INTRODUCTION TO
ENGINEERING THERMODYNAMICS

CLAIRE YU YAN

Introduction to Engineering Thermodynamics

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Dr. Claire Yu Yan is an associate professor of teaching in the School of Engineering, Faculty of Applied Science, University of British Columbia, Okanagan. She teaches core engineering courses in the field of thermofluids, such as thermodynamics, fluid mechanics, fluid machinery, and heating, ventilation, and air conditioning (HVAC). In the past years, Dr. Yan has taught several thousands of students of diverse backgrounds. She has a strong passion for teaching innovations, in particular, open and engagement pedagogies and strives to make sustained contributions to support holistic student success and wellbeing through her teaching practices and scholarship of teaching and learning. Beyond teaching, Dr. Yan is an active contributor to UBC and the broader communities through her outreach program and committee work. Dr. Yan is a registered P.Eng. with EGBC (Engineers and Geoscientists BC), and a member of CEEA (Canadian Engineering Education Association) and ASEE (American Society for Engineering Education).

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Regards,

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Preface

This book aims to help students develop a fundamental understanding of classical thermodynamics and its engineering applications. It features concise explanations of key concepts, step-by-step engineering examples, and interactive practice problems at the end of each section.

The book consists of seven chapters. It is suitable for a one-term, introductory engineering thermodynamics course at the undergraduate level. It may also be used as self-learning materials or a supplement to other thermodynamics books.

Chapter 1: Basic Concepts and Definitions
This chapter introduces basic concepts and definitions in thermodynamics, such as open and closed systems; extensive and intensive properties; equilibrium states, quasi-equilibrium processes, and cycles. It lays a foundation for students to understand the key concepts in the subsequent chapters.

Chapter 2: Thermodynamic Properties of a Pure Substance
This chapter introduces thermodynamic properties, phase diagrams, and thermodynamic tables of pure substances. Students will learn how to determine thermodynamic properties of pure substances by using thermodynamic tables, and how to illustrate states and processes in phase diagrams. This chapter prepares students with essential skills for performing comprehensive analyses of various thermodynamic processes and cycles.

Chapter 3: Ideal and Real Gases
This chapter explains the concepts of “ideal” gas, ideal gas law, real gas, and compressibility factor. Students will learn the difference between an “ideal” and real gases, and at what conditions

the ideal gas model may be used as an approximation for evaluating properties, such as pressure, temperature, and volume of a gas.

Chapter 4: The First Law of Thermodynamics for Closed Systems

This chapter explains the concepts of heat, work, and the first law of thermodynamics for closed systems. Through examples, students will learn how to apply the first law of thermodynamics to engineering problems involving closed systems.

Chapter 5: The First Law of Thermodynamics for a Control Volume

This chapter extends the concept of energy conservation to open systems with a focus on steady-state, steady flows (SSSF). Students will learn how to use the first law of thermodynamics to analyze processes in typical SSSF devices, such as pipes, turbines, compressors, heat exchangers, expansion valves, and mixing chambers.

Chapter 6: Entropy and the Second Law of Thermodynamics

This chapter introduces the concepts of reversible and irreversible processes, Carnot cycles, entropy and entropy generation, and the second law of thermodynamics. Students will learn the applications of the second law in both closed and open systems.

Nomenclature

Symbol	Physical Meaning	Common SI Units
A	Area	m^2
C_p	Constant-pressure specific heat	J/kg-K or kJ/kg-K
C_v	Constant-volume specific heat	J/kg-K or kJ/kg-K
e	Total stored specific energy	J/kg or kJ/kg
E	Total stored energy	J or kJ
F	Force	N or kN
g	Gravitational acceleration	m/s^2
h	Specific enthalpy	J/kg or kJ/kg
H	Enthalpy	J or kJ
k	Heat capacity ratio	–
K	Spring constant	N/m or kN/m
KE	Kinetic energy	J or kJ
m	Mass	kg
PE	Potential energy	J or kJ
P	Pressure	Pa or kPa
Q	Heat	J or kJ
R	Gas constant	J/kg-K or kJ/kg-K
s	Specific entropy	J/kg-K or kJ/kg-K
S	Entropy	J/kg or kJ/kg
t	Time	s
T	Temperature	Kelvin (K)
u	Specific internal energy	J/kg or kJ/kg

U	Internal energy	J or kJ
v	Specific volume	m^3/kg
$\mathbb{V}$	Volume	m^3
V	Velocity	m/s
W	Work	W or kW
x	Quality of saturated liquid-vapor two-phase mixture	–
z	Height	m
Z	Compressibility factor	–

Greek Symbols

Symbol	Physical Meaning	Common SI Units
ρ	Density	kg/m^3
η	Efficiency	–

1. BASIC CONCEPTS AND DEFINITIONS

1.0 Chapter introduction and learning objectives

Classical thermodynamics is a macroscopic approach to the study of thermodynamics. This chapter introduces basic concepts and definitions used in classical thermodynamics. It lays the foundation for a comprehensive analysis of different thermodynamic processes and cycles to be presented in this book.

Learning Objectives

After completing the chapter, you should be able to

1. Explain the basic scope of engineering thermodynamics and its common areas of application
2. Demonstrate an understanding of fundamental concepts, such as system and its surroundings, closed and open systems, extensive and intensive properties, equilibrium state, quasi-equilibrium process, and cycle

1.1 What is thermodynamics about?

You probably have this experience; when you rub your hands quickly for a few minutes, your hands will start to feel warmer. How is this common phenomenon related to thermodynamics? Well, when you rub your hands quickly, your muscles do **work**. This work is then converted to **heat**; therefore, you feel warmer. Heat and work are two forms of energy. Work can be converted to heat, as seen in this daily example. However, can heat be converted to work? Can we use heat to produce work?

Heat engine is a device that produces work continuously by absorbing heat from a high-temperature heat source and rejecting the waste heat to a low-temperature heat sink. Since the 17th century, various heat engines were invented in an attempt to harness work from heat. Figure 1.1.1 illustrates Watt's engine invented by Scottish engineer James Watt in the late 18th century. Watt's engine is one of the most successful early heat engines. Its main components are a boiler (not shown in the figure) and a condenser, each connecting to a piston-cylinder device. The two valves, V and V', control the flow of steam into and out of the cylinder. When valve V opens, valve V' remains closed. Steam from the boiler enters the cylinder, pushing the piston up until it reaches the top of the cylinder. Then valve V' opens, and valve V closes. The steam in the cylinder escapes to the condenser and is condensed, creating a vacuum in the cylinder. Consequently, the piston moves downward under atmospheric pressure. The reciprocating motion of the piston drives the pivoting beam *DEF*, which then powers the pump chained to the beam. Watt's engine demonstrates how heat is converted to work. This conversion relies on the phase change of a working fluid, e.g., water, in the Watt's engine. The boiler in the

Watt's engine is the heat source, where the hot steam is generated; and the condenser is the heat sink, where the hot steam is cooled and condensed to liquid water. All heat engines need a working fluid circulating in a specially-arranged set of equipment, which operates between a high-temperature heat source and a low-temperature heat sink. Figure 1.1.2 is a schematic drawing of a heat engine. The yellow circle represents the heat engine consisting of a set of equipment.

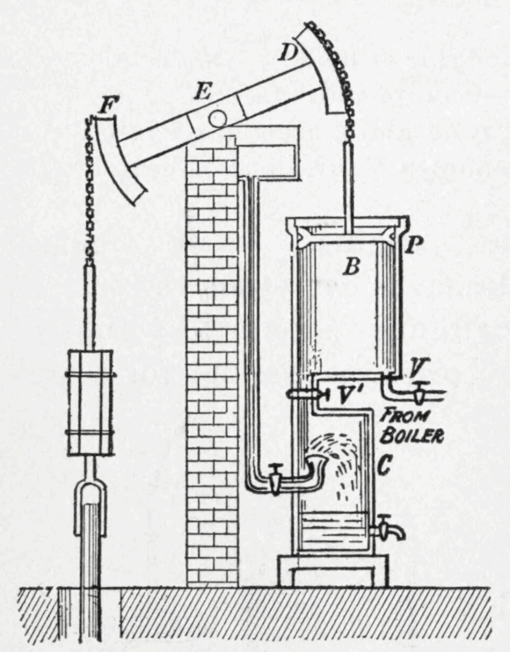

***Figure 1.1.1** Watt's heat engine*

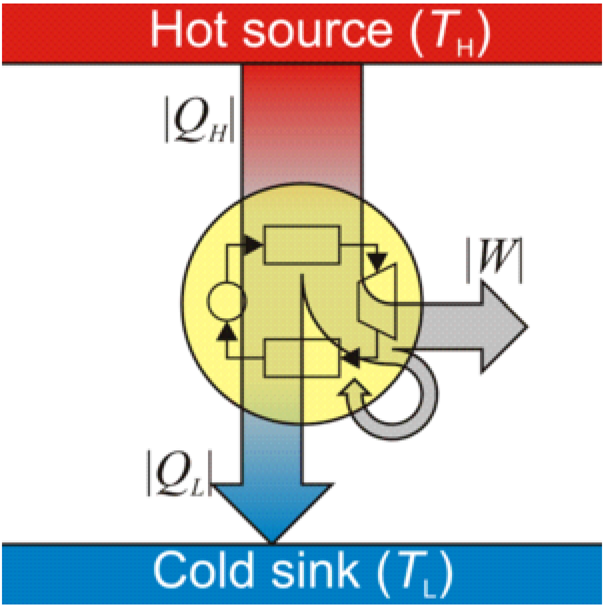

Figure 1.1.2 *Schematic drawing of a heat engine*

By examining Figure 1.1.2, you might notice that a certain amount of heat is not converted to work. It is true that ***not all of the heat from a heat source can be converted to useful work!*** Heat engines and their underlying principles are governed by the two fundamental laws of thermodynamics, the first and second laws of thermodynamics. We will briefly introduce the two laws here and will provide detailed explanations in Chapters 4-6.

- **The first law of thermodynamics** is about energy conservation. Energy can neither be created nor destroyed. It can only be converted between different forms.
- **The second law of thermodynamics** explains why all real processes are irreversible, and how the irreversibility of a process is quantified with the concept of entropy generation. In reality, all processes always occur in the direction of producing

positive entropy generation due to the existence of irreversibilities. From the second law of thermodynamics, we can estimate the theoretical limit of efficiency that a real thermodynamic process or system can possibly achieve.

Thermodynamics emerged in the early 19th century with the inventions of heat engines. It originally focused on the scientific theories of heat-work conversion, and the operations and efficiency improvement of heat engines. Nowadays, the applications of thermodynamics have extended to all fields related to energy conversion and conservation. In engineering fields, the principles of thermodynamics are widely used in the design of thermal systems, such as power plants using different energy sources (e.g., steam, gas, nuclear, hydro, wind, and solar), air conditioning and refrigeration systems, jet engines, biomedical devices, and chemical processes, to name but a few. Figure 1.1.3 is a schematic drawing of a nuclear power plant, whose performance and efficiency are governed by the fundamental principles of thermodynamics.

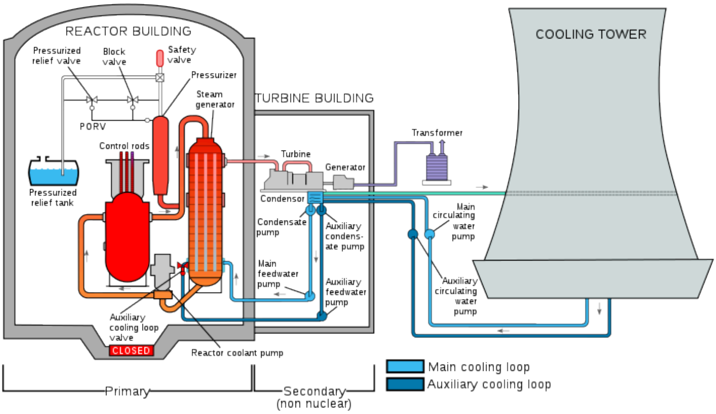

Figure 1.1.3 *Nuclear Power Plant*

Practice Problems

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1.2 System and surroundings

In thermodynamics, a **system** refers to a selected quantity of matter in the case of closed systems or a selected region in space in the case of open systems, see Figure 1.2.1. The rest of the universe outside the system is called **surroundings**, and the surface that separates the system and its surroundings is called **boundary.** A boundary may be fixed or movable, real or imaginary, rigid or flexible.

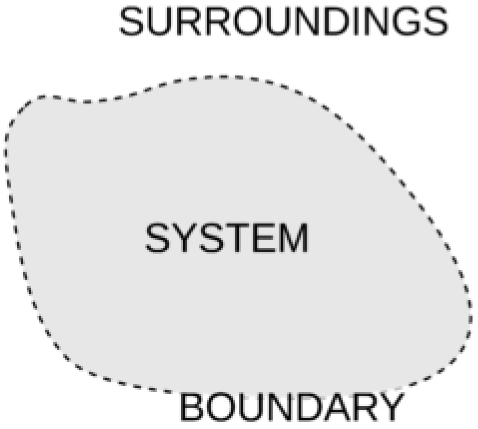

***Figure* 1.2.1** *System and surroundings*

A system interacts with its surroundings through two mechanisms:

1. **Mass transfer**
2. **Energy transfer** (i.e., in the form of heat and work)

A system of a fixed mass is a **closed system**, which can only interact with its surroundings through energy transfer. Mass cannot cross the boundary of a closed system. For example, a sealed bottle of soft drink, Figure 1.2.2, can be modelled as a closed system because there is a fixed amount of liquid in the bottle. When you take the bottle out of your cooler, the liquid will warm up slowly due to the temperature difference between the bottle and the ambient air (surroundings). In other words, the system (the liquid in the bottle) interacts with its surroundings (the ambient air) through energy transfer (in the form of heat transfer). Figure 1.2.3 illustrates a piston-cylinder device, which can also be modelled as a closed system. The amount of the fluid in the cylinder (the system) remains constant as the piston moves. Only the transfer of energy, in the form of heat and work, may happen across the system boundary consisting of the cylinder walls and the lower surface of the piston.

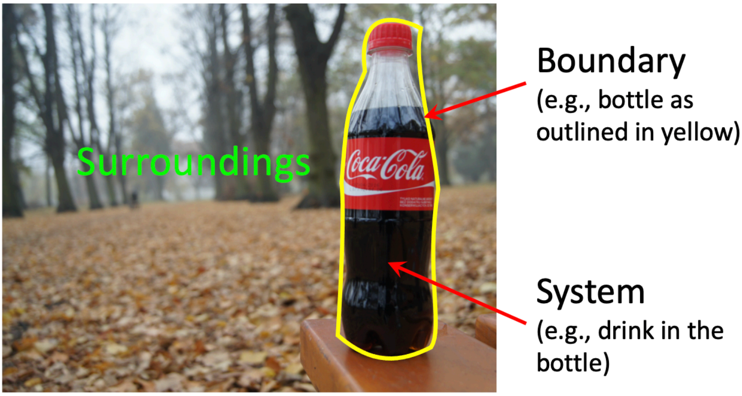

Figure 1.2.2 *A sealed bottle of soft drink as an example of closed systems*

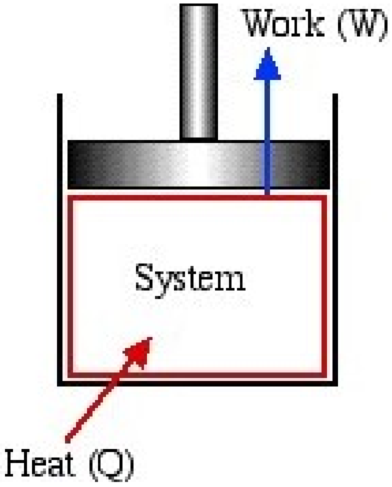

Figure 1.2.3 *Piston cylinder device as an example of closed systems*

An **open system**, also called **control volume**, is a selected region in space. An open system always exchanges mass with its surroundings. It may exchange energy with its surroundings in the form of heat and work, but energy transfer is not a necessary condition for a system to be an open system. In other words, an open system doesn't have to exchange heat or work with its surroundings at all. Figure 1.2.4 illustrates an open system, which typically encloses a device that involves mass flow through its inlet and outlet. Figure 1.2.5 illustrates the outdoor condensing unit of an air conditioner. It may be treated as an open system because the coolant can enter and leave the condensing unit (the system) via its connecting coolant lines.

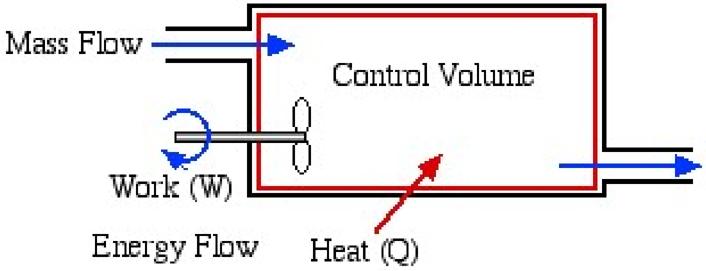

***Figure* 1.2.4** *Open system (also called control volume)*

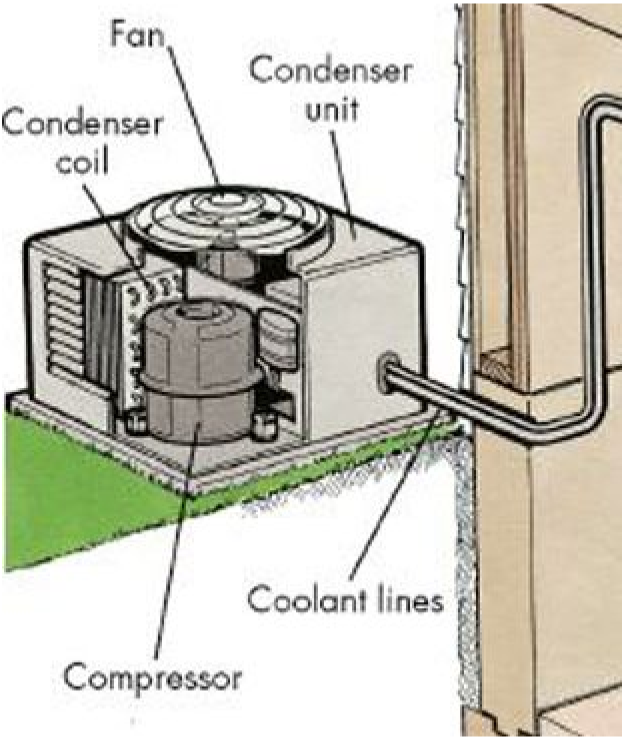

***Figure* 1.2.5** *Outdoor condensing unit of an air conditioner as an example of open systems*

If a system doesn't allow the exchange of mass and energy with its surroundings, it is called an **isolated system**. An isolated system is an idealized, hypothetical system. In reality, no device is absolutely isolated.

Practice Problems

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1.3 Extensive and intensive properties

From the macroscopic perspective, a system is viewed as a continuous, homogeneous matter called **continuum**, which consists of a huge number of interacting molecules distributed throughout the system. The interactions between the molecules are so frequent that the physical or bulk properties of the system do NOT depend on the behaviour of individual molecules. This hypothesis is valid in a wide range of engineering applications. It allows the physical properties of a system, such as pressure, density, and temperature, to be defined as a continuous function at any point of the system.

The following thermodynamic properties are typically used to describe the interactions between a system and its surroundings:

- mass m
- pressure P
- temperature T
- volume $\mathbb{V}$ and specific volume v
- internal energy U and specific internal energy u
- enthalpy H and specific enthalpy h
- entropy S and specific entropy s

These properties can be classified into two categories based on their dependence on the mass of a system. More detailed explanations of their physical meanings can be found in Chapter 2.

- **Extensive properties** depend on the mass of a system. Properties, such as mass m, volume $\mathbb{V}$, internal energy U, enthalpy H, and entropy S are extensive properties. Their values change accordingly as the mass of a system changes.

- **Intensive properties** are independent of the mass of a system. Pressure P, temperature T, specific volume v, specific internal energy u, specific enthalpy h, and specific entropy s are intensive properties.

Let us consider a container of air at 101 kPa and 20°C. If the container is divided into two compartments and all other conditions remain unchanged, see Figure 1.3.1, the air in each compartment is still at 101 kPa and 20°C. The pressure and temperature of the air are not affected by the changing mass in each compartment; therefore, pressure P and temperature T are intensive properties. On the other hand, the mass and volume of the air in each of the compartments are different from the original values in the container. Both of them depend on the mass of the system; therefore, mass and volume are extensive properties.

All **specific properties** are intensive properties, as they refer to the corresponding extensive properties per unit mass, e.g., specific volume $v = \mathbb{V}/m$ and specific internal energy $u = U/m$.

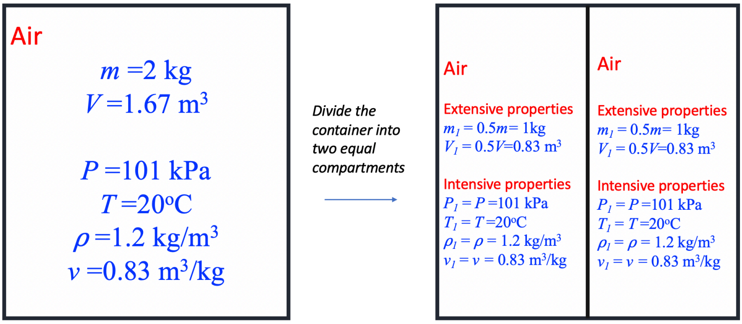

Figure 1.3.1 *Intensive and extensive properties*

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1.4 State, process, and cycle

If a system is isolated from its surroundings or is free from any unbalanced potentials, such as forced flows of mass or energy, the system will eventually reach a uniform condition called **equilibrium**. A system in equilibrium has uniform properties throughout the system. The following equilibrium conditions are commonly considered in thermodynamics.

- A system that features spatially-uniform temperature is in **thermal equilibrium**.
- A system free from chemical reactions is in **chemical equilibrium**.
- If there is no tendency for a system to change its pressure over time, the system is in **mechanical equilibrium**.
- For a system consisting of a mixture of multiple phases, such as liquid water and water vapour, if the composition of the mixture remains constant over time, the system is in **phase equilibrium**.

State refers to the condition of a system, which may be described by a unique set of properties, such as pressure, temperature, and specific volume. The state of a system in equilibrium is called **equilibrium state**. A system may change from one state to another state through a **process**. Let us consider a container of water initially at 10^{o}C and 101 kPa, as an example. We set the water in the container as the system. The water is heated until its temperature reaches 50^{o}C, while its pressure is kept constant 101 kPa. We may say that the water undergoes a constant-pressure, heating process with an initial state of 10^{o}C and 101 kPa and a final state of 50^{o}C and 101 kPa.

Typically, there are many possible paths that a system may take between two states; therefore, ***the exact path of a process is extremely important and must be clearly specified in order to describe a process***! Here are the definitions of some common processes.

- **Isobaric process**: the pressure remains constant in a process.
- **Isochoric process**: the specific volume remains constant in a process.
- **Isothermal process**: the temperature remains constant in a process.
- **Adiabatic process**: no heat transfer occurs between a system and its surroundings in a process.
- **Isentropic process**: the entropy remains constant in a process.

Figure 1.4.1 shows a compression process as the piston moves from the right to the left. States 1 and 2 represent the initial and final states. Each point along the process path represents an equilibrium state. If all states in a process are equilibrium states, the process is called **quasi-equilibrium process**. In this book, we will deal with systems in equilibrium; therefore, all states thereafter refer to equilibrium states, and all processes refer to quasi-equilibrium processes.

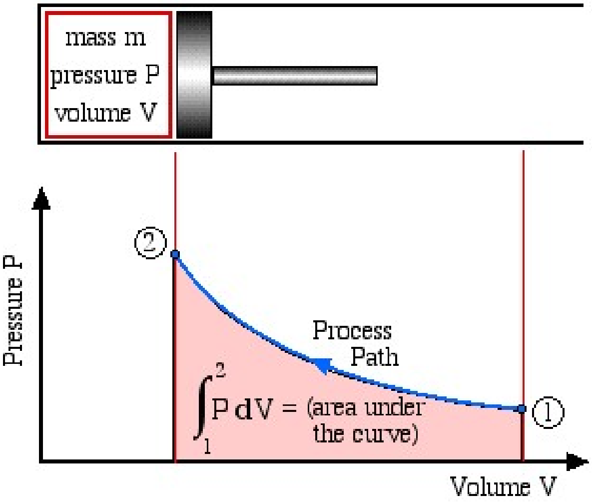

Figure 1.4.1 *Schematic of a process. States 1 and 2 represent the initial and final states; each point along the process path represents an equilibrium state.*

If a system undergoes a series of processes and finally returns to its initial state, we say that the system completes a **cycle**. Thermodynamic cycles are the basis for the operation of thermal equipment. For example, the vapour-compression refrigeration cycle is often used in conventional refrigerators and air conditioners, as shown in Figure 1.4.2. The cycle consists of four main devices: compressor, condenser, expansion valve, and evaporator. A working fluid called refrigerant circulates through these devices connected by tubes. The refrigerant in the cycle experiences phase changes between vapour and liquid, as shown in Figure 1.4.3. Phase diagrams (see details in Chapter 2) are commonly used to analyze a process or a cycle. Figure 1.4.4 illustrates the temperature-specific entropy, $T - s$, diagram for the vapour-compression refrigeration cycle, where the numbered dots represent different states and the lines with arrows represent different processes in this cycle. For example, the number “1” in

Figure 1.4.3 and Figure 1.4.4 refers to the state of the refrigerant at the inlet of the compressor or the exit of the evaporator. The line 1-2 in Figure 1.4.4 refers to the compression process in the compressor.

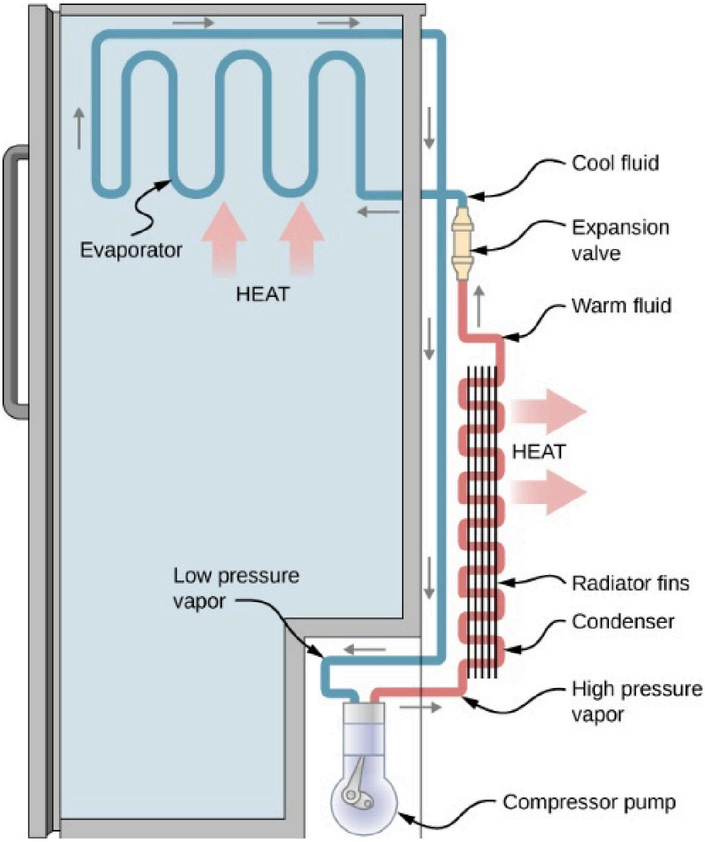

Figure 1.4.2 *Refrigerator working on the vapour compression cycle*

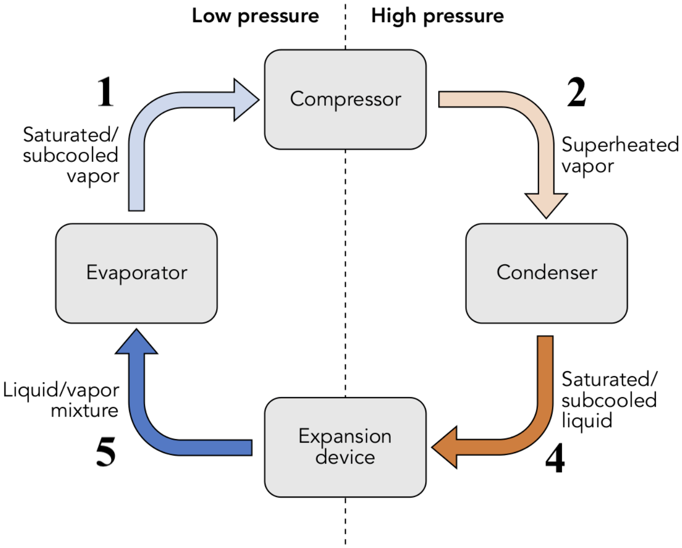

Figure 1.4.3 *Vapour compression cycle*

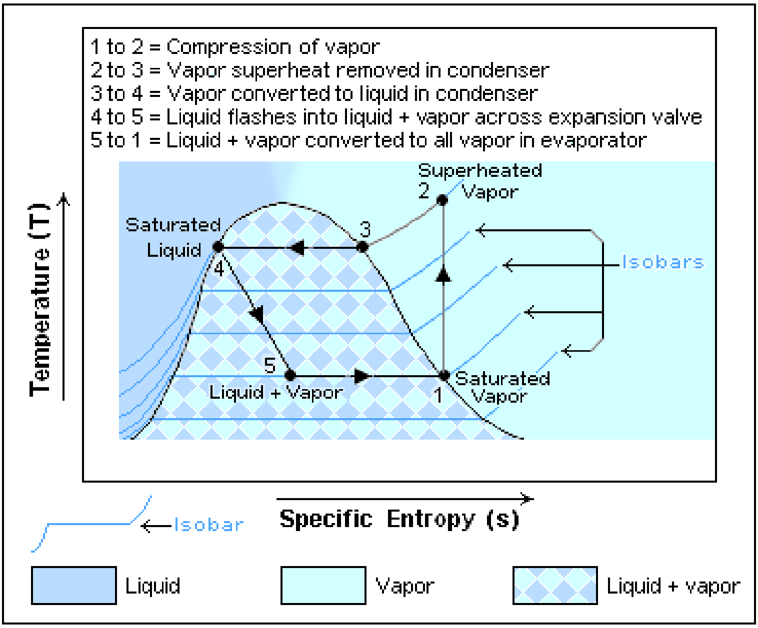

Figure 1.4.4 *Temperature-specific entropy (T-s) diagram of a vapour compression cycle*

Otto cycle is another thermodynamic cycle. It is an ideal cycle that modells the operation of internal combustion engines. Figure 1.4.5 shows the cycle consisting of four strokes. The pressure-volume diagram, Figure 1.4.6, illustrates different processes in this cycle.

1. Intake stroke, line 0-1 in Figure 1.4.6. During the intake stroke, the inlet valve opens and the outlet valve remains closed. Air is drawn into the cylinder as the piston moves to the bottom dead center (BDC).

2. Compression stroke, line 1-2 in Figure 1.4.6. During the compression stroke, both valves remain closed. The air is compressed as the piston moves from BDC to the top dead center (TDC).

3. Ignition and power stroke, line 2-3-4 in Figure 1.4.6. During this stroke, both valves remain closed. The piston is at TDC momentarily while the fuel-air mixture is ignited by the spark. The burning of the fuel-air mixture generates a large force, pushing the piston from TDC to BDC.
4. Exhaust stroke, line 4-1-0 in Figure 1.4.6. During the exhaust stroke, the outlet valve opens and the inlet valve remains closed. The piston remains at BDC momentarily, allowing a certain amount of heat to release to the surroundings. Then the piston moves from BDC towards TDC to reject the exhaust and more heat to the surroundings.

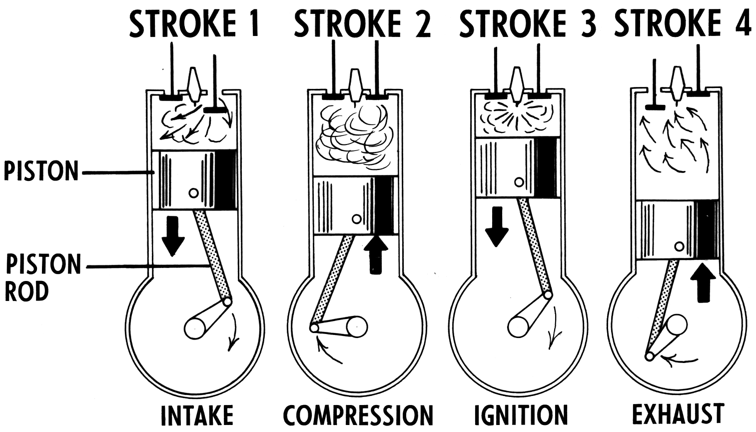

***Figure* 1.4.5** *Four-stroke combustion engine*

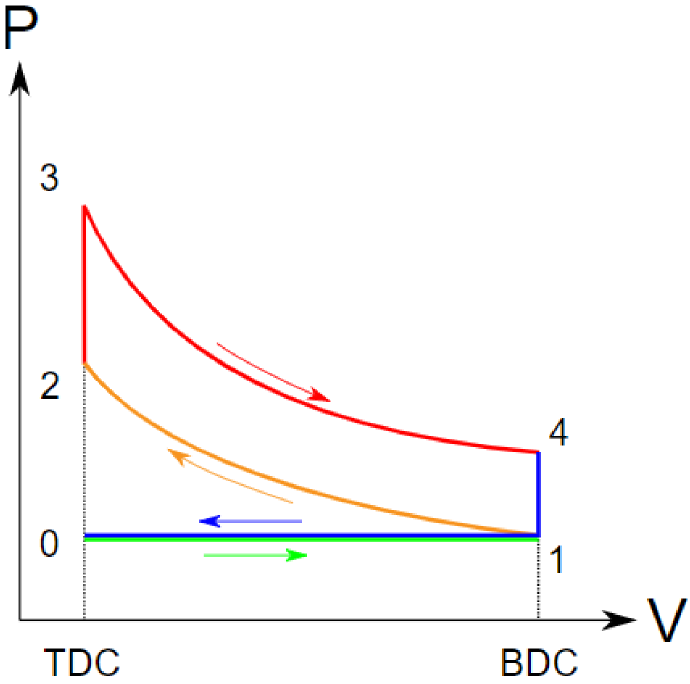

Figure 1.4.6 *Pressure-volume diagram of an Otto cycle*

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1.5 Chapter review

Thermodynamics has a wide range of applications in many engineering fields, in particular, the fields related to energy conversion and conservation. In this chapter, we have introduced some fundamental concepts and definitions used in the study of engineering thermodynamics.

An important skill that students need to develop, when performing thermodynamic analysis on devices, is to identify the system, its surroundings, and their interactions. There are three types of systems: closed system, open system, and isolated system, defined in terms of their ability to transfer mass and energy with the surroundings.

In this book, we consider systems in equilibrium. Each equilibrium state possesses a unique set of thermodynamic properties, which can be classified as extensive and intensive properties. When a system undergoes a process from one equilibrium state to another equilibrium state, its thermodynamic properties will change accordingly. The process path must be clearly specified when describing a process. Students need to understand the definitions of the common processes, including isobaric, isothermal, isochoric, adiabatic, and isentropic processes.

2. THERMODYNAMIC PROPERTIES OF A PURE SUBSTANCE

2.0 Chapter introduction and learning objectives

An equilibrium state in a process has a unique set of thermodynamic properties. In this chapter, we will introduce common thermodynamic properties, phase diagrams, and thermodynamic tables. Through examples, we will demonstrate how to use thermodynamic tables to determine thermodynamic properties of a pure substance at a given state. The International Systems of Units (SI units) will be used for all properties and examples.

Learning Objectives

After learning this chapter, you should be able to

- Demonstrate an understanding of common thermodynamic properties
- Interpret phase diagrams, including $P-T$, $P-v$, and $T-v$ diagrams for pure substances
- Use thermodynamic tables to determine properties of pure substances

2.1 Pure substance

A **pure substance** refers to a matter that has a homogeneous and definite chemical composition. A pure substance may exist in a single phase, such as liquid water, ice, and CO_2 gas. It can also exist as a multi-phase mixture, such as a mixture of liquid water and water vapour in equilibrium.

Air is a common working fluid used in many thermal devices. Can air be treated as a pure substance? Although air is a mixture consisting of roughly 78% of nitrogen, 21% of oxygen, and trace amounts of other gases and moisture, air is homogeneous and has distinct properties without phase changes in its composition in many thermodynamic processes. In such conditions, we can treat air as a pure substance.

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2.2 Thermodynamic properties

Pure substances are widely used in thermodynamic cycles. For example, in the vapour-compression refrigeration cycle, Figure 1.4.2, refrigerant R134a is used as the working fluid, which is circulated through a compressor, condenser, expansion valve, and evaporator via the connecting tubes.

In steam power plants, water is used as the working fluid. Figure 2.2.1 shows a coal-fired steam power plant. Purified water flows vertically up the tube-lined walls of the boiler, where it turns into steam. Steam flows into the boiler drum (17), separating from any remaining water, and then flows into the pendant superheater (19), where its temperature and pressure increase rapidly to around 200 bar and 570°C. The superheated steam is piped to the high-pressure turbine (11), where both its pressure and temperature are reduced. The steam is then returned to the boiler reheater (21). After being reheated, it enters the intermediate-pressure turbine (9) and then the low-pressure turbine (6). The steam, now just above its boiling point, is brought into thermal contact with the cold water in the condenser (8), where it is condensed rapidly back into water. The water then passes a feed pump (7), a de-aerator (12), a feed heater (13), and an economizer (23) before being returned to the boiler drum.

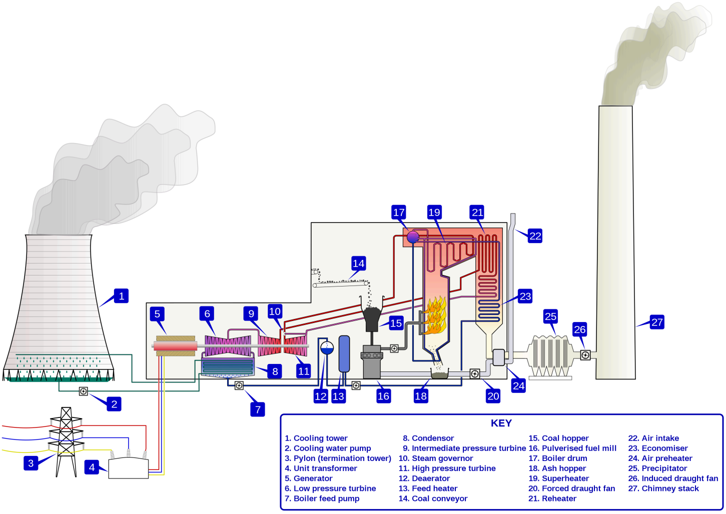

Figure 2.2.1 *Steam power plant: water is used as the working fluid.*

In both cycles, the working fluids, water and R134a, are pure substances. Their properties, such as pressure and temperature, continuously change as they flow through different devices in the cycles. How are the changes of fluid properties related to the overall performance of a device, such as the aforementioned steam power plant and refrigerator? To answer this question, we need to understand the thermodynamic properties of pure substances at different states of a process or a cycle.

The common properties of a pure substance include pressure, temperature, specific volume, density, specific internal energy, specific enthalpy, and specific entropy. These properties are interrelated to each other. Thermodynamic tables or equations of state are commonly used to determine the thermodynamic properties of a pure substance. Appendices A-D provide thermodynamic tables for selected pure substances. The ideal gas equation of state will be introduced in Chapter 3.

2.2.1 Pressure

Consider a normal force exerted by an object on a surface. **Pressure** is defined as the normal force per unit area of the surface.

$$P = F/A$$

where

F: force perpendicular to a surface, in kN or N
A: surface contact area, in m^2
P: pressure, in kPa or Pa

In the SI units, force is usually expressed in Newtons, N, or kilo Newtons, kN, and area in meter squared, m^2; therefore, pressure is expressed in Pascal, 1 Pa = 1 N/m^2, or kilo-Pascal, 1 kPa = 1 kN/m^2. Other common SI units include 1 bar =100 kPa, 1 atm =101.325 kPa, and 1 MPa = 10^6 Pa =1000 kPa.

Pressure depends on the normal force and the contact area. If you stand up tall on a flat horizontal surface, the pressure on your feet is your weight divided by the total contact area of your feet on the surface. If you lift up one foot, the pressure on your other foot will double as the contact area is decreased by half.

Pressure in a fluid acts equally in all directions at a given point in that fluid; therefore, *pressure is a scalar quantity*. It is important to note that although pressure is a scalar quantity, pressure force, $F = PA$, is a vector and is always perpendicular to the contact surface.

Pressure in a fluid is commonly measured by pressure gauges (Figure 2.2.2) or U-tube manometers (Figure 2.2.3). Most pressure gauges are calibrated to show the gauge pressure or vacuum pressure, which is the difference between the **absolute pressure** and the atmospheric pressure.

Gauge pressure:

$$P_{gauge} = P_{abs} - P_{atm}$$

Vacuum pressure:

$$P_{vac} = P_{atm} - P_{abs}$$

where

P_{abs}: absolute pressure of a fluid at a given point in the fluid

P_{atm}: atmospheric pressure

P_{gauge}: gauge pressure. It is commonly used when $P_{abs} > P_{atm}$. P_{gauge} is always a positive number.

P_{vac}: vacuum pressure. It is commonly used when $P_{atm} > P_{abs}$. P_{vac} is always a positive number.

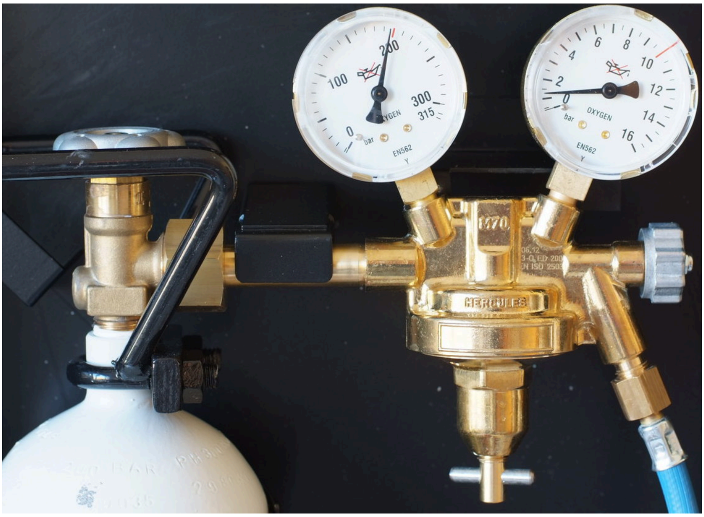

Figure 2.2.2 *Pressure gauge*

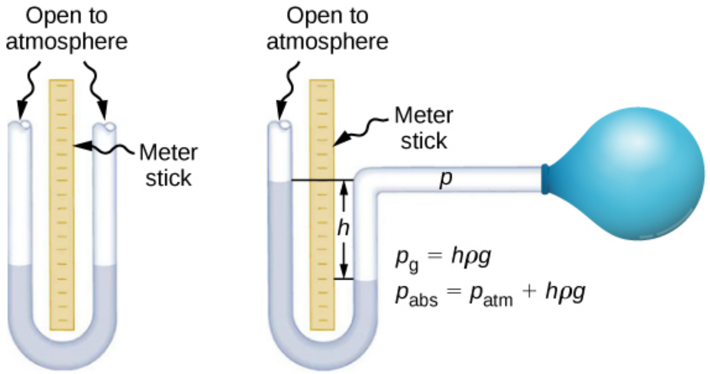

***Figure* 2.2.3** *U-tube Manometer*

Example 1

Consider a piston-cylinder device containing a gas. The mass of the piston is 10 kg. The diameter of the piston is 20 cm. The atmospheric pressure is 1 atm = 101.325 kPa. What is the absolute pressure of the gas in the cylinder assuming the piston is in static equilibrium? The friction between the piston and the cylinder is negligible.

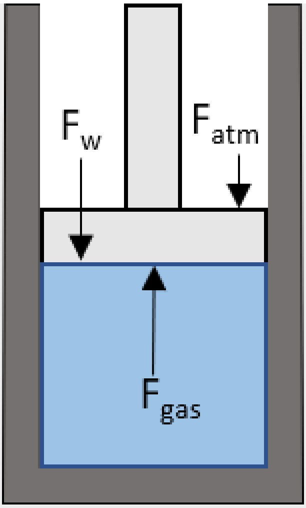

Figure 2.2.e1 *Piston cylinder device showing acting forces*

<u>*Solution*</u>

There are three forces acting on the piston:

- Weight of the piston $F_w = mg$
- Force due to the atmospheric pressure $F_{atm} = P_{atm}A$
- Force due to the gas pressure $F_{gas} = P_{gas}A$

As the piston is in static equilibrium, the resultant force on the piston is zero.

$$\sum F = F_{gas} - (F_w + F_{atm}) = 0$$

Therefore, the absolute pressure of the gas is

$$\begin{aligned} P_{gas} &= P_{atm} + mg/A \\ &= 101.325 + (10 \times 9.8/1000)/(\pi \times 0.1^2) = 104.44\,\text{kPa} \end{aligned}$$

The gauge pressure of the gas is

$$P_{gauge} = P_{gas} - P_{atm} = 3.12\,\text{kPa}$$

2.2.2 Temperature

Temperature is another measurable thermodynamic property, which indicates the hotness or coldness of a body. Thermometer is a common device for temperature measurement. If two bodies have the same temperature readings on a thermometer, the two bodies must have the same temperature. In other words, they are in thermal equilibrium. This seemingly obvious observation is called the *zeroth law of thermodynamics*: *if two bodies are in thermal equilibrium with a third, the three bodies must be in thermal equilibrium with each other.* This law establishes the basis for temperature measurement; it allows us to compare the temperatures of different bodies, no matter if they are in contact or not.

Temperature can be expressed on different temperature scales, such as Celsius or Kelvin scales in the SI units and Fahrenheit or Rankine scales in the Imperial units. For example, a room temperature may be expressed as 20^oC or 68 F. All temperature scales are calibrated based on two easily reproducible temperatures, such as the freezing and boiling temperatures of water at the standard atmospheric pressure.

A temperature expressed in Kelvin in the SI units or in Rankine in the Imperial units is called **absolute temperature**. An absolute temperature of zero indicates the state of the minimum energy of a matter. Absolute temperature, also called thermodynamic temperature, is an important property in thermodynamic analyses. The conversion between the Celsius and Kelvin scales is

$$T\,(K) = T(^\circ C) + 273.15$$
$$\Delta T\,(K) = \Delta T(^\circ C)$$

2.2.3 Density and specific volume

Density is defined as the mass per unit volume of a body. **Specific volume** is the reciprocal of density.

$$\rho = m/\mathbb{V}$$
$$v = \mathbb{V}/m = 1/\rho$$

where

m: mass, in kg
$\mathbb{V}$: volume, in m^3
v: specific volume, in m^3/kg
ρ: density, in kg/m^3

Both temperature and pressure may affect the density and specific volume of a fluid. The density and specific volume of a gas are strong functions of both temperature and pressure. For example, when a gas is compressed, its pressure will increase, causing the density of the gas to increase and the specific volume of the gas to decrease. When a gas is heated, its temperature will increase. Consequently, the density of the gas will decrease and the specific volume of the gas will increase.

On the contrary, the density and specific volume of a liquid depend much more strongly on the temperature than on the pressure; therefore, it is common to assume that the density and specific volume of a liquid are functions of temperature only.

Example 2

A container consists of two sections separated by a membrane. Section A contains 0.5 kg of air. Its volume is 0.75 m^3. Section B contains air of density 0.6 kg/m^3. Its volume is 0.5 m^3. If the membrane is broken, and the air in the two sections mixes and reaches a uniform state, what is the specific volume of the air at the final state?

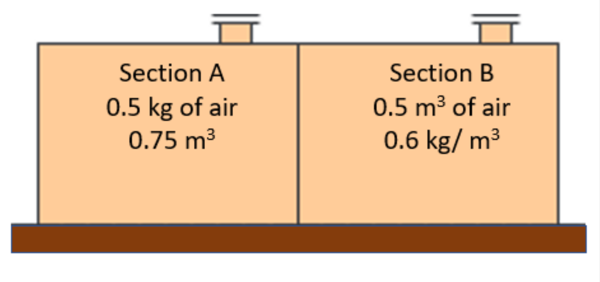

Figure 2.2.e2 *Two sections filled with air*

<u>*Solution*</u>

The masses and volumes in sections A and B at the initial state are

$$m_A = 0.5 \text{ kg}$$
$$\mathbb{V}_A = 0.75 \text{ m}^3$$
$$m_B = \rho \mathbb{V}_B = 0.6 \times 0.5 = 0.3 \text{ kg}$$
$$\mathbb{V}_B = 0.5 \text{ m}^3$$

The two sections reach a uniform state at the final state; therefore,

$$m_{total} = m_A + m_B = 0.5 + 0.3 = 0.8 \text{ kg}$$

$$\mathbb{V}_{total} = \mathbb{V}_A + \mathbb{V}_B = 0.75 + 0.5 = 1.25 \text{ m}^3$$

Therefore,

$$v_{total} = \mathbb{V}_{total}/m_{total} = 1.25/0.8 = 1.5625 \text{ m}^3/\text{kg}$$

2.2.4 Internal energy and specific internal energy

The essence of the first law of thermodynamics is energy conservation. We need to understand different forms of energy in a system. **Internal energy**, kinetic energy, and potential energy constitute the total *stored energy* of a system if the system is free of magnetic, electric, and surface tension effects.

$$E = U + KE + PE$$

where

E: total stored energy, in J or kJ
KE: kinetic energy, in J or kJ
PE: potential energy, in J or kJ
U: internal energy, in J or kJ

Internal energy is associated with the motions and structure of the molecules of a system in a microscopic level. A system, no matter how tiny it is, contains a significantly large number of molecules in various modes of random motions, such as translation, rotation, and vibration. Although it is difficult to predict the behaviour of individual molecules, the statistical average can be calculated by using statistical theory. From a statistical standpoint, the internal energy of a system is considered to be the sum of the kinetic and potential energies of the collection of all molecules in the system. Maxwell explains the relation between the molecular velocity (a microscopic quantity) and the temperature (a macroscopic quantity) of a gas: as the temperature increases, the gas molecules move faster causing the mean microscopic kinetic energy of all molecules to increase; therefore, the internal energy of the system increases. In other words, the internal energy of a substance is strongly associated with the temperature of the substance; therefore, internal energy is a form of thermal energy. For example, the internal energy of 1 kg of water at 500 kPa increases from about 83.88 kJ to 167.47 kJ when its temperature increases from 20^oC to 40^oC. In general, temperature has a dominant effect on the internal energy of a system although other factors, such as pressure, phase, and chemical composition may also have an effect on the internal energy of a system.

From a macroscopic standpoint, a thermodynamic system, as a whole, has both kinetic and potential energies when it is in motion and is positioned at an elevation on Earth. The kinetic and potential energies are two forms of mechanical energy. They must be described with respect to an external reference frame. When a system experiences a change of elevation or velocity, the kinetic and potential energies of the system will convert between each other. For example, a rock falling from a cliff gains kinetic energy but losses potential energy because its elevation decreases with respect to the ground.

In general, the total stored energy of a thermodynamic system consists of the internal, kinetic, and potential energies. However, the kinetic and potential energies are insignificant compared to the internal energy in many cases, and thus can be neglected.

E, U, KE, and PE are extensive properties as their values depend on the mass of a system. The relation of their corresponding intensive properties is expressed as

$$e = u + ke + pe$$

where

$e = E/m$: total specific energy, in J/kg or kJ/kg
$u = U/m$: **specific internal energy**, J/kg or kJ/kg
$ke = KE/m$: specific kinetic energy, J/kg or kJ/kg
$pe = PE/m$: specific potential energy, J/kg or kJ/kg

The internal energy and specific internal energy are important properties for the analysis of closed systems, which will be explained in detail in Chapters 4-6.

2.2.5 Enthalpy and specific enthalpy

Enthalpy is an important thermodynamic property for the analysis of open systems or control volumes, where mass transfer is always accompanied by energy transfer. Enthalpy is calculated as the sum of the internal energy, U, and the flow work, $P\mathbb{V}$, of the fluid flowing into or out of a control volume.

$$H = U + P\mathbb{V}$$

Now, what is flow work and why is it written as $P\mathbb{V}$? Let us consider a fluid element entering a control volume of a pipe section, see Figure 2.2.4. The control volume is illustrated as the red, dashedlined rectangle. The fluid element, just before it enters the control volume, is at an equilibrium state and has an internal energy, U, and a pressure, P. The fluid element carries its internal energy as it flows into the control volume. In addition, the fluid element must overcome the resistance from the downstream flow in the pipe. In other words, a certain amount of work must be done to force the fluid element into the control volume. We call such work essential to maintain the continuous flow of a fluid through a control volume as flow work. Refer to Figure 2.2.4, the flow work can be calculated as

$$W_{flow\ work} = FL = (PA)L = P(AL) = P\mathbb{V}$$

where $F = PA$ is the pressure force acting on the cross-section, or the left control surface, of the control volume. A is the cross-sectional area of the left control surface. L is the distance that the fluid element travels from its original position to where it just enters the control volume completely. $\mathbb{V} = AL$ is the volume of the fluid element.

For flow through an open system or control volume, both the internal energy carried by the fluid throughout the flow and the flow work required to maintain the flow contribute to the change of energy in a control volume. Therefore, it is common to combine the internal energy and the flow work as a new thermodynamic property, enthalpy, H.

Enthalpy, similar to the internal energy, is an extensive property because its value depends on the mass of a system. Its corresponding intensive property is called **specific enthalpy**, or enthalpy per unit mass of a substance.

$$h = H/m = u + Pv$$

where

h: specific enthalpy, in J/kg or kJ/kg

H: enthalpy, in J or kJ
m: mass, in kg
P: pressure, in Pa or kPa
u: specific internal energy, in J/kg or kJ/kg
v: specific volume, in m^3/kg

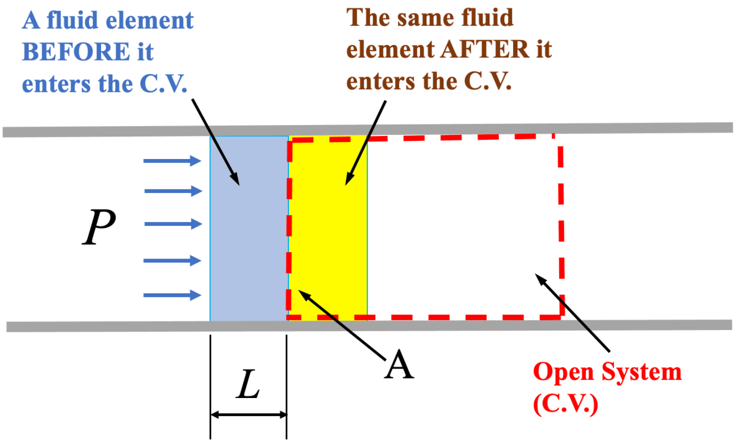

Figure 2.2.4 *Flow work*

2.2.6 Entropy and specific entropy

Some processes occur spontaneously in nature, such as salt dissolving in water. But some processes cannot occur spontaneously in nature. For example, water at 80°C and 10°C can mix in a container and eventually reach a thermal equilibrium. The mixture, however, cannot separate spontaneously into water at 80°C and water at 10°C. In other words, the process of separating the mixture is "irreversible" and cannot happen spontaneously. We may use a thermodynamic property called **entropy** to help us understand the physics of such phenomena.

Entropy is a measure of the lack of order resulting from the dispersal of energy and matter. Every spontaneous process is accompanied by an increase in the entropy (or disorder) of the universe. From a statistics point of view, it is more probable that the dispersal of energy and matter in a process is rather "random" than "orderly". If we consider a fixed amount of gas in an isolated container, as illustrated in Figure 2.2.5. The probability of the molecules having a random distribution is much higher than the probability of them having an "orderly" distribution. Entropy is a concept to represent the degree of the "randomness" or "disorder" of a system. Such "disorder" was explained in statistical theories by early scientists such as Maxwell, Boltzmann, and Gibbs. Nowadays, entropy has been generally recognized as an important thermodynamic property associated with the "quality" aspect of the energy of a system; while energy represents the "quantity" aspect of the energy of the system. Both energy and entropy are of great importance in thermal analysis. The first law of thermodynamics, see Chapters 4 and 5, addresses the concepts of energy and energy conservation. The second law of thermodynamics is closely associated with the concepts of entropy and entropy generation, which explain why a process in nature only occurs in the direction of decreasing the quality of energy or increasing the entropy of the universe. These concepts will be explained in detail as we study the second law of thermodynamics in Chapter 6.

Entropy, S, is an extensive property as its value depends on the mass of a system. The common SI units for entropy include kJ/K and J/K. Its corresponding intensive property is called **specific entropy**, $s = S/m$. Its common SI units are kJ/kgK or J/kgK.

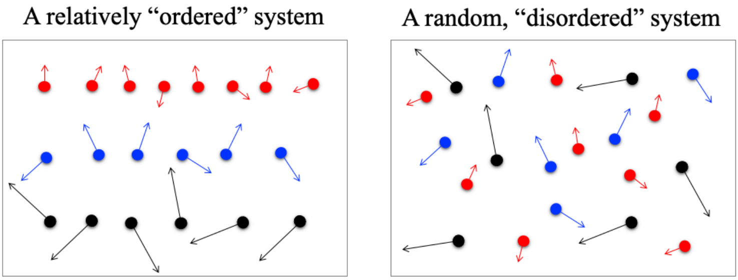

***Figure* 2.2.5** *Illustration of ordered and disordered systems*

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2.3 Phase diagrams

A pure substance may exist in any of the three phases: solid, liquid, and vapour, at certain temperatures and pressures. When its temperature or pressure changes, a substance may transition from one phase to another. For example, liquid water at 1 atm turns into ice when its temperature drops to the freezing point of 0^{o}C. The equilibrium state of a pure substance and its phase transitions are commonly illustrated in **phase diagrams**. Figure 2.3.1 is a three-dimensional $P-v-T$ phase diagram, where the three axes are pressure, specific volume, and temperature, respectively. This phase diagram clearly shows the single phase regions of solid, liquid, and vapour or gas, as well as three two-phase regions, where solid-liquid, liquid-vapour, or solid-vapour coexist in equilibrium. The three-dimensional $P-v-T$ phase diagram can be projected to generate two-dimensional phase diagrams, such as $P-T$, $P-v$, and $T-v$ diagrams. When analyzing processes and cycles, these two-dimensional phase diagrams are commonly used, and therefore will be discussed in detail here.

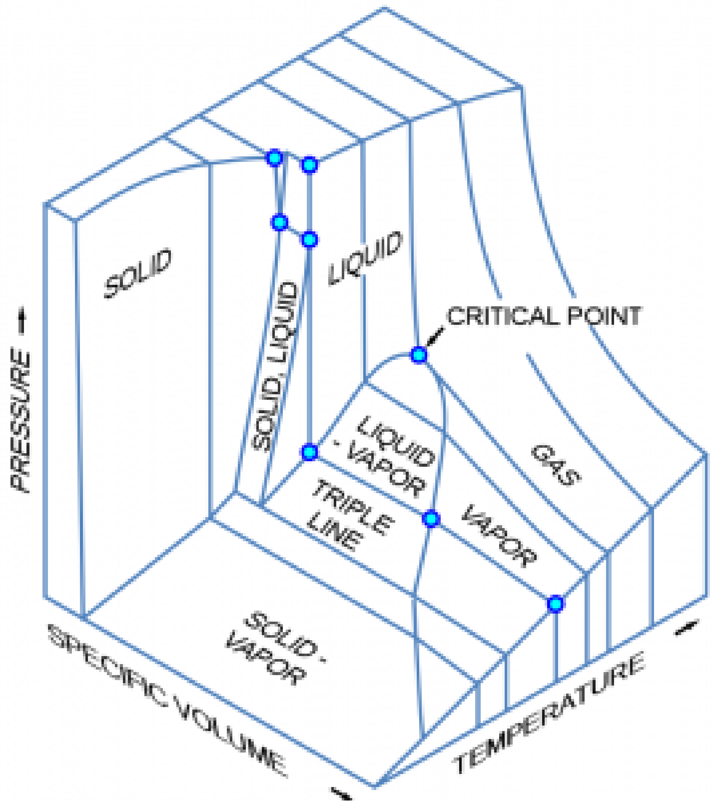

Figure 2.3.1 *Pressure-specific volume-temperature* (P-*v*-T) *diagram*

2.3.1 P-T diagram

Figure 2.3.2 shows a generic pressure-temperature, $P - T$, diagram, from which we can observe three single-phase regions, three curves representing the two-phase mixtures, and two unique points: the triple point and the critical point.

The **single phase** regions are labeled as solid, liquid, and vapour or gas in the $P - T$ diagram. The liquid and vapour phases are often called compressed liquid and superheated vapour, respectively.

In the $P - T$ diagram, the two-phase regions appear as curves separating different single phases. The curve that lies between the liquid and vapour phases is called **vaporization line.** Each point on the vaporization line represents an equilibrium state of saturation; the substance is either a saturated liquid, a saturated vapour, or a two-phase liquid-vapour mixture. The temperature and its corresponding pressure at each point on the vaporization line are called saturation temperature, T_{sat}, and saturation pressure, P_{sat} , respectively. Each saturation temperature corresponds to a unique saturation pressure, and vice versa. A liquid (or vapour) starts to evaporate (or condense) when its temperature and pressure reach T_{sat} and P_{sat}. The saturation properties of selected fluids can be found in the thermodynamic tables in Appendices A-D.

The curve that represents the transition between the solid and liquid phases is called **fusion line**. Each point on the fusion line has a unique set of temperature and pressure called freezing temperature and freezing pressure, respectively. Along the fusion line, the substance may exist as a saturated liquid, a solid, or a two-phase solid-liquid mixture.

The curve below the triple point is called **sublimation line**, across which a substance can change directly from solid to vapour or vice versa without a transition through the liquid phase. Each point on the sublimation line represents an equilibrium state, at which the substance may exist as a saturated vapour, a solid, or a two-phase solid-vapour mixture.

The vaporization, fusion and sublimation lines meet at the **triple point**, at which the three phases, solid, liquid, and vapour, coexist in equilibrium. Appendix F lists the triple points of a selection of pure substances. It is noted, from Figure 2.3.2, that the liquid phase cannot exist below the triple point pressure. When a substance is at a pressure lower than the triple point pressure, it can only transition between the solid and vapour phases.

The critical point in the $P - T$ diagram is where the vaporization line ends. The pressure and temperature at the critical point are called critical pressure, P_c, and critical temperature, T_c , respectively. A state above the critical point has a pressure $P > P_c$ and a temperature $T > T_c$; therefore, it is referred to as a supercritical state. A substance at a supercritical state is called supercritical fluid, which has a unique characteristic: no distinct liquid and gas phases can exist anymore in the supercritical zone. In this book, we will mainly focus on the subcritical zone, where $P < P_c$.

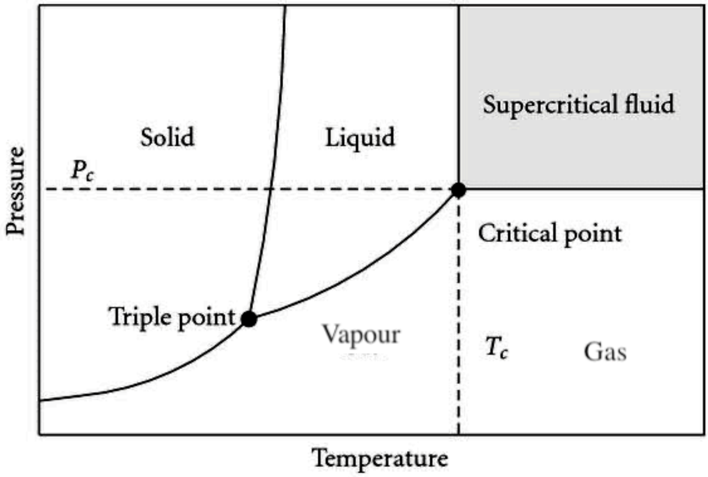

Figure 2.3.2 P-T *diagram*

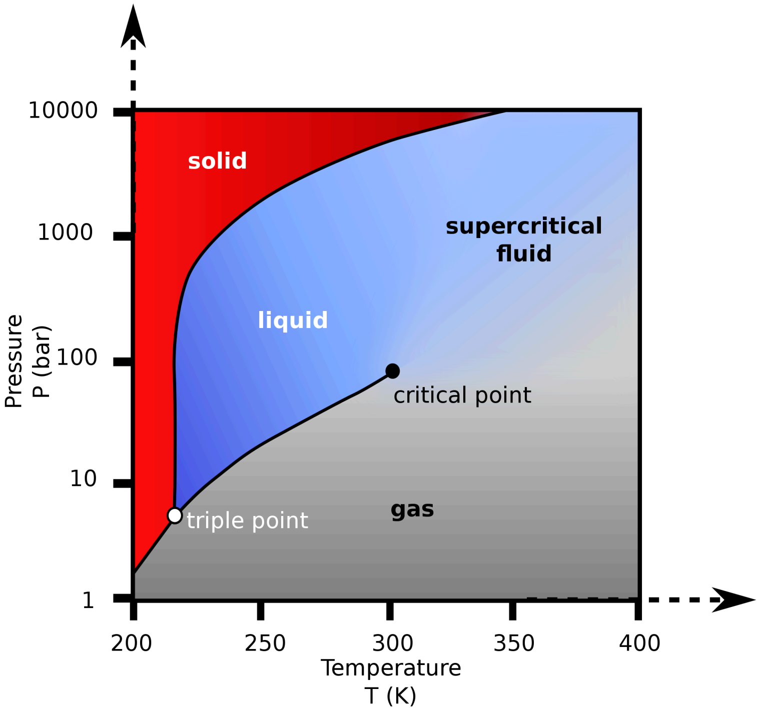

Figure 2.3.3 *P-T diagram for CO2*

Example 1

Review the *P-T* diagram for CO_2, as shown in Figure 2.3.3.

1. What is the lowest pressure for liquid CO_2 to exist?
2. Is CO_2 at 100 bar, 275 K a solid, liquid or gas?
3. CO_2 at 100 bar, 275 K is cooled in an isobaric

process. At approximately what temperature will CO_2 start to change its phase? Will it change to a liquid or a solid? Draw the process line.

Solution:

1. From the *P-T* diagram, the liquid phase can only exist when the pressure is great than the triple point pressure. From Appendix F, Table F1, CO_2 has a triple point pressure of 517 kPa and a triple point temperature of 216.55 K (−56.60 °C), respectively; therefore, the lowest pressure for liquid CO_2 to exist is 517 kPa.
2. From the *P-T* diagram, Figure 2.3.e1, CO_2 at 100 bar, 275 K is in the liquid phase.
3. The isobaric process is shown as the horizontal, yellow line with a constant pressure of 100 bar, see Figure 2.3.e1. At approximately 220 K, the isobaric process line meets the fusion line, and the liquid CO_2 starts to change to solid CO_2.

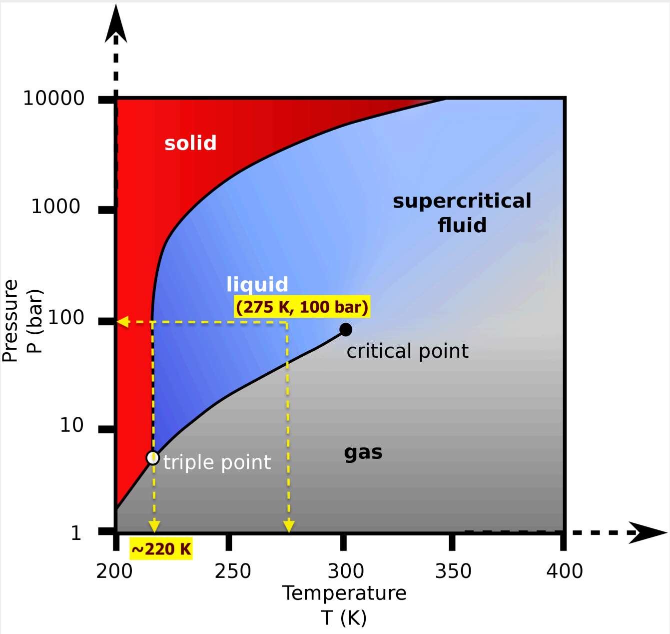

Figure 2.3.e1 *P-T diagram for CO_2, showing the solution to example 1*

2.3.2 T-*v* and P-*v* diagrams

In many thermodynamic cycles, a working fluid experiences phase changes between liquid and vapour in the subcritical zone, such as water in a steam power plant and R134a in a vapour-compression refrigeration system. The liquid-vapour phase change can be illustrated in the $T-v$ and $P-v$ diagrams, as shown in Figures 2.3.4 and 2.3.5. In these diagrams, we can clearly see the three regions: compressed liquid region, saturated liquid-vapour region, and superheated vapour region. The curve that separates the compressed liquid region and saturated liquid-vapour region is called the saturated liquid line. Any point on the saturated liquid line represents a saturated liquid state. In a similar fashion, the curve that lies between the saturated liquid-vapour region and the superheated vapour region is called the saturated vapour line. Any point on the saturated vapour line represents a saturated vapour state. The two saturation lines meet at the critical point.

It is important to note that the liquid state is commonly called compressed liquid or subcooled liquid, and the vapour state is commonly called superheated vapour. In the liquid-vapour, two-phase region, the corresponding isothermal and isobaric processes coincide and remain as horizontal lines. This indicates that, during the phase change process, both temperature and pressure remain constant, i.e., $T = T_{sat}$ and $P = P_{sat}$.

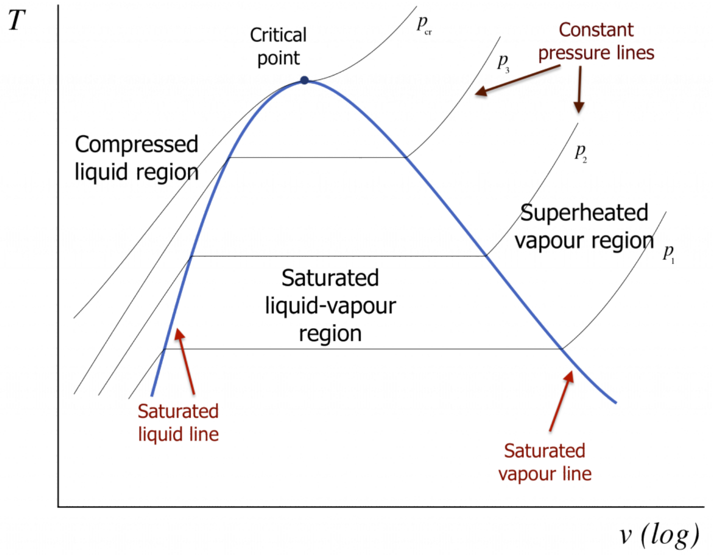

Figure 2.3.4 *T-v diagram*

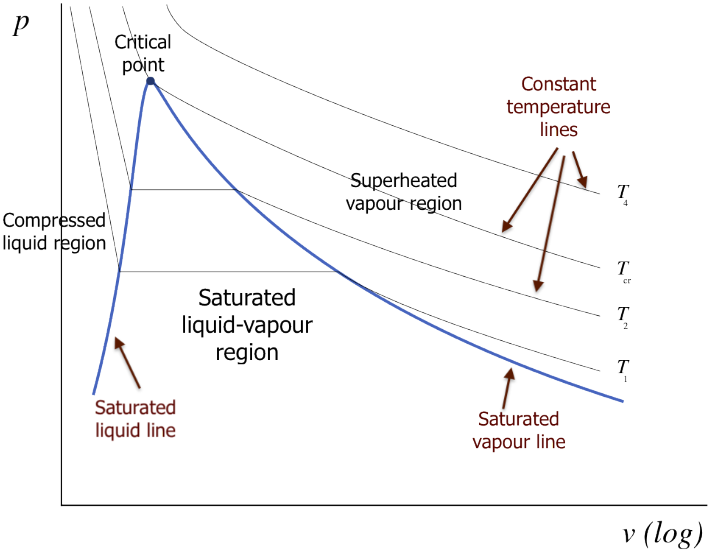

***Figure* 2.3.5** *P-v diagram*

2.3.3 The saturated liquid-vapour two-phase region

For a saturated liquid-vapour, two-phase mixture, we define quality x to denote the mass fraction of the saturated vapour in the mixture.

$$x = \frac{m_g}{m_{mix}}$$

where

m_g: mass of the saturated vapour, in kg or g

m_{mix}: mass of the saturated liquid-vapour mixture, in kg or g

x: quality of the saturated liquid-vapour mixture, dimensionless

With the concept of quality, we can calculate the specific volume, specific internal energy, specific enthalpy, and specific entropy for a saturated liquid-vapour, two-phase mixture by using the following equations.

$$v = v_f + x\left(v_g - v_f\right) = \left(1 - \ x\right)v_f + xv_g$$
$$u = u_f + x(u_g - u_f) = (1 - \ x)u_f + xu_g$$
$$h = h_f + x(h_g - h_f) = (1 - \ x)h_f + xh_g$$
$$s = s_f + x(s_g - s_f) = (1 - \ x)s_f + xs_g$$

where

v, v_f, and v_g: specific volumes of the two-phase mixture, saturated liquid, and saturated vapour, respectively, in m^3/kg

u, u_f, and u_g: specific internal energies of the two-phase mixture, saturated liquid, and saturated vapour, respectively, in kJ/kg

h, h_f, and h_g: specific enthalpies of the two-phase mixture, saturated liquid, and saturated vapour, respectively, in kJ/kg

s, s_f, and s_g: specific entropies of the two-phase mixture, saturated liquid, and saturated vapour, respectively, in kJ/kgK

Practice problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
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2.4 Thermodynamic tables

Thermodynamic tables are commonly used to determine the properties of a substance at a given state. This book includes the tables for four pure substances: water, ammonia, R134a, and carbon dioxide. The data in these tables are obtained from NIST Chemistry WebBook, SRD 69, which consists of the thermophysical properties of various common fluids.

Appendix A: Thermodynamic Properties of Water

- Table A1: Saturated water
- Table A2: Superheated vapour, water
- Table A3: Compressed liquid water

Appendix B: Thermodynamic Properties of Ammonia

- Table B1: Saturated ammonia
- Table B2: Superheated ammonia

Appendix C: Thermodynamic Properties of R134a

- Table C1: Saturated R134a
- Table C2: Superheated R134a

Appendix D: Thermodynamic Properties of Carbon Dioxide

- Table D1: Saturated CO_2
- Table D2: Superheated CO_2

Tables A1, B1, C1, and D1 are the tables for the saturated fluids. They are used to find the properties of the corresponding fluids in saturated liquid, saturated vapour, and two-phase regions. Tables A2, B2, C2, and D2 are the superheated vapour tables for finding the properties of the fluids in the superheated vapour region. Table A3 is the compressed liquid table for water.

In these tables, the specific volume, specific internal energy, specific enthalpy, and specific entropy are tabulated as functions of the pressure and temperature. Among those thermodynamic properties, P, T, and v are measurable properties, and u, h, and s cannot be measured directly; they are calculated with respect to predefined reference states. The reference states for the tables in this book are clearly stated in Appendices A-D.

It is important to note that thermodynamic tables can be generated with respect to different reference states. Because the values of u, h, and s strongly depend on the reference state, it is not surprising to see different values of u, h, and s from different sources. This may cause inconvenience or errors when performing calculations. To avoid unnecessary confusion and to reduce the chance of errors, thermodynamic tables from the same source should be used in calculations because, in most thermodynamic analyses, we are concerned about the *changes* in specific internal energy, specific enthalpy, and specific entropy, or Δu, Δh, and Δs.

How do we determine if a fluid is a compressed liquid, superheated vapour, or two-phase liquid-vapour mixture? How do we find the fluid properties at a certain state? By examining the tables in Appendices A-D, you probably have noticed that all properties in these tables are intensive properties. To determine the fluid properties at a certain state, we will need to know two intensive properties from this list: P, T, v, u, h, s, and x. The following flow charts demonstrate the procedure.

Case 1: **both T and P are given**. You may draw a $P-T$ diagram (see Figure 2.3.2) to help you better understand the flow chart in Figure 2.4.1.

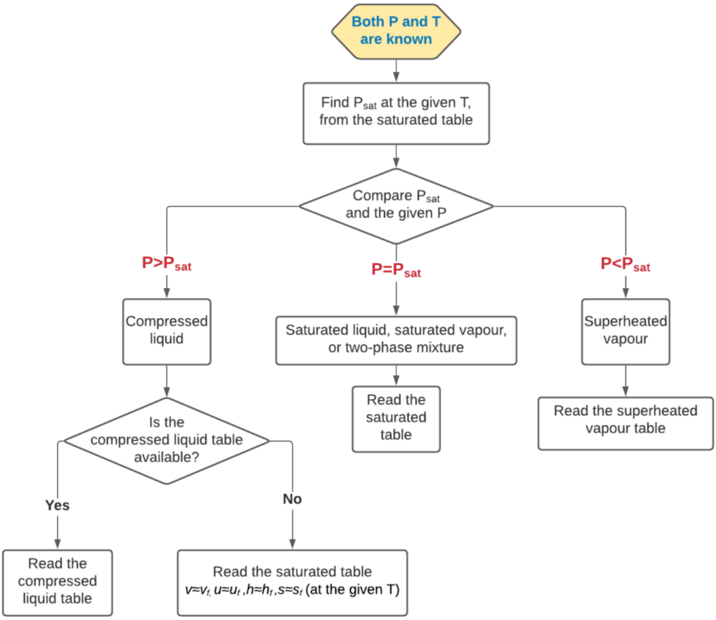

Figure 2.4.1 *Flow chart for determining fluid properties from thermodynamic tables if P and T are known*

Case 2: **both T and x are given**. You may draw a $T - v$ diagram (see Figure 2.3.4) to help you better understand the flow chart in Figure 2.4.2.

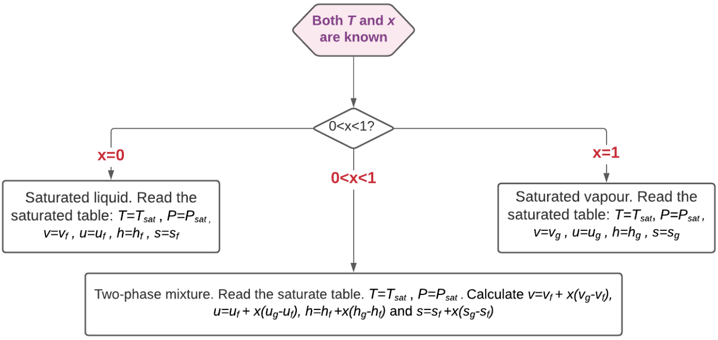

***Figure 2.4.2** Flow chart for determining fluid properties from thermodynamic tables if T and x are known*

Case 3: **both T and v are given**. You may draw a $T-v$ diagram (see Figure 2.3.4) to help you better understand the flow chart in Figure 2.4.3.

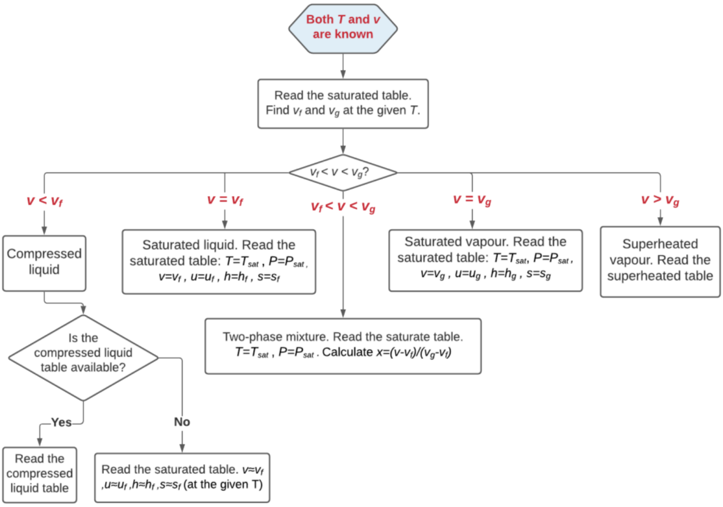

***Figure 2.4.3** Flow chart for determining fluid properties from thermodynamics tables if T and v are known*

Case 4: **temperature T and one of u, h and s are given**. The procedure is exactly the same as for case 3. Replace v with the given u, h, or s in the flow chart, Figure 2.4.3.

The compressed liquid table is presented only for water in the pressure range of 0.5-50 MPa in this book. When the compressed liquid tables are not available for a specific fluid or in a specific range, the saturated liquid properties at the same temperature may be used as an approximation, i.e., $v \approx v_{f@T}$, $u \approx u_{f@T}$, $h \approx h_{f@T}$, and $s \approx s_{f@T}$.

The tables in Appendices A-D are presented with a small temperature increment. Linear interpolations, see Figure 2.4.4, are often used if the given temperature or other properties cannot be found directly from these tables.

$$y = y_0 + (x - x_0)\,\frac{y_1 - y_0}{x_1 - x_0}$$

where (x, y) is the state, at which the property x is known and the property y is to be found. (x_0, y_0) and (x_1, y_1) indicate the properties of two known states, between which the unknown state (x, y) is located. To improve the accuracy, the two states should be selected as close as possible to the unknown state.

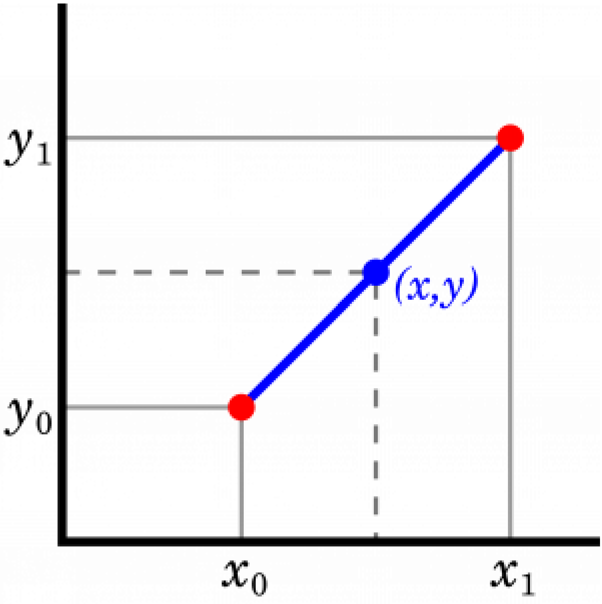

Figure 2.4.4 *Linear interpolation*

The following examples demonstrate how to use these tables to find the properties of a compressed liquid, superheated vapour, and liquid-vapour mixture.

Example 1

Determine the properties of water at T=150°C and P=100 kPa.

<u>*Solution*</u>:

1. Both temperature and pressure are given for water. Use the flow chart for case 1, Figure 2.4.1.
2. From Table A1: at T=150^{o}C, P_{sat} = 0.47617 MPa = 476.17 kPa.
3. Because P=100 kPa < 476.17 kPa, or $P < P_{sat}$, water at this state is a superheated vapour.
4. From Table A2: at T=150^{o}C and P=100 kPa,

   v = 1.93665 m^3/kg, u = 2582.94 kJ/kg

   h = 2776.60 kJ/kg, s = 7.6148 kJ/kgK

Example 2

Determine the properties of ammonia at T=0^{o}C and v =0.2 m^3/kg.

<u>*Solution*</u>:

1. Both T and v are given for ammonia. Use the flow chart for case 3, Figure 2.4.3.
2. From Table B1: at T=0^{o}C, v_f =0.001566 m^3/kg and v_g = 0.289297 m^3/kg.

3. Because $v_f < v < v_g$, ammonia at this state is a liquid-vapour two–phase mixture. Its pressure and quality are

$$P = P_{sat} = 0.42939 \text{ MPa} = 429.39 \text{ kPa}$$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.2 - 0.001566}{0.289297 - 0.001566} = 0.68965$$

From Table B1: at T=0°C,

u_f =342.48 kJ/kg and u_g = 1481.17 kJ/kg

h_f = 343.16 kJ/kg and h_g = 1605.39 kJ/kg

s_f = 1.4716 kJ/kgK and s_g = 6.0926 kJ/kgK

Therefore, the specific internal energy, specific enthalpy, and specific entropy of this two-phase mixture are

$$\begin{aligned} u &= u_f + x(u_g - u_f) \\ &= 342.48 + 0.68965 \times (1481.17 - 342.48) = 1127.78 \text{ kJ/kg} \\ h &= h_f + x(h_g - h_f) \\ &= 343.16 + 0.68965 \times (1605.39 - 343.16) = 1213.66 \text{ kJ/kg} \\ s &= s_f + x(s_g - s_f) \\ &= 1.4716 + 0.68965 \times (6.0926 - 1.4716) = 4.6585 \text{ kJ/kgK} \end{aligned}$$

Example 3

Refrigerant R134a has a specific enthalpy *h* = 420 kJ/kg at T=20°C. Determine the pressure P and specific volume *v* of R134a at this state.

Solution:

1. Refer to case 4 as both *T* and *h* are given for R134a. Because the procedures for cases 3 and 4 are the same, the flow chart for case 3, Figure 2.4.3, is used by replacing *v* with *h*.
2. From Table C1: at *T*=20°C, h_g=409.75 kJ/kg. Because *h* = 420 kJ/kg > h_g , R134a at this state is a superheated vapour.
3. From Table C2:

   At T=20°C and P_1 = 100 kPa: h_1 = 420.31 kJ/kg, v_1 = 0.233731 m^3/kg

   At T=20°C and P_2 = 150 kPa: h_2 = 419.33 kJ/kg, v_2 = 0.154053 m^3/kg

   Because 419.33 kJ/kg < 420 kJ/kg < 420.31 kJ/kg, the pressure of R134a at the given state must be between 100 kPa and 150 kPa. Use linear interpolation to calculate the pressure and specific volume at the given state.

   Pressure

$$\because \frac{P-P_1}{P_2-P_1}=\frac{h-h_1}{h_2-h_1}$$

$$\therefore \frac{P-100}{150-100} = \frac{420-420.31}{419.33-420.31}$$

$$\therefore P = 115.82 \text{ kPa}$$

Specific volume

$$\because \frac{v-v_1}{v_2-v_1} = \frac{h-h_1}{h_2-h_1}$$

$$\therefore \frac{v-0.233731}{0.154053-0.233731} = \frac{420-420.31}{419.33-420.31}$$

$$\therefore v = 0.208527 \text{ m}^3/\text{kg}$$

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
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2.5 Chapter review

An important step in thermodynamic analysis is to predict the thermodynamic properties, in particular, the intensive properties at different states of a process or a cycle. In this chapter, we have introduced common thermodynamic properties of pure substances, and how to use phase diagrams and thermodynamic tables to determine the phase of a fluid and its corresponding properties at a given state. Below are the key takeaways:

- Pure substances may be used as working fluids in thermal devices. A pure substance may exist as a single-phase matter, such as solid, liquid, vapour or gas, or a saturated two-phase mixture.
- The state of a pure substance can be illustrated in the $P-T$, $T-v$, and $P-v$ phase diagrams.
- A state of a pure substance is fixed by **two independent, intensive properties**.
    - For a single-phase fluid, if two intensive properties from the list: $T, P, v, u, h,$ and s are known, then the state is fixed.
    - For a two-phase mixture, if two properties from the list: T, P, v, u, h, s and the quality x are known, then the state is fixed.
- The properties of a fixed state can be extracted from the thermodynamic tables directly or by using linear interpolations.
- $P-T$, $T-v$, and $P-v$ diagrams are of particular importance in thermodynamic analysis. It is highly recommended that students relate the appropriate phase diagrams to the flow charts in Section 2.4, when practicing how to locate a state and how to read thermodynamic tables.

2.6 Key Equations

Pressure, temperature, and specific volume

Pressure	$P = F/A$
Absolute and gauge pressures	$P_{gauge} = P_{abs} - P_{atm}$
Absolute and vacuum pressures	$P_{vac} = P_{atm} - P_{abs}$
Density	$\rho = m/\mathbb{V}$
Specific volume	$v = \mathbb{V}/m = 1/\rho$
Conversion of temperatures in Kelvin and Celsius degrees	$T\,(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15$

Energy, enthalpy, and entropy

Total stored energy in a system	$\begin{aligned} E &= U + KE + PE \\ &= mu + \frac{1}{2}\,mV^2 + mgz \quad (V : velocity) \end{aligned}$
Total stored specific energy in a system	$e = \dfrac{E}{m} = u + \dfrac{1}{2}\,V^2 + gz \quad (V : velocity)$
Enthalpy	$H = U + P\mathbb{V}$
Specific internal energy	$u = U/m$
Specific enthalpy	$h = H/m$ and $h = u + Pv$
Specific entropy	$s = S/m$

Saturated liquid-vapour two-phase mixtures

Quality	$x = \dfrac{m_g}{m_{mix}}$
Specific volume	$v = v_f + x\left(v_g - v_f\right) = (1 - x)\, v_f + x v_g$
Specific internal energy	$u = u_f + x(u_g - u_f) = (1 - x)u_f + x u_g$
Specific enthalpy	$h = h_f + x(h_g - h_f) = (1 - x)h_f + x h_g$
Specific entropy	$s = s_f + x(s_g - s_f) = (1 - x)s_f + x s_g$

Compressed liquid (when the compressed liquid tables are not available)

Specific volume	$v \approx v_{f@T}$
Specific internal energy	$u \approx u_{f@T}$
Specific enthalpy	$h \approx h_{f@T}$
Specific entropy	$s \approx s_{f@T}$

3. IDEAL AND REAL GASSES

3.0 Chapter introduction and learning objectives

Thermodynamic tables are commonly used to evaluate thermodynamic properties of a pure substance. This method is accurate, but may be time consuming for complicated calculations. It would be desirable if, for some special cases, a simple method with relatively good accuracy could be developed for evaluating thermodynamic properties.

This chapter introduces the concepts of "ideal" gas, ideal gas equation of state (EOS), real gas, and compressibility factor. It explains the difference between "ideal" and real gases, and at what conditions the ideal gas model may be used as an approximation for evaluating thermodynamic properties of simple gases.

Learning Objectives

After completing the module, you should be able to

- Understand the limitation of the ideal gas equation of state
- Apply the ideal gas equation of state in solving engineering problems as appropriate
- Explain the difference between "ideal" and real gases
- Calculate the compressibility factor of a pure

substance at a given state
- Identify if a substance may be treated as an ideal or real gas for a given condition based on its compressibility factor

3.1 Ideal gas and ideal gas equation of state

Consider a container of fixed volume filled with a gas. When the container is heated, the gas temperature will increase, causing the gas pressure to increase. The variations of gas pressure and temperature are governed by the equations of state. An **equation of state** (EOS) is an expression that relates pressure, temperature, and specific volume of a gas.

The simplest equation of state is the ***ideal gas equation of state***, which is expressed as

$$Pv = RT \quad \text{or} \quad P\mathbb{V} = mRT$$

where

m: mass, in kg
$\mathbb{V}$: volume, in m^3
v: specific volume, in m^3/kg
T: absolute temperature, in K
P: pressure, in kPa or Pa
R: gas constant in kJ/kgK or J/kgK

A gas which obeys the ideal gas EOS is called an **ideal gas**. The ideal gas model is a hypothetical model. It approximates the $P - v - T$ behaviour of a gas at high temperatures and low pressures in the superheated vapour region.

When a gas is at a state near the saturation region or its critical point, the gas behaviour deviates from the ideal gas model significantly. For example, Figure 3.1.1 shows the $T - v$ diagram for water. Steam in the shaded region is either at a high temperature or a low pressure. The ideal gas model is valid in this region with a relative error of less than 1%. Moving out of the shaded region and towards the saturated vapour line or the critical point, the relative error increases significantly because the ideal gas EOS can no longer represent the gas behaviour in these regions.

A common mistake that students tend to make is to use the ideal gas EOS in all calculations without evaluating its suitability for the given conditions. It is important to note that, although many gasses may be treated as ideal gases in a certain range of pressures and temperatures, ***the ideal gas EOS is NOT valid for gases in all conditions.*** Therefore, it cannot be used without verification. The compressibility factor in Section 3.2 explains how to verify if a gas is an "ideal" or real gas.

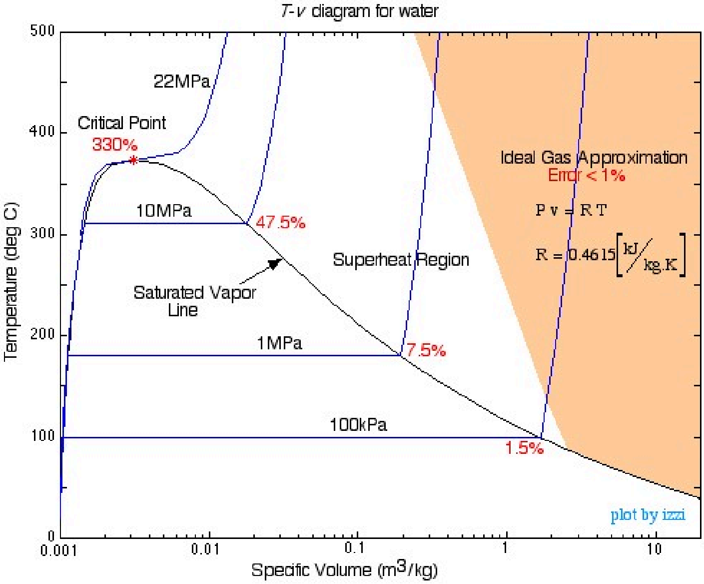

***Figure 3.1.1** T-v diagram for water*

Example 1

Two tanks contain methane. For the given conditions, methane can be treated as an ideal gas.

- Tank 1 has a volume of 0.3 m^3, and is at a temperature of 20°C and a pressure of 300 kPa.
- Tank 2 contains 1.5 kg of methane, and is at a temperature of 30°C and a pressure of 800 kPa.

The partition between the two tanks is removed to allow methane in the tanks to mix and reach equilibrium. What is the equilibrium pressure if the temperature of the two tanks is 25°C at equilibrium?

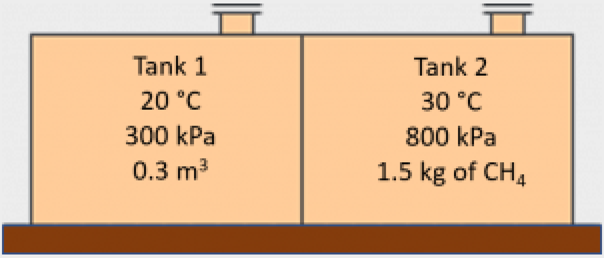

Figure 3.1.e1 *Rigid tanks containing methane*

Solution

Methane is treated as an ideal gas at the given conditions.

From Table G1: R=0.5182 kJ/kgK for methane.

Apply the ideal gas law $P\mathbb{V} = mRT$ to both initial and final conditions of methane in the two tanks.

Tank 1 at the initial condition:

$$P_1\mathbb{V}_1 = m_1RT_1$$

$$m_1 = \frac{P_1\mathbb{V}_1}{RT_1} = \frac{300 \times 0.3}{0.5182 \times (273.15 + 20)} = 0.5925 \text{ kg}$$

Tank 2 at the initial condition:

$$P_2\mathbb{V}_2 = m_2RT_2$$

$$\mathbb{V}_2 = \frac{m_2RT_2}{P_2} = \frac{1.5 \times 0.5182 \times (273.15 + 30)}{800} = 0.2945 \text{ m}^3$$

The two tanks are in equilibrium at the final state.

$$m_3 = m_1 + m_2 = 0.5925 + 1.5 = 2.0925 \text{ kg}$$

$$\mathbb{V}_3 = \mathbb{V}_1 + \mathbb{V}_2 = 0.3 + 0.2945 = 0.5945 \text{ m}^3$$

$$P_3\mathbb{V}_3 = m_3R_3T_3$$

$$P_3 = \frac{m_3RT_3}{\mathbb{V}_3} = \frac{2.0925 \times 0.5182 \times (273.15 + 25)}{0.5925} = 543.8 \text{ kPa}$$

The equilibrium pressure of the two tanks at the final state is 543.8 kPa.

Important note:

- ***The temperature must be expressed in Kelvin when applying the ideal gas EOS.***

Example 2

Consider 1 kg of oxygen in a piston-cylinder device undergoing a thermodynamic cycle consisting of three processes.

- Process 1⟶2: isochoric
- Process 2⟶3: isothermal expansion
- Process 3⟶1: isobaric compression

At state 1, T_1= 300 K, P_1=1.5 atm. At state 2, P_2= 3 atm. Treat oxygen as an ideal gas at the given conditions.

1. Sketch the cycle on a $P - v$ diagram.
2. Determine the temperature, T_2 , at state 2, and the specific volume, v_3, at state 3.

Solution

1. The cycle on a $P - v$ diagram

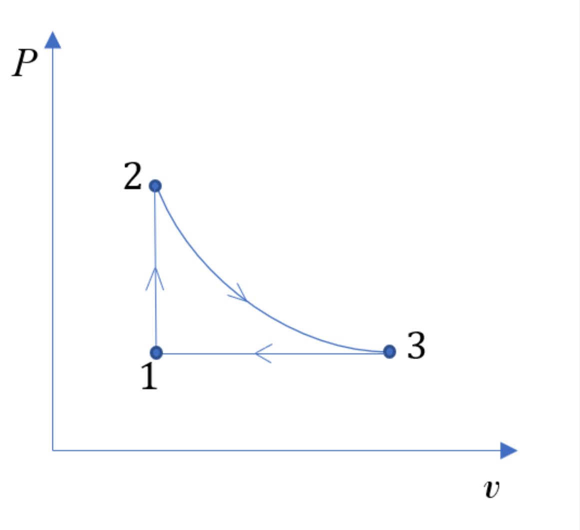

Figure 3.1.e2 *P-v diagram of a cycle consisting of three processes*

2. Oxygen is treated as an ideal gas at the given conditions.

From Table G1: R=0.2598 kJ/kgK for oxygen.

Apply the ideal gas law $Pv = RT$ to the three processes.

Process 1⟶2 is an isochoric process; therefore, the specific volume remains constant in the process, $v_1 = v_2$

$$\because Pv = RT$$

\therefore v_1 = \displaystyle\ frac{RT_1}{P_1}

and v_2 = \displaystyle\frac{RT_2}{P_2}

$$\because v_1 = v_2 \quad \text{and} \quad R = \text{constant}$$

$$\therefore \frac{T_2}{T_1} = \frac{P_2}{P_1}$$

$$\therefore T_2 = T_1 \times \frac{P_2}{P_1} = 300 \times \frac{3}{1.5} = 600 \text{ K}$$

Process 2⟶3 is an isothermal expansion process; therefore, $T_3 = T_2 = 600 \text{ K}$.

Process 3⟶1 is an isobaric compression process; therefore,

$$P_3 = P_1 = 1.5 \text{ atm} = 1.5 \times 101.325 = 152 \text{ kPa}$$

$$v_3 = \frac{RT_3}{P_3} = \frac{0.2598 \times 600}{152} = 1.026 \text{ m}^3/\text{kg}$$

The temperature at state 2 is 600 K and the specific volume at state 3 is 1.026 m^3/kg.

Important note:

- ***The temperature must be expressed in Kelvin when applying the ideal gas EOS.***

Practice Problems

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3.2 Real gas and compressibility factor

The $P-v-T$ relation of a **real gas** doesn't obey the ideal gas EOS. For a gas at a given range of pressures and temperatures, we may calculate its compressibility factor to evaluate the suitability of the ideal gas EOS for the given condition.

The compressibility factor is a dimensionless correction factor to account for the deviation of the real gas behaviour from the "ideal" gas model. It is defined as

$$Z = Pv/RT \quad \text{or} \quad Pv = ZRT$$

where

Z: compressibility factor, dimensionless
v: specific volume, in m^3/kg
T: absolute temperature, in K
P: pressure, in kPa or Pa
R: gas constant, in kJ/kgK or J/kgK

The compressibility factor of an ideal gas is exactly one. For real gases, the compressibility factor may be very different from one. Figures 3.2.1 and 3.2.2 illustrate the compressibility factors of hydrogen and nitrogen, respectively, over a range of pressures and temperatures. It can be seen that the compressibility factor changes with both pressure and temperature. As the pressure approaches zero, the compressibility factor tends to converge to one. In other words, a real gas may behave like an ideal gas at "low" pressures regardless of its temperature.

From Figures 3.2.1 and 3.2.2, it can be seen that the compressibility charts for different gases may vary largely. It is therefore more practical to use a generalized compressibility chart, as shown in Figure 3.2.3, where the pressures and temperatures are normalized with respect to the critical pressure and critical temperature of a gas. The reduced pressure P_r and reduced temperature T_r are defined as

$$\begin{aligned} P_r &= P/P_{crit} \\ T_r &= T/T_{crit} \end{aligned}$$

where

P: pressure, in kPa or Pa
P_{crit}: critical pressure, in kPa or Pa. P_{crit} and P must have the same unit.
P_r: reduced pressure, dimensionless
T: absolute temperature, in K
T_{crit}: critical temperature, in K
T_r: reduced temperature, dimensionless

The compressibility factor is thus plotted as a function of P_r and T_r. The generalized compressibility chart can be viewed as a graphical representation of the gas behaviour over a wide range of pressures and temperatures. It is valid for many substances, especially those that have simple molecular structures. From Figure 3.2.3, it can be seen that the smallest compressibility factor occurs at the critical point, $P_r = 1$ and $T_r = 1$. This indicates that a real gas deviates significantly from the ideal gas behaviour near its critical point. As $P_r \rightarrow 0$ or $T_r \geq 2$, the compressibility factor $Z \rightarrow 1$. In other words, a real gas behaves like an ideal gas at “low” pressures and “high” temperatures.

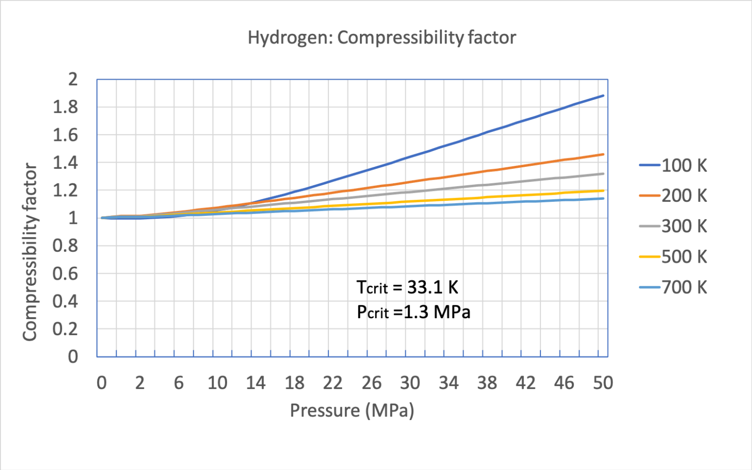

Figure 3.2.1 *Compressibility factor of hydrogen*

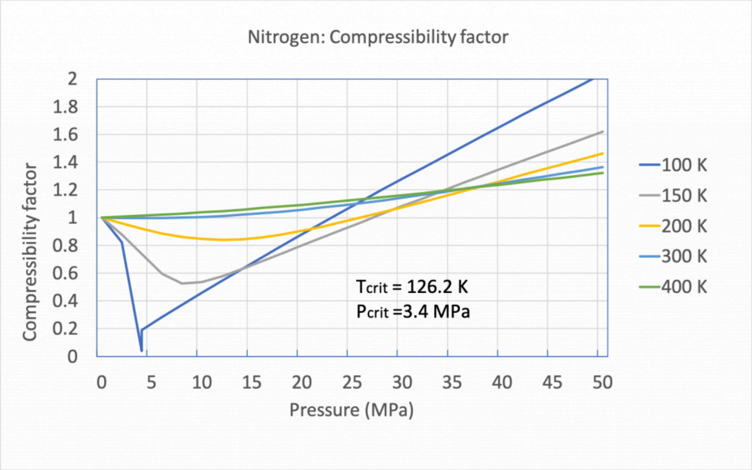

Figure 3.2.2 *Compressibility factor of nitrogen*

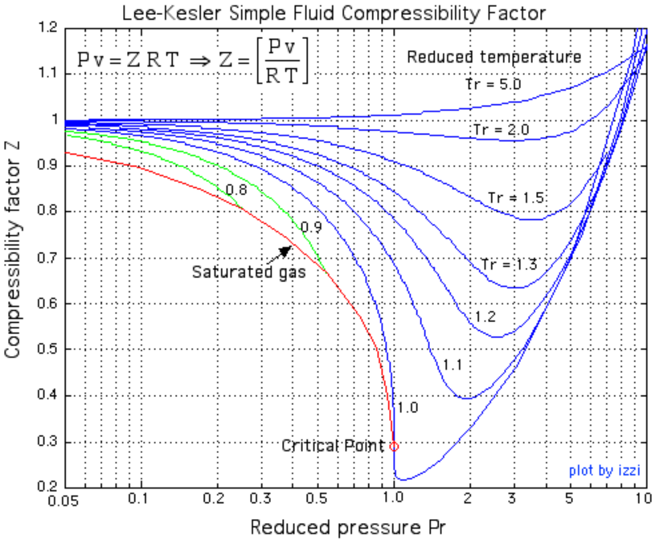

***Figure 3.2.3** Lee-Kesler simple fluid compressibility factor*

Typically, thermodynamic tables or real-gas EOS are used for evaluating the relation of pressure, temperature, and specific volume of real gases. If neither tool is available, the generalized compressibility chart may be used instead as a fast and reasonably accurate tool, especially for substances with simple molecular structures. Below is the procedure of how to use the generalized compressibility chart, followed by examples.

1. Find the critical pressure, P_{crit}, and critical temperature, T_{crit}, of a substance from Appendix E
2. Calculate the reduced pressure, P_r, and reduced temperature, T_r, of the substance at the given condition
3. Estimate the compressibility factor, Z, from Figure 3.2.3.
4. Apply $Pv = ZRT$ to determine the unknown specific volume (if needed)

Example 1

Find the compressibility factor of the following substances at the given conditions. Is it reasonable to treat them as ideal gases at the given conditions?

1. Methane at -50^{o}C, 4.1 MPa
2. Ammonia at 600^{o}C, 500 kPa

<u>*Solution*</u>

1. Methane at -50^{o}C, 4.1 MPa

First, find the critical properties of methane from Table E1.

$$T_{crit} = 190.6 \text{ K}, \; P_{crit} = 4.60 \text{ MPa}$$

Second, calculate the reduced temperature and reduced pressure.

$$T_r = \frac{T}{T_{crit}} = \frac{273.15 - 50}{190.6} = 1.17$$

$$P_r = \frac{P}{P_{crit}} = \frac{4.10}{4.60} = 0.89$$

From Figure 3.2.3, the compressibility factor $Z \approx 0.78 << 1$; therefore, methane at the given condition cannot be treated as an ideal gas.

2. Ammonia at 600^{o}C, 500 kPa

First, find the critical properties of ammonia from Table E1.

$$T_{crit} = 405.4 \text{ K}, P_{crit} = 11.34 \text{ MPa}$$

Second, calculate the reduced temperature and reduced pressure.

$$T_r = \frac{T}{T_{crit}} = \frac{273.15 + 600}{405.4} = 2.15$$

$$P_r = \frac{P}{P_{crit}} = \frac{0.5}{11.34} = 0.04$$

From Figure 3.2.3, the compressibility factor $Z \approx 1$; therefore, ammonia can be treated as an ideal gas at the given condition. Note that the reduced temperature of ammonia is greater than 2 and the reduced pressure is very small, indicating the given state is far away from the critical point.

Example 2

Calculate the specific volume of steam at 3 MPa, 350°C by using three methods: (1) superheated water vapour table, (2) ideal gas EOS, and (3) compressibility factor. How accurate is each of the methods?

Solution

Method 1: use the steam table.

From Table A2: P=3 MPa and T=350$^{\text{o}}$C, therefore, v=0.09056 m^3/kg

Method 2: use the ideal gas EOS alone

From Table G1: R=0.4615 kJ/kgK for steam.

$$\because Pv = RT$$

$$\therefore v = \frac{RT}{P} = \frac{0.4615 \times (273.15 + 350)}{3000} = 0.09586 \text{ m}^3/\text{kg}$$

The relative error in comparison to method 1 is

$$error\% = \frac{|0.09586 - 0.09056|}{0.09056} \times 100\% = 5.85\%$$

Method 3: use the ideal gas EOS corrected by the compressibility factor

From Table E1: P_{crit}=22.06 MPa, T_{crit}=647.1 K for water.

Calculate the reduced pressure and reduced temperature at the given condition:

$$P_r = \frac{P}{P_{crit}} = \frac{3}{22.06} = 0.136$$

$$T_r = \frac{T}{T_{crit}} = \frac{273.15 + 350}{647.1} = 0.963$$

Estimate the compressibility factor from Figure 3.2.3: $Z \approx 0.94$

Calculate the specific volume at the given condition by incorporating the compressibility factor

$$\because Pv = ZRT$$

\begin{align*} \therefore v = \displaystyle\frac{ZRT}{P} &= \displaystyle\frac{0.94\times0.4615\times(273.15 + 350)}{3000} \\&= 0.09011 \ \rm{m^3/kg} \end{align*}

The relative error in comparison to method 1 is

$$error\% = \frac{|0.09011 - 0.09056|}{0.09056} \times 100\% = 0.497\%$$

Comment:

Method 1 gives the most accurate value for specific volume among the three methods, as the steam table is specific for water vapour at different pressures and temperatures. Method 2 assumes steam as an ideal gas. This method is easy to use but gives the least accurate result. Method 3, by correcting the ideal gas EOS with the compressibility factor, improves the accuracy of the calculation.

Practice Problems

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3.3 Chapter Review

The ideal gas EOS is the simplest model to relate the pressure, temperature, and specific volume of a gas. It can be used for thermodynamic analysis if the compressibiliy factor of a gas at a given condition approaches one, or the gas behaves like an "ideal" gas. It is important to note that, despite its simplicity, the ideal gas EOS cannot be used in all conditions. Its suitability at a given state must be verified.

A real gas doesn't obey the ideal gas EOS at a given state. For a real gas, the compressibility factor can be incorporated to improve the accuracy of the prediction of the $P - v - T$ relation. The compressibility factor of a gas may be obtained from the generalized compressibility chart.

3.4 Key Equations

Ideal gas equation of state (mass form)	$Pv = RT$ and $P\mathbb{V} = mRT$ m : mass R : gas constant $\mathbb{V}$: volume v : specific volume T : temperature in Kelvin
Compressibility factor	$Z = Pv/RT$

4. THE FIRST LAW OF THERMODYNAMICS FOR CLOSED SYSTEMS

4.0 Chapter introduction and learning objectives

The first law of thermodynamics is the law of energy conservation: ***the energy can neither be created nor destroyed; it is conserved in a system***. This chapter explains the fundamental concepts of heat and work, and the first law of thermodynamics. Examples are given to illustrate the applications of the first law of thermodynamics in closed systems.

Learning Objectives

After completing the chapter, you should be able to

- Determine the internal energy of real substances by using thermodynamic tables
- Calculate the internal energy of ideal gases by using constant-volume specific heat
- Calculate different forms of work, such as the boundary work in various processes and the spring work due to the deformation of a spring
- Explain the differences between work, heat, and energy stored in a system
- Explain the physical meaning of the first law of thermodynamics
- Apply the first law of thermodynamics to closed systems

4.1 Internal energy in a system

The total energy of a system may consist of internal energy, kinetic energy, potential energy, and other forms of energy. For a system free of magnetic, electric, and surface tension effects, its total energy and corresponding specific energy can be expressed as

$$E = U + KE + PE$$
$$e = u + ke + pe$$

where E, U, KE, and PE represent the total energy, internal energy, kinetic energy, and potential energy of a system, respectively; e, u, ke, and pe are their corresponding specific energies. Recall from Chapter 2, internal energy U is a form of thermal energy. A system at different states may have different internal energies due to different temperature and pressure at each state; therefore, U is a state function. It is important to note that the change in internal energy in a process depends on the initial and final states, not on the path of the process. For example, although the three processes in Figure 4.1.1 undergo different paths, their changes in internal energy, ΔU, between the two states 1 and 2 are the same because the three processes have identical initial states and identical final states.

The first law of thermodynamics gives the relation between the total energy stored in a system and the energy transferred into or out of the system in the form of heat and work. In this chapter, we will firstly introduce the common methods of determining internal energy and work, and then the first law of thermodynamics and its applications to closed systems.

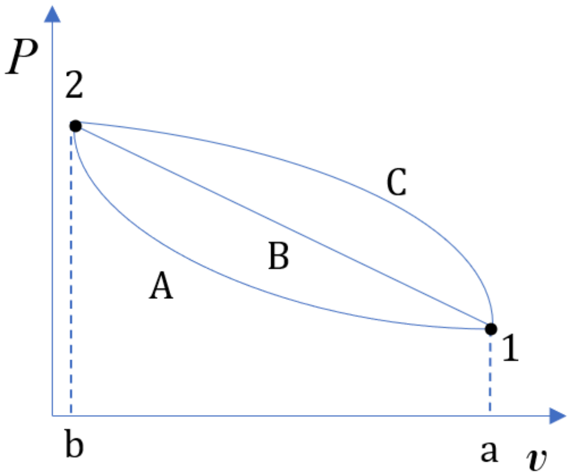

Figure 4.1.1 *P- v diagram showing different process paths with the same initial and final states*

4.1.1 Using thermodynamic tables to determine specific internal energy *u*

For pure substances with available thermodynamic tables, the specific internal energy can be read from the thermodynamic tables, then the internal energy can be found from

$$U = mu$$

where

m: mass of a system, in kg
U: internal energy, in kJ
u: specific internal energy, in kJ/kg

Example 1

Complete the table, and label each state on the *P*-*T*, *T*-*v* and *P*-*v* diagrams.

	Substance	T ^{o}C	*P* kPa	*v* m^3/kg	*u* kJ/kg	*x*	Phase
	1 Water	60	500				
	2 R134a	40		0.1			

<u>*Solution*</u>

1. Water at 60^{o}C and 500 kPa

From Table A1: P_{sat} = 0.01995 MPa = 19.95 kPa at 60^{o}C. The given pressure *P* = 500 kPa > P_{sat} ; therefore, water at the given state is a compressed liquid.

From Table A3: *v* = 0.001017 m^3/kg and *u* = 251.08 kJ/kg for the given state.

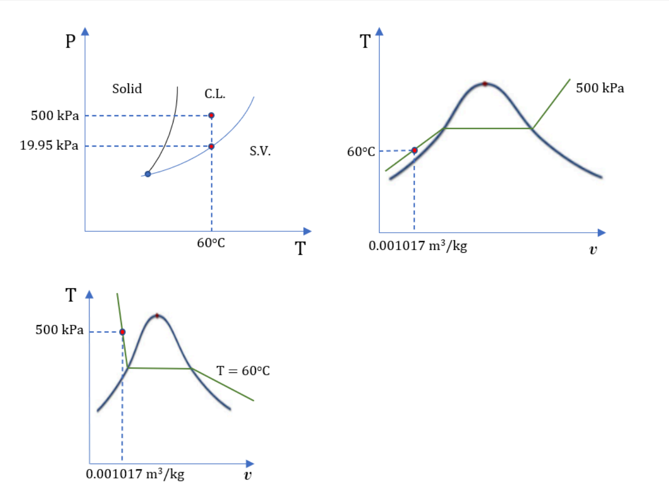

Figure 4.1.e1 *Water is a compressed liquid at the given state, as illustrated on the phase diagrams.*

2. R134a at 40$^{\text{o}}$C and 0.1 m^3/kg

From Table C1: v_g = 0.019966 m^3/kg at 40$^{\text{o}}$C. The given specific volume v = 0.1 m^3/kg > v_g; therefore, R134a at the given state is a superheated vapour.

From Table C2:

v = 0.080629 m^3/kg and u = 410.00 kJ/kg at 40$^{\text{o}}$C and 300 kPa

v = 0.123226 m^3/kg and u = 411.22 kJ/kg at 40$^{\text{o}}$C and 200 kPa

Use linear interpolation to find P and u at the given condition

$$\because \frac{P-300}{200-300}=\frac{0.1-0.080629}{0.123226-0.080629}$$

\therefore P= 254.52\ \rm{kPa}\\

$$\because \frac{u-410.00}{411.22-410.00}=\frac{0.1-0.080629}{0.123226-0.080629}$$

$$\therefore u=410.55\ \mathrm{kJ/kg}$$

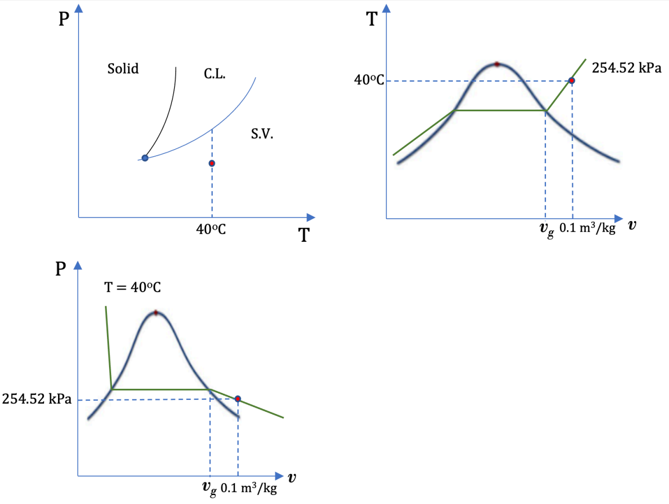

Figure 4.1.e2 *R134a is a superheated vapour at the given state, as illustrated on the phase diagrams.*

In summary, the table below gives the final answers to the question.

	Substance	T °C	P kPa	v m^3/kg	u kJ/kg	x	Pha
1	Water	60	500	0.001017	251.08	n.a.	Compress
2	R134a	40	254.52	0.1	410.55	n.a.	Superh vapou

4.1.2 Constant-volume specific heat

When a substance absorbs heat, its temperature tends to increase. Different substances require different amounts of heat for a given temperature rise. For example, it requires 4.18 kJ of heat to warm up 1 kg of water by 1^{o}C. But it only requires 2.22 kJ of heat to warm up the same amount of gasoline by 1^{o}C. In other words, water and gasoline have different energy storage capacities. **Specific heat**, also called heat capacity, is an important property used to quantify the energy storage capacity of a substance. Specific heat is defined as the energy required to raise the temperature of one unit mass (i.e., 1 kg) of a substance by one degree (i.e., 1^{o}C, or 1 K),

$$C=\left(\frac{1}{m}\frac{\delta Q}{\partial T}\right)$$

where

C: specific heat, in kJ/kgK

m: mass of a substance, in kg

$\dfrac{\delta Q}{\partial T}$: the amount of heat supplied to or extracted from a substance per unit change of temperature, in kJ/K

The specific heat of a substance may be measured in an isochoric or isobaric process; they are therefore called constant-volume specific heat, C_v, and constant-pressure specific heat, C_p, respectively. Both C_v and C_p are properties of a substance. They can be used to calculate the changes of specific internal energy, Δu, and specific enthalpy, Δh, respectively, in a process involving ideal gases, liquids and solids. The constant-volume specific heat is introduced below in detail and the constant-pressure specific heat will be introduced in Chapter 5.

Constant-volume specific heat is defined as the energy required to raise the temperature of one unit mass (i.e., 1 kg) of a substance by one degree (i.e., 1°C, or 1 K) in an isochoric process. Mathematically, it is expressed as,

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v$$

where

C_v: constant-volume specific heat, in kJ/kgK
u: internal energy, in kJ/kg
T: temperature, in K or $^{\circ}$C

The constant-volume specific heat of selected ideal gases can be found in Appendix G, Table G1. For example, oxygen has $C_v =$ 0.658 kJ/kgK. If we heat up 1 kg of oxygen at 300 K in a sealed, rigid tank, it will require 0.658 kJ of heat for the temperature of the oxygen to rise from 300 K to 301 K.

It is important to note that although C_v is typically measured in isochoric processes, it is a property of a substance. The use of C_v is NOT limited to isochoric processes. As can be seen in the next section, for ideal gases C_v can be used to calculate the change in specific internal energy, Δu, in ANY processes.

4.1.3 Using C_v to calculate Δu for ideal gases

A gas behaves like an ideal gas as its compressibility factor $Z \to 1$. The specific internal energy of an ideal gas is a function of temperature only, $u = f(T)$; therefore,

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{du}{dT}\right)_v = f(T)$$

The change in specific internal energy between two states in any process involving ideal gases can be found from

$$\Delta u = u_2 - u_1 = C_v(T_2 - T_1)$$

where

u: specific internal energy, in kJ/kg
T: temperature, in K or $^{\text{o}}$C
C_v: average constant-volume specific heat. Table G1 provides the values of C_v for selected ideal gases.

The above equation provides a convenient way for estimating Δu of ideal gases in a process. Its accuracy depends on the change in temperature in a process. In many cases, especially, those with small temperature variations, this method is reasonably accurate and can be used for ideal gases when the thermodynamic tables are not available. If the thermodynamic tables are available or high accuracy is required for the process analysis, it is preferable to use the thermodynamic tables to determine u at different states first, and then Δu.

Example 2

Two kilograms of air is heated from 10$^{\text{o}}$C to 40$^{\text{o}}$C. Calculate the change in internal energy, ΔU, in this process. Will your answer be different if the process is isochoric or isobaric?

Solution

Air is treated as an ideal gas. From Table G1: $C_v = 0.718\ \mathrm{kJ/kgK}$; therefore,

$$\Delta u = C_v(T_2 - T_1) = 0.718 \times (40 - 10) = 21.54\ \mathrm{kJ/kg}$$

$$\Delta U = m\Delta u = 2 \times 21.54 = 43.08\ \mathrm{kJ}$$

The change in internal energy in this process is 43.08 kJ. As C_v is a property of the substance (e.g., air in this example), the answer will be the same regardless of the type of the process.

Practice Problems

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Practice Problems

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4.2 Heat transfer across a boundary

The total energy stored in a system may change when energy is transferred into or out of the system. For a closed system, the energy transfer is achieved via two mechanisms: heat and work, as illustrated in Figure 1.2.3.

Heat transfer takes place when a temperature difference exists between a system and its surroundings. As heat transfer must cross the system boundary, it is a *boundary phenomenon*. The heat transfer between two states during a process can be written as

$$ {}_1Q_2 = \int_1^2 \delta Q $$

Different from internal energy, heat transfer is NOT a state function. It is a path function because the amount of heat that is absorbed or rejected by a substance in a process depends not only on the initial and final states, but also on the process path. Although heat transfer is NOT a property of a system, it has a significant effect on the changes of properties of the system in a process.

Specific heat transfer refers to the amount of heat transfer per unit mass of a substance. It is defined as

$$ q = \frac{Q}{m} $$

where

m: mass of a system, in kg
Q: amount of heat transfer in a process, in kJ
q: amount of specific heat transfer in a process, in kJ/kg

Practice Problems

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4.3 Work

Work is a form of mechanical energy associated with a force and its resulting displacement. When a force F moves a body from one position to another, it does work on that body over the distance, see Figure 4.3.1.

$$_1W_2 = \int_1^2 F dx$$

The common SI units for work are kJ and J.

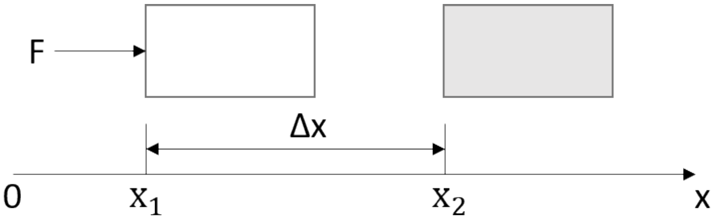

***Figure* 4.3.1** *Work done due to a force acting on a block over a distance*

4.3.1 Boundary work

Work associated with the expansion and compression of a gas is commonly called **boundary work** because it is done at the boundary between a system and its surroundings.

Let us consider a piston-cylinder device, as illustrated in Figure 4.3.2. The gas in the cylinder exerts an upward force, $F = PA$

, where P is the gas pressure, and A is the cross-sectional area of the piston. Upon receiving heat, the gas will tend to expand, pushing the piston up. We will assume the expansion process is quasi-equilibrium, and the piston moves up an infinitesimal distance d. The boundary work done by the gas to the surroundings in this infinitesimal process is $dW = Fd = (PA)d = P(Ad) = P\Delta\mathbb{V}$; therefore, the total boundary work between two states in a process can be written as

$$_1W_2 = \int_1^2 Pd\mathbb{V}$$

where

P: pressure, in kPa or Pa
$\mathbb{V}$: volume, in m^3
$_1W_2$: boundary work, in kJ or J

Specific boundary work refers to the boundary work done by a unit mass of a substance. It can be written as

$$_1w_2 = \int_1^2 Pdv$$

where

P: pressure, in kPa or Pa
v: specific volume, in m^3/kg
$_1w_2$: specific boundary work, in kJ/kg or J/kg

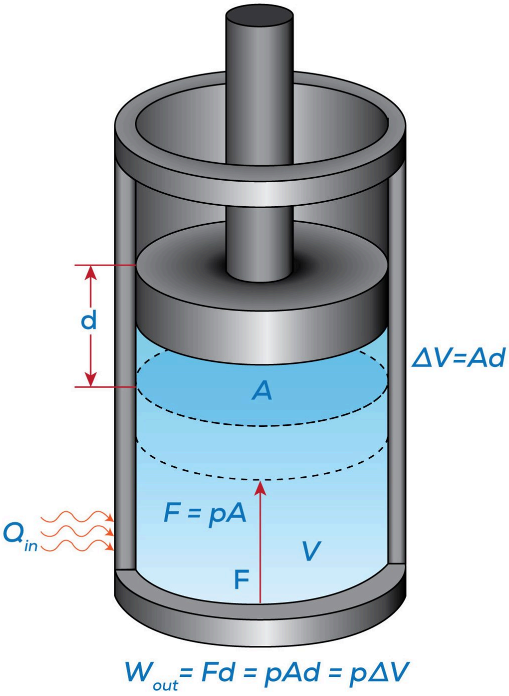

Figure 4.3.2 *Boundary work caused by the expansion of gas in a piston-cylinder device*

From the integral equations for ${}_1W_2$ and ${}_1w_2$, we can tell that the boundary work and specific boundary work between any two states in a process can be illustrated graphically as the area under the process curve in the $P-\mathbb{V}$ and $P-v$ diagrams, respectively. For example, the shaded area in Figure 4.3.3 represents the specific boundary work between states 1 and 2 in the compression process A. The three compression processes, A, B, and C in Figure 4.3.3 have different specific boundary work because of their different paths. By comparing the areas under the process curves, we can tell that process A has the smallest specific boundary work and process C has the largest specific boundary work.

Figure 4.3.3 demonstrates that the boundary work and specific boundary work in a quasi-equilibrium process are path functions; they depend on the initial and final states as well as the process path. Boundary work can be defined as positive or negative. Here is a common sign convention: the boundary work in an expansion process is positive. This is because the change of volume in an expansion process is positive. Likewise, the boundary work in a compression process is negative.

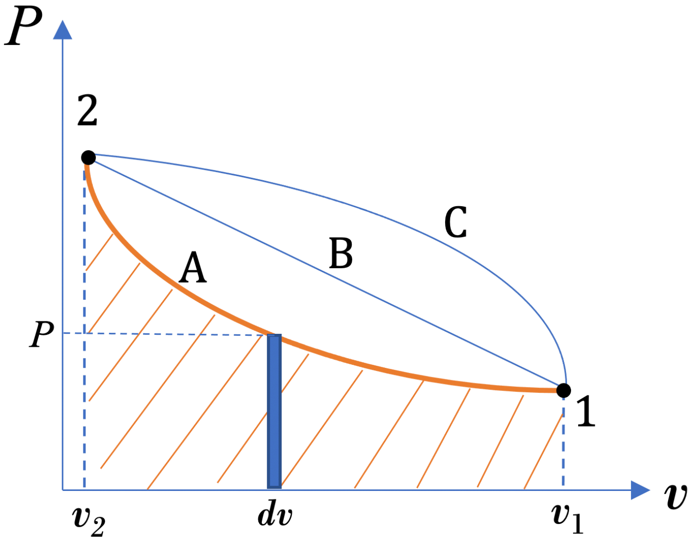

Figure 4.3.3 *P-v diagram showing the specific boundary work as the shaded area under the P-v curve*

Example 1

Consider a **rigid sealed** tank of a volume of 0.3 m^3 containing nitrogen at 10^oC and 150 kPa. The tank is heated until the temperature of the nitrogen reaches 50^oC. Treat nitrogen as an ideal gas.

1. Sketch the process on a $P - \mathbb{V}$ diagram
2. Calculate the boundary work in this process
3. Calculate the change in internal energy in this process

Solution

1. $P - \mathbb{V}$ diagram

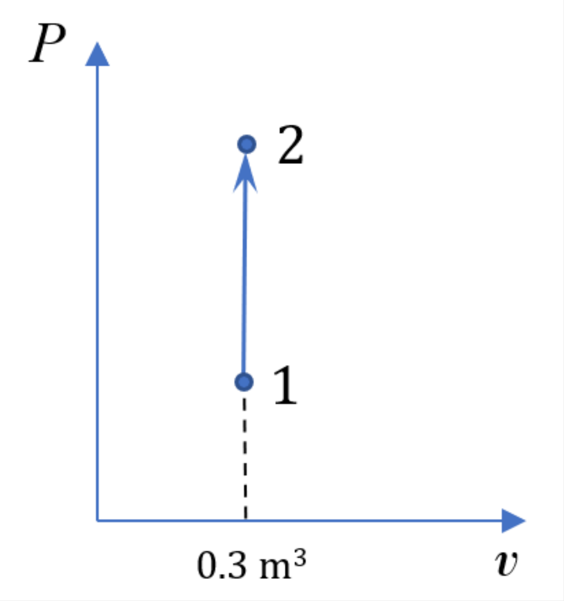

Figure 4.3.e1 *Isochoric process in a rigid sealed tank*

2. The boundary work is zero because the volume of nitrogen remains constant in the process.

$$ {}_1W_2 = \int_1^2 P d\mathbb{V} = 0 $$

3. Change in internal energy in the process

From Table G1:

R=0.2968 kJ/kgK and C_v= 0.743 kJ/kgK

The mass of nitrogen:

$$\because P\mathbb{V} = mRT$$

$$\therefore m = \frac{P\mathbb{V}}{RT} = \frac{150 \times 0.3}{0.2968 \times (273.15 + 10)} = 0.5355 \text{ kg}$$

The change in internal energy:

$$\begin{aligned}\Delta U &= m\Delta u = mC_v\left(T_2 - T_1\right) \\ &= 0.5355 \times 0.743 \times (50 - 10) = 15.9 \text{ kJ}\end{aligned}$$

Nitrogen absorbs 15.9 kJ of heat in this process.

Example 2

Consider 0.2 kg of ammonia in a reciprocating compressor (piston-cylinder device) undergoing an **isobaric** expansion. The initial and final temperatures of the ammonia are 0^{o}C and 30^{o}C, respectively. The pressure remains 100 kPa in the process.

1. Sketch the process on a $P - v$ diagram
2. Calculate the boundary work in this process
3. Calculate the change in internal energy in this process

Solution:

1. $P - v$ diagram

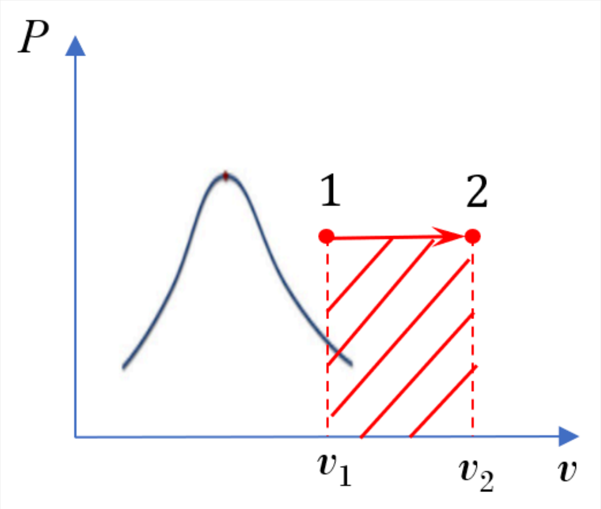

Figure 4.3.e2 *An isobaric expansion process in the superheated vapour region*

2. Boundary work

From Table B2: for the initial state 1 at T = 0$^{\text{o}}$C, P = 100 kPa,

$$v_1 = 1.31365\ \mathrm{m^3/kg}$$
$$u_1 = 1504.29\ \mathrm{kJ/kg}$$

For the final state 2 at T = 30$^{\text{o}}$C, P = 100 kPa,

$$v_2 = 1.46562\ \mathrm{m^3/kg}$$
$$u_2 = 1554.1\ \mathrm{kJ/kg}$$

Graphically, the specific boundary work is the shaded rectangular area under the process line in the P – v diagram.

$$\begin{aligned} {}_1W_2 = m_1w_2 = m\int_1^2 Pdv = mP(v_2 - v_1) \\ = 0.2 \times 100 \times (1.46562 - 1.31365) = 3.0394 \text{ kJ} \end{aligned}$$

3. Change in internal energy

$$\begin{aligned} \Delta U = m\Delta u = m(u_2 - u_1) \\ = 0.2 \times (1554.1 - 1504.29) = 9.962 \text{ kJ} \end{aligned}$$

Example 3

Consider air undergoing an **isothermal** expansion. The initial and final pressures of the air are 200 kPa and 100 kPa respectively. The temperature of the air remains 50°C in the process. Treat air as an ideal gas.

1. Sketch the process on a $P - v$ diagram
2. Calculate the specific boundary work in this process
3. Calculate the change in specific internal energy in this process

Solution:

1. $P - v$ diagram

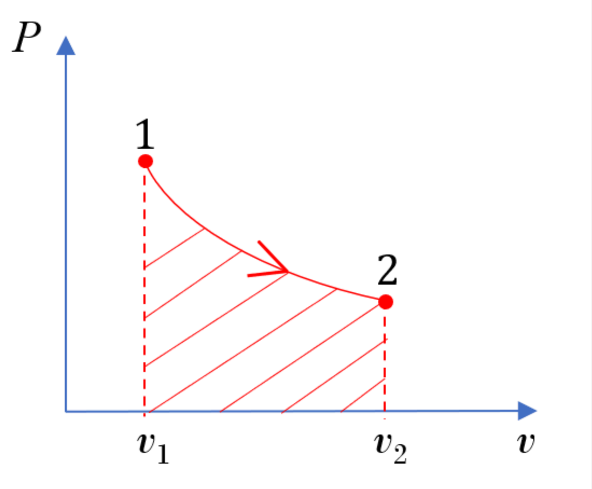

Figure 4.3.e3 *Isothermal expansion process*

2. Specific Boundary work

From Table G1: R = 0.287 kJ/kgK for air. The ideal gas, air, undergoes an isothermal process.

$$\because Pv = RT$$

$$\therefore P = \frac{RT}{v} \text{ and } \frac{v_2}{v_1} = \frac{P_1}{P_2}$$

$$_1W_2 = \int_1^2 Pdv = \int_1^2 \frac{RT}{v}dv$$

$$= RT\int_1^2 \frac{1}{v}dv = RTln\frac{v_2}{v_1} = RTln\frac{P_1}{P_2}$$

$$= 0.287 \times (273.15 + 50)ln\frac{200}{100} = 64.285 \text{ kJ}$$

3. The process is isothermal; therefore, the temperature remains constant and the change in internal energy is zero.

$$\Delta u = C_v\left(T_2 - T_1\right) = 0$$

4.3.2 Polytropic process and its boundary work

A polytropic process refers to any quasi-equilibrium thermodynamic process, which can be described with the following mathematical expression.

$$P\mathbb{V}^n = \text{constant} \qquad \text{or} \qquad Pv^n = \text{constant}$$

where

P: pressure, in kPa or Pa
$\mathbb{V}$: volume, in m^3
v: specific volume, in m^3/kg
n: polytropic exponent, dimensionless

By adjusting n to different values, the above two simple expressions can be used to represent the relations of pressure-volume or pressure-specific volume of various processes that are encountered in real thermal systems, including the isobaric, isochoric, and isothermal processes. Table 4.3.1 lists the polytropic exponents corresponding to an ideal gas undergoing an isobaric, isochoric, and isothermal process, respectively.

Table 4.3.1 Ideal gas equation of state expressed in polytropic relations

Process	Polytropic exponent	Ideal gas equation of state	Polytropic relation
Isobaric	$n = 0$	$P = \text{constant}$	$Pv^0 = \text{constant}$
Isothermal	$n = 1$	$\because T = \text{constant}$ and $Pv = RT$ $\therefore Pv = \text{constant}$	$Pv^1 = \text{constant}$
Isochoric	$n = \infty$	$v = \text{constant}$	$Pv^\infty = \text{constant}$

Figure 4.3.4 shows different polytropic processes of an ideal gas. In many actual thermodynamic processes, the polytropic exponents are typically in the range of $1 < n < k$, where $k = \dfrac{C_p}{C_v}$. C_p and C_v are the constant-pressure and constant-volume specific heats, respectively.

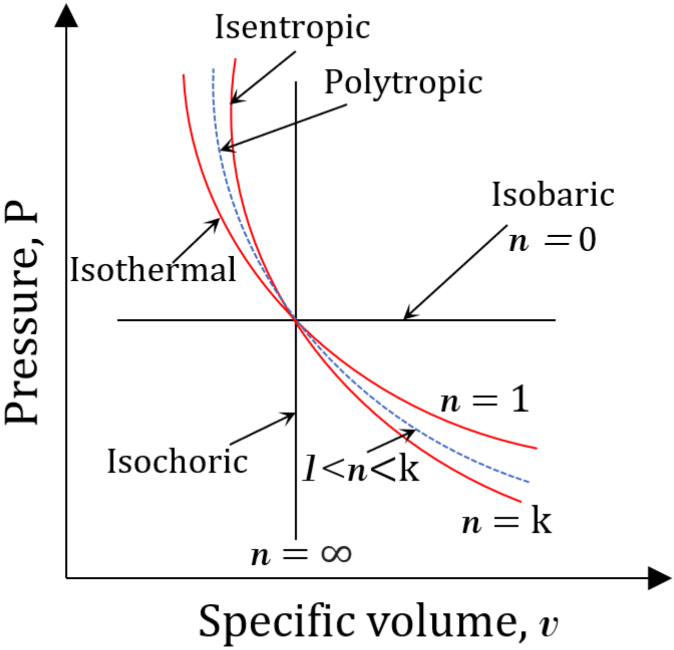

***Figure* 4.3.4** *Different processes of an ideal gas*

The boundary work and corresponding specific boundary work in a polytropic process can be calculated by using the following equations. Detailed derivations are left for the readers to practice.

The following expressions are valid for both real and ideal gases.

If $n \neq 1$,

$$ {}_{1}W_{2} = \frac{P_2\mathbb{V}_2 - P_1\mathbb{V}_1}{1-n} $$

$$ {}_{1}w_{2} = \frac{P_2v_2 - P_1v_1}{1-n} $$

If $n = 1$,

$$ {}_{1}W_{2}={P}_\mathbf{1}\mathbb{V}_\mathbf{1}{ln}{\displaystyle\frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}={P}_\mathbf{2}\mathbb{V}_\mathbf{2}{ln}{\displaystyle\frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}\\ $$

$$ {}_{1}W_{2}={P}_\mathbf{1}\mathbb{V}_\mathbf{1}{ln}{\displaystyle\frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}={P}_\mathbf{2}\mathbb{V}_\mathbf{2}{ln}{\displaystyle\frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}\\ $$

$$ {}_{1}w_{2} = P_1v_1 ln\frac{v_2}{v_1} = P_2v_2 ln\frac{v_2}{v_1} $$

$$ {}_{1}w_{2} = P_1v_1 ln\frac{P_1}{P_2} = P_2v_2 ln\frac{P_1}{P_2} $$

The following two expressions are valid only for ideal gases in an isothermal process ($n = 1$).

If $n = 1$,

$$ \\ {}_{1}W_{2}={mRT}{ln}{\displaystyle\frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}={mRT}{ln}{\displaystyle\frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}\\ $$

$$ {}_{1}w_{2} = RTln\frac{v_2}{v_1} = RTln\frac{P_1}{P_2} $$

where

$_1W_2$: boundary work, in kJ or J

${}_1w_2$: specific boundary work, in kJ/kg or J/kg
P: pressure, in kPa or Pa
$\mathbb{V}$: volume, in m^3
v: specific volume, in m^3/kg
T: absolute temperature, in K
R: gas constant, in kJ/kgK or J/kgK
m: mass, in kg

Example 4

Consider an ideal gas undergoing a **polytropic** process. At the initial state: P_1=200 kPa, v_1=0.05 m^3/kg. At the final state: v_2=0.1 m^3/kg. For n=1.3 and n=1,

1. Sketch the two processes on a $P-v$ diagram. Which process has a larger specific boundary work?
2. Calculate the specific boundary work and verify your answer to the question in part 1.

<u>*Solution:*</u>

1. $P-v$ diagram

   From the $P-v$ diagram, the area under the process line for $n=1$ is greater than that for $n=1.3$; therefore, the isothermal process with $n=1$ has a larger specific boundary work.

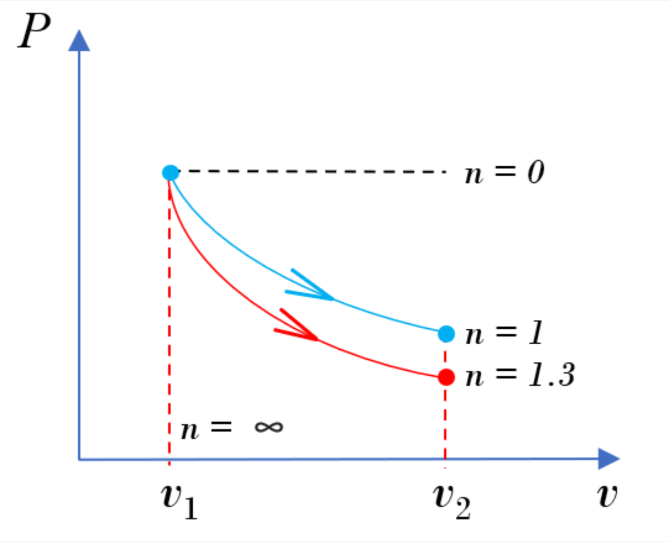

Figure 4.3.e4 *Polytropic processes of an ideal gas illustrated on the P-v diagram*

2. For $n = 1.3$, find the final pressure P_2 first.

$$\because P_1 v_1^n = P_2 v_2^n$$

$$\therefore P_2 = P_1 \left(\frac{v_1}{v_2}\right)^n = 200 \times \left(\frac{0.05}{0.1}\right)^{1.3} = 81.225 \text{ kPa}$$

$$\begin{aligned} {}_1W_2 &= \frac{P_2 v_2 - P_1 v_1}{1-n} \\ &= \frac{81.225 \times 0.1 - 200 \times 0.05}{1 - 1.3} = 6.258 \text{ kJ/kg} \end{aligned}$$

For $n = 1$, the process is isothermal; therefore,

$$
\begin{aligned}
{}_1W_2 &= P_1 v_1 ln\left(\frac{v_2}{v_1}\right) \\
&= 200 \times 0.05 \times ln\frac{0.1}{0.05} = 6.931\ \mathrm{kJ/kg}
\end{aligned}
$$

Compare the specific boundary work in these two processes, the isothermal process (*n*=1) has a larger specific boundary work than the polytropic process with *n* = 1.3. The calculation results are consistent with the observation from the $P - v$ diagram.

4.3.3 Spring work

Spring work is a form of mechanical energy required to compress or expand a spring to a certain distance, see Figure 4.3.5. Spring force and spring work can be expressed as follows:

$$F = Kx$$

$$W_{spring} = \int_1^2 Fdx = \frac{1}{2}K\left(x_2^2 - x_1^2\right)$$

where

F: spring force, in kN or N
K: spring constant, in kN/m or N/m
W_{spring}: spring work, in kJ or J
x_1 and x_2: initial and final displacements, in m, measured from the spring's rest position.

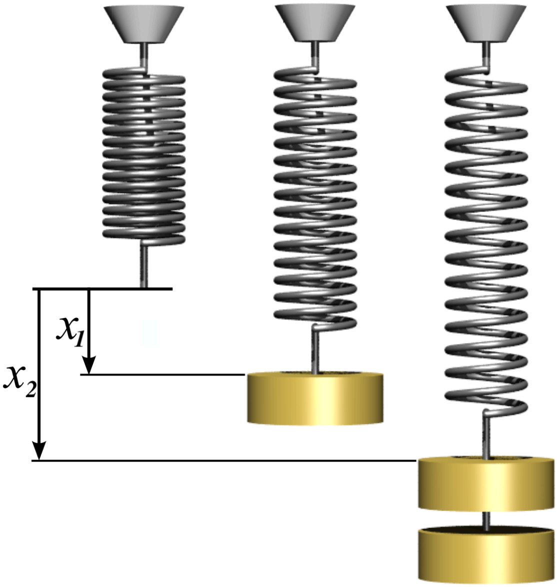

Figure 4.3.5 *Work done by the spring force due to the displacement from the spring's rest position.*

Example 5

A linear spring with spring constant K=100 kN/m is mounted on a piston-cylinder device. At the initial state, the cylinder contains 0.15 m^3 of gas at 100 kPa. The spring is uncompressed. The gas is then heated until its volume expands to 0.2 m^3. The piston's cross-sectional area is 0.1

m^2. Assume the piston is frictionless with negligible weight and the process is quasi-equilibrium,

1. Write an expression of the gas pressure as a function of the gas volume in this process
2. Sketch the process on a $P - \mathbb{V}$ diagram
3. Calculate the total work done by the gas during this expansion process
4. If the spring is not mounted on the piston, the gas in the cylinder will expand isobarically after being heated. To reach the same final volume, 0.2 m^3, how much work must be done by the gas in the expansion process?

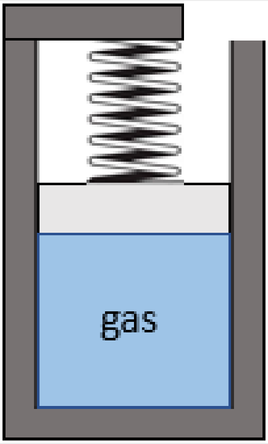

Figure 4.3.e5
Piston-cylinder device with a spring loaded on top of the piston

Solution:

1. Analyze the forces acting on the piston, see below.

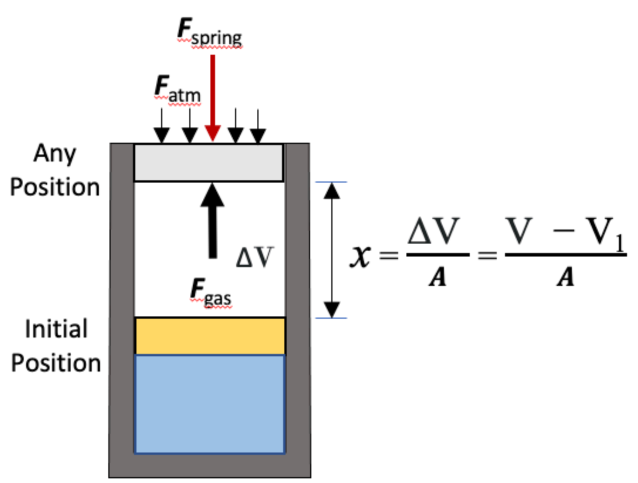

Figure 4.3.e6 *Forces acting on the piston in equilibrium*

Three forces acting on the piston are in equilibrium.

$$\because \sum F = 0 : F_{gas} = F_{spring} + F_{atm}$$

$$\therefore P_{gas} A = Kx + P_{atm} A = K\frac{\mathbb{V} - \mathbb{V}_1}{A} + P_{atm} A$$

$$\therefore P_{gas} = \frac{K(\mathbb{V} - \mathbb{V}_1)}{A^2} + P_{atm}$$

At the initial state: $P_{gas} = 100 \text{ kPa}, \mathbb{V} = \mathbb{V}_1$.

Substitute the two values,

$$\because 100 = \frac{K(\mathbb{V} - \mathbb{V}_1)}{A^2} + P_{atm}$$

$$\therefore P_{atm} = 100 \text{ kPa}$$

with K =100 kN/m, A = 0.1 m^2, P_{atm} = 100 kPa and $\mathbb{V}_1$ = 0.15 m^3, the gas pressure can be expressed as a function of the gas volume.

$$\because P_{gas} = \frac{100(\mathbb{V} - 0.15)}{0.1^2} + 100$$

$$\therefore P_{gas} = 10^4\mathbb{V} - 1400$$

where gas pressure is in kPa and $\mathbb{V}$ is in m^3.

2. $P - \mathbb{V}$ diagram

From part 1, P_{gas} is a linear function of volume $\mathbb{V}$.

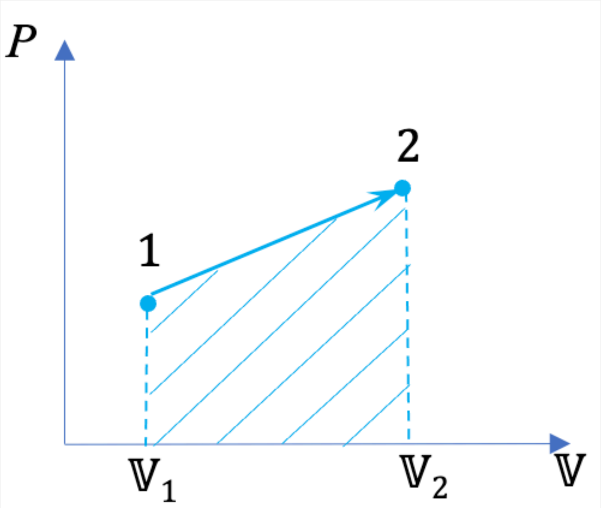

***Figure* 4.3.e7** P-V *diagram showing the process from the initial to the final states*

3. During the expansion process, the gas has to overcome the resistance from the springand. At the same time, the gas pressure and volume increase until the gas reaches the final state. The total work done by the gas is the shaded area of the trapezoid in the $P - \mathbb{V}$ diagram.

At the final state, $\mathbb{V}_2 = 0.2\ m^3$; therefore,

$$P_{gas,2} = 10^4 \times 0.2 - 1400 = 600 \text{ kPa}$$

The total work done by the gas is

$$
\begin{aligned}
{}_1W_2 &= \frac{(P_{gas,2} + P_{gas,1})(\mathbb{V}_2 - \mathbb{V}_1)}{2} \\
&= \frac{(600 + 100)(0.2 - 0.15)}{2} = 17.5 \text{ kJ}
\end{aligned}
$$

4. If the gas expands isobarically from $\mathbb{V}_1$ = 0.15 m^3 to $\mathbb{V}_2$ = 0.2 m^3, the work done by the gas will be

$$
\begin{aligned}
{}_1W_{2a} &= P_{gas,1}(\mathbb{V}_2 - \mathbb{V}_1) \\
&= 100 \times (0.2 - 0.15) = 5 \text{ kJ}
\end{aligned}
$$

${}_1W_{2a}$ is the shaded area of the rectangle. The difference between ${}_1W_2$ and ${}_1W_{2a}$ is the gas work used to overcome the resistance of the spring, as shown in the shaded area of the triangle.

$$W_{spring} = {}_1W_2 - {}_1W_{2a} = 17.5 - 5 = 12.5 \text{ kJ}$$

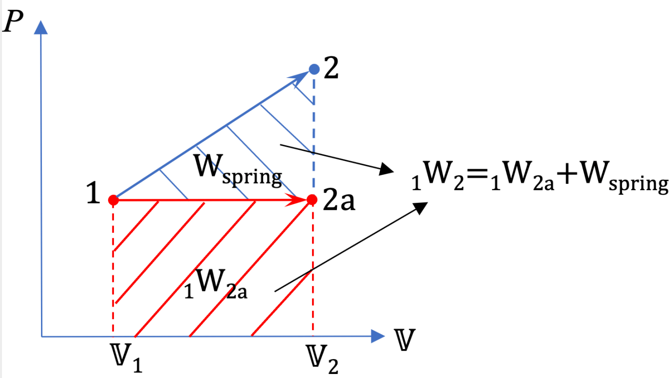

Figure 4.3.e8 *P-V diagram showing the total work and its compositions*

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here: https://pressbooks.bccampus.ca/thermo1/?p=1452#h5p-33

Practice Problems

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4.4 The first law of thermodynamics for closed systems

The first law of thermodynamics is essentially an energy conservation law. Both heat and work are energy transfer mechanisms. They play an important role in the first law of thermodynamics. Table 4.4.1 summarizes the main differences between heat and work, and internal energy.

Table 4.4.1 Comparison of heat, work and internal energy

	Heat and Work	**Internal Energy**
Characteristic	Both heat and work are **energy transfer mechanisms.** They are **not properties** of a system.	Internal energy is a **property** of a system.
Interaction with the system	Both heat and work must cross the boundary between a system and its surroundings.	A system possesses a total energy and an internal energy.
Magnitude	Both heat and work are **path functions**; their magnitudes depend on the states and the specific process path.	Internal energy is a **state function**; its magnitude depends on the state only.

The first law of thermodynamics states that ***the change in the total energy stored in a system equalsthe net energy transferred tothe system in the formof heat and work.***

$$\Delta \text{energy} = +\text{in} - \text{out}$$

The change in the total energy of a system during a process from states 1 to 2 can be expressed as

$$\Delta E = E_2 - E_1 = {}_1Q_2 - {}_1W_2$$

If the changes in the kinetic and potential energies of the system are negligible, i.e., $\Delta KE = \Delta PE = 0$, then the first law of thermodynamics for a closed system can be simplified as

$$\Delta U = U_2 - U_1 = {}_1Q_2 - {}_1W_2$$

where

E: total energy stored in a system

U: internal energy of a system

Q: heat transfer in a process. A common sign convention: positive sign (+) for the heat transfer into a system, and negative sign (-) for the heat transfer out of a system. In short, the sign for heat transfer: in (+), out (-). See Figure 4.4.1.

W: work done by or to a system. A common sign convention: positive sign (+) for the work output (work done by a system to its surroundings), and negative sign (-) for the work input (work done by the surroundings to the system). In short, the sign for work: in (-), out (+). See Figure 4.4.1.

Subscripts 1 and 2 refer to the initial and final states of a process.

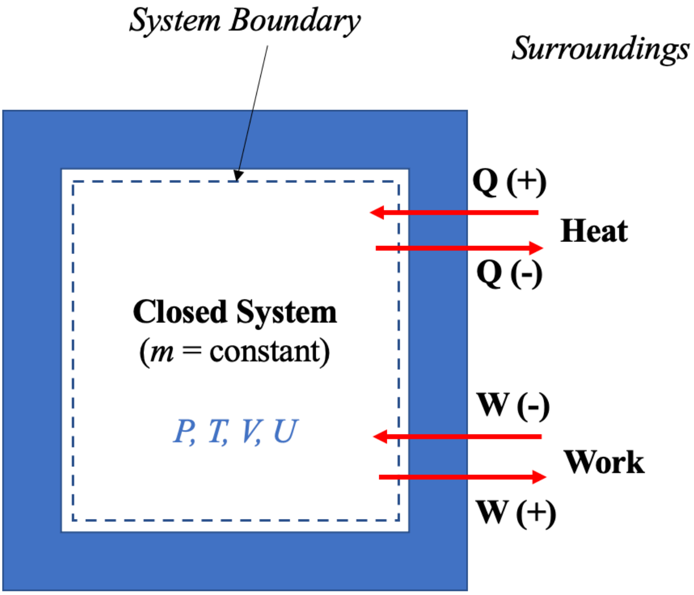

Figure 4.4.1 *A common sign convention for heat and work transfer to a closed system*

The following procedure may be followed when solving problems with the first law of thermodynamics.

1. Sketch the physical system described in the problem and show its main components.
2. Set up an appropriate closed system by drawing the system boundary. How a system is set up may determine if a means of energy transfer can be regarded as heat or work.
3. Indicate the heat and work transferred into or out of the system and their signs, see Figure 4.4.1.
4. Identify the type of the processes (e.g., isobaric, isothermal, isochoric, polytropic, or isentropic). Show the processes on the $P-v$ and $T-v$ diagrams if possible, and list all of the known and unknown states.