Table 6-1: Ideal and typical parameters of a directional coupler.

Parameter	Ideal	Ideal (dB)	Typical
Coupling, C	-	-	3–40 dB
Transmission, T	$\lvert\sqrt{1-1/C^2}\rvert$	$20\log\lvert\sqrt{1-1/C^2}\rvert$	-0.5 dB
Directivity, D	∞	∞	40 dB
Isolation, I	∞	∞	40 dB

wavelength, with longer lengths of line resulting in broader bandwidth operation. Directional couplers are used to sample a traveling wave on one line and to induce a usually much smaller image of the wave on another line. That is, the forward- and backward-traveling waves are separated. Here a prescribed amount of the incident power is coupled out of the system. Thus, for example, a 20 dB microstrip coupler is a pair of coupled microstrip lines in which $1/100$ of the power input is coupled from one microstrip line onto the another.

Referring to Figure 6-7, a coupler is specified in terms of the following parameters (always check the magnitude of the factors, as some papers and books on couplers use the inverse of the C used here):

- Coupling factor:
  $C = V_1^+/V_3^-$ = inverse of the voltage fraction "transferred" (coupled) across to the opposite arm ($C > 1$).
- Transmission factor (inverse of insertion loss):
  $T = V_2^-/V_1^+$ = transmission directly through the "primary" arm of the structure ($T < 1$).
- Directivity factor:
  $D = V_3^-/V_4^-$ = measure of the undesired coupling from Port 1 to Port 4 relative to the signal level at Port 3 ($D > 1$).
- Isolation factor:
  $I = V_1^+/V_4^-$ = isolation between Port 4 and Port 1 ($I > 1$).

It is usual to quote these quantities in decibels. For example, the coupling factor in decibels is $C|_{\rm dB} = 20\log C$. So 20 dB coupling indicates that the coupling factor is 10. An ideal quarter-wave coupler has $D = \infty$ (i.e., infinite directivity) and

$$C = \frac{Z_{0e} + Z_{0o}}{Z_{0e} - Z_{0o}}. \tag{6.29}$$

In decibels the coupling is

$$C|_{\rm dB} = 20\log\left(\frac{Z_{0e} + Z_{0o}}{Z_{0e} - Z_{0o}}\right). \tag{6.30}$$

Typical and ideal parameters of a directional coupler are given in Table 6-1. Since an ideal coupler does not dissipate power, the magnitude of the transmission coefficient is

$$|T| = |\sqrt{1 - 1/C^2}|. \tag{6.31}$$

There are many types of directional couplers, and the phases of the traveling waves at the ports will not necessarily coincide. The microstrip coupler shown in Figure 6-7(b) has maximum coupling when the lines are one-quarter wavelength long.[3] At the frequency where they are one-quarter

[3] This is shown in a detailed derivation provided in Section 11.4 of [1].

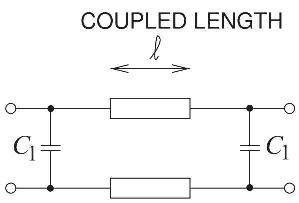


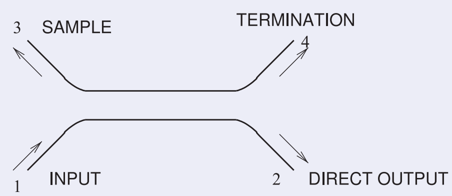
Figure 6-8: Parallel coupled lines with lumped capacitors reducing the length of the coupled lines.

wavelength long, the phase difference between traveling waves entering at Port 1 and leaving at Port 2 will be 90°.

EXAMPLE 6.1 Directional Coupler Isolation

A lossless directional coupler has coupling C = 20 dB, transmission factor 0.8, and directivity 20 dB. What is the isolation? Express your answer in decibels.

Solution:



Coupling factor: $C = V_1^+/V_3^-$
Transmission factor: $T = V_2^-/V_1^+$
Directivity factor: $D = V_3^-/V_4^-$
Isolation factor: $I = V_1^+/V_4^-$

$$D = 20 \text{ dB} = 10 \quad \text{and} \quad C = 20 \text{ dB} = 10,$$

so the isolation is

$$I = \frac{V_1^+}{V_4^-} = \frac{V_3^-}{V_4^-} \cdot \frac{V_1^+}{V_3^-} = D \cdot C = 10 \cdot 10 = 100 = 40 \text{ dB}.$$

6.5.1 Directional Coupler With Lumped Capacitors

Directional couplers using only coupled transmission lines can be large at low frequencies, as the minimum length is approximately one-quarter of a wavelength. This can be a problem at RF and low microwave frequencies, say, below 3 GHz. The length of the line can be reduced by incorporating lumped elements, as shown in Figure 6-8(a).

6.6 Summary

Coupling from one transmission line to a nearby neighbor may often be undesirable. However, the coupling can be controlled and coupled lines have become an important circuit component in distributed microwave circuits. The suite of microwave elements that exploit distributed effects available to a microwave designer is surprisingly large. Coupled lines comprise a large proportion of these elements. This chapter concludes with an example of the broadband response of a pair of coupled microstrip lines.

6.7 References

[1] T. Edwards and M. Steer, *Foundations for Microstrip Circuit Design*. John Wiley & Sons, 2016.

6.8 Exercises

1. Consider the cross section of a coupled transmission line, as shown in Figure 6-1, with even and odd modes both traveling out of the page.
   (a) For an even mode on the coupled line, consider a phasor voltage of 1 V on each of the lines above the ground plane at 0 V. Sketch the directed electric field in the transverse plane, i.e. show the direction of the electric field.
   (b) For the even mode, sketch the directed magnetic fields in the transverse plane (the plane of the cross section).
   (c) For an odd mode on the coupled line, consider a phasor voltage of $+1$ V on the left line and a phasor voltage of -1 V on the right line. Sketch the directed electric fields in the transverse plane (the plane of the cross section).
   (d) For the odd mode, sketch the directed magnetic fields in the transverse plane (the plane of the cross section).
2. An ideal directional coupler is lossless and there are no reflections at the ports. If the coupling factor is 10, what is the the magnitude of the transmission coefficient?
3. A directional coupler has the following characteristics: coupling factor C = 20, transmission factor 0.9, and directivity factor 25 dB. Also, the coupler is matched so that there is no reflection at any of the ports. What is the isolation in decibels?
4. A lossy 6 dB directional coupler is matched so that there is no reflection at any of the ports. The insertion loss (considering the through path) is 2 dB. If 1 mW is input to the directional coupler, what is the power in microwatts dissipated in the directional coupler? Ignore power leaving the isolated port.
5. A matched directional coupler has a coupling factor C of 20, transmission factor 0.9, and directivity of 25 dB. What is the power dissipated in the directional coupler if the input power to Port 1 is 1 W.
6. Consider a pair of parallel microstrip lines separated by a spacing, s, of 100 μm.
   (a) What happens to the coupling factor of the lines as s reduces?
   (b) What happens to the system impedance as s reduces and no other dimensions change?
   (c) In terms of wavelengths, what is the optimum length of the coupled lines for maximum coupling?

6.8.1 Exercises by Section

†challenging

§6.2 $1^{\dagger}$

§6.5 $2, 3, 4, 5, 6^{\dagger}$

6.8.2 Answers to Selected Exercises

2 0.9950

3(b) 187 mW

CHAPTER 7

Microwave Network Analysis

7.1 Introduction

Analog circuits at frequencies up to a few tens of megahertz are characterized by admittances, impedances, voltages, and currents. Above these frequencies it is not possible to measure voltage, current, or impedance directly. It is better to use quantities such as voltage reflection and transmission coefficients that can be quite readily measured and are related to power flow. As well, in RF and microwave circuit design the power of signals and of noise is always of interest. Thus there is a predisposition to focus on measurement parameters that are related to the reflection and transmission of power.

Scattering parameters, S parameters, embody the effects of reflection and transmission. As will be seen, it is easy to convert these to more familiar network parameters such as admittance and impedance parameters. In this chapter S parameters will be defined and related to impedance and admittance parameters, then it will be demonstrated that the use of S parameters helps in the design and interpretation of RF circuits. S parameters have become the most important parameters for RF and microwave engineers and many design methodologies have been developed around them.

This chapter presents microwave circuit theory which is based on the representation of distributed structures such as transmission lines, and other structures that are too large to be considered to be dimensionless, by lumped element circuits.

7.2 Two-Port Networks

In microwave circuits it is generally difficult to do this. Recall that with transmission lines it is not possible to establish a common ground point. However, with transmission lines it was seen that for each signal current there is a signal return current. Thus at radio frequencies, and for circuits that are distributed, ports are used, as shown in Figure 7-1(a), which define the voltages and currents for a two-port network, or just two-port.[1] The network in Figure 7-1(a) has four terminals and two ports. A port voltage is defined as the voltage difference between a pair of terminals with one of the terminals in the pair becoming the reference terminal. The current entering the network at the top terminal of Port 1 is I_1 and there is an equal current leaving the reference terminal. This arrangement clearly makes sense when transmission lines are attached to Ports 1 and 2, as in Figure 7-1(b). With transmission lines at Ports 1 and 2 there will be traveling-wave voltages, and at the ports the traveling-wave components add to give the total port voltage. In dealing with nondistributed circuits it is preferable to use the total port voltages and currents—V_1, I_1, V_2, and I_2, shown in Figure 7-1(a). However, with distributed elements it is preferable to deal with traveling voltages and currents—V_1^+, V_1^-, V_2^+, and V_2^-, shown in Figure 7-1(b). RF and microwave design necessarily requires switching between the two forms.

7.2.1 Reciprocity, Symmetry, Passivity, and Linearity

Reciprocity, symmetry, passivity, and linearity are fundamental properties of networks. A network is linear if the response is linearly dependent on the drive level, and superposition also applies. So if the two-port shown in Figure 7-1(a) is linear, the currents I_1 and I_2 are linear functions of V_1 and V_2. An example of a linear network would be one with resistors and capacitors. A network with a diode would be nonlinear. A passive network has no internal sources of power and a symmetrical two-port has the same characteristics at each of the ports. An example of a symmetrical network is a transmission line with a uniform cross section.

A reciprocal two-port has a response at Port 2 from an excitation at Port 1 that is the same as the response at Port 1 to the same excitation at Port 2. As an example, consider the two-port in Figure 7-1(a) with $V_2 = 0$. If the network is reciprocal, then the ratio I_2/V_1 with $V_2 = 0$ will be the same as the ratio I_1/V_2 with $V_1 = 0$. Networks with resistors, capacitors, and transmission lines are reciprocal. A transistor amplifier is not reciprocal as gain is in one

[1] Even when the term "two-port" is used on its own, the hyphen is used, as it is referring to a two-port network.

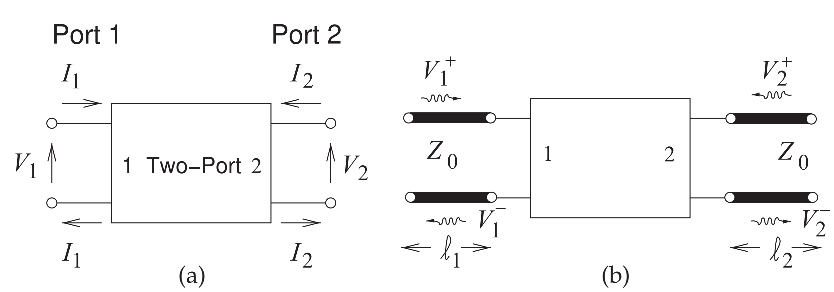


Figure 7-1: A two-port network: (a) port voltages; and (b) with transmission lines at the ports.

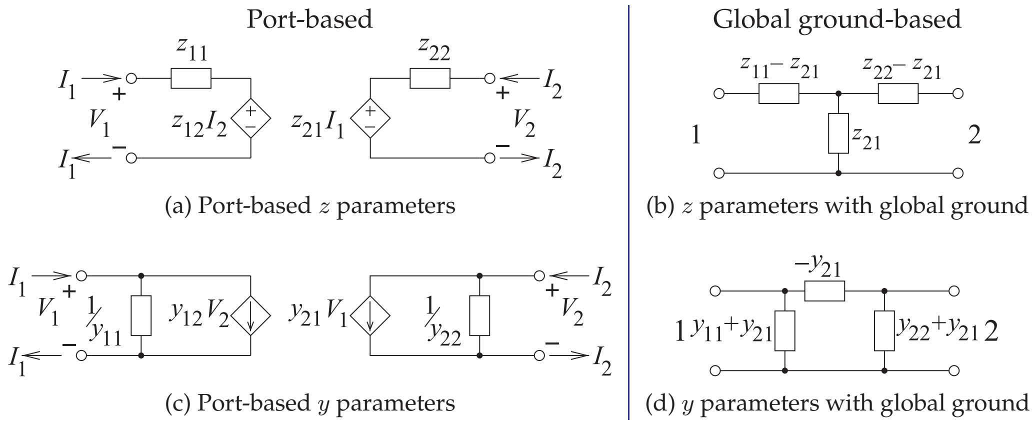


Figure 7-2: The z- and y-parameter equivalent circuits for port-based and global ground representations. The immittances are shown as impedances.

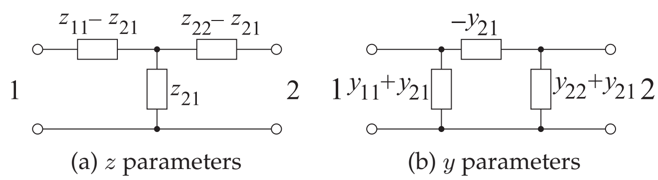


Figure 7-3: Circuit equivalence of the z and y parameters for a reciprocal network (in (b) the elements are admittances).

direction.

7.2.2 Parameters Based on Total Voltage and Current

Here port-based impedance (z), admittance (y), and hybrid (h) parameters will be described. These are similar to the more conventional z, y, and h parameters defined with respect to a common or global ground. The contrasting circuit representations of the parameters are shown in Figure 7-2.

Impedance parameters

With reference to Figure 7-1the port-based impedance parameters, or z parameters, are defined as

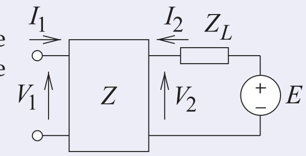
$$V_1 = z_{11}I_1 + z_{12}I_2 \qquad (7.1) \qquad\qquad V_2 = z_{21}I_1 + z_{22}I_2, \qquad (7.2)$$

or in matrix form as $\quad \mathbf{V} = \mathbf{ZI}. \qquad (7.3)$

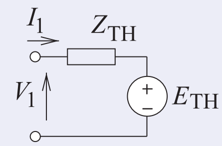
The double subscript on a parameter is ordered so that the first refers to the output and the second refers to the input, so z_{ij} relates the voltage output at Port i to the current input at Port j. If the network is reciprocal, then $z_{12} = z_{21}$, but this simple type of relationship does not apply to all network parameters. The reciprocal circuit equivalence of the z parameters is shown in Figure 7-3(a).

EXAMPLE 7.1 Thevenin equivalent of a source with a two-port

What is the Thevenin equivalent circuit of the source-terminated two-port network on the right.



Original circuit



Thevenin equivalent

Solution:
From the original circuit

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (7.4)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad (7.5)$$

$$V_2 = E - I_2 Z_L \quad (7.6)$$

Substituting Equation (7.6) in Equation (7.5)

$$E = Z_{21} I_1 + (Z_{22} + Z_L) I_2 \quad (7.7)$$

Multiplying Equation (7.4) by $(Z_{22} + Z_L)$ and Equation (7.7) by Z_{12}

$$(Z_{22} + Z_L) V_1 = (Z_{22} + Z_L) Z_{11} I_1 + (Z_{22} + Z_L) Z_{12} I_2 \quad (7.8)$$

$$Z_{12} E = Z_{12} Z_{21} I_1 + Z_{12} (Z_{22} + Z_L) I_2 \quad (7.9)$$

Subtracting Equation (7.9) from Equation (7.8)

$$(Z_{22} + Z_L) V_1 - Z_{12} E = [(Z_{22} + Z_L) Z_{11} -] I_1$$

$$V_1 = \frac{Z_{12} E}{Z_{22} + Z_L} + \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \right) I_1 \quad (7.10)$$

For the Thevenin equivalent circuit $V_1 = E_{\text{TH}} + I_1 Z_{\text{TH}}$ and so

$$Z_{\text{TH}} = \left(Z_{11} - \frac{Z_{12} Z_{21}}{Z_{22} + Z_L} \right) \quad \text{and} \quad E_{\text{TH}} = \frac{Z_{12} E}{Z_{22} + Z_L} \quad (7.11)$$

Admittance parameters

The port-based admittance parameters, or y parameters, are defined as

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad (7.12) \qquad I_2 = y_{21} V_1 + y_{22} V_2, \quad (7.13)$$

or in matrix form as $\mathbf{I} = \mathbf{Y}\mathbf{V}$. (7.14)

Now, for reciprocity, $y_{12} = y_{21}$ and the circuit equivalence of the y parameters is shown in Figure 7-3(b).

7.3 Scattering Parameters

Direct measurement of the z and y parameters requires that the ports be terminated in either short or open circuits. For active circuits this could result in undesired behavior, including oscillation or destruction. Also, at RF it is difficult to realize a good open or short. Since RF circuits are designed with close attention to maximum power transfer conditions, resistive terminations are preferred, as these are closer to the actual operating conditions. Thus the effect of measurement errors will have less impact. This is the way scattering parameters are measured.

The discussion of scattering parameters, S parameters[2], begins by considering the reflection coefficient, which is the S parameter of a one-port network.

[2] For historical reasons a capital "S" is used when referring to S parameters. For most other network parameters, lowercase is used (e.g., z parameters for impedance parameters).

7.3.1 Reflection Coefficient

The reflection coefficient, Γ, of a load Z_L can be determined by separately measuring the forward- and backward-traveling voltages on a transmission line terminated by the load:

$$\Gamma(x) = \frac{V^-(x)}{V^+(x)}. \tag{7.15}$$

Imagine between the source and the load that there is a line of characteristic impedance Z_0 and with infinitesimal length, then Γ at the load is related to the impedance Z_L by

$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{7.16}$$

where Z_0 is the characteristic impedance of the connecting transmission line. This can also be written as

$$\Gamma(0) = \frac{Y_0 - Y_L}{Y_0 + Y_L}, \tag{7.17}$$

where $Y_0 = 1/Z_0$ and $Y_L = 1/Z_L$. More completely, Γ as defined above is called the voltage reflection coefficient sometimes denoted Γ^V.

Recall that the current reflection coefficient $\Gamma^I = -\Gamma^V$.

7.3.2 Two-Port S Parameters

Two-port S parameters are defined in terms of traveling waves on transmission lines with real characteristic impedance Z_0 attached to each of the ports of a network, see Figure 7-1(b):

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ \quad (7.18) \qquad V_2^- = S_{21}V_1^+ + S_{22}V_2^+, \quad (7.19)$$

where S_{ij} are the individual S parameters. In matrix form

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} = \mathbf{S} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}. \tag{7.20}$$

Individual S parameters are determined by measuring the forward- and backward-traveling waves with loads Z_0 at the ports. Since the load is Z_0 it cannot reflect power and so $V_2^+ = 0$, then

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}. \tag{7.21}$$

The remaining parameters are determined similarly and so S_{22} is found as

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} \tag{7.22}$$

and the transmission parameter as

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}. \tag{7.23}$$

Table 7-1: Two-port S parameter conversion chart. The z, and y parameters are normalized to Z_0. Z' and Y' are the actual parameters.

	S	In terms of S
z	$Z'_{11} = z_{11}Z_0 \quad Z'_{12} = z_{12}Z_0$	$Z'_{21} = z_{21}Z_0 \quad Z'_{22} = z_{22}Z_0$
	$\delta_z = (1 + z_{11})(1 + z_{22}) - z_{12}z_{21}$	$\delta_S = (1 - S_{11})(1 - S_{22}) - S_{12}S_{21}$
	$S_{11} = [(z_{11} - 1)(z_{22} + 1) - z_{12}z_{21}]/\delta_z$	$z_{11} = [(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}]/\delta_S$
	$S_{12} = 2z_{12}/\delta_z$	$z_{12} = 2S_{12}/\delta_S$
	$S_{21} = 2z_{21}/\delta_z$	$z_{21} = 2S_{21}/\delta_S$
	$S_{22} = [(z_{11} + 1)(z_{22} - 1) - z_{12}z_{21}]/\delta_z$	$z_{22} = [(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}]/\delta_S$
y	$Y'_{11} = y_{11}/Z_0 \quad Y'_{12} = y_{12}/Z_0$	$Y'_{21} = y_{21}/Z_0 \quad Y'_{22} = y_{22}/Z_0$
	$\delta_y = (1 + y_{11})(1 + y_{22}) - y_{12}y_{21}$	$\delta_S = (1 + S_{11})(1 + S_{22}) - S_{12}S_{21}$
	$S_{11} = [(1 - y_{11})(1 + y_{22}) + y_{12}y_{21}]/\delta_y$	$y_{11} = [(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}]/\delta_S$
	$S_{12} = -2y_{12}/\delta_y$	$y_{12} = -2S_{12}/\delta_S$
	$S_{21} = -2y_{21}/\delta_y$	$y_{21} = -2S_{21}/\delta_S$
	$S_{22} = [(1 + y_{11})(1 - y_{22}) + y_{12}y_{21}]/\delta_y$	$y_{22} = [(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}]/\delta_S$

In the reverse direction,

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0}. \tag{7.24}$$

In the above Z_0 is referred to as the **normalization impedance** or equivalently the **reference impedance**. In some circumstances $Z_{\rm REF}$ is used to denote reference impedance to avoid possible confusion with a transmission line impedance that is not the same as the reference impedance. The S parameters here are also called **normalized S parameters**, and the S parameters are normalized to the same real reference impedance at each port.

The relationships between the two-port S parameters and the common network parameters are given in Table 7-1. It is interesting to note that $S_{21}/S_{12} = z_{21}/z_{12} = y_{21}/y_{12}$. That is, the ratio of the forward to reverse parameters (at least for S, z, and y parameters) are the same and this ratio is one for a reciprocal device. An S parameter is a voltage ratio, so when it is expressed in decibels $S_{ij}|_{\rm dB} = 20\log(S_{ij})$.

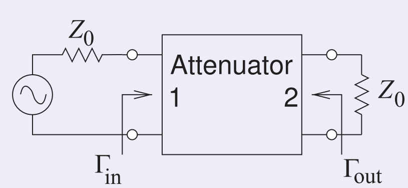
A reciprocal network has $S_{12} = S_{21}$. If unit power flows into a two-port, a fraction, $|S_{11}|^2$, is reflected and a further fraction, $|S_{21}|^2$, is transmitted through the network.

EXAMPLE 7.2 Two-Port S Parameters

What are the S parameters of a 30 dB attenuator?

Solution:

An attenuator is shown with a system impedance of Z_0. An ideal attenuator has no reflection at each of the two ports when the attenuator is embedded in its system impedance. Thus $\Gamma_{\rm in} = 0 = \Gamma_{\rm out}$. Since there is no reflection from the load or the source, this implies that $S_{11} = 0 = S_{22}$.



Since this is a 30 dB attenuator, the power delivered to the load impedance Z_0 is 30 dB below the power available from the source, thus $S_{21} = -30\text{ dB} = 0.0316$. The attenuator is reciprocal and so $S_{12} = S_{21}$. Thus the S parameters of the attenuator are

$$\mathbf{S} = \begin{bmatrix} 0 & 0.0316 \\ 0.0316 & 0 \end{bmatrix}. \tag{7.25}$$

Note that the reference impedance did not need to be known to develop the S parameters.

7.3.3 Input Reflection Coefficient of a Terminated Two-Port Network

A two-port is shown in Figure 7-4 that is terminated at Port 2 in a load with a reflection coefficient Γ_L. The lines at each of the ports are of infinitesimal length (i.e., $\ell_1 \to 0$ and $\ell_2 \to 0$) and are used to make it easier to visualize the separation of the voltage into forward- and backward-traveling components. The aim in this section is to develop a formula for the input reflection coefficient $\Gamma_{\text{in}} = V_1^-/V_1^+$. For the circuit in Figure 7-4 , three equations can be developed:

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ \tag{7.26}$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ \tag{7.27}$$
$$V_2^+ = \Gamma_L V_2^-, \quad \text{i.e.,} \quad V_2^- = V_2^+/\Gamma_L. \tag{7.28}$$

Note that V_2^- is the voltage wave that leaves the two-port but is incident on the load Γ_L. The aim here is to eliminate V_2^+ and V_2^-. Substituting Equation (7.28) into Equation (7.27) leads to

$$V_2^+/\Gamma_L = S_{21}V_1^+ + S_{22}V_2^+ \tag{7.29}$$
$$V_2^+\left(\frac{1 - S_{22}\Gamma_L}{\Gamma_L}\right) = S_{21}V_1^+ \tag{7.30}$$
$$V_2^+ = \left(\frac{S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right)V_1^+. \tag{7.31}$$

Now substituting Equation (7.31) in Equation (7.26) yields

$$V_1^- = S_{11}V_1^+ + S_{12}\left(\frac{S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right)V_1^+ \tag{7.32}$$

and so
$$\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}. \tag{7.33}$$

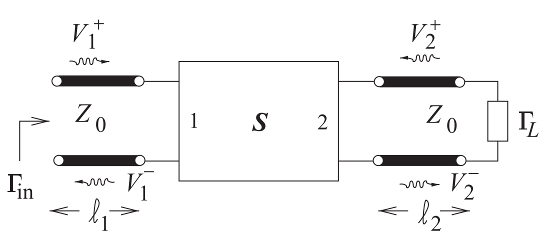


Figure 7-4: A terminated two-port network with transmission lines of infinitesimal length at the ports.

7.3.4 Properties of a Two-Port in Terms of S Parameters

The properties of most interest are whether the two-port network is lossless, passive, or reciprocal.

If a network is lossless, all of the power input to the network must leave the network. The power incident on Port 1 of a network is

$$P_1^+ = \left|\frac{\frac{1}{2}V_1^+}{Z_0}\right|^2 \tag{7.34}$$

and the power leaving Port 1 is

$$P_1^- = \left|\frac{\frac{1}{2}V_1^-}{Z_0}\right|^2. \tag{7.35}$$

This can be repeated for Port 2 and the factor $\frac{1}{2}/Z_0$ appears in all expressions. So cancelling this factor, the condition for the network to be lossless is

$$|S_{11}|^2 + |S_{21}|^2 = 1 \quad \text{and} \quad |S_{12}|^2 + |S_{22}|^2 = 1. \tag{7.36}$$

For a network to be passive, no more power can leave the network than enters it. So the condition for passivity is

$$|S_{11}|^2 + |S_{21}|^2 \le 1 \quad \text{and} \quad |S_{12}|^2 + |S_{22}|^2 \le 1. \tag{7.37}$$

Reciprocity requires that $S_{21} = S_{12}$.

7.4 Return Loss, Substitution Loss, and Insertion Loss

7.4.1 Return Loss

Return loss, also known as **reflection loss**, is a measure of the fraction of available power that is not delivered by a source to a load. If the power incident on a load is P_i and the power reflected by the load is P_r, then the return loss in decibels is [1, 2]

$$\text{RL}_{\text{dB}} = 10 \log \frac{P_i}{P_r}. \tag{7.38}$$

The better the load is matched to the source, the lower the reflected power and hence the higher the return loss. RL is a positive quantity if the reflected power is less than the incident power. If the load has a complex reflection coefficient ρ, then

$$\text{RL}_{\text{dB}} = 10 \log \left|\frac{1}{\rho^2}\right| = -20 \log |\rho| . \tag{7.39}$$

That is, the return loss is the negative of the reflection coefficient expressed in decibels [3].

When generalized to terminated two ports, the return loss is defined with respect to the input reflection coefficient of a terminated two port [4]. The two port in Figure 7-5 has the input reflection coefficient

$$\Gamma_{\text{in}} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{(1 - \Gamma_L S_{22})}, \tag{7.40}$$

where Γ_L is the reflection coefficient of the load. Thus the return loss of a terminated two-port is

$$\mathrm{RL}_{\mathrm{dB}} = -20\log|\Gamma_{\mathrm{in}}| = -20\log\left|S_{11} + \frac{\Gamma_L S_{12} S_{21}}{(1-\Gamma_L S_{22})}\right|. \tag{7.41}$$

If the load is matched, i.e. $Z_L = Z_0^*$ (the system reference impedance), then

$$\mathrm{RL}_{\mathrm{dB}} = -20\log|S_{11}|. \tag{7.42}$$

This return loss is also called the input return loss since the reflection coefficient is calculated at Port 1. The output return loss is calculated looking into Port 2 of the two-port, where now the termination at Port 1 is just the source impedance.

7.4.2 Substitution Loss and Insertion Loss

The substitution loss is the ratio of the power, ${}^iP_L$, delivered to the load by an initial two-port identified by the leading superscript 'i', and the power delivered to the load, ${}^fP_L$, with a substituted final two-port identified by the leading superscript 'f'. In terms of scattering parameters with reference impedances Z_{REF}, the substitution loss in decibels is (using the results in [5] and noting that Γ_S and Γ_L are referred to Z_{REF})

$$L_S|\,\mathrm{dB} = \frac{{}^iP_L}{{}^fP_L} = 10\log\left|\frac{{}^iS_{21}}{{}^fS_{21}}\frac{[(1-{}^fS_{11}\Gamma_S)(1-{}^fS_{22}\Gamma_L)-{}^fS_{12}{}^fS_{21}\Gamma_S\Gamma_L]}{[(1-{}^iS_{11}\Gamma_S)(1-{}^iS_{22}\Gamma_L)-{}^iS_{12}{}^iS_{21}\Gamma_S\Gamma_L]}\right|^2. \tag{7.43}$$

Insertion loss is a special case of substitution loss with a particular type of initial two-port networks.

Insertion Loss with an Ideal Adaptor

Insertion loss is the substitution loss when the initial two-poort is a direct connection so ${}^iS_{11} = 0 = {}^iS_{22}$ (for no reflection), ${}^iS_{12}{}^iS_{21} = 1$ (for no loss in the adaptor and there is no phase shift), and ${}^iS_{12} = {}^iS_{21}$ (for reciprocity), insertion loss in decibels is, using Equation (7.43):

$$\mathrm{IL}|_{\mathrm{dB}} = 10\log\left[\left|\frac{(1-{}^fS_{11}\Gamma_S)(1-{}^fS_{22}\Gamma_L)-{}^fS_{12}{}^fS_{21}\Gamma_S\Gamma_L}{{}^fS_{12}\,(1-\Gamma_S\Gamma)}\right|^2\right] \tag{7.44}$$

Attenuation is defined as the insertion loss without source and load

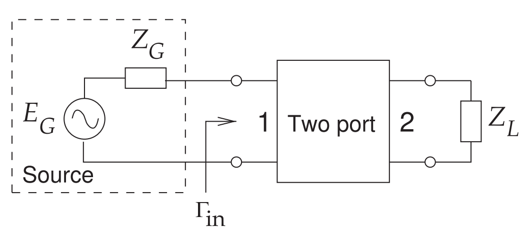


Figure 7-5: Terminated two-port used to define return loss.

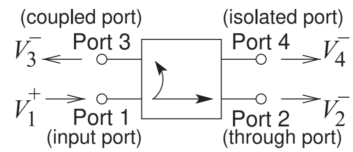


Figure 7-6: Schematic of a directional coupler.

reflections ($\Gamma_S = 0 = \Gamma_L$) [6], and Equation (7.44) becomes

$$A|\,\text{dB} = 10\log\left(\frac{Z_{02}}{Z_{01}}\frac{1}{|S_{21}|^2}\right) \qquad (= \text{IL with } \Gamma_S = 0 = \Gamma_L) \tag{7.45}$$

7.5 Scattering Parameters and Directional Couplers

Directional couplers were described in Section 6.5, but without the use of S parameters. A directional coupler with ports defined as in Figure 7-6, and with the ports matched (so that $S_{11} = 0 = S_{22} = S_{33} = S_{44}$), has the following scattering parameter matrix:

$$\mathbf{S} = \begin{bmatrix} 0 & T & 1/C & 1/I \\ T & 0 & 1/I & 1/C \\ 1/C & 1/I & 0 & T \\ 1/I & 1/C & T & 0 \end{bmatrix}. \tag{7.46}$$

There are many types of directional couplers, and the phases of the traveling waves at the ports will not necessarily be in phase as Equation (7.46) implies. When the phase difference between traveling waves entering at Port 1 and leaving at Port 2 is $90°$, Equation (7.46) becomes

$$\mathbf{S} = \begin{bmatrix} 0 & -\jmath T & 1/C & 1/I \\ -\jmath T & 0 & 1/I & 1/C \\ 1/C & 1/I & 0 & -\jmath T \\ 1/I & 1/C & -\jmath T & 0 \end{bmatrix}. \tag{7.47}$$

EXAMPLE 7.3 Identifying Ports of a Directional Coupler

A directional coupler has the following S parameters:

$$S = \begin{bmatrix} 0 & 0.9 & 0.001 & 0.1 \\ 0.9 & 0 & 0.1 & 0.001 \\ 0.001 & 0.1 & 0 & 0.9 \\ 0.1 & 0.001 & 0.9 & 0 \end{bmatrix}.$$

(a) What are the through (i.e., transmission) paths? Identify two paths. That is, identify the pairs of ports at the ends of the through paths.
First note that the assignment of ports to a directional coupler is arbitrary. So the S parameters need to be considered to figure out how the ports are related. The S parameters relate the forward-traveling waves to the backward-traveling waves and this leads to the required understanding, thus

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = S \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix} = \begin{bmatrix} 0 & 0.9 & 0.001 & 0.1 \\ 0.9 & 0 & 0.1 & 0.001 \\ 0.001 & 0.1 & 0 & 0.9 \\ 0.1 & 0.001 & 0.9 & 0 \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \\ V_4^+ \end{bmatrix}.$$

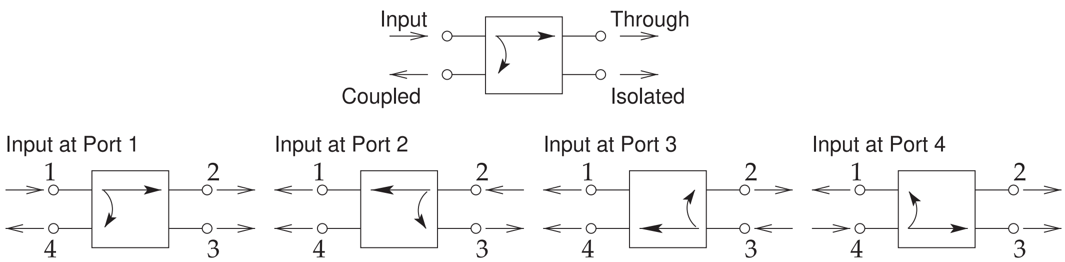


Figure 7-7: Directional coupler schematic drawn with each of the four possible input ports.

Writing the S parameters out this way makes it easier to identify the largest backward-traveling waves for each of the inputs at Ports 1, 2, 3, and 4. The backward-traveling wave will leave the directional coupler and the inputs will be forward-traveling waves. Consider Port 1, the largest backward-traveling wave is at Port 2, and so Ports 1 and 2 define one of the through paths. The other through path is between Ports 3 and 4.
So the through paths are 1–2 and 3–4.

(b) What is the coupled port for the signal entering Port 1?
The coupled port is identified by the port with the largest backward-traveling signal not including the port at the other end of the through path. For Port 1 the coupled port is Port 4.

(c) What is the coupling factor?

$$C = \frac{V_1^+}{V_4^-} = \frac{1}{0.1} = 10 = 20 \text{ dB}.$$

(d) What is the isolated port for the signal entering Port 1?
The isolated port is Port 3. The backward-traveling wave at this port is the smallest given an input at Port 1.

(e) What is the isolation factor?

$$I = \frac{V_1^+}{V_3^-} = \frac{1}{0.001} = 1000 = 60 \text{ dB}.$$

(f) What is the directivity factor?
The directivity factor indicates how much stronger the signal is at the coupled port compared to the isolated port for a signal at the input. For an input at Port 1, the directivity factor is

$$D = \frac{V_4^-}{V_3^-} = \frac{0.1}{0.001} = 100 = 40 \text{ dB}.$$

As a check $D = I/C = 1000/10 = 100$.

(g) Draw a schematic of the directional coupler.
There are four ways to draw it depending on the input port chosen, see Figure 7-7.

7.6 Summary

There are several network parameters used with RF and microwave circuits. Which is used depends on which makes the task of visualizing circuit operation more clear, which makes analyzing circuits more convenient, and which enables different circuits to be made electrically equivalent.

Scattering parameters are parameters that are almost exclusively used by RF and microwave engineers. They describe power flow and traveling waves and are essential to describing distributed circuits. Much of RF and microwave engineering is concerned with managing the signal-to-noise power ratio and with power efficiency. It is therefore natural to work with parameters that directly relate to power flow. RF and microwave design is characterized by conceptual insight and it is essential to use parameters and graphical representations that are close to the physical world. Scattering parameters have very natural graphical representations, as will be seen in the next chapter.

7.7 References

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[4] T. Otoshi, "Maximum and minimum return losses from a passive two-port network terminated with a mismatched load," *IEEE Trans. on Microwave Theory and Techniques*, vol. 42, no. 5, pp. 787–792, May 1994.

[5] M. Steer, *Microwave and RF Design, Networks*, 3rd ed. North Carolina State University, 2019.

[6] R. Beatty, "Insertion loss concepts," *Proc. of the IEEE*, vol. 52, no. 6, pp. 663–671, Jun. 1964.

7.8 Exercises

1. A load has a reflection coefficient of $0.5 - \jmath 0.1$ in a $75\ \Omega$ reference system. What is the reflection coefficient in a $50\ \Omega$ reference system?

2. The $50\ \Omega$ S parameters of a two-port are $S_{11} = 0.5+\jmath 0.5$, $S_{12} = 0.95+\jmath 0.25$, $S_{21} = 0.15-\jmath 0.05$, and $S_{22} = 0.5 - \jmath 0.5$. Port 1 is connected to a $50\ \Omega$ source with an available power of 1 W and Port 2 is terminated in $50\ \Omega$. What is the power reflected from Port 1?

3. The scattering parameters of a certain two-port are $S_{11} = 0.5 + \jmath 0.5$, $S_{12} = 0.95 + \jmath 0.25$, $S_{21} = 0.15 - \jmath 0.05$, and $S_{22} = 0.5 - \jmath 0.5$. The system reference impedance is $50\ \Omega$.
   (a) Is the two-port reciprocal? Explain.
   (b) Consider that Port 1 is connected to a $50\ \Omega$ source with an available power of 1 W. What is the power delivered to a $50\ \Omega$ load placed at Port 2?
   (c) What is the reflection coefficient of the load required for maximum power transfer at Port 2?

4. In characterizing a two-port, power could only be applied at Port 1. The signal reflected was measured and the signal at a $50\ \Omega$ load at Port 2 was also measured. This yielded two S parameters referenced to $50\ \Omega$: $S_{11} = 0.3 - \jmath 0.4$ and $S_{21} = 0.5$.
   (a) If the network is reciprocal, what is S_{12}?
   (b) Is the two-port lossless?
   (c) What is the power delivered into the $50\ \Omega$ load at Port 2 when the available power at Port 1 is 0 dBm?

5. A matched lossless transmission line has a length of one-quarter wavelength. What are the scattering parameters of the two-port?

6. A connector has the scattering parameters $S_{11} = 0.05$, $S_{21} = 0.9$, $S_{12} = 0.9$, and $S_{22} = 0.04$ and the reference impedance is $50\ \Omega$. What is the return loss in dB of the connector at Port 1 in a $50\ \Omega$ system?

7. The scattering parameters of an amplifier are $S_{11} = 0.5$, $S_{21} = 2.$, $S_{12} = 0.1$, and $S_{22} = -0.2$ and the reference impedance is $50\ \Omega$. If the amplifier is terminated at Port 2 in a resistance of $25\ \Omega$, what is the return loss in dB at Port 1?

8. A two-port network has the scattering parameters $S_{11} = -0.5$, $S_{21} = 0.9$, $S_{12} = 0.8$, and $S_{22} = 0.04$ and the reference impedance is $50\ \Omega$.
   (a) What is the return loss in dB of the connector at Port 1 in a $50\ \Omega$ system?
   (b) Is the two-port reciprocal and why?

9. A two-port network has the scattering parameters $S_{11} = -0.2$, $S_{21} = 0.8$, $S_{12} = 0.7$, and $S_{22} = 0.5$ and the reference impedance is $75\ \Omega$.
   (a) What is the return loss in dB of the connector at Port 1 in a $75\ \Omega$ system?
   (b) Is the two-port reciprocal and why?

10. A cable has the scattering parameters $S_{11} = 0.1$, $S_{21} = 0.7$, $S_{12} = 0.7$, and $S_{22} = 0.1$. At Port 2 is a 55 Ω load and the S parameters and reflection coefficients are referred to 50 Ω.
    (a) What is the load's reflection coefficient?
    (b) What is the input reflection coefficient of the terminated cable?
    (c) What is the return loss, at Port 1 and in dB, of the cable terminated in the load?
11. A cable has the 50 Ω scattering parameters $S_{11} = 0.05$, $S_{21} = 0.5$, $S_{12} = 0.5$, and $S_{22} = 0.05$. What is the insertion loss of the cable if the source at Port 1 has a 50 Ω Thevenin impedance and the termination at Port 2 is 50 Ω? Express your answer in decibels.
12. A 1 m long cable has the 50 Ω scattering parameters $S_{11} = 0.1$, $S_{21} = 0.7$, $S_{12} = 0.7$, and $S_{22} = 0.1$. The cable is used in a 55 Ω system. Express your answers in decibels.
    (a) What is the return loss of the cable in the 55 Ω system? (Hint see Section sec:input:terminated:two:port and consider finding Z_in.]
    (b) What is the insertion loss of the cable in the 55 Ω system? Follow the procedure in Example 7.0
    (c) What is the return loss of the cable in a 50 Ω system?
    (d) What is the insertion loss of the cable in a 50 Ω system?
13. A 1 m long cable has the 50 Ω scattering parameters $S_{11} = 0.05$, $S_{21} = 0.5$, $S_{12} = 0.5$, and $S_{22} = 0.05$. The Thevenin equivalent impedance of the source and terminating load impedances of the cable are 50 Ω. Express your answers in decibels.
    (a) What is the return loss of the cable?
    (b) What is the insertion loss of the cable?
14. A lossy directional coupler has the following 50 Ω S parameters:
$$S = \begin{bmatrix} 0 & -0.95\jmath & 0.005 & 0.1 \\ -0.95\jmath & 0 & 0.1 & 0.005 \\ 0.005 & 0.1 & 0 & -0.95\jmath \\ 0.1 & 0.005 & -0.95\jmath & 0 \end{bmatrix}.$$
    (a) What are the through (transmission) paths (identify two paths)? That is, identify the pairs of ports at the ends of the through paths.
    (b) What is the coupling in decibels?
    (c) What is the isolation in decibels?
    (d) What is the directivity in decibels?
15. A directional coupler has the following characteristics: coupling factor C = 20, transmission factor 0.9, and directivity factor 25 dB. Also, the coupler is matched so that $S_{11} = 0 = S_{22} = S_{33} = S_{44}$.
    (a) What is the isolation factor in decibels?
    (b) Determine the power dissipated in the directional coupler if the input power to Port 1 is 1 W.
16. A lossy directional coupler has the following 50 Ω S parameters:
$$S = \begin{bmatrix} 0 & 0.25 & -0.9\jmath & 0.01 \\ 0.25 & 0 & 0.01 & -0.9\jmath \\ -0.9\jmath & 0.01 & 0 & 0.25 \\ 0.01 & -0.9\jmath & 0.25 & 0 \end{bmatrix}.$$
    (a) Which port is the input port (there could be more than one answer)?
    (b) What is the coupling in decibels?
    (c) What is the isolation in decibels?
    (d) What is the directivity factor in decibels?
17. A directional coupler using coupled lines has a coupling factor of 3.38, a transmission factor of $-\jmath0.955$, and infinite directivity and isolation. The input port is Port 2 and the through port is Port 2. Write down the 4×4 S parameter matrix of the coupler.

7.8.1 Exercises by Section

[†]challenging

§7.3 $1, 2, 3^\dagger, 4^\dagger, 5^\dagger$ §7.4 $6, 7, 8, 9, 10, 11, 12^\dagger, 13$ §7.5 $14^\dagger, 15^\dagger, 16^\dagger, 17^\dagger$

7.8.2 Answers to Selected Exercises

1	$0.638 - \jmath0.079$	6	26 dB	15(b)	187 mW
3(d)	$50 + \jmath100$ Ω	12(b)	2.99 dB	16(d)	28 dB

CHAPTER 8

Graphical Network Analysis

8.1 Introduction

This chapter introduces the Smith chart, most widely used graphical technique for describing and solving problems using S parameters. The Smith chart is an annotated polar plot of S parameters and is one of the most powerful tools in RF and microwave engineering and is used to present measured results, to conceptualize designs, and to intuitively solve problems involving distributed networks.

8.2 Polar Representations of Scattering Parameters

Scattering parameters are most naturally represented in polar form with the square of the magnitude relating to power flow. In this section a greater rationale for representing S parameters on a polar plot is presented and this serves as the basis for a more complicated representation of S parameters on a Smith chart to be described in the next section.

8.2.1 Shift of Reference Planes as a S Parameter Rotation

A polar plot is a natural way to present S parameters graphically. Adding additional lengths of the lines at each port rotates the S parameters. Consider the two-port in Figure 8-1. Here the original two-port with scattering

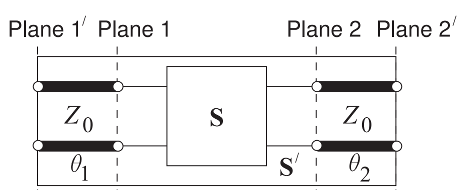


Figure 8-1: A two-port with scattering parameter matrix **S** augmented by lines at each port with the lines having the reference characteristic impedance Z_0 and electrical length θ_n. The scattering parameter matrix of the augmented two-port is **S**′.

parameter matrix $\mathbf{S}$ is augmented by lines at each port with each having a characteristic impedance equal to the reference impedance. The S parameter matrix of the augmented two-port, $\mathbf{S}'$, are the same as the original S parameter matrix but phase-shifted. That is

$$\mathbf{S}' = \begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} S_{11}\mathrm{e}^{-\jmath 2\Theta 1} & S_{12}\mathrm{e}^{-\jmath(\Theta 1+\Theta 2)} \\ S_{21}\mathrm{e}^{-\jmath(\Theta 1+\Theta 2)} & S_{22}\mathrm{e}^{-\jmath 2\Theta 2} \end{bmatrix}. \quad (8.1)$$

The shift in reference planes simply rotates the S parameters. This is one of the main reasons why S parameters are commonly plotted on a polar plot.

8.2.2 Polar Plot of Reflection Coefficient

The polar plot of reflection coefficient is simply the polar plot of a complex number. Figure 8-2 is used in plotting reflection coefficients and is a polar plot that has a radius of one. So a reflection coefficient with a magnitude of one is on the unit circle. The center of the polar plot is zero so the reflection coefficient of a matched load, which is zero, is plotted at the center of the circle. Plotting a reflection coefficient on the polar plot enables convenient interpretation of the properties of a reflection. The graph has additional notation that enables easy plotting of an S parameter on the graph. Conversely, the magnitude and phase of an S parameter can be easily read from the graph. The horizontal label going from 0 to 1 is used in determining magnitude. The notation arranged on the outer perimeter of the polar plot is used to read off angle information. Notice the additional notation “ANGLE OF REFLECTION COEFFICIENT IN DEGREES” and the scale relates to the actual angle of the polar plot. Verify that the 90° point is just where one would expect it to be.

Figure 8-3 annotates the polar plot of reflection coefficient with real and imaginary axes and shows the location of the short circuit and open circuit points. Note that the reflection coefficient of an inductive impedance is in the top half of the polar plot while the reflection coefficient of a capacitive impedance is in the bottom half of the polar plot.

The nomograph shown in Figure 8-4 aids in interpretation of polar reflection coefficient plots. The nomograph relates the reflection coefficient (RFL. COEFFICIENT), ρ (ρ was originally used instead of Γ and is still used with the Smith chart); the return loss (RTN. LOSS) (in decibels); and the standing wave ratio (SWR); and the standing wave ratio (in decibels) as $20\ \log(\mathrm{SWR})$. When printed together with the reflection coefficient polar plot (Figures 8-2 and 8-4 combined) the nomograph is scaled properly, but it is expanded here so that it can be read more easily. So with the aid of a compass with one point on the zero point of the polar plot and the other at the reflection coefficient (as plotted on the polar plot), the magnitude of the reflection coefficient is captured. The compass can then be brought down to the nomograph to read ρ, the return loss, and VSWR directly.

8.3 Smith Chart

The Smith chart is a powerful graphical tool and mastering the Smith chart is essential to entering the world of RF and microwave circuit design as all practitioners use this as if it is well understood by others. It takes effort to master but fundamentally it is quite simple combining a polar plot used for

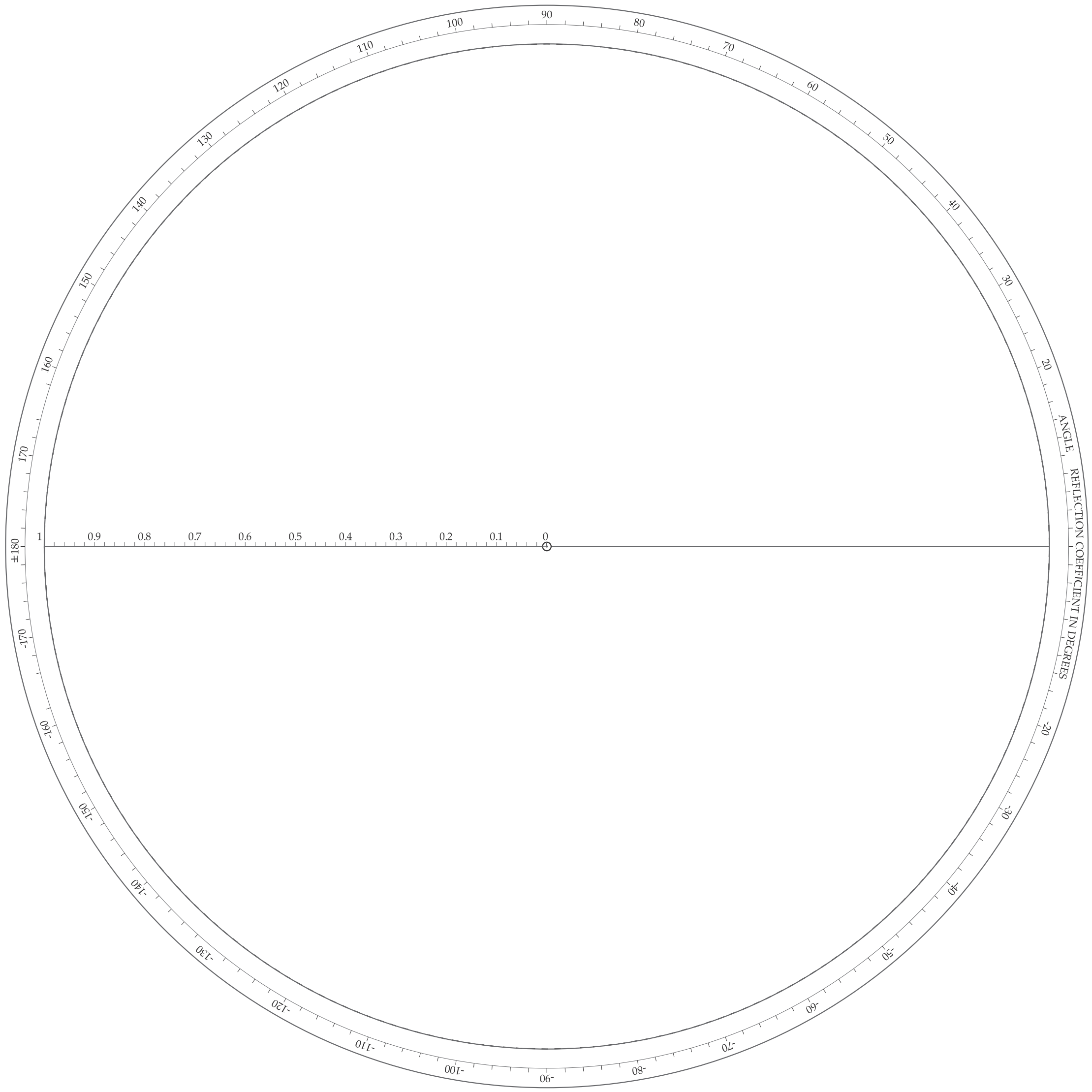


Figure 8-2: Polar chart for plotting reflection coefficient and transmission coefficient.

plotting S parameters directly, curves that enable normalized impedances and admittances to be plotted directly, and scales that enable electrical lengths in terms of wavelengths and degrees to be read off. The chart has many numbers printed in quite small font and with signs dropped off as there is limited room.

The Smith chart was invented by Phillip Smith and presented in close to its current form in 1937, see [1–4]. Once nomographs and graphical calculators were common engineering tools mainly because of limited computing resources. Only a few have survived in electrical engineering usage, with Smith charts being overwhelmingly the most important.

This section first presents the impedance Smith chart and then the admittance Smith chart before introducing a combined Smith chart which is the form needed in design. A number of examples are presented to

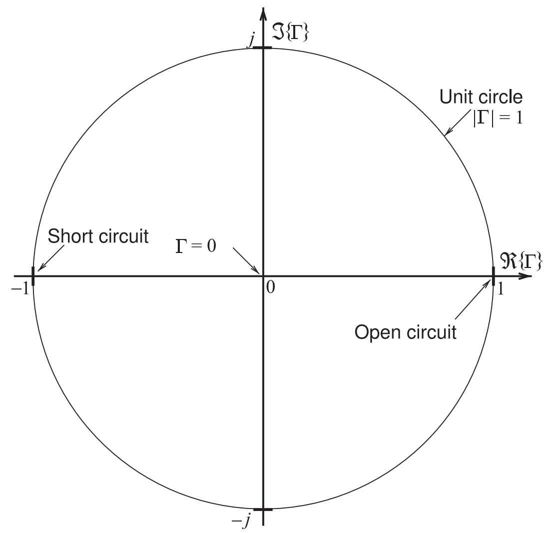


Figure 8-3: Annotated polar plot of reflection coefficient with real, $\Re$, and imaginary, $\Im$, axes. The short circuit $\Gamma = -1$ and open circuit $\Gamma = +1$ are indicated. The reflection coefficient is referenced to a reference impedance Z_{REF}. Thus an impedance Z_L has the reflection coefficient $\Gamma = (Z_L - Z_{\rm REF})/(Z_L + Z_{\rm REF}))$. An interesting observation is that the angle of Γ when Z_L is inductive, i.e. has a positive reactance, has a positive angle between $0°$ and $180°$ and so Γ is in the top half of the polar plot. Similarly the angle of Γ when Z_L is capacitive, i.e. has a negative reactance, has a negative angle between $0°$ and $-180°$ and so Γ is in the bottom half of the polar plot.

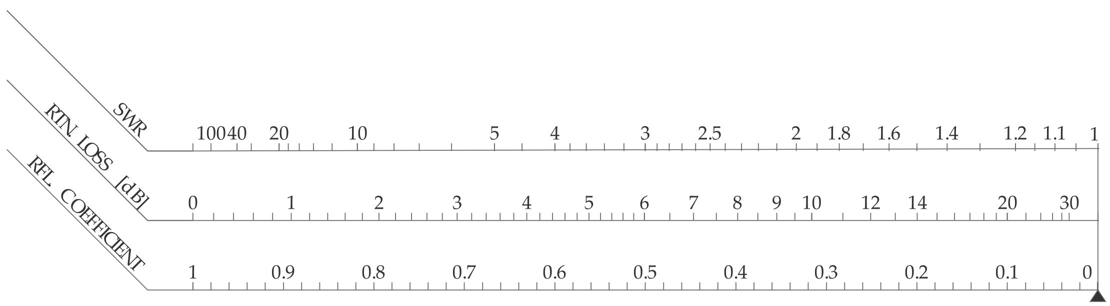


Figure 8-4: Nomograph relating the reflection coefficient (RFL. COEFFICIENT), ρ; the return loss (RTN. LOSS) (in decibels); and the standing wave ratio (SWR).

illustrate how a Smith chart is read and used to implement simple designs. The Smith chart presents a large amount of information in a confined space and interpretation, such as applying appropriate signs, is required to extract values. The Smith chart is a 'back-of-the-envelope' tool that RF and microwave circuit designers use to sketch out designs.

8.3.1 Impedance Smith Chart

The reflection coefficient, Γ, is related to a load, Z_L, by

$$\Gamma = \frac{Z_L - Z_{\rm REF}}{Z_L + Z_{\rm REF}}, \tag{8.2}$$

where $Z_{\rm REF}$ is the system reference impedance. With normalized load impedance $z_l = r + \jmath x = Z_L/Z_{\rm REF}$, this becomes

$$\Gamma = \frac{r + \jmath x - 1}{r + \jmath x + 1}. \tag{8.3}$$

Commonly in network design reactive elements are added either in shunt or in series to an existing network. If a reactive element is added in series then the input reactance, x, is changed while the input resistance, r, is held constant. So superimposing the loci of Γ (on the S parameter polar plot) with fixed values of r, but varying values of x (x varying from $-\infty$ to ∞), proves useful, as will be seen. Also, plotting the loci of Γ with fixed values of x and varying values of r (r varying from 0 to ∞) is also useful. The combination of the reflection/transmission polar plots, the nomographs, and the r and x loci is called the impedance Smith chart, see Figure 8-5. This is still a polar plot of reflection coefficient and the arcs and circles of constant and resistance enable easy conversion between reflection coefficient and impedance.

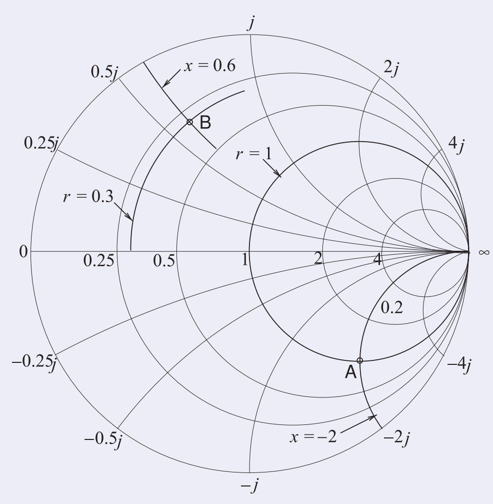
The full impedance Smith chart shown in Figure 8-5 is daunting so discussion will begin with the less dense form of the impedance Smith chart shown in Figure 8-6(a) which is annotated in Figure 8-6(b). Referring to Figure 8-6(b), the unit circle corresponds to a reflection coefficient magnitude of one and hence a pure reactance. Note that there are lines of constant resistance and arcs of constant reactance. All points in the top half of the Smith chart have positive reactances and so all reflection coefficient points plotted in the top half of the Smith chart indicate inductive impedances. All points in the bottom half of the Smith chart have negative reactances and so all reflection coefficient points plotted in the bottom half of the Smith chart indicate capacitive impedances. The horizontal line across the middle of the Smith chart indicates pure resistance just as the unit circle indicates a pure reactance.

One big difference between the less dense form of the impedance Smith chart, Figure 8-6(a), and the full impedance Smith chart of Figure 8-5 is that the signs of the reactances are missing in the full impedance Smith chart. This is simply because there is not enough room and the user must add the appropriate sign when reading the chart. Thus it is essential that the user keep the annotations in Figure 8-6(b) in mind. Yet another factor that makes it difficult to develop essential Smith chart skills.

EXAMPLE 8.1 Impedance Plotting

Plot the normalized impedances $z_A = 1 - 2\jmath$ and $z_B = 0.3 + 0.6\jmath$ on an impedance Smith chart.

Solution:
The impedance $z_A = 1 - 2\jmath$ is plotted as Point A to the right. To plot this first identify the circle of constant normalized resistance $r = 1$, and then identify the arc of constant normalized reactance $x = -2$. The intersection of the circle and arc locates z_A at point A. The reader is encouraged to do this with the full impedance Smith chart as shown in Figure 8-5. Recall that signs of reactances are missing on the full chart. As an exercise read off the reflection coefficient (the answer is $0.5 - \jmath 0.5 = 0.707\angle -45°$).



The impedance $z_B = 0.3 + 0.6\jmath$ is plotted as Point B which is at the intersection of the circle $r = 0.3$ and the arc $x = +0.6$. Interpolation is required to identify the required circle and arc. The reader should do this with the full impedance Smith chart. As an exercise read off the reflection coefficient (the answer is $-0.268 + \jmath 0.585 = 0.644\angle 115°$).

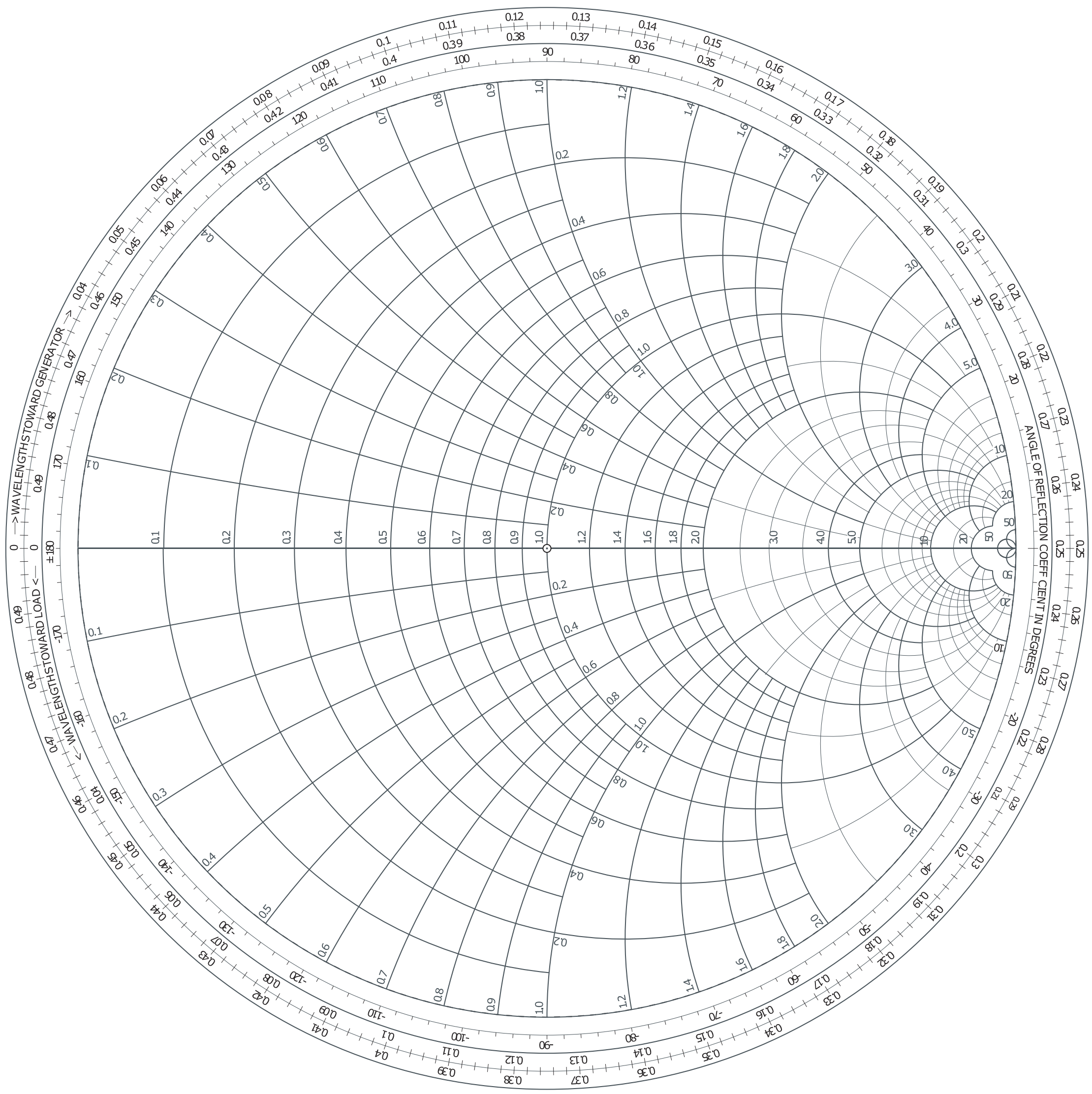
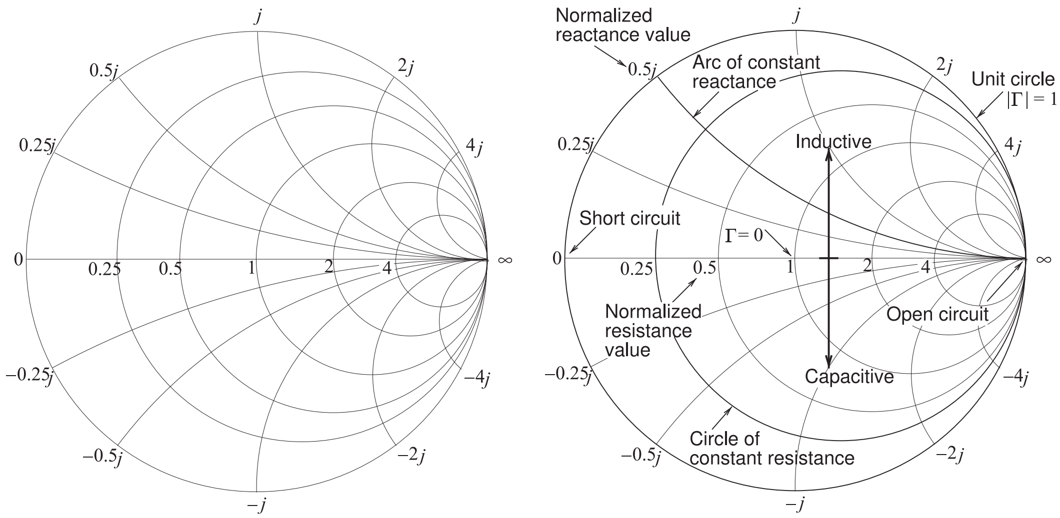


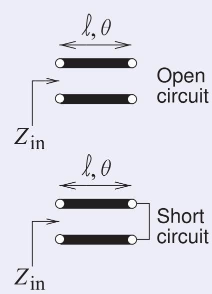
Figure 8-5: Impedance Smith chart. Also called a normalized Smith chart since resistances and reactances have been normalized to the system reference impedance $Z_{\rm REF}$: $z = Z/Z_{\rm REF}$.
A point plotted on the Smith chart represents a complex number A. The magnitude of A is obtained by measuring the distance from the origin of the polar plot (the same as the origin of the Smith chart in the center of the unit circle) to point A, say using a ruler, and comparing that to the measurement of the radius of the unit circle which corresponds to a complex number with a magnitude of 1. The angle in degrees of the complex number A is read from the innermost circular scale. The technique used is to draw a straight line from the origin through point A out to the circular scale.



(a) Impedance Smith chart (b) Annotated impedance Smith chart

Figure 8-6: Normalized impedance Smith charts.

EXAMPLE 8.2 Impedance Synthesis



Use a length of terminated transmission line to realize an impedance of $Z_{\text{in}} = \jmath 140\ \Omega$.

Solution:
The impedance to be synthesized is reactive so the termination must also be lossless. The simplest termination is either a short circuit or an open circuit. Both cases will be considered. Choose a transmission line with a characteristic impedance, Z_0, of 100 Ω so that the desired normalized input impedance is $\jmath 140\ \Omega/Z_0 = 1.4\jmath$, plotted as point B in Figure 8-7.

First, the short-circuit case. In Figure 8-7, consider the path AB. The termination is a short circuit and the impedance of this load is Point A with a reference length of $\ell_A = 0\ \lambda$ (from the outermost circular scale). The corresponding reflection coefficient reference angle from the scale is $\theta_A = 180°$ (from the innermost circular scale labeled 'WAVELENGTHS TOWARDS THE GENERATOR' which is the same as 'wavelengths away from the load'). As the line length increases, the input impedance of the terminated line follows the clockwise path to Point B where the normalized input impedance is $\jmath 1.4$. (To verify your understanding that the locus of the refection coefficient rotates in the clockwise direction, i.e. increasingly negative angle as the line length increases, see Section 3.3.3 .) At Point B the reference line length $\ell_B = 0.1515\ \lambda$ and the corresponding reflection coefficient reference angle from the scale is $\theta_B = 71.2°$. The reflection coefficient angle and length in terms of wavelengths were read directly off the Smith chart and care needs to be taken that the right sign and correct scale are used. A good strategy is to correlate the scales with the easily remembered properties at the open-circuit and short-circuit points. Here the line length is

$$\ell = \ell_B - \ell_A = 0.1515\lambda - 0\lambda = 0.1515\lambda, \tag{8.4}$$

and the electrical length is half of the difference in the reflection coefficient angles,

$$\theta = \tfrac{1}{2}\,|\theta_B - \theta_A| = \tfrac{1}{2}\,|71.2° - 180°| = 54.4°, \tag{8.5}$$

corresponding to a length of $(54.4°/360°)\lambda = 0.1511\lambda$ (the discrepancy with the previously determine line length of 0.1515λ is small). This is as close as could be expected from using the scales. So the length of the stub with a short-circuit termination is 0.1515λ.

For the open-circuited stub, the path begins at the infinite impedance point $\Gamma = +1$ and rotates clockwise to Point A (this is a $90°$ or 0.25λ rotation) before continuing on to Point B. For the open-circuited stub,

$$\ell = 0.1515\lambda + 0.25\lambda = 0.4015\lambda. \tag{8.6}$$

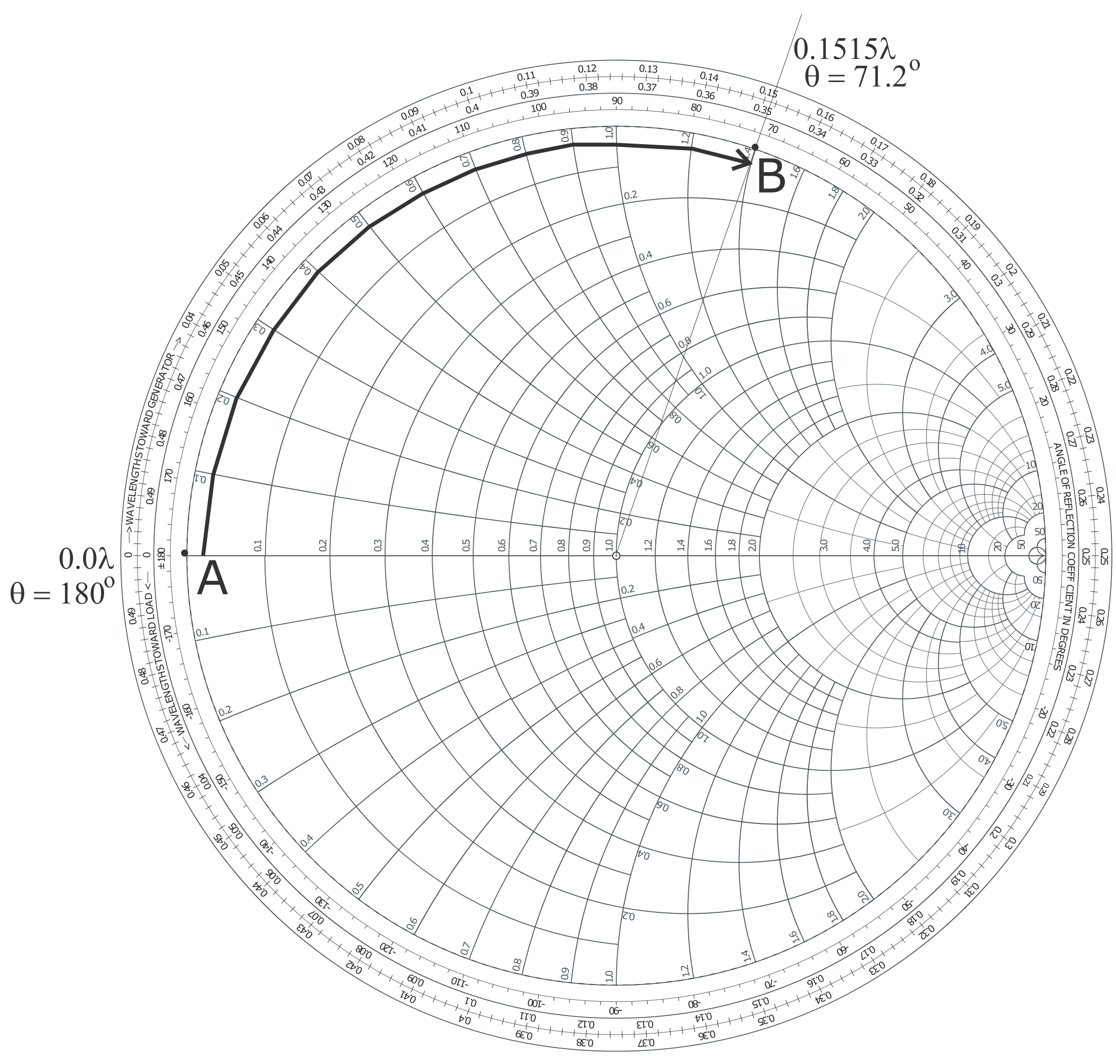


Figure 8-7: Design of a short-circuit stub with a normalized input impedance of $\jmath1.4$. The path AB is actually on the unit circle but has been displaced here to avoid covering numbers. The electrical length in wavelengths has been read from the outermost circular scale, and the angle, Θ, in degrees refers to the angle of the polar plot (and is twice the electrical length).

8.3.2 Admittance Smith Chart

The admittance Smith chart has loci for discrete constant susceptances ranging from $-\infty$ to ∞, and for discrete constant conductance ranging from 0 to ∞, see Figure 8-8. A less dense form is shown in Figure 8-9(a). This chart looks like the flipped version of the impedance Smith chart but it is the same polar plot of a reflection coefficient so that the positions of the open and short circuit remain the same as do the capacitive and inductive halves of the Smith chart. In the full version of the admittance Smith chart, Figure 8-8, signs have been dropped as there is not room for them. Thus interpreting admittances from the chart requires that the user separately determine the signs of susceptances. The less dense version, Figure 8-9(a),

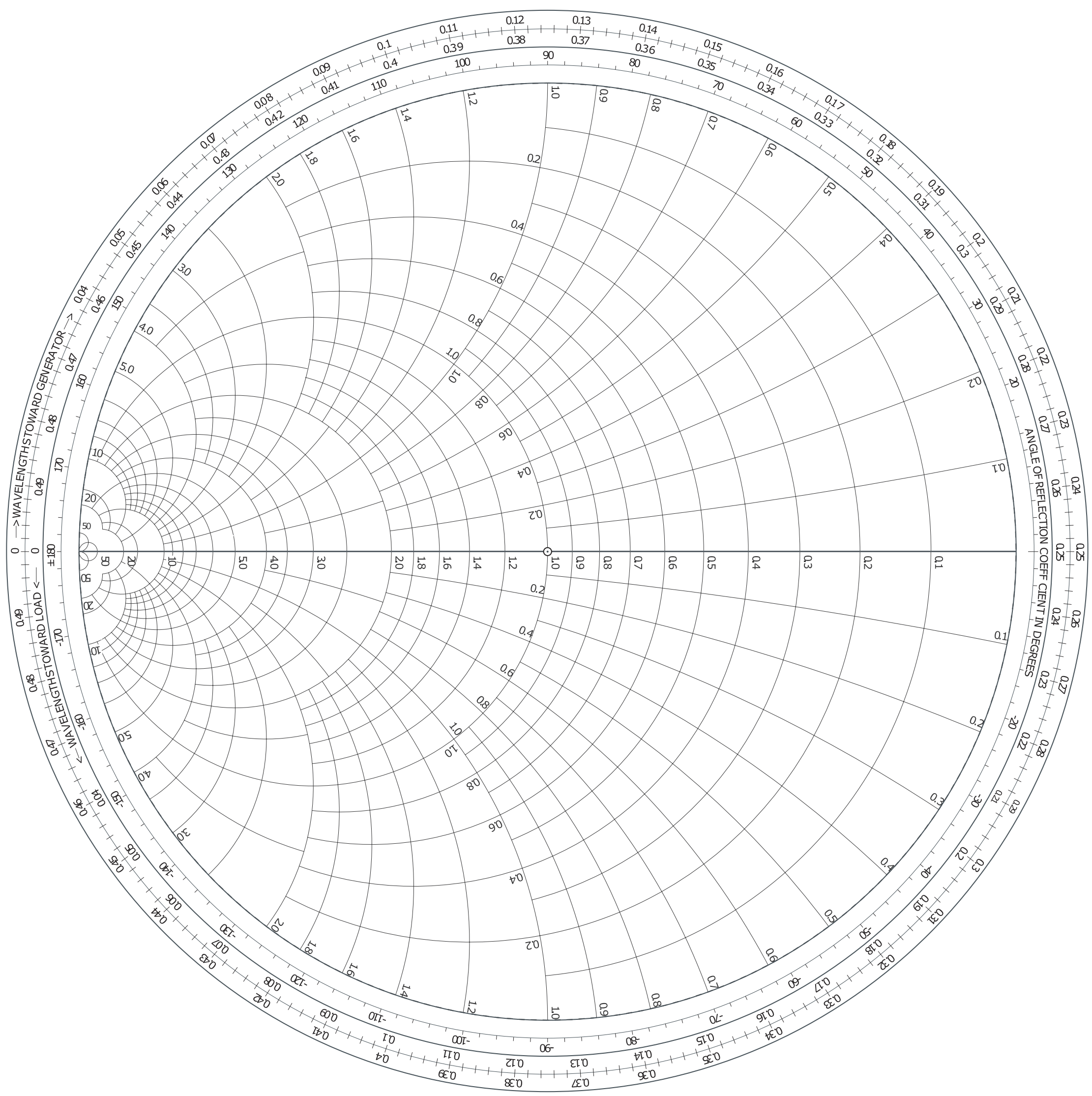
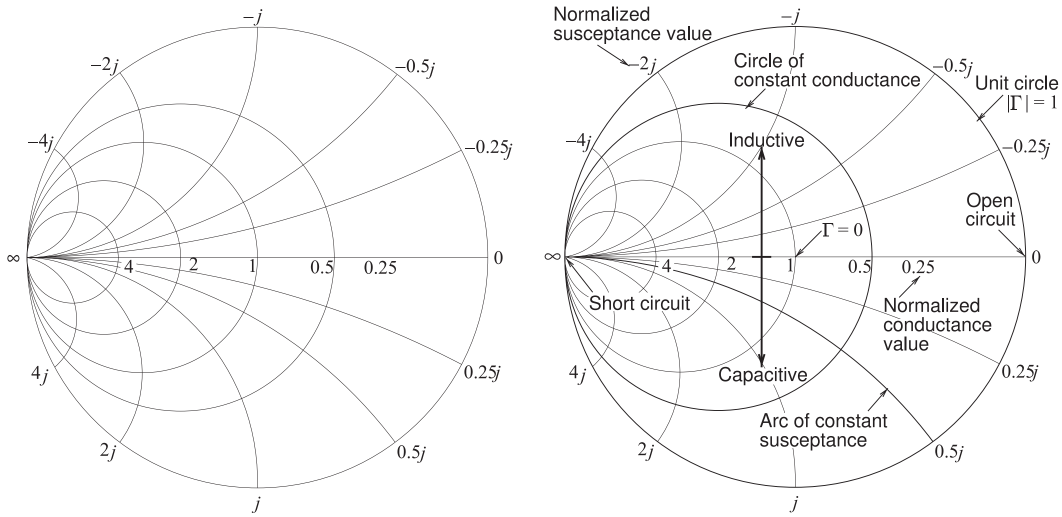


Figure 8-8: Admittance Smith chart.



2 (a) Admittance Smith chart (b) Annotated admittance Smith chart

Figure 8-9: Normalized admittance Smith chart.

retains the signs making it easier to follow some of the discussions and examples. It is important that the user readily understand the annotations on the less dense form of the Smith chart, see Figure 8-9(b).

8.3.3 Combined Smith Chart

The combination of the reflection/transmission polar plots, nomographs, and the impedance and admittance Smith chart leads to the combined Smith chart (see Figure 8-10). This color Smith chart is the preferred version for use in design and the separate impedance and admittance versions of the Smith chart are rarely used. The combined Smith charts is rich with information and care is required to identify the lines that correspond to admittances (specifically lines of constant normalized conductance and constant normalized susceptance), and the lines that correspond to impedances (constant normalized resistances and constant normalized reactances). The signs of the reactances and susceptible are missing and left to the user to add them depending on whether a reflection coefficient point is capacitive (in the lower half of the Smith chart and hence susceptances are positive and reactances are negative) or whether a point is inductive (in the upper half of the Smith chart and hence susceptances are negative and reactances are positive). A less dense version of the combined Smith chart, with the addition of signs, is shown in Figure 8-11(a).

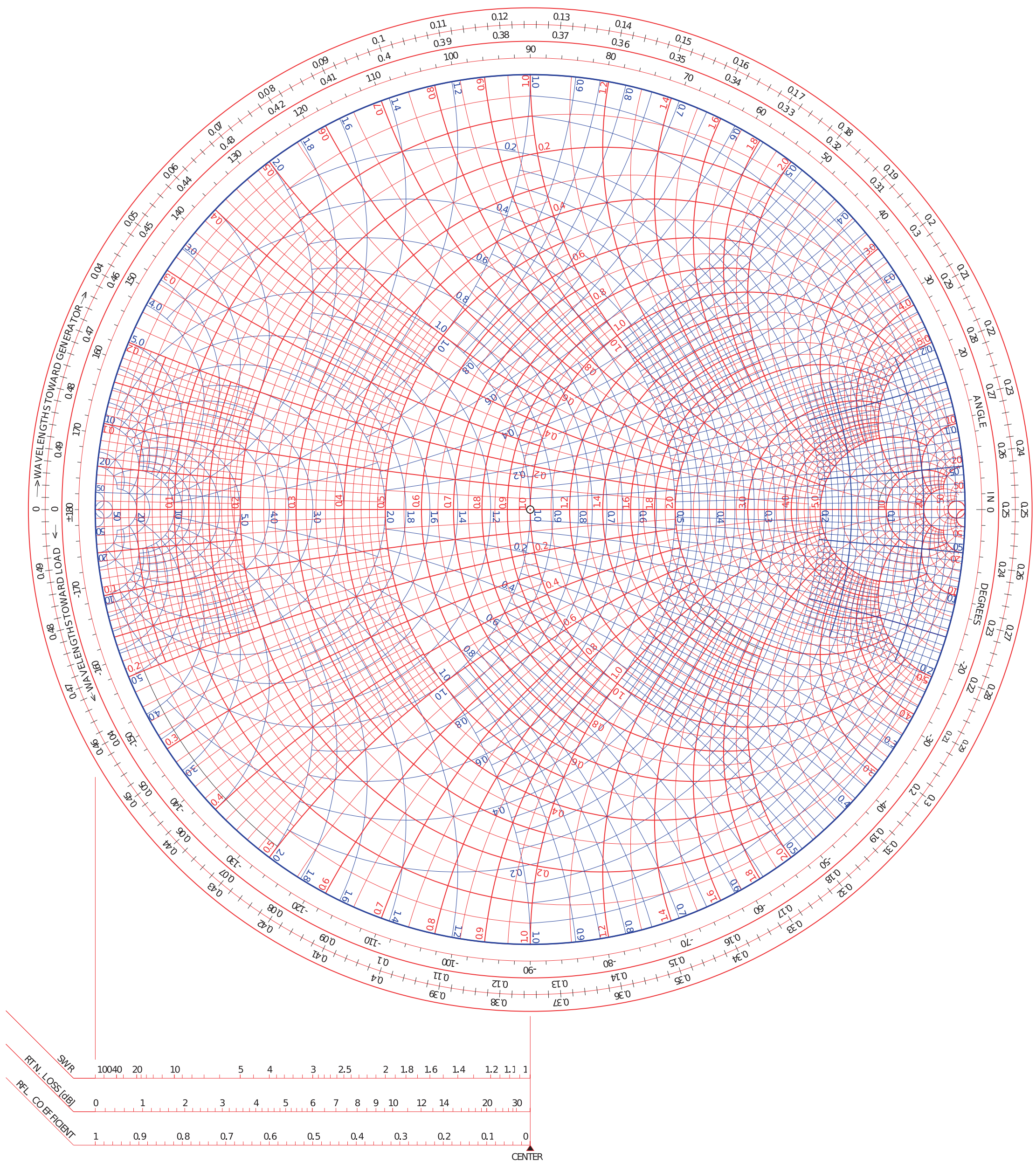
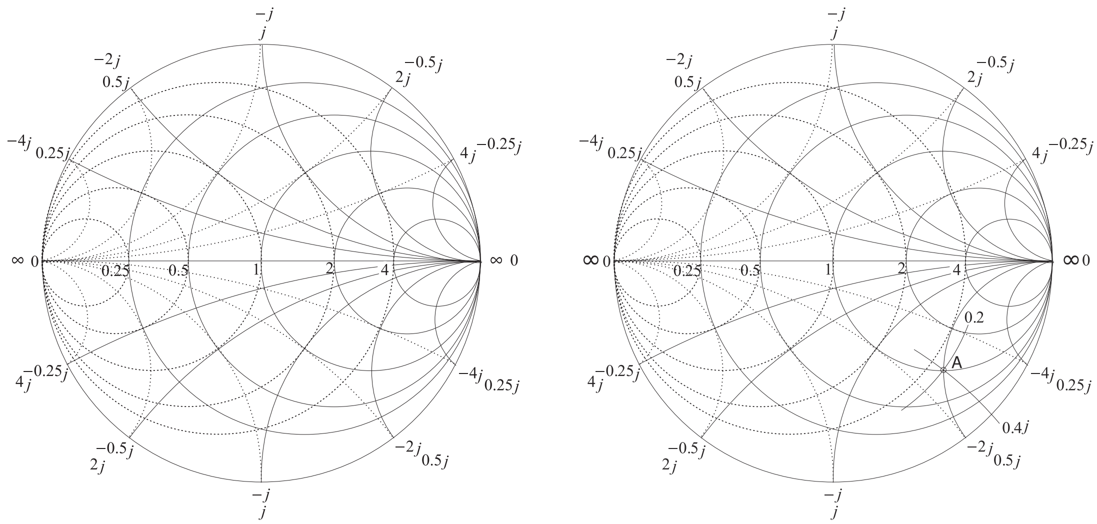


Figure 8-10: Normalized combined Smith chart combining impedance and admittance Smith charts.



(a) Combined impedance and admittance Smith chart

(b) Smith chart used in Example 8.3

Figure 8-11: Combined impedance and admittance Smith chart.

EXAMPLE 8.3 Impedance to Admittance Conversion

Use a Smith chart to convert the impedance $z = 1 - 2\jmath$ to an admittance.

Solution:

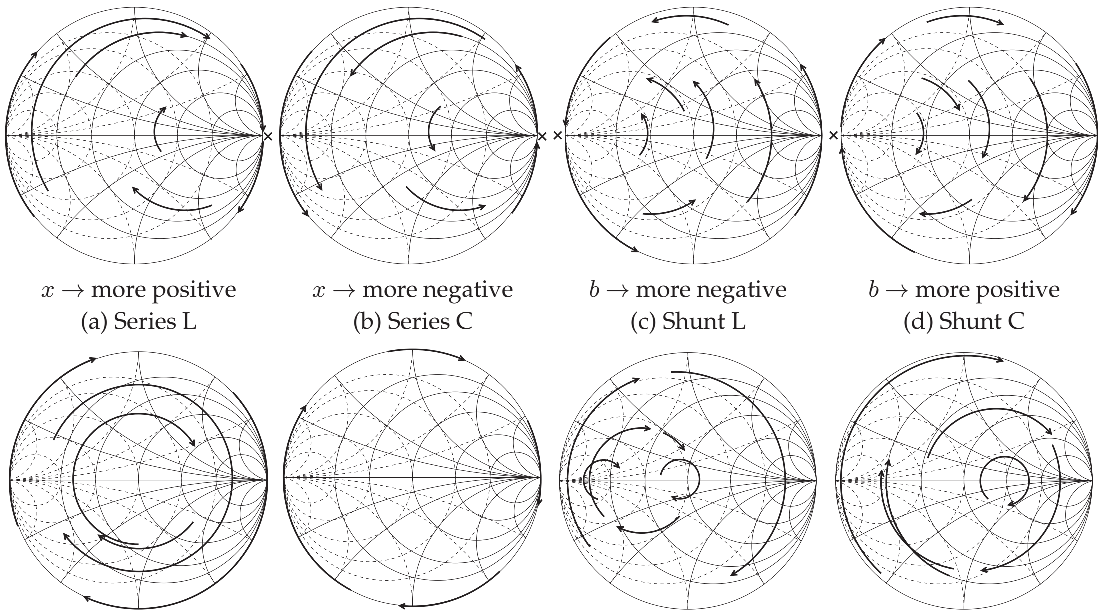
The impedance $z = 1 - 2\jmath$ is plotted as Point A in Figure 8-11(b). To read the admittance from the chart, the lines of constant conductance and constant susceptance must be interpolated from the arcs and circles provided. The interpolations are shown in the figure, indicating a conductance of 0.2 and a susceptance of $0.4\jmath$. Thus

$$y = 0.2 + 0.4\jmath.$$

(This agrees with the calculation: $y = 1/z = 1/(1 - 2\jmath) = 0.2 + 0.4\jmath$.)

Adding Reactance and Susceptance

A good amount of microwave design, such as designing a matching network for maximum power transfer, involves beginning with a load impedance plotted on a Smith chart and inserting series and shunt reactances, and transmission lines, to transform the impedance to another value. The preferred view of the design process is that of a reflection coefficient that gradually evolves from one value to another. That is, in the case of a series reactance, the effect is that of a reflection coefficient gradually changing as the reactances increase from zero to its actual value. The path traced out by the gradually evolving reflection coefficient value is called a locus. The loci of common circuit elements added to various loads are shown in Figure 8-12. For each locus the load is at the start of the arrow with the value of the element increasing from zero to its actual final value at the arrow head.



$x \rightarrow$ more positive (a) Series L

$x \rightarrow$ more negative (b) Series C

$b \rightarrow$ more negative (c) Shunt L

$b \rightarrow$ more positive (d) Shunt C

ℓ increasing (e)Transmission line $Z_0 = Z_{\text{REF}}$

ℓ increasing (f) Line with reactive load, Z_0 has any value

ℓ increasing (g) Transmission line $Z_0 < Z_{\text{REF}}$

ℓ increasing (h) Transmission line $Z_0 > Z_{\text{REF}}$

Figure 8-12: Reflection coefficient loci of a load augmented by elements as indicated. The original load is at the start of the arrowed arc. In (a) and (b) each locus is with respect to increasing $|x|$ (normalized reactance magnitude), in (c) and (d) the locus is with respect to increasing $|b|$ (normalized susceptance magnitude), and in (e)–(h) with respect to increasing length ℓ. In (e)–(h) the loci are parts of circles centered on the $x = 0$ line. x indicates that the locus cannot cross infinity (open circuit for (a) and (b), short circuit for (c) and (d)).

8.3.4 Smith Chart Manipulations

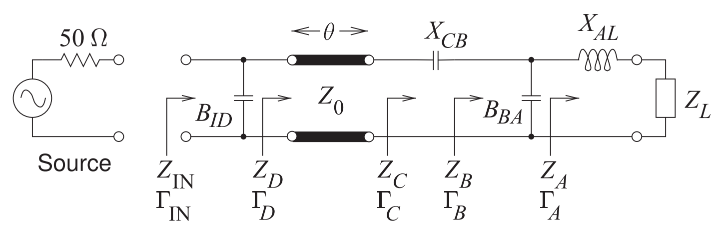
Microwave design often proceeds by taking a known load and transforming it into another impedance, perhaps for maximum power transfer. Smith charts are used to show these manipulations. In design the manipulations required are not known up front and the Smith chart enables identification of those required. Except for particular situations, lossless elements such as reactances and transmission lines are used. The few situations where resistances are introduced include introducing stability in oscillators and amplifiers, and deliberately reduce signals levels. Most of the time introducing resistances unnecessarily increases noise and reduces signal levels thus reducing critical signal-to-noise ratios.

The circuit in Figure 8-13 will be used to illustrate the manipulations on a Smith chart. The most common view is to consider that the Smith chart is a plot of the reflection coefficient at various stages in the circuit, i.e., Γ_A, Γ_B, Γ_C, Γ_D, and Γ_{IN}. Additionally the load, Z_L, is plotted and reactances, susceptances, or transmission lines transform the reflection coefficient from one stage to the next. Yet another concept is the idea that the effect of an element is regarded as gradually increasing from a negligible value up to the final actual value and in so doing tracing out a locus which ends in an arrow head. This is the approach nearly all RF and microwave engineers use. The manipulations corresponding to the circuit are illustrated in Figure 8-14. The individual steps are identified in Figure 8-15. The first few steps are confined to the top half of the Smith chart which is repeated on a larger scale in Figure 8-16 for the first three steps.

Step 1 L, a, b, c, d, e, f
The first step is to plot the load Z_L on a Smith chart and this is the Point L in Figures 8-14–8-16. The reference impedance Z_{REF} is chosen to be $50\ \Omega$, the same as the characteristic impedance of the transmission line in the circuit. This is a common choice because then the locus of reflection coefficient variation introduced by the line will be a circle centered at the origin of the Smith chart. To plot L derive the normalized load impedance $z_L = Z_L/Z_{\text{REF}} = 0.3 + \jmath 0.6$ (future numerical values are given in Figure 8-13).

To plot z_L the normalized resistance circle for 0.3 and the normalized reactance arc for $+0.3$ must be located. The resistance circle is identified from the scale on the horizontal axis of the Smith chart, see the circled value labeled 'a'. Locating the $+0.3$ reactance arc is not as direct as the signs of reactance are missing on the Smith chart. Referring back to Figure 8-6 it is noted that an inductive impedance is in the top half of the Smith chart and so positive reactances are in the top half. Thus the $+0.3$ reactance arc is in the top half of the Smith chart and the correct arc is identified by 'c'. From the curves identified by 'a' and 'c' the arcs 'b' and 'd' are drawn with the impedance z_L, i.e. point L, at the intersection of the arcs.

It is instructive to determine Γ_L, the reflection coefficient at L. On the Smith chart the reflection coefficient vector Γ_L is drawn from the origin to the point L. Γ_L is evaluated by separately determining its magnitude and angle. To determine the magnitude measure the length of the $\Gamma_L|$ vector either using a ruler, a compass, or marking the edge of a piece of paper. Then this length can be compared against the reflection coefficient scale shown at the bottom of Figure 8-14 yielding $|\Gamma_L| = 0.644$. Alternatively the distance from the origin to the unit circle can be measured using a ruler and the ratio of the $|\Gamma_L|$ measurement and of the unit circle measurement taken as the value of $|\Gamma_L|$. The angle of Γ_L is read by extending the Γ_L vector out to the inner most circular scale on the Smith chart. This extension is labeled as e. The angle is



Circuit elements	Normalized	Derived values	
$Z_L = 15 + \jmath 30\ \Omega$	$z_L = 0.3 + \jmath 0.6$	$\Gamma_L = 0.644\angle 115°$	$z_A = 0.300 + \jmath 1.200$
$X_{AL} = 30\ \Omega$	$x_{AL} = 0.600$	$\Gamma_A = 0.785\angle 115°$	$y_A = 0.196 - \jmath 0.784$
$B_{BA} = 4.6$ mS	$b_{BA} = 0.230$	$\Gamma_B = 0.741\angle 59°$	$y_B = 0.196 - \jmath 0.554$
$X_{CB} = -180\ \Omega$	$x_{CB} = -3.600$	$\Gamma_C = 0.805\angle -50°$	$z_B = 0.567 + \jmath 1.603$
$Z_0 = 50\ \Omega$	$Z_0 = 1$	$\Gamma_D = 0.805\angle 144°$	$z_C = 0.567 - \jmath 1.997$
$\Theta = 83°$	$\Theta = 83°$	$\Gamma_{\text{IN}} = 0$	$y_D = 0.998 - \jmath 2.71$
$B_{ID} = 54.2$ mS	$b_{ID} = 2.71$		$z_{\text{IN}} = 1$, $y_{\text{IN}} = 1$

Figure 8-13: Circuit used in illustrating Smith chart manipulations. $Z_{\text{REF}} = 50\ \Omega$, $Y_{\text{REF}} = 1/Z_{\text{REF}} = 20$ mS, $z = Z/Z_{\text{REF}}$, $y = Y/Y_{\text{REF}}$, $Z_{\text{IN}} = Z_{\text{REF}} = 50\ \Omega$. In the circuit B indicates susceptance and X indicates reactance.

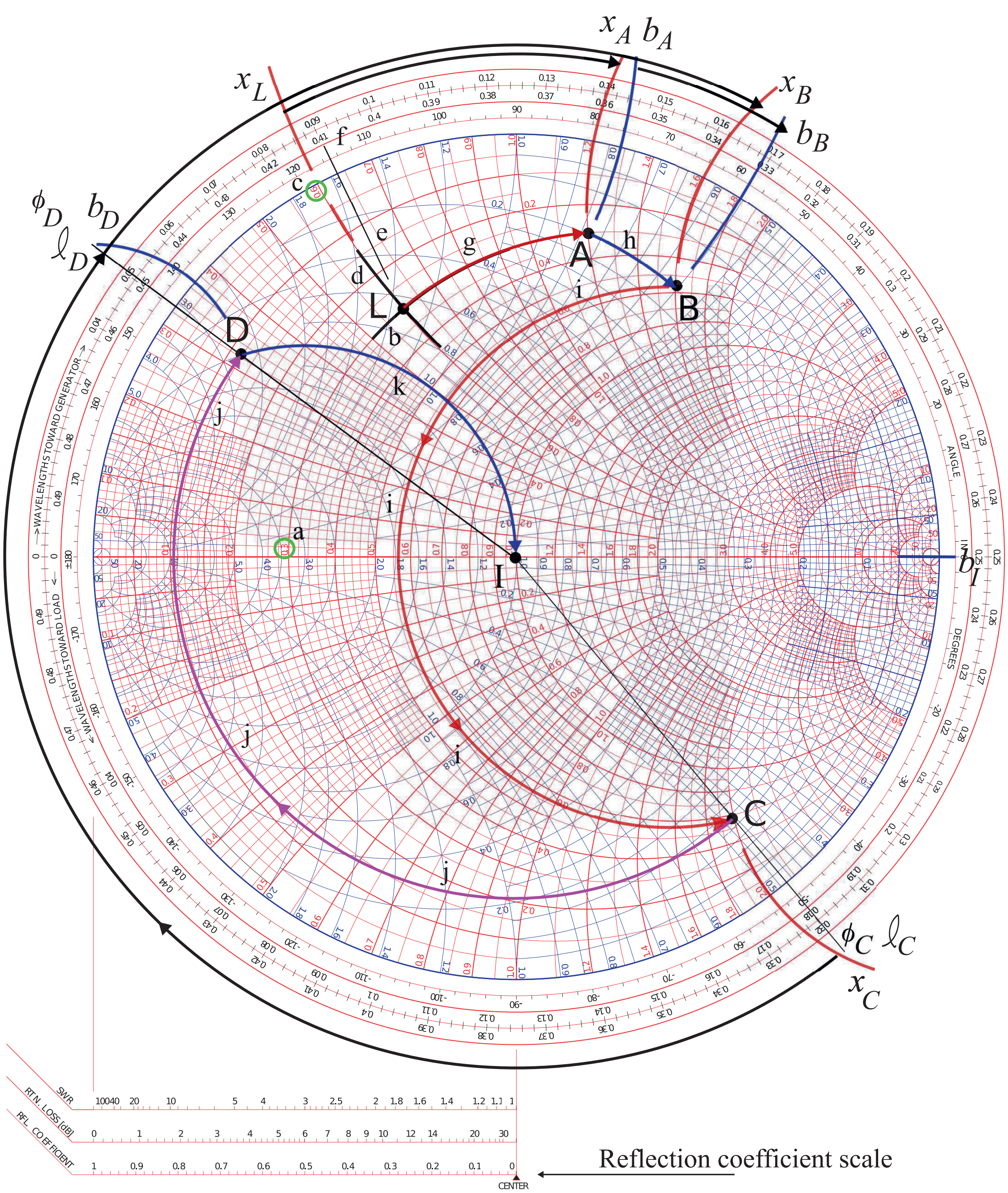
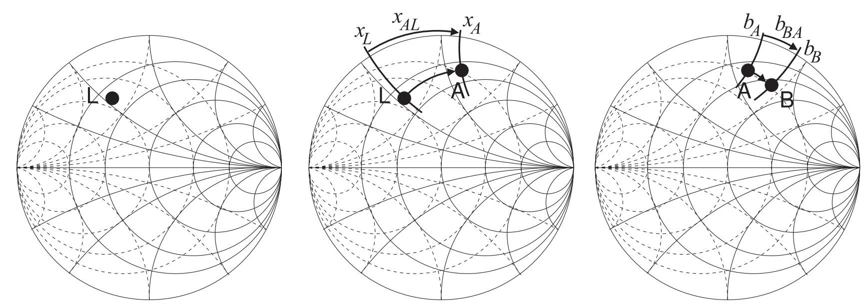
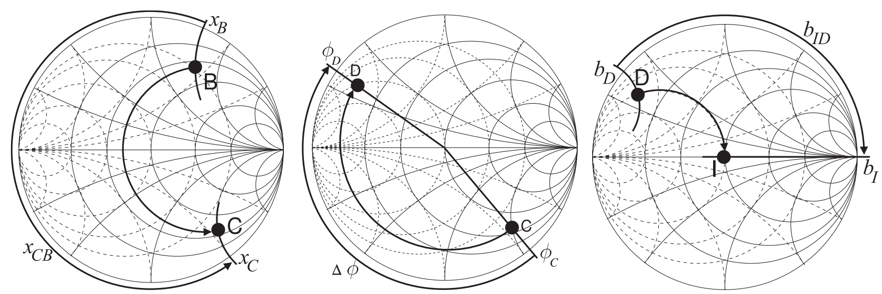


Figure 8-14: Smith chart manipulations corresponding to the circuit in Figure 8-13 with circuit elements added one at a time beginning with the load impedance at Point L.



Step 1	Step 2	Step 3
Locate Z_L	$x_A = x_L + x_{AL}$	$b_B = b_A + b_{BA}$
$z_L = 0.3 + \jmath 0.6$	$z_A = z_L + \jmath x_{AL}$	$y_B = y_A + \jmath b_{BA}$
	$x_{AL} = 0.600$	$b_{BA} = 0.230$
	$x_L = 0.600, x_A = 1.200$	$b_A = -0.784, b_B = -0.554$



Step 4	Step 5	Step 6
$x_C = x_B + x_{CB}$	$\Gamma_D = \Gamma_C \angle \Delta\phi, \Delta\phi = -2\Theta$	$b_I = b_D + b_{ID}$
$z_C = z_B + x_{CB}$	$\Theta = 46.6^\circ, \Delta\phi = 93.2^\circ$	$z_I = z_D + \jmath b_{ID} = 1$
$x_{CB} = -3.600$	$\phi_C = -50^\circ, \phi_D = -144^\circ$	$b_{ID} = 2.717$
$x_B = 1.603, x_C = -1.997$		$b_D = -2.717, b_I = 0$

Figure 8-15: Steps in the Smith chart manipulations shown in Figures 8-14 and 8-16.

read at point 'f' as 115°. Thus $\Gamma_L = 0.644\angle 115^\circ$.

Step 2 Add the series reactance X_{AL}, path L to A, g
In step 2 an inductor with reactance X_{AL} is in series with Z_L and the reflection coefficient transitions from point L to point A. Now $x_L = \Im\{z_L\} = 0.6$. To this $x_{AL} = 0.6$ is added so that the normalized reactance at A is $x_A = x_L + x_{AL} = 0.6 + 0.6 = 1.2$. The locus of this operation is the arc, 'g', extending from L with gradually increasing series reactance until the full value x_{AL}, at the arrowhead of the locus, is obtained. The procedure is to identify the arc of $x_A = +1.2$ reactance and then follow the circle of constant resistance (since the resistance is not changing) up to this arc. This operation traces out the locus 'g'.

Step 3 Add the shunt susceptance B_{BA}, path A to B, h
In step 3 a capacitor with susceptance X_{BA} is in shunt with Z_A and the reflection coefficient transitions from point A to point B. The locus of this operation follows a circle of constant conductance with only they susceptance changing. The final value must be calculated as $b_B = b_A + b_{BA}$. The susceptance b_B is read from the graph as -0.784. This value is read by following the arc of constant susceptance out to the unit circle where a value of 0.784 is read. Note that the arc must be interpolated and that the user must realize that the susceptance is negative in the top half of the

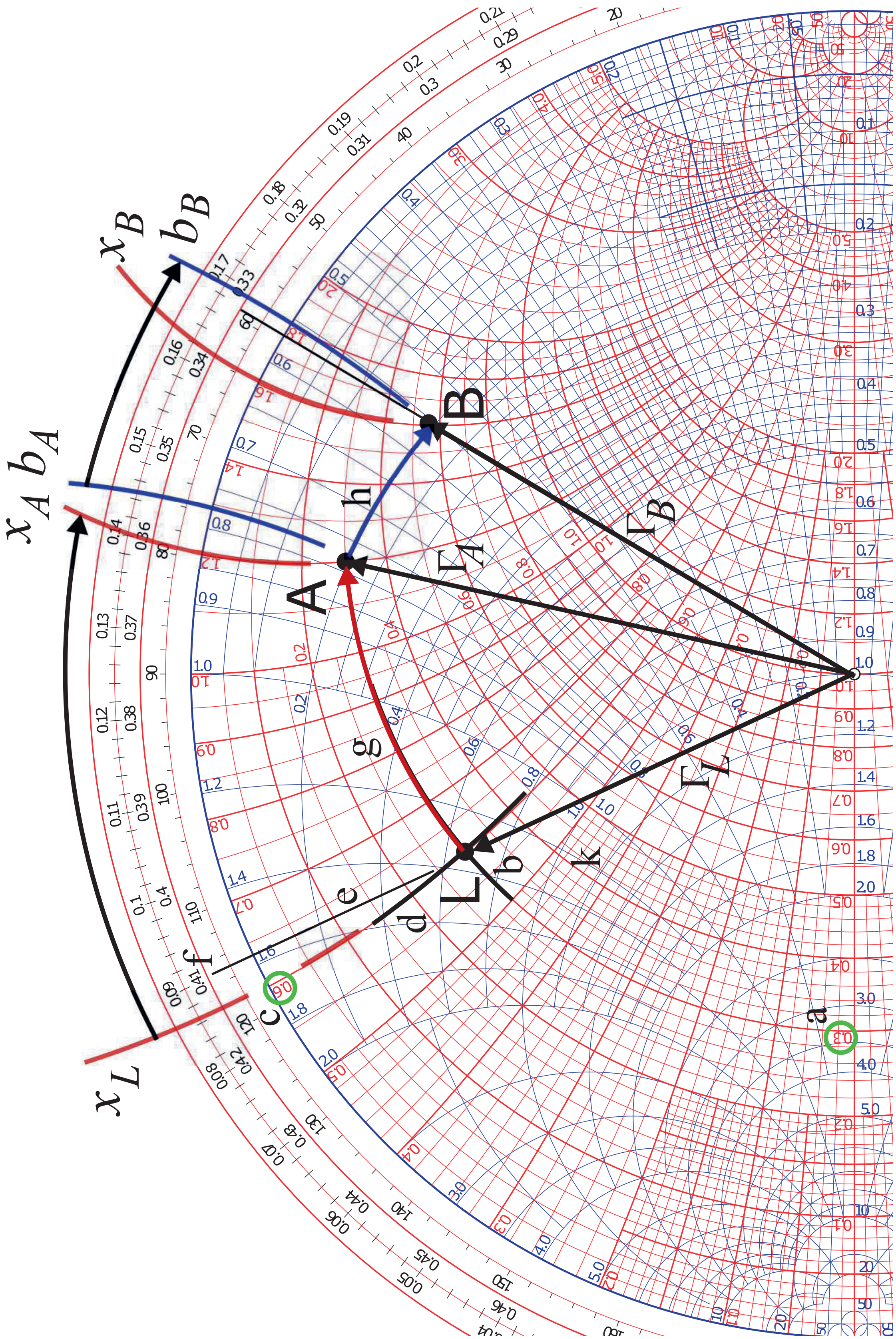


Figure 8-16: The first three steps in the Smith chart manipulations in Figure 8-14.

Smith chart so the susceptance must be negative even though a minus sign is not shown on the Smith chart. Thus the correct reading for x_A is -0.784. Now $b_{BA} = +0.230$ so the locus, 'h', of the transition follows the circle of constant conductance ending at the (interpolated arc) with susceptance $b_B = b_a + b_{BA} = -0.784 + 0.230 = -0.554$.

Step 4 Add the series reactance X_{CB}, path B to C, i
In step 4 a capacitor with reactance X_{CB} is in series with Z_B and the reflection coefficient transitions from point B to point C. Now x_B is read from the graph as $+1.603$. To this add $x_{CB} = -3.600$ so that the normalized reactance at C is $x_C = x_B + x_{CB} = 1.603 + (-3.600) = -1.997$. The locus from B to C, path 'i' begins at B and follows a circle of constant resistance up to the arc with normalized reactance $x_C = -1.997$. This reactance arc is in the bottom half of the Smith chart as reactances are negative there even though the signs are missing on the labels of the reactance arcs in the bottom half of the chart. The locus of this operation is the arc, 'g', from L with gradually increasing series reactance until the full value z_{CB}, at the arrowhead of the locus, is obtained. The procedure is to identify the arc of $x_C = -1.997$ reactance and then follow the circle of constant resistance (since the resistance is not changing) up to this arc. This operation traces out the locus 'i'.

Step 5 Insert the transmission line, path C to D, j
Step 5 illustrates a different type of manipulation as now there is a transmission line and the reflection coefficient transitions from Γ_C, i.e. point C to the input reflection coefficient of the line Γ_D. The locus of this transition must be clockwise, i.e. having increasingly negative angle (as discussed in Section 3.3.3 of [5]). The electrical length of the transmission line is $\Theta = 83°$ and the reflection coefficient changes by the negative of twice this amount, $\phi_{DC} = -2\Theta = -166°$. The locus is drawn in Figure 8-14 with the transmission line gradually increasing in length tracing out a circle which, since $Z_0 = Z_{\rm REF}$, is centered at the origin of the Smith cart. The procedure is to find the scale value of the angle at point C which is read by drawing a line from the origin through C intersecting the reflection coefficient angle scale (the innermost circular scale) yielding $\phi_C = -50.4°$ and so $\phi_D = \phi_C + \phi_{DC} = -50.4° - 166° + 360°) = +143.6°$. The locus from C to D is drawn by first drawing a line from the origin to the $\phi_D = -143.6°$ point on the angle scale. Then point D will be at the intersection of this line and a circle of constant radius drawn through C. The locus is shown as path 'j' in Figure 8-14.

Step 6 Add the shunt susceptance B_{ID}, path D to I, k
The final step is to add the shunt capacitor with susceptance B_{ID} to Z_D. Following the previous procedure b_D is read as -2.71 to which $b_{ID} = 2.71$ is added so that $b_I = 0$ which is just the horizontal line across the middle of the Smith chart. This line corresponds to zero reactance and zero susceptance. The locus, path 'k', extends from D to I following the circle of constant conductance. The final result is that $z_{\rm IN} = 1.00$ and $Z_{\rm IN} = z_{\rm IN} \cdot Z_0 = 50\ \Omega$.

Summary

The Smith chart manipulations considered here modified the reflection coefficient of a load by adding shunt and series reactive elements. The final result of the circuit manipulations is that the input impedance is $Z_{\rm IN} =$

$50\ \Omega$. If the source has a Thevenin source impedance of $50\ \Omega$ then there is maximum power transfer to the circuit. Since all of the elements in the circuit manipulations are lossless this means that there is maximum power transfer from the source to the load Z_L. Of course fewer circuit manipulations could have been used to achieve this result. Note that manipulations resulting from adding series and/or shunt resistances were not considered. It is rare that this would be desired as that simply means that power is absorbed in the resistance and additional noise is added to the circuit.

EXAMPLE 8.4 Reflection Coefficient of a Shorted Microstrip Line on a Smith Chart

AWR Design Environment Project File: RFDesign_Shorted_Microstrip_Line_Smith.emp

A shorted microstrip line on an alumina substrate is shown in Figure 8-17. The line has low loss and so Γ is always close to 1. The microstrip line was designed to be 50 Ω and Γ of the low loss line is plotted on a 50 Ω Smith chart in Figure 8-18(a). Plotting Γ on a 50 Ω Smith chart indicates that the reflection coefficient was calculated with respect to 50 Ω. At a very low frequency, 0.1 GHz is the lowest frequency here, the locus of the reflection coefficient is very close to $\Gamma = -1$, identifying a short circuit. As the electrical length increases, in this case the frequency increases as the physical length of the line is fixed, the locus of the reflection coefficient moves clockwise, hugging the unity reflection coefficient circle. At the highest frequency, 30 GHz, the reflection coefficient is less than 1 and the locus starts moving in from the unity circle. It is interesting to see what happens with a high-loss line, and this is achieved by changing the loss tangent, $\tan\delta$, of the substrate from 0.001 to 0.1. The locus of the reflection coefficient of the high-loss line is shown in Figure 8-18(b). Loss increases as the electrical length of the line increases and the locus of the reflection coefficient traces out a clockwise inward spiral.

8.3.5 An Alternative Admittance Chart

Design often requires switching between admittance and impedance. So it is convenient to use the colored combined Smith chart shown in Figure 8-10. Monochrome charts were once the only ones available and an impedance Smith chart was used for admittance-based calculations provided that reflection coefficients were rotated by $180°$. This form of the Smith chart is not used now and is very confusing.

8.3.6 Summary

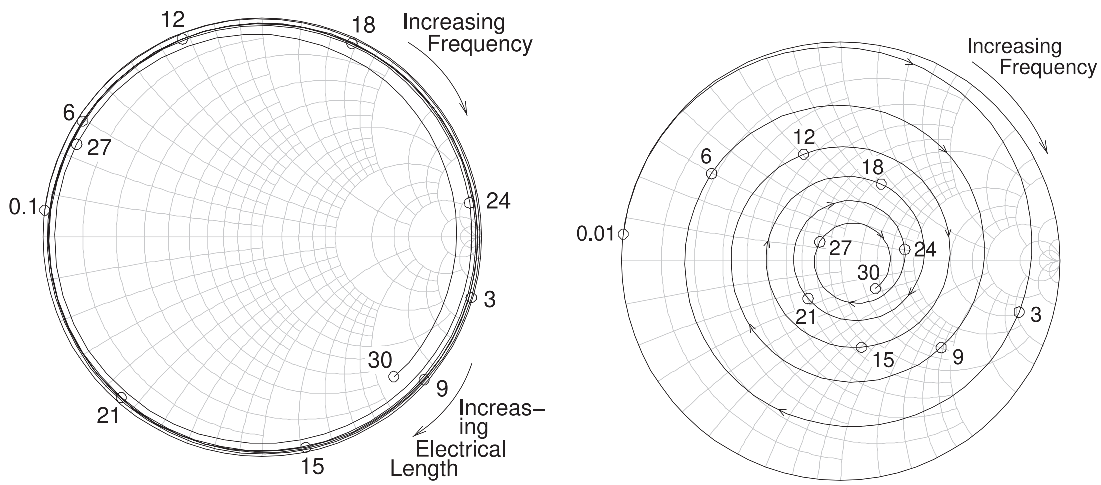
The Smith chart is the most powerful of tools used in RF and Microwave Design. Design using Smith charts will be considered in other chapters but at this stage the reader should be totally conversant with the techniques described in this section.

8.4 Summary

Graphical representations of power flow enable RF and microwave engineers to quickly ascertain circuit performance and arrive at qualitative design decisions. Humans are very good at processing graphical information and seeing patterns, anomalies, and the path from one point to another.

Figure 8-17: A 50 Ω shorted gold microstrip line with width w = 500 μm, length ℓ = 1 cm on a 600 μm thick alumina substrate with relative permittivity ε_r = 9.8 and loss tangent $\tan\delta = 0.001$.

The Smith chart is a richly annotated polar plot for representing reflection and transmission coefficients, and more generally, scattering parameters. The Smith chart representation of scattering parameter data aligns very well with the intuitive understanding of an RF designer. The experienced RF designer is intrinsically familiar with the Smith chart and prefers that circuit performance during design be represented on one. Representing something as simple as an extension of a line length to a two-port is quite complex if described using network parameters other than scattering parameters. However, with scattering parameters this extension results in a change of the angle of a scattering parameter, or on a Smith chart an arc. Scattering parameters relate directly to power flow. So from a Smith chart an experienced designer can ascertain the effect of circuit design on power flow, which then relates to signal-to-noise ratio and power gain.



(a) $\tan \delta = 0.001$ (b) $\tan \delta = 0.1$

Figure 8-18: Smith chart plot of the reflection coefficient of the shorted 1 cm-long microstrip line in Figure 8-17: (a) a low-loss substrate with $\tan \delta = 0.001$; and (b) a high-loss substrate with $\tan \delta = 0.1$. Frequencies are marked in gigahertz.

8.5 References

[1] P. Smith, *Electronic Applications of the Smith Chart: In Waveguide, Circuit, and Component Analysis*. SciTech Publishing, 2000.

[2] ——, "Transmission line calculator," *Electronics*, 1939.

[3] ——, "An improved transmission-line calculator," *Electronics*, 1944.

[4] ——, "R.F. transmission line nomographs," *Electronics*, vol. 22, pp. 112–117, Feb. 1949.

[5] M. Steer, *Microwave and RF Design, Transmission Lines*, 3rd ed. North Carolina State University, 2019.

8.6 Exercises

1. In the distribution of signals on a cable TV system a 75 Ω coaxial cable is used, with a loss of 0.1 dB/m at 1 GHz. If a subscriber disconnects a television set from the cable so that the load impedance looks like an open circuit, estimate the input impedance of the cable at 1 GHz and 1 km from the subscriber. An answer within 1% is required. Estimate the error of your answer. Indicate the input impedance on a Smith chart, drawing the locus of the input impedance as the

line is increased in length from nothing to 1 km. (Consider that the dielectric filling the line has $\epsilon_r = 1$.)

2. A resistive load has a reflection coefficient with a magnitude of 0.7. If a transmission line is placed in series with the load, on a polar plot sketch the locus of the input reflection coefficient looking into the input of the terminated line as the line increases in electrical length from zero to $\lambda/2$. By reading the Smith chart, determine the normalized input impedance of the line when it has an electrical length of $\pi/2$.
3. A complex load has a reflection coefficient of $0.5+\jmath0.5$. If a transmission line is placed in series with the load, on a polar plot sketch the locus of the input reflection coefficient looking into the input of the terminated line as the line increases in electrical length from zero to $\pi/2$.
4. A resistive load has a reflection coefficient of -0.5. If a transmission line is placed in series with the load, on a polar plot sketch the locus of the input reflection coefficient looking into the input of the terminated line as the line increases in electrical length from 0 to $3\lambda/8$.
5. S_{21} of a two-port is 0.5. If a transmission line is placed in series with Port 1, on a polar plot sketch the locus of S_{21} of the augmented two-port as the electrical length of the line increases from zero to $\lambda/2$.
6. A load has an impedance $Z = 115 - \jmath20\ \Omega$.
   (a) What is the reflection coefficient, Γ_L, of the load in a 50 Ω reference system?
   (b) Plot the reflection coefficient on a polar plot of reflection coefficient.
   (c) If a one-eighth wavelength long lossless 50 Ω transmission line is connected to the load, what is the reflection coefficient, Γ_{in}, looking into the transmission line? (Again, use the 50 Ω reference system.) Plot Γ_{in} on the polar reflection coefficient plot of part (b). Clearly identify Γ_{in} and Γ_L on the plot.
   (d) On the Smith chart, identify the locus of Γ_{in} as the length of the transmission line increases from 0 to $\lambda/8$ long. That is, on the Smith chart, plot Γ_{in} as the length of the transmission line varies.
7. A load has a reflection coefficient with a magnitude of 0.5. If a transmission line is placed in series with the load, on a polar plot sketch the locus of the input reflection coefficient looking into the input of the terminated line as the line increases in electrical length from zero to $\lambda/2$. What is the normalized input impedance of the line when it has an electrical length of $\lambda/2$?
8. A resistive load has a reflection coefficient with a magnitude of 0.7. If a transmission line is placed in series with the load, on a polar plot sketch the locus of the input reflection coefficient looking into the input of the terminated line as the line increases in electrical length from zero to $\lambda/4$. By reading the Smith chart, determine the normalized input impedance of the line when it has an electrical length of $\lambda/4$.

Problems 9–15 refer to the normalized Smith chart in Figure 8-20 with reference impedance $Z_{\text{REF}} = 50\ \Omega$ and reflection coefficient Γ, voltage reflection coefficient ${}^V\Gamma$, current reflection coefficient ${}^I\Gamma$, and normalized impedance $z = r + \jmath x$ and admittance $y = g + \jmath b$. Γ should be given in magnitude-angle format.

9. (a) What is ${}^V\Gamma$ at A? (b) What is ${}^I\Gamma$ at A? (c) What is r at B? (d) What is z at C? (e) What is y at D? (f) What is $|\Gamma|$ at F? (g) What is b at I? (h) What is Γ at P? (i) What is Γ at D? (j) What is Γ at T?
10. (a) What is z at A? (b) What is y at I? (c) What is z at E? (d) What is y at Z? (e) What is y at H? (f) What is $|\Gamma|$ at W? (g) What is b at F? (h) What is x at K? (i) What is Γ at K? (j) What is Γ at R?
11. (a) What is y at A? (b) What is y at I? (c) What is z at G? (d) What is y at O? (e) What is y at V? (f) What is Γ at B? (g) What is x at C? (h) What is Γ at G? (i) What is z at L, label this z_L? (j) Use the Smith chart to find z_{in} of a 50 Ω $\lambda/8$-long line with load z_L?
12. (a) What is ${}^V\Gamma$ at M? (b) What is ${}^I\Gamma$ at M? (c) What is r at W? (d) What is z at Y? (e) What is y at V? (f) What is $|\Gamma|$ at B? (g) What is b at K? (h) What is Γ at V? (i) What is Γ at P? (j) What is Γ at N?
13. (a) What is z at M? (b) What is y at K? (c) What is z at S? (d) What is y at R? (e) What is g at B? (f) What is $|\Gamma|$ at F? (g) What is b at B? (h) What is x at I? (i) What is Γ at I? (j) What is Γ at Q?
14. (a) What is g at M? (b) What is r at K? (c) What is y at S? (d) What is z at R? (e) What is y at Z? (f) What is Γ at W? (g) What is x at Y? (h) What is Γ at T? (i) What is z at O, label this z_O? (j) Using the Smith chart find z_{in} of a $3\lambda/8$-long 50 Ω line with load z_O?
15. (a) What is g at P? (b) What is y at J? (c) What is Γ at L? (d) What is z at N? (e) What is y at S? (f) What is $|\Gamma|$ at U? (g) What is ${}^V\Gamma$ at X? (h) What is ${}^I\Gamma$ at X? (i) What is g at B? (j) What is x at I?

16. Design a short-circuited stub to realize a normalized susceptance of 2.15. Show the locus of the stub as its length increases from zero to its final length. What is the minimum length of the stub in terms of wavelengths?
17. Design a short-circuited stub to realize a normalized susceptance of -0.56. Show the locus of the stub as its length increases from zero to its final length. What is the minimum length of the stub in terms of wavelengths?
18. Design a short-circuited stub to realize a normalized susceptance of -2.2. Show the locus of the stub as its length increases from zero to its final length. What is the minimum length of the stub in terms of wavelengths?
19. A 75 Ω transmission line is terminated in a load with a reflection coefficient, Γ, normalized to 75 Ω, of $0.5\angle 45^\circ$. If Γ at the input of the line is $0.5\angle{-135^\circ}$, what is the minimum electrical length of the line in degrees.
20. An open-circuited 75 Ω transmission line has an input reflection coefficient with an angle of 40° what is the electrical length of the line in degrees? If there is more than one answer provide at least two correct answers.
21. Repeat Example 8.1 using the full impedance Smith chart of Figure 8-5.
22. Plot the normalized impedances $z_A = 0.5 + \jmath 0.5$, $z_A = 0.5 + \jmath 0.5$, and $z_B = 0.185 - \jmath 1.05$ on the full impedance Smith chart of Figure 8-5. [Parallels Example 8.1]
23. A 50 Ω lossy transmission line is shorted at one end. The line loss is 2 dB per wavelength. Note that since the line is lossy the characteristic impedance will be complex, but close to 50 Ω, since it is only slightly lossy. There is no way to calculate the actual characteristic impedance with the information provided. That is, problems must be solved with small inconsistencies. Microwave engineers do the best they can in design and always rely on measurements to calibrate results.
    (a) What is the reflection coefficient at the load (in this case the short)?
    (b) Consider the input reflection coefficient, Γ_{in}, at a distance ℓ from the load. Determine Γ_{in} for ℓ going from 0.1λ to λ in steps of 0.1λ.
    (c) On a Smith chart plot the locus of Γ_{in} from $\ell = 0$ to λ.
    (d) Calculate the input impedance, Z_{in}, when the line is $3\lambda/8$ long using the telegrapher's equation.
    (e) Repeat part (d) using a Smith chart.
24. In Figure 8-19 the results of several different experiments are plotted on a Smith chart. Each experiment measured the input reflection coefficient from a low frequency (denoted by a circle) to a high frequency (denoted by a square) of a one-port. Determine the load that was measured. The loads that were measured are one of those shown below.

Load	Description
i	An inductor
ii	A capacitor
iii	A reactive load at the end of a transmission line
iv	A resistive load at the end of a transmission line
v	A parallel connection of an inductor, a resistor, and a capacitor going through resonance and with a transmission line offset
vi	A series connection of a resistor, an inductor and a capacitor going through resonance and with a transmission line offset
vii	A series resistor and inductor
viii	An unknown load and not one of the above

    For each of the measurements below indicate the load or loads using the load identifier above (e.g., i, ii, etc.).
    (a) What load(s) is indicated by curve A?
    (a) What load(s) is indicated by curve A?
    (b) What load(s) is indicated by curve B?
    (c) What load(s) is indicated by curve C?
    (d) What load(s) is indicated by curve D?
    (e) What load(s) is indicated by curve E?
    (f) What load(s) is indicated by curve F?
25. Design an open-circuited stub with an input impedance of $+\jmath 75$ Ω. Use a transmission line with a characteristic impedance of 75 Ω. [Parallels Example 8.2]
26. Design a short-circuited stub with an input impedance of $-\jmath 50$ Ω. Use a transmission line with a characteristic impedance of 100 Ω. [Parallels Example 8.2]
27. A load has an impedance $Z_L = 25 - \jmath 100$ Ω.
    (a) What is the reflection coefficient, Γ_L, of the load in a 50 Ω reference system?
    (b) If a one-quarter wavelength long 50 Ω transmission line is connected to the load, what is the reflection coefficient, Γ_{in}, looking into the transmission line?
    (c) Describe the locus of Γ_{in}, as the length of the transmission line is varied from zero length

to one-half wavelength long. Use a Smith chart to illustrate your answer

28. A network consists of a source with a Thevenin equivalent impedance of 50 Ω driving first a series reactance of −50 Ω followed by a one-eighth wavelength long transmission line with a characteristic impedance of 40 Ω and an element with a reactive impedance of $\jmath 25\ \Omega$ in shunt with a load having an impedance $Z_L = 25 - \jmath 50\ \Omega$. This problem must be solved graphically and no credit will be given if this is not done.
    (a) Draw the network.
    (b) On a Smith chart, plot the locus of the reflection coefficient first for the load, then with the element in shunt, then looking into the transmission line, and finally the series element. Use letters to identify each point on the Smith chart. Write down the reflection coefficient at each point.
    (c) What is the impedance presented by the network to the source?

8.6.1 Exercises by Section

†challenging

§8.3 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23†, 24†, 25†, 26†, 27†, 28†

8.6.2 Answers to Selected Exercises

1 $\Gamma_{\text{IN}} = 10^{-10}$

23(c) ii & iii

24(d) $8.37 - \jmath 49.3\ \Omega$

27 $0.825\angle{-50.9^\circ}$

28 $\approx 250 - \jmath 41\ \Omega$

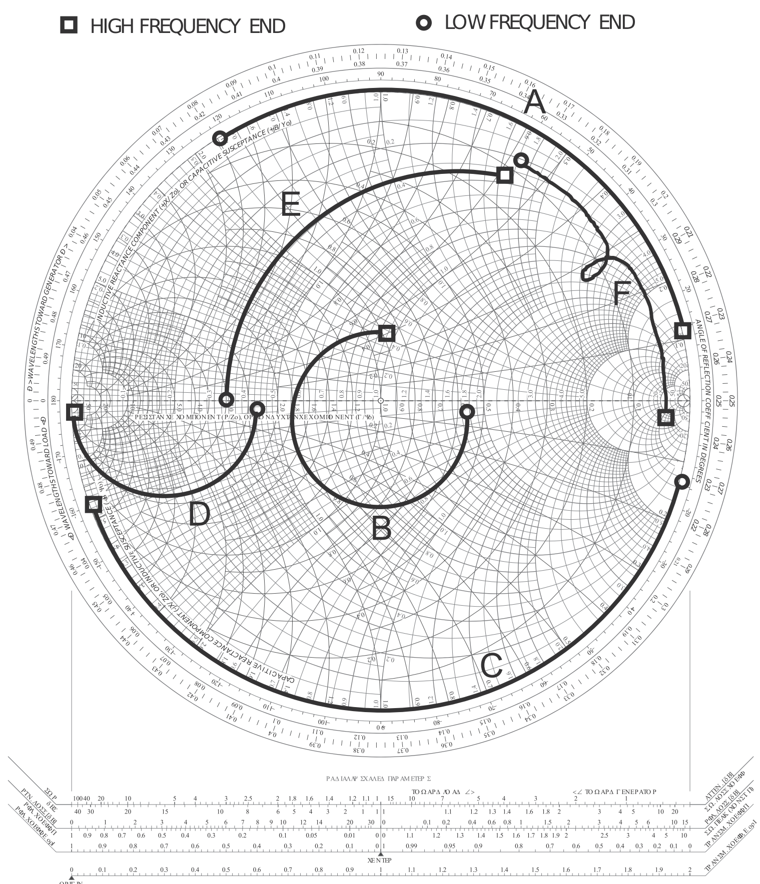


Figure 8-19: The locus of various loads plotted on a Smith chart.

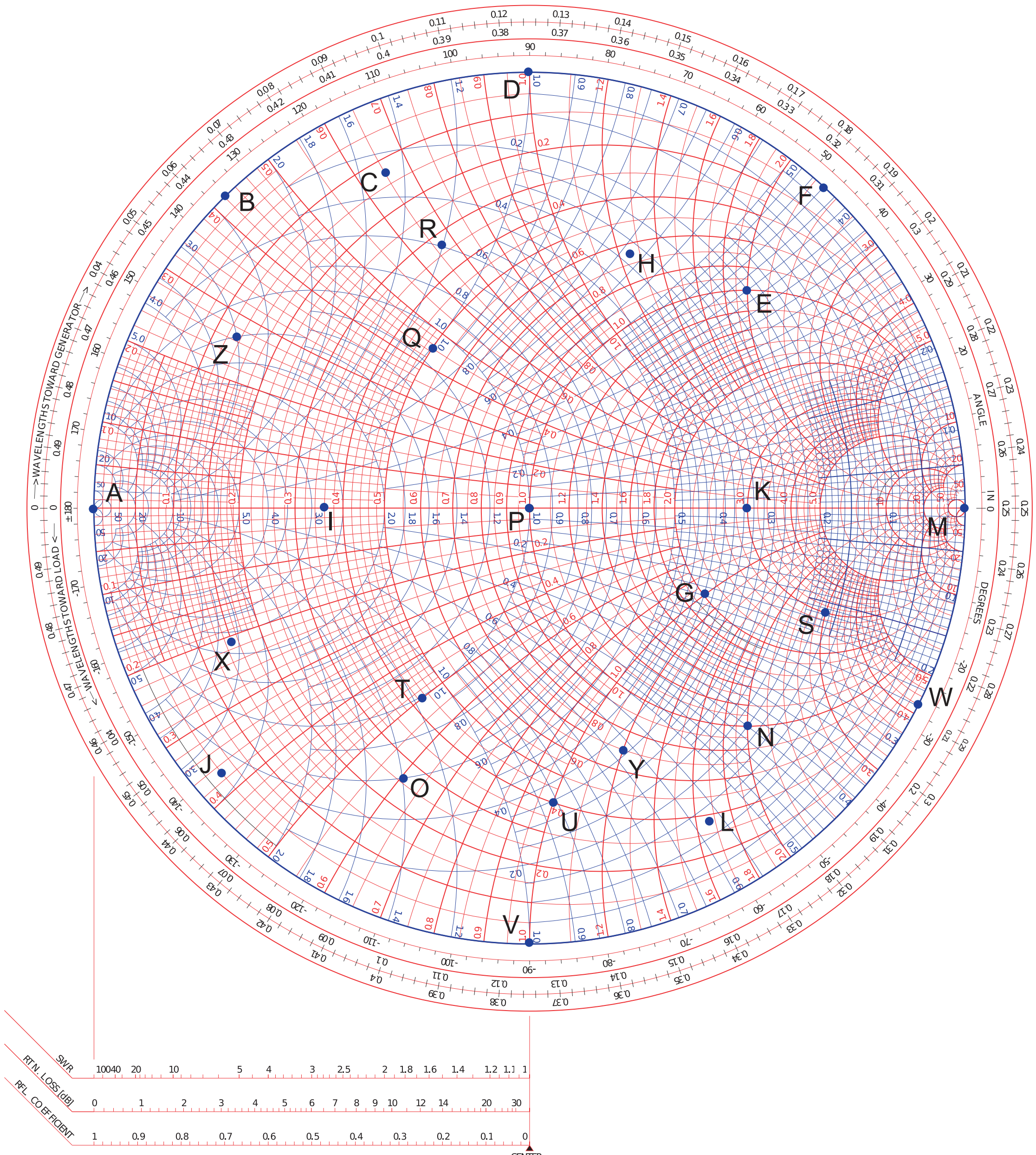

A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
ANGLE
DEGREES
IN
SWR
RTN. LOSS [dB]
RFL. COEFFICIENT
CENTER

Figure 8-20:

CHAPTER 9

Passive Components

9.1 Introduction

This chapter introduces a wide variety of passive components. It is not possible to be comprehensive, as there is an enormous catalog of microwave elements and scores of variations, and new concepts are introduced every year. At microwave frequencies distributed components can be constructed that have features with particular properties related to coupling, to traveling waves, and to storage of EM energy. Sometimes it is possible to develop lumped-element equivalents of the distributed elements by using the LC ladder model of a transmission line thus realizing lumped-element circuits that would be difficult to imagine otherwise.

9.2 Q Factor

RF inductors and capacitors also have loss and parasitic elements. With inductors there is both series resistance and shunt capacitance mainly from interwinding capacitance, while with capacitors there will be shunt resistance and series inductance. A practical inductor or capacitor is limited to operation below the self-resonant frequency determined by the inductance and capacitance itself resonating with its reactive parasitics. The impact of loss is quantified by the Q factor (the quality factor). Q is loosely related to bandwidth in general and the strict relationship is based on the response of a series or parallel connection of a resistor (R), an inductor (L), and a capacitor (C). The response of an RLC network is described by a second-

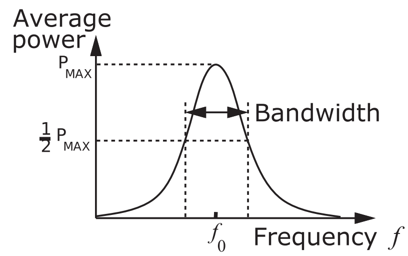


Figure 9-1: Transfer characteristic of a resonant circuit. (The transfer function is V/I for the parallel resonant circuit of Figure 9-2(a) and I/V for the series resonant circuit of Figure 9-2(b).)

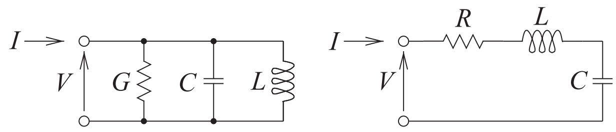


Figure 9-2: Second-order resonant circuits.

order differential equation with the conclusion that the 3 dB fractional bandwidth of the response (i.e., when the power response is at its half-power level below its peak response) is $1/Q$. (The fractional bandwidth is $\Delta f/f_0$ where $f_0 = f_r$ is the resonant frequency at the center of the band and Δf is the 3 dB bandwidth.) This is not true for any network other than a second-order circuit, but as a guiding principle, networks with higher Qs will have narrower bandwidths.

9.2.1 Definition

The Q factor of a component at frequency f is defined as the ratio of $2\pi f$ times the maximum energy stored to the energy lost per cycle. In a lumped-element resonant circuit, stored energy is transferred between an inductor, which stores magnetic energy, and a capacitor, which stores electric energy, and back again every period. Distributed resonators function the same way, exchanging energy stored in electric and magnetic forms, but with the energy stored spatially. The quality factor is

$$Q = 2\pi f_0 \left(\frac{\text{average energy stored in the resonator at } f_0}{\text{power lost in the resonator}} \right), \tag{9.1}$$

where f_0 is the resonant frequency.
A simple response is shown in Figure 9-1 for a parallel resonant circuit with elements L, C, and $G = 1/R$ (see Figure 9-2(a)),

$$Q = \omega_r C/G = 1/(\omega_r L G), \tag{9.2}$$

where $f_r = \omega_r/(2\pi)$ is the resonant frequency and is the frequency at which the maximum amount of energy is stored in a resonator. The conductance, G, describes the energy lost in a cycle. For a series resonant circuit (Figure 9-2(b)) with L, C, and R elements,

$$Q = \omega_r L/R = 1/(\omega_r C R). \tag{9.3}$$

These second-order resonant circuits have a bandpass transfer characteristic (see Figure 9-1) with Q being the inverse of the fractional bandwidth of the resonator. The fractional bandwidth, $\Delta f/f$, is measured at the half-power points as shown in Figure 9-1. (Δf is also referred to as the two-sided -3 dB

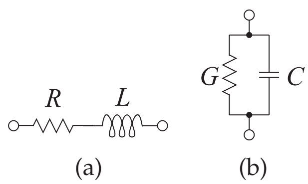


Figure 9-3: Loss elements of practical inductors and capacitors: (a) an inductor has a series resistance R; and (b) for a capacitor, the dominant loss mechanism is a shunt conductance $G = 1/R$.

bandwidth.) Then

$$Q = f_r/\Delta f. \tag{9.4}$$

Thus the Q is a measure of the sharpness of the bandpass frequency response. The determination of Q using the measurement of bandwidth together with Equation (9.4) is often not very precise, so another definition that uses the much more sensitive phase change at resonance is preferred when measurements are being used. With ϕ being the phase (in radians) of the transfer characteristic, the definition of Q is now

$$Q = \frac{\omega_r}{2}\left|\frac{d\phi}{d\omega}\right|. \tag{9.5}$$

Equation (9.5) is another equivalent definition of Q for parallel RLC or series RLC resonant circuits. It is meaningful to talk about the Q of circuits other than three-element RLC circuits, and then its meaning is always a ratio of the energy stored to the energy dissipated per cycle. The Q of these structures can no longer be determined by bandwidth or by the rate of phase change.

9.2.2 Q of Lumped Elements

Q is also used to characterize the loss of lumped inductors and capacitors. Inductors have a series resistance R, and the main loss mechanism of a capacitor is a shunt conductance G (see Figure 9-3).
The Q of an inductor at frequency $f = \omega/(2\pi)$ with a series resistance R and inductance L is

$$Q_{\text{INDUCTOR}} = \frac{\omega L}{R}. \tag{9.6}$$

Since R is approximately constant with respect to frequency for an inductor, the Q will vary with frequency.
The Q of a capacitor with a shunt conductance G and capacitance C is

$$Q_{\text{CAPACITOR}} = \frac{\omega C}{G}. \tag{9.7}$$

G is due mainly to relaxation loss mechanisms of the dielectric of a capacitor and so varies linearly with frequency but also it is usually very small. Thus the Q of a capacitor is almost constant with respect to frequency. For microwave components $Q_{\text{CAPACITOR}} \gg Q_{\text{INDUCTOR}}$, and both are smaller than the Q of most transmission line networks. Thus, if the length of a transmission line is not too long for an application, transmission line networks are preferred. If lumped elements must be used, the use of inductors should be minimized.

9.2.3 Loaded Q Factor

The Q of a component as defined in the previous section is called the unloaded Q, Q_U. However, if a component is to be measured or used in any way, it is necessary to couple energy in and out of it. The Q is reduced and thus the resonator bandwidth is increased by the power lost to the external circuit so that the loaded Q, the Q that is measured, is

$$Q_L = 2\pi f_0 \left(\frac{\text{average energy stored in the resonator at} f_0}{\text{power lost in the resonator and to the external circuit}} \right)$$

$$= \frac{1}{1/Q + 1/Q_X}, \tag{9.8}$$

and $$Q_X = \left(\frac{1}{Q_L} - \frac{1}{Q_U} \right)^{-1}. \tag{9.9}$$

where Q_X is called the external Q. Q_L accounts for the power extracted from the resonant circuit. If the loading is kept very small, $Q_L \approx Q_U$.

9.2.4 Summary of the Properties of Q

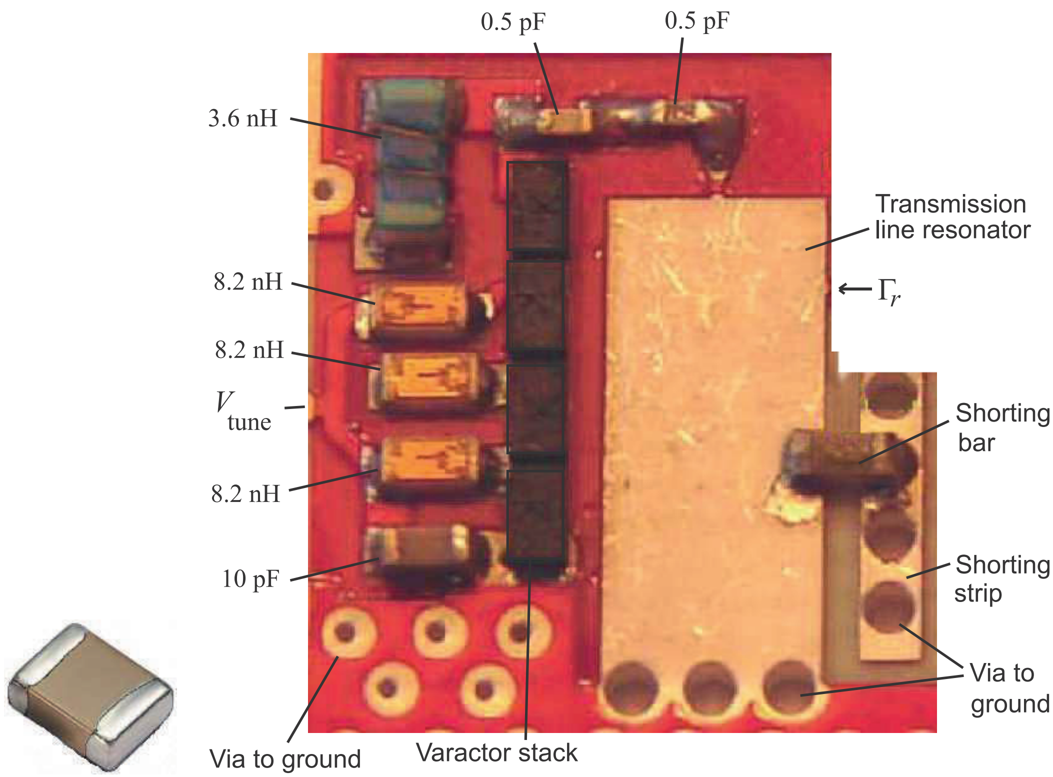
In summary:

(a) Q is properly defined and related to the energy stored in a resonator for a second-order network, one with two reactive elements of opposite types.
(b) Q is not well defined for networks with three or more reactive elements.
(e) It is only used (as defined or some approximation of it) for guiding the design.

9.3 Surface-Mount Components

The majority of the RF and microwave design effort goes into developing modules and interconnecting modules on circuit boards. With these the most common type of component to use is a surface-mount component. Figure 9-4(a) shows a two-terminal element, such as a resistor or capacitor, in the form of a surface-mount component. Figure 9-4(b and c) show the use of surface-mount components on a microwave circuit board.

A two-terminal surface-mount resistor or capacitor is commonly called a chip resistor or chip capacitor. These can be very small, and the smaller the component often the higher the operating frequency due to reduced parasitic capacitance or inductance. Common sizes of two-terminal chip components are listed in Table 9-1. The resonance when a chip capacitor (inductor) resonates with the parasitic inductance (capacitance) is called self-resonance and the resonant frequency is called the self-resonance frequency. Figure 9-5(a) shows an inductor in a surface-mount package and details are shown in Figure 9-5(b) and the inductor is useable as an inductor at a frequency backed-off from the SRF, see Table 9-2.



(a) (b)

Figure 9-4: Circuit board showing the use of surface-mount components: (a) chip resistor or capacitor with metal terminals at the two ends; and (b) a populated RF microstrip circuit board of a 5 GHz tunable resonator [1]. The larger components have dimensions 1.6 mm × 0.8mm. The shorting bar is a 0 Ω chip resistor.

Designation	Size (inch × inch)	Metric designation	Size (mm × mm)
01005	0.016 × 0.0079	0402	0.4 × 0.2
0201	0.024 × 0.012	0603	0.6 × 0.3
0402	0.039 × 0.020	1005	1.0 × 0.5
0603	0.063 × 0.031	1608	1.6 × 0.8
0805	0.079 × 0.049	2012	2.0 × 1.25
1008	0.098 × 0.079	2520	2.5 × 2.0
1206	0.13 × 0.063	3216	3.2 × 1.6
1210	0.13 × 0.098	3225	3.2 × 2.5
1806	0.18 × 0.063	4516	4.5 × 1.6
1812	0.18 × 0.13	4532	4.5 × 3.2
2010	0.20 × 0.098	5025	5.0 × 2.5
2512	0.25 × 0.13	6432	6.4 × 3.2
2920	0.29 × 0.20	–	7.4 × 5.1

Table 9-1: Sizes and designation of two-terminal surface mount components. Note the designation of a surface-mount component refers (approximately) to its dimensions in hundredths of an inch.

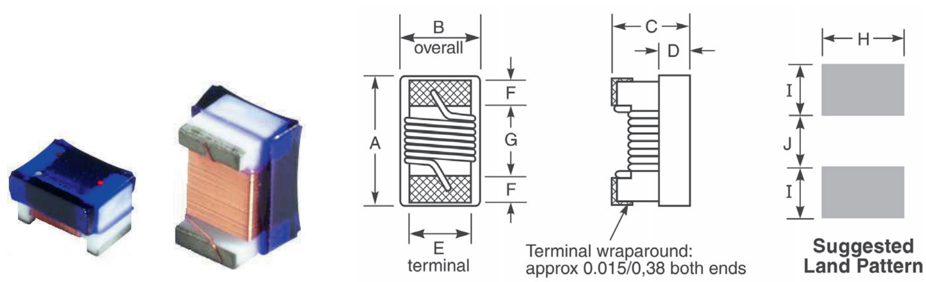


Figure 9-5: Chip inductor: (a) in an 0603 surface-mount package; and (b) schematic showing sizes and pads to be provided on a circuit board (A=64 mils (1.63 mm), B=33 mils (0.84 mm), C=24 mils (0.61 mm), D=13 mils (0.33 mm), E=30 mils (0.76 mm), F=25 mils (0.64 mm), G=25 mils (0.64 mm), and H=40 mils (1.02 mm)). Copyright Coilcraft, Inc., used with permission [2].

9.4 Terminations and Attenuators

9.4.1 Terminations

Terminations are used to completely absorb a forward-traveling wave and the reflection coefficient of a termination is ideally zero. If a transmission line has a resistive characteristic impedance $R_0 = Z_0$, then terminating the line in a resistance R_0 will fully absorb the forward-traveling wave and there will be no reflection. The line is then said to be matched. At RF and microwave frequencies some refinements to this simple circuit connection are required. On a transmission line the energy is contained in the EM fields. For the coaxial line, a simple resistive connection between the inner and outer conductors would not terminate the fields and there would be some reflection. Instead, coaxial line terminations generally comprise a disk of resistive material (see Figure 9-6(a)). The total resistance of the disk from the inner to the outer conductor is the characteristic resistance of the line, however, the resistive material is distributed and so creates a good termination of the fields.

Terminations are a problem with microstrip, as the characteristic impedance varies with frequency, is in general complex, and the vias that would be required if a lumped resistor was used has appreciable inductance at frequencies above a few gigahertz. A high-quality termination is realized using a section of lossy line as shown in Figure 9-6(b). Here lossy material is deposited on top of an open-circuited microstrip line. This increases the loss of the line appreciably without significantly affecting the characteristic

Table 9-2: Parameters of the inductors in Figure 9-5. $L_{\rm nom}$ is the nominal inductance, SRF is the self-resonance frequency, $R_{\rm DC}$ is the inductor's series resistance, and $I_{\rm max}$ is the maximum RMS current supported.

$L_{\rm nom}$ (nH)	900 MHz L (nH)	900 MHz Q	1.7 GHz L (nH)	1.7 GHz Q	SRF (GHz)	$R_{\rm DC}$ (Ω)	$I_{\rm max}$ (mA)
1.0	0.98	39	0.99	58	16.0	0.045	1600
2.0	1.98	46	1.98	70	12.0	0.034	1900
5.1	5.12	68	5.18	93	5.50	0.050	1400
10	10.0	67	10.4	85	3.95	0.092	1100
20	20.2	67	21.6	80	2.90	0.175	760
56	59.4	54	75.4	48	1.75	0.700	420

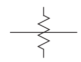
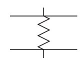
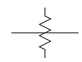
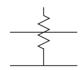
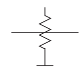
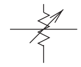
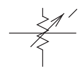
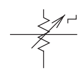
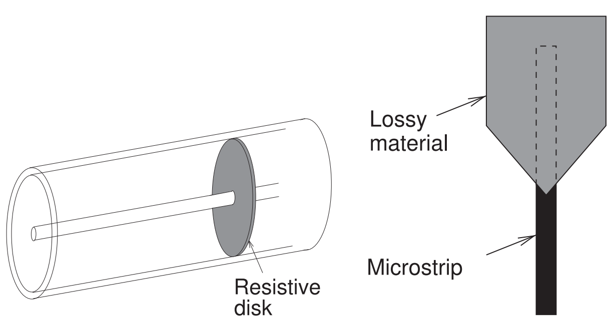
Component	Symbol	Alternate
Attenuator, fixed		
Attenuator, balanced		
Attenuator, unbalanced		
Attenuator, variable		
Attenuator, continuously variable		
Attenuator, stepped variable		

Table 9-3: IEEE standard symbols for attenuators [3].

impedance of the line. If the length of the lossy line is sufficiently long, say one wavelength, the forward-traveling wave will be totally absorbed and there will be no reflection. Tapering the lossy material, as shown in Figure 9-6(b), reduces the discontinuity between the lossless microstrip line and the lossy line by ensuring that some of the power in the forward-traveling wave is dissipated before the maximum impact of the lossy material occurs. Thus a matched termination is achieved without the use of a via.

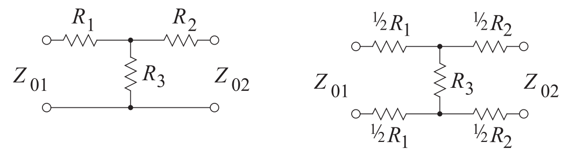
9.4.2 Attenuators

An attenuator is a two-port network that reduces the amplitude of a signal and it does this by absorbing power and without distorting the signal. The input and output of the attenuator are both matched, so there are no reflections. An attenuator may be fixed, continuously variable, or discretely variable. The IEEE standard symbols for attenuators are shown in Table 9-3. When the attenuation is fixed, an attenuator is commonly called a **pad**.
Balanced and unbalanced resistive pads are shown in Figures 9-7 and 9-8 together with their design equations. The attenuators in Figure 9-7 are T or Tee attenuators, where Z_{01} is the system impedance to the left of the pad and Z_{02} is the system impedance to the right of the pad. The defining characteristic is that the reflection coefficient looking into the pad from the



(a) (b)

Figure 9-6: Terminations: (a) coaxial line resistive termination; (b) microstrip matched load.



(a) Unbalanced (b) Balanced

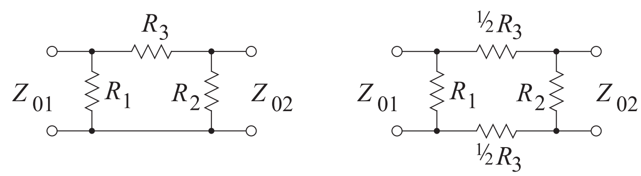
$$R_1 = \frac{Z_{01}(K+1) - 2\sqrt{KZ_{01}Z_{02}}}{K-1} \qquad R_2 = \frac{Z_{02}(K+1) - 2\sqrt{KZ_{01}Z_{02}}}{K-1}$$

$$R_3 = \frac{2\sqrt{KZ_{01}Z_{02}}}{K-1} \tag{9.10}$$

If $Z_{01} = Z_{02} = Z_0$, then

$$R_1 = R_2 = Z_0\left(\frac{\sqrt{K}-1}{\sqrt{K}+1}\right) \quad \text{and} \quad R_3 = \frac{2Z_0\sqrt{K}}{K-1} \tag{9.11}$$

Figure 9-7: T (Tee) attenuator. K is the (power) attenuation factor, e.g. a 3 dB attenuator has $K = 10^{3/10} = 1.995$.



(a) Unbalanced (b) Balanced

$$R_1 = \frac{Z_{01}(K-1)\sqrt{Z_{02}}}{(K+1)\sqrt{Z_{02}} - 2\sqrt{KZ_{01}}} \qquad R_2 = \frac{Z_{02}(K-1)\sqrt{Z_{01}}}{(K+1)\sqrt{Z_{01}} - 2\sqrt{KZ_{02}}} \tag{9.12}$$

$$R_3 = \frac{(K-1)}{2}\sqrt{\frac{Z_{01}Z_{02}}{K}} \tag{9.13}$$

If $Z_{01} = Z_{02} = Z_0$, then

$$R_1 = R_2 = Z_0\left(\frac{\sqrt{K}+1}{\sqrt{K}-1}\right) \quad \text{and} \quad R_3 = \frac{Z_0(K-1)}{2\sqrt{K}}. \tag{9.14}$$

If $R_1 = R_2$, then $Z_{01} = Z_{02} = Z_0$

$$Z_0 = \sqrt{\frac{R_1^2 R_3}{2R_1 + R_3}}, \quad \text{and} \quad K = \left(\frac{R_1 + Z_0}{R_1 - Z_0}\right)^2. \tag{9.15}$$

Figure 9-8: Pi (Π) attenuator. K is the (power) attenuation factor, e.g. a 20 dB attenuator has $K = 10^{20/10} = 100$.

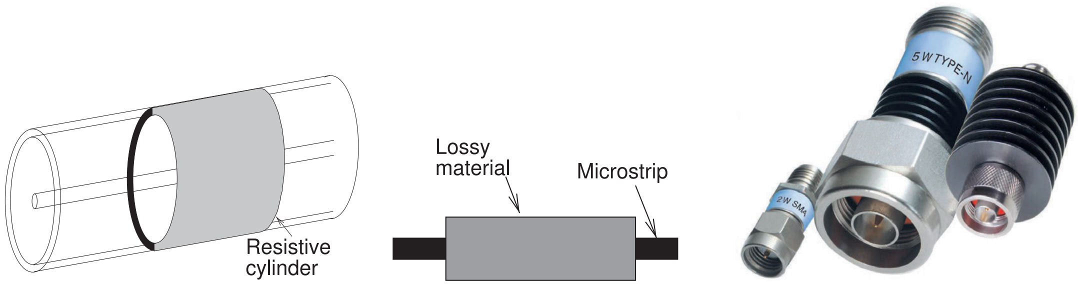
left is zero when referred to Z_{01}. Similarly, the reflection coefficient looking into the right of the pad is zero with respect to Z_{02}. The attenuation factor is

$$K = \frac{\text{Power in}}{\text{Power out}}. \tag{9.16}$$

In decibels, the attenuation is

$$K|_{\text{dB}} = 10\log_{10} K = (\text{Power in})|_{\text{dBm}} - (\text{Power out})|_{\text{dBm}}. \tag{9.17}$$

If the left and right system impedances are different, then there is a minimum



(a) Coaxial attenuator (b) Microstrip attenuator (c) Precision fixed coaxial attenuators

Figure 9-9: Distributed attenuators. The attenuators in (c) have power handling ratings of 2W, 5W and 20 W (left to right). Copyright 2012 Scientific Components Corporation d/b/a Mini-Circuits, used with permission [4].

attenuation factor that can be achieved:

$$K_{\mathrm{MIN}} = \frac{2Z_{01}}{Z_{02}} - 1 + 2\sqrt{\frac{Z_{01}}{Z_{02}}\left(\frac{Z_{01}}{Z_{02}-1}\right)}. \tag{9.18}$$

This limitation comes from the simultaneous requirement that the pad be matched. If there is a single system impedance, $Z_0 = Z_{01} = Z_{02}$, then $K_{\mathrm{MIN}} = 1$, and so any value of attenuation can be obtained.

Lumped attenuators are useful up to 10 GHz above which the size of resistive elements becomes large compared to a wavelength. Also, for planar circuits, vias are required, and these are undesirable from a manufacturing standpoint, and electrically they have a small inductance. Fortunately attenuators can be realized using a lossy section of transmission line, as shown in Figure 9-9. Here, lossy material results in a section of line with a high-attenuation constant. Generally the lossy material has little effect on the characteristic impedance of the line, so there is little reflection at the input and output of the attenuator. Distributed attenuators can be used at frequencies higher than lumped-element attenuators can, and they can be realized with any transmission line structure.

EXAMPLE 9.1 Pad Design

Design an unbalanced 20 dB pad in a 75 Ω system.

Solution:

There are two possible designs using resistive pads. These are the unbalanced Tee and Pi pads shown in Figures 9-7 and 9-8. The Tee design will be chosen. The K factor is

$$K = 10^{(\,K|_{\mathrm{dB}}/10)} = 10^{(20/10)} = 100. \tag{9.19}$$

Since $Z_{01} = Z_{02} = 75\ \Omega$, Equation (9.11) yields

$$R_1 = R_2 = 75\left(\frac{\sqrt{100}-1}{\sqrt{100}+1}\right) = 75\left(\frac{9}{11}\right) = 61.4\ \Omega \tag{9.20}$$

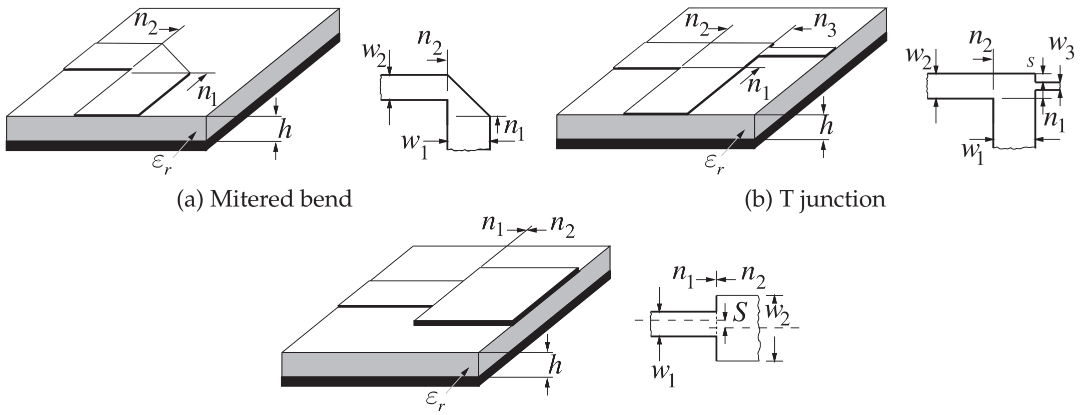
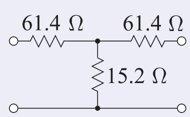


Figure 9-10: Microstrip discontinuities.

$$R_3 = \frac{2 \cdot 75\sqrt{100}}{100 - 1} = 150\left(\frac{10}{99}\right) = 15.2\ \Omega. \tag{9.21}$$

The final design is



9.5 Transmission Line Stubs and Discontinuities

Interruptions of the magnetic or electric field create regions where additional magnetic energy or electric energy is stored. If the additional energy stored is predominantly magnetic, the discontinuity will introduce an inductance. If the additional energy stored is predominantly electric, the discontinuity will introduce a capacitance. Such discontinuities occur with all transmission lines. In some cases transmission line discontinuities introduce undesired parasitics, but they also provide an opportunity to effectively introduce lumped-element components.

The simplest microwave circuit element is a uniform section of transmission line that can be used to introduce a time delay or a frequency-dependent phase shift. More commonly it is used to interconnect other components. Other line segments including bends and junctions are shown in Figure 9-10.

9.5.1 Open

Many transmission line discontinuities arise from fringing fields. One element is the microstrip open, shown in Figure 9-11. The fringing fields at the end of the transmission line in Figure 9-11(a) store energy in the electric field, and this can be modeled by the fringing capacitance, C_F, shown in Figure 9-11(b). This effect can also be modeled by an extended transmission line, as shown in Figure 9-11(c). For a typical microstrip line with ε_r = 9.6, h = 600 μm, and w/h = 1, C_F is approximately 36 fF. However, C_F varies

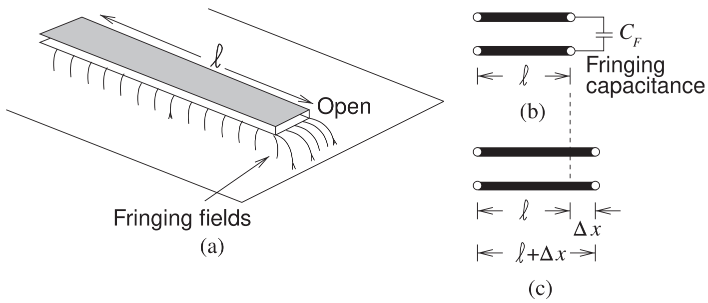


Figure 9-11: An open on a microstrip transmission line: (a) microstrip line showing fringing fields at the open; (b) fringing capacitance model of the open; and (c) an extended line model of the open with Δx being the extra transmission line length that captures the open.

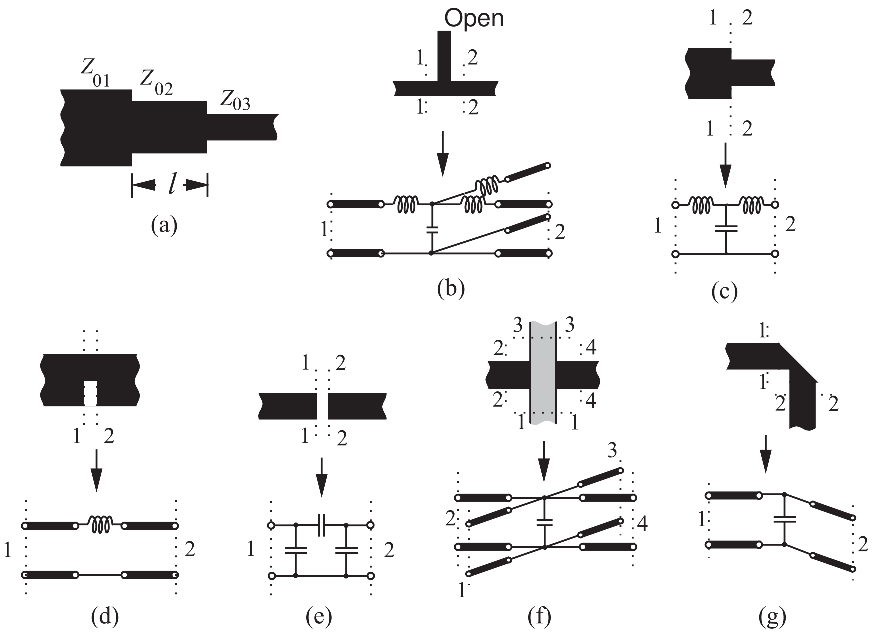
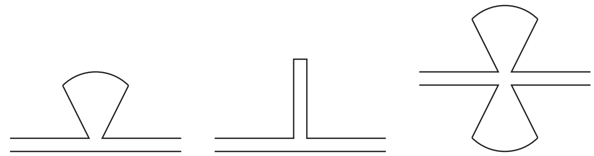


Figure 9-12: Microstrip discontinuities: (a) quarter-wave impedance transformer; (b) open microstrip stub; (c) step; (d) notch; (e) gap; (f) crossover; and (g) bend.

with frequency, and the extended length is a much better approximation to the effect of fringing [5]. For the same dimensions, the length section is approximately $0.35h$ and almost independent of frequency [6]. As with many fringing effects, a capacitance or inductance can be used to model the effect of fringing, but generally a distributed model is better.

9.5.2 Discontinuities

Several microstrip discontinuities and their equivalent circuits are shown in Figure 9-12. Discontinuities (Figure 9-12(b–g)) are modeled by capacitive elements if the E field is affected and by inductive elements if the H field (or current) is disturbed. The stub shown in Figure 9-12(b), for example, is best modeled using lumped elements describing the junction as well as the transmission line of the stub itself. Current bunches at the right angle bends from the through line to the stub. The current bunching leads to excess energy being stored in the magnetic field, and hence an inductive effect. There is also excess charge storage in the parallel plate region bounded by the left- and right-hand through lines and the stub. This is modeled by a capacitance.



(a) (b) (c)

Figure 9-13: Microstrip stubs: (a) radial shunt-connected stub; (b) conventional shunt stub; and (c) butterfly radial stub.

9.5.3 Impedance Transformer

Impedance transformers interface two lines of different characteristic impedance. The smoothest transition and the one with the broadest bandwidth is a tapered line. This element can be long and then a quarter-wave impedance transformer (see Figure 9-12(a)) is sometimes used, although its bandwidth is relatively small and centered on the frequency at which $l = \lambda_g/4$. Ideally $Z_{0,2} = \sqrt{Z_{0,1}Z_{0,3}}$.

9.5.4 Planar Radial Stub

The use of a radial stub (Figure 9-13(a)), as opposed to the conventional microstrip stub (Figure 9-13(b)), can improve the bandwidth of many microstrip circuits. A major advantage of a radial stub is that the input impedance presented to the through line has broader bandwidth than that obtained with the conventional stub. When two shunt-connected radial stubs are introduced in parallel (i.e., one on each side of the microstrip feeder line) the resulting configuration is termed a "butterfly" stub (see Figure 9-13(c)).

9.6 Magnetic Transformer

In this section the use of magnetic transformers in microwave circuits will be discussed. Magnetic transformers can be used directly up to a few hundred megahertz or so, but the same transforming properties can be achieved using coupled transmission lines.

9.6.1 Properties of a Magnetic Transformer

A magnetic transformer (see Figure 9-14) magnetically couples the current in one wire to current in another. The effect is amplified using coils of wires and using a core of magnetic material (material with high permeability) to create greater magnetic flux density. When coils are used, the symbol shown in Figure 9-14(a) is used, with one of the windings called the primary winding and the other called the secondary winding. If there is a **magnetic core** around which the coils are wound, then the symbol shown in Figure 9-14(b) is used, with the vertical lines indicating the core. However, even if there is a core, the simpler transformer symbol in Figure 9-14(a) is more commonly used. Magnetic cores are useful up to several hundred megahertz and rely on the alignment of magnetic dipoles in the core material. Above a few hundred megahertz the magnetic dipoles cannot react quickly enough and so the core looks like an open circuit to magnetic flux. Thus the core is not useful for magnetically coupling signals above a few hundred megahertz. As mentioned, the dots above the coils in Figure 9-14(a) indicate the polarity of the magnetic flux with respect to the currents in the coils so that, as shown, V_1 and V_2 will have the same sign. Even if the magnetic polarity is not specifically shown, it is implied (see Figure 9-14(c)). There are two ways of showing inversion of the magnetic polarity, as shown in Figure 9-14(d), where a negative number of windings indicates opposite magnetic polarity. The interest in using magnetic transformers in high-frequency circuit design is that configurations of magnetic transformers can be realized using coupled transmission lines to extend operation to hundreds of gigahertz. The transformer is easy to conceptualize, so it is convenient to first develop circuits using the transformer and then translate it to transmission line form. That is, in "back-of-the-envelope" microwave design, transformers can be used to indicate coupling, with the details of the coupling left until later when the electrical design is translated into a physical design.

The following notation is used with a magnetic transformer:

L_1, L_2: the self-inductances of the two coils

M: the mutual inductance

k: the coupling factor,

$$k = \frac{M}{\sqrt{L_1 L_2}}. \tag{9.22}$$

Referring to Figure 9-14(e), the voltage transformer ratio is

$$V_2 = nV_1, \tag{9.23}$$

where n is the ratio of the number of secondary to primary windings. An ideal transformer has "perfect coupling," indicated by $k = 1$, and the self-inductances are proportional to the square of the number of windings, so

$$\frac{V_2}{V_1} = \sqrt{\frac{L_2}{L_1}}. \tag{9.24}$$

The general equation relating the currents of the circuit in Figure 9-14(e) is

$$RI_2 + \jmath\omega L_2 I_2 + \jmath\omega M I_1 = 0, \tag{9.25}$$

and so

$$\frac{I_1}{I_2} = -\frac{R + \jmath\omega L_2}{\jmath\omega M}. \tag{9.26}$$

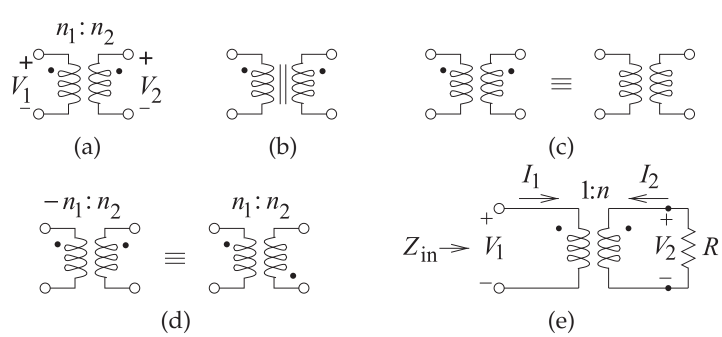


Figure 9-14: Magnetic transformers: (a) a transformer as two magnetically coupled windings with n_1 windings on the primary (on the left) and n_2 windings on the secondary (on the right) (the dots indicate magnetic polarity so that the voltages V_1 and V_2 have the same sign); (b) a magnetic transformer with a magnetic core; (c) identical representations of a magnetic transformer with the magnetic polarity implied for the transformer on the right; (d) two equivalent representations of a transformer having opposite magnetic polarities (an **inverting transformer**); and (e) a magnetic transformer circuit.

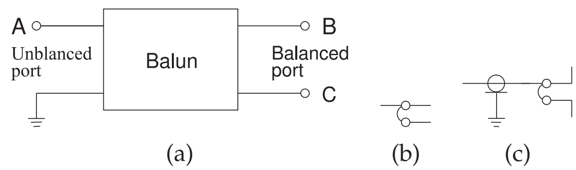


Figure 9-15: A balun: (a) as a two-port with four terminals; (b) IEEE standard schematic symbol for a balun [3]; and (c) an (unbalanced) coaxial cable driving a dipole antenna through a balun.

If $R \ll \omega L_2$, then the current transformer ratio is

$$\frac{I_1}{I_2} \approx -\frac{L_2}{M} = -\sqrt{\frac{L_2}{L_1}}. \tag{9.27}$$

Notice that combining Equations (9.24) and (9.27) leads to calculation of the transforming effect on impedance. On the Coil 1 side, the input impedance is

$$Z_{\text{in}} = \frac{V_1}{I_1} = -\frac{V_2}{I_2}\left(\frac{L_1}{L_2}\right) = R\frac{L_1}{L_2}. \tag{9.28}$$

In practice, however, there is always some magnetic field leakage—not all of the magnetic field created by the current in Coil 1 goes through (or links) Coil 2—and so $k < 1$. Then from Equations (9.24)–(9.28),

$$V_1 = \jmath\omega L_1 I_1 + \jmath\omega M I_2 \tag{9.29}$$
$$0 = R I_2 + \jmath\omega L_2 I_2 + \jmath\omega M I_1. \tag{9.30}$$

Again, assuming that $R \ll \omega L_2$, a modified expression for the input impedance is obtained that accounts for nonideal coupling:

$$Z_{\text{in}} = R\frac{L_1}{L_2} + \jmath\omega L_1(1 - k^2). \tag{9.31}$$

Imperfect coupling, $k < 1$, causes the input impedance to be reactive and this limits the bandwidth of the transformer. Stray capacitance is another factor that impacts the bandwidth of the transformer.

9.7 Baluns

A balun [7, 8] is a structure that joins balanced and unbalanced circuits. The word itself (balun) is a contraction of balanced-to-unbalanced transformer. Representations of a balun are shown in Figure 9-15. A situation when a balun is required is with an antenna. Many antennas do not operate correctly if part of the antenna is at the same potential electrically as the ground. Instead, the antenna should be electrically isolated from the ground (i.e., balanced). The antenna would usually be fed by a coaxial cable with its outer conductor connected to ground.

The schematic of a magnetic transformer used as a balun is shown in Figure 9-16(a) with one terminal of the unbalanced port grounded. Figure 9-15(b) is the standard schematic symbol for a balun and its use with a dipole antenna is shown in Figure 9-15(c). The second port is floating and is not referenced to ground. An example of the use of a balun is shown in Figure 9-16(b), where the output of a single-ended amplifier is unbalanced, being referred to ground. A balun transitions from the unbalanced transistor output to a balanced output.

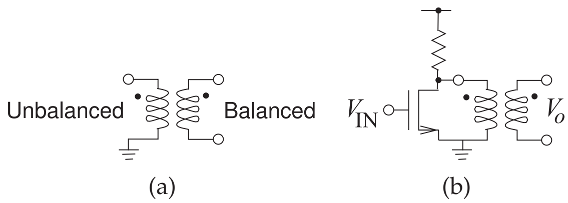


Figure 9-16: Balun: (a) schematic representation as a transformer showing unbalanced and balanced ports; and (b) connected to a single-ended unbalanced amplifier yielding a balanced output.

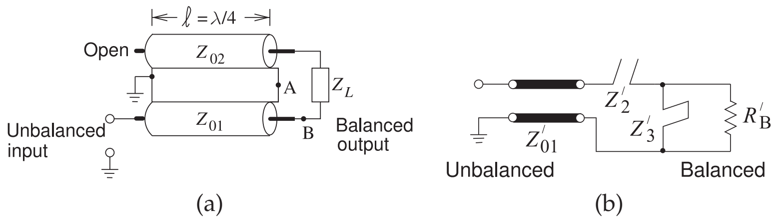


Figure 9-17: Marchand balun: (a) coaxial form of the Marchand balun; and (b) its equivalent circuit.

9.7.1 Marchand Balun

The most common form of microwave balun is the Marchand balun [7, 9–12]. An implementation of the Marchand balun using coaxial transmission lines is shown in Figure 9-17(a) [13]. The Z_2 line acts as both a series stub and a shunt stub. Thus the model of the Marchand balun is as shown in Figure 9-17(b).

9.8 Wilkinson Combiner and Divider

A combiner is used to combine power from two or more sources. A typical use is to combine power from two high power amplifiers to obtain a higher power than would be otherwise be available. Dividers divide power so that the power from an amplifier can be routed to different parts of a circuit.

The Wilkinson divider can be used as a combiner or divider that divides input power among the output ports [14]. Figure 9-18(a) is a two-way divider that splits the power at Port 1 equally between the two output ports at Ports 2 and 3. A particular insight that Wilkinson brought was the introduction of the resistor between the output ports and this acts to suppress any imbalance between the output signals due to nonidealities. If the division is exact, no current will flow in the resistor. The circuit works less well as a general purpose combiner. Ideally power entering Ports 2 and 3 would combine losslessly and appear at Port 1. A typical application is to combine the power at the output of two matched transistors where the amplitude and the phase of the signals can be expected to be closely matched. However, if the signals are not identical, the portion that is mismatched will be absorbed in the resistor. The bandwidth of the Wilkinson combiner/divider is limited by the one-quarter wavelength long lines.

The S parameters of the two-way Wilkinson power divider with an equal split of the output power are therefore

$$\mathbf{S} = \begin{bmatrix} 0 & -\jmath/\sqrt{2} & -\jmath/\sqrt{2} \\ -\jmath/\sqrt{2} & 0 & 0 \\ -\jmath/\sqrt{2} & 0 & 0 \end{bmatrix}. \tag{9.32}$$

Figure 9-18(b) is a compact representation of the two-way Wilkinson divider, and a three-way Wilkinson divider is shown in Figure 9-18(d). This pattern can be repeated to produce N-way power dividing. The lumped-element version of the Wilkinson divider shown in Figure 9-18(c) is based on the LC model of a one-quarter wavelength long transmission line segment. With a $50\,\Omega$ system impedance and center frequency of 400 MHz, the elements of the lumped element are (from Section 22.7.2 of [15]) L = 28.13 nH, C_1 = 11.25 pF, C_2 = 5.627 pF, and R = 100 Ω.

Figure 9-19(a) is the layout of a direct microstrip realization of a Wilkinson divider. A high-performance microstrip layout is shown in Figure 9-19(b), where the transmission lines are curved to bring the output ports near each other so that a chip resistor can be used.

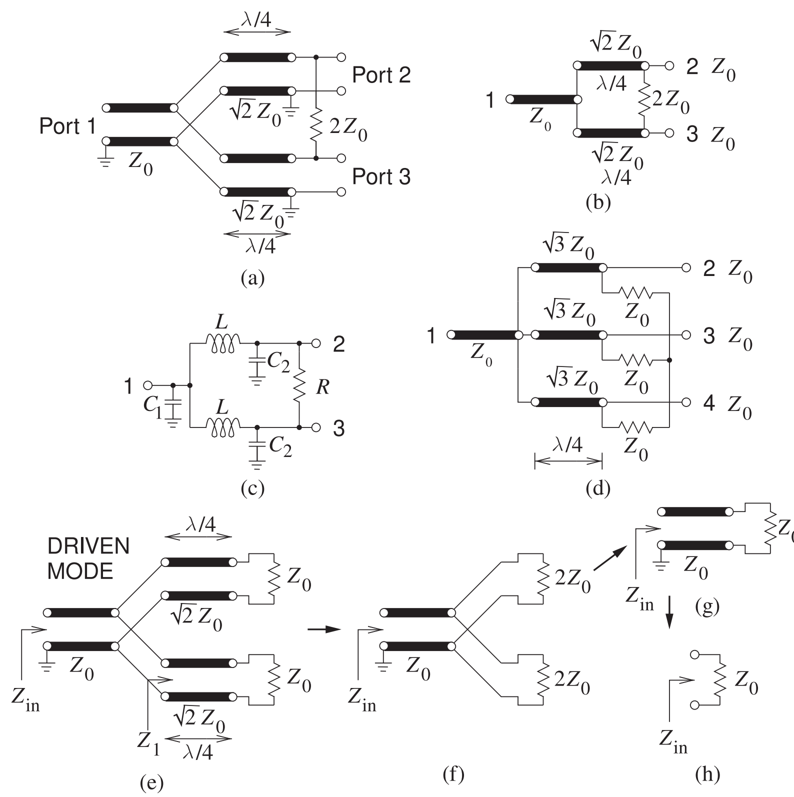


Figure 9-18: Wilkinson divider: (a) two-way divider with Port 1 being the combined signal and Ports 2 and 3 being the divided signals; (b) less cluttered representation; (c) lumped-element implementation; (d) three-way divider with Port 1 being the combined signal and Ports 2, 3, and 4 being the divided signals; and (e)–(h) steps in the derivation of the input impedance.

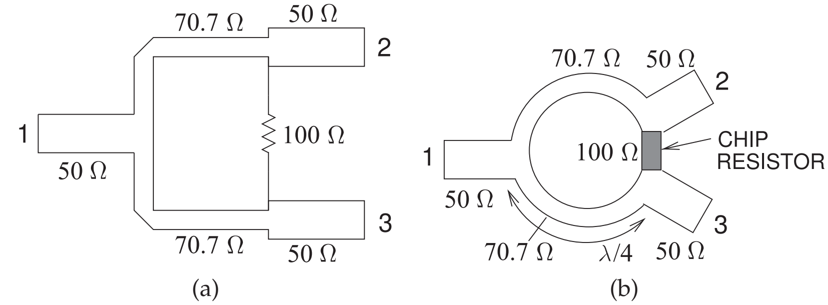


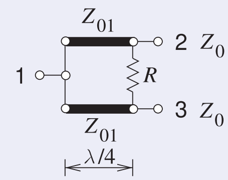
Figure 9-19: Wilkinson combiner and divider: (a) microstrip realization; and (b) higher performance microstrip implementation.

EXAMPLE 9.2 Lumped-Element Wilkinson Divider

Design a lumped-element 2-way Wilkinson divider in a 60 Ω system. The center frequency of the design should be 10 GHz.

Solution:

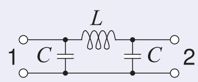
The design begins with the transmission line form of the Wilkinson divider which will be converted to a lumped-element form latter. The design parameters are $Z_0 = 60\ \Omega$, $f = 10$ GHz, $\omega = 2\pi 10^{10} = 2.283 \cdot 10^{10}$ and so



$$Z_{01} = \sqrt{2} Z_0 = 84.85\ \Omega, \quad R = 2Z_0 = 120\ \Omega.$$

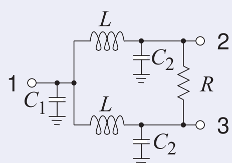
,

The next stage is to convert the transmission lines to lumped elements. A broadband design of a quarter-wavelength transmission line is presented in Section 22.7.2 of [15]. That is, each of the quarter-wave lines has the model



with $L = Z_{01}/\omega = 84.85/\omega = 954.9$ pH,
$C = 1/(Z_{01}\omega) = 1/(84.85\omega) = 265.3$ fF.

So the final lumped element design is



with
$C_1 = 2C = 530.6$ fF,
$C_2 = C = 265.3$ fF,
$L = 954.9$ pH,
$R = 120\ \Omega$.

9.9 Summary

This chapter introduced the richness of microwave components available to the RF designer. Microwave lumped-element R, L, and C components are carefully constructed so that they function as intended up to 10 or so. They are usually in surface-mount form so that they can be integrated in design while minimizing the parasitic effects introduced by leads. To a limited extent, transmission line discontinuities can be used as lumped elements. Even if the transmission line discontinuities are not specifically introduced for this purpose, their lumped-element equivalent circuits must be included in circuit analysis. Transmission line stubs are widely used to introduce capacitance and inductance in circuits. In most transmission line technologies only shunt stubs are available, and thus there is a strong preference for shunt elements in circuit designs.

9.10 References

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9.11 Exercises

1. A spiral inductor is modeled as an ideal inductor of 10 nH in series with a 5 Ω resistor. What is the Q of the spiral inductor at 1 GHz?
2. Consider the design of a 50 dB resistive T attenuator in a 75 Ω system. [Parallels Example 9.1]
   (a) Draw the topology of the attenuator.
   (b) Write down the design equations.
   (c) Complete the design of the attenuator.
3. Consider the design of a 50 dB resistive Pi attenuator in a 75 Ω system. [Parallels Example 9.1]
   (a) Draw the topology of the attenuator.
   (b) Write down the design equations.
   (c) Complete the design of the attenuator.
4. A 20 dB attenuator in a 17 Ω system is ideally matched at both the input and output. Thus there are no reflections and the power delivered to the load is reduced by 20 dB from the applied power. If a 5 W signal is applied to the attenuator, how much power is dissipated in the attenuator?
5. A resistive Pi attenuator has shunt resistors of $R_1 = R_2 = 294\ \Omega$ and a series resistor $R_3 = 17.4\ \Omega$. What is the attenuation (in decibels) and the characteristic impedance of the attenuator?
6. Design a resistive Pi attenuator with an attenuation of 10 dB in a 100 Ω system.
7. Design a 3 dB resistive Pi attenuator in a 50 Ω system.
8. A resistive Pi attenuator has shunt resistors $R_1 = R_2 = 86.4\ \Omega$ and a series resistor $R_3 = 350\ \Omega$. What is the attenuation (in decibels) and the system impedance of the attenuator?
9. A balun can be realized using a wire-wound transformer, and by changing the number of windings on the transformer it is possible to achieve impedance transformation as well as balanced-to-unbalanced functionality. A 500 MHz balun based on a magnetic transformer is required to achieve impedance transformation from an unbalanced impedance of 50 Ω to a balanced impedance of 200 Ω. If there are 20 windings on the balanced port of the balun transformer, how many windings are there on the unbalanced port of the balun?

9.11.1 Exercises by Section

†challenging

§9.2 1, 2, 3, 4 §9.4 $5^{\dagger}$, 6, 7, $8^{\dagger}$ §9.7 $9^{\dagger}$

9.11.2 Answers to Selected Exercises

1 12.57
2 R_1=R_2=74.5 Ω
3 75.48 Ω
4 4.95 W

CHAPTER 10

Impedance Matching

10.1 Introduction

The maximum transfer of signal power is one of the prime objectives in RF and microwave circuit design. Power traverses a network from a source to a load generally through a sequence of two-port networks. Maximum power transfer requires that the Thevenin equivalent impedance of a source be matched to the impedance seen from the source. That is, the source should be presented with the complex conjugate of the source impedance. This is achieved by designing what is called a matching network inserted between the source and load. Design of the matching network is called impedance matching.

Section 10.2 describes two common matching objectives. Then design approaches for impedance matching are presented first with an algorithmic approach in Sections 10.3–10.6 and then a graphical approach based on using a Smith chart in Sections 10.7 and 10.9.

10.2 Matching Networks

Matching networks are constructed using lossless elements such as lumped capacitors, lumped inductors and transmission lines and so have, ideally, no loss and introduce no additional noise. This section discusses matching objectives and the types of matching networks.

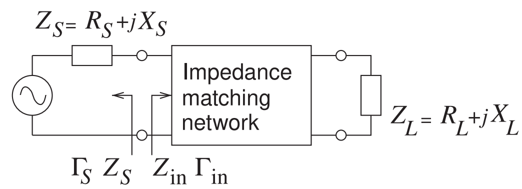


Figure 10-1: A source with Thevenin equivalent impedance Z_S and load with impedance Z_L interfaced by a matching network presenting an impedance Z_{in} to the source.

10.2.1 Matching for Zero Reflection or for Maximum Power Transfer

With RF circuits the aim of matching is to achieve maximum power transfer. With reference to Figure 10-1 the condition for maximum power transfer is $Z_{\text{in}} = Z_S^*$. which is equivalent to $\Gamma_{\text{in}} = \Gamma_S^*$. The proof is as follows:

$$\Gamma_{\text{in}} = \left(\frac{Z_{\text{in}} - Z_{\text{REF}}}{Z_{\text{in}} + Z_{\text{REF}}}\right), \tag{10.1}$$

and for maximum power transfer $Z_{\text{in}} = Z_S^*$, so

$$\Gamma_{\text{in}}^* = \frac{Z_{\text{in}} - Z_{\text{REF}}}{Z_{\text{in}} + Z_{\text{REF}}} = \left(\frac{Z_S^* - Z_{\text{REF}}}{Z_S^* + Z_{\text{REF}}}\right)^* = \frac{(Z_S^* - Z_0)^*}{(Z_S^* + Z_0)^*}$$
$$= \frac{(Z_S^*)^* - Z_{\text{REF}}^*}{(Z_S^*)^* + Z_{\text{REF}}^*} = \frac{Z_S - Z_{\text{REF}}^*}{Z_S + Z_{\text{REF}}^*} = \Gamma_S. \tag{10.2}$$

If Z_{REF} is real, $Z_{\text{REF}}^* = Z_{\text{REF}}$ and then the condition for maximum power transfer is

$$\Gamma_{\text{in}}^* = \frac{Z_S - Z_{\text{REF}}}{Z_S + Z_{\text{REF}}} = \Gamma_S. \tag{10.3}$$

Thus, provided that Z_{REF} is real, the condition for maximum power transfer in terms of reflection coefficients is $\Gamma_{\text{in}}^* = \Gamma_S$ or $\Gamma_{\text{in}} = \Gamma_S^*$.

10.2.2 Types of Matching Networks

Up to a few gigahertz, lumped inductors and capacitors can be used in matching networks. Above a few gigahertz parasitics result in self-resonance. Lumped elements are also lossy. Segments of transmission lines are also used in matching networks as the loss of a transmission line component is always much less than the loss of a lumped inductor. However the length of a transmission line segement is up to $\lambda/4$ which is far too large to fit in consumer wireless products operating below a few gigahertz.
An impedance matching network may consist of

(a) Lumped elements only. These are the smallest networks, but have the most stringent limit on the maximum frequency of operation. The relatively high resistive loss of an inductor is the main limiting factor limiting performance. The self resonant frequency of an inductor limits operation to low microwave frequencies.
(b) Distributed elements (microstrip or other transmission line circuits) only. These have excellent performance, but their size restricts their use in systems to above a few gigahertz.
(c) A hybrid design combining lumped and distributed elements, primarily small sections of lines with capacitors. These lines are shorter than in a design with distributed elements only, but the hybrid design has higher performance than a lumped-element-only design.

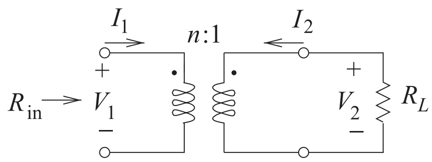


Figure 10-2: A transformer as a matching network. Port 1 is on the left or primary side and Port 2 is on the right or secondary side.

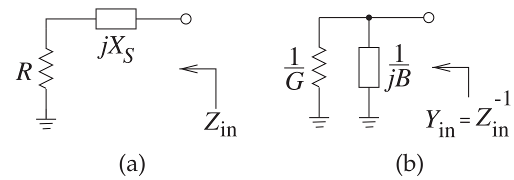


Figure 10-3: Matching using a series reactance: (a) the series reactive element; and (b) its equivalent shunt circuit.

10.3 Impedance Transforming Networks

Transformers and reactive elements considered in this section can be used to losslessly transform impedance levels. his is a basic aspect of network design.

10.3.1 The Ideal Transformer

The ideal transformer shown in Figure 10-2 can be used to match a load to a source if the source and load impedances are resistances. This will be shown by starting with the constitutive relations of the transformer:

$$V_1 = nV_2 \quad \text{and} \quad I_1 = -I_2/n. \tag{10.4}$$

Here n is the transformer ratio. For a wire-wound transformer, n is the ratio of the number of windings on the primary side, Port 1, to the number of windings on the secondary side, Port 2. Thus the input resistance, R_{in}, is related to the load resistance, R_L, by

$$R_{\text{in}} = \frac{V_1}{I_1} = -n^2\frac{V_2}{I_2} = n^2 R_{\text{L}}. \tag{10.5}$$

The matching problem with purely resistive load and source impedances is solved by choosing the appropriate winding ratio, n. However, resistive-only problems are rare at RF, and so other matching circuits must be used.

10.3.2 A Series Reactive Element

Matching using lumped elements is based on the impedance and admittance transforming properties of series and shunt reactive elements. Even a single reactive element can achieve limited impedance matching. Consider the series reactive element shown in Figure 10-3(a). Here the reactive element, X_S, is in series with a resistance R. The shunt equivalent of this network is shown in Figure 10-3(b) with a shunt susceptance of B. In this transformation the resistance R has been converted to a resistance $R_P = 1/G$. The mathematics describing this transformation is as follows. The input admittance of the series connection (Figure 10-3(a)) is

$$Y_{\text{in}}(\omega) = \frac{1}{Z_{\text{in}}(\omega)} = \frac{1}{R + \jmath X_S} = \frac{R}{R^2 + X_S^2} - \jmath\frac{X_S}{R^2 + X_S^2}. \tag{10.6}$$

Thus the elements of the equivalent shunt network, Figure 10-3(b), are

$$G = \frac{R}{R^2 + X_S^2} \quad \text{and} \quad B = -\frac{X_S}{R^2 + X_S^2}. \tag{10.7}$$

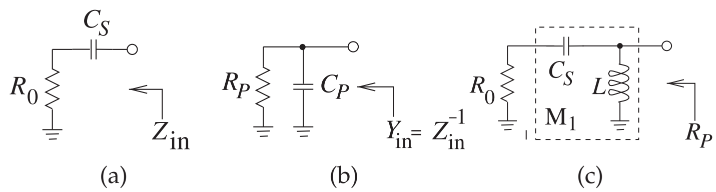


Figure 10-4: Impedance transformation by a series reactive element: (a) a resistor with a series capacitor; (b) its equivalent shunt circuit; and (c) an LC network.

The "resistance" of the network, R, has been transformed to a new value,

$$R_P = G^{-1} = \frac{R^2 + X_S^2}{R} > R. \tag{10.8}$$

This is an important start to matching, as X_S can be chosen to convert R (a load, for example) to any desired resistance value (such as the resistance of a source). However there is still a residual reactance that must be removed to complete the matching network design. Before moving on to the solution of this problem consider the following example.

EXAMPLE 10.1 Capacitive Impedance Transformation

Consider the impedance transforming properties of the capacitive series element in Figure 10-4(a). Show that the capacitor can be adjusted to obtain any positive shunt resistance.

Solution:

The concept here is that the series resistor and capacitor network has an equivalent shunt circuit that includes a capacitor and a resistor. By adjusting C_S any value can be obtained for R_P. From Equation (10.8),

$$R_P = \frac{R_0^2 + (1/\omega^2 C_S^2)}{R_0} = \frac{1 + \omega^2 C_S^2 R_0^2}{\omega^2 C_S^2 R_0} \tag{10.9}$$

and the susceptance is $$B = \frac{(1/\omega C_S)}{R_0^2 + 1/\omega^2 C_S^2} = \omega \frac{C_S}{1 + \omega^2 C_S^2 R_0^2}. \tag{10.10}$$

Thus $$C_P = \frac{B}{\omega} = \frac{C_S}{1 + \omega^2 C_S^2 R_0^2}. \tag{10.11}$$

To match R_0 to a resistive load R_P ($> R_0$) at a radian frequency ω_d, then, from Equation (10.9), the series capacitance required, i.e. the design equation for C_S, comes from

$$\omega_d C_S = 1/\sqrt{R_0 R_P - R_0^2}, \tag{10.12}$$

To complete the matching design, use a shunt inductor L, as shown in Figure 10-4(c), where $\omega_d C_P = 1/(\omega_d L)$. The equivalent impedance in Figure 10-4(c) is a resistor of value R_P, with a value that can be adjusted by choosing C_S which then requires L to be adjusted.

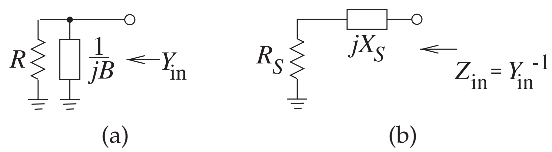


Figure 10-5: A resistor with (a) a shunt parallel reactive element where B is a susceptance, and (b) its equivalent series circuit.

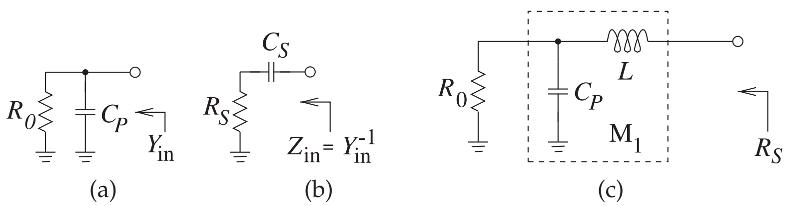


Figure 10-6: Parallel-to-series transformation: (a) resistor with shunt capacitor; (b) its equivalent series circuit; and (c) the transforming circuit with added series inductor.

10.3.3 A Parallel Reactive Element

The dual of the series matching procedure is the use of a parallel reactive element, as shown in Figure 10-5(a). The input admittance of the shunt circuit

$$Y_{\text{in}} = \frac{1}{R} + \jmath B. \tag{10.13}$$

This can be converted to a series circuit by calculating $Z_{\text{in}} = 1/Y_{\text{in}}$:

$$Z_{\text{in}} = \frac{R}{1 + \jmath BR} = \frac{R}{1 + B^2R^2} - \jmath\frac{BR^2}{1 + B^2R^2}. \tag{10.14}$$

So $$R_S = \frac{R}{1 + B^2R^2} \quad \text{and} \quad X_S = \frac{-BR^2}{1 + B^2R^2}. \tag{10.15}$$

Notice that $R_S < R$.

EXAMPLE 10.2 Parallel Tuning

As an example of the use of a parallel reactive element to tune a resistance value, consider the circuit in Figure 10-6(a) where a capacitor tunes the effective resistance value so that the series equivalent circuit (Figure 10-6(b)) has elements

$$R_S = \frac{R_0}{1 + \omega^2 C_P^2 R_0^2} \quad \text{and} \quad X_S = -\frac{\omega C_P R_0^2}{1 + \omega^2 C_P^2 R_0^2} = -\frac{1}{\omega C_S}. \tag{10.16}$$

So $$C_S = \frac{1 + \omega^2 C_P^2 R_0^2}{\omega^2 C_P R_0^2}. \tag{10.17}$$

Now consider matching R_0 to a resistive load R_S, which is less than R_0 at a given frequency ω_d. This requires that

$$\omega_d C_P = \sqrt{1/(R_S R_0) - 1/R_0^2}.$$

To complete the design, use a series inductor to remove the reactive effect of the capacitor, as shown in Figure 10-6(C). The value of the inductor required is found from

$$\omega_d L = \frac{1}{\omega_d C_S}, \quad \text{that is,} \quad L = \frac{1}{\omega_d^2 C_S}. \tag{10.18}$$

10.4 The L Matching Network

The examples in the previous two sections suggest the basic concept behind lossless matching of two different resistance levels using an L network:

Step 1: Use a series (shunt) reactive element to transform a smaller (larger) resistance up (down) to a larger (smaller) value with a real part equal to the desired resistance value.

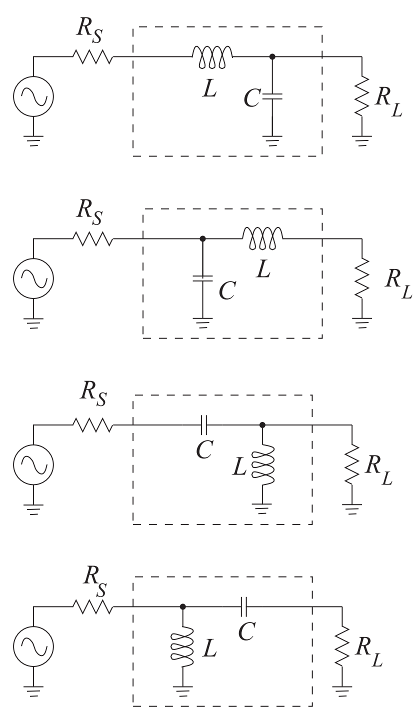
Step 2: Use a shunt (series) reactive element to resonate with (or cancel) the imaginary part of the impedance that results from Step 1.

So a resistance can be transformed to any resistive value by using an LC transforming circuit. A summary of the L matching networks is given in Figure 10-7. The two possible cases, $R_S < R_L$ and $R_L < R_S$, will be considered in the following subsections.

10.4.1 Design Equations for $R_S < R_L$

Consider the matching network topology of Figure 10-8. Here

$$Z_{\text{in}} = \frac{R_L(\jmath X_P)}{R_L + \jmath X_P} = \frac{R_L X_P^2}{R_L^2 + X_P^2} + \jmath \frac{X_P R_L^2}{R_L^2 + X_P^2} \tag{10.19}$$



(a) Lowpass $R_S < R_L$.
$Q = \sqrt{R_L/R_S - 1}$
$Q = |X_L|/R_S = R_L/|X_C|$

(b) Lowpass $R_S > R_L$.
$Q = \sqrt{R_S/R_L - 1}$
$Q = |X_L|/R_L = R_S/|X_C|$

(c) Highpass $R_S < R_L$.
$Q = \sqrt{R_L/R_S - 1}$
$Q = |X_C|/R_S = R_L/|X_L|$

(d) Highpass $R_S > R_L$.
$Q = \sqrt{R_S/R_L - 1}$
$Q = |X_C|/R_L = R_S/|X_L|$

Figure 10-7: L matching networks consisting of one shunt reactive element and one series reactive element. (R_S is matched to R_L.) X_C is the reactance of the capacitor C, and X_L is the reactance of the inductor L. Note that with a two-element matching network the Q and thus bandwidth of the match is fixed.

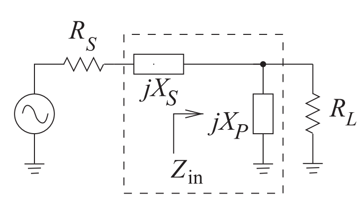


Figure 10-8: Two-element matching network topology for $R_S < R_L$. X_S is the series reactance and X_P is the parallel reactance.

and the matching objective is $Z_{\text{in}} = R_S - \jmath X_S$ so that

$$R_S = \frac{R_L X_P^2}{R_L^2 + X_P^2} \quad \text{and} \quad X_S = \frac{-X_P R_L^2}{X_P^2 + R_L^2}. \tag{10.20}$$

From these $$\frac{R_S}{R_L} = \frac{1}{(R_L/X_P)^2 + 1} \quad \text{and} \quad -\frac{X_S}{R_S} = \frac{R_L}{X_P}. \tag{10.21}$$

Introducing the quantities

$$Q_S = \text{the } Q \text{ of the series leg} = |X_S/R_S| \tag{10.22}$$
$$Q_P = \text{the } Q \text{ of the shunt leg} = |R_L/X_P| \tag{10.23}$$

leads to the final design equations for $R_S < R_L$:

$$|Q_S| = |Q_P| = \sqrt{\frac{R_L}{R_S} - 1}. \tag{10.24}$$

The L matching network principle is that X_P and X_S will be either capacitive or inductive and they will have the opposite sign (i.e., the L matching network comprises one inductor and one capacitor). Also, once R_S and R_L are given, the Q of the network and thus bandwidth is defined; with the L network, the designer does not have a choice of circuit Q.

EXAMPLE 10.3 Matching Network Design

Design a circuit to match a 100 Ω source to a 1700 Ω load at 900 MHz. Assume that a DC voltage must also be transferred from the source to the load.

Solution:

Here $R_S < R_L$, and so the topology of Figure 10-9(a) can be used and there are two versions, one with a series inductor and one with a series capacitor. The series inductor version (see Figure 10-9(b)) is chosen as this enables DC bias to be applied. From Equations (10.22)–(10.24) the design equations are

$$|Q_S| = |Q_P| = \sqrt{\frac{1700}{100} - 1} = \sqrt{16} = 4, \quad \frac{X_S}{R_S} = 4, \quad \text{and} \quad X_S = 4 \cdot 100 = 400. \tag{10.25}$$

This indicates that $\omega L = 400\ \Omega$, and so the series element is

$$L = \frac{400}{2\pi \cdot 9 \cdot 10^8} = 70.7\ \text{nH}. \tag{10.26}$$

For the shunt element next to the load, $|R_L/X_C| = 4$, and so

$$|X_C| = \frac{R_L}{4} = \frac{1700}{4} = 425. \tag{10.27}$$

Thus $1/\omega C = 425$ and $$C = \frac{1}{2\pi \cdot 9 \cdot 10^8 \cdot 425} = 0.416\ \text{pF}. \tag{10.28}$$

The final matching network design is shown in Figure 10-9(c).

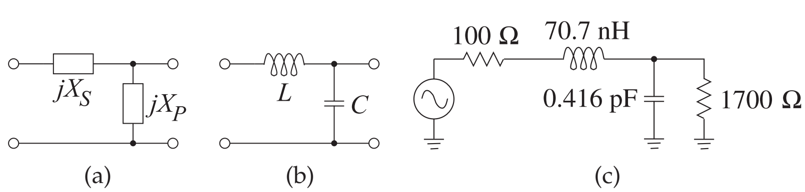


Figure 10-9: Matching network development for Example 10.3.

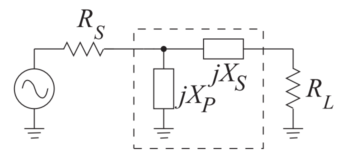


Figure 10-10: Two-element matching network topology for $R_S > R_L$.

10.4.2 L Network Design for $R_S > R_L$

For $R_S > R_L$, the topology shown in Figure 10-10 is used. The design equations for the L network for $R_S > R_L$ are similarly derived and are

$$|Q_S| = |Q_P| = \sqrt{\frac{R_S}{R_L} - 1} \tag{10.29}$$

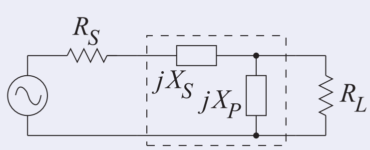
$$-Q_S = Q_P, \quad Q_S = \frac{X_S}{R_L}, \quad \text{and} \quad Q_P = \frac{R_S}{X_P}. \tag{10.30}$$

EXAMPLE 10.4 Two-Element Matching Network

Design a passive two-element matching network that will achieve maximum power transfer from a source with an impedance of 50 Ω to a load with an impedance of 75 Ω. Choose a matching network that will not allow DC to pass.

Solution:

$R_L > R_S$, so, from Figure 10-7, the appropriate matching network topology is



This topology can be either high pass or low pass depending on the choice of X_S and X_P. Design proceeds by finding the magnitudes of X_S and X_P. In two-element matching the circuit Q is fixed. With $R_L = 75\ \Omega$ and $R_S = 50\ \Omega$.

The Q of the matching network is the same for the series and parallel elements:

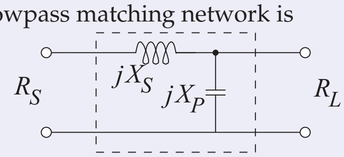
$$|Q_S| = \frac{|X_S|}{R_S} = \sqrt{\frac{R_L}{R_S} - 1} = 0.7071 \quad \text{and} \quad |Q_P| = \frac{R_L}{|X_P|} = |Q_S| = 0.7071,$$

therefore $|X_S| = R_S \cdot |Q_S| = 50 \cdot 0.7071 = 35.35\ \Omega$. Also

$$|X_P| = R_L/|Q_P| = 75/0.7071 = 106.1\ \Omega.$$

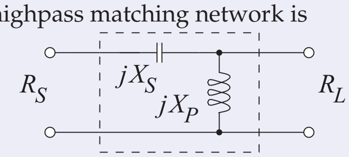
Specific element types can now be assigned to X_S and X_P, and note that they must be of opposite type.

The lowpass matching network is



$X_S = +35.35\ \Omega, \quad X_P = -106.1\ \Omega.$

The highpass matching network is



$X_S = -35.35\ \Omega, \quad X_P = +106.1\ \Omega.$

This highpass design satisfies the design criterion that DC is not passed, as DC is blocked by the series capacitor.

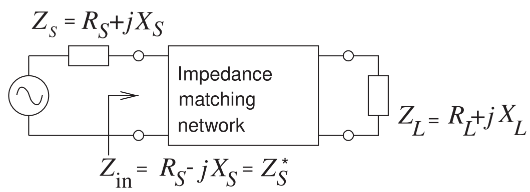


Figure 10-11: A matching network matching a complex load to a source with a complex Thevenin impedance.

10.5 Dealing with Complex Sources and Loads

This section presents strategies for dealing with complex loads. In the algorithmic matching approach design proceeds first by ignoring the complex load and source and then accounting for them either through topology choice or canceling their effect through resonance.

10.5.1 Matching

Input and output impedances of transistors, mixers, antennas, etc., contain both resistive and reactive components. Thus a realistic impedance matching problem looks like that shown in Figure 10-11. The matching approaches that were presented in the previous sections can be directly applied if X_S and X_L are treated as stray reactances that need to be either canceled or, ideally, used as part of the matching network. There are two basic approaches to handling complex impedances:

(a) Absorption: Absorb source and load reactances into the impedance matching network itself. This is done through careful placement of each matching element such that capacitors are placed in parallel with source and load capacitances, and inductors in series with source and load inductances. The stray values are then subtracted from the L and C values for the matching network calculated on the basis of the resistive parts of Z_S and Z_L only. The new (smaller) values, L' and C', constitute the elements of the matching network. Sometimes it is necessary to perform a series-to-parallel, or parallel-to-series, conversion of the source or load impedances so that the reactive elements are in the correct series or shunt arrangement for absorption.

(b) Resonance: Resonate source and load reactances with an equal and opposite reactance at the frequency of interest.

The presence of reactance in a load indicates energy storage, and therefore bandwidth limiting. In the above approaches to handling a reactive load, the resonance approach could easily result in a narrowband matching solution. The major problem in matching is often to obtain sufficient bandwidth. What is sufficient will vary depending on the application. To maximize bandwidth the general goal is to minimize the total energy storage. Roughly the total energy stored will be proportional to the sum of the magnitudes of the reactances in the circuit. Of course, the actual energy storage depends on the voltage and current levels, which will themselves vary in the circuit. A good approach leading to large bandwidths is to incorporate the load reactance into the matching network. Thus the choice of appropriate matching network topology is critical. However, if the source and load reactance value is larger than the calculated matching network element value, then absorption on its own cannot be used. In this situation resonance must be combined with absorption. The majority of impedance matching designs are based on a combination of resonance and absorption.

EXAMPLE 10.5 Matching Network Design Using Resonance

For the configuration shown in Figure 10-12, design an impedance matching network that will block the flow of DC current from the source to the load. The frequency of operation is 1 GHz. Design the matching network, neglecting the presence of the 10 pF capacitance at the load. Since $R_S = 50\ \Omega < R_L = 500\ \Omega$, and from Figure 10-7, consider the topologies of Figures 10-13(a) and 10-13(b). The design criterion of blocking flow of DC from the source to the load narrows the choice to the topology of Figure 10-13(b).

Solution:

Step 1: $$|Q_S| = \left|\frac{X_S}{R_S}\right| = |Q_P| = \left|\frac{R_L}{X_P}\right| = \sqrt{\frac{R_L}{R_S} - 1} = 3 \quad \text{and} \quad Q_P = \frac{R_L}{X_P}. \tag{10.31}$$

So $X_P = \omega L = R_L/Q_P = 500/3$. Reducing this gives

$$\omega L = \frac{500}{3}, \quad \text{and so} \quad L = \frac{500}{3 \times 2\pi \times 10^9} = 26.5\ \text{nH}. \tag{10.32}$$

Similarly $-X_S/R_S = 3$ and so

$$\frac{1/(\omega\, C)}{R_S} = 3 \quad \text{or} \quad C = \frac{1}{3\omega R_S} = \frac{1}{3 \times 2\pi \times 10^9 \times 50} = 1.06\ \text{pF}. \tag{10.33}$$

Step 2:

Resonate the 10 pF capacitor using an inductor in parallel:

$$\left(\omega L'\right)^{-1} = \omega \times 10 \times 10^{-12} \tag{10.34}$$

$$L' = \frac{1}{\omega^2 10^{-11}} = \frac{1}{(2\pi)^2\, 10^{18} \times 10^{-11}} = 2.533\ \text{nH}. \tag{10.35}$$

Thus Figure 10-13(c) is the required matching network. Two inductors are in parallel and the circuit can be simplified to that shown in Figure 10-14, where

$$L_X = (26.5\ \text{nH} \parallel 2.533\ \text{nH}) = \frac{2.533 \times 26.5}{2.533 + 26.5}\ \text{nH} = 2.312\ \text{nH}. \tag{10.36}$$

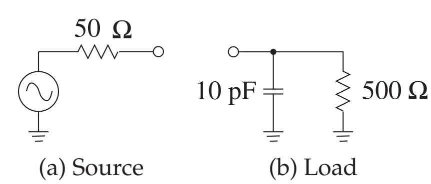


Figure 10-12: Matching problem considered in Example 10.5.

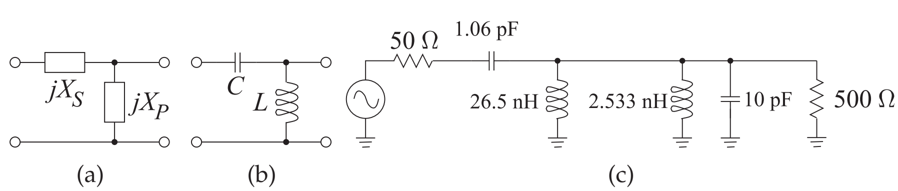


Figure 10-13: Matching network topology used in Example 10.5: a) and (b) topology; and (c) intermediate matching network.

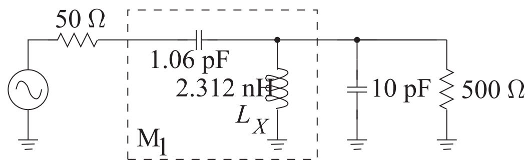


Figure 10-14: Final matching network in Example 10.5.

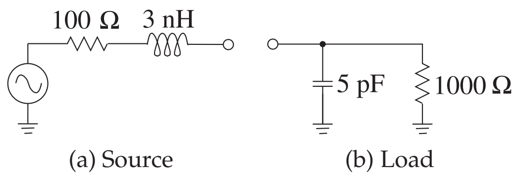


Figure 10-15: Matching problem in Example 10.6.

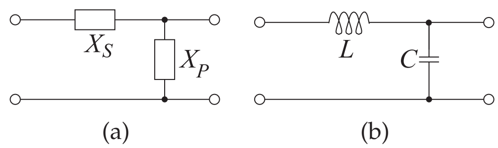


Figure 10-16: Topologies referred to in Example 10.6.

EXAMPLE 10.6 Matching Network Design Using Resonance and Absorption

For the source and load configurations shown in Figure 10-15, design a lowpass impedance matching network at f = 1 GHz.

Solution:

Since $R_S < R_L$, use the topology shown in Figure 10-16(a). For a lowpass response, the topology is that of Figure 10-16(b). Notice that absorption is the natural way of handling the 3 nH at the source and the 5 pF at the load. The design process is as follows:

Step 1:

Design the matching network, neglecting the reactive elements at the source and load:

$$|Q_S| = |Q_P| = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{10 - 1} = 3 \tag{10.37}$$

$$\frac{X_S}{R_S} = 3, \quad X_S = 3 \times 100, \quad \omega L = 300 \quad \text{and} \quad L = \frac{300}{2\pi \times 10^9} = 47.75 \text{ nH} \tag{10.38}$$

$$\frac{R_P}{X_P} = -3 \quad \text{and} \quad \frac{1000}{-(1/\omega C)} = -3 \quad \text{and} \quad C = \frac{3}{1000 \times 2\pi \times 10^9} = 0.477 \text{ pF.} \tag{10.39}$$

This design is shown in Figure 10-17(a). This is the matching network that matches the 100 Ω source resistance to the 1000 Ω load with the source and load reactances ignored.

Step 2:

Figure 10-17(b) is the interim matching solution. The source inductance is absorbed into the matching network, reducing the required series inductance of the matching network. The capacitance of the load cannot be fully absorbed. The design for the resistance-only case requires a shunt capacitance of 0.477 pF, but 5 pF is available from the load. Thus there is an excess capacitance of 4.523 pF that must be resonated out by the inductance L'':

$$\frac{1}{\omega L''} = \omega 4.523 \times 10^{-12}. \quad \text{So} \quad L'' = \frac{1}{(2\pi)^2 \times 10^{18} \times 4.523 \times 10^{-12}} = 5.600 \text{ nH.} \tag{10.40}$$

The final matching network design (Figure 10-17(c)) fully absorbs the source inductance into the matching network, but only partly absorbs the load capacitance.

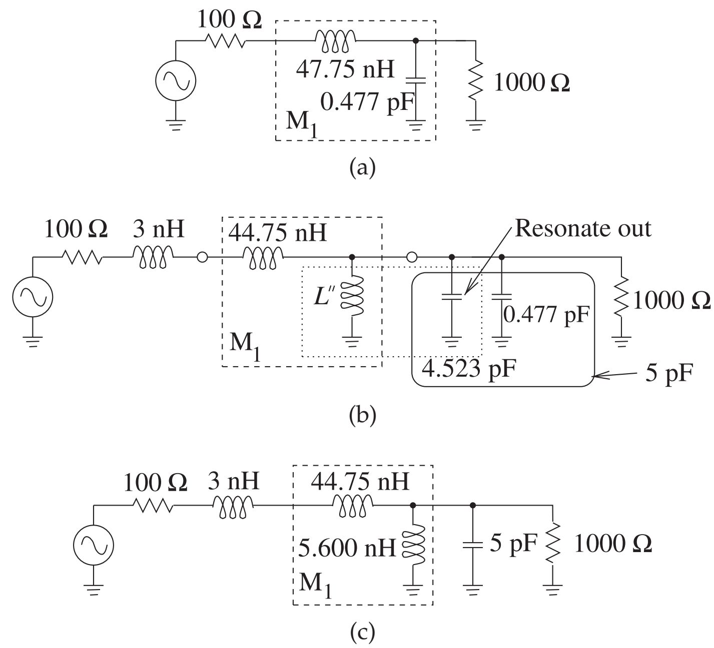


Figure 10-17: Evolution of the matching network in Example 10.6: (a) matching network design considering only the source and load resistors; (b) matching network with the reactive parts of the source and load impedances included; and (c) final design.

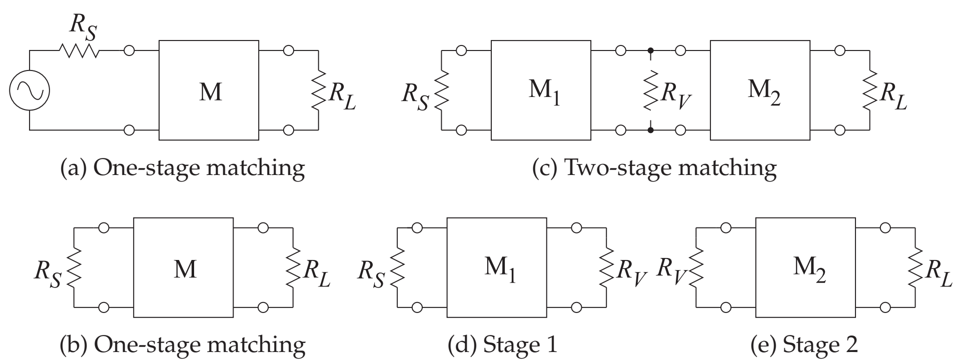


Figure 10-18: Matching in stages: (a) matching network M matching R_S to R_L; (b) without an explicit source; (c) two-stage matching with a virtual resistor R_V; (d) matching R_S to R_V; and (e) matching R_V to R_L.

10.6 Multielement Matching

The bandwidth of a matching network can be controlled by using multiple matching stages either making the matching bandwidth wider or narrower. This concept is elaborated on in this and several design approaches presented.

10.6.1 Design Concept for Manipulating Bandwidth

The concept for manipulating matching network bandwidth is to do the matching in stages as shown in Figure 10-18. Figure 10-18(a) shows the one-stage matching problem using the common identification of the matching network as 'M'. The one-stage matching problem is repeated in Figure 10-

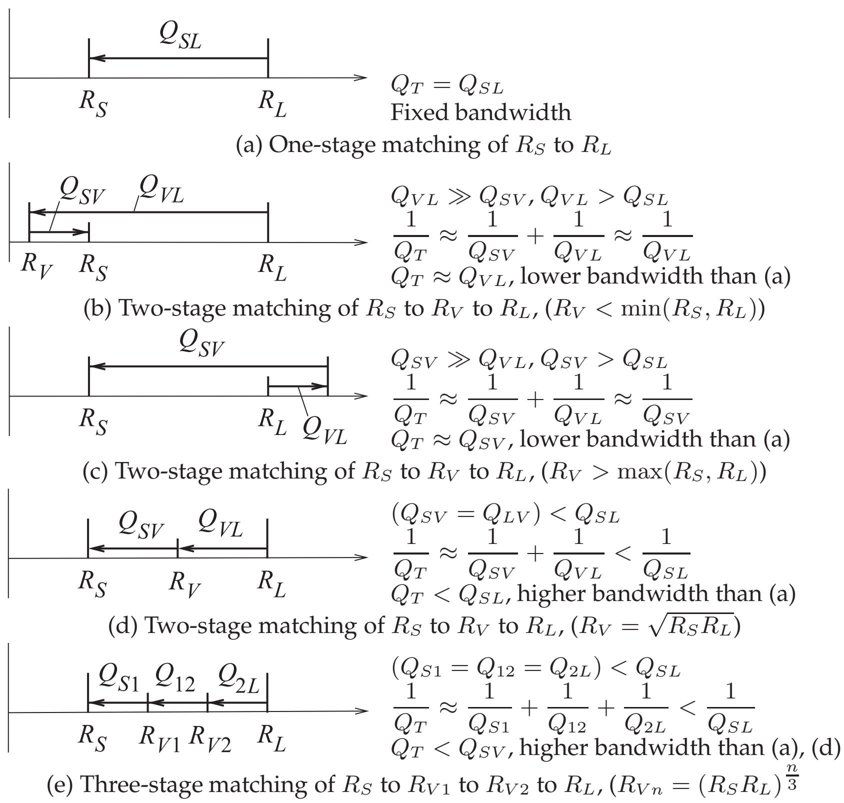


Figure 10-19: Effect of multi-stage matching on total circuit Q, Q_T, and matching bandwidth (which is approximately inversely proportional to Q_T.)

18(b) without explicitly showing the source generator. A two-stage matching problem is shown in Figure 10-18(c) with the introduction of a virtual resistor R_V between the first, M_1, and second, M_2, stage matching networks. R_V is shown as a virtual connection as it is not actually inserted in the circuit. Instead this is a short-hand way of indicating the matching problem to be done in two stages as shown in Figure 10-18(d and e) with the first stage matching the source resistor R_S to R_V and the second stage matching R_V to the load resistor R_L. After M_1 has been designed the resistance looking into the right-hand port of M_1, see Figure 10-18(d), will be R_V so R_V is the Thevenin equivalent source resistance to M_2. Similarly the input impedance looking into the left-hand port of M_2 is R_V so R_V is the effective load resistor of M_1. Of course these are the impedances at the center frequency and away from the center frequency of the match the input impedances will be complex.

The concept behind multi-stage matching network design is shown in Figure 10-19 where the standard one-stage match is shown in Figure 10-19(a). While this is shown for $R_L > R_S$ the concept holds for $R_L < R_S$. The arrows follow the that design begins with the load and ends at the source. With the one stage match the circuit Q is fixed and designated here as the total circuit Q, Q_T being the same as the Q of the one-stage R_L to R_S matching network, Q_{SL}. The two-stage match that reduces bandwidth (compared to the one-stage match) is shown in Figure 10-19(b). The total Q, Q_T, of the second stage is higher than for the one-stage design because the ratio of R_L to R_V is greater than the ratio of R_V to R_S. Bandwidth can also be reduced relative to the one-stage match by assigning R_V to be greater than both R_L

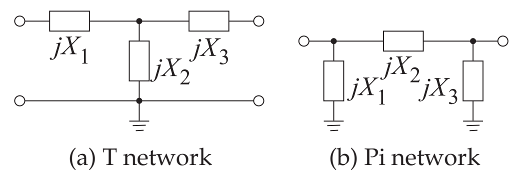


Figure 10-20: Two three-element matching networks.

and R_S, see Figure 10-19(c).

Choosing R_V to be between R_S and R_L will result in a circuit with lower Q_T and the bandwidth of the match will increase, see Figure 10-19(d). The maximum bandwidth for a two-stage match is to choose R_V as the geometric mean of R_S and R_L. This concept can be extended to multiple stages as shown for a three-stage match in Figure 10-19(e).

This section presents various matching network designs for manipulating bandwidth and all are based on the concept of choosing a virtual resistor.

10.6.2 Three-Element Matching Networks

With the L network (i.e., two-element matching), the circuit Q is fixed once the source and load resistances, R_S and R_L, are fixed:

$$Q = \sqrt{\frac{R_L}{R_S} - 1}, \quad (R_L > R_S). \tag{10.41}$$

Thus the designer does not have a choice of circuit Q. Breaking the matching problem into parts enables the circuit Q to be controlled. Introducing a third element in the matching network provides the extra degree of freedom in the design for adjusting Q, and hence bandwidth.

Two three-element matching networks, the T network and the Pi network, are shown in Figure 10-20. Which network is used depends on

(a) the realization constraints associated with the specific design, and
(b) the nature of the reactive parts of the source and load impedances and whether they can be used as part of the matching network.

The three-element matching network comprises 2 two-element (or L) matching networks and is used to increase the overall Q and thus narrow bandwidth. Given R_S and R_L, the circuit Q established by an L matching network is the minimum circuit Q available in the three-element matching arrangement. With three-element matching, the Q can only increase, so three-element matching is used for narrowband (high-Q) applications. However, lower Q can be obtained with more than three elements. The next subsections consider matching with more than three elements.

10.6.3 The Pi Network

The Pi network may be thought of as two back-to-back L networks that are used to match the load and the source to a virtual resistance, R_V, placed at the junction between the two networks, as shown in Figure 10-21(b). The design of each section of the Pi network is as for the L network matching. R_S is matched to R_V and R_V is matched to R_L.

R_V must be selected smaller than R_S and R_L since it is connected to the series arm of each L section. Furthermore, R_V can be any value that is smaller than the smaller of R_S, R_L. However, it is customarily used as the design parameter for specifying the desired Q.

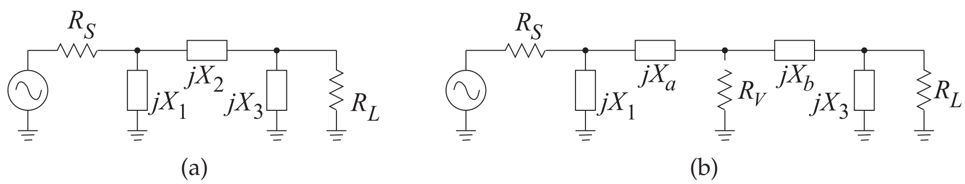


Figure 10-21: Pi matching networks: (a) view of a Pi network; and (b) as two back-to-back L networks with a virtual resistance, R_V, between the networks.

As a useful design approximation, the loaded Q of the Pi network can be taken as the Q of the L section with the highest Q:

$$Q = \sqrt{\frac{\max(R_S, R_L)}{R_V} - 1}. \tag{10.42}$$

Given R_S, R_L, and Q, the above equation yields the value of R_V.

EXAMPLE 10.7 Three-Element Matching Network Design

Design a Pi network to match a 50 Ω source to a 500 Ω load. The desired Q is 10. A suitable matching network topology is shown in Figure 10-22 together with the virtual resistance, R_V, to be used in design.

Solution: $R_S = 50\ \Omega$ and $R_L = 500\ \Omega$ so $\max(R_S, R_L) = 500\ \Omega$ and so the virtual resistor is

$$R_V = \frac{\max(R_S, R_L)}{Q^2 + 1} = \frac{500}{101} = 4.95\ \Omega. \tag{10.43}$$

Design proceeds by separately designing the L networks to the left and right of R_V. For the L network on the left,

$$Q_{\text{left}} = \sqrt{\frac{50}{4.95} - 1} = 3.017. \quad \text{so} \quad Q_{\text{left}} = \frac{|X_a|}{R_V} = \frac{R_S}{|X_1|} = 3.017, \tag{10.44}$$

Note that X_1 and X_a must be of opposite types (one is capacitive and the other is inductive). The left L network has elements

$$|X_a| = 14.933\ \Omega \quad \text{and} \quad |X_1| = 16.6\ \Omega. \tag{10.45}$$

For the L network on the right of R_V,

$$Q_{\text{right}} = Q = 10, \quad \text{thus} \quad \frac{|X_b|}{R_V} = \frac{R_L}{X_3} = 10. \tag{10.46}$$

X_b, X_3 are of opposite types, and

$$|X_b| = 49.5\ \Omega \quad \text{and} \quad |X_3| = 50\ \Omega. \tag{10.47}$$

The resulting Pi network is shown in Figure 10-23 with the values

$$|X_1| = 16.6\ \Omega, \quad |X_3| = 50\ \Omega, \quad |X_a| = 14.933\ \Omega, \quad \text{and} \quad |X_b| = 49.5\ \Omega. \tag{10.48}$$

Note that the pair X_a, X_1 are of opposite types and similarly X_b, X_3 are of opposite types. So there are four possible realizations, as shown in Figure 10-24.

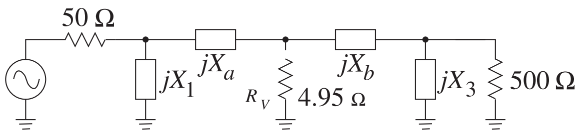


Figure 10-22: Matching network problem of Example 10.7.

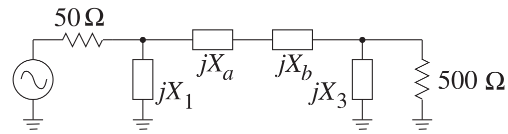


Figure 10-23: Final matching network in Example 10.7.

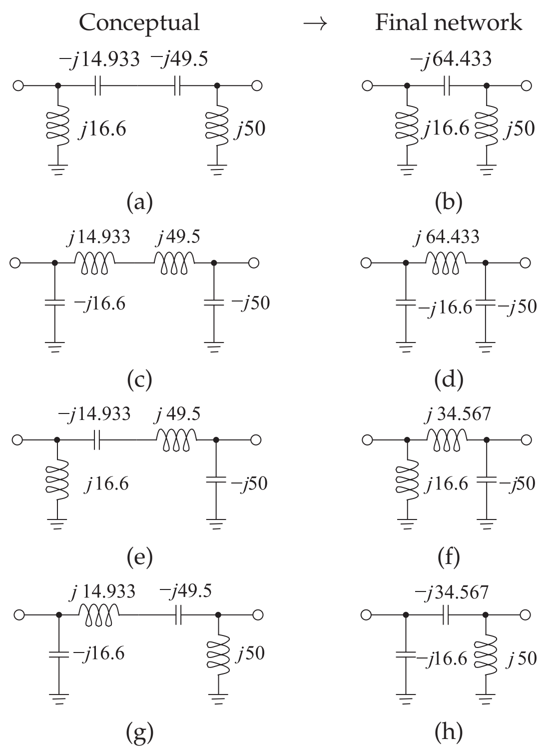


Figure 10-24: Four possible Pi matching networks: (a), (c), (e), and (g) conceptual circuits; and (b), (d), (f), and (h), respectively, their final reduced Pi networks.

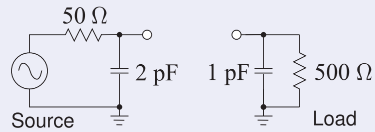
In the previous example there were four possible realizations of the three-element matching network, and this is true in general. The specific choice of one of the four possible realizations will depend on specific application-related factors such as

(a) elimination of stray reactances,
(b) the need to pass or block DC current, and
(c) the need for harmonic filtering.

It is fortunate that it may be possible to achieve multiple functions with the same network.

EXAMPLE 10.8 Three-Element Matching with Reactive Source and Load

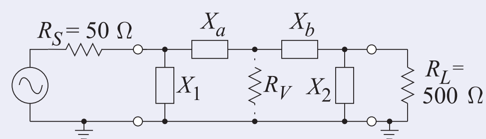
Design a Pi network to match the source to the load shown. The design frequency is 900 MHz and the desired Q is 10.



Solution:

The design objective is to arrive at an overall network which has a Q of 10. To achieve this it is necessary to absorb the source and load reactances into the matching network. If they were resonated instead, the overall Q of the network can be expected to higher than the Q of the L matching network on its own.

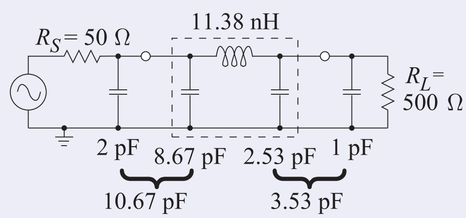
Design begins by considering the matching of $R_S = 50\ \Omega$ to $R_L = 500\ \Omega$. Since the Q is specified, three (or more) matching elements must be used. The design starting point is shown on the right:



The virtual resistor $R_V = \max(R_s, R_L)/(1+Q^2) = (500\ \Omega)/(1+100) = 4.95\ \Omega$. The left subnetwork with X_1 and X_a has $Q_{\text{LEFT}} = \sqrt{R_S/R_V - 1} = \sqrt{50/4.95 - 1} = 3.017$. The right subnetwork with X_2 and X_b has $Q_{\text{RIGHT}} = \sqrt{R_L/R_V - 1} = \sqrt{500/4.95 - 1} = 10.001$.

Note that Q_{RIGHT} is almost exactly the desired Q of the network and Q_{LEFT} will have little effect on the Q of the overall circuit. Now $Q_{\text{LEFT}} = |X_a|/R_V = R_S/|X_1|$, so $|X_a| = 14.9\ \Omega$ and $|X_1| = 16.57\ \Omega$. $Q_{\text{RIGHT}} = |X_b|/R_V = R_S/|X_2|$, so $|X_b| = 49.5\ \Omega$ and $|X_2| = 50.0\ \Omega$.

X_1 must be chosen to be a capacitor $C_1 = 10.67$ pF so that the 2 pF source capacitance can be absorbed. Similarly X_2 is a capacitor $C_2 = 3.53$ pF. X_a and X_b are both inductors that combine in series for a total inductance $L_3 = 11.38$ nH. This leads to the final design shown on the right where the matching network is in the dashed box.



10.6.4 Matching Network Q Revisited

To demonstrate that the circuit Q established by an L matching network is the minimum circuit Q for a network having at most three elements, consider the design equations for $R_S > R_L$. Referring to Figure 10-25,

$$X_1 = \frac{R_S}{Q},\ X_3 = R_L\left(\frac{R_S/R_L}{Q^2+1-R_S/R_L}\right)^{\frac{1}{2}},\ X_2 = \frac{QR_S + R_SR_L/X_3}{Q^2+1}. \quad (10.49)$$

Notice that the denominator of X_3 can be written as

$$Q^2 + 1 - \frac{R_S}{R_L} = \left(Q + \sqrt{\frac{R_S}{R_L} - 1}\right)\left(Q - \sqrt{\frac{R_S}{R_L} - 1}\right). \quad (10.50)$$

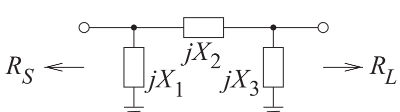
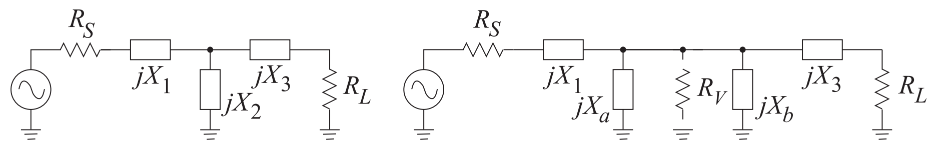
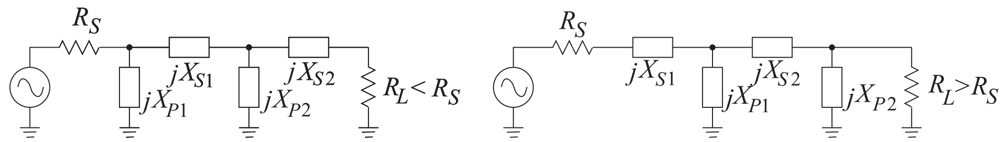


Figure 10-25: A three-element matching network.



(a) T network (b) Virtual intermediate matching network view

Figure 10-26: T network design approach.



(a) (b)

Figure 10-27: Broadband matching networks.

Then for a real solution we must have

$$Q \geq \sqrt{\frac{R_S}{R_L} - 1}, \tag{10.51}$$

and so $Q \geq Q_{\text{L network}}$. For $Q = Q_{\text{L network}},\quad X_3 \to \infty$ and the Pi network reduces to an L network that has two elements. Thus it is not possible to have a lower Q with a three-element matching network than the Q of a two-element matching network. Thus a three-element matching network must have lower bandwidth than that of a two-element matching network.

10.6.5 The T Network

The T network may be thought of as two back-to-back L networks that are used to match the load and the source to a virtual resistance, R_V, placed at the junction between the two L networks (see Figure 10-26). R_V must be selected to be larger than both R_S and R_L since it is connected to the shunt leg of each L section. R_V is chosen according to the equation

$$Q = \sqrt{\frac{R_V}{\min(R_S, R_L)} - 1}, \tag{10.52}$$

where Q is the desired loaded Q of the network. Each L network is calculated in exactly the same manner as was done for the Pi network matching. That is, R_S is matched to R_V and R_V is matched to R_L. Once again there will be four possible designs for the T network, given R_S, R_L, and Q.

10.6.6 Broadband (Low Q) Matching

L network matching does not allow the circuit Q, and hence bandwidth, to be selected. However, Pi network and T network matching allows the circuit Q to be selected independent of the source and load impedances, provided that the chosen Q is larger than that which can be obtained with an L network. Thus the Pi and T networks result in narrower bandwidth designs.

One design solution for broadband matching is to use two (or more) series-connected L sections (see Figure 10-27). Design is still based on the concept of a virtual resistor, R_V, placed at the junction of the two L networks (as in Figure 10-28), but now R_V is chosen to be between R_S and R_L:

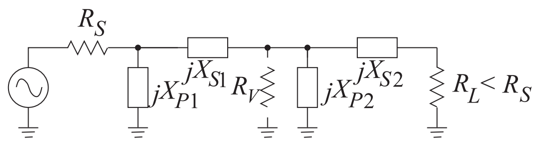


Figure 10-28: Matching network with two L networks.

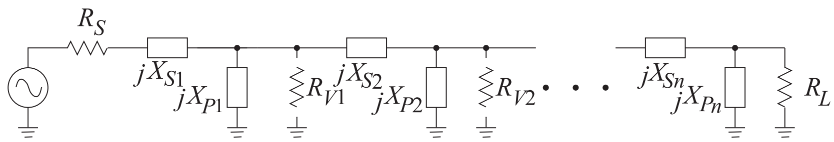


Figure 10-29: Cascaded L networks for broadband matching.

$$R_{\min} \leq R_V \leq R_{\max}, \tag{10.53}$$

where $R_{\min} = \min(R_L, R_S)$ and $R_{\max} = \max(R_L, R_S)$. Then one of the two networks will have

$$Q_1 = \sqrt{\frac{R_V}{R_{\min}} - 1} \quad \text{and the other} \quad Q_2 = \sqrt{\frac{R_{\max}}{R_V} - 1}. \tag{10.54}$$

The maximum bandwidth (minimum Q) available is obtained when

$$Q_1 = Q_2 = \sqrt{\frac{R_V}{R_{\min}} - 1} = \sqrt{\frac{R_{\max}}{R_V} - 1}\,. \tag{10.55}$$

That is, the maximum matching bandwidth is obtained when R_V is the geometric mean of R_S and R_L:

$$R_V = \sqrt{R_L R_S}. \tag{10.56}$$

Even wider bandwidths can be obtained by cascading more than two L networks, as shown in Figure 10-29. In this circuit

$$R_S < R_{V1} < R_{V2} \ldots < R_{Vn-1} < R_L. \tag{10.57}$$

For optimum bandwidth the ratios should be equal,

$$\frac{R_{V_1}}{R_S} = \frac{R_{V_2}}{R_{V_1}} = \frac{R_{V_3}}{R_{V_2}} = \cdots = \frac{R_L}{R_{V_{n-1}}}, \tag{10.58}$$

and the Q is given by

$$Q = \sqrt{\frac{R_{V_1}}{R_S} - 1} = \sqrt{\frac{R_{V_2}}{R_1} - 1} = \cdots = \sqrt{\frac{R_L}{R_{V_{n-1}}} - 1}. \tag{10.59}$$

If there are N L networks used in the match, the maximum bandwidth will be obtained if the ith virtual resistor is

$$R_{Vi} = (R_S R_L)^{i/N}, \quad i = 1, \ldots, (N-1). \tag{10.60}$$

EXAMPLE 10.9 Two-Section Matching Network Design

Consider matching a 10 Ω source to a 1000 Ω load using two L matching networks and designing for a Q of 3. How many matching sections are required?

Solution:

Here the approximate Qs achieved with a single L matching network and with an optimum two-section design are compared. For a single L network design

$$Q = \sqrt{\frac{R_L}{R_S} - 1} = 9.95. \tag{10.61}$$

Now consider an optimum two-section design:

$$R_V = \sqrt{R_S R_L};\ Q_2 = \sqrt{\frac{R_L}{R_V} - 1} = \sqrt{\sqrt{\frac{R_L}{R_S}} - 1} = 3. \tag{10.62}$$

Thus the Q is 3 compared to the Q of an L section of 9.95. If the fractional bandwidth is inversely proportional to Q, then the bandwidth of the two-section design is 9.95/3 = 3.32 times more than that of the L section.

Now consider how many sections are required to obtain a Q of 2:

$$(1 + Q^2) = \frac{R_{V_1}}{R_S} = \frac{R_{V_2}}{R_{V_1}} = \ldots = \frac{R_L}{R_{V_{n-1}}} \Rightarrow \tag{10.63}$$

$$\left(1 + Q^2\right)^n = \frac{R_L}{R_S} \Rightarrow n \ln\left(1 + Q^2\right) = \ln\frac{R_L}{R_S} \Rightarrow n = \frac{\ln(R_L/R_S)}{\ln\left(1 + Q^2\right)}. \tag{10.64}$$

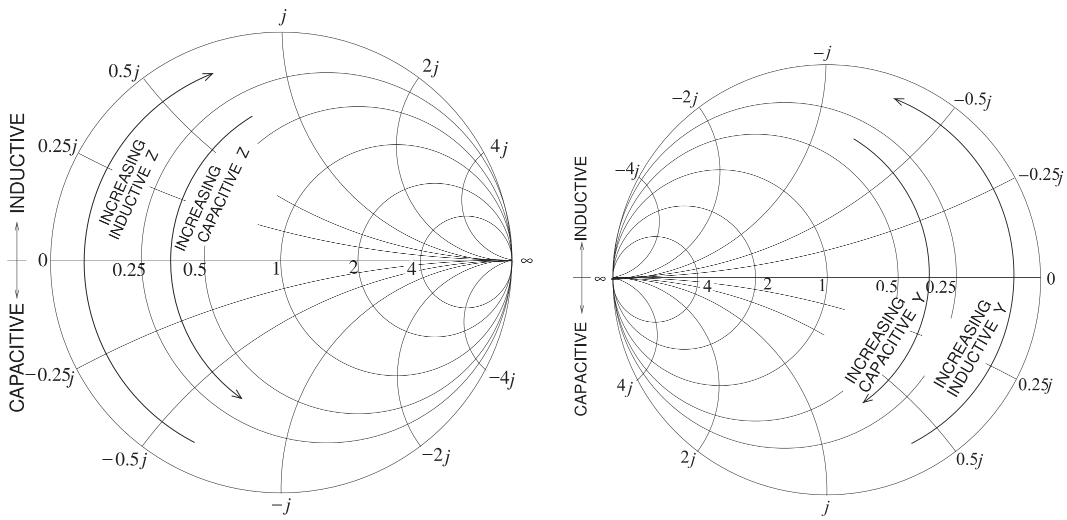
For $Q = 2$ and $R_L/R_S = 100$, $n = 2.86$, which rounds to $n = 3$, and three sections are required.

10.7 Impedance Matching Using Smith Charts

The lumped-element matching networks presented up to now can also be developed using Smith charts which provide a fairly intuitive approach to network design. With experience it will be found that this is the preferred approach to developing designs, as trade-offs can be captured graphically. Smith chart-based design will be presented using examples.

10.7.1 Two-Element Matching

The examples here build on the preceding lumped-element matching network design and now use the Smith. Capacitive and inductive regions on the Smith chart are shown in Figure 10-30. In the design examples presented here, circles of constant resistance or constant conductance are followed and these correspond to varying reactance or susceptance, respectively.



(a) Impedance Smith chart (b) Admittance Smith chart

Figure 10-30: Inductive and capacitive regions on Smith charts. Increasing capacitive impedance (Z) indicates smaller capacitance; increasing inductive admittance (Y) indicates smaller inductance.

EXAMPLE 10.10 Two-Element Matching Network Design Using a Smith Chart

Develop a two-element matching network to match a source with an impedance of R_S = 25 Ω to a load $R_L = 200\ \Omega$ (see Figure 10-31).

Solution:

The design objective is to present conjugate matched impedances to the source and load. However, since here the source and load impedances are real, the design objective is $Z_1 = R_S$ and $Z_2 = R_L$. The load and source resistances are plotted on the Smith chart in Figure 10-33(a) after choosing a normalization impedance of $Z_0 = 50\ \Omega$ (and so $r_S = R_S/Z_0 = 0.5$ and $r_L = R_L/Z_0 = 4$). The normalized source impedance, r_S, is Point A, and the normalized load impedance, r_L, is Point C. The matching network must be lossless, which means that the design must follow lines of constant resistance (on the impedance part of the Smith chart) or constant conductance (on the admittance part of the Smith chart). So Points A and C must be on the above circles and the circles must intersect if a design is possible. The design can be viewed as moving back from the source toward the load or moving back from the load toward the source. (The views result in identical designs.) Here the view taken is moving back from the source toward the load.

One possible design is shown in Figure 10-33(a). From Point A, the line of constant resistance is followed to Point B (there is increasing series reactance along this path). From Point B, the locus follows a line of constant conductance to the final point, Point C. There is also an alternative design that follows the path shown in Figure 10-33(b). There are only two designs that have a path from A to B following just two arcs. At this point two designs have been outlined. The next step is assigning element values.

The design shown in Figure 10-33(a) begins with r_S followed by a series reactance, x_S, taking the locus from A to B. Then a shunt capacitive susceptance, b_P, takes the locus from B to C and r_L. At Point A the reactance $x_A = 0$, at Point B the reactance $x_B = 1.323$. This value is read off the Smith chart, requiring that an arc as shown be interpolated between the arcs provided. It should be noted that not all versions of Smith charts include negative signs, as the chart becomes too complicated. Thus the user needs to be aware and add signs where appropriate. The normalized series reactance is

$$x_S = x_B - x_A = 1.323 - 0 = 1.323, \tag{10.65}$$

that is,
$$X_S = x_s Z_0 = 1.323 \times 50 = 66.1\ \Omega. \tag{10.66}$$

A shunt capacitive element takes the locus from Point B to Point C and

$$b_P = b_C - b_B = 0 - (-0.661) = 0.661, \tag{10.67}$$

so
$$B_P = b_P/Z_0 = 0.661/50 = 13.22\ \text{mS} \quad \text{or} \quad X_P = -1/B_P = -75.6\ \Omega. \tag{10.68}$$

The final design is shown in Figure 10-32.

One of the advantages of using the Smith chart is that the design progresses in stages, with the structure of the design developed before actual numerical values are calculated. Of course, it is difficult to extract accurate values from a chart, so designs are regularly roughed out on a Smith chart and refined using CAD tools. Example 10.10 matched a resistive source to a resistive load. The next example considers the matching of complex load

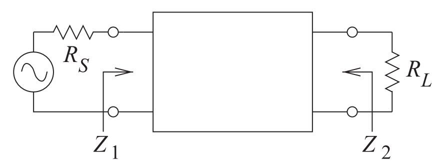
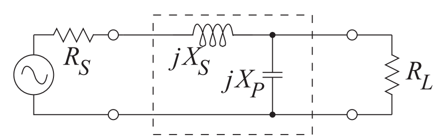
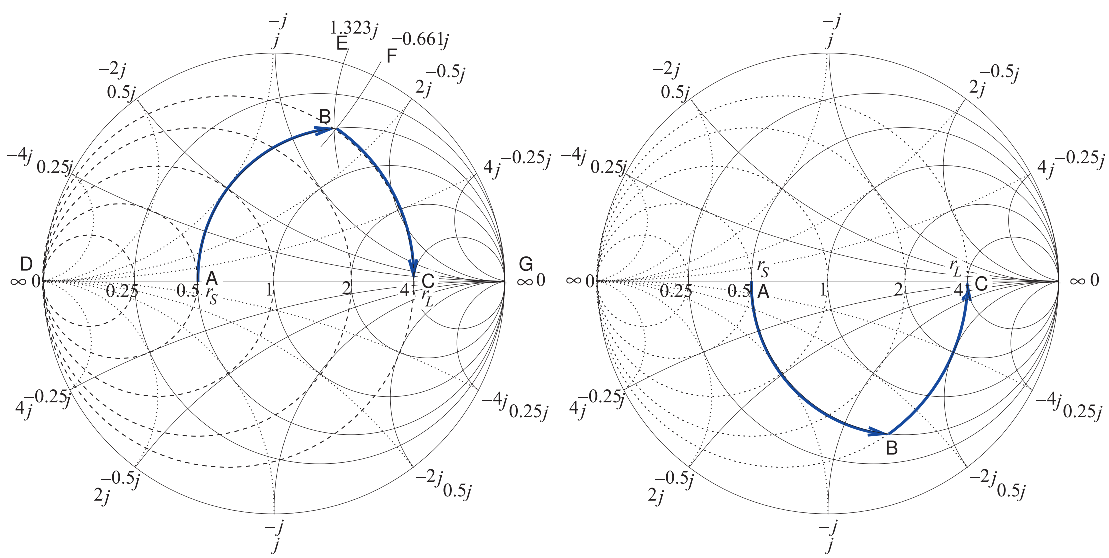


Figure 10-31: Design objectives for Example 10.10. $R_S = 15\ \Omega$, $R_L = 200\ \Omega$.



$X_S = 66.1\ \Omega$. $X_P = -75.6\ \Omega$

Figure 10-32: Final design for Example 10.10 using the path shown in Figure 10-33(a).



(a) Design 1 (b) Design 2

Figure 10-33: Alternative designs for Example 10.10. The normalization impedance is $50\ \Omega$.

and source impedances. In the earlier algorithmic approach to matching network design absorption and resonance were introduced as strategies for dealing with complex terminations. Design was not always straightforward. This complication disappears with a Smith chart-based design, as it is conceptually not much different from the resistive problem of Example 10.10.

EXAMPLE 10.11 Matching Network Design With Complex Impedances

Develop a two-element matching network to match a source with an impedance of $Z_S = 12.5 + 12.5\jmath\ \Omega$ to a load $Z_L = 50 - 50\jmath\ \Omega$, as shown in Figure 10-34.

Solution:

The design objective is to present conjugate matched impedances to the source and load; that is, $Z_1 = Z_S^*$ and $Z_2 = Z_L^*$. The choice here is to design for Z_1; that is, elements will be inserted in front of Z_L to produce the impedance Z_1. The normalized source and load impedances are plotted in Figure 10-35(a) using a normalization impedance of $Z_0 = 50\ \Omega$, so $z_S = Z_S/Z_0 = 0.25 + 0.25\jmath$ (Point S) and $z_L = Z_L/Z_0 = 1 - \jmath$ (Point C).

The impedance to be synthesized is $z_1 = Z_1/Z_0 = z_S^* = 0.25 - 0.25\jmath$ (Point A). The matching network must be lossless, which means that the lumped-element design must follow lines of constant resistance (on the impedance part of the Smith chart) or constant conductance (on the admittance part of the Smith chart). Points A and C must be on the above circles and the circles must intersect if a design is possible.

The design can be viewed as moving back from the load impedance toward the conjugate of the source impedance. The direction of the impedance locus is important. One possible design is shown in Figure 10-35(a). From Point C the line of constant conductance is followed to Point B (there is increasing positive [i.e., capacitive] shunt susceptance along this path). From Point B the locus follows a line of constant resistance to the final point, Point A.

The design shown in Figure 10-35(a) begins with a shunt susceptance, b_P, taking the locus from Point C to Point B and then a series inductive reactance, x_S, taking the locus to Point A. At Point C the susceptance $b_C = 0.5$, at Point B the susceptance $b_B = 1.323$. This value is read off the Smith chart, requiring that an arc of constant susceptance, as shown, be interpolated between the constant susceptance arcs provided. The normalized shunt susceptance is

$$b_P = b_B - b_C = 1.323 - 0.5 = 0.823, \tag{10.69}$$

that is,

$$B_P = b_P/Z_0 = 0.823/(50\ \Omega) = 16.5\ \text{mS or } X_P = -1/B_P = -60.8\ \Omega. \tag{10.70}$$

A series reactive element takes the locus from Point B to Point A, so

$$x_S = x_A - x_B = -0.25 - (-0.661) = 0.411, \tag{10.71}$$

so

$$X_S = x_S Z_0 = 0.411 \times 50\ \Omega = 20.6\ \Omega. \tag{10.72}$$

The final design is shown in Figure 10-36.

There are only two designs that have a path from Point C to Point A following just two arcs. In Design 1, shown in Figure 10-35(a), Path CBA is much shorter than Path CHA for Design 2 shown in Figure 10-35(b). The path length is an approximate indication of the total reactance required, and the higher the reactance, the greater the energy storage and hence the narrower the bandwidth of the design. (The actual relative bandwidth depends on the voltage and current levels in the network; the path length criteria, however, is an important rule of thumb.) Thus Design 1 can be expected to have a much higher bandwidth than Design 2. Since designing broader bandwidth is usually an objective, a design requiring a shorter path on a Smith chart is usually preferable.

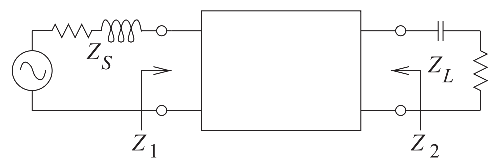
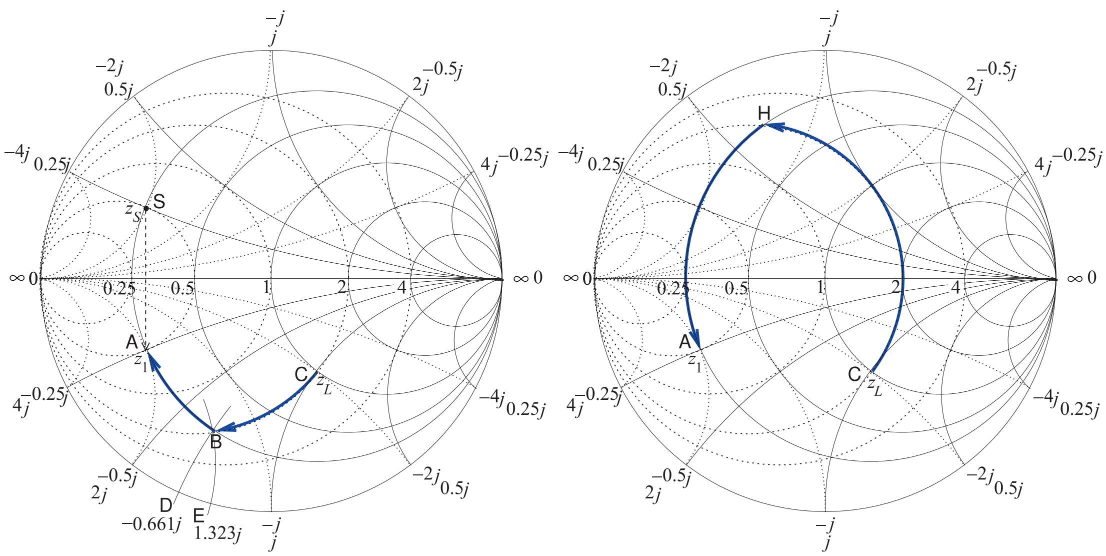


Figure 10-34: Design objectives for Example 10.11.



(a) Design 1 (b) Design 2

Figure 10-35: Smith chart-based designs used in Example 10.11. (50 Ω normalization used.)

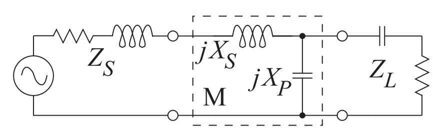


Figure 10-36: Final circuit for Design 1 of Example 10.11. $X_S = 20.6\ \Omega, X_P = -60.8\ \Omega$.

10.8 Distributed Matching

Matching using lumped elements leads to series and shunt lumped elements. The shunt elements can be implemented using shunt transmission lines, as a short length (less than one-quarter wavelength long) of short-circuited transmission line looks like an inductor and a short section of open-circuited transmission line looks like a capacitor. However, in microstrip it is not possible to realize the series elements as lengths of transmission lines. The solution is to use lengths of transmission line together with shunt elements. If space is not at a premium, this is an optimum solution, as transmission lines have much lower loss than a lumped inductor. The series transmission lines rotate the reflection coefficient on the Smith chart.

As with all matching design, using transmission lines begins with a topology in mind. Several topologies are shown in Figure 10-37. Figure 10-37(a) is the top view of a microstrip matching network with a series transmission line and stub realized as an open-circuited transmission line. Figure 10-37(b) is a shorthand schematic for this circuit. Matching network design then becomes a problem of choosing the lengths and characteristic impedances of the lines.

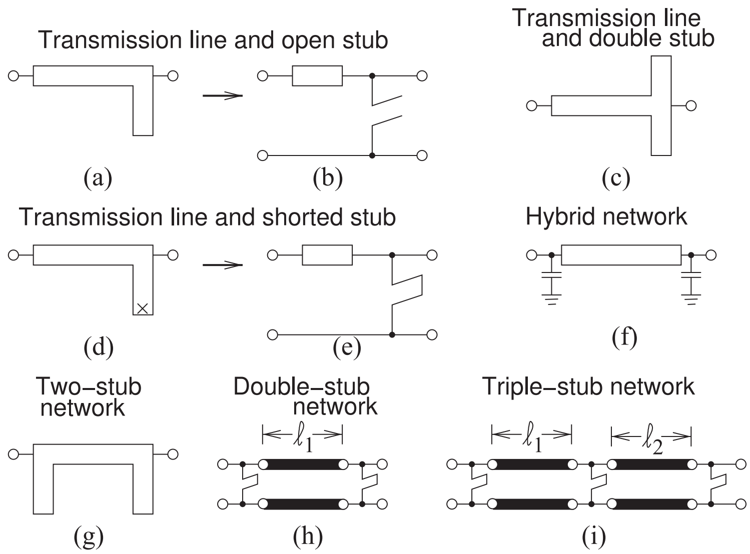


Figure 10-37: Matching networks with transmission line elements.

The stub here is used to realize a capacitive shunt element. This network corresponds to two-element matching with a shunt capacitor. The value of the shunt capacitance can be increased using a dual stub, as shown in Figure 10-37(c), where the capacitive input impedances of each stub are in parallel. The dual circuit to that in Figure 10-37(a) is shown in Figure 10-37(d) together with its schematic representation in Figure 10-37(e). This circuit has a short-circuited stub that realizes a shunt inductance.

Mixing lumped capacitors with a transmission line element, as shown in Figure 10-37(f), realizes a much more space-efficient network design. There are many variations to stub-based matching network design, including the two-stub design in Figure 10-37(g).

A common situation encountered in the laboratory is the matching of circuits that are in development. Laboratory items available for matching include the **stub tuner**, shown in Figure 10-38(a), and the **double-stub tuner**, shown in Figure 10-38(b). With the double-stub tuner the length of the series transmission line is fixed, but stubs can have variable length using lengths of transmission lines with sliding short circuits. Not all impedances can be matched using a double stub tuner, however. A triple-stub tuner can match all impedances presented to it [1]. The **double-slug tuner** shown in Figure 10-38(c) has dielectric slugs each of which introduces a short section of lower impedance line. The slugs are moved up and down the line and avoid the rapid changes in impedances that occur with the stub tuners and as a result the double-slug tuner provides a broader bandwidth match than does the double stub tuner.. The **slide-screw slug tuner** shown in Figure 10-38(d) can achieve a broadband match. Here a metal slug can be lowered into the slabline changing the impedance of a section of transmission line and mostly affects the magnitude of the reflection coefficient while moving the metal slug along the line mostly affects the phase. This is the type of tuner incorporated in computer-controlled automated tuners.

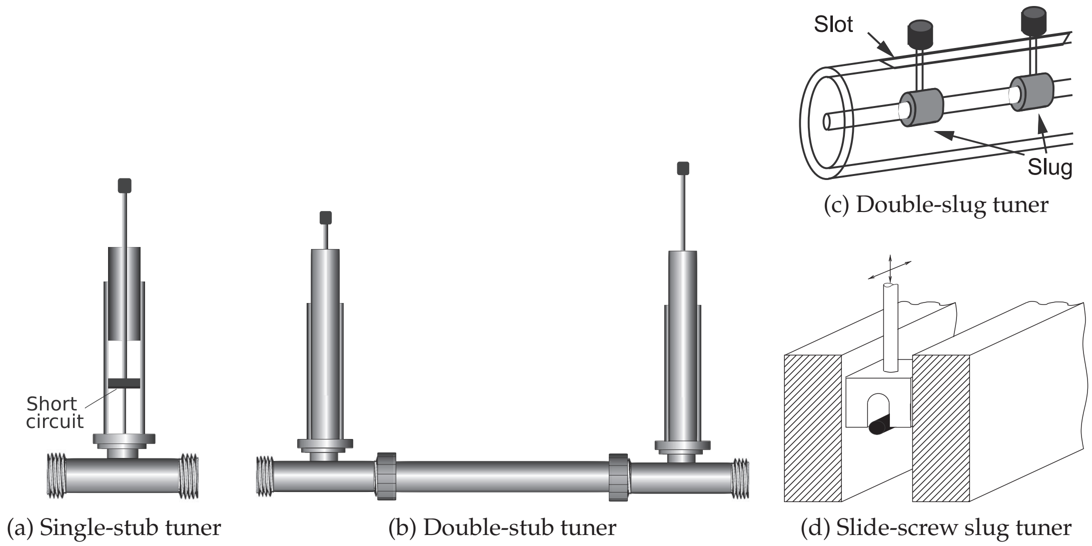
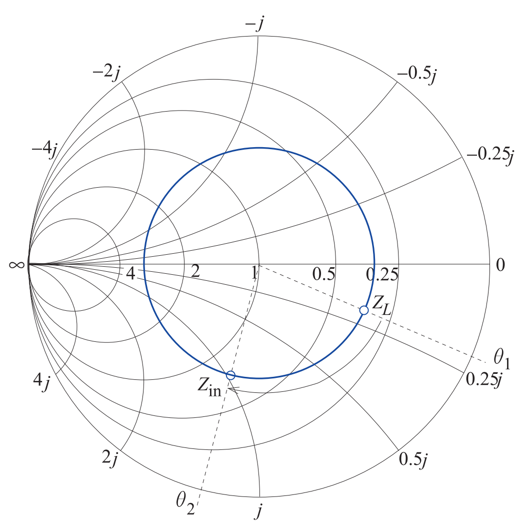


Figure 10-38: Laboratory tuners.



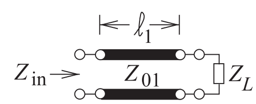


Figure 10-39: Rotation of the input impedance of a transmission line on a Smith chart normalized to Z_{01} as the line length increases.

10.8.1 Stub Matching

In this section matching using one series transmission line and one stub will be considered. This corresponds to the microstrip circuit topologies shown in Figures 10-37(a and d). First, consider the terminated transmission line shown in Figure 10-39. When the length, ℓ_1, of the line is zero, the input impedance of the line, Z_{in}, equals Z_L. How it changes is best described by considering the input reflection coefficient, Γ_{in}, of the line. If the reflection coefficient is normalized to Z_{01}, then the magnitude of Γ_{in} and its phase varies as twice the electrical length of the line. This situation is shown on the Smith chart in Figure 10-39, where Z_L is chosen arbitrarily. The input

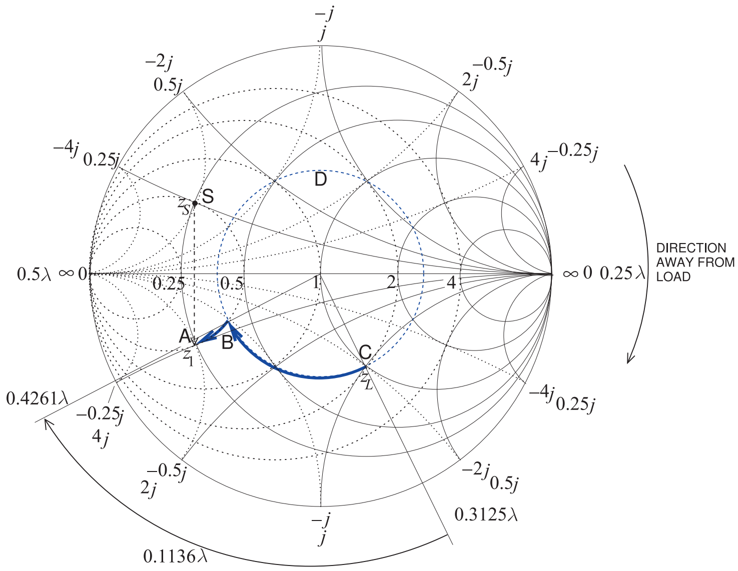


Figure 10-40: Design for Example 10.12.

reflection coefficient of the line rotates in a clockwise direction as the length of the line increases. One way of remembering this is to consider an open-circuited line. When the line length is zero, $Y_{\text{in}} = 0$ and $\Gamma_{\text{in}} = +1$. A short length of this line is capacitive so that its reflection coefficient will be in the bottom half of the Smith chart. A length of line can be used to rotate the impedance to an appropriate point to follow a line of constant conductance to the desired input impedance.

EXAMPLE 10.12 Matching Network Design With a Transmission Line and a Single Stub

Design a two-element matching network to match a source with an impedance $Z_S = 12.5 + 12.5\jmath\ \Omega$ to a load $Z_L = 50 - 50\jmath\ \Omega$, as shown in Figure 10-34. This example repeats the design in Example 10.11, but now using a transmission line.

Solution:

As in Example 10.11, choose $Z_0 = 50\ \Omega$ and the design path is from $z_L = Z_L/Z_0 = 1 - \jmath$ to z_s^*, where $z_s = 0.25 + 0.25\jmath$. One possible design solution is indicated in Figure 10-40. The line length, ℓ (taking the locus from Point C to Point B), is

$$\ell = 0.4261\lambda - 0.3125\lambda = 0.1136\lambda,$$

and the normalized shunt susceptance, b_P (taking the locus from Point B to Point C), is

$$b_P = b_A - b_B = 2 - 1 = 1.$$

Thus $X_P = (-1/b_P) \times 50\ \Omega = -50\ \Omega$. The final design is shown in Figure 10-41(a). The stub design of Figure 10-41(b) follows the procedure described in Example 6.5.

10.8.2 Hybrid Lumped-Distributed Matching

A lossless matching network can have transmission lines as well as inductors and capacitors. If the system reference or normalization impedance is the characteristic impedance of a transmission line, then the locus of the input impedance (or reflection coefficient) of the line with respect to the length of

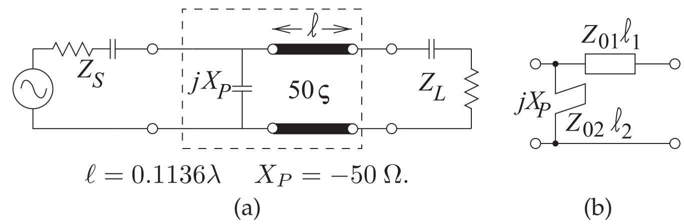


Figure 10-41: Single-stub matching network design of Example 10.12: (a) electrical design; and (b) electrical design with a shunt stub.

the line is an arc on a circle centered at the origin of the Smith chart. The direction of the arc is clockwise as the electrical length of the line moves away from the load. So a hybrid matching network is possible that combines a length of transmission line with a lumped element (preferably a capacitor rather than an a inductor as the inductor would have a relatively lower Q).

10.9 Matching Options Using the Smith Chart

The purpose of this section is to use the Smith chart to present several design options for matching a source to a load, see Figure 10-42. The designs here provide another view of design using the Smith chart.

10.9.1 Locating the Design Points

The first design choice to be made is the reference impedance to use. Here $Z_{\rm REF} = 50\ \Omega$ will be chosen largely because this is in the center of the design space for microstrip lines. Generally the characteristic impedance, Z_0, of a microstrip line needs to be between 20 Ω and 100 Ω. A microstrip line with $Z_0 < 20\ \Omega$ will be wide and there is a possibility of multimoding due to transverse resonance. Also a 20 Ω line is about six times wider than a 50 Ω line and so takes up a lot of room and there is a good chance that it could be close to other microstrip lines or perhaps the wall of an enclosure. This is based on the rule of thumb (developed in Example 3.4 of [2].) that $Z_0 \propto \sqrt{h/w}$ where h is the substrate thickness and w is the strip width. The thickness is usually fixed. If $Z_0 > 100\ \Omega$ the characteristic impedance is getting close to the wave impedance of free space or of the dielectric of the substrate. As such it is likely that field lines are not tightly constrained by the metal of the strip and the fields can more likely radiate. Then radiation loss can be high or coupling to a neighboring microstrip can be high.

The normalized source and load impedances are $z_S = Z_S/Z_{\rm REF} = [(29.36 - \jmath 12.05)\ \Omega]/(50\ \Omega) = 0.587 - \jmath 0.241$ and $z_L = Z_L/Z_{\rm REF} = [(132.7 - \jmath 148.8)\ \Omega/(50\ \Omega) = 2.655 - \jmath 2.976 = r_L + \jmath x_L$, respectively. Maximum power transfer requires that the input impedance of the matching network terminated in Z_L be $Z_1 = Z_S^*$, i.e. $z_1 = z_S^* = 0.587 + \jmath 0.241 = r_1 + \jmath x_1$. These impedances are plotted on the normalized Smith chart in Figure 10-43.

The normalized load impedance is Point L. To locate this point the arcs corresponding to the real and imaginary parts of z_L are considered

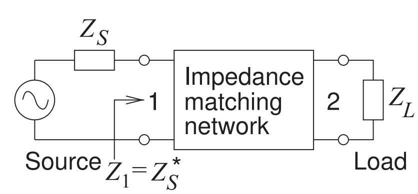


Figure 10-42: Matching problem with the matching network between the source and load designed for maxium power transfer. $Z_S = R_S + \jmath X_S = 29.36 - \jmath 12.05$, $Z_1 = R_1 + \jmath X_1 = Z_S^* = R_S - \jmath X_S = 29.36 + \jmath 12.05$, and $Z_L = R_L + \jmath X_L = 32.7 - \jmath 148.8$.

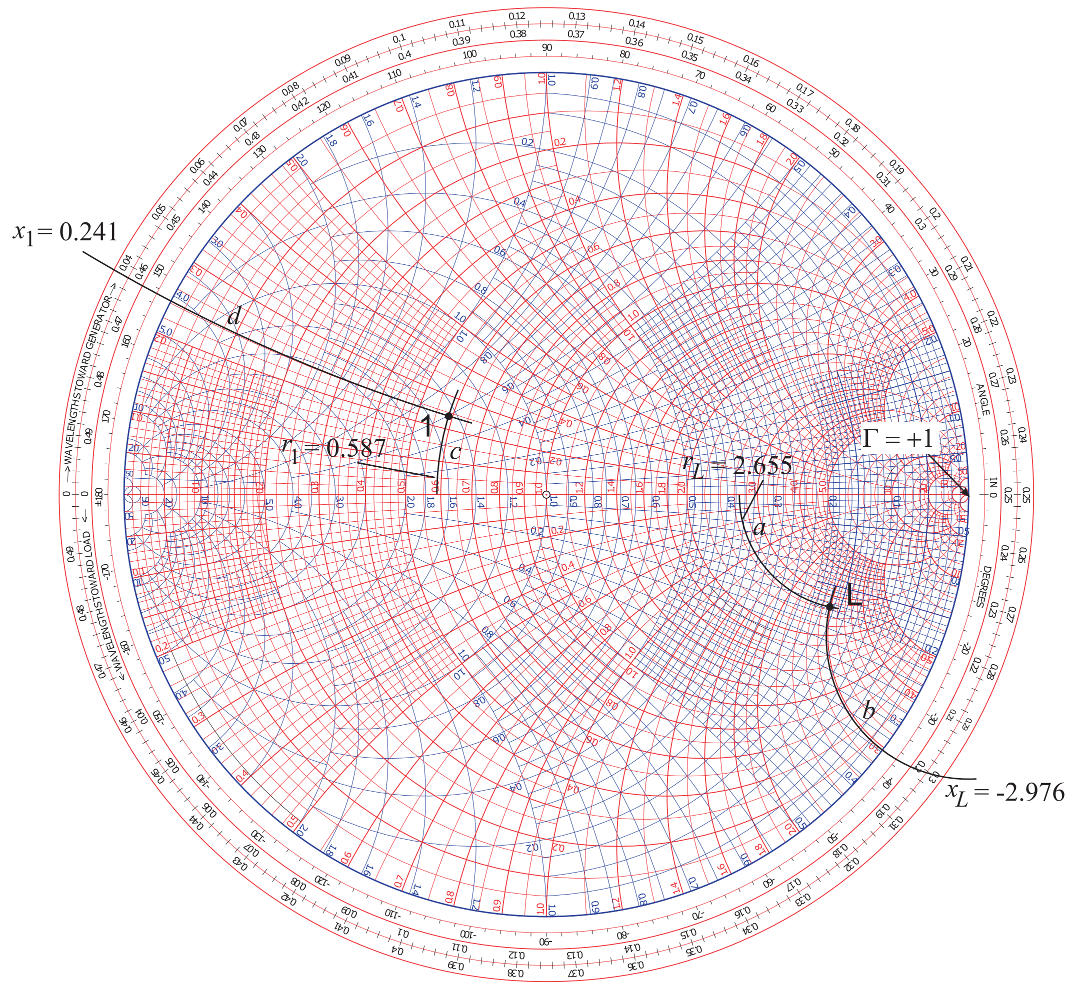


Figure 10-43: Locating Z_L at Point L and $Z_1 = Z_S^*$ at Point 1.

separately. The resistive part of z_L is $r_L = 2.655$ and the resistance labels are located on the horizontal axis (or equator) of the Smith chart. There are two sets of labels, one for normalized resistance, r (which is above the horizontal axis), and one for normalized conductance, g (which is below the horizontal axis). The way to remember which is which is to realize that the infinite impedance point is at the $\Gamma = +1$ (open circuit) location on the right of the graph. At the origin (center) of the Smith chart $r = 1 = g$ and to the right of the center the values of r should be greater than one. The closest r labels to $r_L = 2.655$ are $r = 2.0$ and $r = 3.0$. There are five divisions so the unlabeled curves correspond to 2.2, 2.4, ... The arc corresponding to $r = 2.655$ must be interpolated and this interpolation is shown as the Path 'a'.

The imaginary part of z_L is $x_L = -2.976$. The labels for the arcs of constant reactance are given adjacent to the unit circle. There are two sets of labels, one for reactance and one for susceptance. To recall which is which, the point of infinite impedance can be used and the required reactance labels should

increase towards the $\Gamma = +1$ point. Recall that the Smith chart does not include signs of reactances (there is not enough room) so note must be made that positive reactances are in the top half of the Smith chart and negative reactances are in the bottom half. Since x_L is negative it will be in the bottom half of the Smith chart. The closest labels are $x = 2.0$ (this is actually -2.0) and $x = 3.0$ (this is actually -3.0) so the arc for $x = -2.976$ is interpolated as the Path 'b'. Point L, i.e. z_L, is located at the intersection of Paths 'a' and'b'.
The normalized impedance z_1 is located similarly at Point 1 by finding the point of intersection of the $r_1 = 0.587$ arc, Path 'c', and the $x_1 = +0.241$ arc, Path 'd'.

10.9.2 Design Options

By convention design follows a process of beginning with z_L and adding series and shunt elements in front of it evolving the impedance (or reflection coefficient) until the input impedance is $z_1 = z_s^*$. Two electrical designs are shown in Figure 10-44 and the corresponding lumped-element and microstrip topologies are shown in Figure 10-45. The subscript on the circuit elements correspond to the paths on the Smith chart in Figure 10-44 . The designs will be elaborated in the following subsections.

10.9.3 Design 1, Hybrid Design

Design 1 on its own is shown in Figure 10-46. The concept here is to use a transmission line and a shunt to go from the load Point L to the Point 1. The reason why a shunt element is chosen and not a series element is that the shunt element can be implemented as a stub line and a series element, i.e. a series stub, cannot be implemented in microstrip. A lumped element limits a design to the low microwave range as losses become prohibitively large especially for inductors. Also if a microstrip line is going to be used anyway then a decision has already been made that there is enough room to implement a transmission-line based design and so the shunt lumped element can reasonably bereplaced by a stub.
Design follows trial and error. The first attempt, and the one that works here, is to draw a circle through L centered on the origin at Point O. This circle describes a transmission line whose characteristic impedance is the same as the reference impedance of the Smith chart, here $50\ \Omega$. The next step is to draw a circle of constant conductance through Point 1. The combination path from L to 1 needs to lie on these circles and the intermediate point will be where these circles intersect. One other constraint is that with a transmission line the locus (as the line length increases) of the input reflection coefficient of the line must rotate in the clockwise direction. It is seen that there are two points of intersection and the first of these, at A, is chosen in design. So the electrical design is defined by the directed Paths 'g' and 'h'. Path 'g' defines the properties of the transmission line and Path 'h' defines the properties of the shunt element. As 'h' is directed towards the infinite inductive susceptance point, Path 'h' defines an inductor. The topology of this design is shown in Figure 10-45(a).
The characteristic impedance of the transmission line (defined by Path 'g') is $Z_{0g} = 50\ \Omega$ and the electrical length of the line is defined by the angle subtended by the arc 'g'. The electrical length of the line is determined from

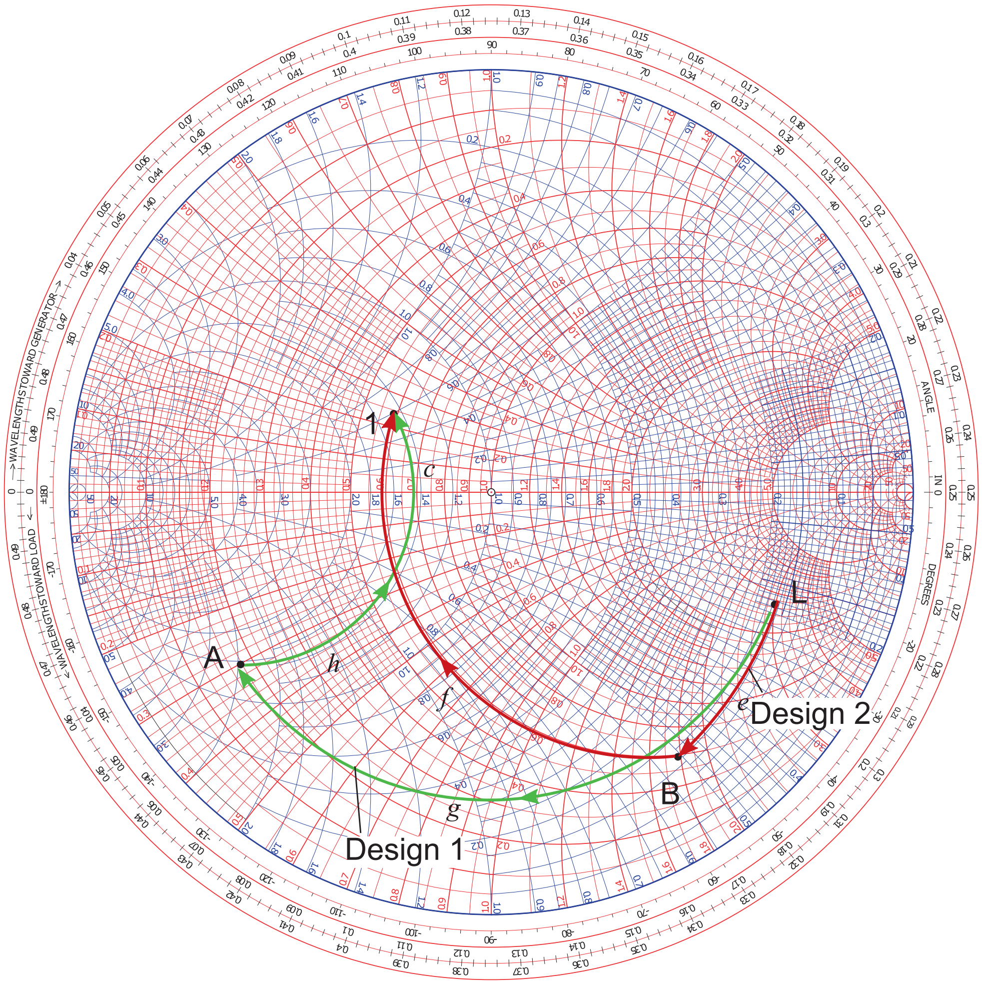


Figure 10-44: Two matching network electrical designs matching a load impedance Z_L at Point L to a source Z_S showing $Z_1 = Z_S^*$ at Point 1.

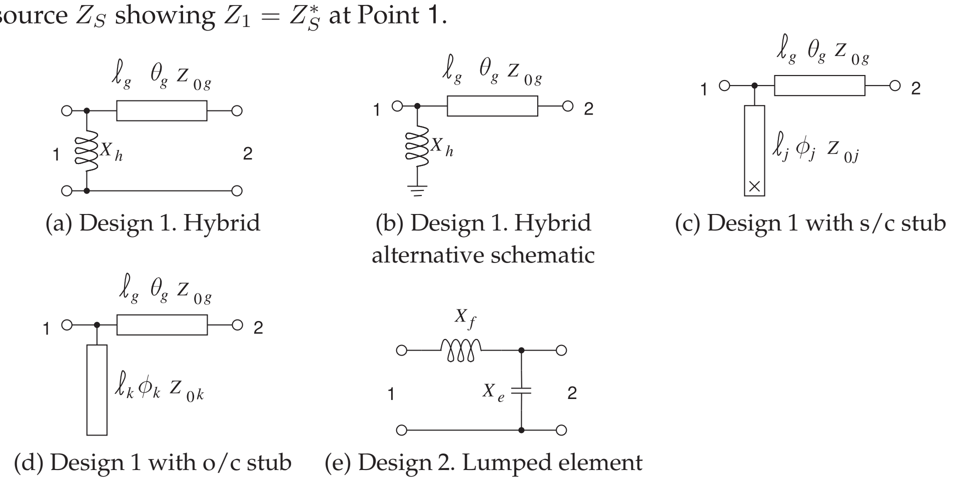


Figure 10-45: Matching network topologies using lumped elements and microstrip lines. In the stub layouts X is a via to the ground plane implementing a short circuit (s/c) and an open circuit o/c simply does not show a connection to the microstrip ground plane.

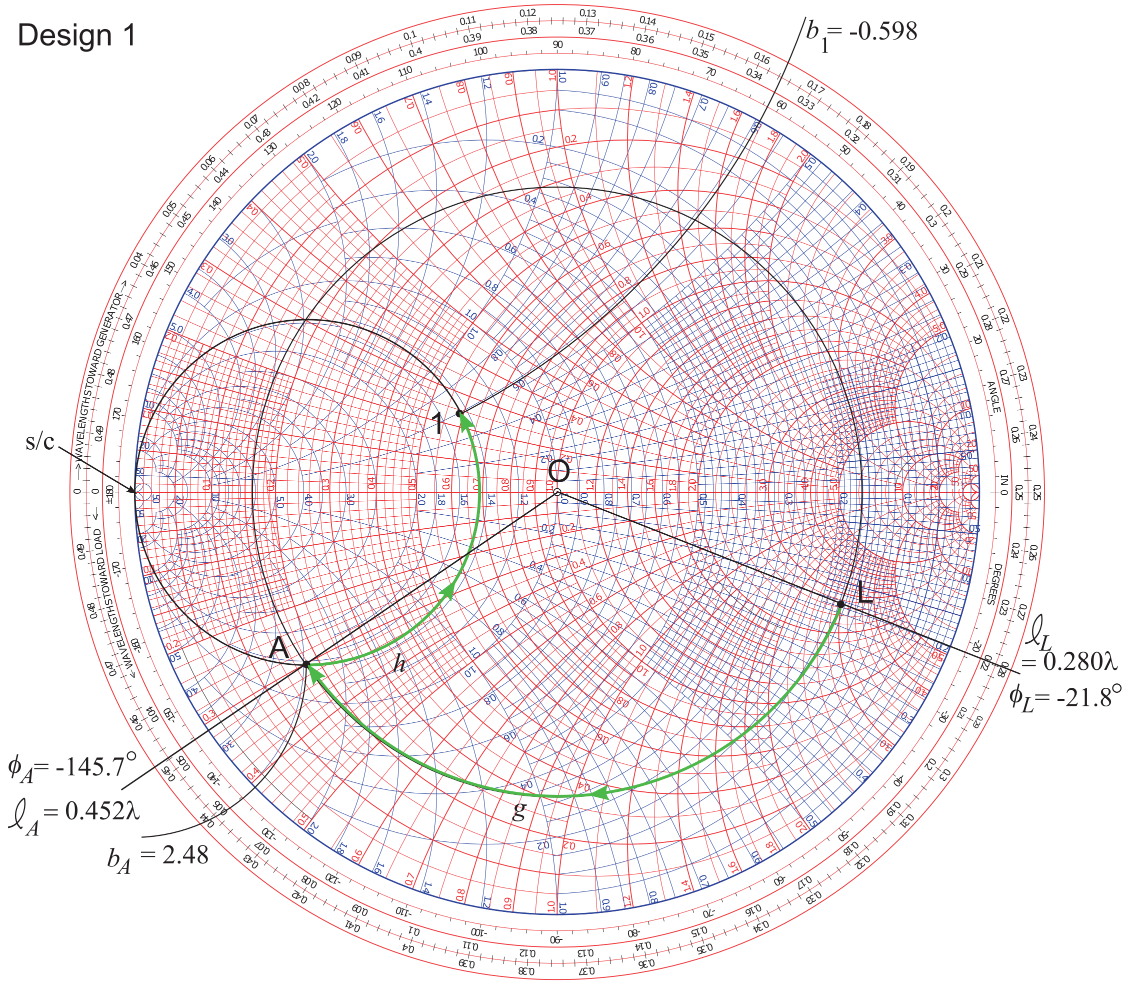


Figure 10-46: Design 1. Hybrid design combining a transmission line with a lumped element in shunt. The design is identified by paths 'g' and 'h'.

the outermost circular scale which is labeled 'WAVELENGTHS TOWARDS GENERATOR.' A line drawn from O through L intersecting the scale has a scale reading of $\ell_L = 0.280\lambda$. Then the scale reading at A is similarly found as $\ell_A = 0.452\lambda$ and so the line length is $\ell_g = \ell_A - \ell_L = 0.452\lambda - 0.280\lambda = 0.172\lambda$. Another way of determining the electrical length of the line is from the change in reflection coefficient angle. For L the reflection coefficient angle is $\phi_L = -21.8°$ read from the innermost circular scale. This angle is just the angle from the polar plot. Then the angle at A is read as $\phi_A = -145.7°$. The difference is $|\phi_A - \phi_L| = |-145.7 - (-21.8°)| = 124.9°$. The electrical length of the line is half the change in reflection coefficient angle and so the electrical length of the line is $\theta_g = \frac{1}{2}124.9° = 62.5°$. Now λ corresponds to an electrical length of $360°$ so θ_g corresponds to $62.5/360\lambda = 0.174\lambda$ corresponding to the previously determined length of 0.172λ which is very good agreement given that these were derived from graphical readings.

Path 'h' defines a shunt inductor and a circle of constant conductance is followed with only the susceptance changing. The susceptance indicated by Path 'h' is $b_h = b_1 - b_A$. To obtain b_A extend the circle of constant susceptance through A out to the unit circle. The extended circle crosses the unit circle between the susceptance labels 2.0 and 3.0. A check is that susceptance is positive in the bottom half of the Smith chart so the signs of the labels do not need to be adjusted. There are two scales adjacent to the unit circle, one for normalized susceptance and one for normalized reactance. The intersection is close to the infinite susceptance point at the s/c (short-circuit) so the values that are becoming very large towards s/c are used. Interpolation results in the reading $b_A = 2.48$. A similar process applied to Point 1 results in $b_1 = -0.598$ where the negative sign has been applied to the scale reading since Point 1 is in the top half of the Smith chart. Thus $b_h = b_1 - b_A = -0.598 - 2.48 = -3.08$ and so the normalized reactance of the shunt element is $x_h = -1/b_h = 0.325$. The un-normalized reactance of the shunt element is $X_h = x_h Z_{\text{REF}} = 16.2\ \Omega$.

The final Design 1 hybrid layout is shown in Figure 10-45(a) with $X_h = 16.2\ \Omega$, $Z_{0g} = 50\ \Omega$, and $\ell_g = 0.172\lambda$. That is all that is needed to define the electrical design, providing the electrical length in degrees, $\theta_g = 62.5°$ is redundant but provided anyway. The transmission line in Figure 10-45(a) is shown as as the top view of the strip of a microstrip line as is commonly done. A more common way of representing this schematic is shown in Figure 10-45(b) where the ground connections at Ports 1 and 2 have been removed and the ground connection of the inductor shown separately.

10.9.4 Design 1 with an Open-Circuited Stub

In the previous section Design 1 was left as a hybrid design with a transmission line and a lumped-element inductor. In this section the lumped-element inductor is implemented as an open-circuited stub, see Figure 10-45(d). Recall that the $50\ \Omega$-normalized susceptance of the inductor is $b_h = -3.08$. If the stub is also implemented as a $50\ \Omega$ line then b_h can be used unchanged. Point C in Figure 10-47 corresponds to the normalized admittance $0 - \jmath 3.08$. The unit circle is the zero conductance circle (and is also the zero resistance circle) and the susceptance is read from the scale adjacent to the unit circle again noting that susceptances in the top half of the Smith chart need to incorporate a negative sign, and the susceptance scale is identified by the susceptance values becoming larger approaching the s/c point. Point C also corresponds to $x_h = -1/b_h = 3.25$ and indeed this is the value read from the normalized reactance scale.

A transmission line needs to be designed to have a normalized input susceptance of $b_h = -3.08$. Choosing an open circuit, o/c, termination the point corresponding to o/c is as identified in the figure. At the o/c point the length scale reads $\ell_{\text{o/c}} = 0.250\lambda$. The locus rotates in the clockwise direction up to Point C where the direct electrical reading reading is $\ell_C = 0.050\lambda$. Using this directly to determine the line length $\ell_k = \ell_C - \ell_{\text{o/c}} = 0.050\lambda - 0.250\lambda = -0.20\lambda$ which indicates that the stub has a negative length. Clearly an erroneous result. This apparent discrepancy comes about because the length scale resets at the short circuit point where the length scale abruptly goes from 0.5λ to 0λ. Thus the corrected ℓ_C reading needs to have an additional 0.5λ. Thus the corrected value of $\ell_C = (0.5 + 0.050)\lambda = 0.550\lambda$

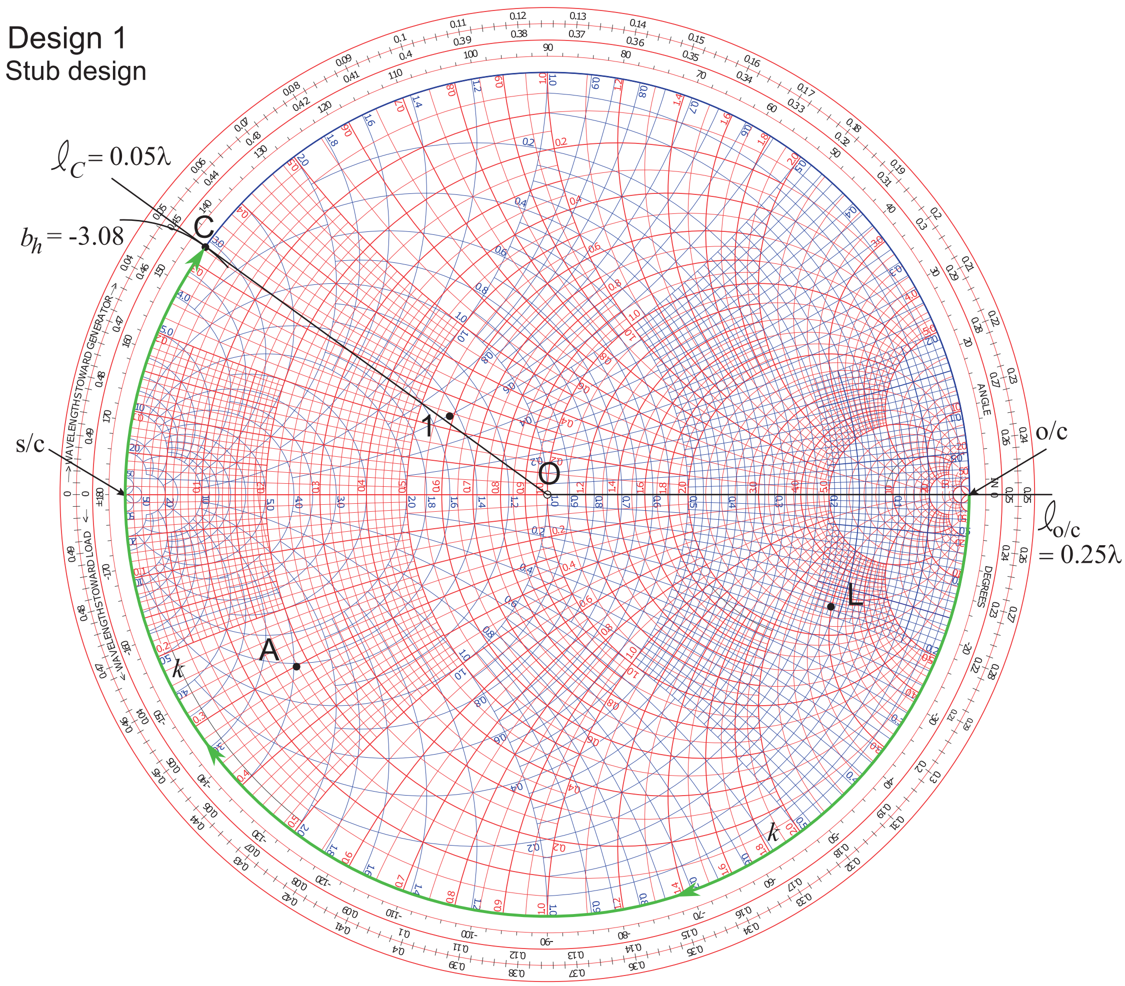


Figure 10-47: Design 1. Design of an open-circuit stub having normalized input susceptance b_h.

and $\ell_k = \ell_C - \ell_{o/c} = 0.550\lambda - 0.250\lambda = 0.300\lambda$.
Thus the final design is as shown in Figure 10-45(d) with $Z_{0k} = 50\ \Omega$, and $\ell_g = 0.300\lambda$, $Z_{0g} = 50\ \Omega$, and $\ell_g = 0.172\lambda$.
The stub could also have been implemented as a short-circuit stub as shown in Figure 10-45(c). Now the beginning of the line would be at the s/c point and the line length would be 0.050λ

10.9.5 Design 2, Lumped-Element Design

Design 2 is a lumped-element design and the Smith-chart-based electrical design is shown in Figure 10-48 resulting in the schematic shown in Figure 10-45(e). Design proceeds by identifying where circles of constant conductance and constant resistance passing through the Points L and 1 intersect. One solution is shown in Figure 10-48. A circle of constant conductance passes through L and part of a circle of constant resistance passes through 1. If the circle had continued there would have been a

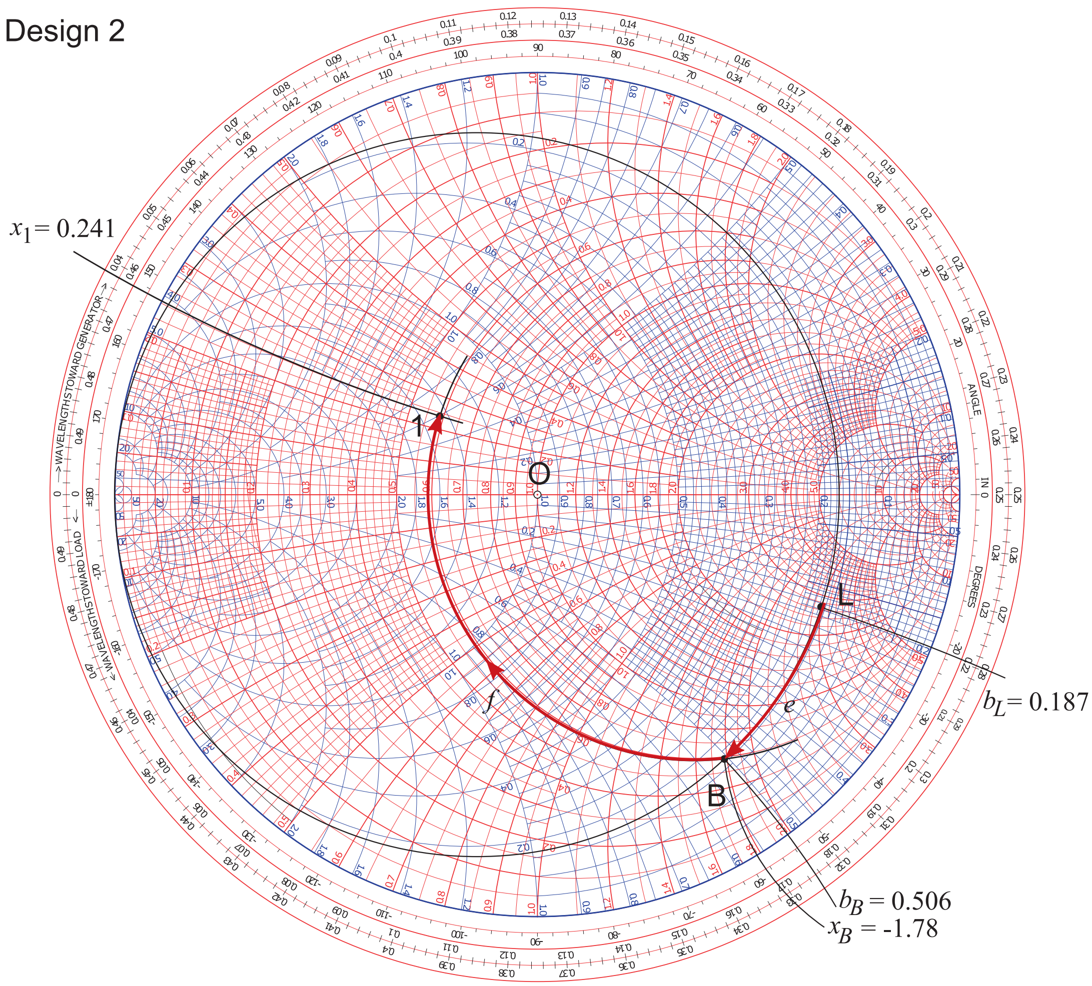


Figure 10-48: Design 2.

second intersection with the circle through L. Both of these intersections mean that there is a shunt element adjacent to the load and series element adjacent to the source. Recall that in a lumped-element design that under no circumstances can the locus a lumped element pass through the short circuit or open-circuit points (the susceptance and reactance infinity points respectively).

Returning to the actual design shown in Figure 10-48. The first intersection of the two circles is Point B so that the design is specified by the Paths ‘e’ and ′f′. Design has largely been completed by identifying these paths and the next stage is determining the circuit elements that correspond to these paths. Path ‘e’ follows a circle of constant conductance and so indicates a shunt susceptance and the direction of the locus indicates a capacitance. The value of this normalized susceptance is $b_e = b_B - b_L = 0.506 - 0.187 = 0.319$. (Remember to check the signs of the readings.) Path ‘f’ identifies a series inductor with a reactance $x_f = x_1 - x_B = 0.241 - (-1.78) = 2.02$. The final design is shown in Figure 10-45(e) with $X_e = Z_0/b_e = 50/0.319\ \Omega = 158\ \Omega$ and $X_f = Z_0 x_f = 50 \cdot 2.02\ \Omega = 101\ \Omega$.

10.9.6 Summary

This section presented two designs for a matching network. One of the particular benefits of using the Smith chart is identifying topologies and initial design values. Design can then transfer to a microwave circuit simulator. The Smith chart enables back-of-the-envelope design studies. While with experience it is possible to complete many of these steps with a computer-based Smith chart tool, even experienced designers doodle with a printed Smith chart when exploring design options.

10.10 Summary

This chapter presented techniques for impedance matching that achieve maximum power transfer from a source to a load. The simplest matching network uses a series and a shunt element, a two-element matching network, to realize a single-frequency match. This type of impedance matching network uses lumped elements and can be used up to a few gigahertz. Performance is limited by the self-resonant frequency of lumped elements and by their loss, particularly that of inductors. The shunt element can be replaced by a shunt stub, but in most transmission line technologies, including microstrip, the series element cannot be implemented as a stub. Matching networks can also be realized using transmission line segments only, principally shunt stubs and cascaded transmission lines. A tunable double-stub matching network, which uses two stubs separated by a transmission line, is standard equipment in microwave laboratories and facilitates matching of a circuit under development.

The bandwidth of a matching network is set by the maximum allowable reflection coefficient of the terminated network. Two-element matching nearly always results in a narrow match and for typical communications applications often achieves acceptable matching over bandwidths of only 1%–3%. The most significant determinant of the quality of the match that can be achieved is the ratio of the source and load resistances, as well as the reactive energy storage of the source and load.

An important concept in matching network design is a technique for controlling bandwidth. The concept is based on matching to an intermediate resistance, typically designated as R_v. Compared to a two-element network, increased bandwidth is obtained if R_v is the geometric mean of the source and load resistances. This new network consists of two two-element matching networks. If R_v is greater or less than both the source and load resistances, then the bandwidth of the matching network is reduced. The matching network synthesis problem can also be addressed using filter design techniques, and this enables simultaneous control over the quality and bandwidth of the match. It is always a good idea to have no more bandwidth in the system than is needed, as this minimizes the propagation of noise.

10.11 References

[1] R. Collins, *Foundations for Microwave Engineering*. McGraw Hill, 1966.
[2] M. Steer, *Microwave and RF Design, Transmission Lines*, 3rd ed. North Carolina State University, 2019.

10.12 Exercises

1. Consider the design of a magnetic transformer that will match the 3 Ω output resistance of a power amplifier (this is the source) to a 50 Ω load. The secondary of the transformer is on the load side.
   (a) What is the ratio of the number of primary turns to the number of secondary turns for ideal matching?
   (b) If the transformer ratio could be implemented exactly (the ideal situation), what is the reflection coefficient normalized to 3 Ω looking into the primary of the transformer with the 50 Ω load?
   (c) What is the ideal return loss of the loaded transformer (looking into the primary)? Express your answer in dB.
   (d) If there are 100 secondary windings, how many primary windings are there in your design? Note that the number of windings must be an integer. (This practical situation will be considered in the rest of the problem.)
   (e) What is the input resistance of the transformer looking into the primary?
   (f) What is the reflection coefficient normalized to 3 Ω looking into the primary of the transformer with the 50 Ω load?
   (g) What is the actual return loss (in dB) of the loaded transformer (looking into the primary)?
   (h) If the maximum available power from the amplifier is 20 dBm, how much power (in dBm) is reflected at the input of the transformer?
   (i) Thus, how much power (in dBm) is delivered to the load ignoring loss in the transformer?

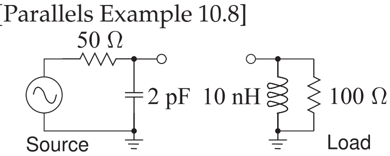
2. Consider the design of a magnetic transformer that will match a 50 Ω output resistance to the 100 Ω load presented by an amplifier. The secondary of the transformer is on the load (amplifier) side.
   (a) What is the ratio of the number of primary turns to the number of secondary turns for ideal matching?
   (b) If the transformer ratio could be implemented exactly (the ideal situation), what is the reflection coefficient normalized to 50 Ω looking into the primary of the transformer with the load?
   (c) What is the ideal return loss of the loaded transformer (looking into the primary)? Express your answer in dB.
   (d) If there are 20 secondary windings, how many primary windings are there in your design? Note that the number of windings must be an integer? (This situation will be considered in the rest of the problem.)
   (e) What is the input resistance of the transformer looking into the primary?
   (f) What is the reflection coefficient normalized to 50 Ω looking into the primary of the loaded transformer?
   (g) What is the actual return loss (in dB) of the loaded transformer (looking into the primary)?
   (h) If the maximum available power from the source is -10 dBm, how much power (in dBm) is reflected from the input of the transformer?
   (i) Thus, how much power (in dBm) is delivered to the amplifier ignoring loss in the transformer?

3. Consider the design of an L-matching network centered at 1 GHz that will match the 2 Ω output resistance of a power amplifier (this is the source) to a 50 Ω load. [Parallels Example 10.3 but note the DC blocking requirement below.]
   (a) What is the Q of the matching network?
   (b) The matching network must block DC current. Draw the topology of the matching network.
   (c) What is the reactance of the series element in the matching network?
   (d) What is the reactance of the shunt element in the matching network?
   (e) What is the value of the series element in the matching network?
   (f) What is the value of the shunt element in the matching network?
   (g) Draw and label the final design of your matching network including the source and load resistances.
   (h) Approximately, what is the 3 dB bandwidth

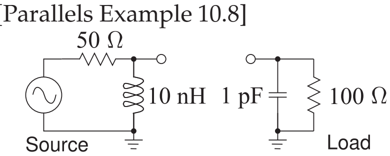
of the matching network?

4. Consider the design of an L-matching network centered at 100 GHz that will match a source with a Thevenin resistance of 50 Ω to the input of an amplifier presenting a load resistance of 100 Ω to the matching network. [Parallels Example 10.4 but note the DC blocking requirement below.]
   (a) What is the Q of the matching network?
   (b) The matching network must block DC current. Draw the topology of the matching network.
   (c) What is the reactance of the series element in the matching network?
   (d) What is the reactance of the shunt element in the matching network?
   (e) What is the value of the series element in the matching network?
   (f) What is the value of the shunt element in the matching network?
   (g) Draw and label the final design of your matching network including the source and load resistance.
   (h) Approximately, what is the 3 dB bandwidth of the matching network?

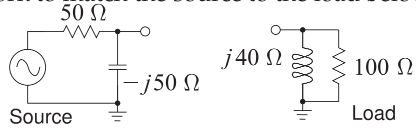
5. Design a Pi network to match the source configuration to the load configuration below. The design frequency is 900 MHz and the desired Q is 10. [Parallels Example 10.8]



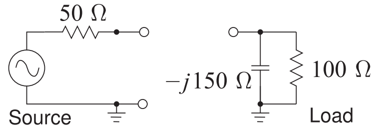
6. Design a Pi network to match the source configuration to the load configuration below. The design frequency is 900 MHz and the desired Q is 10. [Parallels Example 10.8]



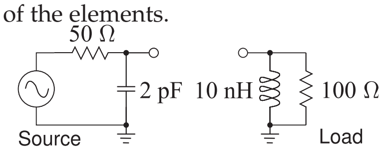
7. Develop the electrical design of an L-matching network to match the source to the load below.



8. Develop the electrical design of an L-matching network to match the source to the load below.



9. Design a lowpass lumped-element matching network to match the source and load shown below. The design frequency is 1 GHz. You must use a Smith Chart and clearly show your working and derivations. You must develop the final values of the elements.



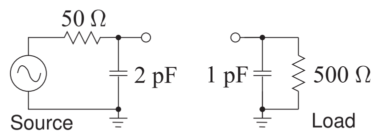
10. Consider the design of an L-matching network centered at 100 GHz that will match a source with a Thevenin resistance of 50 Ω to the input of an amplifier presenting a load resistance of 200 Ω to the matching network. [Parallels Example 10.4 but note the DC blocking requirement below.]
   (a) What is the Q of the matching network?
   (b) The matching network must block DC current. Draw the topology of the matching network.
   (c) What is the reactance of the series element in the matching network?
   (d) What is the reactance of the shunt element in the matching network?
   (e) What is the value of the series element in the matching network?
   (f) What is the value of the shunt element in the matching network?
   (g) Draw and label the final design of your matching network including the source and load resistance.
   (h) Approximately, what is the 3 dB bandwidth of the matching network?

11. Design a two-element matching network to interface a source with a 25 Ω Thevenin equivalent impedance to a load consisting of a capacitor in parallel with a resistor so that the load admittance is $Y_L = 0.02 + \jmath 0.02$ S. Use the absorption method to handle the reactive load.

12. Design a matching network to interface a source with a 25 Ω Thevenin equivalent impedance to a load consisting of a capacitor in parallel with a resistor so that the load admittance is $Y_L = 0.01 + \jmath 0.01$ S.
   (a) If the complexity of the matching network is not limited, what is the minimum Q that could possibly be achieved in the complete network consisting of the matching network and the source and load impedances?
   (b) Outline the procedure for designing the matching network for maximum bandwidth if only four elements can be used in the network. You do not need to design the net-

work.

13. Design a Pi network to match the source configuration to the load configuration below. The design frequency is 900 MHz and the desired Q is 10.



14. Design a passive matching network that will achieve maximum bandwidth matching from a source with an impedance of 2 Ω (typical of the output impedance of a power amplifier) to a load with an impedance of 50 Ω. The matching network can have a maximum of three reactive elements. You need only calculate reactances and not the capacitor and inductor values.

15. Design a passive matching network that will achieve maximum bandwidth matching from a source with an impedance of 20 Ω to a load with an impedance of 125 Ω. The matching network can have a maximum of four reactive elements. You need only calculate reactances and not the capacitor and inductor values.
    (a) Will you use two, three, or four elements in your matching network?
    (b) With a diagram, and perhaps equations, indicate the design procedure.
    (c) Design the matching network. It is sufficient to use reactance values.

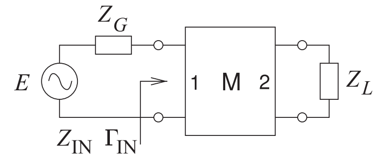
16. Design a passive matching network that will achieve maximum bandwidth matching from a source with an impedance of 60 Ω to a load with an impedance of 5 Ω. The matching network can have a maximum of four reactive elements. You need only calculate reactances and not the capacitor and inductor values.
    (a) Will you use two, three, or four elements in your matching network?
    (b) With a diagram and perhaps equations, indicate the design procedure.
    (c) Design the matching network. It is sufficient to use reactance values.

17. Design a T network to match a 50 Ω source to a 1000 Ω load. The desired loaded Q is 15.

18. Repeat Example 10.2 with an inductor in series with the load. Show that the inductance can be adjusted to obtain any positive shunt resistance value.

19. Design a three-lumped-element matching network that interfaces a source with an impedance of 5 Ω to a load with an impedance consisting of a resistor with an impedance of 10 Ω. The network must have a Q of 6.

20. A two-port matching network is shown below with a generator and a load. The generator impedance is 40 Ω and the load impedance is $Z_L = 50 - \jmath 20$ Ω. Use a Smith chart to design the matching network.



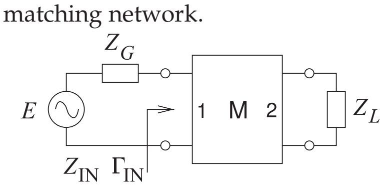
    (a) What is the condition for maximum power transfer from the generator? Express your answer using impedances.
    (b) What is the condition for maximum power transfer from the generator? Express your answer using reflection coefficients.
    (c) What system reference impedance are you going to use to solve the problem?
    (d) Plot Z_L on the Smith chart and label the point. (Remember to use impedance normalization if required.)
    (e) Plot Z_G on the Smith chart and label the point.
    (f) Design a matching network using only transmission lines. Show your work on the Smith chart. You must express the lengths of the lines in terms of electrical length (either degrees or wavelengths). Characteristic impedances of the lines are required. (You will therefore have a design that consists of one stub and one other length of transmission line.)

21. Use a lossless transmission line and a series reactive element to match a source with a Thevenin equivalent impedance of $25 + \jmath 50$ Ω to a load of 100 Ω. (That is, use one transmission line and one series reactance only.)
    (a) Draw the matching network with the source and load.
    (b) What is the value of the series reactance in the matching network (you can leave this in ohms)?
    (c) What is the length and characteristic impedance of the transmission line?

22. Consider a load $Z_L = 80 + \jmath 40$ Ω. Use the Smith chart to design a matching network consisting of only two transmission lines that will match the load to a generator of 40 Ω.
    (a) Draw the matching network with transmission lines. If you use a stub, it should be a short-circuited stub.

(b) Indicate your choice of characteristic impedance for your transmission lines. What is the normalized load impedance? What is the normalized source impedance?
(c) Briefly outline the design procedure you will use. You will need to use Smith chart sketches.
(d) Plot the load and source on a Smith chart.
(e) Complete the design of the matching network, providing the lengths of the transmission lines.

23. A two-port matching network is shown below with a generator and a load. The generator impedance is 40 Ω and the load impedance is $Z_L = 20 - \jmath 50$ Ω. Use a Smith chart to design the matching network.



(a) What is the condition for maximum power transfer from the generator? Express your answer using impedances.
(b) What is the condition for maximum power transfer from the generator? Express your answer using reflection coefficients.
(c) What system reference impedance are you going to use to solve the problem?
(d) Plot Z_L on a Smith chart and label the point. (Remember to use impedance normalization if required.)
(e) Plot Z_G on a Smith chart and label the point.
(f) Design a matching network using only transmission lines and show your work on a Smith chart. You must express the lengths of the lines in terms of electrical length (either degrees or wavelengths long). Characteristic impedances of the lines are required. (You will therefore have a design that consists of one stub and one other length of transmission line.)

24. Use a Smith chart to design a microstrip network to match a load $Z_L = 100 - \jmath 100$ Ω to a source $Z_S = 34 - \jmath 40$ Ω). Use transmission lines only and do not use short-circuited stubs. Use a reference impedance of 40 Ω.
(a) Draw the matching network problem labeling impedances and the impedance looking into the matching network from the source as Z_1.
(b) What is the condition for maximum power transfer in terms of impedances?
(c) What is the condition for maximum power transfer in terms of reflection coefficients?
(d) Identify, i.e. draw, at least two suitable microstrip matching networks.
(e) Develop the electrical design of the matching network using the Smith chart using 40 Ω lines only. You only need do one design.
(f) Draw the microstrip layout of the matching network identify critical parameters such characteristic impedances and electrical length. Ensure that you identify which is the source side and which is the load side. You do not need to determine the widths of the lies or their physical lengths.

25. Repeat exercise 36 but now with $Z_L = 10 - \jmath 40$ Ω and $Z_S = 28 - \jmath 28$ Ω).

26. Use a Smith chart to design a two-element lumped-element lossless matching network to interface a source with an admittance $Y_S = 6 - \jmath 12$ mS to a load with admittance $Y_L = 70 - \jmath 50$ mS.

27. Use a Smith chart to design a two-element lumped-element lossless matching network to interface a load $Z_L = 50 + \jmath 50$ Ω to a source $Z_S = 10$ Ω.

10.12.1 Exercises by Section

†challenging

§10.3 1, 2
§10.4 3, 4
§10.5 5, 6, 7, 8, 9, 10, 11†, 12†, 13†
§10.6 14†, 15†, 16†, 17†, 18†, 19†
§10.7 20†, 21†, 22†, 23†
§10.9 24, 25, 26, 27

10.12.2 Answers to Selected Exercises

15(c) $Q = 1.22467$
18 $C = 1/(\omega_d^2 L_P)$
20(d) $1.25 - \jmath 0.5$
21(b) $-\jmath 50$ Ω
23(f) 40 Ω, 0.085 λ-long line before load, 40 Ω, 0.076 λ-long shorted stub

CHAPTER 11

RF and Microwave Modules

11.1 Introduction to Microwave Modules

Most microwave design uses modules such amplifiers, integrated circuits (ICs), filters, frequency multipliers, and passive components to create systems. Economics necessitate that since modules are expensive to design that they be developed for multiple applications. In system design modules are chosen for their dynamic range, noise performance, DC power consumption, and cost. Foremost the system designer must have knowledge of available modules and be prepared to design a module itself if this results in competitive performance or better manages cost. Modules are interconnected by transmission lines, and bias settings and matching networks must be designed. The system frequency plan must be developed that trades-off cost and performance while minimizing interference. This chapter begins with examples of module usage and then develops metrics that characterize microwave modules. These include characterizations of nonlinear distortion, noise, and dynamic range.

11.2 RF System as a Cascade of Modules

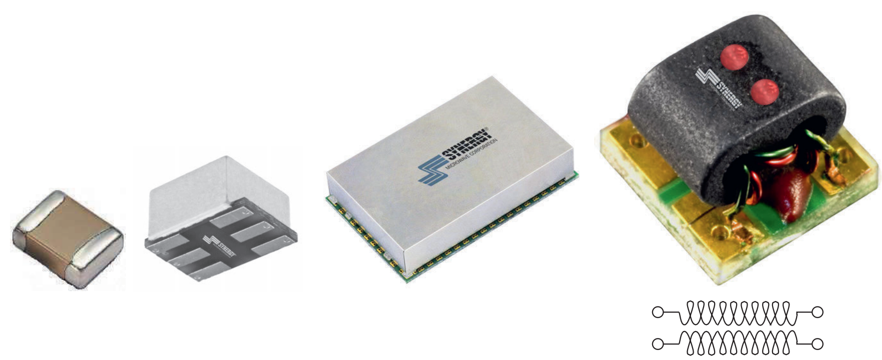
Most RF and microwave engineers work at the circuit board level and begin system design using modules. Some companies develop some of their own proprietary modules, thus providing a competitive advantage, but still use many modules developed by others. Examples of commercially available modules are shown in Figure 11-1. Modules can range in complexity from the simple surface mount resistor of Figure 11-1(a) and the transformer of Figure 11-1(d), up to the mixer and synthesizer modules shown in Figure 11-1(b and c). Modules can have very good performance as it is cost effective to put considerable design effort into a module that can be used in many applications and thus design costs shared.
Modules comprising a receiver are shown in Figure 11-2. Beginning with the bandpass filter after the antenna, each module contributes noise and nonlinear distortion. The system design objectives are generally to maximize dynamic range, the region between the signal being sufficiently above the noise level to be detected but before nonlinear distortion introduces spurious signals that limit the detectability of signals. At the same time power consumption and time to market must be minimized.

11.2.1 A 15 GHz Receiver Subsystem

An example of a microwave subsystem is the 15 GHz receiver shown in Figure 11-3 with details of the frequency conversion section shown in Figure 11-4. This subsystem is itself a module used in a point-to-point microwave link. The amplifier, frequency multiplier, mixer, circulator, and waveguide adaptor modules are available as off-the-shelf components from companies that specialize in particular types of modules.

11.3 Amplifiers

Amplifier modules must be optimized for low noise, moderate to high gain, high efficiency, stability, low distortion, and particular output power. Usually



(a) Resistor (b) Mixer (c) Synthesizer (d) Transformer

Figure 11-1: Modules in surface-mount packages. Copyright Synergy Microwave Corporation, used with permission [1].

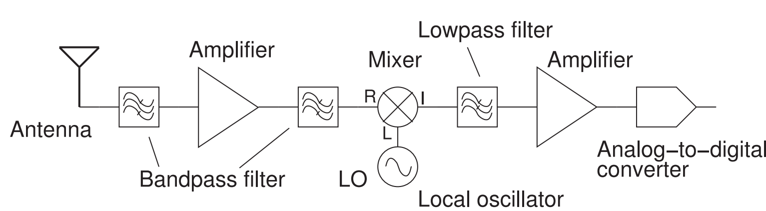
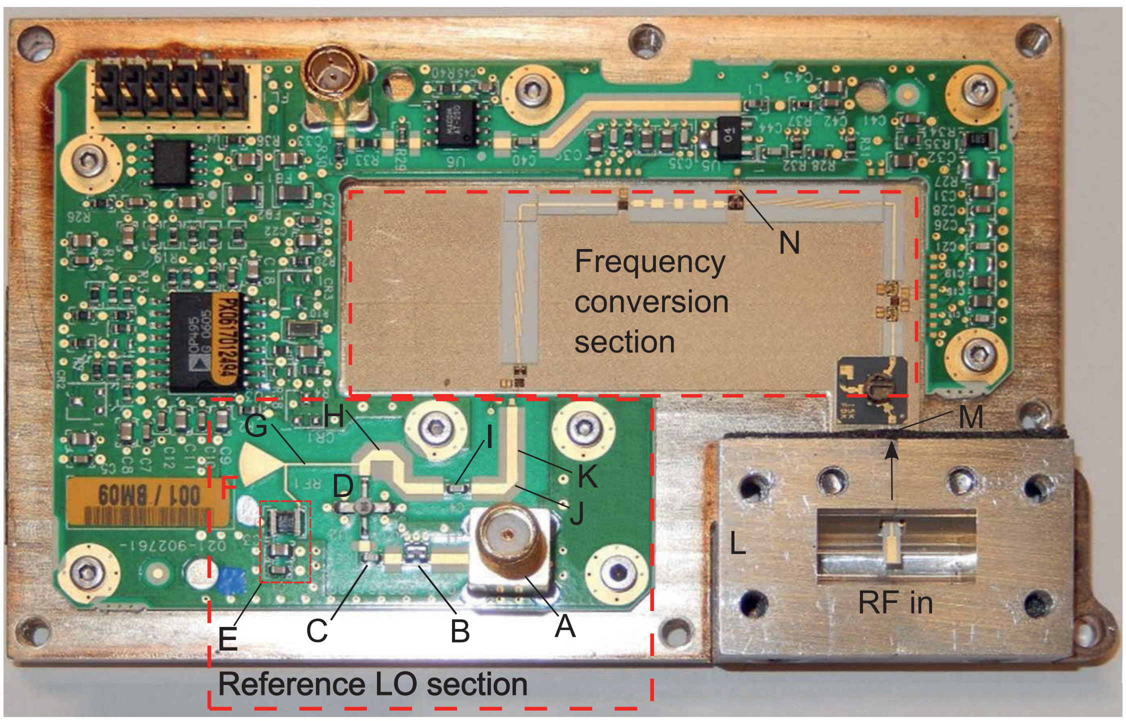


Figure 11-2: Receiver as a cascade of modules.

an application note provided by the amplifier module vendor indicates how to do this for their module. Amplifiers require input and output matching networks as shown in Figure 11-5. The DC bias control circuit is fairly standard and the lowpass filters (in the bias circuits) are often incorporated into the input and output matching networks. Manufacturers of amplifier modules provide substantial information, including S parameters and, in many cases, reference designs.

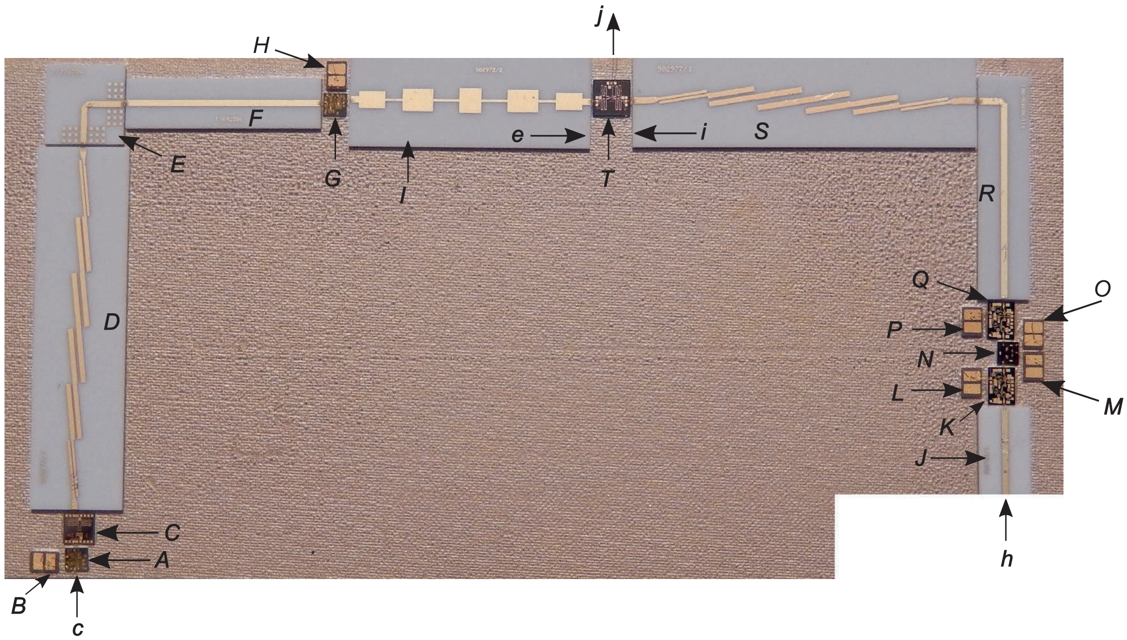
Some amplifier modules come with input and output matching networks embodied in the module package. Since matching networks have limited bandwidth, incorporating them in the module sets the operating frequency range. Alternatively the designer can design the input and output matching networks and thus control the operating frequency.

Usually the module manufacturer provides a reference design and often



A	SMA connector, Reference LO in	**H**	50 Ω transmission line
B	Attenuator	**I**	DC blocking capacitor
C	DC blocking capacitor	**J**	Mitered bend
D	Reference LO amplifier	**K**	50 Ω transmission line
E	Bias line	**L**	Waveguide-to-microstrip adaptor, RF in
F	Radial stub	**M**	Interface to frequency conversion section
G	High-impedance transmission line	**N**	Interface to IF section

Figure 11-3: A 14.4–15.35 GHz receiver consisting of cascaded modules interconnected by microstrip lines. Surrounding the microwave circuit are DC conditioning and control circuitry. RF in is 14.4 GHz to 15.35 GHz, LO in is 1600.625 MHz to 1741.875 MHz. The frequency of the IF is 70–1595 MHz. Detail of the frequency conversion section is shown in Figure 11-4.



A	Reference LO amplifier	*K*	RF amplifier
B	Power supply decoupling capacitor	*L*	Power supply decoupling capacitor
C	×2 frequency multiplier	*M*	Power supply decoupling capacitor
D	Edge-coupled PCL bandpass filter	*N*	Voltage variable attenuator
E	Microstrip bend	*O*	Power supply decoupling capacitor
F	Microstrip transmission line	*P*	Power supply decoupling capacitor
G	×2 frequency multiplier	*Q*	RF amplifier
H	Power supply decoupling capacitor	*R*	Microstrip transmission line and bend
I	stepped impedance lowpass filter	*S*	Edge-coupled PCL bandpass filter
J	Microstrip transmission line	*T*	Subharmonic mixer

Figure 11-4: Frequency conversion section of the receiver module. The reference LO is applied at *c*, the RF is applied at *h* following the isolator. The IF is output at *j*.

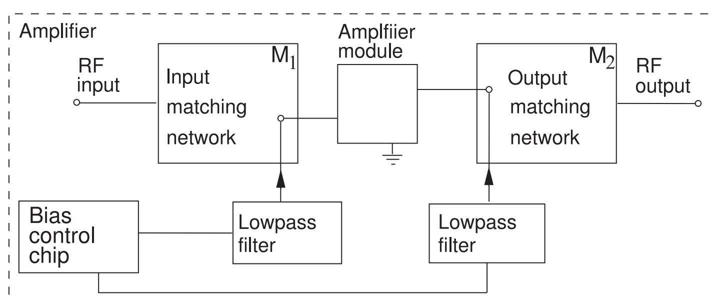
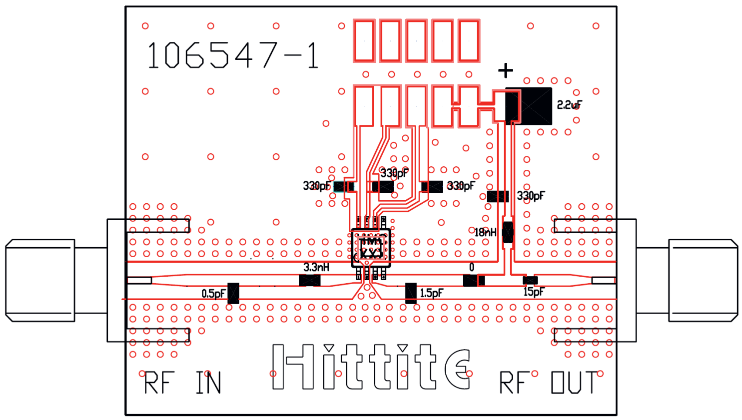
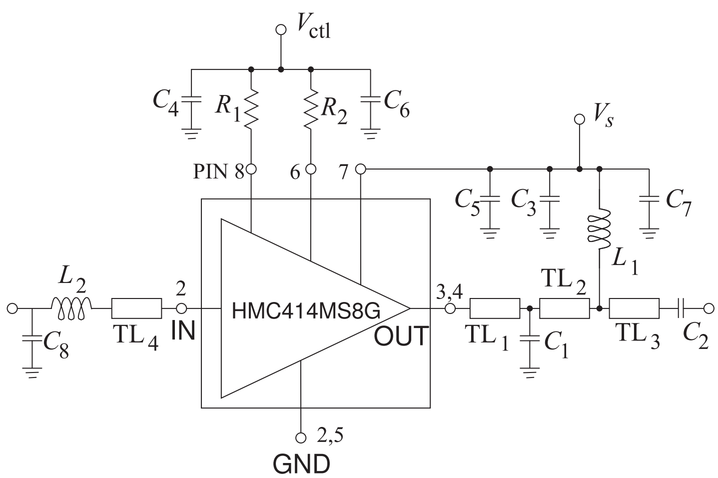


Figure 11-5: Block diagram of an RF amplifier including biasing networks.

an evaluation board, e.g. see the evaluation board in Figure 11-6 for a power amplifier. This module does not include input and output matching networks but these are provided on the evaluation board.



(a) Evaluation board



(b) Schematic of the evaluation board

Figure 11-6: Evaluation board for the HMC414MS8G GaAs InGaP HBT MMIC power amplifier module operating between 2.2 and 2.8 GHz. The amplifier provides 20 dB of gain and +30 dBm of saturated power at 32% PAE from a +5V supply voltage. The amplifier can also operate with a 3.6 V supply selectable by the resistors R_1 and R_2. Copyright Hittite Microwave Corporation, used with permission [2].

11.4 Filters

A microwave filter can consist solely of lumped elements, solely of distributed elements, or a mix. Loss of lumped elements, particularly above a few gigahertz, means that the performance of distributed filters nearly always exceeds that of lumped-element filters. However, since the basic component of a distributed filter is a one-quarter wavelength long transmission line, distributed filters can be prohibitively large below a few gigahertz. The basic types of responses required at RF as follows:

(a) Lowpass—providing maximum power transfer at frequencies below the corner frequency, f_0. See Figure 11-7(a).
(b) Highpass—passing signals at frequencies above f_0. Below f_0, transmission is blocked. See Figure 11-7(b).
(c) Bandpass—passing signals at frequencies between lower and upper corner frequencies (defining the passband) and blocking transmission outside the band. This is the most common type. See Figure 11-7(c).
(d) Bandstop (or notch)—which blocks signals between lower and upper corner frequencies (defining the stopband). See Figure 11-7(d).
(e) Allpass—which equalizes a signal by adjusting the phase generally to

correct for phase distortion elsewhere. See Figure 11-7(e).

In the passband, filters can be treated as though they are attenuators with typical losses, or attenuation, of 0.1 dB for a large basestation filter to 3 dB for a miniature filter in a handset. Out-of-band losses are typically at least 40 dB for a handset filter to 120 dB or more for a basestation filter.

11.5 Noise

Amplifiers, filters, and mixers in an RF front end process (e.g., amplify, filter, and mix) input noise the same way as an input signal. In addition, these modules contribute **excess noise** of their own. Without loss of generality, the following discussion considers noise with respect to the amplifier shown in Figure 11-8(a), where v_s is the input signal. The noise signal, with source designated by v_n, is uncorrelated and random, and described as an RMS voltage or by its noise power.

11.5.1 Noise Figure

The most important noise-related metric is the SNR. Denoting the noise power input to the amplifier as N_i, and denoting the signal power input to the amplifier as S_i, the input signal-to-noise power ratio is $\mathrm{SNR}_i = S_i/N_i$. If the amplifier is noise free, then the input noise and signal powers are amplified by the power gain of the amplifier, G. Thus the output noise power is $N_o = GN_i$, the output signal power is $S_o = GS_i$, and the output SNR is

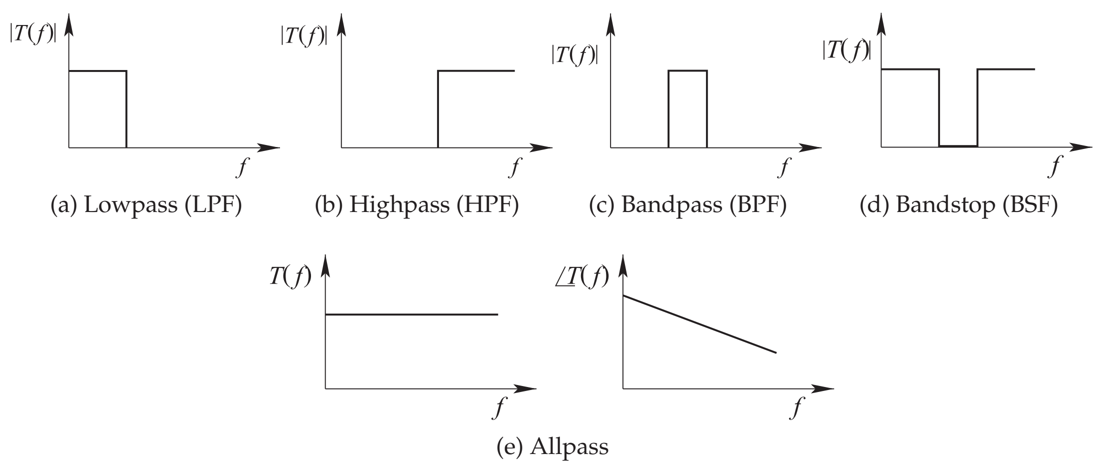
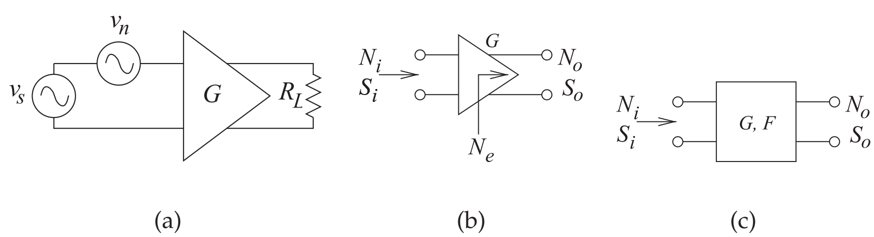


Figure 11-7: Ideal filter transfer function, $T(f)$, responses.

Figure 11-8: Noise and two-ports: (a) amplifier; (b) amplifier with excess noise; and (c) noisy two-port network.



$\mathrm{SNR}_o = S_o/N_o = \mathrm{SNR}_i$.
In practice, an amplifier is noisy, with the addition of excess noise, N_e, indicated in Figure 11-8(b). The excess noise originates in different components in the amplifier and is either referenced to the input or to the output of the amplifier. Most commonly it is referenced to the output so that the total output noise power is $N_o = GN_i + N_e$. In the absence of a qualifier, the **excess noise** should be assumed to be referred to the output. N_e is not measured directly. Instead, the ratio of the SNR at the input to that at the output is measured and called the **noise factor**, F:

$$F = \frac{\mathrm{SNR}_i}{\mathrm{SNR}_o}, \tag{11.1}$$

If the circuit is noise free, then $\mathrm{SNR}_o = \mathrm{SNR}_i$ and $F = 1$. If the circuit is not noise free, then $\mathrm{SNR}_o < \mathrm{SNR}_i$ and $F > 1$. With the excess noise, referred to the output,

$$F = \frac{\mathrm{SNR}_i}{\mathrm{SNR}_o} = \frac{\mathrm{SNR}_i}{1}\frac{1}{\mathrm{SNR}_o} = \frac{S_i}{N_i}\frac{N_o}{S_o} = \frac{S_i}{N_i}\frac{GN_i + N_e}{GS_i} = 1 + \frac{N_e}{GN_i}. \tag{11.2}$$

One of the conclusions that can be drawn from this is that F depends on the available noise power at the input of the circuit. As reference, the available noise power, N_R, from a resistor at **standard temperature**, T_0 (290 K), and over bandwidth, B (in Hz), is used,

$$N_i = N_R = kT_0B, \tag{11.3}$$

where k ($= 1.381 \times 10^{-23}$ J/K) is **the Boltzmann constant**. If the input of an amplifier is connected to this resistor and all of the available noise power is delivered to the amplifier, then

$$F = 1 + \frac{N_e}{GN_i} = 1 + \frac{N_e}{GkT_0B}. \tag{11.4}$$

When expressed in decibels, the **noise figure** (**NF**) is used:

$$\mathrm{NF} = 10\log_{10} F = \mathrm{SNR}_i|_{\mathrm{dB}} - \mathrm{SNR}_o|_{\mathrm{dB}}.$$

where the SNR is expressed in decibels. Rearranging Equation (11.4), the output-referred excess noise power is

$$N_e = (F-1)GkT_0B. \tag{11.5}$$

Also from Equation (11.4), the output noise is

$$N_o = GN_i + N_e = FGN_i. \tag{11.6}$$

That is, $$N_o|_{\mathrm{dBm}} = \mathrm{NF} + G|_{\mathrm{dB}} + N_i|_{\mathrm{dBm}}. \tag{11.7}$$

EXAMPLE 11.1 Noise Figure of an Attenuator

What is the noise figure of a 20 dB attenuator in a 50 Ω system?

Solution:

The appropriate circuit model to use in the analysis consists of the attenuator driven by a generator with a 50 Ω source impedance, and the attenuator drives a 50 Ω load. Also, the input impedance of the terminated attenuator is 50 Ω, as is the impedance looking into the output of the attenuator when it is connected to the source. The key point is that the noise coming from the source is the noise thermally generated in the 50 Ω source impedance, and this noise is equal to the noise that is delivered to the load. So the input noise, N_i, is equal to the output noise:

$$N_o = N_i. \tag{11.8}$$

The input signal is attenuated by 20 dB (= 100), so $S_o = S_i/100,$ (11.9)

and thus the noise factor is $F = \frac{SNR_i}{SNR_o} = \frac{S_i}{N_i}\frac{N_o}{S_o} = \frac{S_i}{N_i}\frac{N_i}{S_i/100} = 100$ (11.10)

and the noise figure is NF $= 20$ dB. (11.11)

THat is, the noise figure of an attenuator (or a filter) is just the loss of the component. This is not true for amplifiers of course, as there are other sources of noise, and the output impedance of a transistor is not a thermal resistance.

11.5.2 Noise of a Cascaded System

Section 11.5.1 developed the noise factor and noise figure measures for a two-port. This can be generalized for a system. Considering the second stage of the cascade in Figure 11-9, the excess noise at the output of the second stage, due solely to the noise generated internally in the second stage, is

$$N_{2e} = (F_2 - 1)kT_0BG_2. \tag{11.12}$$

Then the total noise power at the output of the two-stage cascade is

$$\begin{aligned} N_{2o} &= (F_2 - 1)kT_0BG_2 + N_{o,1}G_2 \\ &= (F_2 - 1)kT_0BG_2 + F_1kT_0BG_1G_2. \end{aligned} \tag{11.13}$$

This assumes (correctly) that the excess noise added in one stage is uncorrelated to the other sources of noise. Thus noise powers can be added. Generalizing this result the total system noise factor:

$$F^{\mathrm{T}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \frac{F_4 - 1}{G_1G_2G_3} + \cdots . \tag{11.14}$$

This equation is known as **Friis's formula** [3].

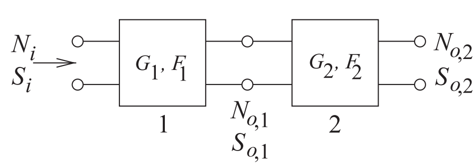


Figure 11-9: Cascaded noisy two-ports.

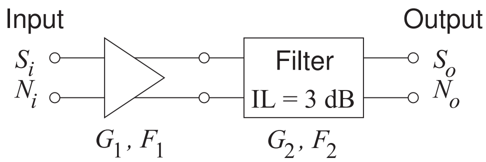


Figure 11-10: Differential amplifier followed by a differential filter.

EXAMPLE 11.2 Noise Figure of Cascaded Stages

Consider the cascade of a differential amplifier and a filter shown in Figure 11-10.

(a) What is the midband gain of the filter in decibels? Note that IL is insertion loss.
(b) What is the midband noise figure of the filter?
(c) The amplifier has a gain G_1 = 20 dB and a noise figure of 2 dB. What is the overall gain of the cascade system in the middle of the band?
(d) What is the noise factor of the cascade system?
(e) What is the noise figure of the cascade system?

Solution:

(a) $G_2 = 1/\mathrm{IL}$, thus $G_2 = -3$ dB.
(b) For a passive element, $\mathrm{NF}_2 = \mathrm{IL} = 3$ dB.
(c) $G_1 = 20$ dB and $G_2 = -3$ dB, so $G_{\mathrm{TOTAL}} = G_1|_{\mathrm{dB}} + G_2|_{\mathrm{dB}} = 17$ dB.
(d) $F_1 = 10^{\mathrm{NF}_1/10} = 10^{2/10} = 1.585$, $F_2 = 10^{\mathrm{NF}_2/10} = 10^{3/10} = 1.995$, $G_1 = 10^{20/10} = 100$, and $G_2 = 10^{-3/10} = 0.5$. Using Friis's formula

$$F_{\mathrm{TOTAL}} = F_1 + \frac{F_2 - 1}{G_1} = 1.585 + \frac{1.995 - 1}{100} = 1.594. \tag{11.15}$$

(e) $\mathrm{NF}_{\mathrm{TOTAL}} = 10\log_{10}(\mathrm{F}_{\mathrm{TOTAL}}) = 10\log_{10}(1.594) = 2.03$ dB.

EXAMPLE 11.3 Noise Figure of a Two-Stage Amplifier

Consider a room-temperature (20°C) two-stage amplifier where the first stage has a gain of 10 dB and the second stage has a gain of 20 dB. The noise figure of the first stage is 3 dB and the second stage is 6 dB. The amplifier has a bandwidth of 10 MHz.

(a) What is the noise power presented to the amplifier in 10 MHz?
(b) What is the total gain of the amplifier?
(c) What is the total noise factor of the amplifier?
(d) What is the total noise figure of the amplifier?
(e) What is the noise power at the output of the amplifier in 10 MHz?

Solution:

(a) Noise power of a resistor at room temperature is -174 dBm/Hz (or more precisely -173.86 dBm/Hz at 293 K). In 10 MHz the input noise power is $N_i = -173.86\ \mathrm{dBm} + 10\log(10^7) = -173.86 + 70\ \mathrm{dBm} = -103.86\ \mathrm{dBm}$.
(b) Total gain $G^{\mathrm{T}} = G_1 G_2 = 10\ \mathrm{dB} + 20\ \mathrm{dB} = 30\ \mathrm{dB} = 1000$.
(c) $F_1 = 10^{\mathrm{NF}_1/10} = 10^{3/10} = 1.995$, $F_2 = 10^{\mathrm{NF}_2/10} = 10^{6/10} = 3.981$. Using Friis's formula, the total noise figure is $F^{\mathrm{T}} = F_1 + \dfrac{F_2 - 1}{G_1} = 1.995 + \dfrac{3.981 - 1}{10} = 2.393$.
(d) The total noise figure is $\mathrm{NF}^{\mathrm{T}} = 10\log_{10}(\mathrm{F}^{\mathrm{T}}) = 10\log_{10}(2.393) = 3.79$ dB.
(e) Output noise power in 10 MHz bandwidth is $N_o = F^{\mathrm{T}} k T_0 B G^{\mathrm{T}} = (2.393)\cdot(1.3807\cdot 10^{-23}\cdot \mathrm{J}\cdot\mathrm{K}^{-1})\cdot(293\ \mathrm{K})\cdot(10^7\cdot\mathrm{s}^{-1})(1000) = 9.846\cdot 10^{-11}\ \mathrm{W} = -70.07\ \mathrm{dBm}$. Alternatively, $N_o|_{\mathrm{dBm}} = N_i|_{\mathrm{dBm}} + G^{\mathrm{T}}\big|_{\mathrm{dB}} + \mathrm{NF}^{\mathrm{T}} = -103.86\ \mathrm{dBm} + 30\ \mathrm{dB} + 3.79\ \mathrm{dB} = -70.07\ \mathrm{dBm}$.

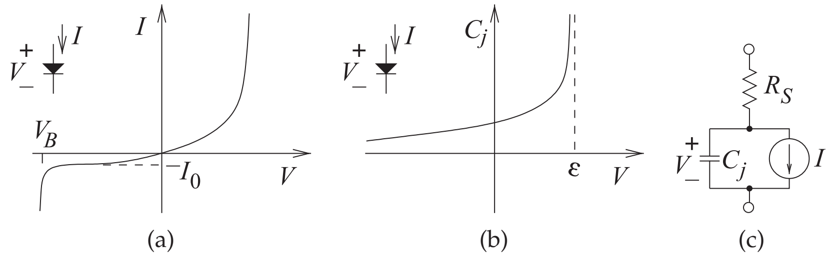


Figure 11-11: Characteristics of a pn junction diode or a Schottky diode: (a) current-voltage characteristic; (b) capacitance-voltage characteristic; and (c) diode model.

11.6 Diodes

Diodes are two-terminal devices that have nonlinear current-voltage characteristics. The most common diodes are listed in Table 11-1.

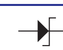
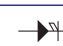
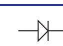
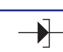
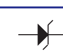
Junction and Schottky Diodes: A junction diode has an asymmetric current-voltage characteristic, see Figure 11-11(a),

$$I = I_0 \left[\exp\left(\frac{qV}{nkT}\right) - 1 \right], \tag{11.16}$$

where $q(= -e)$ is the absolute value of the charge of an electron, k is the Boltzmann constant ($1.37 \cdot 10^{-23}$ J/K), and T is the absolute temperature (in kelvin). I_0 is the reverse saturation current and is small, with values ranging from 1 pA to 1 nA. The quantity n is the diode ideality factor, with $n = 2$ for a graded junction pn junction diode, and $n = 1.0$ when the interface between p-type and n-type semiconductor materials is abrupt. The abrupt junction is most closely realized by a Schottky diode, where a metal forms one side of the interface. When the applied voltage is sufficiently positive to cause a large current to flow, the diode is said to be forward biased. When the voltage is negative, the current flow is negligible and the diode is said to be reverse biased. In a diode, charge is separated over distance and so a diode has junction capacitance modeled as

$$C_j(V) = \frac{C_{j0}}{(1 - (V/\phi))^{\gamma}}, \tag{11.17}$$

Table 11-1: IEEE standard symbols for diodes and a rectifier [4]. ([1]In the direction of anode (A) to cathode (K). [2]Use symbol for general diode unless it is essential to show the intrinsic region.)

Component	Symbol
Diode, general (including Schottky)[1]	
IMPATT diode[1]	
Gunn diode	
PIN diode[1,2]	
Light emitting diode (LED)[1]	
Rectifier[1]	
Tunnel diode[1]	
Varactor diode[1]	 or 
Zener diode[1]	

where ϕ is the built-in potential difference across the diode. This capacitance profile is shown in Figure 11-11(b). The built-in potential is typically 0.6 V for silicon diodes and 0.75 V for GaAs diodes. The doping profile can be adjusted so that γ can be less than the ideal $\frac{1}{2}$ of an abrupt junction diode. Current must flow through bulk semiconductor before reaching the active region of the semiconductor diode, and so there will be a resistive voltage drop. Combining effects leads to the equivalent circuit of a pn junction or Schottky diode, shown in Figure 11-11(c).

Varactor Diode: A varactor diode is a pn junction diode operated in reverse bias and optimized for good performance as a tunable capacitor. A common application of a varactor diode is as the tunable element in a voltage-controlled oscillator (VCO) where the varactor, with voltage-dependent capacitance, C, is part of a resonant circuit.

PIN Diode: A PIN diode is a variation on a pn junction diode with a region of intrinsic semiconductor (the I in PIN) between the p-type and n-type semiconductor regions. The properties of the PIN diode depend on whether there are carriers in the intrinsic region. The PIN diode has the current-voltage characteristics of a pn junction diode at low frequencies; however, at high frequencies it looks like a linear resistor, as carriers in the intrinsic region move slowly. When a forward DC voltage is applied to the PIN diode, the intrinsic region floods with carriers, and at microwave frequencies the PIN diode is then modeled as a low-value resistor. At high frequencies there is not enough time to remove the carriers in the intrinsic region, so even if the total voltage (DC plus RF) across the PIN diode is negative, there are carriers in the intrinsic region throughout the RF cycle. If the DC voltage is negative, carriers are removed from the intrinsic region and the diode looks like a large-value resistor at RF. The PIN diode is used as a microwave switch controlled by a DC voltage.

Zener Diode: Zener diodes are pn junction or Schottky diodes that have been specially designed to have sharp reverse breakdown characteristics. They can be used to establish a voltage reference or, used as a limiter diode, to provide protection of more sensitive circuitry. As a limiter, they are found in communication devices in a back-to-back configuration to limit the voltages that can be applied to sensitive RF circuitry.

11.7 Switch

In many cellular systems a phone does not transmit and receive simultaneously. Consequently a switch can be used to alternately connect a transmitter and a receiver to an antenna. An ideal switch is shown in Figure 11-12(a), where an input port, $\mathrm{RF_{IN}}$, and an output port, $\mathrm{RF_{OUT}}$, are shown. At microwave frequencies, realistic switches must be modeled with parasitics and with finite on and off resistances. A realistic model applicable to many switch types is shown in Figure 11-12(b). The capacitive parasitics, the C_Ps, limit the frequency of operation and the on resistance, R_{ON}, causes loss. Ideally the off resistance, R_{OFF}, is very large, however, the parasitic shunt capacitance, C_{OFF}, is nearly always more significant. The result is that

Table 11-2: Typical properties of small microwave switches. (Sources: [1]Radant MEMS, [2]RF Micro Devices, and [3]Tyco Electronics.)

Switch type	Power handling	Insertion loss	Operating frequency	Actuation voltage	Response time
MEMS[1]	0.5 W	0.5 dB	to 10 GHz	90 V	10 μs
MEMS[1]	4 W	0.8 dB	to 35 GHz	110 V	10 μs
pHEMT[2]	10 W	0.3 dB	to 6.5 GHz	5 V	0.5 μs
pHEMT[2]	0.3 W	1.1 dB	to 25 GHz	5 V	0.5 μs
PIN[3]	13 W	0.35 dB	to 2 GHz	12 V	0.5 μs
PIN[3]	10 W	0.4 dB	to 6 GHz	12 V	0.5 μs

at high frequencies there is an alternative capacitive connection between the input and output through C_{OFF}. The on resistance of the switch introduces voltage division that can be seen by comparing the ideal connection shown in Figure 11-12(c) and the more realistic connection shown in Figure 11-12(d). There are four main types of microwave switches: mechanical (rarely used except in instrumentation), PIN diode, FET, and **microelectromechanical system** (**MEMS**), see Figures 11-12(e–g) and Table 11-2.

A MEMS switch is fabricated using photolithographic techniques similar to those used in semiconductor manufacturing. They are essentially miniature mechanical switches with a voltage used to control the position of a shorting arm, see Figure 11-13.

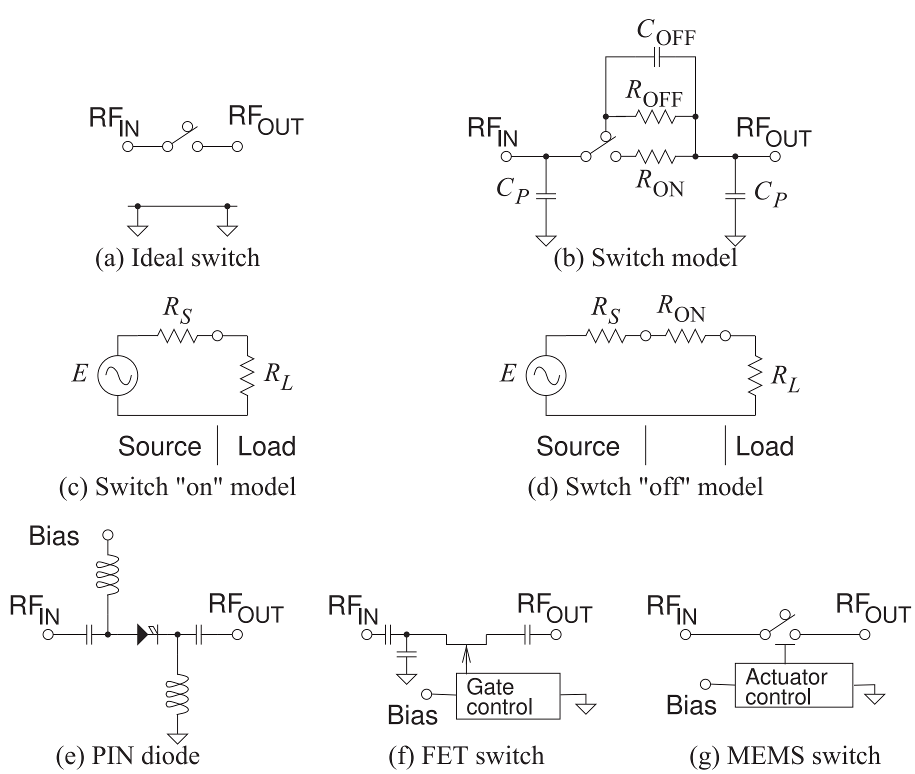


Figure 11-12: Microwave switches: (a) ideal switch connecting RF_{IN} and RF_{OUT} ports; (b) model of a microwave switch; (c) ideal circuit model with switch on and with source and load; (d) realistic low-frequency circuit model with switch on; (e) switch realized using a PIN diode; (f) switch realized using an FET; and (g) switch realized using a MEMS switch.

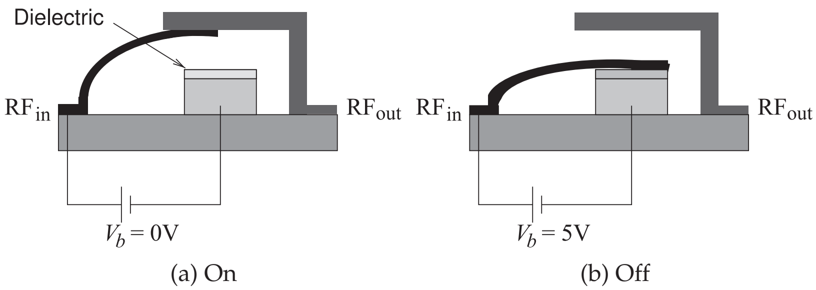


Figure 11-13: RF MEMS switch: (a) the $\mathrm{RF_{in}}$ line in contact with the $\mathrm{RF_{out}}$ line; and (b) the cantlever beam electrostatically attracted to the pedestal and there is no RF connection.

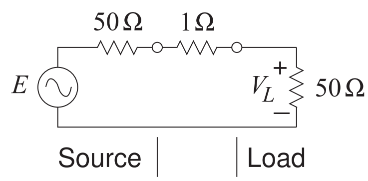


Figure 11-14: Model used in calculating the loss of a switch in a 50 Ω system.

EXAMPLE 11.4 Insertion Loss of a Switch

What is the insertion loss of a switch with a 1 Ω on resistance when it used in a 50 Ω system?

Solution:

The model to be used for evaluating the insertion loss of the switch is shown in Figure 11-14. The insertion loss is found by first determining the available power from the source and then the actual power delivered to the load. The available input power is calculated by first ignoring the 1 Ω switch resistance. Then there is maximum power transfer from the source to the load. The available input power is

$$P_{Ai} = \frac{1}{2}\frac{\left(\frac{1}{2}E\right)^2}{50} = \frac{E^2}{400}, \tag{11.18}$$

where E is the peak RF voltage at the Thevenin equivalent source generator. The power delivered to the 50 Ω load is found after first determining the peak load voltage:

$$V_L = \frac{50}{50+1+50}E = \frac{50}{101}E. \tag{11.19}$$

Thus the power delivered to the load is

$$P_D = \frac{1}{2}\frac{V_L^2}{50} = \frac{1}{100}\left(\frac{50}{101}\right)^2 E^2. \tag{11.20}$$

The insertion loss is

$$\mathrm{IL} = \frac{P_{Ai}}{P_D} = \frac{E^2}{400}\frac{100}{E^2}\left(\frac{101}{50}\right)^2 = 1.020 \;= 0.086\ \mathrm{dB}. \tag{11.21}$$

11.8 Ferrite Components: Circulators and Isolators

Circulators and isolators are nonreciprocal devices that preferentially route microwave signals. The essential element is a disc (puck) of ferrite which when magnetized supports a preferred direction of propagation due to the gyromagnetic effect.

11.8.1 Gyromagnetic Effect

An electron has a quantum mechanical property called spin (there is not a spinning electron) creating a magnetic moment m, see Figure 11-15(a). In most materials the electron spin occurs in pairs with the magnetic moment of one electron canceled by the oppositely-directed magnetic moment of another electron. However, in some materials the spin does not occur in pairs and there is a net magnetic moment.

A most interesting microwave property occurs when a magnetic material is biased by a DC magnetic field resulting in the gyromagnetic effect. This affects the way an RF field propagates and this is described by a nine element permeability called a tensor. The permeability of a magnetically biased magnetic material is:

$$[\mu] = \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu & \jmath\kappa \\ 0 & -\jmath\kappa & \mu \end{bmatrix}. \tag{11.22}$$

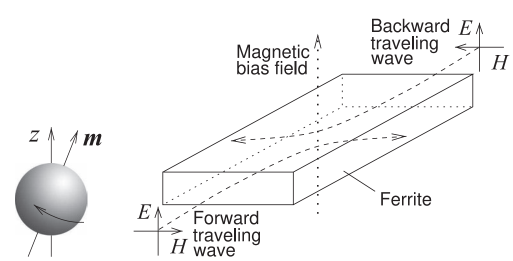
That is,

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu & \jmath\kappa \\ 0 & -\jmath\kappa & \mu \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}. \tag{11.23}$$

Thus a z-directed H field produces a y-directed B field. The effect on propagation of an EM field is shown in Figure 11-15(b). The EM wave does not travel in a straight line and instead curves, in this case, to the right. Thus forward- and backward-traveling waves diverge from each other and propagation is not reciprocal. This separates forward- and backward-traveling waves.

11.8.2 Circulator

A circulator exploits the gyromagnetic effect. Referring to the schematic of a circulator shown in Figure 11-16(a), where the arrows indicate that the signal that enters Port 1 of the circulator leaves the circulator at Port 2 and not at Port 3. Similarly power that enters at Port 2 is routed to Port 3, and



(a) (b)

Figure 11-15: Magnetic moments: (a) electron spin; and (b) Gyromagnetic effect impact on the propagation of EM waves in a magnetic material with an externally applied magnetic bias field.

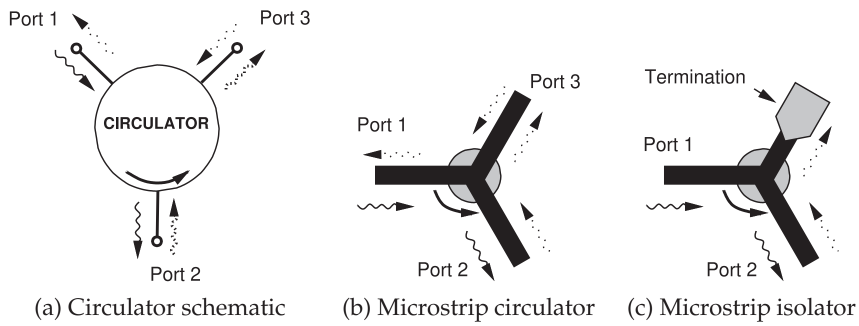


Figure 11-16: Ferrite components with a magnetized ferrite puck in the center in (b) and (c).

power entering at Port 3 is routed to Port 1. In terms of S parameters, an ideal circulator has the scattering matrix

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} \\ S_{21} & 0 & 0 \\ 0 & S_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{11.24}$$

A microstrip circulator is shown in Figure 11-16(b), where a disc of magnetized ferrite can be placed on top of a microstrip Y junction to realize a preferential direction of propagation of the EM fields. In the absence of the biasing magnetic field, the circulation function does not occur.

11.8.3 Circulator Isolation

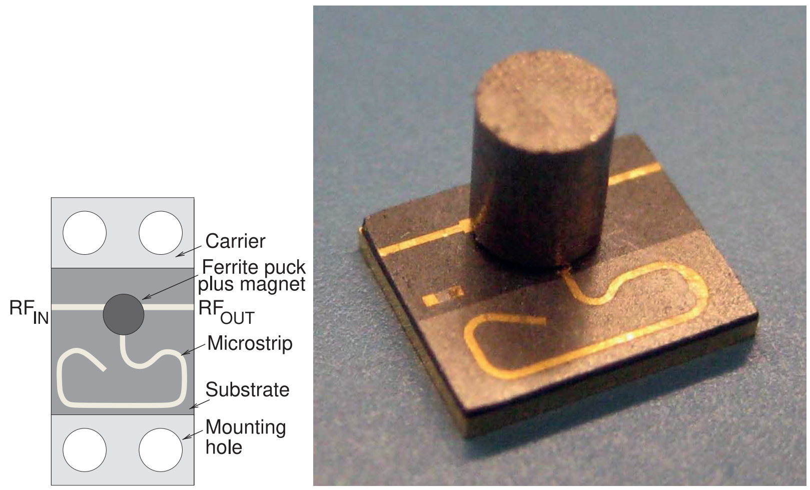
The isolation of a circulator is the insertion loss from what is the output port to the input port, i.e. in the reverse direction. Referring to the circulator in Figure 11-16(a), if port 1 is the input port there are two output ports and so there are two isolations equal to the return loss from port 3 to port 2, and from port 2 to port 1. The smaller of these is the quoted isolation. If this is an ideal circulator and port 2 is perfectly matched, then the isolation would be infinite. Then the S parameters of the circulator are

$$\mathbf{S} = \begin{bmatrix} \Gamma & \alpha & T \\ T & \Gamma & \alpha \\ \alpha & T & \Gamma \end{bmatrix} \approx \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{11.25}$$

where T is the transmission factor, Γ is the reflection coefficient at the ports, and α is the leakage.

11.8.4 Isolator

Isolators are devices that allow power flow in only one direction. Figures 11-16(c) and 11-17 show microstrip isolators based on three-port circulators. Power entering Port 1 as a traveling wave is transferred to the ferrite and emerges at Port 2. Virtually none of the power emerges at Port 3. A traveling wave signal applied at Port 2 appears at Port 3, where it is absorbed in a termination created by resistive material placed on top of the microstrip. The resistive material forms a lossy transmission line and no power is reflected. Thus power can travel from Port 1 to Port 2, but not in the reverse direction.



(a) Layout with carrier (b) Image

Figure 11-17: A microstrip isolator operating from 29 to 31.5 GHz. Isolator in (b) has the dimensions $5 \text{ mm} \times 6$ mm and is 6 mm high. The isolator supports 2 W of forward and reverse power with an isolation of 18 dB and insertion loss of 1 dB. Renaissance 2W9 series, copyright Renaissance Electronics Corporation, used with permission.

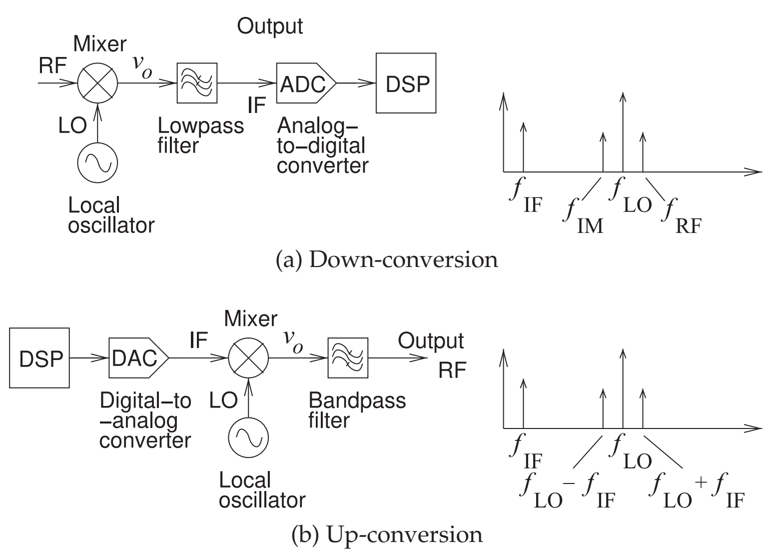


Figure 11-18: Frequency conversion using a mixer.

11.9 Mixer

Frequency conversion or mixing is the process of converting information centered at one frequency (present in the form of a modulated carrier) to another frequency. The second frequency is either higher, in the case of frequency **up-conversion**, where it is more easily transmitted; or lower when mixing is called frequency **down-conversion**, where it is more easily captured, see Figure 11-18.

Conversion loss, L_C**:** This is the ratio of the available power of the input signal, $P_{\text{in}}(\text{RF})$, to that of the output signal after mixing, $P_{\text{out}}(\text{IF})$:

$$L_C = \frac{P_{\text{in}}(\text{RF})}{P_{\text{out}}(\text{IF})}. \tag{11.26}$$

EXAMPLE 11.5 Mixer Calculations

A mixer has an LO of 10 GHz. The mixer is used to down-convert a signal at 10.1 GHz and has a conversion loss, L_c of 3 dB and an image rejection of 20 dB. A 100 nW signal is presented to the mixer at 10.1 GHz. What is the frequency and output power of the down-converted signal at the IF?

Solution:

The IF is at 100 MHz. $L_c = 3$ dB = 2 and from Equation (11.26) the output power at IF of the intended signal is

$$P_{\text{out}} = P_{\text{in}}(\text{RF})/L_c = 100\ \text{nW}/2 = 50\ \text{nW} = -43\ \text{dBm}. \tag{11.27}$$

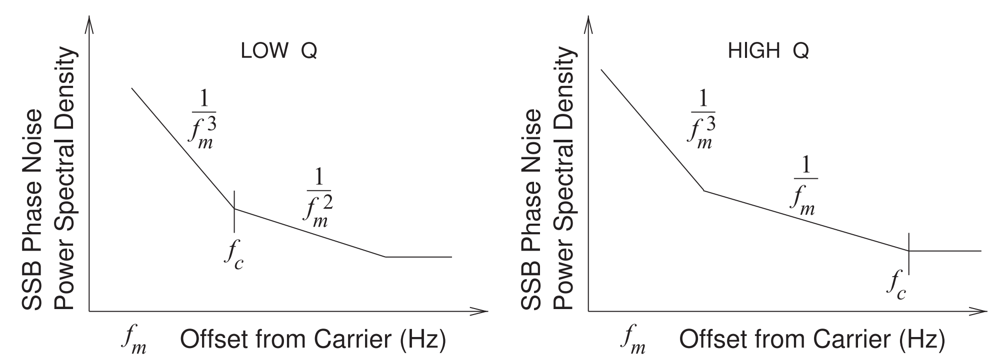
11.10 Local Oscillator

In an oscillator noise close to the oscillation center frequency is called flicker noise or $1/f$ noise and in offsets below a few tens of megahertzis much larger than thermal noise and so a big concern in microwave systems. The noise manifests itself as random fluctuations of amplitude and phase of the carrier. The amplitude fluctuations are quenched by saturation in the oscillator and so are not of concern. Thus the close-in noise of concern is just phase noise.

The phase noise of an oscillator with a low Q feedback loop is shown in Figure 11-19(a) and in Figure 11-19(b) for a high Q loop. The physical origin of the straight line phase regions is nt understood.

Phase noise is expressed as the ratio of the phase noise power in a 1 Hz bandwidth of a single sideband (SSB) to the total signal power. This is measured at a frequency f_m offset from the carrier and denoted $\mathcal{L}(f_m)$ with the units of dBc/Hz (i.e., decibels relative to the carrier power per hertz).

The phase noise that is important in RF and microwave oscillators (having relatively low Q) is usually dominated by a $1/f_m^2$ shape. Then the phase noise



(a) Closed loop (b) Closed loop

Figure 11-19: Log-log plot of oscillator noise spectra: (a) closed-loop noise with low-Q loop; and (b) closed-loop noise with high-Q loop.

at 1 MHz (a common frequency for comparing the phase noise performance of different oscillators) is related to the phase noise measured at f_m by

$$\mathcal{L}(1\text{ MHz}) = \mathcal{L}(f_m) - 10\log\left(\frac{1\text{ MHz}}{f_m}\right)^2. \tag{11.28}$$

11.11 Frequency Multiplier

A microwave frequency multiplier uses a nonlinear element to generate harmonics and a bandpass filter selects the appropriate harmonic for the output, see Figure 11-20. If the input signal is

$$x(t) = A\cos(\omega_t + \phi) \tag{11.29}$$

then the output at the n harmonic is

$$y(t) = A_n\cos(n\omega_t + n\phi). \tag{11.30}$$

11.12 Summary

The use of modules has become increasingly important in microwave engineering. A wide variety of passive and active modules are available and high-performance systems can be realized enabling many microwave systems of low to medium volumes to be realized cost effectively and with stellar performance. Module vendors are encouraged by the market to develop competitive modules that can be used in a wide variety of applications.

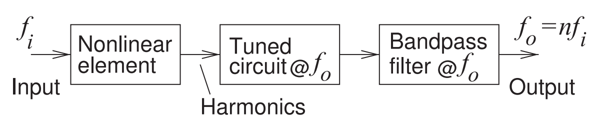


Figure 11-20: Frequency multiplier using a nonlinear tuned circuit.

11.13 References

[1] http://www.synergymwave.com.
[2] http://www.hittite.com.
[3] H. Friis, "Noise figures of radio receivers," *Proc. of the IRE*, vol. 32, no. 7, pp. 419–422, Jul. 1944.
[4] IEEE Standard 315-1975, Graphic Symbols for Electrical and Electronics Diagrams (Including Reference Designation Letters), Adopted Sept. 1975, Reaffirmed Dec. 1993. Approved by American National Standards Institute, Jan. 1989. Approved adopted for mandatory use, Department of Defense, United States of America, Oct. 1975. Approved by Canadian Standards Institute, Oct. 1975.

11.14 Exercises

1. An amplifier consists of three cascaded stages with the following characteristics:

	Stage 1	Stage 2	Stage 3
Gain (dB)	−3	15	5
NF (dB)	3	2	2

   (a) What is the overall gain of the amplifier?
   (b) What is the overall noise figure of the amplifier?

2. What is the available noise power of a 50 Ω resistor in a 10 MHz bandwidth. The resistor is at standard temperature.

3. A 50 Ω resistor a 20 Ω resistor are in shunt. If both resistors have a temperature of 300 K, what is the total available noise power spectral den-

sity of the shunt resistors?

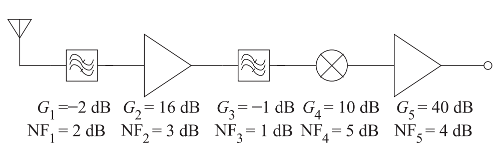
4. A 2 GHz amplifier in a 50 Ω system has a bandwidth of 10 MHz, a gain of 40 dB, and a noise figure of 3 dB. The amplifier is driven by a circuit with a Thevenin equivalent resistance of 50 Ω held at 290 K (standard temperature). What is the available noise power at the output of the amplifier?

5. A 30 dB attenuator is terminated at Port 2 in a matched resistor and both are at 290 K. What is the noise temperature at Port 1 of the attenuator?

6. A receive amplifier with a gain of 30 dB, a noise figure of 2 dB, and bandwidth of 5 MHz is connected to an antenna which has a noise temperature of 20 K. [Parallels Example 11.2]
   (a) What is the available noise power presented to the input of the amplifier in the 5 MHz bandwidth (recall that the antenna noise temperature is 20 K?
   (b) If instead the input of the amplifier is connected to a resistor held at standard temperature, what is the available noise power presented to the input of the amplifier in the 5 MHz bandwidth?
   (c) What is the noise factor of the amplifier?
   (d) What is the excess noise power of the amplifier referred to the its output?
   (e) What is the effective noise temperature of the amplifier when the amplifier is connected to the antenna with a noise temperature of 20 K. That is, what is the effective noise temperature of the resistor in the Thevenin equivalent circuit of the amplifier output?

7. A receive amplifier has a bandwidth of 5 MHz, a 1 dB noise figure, a linear gain of 20 dB. The minimum acceptable SNR is 10 dB.
   (a) What is the output noise power in dBm?
   (b) What is the minimum detectable output signal in dBm?
   (c) What is the minimum detectable input signal in dBm?

8. The system shown below is a receiver with bandpass filters, amplifiers, and a mixer. [Parallels Example 11.2]



   (a) What is the total gain of the system?
   (b) What is the noise factor of the first filter?
   (c) What is the system noise factor?
   (d) What is the system noise figure?

9. An amplifier consists of three cascaded stages with the following characteristics:

	Stage 1	Stage 2	Stage 3
Gain	10 dB	15 dB	30 dB
NF	0.8 dB	2 dB	2 dB

What is the noise figure (NF) and gain of the cascade amplifier?

10. The first stage of a two-stage amplifier has a linear gain of 40 dB and a noise figure if 3 dB. The second stage has a gain of 10 dB and a noise figure of 5 dB.
   (a) What is the overall gain of the amplifier?
   (b) What is the overall noise figure of the amplifier?

11. A subsystem consists of a matched filter with an insertion loss of 2 dB then an amplifier with a gain of 20 dB and a noise figure, NF, of 3 dB.
   (a) What is the overall gain of the subsystem?
   (b) What is NF of the filter?
   (c) What is NF of the subsystem?

12. A subsystem consists of a matched amplifier with a gain of 20 dB and a noise figure of 2 dB, followed by a 2 dB attenuator, and then another amplifier with a gain of 10 dB and NF of 3 dB.
   (a) What is the overall gain of the subsystem?
   (b) What is NF of the attenuator?
   (c) What is NF of the subsystem?

13. A connector used in a 50 Ω system introduces a series resistance of 0.5 Ω. What is the insertion loss of the connector?

14. A microwave switch is used in a 75 Ω system and has a 5 Ω on resistance. The reactive parasitics of the switch are negligible.
   (a) What is the insertion loss of the switch in the on state?
   (b) What is the return loss of the switch in the on state?

15. A microwave switch is used in a 50 Ω system and has a 5 Ω on resistance. The reactive parasitics of the switch are negligible.
   (a) What is the insertion loss of the switch in the on state?
   (b) If the available power of the source is 50 W, what is the power dissipated by the switch?

16. A microwave switch is used at 1 GHz in a 50 Ω system and it has a 2 Ω on resistance and a 2 kΩ off resistance. The reactive parasitics of the switch are negligible.
   (a) What is the insertion loss of the switch?

(b) What is the isolation of the switch (i.e., what is the insertion loss of the switch when it is in the off state)?

17. The RF front end of a communications unit consists of a switch, then an amplifier, and then a mixer. The switch has a loss of 0.5 dB, the amplifier has a gain of 20 dB, and the mixer has a conversion gain of 3 dB. What is the overall gain of the cascade?

18. A three-port circulator has the S parameters

$$\begin{bmatrix} 0 & 0 & 0.5 \\ 20.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}.$$

Port 3 is terminated in a matched load creating a two-port network.

(a) Find the S parameters of the two-port.
(b) What is the return loss in dB at Port 1 if Port 2 is terminated in a matched load?
(c) What is the insertion loss in dB for a signal applied at Port 1 and leaving at Port 2 with matched source and load impedances?
(d) What is the insertion loss in dB for a signal applied at Port 2 and leaving at Port 1 with matched source and load impedances?

19. Two isolators are used in cascade. Each isolator has an isolation of 20 dB. The isolators are matched so that their input and output reflection coefficients are zero. Determine the isolation of the cascaded isolator system?

20. A three-port circulator in a 50-Ω system has the S parameters

$$\begin{bmatrix} 0.1 & 0.01 & 0.5 \\ 0.5 & 0.1 & 0.01 \\ 0.01 & 0.5 & 0.1 \end{bmatrix}.$$

If port 3 is terminated in a matched load to create a two-port network

(a) Find the S parameters of the two-port.
(b) What is the return loss in dB at Port 1 if Port 2 is terminated in 50-Ω?
(c) What is the insertion loss in dB for a signal applied at Port 2 and leaving at Port 1 with 50-Ω source and load impedances?
(d) What is the insertion loss in dB for a signal applied at Port 1 and leaving at Port 2 with 50-Ω source and load impedances?
(e) What is is the name of this network?

21. A mixer in a receiver has a conversion loss of 16 dB. If the applied RF signal has an available power of 100 μW, what is the available power of the IF at the output of the mixer?

22. The RF signal applied to the input of a mixer has a power of 1 nW and the output of the mixer at the IF has a power level of 100 pW. What is the conversion loss of the mixer in decibels?

23. A mixer in a receiver has a conversion gain of 10 dB. If the applied RF signal has a power of 100 μW, what is the available power of the IF at the output of the mixer?

24. A mixer in a receiver has a conversion loss of 6 dB. If the applied RF signal has a power of 1 μW, what is the available power of the IF at the output of the mixer?

25. The phase noise of an oscillator was measured as −125 dBc/Hz at 100 kHz offset. What is the normalized phase noise at 1 MHz offset, assuming that the phase noise power varies as the square of the inverse of frequency?

26. The phase noise of an oscillator was measured as −125 dBc/Hz at 100 kHz offset. What is the normalized phase noise at 1 MHz offset, assuming that the phase noise power varies inversely with frequency offset?

11.14.1 Exercises by Section

†challenging,

§11.5	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12	§11.8	18, 19, 20	§11.10	25†, 26
§11.7	13, 14†, 15†, 16†, 17†	§11.9	21, 22, 23, 24		

11.14.2 Answers to Selected Exercises

8(d)	5.17 dB	14(b)	29.8 dB	19	40 dB
10(b)	3 dB	15(a)	0.424 dB	21	−26 dBm
12(b)	60 dB	17	22.5 dB	23	0 dBm

Index

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