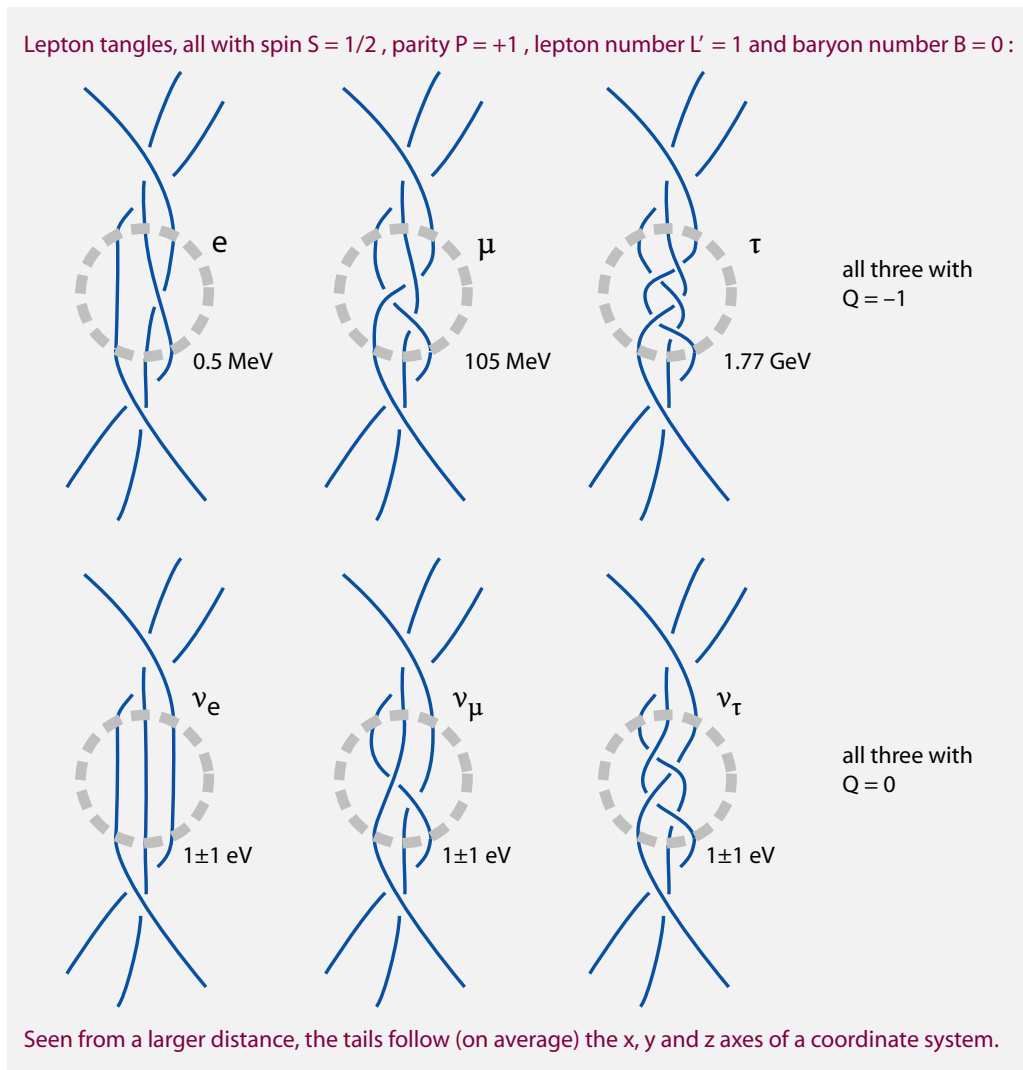


FIGURE 110 The simplest candidate tangles for the leptons. Antileptons are mirror tangles. The experimental mass values are also given.

creases roughly proportionally to l . Each lepton mass value will again be given by the frequency of crossing switches due to rotations, including the belt trick, and of tail braiding. We thus expect a general relation of the type

$$m_l \sim e^{bL_l} \quad (205)$$

where b is a number of order 1 that takes into account the shape of the tangle core. Such a relation is in general agreement with the observed ratios between lepton masses. Research on these issues is ongoing; calculations of ropelengths and other geometric properties of the lepton tangles will allow a more detailed analysis. The most important challenges are, first, to deduce the correct mass sequence among the muon neutrino and the

electron, and second, to estimate the neutrino masses. Indeed, estimating the mass effect due to electric charge is still a subject of research.

Page 329 We note that the lepton mass generation mechanism of the strand model differs from other proposals in the research literature. It agrees with the Higgs mechanism but goes beyond it. For neutrinos, the mechanism contradicts the see-saw mechanism but confirms the Yukawa mechanism directly. From a distance, the mass mechanism of the strand model also somewhat resembles conformal symmetry breaking.
Ref. 246

PREDICTIONS ABOUT ABSOLUTE MASS VALUES

In nature, the masses of elementary particles are much lower than the Planck mass: the observed values lie between about 10^{-30} for neutrinos and 10^{-17} for the top quark.

As we just saw, the strand model predicts mass *sequences* and *ratios* of masses that corroborate or at least do not contradict observations. The next step is to determine *absolute* mass values from the strand model. So far we only found that elementary particle masses are much smaller than a Planck mass. But to validate the strand model, we need more precise statements.

In general, the strand model reduces mass determination to the calculation of the details of a process: How often do shape changes of strands in a particle lead to crossing switches? Strand shape change leads to the braiding of strand tails, to core rotation and, for fermions, to the belt trick.

Ref. 246, Ref. 247 We note that in the past, various researchers have reached the conclusion that all elementary particle masses should be due to a common process or energy scale. Among theoretical physicists, the breaking of conformal symmetry has always been a candidate for the associated process. Among experimental physicists, the Higgs mechanism – now confirmed by experiment – is the favourite common explanation of all elementary particle masses. The strand model, with hindsight, agrees with both approaches. Indeed, in the strand model, tail braiding effects are related to the Higgs boson; we can also say that tails braiding effects are the strand model's version of the Yukawa coupling terms. At the same time we can also argue that tangles break the conformal symmetry of vacuum. With a bit of distance, we can thus say that the strand model agrees with both theoretical expectations.
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We now continue with the quest for absolute mass estimates. In the strand model, *absolute* mass values are *not* purely geometric quantities that can be deduced directly from the shapes of tangle knots. Particle masses are due to *dynamical* processes. Absolute mass values are due to strand fluctuations; and these fluctuations are influenced by the core ropelength and tightness, by tail braiding and by core rotation, including the belt trick. All these aspects lead to crossing switches that determine the mass.

To determine absolute particle mass values, we need to determine the ratio between the particle mass and the *Planck mass*. This means to determine the ratio between the crossing switch probability for a given particle and the crossing switch probability for a Planck mass, namely one switch per Planck time.*

* What is a Planck mass? In the strand model, a Planck mass corresponds to a structure that produces one crossing switch for every Planck time, constantly, without interruption. But the strand model predicts that such structures do *not* appear as localized particles, because every localized particle – i.e., every tangle – has, by construction, a much smaller number of induced crossing switches per time. Following the strand

Energy is action per time. Mass is a form of energy. In other words, the *absolute* mass of a particle is given by the average number of crossing switches it induces over time:

- ▷ Mass is crossing switch rate.

More precisely, the crossing switch rate of a particle at rest is its gravitational mass, and the crossing switch rate induced by accelerated motion is its inertial mass. Let us explore the relation.

Assuming that mass is determined by the crossing switch rate, we deduce that particle mass values are determined by particle topology, are fixed, are discrete, are positive, increase with tangle core complexity, are identical for particle and antiparticles, and are constant over time. Because all these properties match observations, the crossing switch rate indeed realizes all qualitative requirements for absolute particle mass values. Thus we can proceed with the hope to learn more. In order to calculate absolute particle masses, we thus need to determine the number of crossing switches per time that every particle tangle induces.

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One general way to perform a particle mass calculation is to use a large computer, insert a strand model of the vacuum plus the strand model of the particle, and count the number of crossing switches per time. The basis for *one* such approach, using the analogy of the evolution of a polymer in liquid solution, is shown in [Figure III](#). Also the change of strand length has to be taken into account. By determining, for a given core topology, the average frequency with which crossing switches appear for a tethered core, we can determine the masses of the leptons, quarks and bosons. Crossing switches can appear because of the belt trick and the related core rotation, or because of tail braiding. Other processes might also play a role. In such a calculation, the mass scale is set indirectly, through the time scale of the fluctuation spectrum. This is tricky but seems feasible. One would first need to find the parameter space and the fluctuation spectrum for which the polymer tangle follows the Schrödinger equation. Calculations with different tangles should then yield the different mass values. This *computer* approach to particle masses is still a topic of research.

A second, more precise computer program would also model the vacuum itself with strands. In this approach the particle mass appears when the behaviour of a tangle moving through a strand vacuum is explored. At present, this approach is not even a topic of research; it will require an intensive dedicated effort.

A further general way to determine particle masses is to search for *analytical approximations*. This is a fascinating conceptual and mathematical challenge. The main issue is to clarify which crossing switches contribute most to particle mass.

A first analytical attempt is the following. We assume that the inertial mass for a moving fermion is mainly determined by the frequency of the belt trick, and only negligibly

model, elementary particles with Planck mass *do not exist*. This conclusion agrees with observation. But the strand model also implies that black holes with a Planck mass *do not exist*. Indeed, such Planck-scale black holes, apart from being extremely short-lived, have no simple strand structure. We can state that a Planck mass is never localized. Given these results, we cannot use a model of a *localized* Planck mass as a unit or a benchmark to determine particle masses.

The impossibility is also encountered in everyday life. Even though the Planck mass defines the mass unit in nature, no mass measurement in any laboratory is performed by using this unit as a standard.

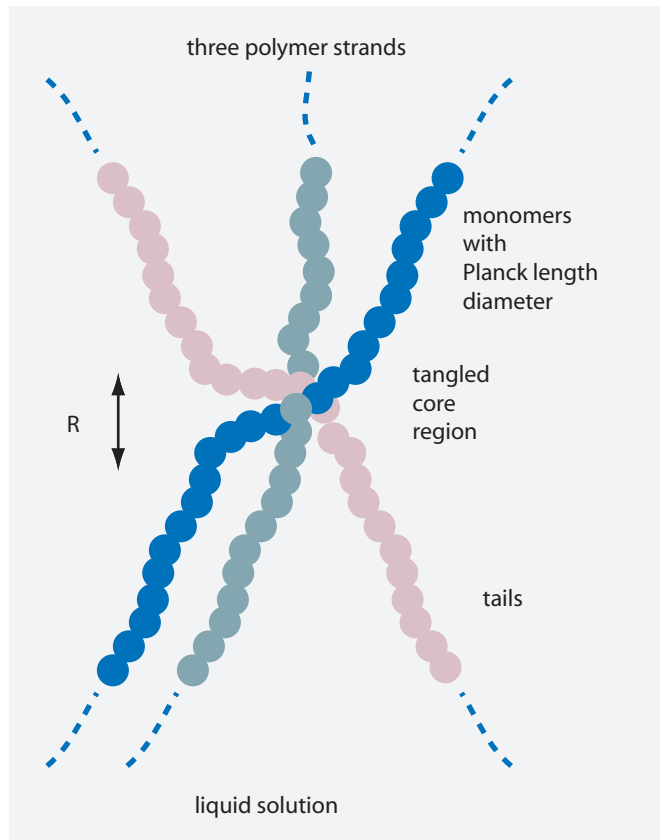


FIGURE 111 Determining lepton mass values with the help of a polymer analogy of strands. The probability of the belt trick for the tangle core yields an estimate for the mass of the elementary particle with the same topology.

by tail braiding. If the core has a diameter of, say, three Planck lengths, then the probability of the belt trick for a particle with six tails will easily be in the range

$$e^{-30} \tag{206}$$

or less. This value, about 10^{-13} , would also be the order of magnitude for the mass estimate in Planck units. Such an estimate is only very rough, and the exponent can be quite different. Nevertheless, we do get an explanation for the large difference between the Planck mass and the typical fermion mass. A more precise analytical approximation for the belt trick probability – not an impossible feat – could therefore solve the so-called *mass hierarchy problem*.

- ▷ What is the numerical probability of the belt trick for a tethered core with fluctuating tails?

Challenge 204 ny

So far, several experts on polymer evolution have failed to provide even the crudest estimate for the probability of the belt trick in a polymer-tethered ball. Can you provide one? And does this estimate yield a good approximation to inertial mass?

Challenge 205 r

A second analytical approximation for the mass value arises when we explore the

probability of tail braiding – i.e., the strand model of the Higgs mechanism or Yukawa coupling. In this approximation, we assume that tail braiding is the main contribution to all the crossing switches that determine mass. Starting from this assumption yields the conclusion that the braiding probability will strongly depend on the relative angles among the tails. Of course, the braiding probability will *also* depend on the core of the tangle, as it determines a part of the crossing switches; but in a first approximation we concentrate at tail braiding.

- ▷ What is the probability of braiding for *three* tails with given directions to spatial infinity?

Challenge 206 ny

That is the challenge to solve.

Let us start with the special case of three tails that are parallel and touching each other. The probability that a Higgs braid – with six crossings – will appear is given by the sixth power of the probability for a two-strand twist to appear, divided by three. And a two-strand twist would have a probability of about e^{-3} at most. That yields a Higgs braid probability estimate of $2 \cdot 10^{-11}$. This would also be an upper limit of the Higgs mass in Planck units for the unrealistic assumption of parallel tails.

Challenge 207 e

For diverging tails, in contrast to parallel ones, the crossing switch probability can easily be the square or the cube of that value. The resulting mass estimates for particle tangles, $\cdot 10^{-22}$ to $\cdot 10^{-33}$, are of the same order of magnitude as the observed mass values.

Challenge 208 r

Can you find a better estimate for the braiding probability in this situation?

All these attempts lead us to the approximation that the particle mass is determined by two effects: the effects of core rotation and the related belt trick on the one hand, and the effects of the tails on the other hand. Are these two effects on particle mass *independent*? If so, particle mass values would be given by the *sum* of two probabilities: a switch probability due to core motion and a switch probability due to tail motion.

If the core and tail processes on mass are not independent, but both required to occur in combination – as seems to be the case – then the particle mass value is given by the *product* of two probabilities: a switch probability due to the core and a switch probability due to the tails. In this case, we get the important conjecture that among particles that share the same tail structure *and* the same tail geometry, i.e., the same tail orientation at spatial infinity, mass ratios can be estimated by comparing tangle cores only. This justifies the above discussions on mass ratios, where this conjecture was intuitively assumed.

Ref. 248

In particular, for the product situation, particle mass is proportional to the crossing switch rate of the core. This proportionality is interesting. Generally speaking, there is a roughly linear relation between ropelength and (average) crossing number; thus a roughly linear or power relation between ropelength and crossing switch rate in the core is expected. Therefore, within a particle family, particle mass should increase with core ropelength: mass should increase with some power of ropelength.

In nature, neutrinos are light, electrons have higher mass, the light quarks a still higher one, and the W and Z bosons an even higher one. What is the origin of these mass differences? In other words, what is the origin of this so-called *mass hierarchy*?

In the strand model, the two influences on particle mass are the size and shape of the tangle core and the structure of the tails. So far, comparisons across particle types are possible only for those particles that share the same tail structure. Now, quarks, leptons

and the massive bosons all differ in their tail structure. We thus find that simple hand-waving mass comparisons are only possible among different generations of the *same* particle type. We have no way yet to compare, with numerical estimates, particle masses from different particle families.

In short, the strand model implies that particle masses are much smaller than the Planck mass by many orders of magnitude. The strand model also implies that more complex tangles lead to higher mass values. However, finding an accurate *numerical* or *analytical* approximation for absolute particle mass values is still an open issue. Obviously, this challenge is a subject of research. In summary, so far we can only state: the mass hierarchy is an unavoidable consequence of the tangles of the elementary particles.

OPEN ISSUES ABOUT MASS CALCULATIONS

Calculating absolute particle masses from tangle fluctuations, either numerically or with an analytical approximation, will allow the final check of the statements in this section.

Challenge 209 e The strand model predicts that the resulting values will match experiments.

* *

Because the strand model predicts a lack of new physics beyond the standard model of particle physics, the calculation of neutrino masses, and thus their mass sequence, is one of the few possible *predictions* – in contrast to retrodictions – that are left over in the strand model. The strand model suggests a conjecture about neutrino masses: assuming that the neutrino tangles of [Figure 110](#) are correct, then the mass of the electron neutrino is influenced more strongly by tail braiding than by the belt trick.

* *

Challenge 210 s Is the mass of a tangle related to the vacuum density of strands?

* *

Challenge 211 s Do particle masses depend on the cosmological constant?

* *

The mass of elementary particles does not depend on the spin direction. In particular, the W and Z bosons have equal longitudinal and transversal mass. The strand model does not allow an influence of spin orientation on mass.

* *

Challenge 212 s Can the concept of *total curvature* help to calculate particle masses?

* *

Challenge 213 d Does the effect of tail braiding confirm the conjecture that every experiment is described by a small energy scale, determining the resolution or precision, and a large energy scale, less obvious, that determines the accuracy?

* *

If tail braiding is due to the weak interaction, and if the Higgs is a tail-braided vacuum,

can we deduce that the Higgs interaction is a higher order of the weak interaction? Can we deduce a concrete experimental prediction from this relation?

FINE-TUNING AND NATURALNESS

It has become fashionable, since about a decade, to state that the standard model of elementary particle physics is ‘fine-tuned’. The term expresses several ideas. Above all, the extremely low value of the vacuum energy is not obvious when all the zero-point field contributions from the various elementary particles of the standard model are included. A low vacuum energy seems only possible if the masses and the particle types of the standard model are somehow interrelated. In other words, the term ‘fine tuning’ expresses, above all, the lack of understanding of the origin of the masses, mixings and coupling constants of elementary particles.

The term ‘fine tuning’ is generally used to state that the universe would be very different if the fundamental constants would be different. Generally speaking, this statement has no deep content. The term ‘fine tuning’ just expresses that particle masses are *not* parameters that can be varied at will. In common usage, ‘parameters’ are variable constants; but the low value of the vacuum energy – as well as many other observations – shows that the masses of elementary particles *cannot* be varied without destroying the validity of the standard model of particle physics.

Some people suggest that ‘fine-tuning’ implies that the standard model of particle physics is ‘unnatural’. The strand model – but also common sense – show that this suggestion is false. The strand model *naturally* has a low vacuum energy, because the unknotted strands of flat space *naturally* have a zero energy density.

In fact, the expression ‘fine-tuning’ means that the fundamental constants in the standard model are not a random choice, but that there is an explanation for these constants that is waiting to be discovered. And indeed, the strand model *naturally* explains that the values for the masses, mixings and coupling constants are not variable, but *fixed*. If the standard model would not be ‘fine-tuned’, it would not describe nature. Any correct description of the world must be ‘fine-tuned’.

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In summary, the term ‘fine-tuned’ is equivalent to the terms ‘unmodifiable’ or ‘hard to vary’ that were discussed above. The term ‘fine-tuning’ highlights the complete lack of alternatives to the world as we observe it; this is one of the wonders of nature. And the strand model makes this wonder apparent in the microscopic domain.

SUMMARY ON ELEMENTARY PARTICLE MASSES AND MILLENNIUM ISSUES

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The strand model implies that masses are dynamic quantities fixed by geometric and topological properties of specific tangle families. As a result, strands explain why the masses of elementary particles are not free parameters, but fixed constants, and why they are much smaller than the Planck mass by many orders of magnitude. Strands also reproduce all known qualitative properties of particle masses.

Strands provide estimates for a number of elementary particle mass ratios, such as m_W/m_Z and m_{Higgs}/m_W . Most quark and lepton mass sequences and first rough estimates of mass ratios agree with the experimental data. All hadron mass sequences are predicted correctly. The strand model also promises to calculate absolute mass values, including their change or ‘running’ with energy. Such future calculations will allow either

improving the match with observations or refuting the strand model.

The results are encouraging for two reasons. First of all, no other unified model that agrees with experiment explains the qualitative properties of mass and the mass sequences. Secondly, no research on statistical tangles exists; this suggests that the understanding of the parameters of nature is missing because results in this research field are still missing.

Page 161 In the millennium list of open issues we have thus seen how to proceed for settling several further items – though we have not settled them yet. Because a few even more interesting challenges are awaiting us, we continue nevertheless. In the next leg, we investigate the mixing of elementary particle states.

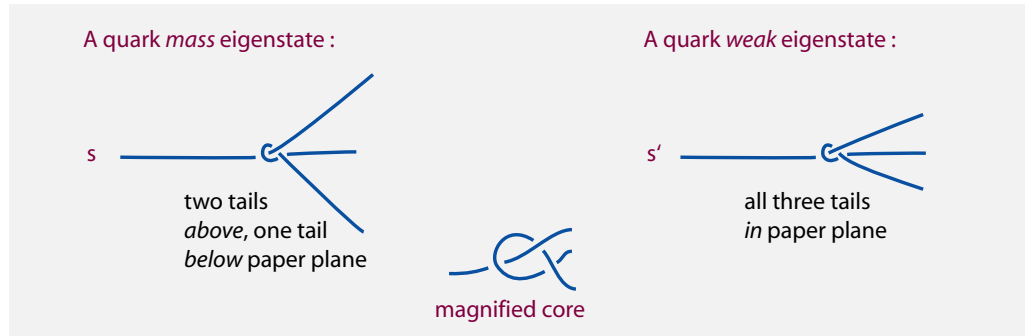


FIGURE 112 Tail shifting leads to quark mixing: mass eigenstates and weak eigenstates differ.

MIXING ANGLES

In nature, the *mass* eigenstates for fermions differ from their *weak* eigenstates: quarks mix among themselves, and so do neutrinos. Quarks also show CP violation. These effects are described by two so-called *mixing matrices*. The two mixing matrices thus contain fundamental constants of nature. For the strand model to be correct, it must allow calculating the measured values of all components of both mixing matrices.

QUARK MIXING – THE EXPERIMENTAL DATA

In nature, the quark mass eigenstates and their weak eigenstates differ. This difference was discovered in 1963 by Nicola Cabibbo and is called *quark mixing*. The values of the elements of the quark mixing matrix have been measured in many experiments, and more experiments aiming to increase the measurement precision are under way.

Ref. 229

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The quark *mixing matrix* is defined by

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (V_{ij}) \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (207)$$

where, by convention, the states of the +2/3 quarks u , c and t are unmixed. Unprimed quark names represent strong (and electromagnetic) eigenstates, primed quark names represent weak eigenstates. In its standard parametrization, the mixing matrix reads

Ref. 229

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (208)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ and i and j label the generation ($1 \leq i, j \leq 3$). The mixing matrix thus contains three mixing angles, θ_{12} , θ_{23} and θ_{13} , and one phase, δ_{13} . In the limit $\theta_{23} = \theta_{13} = 0$, i.e., when only *two* generations mix, the only remaining parameter is the angle θ_{12} , called the *Cabibbo angle*; this angle is Cabibbo's original discovery. The last parameter, the so-called *CP-violating phase* δ , lies between 0 and 2π , is measured to

Challenge 214 e be different from zero; it expresses the observation that CP invariance is violated in the case of the weak interactions. The CP-violating phase only appears in the third column of the matrix; therefore CP violation requires the existence of (at least) three generations.

Ref. 229 The present 90 % confidence values for the measured *magnitude* of the complex quark mixing matrix elements are

$$|V| = \begin{pmatrix} 0.97427(14) & 0.22536(61) & 0.00355(15) \\ 0.22522(61) & 0.97343(15) & 0.0414(12) \\ 0.00886(33) & 0.0405(12) & 0.99914(5) \end{pmatrix}. \quad (209)$$

All these numbers are unexplained constants of nature, like the particle masses. Within experimental errors, the matrix V is unitary.

A huge amount of experimental work lies behind this short summary. The data have been collected over many years, in numerous scattering and decay experiments, by thousands of researchers. Nevertheless, this short summary represents all the data that any unified description has to reproduce about quark mixing.

QUARK MIXING – EXPLANATIONS

In the standard model of particle physics, the quark mixing matrix is usually seen as due to the coupling between the vacuum expectation value of the Higgs field and the left-handed quark doublets or the right handed quark singlets. However, this description does not lead to a numerical prediction.

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A slightly different description of quark mixing is given in the strand model. In the strand model, the Higgs field and its role as mass generator and unitarity maintainer is a special case of the process of tail braiding. And braiding is related to the weak interaction. Because the various quarks are differently tangled rational tangles, tail braiding can reduce or increase the crossings in a quark tangle, and thus change quark flavours. We thus deduce from the strand model that quark mixing is an automatic result of the strand model and related to the weak interaction. We also deduce that quark mixing is due to the *same* process that generates quark masses, as expected. But we can say more.

In the strand model, the *mass eigenstate* – and colour eigenstate – is the tangle shape in which colour symmetry is manifest and in which particle position is defined. The mass eigenstates of quarks correspond to tangles whose three colour-tails point in three directions that are equally distributed in space. The shape in which the tails point in three, equally spaced directions is the shape that makes the SU(3) representation under core slides manifest.

In contrast, the *weak eigenstates* are those shapes that makes the SU(2) behaviour of core pokes manifest. For a quark, the weak eigenstate appears to be that shape of a tangle for which all tails lie in a plane; for such plane configuration, the tails and the core mimic a belt and its buckle, the structure that generates SU(2) behaviour. The two types of eigenstates are illustrated in [Figure 112](#).

In the strand model, masses are dynamical effects related to tangle shape. In the case of quarks, the two configurations just mentioned will thus behave differently. We call the transformation from a mass eigenstate to a weak eigenstate or back *tail shifting*. Tail shifting is a deformation: the tails as a whole are rotated and shifted. On the other hand,

tail shifting can also lead to untangling of a quark tangle; in other words, tail shifting can lead to tail braiding and thus can transform quark flavours. The process of tail shifting can thus explain quark mixing. (Tail shifting also explains the existence of neutrino mixing, and the lack of mixing for the weak bosons.)

Tail shifting can thus be seen as a *partial* tail braiding; as such, it is due to the weak interaction. This connection yields the following predictions:

- Tail shifting, both with or without tail braiding at the border of space, is a generalized deformation. Therefore, it is described by a unitary operator. The first result from the strand model is thus that the quark mixing matrix is unitary. This is indeed observed.
- For quarks, tail braiding is a process with small probability. As a consequence, the quark mixing matrix will have its highest elements on the diagonal. This is indeed observed.
- Tail shifting also naturally predicts that quark mixing will be higher between neighbouring generations, such as 1 and 2, than between distant generations, such as 1 and 3. This is also observed.
- The connection between mixing and mass also implies that the 1–2 mixing is stronger than the 2–3 mixing, as is observed.
- Finally, tail shifting predicts that the numerical values in the quark mixing matrix can be deduced from the difference between the shapes of the two kinds of tangles shown in [Figure 112](#). In particular, tail shifting also predicts that the quark mixing angles change, or run, with energy. In addition, the effect is predicted to be small. On the other hand, so far there is no reliable experimental data on the effect.

Performing a precise calculation of mixing angles and their running with energy is still a subject of research.

A CHALLENGE

Ref. 249 Can you deduce the approximate expression

$$\tan \theta_{u \text{mix}} = \sqrt{\frac{m_u}{m_c}} \quad (210)$$

Challenge 215 r for the mixing of the up quark from the strand model?

CP VIOLATION IN QUARKS

The CP violating phase δ for quarks is usually expressed with the *Jarlskog invariant*, defined as $J = \sin \theta_{12} \sin \theta_{13} \sin \theta_{23}^2 \cos \theta_{12} \cos \theta_{13} \cos \theta_{23} \sin \delta$. This involved expression is independent of the definition of the phase angles and was discovered by Cecilia Jarlskog, an important Swedish particle physicist. Its measured value is $J = 3.06(21) \cdot 10^{-5}$.

Ref. 229

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Because the strand model predicts three quark generations, the quark model implies the possibility of CP violation. In the section on mesons we have seen that the strand model actually predicts the existence CP violation. In particular, [Figure 97](#) shows that with the help of tail shifting, K^0 and \bar{K}^0 mesons mix, and that the same happens with certain other neutral mesons. [Figure 98](#) shows a further example. As just mentioned,

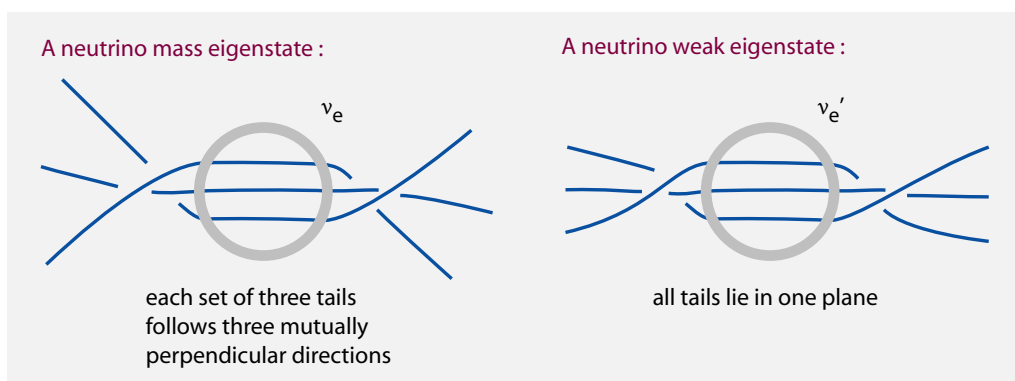


FIGURE 113 Tail shifting leads to neutrino mixing: mass eigenstates and weak eigenstates differ.

the possibility of tail shifting implies that CP violation is small, but non-negligible – as is observed.

The strand model thus predicts that the quark mixing matrix has a non-vanishing CP-violating phase. The value of this phase is predicted to follow from the geometry of the quark tangles, as soon as their shape fluctuations are properly accounted for. This topic is still a subject of research.

NEUTRINO MIXING

Ref. 250 The observation, in 1998, of neutrino mixing is comparably recent in the history of particle physics, even though the important physicist Bruno Pontecorvo predicted the effect already in 1957. Again, the observation of neutrino mixing implies that also for neutrinos the mass eigenstates and the weak eigenstates differ. The values of the mixing matrix elements are only known with limited accuracy so far, because the extremely small neutrino mass makes experiments very difficult:

Ref. 229

$$U = \begin{pmatrix} 0.82(1) & 0.54(2) & -0.15(3) \\ -0.35(6) & 0.70(6) & 0.62(6) \\ 0.44(6) & -0.45(6) & 0.77(6) \end{pmatrix}. \quad (211)$$

Again, all these numbers are unexplained constants of nature. Within experimental errors, the matrix U is unitary. The mixing among the three neutrino states is strong, in contrast to the situation for quarks. Neutrino masses are known to be positive. Present measurements only yield values of the order of 1 ± 1 eV.

In the strand model, the lepton mass eigenstates correspond to tangles whose tails point along the three coordinate axes. In contrast, the weak eigenstates again correspond to tangles whose tails lie in a plane. The two kinds of eigenstates are illustrated in Figure 113. Again, the transition between the two eigenstates is due to tail shifting, a special kind of strand deformation.

We thus deduce that neutrino mixing, like quark mixing, is an automatic result of the strand model and is related to the weak interaction. Given that the neutrino masses are small and similar, and that neutrinos do not form composites, the strand model predicts

that the mixing values are large. This is a direct consequence of the leather trick, which in the case of similar masses, mixes neutrino tangles with 0, 2, 4, 6, 8, 10 etc. crossings and thus leads to large mixings between *all* generations, and not only between neighbouring generations. In the strand model, the large degree of neutrino mixing is thus seen as a consequence of their low and similar masses, and of their existence as free particles.

Again, the strand model predicts a *unitary* mixing matrix for neutrinos. The strand model also predicts that the geometry of the neutrino tangles and their fluctuations will allow us to calculate the mixing angles. More precise predictions are still subject of research.

CP VIOLATION IN NEUTRINOS

The strand model predicts that the three neutrinos are massive Dirac particles, not Majorana particles. This has not yet been verified by experiment. The strand model thus predicts that the neutrino mixing matrix has *only one* CP-violating phase. (It would have three such phases if neutrinos were Majorana particles.) The value of this phase is predicted to follow from the neutrino tangles and a proper accounting of their fluctuations. Also this calculation is still a subject of research.

On the one hand, the strand model suggests the appearance of CP violation in neutrinos. On the other hand, it is unclear when the value of the CP-violating phase will be measured. This is one of the hardest open challenge of experimental particle physics.

The mechanism of CP violation has important consequences in cosmology, in particular for the matter–antimatter asymmetry. Since the strand model predicts the absence of the see-saw mechanism, the strand model rules out leptogenesis, an idea invented to explain the lack of antimatter in the universe. The strand model is more on the line with electroweak baryogenesis.

Ref. 251

Ref. 252

OPEN CHALLENGE: CALCULATE MIXING ANGLES AND PHASES AB INITIO

Calculating the mixing angles and phases ab initio, using the statistical distribution of strand fluctuations, is possible in various ways. In particular, it is interesting to find the relation between the probability for a tail shift and for a tail braiding. This will allow checking the statements of this section.

Challenge 216 ny

Because the strand model predicts a lack of new physics beyond the standard model of particle physics, the calculation of neutrino mixing angles is one of the few possible *predictions* that are left over in fundamental physics. Since the lepton tangles are still tentative, a careful investigation is necessary.

SUMMARY ON MIXING ANGLES AND THE MILLENNIUM LIST

The strand model implies that mixing angles for quarks and neutrinos are properties of their tangle families. The existence of mixing is due to the shape of tangles and their fluctuations. As a result, strands explain why mixing angles are not free parameters, but discrete constants of nature. The strand model also predicts that mixing angles are constant during the evolution of the universe.

We have shown that tangles of strands predict non-zero mixing angles for quarks and neutrinos, as well as CP-violation in both cases. The strand model also predicts that

the mixing angles of quarks and neutrinos can be calculated from strand fluctuations. Strands predict that mixing matrices are unitary and that they run with energy. Strands also predict a specific sequence of magnitudes among matrix elements; the few predictions so far agree with the experimental data. Finally, the strand model rules out leptogenesis.

Page 161 We have thus partly settled four further items from the millennium list of open issues. All qualitative aspects and some sequences are reproduced correctly, but no hard quantities. The result is somewhat disappointing, but it is also encouraging. No other explanation for quark and neutrino mixing is known at present. Future calculations will allow either improving the checks or refuting the strand model. We leave this topic unfinished because we can now proceed to the most interesting topic left: understanding the coupling constants.

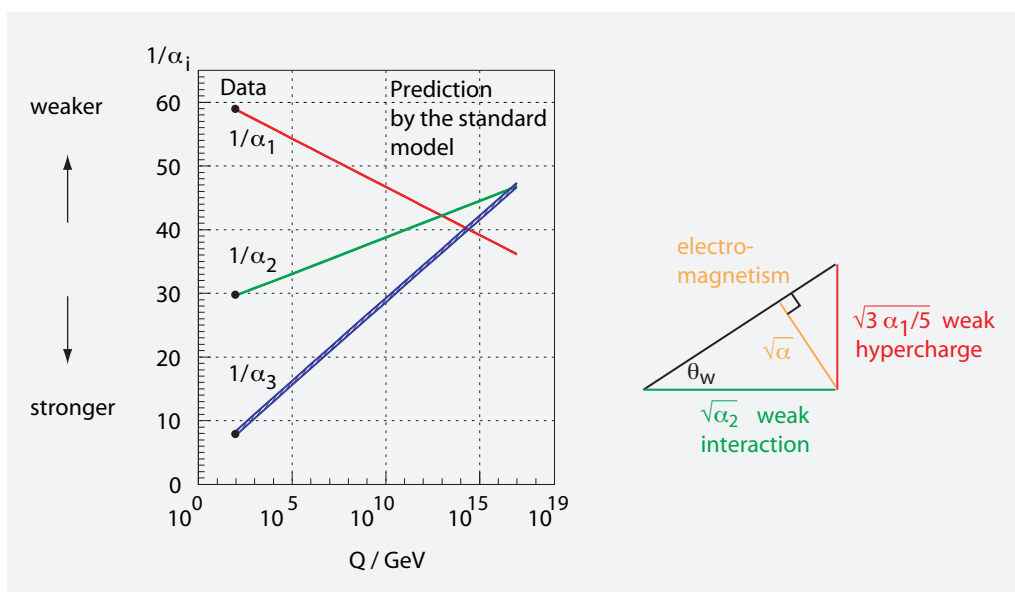


FIGURE 114 Left: How the three coupling constants (squared) change with energy, as predicted by the standard model of particle physics; the graph shows the constants $\alpha_1 = \frac{5}{3}\alpha/\cos^2\theta_W$ for the electromagnetic interaction (more precisely, the weak hypercharge – the factor 5/3 is useful in grand unification), $\alpha_2 = \alpha/\sin^2\theta_W$ for the weak interaction, and $\alpha_3 = \alpha_s$ for the strong coupling constant. The three points are the data points for the highest energies measured so far; at lower energies, data and calculation match within experimental errors (courtesy Wim de Boer). Right: The relation between the coupling constants α for the electromagnetic $U(1)_{EM}$, α_2 for the weak $SU(2)$, α_1 for the weak hypercharge $U(1)_Y$ gauge group and the weak mixing angle θ_W .

COUPLING CONSTANTS AND UNIFICATION

In nature, electric, weak and strong charge are *quantized*. No experiment has ever found even the smallest deviation from charge quantization. *All charges in nature are integer multiples of a smallest charge unit.* Specifically, the electric charge of every free particle is observed to be an integer multiple of the positron electric charge. We call the integer the *electric charge quantum number*.

In nature, the *strength* of a gauge interaction for a unit charge is described by its coupling constant. The coupling constant gives the probability with which a unit charge emits a virtual gauge boson, or, equivalently, the average phase change produced by the absorption of a gauge boson. There are three charge types and three coupling constants: for the electromagnetic, for the weak and for the strong interaction. All particles with a given charge type and value share the same coupling constant, even if their masses differ. The three coupling constants depend on energy. The known data and the change with energy predicted by the standard model of particle physics are shown in [Figure 114](#).

In nature, the *fine structure constant* α , i.e., the electromagnetic coupling constant, at the lowest possible energy, 0.511 MeV, has the well-known measured value

$$\alpha = 1/137.035\,999\,139(31). \quad (212)$$

Equivalently, the electromagnetic coupling of the positron can also be described by the equivalent number

$$\sqrt{\alpha} = 1/11.706\,237\,6167(13) = 0.085424543114(10), \quad (213)$$

which is also called the *electric charge unit* (at low energy). Quantum electrodynamics predicts the precise change with energy of this charge unit; the experiments performed so far, up to over 100 GeV, agree with this prediction. Quantum electrodynamics also predicts that the charge unit, when extrapolated right up to the Planck energy, would have a value of $1/10.2(1)$. These predictions are shown, in a common, but somewhat scrambled way, in [Figure 114](#).

Explaining the value of α , which determines all colours and all material properties in nature, is the most famous millennium issue. If the strand model cannot reproduce every observation about α and other coupling constants, it is wrong. In particular, we thus need to understand, using the strand model, the quantization of charges on the one hand, and the mysterious value of the charge unit – either at low energy or at Planck energy – on the other hand.

INTERACTION STRENGTHS AND STRANDS

In the strand model, *all three gauge interactions are due to shape changes of tangle cores*. We first classify the possible shape changes. Given a tangle core, the following shape changes can occur:

- *Small* changes of core shape do not produce any crossing switch. Small shape changes thus have no physical significance: for a given observer, they leave all observables unchanged.
- *Twist* shape changes of a strand segment in the core produce an *electric field*, if the particle is charged. More precisely, the electric field around a particle is the difference between the average number p_{tr} of right twists and the average number p_{tl} of inverse, left twists that a particle tangle produces per unit time.
- *Poke* shape changes of a strand segment in the core produce a *weak interaction field*. More precisely, the weak field is the asymmetry among the probabilities p_{px} , p_{py} and p_{pz} for the three fundamental poke types and their inverses.
- *Slide* shape changes of a strand segment in the core produce a *colour field*, if the particle has colour. More precisely, the colour field is the asymmetry among the probabilities p_{s1} to p_{s8} for the eight fundamental slide types and their inverses.
- A combination of these moves can also appear.

In the strand model, the fluctuation probabilities for each Reidemeister move – twist, poke or slide – determine the coupling constants. We thus need to determine these probability values. We can directly deduce a number of conclusions, without any detailed calculation:

- The coupling constants are not free parameters, but are specified by the geometric, *three-dimensional shape* of the particle tangles.
- By relating coupling constants to shape fluctuation probabilities, the strand model

predicts that coupling constants are *positive* numbers and *smaller than 1* for all energies. This is indeed observed.

- A still stricter bound for coupling constants can also be deduced. The sum of all possible fluctuations for a particular tangle has unit probability. We thus have

$$1 = p_{\text{small}} + p_{\text{tr}} + p_{\text{tl}} + \sum_{w=x,y,z} (p_{pw} + p_{p-w}) + \sum_{g=1}^8 (p_{sg} + p_{s-g}) + p_{\text{combination}}. \quad (214)$$

The strand model thus predicts that the *sum* of the three charge units must be *strictly smaller than 1*, for every energy value. This is easily checked, both with the data and with the prediction of quantum field theory. In quantum field theory, the three (modified) coupling constants are given, as a function of energy, in the popular graph shown in [Figure 114](#). The values are a combination of experimental data – for low energies – and theoretical extrapolations – for high energies. In this popular graph, the electromagnetic coupling is traditionally multiplied by $5/(3 \cos^2 \theta_W)$. (This is done in order to test grand unification; we keep the traditional factor, even though grand unification is shown by experiment and predicted by the strand model not to apply to nature.) The graph allows us to confirm that the sum of the three unmodified charge units is indeed smaller than 1 for all energy values, as predicted by the strand model.

- The strand model also predicts that the three coupling constants are related by small numbers, as the corresponding fluctuations differ only in the number of involved strands. This is also observed, as [Figure 114](#) shows – especially if we remember that the couplings are the square roots of the values shown in the graph, corrected for the traditional factor.
- The strand model further predicts that the coupling constants are independent of time and space, and that in particular, they do not depend on the age of the universe. This is also observed, despite occasional claims to the contrary.
- Finally, strand model predicts that the coupling constants are the same for particles and antiparticles, as is observed.

Ref. 253

In summary, the strand model implies, like quantum field theory, that *coupling constants are probabilities*. The obvious consequences are

- ▷ $\alpha < 1$,
- ▷ $\alpha_w < 1$,
- ▷ $\alpha_s < 1$
- ▷ $\alpha + \alpha_w + \alpha_s < 1$ and $\sqrt{\alpha} + \sqrt{\alpha_w} + \sqrt{\alpha_s} < 1$.

These properties are valid both in quantum field theory and in the strand model. Despite the agreement with experiment, we have not deduced any new result yet – except one.

STRANDS IMPLY UNIFICATION

In fact, one new point is made by the strand model. Each gauge interaction is due to a different Reidemeister move. However, given a specific tangle core deformation, different observers will classify the deformation as a different Reidemeister move. Indeed, *every*

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Reidemeister move can be realized by the *same* deformation of a single strand: for each Reidemeister move, it is sufficient to add a curved section to a straight strand segment. Such a deformation can look like a type I Reidemeister move for one observer, like a type II move for another, and like a type III move for a third observer.

Because all interactions follow from the same kind of strand deformation of tangle cores, the strand model thus provides *unification* of the interactions. This result is new: in fact, this kind of strand unification of the interactions differs completely from any other approach ever proposed. In contrast to other approaches, strand unification does *not* require that the three coupling constants have the same value at high energy.

A given shape deformation thus has five probabilities associated to it: the probabilities describe what percentage of observers sees this deformation as a type I move, as a type II move, as a type III move, as a combination of such moves, or as no move at all, i.e., as a small move without any crossing switch. On the other hand, at energies measurable in the laboratory, the moves can *almost always* be distinguished, because for a given reaction, usually all probabilities but one practically vanish, due to the time averaging and spatial scales involved.* In short, at energies measurable in the laboratory, the three gauge interactions almost always differ.

Challenge 217 e

CALCULATING COUPLING CONSTANTS

The strand model predicts that the calculation of the three coupling constants is a problem of tangle geometry and fluctuation statistics. Thus it can be approached, at each energy scale, either *analytically* or with *computer calculations*. The calculations need to determine the probabilities of the corresponding Reidemeister moves. If the results do not agree with the experimental values, the strand model is false. We note that there is no freedom to tweak the calculations towards the known experimental results.

In particular, in the strand model, one way to proceed is the following. The (square root of the) fine structure constant is the probability for the emission of twists by a fluctuating chiral tangle.

- ▷ The strand model predicts that the fine structure constant can be calculated by determining the probability of twists, i.e., Reidemeister I moves, in the *fluctuating tangle shapes* of a given particle with nonzero electrical charge.

In other words, the strand model must show that the probability of the first Reidemeister move in chiral particle tangles is *quantized*. This probability must be an integer multiple of a unit that is common to all tangles; and this coupling unit must be the fine structure constant. Any check for the existence of a coupling unit requires the calculation of twist appearance probabilities for *each* chiral particle tangle. The strand model is only correct if all particles with the *same* electric charge yield the *same* twist emission probability.

Instead of emission, also absorption can be used to calculate the fine structure constant:

* The strand model thus predicts that at extremely high energy, meaning near the Planck energy, for each gauge interaction, also particles with zero charge can interact. At Planck energy, when horizons form, the time averaging is not perfect, and interactions become possible even with zero charge.

- ▷ The strand model predicts that the fine structure constant can be calculated from the average angle that a tangle core rotates when absorbing a photon.

We will pursue this alternative shortly.

So far, there do not seem to exist any analytical tool that permits the calculation of shape deformation probabilities. Thus, at present, computer calculations seem to be the only possible choice. Of all existing software programs, the most adapted to calculating fluctuation probabilities are the programs that simulate the dynamics of tangled polymers; but also the programs that simulate the dynamics of cosmic strings or the dynamics of helium vortices are candidates. The main issue, apart from a large computer time, is the correct and self-consistent specification of the shape fluctuation distribution at each energy scale.

Challenge 218 r

In summary, using the strand model we expect to be able to calculate the electromagnetic coupling constant and to understand its validity across all elementary particles. The same expectation obviously also holds for the two nuclear interactions. If any of the expectations on tangle interactions are found to be incorrect, the strand model is false. *The strand model must yield quantized tangle equivalence classes for the electromagnetic, weak and colour charge.* Even though the calculation issues are still subject of research, there are encouraging hints that these expectations will be validated.

FIRST HINT: THE ENERGY DEPENDENCE OF PHYSICAL QUANTITIES

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In nature, all effective charges, i.e., the coupling constants, change with energy. One also says that they *run* with energy. Figure 114 shows the details. Running also occurs for masses and mixing angles. All other intrinsic particle properties, such as spin, parities and all other quantum numbers, are found *not* to change with energy. For the coupling constants, the measured changes between everyday energy and about 100 GeV agree with the prediction from quantum field theory.*

The strand model predicts

- ▷ Coupling constants, masses and mixing angles change with energy because they are quantities that *depend* on the average *geometrical details*, and in particular, on the *scale* of the underlying particle tangles.

More precisely, the running quantities depend on the fluctuations of the geometric tangle shapes, and these fluctuations depend somewhat on the spatial and thus the energy scale under consideration. We note that the strand model predicts a running *only* for these three types of observables; all the other observables – spin, parities or other quantum numbers – are predicted to depend on the *topology* of the particle tangles, and thus to be *independent* of energy. This prediction agrees with observation. Therefore, we can now

Ref. 225
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* In the standard model of particle physics, the running of the electromagnetic and weak coupling constants – the slope in Figure 114 – depends on the number of existing Higgs boson types. The (corrected) strand model predicts that this number is one. Measuring the running of the constants thus allows checking the number of Higgs bosons. Unfortunately, the difference is small; for the electromagnetic coupling, the slope changes by around 2% if the Higgs number changes by one. But in future, such a measurement accuracy might be possible.

explore the details of the running.

SECOND HINT: THE RUNNING OF THE COUPLING CONSTANTS AT LOW ENERGY

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The strand model proposes a new view on the screening and antiscreening effects that are part of quantum field theory. In the strand model, screening effects are consequences of the statistics of shape deformations for loose tangle cores that are embedded into the strands that form the vacuum. Since these statistical effects can in principle be calculated, it is expected that such calculations can be compared with the predictions of quantum field theory shown in [Figure 114](#). This check is in progress. A few results, however, can be deduced without any calculations at all.

In the strand model, the electromagnetic interaction is due to the first Reidemeister move, the twist. For a charged particle – thus one with a chiral tangle core – the average difference in the occurrence of right and left twists determines the effective charge. It is expected that this difference *decreases* when the strand core is loose, because the loose strands are more similar to those of the surrounding vacuum, so that the differences due to the chirality of the tangle will be washed out. In the language of quantum field theory, the virtual particle-antiparticle pairs – created by the fluctuations of the vacuum strands – screen the central, naked charge. The screening is reduced when the energy is increased, and thus when the scales are reduced. In other words, the strand model predicts that the electromagnetic coupling *increases with energy*, as is observed:

$$\triangleright \frac{d\alpha}{dE} > 0 .$$

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Also for the two nuclear interactions, the washing out effect for loose tangle cores by the vacuum does occur as predicted by quantum field theory. In the weak interaction, the *antiscreening* of the weak charge appears in this way. In the strong interaction, both virtual quark–antiquark pairs and virtual gluon pairs can appear from the strands that make up the vacuum. Virtual quark–antiquark pairs lead to screening, as virtual electron–antielectron pairs do for the electromagnetic interaction. In addition, however, we have seen that the strand model of mesons implies that virtual gluon pairs lead to antiscreening. (In contrast, virtual photon pairs do not lead to such an effect.) Because the strand model fixes the number of quark and gluons, the strand model is consistent with the result that the screening of the colour charge by quark pairs is overcompensated by the antiscreening of the virtual gluon pairs.

In other words, the strand model reproduces the observed signs for the slopes of the coupling constants in [Figure 114](#), for the same reason that it reproduces the quantum field theoretic description of the three gauge interactions. The predicted running could also be checked quantitatively, by taking statistical averages of tangle fluctuations of varying dimension. This is a challenge for future research.

THIRD HINT: PREDICTIONS AT LOW ENERGY, INDEPENDENT OF PARTICLE CONTENT

As we just saw, the complete explanation of the running of the couplings depends on the explicit boson and fermion content of nature and on the fact that the strand model reproduces quantum field theory. Interestingly, the strand model also proposes a simpler, though less precise explanation of the running.

At energies much smaller than the Planck energy, such as everyday energies, the strand model implies that the average size of the tangle core is of the order of the position uncertainty of a particle. In other words, any thickness of the strands – real or effective – can be neglected at low energies. Therefore, at low energies, the average strand length within a particle tangle core is also of the order of the de Broglie wavelength. Low, everyday energy thus implies *large, loose* and *spherical/ellipsoidal* tangle cores.

At low energies, shape fluctuations can lead to any of the three Reidemeister moves. The probabilities of such shape deformations will scale with some power of the average strand length within the tangle core. In other words, coupling constants depend on energy. But how exactly?

We note directly that *higher* Reidemeister moves, which involve larger numbers of strand segments, will scale with *larger* power values. In particular, the longer the strand in the core – i.e., the lower the energy – the more the relative probability for the higher Reidemeister moves will increase.

In summary, the strand model predicts that when a tangle is loose and long, i.e., when energies are low, the strong nuclear interaction, due to the third Reidemeister move, is the strongest gauge interaction, followed by the weak nuclear interaction, due to the second Reidemeister move, in turn followed by the electromagnetic interaction:

$$\triangleright \alpha_{\text{em}} < \alpha_w < \alpha_s .$$

The prediction matches observations. Unfortunately, this argument is not reliable. If the strand number were the *only* cause of the running, the argument would imply that the three slopes for the running of the three coupling constants should behave like 3:2:1. However, the graph of [Figure 114](#) shows otherwise, even if the difference between the electromagnetic coupling and the weak hypercharge coupling is taken into account. Indeed, the running of the coupling constants is not due to strand number only, but also to the explicit boson and fermion content of nature, as we just saw.

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THE RUNNING OF THE COUPLING CONSTANTS UP TO PLANCK ENERGY

At energies near the Planck energy, quantum field theory is modified: effects due to the strand diameter start to play a role. Near Planck energy, tangles get tighter and tighter and fluctuations get weaker, because there is less room for them. In other words, near Planck energy tangles tend to approach the structure of horizons. Therefore, near the Planck energy, the strand model predicts deviations from the energy dependence of the coupling constants that is predicted by quantum field theory. So far, estimating such deviations has not been possible.

Another calculation might seem more promising: to calculate the coupling constants near Planck energy. It could be argued that the approach to calculate the low-energy

coupling constants from Planck-energy values seems unsatisfactory, due to the approximations and extrapolations involved. But it is possible if we are convinced that quantum field theory is correct up to Planck energy. And this is just what the strand model predicts. Such a Planck-scale calculation might then allow us to estimate the low-energy coupling constants from their Planck energy values. However, so far, also this approach has not led to success, despite a number of attempts. The challenge seems to be to understand core deformation for case of tight tangle cores. We keep this option in mind.

LIMITS FOR THE FINE STRUCTURE CONSTANT DO NOT PROVIDE EXPLANATIONS

When searching for ways to determine the fine structure constant, we need to be careful. Here is an example that explains why.

Ref. 258 Numerous observations of nature imply a limit on the fine structure constant. A pretty one appeared in a post on the internet in 2017. The *electrostatic repulsion* between two electrons at a given distance must be larger than the *radiation force* between two small neutral black holes at that same distance. In other words,

$$\frac{e^2}{4\pi\epsilon_0} \frac{1}{r^2} > \frac{L_{\text{bh}}}{c} \frac{\pi R_{\text{bh}}^2}{4\pi r^2}. \quad (215)$$

Vol. V, page 149 Here it is assumed that thermal radiation from one black hole acts on the cross section of the other black hole by pushing it away. Multiplying both sides by $r^2/\hbar c$ and inserting the expressions for the black hole luminosity L_{bh} and the black hole radius R_{bh} gives

$$\alpha > \frac{1}{15\,320\,\pi}. \quad (216)$$

The bound is not tight, but is obviously correct.

Ref. 259 Various researchers are looking for observations that give the best possible bound for α . Such a search can indeed yield much better bounds. However, such a search cannot *explain* the value of α . We can indeed use thermodynamics, gravity or other observed properties to deduce *observational limits* on α . Many formulae of physics contain α in a more or less obvious way. Maybe, one day, known physics will be able to yield very tight upper and lower bounds for α . Still, *the explanation of the value of α would still lack*.

To explain the fine structure constant α , we need an approach based on the final theory, not one based on known, millennium physics, such as expression (215). Millennium physics can *measure* α , but *cannot explain* it. To explain the fine structure constant, a final theory is needed. In our case, we need to check whether we can calculate α with strands. Therefore, we now explore tangle topology, tangle shapes and tangle motion with this aim in mind.

CHARGE QUANTIZATION AND TOPOLOGICAL WRITHE

In the strand model, electric charge is related to the *chirality* of a tangle. Only chiral tangles are electrically charged. The strand model thus implies that a topological quantity for tangles – defined for each tangle in the tangle family corresponding to a specific

elementary particle – must represent electric charge. Which quantity could this be?

The first candidate for charge in the strand model is provided by knot theory:

- ▷ The usual topological quantity to determine chirality of knots and tangles is the *topological writhe*.

To determine its value, we draw a *minimal projection*, i.e., a two-dimensional knot or tangle diagram with the *smallest* number of crossings possible. We then count the right-handed crossings and subtract the number of left-handed crossings. This difference, an integer, is the topological writhe. Topological writhe is thus a two-dimensional concept and does not depend on the shape of a knot or tangle. We note:

- The topological writhe of the W boson tangles is +3 or –3, depending on which mirror image we look at; the topological writhe of the Z boson and Higgs boson tangles vanishes. The topological writhe of any unknotted strand also vanishes. In this way, if we define the electric charge quantum number as *one third* of the topological writhe, we recover the correct electric charge quantum number of the weak and all other gauge bosons. We note that the leather trick does not change this result, so that all family members of a particle share the same topological writhe.
- Page 319 – The tangles of the quarks show that if we define the electric charge quantum number as *one third* of the topological writhe, we recover the correct electric charge quantum number of all quarks. The leather trick has no effect on this definition.
- Page 325 – The tangles of the leptons show that if we define the electric charge quantum number as the topological writhe of the *centre region* only, we recover the correct electric charge quantum number of all leptons. Again, the leather trick does not change this result.

In other terms, the electric charge quantum number can be reproduced with the help of topological writhe. And indeed, the electric charge of massless bosons, i.e., photons and gravitons, vanishes.

Let us sum up. In nature, electric charge is quantized. The strand model describes charged particles with the help of fluctuating alternating tangles, and charge quantization is a topological effect that results because all particles are made of strands. In particular,

- ▷ The *electric charge quantum number* behaves similarly to topological writhe (times one third or times one): it is quantized, has two possible signs, vanishes for achiral tangles, is a topological invariant – and thus is conserved.

In short, a topological quantity, namely topological writhe, reproduces the electric charge quantum number in the strand model. Three issues remain. First, given that every particle is described by a tangle family with an infinite number of members, how is the electric charge, i.e., the topological writhe of the other tangle family members accounted for? It is not hard to see that family members do not change topological writhe. The second issue is more thorny: why is the charge definition different for leptons? We skip this problem for the time being. The third issue is the central one: What is the origin of the peculiar value of the charge unit, whose square has the value

Challenge 219 e

Challenge 220 ny

Ref. 5 $\alpha = 1/137.035\,999\,139(31)$ at low energy?

CHARGE QUANTIZATION AND LINKING NUMBER

An alternative conjecture for charge quantization is the following:

- ▷ Electric charge, i.e., twist emission probability, might be proportional to the *linking number* of ribbons formed by strand pairs.

The following arguments speak in favour of this conjecture.

- In knot theory, a ribbon is the strip associated to and limited by two strands.
- The *linking number* of a ribbon is the number of times that the two edges of a ribbon wind around each other. The linking number is a topological invariant and an integer.
- In particle tangles, only wound up, i.e., linked ribbons should lead to (net) boson emission. For tangles made of three strands, we define a total linking number as the sum of all three possible linking numbers.
- The linking number of the Higgs boson strand pairs is zero; that of the Z boson strand pairs is the sum of 1, 0 and -1, thus also zero. The linking number for the W boson is 3 or -3, that of the quarks is 1, -1, 2 or -2. We thus conjecture that the charge quantum number is one third of the total linking number.
- Massless bosons, i.e., photons, gluons and gravitons, have no electric charge.

In short, linking number, an integer, might be a better topological quantity to explain electric charge quantization than topological writhe. On the other hand, it might well be that linking number, being a quantity that depends on *two* strands, is related to the *weak charge* rather than to the electric charge.

If the conjectured relation between linking number and electric or weak charge is correct, it might lead to a calculation of the corresponding coupling constant, once the tangle shape or, better, once the tangle dynamics is included in the proper way. For example, the photon emission probability could depend on the *writhe* or on the *twist* of the (averaged) ribbons. Both these properties might lead to virtual photon emission. (The sum of writhe and twist of a ribbon is given by the linking number, as explained by Calugareanu's theorem.)

In this and any topological definition of electric charge, we face two slight hurdles: First, we have to watch out for the graviton: it is uncharged. Secondly, we have to explain why the strand model for the simplest family member of the d quark is not chiral. Both hurdles can be overcome.

Challenge 221 e

If the linking of *two* strands is connected to weak charge, it might well be that a similar quantity defined for *three* strands is related to colour charge. All these possibilities are topic of research.

HOW TO CALCULATE COUPLING CONSTANTS

The strand model suggests that crossing number and linking number somehow define electric and weak charge. In simple words, the model suggests that quantization of all charge types is a topological effect; quantization is due to the multiple ways in which

strands cross inside tangles.

Coupling constants describe the probability of interaction with gauge bosons. Experiments show that these quantities are slightly *scale*-dependent, since they run with energy. But in the strand model, coupling constants are not really *shape*-dependent: electrons, muons and antiprotons have the same electric charge and fine structure constant values despite being described by different tangles. Coupling constants do not depend on the *kind* of tangle. Experiments show that they just depend somewhat on its size. In short,

- ▷ We need a definition of each coupling constant that is *tangle-independent* and *shape-independent*, and only depends on a topological invariant of tangles.

In fact, this conclusion eliminates many speculations, including a number of calculation approaches that were included in this chapter in previous editions. We are left with just a few options. To explore them, we start with an overview.

COUPLING CONSTANTS IN THE STRAND MODEL

In experiments, there are the following gauge interactions with their charges:

1. The electromagnetic interaction with electric charge and U(1) symmetry.
2. The weak interaction with weak isospin and SU(2) symmetry.
3. The strong interaction with colour and SU(3) symmetry.

In the strand model, the *gauge interactions* are modelled as transfers of Reidemeister moves:

1. The electromagnetic interaction is twist transfer and the electric charge is preferred twist transfer to or from a massive particle. Twists can be added and form a circle: they form a U(1) Lie group. They change the tangle phase by exchanging one observable crossing.
2. The weak interaction is poke transfer and the weak isospin is preferred poke transfer to or from a massive particle. Pokes exist in three linearly independent directions and their generators behave like the belt trick: they generate an SU(2) Lie group. They change the tangle phase by exchanging two observable crossings.
3. The strong interaction is slide transfer and the colour charge is preferred slide transfer to or from a massive particle. Slides can be added, its generators have a Z_3 symmetry and they form an SU(3) Lie group. They change the tangle phase by exchanging two or three? crossings.

In the strand model, *neutral particles* are those that cannot receive Reidemeister moves or that receive them all in equal way:

1. Electromagnetism: Neutral ‘tangles’ are made of one strand (e.g., the photon) or are achiral (e.g., the Z and the neutrinos).
2. Weak interaction: Neutral tangles are made of one strand (e.g., the photon) or of two straight or unpokeable strand pairs (e.g., the Z, the right-handed leptons and quarks).
3. Strong interaction: Neutral tangles are made of one strand or of three strands.

In the strand model, *charged particles* are specific tangles:

1. Electric charge is due to the observability of crossings during photon emission or absorption, i.e., when twists are applied. Particles with electric charge, i.e., with preferred twist transfer, have a global asymmetry, global twistedness, namely topological chirality. Locally, electrically charged particles have crossings; electric charge is positive or negative. Charge is $1/3$ of the signed crossing number. Examples are the charged leptons, the quarks and the W boson.
2. Weak charge is thus due to the observability of crossings during W or Z emission or absorption, i.e., when pokes are applied. Particles with weak isospin, i.e., with preferred poke transfer, have a global asymmetry that prevents all pokes to act equally effectively: For fermions, such an asymmetry arises when tangle twistedness and the belt trick have the same sign; thus all left-handed fermions and right-handed antifermions have weak isospin. Locally, weakly charged fermions behave like a belt buckle that rotates in the appropriate direction. Due to their tangle topology, some fermions have positive, others negative weak isospin. For the W boson, the asymmetry is built into the tangle; due to the tangle structure, the W and its antiparticle have plus or minus twice the weak isospin of fermions.
3. Colour, strong charge, is due to the observability of crossings during gluon emission or absorption, i.e., when slides are applied. Particles with colour charge, i.e., with preferred slide transfer, have a global asymmetry that prevents all slides to act equally effectively: Coloured particles are made of exactly two strands with tails in tetrahedron skeleton directions. Only two-stranded tangles allow certain slides and prevent others. Therefore only quarks have colour charge. Locally, red, blue and green colours correspond to three directions in one plane that differ by an angle of $2\pi/3$.

Coupling strength is the ease of crossing rotation, of poke creation, and of slide induction. These connections allow calculating the coupling strength values.

DEDUCING α FROM PRECESSION

In nature, magnetic fields rotate charged particles. In the strand model, as shown in [Page 230](#) [Figure 52](#), magnetic fields are made of moving twists. In fact, from the strand definition of the electromagnetic interaction and the electric charge and from the drawing in [Page 225](#) [Figure 49](#), we deduce:

- ▷ Moving twists rotate crossings.

We note that this description differs slightly from a pure twist transfer. But this formulation is the key to calculating α .

We assume that the typical, average crossing is lying in the paper plane, as in the drawing of the fundamental principle. For an average crossing, the two strands lie along the x and y axes. When a photon, i.e., a twist, arrives along the diagonal in the first quadrant, it rotates the crossing completely, by one turn. If the twist arrives at a different angle, its effect is lowered. We approximate this angle effect with simple trigonometry: we assume that the angular projection describes the reduction of the effect with the incoming angle of the twist.

For the incident photon, we call γ the angle from the y-axis and β the angle out of

the paper plane. The average rotation angle induced by an absorbed photon or twist on a charged particle with *three* crossings, corresponding to *one* elementary charge, is approximately given by

$$\sqrt{\alpha_{\text{calc}}} = 3 \int_{\gamma=0}^{\pi/2} \int_{\beta=-\pi/2}^{\pi/2} \sin \gamma (\cos \gamma \sin \gamma \cos \beta)^4 d\beta d\gamma \approx 0.089 . \quad (217)$$

Here, $\sin \gamma$ is from the volume element in spherical coordinates and all the other terms arise from the trigonometric approximation. In particular, the fourth power arises from the two tails plus the squaring required to get probabilities. However, the expression remains open to dispute.

A better approximation averages over the angle δ between the strands at the crossing. The average over δ is realized by including $\frac{1}{\pi} \int_{\delta=0}^{\pi}$, and modifying the integrand and one limit:

$$\sqrt{\alpha_{\text{calc}}} = \frac{3}{\pi} \int_{\delta=0}^{\pi} \int_{\gamma=0}^{\delta} \int_{\beta=-\pi/2}^{\pi/2} \sin \gamma \left(\frac{1}{2} \cos(2(\delta - \gamma)) \cos \beta \right)^4 d\beta d\gamma d\delta \approx 0.083 . \quad (218)$$

The resulting values of 0.089 and 0.083 are not acceptable approximations to reality, in which $\sqrt{\alpha'} = 0.08542454311(1)$ at low energy and $\sqrt{\alpha'} = 0.10(1)$ at Planck energy. Neither are the values good approximations to the hypercharge coupling, which changes from $\sqrt{\alpha_1'} = 0.13(1)$ at low energy (100 GeV) to $\sqrt{\alpha_1'} = 0.17(1)$ at Planck energy. We need a better approximation for the value of the electromagnetic coupling strength.

DEDUCING THE WEAK COUPLING

Weak fields deform strand (crossing) pairs by adding or transferring generalized pokes. Weak fields are collections of pokes; pokes represent virtual weak bosons. The weak isospin, the weak charge, is related to the orientation of the strand pairs. The weak interaction occurs through an incoming poke that deforms a strand pair:

- ▷ A moving poke rotates a pair of strands.

This process is the key to calculating α_w . We note that there is a certain similarity to the setting used for calculating the electromagnetic coupling: in both cases, the incoming boson acts on a target consisting of two strands. This similarity is the reason for electroweak mixing.

We calculate the coupling constant for a single belt buckle, assuming parallel strands. The average rotation angle induced by one incoming weak (unbroken) boson (out of the three possible cases) is one full turn when the impact is perpendicular to the two strands and to the plane defined by them. For a general incidence angle the induced rotation angle is lower. We again use trigonometrical projection to approximate the induced crossing rotation angle in the general case, with the same issues as in the previous case. We call γ the angle from ideal incidence, and β the longitude. The average angle is

then given by

$$\sqrt{\alpha_w \text{ calc}} = \int_{\gamma=0}^{\pi/2} \int_{\beta=0}^{2\pi} \sin \gamma (\cos^2 \gamma \cos^2 \beta)^4 d\gamma d\beta \approx 0.19 . \quad (219)$$

If we need to average over the different angles between the strands that make up the pair experiencing the poke, we get a different value.

The calculated value of the weak coupling is not an acceptable approximation to reality, in which $\sqrt{\alpha_w} = 0.18$ at the (low) energy of 100 GeV and $\sqrt{\alpha_w} = 0.14$ at Planck energy. We need a better approximation.

DEDUCING THE STRONG COUPLING

Strong fields deform specific three-strand configurations by adding generalized slides. The generalized slides are due to gluons. Strong colour is related to the order and orientation of the strands in these specific three-strand configurations. In short:

- ▷ Incoming, moving slides deform three-strand configurations.

This is the key to calculating α_s .

We assume that one of eight possible gluons is incident. In an average triple strand configuration, the three strands are oriented in a way that in the paper plane they look like three symmetrically arranged rays. One ray lies along the y axis. When a gluon arrives, it performs a slide. For an incident gluon, we call γ the angle from the y-axis to the next strand and β the angle out of the paper. In the trigonometric approximation, the average slide angle induced on a coloured particle is given by

$$\sqrt{\alpha_s \text{ calc}} = \int_{\gamma=0}^{\pi/2} \int_{\beta=0}^{2\pi} \sin \gamma (\cos^3 \gamma \cos^3 \beta)^4 d\beta d\gamma \approx 0.11 . \quad (220)$$

This is not an acceptable approximation to reality, in which $\sqrt{\alpha_s} = 0.7(1)$ at the (low) energy of 1 GeV and $\sqrt{\alpha_s} = 0.13(1)$ at Planck energy. We need a better approximation for the strong coupling.

OPEN CHALLENGE: CALCULATE COUPLING CONSTANTS WITH PRECISION

The approximations used above for estimating the coupling constants can be dismissed as mere educated guesses. Despite this objection, these guesses show that a determination of the coupling constants from the strand model is within reach, and that it can be realized with limited effort. It is sufficient to improve the three approximations; this can be realized by using computer simulations for the transfer of Reidemeister moves or by finding an improved analytical model.

Calculating all three coupling constants ab initio with high precision will allow checking the statements of this section in an independent manner and, above all, will allow testing the strand model. The calculations should be performed at different energies, to

Challenge 222 r

confirm the energy dependence of the couplings.*

Page 350 In order to reach highest precision, the effects of the various tangle family members have to be taken into account, because in the strand model, each particle is described by a family of tangles. On the other hand, the strand model predicts that family members have a small effect on the coupling constant, so that the family issue can be neglected in the beginning.

Ref. 260 In the case of the nuclear coupling constants, Arnold's results on plane curves may help in the estimations and calculations.

ELECTRIC DIPOLE MOMENTS

Experimental physicists are searching for electric dipole moments of elementary particles. No non-zero value has been detected yet. The idea of electric dipole moment is based on a non-spherical distribution of electric charge in space.

In the strand model, particles are tangles. As a consequence, the electric charge distribution – the distribution of the crossings in a tangle – is intrinsically a slightly non-spherical quantity, thus a quantity unequally distributed in space. However, it is only non-local on a scale of the order of a Planck length. In other terms, the electric dipole moment d of elementary particles is predicted to be

$$\triangleright d = f e l_{\text{Pl}},$$

Ref. 261 where the factor f arises from averaging the tangle and is of order one. Similar values are predicted by the standard model in the absence of supersymmetry and grand unification. However, the sensitivity of measurements has not reached these values yet, by several orders of magnitude.

We note that the strand model predicts that the dipole moment changes, or 'runs', with energy. This follows from the shape-dependence of the dipole moment. Such a dependence is also predicted by quantum field theory.

In summary, we expect that up to a region close to a Planck length, the *strand* model should not yield dipole moments that differ in order of magnitude from those predicted by the *standard* model of particle physics. In the future, more precise calculations and measurements could allow testing the strand model using dipole moments.

FIVE KEY CHALLENGES ABOUT COUPLING STRENGTHS

There are many ways to evaluate candidates for unified models. A concrete evaluation focuses on four key challenges about coupling constants. These challenges must be resolved by any candidate model in order to be of interest.

1. So far, we explained particle charges with topological properties of the tangle models of the particles, and we explained coupling strengths with the transfer of crossings, pokes and slides. This allowed deducing a rough approximation of coupling constants. By doing so, we have settled a first key challenge:

* In particular, the influence of the effective strand diameter on the fine structure constant should be explored.

- ▷ The strand model explains why the fine structure constant, or equivalently, the electric charge, is the *same* for electrons and protons.

Deducing this equality is a key challenge for any unified model. In fact, all coupling constants must be independent of particle type. This is the case in the strand model.

2. The second key challenge was the energy-dependence of the coupling constants. The strand model predicts that coupling constants run with energy in exactly the way that is predicted by QED, QCD and electroweak theory. We could also argue that this is not a real challenge for any unified model that reproduces these theories. In the strand model, the running of the electromagnetic coupling constant can be seen as a consequence of the gradual tightening of tangles with energy. For a typical electrically charged particle at low energy, the tangle is very loose; therefore:

- ▷ The Planck scale number of crossings is shielded by an additional cloud of crossings created by the loose strands of the tangle.

In this way, the strand model explains the running of the fine structure constant in exactly the same way as QED.

3. The third key challenge has only been touched upon very briefly:

- ▷ Any unified model needs to clarify the relation between the hypercharge, the electric charge and the weak isospin (the ‘weak charge’).

The strand model explains electromagnetism as acting on crossings and the weak interaction as acting on parallel strands. This general statement contains the required explanation; but the details still need to be worked out. It is expected that in electromagnetism, a *single* crossing is rotated, mainly by rotating *one* strand around the other. In contrast, in the weak interaction, *two* strands are rotated together, producing a switching *two* crossings. The number of crossings differs between electromagnetism and the weak interaction, but the total number of involved strands is two in both cases. As a result of this similarity, the two interactions mix. The final explanation of electroweak mixing might even allow to deduce an intuitive geometric meaning of θ_w , the weak mixing angle or Weinberg angle.

4. The fourth key challenge, related to the previous one, still needs to be explored in more detail:

- ▷ Any unified model must explain why the mass ratio of the intermediate weak vector bosons is related to the coupling ratio of the weak and the electromagnetic interaction as

$$\left(\frac{m_W}{m_Z}\right)^2 + \frac{\alpha}{\alpha_w} = 1. \quad (221)$$

The strand model strongly suggests that it can explain the relation, but the detailed ar-

gument must yet be provided. Using more drastic language, we can repeat what many have said already in the past: explaining the electroweak mixing expression (221) is the key challenge for any unified model.

In the strand model, the two electroweak coupling constants are measures for interaction probabilities of crossings with twists and with pokes. In contrast, masses are interaction probabilities of crossings with spatial curvature. Why are they related by expression (221)? Here is a short brainstorm on the issue.

In the strand model, mass appears by tail braiding. Tail braiding adds crossings, and in this way adds mass. Added crossings also imply added weak and sometimes electric charges. The Z boson arises from vacuum by different tail braidings than the W. The W arises by the braiding of two tail pairs at 90 degrees; the Z arises by braiding one tail pair at 90 degrees.

In case of the W and the Z bosons, the Z tangle produces a larger disturbance of the vacuum than the W; therefore it is more massive than the W.

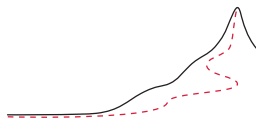
At which angle does a clasp start to form a “enclosed space in between”? How does this space change with scale, given that scale might change the clasp angle? This question might be related to the running of masses, mixing angles or coupling constants. In particular, we should answer the following question: Which physical observable does this enclosed space influence? Mass, couplings, or mixings? Is mass more related to ropelength or more related to the enclosed space?

5. The fifth key challenge is, of course, the precise calculation of the coupling constants.

SUMMARY ON COUPLING CONSTANTS

The strand model implies that coupling constants are geometric properties of tangle families that correspond to charged particles. As a result, strands explain why the coupling constants are not free parameters in nature, but fixed constants. Strands predict that coupling constants are the same for similarly charged particles, that they run with energy and that they are constant during the macroscopic evolution of the universe; all this is observed. Strands predict small electric dipole moments for elementary particles, compatible with present measurement limits. Strands also predict the correct sequence of the coupling constants at low energy and the correct sign of their running with energy. Strands thus reproduce all qualitative properties of coupling constants. No other unified model achieves this yet.

The strand model proposes several ways to calculate coupling constants ab initio, using tangle shapes. First estimates of the fine structure constant, based on the *knotted* particle models of 2010, deviated from experiment by 40%. However, those particle models turned out to be mistaken. Calculations based on the new *tangled* particle models are ongoing; the accuracy seems better than the first estimates, but is still not satisfying. Improved calculations will allow to confirm or to refute the strand model.



EXPERIMENTAL PREDICTIONS OF THE STRAND MODEL

“Es gibt viele Theorien,
die sich jedem Test entziehen.
Diese aber kann man checken,
elend wird sie dann verrecken.**”
Anonymous

Around the world, numerous researchers are involved in experiments that are searching for new effects. They are searching for new observations that are unexplained by the standard model of particle physics or by the conventional view of cosmology. At the same time, all these experiments are testing the strand model presented here. In fact, most people working on these experiments have not heard about the strand model, so that there is not even the danger of unconscious bias.

To simplify the check with experiments, the most important predictions of the strand model that we deduced in our adventure are listed in [Table 16](#).

TABLE 16 The main predictions of the strand model that follow from the fundamental principle. The typeface distinguishes predictions that are unsurprising, that are *unconfirmed* or *unique* to the strand model, and those that are both **unconfirmed and unique**.

	EXPERIMENT	PREDICTION (MOST FROM 2008/2009)	STATUS (2017)
Page 36	Planck units ($c, \hbar, k, c^4/4G$)	are limit values.	None has been exceeded, but more checks are possible.
Page 327	Higgs boson	2008/9: does not exist.	Falsified.
Page 329		2012: does exist.	Verified.
Page 380	<i>Running of the coupling constants</i>	2008/9: <i>implies no Higgs.</i>	<i>No data yet.</i>
Page 380		2012: <i>implies one Higgs.</i>	<i>No data yet.</i>
Page 327	<i>Longitudinal W and Z boson scattering</i>	2008/9: <i>show non-local effects at the Large Hadron Collider.</i>	<i>No data yet.</i>
		2012: <i>show no non-local effects at the Large Hadron Collider.</i>	<i>None found yet.</i>
Page 329			

** No adequate translation is possible of this rhyme claiming that any theory that can be tested is bound to die miserably.

TABLE 16 (Continued) The main predictions of the strand model that follow from the fundamental principle. The typeface distinguishes predictions that are unsurprising, that are *unconfirmed or unique* to the strand model, and those that are both **unconfirmed and unique**.

	EXPERIMENT	PREDICTION (MOST FROM 2008/2009)	STATUS (2017)
Page 327	<i>Longitudinal W and Z boson scattering</i>	<i>is unitary at the LHC.</i>	<i>Obvious.</i>
	W boson g-factor	is near to 2.	Is observed.
Page 350	<i>Unknown fermions (supersymmetric particles, magnetic monopoles, dyons, heavy neutrinos etc.)</i>	<i>do not exist.</i>	<i>None found yet.</i>
Page 312	<i>Unknown bosons (other gauge bosons, supersymmetric particles, axions etc.)</i>	<i>do not exist.</i>	<i>None found yet.</i>
Page 274, page 316	<i>Unknown interactions, energy scales and symmetries (grand unification, supersymmetry, quantum groups, technicolour etc.)</i>	<i>do not exist.</i>	<i>None found yet.</i>
Page 311	Particle masses, mixing angles and coupling constants	are calculable by modifying existing software packages.	Most not yet calculated; approximations very encouraging.
Page 311	Particle masses, mixing angles and coupling constants	are constant in time.	Is observed.
	Particle masses, mixing angles, coupling constants and g-factors	are identical for antimatter.	Is observed.
Page 370	Mixing matrix for quarks	is unitary.	Is observed.
Page 373	<i>Mixing matrix for neutrinos</i>	<i>is unitary.</i>	<i>No data yet.</i>
Page 338	<i>Neutrinos</i>	<i>are Dirac particles.</i>	<i>No data yet.</i>
Page 374	<i>Neutrinos</i>	<i>violate CP symmetry.</i>	<i>No data yet.</i>
Page 326	Neutrino-less double beta decay	does not exist.	Not yet found.
Page 274, page 326	Electric dipole moments of elementary particles, magnetic dipole moment of neutrinos	have extremely small, calculable values.	No data yet.
Page 341	Tetraquarks	exist.	Likely.
Page 322, page 338	<i>Glueballs</i>	<i>probably do not exist; if they do, the spectrum can be compared to the strand model.</i>	<i>Not yet observed.</i>
Page 274, page 326	Proton decay and other rare decays, neutron-antineutron oscillations	occur at extremely small, standard model rates.	Not yet observed.
Page 339	Neutron decay	follows the standard model.	No deviations found.
Page 339	Neutron charge	vanishes.	None observed.
Page 335	<i>Hadron masses and form factors</i>	<i>can be calculated ab initio.</i>	<i>Not yet calculated; value sequences and signs correct.</i>

TABLE 16 (Continued) The main predictions of the strand model that follow from the fundamental principle. The typeface distinguishes predictions that are unsurprising, that are *unconfirmed or unique* to the strand model, and those that are both **unconfirmed and unique**.

	EXPERIMENT	PREDICTION (MOST FROM 2008/2009)	STATUS (2017)
Page 351	Dark matter	is conventional matter plus black holes.	Partly confirmed by black hole mergers and lack of other results.
Page 312	<i>Standard model of particle physics</i>	<i>2008/9: is essentially correct, with deviations for the scattering of longitudinal vector bosons at LHC energy.</i> <i>2012: is correct for all measurable energies.</i>	<i>Not yet falsified, but deviations not yet observed.</i> <i>All data agrees.</i>
Page 146	Additional dimensions	do not exist.	Not observed.
Page 146	Non-commutative space-time	does not exist.	Not observed.
Page 290	General relativity	is correct at all accessible energies.	No deviation found.
Page 290	Short-distance deviations from universal gravitation and modified gravity	do not exist.	All data agrees.
Page 289	Space-time singularities, cosmic strings, wormholes, time-like loops, negative energy regions, domain walls	do not exist.	None observed.
Page 295, page 275	Quantum gravity effects	will not be found.	None observed yet.
Page 302	Behind a horizon	nothing exists.	Nothing observed.
Page 305	Cosmological constant (dark energy)	is small and positive.	Is observed.
Page 305	<i>Cosmological constant (dark energy)</i>	<i>decreases with time squared.</i>	<i>Data are inconclusive.</i>
Page 307	<i>Cosmic matter density</i>	<i>decreases with time squared.</i>	<i>Data are inconclusive.</i>
Page 304	Cosmic inflation	did not occur.	Data not in contrast.
Page 374	Leptogenesis	did not occur.	Data are inconclusive.
Page 307	Cosmic topology	is trivial.	As observed.
Page 351	Vacuum	is stable and unique.	As observed.
	In summary: all motion	results from strands.	Not yet falsified.

In this list, the most interesting predictions of the strand model are the *numerical* predictions on the decay of the cosmological constant, the limited spectrum of possible elementary particles – including the existence of just three generations – the various mass ratios and mass sequences – including the Z/W and Higgs/W mass ratios – and the relative strength of the three gauge interactions. Above all, there is the possibility to calculate all fundamental constants in the foreseeable future.

The strand model reproduces the quark model, gauge theory, wave functions and general relativity; at the same time, the model predicts the lack of measurable deviations. The strand model solves conceptual problems such as the dark matter problem, inflation, confinement, the strong CP problem and the anomaly issue; by doing so, the strand

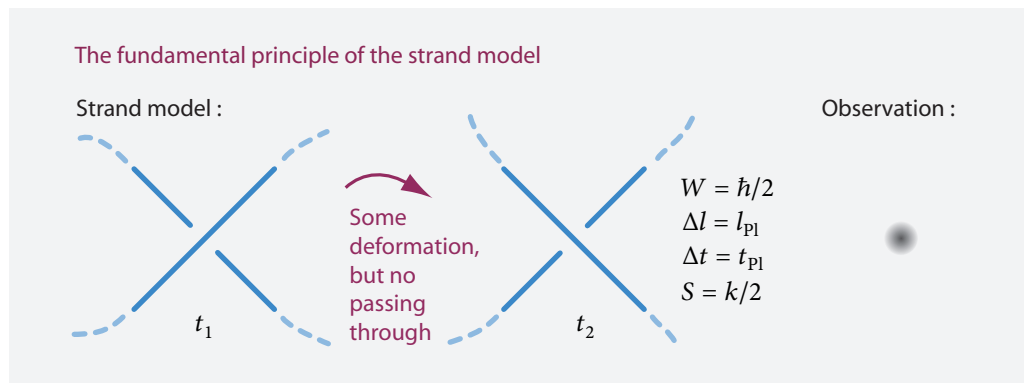


FIGURE 115 The fundamental principle of the strand model: *Planck units are defined by a crossing switch in three spatial dimensions.* With this principle, as shown in the previous chapters, the fundamental principle implies general relativity and the standard model of particle physics.

model predicts the lack of unknown effects in these domains.

The strand model deduces all its experimental predictions from a single and simple fundamental principle: *events and Planck units are due to crossing switches of strands.* Provided there are no errors of reasoning, there is no way to change the predictions summarized here. The strand model is both simple and unmodifiable.

Naturally, errors of reasoning in the preceding chapters are well possible. A few have occurred in the past. The exploration was performed at high speed – possibly too high. If any experiment ever contradicts a prediction of the strand model, the model is doomed. When the above experimental predictions were first deduced in 2008 and 2009, they were quite unpopular. Practically all other attempts at unification predicted the existence of yet undiscovered particles and effects. However, so far, experiment does not confirm these other attempts; in fact, no prediction of the strand model has been falsified yet.

FINAL SUMMARY ABOUT THE MILLENNIUM ISSUES

In our adventure, we have argued that Planck's natural units should be modelled with the fundamental principle for strands, which is shown again in [Figure 115](#). As we discovered, the fundamental principle explains the following measured properties of nature:

- Strands explain the principle of least action and the invariance of c , \hbar , G and k .
- Strands explain the three dimensions of space, the existence of gravitation, curvature and horizons, the equations of general relativity, the value of black hole entropy and the observations of modern cosmology.
- Strands explain all the concepts used in the Lagrangian of the standard model of particle physics, including wave functions, the Dirac equation and the finite, discrete and small mass of elementary particles.
- Strands explain the existence of electromagnetism and of the two nuclear interactions, with their gauge groups and all their other observed properties.
- Strands describe the observed gauge and Higgs bosons, their charges, their quantum

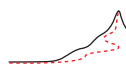
- numbers and their mass ranges.
- Strands explain the three generations of quarks and leptons, their charges and quantum numbers, their mixing, their mass sequences, as well as their confinement properties.
- Strands explain the quark model of hadrons, including CP violation, mass sequences, signs of quadrupole moments, the lack of unobserved hadrons, common Regge slopes and the existence of tetraquarks.
- Strands *do not allow* arbitrary values for masses, coupling constants, mixing angles and CP violating phases.
- Strands *enable* calculations of particle masses, their coupling constants, their mixing angles and the CP violating phases. First rough estimates of these values agree with the (much more precise) experimental data. Computer calculations will allow us to improve these checks in the near future.
- Strands *predict* the lack of unknown dark matter and of unknown inflation mechanisms.
- Finally, strands predict that nature *does not hide* any unknown elementary particle, fundamental interaction, fundamental symmetry or additional dimension. In particular, strands predict that no additional mathematical or physical concepts are required for a final theory.

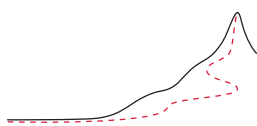
All these results translate to specific statements on experimental observations. So far, there is no contradiction between the strand model and experiments. These results allow us to sum up our adventure in three statements:

1. *Strands solve all open issues.* With one simple fundamental principle, the strand model solves or at least proposes a way to solve *all* issues from the millennium list of open issues in fundamental physics. All fundamental constants can be calculated.
2. *Strands agree with all observations.* In particular, the strand model implies that general relativity, quantum theory and the standard model of elementary particles are a *precise* description of motion for all practical purposes.
3. *Nothing new will be discovered in fundamental physics.* Unexpectedly but convincingly, strands predict that general relativity, quantum theory and the standard model of elementary particles are a *complete* description of motion for all practical purposes.

We have not yet literally reached the top of Motion Mountain – because certain numerical predictions of the fundamental constants are not yet precise enough – but if no cloud has played a trick on us, we have seen the top from nearby. In particular, we finally know the origin of colours.

The last leg, the accurate calculation of the constants of the standard model of particle physics, is still under way. The drive for simplicity and the spirit of playfulness that we invoked at the start have been good guides.





CHAPTER 14

THE TOP OF MOTION MOUNTAIN

“All things are full of gods.

Thales**”

Who am I? Where do I come from? What shall I do? Where does the world come from? Can the whole world really come to a sudden end? What will happen in the future? What is beauty? All these questions have a common aspect: they are questions about motion. But what is motion? Our search for an answer led us to study motion in all its details. In this quest, every increase in the precision of our description of motion was a step towards the peak of Motion Mountain. Now that we arrived there, we can savour what we have achieved and recall the emotions that we have experienced.

In our ascent, we have learned how we move, how we experience our environment, how we grow, what parts we are made of, and how our actions and our convictions about them can be understood. We have learned a lot about the history and a bit about the future of matter, of radiation and of space. We have experienced and understood the many ways in which beauty appears in nature: as colours, as shapes, as rhythms and most of all: as simplicity.

Savouring our achievement means that first of all, we now can look back to where we came from. Then we enjoy the view we are offered and look out for what we could not see before. After that, we search for what is still hidden from our sight. And finally, we take a different path back down to where we live.

OUR PATH TO THE TOP

“The labour we delight in physics pain.

William Shakespeare, *Macbeth*.”

Our walk had a simple aim: to talk accurately about all motion. This 2500 year old quest drove us to the top of this mountain. We can summarize our path in three legs: everyday life, general relativity plus quantum theory, and unification.

** Thales of Miletus (c. 624 – c. 546 BCE) was the first known philosopher, mathematician and scientist.

EVERYDAY LIFE: THE RULE OF INFINITY

Ref. 1, Ref. 3 *Galilean physics* is the description of everyday life. We all learned Galilean physics before secondary school. Galilean physics is the exploration and description of the motion of stones, water, trees, heat, the weather, electricity and light. To achieve this description of our environment, our first and main act in life is to partition experience into experiences. In other words, our first intellectual act is the invention of *parts*; we invented the *plural*.

The act of partitioning allows us to define sequences among our experiences, and thus to define the concept of *time*. The concept of *space* arises similarly by our possibility to distinguish observations that occur at the same time. By comparing parts with other parts, we define *measurement*. Using all of this, we become able to define *velocity*, *mass* and *electric charge*, among others. These allow us to introduce *action*, the quantity that quantifies change.

For a simple description of observations, we assume that division is possible without end: thus we introduce the infinitely small. We also assume that widening our scope of observation is possible without end. Thus we introduce the infinitely large. Defining parts thus leads us to introduce infinity.

Using parts and, with them, the infinitely small and the infinitely large, we found, in volumes I and III, that everyday motion has six main properties: it is continuous, conserved, relative, reversible, mirror-invariant and lazy. Motion is lazy – or efficient – because it produces as little change as possible.

Nature minimizes change. This is Galilean physics, the description of everyday motion, in one statement. It allows us to describe all our everyday experiences with stones, fluids, stars, electric current, heat and light. The idea of change-minimizing motion is based on a concept of motion that is continuous and predictable, and a concept of nature that contains the infinitely small and the infinitely large.

RELATIVITY AND QUANTUM THEORY: THE ABSENCE OF INFINITY

“ Vorhin haben wir gesehen, daß in der Wirklichkeit das Unendliche nirgends zu finden ist, was für Erfahrungen und Beobachtungen und welcherlei Wissenschaft wir auch heranziehen.* ”
David Hilbert

Ref. 2, Ref. 4 The idea that nature offers an infinite range of possibilities is often voiced with deep personal conviction. However, the results of relativity and quantum theory show the opposite. In nature, speeds, forces, sizes, ages and actions are limited. No quantity in nature is infinitely large or infinitely small. No quantity in nature is defined with infinite precision. There never are infinitely many examples of a situation; the number of possibilities is always finite. The world around us is not infinite; neither its size, nor its age, nor its content. *Nature is not infinite*. This is general relativity and quantum theory in one statement.

Relativity and quantum theory show that the idea of infinity appears only in *approximate* descriptions of nature; it disappears when talking with precision. Nothing in nature

Ref. 262 * ‘Above we have seen that in the real world, the infinite is nowhere to be found, whatever experiences and observations and whatever knowledge we appeal to.’

is infinite. For example, we found in volume II that the sky is dark at night (also) because space is not infinite. And we found, in volumes IV and V, that quantum theory contains probabilities because there is a smallest action value in nature. In fact, the statement that a quantity is infinitely large or infinitely small cannot be confirmed or reproduced by any experiment. Worse, such a statement is falsified by every measurement. In short, we found that infinity is a fantasy of the human mind. In nature, it does not appear. *Infinity about nature is always a lie.*

Ref. 4 The number of particles, their possible positions, the states they can have, our brain, our creativity, our possible thoughts: all this is not infinite. Nevertheless, quantum theory and relativity changed the world: they allowed building ultrasound imaging, magnetic resonance imaging, lasers, satellite navigation systems, music players and the internet.

Despite the vast progress due to modern physics and the related technologies, one result remains: nothing in our environment is infinite – neither our life, nor our experiences, nor our memories, not even our dreams or our fantasies. Neither the information necessary to describe the universe, nor the paper to write down the formulae, nor the necessary ink, nor the time necessary to understand the formulae is infinite. Nature is not infinite. On the other hand, we also know that the illusion of the existence of infinity in nature is one the most persistent prejudices and myths ever conceived. Why did we use it in the first place?

The habit to use infinity to describe the world has many emotional reasons. For some, it reflects the deep-rooted experience of smallness that we carry within us as a remnant our personal history, when the world seemed so large and powerful. For others, the idea of our smallness allows us to deny somehow the responsibility for our actions or the existence of death. For others again, the idea of a finite universe often, at a first glance, produces deception, disbelief and discouragement. The absence of infinity means that we cannot achieve everything we want, and that our dreams and our possibilities are limited. Clinging to the idea of infinity is a way to avoid confronting this reality.

Challenge 223 e However, once we face and accept the absence of infinity, we make a powerful experience. We gain in strength. We are freed from the power of those who use this myth to put themselves above others. It is an illuminating experience to reread all those sentences on nature, on the world and on the universe containing the term ‘infinite’, knowing that they are incorrect, and then clearly experience the manipulations behind them. The desire to make others bow to what is called the infinite is a common type of human violence.

At first, the demise of infinity might also bring panic fear, because it can appear as a lack of guidance. But at closer inspection, the absence of infinity brings strength. Indeed, the elimination of infinity takes from people one of the deepest fears: the fear of being weak and insignificant.

Moreover, once we face the limits of nature, we react like in all those situations in which we encounter a boundary: the limit becomes a challenge. For example, the experience that all bodies unavoidably fall makes parachuting so thrilling. The recognition that our life is finite produces the fire to live it to the full. The knowledge of death gives meaning to our actions. In an infinite life, every act could be postponed without any consequence. The disappearance of infinity generates creativity. A world without limits is discouraging and depressing. Infinity is empty; limits are a source of strength and pour passion into our life. Only the limits of the world ensure that every additional step

in life brings us forward. Only in a limited universe is progress possible and sensible. Who is wiser, the one who denies limits, or the one who accepts them? And who lives more intensely?

UNIFICATION: THE ABSENCE OF FINITUDE

“Pray be always in motion. Early in the morning go and see things; and the rest of the day go and see people. If you stay but a week at a place, and that an insignificant one, see, however, all that is to be seen there; know as many people, and get into as many houses as ever you can.”
Philip Stanhope,* *Letters to his Son on the Fine Art of Becoming a Man of the World and a Gentleman.*

The last part of our adventure, described in this volume, produced an unexpected result. Not only is nature not infinite; nature is not finite either. None of the quantities which were supposed to be finite turn out to be so. Finitude turns out to be an approximation, or better, an illusion, though a subtle one. *Nature is not finite.* This is the unification of physics in one statement.

Page 127

Precise observation shows that nothing in nature can be counted. If nature were finite it would have to be (described by) a set. However, the exploration of Planck scales shows that such a description is intrinsically incomplete and inaccurate. Indeed, a description of nature by a set can never explain the number of its elements, and thus cannot explain finitude itself. In other words, any approach that tries to describe nature as finite is a belief, and is never correct. *Finitude is a lie.*

We thus lost our security of thought a second time. Nature is neither infinite nor finite. We explored the possibilities left over and found that only one option is left: *Nature is indivisible.* In other words, all parts that we experience are approximations. Both finitude and infinity are approximation of nature. All distinctions are approximate. This central conclusion solved the remaining open issues about motion. *Nature has no parts.*

The impossibility to count and the lack of parts imply that nature is not a computer, not an automaton, nor a physical system. *Nature is not discrete.*

Recognizing all distinctions as being approximate abolishes the distinction between the permanent aspects of nature ('objects', described by mass, charge, spin, etc.) and the changing aspects ('states', described by position, momentum, energy). Taking all distinctions as approximate introduces extended constituents: fluctuating strands. Looking even closer, these extended constituents are all the same one. Space, formally only used to describe states, also acquires changing aspects: it is made from fluctuating strands. Also properties like mass or charge, which formally were seen as static, become aspects of the ever changing interplay between these fundamental constituents. Describing nature as one fluctuating strand allows us to avoid finitude and to answer all questions left open by quantum theory and general relativity.

In a sense, the merging of objects and states is a resolution of the contrasting views on motion of the Greek thinkers Parmenides – 'there is no motion', i.e., in physical language, 'there are no states, there is only permanence' – and Heraclitus – 'everything

* Philip D. Stanhope (b. 1694 London, d. 1773 London) was a statesman and writer.

moves', i.e., in physical language 'there is no permanence, there are only states'. Both turn out to be right.

We can thus sum up the progress during our adventure of physics in the following table:

TABLE 17 The progress of physics.

Step 1	Galilean Physics	Nature is continuous.	We live in Galilean space.
Step 2	Relativity	Nature has no infinitely large.	We live in Riemannian space.
Step 3	Quantum field theory	Nature has no infinitely small.	We live in a Hilbert/Fock space.
Step 4	Unification	Nature is not finite. Nature has no parts.	We do not live in any space; we are space.

In summary, we are made of space. More precisely, we are made of the same constituents as space. In fact, the fascination of this result goes further than that.

NEW SIGHTS

“ Nel suo profondo vidi che s'interna,
legato con amore in un volume,
ciò che per l'universo si squaderna:

sustanze e accidenti e lor costume
quasi conflati insieme, per tal modo
che ciò ch' i' dico è un semplice lume.

La forma universal di questo nodo
credo ch' i' vidi, perché più di largo,
dicendo questo, mi sento ch' i' godo.*

Dante, *La (Divina) Commedia*, Paradiso,
XXXIII, 85-93.

Modelling nature as a complicated web of fluctuating strands allowed us to describe at the same time empty space, matter, radiation, horizons, kefir, stars, children and all our other observations. All everyday experiences are consequence of everything in nature being made of one connected strand. This result literally widens our horizon.

* 'In its depth I saw gathered, bound with love into one volume, that which unfolds throughout the universe: substances and accidents and their relations almost joined together, in such a manner that what I say is only a simple image. The universal form of that knot, I think I saw, because, while I am telling about it, I feel deep joy.' This is, in nine lines, Dante's poetic description of his deepest mystical experience: the vision of god. For Dante, god, at the depth of the light it emanates, is a knot. That knot spreads throughout the universe, and substances and accidents – physicists would say: particles and states – are aspects of that knot. Dante Alighieri (b. 1265 Florence, d. 1321 Ravenna) was one of the founders and the most important poet of the Italian language. Most of the Divine Comedy, his magnum opus, was written in exile, after 1302, the year when he had been condemned to death in Florence.

THE BEAUTY OF STRANDS

“Someday, surely, we will see the principle underlying existence itself as so simple, so beautiful, so obvious, that we will all say to each other, “Oh, how could we all have been so blind, so long.””
John Wheeler, *A Journey Into Gravity And Spacetime*.

Describing everything as connected does not come natural to us humans. After all, in our life, we perform only one act: to partition. We define pluralities. There is no way we can avoid doing this. To observe, to think, to talk, to take a decision, to move, to suffer, to love or to enjoy life is impossible without partitioning.

Our walk showed us that there are limits to the ability to distinguish. Any kind of partitioning is always approximate. In fact, most people can summarize their personal experience by saying that they learned to make finer and finer distinctions. However, talking with highest precision about a part of the world inevitably leads to talk about the whole universe. The situation resembles a person who gets a piece of rope in his hand, and by following it, discovers a large net. He continues to pull and finally discovers that everything, including himself, is part of the net.

For the strand model, the term ‘theory of everything’ is therefore not acceptable. Nature cannot be divided into ‘things’. In nature, things are never separable. There is no way to speak of ‘every’ thing; there are no sets, no elements and no parts in nature. A theory describing all of nature cannot be one of ‘everything’, as ‘things’ are only approximate entities: properly speaking, they do not exist. The strand model is not a theory of everything; it is a *final theory*.

The strand model shows that nature is not made of related parts. Nature is made of relations only. Parts only exist approximately. The strand model also shows: being in motion is intrinsic to being a part. Parts, being approximate, are always in motion. As soon as we divide, we observe motion. The act of dividing, of partitioning, of defining parts is the very one which produces order out of chaos. Strands force us to rethink this habit.

Despite being so tough to grasp, strands yield a precise description of motion that unifies quantum field theory and general relativity. The strand model for the unification of motion is both simple and powerful. There are no free parameters. There are no questions left. Our view from the top of the mountain is thus complete. No uncertainty, no darkness, no fear and no insecurity are left over. Only wonder remains.

CAN THE STRAND MODEL BE GENERALIZED?

“Die Natur kann besser Physik als der beste Physiker.*”
Carl Ramsauer

Page 163

As mentioned above, mathematical physicists are fond of *generalizing* models. Despite this fondness, we required that any final, unified description must be unique: any final, unified description must be impossible to reduce, to modify or to generalize. In par-

* ‘Nature knows physics better than the best physicist.’ Carl Ramsauer (b. 1879 Oldenburg, d. 1955 Berlin), influential physicist, discovered that electrons behave as waves.

ticular, a final theory must neither be a generalization of particle physics nor of general relativity. Let us check this.

The strand model is not a generalization of general relativity: the definitions of curvature, of gravitons and of horizons differ radically from general relativity's approach. The strand model is also not a generalization of particle physics: the definitions of particle and of interactions differ radically from the concepts of quantum field theory. Indeed, we have shown that quantum field theory and general relativity are *approximations* to the strand model; they are neither special cases nor reductions of the strand model.

But what about the other requirements for a unified theory? Can the strand model be modified or generalized? We have seen that the model does not work in more spatial dimensions, does not work with more families of quarks, does not work with more interactions, and does not work with other evolution equations in general relativity or particle physics. The strand model does not work with other fundamental constituents, such as bifurcating entities, membranes, bands, or networks. (Though it does work with the equivalent *funnels*, as explained earlier on, but that description is equivalent to the one with strands.) The strand model does not work with any modified fundamental principle. Obviously, exploring all possible variations and modifications remains a challenge for the years to come. If an actual modification of the strand model can be found, the strand model instantly loses its value: in that case, it would need to be shelved as a failure. Only a *unique* unified model can be correct.

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Challenge 224 r

In summary, one of the beautiful aspects of the strand model is its radical departure from twentieth-century physics in its basic concepts, combined with its almost incredible uniqueness. No generalization, no specialization and no modification of the strand model seems possible. In short, the strand model qualifies as a unified, final theory.

Ref. 155

What is a requirement to one person, is a criticism to another. A number of researchers deeply dislike the strand model precisely because it doesn't generalize previous theories and because it cannot be generalized. This attitude deserves respect, as it is born from the admiration for several ancient masters of physics. However, the strand model points into a different direction.

WHAT IS NATURE?

“Nature is what is whole in each of its parts.”
Hermes Trismegistos, *Book of Twenty-four Philosophers*.

Ref. 263

At the end of our long adventure, we discovered that nature is not a set: everything is connected. Nature is only *approximately* a set. The universe has no topology, because space-time is not a manifold. Nevertheless, the approximate topology of the universe is that of an open Riemannian space. The universe has no definite particle number, because the universe is not a container; the universe is made of the same stuff of which particles are made. Nevertheless, the approximate particle density in the universe can be deduced.

In nature, everything is connected. This observation is reflected in the conjecture that all of nature is described by a single strand.

We thus arrive at the (slightly edited) summary given around the year 1200 by the author who wrote under the pen name Hermes Trismegistos: *Nature is what is whole in*

each of its parts. But in contrast to that author, we now also know how to draw testable conclusions from the statement.

QUANTUM THEORY AND THE NATURE OF MATTER AND VACUUM

“In everything there is something of everything.”
Anaxagoras of Clazimenes (500
–428 BCE Lampsacus)

The strand model shows that as soon as we separate the universe into space-time and the rest, i.e., as soon as we introduce the coordinates x and t , quantum mechanics appears automatically. More precisely, *quantum effects are effects of extension*. Quantum theory appears when we realize that observations are composed of smallest events due to crossing switches, each with a change given by the quantum of action. All events and observations appear through the fluctuations of the strand that composes nature.

We found that *matter is made of tangled strands*. In fact, the correct way would be to say: matter is made of tangled strand *segments*. This connection leads to Schrödinger’s equation and to Dirac’s equation.

Insofar as matter is of the same fabric as the vacuum, we can rightly say that *everything is made of vacuum* and that *matter is made of nothing*. But the most appropriate definition arises when we realize that matter is not made from something, but that matter is a certain aspect of the *whole* of nature. Unification showed that every single elementary particle results from an arrangement of strands that involves the whole of nature, or, if we prefer, the entire universe. In other words, we can equally say: *matter is made of everything*.

We can also turn the equivalence of matter and vacuum around. Doing so, we arrive at the almost absurd statement: *vacuum is made of everything*.

“Der heutigen Physik liegt die Frage nicht mehr ferne, ob nicht etwa alles, was ist, aus dem Äther geschaffen sei. Diese Dinge sind die äußersten Ziele unserer Wissenschaft, der Physik.*”
Heinrich Hertz

COSMOLOGY

The strand model also showed us how to deduce general relativity. The strand model clarified the fabric of horizons and explained the three dimensions of space. Most fascinating is the idea of a universe as the product of a single strand. A single strand implies that there was nothing before the big bang, and that there is nothing outside the night sky. For example, the strand model implies that there is no ‘multiverse’ and that there are no hidden worlds of any kind. And the fluctuating strand explains all observations of our universe.

Page 8 The cosmological constant is not constant; it only measures the present age and size of the universe. Therefore, the constant does not need to appear in [Figure 1](#). In other words, the cosmological constant simply measures the time from the big bang to the present.

* ‘Modern physics is not far from the question whether everything that exists could possibly be made from aether. These things are the extreme goals of our science, physics.’ Hertz said this in a well-known speech he gave in 1889. If we recall that ‘aether’ was the term of the time for ‘vacuum’, the citation is particularly striking.

The ‘big bang’ is the name for what we observe if we try to make observations approaching the limits of nature. The ‘big bang’ appears automatically from the strand model whenever we observe nature at the most distant times, the largest distances or at the largest energies: ‘big bang’ is the name for Planck scale physics.

The universe consists of a single strand. There are many particles in nature, because the strand is tangled up in complicated ways. What we call the ‘horizon’ of the universe is the place where new tangles appear.

The belief that the big bang or the horizon are examples of creation is incorrect. What happened at the big bang still happens at the horizon today. Both the black sky at night and the big bang are nature’s way to tell us: ‘Galilean physics is approximate! Quantum theory is approximate! General relativity is approximate!’

MUSINGS ABOUT UNIFICATION AND STRANDS

“Continuing motion masters coldness.
Continuing rest masters heat.
Motion based on rest:
Measure of the all-happening for the single one.”
Lao Tse,* *Tao Te King*, XXXV.

All is made from one sort of thing: all is one substance. This idea, *monism*, sounds a lot like what the influential philosopher Baruch Spinoza (b. 1632 Amsterdam, d. 1677 The Hague) held as conviction. Monism, though mixed up with the idea of god, is also the basis of the philosophical ideas that Gottfried Wilhelm Leibniz (b. 1646 Leipzig, d. 1716 Hannover) presents in his text *La Monadologie*.

* *

Ref. 264 Any complete theory of motion, also the strand model, is built on a single statement about nature: The *many* exists only approximately. Nature is approximately multiple. The etymological meaning of the term ‘multiple’ is ‘it has many folds’; in a very specific sense, nature thus has many folds.

* *

Any precise description of nature is free of arbitrary choices, because the divisions that we have to make in order to think are all common to everybody, and logically inescapable. Because physics is a consequence of this division, it is also ‘theory-free’ and ‘interpretation-free’. This consequence of the final theory will drive most philosophers up the wall.

* *

For over a century, physics students have been bombarded with the statement: ‘Symmetries are beautiful.’ Every expert on beauty, be it a painter, an architect, a sculptor, a musician, a photographer or a designer, fully and completely disagrees, and rightly so. Beauty has no relation to symmetry. Whoever says the contrary is blocking out his experiences of a beautiful landscape, of a beautiful human figure or of a beautiful work of art.

* Lao Tse (sixth century BCE) was an influential philosopher and sage.

The correct statement is: ‘Symmetries simplify descriptions.’ Symmetries simplify physical theories. That is the background for the statement of Werner Heisenberg: ‘In the beginning there was symmetry.’ On the other hand, the strand model shows that even this statement is incorrect. In fact, neither the search for beauty nor the search for symmetry were the right paths to advance towards unification. Such statements have always been empty marketing phrases. In reality, the progress of fundamental theoretical physics was always driven by the search for *simplicity*.

* *

Strands unify physics. In particular, strands extend our views on quantum theory and mathematical physics, on particle physics and field theory, on axiomatic physics and algebraic physics, on polymer physics and gauge theory, on general relativity and cosmology. It will take several years before all these extensions will have been explored.

* *

The description of nature with strands is surprisingly simple, mainly because it uses so few basic concepts. Is this result astonishing? In our daily life, we describe our experiences with the help of a few thousand words, e.g. taking them from the roughly 350 000 words which make up the English language, or from a similar number from another language. This set is sufficient to talk about everything, from love to suffering, from beauty to happiness. And these terms are constructed from no more than about 35 basic concepts, as we have seen already. We should not be too surprised that we can in fact talk about the whole universe using only a few basic concepts: the act and the results of (approximate) distinction, or more specifically, a basic event – the crossing switch – and its observation.

* *

Almost all discoveries in physics were made at least 30 years too late. The same is true for the strand model. If we compare the strand model with what many physicists believed in the twentieth century, we can see why: researchers had too many wrong ideas about unification. All these wrong ideas can be summarized in the following statement:

– ‘Unification requires generalization of existing theories.’

This statement is subtle: it was rarely expressed explicitly but widely believed. But the statement is wrong, and it led many astray. On the other hand, the development of the strand model also followed a specific guiding idea, namely:

– ‘Unification requires simplification.’

Hopefully this guiding idea will not become a dogma itself: in many domains of life, simplification means not to pay attention to the details. This attitude does a lot of harm.

* *

The strand model shows that achieving unification is not a feat requiring difficult abstraction. Unification was not hidden in some almost inaccessible place that can be reached only by a few select, well-trained research scientists. No, unification is accessible to everyone who has a basic knowledge of nature and of physics. No Ph.D. in theoretical physics is



FIGURE 116 Motion Mountain does not resemble Cerro Torre, but a gentle hill (© Davide Brighenti, Myriam70)

needed to understand or to enjoy it. The knowledge presented in the previous volumes of this series is sufficient.

When Andrew Wiles first proved Fermat's last theorem after three centuries of attempts by the brightest and the best mathematicians, he explained that his search for a proof was like the exploration of a dark mansion. And seen the conceptual difficulties he had to overcome, the analogy was fitting. Recalling how many more people have already searched for unification without success, the first reaction is to compare the search for unification to the exploration of something even bigger, such as a complex dark cave system. But that analogy was not helpful. In contrast to the proof of Fermat's theorem, the goal of the quest for unification turned out to be simple and lying out in the open. Researchers had simply overlooked it, because they were convinced that the goal was complex, hidden in the dark and hard to reach. It was not.

The adventure of climbing Motion Mountain is thus not comparable to climbing Cerro Torre, which might be the toughest and most spectacular challenge that nature offers to mountain climbers. [Figure 116](#) gives an impression of the peak. Motion Mountain does not resemble this peak at all. Neither does Motion Mountain resemble the Langtang Lirung peak in the Nepalese Himalayas shown on the cover of this volume. Climbing Motion Mountain is more like walking up a gentle green hill, alone, with a serene mind, on a sunny day, while enjoying the surrounding beauty of nature.

* *

Page 84 The strand model settles all questions about *determinism*. Quantum theory and general relativity are deterministic. Nevertheless, when both descriptions are combined, time turns out to be an approximate, low-energy concept. The same applies to determinism. Even though nature is deterministic for all practical purposes and shows no surprises, determinism shares the fate of all its conceivable opposites, such as fundamental randomness, indeterminism of all kinds, existence of wonders, creation out of nothing, or

divine intervention: determinism is an *incorrect* description of nature at the Planck scale – like all its alternatives.

* *

Challenge 225 e

The strand model also settles most so-called *really big questions* that John Wheeler used to ask: Why the quantum? How come existence? It from bit? A "participatory universe"? What makes "meaning"? Enjoy the exploration.

* *

Any unified model of nature encompasses a lot of ideas, issues and knowledge. Due to the sheer amount of material, publishing it in a journal will be challenging.

* *

Ref. 266 The strand model is so simple that it fits on a tombstone or on a T-shirt. This would surely be god's favourite T-shirt. It is available at www.motionmountain.net/gfts.html.

* *

Historically, the strand model evolved from an exploration, started in the 1990s, of the maximum force in nature, the belt trick and the entropy of black holes. After the first six chapters of the present volume were completed in 2002, meditating on their implications led to the strand model and its fundamental principle.

Page 8

Above all, it was the description of general relativity with the help of the maximum force that triggered the search for a unified description that was purely based on Planck units. Another essential point was the drive to search for a final theory directly, from its requirements ('top down' in [Figure 1](#)), and *not* from the unification of quantum theory and general relativity ('bottom up'). In the years from 2002 to 2007, most of the ideas of the strand model took shape, mainly in Munich's underground trains, while commuting between home and work. In those years, it appeared that strands could explain the Dirac equation, the entropy of black holes, general relativity and the particle spectrum with the three particle generations. While walking in the woods and fields around Munich during 2008 and 2009, it appeared that strands explain the three gauge interactions, predict (with almost complete certainty) the lack of a Higgs boson – a big mistake due to faulty reasoning, as turned out in 2012 – and of any new physical effects beyond the standard model, and allow calculating the unexplained constants of particle physics. The model thus yielded all its main predictions before the accelerator experiments at the Large Hadron Collider at CERN in Geneva were switched on in autumn 2010. Thus much of the work was done in a haste – future will show what is of lasting value.

Page 329

In 2012, the discovery of the Higgs boson, and in 2014, the comments by Sergei Fadeev led to an improvement and simplification of the strand model, eliminating knotted strands. In 2016 and 2017, the experimental results at the LHC, of dark matter searches, and of the LIGO observatory confirmed the lack of deviations from the standard model of particle physics and from general relativity, as predicted by the strand model.

* *

Many researchers believed during all their life that the final theory is something useful, important and valuable. This common belief about the importance and seriousness of

the quest has led, over the past decades, to an increasingly aggressive atmosphere among these researchers. This unprofessional atmosphere, combined with the dependence of researchers on funding, has delayed the discovery of the final theory by several decades.

In fact, the final theory is not useful: it adds nothing of practical relevance to the combination of the standard model and general relativity. The final theory is also not important: it has no application in everyday life or in industry and does not substantially change our view of the world; it just influences teaching – somewhat. Finally, the final theory is not valuable: it does not help people in their life or make them happier. In short, the final theory is what all fundamental theoretical research is: entertaining ideas.

Even if the strand model were to be replaced by another model, the conclusion remains: the final theory is not useful, not important and not valuable. But it is enjoyable.

* *

The strand model will take a long time to get accepted. The first reason is obvious: *The strand model contradicts thinking habits* in many research fields. Researchers working on the foundations of quantum theory, on general relativity, on cosmic strings, on mathematical physics, on classical and quantum field theory, on polymer physics, on shape deformations, on quantum gravity, on strings, on the visualization of quantum mechanics, on knot theory, on higher dimensions, on supersymmetry, on the axiomatization of physics, on group theory, on the foundation of physics, on quantum optics and on particle physics have to give up many life-long thinking habits. So do all other physicists. *Strands supersede particles and points.*

There is also a second reason for the slow acceptance of the model presented here: *The strand model, in its simplicity, is only a small step away from present research.* Many researchers are finding out how close they have been to the ideas of the strand model, and for how long they were overlooking or ignoring such a simple option. The simplicity of the fundamental principle contrasts with the expectation of most researchers, namely that the final theory is complicated, difficult and hard to discover. In fact, the opposite is true. *Strands are based on Planck units and provide a simple, almost algebraic description of nature.*

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In summary, for many researchers and for many physicists, there is a mixture of confusion, anger and disappointment. It will take time before these feelings subside and are replaced by the fascination provided by the strand model.

“Only boring people get bored.”
Anonymous

THE ELIMINATION OF INDUCTION

“Cum iam profeceris tantum, ut sit tibi etiam tui reverentia, licebit dimittas pedagogum.*”
Seneca

The final theory of motion has a consequence worth mentioning in detail: its lack of infinity and its lack of finitude eliminate the necessity of induction. This conclusion is of

* ‘When you have profited so much that you respect yourself you may let go your tutor.’ Seneca, the influential Roman poet and philosopher, writes this in his *Epistulae morales ad Lucilium*, XXV, 6.

importance for general discussions on man's grasp of nature.

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In physics, as in the other natural sciences, there is a tradition to state that a certain description of nature – once confusingly called a ‘law’ – is valid in *all* cases. In these statements, ‘all’ means ‘for all values of the quantities appearing’. As a concrete example, the ‘law’ of universal gravitation is always claimed to be the same here and today, as well as at *all* other places and times, such as on the other end of the universe and in a few thousand years. The full list of such all-claims is part of the millennium list of open issues in twentieth-century physics. For many decades, the habit of claiming general validity from a limited and finite number of experiences, also called *induction*, has been seen, and rightly so, as a logically dubious manoeuvre, tolerated only because it works. But the developments described in this text show that this method is indeed justified.

First of all, a claim of generality is not that enormous as it may seem, because the number of events that can be distinguished in nature is finite, not infinite. The preceding sections showed that the maximal number N of events that can be distinguished in the universe is of the order of $N = (T_0/t_{\text{pl}})^4 = 10^{244\pm 2}$, T_0 being the age of the universe and t_{pl} the Planck time. This is a big, but certainly finite number.

The unified description of nature has thus first reduced the various all-claims from an apparently infinite to a finite number of cases, though still involving astronomically large numbers. This reduction results from the recognition that infinities do not appear in the description of nature. We now know that when talking about nature, ‘all’ cases never means an infinite number.

A second, important result is achieved by the description of nature with strands. In any all-claim about fundamental motion, the checking of each of the large number of possibilities is not necessary any more, because all events result from a single entity, in which we introduce distinctions with our senses and our brain. And the distinctions we introduce imply automatically that the symmetries of nature – the ‘all-claims’ or ‘inductions’ – that are used in the description of motion are correct. Nature does not contain separate parts. Therefore, there is no way that separate parts can behave differently. Induction is a consequence of the unity of nature.

Ultimately, the possibility to *verify* statements of nature is due to the fact that all the aspects of our experience are *related*. Complete separation is impossible in nature. The verification of all-claims is possible because the strand model achieves the full description of how all ‘parts’ of nature are related.

The strand model shows that we can talk and think about nature because we are a part of it. The strand model also shows that induction works because everything in nature is related to everything else: nature is one.

WHAT IS STILL HIDDEN?

“That which eludes curiosity can be grasped in action.”
Traditional saying.

Where do we come from? Where does the world come from? What will future bring? What is death? All these questions are questions about motion – and its meaning. To all such questions, the strand model does not provide answers. We are a collection of tangled strands. We are everything and nothing. The strand(s) we are made of will con-

tinue to fluctuate. Birth, life and death are aspects of tangled strands. The universe is a folded strand that grows in complexity.

Obviously, abstract statements about tangles do not help in any human quest. Indeed, we aimed at achieving a precise description of moving particles and bending space. Studying them was a sequence of riddles; but solving these riddles does not provide meaning, not even at the top of Motion Mountain. From the top we cannot see the evolution of complicated systems; in particular, we cannot see or describe the evolution of life, the biological evolution of species, or the growth of a human beings. Nor can we understand why we are climbing at all.

Challenge 226 s

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In short, from the top of Motion Mountain we cannot see the details down in the valleys of human relations or experience; strands do not provide advice or meaning. Remaining too long on the top is of no use. To find meaning, we have to descend back down to real life.

A RETURN PATH: JE RÊVE, DONC JE SUIS

“ I hate reality. But it is the only place where one can get a good steak. ”

Woody Allen

Enjoying life and giving it meaning requires to descend from the top of Motion Mountain. The return path can take various different directions. From a mountain, the most beautiful and direct descent might be the use of a paraglider. After our adventure, we take an equally beautiful way: we leave reality.

The usual trail to study motion, also the one of this text, starts from our ability to talk about nature to somebody else. From this ability we deduced our description of nature, starting from Galilean physics and ending with the strand model. The same results can be found by requiring to be able to talk about nature to ourselves. Talking to oneself is an example of thinking. We should therefore be able to derive all physics from René Descartes' sentence 'je pense, donc je suis' – which he translated into Latin as 'cogito ergo sum'. Descartes stressed that this is the only statement of which he is completely sure, in opposition to his observations, of which he is not. He had collected numerous examples in which the senses provide unreliable information.

Ref. 267

However, when talking to ourselves, we can make more mistakes than when asking for checks from others. Let us approach this issue in a radically different way. We directly proceed to that situation in which the highest freedom is available and the largest number of mistakes are possible: the world of dreams. If nature would only be a dream, could we deduce from it the complete set of physical knowledge? Let us explore the issue.

- Dreaming implies the use of distinctions, of memory and of sight. Dreams contain *parts* and *motion*.
- Independently on whether dreams are due to previous observations or to fantasies, through memory we can define a sequence among them. The order relation is called *time*. The dream aspects being ordered are called *events*. The set of all (dream) events forms the (dream) *world*.
- In a dream we can have several independent experiences at the same time, e.g. about

Ref. 268

thirst and about hunger. Sequences thus do not provide a complete classification of experiences. We call the necessary additional distinction *space*. Dream space has three dimensions.* Dreaming thus means to use space and time.

- We can distinguish between dream contents. Distinguishing means that we can count items in dreams. Counting means that we have a way to define measurements. Dreams are thus characterized by something which we can call ‘observables’. Dream experiences at a given instant of time are characterized by a *state*.
- Because we can describe dreams, the dream contents exist independently of dream time. We can also imagine the same dream contents at different places and different times in the dream space. There is thus an invariance of dream concepts in space and time. There are thus symmetries in dream space.
- Dream contents can interact. Dreams appear to vary without end. Dreams seem to be infinite.

In other words, a large part of the world of dreams is described by a modified form of *Galilean physics*. We note that the biggest difference between dreams and nature is the lack of conservation. In dreams, observations can appear, disappear, start and stop. We also note that instead of dreams, we could equally explore cinema *films*. Films, like dreams, are described by a modified form of Galilean physics. And films, like dreams, do not follow conservation laws. But dreams teach us much more.

Challenge 227 s

- Dreams show that space can warp.
- Dream motion, as you may want to check, shows a maximum speed.
- Dreams show a strange limit in distance. There is a boundary to our field of vision, even though we do not manage to see it.

Pondering these issues shows that there are *limits* to dreams. In summary, the world of dreams has a maximum size, a maximum speed and three dimensions that can warp. The world of dreams and of films is described by a simple form of *general relativity*.

- Both the number of items we can dream of at the same time and the memory of previous dreams is finite.
- Dreams have colours.
- There are pixels in dreams, though we do not experience them directly. But we can do so indirectly: The existence of a highest number of things we can dream of at the same time implies that dream space has a smallest scale.

In summary, the world of dreams has something similar to a minimum change. The world of dreams and that of films is described by a simple form of *quantum theory*. The difference with nature is that in dreams and films, space is discrete from the outset. But there is still more to say about dreams.

- There is no way to say that dream images are made of mathematical points, as there is nothing smaller than pixels.
- In dreams, we cannot clearly distinguish objects (‘matter’) and environment (‘space’); they often mix.

* Though a few mathematicians state that they can *think* in more than three spatial dimensions, all of them *dream* in three dimensions.

- In dreams, fluctuations appear both for images as well as for the background.
- In dreams, sharp distinctions are impossible. Dream space-time cannot be a set.
- Dream motion appears when approximate conservation (over time) is observed.
- In dreams, dimensionality at small distances is not clear; two and three dimensions are mixed up there.

In summary, the world of dreams seems to behave as if points and point particles do not exist; and since quantum theory and general relativity hold, the world of dreams seems to be described by extended constituents! We thus conclude this short exploration of the physics of dreams with a fascinating *conjecture*: even if nature would be a dream, an illusion or a fantasy, we might still get most of the results that we discovered in our ascent of Motion Mountain. (What differences with modern physics would be left?) Speaking with tongue in cheek, the fear of our own faults of judgement, so rightly underlined by Descartes and many others after him, might not apply to fundamental physics.

Challenge 228 s

WHAT IS THE ORIGIN OF COLOURS?

All colours around us are determined by the fine structure constant α – the coupling constant for the electromagnetic interaction at low energy – with its measured value Ref. 5 $1/137.035999139(31)$. The fine structure constant is also essential to describe most everyday devices and machines, as well as all human thoughts and movements. The constant is an aspect of every electric charge in nature.

The strand model showed us that electrical charge is a property of tangles of strands. In particular, the strand model showed:

- ▷ The fine structure constant describes the probability that a fluctuation adds a twist to the chiral tangles of electrically charged particles.

We have not yet deduced an accurate value for the fine structure constant, but we seem to have found out how to do so.

In short, we seem to glimpse the origin of all colours – and thus of all beauty around us. Strands provide a beautiful explanation for beauty.

SUMMARY: WHAT IS MOTION?

« Deep rest is motion in itself. Its motion rests in itself. »
Lao Tse, *Tao Te King*, VI.

We can now answer the question that drove us through our adventure:

- ▷ **Motion** is the observation of crossing switches of the one, unobservable, tangled and fluctuating strand that describes all of nature.

Nature's strand forms particles, horizons and space-time: these are the parts of nature.

Particles are tangles of strands; horizons and space-time are weaves of strands. The parts of nature move. The parts move because their strands fluctuate.

Motion appears because all parts in nature are approximate. Indeed, the observation of crossing switches and the description of strand segments fluctuating in a background space result and are possible because we approximate from the one strand that makes up nature to the many parts inside nature. The one strand (approximately) forms the many elementary particles inside us. Strand segments and particles (approximately) lead us to introduce background space, matter and radiation. Introducing background space implies observing motion. Motion thus appears automatically when approximate parts of nature, such as humans, animals or machines, describe other approximate parts of nature, such as other bodies or systems.

The observation of motion is due to our introduction of the plural. Motion results from our forced use of *many (approximate) parts* to describe the *unity* of nature. The observation of motion results from approximations. All these approximate distinctions are unavoidable and are due to the limitations of our experience.

Motion appears as soon as we divide the world into parts and then follow these parts. Dividing nature into parts is not a conscious act; our human nature – our senses and our brain – *force* us to perform it. And whenever we experience or talk about parts of the universe, we find motion. Our senses and our brain are made to distinguish and to divide – and cannot do otherwise. We need to distinguish in order to survive, to think and to enjoy life. In a sense, we can say that motion appears as a logical consequence of our limitations; the fundamental limitation is the one that makes us distinguish and introduce parts, including points and sets.

Motion is an ‘artefact’ of locality. Locality is an approximation and is due to our human nature. Distinction, localization and motion are inextricably linked.

Motion is low energy concept. Motion does not exist at Planck scales, i.e., at the limits of nature.

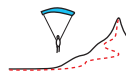
Motion is an illusion. Motion is an artefact due to our limitations. We can thus say, in a certain sense, that motion is an illusion. We seem to confirm what Zeno of Elea stated 2500 years ago. But in contrast to Zeno’s pessimistic view, we now have a fascinating spectrum of results and tools at our disposition. They allow us to describe motion and nature with high precision. Most of all, these tools allow us to change ourselves and our environment for the better.

Vol. I, page 14

Ref. 269

“ All the great things that have happened in the world first took place in a person’s imagination, and how tomorrow’s world will look like will largely depend on the power of imagination of those who are just learning to read right now.

Astrid Lindgren*



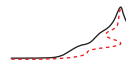
* Astrid Lindgren (b. 1907 Näs, d. 2002 Stockholm) was a beloved writer of children books.



POSTFACE

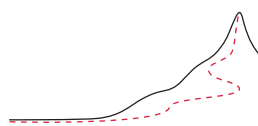
Perhaps once you will read Plato's *Phaedrus*, one of the beautiful philosophical Greek texts. In it, Socrates is made to say that he almost never left the city walls because to him, as a 'lover of learning, trees and the open country do not teach anything, whereas men in the town do.' This is a veiled critique of Democritus, the most important and famous philosopher in Greece during Plato's time. Democritus was the natural philosopher par excellence, and arguably had learned from nature – with its trees and open country – more than anybody else after him.

After this adventure you can decide for yourself which of these two approaches is more congenial to you. It might be useful to know that Aristotle refused to choose and cultivated them both. There is no alternative in life to following one's own mind, and to enjoy doing so. If you enjoyed this particular trip, show it to your friends. For yourself, after this walk, sense intensively the pleasure of having accomplished something important. Many before you did not have the occasion. Enjoy the beauty of the view offered. Enjoy the vastness of horizon it provides. Enjoy the impressions that it creates inside you. Collect them and rest. You will have a treasure that will be useful in many occasions. Then, when you feel the desire of going further, get ready for another of the adventures life has to offer.



Plato's *Phaedrus*, written around 380 BCE, is available in many pocket editions. Do not waste your time learning ancient Greek to read it; the translated versions are as beautiful as the original.

Plato's lifelong avoidance of the natural sciences had two reasons. First of all, he was jealous of Democritus. Plato never even cites Democritus in his texts. Democritus was the most prolific, daring, admired and successful philosopher of his time (and maybe of all times). Democritus was a keen student of nature. His written works did not survive, because his studies were not congenial to the followers of christianity, and thus they were not copied by the monks in the Middle Ages. The loss of these texts is related to the second reason that kept Plato away from the natural sciences: he wanted to save his life. Plato had learned one thing from men in the town: talking about nature is dangerous. Starting around his lifetime, for over 2000 years people practising the natural sciences were regularly condemned to exile or to death for impiety. Fortunately, this is only rarely the case today. But such violence still occurs, and we can honour the dangers that those preceding us had to overcome in order to allow us enjoying this adventure.



APPENDIX A

KNOT AND TANGLE GEOMETRY

The following table provides a terse summary of the mathematics of knot shapes.

TABLE 18 Important properties of knot, links and tangles.

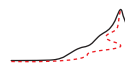
CONCEPT	DEFINING PROPERTY	OTHER PROPERTIES
Knot / link / tangle	one closed / several closed / one or several open curves, all in 3d and without intersections	ropelength is integral of arclength; ropelength is shape-dependent.
<i>Ideal</i> knot, link, tangle (shape)	tightest possible knot, link or tangle (shape) assuming a rope of constant diameter that is infinitely flexible and infinitely slippery	at present, all non-trivial ideal shapes are only known approximately; most ideal knots (almost surely) have kinks.
<i>Ribbon</i> or framing	short perpendicular (or non-tangent) vector attached at each point of a curve	
<i>Curvature</i> of a curve	inverse curvature radius of ‘touching’ circle	measures departure from straightness, i.e., local bending of a curve.
<i>Normal vector</i> or curvature vector	local vector normal to the curve, in direction of the centre of the ‘touching’ circle, with length given by the curvature	is given by the second and first derivatives of the curve.
<i>Binormal vector</i>	local unit vector normal to the tangent and to the normal/curvature vector	
<i>Torsion</i>	local speed of rotation of the binormal vector; positive (negative) for right-handed (left-handed) helix	measures departure from flatness, i.e., local twisting or local handedness of a curve; essentially a third derivative of the curve.
<i>Frenet frame</i> at a curve point	‘natural’ local orthogonal frame of reference defined by <i>unit</i> tangent, <i>unit</i> normal/curvature and binormal vector	the Frenet frame differs at each curve point, the Frenet frame is <i>not</i> uniquely defined if the curve is locally straight.

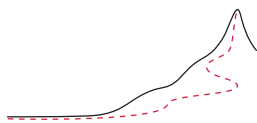
TABLE 18 (Continued) Important properties of knot, links and tangles.

CONCEPT	DEFINING PROPERTY	OTHER PROPERTIES
'Natural' framing or <i>Frenet ribbon</i>	defined by the local normal, i.e., local curvature vector	for a closed curve, it is always closed and two-sided, and thus never a Moebius band.
<i>Linking number</i> between two closed curves	sloppily, number of times that two curves wind around each other, or, equivalently, half the number of times that the curves 'swap' position	topological invariant, i.e., shape-independent; $\text{Lk}(K1, K2) = \frac{1}{4\pi} \oint_{K2} \oint_{K1} \frac{r_{12} (dr_1 \times dr_2)}{r_{12}^3}$
Linking number for a closed two-sided ribbon	number of times that the edges wind around each other	topological invariant, i.e., shape-independent; always an integer.
<i>Self-linking number</i> or 'natural' linking number for a knot	number of times that the edges of the natural/Frenet ribbon wind around each other	not a topological invariant, because of existence of inflection points.
Link integral for an open curve	generalization of the linking number for knots to open curves	usually not an integer.
<i>Twist</i> of a ribbon, open or closed	$\text{Tw}(R)$ is the total angle, in units of 2π , by which the ribbon rotates around the central axis of the ribbon; sloppily said, it measures the <i>local</i> helicity; this type of twist has no relation to the first Reidemeister move	vanishes for ribbons that are everywhere flat.
<i>Twist</i> of a curve or knot	$\text{Tw}(K)$ is the total angle, in units of 2π , by which the Frenet frame rotates around the tangent direction, or equivalently, (total) twist of the Frenet ribbon, also called the <i>total torsion</i> of the curve; this type of twist has no relation to the first Reidemeister move	not an integer even in case of knots; depends on curve/knot shape; is different from zero for chiral curves/knots; is zero for achiral curves/knots that have a rigid reflective symmetry; twist and torsion are only equal if the twist is defined with the Frenet ribbon – with other framings they differ.
<i>Signed crossing number</i>	sum of positive minus sum of negative crossings in a given oriented 2d projection of a curve or knot (sometimes called '2d-writhe')	always an integer; depends on shape.
<i>2d-writhe</i> of a knot, or <i>topological writhe</i> , or <i>Tait number</i>	signed crossing number for a <i>minimal</i> crossing number diagram/projection (sometimes the term '2d-writhe' is used for the signed crossing number of <i>any</i> configuration)	is shape-invariant; is always an integer; differs from 0 for all chiral knots; has the value 3 for the trefoil, 0 for the figure-eight knot, 5 for the 5_1 and 5_2 knots, 2 for the 6_1 knot, 7 for the 7_1 and 7_2 knots, 4 for the 8_1 knot, and 9 for the 9_2 knot.

TABLE 18 (Continued) Important properties of knot, links and tangles.

CONCEPT	DEFINING PROPERTY	OTHER PROPERTIES
Writhing number or 3d-writhe of a knot	$Wr(K)$ is the average, over all projection directions, of the signed crossing number; sloppily said, it measures how wrapped, coiled and chiral a knot is, i.e., it measures the <i>global</i> helicity	depends on knot shape; usually is not an integer; is different from zero for chiral knots; is zero for achiral knots that have a rigid reflective symmetry; $Wr(K) = \frac{1}{4\pi} \oint_K \oint_K \frac{r_{12} (dr_1 \times dr_2)}{r_{12}^3}$; uses no ribbon and thus is independent of the ribbon shape attached to the knot.
Writhe of ideal, alternating knots and of odd-component links	the value is quasi-quantized for alternating knots with small crossing numbers (< 11) in values that differ from $m4/7$ by only a few per cent	is additive under knot addition for knots with small crossing numbers (< 11) within less than 1%.
Writhe of ideal, alternating even-component links	the value is quasi-quantized for alternating links with small crossing numbers (< 11) in values that differ from $2/7 + m4/7$ by only a few per cent	
Writhe of a ribbon	sloppily said, measures how wrapped, coiled and chiral a ribbon is, i.e., measures its <i>global</i> helicity	
Writhe of an open curve		vanishes for plane curves.
Calugareanu's theorem	for any knot K and any ribbon G attached to it, $Lk(K, G) = Tw(K, G) + Wr(K)$	for applying the theorem to <i>open</i> curves, a (standardized) closing of curves is required.





CHALLENGE HINTS AND SOLUTIONS

Challenge 2, page 28: Take $\Delta f \Delta t \geq 1$ and substitute $\Delta l = c/\Delta f$ and $\Delta a = c/\Delta t$.

Challenge 16, page 43: Yes. But we can also argue its opposite, namely that matter appears when space is compressed too much. Both viewpoints are correct.

Challenge 22, page 46: The strictest upper limits are those with the smallest exponent for length, and the strictest lower limits are those with the largest exponent of length.

Challenge 24, page 48: To my knowledge, no such limits have been published. Do it yourself!

Challenge 25, page 48: The system limits cannot be chosen in other ways; after the limits have been corrected, the limits given here should still apply.

Challenge 28, page 49: Just insert numbers to check this.

Challenge 30, page 50: No.

Challenge 31, page 51: This is a trick question due to two issues. First, is the cosmological constant the same for all observers in the universe that are, like ourselves, more or less at rest with respect to the background radiation? Most researchers would agree that this is the case. Secondly, is the cosmological constant the same for extremely rapid observers, observers that move at extremely high energy with respect to the background radiation? Enjoy finding out.

Challenge 33, page 52: If you ever write such a table, publish it and send me a copy. I will include it in the text.

Challenge 36, page 65: Sloppily speaking, such a clock is not able to move its hands in a way that guarantees precise time reading.

Challenge 40, page 82: The final energy E produced by a proton accelerator increases with its radius R roughly as $E \sim R^{1.2}$; as an example, CERN's LHC achieved about 13 TeV for a radius of 4.3 km. Thus we would get a radius of more than 100 light years for a Planck energy accelerator. Building an accelerator achieving Planck energy is impossible.

Nature has no accelerator of this power, but gets near it. The maximum measured value of cosmic rays, 10^{22} eV, is about one millionth of the Planck energy. The mechanism of acceleration is still obscure. Neither black holes nor the cosmic horizon seem to be sources, for some yet unclear reasons. This issue is still a topic of research.

Challenge 41, page 82: The Planck energy is $E_{\text{Pl}} = \sqrt{\hbar c^5/G} = 2.0$ GJ. Car fuel delivers about 43 MJ/kg. Thus the Planck energy corresponds to the energy of 47 kg of car fuel, about a tankful.

Challenge 42, page 83: Not really, as the mass error is equal to the mass only in the Planck case.

Challenge 43, page 83: It is improbable that such deviations can be found, as they are masked by the appearance of quantum gravity effects. However, if you do think that you have a prediction for a deviation, publish it, and send the author an email.

Challenge 44, page 83: The minimum measurable distance is the same for single particles and systems of particles.

Challenge 45, page 83: There is no gravitation at those energies and there are no particles. There is thus no paradox.

Challenge 46, page 84: The issue is still being debated; a good candidate for a minimum momentum of a single particle is given by \hbar/R , where R is the radius of the universe. Is this answer satisfying?

Challenge 47, page 85: All mentioned options could be valid at the same time. The issue is not closed and clear thinking about it is not easy.

Challenge 48, page 85: The precise energy scale is not clear. The scale is either the Planck energy or within a few orders of magnitude from it; the lowest possible energy is thus around a thousandth of the Planck energy.

Challenge 50, page 87: If you can think of an experiment, publish the proposal, and send the author an email.

Vol. I, page 257

Challenge 51, page 90: The table of aggregates shows this clearly.

Challenge 52, page 91: The cosmic background radiation is a clock in the widest sense of the term.

Challenge 53, page 92: The way to deduce cosmological limits is presented in detail in the section starting on page 45.

Challenge 64, page 100: Also measurement errors at Planck scales prevent the determination of topology at those scales.

Challenge 66, page 102: The measurement error is as large as the measurement result.

Challenge 70, page 104: You will not find one.

Challenge 72, page 105: If you find one, publish it, and send the author an email.

Challenge 74, page 106: For the description of nature this is a contradiction. Nevertheless, the term ‘universe’, ‘set of all sets’ and other mathematical terms, as well as many religious concepts are of this type.

Challenge 75, page 107: No, for the reasons mentioned earlier on: fundamental measurement errors for horizon measurements, as well as many other effects, prevent this. The speculation is another example of misguided fantasy about extremal identity.

Challenge 76, page 108: The physical concepts most related to ‘monad’ are ‘strand’ and ‘universe’, as shown in the second half of this text.

Vol. II, page 257

Challenge 77, page 108: The macroscopic content of the universe may be observer-dependent. But to speak about many universes (Many ‘everything’s?) or a ‘multiverse’ (What is more than everything? Why only one multiverse?) is pure nonsense.

Challenge 80, page 108: True only if it were possible to do this. Because particles and space are indistinguishable, removing particles means to remove everything. (The strand model visualizes this connection most clearly.)

Vol. III, page 318

Challenge 82, page 108: True. Existence is the ability to interact. If the ability disappears, existence disappears. In other words, ‘existence’ is a low-energy concept.

Challenge 83, page 110: If you find a sensible statement about the universe, publish it! And send it to the author as well. The next challenge shows one reason why this issue is interesting. In addition, such a statement would contradict the conclusions on the combined effects of general relativity and quantum theory.

Challenge 84, page 110: Plotinus in the *Enneads* has defined ‘god’ in exactly this way. Later, Augustine in *De Trinitate* and in several other texts, and many subsequent theologians have taken up this view. (See also Thomas Aquinas, *Summa contra gentiles*, I, 30.) The idea they propose is

simple: it is possible to clearly say what ‘god’ is *not*, but it is impossible to say what ‘god’ *is*. This statement is also part of the official *Roman Catholic Catechism*: see part one, section one, chapter one, IV, 43, found at www.vatican.va/archive/ENG0015/_PC.HTM. Similar statements are found in Judaism, Hinduism and Buddhism.

Challenge 229 e

In other terms, theologians admit that ‘god’ cannot be defined, that the term has no properties or content, and that therefore the term cannot be used in any positive sentence. The aspects common to ‘universe’ and to ‘god’ suggest the conclusion that both are the same. Indeed, the analogy between the two concepts can be expanded to a proof: both concepts have the same content, the same boundary, and the same domain of application. (This is an intriguing and fascinating exercise.) In fact, this might be the most interesting of all proofs of the existence of ‘god’, as it lacks all the problems that the more common ‘proofs’ have. Despite its interest, this proof of equivalence is not found in any book on the topic yet. The reason is twofold. First, the results of modern physics – showing that the concept of universe has all these strange properties – are not common knowledge yet. Secondly, the result of the proof, the identity of ‘god’ and ‘universe’ – also called *pantheism* – is a heresy for most religions. It is an irony that the catholic catechism, together with modern physics, can be used to show that pantheism is correct, because any catholic who defends pantheism (or other heresies following from modern physics) incurs automatic excommunication, *latae sententiae*, without any need for a formal procedure.

Challenge 230 e

If one is ready to explore the identity of universe and ‘god’, one finds that a statement like ‘god created the universe’ translates as ‘the universe implies the universe’. The original statement is thus not a lie any more, but is promoted to a tautology. Similar changes appear for many other – but not all – statements using the term ‘god’. (The problems with the expression ‘in the beginning’ remain, though.) In fact, one can argue that statements about ‘god’ are only sensible and true if they remain sensible and true after the term has been exchanged with ‘universe’. Enjoy the exploration of such statements.

Challenge 86, page 112: If you find one, publish it and send it also to me. The conjecture is that no such effects exist.

Challenge 88, page 113: In fact, no length below the Planck length itself plays any role in nature.

Challenge 90, page 114: You need quantum humour, because the result obviously contradicts a previous one given on page 93 that includes general relativity.

Challenge 93, page 122: The number of spatial dimensions must be given first, in order to talk about spheres.

Challenge 94, page 126: This is a challenge to you to find out. It is fun, it may yield a result in contradiction with the arguments given so far (publish it in this case), or it may yield an independent check of the results of the section.

Challenge 96, page 130: This issue is open and still a subject of research. The conjecture of the author is that the answer is negative. If you find an alternative, publish it, and send the author an email.

Challenge 98, page 135: The lid of a box must obey the indeterminacy relation. It cannot be at perfect rest with respect to the rest of the box.

Challenge 100, page 136: No, because the cosmic background is not a Planck scale effect, but an effect of much lower energy.

Challenge 101, page 136: Yes, at Planck scales all interactions are strand deformations; therefore collisions and gravity are indistinguishable there.

Challenge 102, page 136: No. Time is continuous only if *either* quantum theory and point particles *or* general relativity and point masses are assumed. The argument shows that only the combination of *both* theories with continuity is impossible.

Challenge 103, page 136: You should, because at Planck scales nature's inherent measurement errors cannot clearly distinguish between different measurement results.

Challenge 104, page 136: We still have the chance to find the best approximate concepts possible. There is no reason to give up.

Challenge 105, page 136: Here are a few thoughts. A beginning of the big bang does not exist; something similar is given by that piece of continuous entity which is encountered when going backwards in time as much as possible. This has several implications.

- Going backwards in time as far as possible – towards the 'beginning' of time – is the same as zooming to smallest distances: we find a single strand of the amoeba.
- In other words, we speculate that the whole world is one single piece, fluctuating, and possibly tangled, knotted or branched.
- Going far away into space – to the border of the universe – is like taking a snapshot with a short shutter time: strands everywhere.
- Whenever we sloppily say that extended entities are 'infinite' in size, we only mean that they reach the horizon of the universe.

In summary, no starting point of the big bang exists, because time does not exist there. For the same reason, no initial conditions for particles or space-time exist. In addition, this shows that the big bang involved no creation, because without time and without possibility of choice, the term 'creation' makes no sense.

Challenge 106, page 136: The equivalence follows from the fact that all these processes require Planck energy, Planck measurement precision, Planck curvature, and Planck shutter time.

Page 363 **Challenge 107**, page 136: No, as explained later on in the text.

Challenge 108, page 137: Probably there is nothing wrong with the argument. For example, in the strand model, all observables are composed of fundamental events, and so, in some way, all observables are fundamentally indistinguishable.

Challenge 109, page 137: If not, force yourself. Brainstorming is important in life, as is the subsequent step: the checking of the speculations.

Challenge 114, page 149: The author would like to receive a mail on your reasons for disagreement.

Challenge 115, page 151: Let the author know if you succeed. And publish the results.

Challenge 116, page 151: Energy is action per time. Now, the Planck constant is the unit of action, and is defined by a crossing switch. A system that continuously produces a crossing switch for every Planck time running by thus has Planck energy. An example would be a tangle that is rotating extremely rapidly, once per Planck time, producing a crossing switch for every turn.

Momentum is action per length. A system that continuously produces a crossing switch whenever it advances by a Planck length has Planck momentum. An example would be a tangle configuration that lets a switch hop from one strand to the next under tight strand packing.

Force is action per length and time. A system that continuously produces a crossing switch for every Planck time that passes by and for every Planck length it advances through exerts a Planck force. A tangle with the structure of a screw that rotates and advances with sufficient speed would be an example.

Challenge 120, page 160: Yes; the appearance of a crossing does not depend on distance or on the number of strands in between.

Challenge 121, page 160: No; more than three dimensions do not allow us to define a crossing switch.

Challenge 122, page 160: If so, let the author know. If the generalization is genuine, the strand model is not correct.

Challenge 134, page 187: The magnitude at a point should be related to the vectorial sum of all inverse shortest crossing distances at that point.

Challenge 140, page 198: This algebraic transformation is shown in all textbooks that treat the Pauli equation. It can also be checked by writing the two equations out component by component.

Challenge 143, page 217: Yes, as can easily be checked by rereading the definitions with the spinor tangle description in mind.

Challenge 146, page 218: No contradiction is known.

Challenge 147, page 218: In the relativistic case, local space curvature is also taken into account.

Challenge 149, page 218: Find out, publish the result, and let the author know.

Challenge 150, page 218: If the strand interpenetration is allowed *generally*, quantum theory is impossible to derive, as the spinor behaviour would not be possible. If strand interpenetration were allowed only *under certain conditions* (such as only for a strand with itself, but not among two different strands), quantum theory might still be possible. A similar process lies at the basis of mass generation, as shown in the section on the weak interaction.

Challenge 152, page 218: The belt trick would imply that a wheel rolls over its own blood supply at every second rotation.

Challenge 160, page 241: If you find one, publish it!

Challenge 171, page 271: No slide is possible, thus no crossing change appears; thus the situation has no observable effects. If we deform one slide before the slide – which is possible – we get back the situation already discussed above.

Challenge 176, page 272: For the Wightman axioms, this seems to be the case; however, a formal proof is still missing. The same is expected for the Haag–Kastler axioms.

Challenge 186, page 299: A black hole has at least one crossing, thus at least a Planck mass.

Challenge 189, page 306: The present consensus is no.

Challenge 194, page 323: These tangles are not rational. In the renewed strand model of 2015, they cannot form; they are not allowed and do not represent any particle.

Challenge 196, page 343: Tail braiding leads to tangledness, which in turn is the basis for core rotation. And core rotation is kinetic energy, not rest mass.

Challenge 198, page 344: The issue is topic of research; for symmetry reasons it seems that a state in which each of the six quarks has the same bound to the other five quarks cannot exist.

Challenge 205, page 365: If you find such an estimate, publish it and send it to the author. A really good estimate also answers the following question: why does particle mass increase with core complexity? A tangle with a complex core, i.e., with a core of large ropelength, has a large mass value. Any correct estimate of the mass must yield this property. But a more complex knot will have a smaller probability for the belt trick. We seem to be forced to conclude that particle mass is not due to the belt trick alone.

Challenge 208, page 366: If you find such an estimate, publish it and send it to the author.

Challenge 210, page 367: Probably not.

Challenge 211, page 367: Probably not.

Challenge 212, page 367: Probably not.

Challenge 213, page 367: Find out – and let the author know.

Challenge 215, page 372: This would be an interesting result worth a publication.

Challenge 218, page 380: If you plan such a calculation, the author would be delighted to help.

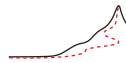
Challenge 222, page 389: Take up the challenge!

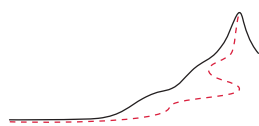
Challenge 224, page 404: There is a good chance, however, that such alternatives can be eliminated rather quickly. If you cannot do so, do publish the argument, and let the author know.

Challenge 226, page 412: Nobody can really answer ‘why’-questions about human actions. Climbing, like every other passion, is also a symbolic activity. Climbing can be a search for adventure, for meaning, for our mother or father, for ourselves, for happiness, or for peace.

Challenge 227, page 413: Also in dreams, speeds can be compared; and also in dreams, a kind of causality holds (though not a trivial one). Thus there is an invariant and therefore a maximum speed.

Challenge 228, page 414: Probably none. The answer depends on whether the existence of strands can be deduced from dreams. If strands can be deduced from dreams, all of physics follows. The conjecture is that this deduction is possible. If you find an argument against or in favour of this conjecture, let the author know.





BIBLIOGRAPHY

“The only end of writing is to enable the readers better to enjoy life, or better to endure it.”
Samuel Johnson*

- 1 See the first volume of the Motion Mountain series, *Fall, Flow and Heat*, available as free download at www.motionmountain.net. Cited on pages 17 and 399.
- 2 See the second volume of the Motion Mountain series, *Relativity*, available as free download at www.motionmountain.net. Cited on pages 17, 18, 399, and 428.
- 3 See the third volume of the Motion Mountain series, *Light, Charges and Brains*, available as free download at www.motionmountain.net, as well as the mentioned fourth and fifth volumes. Cited on pages 17 and 399.
- 4 See the fourth and fifth volumes of the Motion Mountain series, *The Quantum of Change* and *Pleasure, Technology and the Stars*, available as free download at www.motionmountain.net. Cited on pages 18, 399, 400, and 427.
- 5 The most precise value of the fine structure constant is determined from a weighted world average of high-precision measurements by a special international scientific committee called CODATA. Its website is www.codata.org/committees-and-groups/fundamental-physical-constants. The site also provides the latest official publication with the values of the fundamental constants. The most recent value of the fine structure constant is published at physics.nist.gov/cgi-bin/cuu/Value?alphinv and physics.nist.gov/cgi-bin/cuu/Value?alph. Cited on pages 18, 225, 376, 385, and 414.
- 6 See for example, the book by ROBERT LAUGHLIN, *A Different Universe: Reinventing Physics from the Bottom Down* Basic Books, 2005. Of the numerous books that discuss the idea of a final theory, this is the only one worth reading, and the only one cited in this bibliography. The opinions of Laughlin are worth pondering. Cited on page 21.
- 7 Many physicists, including Steven Weinberg, regularly – and incorrectly – claim in interviews that the measurement problem is not solved yet. Cited on page 21.
- 8 Undocumented sentences to this effect are regularly attributed to Albert Einstein. Because Einstein was a pantheist, as he often explained, his statements on the ‘mind of god’ are not really to be taken seriously. They were all made – if at all – in a humorous tone. Cited on page 21.
- 9 For an example for the inappropriate fear of unification, see the theatre play *Die Physiker* by the Swiss author FRIEDRICH DÜRRENMATT. Several other plays and novels took over this type of disinformation. Cited on page 21.

* This is a statement from the brilliant essay by the influential writer SAMUEL JOHNSON, *Review of Soame Jenyns’ “A Free Enquiry Into the Nature and Origin of Evil”*, 1757. See www.samueljohnson.com.

- 10 Exploring the spirit of play is the subject of research of the famous National Institute for Play, founded by Stuart Brown, and found at www.nifplay.org. Cited on page 22.
- 11 See e.g. the 1922 lectures by Lorentz at Caltech, published as H. A. LORENTZ, *Problems of Modern Physics*, edited by H. Bateman, Ginn and Company, 1927, page 99. Cited on page 27.
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- The *concept* of a maximum force was first proposed, most probably, by Venzo de Sabbata and C. Sivaram in 1993. Also this physics discovery was thus made much too late. In 1995, Corrado Massa took up the idea. Independently, Ludwik Kostro in 1999, Christoph Schiller just before 2000 and Gary Gibbons in the years before 2002 arrived at the same concept. Gary Gibbons was inspired by a book by Oliver Lodge; he explains that the maximum force value follows from general relativity; he does not make a statement about the converse, nor do the other authors. The statement of maximum force as a *fundamental principle* seems original to Christoph Schiller.

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- 21 See for example WOLFGANG RINDLER, *Relativity – Special, General and Cosmological*, Oxford University Press, 2001, p. 70 ss, or RAY D’INVERNO, *Introducing Einstein’s Relativity*, Clarendon Press, 1992, p. 36 ss. Cited on page 33.
- 22 T. JACOBSON, *Thermodynamics of spacetime: the Einstein equation of state*, Physical Review Letters 75, pp. 1260–1263, 1995, preprint at arxiv.org/abs/gr-qc/9504004; this deep article remains fascinating to this day. Even the author was scared to draw all the possible conclusions. The general concepts are explained, almost without formulae, in L. SMOLIN, *On the nature of quantum fluctuations and their relation to gravitation and the principle of inertia*, Classical and Quantum Gravity 3, pp. 347–359, 1986. Cited on pages 33 and 290.
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- 113 See for example the speculative model of vacuum as composed of Planck-size spheres proposed by F. WINTERBERG, *Zeitschrift für Naturforschung* 52a, p. 183, 1997. Cited on page 122.
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- 122** J. B. HARTLE, & S. W. HAWKING, *Path integral derivation of black hole radiance*, *Physical Review D* 13, pp. 2188–2203, 1976. See also A. STROMINGER & C. VAFA, *Microscopic origin of Bekenstein–Hawking entropy*, *Physics Letters B* 379, pp. 99–104, 1996, preprint at arxiv.org/abs/hep-th/9601029. For another derivation of black hole entropy, see G. T. HOROWITZ & J. POLCHINSKI, *A correspondence principle for black holes and strings*, *Physical Review D* 55, pp. 6189–6197, 1997, preprint at arxiv.org/abs/hep-th/9612146. Cited on pages 131 and 140.
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- 132** P. F. MENDE, *String theory at short distance and the principle of equivalence*, preprint at arxiv.org/abs/hep-th/9210001. Cited on page 138.

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- 136** A good introduction into his work is the paper D. KREIMER, *New mathematical structures in renormalisable quantum field theories*, *Annals of Physics* 303, pp. 179–202, 2003, erratum *ibid.* 305, p. 79, 2003, preprint at arxiv.org/abs/hep-th/0211136. Cited on page 139.
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- 143** S. K. NG, *On a knot model of the π^+ meson*, preprint at arxiv.org/abs/hep-th/0210024, and S. K. NG, *On a classification of mesons*, preprint at arxiv.org/abs/hep-ph/0212334. Cited on pages 139 and 345.
- 144** For a good introduction to superstrings, see the lectures by B. ZWIEBACH, *String theory for pedestrians*, agenda.cern.ch/fullAgenda.php?ida=a063319. For an old introduction to superstrings, see the famous text by M. B. GREEN, J. H. SCHWARZ & E. WITTEN, *Superstring Theory*, Cambridge University Press, volumes 1 and 2, 1987. Like all the other books on superstrings, they contain no statement that is applicable to or agrees with the strand model. Cited on pages 139 and 346.
- 145** See A. SEN, *An introduction to duality symmetries in string theory*, in *Les Houches Summer School: Unity from Duality: Gravity, Gauge Theory and Strings (Les Houches, France, 2001)*,

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- 146** Brian Greene regularly uses the name *string conjecture*. For example, he did so in a podium discussion at TED in 2009; the video of the podium discussion can be downloaded at www.ted.org. Cited on page 140.
- 147** L. SUSSKIND, *Some speculations about black hole entropy in string theory*, preprint at arxiv.org/abs/hep-th/9309145. G. T. HOROWITZ & J. POLCHINSKI, *A correspondence principle for black holes and strings*, Physical Review D 55, pp. 6189–6197, 1997, preprint at arxiv.org/abs/hep-th/9612146. Cited on pages 140 and 442.
- 148** F. WILCZEK, *Getting its from bits*, Nature 397, pp. 303–306, 1999. Cited on page 141.
- 149** M. R. DOUGLAS, *Understanding the landscape*, preprint at arxiv.org/abs/hep-th/0602266; his earlier papers also make the point. For the larger estimate, see W. TAYLOR & Y. - N. WANG, *The F-theory geometry with most flux vacua*, preprint at arxiv.org/abs/1511.03209. Cited on page 141.
- 150** The difficulties of the string conjecture are discussed in the well-known internet blog by PETER WOIT, *Not even wrong*, at www.math.columbia.edu/~woit/blog. Several Nobel Prize winners for particle physics dismiss the string conjecture: Martin Veltman, Sheldon Glashow, Burton Richter, Richard Feynman and since 2009 also Steven Weinberg are among those who did so publicly. Cited on pages 142 and 165.
- 151** The present volume was originally started with the aim to clarify the basic principles of string theory and to simplify it as much as possible. In particular, the first six chapters and the last chapter were conceived, structured and written with that aim. They are older than the strand model. Later on, the project took an unexpected direction, as explained in Ref. 19. Cited on page 142.
- 152** Searches for background-free approaches are described by E. WITTEN, *Quantum background independence in string theory*, preprint at arxiv.org/abs/hep-th/9306122 and E. WITTEN, *On background-independent open string theory*, preprint at arxiv.org/abs/hep-th/9208027. Cited on page 143.
- 153** In fact, no other candidate model that fulfils all requirements for the final theory is available in the literature so far. This might change in the future, though. Cited on page 149.
- 154** S. CARLIP, *The small scale structure of spacetime*, preprint at arxiv.org/abs/1009.1136. This paper deduces the existence of fluctuating lines in vacuum from a number of arguments that are completely independent of the strand model. Steven Carlip has dedicated much of his research to the exploration of this topic. One summary is S. CARLIP, *Spontaneous dimensional reduction in quantum gravity*, preprint at arxiv.org/abs/1605.05694; its is also instructive to read his review S. CARLIP, *Dimension and dimensional reduction in quantum gravity*, preprint at arxiv.org/abs/1705.05417. With the strand model in the back of one's mind, these results are even more fascinating. Cited on pages 160 and 298.
- 155** David Deutsch states that any good explanation must be 'hard to vary'. This must also apply to a unified model, as it claims to explain everything that is observed. See D. DEUTSCH, *A new way to explain explanation*, video talk at www.ted.org. Cited on pages 164 and 404.
- 156** L. BOMBELLI, J. LEE, D. MEYER & R. D. SORKIN, *Space-time as a causal set*, Physical Review Letters 59, pp. 521–524, 1987. See also the review by J. HENSON, *The causal set approach to quantum gravity*, preprint at arxiv.org/abs/gr-qc/0601121. Cited on pages 165 and 297.
- 157** D. FINKELSTEIN, *Homotopy approach to quantum gravity*, International Journal of Theoretical Physics 47, pp. 534–552, 2008. Cited on pages 165 and 297.

- 158** L. H. KAUFFMAN & S. J. LOMONACO, *Quantum knots*, preprint at arxiv.org/abs/quant-ph/0403228. See also S. J. LOMONACO & L. H. KAUFFMAN, *Quantum knots and mosaics*, preprint at arxiv.org/abs/quant-ph/0805.0339. Cited on page 165.
- 159** IMMANUEL KANT, *Critik der reinen Vernunft*, 1781, is a famous but long book that every philosopher pretends to have read. In his book, Kant introduced the ‘a priori’ existence of space and time. Cited on page 167.
- 160** The literature on circularity is rare. For two interesting exceptions, see L. H. KAUFFMAN, *Knot logic*, downloadable from www2.math.uic.edu/~kauffman, and L. H. KAUFFMAN, *Reflexivity and eigenform*, *Constructivist Foundations* 4, pp. 121–137, 2009. Cited on page 168.
- 161** Information on the belt trick is scattered across many books and few papers. The best source of information on this topic are websites. For belt trick visualizations see www.evl.uic.edu/hypercomplex/html/dirac.html, www.evl.uic.edu/hypercomplex/html/handshake.html, or www.gregegan.net/APPLETS/21/21.html. For an excellent literature summary and more movies, see www.math.utah.edu/~palais/links.html. None of these sites or the cited references seem to mention that there are *many* ways to perform the belt trick; this seems to be hidden knowledge. In September 2009, Greg Egan took up my suggestion and changed his applet to show an additional version of the belt trick. Cited on pages 175 and 177.
- 162** There is an interesting exploration behind this analogy between a non-dissipative system – a free quantum particle moving in vacuum – and a dissipative system – a macroscopic body drawn through a viscous liquid, say honey. The first question is to discover why this analogy is possible at all. (A careful distinction between the cases with spin 0, spin 1 and spin 1/2 are necessary.) The second question is the exploration of the motion of bodies of general shape in viscous fluids at low Reynolds numbers and under constant force. For the best overview of this question, see the beautiful article by O. GONZALEZ, A. B. A. GRAF & J. H. MADDOKS, *Dynamics of a rigid body in a Stokes fluid*, *Journal of Fluid Mechanics* 519, pp. 133–160, 2004. Cited on pages 194 and 357.
- 163** D. BOHM, R. SCHILLER & J. TIOMNO, *A causal interpretation of the Pauli equation (A)*, *Supplementi al Nuovo Cimento* 1, pp. 48 – 66, 1955, and D. BOHM & R. SCHILLER, *A causal interpretation of the Pauli equation (B)*, *Supplementi al Nuovo Cimento* 1, pp. 67–91, 1955. The authors explore an unusual way to interpret the wavefunction, which is of little interest here; but doing so, they give and explore the description of Pauli spinors in terms of Euler angles. Cited on page 197.
- 164** RICHARD P. FEYNMAN, *QED – The Strange Theory of Light and Matter*, Princeton University Press 1988. This is one of the best summaries of quantum theory ever written. Every physicist should read it. Cited on pages 198, 203, 215, 221, and 439.
- 165** S. KOCHEN & E. P. SPECKER, *The problem of hidden variables in quantum mechanics*, 17, pp. 59–87, 1967. This is a classic paper. Cited on page 201.
- 166** A. ASPECT, J. DALIBARD & G. ROGER, *Experimental tests of Bell’s inequalities using time-varying analyzers*, *Physical Review Letters* 49, pp. 1804–1807, 1982, Cited on page 205.
- 167** L. KAUFFMAN, *New invariants of knot theory*, *American Mathematical Monthly* 95, pp. 195–242, 1987. See also the image at the start of chapter 6 of LOUIS H. KAUFFMAN, *On Knots*, Princeton University Press, 1987. Cited on page 206.
- 168** The details on the speed of photons are explained in any textbook on quantum electrodynamics. The issue is also explained by Feynman in [Ref. 164](#) on page 89. Cited on page 210.
- 169** J. -M. LÉVY-LEBLOND, *Nonrelativistic particles and wave equations*, *Communications in Mathematical Physics* 6, pp. 286–311, 1967. See also A. GALINDO &

- C. SÁNCHEZ DEL RÍO, *Intrinsic magnetic moment as a nonrelativistic phenomenon*, American Journal of Physics 29, pp. 582–584, 1961, and V. I. FUSHCHICH, A. G. NIKITIN & V. A. SALOGUB, *On the non-relativistic motion equations in the Hamiltonian form*, Reports on Mathematical Physics 13, pp. 175–185, 1978. Cited on page 211.
- 170 L. LERNER, *Derivation of the Dirac equation from a relativistic representation of spin*, European Journal of Physics 17, pp. 172–175, 1996. Cited on pages 211 and 213.
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WRITHE	2	3	4	5
CROSSING NUMBER				
3	0	0.485	0	0
4	0.00046	0	0.392	0
5	0	0.045	0	0
6	0	0	0.076	0
7	0	0.00022	0	0
8	0	0	0.00011	0
9	0	0.000007	0	0
10	0	0	0.000002	0
11	0	0	0	0.000004
12	0	0	0	0
13	0	0	0	0.000002

These are the probabilities of knot orientations with a given writhe and crossing number for the tight open trefoil knot 3_1 . The smaller numbers are expected to be calculation artefacts.

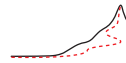
WRITHE	- 1	0	1
CROSSING NUMBER			
4	0	0.561	0
5	0.001	0	0.00083
6	0	0.296	0
7	0.0004	0	0.00065
8	0	0.083	0
9	0.00038	0	0.00038
10	0	0.043	0
11	0.000087	0	0.0001
12	0	0.013	0
13	0	0	0.00004
14	0	0.00008	0
15	0	0	0
16	0	0.000005	0

These are the probabilities of knot orientations with a given writhe and crossing number for the tight open figure-eight knot 4_1 . The limits to the precision of the calculation are clearly noticeable.

The lack of precision is due to the lack of precision of the available tight knot shapes. The probabilities for random *changes* in orientation are then deduced from the values in these tables. Because the knots are tight, it is a good approximation to assume that Reidemeister I and Reidemeister II moves can be distinguished without ambiguity – in contrast to the loose knot case, where this is not possible – from the writhe and crossing numbers of the start and end orientation. No citations.

- 258** The poster on www.physicsoverflow.org referred to J. P. LESTONE, *Physics based calculation of the fine structure constant*, preprint at arxiv.org/abs/physics/0703151. The preprint has never been published. Cited on page 383.
- 259** For a highly questionable, but still intriguing argument based on black hole thermodynamics that claims to deduce the limit $\alpha > \ln 3/48\pi \approx 1/137.26$, see S. HOD, *Gravitation, thermodynamics, and the fine-structure constant*, International Journal of Modern Physics D 19, pp. 2319–2323, 2010. It might well be that similar or other arguments based on textbook physics will yield more convincing or even better limits in the future. Cited on page 383.
- 260** V. ARNOLD, *Topological Invariants of Plane Curves and Caustics*, American Mathematical Society, 1994. Cited on page 390.
- 261** See M. POSPELOV & A. RITZ, *Electric dipole moments as probes of new physics*, preprint at arxiv.org/abs/hep-ph/0504231. Cited on page 390.
- 262** D. HILBERT, *Über das Unendliche*, Mathematische Annalen 95, pp. 161–190, 1925. Cited on page 399.
- 263** The *Book of Twenty-four Philosophers*, c. 1200, is attributed to the god Hermes Trismegistos, but was actually written in the middle ages. The text can be found in F. HUDRY, ed., *Liber viginti quattuor philosophorum*, Turnholt, 1997, in the series *Corpus Christianorum, Continuatio Mediaevalis*, CXLIII a, tome III, part 1, of the Hermes Latinus edition project headed by P. Lucentini. There is a Spinozian cheat in the quote: instead of ‘nature’, the original says ‘god’. The reason why this substitution is applicable is given above. Cited on page 404.

- 264** As a disappointing example, see GILLES DELEUZE, *Le Pli – Leibniz et le baroque*, Les Editions de Minuit, 1988. In this unintelligible, completely crazy book, the author pretends to investigate the implications of the idea that the *fold* (in French ‘le pli’) is the basic entity of matter and ‘soul’. Cited on page 406.
- 265** WERNER HEISENBERG, *Der Teil und das Ganze*, Piper, 1969. The text shows well how boring the personal philosophy of an important physicist can be. Cited on page 407.
- 266** John Barrow wrote to the author saying that he might have been the first to have used the T-shirt image, in the 1988 Gifford Lectures at Glasgow that were a precursor to his book JOHN D. BARROW, *Theories of Everything: The Quest for Ultimate Explanation*, 1991. He added that one can never be sure, though. Cited on page 409.
- 267** RENÉ DESCARTES, *Discours de la méthode*, 1637. He used and discussed the sentence again in his *Méditations métaphysiques* 1641, and in his *Les principes de la philosophie* 1644. These books influenced many thinkers in the subsequent centuries. Cited on page 412.
- 268** D. D. KELLY, *Sleep and dreaming*, in *Principles of Neural Science*, Elsevier, New York, 1991. The paper summarises experiments made on numerous humans and shows that even during dreams, people’s estimate of time duration corresponds to that measured by clocks. Cited on page 412.
- 269** Astrid Lindgren said this in 1977, in her speech at the fiftieth anniversary of Oetinger Verlag, her German publisher. The German original is: ‘Alles was an Großem in der Welt geschah, vollzog sich zuerst in der Phantasie eines Menschen, und wie die Welt von morgen aussehen wird, hängt in großem Maß von der Einbildungskraft jener ab, die gerade jetzt lesen lernen.’ The statement is found in ASTRID LINDGREN, *Deshalb brauchen Kinder Bücher*, Oetinger Almanach Nr. 15, p. 14, 1977. Cited on page 415.





CREDITS

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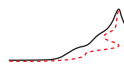
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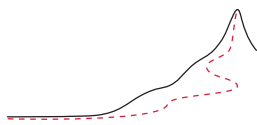
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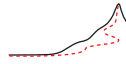
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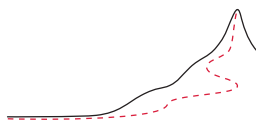
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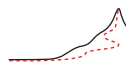
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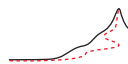
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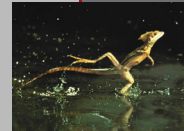


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Christoph Schiller, PhD Université Libre de Bruxelles, is a physicist and physics popularizer. This entertaining book is for students, teachers and anybody interested in modern research about fundamental physics.

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