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## 4.8 Exercises

1. In Section 4.3.1, the $S$ parameters of a reciprocal error network were determined by applying three loads—$Z_1$, $Z_2$, and $Z_3$—and measuring the respective input reflection coefficients. If $Z_1$ is a matched load, $Z_2$ is a short circuit, and $Z_3$ is an open, the $S$ parameters of Equations (4.1), (4.2), and (4.3) are found. Use SFG theory to derive these results.

2 The $S$ parameters of a line with a physical length of 20 cm was measured at 1 GHz in a $Z_{\text{ref}} = 50\,\Omega$ system and found to be $S_{11} = S_{22} = 0.1$ and $S_{21} = S_{12} = -0.9$. What are the characteristic impedance, attenuation constant, and phase constant of the line at 1 GHz. It is known that the line is less than a wavelength long.

3 At 10 GHz the propagation constant of a line is $\gamma = 4.6 + \jmath 400$ and the characteristic impedance is $Z_0 = 60 - \jmath 0.5$. What are the $R, L, G$ and $C$ parameters of the line?

4 At 100 GHz, the propagation constant of a line is $\gamma = 30 + \jmath 600$ and the characteristic impedance is $Z_0 = 27 + \jmath 0.7$. What are the $R, L, G$ and $C$ parameters of the line?

5 At 1 GHz, the propagation constant of a line is $\gamma = 2.5 + \jmath 36$ and the characteristic impedance is $Z_0 = 105\jmath$. What are the $R, L, G$ and $C$ parameters of the line?

6 The $S$ parameters of a line with a physical length of 2 mm was measured at 10 GHz in a $Z_{\text{ref}} = 50\,\Omega$ system and found to be $S_{11} = S_{22} = 0.1 - \jmath 0.001$ and $S_{21} = S_{12} = -0.7 + \jmath 0.3$. It is known that the line is less than a wavelength long. For the line find the following at 10 GHz:

   (a) Characteristic impedance.
   (b) Why is it important to know the approximate length of the line in terms of wavelengths?
   (b) Complex propagation constant.
   (c) Attenuation constant, .
   (c) R, L, G, and C parameters.

7 Repeat 6 but now the line is between one and two wavelengths long.

8. The properties of a 5 mm long microstrip line on an unknown substrate are to be determined by terminating the line in a known impedance and measuring $\Gamma_{\text{in}}$, the reflection coefficient at the input of the line. At 10 GHz the load has a reflection coefficient $\Gamma_L = 0.9\angle 0°$ and $\Gamma_{\text{in}} = 0.9\angle 170°$. When the frequency is swept, on a Smith chart $\Gamma_{\text{in}}$ traces out a circle centered at the origin. All measurements are referenced to 50 Ω. It is known that the substrate is not magnetic and so the relative permeability of the substrate is one.

   (a) At 10 GHz what is the electrical length of the line in degrees? (Assume that the line is less than a half-wavelength long.)
   (b) What is the electrical length of the line in fractions of a wavelength?
   (c) Since the line is 5 mm long, what is the guide wavelength, $\lambda_g$, of the line?
   (d) What is the free space wavelength, $\lambda_0$?
   (e) What is the relationship between $\lambda_g$, $\lambda_0$, and the line's effective relative permittivity $\varepsilon_e$?
   (f) What is $\varepsilon_e$?
   (g) What is the characteristic impedance of the line?
   (h) What is the loss of the line in terms of dB per meter?

(i) If there was no substrate, i.e. $\epsilon_r = \epsilon_0$, what would the electrical length of the line be in terms of $\lambda_0$?

9 A long slightly lossy line has a frequency-independent input reflection coefficient located at the point $\Gamma_{\text{in}} = 0.8$ on a Smith chart. What is the characteristic impedance of the line?

10 A long slightly lossy line has a frequency-independent input reflection coefficient located at the point $\Gamma_{\text{in}} = -0.7$ on a Smith chart. What is the characteristic impedance of the line?

11 Port 2 of a transmission line with characteristic impedance $Z_{01} = 75\ \Omega$ is terminated in $75\ \Omega$ and the input reflection coefficient $\Gamma_{\text{in}}$ at Port 1 is measured and plotted on a $50\ \Omega$ Smith chart. As the frequency is varied $\Gamma_{\text{in}}$ traces out a circle. What is the center and radius of that circle.

12 Port 2 of a transmission line with characteristic impedance $Z_{01} = 75\ \Omega$ is left open and the input reflection coefficient $\Gamma_{\text{in}}$ at Port 1 is measured and plotted on a $50\ \Omega$ Smith chart. As the frequency is varied $\Gamma_{\text{in}}$ traces out a circle. What is the center (use polar coordinates) and radius of that circle.

13 The input reflection coefficient $\Gamma_{\text{in}}$ of a transmission line with unknown characteristic impedance $Z_{01}$ and is measured using a VNA in a $50\ \Omega$ system but the load terminating the line is unkown. On a $50\ \Omega$ Smith chart the locus of $\Gamma_{\text{in}}$ with respect to frequency is a circle centered at 0.7 on the horizontal axis of the Smith chart with a radius of 0.3. What is $Z_{01}$?

14 The input reflection coefficient $\Gamma_{\text{in}}$ of a transmission line with unknown characteristic impedance $Z_{01}$ and is measured using a VNA in a $50\ \Omega$ system but the load terminating the line is unknown. On a $50\ \Omega$ Smith chart the locus of $\Gamma_{\text{in}}$ with respect to frequency is a circle centered at 1.2 on the horizontal axis of the Smith chart with a radius of 0.25. What is $Z_{01}$?

### 4.8.1 Exercises by Section

†challenging

§4.3 1† §4.4 2, 3†, 4, 5, 6, 7, 8 §4.5 9, 10, 11, 12, 13, 14

### 4.8.2 Answers to Selected Exercises

4 $R = 476.0\ \Omega/\text{m}$, $G = 21.11\ \text{mS/m}$,
$L = 381.9\ \text{nH/m}$, $C = 106.1\ \text{pF/m}$

CHAPTER 5

# Passive Components



## 5.1 Introduction

This chapter introduces a wide variety of passive components. It is not possible to be comprehensive, as there is an enormous catalog of microwave elements and scores of variations, and new concepts are introduced every year. At microwave frequencies distributed components can be constructed that have features with particular properties related to coupling, to traveling waves, and to storage of EM energy. Sometimes it is possible to develop lumped-element equivalents of the distributed elements by using the LC ladder model of a transmission line thus realizing lumped-element circuits that would be difficult to imagine otherwise.

## 5.2 $Q$ Factor

RF inductors and capacitors also have loss and parasitic elements. With inductors there is both series resistance and shunt capacitance mainly from interwinding capacitance, while with capacitors there will be shunt resistance and series inductance. A practical inductor or capacitor is limited

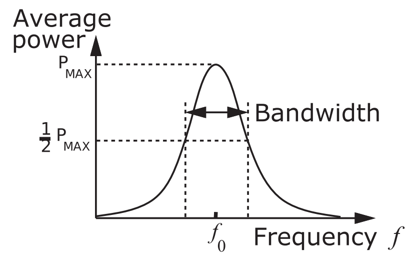



**Figure 5-1:** Transfer characteristic of a resonant circuit. (The transfer function is $V/I$ for the parallel resonant circuit of Figure 5-2(a) and $I/V$ for the series resonant circuit of Figure 5-2(b).)

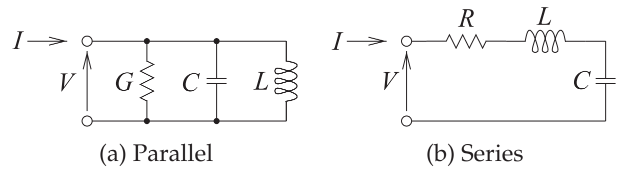



**Figure 5-2:** Second-order resonant circuits.

to operation below the self-resonant frequency determined by the inductance and capacitance itself resonating with its reactive parasitics. The impact of loss is quantified by the $Q$ factor (the quality factor). $Q$ is loosely related to bandwidth in general and the strict relationship is based on the response of a series or parallel connection of a resistor ($R$), an inductor ($L$), and a capacitor ($C$). The response of an $RLC$ network is described by a second-order differential equation with the conclusion that the 3 dB fractional bandwidth of the response (i.e., when the power response is at its half-power level below its peak response) is $1/Q$. (The fractional bandwidth is $\Delta f/f_0$ where $f_0 = f_r$ is the resonant frequency at the center of the band and $\Delta f$ is the 3 dB bandwidth.) This is not true for any network other than a second-order circuit, but as a guiding principle, networks with higher $Q$s will have narrower bandwidths.

### 5.2.1 Definition

The $Q$ factor of a component at frequency $f$ is defined as the ratio of $2\pi f$ times the maximum energy stored to the energy lost per cycle. In a lumped-element resonant circuit, stored energy is transferred between an inductor, which stores magnetic energy, and a capacitor, which stores electric energy, and back again every period. Distributed resonators function the same way, exchanging energy stored in electric and magnetic forms, but with the energy stored spatially. The quality factor is

$$Q = 2\pi f_r \left( \frac{\text{average energy stored in the resonator at } f_r}{\text{power lost in the resonator}} \right), \tag{5.1}$$

where $f_1 = \omega_r/(2\pi)$ is the resonant frequency.
A simple response is shown in Figure 5-1. For a parallel resonant circuit with elements $L$, $C$, and $G = 1/R$ (see Figure 5-2(a)),

$$Q = \omega_r C/G = 1/(\omega_r LG), \tag{5.2}$$

where $f_r = \omega_r/(2\pi)$ is the resonant frequency and is the frequency at which the maximum amount of energy is stored in a resonator. The conductance, $G$, describes the energy lost in a cycle. For a series resonant circuit (Figure 5-2(b)) with $L$, $C$, and $R$ elements,

$$Q = \omega_r L/R = 1/(\omega_r CR). \tag{5.3}$$

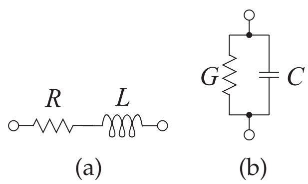



**Figure 5-3:** Loss elements of practical inductors and capacitors: (a) an inductor has a series resistance $R$; and (b) for a capacitor, the dominant loss mechanism is a shunt conductance $G = 1/R$.

These second-order resonant circuits have a bandpass transfer characteristic (see Figure 5-1) with $Q$ being the inverse of the fractional bandwidth of the resonator. The fractional bandwidth, $\Delta f/f_r$, is measured at the half-power points as shown in Figure 5-1. ($\Delta f$ is also referred to as the two-sided $-3$ dB bandwidth.) Then

$$Q = f_r/\Delta f. \tag{5.4}$$

Thus the $Q$ is a measure of the sharpness of the bandpass frequency response. The determination of $Q$ using the measurement of bandwidth together with Equation (5.4) is often not very precise, so another definition that uses the much more sensitive phase change at resonance is preferred when measurements are being used. With $\phi$ being the phase (in radians) of the transfer characteristic, the definition of $Q$ is now

$$Q = \frac{\omega_r}{2}\left|\frac{d\phi}{d\omega}\right|. \tag{5.5}$$

Equation (5.5) is another equivalent definition of $Q$ for parallel $RLC$ or series $RLC$ resonant circuits. It is meaningful to talk about the $Q$ of circuits other than three-element $RLC$ circuits, and then its meaning is always a ratio of the energy stored to the energy dissipated per cycle. The $Q$ of these structures can no longer be determined by bandwidth or by the rate of phase change.

### 5.2.2 $Q$ of Lumped Elements

$Q$ is also used to characterize the loss of lumped inductors and capacitors. Inductors have a series resistance $R$, and the main loss mechanism of a capacitor is a shunt conductance $G$ (see Figure 5-3).

The $Q$ of an inductor at frequency $f = \omega/(2\pi)$ with a series resistance $R$ and inductance $L$ is

$$Q_{\text{INDUCTOR}} = \frac{\omega L}{R}. \tag{5.6}$$

Since $R$ is approximately constant with respect to frequency for an inductor, the $Q$ will vary with frequency.

The $Q$ of a capacitor with a shunt conductance $G$ and capacitance $C$ is

$$Q_{\text{CAPACITOR}} = \frac{\omega C}{G}. \tag{5.7}$$

$G$ is due mainly to relaxation loss mechanisms of the dielectric of a capacitor and so varies linearly with frequency. Thus the $Q$ of a capacitor is almost constant with respect to frequency. For microwave components invariably $Q_{\text{CAPACITOR}} \gg Q_{\text{INDUCTOR}}$, and $Q_{\text{INDUCTOR}}$ is smaller than the $Q$ of transmission line networks. Thus, if the length of a transmission line is not too long, transmission line networks are preferred. If lumped elements must be used, the use of inductors should be minimized.

### 5.2.3 Loaded $Q$ Factor

The $Q$ of a component as defined in the previous section is called the unloaded $Q$, $Q_U$. However if a component is to be measured or used in any way, it is necessary to couple energy in and out of it. The $Q$ is reduced and thus the resonator bandwidth is increased by the power lost to the external circuit so that the loaded $Q$, then the loaded $Q$ is

$$Q_L = 2\pi f_0 \left( \frac{\text{average energy stored in the resonator at} f_0}{\text{power lost in the resonator and to the external circuit}} \right)$$
$$= \frac{1}{1/Q + 1/Q_X}, \tag{5.8}$$

where $Q_X$ is called the external $Q$. $Q_L$ accounts for the power extracted from the resonant circuit.
So a parallel $LCG$ circuit with elements $L_r$, $C_r$, and $G_r$ (at resonance) loaded by a shunt conductance $G_l$ has

$$Q_U = \omega_r C_r / G_r = 1/(\omega_r L_r G_r) \tag{5.9}$$

and

$$Q_L = \omega_r C_r / (G_r + G_l). \tag{5.10}$$

Thus

$$\frac{1}{Q_L} = \frac{1}{Q_U} + \frac{1}{Q_X} \tag{5.11}$$

or

$$Q_X = \left( \frac{1}{Q_L} - \frac{1}{Q_U} \right)^{-1}. \tag{5.12}$$

$Q_X$ is called the external $Q$, and it describes the effect of loading. $Q_L$ is the $Q$ that would actually be measured. $Q_U$ normally needs to be determined, but if the loading is kept very small, $Q_L \approx Q_U$.

### 5.2.4 Summary of the Properties of $Q$

In summary:

(a) $Q$ is properly defined and related to the energy stored in a resonator for a second-order network, one with two reactive elements of opposite types.
(b) $Q$ is not well defined for networks with three or more reactive elements.
(c) However, $Q$ is a frequently used parameter in the design equations for more complex networks than second-order ones.
(d) For complex networks the $Q$ is defined as the ratio of a reactance to a resistance when looking into one end of the network at one frequency. This value of $Q$ should not be used to deduce the bandwidth of the network.
(e) It is only used (as defined or some approximation of it) for guiding the design.

## 5.3 Integrated Lumped Elements

This section considers lumped-elements used in integrated circuits operating at microwave frequencies. Lumped elements such as capacitors, inductors and resistors can rarely be regarded as pure elements at microwave frequencies. Inductors and capacitors have significant loss and all of the elements store energy in both electric and magnetic forms.

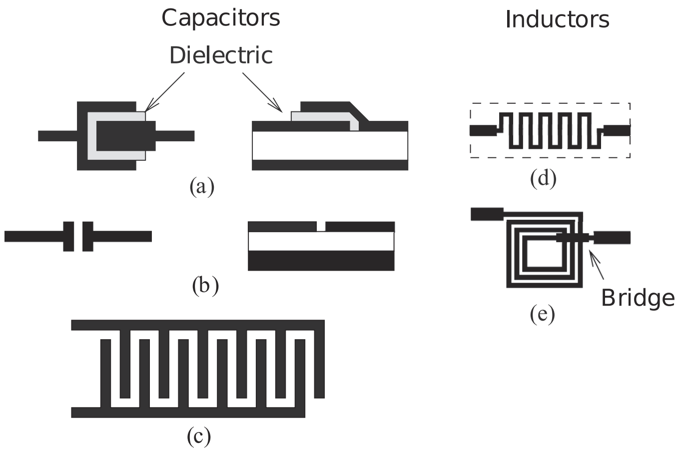


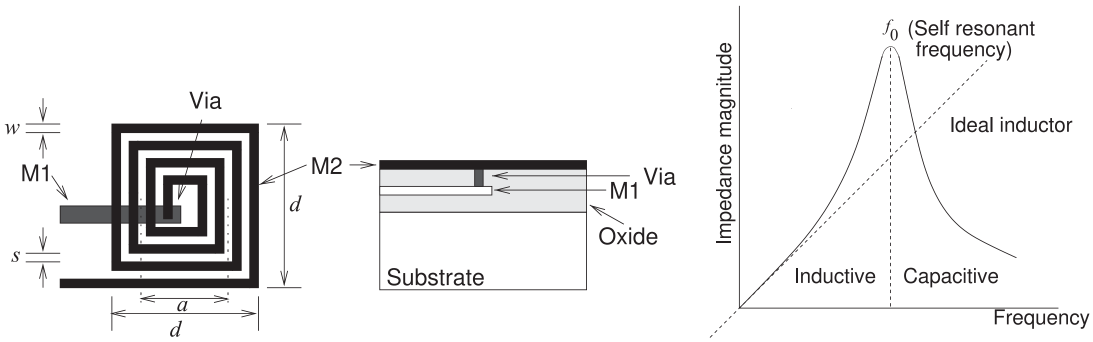

**Figure 5-4:** Monolithic lumped elements: (a) parallel plate capacitor; (b) gap capacitor; (c) interdigitated capacitor; (d) meander line inductor; and (e) spiral inductor.

### 5.3.1 On-Chip Capacitors

There are three primary forms of on-chip capacitor:

(a) Metal-dielectric-metal capacitor—using interconnect metalization.
(b) Metal-dielectric-semiconductor capacitor—essentially a MOS transistor.
(c) Semiconductor junction capacitor—either the capacitance of a reverse-biased $pn$ junction or Schottky barrier.

In silicon technology it is common to refer to the first capacitor type as a **metal-oxide-metal** (**MOM**) capacitor or as a **metal-insulator-metal** (**MIM**) capacitor. A MOM capacitor can be realized as a parallel plate capacitance (see Figure 5-4(a)), and multiple levels of metalization can be used to increase the capacitance density. Relatively low capacitance values of up to 500 $\text{fF}/\mu\text{m}^2$ are typically available.
An alternative MOM capacitance is available using lateral arrangements of conductors on the same layer (see Figure 5-4(b)); that is, adjacent metal structures are separated by a small horizontal gap. Again, there are two distinct metal connections, and a smaller metal separation can be obtained using photolithography than that possible using dielectric separation. However the capacitance density is only increased by a factor of about three. Higher values can be obtained using the **interdigitated capacitor** (**IDC**) of Figure 5-4(c). Both types of MOM capacitance, parallel plate and lateral, are geometrically defined, are voltage independent, have very low temperature coefficients, and have initial fabrication tolerances of 20%–30%.
The second type of capacitor in MOS technology is referred to as a **metal-oxide-semiconductor** (**MOS**) capacitor. MOS capacitors use a MOS transistor with a parallel-plate capacitance between the gate of a MOS transistor and a heavily inverted channel. The drain and source are connected in this configuration and the separation between the conductors is thin, being the gate oxide thickness. This leads to high values of capacitance, although with a weak voltage dependence. Junction capacitance is realized as the capacitance of a reverse-biased semiconductor junction. This capacitance can be quite large, but has a strong voltage dependence. This voltage dependence can be utilized to realize tunable circuits (e.g., a voltage-controlled oscillator).




(a) Plan view (b) Side view (c) Impedance magnitude

**Figure 5-5:** An on-chip spiral inductor.

### *5.3.2 Planar Inductors*

Inductors are important components in RF and microwave circuits. In addition to their role in matching networks, they are used to provide bias to active devices while effectively blocking RF signals from the bias circuitry. Inductors of up to 10 nH can be fabricated in compact form, with the spiral inductor of Figure 5-4(e) being typical. Bond wires can also be used to realize small inductances in the 0.5–1 nH range. One of the advantages of having a portion of a large inductance on-chip is reduced sensitivity to die attach (bondwire, etc.) connections used to connect to an external inductance. Small values of inductance can be realized by the meander line inductor of Figure 5-4(d). This is based on a high-impedance length of line (narrow line in microstrip) appearing inductive.

An on-chip **spiral inductor** is shown in Figure 5-5. An approximate expression for the inductance of this structure was developed by Wheeler [1–3]:

$$L \approx \frac{9.4\mu_0 n^2 a^2}{11d - 7a}, \tag{5.13}$$

where $a$ is the mean radius of the spiral and $n$ is the number of turns. This formula was derived for circular coils, but its accuracy for square spirals is within 5% of values obtained using EM simulation [4]. It is therefore a very useful formula in the early stages of design, but EM analysis is required to obtain the necessary accuracy and frequency dependence of the inductor.

Fields produced by a spiral inductor penetrate the substrate, and as the ground plane is located at a short distance away, the eddy currents on the ground plane reduce the inductance that would otherwise be obtained. The eddy current in the ground conductor rotates in a direction opposite to that of the current on the spiral itself. As a result, the flux of the image inductor in the ground is in the opposite direction to that produced by the spiral itself, with the consequent effect that the effective total inductance is reduced. By creating a broken conductor pattern, the ground inductance is largely eliminated as the eddy currents cannot flow [5].

All inductors have appreciable resistive loss of conductors, and for inductors on semiconductor substrates, loss due to induced substrate currents is

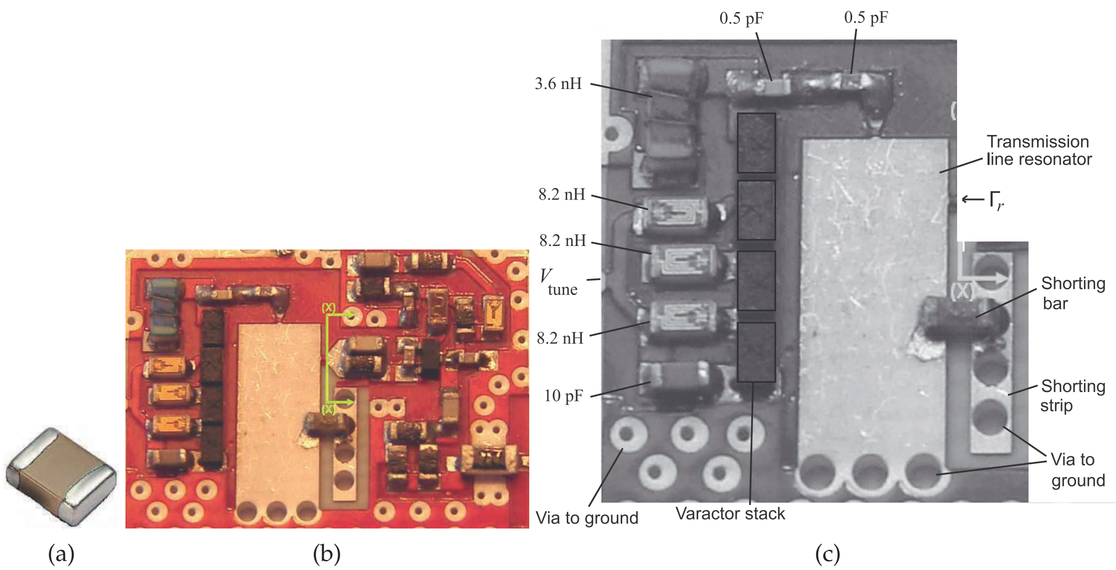



**Figure 5-6:** Circuit board showing the use of surface-mount components: (a) chip resistor or capacitor with metal terminals at the two ends; (b) populated RF microstrip circuit board of a 5 GHz voltage-controlled oscillator [7] (the larger components have dimensions 1.6 mm $\times$ 0.8mm); and (c) identification of several components including a varactor stack with four varactor diodes and a shorting bar that is a 0 $\Omega$ resistor.

important and often dominates. Loss in the substrate is particularly large in silicon substrates due to the finite conductivity of the substrate and the resulting current flow. These induced currents follow a path under the conductors of the spiral and, just as with ground plane eddy currents, lowers the inductance achieved. However, the resistance of the lines is unchanged. Thus on silicon it is difficult to achieve very high $Q$s (the ratio of stored energy to energy dissipated per cycle).

Lumped inductors are based on coils of conductor, and there is parasitic capacitance between the windings [6]. As a result there will be a frequency where the capacitance and inductance resonate at what is called the self-resonant frequency. The impedance of a realistic inductor is shown in Figure 5-5(c). If the practical inductor was purely inductive, then its impedance would increase linearly with frequency. However, because of resonance, the effective inductance increases just before resonance causing the impedance of the practical inductor to increase more rapidly than linearly. This is seen in Figure 5-5(c), and this effect is often used in narrowband microwave circuits.

## 5.4 Surface-Mount Components

The majority of the RF and microwave design effort goes into developing modules and interconnecting modules on circuit boards. With these the most common type of component to use is surface-mount. Figure 5-6(a) shows a two-terminal element, such as a resistor or capacitor, in the form of a surface-mount component. Figure 5-6(b and c) show the use of surface-mount components on a microwave circuit board.

**Table 5-1:** Sizes and designation of two-terminal surface mount components. Note the designation of a surface-mount component refers (approximately) to its dimensions in hundredths of an inch.

| Designation | Size (inch × inch) | Metric designation | Size (mm × mm) |
|---|---|---|---|
| 01005 | $0.016 \times 0.0079$ | 0402 | $0.4 \times 0.2$ |
| 0201 | $0.024 \times 0.012$ | 0603 | $0.6 \times 0.3$ |
| 0402 | $0.039 \times 0.020$ | 1005 | $1.0 \times 0.5$ |
| 0603 | $0.063 \times 0.031$ | 1608 | $1.6 \times 0.8$ |
| 0805 | $0.079 \times 0.049$ | 2012 | $2.0 \times 1.25$ |
| 1008 | $0.098 \times 0.079$ | 2520 | $2.5 \times 2.0$ |
| 1206 | $0.13 \times 0.063$ | 3216 | $3.2 \times 1.6$ |
| 1210 | $0.13 \times 0.098$ | 3225 | $3.2 \times 2.5$ |
| 1806 | $0.18 \times 0.063$ | 4516 | $4.5 \times 1.6$ |
| 1812 | $0.18 \times 0.13$ | 4532 | $4.5 \times 3.2$ |
| 2010 | $0.20 \times 0.098$ | 5025 | $5.0 \times 2.5$ |
| 2512 | $0.25 \times 0.13$ | 6432 | $6.4 \times 3.2$ |
| 2920 | $0.29 \times 0.20$ | – | $7.4 \times 5.1$ |

**Table 5-2:** Parameters of the inductors in Figure 5-7(a). $L_{\rm nom}$ is the nominal inductance, SRF is the self-resonance frequency, $R_{\rm DC}$ is the inductor's series resistance, and $I_{\rm max}$ is the maximum RMS current supported.

| $L_{\rm nom}$ | 900 MHz | | 1.7 GHz | | SRF | $R_{\rm DC}$ | $I_{\rm max}$ |
|---|---|---|---|---|---|---|---|
| (nH) | $L$ (nH) | Q | $L$ (nH) | Q | (GHz) | (Ω) | (mA) |
| 1.0 | 0.98 | 39 | 0.99 | 58 | 16.0 | 0.045 | 1600 |
| 2.0 | 1.98 | 46 | 1.98 | 70 | 12.0 | 0.034 | 1900 |
| 5.1 | 5.12 | 68 | 5.18 | 93 | 5.50 | 0.050 | 1400 |
| 10 | 10.0 | 67 | 10.4 | 85 | 3.95 | 0.092 | 1100 |
| 20 | 20.2 | 67 | 21.6 | 80 | 2.90 | 0.175 | 760 |
| 56 | 59.4 | 54 | 75.4 | 48 | 1.75 | 0.700 | 420 |

A two-terminal surface-mount resistor or capacitor is commonly called a chip resistor or chip capacitor. These can be very small, and the smaller the component often the higher the operating frequency due to reduced parasitic capacitance or inductance. Common sizes of two-terminal chip components are listed in Table 5-1. With a chip resistor or chip capacitor, the parasitic inductance determines the maximum operating frequency with the self-resonant frequency, in the case of a chip capacitor, being when the capacitance resonates with the parasitic inductance. The usable maximum frequency is below the self-resonant frequency.

Figure 5-7(a) shows an inductor in a surface-mount package and details are shown in Figure 5-7(b). This inductor is wound on a dielectric former, which, unfortunately, increases the inductor's parasitic capacitance. The resonance of this capacitance with the inductance establishes the self-resonant frequency (SRF) of the inductor. The inductor is useable as an inductor at a frequency backed-off from the SRF. The parasitic capacitance is reduced if the inductor has an air core, as for the inductors shown in Figure 5-7(c), with details shown in Figure 5-7(d). The air enables the inductor performance to be improved if the size remains the same or the inductor is smaller for comparable performance. The performance of the two types of inductors is listed in Tables 5-2 and 5-3.

## 5.5 Terminations and Attenuators

### 5.5.1 Terminations

Terminations are used to completely absorb a forward-traveling wave and the defining characteristic is that the reflection coefficient of a termination is ideally zero. If a transmission line has a resistive characteristic impedance

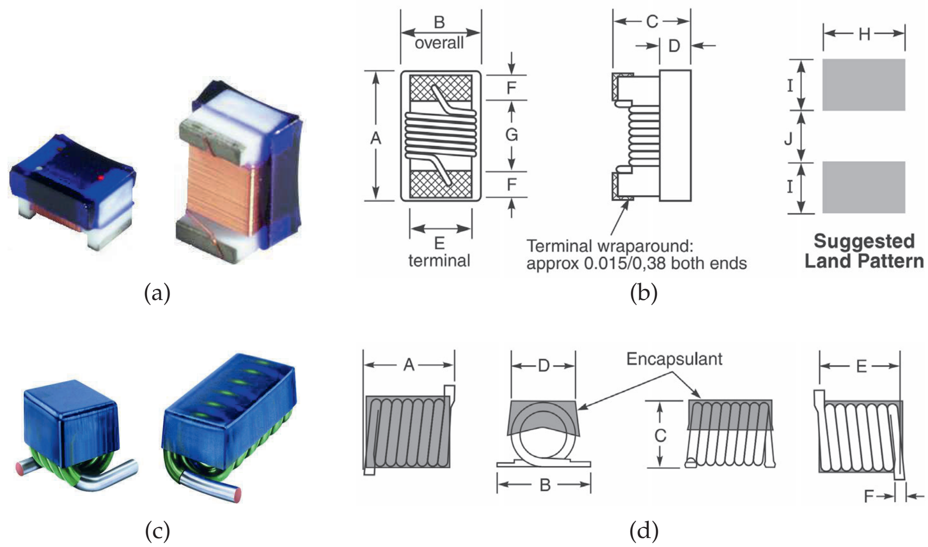



**Figure 5-7:** Chip inductors: (a) inductor in an 0603 surface-mount package; (b) schematic showing sizes and pads to be provided on a circuit board (A=64 mils (1.63 mm), B=33 mils (0.84 mm), C=24 mils (0.61 mm), D=13 mils (0.33 mm), E=30 mils (0.76 mm), F=25 mils (0.64 mm), G=25 mils (0.64 mm), and H=40 mils (1.02 mm)); (c) 0201 surface-mount air-core inductor; and (d) detail (A=23 mils (0.58 mm), B=18 mils (0.46 mm), C=17.7 mils (0.45 mm), D=4 mils (0.1 mm), E=15 mils (0.38 mm), F=9 mils (0.23 mm), G=7 mils (0.18 mm), and H=18 mils (0.46 mm)). Copyright Coilcraft, Inc., used with permission [8].

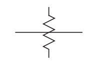
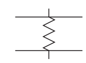
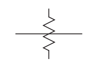
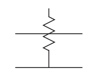
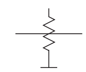
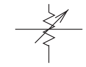
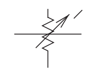
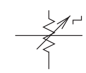
| $L_{\text{nom}}$ | 900 MHz | | 1.7 GHz | | SRF | $R_{\text{DC}}$ | $I_{\text{max}}$ |
|---|---|---|---|---|---|---|---|
| (nH) | $L$ (nH) | Q | $L$ (nH) | Q | (GHz) | (Ω) | (mA) |
| 0.5 | 0.5 | 29 | 0.49 | 43 | 23.5 | 0.020 | 1250 |
| 1.2 | 1.16 | 42 | 1.16 | 60 | 17.9 | 0.042 | 870 |
| 2.3 | 2.28 | 45 | 2.28 | 64 | 16.5 | 0.070 | 670 |
| 5.2 | 5.21 | 36 | 5.21 | 55 | 10.0 | 0.170 | 430 |
| 9.6 | 9.62 | 38 | 9.64 | 53 | 6.2 | 0.400 | 280 |
| 14.0 | 14.13 | 37 | 14.37 | 51 | 5.1 | 0.440 | 270 |

**Table 5-3:** Parameters of the inductors in Figure 5-7(c). $L_{\text{nom}}$ is the nominal inductance, SRF is the self-resonance frequency, $R_{\text{DC}}$ is the inductor's series resistance, and $I_{\text{max}}$ is the maximum RMS current supported.

$R_0 = Z_0$, then terminating the line in a resistance $R_0$ will fully absorb the forward-traveling wave and there will be no reflection. The line is then said to be matched. At RF and microwave frequencies some refinements to this simple circuit connection are required. On a transmission line the energy is contained in the EM fields. For the coaxial line, a simple resistive connection between the inner and outer conductors would not terminate the fields and there would be some reflection. Instead, coaxial line terminations generally comprise a disk of resistive material (see Figure 5-8(a)). The total resistance of the disk from the inner to the outer conductor is the characteristic resistance of the line, however, the resistive material is distributed and so creates a good termination of the fields guided by the coaxial conductors.

Terminations are a problem with microstrip, as the characteristic impedance varies with frequency, is in general complex, and the vias that would be required if a lumped resistor was used has appreciable inductance at

**Table 5-4:** IEEE standard symbols for attenuators [9].

| Component | Symbol | Alternate |
|---|---|---|
| Attenuator, fixed |  | |
| Attenuator, balanced |  |  |
| Attenuator, unbalanced |  |  |
| Attenuator, variable |  | |
| Attenuator, continuously variable |  | |
| Attenuator, stepped variable |  | |

frequencies above a few gigahertz. A high-quality termination is realized using a section of lossy line as shown in Figure 5-8(b). Here lossy material is deposited on top of an open-circuited microstrip line. This increases the loss of the line appreciably without significantly affecting the characteristic impedance of the line. If the length of the lossy line is sufficiently long, say one wavelength, the forward-traveling wave will be totally absorbed and there will be no reflection. Tapering the lossy material, as shown in Figure 5-8(b), reduces the discontinuity between the lossless microstrip line and the lossy line by ensuring that some of the power in the forward-traveling wave is dissipated before the maximum impact of the lossy material occurs. Thus a matched termination is achieved without the use of a via.

### 5.5.2 Attenuators

An attenuator is a two-port network used to reduce the amplitude of a signal and it does this by absorbing power and without distorting the signal. The input and output of the attenuator are both matched, so there are no reflections. An attenuator may be fixed, continuously variable, or discretely variable. The IEEE standard symbols for attenuators are shown in Table 5-4. When the attenuation is fixed, an attenuator is commonly called a **pad**. Resistive pads can be used to minimize the effect of shorts and opens on

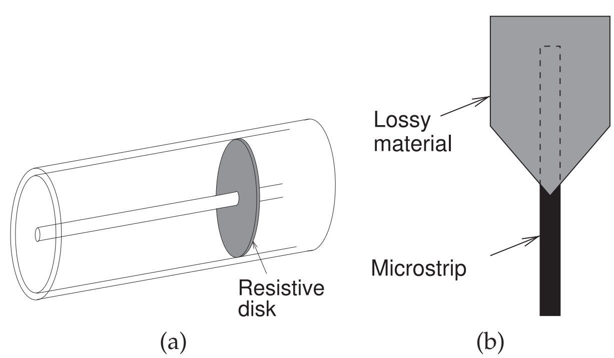
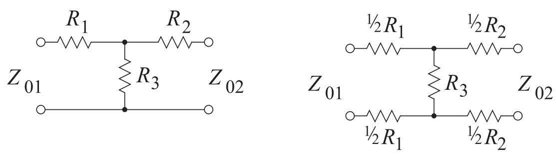



**Figure 5-8:** Terminations: (a) coaxial line resistive termination; (b) microstrip matched load.




(a) Unbalanced (b) Balanced

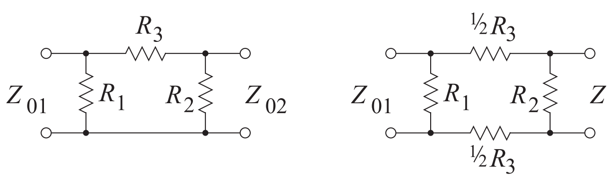
$$R_1 = \frac{Z_{01}(K+1) - 2\sqrt{KZ_{01}Z_{02}}}{K-1} \qquad R_2 = \frac{Z_{02}(K+1) - 2\sqrt{KZ_{01}Z_{02}}}{K-1}$$

$$R_3 = \frac{2\sqrt{KZ_{01}Z_{02}}}{K-1} \tag{5.14}$$

If $Z_{01} = Z_{02} = Z_0$, then

$$R_1 = R_2 = Z_0\left(\frac{\sqrt{K}-1}{\sqrt{K}+1}\right) \quad \text{and} \quad R_3 = \frac{2Z_0\sqrt{K}}{K-1} \tag{5.15}$$

**Figure 5-9:** T (Tee) attenuator. $K$ is the (power) attenuation factor, e.g. a 3 dB attenuator has $K = 10^{3/10} = 1.995$.




(a) Unbalanced (b) Balanced

$$R_1 = \frac{Z_{01}(K-1)\sqrt{Z_{02}}}{(K+1)\sqrt{Z_{02}} - 2\sqrt{KZ_{01}}} \qquad R_2 = \frac{Z_{02}(K-1)\sqrt{Z_{01}}}{(K+1)\sqrt{Z_{01}} - 2\sqrt{KZ_{02}}} \tag{5.16}$$

$$R_3 = \frac{(K-1)}{2}\sqrt{\frac{Z_{01}Z_{02}}{K}} \tag{5.17}$$

If $Z_{01} = Z_{02} = Z_0$, then

$$R_1 = R_2 = Z_0\left(\frac{\sqrt{K}+1}{\sqrt{K}-1}\right) \quad \text{and} \quad R_3 = \frac{Z_0(K-1)}{2\sqrt{K}}. \tag{5.18}$$

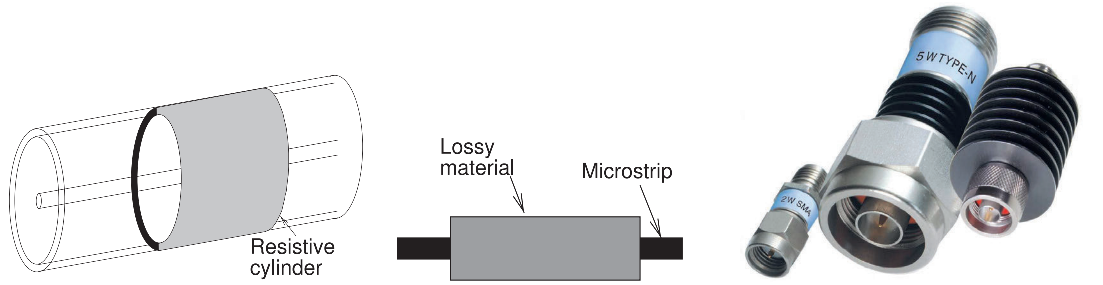
If $R_1 = R_2$, then $Z_{01} = Z_{02} = Z_0$

$$Z_0 = \sqrt{\frac{R_1^2 R_3}{2R_1 + R_3}}, \quad \text{and} \quad K = \left(\frac{R_1 + Z_0}{R_1 - Z_0}\right)^2. \tag{5.19}$$

**Figure 5-10:** Pi (Π) attenuator. $K$ is the (power) attenuation factor, e.g. a 20 dB attenuator has $K = 10^{20/10} = 100$.

the integrity of an RF circuit. An example of attenuator use in this situation is in a cable TV system, where it is critical that the integrity of the system is not compromised by a consumer disconnecting appliances from a cable outlet.

Balanced and unbalanced resistive pads are shown in Figures 5-9 and 5-10 together with their design equations. The attenuators in Figure 5-9 are T or Tee attenuators, where $Z_{01}$ is the system impedance to the left of the pad and $Z_{02}$ is the system impedance to the right of the pad. The defining characteristic is that the reflection coefficient looking into the pad from the left is zero when referred to $Z_{01}$. Similarly, the reflection coefficient looking




(a) Coaxial attenuator (b) Microstrip attenuator (c) Precision fixed coaxial attenuators

**Figure 5-11:** Distributed attenuators. The attenuators in (c) have power handling ratings of 2W, 5W and 20 W (left to right). Copyright 2012 Scientific Components Corporation d/b/a Mini-Circuits, used with permission [11].

from the right of the pad is zero with respect to $Z_{02}$. The attenuation factor is

$$K = \frac{\text{Power in}}{\text{Power out}}. \tag{5.20}$$

In decibels, the attenuation is

$$K|_{\text{dB}} = 10\log_{10} K = (\text{Power in})|_{\text{dBm}} - (\text{Power out})|_{\text{dBm}}. \tag{5.21}$$

If the left and right system impedances are different, then there is a minimum attenuation factor that can be achieved [10]:

$$K_{\text{MIN}} = \begin{cases} \left[2Z_{01} - Z_{02} + 2\sqrt{Z_{01}\left(Z_{01} - Z_{02}\right)}\right]/Z_{02} & , Z_{01} \geq Z_{02} \\ \left[2Z_{02} - Z_{01} + 2\sqrt{Z_{02}\left(Z_{02} - Z_{01}\right)}\right]/Z_{01} & , Z_{01} \leq Z_{02} \end{cases} \tag{5.22}$$

This limitation comes from the simultaneous requirement that the pad be matched. If there is a single system impedance, $Z_0 = Z_{01} = Z_{02}$, then $K_{\text{MIN}} = 1$, and so any value of attenuation can be obtained.

Lumped attenuators are useful up to 10 GHz above which the size of resistive elements becomes large compared to a wavelength. Also, for planar circuits, vias are required, and these are undesirable from a manufacturing standpoint, and electrically they have a small inductance. Fortunately attenuators can be realized using a lossy section of transmission line, as shown in Figure 5-11. Here, lossy material results in a section of line with a high-attenuation constant. Generally the lossy material has little effect on the characteristic impedance of the line, so there is little reflection at the input and output of the attenuator. Distributed attenuators can be used at frequencies higher than lumped-element attenuators can, and they can be realized with any transmission line structure.

Another example of the use of attenuators in combining the output of two sources is shown in Figure 5-12. This is a common situation in measurements where the outputs of two instrumentation sources are to be combined. The attenuators reduce the level of the signal presented to the output of one source by the other. If the level of the second signal is high, most sources would produce nonlinear distortion, including nonlinear mixing products.

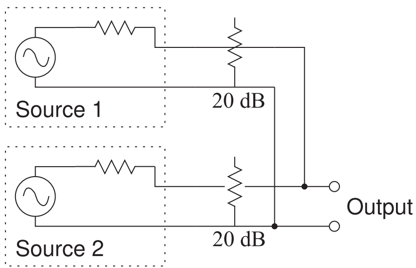



**Figure 5-12:** The use of attenuators to isolate the outputs of two sources that are combined.

**EXAMPLE 5.1** Pad Design

Design an unbalanced 20 dB pad in a 75 Ω system.

**Solution:**

There are two possible designs using resistive pads. These are the unbalanced Tee and Pi pads shown in Figures 5-9 and 5-10. The Tee design will be chosen. The $K$ factor is

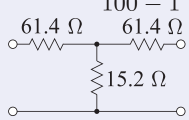
$$K = 10^{(K|_{\text{dB}}/10)} = 10^{(20/10)} = 100. \tag{5.23}$$

Since $Z_{01} = Z_{02} = 75\ \Omega$, Equation (5.15) yields

$$R_1 = R_2 = 75\left(\frac{\sqrt{100}-1}{\sqrt{100}+1}\right) = 75\left(\frac{9}{11}\right) = 61.4\ \Omega \tag{5.24}$$

$$R_3 = \frac{2\cdot 75\sqrt{100}}{100-1} = 150\left(\frac{10}{99}\right) = 15.2\ \Omega. \tag{5.25}$$

The final design is




## 5.6 Transmission Line Stubs and Discontinuities

Interruptions of the magnetic or electric field create regions where additional magnetic energy or electric energy is stored. If the additional energy stored is predominantly magnetic, the discontinuity will introduce an inductance. If the additional energy stored is predominantly electric, the discontinuity will introduce a capacitance. Such discontinuities occur with all transmission lines. In some cases transmission line discontinuities introduce undesired parasitics, but they also provide an opportunity to effectively introduce lumped-element components. In this section microstrip discontinuities will be considered, but the principles apply to all transmission line structures.

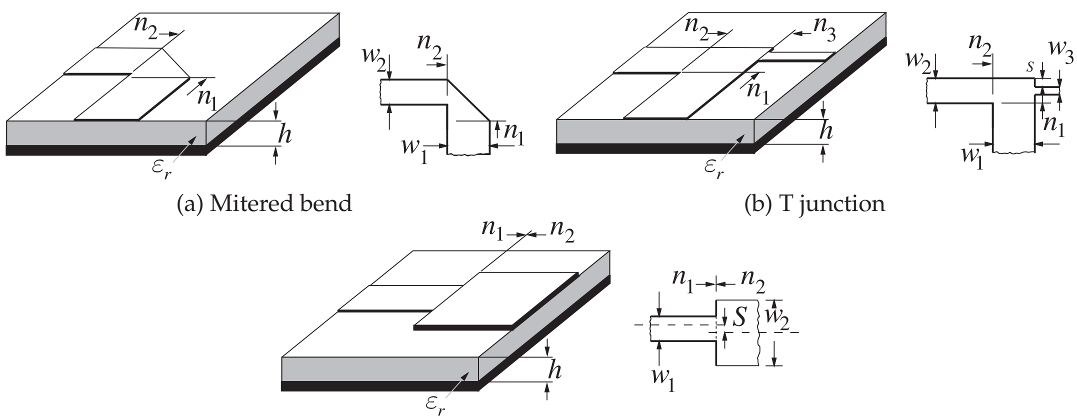
The simplest microwave circuit element is a uniform section of transmission line that can be used to introduce a time delay or frequency-dependent phase shift. More commonly it is used to interconnect other components. Line segments including bends and junctions are shown in Figure 5-13.

### *5.6.1 Open*

Many transmission line discontinuities arise from fringing fields. One element is the microstrip open, shown in Figure 5-14. The fringing fields at the end of the transmission line in Figure 5-14(a) store energy in the electric field, and this can be modeled by the fringing capacitance, $C_F$, shown in Figure 5-14(b). This effect can also be modeled by an extended transmission line, as shown in Figure 5-14(c). For a typical microstrip line with $\varepsilon_r$ = 9.6, $h$ = 600 μm, and $w/h$ = 1, $C_F$ is approximately 36 fF. However, $C_F$ varies with frequency, and the extended length is a much better approximation to the effect of fringing [12]. For the same dimensions, the length of the extension is approximately $0.35h$ and almost independent of frequency [13]. As with many fringing effects, a capacitance or inductance can be used to model the effect of fringing, but generally a distributed model is better.

### 5.6.2 Discontinuities

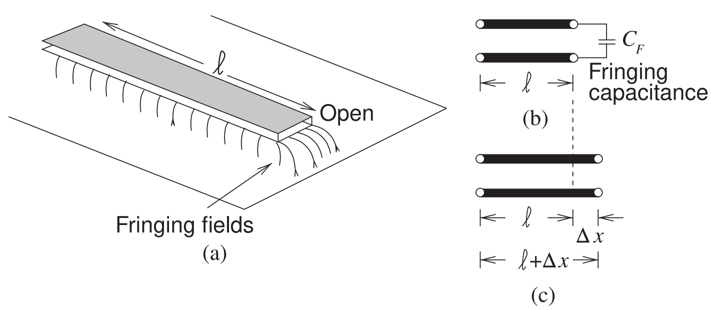
Several microstrip discontinuities and their equivalent circuits are shown in Figure 5-15. Discontinuities (Figure 5-15(b–g)) are modeled by capacitive elements if the $E$ field is affected and by inductive elements if the $H$ field (or current) is disturbed. The stub shown in Figure 5-15(b), for example, is best modeled using lumped elements describing the junction as well as the transmission line of the stub itself. Current bunches at the right angle bends from the through line to the stub. The current bunching leads to excess energy being stored in the magnetic field, and hence an inductive effect. There is also excess charge storage in the parallel plate region bounded by the left- and right-hand through lines and the stub. This is modeled by a capacitance.




(c) Impedance step

**Figure 5-13:** Microstrip discontinuities.

**Figure 5-14:** An open on a microstrip transmission line: (a) microstrip line showing fringing fields at the open; (b) fringing capacitance model of the open; and (c) an extended line model of the open with $\Delta x$ being the extra transmission line length that captures the open.



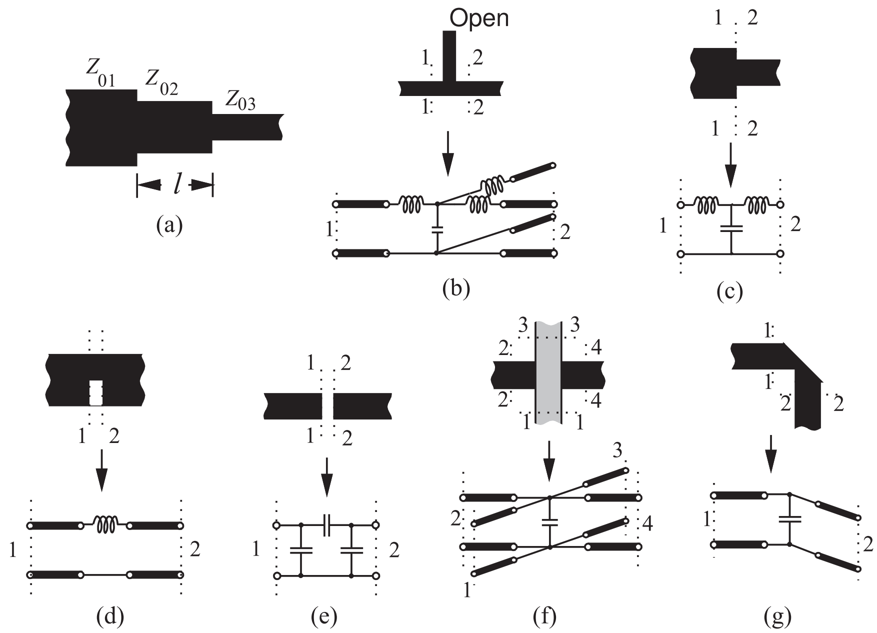



**Figure 5-15:** Microstrip discontinuities: (a) quarter-wave impedance transformer; (b) open microstrip stub; (c) step; (d) notch; (e) gap; (f) crossover; and (g) bend.

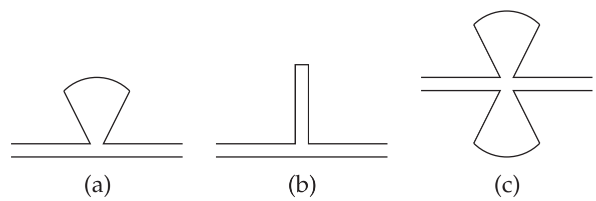



**Figure 5-16:** Microstrip stubs: (a) radial shunt-connected stub; (b) conventional shunt stub; and (c) butterfly radial stub.

### 5.6.3 Impedance Transformer

Impedance transformers interface two lines of different characteristic impedance. The smoothest transition and the one with the broadest bandwidth is a tapered line. This element can be long and then a quarter-wave impedance transformer (see Figure 5-15(a)) is sometimes used, although its bandwidth is relatively small and centered on the frequency at which $l = \lambda_g/4$. Ideally $Z_{0,2} = \sqrt{Z_{0,1}Z_{0,3}}$. This topic is considered further in Section 7.5 where the design of tapered lines and multi-stage quarter-wave transformers are considered in detail.

### 5.6.4 Planar Radial Stub

The use of a radial stub (Figure 5-16(a)), as opposed to the conventional microstrip stub (Figure 5-16(b)), can improve the bandwidth of many microstrip circuits. A major advantage of a radial stub is that the input impedance presented to the through line generally has broader bandwidth than that obtained with the conventional stub. When two shunt-connected radial stubs are introduced in parallel (i.e., one on each side of the microstrip feeder line) the resulting configuration is termed a "butterfly" stub (see Figure 5-16(c)). Critical design parameters include the radius, $r$, and the angle of the stub.

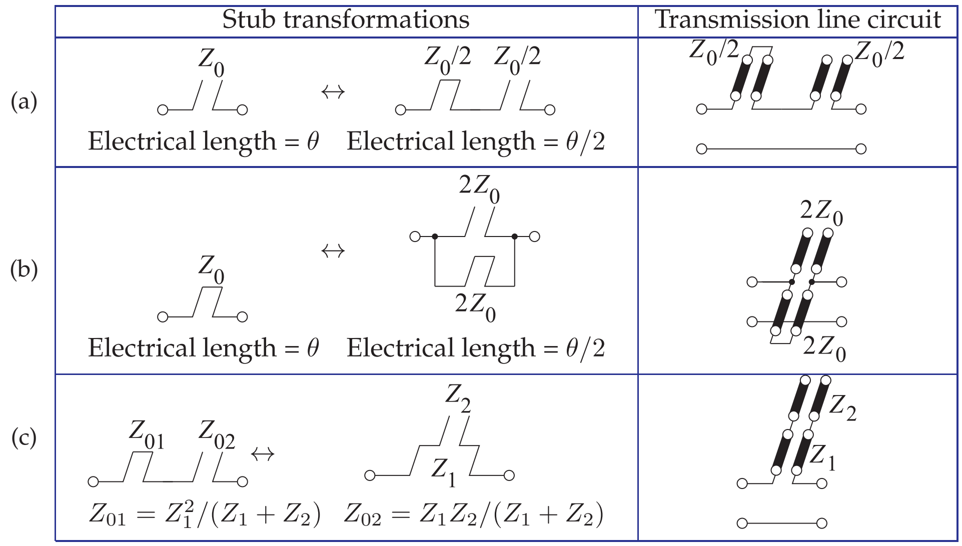



**Figure 5-17:** Stub network transformations: (a) open- and (b) short-circuited stubs after application of the half-angle transform [14]; and (c) equivalence between series connection of open- and short-circuited stubs and stepped impedance transmission line.

### 5.6.5 Stub Transformations

The stub transformations in Figure 5-17 enable the lengths of stubs to be reduced. The first two stub transformations (Figure 5-17(a and b)) are called half-angle transformations and enable a stub to be replaced by two shorter stubs. The third stub transformation (Figure 5-17(c)) enables two stubs to be replaced by two cascaded transmission lines.

Figure 5-17(a) is a half-angle transform of a series open-circuited stub. Mathematically the transform is described as follows. Consider the input impedance of an open-circuited stub [14] with electrical length $\theta = \beta\ell$ at frequency $f$ and which is a quarter-wavelength long at $f_0$ (i.e., $\theta = \pi/2$ at $f_0$) (from Section 2.4.4 of [15] and using Equation ((1.105)) of [15]):

$$Z_{oc} = -\jmath Z_0 \cot(\theta) = \jmath (Z_0/2) \tan(\theta/2) - \jmath (Z_0/2) / \tan(\theta/2). \tag{5.26}$$

In general, $\theta = (\pi/2)(f/f_0)$. The right-hand side of Equation (5.26) describes the series connection of short- and open-circuited stubs having characteristic impedances of $Z_0/2$ and half the original electrical length. This implies that the resulting transmission line resonators are one-quarter wavelength long at $2f_0$ (i.e., they are one-eighth wavelength long at $f_0$). The half-angle transformation applies at all frequencies and not just frequencies near $f_0$.

Through a similar analytical treatment, the short-circuited stub has the equivalence shown in Figure 5-17(b). The series short-circuited stub on the left in Figure 5-17(b) has the series admittance

$$Y_T = \frac{1}{\jmath Z_0 \tan(\theta)} = \frac{1}{\jmath Z_0} \cot(\theta) \tag{5.27}$$

and the stubs in the transformation have an input impedance

$$Z_{oc} = -\jmath\tfrac{1}{2}Z_0\cot(\theta/2) \quad \text{and} \quad Z_{sc} = \jmath\tfrac{1}{2}Z_0\tan(\theta/2). \tag{5.28}$$

These stubs are in parallel so that the total input admittance is

$$\begin{aligned} Y_T &= \frac{1}{Z_{oc}} + \frac{1}{Z_{sc}} = \frac{2}{-\jmath Z_0\cot(\theta/2)} + \frac{1}{\jmath Z_0\tan(\theta/2)} \\ &= \frac{2}{\jmath Z_0}\left[-\tan(\theta/2) + \frac{1}{c}ot(\theta/2)\right] = \frac{1}{\jmath Z_0}\cot(\theta). \end{aligned} \tag{5.29}$$

Examining Equations (5.27) and (5.29) it is seen that they are identical, thus Figure 5-17(b) represents another half-angle transformation.

For the series open- and short-circuited stubs in Figure 5-17(c), the input impedances of the open- and short-circuited stubs are

$$Z_{oc} = -\jmath Z_{02}\cot(\theta) \quad \text{and} \quad Z_{sc} = \jmath Z_{01}\tan(\theta), \tag{5.30}$$

respectively, so that the total series impedance is

$$Z_T = Z_{oc} + Z_{sc} = -\jmath Z_{02}\cot(\theta) + \jmath Z_{01}\tan(\theta). \tag{5.31}$$

For the transformation on the right, the impedance looking into the open circuit (outer) stub is

$$Z_x = -\jmath Z_2\cot(\theta) \tag{5.32}$$

and the total impedance is

$$Z_T = Z_1\left[\frac{-\jmath Z_2\cot(\theta) + \jmath Z_1\tan(\theta)}{Z_1 + Z_2\cot(\theta)\tan(\theta)}\right] = \frac{-\jmath Z_1 Z_2\cot(\theta)}{Z_1 + Z_2} + \frac{\jmath Z_1^2\tan(\theta)}{Z_1 + Z_2}. \tag{5.33}$$

Equating Equations (5.31) and (5.33) and collecting terms,

$$Z_{02} = Z_1Z_2/(Z_1 + Z_2) \quad \text{and} \quad Z_{01} = Z_1^2/(Z_1 + Z_2), \tag{5.34}$$

and the transmission line segments (of characteristic impedance $Z_{01}$, $Z_{02}$, $Z_1$ and $Z_2$) all have the same electrical length.

In summary, the stub transformations hold no matter what the lengths of the stubs. The half-angle stub transformations enable transmission line circuits to be miniaturized.

## 5.7 Resonators

Near resonance, the response of a microwave resonator is very similar to the resonance response of a parallel or series $LC$ resonant circuit, shown in Figure 5-18(e and f). These equivalent circuits can be used over a narrow frequency range of perhaps 5% fractional bandwidth.

Several types of resonators are shown in Figure 5-18. Figure 5-18(a) is a rectangular cavity resonator coupled to an external coaxial line by a small coupling loop. Figure 5-18(b) is a microstrip patch reflection resonator. This resonator has large coupling to the external circuit. The coupling can be reduced and photolithographically controlled by introducing a gap, as shown in Figure 5-18(c), to create what is called a microstrip gap-coupled transmission line reflection resonator. The $Q$ of a resonator can be dramatically increased by using a low-loss, high-dielectric constant material, as shown in Figure 5-18(d), for a dielectric transmission resonator in microstrip.

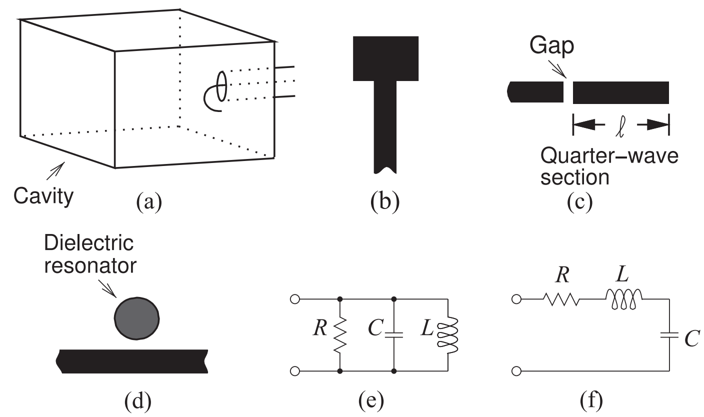



**Figure 5-18:** Microwave resonators: (a) rectangular cavity resonator, (b) microstrip patch resonator (c) microstrip gap-coupled reflection resonator, (d) dielectric transmission resonator in microstrip, (e) parallel equivalent circuits, and (f) series equivalent circuits.

### 5.7.1 Dielectric Resonators

Any dielectric structure can store EM energy, with the resonant frequency dependent on both the permittivity and the physical dimensions. At the resonance frequency, all resonators store the maximum amount of energy. Two resonators that store particularly large amounts of energy are the cavity resonator shown in Figure 5-18(a) and the dielectric resonator shown in Figure 5-18(d).

Microwave ceramics can have very low loss and high permittivity (which results in small wavelengths and hence small size) and are commonly used in microwave resonators. Low-loss ceramic materials with permittivities in the range of 21 to 150 are generally available and unloaded $Q$ factors are in the 5,000 to 10,000 range. Physical dimensions generally depend upon the resonant frequencies desired.

The general theoretical expression for the resonant frequencies, applying to a cylindrical dielectric resonator of radius $a$ and height $d$, is

$$f_{mnl} = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{\chi_{mn}}{\pi a}\right)^2 + \left(\frac{l}{d}\right)^2}, \tag{5.35}$$

where the integer $l$ denotes the number of half wavelengths in the vertical direction and $\chi_{mn}$ is the $m$th extremum of the Bessel function $J_n$ for a TM mode (or alternatively the $m$th zero for a TE mode).

Figure 5-18(d) shows a common use of a dielectric resonator, often called a **puck**, coupling to the fields of a microstrip line. The puck has a resonance defined by the radius of the high permittivity puck. There can be several modes of resonance, with some modes introducing a parallel resonant circuit (Figure 5-18(e)) in shunt across the line and others introducing a series resonant circuit in shunt across the line (Figure 5-18(f)).

## 5.8 Magnetic Transformer

In this section the use of magnetic transformers in microwave circuits will be discussed. Magnetic transformers can be used directly up to a few hundred megahertz or so, but the same transforming properties can be achieved using coupled transmission lines.

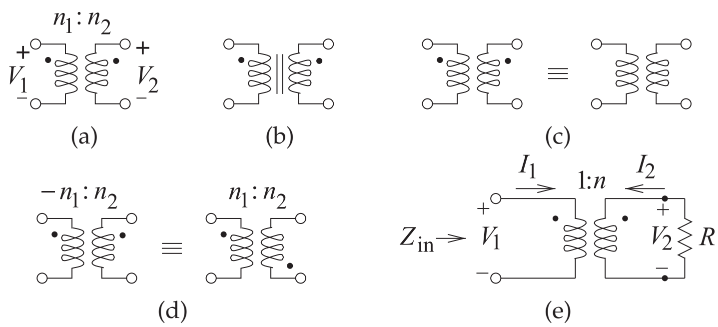



**Figure 5-19:** Magnetic transformers: (a) a transformer as two magnetically coupled windings with $n_1$ windings on the primary (on the left) and $n_2$ windings on the secondary (on the right) (the dots indicate magnetic polarity so that the voltages $V_1$ and $V_2$ have the same sign); (b) a magnetic transformer with a magnetic core; (c) identical representations of a magnetic transformer with the magnetic polarity implied for the transformer on the right; (d) two equivalent representations of a transformer having opposite magnetic polarities (an **inverting transformer**); and (e) a magnetic transformer circuit.

### 5.8.1 Properties of a Magnetic Transformer

A magnetic transformer (see Figure 5-19) magnetically couples the current in one wire to current in another. The effect is amplified using coils of wires and using a core of magnetic material (material with high permeability) to create greater magnetic flux density. When coils are used, the symbol shown in Figure 5-19(a) is used, with one of the windings called the primary winding and the other called the secondary winding. If there is a **magnetic core** around which the coils are wound, then the symbol shown in Figure 5-19(b) is used, with the vertical lines indicating the core. However, even if there is a core, the simpler transformer symbol in Figure 5-19(a) is more commonly used. Magnetic cores are useful up to several hundred megahertz and rely on the alignment of magnetic dipoles in the core material. Above a few hundred megahertz the magnetic dipoles cannot react quickly enough and so the core looks like an open circuit to magnetic flux. Thus the core is not useful for magnetically coupling signals above a few hundred megahertz. As mentioned, the dots above the coils in Figure 5-19(a) indicate the polarity of the magnetic flux with respect to the currents in the coils so that, as shown, $V_1$ and $V_2$ will have the same sign. Even if the magnetic polarity is not specifically shown, it is implied (see Figure 5-19(c)). There are two ways of showing inversion of the magnetic polarity, as shown in Figure 5-19(d), where a negative number of windings indicates opposite magnetic polarity.
The interest in using magnetic transformers in high-frequency circuit design is that configurations of magnetic transformers can be realized using coupled transmission lines to extend operation to hundreds of gigahertz. The transformer is easy to conceptualize, so it is convenient to first develop a circuit using the transformer and then translate it to transmission line form. That is, in "back-of-the-envelope" microwave design, transformers can be used to indicate coupling, with the details of the coupling left until later when the electrical design is translated into a physical design. Restrictions must be followed as not all transformer configurations can be translated this

way. Transformer characteristics are developed below.

The following notation is used with a magnetic transformer:

$L_1, L_2$: the self-inductances of the two coils
$M$: the mutual inductance
$k$: the coupling factor,

$$k = \frac{M}{\sqrt{L_1 L_2}}. \tag{5.36}$$

Referring to Figure 5-19(e), the voltage transformer ratio is

$$V_2 = nV_1, \tag{5.37}$$

where $n$ is the ratio of the number of secondary to primary windings. An ideal transformer has "perfect coupling," indicated by $k = 1$, and the self-inductances are proportional to the square of the number of windings, so

$$\frac{V_2}{V_1} = \sqrt{\frac{L_2}{L_1}}. \tag{5.38}$$

The general equation relating the currents of the circuit in Figure 5-19(e) is

$$RI_2 + \jmath\omega L_2 I_2 + \jmath\omega M I_1 = 0, \tag{5.39}$$

and so

$$\frac{I_1}{I_2} = -\frac{R + \jmath\omega L_2}{\jmath\omega M}. \tag{5.40}$$

If $R \ll \omega L_2$, then the current transformer ratio is

$$\frac{I_1}{I_2} \approx -\frac{L_2}{M} = -\sqrt{\frac{L_2}{L_1}}. \tag{5.41}$$

Notice that combining Equations (5.38) and (5.41) leads to calculation of the transforming effect on impedance. On the Coil 1 side, the input impedance is (referring to Figure 5-19(e))

$$Z_{\text{in}} = \frac{V_1}{I_1} = -\frac{V_2}{I_2}\left(\frac{L_1}{L_2}\right) = R\frac{L_1}{L_2}. \tag{5.42}$$

In practice, however, there is always some magnetic field leakage—not all of the magnetic field created by the current in Coil 1 goes through (or links) Coil 2—and so $k < 1$. Then from Equations (5.38)–(5.42),

$$V_1 = \jmath\omega L_1 I_1 + \jmath\omega M I_2 \tag{5.43}$$
$$0 = RI_2 + \jmath\omega L_2 I_2 + \jmath\omega M I_1. \tag{5.44}$$

Again, assuming that $R \ll \omega L_2$, a modified expression for the input impedance is obtained that accounts for nonideal coupling:

$$Z_{\text{in}} = R\frac{L_1}{L_2} + \jmath\omega L_1(1 - k^2). \tag{5.45}$$

Imperfect coupling, $k < 1$, causes the input impedance to be reactive and this limits the bandwidth of the transformer. Stray capacitance is another factor that impacts the bandwidth of the transformer.

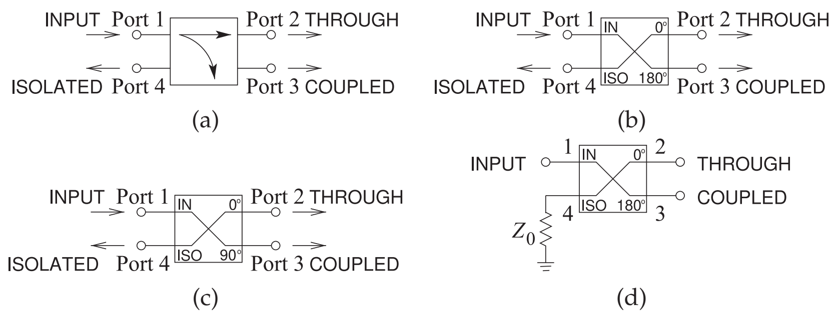



**Figure 5-20:** Commonly used symbols for hybrids: (a) through and coupled ports; (b) 180° hybrid; (c) 90° hybrid; and (d) hybrid with the isolated port terminated in a matched load.

## 5.9 Hybrids

Hybrids are transformers that combine or divide microwave signals among a number of inputs and outputs. They can be implemented using distributed or lumped elements and are not restricted to RF and microwave applications. For example, a magnetic transformer is a hybrid and can be used to combine the outputs of two or more transistor stages, or to obtain appropriate impedance levels. A 3 dB directional coupler based on parallel coupled lines is also a type of hybrid. Specific implementations will be considered later in this chapter, but for now the focus is on their idealized characteristics. The symbols commonly used for hybrids are shown in Figure 5-20.

A hybrid is a special type of four-port junction with the property that if a signal is applied at any port, it emerges from two of the other ports at half power, while there is no signal at the fourth or isolated port. The two outputs have specific phase relationships and all ports are matched. Only two fundamental types of hybrids are used: 180° and 90° hybrids. A hybrid is a type of directional coupler, although the term directional coupler, or just coupler, is most commonly used to refer to devices where only a small fraction of the power of an input signal is sampled. Also, a balun is a special type of hybrid.

### 5.9.1 Quadrature Hybrid

The ideal **90° hybrid**, or **quadrature hybrid**, shown in Figure 5-20(c), has the scattering parameters

$$S_{90^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -\jmath & 1 & 0 \\ -\jmath & 0 & 0 & 1 \\ 1 & 0 & 0 & -\jmath \\ 0 & 1 & -\jmath & 0 \end{bmatrix}. \tag{5.46}$$

The $90^\circ$ phase difference between the through and coupled ports is indicated by $-\jmath$. The actual phase shift, that is, $+90^\circ$ or $-90^\circ$ (indicated by $\jmath$), between the input and output ports depends on the specific hybrid implementation. Another quadrature hybrid could have the parameters

$$S_{90^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \jmath & 1 & 0 \\ \jmath & 0 & 0 & 1 \\ 1 & 0 & 0 & \jmath \\ 0 & 1 & \jmath & 0 \end{bmatrix}. \tag{5.47}$$

Recall the requirement of quadrature modulators that the local oscillator be supplied as two components equal in magnitude but $90^\circ$ out of phase. A quadrature hybrid is just the circuit that can do this.

To convince yourself that Equation (5.47) describes a network that splits the power, consider the power flow implied by Equation (5.47). The fraction of power transmitted from Port $i$ to Port $j$ is described by $|S_{ji}|^2$. Consider the power that enters Port 1. No power is reflected for an ideal hybrid, as the input at Port 1 is matched and $S_{11} = 0$. Port 4 should be isolated so no power will come out of Port 4, and so $S_{41} = 0$. The power should be split between Ports 2 and 3, and these should be equal to half the power entering Port 1. From Equation (5.47),

$$|S_{21}|^2 = \left(\frac{1}{\sqrt{2}}|\jmath|\right)^2 = \tfrac{1}{2} \quad \text{and} \quad |S_{31}|^2 = \left(\frac{1}{\sqrt{2}}|1|\right)^2 = \tfrac{1}{2}. \tag{5.48}$$

Thus the power entering Port 1 is split, with half going to Port 2 and half to Port 3.

### 5.9.2 180° Hybrid

The scattering parameters of the **180° hybrid**, shown in Figure 5-20(b), are

$$S_{180^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \tag{5.49}$$

and this defines the operation of the hybrid. In terms of the root power waves $a$ and $b$, the outputs at the ports are

$$\begin{array}{ll} b_1 = (a_2 - a_3)/\sqrt{2} & b_2 = (a_1 + a_4)/\sqrt{2} \\ b_3 = (-a_1 + a_4)/\sqrt{2} & b_4 = (a_2 + a_3)/\sqrt{2} \end{array}. \tag{5.50}$$

Imperfections in fabricating the hybrid will result in nonzero scattering parameters at the ports where ideally $S_{ji} = 0$, so that it is best to terminate

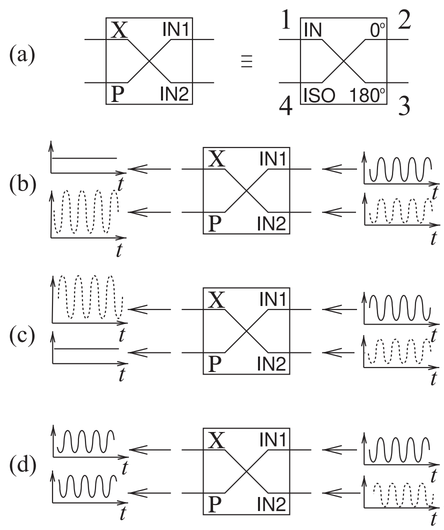



**Figure 5-21:** A $180^\circ$ hybrid as a combiner (with output at the sum port, $\Sigma$) and as a comparator (with output at the difference port, $\triangle$): (a) equivalence between a $180^\circ$ hybrid and a sum-and-difference hybrid; (b) the two inputs are in phase; (c) the two inputs are $180^\circ$ out of phase; and (d) the two inputs are $90^\circ$ out of phase.

the isolated port of the hybrid, as shown in Figure 5-20(d), to ensure that there is no reflected signal (i.e., no $a_4$) entering the hybrid at the isolated port. Equation (5.50) shows that ports are interchangeable. That is, if a signal is applied to Port 3, then it is split between Ports 1 and 4, and Port 2 becomes the isolated port.

A hybrid can be used in applications other than splitting the input signal into a through and a coupled component. An example of another use is in a system that combines or compares two signals, as in Figure 5-21. Here a $180^\circ$ hybrid with the scattering parameters of Equation (5.49) is shown. With a signal $x(t)$ applied to Port 2 in Figure 5-20(b), and another signal $y(t)$ applied to Port 3, the output at Port 4 is the sum signal, $x(t) + y(t)$, and the output at Port 1 is the difference signal, $x(t) - y(t)$. More common names for the output ports (when the $180^\circ$ hybrid is used as a combiner) are to call Port 4 the sigma or sum port, often designated using the symbol $\Sigma$. Port 1 is called the difference or delta port, $\triangle$. Notice that if a signal is applied to the $\triangle$ port it will generate out-of-phase outputs.

## 5.9.3 Magnetic Transformer Hybrid

In the hybrid circuits of Figures 5-22 and 5-23, the number of windings of each coil (there are three in each structure) is the same. The impedance levels given are those required for maximum power transfer and indicate the impedance transformations of the structures. Considering Figure 5-23 and equating powers,

$$\tfrac{1}{2}\frac{(2V)^2}{Z_2} = \tfrac{1}{2}\frac{V^2}{Z_0}, \quad \frac{4}{Z_2} = \frac{1}{Z_0}, \quad \text{and so} \quad Z_2 = 4Z_0. \tag{5.51}$$

That is, if $Z_2 = 4Z_0$, the impedance seen looking into Port 4 (of the transformer in Figure 5-23) is $Z_0$, and so the transformer has provided matching to the large load. For example, if $Z_2 = 200\ \Omega$ (perhaps the input of a transistor), the transformer matches this to $50\ \Omega$.

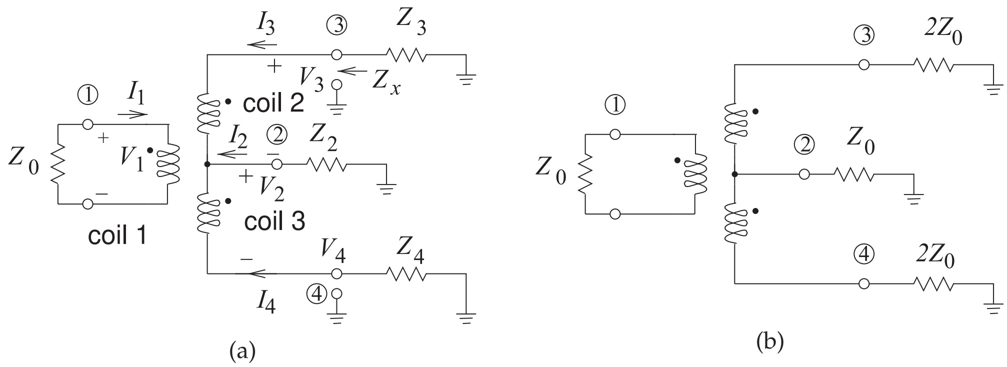



**Figure 5-22:** Magnetic transformer hybrid with each coil having the same number of windings: (a) showing general loading impedance; and (b) optimum loading to function as a $180°$ hybrid with Port 2 (terminal 2 to ground) isolated and the balanced input at Port 1. For example, an antenna could be attached to Port 1. A common flux passes through the three coils.

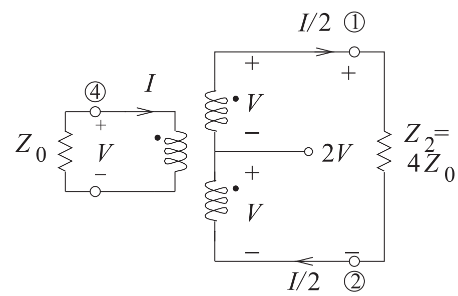



**Figure 5-23:** The magnetic transformer of Figure 5-22 as an impedance transforming balun.

## EXAMPLE 5.2 Magnetic Transformer Hybrid

Consider the magnetic transformer hybrid of Figure 5-22(a) with $Z_0$ real. Determine what type of hybrid this is and calculate the impedance transformations. Assume ideal coupling $(k = 1)$.

**Solution:**

Since the coupling is ideal and each coil has the same number of windings,

$$(V_3 - V_2) = V_1 \quad \text{and} \quad (V_2 - V_4) = V_1. \tag{5.52}$$

The current levels in the transformer depend on the attached circuitry. For this circuit to function as a hybrid with Port 2 isolated, the current $I_2$ must be zero so that $I_3$ and $I_4$ are equal in magnitude but $180°$ out of phase. The loading at Ports 3 and 4 must be the same. Now $V_2 = 0$, since $I_2 = 0$, and so Equation (5.52) become

$$V_3 = V_1 \quad \text{and} \quad V_4 = -V_1, \tag{5.53}$$

so this circuit is a $180°$ hybrid.

To determine loading conditions at Ports 2, 3, and 4 the following transformer equation is used:

$$V_3 = \jmath\omega L_2 I_3 + \jmath\omega M I_1 - \jmath\omega M I_4, \tag{5.54}$$

where $L_2$ is the inductance of Coil 2 and $M$ is the mutual inductance. Now $I_1 = -V_1/Z_0$, $L_2 = L_3 = M$ since coupling is ideal, and $V_3 = V_1$. So Equation (5.54) becomes

$$V_3 = \jmath\omega M(2I_3 - V_3/Z_0). \quad \text{That is,} \quad V_3(1 + \jmath\omega M/Z_0) = \jmath 2\omega M I_3. \tag{5.55}$$

If $\jmath\omega M \gg |Z_0|$, this reduces to $\qquad Z_x = V_3/I_3 = 2Z_0. \qquad (5.56)$

For maximum power transfer $Z_3 = Z_x^*$, but since $Z_0$ is real, $Z_3 = Z_x = 2Z_0$. From symmetry, $Z_4 = Z_3$. Also, $I_3 = -I_4 = I_1/2$.

(The above result can also be obtained by considering only maximum power transfer considerations. The argument is as follows. An impedance $Z_0$ is attached to Coil 1. Maximum power transfer to the transformer through the coil requires that the input impedance be $Z_0$ (since it is real). In the ideal hybrid operation the power is split evenly between the powers delivered to the loads at Ports 3 and 4, since $V_1 = (V_3 - V_2) = (V_2 - V_4)$ and the power delivered to Coil 1 is $V_1^2/(2Z_0)$. The power delivered to $Z_3$ (and $Z_4$) is $(V_3 - V_2)^2/(2Z_3) = V_1^2/(2Z_3) = V_1^2/(4Z_0)$. That is, $Z_3 = 2Z_0 = Z_4$. It is a small extrapolation to say that $Z_3 = 2Z_0^* = Z_4$.)

The problem is not yet finished, as $Z_2$ must be determined. For hybrid operation, a signal applied to Port 2 should not have a response at Port 1. So the current at Port 2, $I_2$, should be split between Coils 2 and 3 so that $I_3 = -I_2/2 = I_4$. Thus

$$V_4 = -I_4(2Z_0) = (I_2/2)(2Z_0) = I_2 Z_0 = V_3. \tag{5.57}$$

Now $\qquad V_3 - V_2 = V_1 = 0 = V_2 - V_4, \quad \text{and so} \quad V_2 = V_4 = I_2 Z_0. \qquad (5.58)$

That is, $\qquad Z_2 = V_2/I_2 = Z_0. \qquad (5.59)$

The final $180^\circ$ hybrid circuit is shown in Figure 5-22(b) with the loading conditions for matched operation as a hybrid.

In the example above it is seen that the number of windings in the coils are the same so that the current in Coils 2 and 3 (of the transformer in Figure 5-22) is half that in Coil 1. The general rule is that with an ideal transformer, the sum of the amp-turns around the magnetic circuit must be zero. The precise way the sum is calculated depends on the direction of the windings indicated by the "dot" convention. A generalization of the rule for the transformer shown in Figure 5-22 is

$$n_1 I_1 - n_2 I_2 - n_3 I_3 = 0, \tag{5.60}$$

where $n_j$ is the number of windings of Coil $j$ with current $I_j$. The example serves to illustrate the type of thinking behind the development of many RF circuits. Considering maximum power transfer provided an alternative, simpler start to the solution of the problem than one that used the full transformer equations.

## 5.10 Baluns

A balun [16, 17] is a structure that joins balanced and unbalanced circuits. The word itself (balun) is a contraction of <u>bal</u>anced-to-<u>un</u>balanced

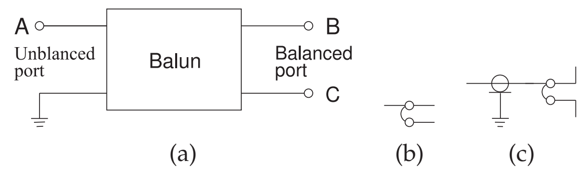



**Figure 5-24:** A balun: (a) as a two-port with four terminals; (b) IEEE standard schematic symbol for a balun [9]; and (c) an (unbalanced) coaxial cable driving a dipole antenna through a balun.

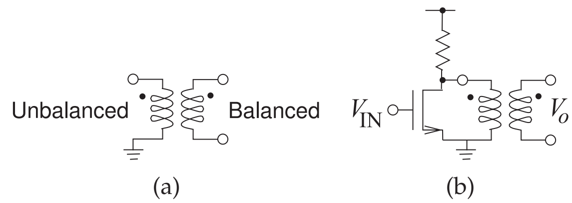



**Figure 5-25:** Balun: (a) schematic representation as a transformer showing unbalanced and balanced ports; and (b) connected to a single-ended unbalanced amplifier yielding a balanced output.

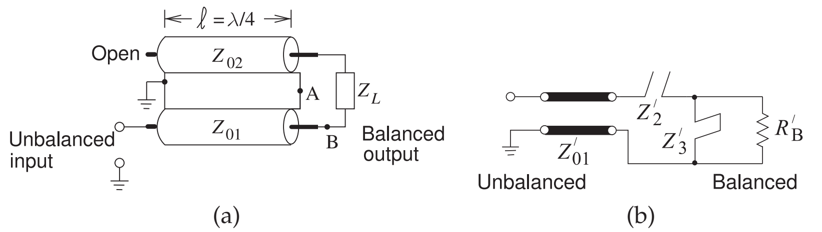



**Figure 5-26:** Marchand balun: (a) coaxial form of the Marchand balun; and (b) its equivalent circuit.

transformer. Representations of a balun are shown in Figure 5-24. A situation when a balun is required is with an antenna. Many antennas do not operate correctly if part of the antenna is at the same potential electrically as the ground. Instead, the antenna should be electrically isolated from the ground (i.e., balanced). The antenna would usually be fed by a coaxial cable with its outer conductor connected to ground, and so a balun is required between the cable and the antenna.

A balun is a key component of many RF and microwave communications systems [18, 19]. Baluns are used in balanced circuits, such as antennas, double-balanced mixers, push-pull amplifiers, and frequency doublers [20]. Another application of a balun is in a system using RFICs, where a balun transforms the differential outputs of an RFIC to unbalanced microwave circuitry. The schematic of a magnetic transformer used as a balun is shown in Figure 5-25(a) with one terminal of the unbalanced port grounded. Figure 5-24(b) is the standard schematic symbol for a balun and its use with a dipole antenna is shown in Figure 5-24(c). The second port is floating and is not referenced to ground. An example of the use of a balun is shown in Figure 5-25(b), where the amplifier is called a single-ended amplifier and its output is unbalanced, being referred to ground. A balun transitions from the unbalanced transistor output to a balanced output.

### *5.10.1 Marchand Balun*

The most common form of microwave balun is the Marchand balun [16, 20–23]. An implementation of the Marchand balun using coaxial transmission lines is shown in Figure 5-26(a) [24]. It can also be realized in planar form [16]. The $Z_{02}$ line acts as both a series stub and a shunt stub. Thus the model of the Marchand balun is as shown in Figure 5-26(b).

The drawback of the conventional Marchand balun is that the center frequency of the balun is actually the resonant frequency of the transmission

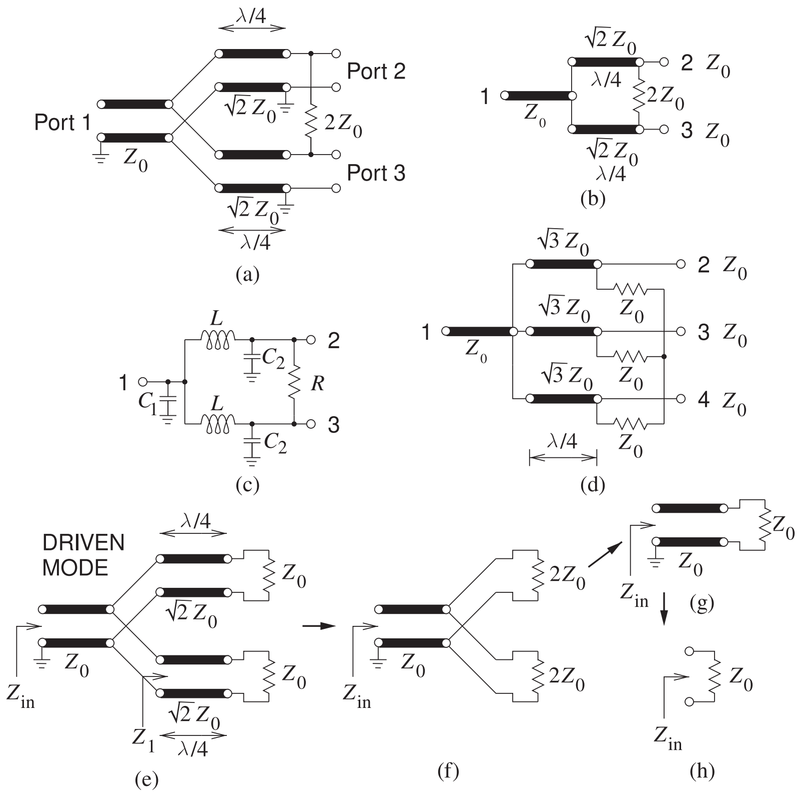



**Figure 5-27:** Wilkinson divider: (a) two-way divider with Port 1 being the combined signal and Ports 2 and 3 being the divided signals; (b) less cluttered representation; (c) lumped-element implementation; (d) three-way divider with Port 1 being the combined signal and Ports 2, 3, and 4 being the divided signals; and (e)–(h) steps in the derivation of the input impedance.

line resonators forming the balun. Thus the overall size of a conventional Marchand balun can be large, with the size determined by the transmission line sections, which are one-quarter wavelength long at the balun center frequency. Network synthesis can lead to miniaturized baluns by shifting the one-quarter wavelength frequency up while maintaining the balun center frequency [23].

## 5.11 Combiners and Dividers

Combiners are used to combine power from two or more sources. A typical use is to combine power from two high power amplifiers to obtain a higher power than would be otherwise be available. Dividers divide power so that the power from an amplifier can be routed to different parts of a circuit.

### *5.11.1 Wilkinson Combiner and Divider*

The Wilkinson divider can be used as a combiner or divider that divides input power among the output ports [25]. Figure 5-27(a) is a two-way divider that splits the power at Port 1 equally between the two output ports at Ports 2 and 3. A particular insight that Wilkinson brought was the introduction of the resistor between the output ports and this acts to suppress any imbalance between the output signals due to nonidealities. If the division is exact, no current will flow in the resistor. The circuit works less well as a general purpose combiner. Ideally power entering Ports 2 and 3 would combine losslessly and appear at Port 1. A typical application is to combine the power at the output of two matched transistors where the amplitude and the phase

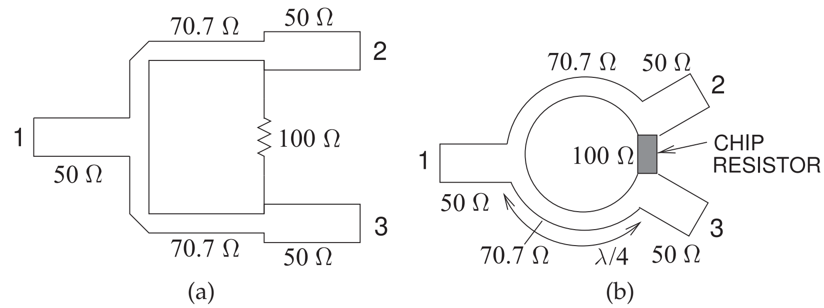



**Figure 5-28:** Wilkinson combiner and divider: (a) microstrip realization; and (b) higher performance microstrip implementation.

of the signals can be expected to be closely matched. However, if the signals are not identical, power will be absorbed in the resistor. The bandwidth of the Wilkinson combiner/divider is limited by the one-quarter wavelength long lines. However, the bandwidth is relatively large, approaching ±50% [25]. Arbitrary power ratios can also be obtained [26–28].

The operation of the Wilkinson divider can be seen by deriving the input impedance of the two-way Wilkinson divider driven at Port 1 (see Figure 5-27(e)). Since the Wilkinson divider is driven, the signals at Ports 2 and 3 will be identical, so it is as though the resistor in the Wilkinson divider is not there. The input impedance of one of the one-quarter wavelength long sections in Figure 5-27(e) is

$$Z_1 = \frac{\left(\sqrt{2}Z_0\right)^2}{Z_0} = 2Z_0, \tag{5.61}$$

and so the Wilkinson model reduces to that in Figure 5-27(f). The two $2Z_0$ resistors are in parallel, resulting in the further model reductions in Figures 5-27(g) and 5-27(h). Thus Port 1 is matched. A similar analysis shows that Ports 2 and 3 are matched (have an input impedance of $Z_0$). The $S$ parameters of the two-way Wilkinson power divider with an equal split of the output power are therefore

$$\mathbf{S} = \begin{bmatrix} 0 & -\jmath/\sqrt{2} & -\jmath/\sqrt{2} \\ -\jmath/\sqrt{2} & 0 & 0 \\ -\jmath/\sqrt{2} & 0 & 0 \end{bmatrix}. \tag{5.62}$$

Figure 5-27(b) is a compact representation of the two-way Wilkinson divider, and a three-way Wilkinson divider is shown in Figure 5-27(d). This pattern can be repeated to produce $N$-way power dividing. The lumped-element version of the Wilkinson divider shown in Figure 5-27(c) is based on the $LC$ model of a one-quarter wavelength long transmission line segment. With a $50\ \Omega$ system impedance and center frequency of 400 MHz, the elements of the lumped element are (from Figure 2-37(c)) $L$ = 28.13 nH, $C_1$ = 11.25 pF, $C_2$ = 5.627 pF, and $R$ = 100 Ω.

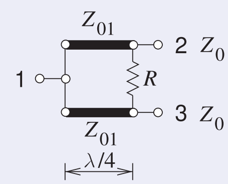
Figure 5-28(a) is the layout of a direct microstrip realization of a Wilkinson divider. The obvious problem is how to incorporate the resistor. As long as the resistor is placed symmetrically this is not as severe a problem as it would initially seem, as power is not dissipated in the resistor unless there is an imbalance. A higher-performance microstrip layout is shown in Figure 5-28(b), where the transmission lines are curved to bring the output ports near each other so that a chip resistor can be used.

**EXAMPLE 5.3** Lumped-Element Wilkinson Divider

Design a lumped-element 2-way Wilkinson divider in a 60 Ω system. The center frequency of the design should be 10 GHz.

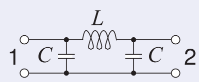
**Solution:**

The design begins with the transmission line form of the Wilkinson divider which will be converted to a lumped-element form latter. The design parameters are $Z_0 = 60\ \Omega$, $f = 10$ GHz, $\omega = 2\pi 10^{10} = 2.283 \cdot 10^{10}$ rads/s and so



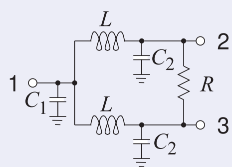

$$Z_{01} = \sqrt{2} Z_0 = 84.85\ \Omega, \quad R = 2Z_0 = 120\ \Omega.$$

The next stage is to convert the transmission lines to lumped elements. A broadband design of a quarter-wavelength transmission line was presented in Figure 2-37(b) of [15]. That is, each of the quarter-wave lines has the model




with $L = Z_{01}/\omega = 84.85/\omega = 954.9$ pH,
$C = 1/(Z_{01}\omega) = 1/(84.85\omega) = 265.3$ fF.

So the final lumped element design is




with
$C_1 = 2C = 530.6$ fF,
$C_2 = C = 265.3$ fF,
$L = 954.9$ pH,
$R = 120\ \Omega$.

### 5.11.2 Chireix Combiner

One of the problems with the Wilkinson structure when it is used as a combiner is that it is only efficient at combining when the two signals to be combined are in phase. A better combiner is the Chireix combiner [29] shown in Figure 5-29. This combiner is often used when combining the outputs of two amplifiers achieving efficient combining even when the signals to be combined are not identical [30–33].

## 5.12 Transmission Line Transformer

One of the challenges in RF engineering is achieving broadband operation of transformers from megahertz up to several gigahertz. In this section, several structures are presented that operate as magnetic transformers at frequencies below several hundred megahertz but as coupled transmission line structures at high frequencies. A transformer that achieves this and,

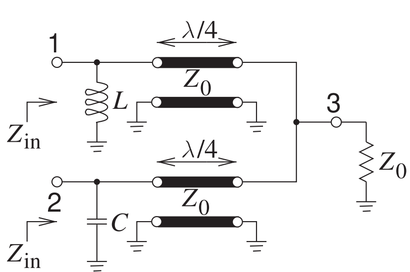
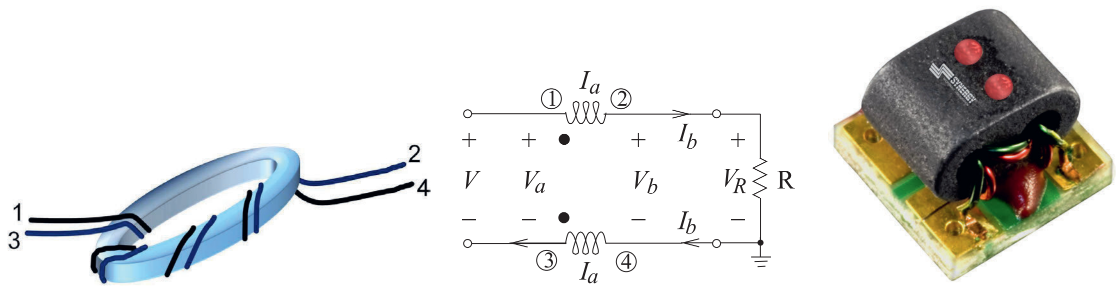



**Figure 5-29:** Chireix combiner.




(a) (b) (c)

**Figure 5-30:** A broadband RF balun as coupled lines wound around a ferrite core: (a) physical realization (the wires 1–2 and 3–4 form a single transmission line); (b) equivalent circuit using a wire-wound transformer (the number of primary and secondary windings are equal); and (c) packaged as a module (Model TM1-9 with a frequency range of 100–5000 MHz. Copyright Synergy Microwave Corporation, used with permission [34]).

(a) Top view (b) Side view (c) With heightened contrast

**Figure 5-31:** Open-core surface-mount transformer. Copyright 2012 Scientific Components Corporation d/b/a Mini-Circuits, used with permission [11].

in this configuration, realizes a balun is shown in Figure 5-30(a). Below a few hundred megahertz this functions as a magnetic transformer. Above this frequency the ferrite core cannot respond to the signal and the magnetic circuit through the core appears as an open circuit. Then, since the transformer is not operating as a magnetic transformer anymore, magnetic leakage is inconsequential. At high frequencies the wire transmission lines are closely coupled and appear as a perfect coupler if the lines (or wires here) are long enough. The equivalent circuit of this structure is shown in Figure 5-30(b). At high frequencies, stray capacitances between the windings becomes important but become part of the transmission line capacitance. Many types of transformers operating from several megahertz to several gigahertz can be realized using the same principle and are available as surface-mount components (see Figure 5-30(c)).

High-frequency coupling is enhanced by twisting the conductors as shown in Figure 5-31. The combination of magnetic coupling at low frequencies with transmission line coupling at high frequencies yields a transformer that operates over very wide bandwidths.

### 5.12.1 Transmission Line Transformer as a Balun

The schematic of a broadband 1:1 RF balun is shown in Figure 5-30(b) and a realization of it is shown in Figure 5-30(a and c). The 1:1 designation indicates that there is no impedance transformation. The circuit equations describing this balun are (from the $ABCD$ parameters of a transmission line; see Table 2-1)

$$V_a = V_b\cos(\beta\ell) + \jmath I_b Z_0 \sin(\beta\ell) \quad \text{and} \quad I_a = I_b\cos(\beta\ell) + \jmath\frac{V_b}{Z_0}\sin(\beta\ell) \tag{5.63}$$

and the load resistance creates a third equation,

$$V_b = I_b R. \tag{5.64}$$

The aim in the following is the development of a design equation that describes the essential properties of the structure. Substituting Equation (5.64) in Equation (5.63) leads to

$$V_a = V_b\left[\cos(\beta\ell) + \jmath\frac{Z_0}{R}\sin(\beta\ell)\right] \quad \text{and} \quad I_a = \frac{V_b}{R}\left[\cos(\beta\ell) + \jmath\frac{R}{Z_0}\sin(\beta\ell)\right]. \tag{5.65}$$

Choosing $Z_0 = R$ yields (since $\mathrm{e}^{\jmath\beta\ell} = \cos(\beta\ell) + \jmath\sin(\beta\ell)$)

$$V_a = V_b\mathrm{e}^{\jmath\beta\ell} \quad \text{and} \quad I_a = (V_b/R)\mathrm{e}^{\jmath\beta\ell}, \tag{5.66}$$

and so

$$Z_{\text{in}} = V_a/I_a = R. \tag{5.67}$$

This analysis is idealized, as parasitics are eliminated (mainly parasitic capacitances), but the above equation indicates that the essential function of the structure is that of a balun with no impedance transformation.
The transformer arrangement shown in Figure 5-30(b) is of particular interest as it can be realized using coupled transmission lines.

### 5.12.2 4:1 Impedance Transformer at High Frequencies

By changing the connection of the transformer terminals it is possible to achieve impedance transformation. A 4:1 impedance transformer is shown in Figure 5-32(a). A specific arrangement of the primary and secondary windings puts this into what is called the transmission line form, shown in Figure 5-33. At high frequencies port conditions are enforced so that the currents at Ports 1 and 3 (and at Ports 2 and 4) are matched, as shown in Figure 5-33. Showing that $Z_{\text{in}} = R$ begins with

$$\begin{aligned} &V_a = V_b\cos(\beta\ell) + \jmath I_b Z_0\sin(\beta\ell), \quad I_a = I_b\cos(\beta\ell) + \jmath\tfrac{V_b}{Z_0}\sin(\beta\ell), \\ &V_{\text{in}} = V_a + R(I_a + I_b), \qquad \text{and} \qquad V_b = (I_a + I_b)R. \end{aligned} \tag{5.68}$$

So the aim is to find $Z_{\text{in}} = V_{\text{in}}/I_a$, as this defines the required electrical function. Now

$$V_{\text{in}} = V_{\text{a}} + V_b = V_b[1 + \cos(\beta\ell)] + \jmath I_b Z_0\sin(\beta\ell), \tag{5.69}$$

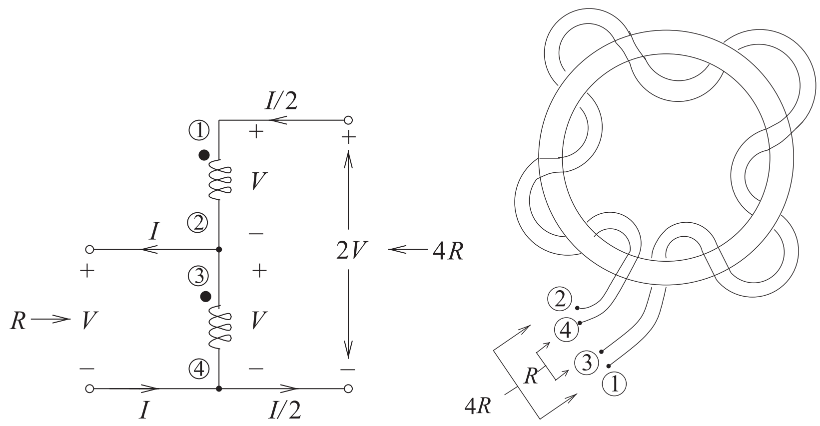



**Figure 5-32:** A 4:1 impedance transformer: (a) Schematic (the coils have the same number of windings); and (b) realization as transformer with twisted coupled wires twisted around a magnetic core.

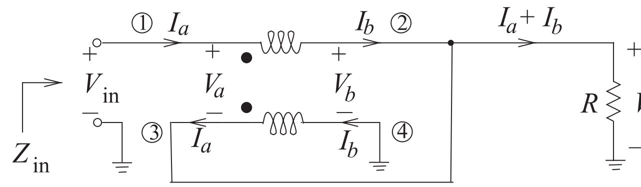



**Figure 5-33:** A transmission line form of the 4:1 impedance transformer of Figure 5-32, after [35]. (The number of primary and secondary windings are the same.)

and using the equations above

$$I_a = I_b \cos(\beta\ell) + \jmath(I_a + I_b)\frac{R}{Z_0}\sin(\beta\ell)$$

$$I_b = I_a \frac{Z_0 - \jmath R \sin(\beta\ell)}{Z_0 \cos(\beta\ell) + \jmath R \sin(\beta\ell)}. \quad (5.70)$$

Thus

$$Z_{in} = \frac{V_{in}}{I_a} = Z_0 \frac{2R[1 + \cos(\beta\ell)] + \jmath Z_0 \sin(\beta\ell)}{Z_0 \cos(\beta\ell) + \jmath R \sin(\beta\ell)}. \quad (5.71)$$

At very low frequencies the electrical length of the transmission line, $\beta\ell$, is negligibly small and $Z_{in} = 4R$. To see what happens when the length of the transmission line has a significant effect, consider the special case when $Z_0 = 2R$, then

$$Z_{in} = 2R(1 + e^{-\jmath\beta\ell}). \quad (5.72)$$

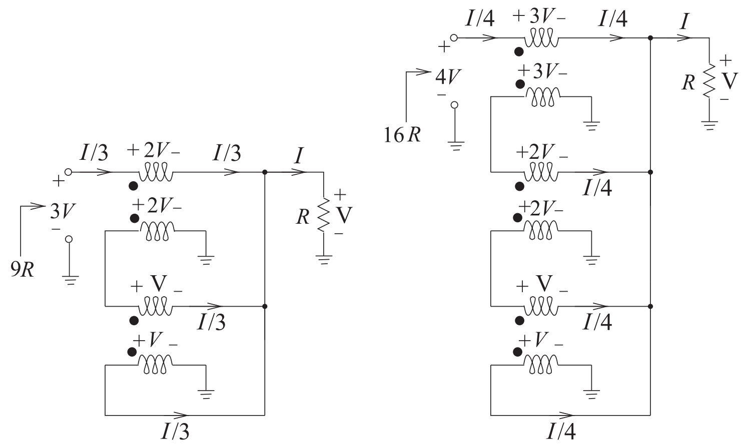
For a short line, that is, $\ell < 0.1\lambda$ or $\beta\ell < 0.2\pi$, Equation (5.72) can be approximated as

$$Z_{in} \approx 2R[1 + 1 - \jmath\beta\ell] = 4R - \jmath R(2\beta\ell). \quad (5.73)$$

The imaginary component, $-\jmath R2\beta\ell$, is a capacitance and it must be resonated out (e.g., by a series inductor) to obtain the required resistance transformation.

The general approach described above can be used to design transformers with higher impedance ratios. Two more, a 9:1 transformer and a 16:1 transformer, are shown in Figure 5-34.

A practical broadband transmission line realization of the 4:1 transformer is shown in Figure 5-32(b), where the transmission line is a pair of twisted wires.




(a) A 9:1 transformer (b) A 16:1 transformer

**Figure 5-34:** High-order impedance transformers.

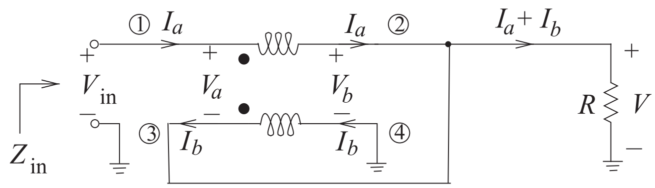



**Figure 5-35:** Low-frequency schematic of the 4:1 impedance transformer shown in Figure 5-32(a).

### *5.12.3 4:1 Impedance Transformer at Low Frequencies*

In the previous section it was shown that the impedance transformer in Figure 5-32(a) acts as a 4:1 impedance transformer at high frequencies where the structure can be considered to be a transmission line. In this section it will be shown that the transformer is also a 4:1 impedance transformer at low frequencies where the structure can be considered to be a wire-wound transformer.

At low frequencies, the port conditions[1] at the ends of the transmission line are not enforced and so a better low frequency representation of the transformer currents is shown in Figure 5-35. If at low frequencies the transformer of Figure 5-33 has a self-inductance of $L_s$ and a mutual inductance of $M$, then the circuit equations for the transformer are

$$V_{\text{in}} - V_b = \jmath\omega L_s I_a - \jmath\omega M I_b \tag{5.74}$$

$$V_b = -\jmath\omega L_s I_b + \jmath\omega M I_a \tag{5.75}$$

$$V_b = (I_a + I_b)R. \tag{5.76}$$

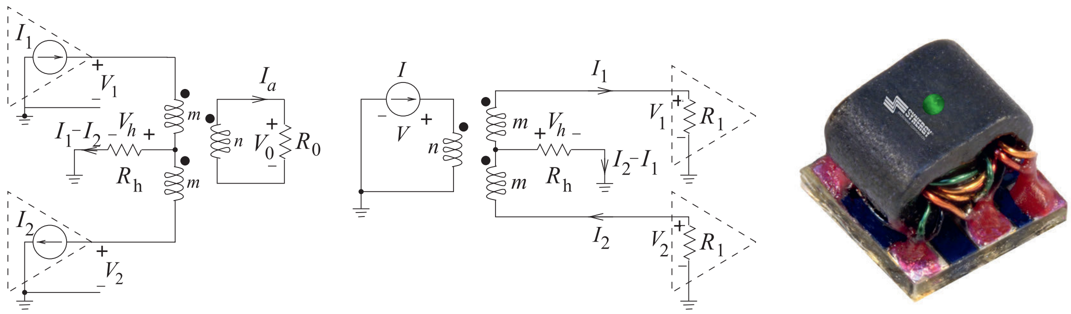
Equating Equations (5.75) and (5.76) and rearranging,

$$I_b = -\left(\frac{R - \jmath\omega M}{R + \jmath\omega L_s}\right) I_a. \tag{5.77}$$

Combining Equations (5.74) and (5.76) and rearranging,

$$V_{\text{in}} = (R + \jmath\omega L_s)\, I_a + (R - \jmath\omega M)\, I_b. \tag{5.78}$$

[1] Currents at the pair of terminals of a port are equal in magnitude and opposite in direction.




(a) (b) (c)

**Figure 5-36:** A 180° hybrid transformer: (a) used as a combiner; (b) used as a splitter; and (c) center-tapped transformer as a surface-mount component (Model TM1-2 with a frequency range of 20–1200 MHz. Copyright Synergy Microwave Corporation, used with permission [34]).

Replacing $I_b$ in Equation (5.78) using Equation (5.77) yields

$$V_{\text{in}} = \left[(R + \jmath\omega L_s) - \frac{(R - \jmath\omega M)^2}{R + \jmath\omega L_s}\right] I_a. \tag{5.79}$$

Thus the input impedance is

$$Z_{\text{in}} = \frac{V_{\text{in}}}{I_a} = \frac{R^2 + \jmath 2R\omega L_s - (\omega L_s)^2 - R^2 + \jmath 2R\omega M + (\omega M)^2}{R + \jmath\omega L_s}. \tag{5.80}$$

If ideal coupling is assumed (i.e., $M = L_s$) then Equation (5.80) reduces to

$$Z_{\text{in}} = \frac{\jmath 4R\omega L_s}{R + \jmath\omega L_s}. \tag{5.81}$$

If $\jmath\omega L_s \gg R$, then

$$Z_{\text{in}} \approx 4R. \tag{5.82}$$

Thus the transformer is a 4:1 impedance transformer both at low frequencies when the structure acts as a magnetic transformer and, as seen in Section 5.12.2, at higher frequencies when the structure acts like a transmission line. Design of the impedance transformer is then directed at managing the frequency transition region and ensuring that the required impedance transformation occurs there as well. In practice, very broadband operation is not difficult to achieve.

### 5.12.4 Hybrid Transformer Used as a Combiner

In Figure 5-36(a) a 180° hybrid transformer is used to combine the outputs of two power amplifiers that are driven 180° out of phase with respect to each other. This is commonly done when the power available from just one solid-state transistor amplifier is not sufficient to meet power requirements. Since the sum of the amp-turns of an ideal transformer must be zero,

$$nI_o = m(I_1 + I_2) \tag{5.83}$$

and $I_1 = I_2$ since the two amplifiers are driven identically but $180°$ out of phase. Then $V_h = 0, V_1 = V_2 = (m/n)V_o$, and both power amplifiers deliver equal power to the load. Also, each amplifier sees a load impedance

$$R_{\text{in}} = \frac{V_1}{I_1} = \left(\frac{m}{n}\right)^2 2R_0. \tag{5.84}$$

**EXAMPLE 5.4** Transformer Design

The Thevenin equivalent output impedance of each amplifier in Figure 5-36(a) is 1 Ω and the system impedance, $R_0$, is 50 Ω. Choose the transformer windings for maximum power transfer.

**Solution:**

For maximum power transfer, $R_{\text{in}} = 1$ Ω, and so from Equation (5.84),

$$1 = \left(\frac{m}{n}\right)^2 \cdot 2 \cdot 50 \quad \text{and} \quad \frac{m}{n} = 0.1. \tag{5.85}$$

Thus with a 10:1 winding ratio the required impedance transformation can be achieved.

### 5.12.5 Hybrid Transformer Used as a Power Splitter

The hybrid transformer can also be used to split power from a source to drive two loads. The circuit of Figure 5-36(b) splits power from a current source driver into two loads. With the number of primary and secondary windings equal (i.e., $m = n$), the circuit equations are

$$I = I_1 + I_2, \quad V_1 = V + V_{\text{h}}, \quad \text{and} \quad V_2 = V_{\text{h}} - V. \tag{5.86}$$

Thus
$$V_1 - V_{\text{h}} = V_{\text{h}} - V_2. \tag{5.87}$$

Using $V_1 = R_1 I_1$, $V_2 = -I_2 R_2$, and $V_h = (I_2 - I_1)R_h$, the desired electrical characteristics of the splitter are obtained:

$$\frac{I_2}{I_1} = \frac{R_1 + 2R_{\text{h}}}{R_2 + 2R_{\text{h}}}. \tag{5.88}$$

Several observations can be made about the performance of the power splitter. If $R_1 = R_2$, then $I_1 = I_2$ for any value of $R_h$. Conversely, if $R_1 \neq R_2$, $I_1 \neq I_2$ for a finite $R_h$. To obtain equal drive currents in both power amplifiers in spite of variations in $R_1$ and $R_2$, the center tap of the transformer needs to be omitted so that $R_h = \infty$.

### 5.12.6 Broadband Hybrid Combiner

A broadband hybrid combiner is shown in Figure 5-37. In what follows, it is shown that this combiner has the property of accommodating mismatches of the amplifiers. The development begins by assuming that the transformers have an equal number of turns on each winding. These two transformers are used to make a broadband (transmission line transformer) hybrid coupler. The circuit equations are

$$I_1 = I_a + I_b \quad \text{and} \quad I_2 = I_a - I_b, \tag{5.89}$$

$$I_a = \frac{1}{2}(I_1 + I_2) \quad \text{and} \quad I_b = \frac{1}{2}(I_1 - I_2), \tag{5.90}$$

$$V_1 = \frac{V_o}{2} + V_h \quad \text{and} \quad V_2 = \frac{V_o}{2} - V_h. \tag{5.91}$$

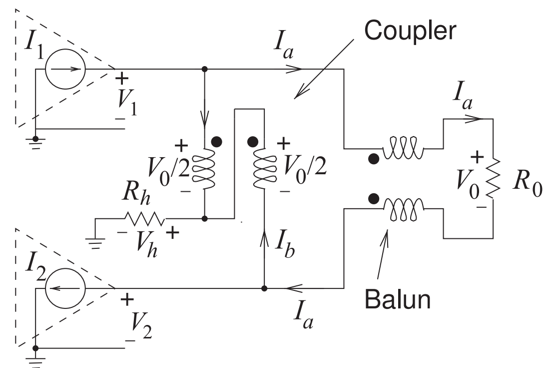



**Figure 5-37:** Broadband hybrid power combiner.

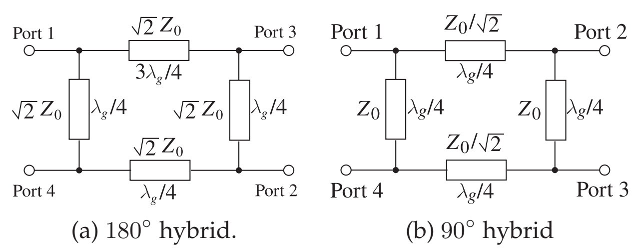



**Figure 5-38:** Topologies of ring-type hybrids.

Thus

$$\frac{V_1}{I_1} = R_h \left(1 - \frac{I_2}{I_1}\right) + \frac{R_0}{2}\left(1 - \frac{I_2}{I_1}\right) \tag{5.92}$$

$$\frac{V_2}{I_2} = \frac{R_0}{2}\left(1 + \frac{I_1}{I_2}\right) - R_h \left(\frac{I_1}{I_2} - 1\right). \tag{5.93}$$

A special situation is when $R_h = R_0/2$, and then $V_2/I_2 = R_0$ and $V_1/I_1 = R_0$. Thus each of the amplifiers sees a constant load resistance, $R_0$, even if the amplifiers are mismatched, resulting, for example, when the amplifiers have slightly different gains.

## 5.13 Transmission Line-Based Hybrids

Hybrids can be realized in a variety of transmission line structures. A few of these, but representative ones, are discussed here. Hybrids can also be realized using lumped-elements by modeling the transmission line segments.

### 5.13.1 Branch-Line Hybrids Based on Transmission Lines

A branch-line hybrid is based on transmission line segments that introduce phase delay. Two such hybrids are shown in Figure 5-38 where the different signal paths result in constructive and destructive interference of signals. This is a very different way of realizing the hybrid function than that obtained using magnetic transformers. The operation of the $180°$ hybrid (Figure 5-38(a)) can be verified approximately by counting the total number of one-quarter wavelength ($90°$) phase shifts in each path. It is not so easy to verify the operation of the $90°$ hybrid. The various characteristic impedances of the transmission line segments adjust the levels of the signals. The operation of the $90°$ hybrid is examined in the example in this section. (A complete analysis could be done using signal flow graphs.)

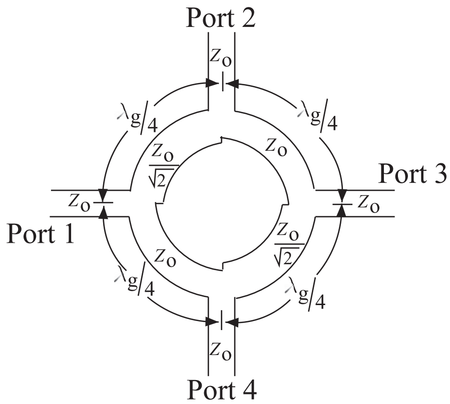



**Figure 5-39:** Planar implementation of the 3 dB ring-type branch-line hybrid with the topology of Figure 5-38(b).

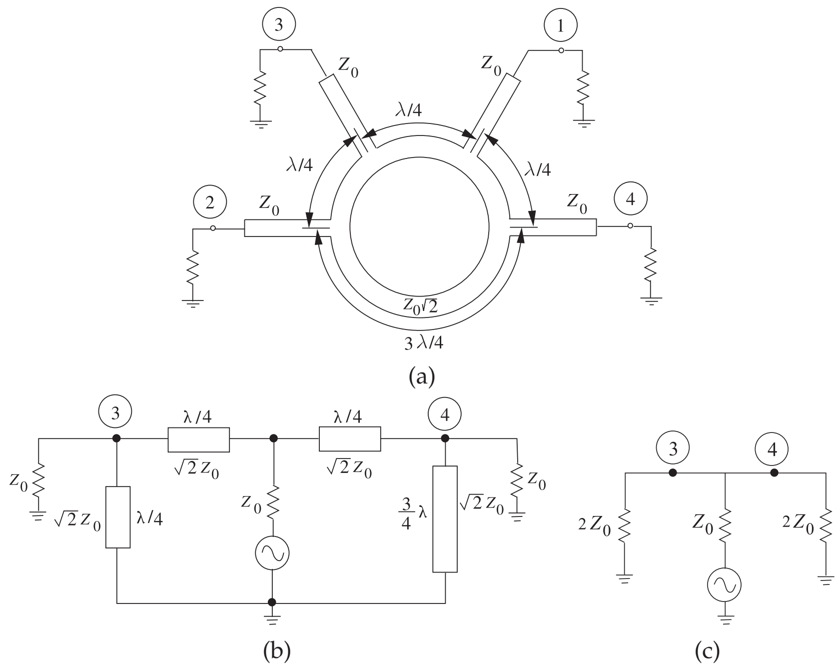



**Figure 5-40:** Rat-race hybrid with input at Port 1, outputs at Ports 3 and 4, and virtual ground at Port 2: (a) implementation as a planar circuit; (b) transmission line model; and (c) equivalent circuit model.

These branch-line hybrids may be formed into a ring shape, as shown in Figure 5-39. It is also worth considering the so-called rat-race or hybrid-ring circuit shown in Figure 5-40(a). Output signals from Ports 2 and 4 differ in phase by 180° (in contrast to the branch-line coupler, where the phase difference is 90°). An interesting and important design feature arises when considering the quarter-wave transformer action of this coupler. Only Ports 2 and 4 exhibit this action, because Port 3 is half-wave separated from the input feeding Port 1. Thus the net effective load on the inner ring lines feeding Ports 2 and 4 amounts to $2Z_0$ (two $Z_0$ loads appearing, equivalently, in series). Now the characteristic impedance $Z_0$ of any quarter-wave transforming line between two impedances, $Z_{01}$ and $Z_{02}$, is equal to $\sqrt{Z_{01}Z_{02}}$. In this case the two impedances are $Z_0$ and $2Z_0$, respectively, so (from Section 2.4.6 of [15]) the input impedance of the intervening quarter-wave line is

$$Z_0' = \sqrt{Z_0 \cdot 2Z_0} = \sqrt{2}Z_0. \tag{5.94}$$

Thus the characteristic impedance of the line forming the ring itself is $\sqrt{2}$ times that of the feeder line impedances. So when the impedance of all ports is 50 Ω, the ring characteristic impedance is 70.7 Ω.

**EXAMPLE 5.5** Rat-Race Hybrid

In this example the rat-race circuit shown in Figure 5-40(a) is considered. One of the functions of this circuit is that with an input at Port 1, the power of this signal is split between Ports 3 and 4. At the same time, no signal appears at Port 2. This example is an exercise in exploiting the impedance transformation properties of the transmission line.

From Figure 5-40(a) it is seen that each port is separated from the other port by a specific electrical length. Intuitively one can realize that there will be various possible outputs for excitation from different ports. Each case will be studied.

When Port $1$ of the hybrid is excited or driven, the outputs at Ports $3$ and $4$ are in phase, as both are distanced from Port $1$ by an electrical length of $\lambda_g/4$, while Port $2$ remains isolated, as the electrical length of the two paths from Port $1$ to Port $2$ differ by an even multiple of $\lambda_g/2$. Thus Port $2$ will be an electrical short circuit to the signal at Port $1$.

In a similar way, a signal excited at Port $2$ will result in outputs at Ports $3$ and $4$, though with a phase difference of $180^\circ$ between the two output ports and Port $1$ remains isolated.

Finally, a signal excited at Ports $3$ and $4$ will result in the sum of the two signals at Port $1$ and the difference of two signals at Port $2$. This combination of output is again due to varying electrical length between every port and every other port in the rat-race hybrid. The equivalent transmission line model and equivalent circuit of the rat-race hybrid are shown in Figures 5-40(b and c), respectively.

### 5.13.2 Lumped-Element Hybrids

Hybrids are developed based on transmission line principles, but it is possible to create hybrids using lumped-element equivalents of transmission line segments. In some cases the full circle of design can begin with the magnetic transformer conceptualization, followed by a transmission line realization, and then a lumped-element approximation. At least this is valid over a bandwidth centered at a particular frequency, $f = \omega_0/(2\pi)$. The lumped-element equivalent of the $180^\circ$ hybrid in Figure 5-38(a) is shown in Figure 5-41(a) with

$$\omega_0 L = \frac{1}{\omega_0 C} = \sqrt{2} Z_0. \tag{5.95}$$

This result comes from the broadband equivalent circuit of a $\lambda/4$ line shown in Figure 2-37. The lumped-element quadrature hybrid of Figure 5-41(b), based on Figure 5-38(b), has

$$\omega_0 Z_0 C_a = 1, \quad C_a + C_b = \frac{1}{{\omega_0}^2 L}, \quad \text{and} \quad \omega_0 L = \frac{Z_0}{\sqrt{2}}, \tag{5.96}$$

where $Z_0$ is the port impedance.

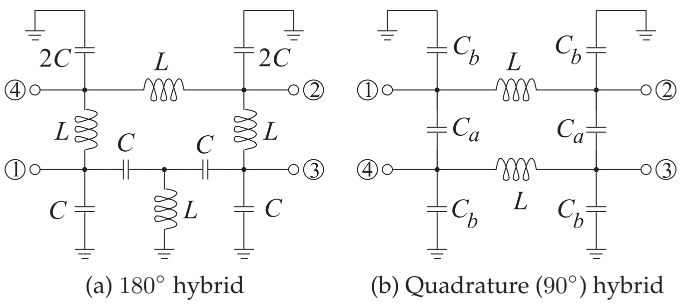



**Figure 5-41:** Lumped-element hybrids.

## 5.14 Summary

This chapter introduced the richness of microwave components available to the RF designer. Microwave lumped-element $R$, $L$, and $C$ components are carefully constructed so that they function as intended up to 20 GHz or so. They are usually in surface-mount form so that they can be integrated in design without the parasitic effects of leads. To a limited extent, transmission line discontinuities can be used as small-valued lumped elements. Even if the transmission line discontinuities are not specifically introduced for this purpose, their lumped-element equivalent circuits must be included in circuit analysis. Transmission line stubs are widely used to introduce capacitance and inductance in circuits. In most transmission line technologies only shunt stubs are available, and thus there is a strong preference for shunt elements in circuit designs. Many functionalities can be developed using the interactions of forward- and backward-traveling waves on transmission lines. Classic examples seen in this chapter are the combiners and hybrids. Sometimes the functionality can only be visualized using transmission line structures. Then coming full circle, lumped-element equivalents can be realized.

There are many more passive components in the repertoire of the RF designer. However, this chapter reviewed the most important and introduced the concepts that can be used to analyze other structures and to invent new ones. Each year new variants of microwave elements are developed and documented in patents and publications. Microwave engineers monitor these developments, especially in their field of expertise.

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## 5.16 Exercises

1. The Thevenin equivalent output impedance of each of the amplifiers in Figure 5-36(a) is 5 Ω, and the system impedance, $R_0$, is 75 Ω. Choose the transformer windings for maximum power transfer. [Parallels Example 5.4]

2. A spiral inductor is modeled as an ideal inductor of 10 nH in series with a 5 Ω resistor. What is the $Q$ of the spiral inductor at 1 GHz?

3. Consider the design of a 50 dB resistive T attenuator in a 75 Ω system. [Parallels Example 5.1]
   (a) Draw the topology of the attenuator.
   (b) Write down the design equations.
   (c) Complete the design of the attenuator.

4. Consider the design of a 50 dB resistive Pi attenuator in a 75 Ω system. [Parallels Example 5.1]
   (a) Draw the topology of the attenuator.
   (b) Write down the design equations.
   (c) Complete the design of the attenuator.

5. A 20 dB attenuator in a 17 Ω system is ideally matched at both the input and output. Thus there are no reflections and the power delivered to the load is reduced by 20 dB from the applied power. If a 5 W signal is applied to the attenuator, how much power is dissipated in the attenuator?

6. A resistive Pi attenuator has shunt resistors of $R_1 = R_2 = 294$ Ω and a series resistor $R_3 = 17.4$ Ω. What is the attenuation (in decibels) and the characteristic impedance of the attenuator?

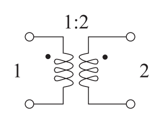
7. A resistive Pi attenuator in a system with characteristic impedance $Z_0$ has equal shunt resistors of $R_1 = R_2$ and a series resistor $R_3$. Show that $Z_0 = \sqrt{(R_1^2 R_3)/(2R_1 + R_3)}$ and the attenuation factor $K = \sqrt{(R_1 + Z_0)/(R1 - Z0)}$. [Start with Equation (5.15).]

8. Design a resistive Pi attenuator with an attenuation of 10 dB in a 100 Ω system.

9. Design a 3 dB resistive Pi attenuator in a 50 Ω system.

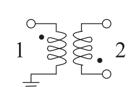
10. A resistive Pi attenuator has shunt resistors $R_1 = R_2 = 86.4$ Ω and a series resistor $R_3 = 350$ Ω. What is the attenuation (in decibels) and the system impedance of the attenuator?

11. Derive the 50 Ω scattering parameters of the ideal transformer shown below where the number of windings on the secondary side (Port 2) is twice the number of windings on the primary side (Port 1).




   (a) What is $S_{11}$? [Hint: Terminate Port 2 in 50 Ω and determine the input reflection coefficient.]
   (b) What is $S_{21}$?
   (c) What is $S_{22}$?
   (d) What is $S_{12}$?

12. Derive the two-port 50 Ω scattering parameters of the magnetic transformer below. The primary (Port 1) has 10 turns, the secondary (Port 2) has 25 turns.




   (a) What is $S_{11}$?
   (b) What is $S_{21}$?
   (c) What is $S_{22}$?
   (d) What is $S_{12}$?

13. An ideal quadrature hybrid has the scattering parameters

$$S_{90^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & -\jmath & 1 & 0 \\ -\jmath & 0 & 0 & 1 \\ 1 & 0 & 0 & -\jmath \\ 0 & 1 & -\jmath & 0 \end{bmatrix}.$$

Draw the signal flow graph of the hybrid, labeling each of the edges and assigning $a_1, b_1$, etc., to the nodes. (Do not start with the SFG of a generic 4-port network.)

14. An ideal 180° hybrid has the scattering parameters

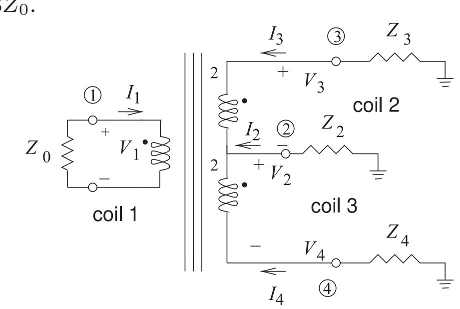
$$S_{180^\circ} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

Draw the signal flow graph of the hybrid, labeling each of the edges and assigning $a_1$, $b_1$, etc., to the nodes. (Note, do not start with the SFG of a generic 4-port network.)

15. A signal is applied to Ports 2 and 3 of a 180° hybrid, as shown in Figure 5-20(b). If the signal consists of a differential component of 0 dBm and a common mode component of 10 dBm:
    (a) Determine the power delivered at Port 1.
    (b) Determine the power delivered at Port 4. Assume that the hybrid is lossless.

16. Silicon RFICs use differential signal paths to minimize the introduction of substrate noise. As well, differential amplifiers are an optimum topology in current-biased circuits. Off-chip signals are often on microstrip lines and so the source and load, being off-chip, are not differential. The off-chip circuits are then called single-ended. Using 180° hybrids, diagrams, and explanations, outline a system architecture accommodating this mixed differential and single-ended environment.

17. Consider the hybrid shown in the figure below. If the number of windings of Coils 2 and 3 are twice the number of windings of Coil 1, show that for matched hybrid operation $2Z_2 = Z_3 = 8Z_0$.




18. The balun of Figure 5-23 transforms an unbalanced system with a system impedance of $Z_0$ to a balanced system with an impedance of $4Z_0$. The actual impedance transformation is determined by the number of windings of the coils. Design a balun of the type shown in Figure 5-23 that transforms an unbalanced 50 Ω system to a balanced 377 Ω system. [Hint: Find the ratio of the windings of the coils.]

19. A balun can be realized using a wire-wound transformer, and by changing the number of windings on the transformer it is possible to achieve impedance transformation as well as balanced-to-unbalanced functionality. A 500 MHz balun based on a magnetic transformer is required to achieve impedance transformation from an unbalanced impedance of 50 Ω to a balanced impedance of 200 Ω. If there are 20 windings on the balanced port of the balun transformer, how many windings are there on the unbalanced port of the balun?

20. Design a lumped-element two-way power splitter in a 75 Ω system at 1 GHz. Base your design on a Wilkinson power divider.

21. Design a three-way power splitter in a 75 Ω system. Base your design on a Wilkinson power divider using transmission lines and indicate lengths in terms of wavelengths.

22. Design a lumped-element three-way power splitter in a 75 Ω system at 1 GHz. Base your design on a Wilkinson power divider.

23. A resistive power splitter is a three-port device that takes power input at Port 1 and delivers power at Ports 2 and 3 that are equal; that is, $S_{21} = S_{31}$. However, the sum of the power at Ports 2 and 3 will not be equal to the input power due to loss. Design a 75 Ω resistive three-port power splitter with matched inputs, $S_{11} = 0 = S_{22} = S_{33}$. That is, draw the resistive circuit and calculate its element values.

24. Design a balun based on a magnetic transformer if the balanced load is 300 Ω and the unbalanced impedance is 50 Ω.
    (a) Draw the schematic of the balun with the load and indicate the ratio of windings.
    (b) If the number of windings on the unbalanced side of the transformer is 20, how many windings are on the unbalanced side?

25. Develop the electrical design of a rat-race hybrid at 30 GHz in a 50 Ω system.

26. Develop the electrical design of a rat-race hybrid at 30 GHz in a 100 Ω system.

27. Design a lumped-element 180° hybrid at 1900 MHz using 1 nH inductors.

28. Design a 90° lumped-element hybrid at 1900 MHz using 1 nH inductors.

29. Design a 90° lumped-element hybrid at 500 MHz for a 75 Ω system.

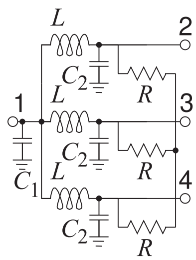
30. Design a lumped-element 180° hybrid at 1900 MHz matched to 50 Ω source and load impedances.

### 5.16.1 Exercises by Section

†challenging, ‡very challenging

| | |
|---|---|
| §5.1 | 1 |
| §5.2 | 2, 3, 4, 5 |
| §5.5 | 6†, 7†, 8, 9, 10† |
| §5.8 | 11†, 12† |
| §5.9 | 13†, 14†, 15†, 16†, 17‡ |
| §5.10 | 18†, 19† |
| §5.11 | 20†, 21†, 22† |
| §5.12 | 23† |
| §5.13 | 24‡, 25†, 26†, 27, 28, 29, 30 |

### 5.16.2 Answers to Selected Exercises

2 12.57
3 $R_1$=$R_2$=74.5 Ω
4 75.48 Ω
5 4.95 W
11(c) 0.6
12 −0.6897
15 10 dBm
22


$L$=20.7 nH, $C_1$=3.68 pF
$C_2$=1.23 pF, $R$=75 Ω
24 ratio of windings is 2.45
30 Fig. 5-41(a)
$L$=5.92 nH, $C$=1.19 pF

CHAPTER 6

# Impedance Matching



## 6.1 Introduction

The maximum transfer of signal power is one of the prime objectives in RF and microwave circuit design. Power traverses a network from a source to a load generally through a sequence of two-port networks. Maximum power transfer requires that the Thevenin equivalent impedance of a source be matched to the impedance seen from the source. That is, the source should be presented with the complex conjugate of the source impedance. This is achieved by designing what is called a matching network inserted between the source and load. As long as the load has some nonzero real part (and so can dissipate power), a matching network can always be found. Design of the matching network is called impedance matching.

Section 6.2 describes two common matching objectives. Then design approaches for impedance matching are presented first with an algorithmic approach in Sections 6.3–6.6 and then a graphical approach based on using a Smith chart in Sections 6.7–6.9.

## 6.2 Matching Networks

Matching networks are constructed using lossless elements such as lumped capacitors, lumped inductors and transmission lines and so have, ideally, no loss and introduce no additional noise. This section discusses matching objectives and the types of matching networks.

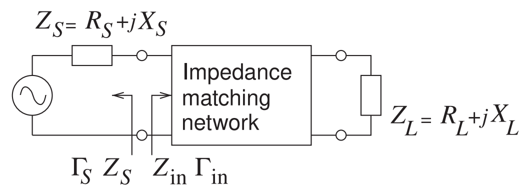



**Figure 6-1:** A source with Thevenin equivalent impedance $Z_S$ and load with impedance $Z_L$ interfaced by a matching network presenting an impedance $Z_{\text{in}}$ to the source.

**Table 6-1:** Matching conditions with reference to Figure 6-1. $\Gamma_{\text{in}}$, $\Gamma_S$, must be with respect to a real $Z_{\text{REF}}$.

| Reflection-less match | Maximum power transfer |
|---|---|
| $Z_{\text{in}} = Z_S$ | $Z_{\text{in}} = Z_S^*$,<br>$\Gamma_{\text{in}} = \Gamma_S^*$ |

### 6.2.1 Matching for Zero Reflection or for Maximum Power Transfer

With RF circuits the aim of matching is to achieve maximum power transfer. With reference to Figure 6-1 the condition for maximum power transfer is $Z_{\text{in}} = Z_S^*$ (see Section 2.6.2 of [1]). An alternative matching objective, used most commonly with digital circuits, is a reflection-less match. The reflection-less match and the maximum power transfer match are only equivalent if $Z_S$ and $Z_L$ are real. Nearly always in RF design the matching objective is maximum power transfer, and this is assumed unless the reflection-less match is specifically indicated.

The maximum power transfer matching condition can also be specified in terms of reflection coefficients with respect to an real reference impedance $Z_{\text{REF}}$. (In the definition of reflection coefficient $Z_{\text{REF}}$ can be complex, but many of the manipulations in this book only apply when $Z_{\text{REF}}$ is real and this should be assumed unless it is specifically stated that it can be complex.) The condition for maximum power transfer is $Z_{\text{in}} = Z_S^*$ which is equivalent to $\Gamma_{\text{in}} = \Gamma_S^*$. The proof is as follows:

$$\Gamma_{\text{in}} = \left(\frac{Z_{\text{in}} - Z_{\text{REF}}}{Z_{\text{in}} + Z_{\text{REF}}}\right), \tag{6.1}$$

and for maximum power transfer $Z_{\text{in}} = Z_S^*$, so

$$\Gamma_{\text{in}}^* = \frac{Z_{\text{in}} - Z_{\text{REF}}}{Z_{\text{in}} + Z_{\text{REF}}} = \left(\frac{Z_S^* - Z_{\text{REF}}}{Z_S^* + Z_{\text{REF}}}\right)^* = \frac{(Z_S^* - Z_0)^*}{(Z_S^* + Z_0)^*}$$
$$= \frac{(Z_S^*)^* - Z_{\text{REF}}^*}{(Z_S^*)^* + Z_{\text{REF}}^*} = \frac{Z_S - Z_{\text{REF}}^*}{Z_S + Z_{\text{REF}}^*} = \Gamma_S. \tag{6.2}$$

If $Z_{\text{REF}}$ is real, $Z_{\text{REF}}^* = Z_{\text{REF}}$ and then the condition for maximum power transfer is

$$\Gamma_{\text{in}}^* = \frac{Z_S - Z_{\text{REF}}}{Z_S + Z_{\text{REF}}} = \Gamma_S. \tag{6.3}$$

Thus, provided that $Z_{\text{REF}}$ is real, the condition for maximum power transfer in terms of reflection coefficients is $\Gamma_{\text{in}}^* = \Gamma_S$ or $\Gamma_{\text{in}} = \Gamma_S^*$. The matching conditions are summarized in Table 6-1.

### 6.2.2 Types of Matching Networks

Up to a few gigahertz, lumped inductors and capacitors can be used in matching networks. Above a few gigahertz, distributed parasitics (losses and additional capacitive or inductive effects) can render lumped-element networks impractical. Inductors in particular have high loss and high parasitic capacitances at microwave frequencies. Lumped capacitors are useful circuit elements at much higher frequencies than are lumped inductors. Segments of transmission lines are used in matching networks and are used instead of lumped elements when loss must be kept to a minimum, power levels are high, or the parasitics of lumped elements render them unusable. This is because the loss of an appropriate transmission line component is always much less than the loss of a lumped inductor.

Many factors affect the selection of the components of a matching network. The most important of these are size, complexity, bandwidth, and adjustability. The impact on a circuit of a distributed element is directly related to its length compared to one-quarter of a wavelength ($\lambda/4$). At 1 GHz a circuit board, for example, with a relative permittivity of 4 has $\lambda/4 = 3.75$ cm. This is far too large to fit in consumer wireless products operating at 1 GHz.

Performance of lumped-element capacitors and inductors refers to both the self-resonant frequency of an element and its loss. Inductors are a particular case in point. An actual inductor must be modeled using capacitive elements, capturing inter-winding capacitance as well as the primary inductive element. At some frequency the inductive and capacitive elements will resonate and this is called the self-resonant frequency of the element. The self-resonant frequency is the maximum operating frequency of the element.

With regards to complexity, the simplest matching network design is most often preferred. A simpler matching network is usually more reliable, less lossy, and cheaper than a complex design. The pursuit of small size and/or higher bandwidth, however, may necessitate that the simplest circuit not be selected. Any type of matching network can ideally give, ignoring resistive loses, a perfect match at a single frequency. Away from this center frequency the match will not be ideal. From experience, achieving a reasonable match over a 5% fractional bandwidth based on a single-frequency match can usually be achieved with simple matching networks..

An impedance matching network may consist of

(a) Lumped elements only. These are the smallest networks, but have the most stringent limit on the maximum frequency of operation. The relatively high resistive loss of an inductor is the main limiting factor limiting performance. The self resonant frequency of an inductor limits operation to low microwave frequencies.
(b) Distributed elements (microstrip or other transmission line circuits) only. These have excellent performance, but their size restricts their use in systems to above a few gigahertz.
(c) A hybrid design combining lumped and distributed elements, primarily small sections of lines with capacitors. These lines are shorter than in a design with distributed elements only, but the hybrid design has higher performance than a lumped-element-only design.
(d) Adhoc solutions (suggested by input impedance behavior and features of various components).

This chapter concentrates on matching network design. One emphasis here is on the development of design equations and synthesis of desired results. The second emphasis is on graphical techniques for matching network design based on using A Smith chart. This is particularly power as it enables

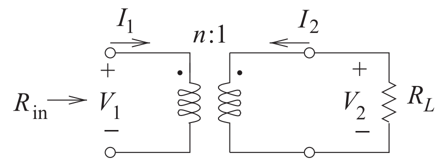



**Figure 6-2:** A transformer as a matching network. Port 1 is on the left or primary side and Port 2 is on the right or secondary side.

typologies to traded-off. An alternative design approach used by some designers is to choose a circuit topology and then use a circuit optimizer to arrive at circuit values that yield the desired characteristics. This is sometimes a satisfactory design technique, but is not a good solution for new designs as it does not provide insight and does not help in choosing new topologies.

### 6.2.3 Summary

There are many design choices in the type of matching network to be developed but the common guidelines are to minimize losses and to keep the matching network compact. These objectives are not always compatible. Matching network design in this chapter is based on perfect matching at one frequency and design decisions are made to maximize, or otherwise manipulate, the bandwidth of the match. True broadband matching network design is more like filter design.

## 6.3 Impedance Transforming Networks

Transformers and reactive elements considered in this section can be used to losslessly transform impedance levels. his is a basic aspect of network design.

### 6.3.1 The Ideal Transformer

The ideal transformer shown in Figure 6-2 can be used to match a load to a source if the source and load impedances are resistances. This will be shown by starting with the constitutive relations of the transformer:

$$V_1 = nV_2 \quad \text{and} \quad I_1 = -I_2/n. \tag{6.4}$$

Here $n$ is the transformer ratio. For a wire-wound transformer, $n$ is the ratio of the number of windings on the primary side, Port 1, to the number of windings on the secondary side, Port 2. Thus the input resistance, $R_{\text{in}}$, is related to the load resistance, $R_L$, by

$$R_{\text{in}} = \frac{V_1}{I_1} = -n^2\frac{V_2}{I_2} = n^2 R_{\text{L}}. \tag{6.5}$$

The matching problem with purely resistive load and source impedances is solved by choosing the appropriate winding ratio, $n$. However, resistive-only problems are rare at RF, and so other matching circuits must be used.

### 6.3.2 A Series Reactive Element

Matching using lumped elements is based on the impedance and admittance transforming properties of series and shunt reactive elements. Even a single reactive element can achieve limited impedance matching. Consider the series reactive element shown in Figure 6-3(a). Here the reactive element, $X_S$,

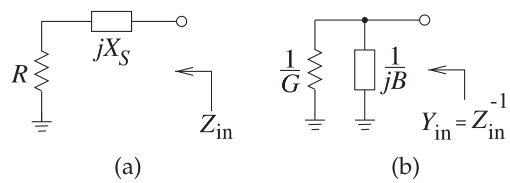



**Figure 6-3:** Matching using a series reactance: (a) the series reactive element; and (b) its equivalent shunt circuit.

is in series with a resistance $R$. The shunt equivalent of this network is shown in Figure 6-3(b) with a shunt susceptance of $B$. In this transformation the resistance $R$ has been converted to a resistance $R_P = 1/G$. The mathematics describing this transformation is as follows. The input admittance of the series connection (Figure 6-3(a)) is

$$Y_{\text{in}}(\omega) = \frac{1}{Z_{\text{in}}(\omega)} = \frac{1}{R + \jmath X_S} = \frac{R}{R^2 + X_S^2} - \jmath \frac{X_S}{R^2 + X_S^2}. \tag{6.6}$$

Thus the elements of the equivalent shunt network, Figure 6-3(b), are

$$G = \frac{R}{R^2 + X_S^2} \quad \text{and} \quad B = -\frac{X_S}{R^2 + X_S^2}. \tag{6.7}$$

The "resistance" of the network, $R$, has been transformed to a new value,

$$R_P = G^{-1} = \frac{R^2 + X_S^2}{R} > R. \tag{6.8}$$

This is an important start to matching, as $X_S$ can be chosen to convert $R$ (a load, for example) to any desired resistance value (such as the resistance of a source). However there is still a residual reactance that must be removed to complete the matching network design. Before moving on to the solution of this problem consider the following example.

## EXAMPLE 6.1 Capacitive Impedance Transformation

Consider the impedance transforming properties of the capacitive series element in Figure 6-4(a). Show that the capacitor can be adjusted to obtain any positive shunt resistance.

**Solution:**

The concept here is that the series resistor and capacitor network has an equivalent shunt circuit that includes a capacitor and a resistor. By adjusting $C_S$ any value can be obtained for $R_P$. From Equation (6.8),

$$R_P = \frac{R_0^2 + (1/\omega^2 C_S^2)}{R_0} = \frac{1 + \omega^2 C_S^2 R_0^2}{\omega^2 C_S^2 R_0} \tag{6.9}$$

and the susceptance is
$$B = \frac{(1/\omega C_S)}{R_0^2 + 1/\omega^2 C_S^2} = \omega \frac{C_S}{1 + \omega^2 C_S^2 R_0^2}. \tag{6.10}$$

Thus
$$C_P = \frac{B}{\omega} = \frac{C_S}{1 + \omega^2 C_S^2 R_0^2}. \tag{6.11}$$

To match $R_0$ to a resistive load $R_P$ ($> R_0$) at a radian frequency $\omega_d$, then, from Equation (6.9), the series capacitance required, i.e. the design equation for $C_S$, comes from

$$\omega_d C_S = 1/\sqrt{R_0 R_P - R_0^2}, \tag{6.12}$$

To complete the matching design, use a shunt inductor $L$, as shown in Figure 6-4(c), where $\omega_d C_P = 1/(\omega_d L)$. The equivalent impedance in Figure 6-4(c) is a resistor of value $R_P$, with a value that can be adjusted by choosing $C_S$ which then requires $L$ to be adjusted.

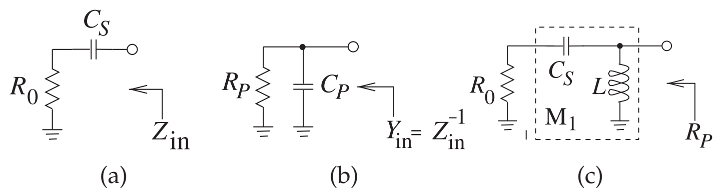



**Figure 6-4:** Impedance transformation by a series reactive element: (a) a resistor with a series capacitor; (b) its equivalent shunt circuit; and (c) an $LC$ network.

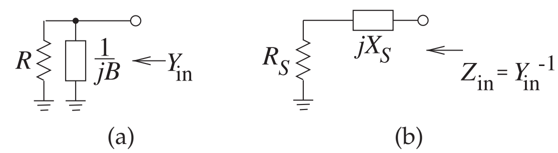



**Figure 6-5:** A resistor with (a) a shunt parallel reactive element where $B$ is a susceptance, and (b) its equivalent series circuit.

### 6.3.3 A Parallel Reactive Element

The dual of the series matching procedure is the use of a parallel reactive element, as shown in Figure 6-5(a). The input admittance of the shunt circuit

$$Y_{\text{in}} = \frac{1}{R} + \jmath B. \tag{6.13}$$

This can be converted to a series circuit by calculating $Z_{\text{in}} = 1/Y_{\text{in}}$:

$$Z_{\text{in}} = \frac{R}{1+\jmath BR} = \frac{R}{1+B^2R^2} - \jmath\frac{BR^2}{1+B^2R^2}. \tag{6.14}$$

$$\text{So} \quad R_S = \frac{R}{1+B^2R^2} \quad \text{and} \quad X_S = \frac{-BR^2}{1+B^2R^2}. \tag{6.15}$$

Notice that $R_S < R$.

**EXAMPLE 6.2** Parallel Tuning

As an example of the use of a parallel reactive element to tune a resistance value, consider the circuit in Figure 6-6(a) where a capacitor tunes the effective resistance value so that the series equivalent circuit (Figure 6-6(b)) has elements

$$R_S = \frac{R_0}{1+\omega^2C_P^2R_0^2} \quad \text{and} \quad X_S = -\frac{\omega C_P R_0^2}{1+\omega^2C_P^2R_0^2} = -\frac{1}{\omega C_S}. \tag{6.16}$$

$$\text{So} \qquad C_S = \frac{1+\omega^2C_P^2R_0^2}{\omega^2C_PR_0^2}. \tag{6.17}$$

Now consider matching $R_0$ to a resistive load $R_S$, which is less than $R_0$ at a given frequency $\omega_d$. This requires that

$$\omega_d C_P = \sqrt{1/(R_SR_0) - 1/R_0^2}.$$

To complete the design, use a series inductor to remove the reactive effect of the capacitor, as shown in Figure 6-6(C). The value of the inductor required is found from

$$\omega_d L = \frac{1}{\omega_d C_S}, \quad \text{that is,} \quad L = \frac{1}{\omega_d^2 C_S}. \tag{6.18}$$

## 6.4 The L Matching Network

The examples in the previous two sections suggest the basic concept behind lossless matching of two different resistance levels using an L network:

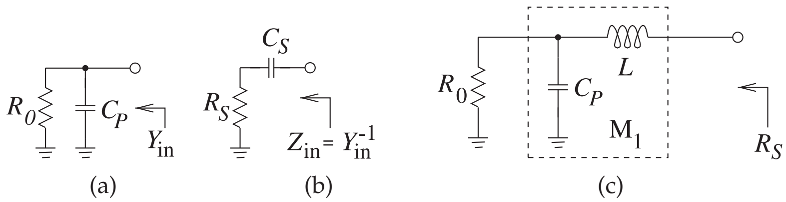
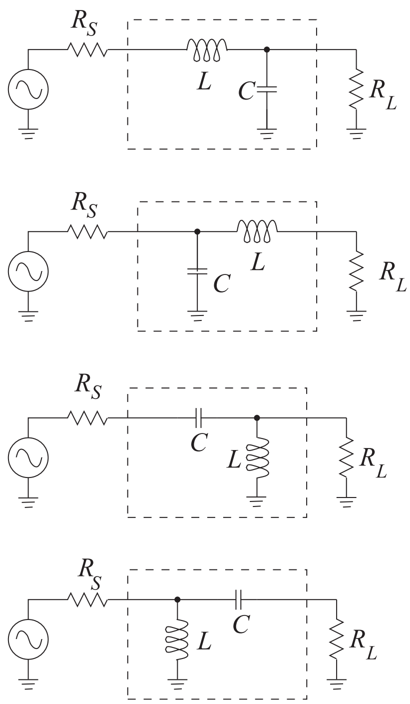



**Figure 6-6:** Parallel-to-series transformation: (a) resistor with shunt capacitor; (b) its equivalent series circuit; and (c) the transforming circuit with added series inductor.




(a) Lowpass $R_S < R_L$.
$Q = \sqrt{R_L/R_S - 1}$
$Q = |X_L|/R_S = R_L/|X_C|$

(b) Lowpass $R_S > R_L$.
$Q = \sqrt{R_S/R_L - 1}$
$Q = |X_L|/R_L = R_S/|X_C|$

(c) Highpass $R_S < R_L$.
$Q = \sqrt{R_L/R_S - 1}$
$Q = |X_C|/R_S = R_L/|X_L|$

(d) Highpass $R_S > R_L$.
$Q = \sqrt{R_S/R_L - 1}$
$Q = |X_C|/R_L = R_S/|X_L|$

**Figure 6-7:** L matching networks consisting of one shunt reactive element and one series reactive element. ($R_S$ is matched to $R_L$.) $X_C$ is the reactance of the capacitor $C$, and $X_L$ is the reactance of the inductor $L$. Note that with a two-element matching network the $Q$ and thus bandwidth of the match is fixed.

Step 1: Use a series (shunt) reactive element to transform a smaller (larger) resistance up (down) to a larger (smaller) value with a real part equal to the desired resistance value.

Step 2: Use a shunt (series) reactive element to resonate with (or cancel) the imaginary part of the impedance that results from Step 1.

So a resistance can be transformed to any resistive value by using an $LC$ transforming circuit.
Formalizing the matching approach described above, note that there are four possible two-element L matching networks (see Figure 6-7). The two possible cases, $R_S < R_L$ and $R_L < R_S$, will be considered in the following subsections.

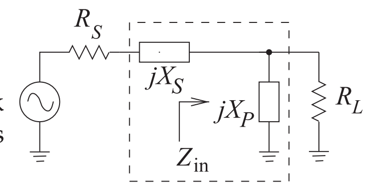



**Figure 6-8:** Two-element matching network topology for $R_S < R_L$. $X_S$ is the series reactance and $X_P$ is the parallel reactance.

### 6.4.1 Design Equations for $R_S < R_L$

Consider the matching network topology of Figure 6-8. Here

$$Z_{\text{in}} = \frac{R_L(\jmath X_P)}{R_L + \jmath X_P} = \frac{R_L X_P^2}{R_L^2 + X_P^2} + \jmath \frac{X_P R_L^2}{R_L^2 + X_P^2} \tag{6.19}$$

and the matching objective is $Z_{\text{in}} = R_S - \jmath X_S$ so that

$$R_S = \frac{R_L X_P^2}{R_L^2 + X_P^2} \quad \text{and} \quad X_S = \frac{-X_P R_L^2}{X_P^2 + R_L^2}. \tag{6.20}$$

From these $$\frac{R_S}{R_L} = \frac{1}{(R_L/X_P)^2 + 1} \quad \text{and} \quad -\frac{X_S}{R_S} = \frac{R_L}{X_P}. \tag{6.21}$$

Introducing the quantities

$$Q_S = \text{the } Q \text{ of the series leg} = |X_S/R_S| \tag{6.22}$$

$$Q_P = \text{the } Q \text{ of the shunt leg} = |R_L/X_P| \tag{6.23}$$

leads to the final design equations for $R_S < R_L$:

$$|Q_S| = |Q_P| = \sqrt{\frac{R_L}{R_S} - 1}. \tag{6.24}$$

The L matching network principle is that $X_P$ and $X_S$ will be either capacitive or inductive and they will have the opposite sign (i.e., the L matching network comprises one inductor and one capacitor). Also, once $R_S$ and $R_L$ are given, the $Q$ of the network and thus bandwidth is defined; with the L network, the designer does not have a choice of circuit $Q$.

**EXAMPLE 6.3** Matching Network Design

Design a circuit to match a 100 Ω source to a 1700 Ω load at 900 MHz. Assume that a DC voltage must also be transferred from the source to the load.

**Solution:**

Here $R_S < R_L$, and so the topology of Figure 6-9(a) can be used and there are two versions, one with a series inductor and one with a series capacitor. The series inductor version (see Figure 6-9(b)) is chosen as this enables DC bias to be applied. From Equations (6.22)–(6.24) the design equations are

$$|Q_S| = |Q_P| = \sqrt{\frac{1700}{100} - 1} = \sqrt{16} = 4, \quad \frac{X_S}{R_S} = 4, \quad \text{and} \quad X_S = 4 \cdot 100 = 400. \tag{6.25}$$

This indicates that $\omega L = 400\ \Omega$, and so the series element is

$$L = \frac{400}{2\pi \cdot 9 \cdot 10^8} = 70.7\ \text{nH}. \tag{6.26}$$

For the shunt element next to the load, $|R_L/X_C| = 4$, and so

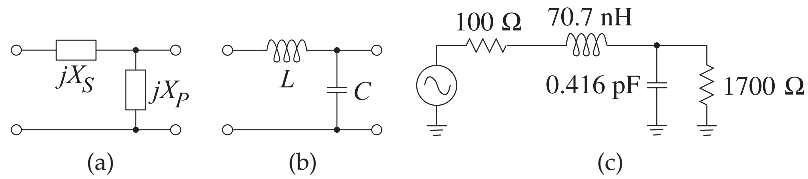



**Figure 6-9:** Matching network development for Example 6.3.

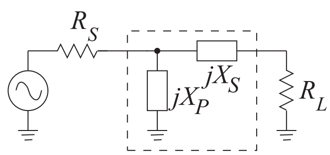



**Figure 6-10:** Two-element matching network topology for $R_S > R_L$.

$$|X_C| = \frac{R_L}{4} = \frac{1700}{4} = 425. \tag{6.27}$$

$$\text{Thus } 1/\omega C = 425 \text{ and } \quad C = \frac{1}{2\pi \cdot 9 \cdot 10^8 \cdot 425} = 0.416 \text{ pF}. \tag{6.28}$$

The final matching network design is shown in Figure 6-9(c).

## 6.4.2 L Network Design for $R_S > R_L$

For $R_S > R_L$, the topology shown in Figure 6-10 is used. The design equations for the L network for $R_S > R_L$ are similarly derived and are

$$|Q_S| = |Q_P| = \sqrt{\frac{R_S}{R_L} - 1} \tag{6.29}$$

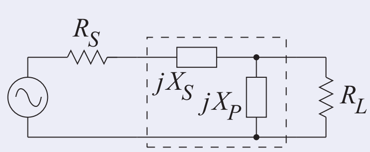
$$-Q_S = Q_P, \quad Q_S = \frac{X_S}{R_L}, \quad \text{and} \quad Q_P = \frac{R_S}{X_P}. \tag{6.30}$$

### EXAMPLE 6.4 Two-Element Matching Network

Design a passive two-element matching network that will achieve maximum power transfer from a source with an impedance of 50 Ω to a load with an impedance of 75 Ω. Choose a matching network that will not allow DC to pass.

**Solution:**

$R_L > R_S$, so, from Figure 6-7, the appropriate matching network topology is




This topology can be either high pass or low pass depending on the choice of $X_S$ and $X_P$. Design proceeds by finding the magnitudes of $X_S$ and $X_P$. In two-element matching the circuit $Q$ is fixed. With $R_L = 75$ Ω and $R_S = 50$ Ω.

The $Q$ of the matching network is the same for the series and parallel elements:

$$|Q_S| = \frac{|X_S|}{R_S} = \sqrt{\frac{R_L}{R_S} - 1} = 0.7071 \quad \text{and} \quad |Q_P| = \frac{R_L}{|X_P|} = |Q_S| = 0.7071,$$

therefore $|X_S| = R_S \cdot |Q_S| = 50 \cdot 0.7071 = 35.35$ Ω. Also

$$|X_P| = R_L/|Q_P| = 75/0.7071 = 106.1 \text{ Ω}.$$

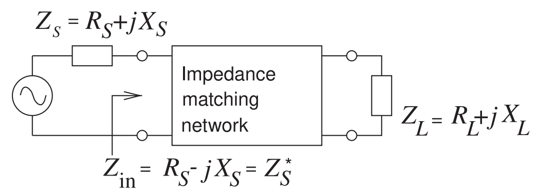
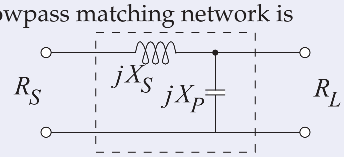



**Figure 6-11:** A matching network matching a complex load to a source with a complex Thevenin impedance.

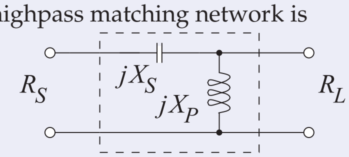
Specific element types can now be assigned to $X_S$ and $X_P$, and note that they must be of opposite type.

The lowpass matching network is




$X_S = +35.35\ \Omega, \quad X_P = -106.1\ \Omega.$

The highpass matching network is




$X_S = -35.35\ \Omega, \quad X_P = +106.1\ \Omega.$

This highpass design satisfies the design criterion that DC is not passed, as DC is blocked by the series capacitor.

## 6.5 Dealing with Complex Sources and Loads

This section presents strategies for dealing with complex loads. In the algorithmic matching approach design proceeds first by ignoring the complex load and source and then accounting for them either through topology choice or canceling their effect through resonance.

### 6.5.1 Matching

Input and output impedances of transistors, mixers, antennas, etc., contain both resistive and reactive components. Thus a realistic impedance matching problem looks like that shown in Figure 6-11. The matching approaches that were presented in the previous sections can be directly applied if $X_S$ and $X_L$ are treated as stray reactances that need to be either canceled or, ideally, used as part of the matching network. There are two basic approaches to handling complex impedances:

(a) Absorption: Absorb source and load reactances into the impedance matching network itself. This is done through careful placement of each matching element such that capacitors are placed in parallel with source and load capacitances, and inductors in series with source and load inductances. The stray values are then subtracted from the $L$ and $C$ values for the matching network calculated on the basis of the resistive parts of $Z_S$ and $Z_L$ only. The new (smaller) values, $L'$ and $C'$, constitute the elements of the matching network. Sometimes it is necessary to perform a series-to-parallel, or parallel-to-series, conversion of the source or load impedances so that the reactive elements are in the correct series or shunt arrangement for absorption.

(b) Resonance: Resonate source and load reactances with an equal and opposite reactance at the frequency of interest.

The very presence of a reactance in a load indicates energy storage, and therefore bandwidth limiting of some kind. In the above approaches to handling a reactive load, the resonance approach could easily result in a narrowband matching solution. The major problem in matching is often to obtain sufficient bandwidth. What is sufficient will vary depending on the application. To maximize bandwidth the general goal is to minimize the total energy storage. Roughly the total energy stored will be proportional to the sum of the magnitudes of the reactances in the circuit. Of course, the actual energy storage depends on the voltage and current levels, which will themselves vary in the circuit. A good approach leading to large bandwidths is to incorporate the load reactance into the matching network. Thus the choice of appropriate matching network topology is critical. However, if the source and load reactance value is larger than the calculated matching network element value, then absorption on its own cannot be used. In this situation resonance must be combined with absorption. Overall, the majority of impedance matching designs are based on some combination of resonance and absorption.

### EXAMPLE 6.5 Matching Network Design Using Resonance

For the configuration shown in Figure 6-12, design an impedance matching network that will block the flow of DC current from the source to the load. The frequency of operation is 1 GHz. Design the matching network, neglecting the presence of the 10 pF capacitance at the load. Since $R_S = 50\ \Omega < R_L = 500\ \Omega$, and from Figure 6-7, consider the topologies of Figures 6-13(a) and 6-13(b). The design criterion of blocking flow of DC from the source to the load narrows the choice to the topology of Figure 6-13(b).

**Solution:**

Step 1:

$$|Q_S| = \left|\frac{X_S}{R_S}\right| = |Q_P| = \left|\frac{R_L}{X_P}\right| = \sqrt{\frac{R_L}{R_S} - 1} = 3 \quad \text{and} \quad Q_P = \frac{R_L}{X_P}. \tag{6.31}$$

So $X_P = \omega L = R_L/Q_P = 500/3$. Reducing this gives

$$\omega L = \frac{500}{3}, \quad \text{and so} \quad L = \frac{500}{3 \times 2\pi \times 10^9} = 26.5\ \text{nH}. \tag{6.32}$$

Similarly $-X_S/R_S = 3$ and so

$$\frac{1/(\omega\, C)}{R_S} = 3 \quad \text{or} \quad C = \frac{1}{3\omega R_S} = \frac{1}{3 \times 2\pi \times 10^9 \times 50} = 1.06\ \text{pF}. \tag{6.33}$$

Step 2:

Resonate the 10 pF capacitor using an inductor in parallel:

$$(\omega L')^{-1} = \omega \times 10 \times 10^{-12} \tag{6.34}$$

$$L' = \frac{1}{\omega^2 10^{-11}} = \frac{1}{(2\pi)^2\, 10^{18} \times 10^{-11}} = 2.533\ \text{nH}. \tag{6.35}$$

Thus Figure 6-13(c) is the required matching network. Two inductors are in parallel and the circuit can be simplified to that shown in Figure 6-14, where

$$L_X = (26.5\ \text{nH} \parallel 2.533\ \text{nH}) = \frac{2.533 \times 26.5}{2.533 + 26.5}\ \text{nH} = 2.312\ \text{nH}. \tag{6.36}$$

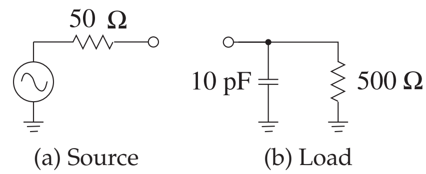



**Figure 6-12:** Matching problem considered in Example 6.5.

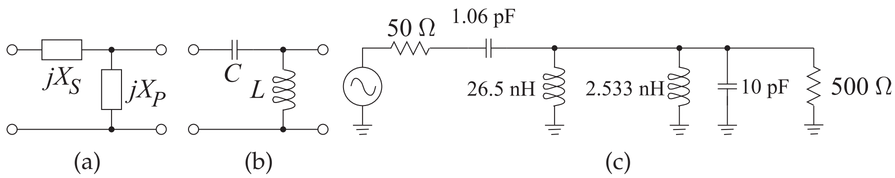



**Figure 6-13:** Matching network topology used in Example 6.5: a) and (b) topology; and (c) intermediate matching network.

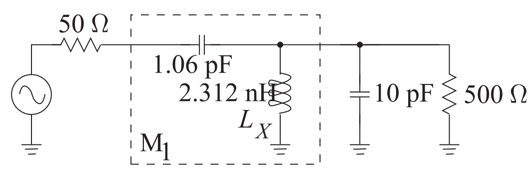



**Figure 6-14:** Final matching network in Example 6.5.

## EXAMPLE 6.6 Matching Network Design Using Resonance and Absorption

For the source and load configurations shown in Figure 6-15, design a lowpass impedance matching network at $f = 1$ GHz.

**Solution:**

Since $R_S < R_L$, use the topology shown in Figure 6-16(a). For a lowpass response, the topology is that of Figure 6-16(b). Notice that absorption is the natural way of handling the 3 nH at the source and the 5 pF at the load. The design process is as follows:

Step 1:

Design the matching network, neglecting the reactive elements at the source and load:

$$|Q_S| = |Q_P| = \sqrt{\frac{R_L}{R_S} - 1} = \sqrt{10 - 1} = 3 \tag{6.37}$$

$$\frac{X_S}{R_S} = 3, \quad X_S = 3 \times 100, \quad \omega L = 300 \quad \text{and} \quad L = \frac{300}{2\pi \times 10^9} = 47.75 \text{ nH} \tag{6.38}$$

$$\frac{R_P}{X_P} = -3 \quad \text{and} \quad \frac{1000}{-(1/\omega C)} = -3 \quad \text{and} \quad C = \frac{3}{1000 \times 2\pi \times 10^9} = 0.477 \text{ pF}. \tag{6.39}$$

This design is shown in Figure 6-17(a). This is the matching network that matches the 100 Ω source resistance to the 1000 Ω load with the source and load reactances ignored.

Step 2:

Figure 6-17(b) is the interim matching solution. The source inductance is absorbed into the matching network, reducing the required series inductance of the matching network. The capacitance of the load cannot be fully absorbed. The design for the resistance-only case requires a shunt capacitance of 0.477 pF, but 5 pF is available from the load. Thus there is an excess capacitance of 4.523 pF that must be resonated out by the inductance $L''$:

$$\frac{1}{\omega L''} = \omega 4.523 \times 10^{-12}. \quad \text{So} \quad L'' = \frac{1}{(2\pi)^2 \times 10^{18} \times 4.523 \times 10^{-12}} = 5.600 \text{ nH}. \tag{6.40}$$

The final matching network design (Figure 6-17(c)) fully absorbs the source inductance into the matching network, but only partly absorbs the load capacitance.

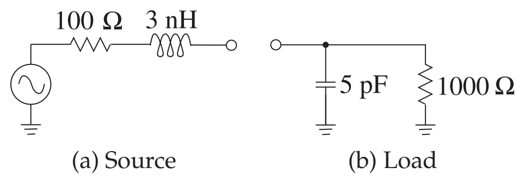



Figure 6-15: Matching problem in Example 6.6.

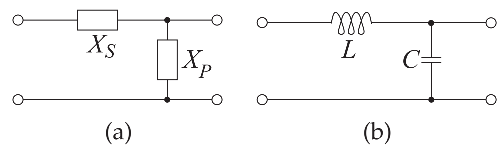



Figure 6-16: Topologies referred to in Example 6.6.

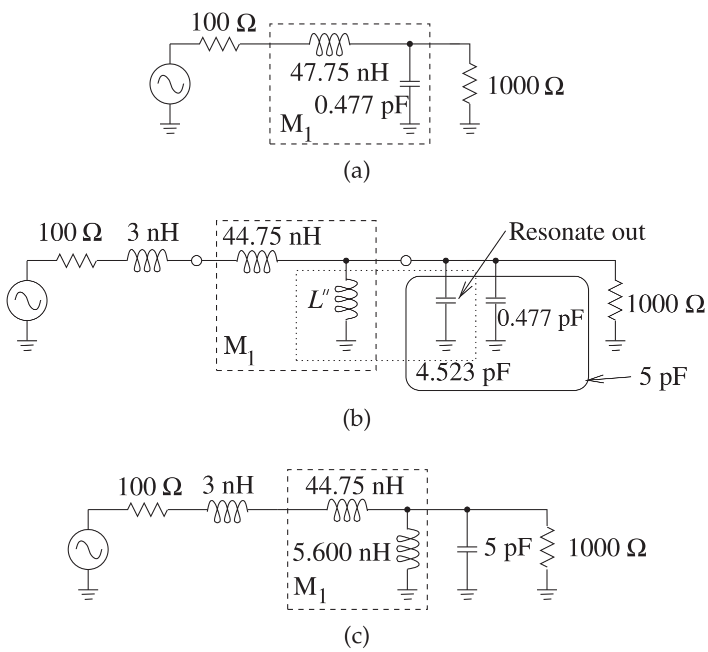



Figure 6-17: Evolution of the matching network in Example 6.6: (a) matching network design considering only the source and load resistors; (b) matching network with the reactive parts of the source and load impedances included; and (c) final design.

## 6.6 Multielement Matching

The bandwidth of a matching network can be controlled by using multiple matching stages either making the matching bandwidth wider or narrower. This concept is elaborated on in this and several design approaches presented.

### 6.6.1 Design Concept for Manipulating Bandwidth

The concept for manipulating matching network bandwidth is to do the matching in stages as shown in Figure 6-18. Figure 6-18(a) shows the one-stage matching problem using the common identification of the matching network as 'M'. The one-stage matching problem is repeated in Figure 6-18(b) without explicitly showing the source generator. A two-stage matching problem is shown in Figure 6-18(c) with the introduction of a virtual resistor $R_V$ between the first, $M_1$, and second, $M_2$, stage matching networks. $R_V$ is shown as a virtual connection as it is not actually inserted in the circuit. Instead this is a short-hand way of indicating the matching problem to be done in two stages as shown in Figure 6-18(d and e) with the first stage matching the source resistor $R_S$ to $R_V$ and the second stage matching $R_V$

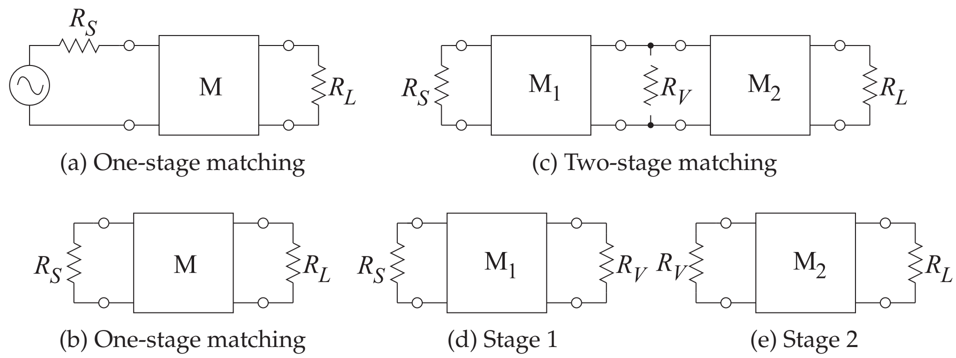



**Figure 6-18:** Matching in stages: (a) matching network M matching $R_S$ to $R_L$; (b) without an explicit source; (c) two-stage matching with a virtual resistor $R_V$; (d) matching $R_S$ to $R_V$; and (e) matching $R_V$ to $R_L$.

to the load resistor $R_L$. After $M_1$ has been designed the resistance looking into the right-hand port of $M_1$, see Figure 6-18(d), will be $R_V$ so $R_V$ is the Thevenin equivalent source resistance to $M_2$. Similarly the input impedance looking into the left-hand port of $M_2$ is $R_V$ so $R_V$ is the effective load resistor of $M_1$. Of course these are the impedances at the center frequency and away from the center frequency of the match the input impedances will be complex.

The concept behind multi-stage matching network design is shown in Figure 6-19 where the standard one-stage match is shown in Figure 6-19(a). While this is shown for $R_L > R_S$ the concept holds for $R_L < R_S$. The arrows follow the convention that design begins with the load and ends at the source. With the one stage match the circuit $Q$ is fixed designated here as the total circuit $Q$, $Q_T$ being the same as the $Q$ of the one-stage $R_L$ to $R_S$ matching network, $Q_{SL}$. The two-stage match that reduces bandwidth (compared to the one-stage match) is shown in Figure 6-19(b). The total $Q$, $Q_T$, of the second stage is higher than for the one-stage design because the ratio of $R_L$ to $R_V$ is greater than the ratio of $R_V$ to $R_S$. Bandwidth can also be reduced relative to the one-stage match by assigning $R_V$ to be greater than both $R_L$ and $R_S$, see Figure 6-19(c).

Choosing $R_V$ to be between $R_S$ and $R_L$ will result in a circuit with lower $Q_T$ and the bandwidth of the match will increase, see Figure 6-19(d). The maximum bandwidth for a two-stage match is to choose $R_V$ as the geometric mean of $R_S$ and $R_L$. This concept can be extended to multiple stages as shown for a three-stage match in Figure 6-19(e).

This section presents various matching network designs for manipulating bandwidth and all are based on the concept of choosing a virtual resistor.

### 6.6.2 Three-Element Matching Networks

With the L network (i.e., two-element matching), the circuit $Q$ is fixed once the source and load resistances, $R_S$ and $R_L$, are fixed:

$$Q = \sqrt{\frac{R_L}{R_S} - 1}, \quad (R_L > R_S). \tag{6.41}$$

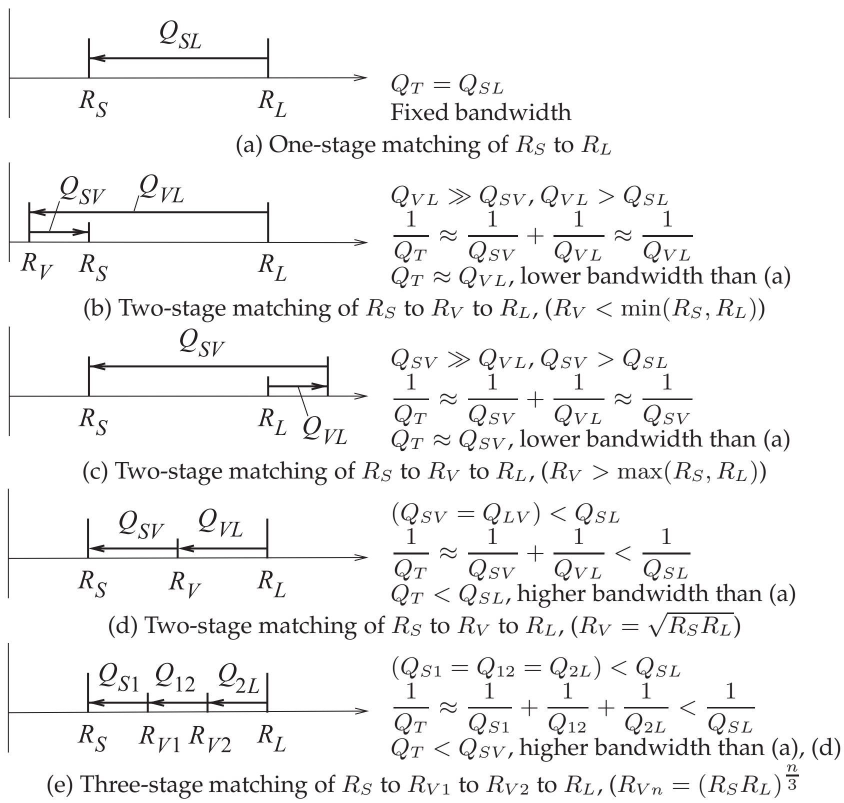



**Figure 6-19:** Effect of multi-stage matching on total circuit $Q$, $Q_T$, and matching bandwidth (which is approximately inversely proportional to $Q_T$.)

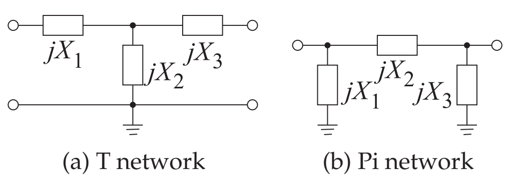



**Figure 6-20:** Two three-element matching networks.

Thus the designer does not have a choice of circuit $Q$. Breaking the matching problem into parts enables the circuit $Q$ to be controlled. Introducing a third element in the matching network provides the extra degree of freedom in the design for adjusting $Q$, and hence bandwidth.

Two three-element matching networks, the T network and the Pi network, are shown in Figure 6-20. Which network is used depends on

(a) the realization constraints associated with the specific design, and
(b) the nature of the reactive parts of the source and load impedances and whether they can be used as part of the matching network.

The three-element matching network comprises 2 two-element (or L) matching networks and is used to increase the overall $Q$ and thus narrow bandwidth. Given $R_S$ and $R_L$, the circuit $Q$ established by an L matching network is the minimum circuit $Q$ available in the three-element matching arrangement. With three-element matching, the $Q$ can only increase, so three-element matching is used for narrowband (high-$Q$) applications. However, lower $Q$ can be obtained with more than three elements. The next subsections consider matching with more than three elements.

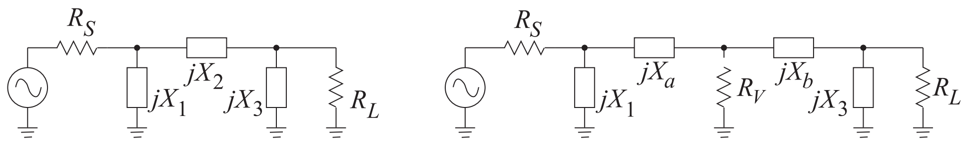



**Figure 6-21:** Pi matching networks: (a) view of a Pi network; and (b) as two back-to-back L networks with a virtual resistance, $R_V$, between the networks.

### *6.6.3 The Pi Network*

The Pi network may be thought of as two back-to-back L networks that are used to match the load and the source to a virtual resistance, $R_V$, placed at the junction between the two networks, as shown in Figure 6-21(b). The design of each section of the Pi network is as for the L network matching. $R_S$ is matched to $R_V$ and $R_V$ is matched to $R_L$.
$R_V$ must be selected smaller than $R_S$ and $R_L$ since it is connected to the series arm of each L section. Furthermore, $R_V$ can be any value that is smaller than the smaller of $R_S, R_L$. However, it is customarily used as the design parameter for specifying the desired $Q$.
As a useful design approximation, the loaded $Q$ of the Pi network can be taken as the $Q$ of the L section with the highest $Q$:

$$Q = \sqrt{\frac{\max(R_S, R_L)}{R_V} - 1}. \tag{6.42}$$

Given $R_S$, $R_L$, and $Q$, the above equation yields the value of $R_V$.

**EXAMPLE 6.7** Three-Element Matching Network Design

Design a Pi network to match a 50 Ω source to a 500 Ω load. The desired $Q$ is 10. A suitable matching network topology is shown in Figure 6-22 together with the virtual resistance, $R_V$, to be used in design.

**Solution:** $R_S = 50\ \Omega$ and $R_L = 500\ \Omega$ so $\max(R_S, R_L) = 500\ \Omega$ and so the virtual resistor is

$$R_V = \frac{\max(R_S, R_L)}{Q^2 + 1} = \frac{500}{101} = 4.95\ \Omega. \tag{6.43}$$

Design proceeds by separately designing the L networks to the left and right of $R_V$. For the L network on the left,

$$Q_{\text{left}} = \sqrt{\frac{50}{4.95} - 1} = 3.017. \quad \text{so} \quad Q_{\text{left}} = \frac{|X_a|}{R_V} = \frac{R_S}{|X_1|} = 3.017, \tag{6.44}$$

Note that $X_1$ and $X_a$ must be of opposite types (one is capacitive and the other is inductive). The left L network has elements

$$|X_a| = 14.933\ \Omega \quad \text{and} \quad |X_1| = 16.6\ \Omega. \tag{6.45}$$

For the L network on the right of $R_V$,

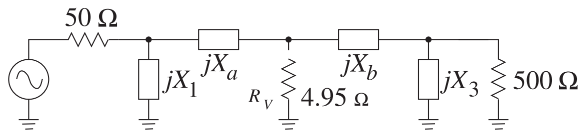



**Figure 6-22:** Matching network problem of Example 6.7.

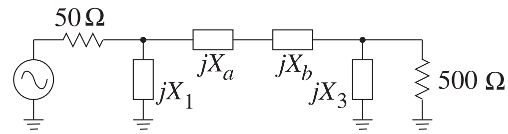



**Figure 6-23:** Final matching network in Example 6.7.

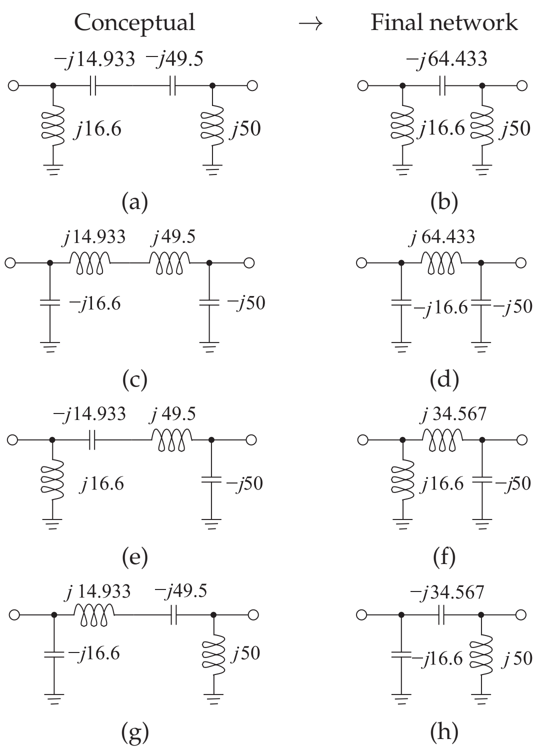



**Figure 6-24:** Four possible Pi matching networks: (a), (c), (e), and (g) conceptual circuits; and (b), (d), (f), and (h), respectively, their final reduced Pi networks.

$$Q_{\text{right}} = Q = 10, \quad \text{thus} \quad \frac{|X_b|}{R_V} = \frac{R_L}{X_3} = 10. \tag{6.46}$$

$X_b$, $X_3$ are of opposite types, and

$$|X_b| = 49.5\ \Omega \quad \text{and} \quad |X_3| = 50\ \Omega. \tag{6.47}$$

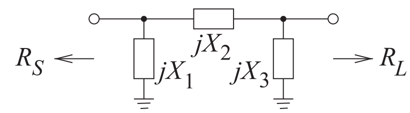
The resulting Pi network is shown in Figure 6-23 with the values

$$|X_1| = 16.6\ \Omega, \quad |X_3| = 50\ \Omega, \quad |X_a| = 14.933\ \Omega, \quad \text{and} \quad |X_b| = 49.5\ \Omega. \tag{6.48}$$

Note that the pair $X_a$, $X_1$ are of opposite types and similarly $X_b$, $X_3$ are of opposite types. So there are four possible realizations, as shown in Figure 6-24.

In the previous example there were four possible realizations of the three-element matching network, and this is true in general. The specific choice of one of the four possible realizations will depend on specific application-related factors such as

**Figure 6-25:** A three-element matching network.

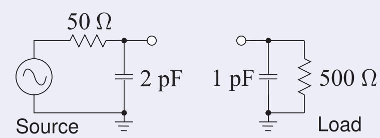



(a) elimination of stray reactances,
(b) the need to pass or block DC current, and
(c) the need for harmonic filtering.

It is fortunate that it may be possible to achieve multiple functions with the same network.

## EXAMPLE 6.8 Three-Element Matching with Reactive Source and Load

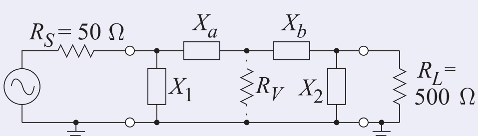
Design a Pi network to match the source to the load shown. The design frequency is 900 MHz and the desired $Q$ is 10.




**Solution:**

The design objective is to arrive at an overall network which has a $Q$ of 10. To achieve this it is necessary to absorb the source and load reactances into the matching network. If they were resonated instead, the overall $Q$ of the network can be expected to higher than the $Q$ of the L matching network on its own.

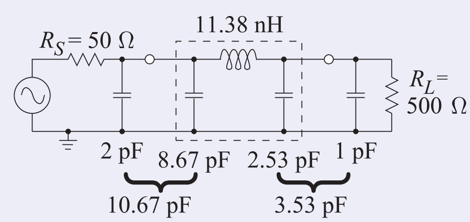
Design begins by considering the matching of $R_S = 50\ \Omega$ to $R_L = 500\ \Omega$. Since the $Q$ is specified, three (or more) matching elements must be used. The design starting point is shown on the right:




The virtual resistor $R_V = \max(R_s, R_L)/(1 + Q^2) = (500\ \Omega)/(1 + 100) = 4.95\ \Omega$. The left subnetwork with $X_1$ and $X_a$ has $Q_{\text{LEFT}} = \sqrt{R_S/R_V - 1} = \sqrt{50/4.95 - 1} = 3.017$. The right subnetwork with $X_2$ and $X_b$ has $Q_{\text{RIGHT}} = \sqrt{R_L/R_V - 1} = \sqrt{500/4.95 - 1} = 10.001$.

Note that $Q_{\text{RIGHT}}$ is almost exactly the desired $Q$ of the network and $Q_{\text{LEFT}}$ will have little effect on the $Q$ of the overall circuit. Now $Q_{\text{LEFT}} = |X_a|/R_V = R_S/|X_1|$, so $|X_a| = 14.9\ \Omega$ and $|X_1| = 16.57\ \Omega$. $Q_{\text{RIGHT}} = |X_b|/R_V = R_S/|X_2|$, so $|X_b| = 49.5\ \Omega$ and $|X_2| = 50.0\ \Omega$.

$X_1$ must be chosen to be a capacitor $C_1 = 10.67$ pF so that the 2 pF source capacitance can be absorbed. Similarly $X_2$ is a capacitor $C_2 = 3.53$ pF. $X_a$ and $X_b$ are both inductors that combine in series for a total inductance $L_3 = 11.38$ nH. This leads to the final design shown on the right where the matching network is in the dashed box.




### 6.6.4 Matching Network $Q$ Revisited

To demonstrate that the circuit $Q$ established by an L matching network is the minimum circuit $Q$ for a network having at most three elements, consider the design equations for $R_S > R_L$. Referring to Figure 6-25,

$$X_1 = \frac{R_S}{Q},\ X_3 = R_L \left( \frac{R_S/R_L}{Q^2 + 1 - R_S/R_L} \right)^{\frac{1}{2}},\ X_2 = \frac{QR_S + R_S R_L/X_3}{Q^2 + 1}. \quad (6.49)$$

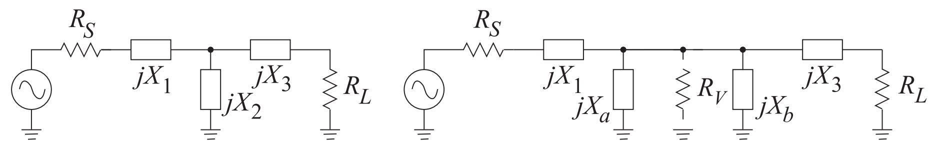



**Figure 6-26:** T network design approach.

Notice that the denominator of $X_3$ can be written as

$$Q^2 + 1 - \frac{R_S}{R_L} = \left( Q + \sqrt{\frac{R_S}{R_L} - 1} \right) \left( Q - \sqrt{\frac{R_S}{R_L} - 1} \right). \tag{6.50}$$

Then for a real solution we must have

$$Q \geq \sqrt{\frac{R_S}{R_L} - 1}, \tag{6.51}$$

and so $Q \geq Q_{\text{L network}}$. For $Q = Q_{\text{L network}}$, $X_3 \rightarrow \infty$ and the Pi network reduces to an L network that has two elements. Thus it is not possible to have a lower $Q$ with a three-element matching network than the $Q$ of a two-element matching network. Thus a three-element matching network must have lower bandwidth than that of a two-element matching network.

### 6.6.5 The T Network

The T network may be thought of as two back-to-back L networks that are used to match the load and the source to a virtual resistance, $R_V$, placed at the junction between the two L networks (see Figure 6-26). $R_V$ must be selected to be larger than both $R_S$ and $R_L$ since it is connected to the shunt leg of each L section. $R_V$ is chosen according to the equation

$$Q = \sqrt{\frac{R_V}{\min(R_S, R_L)} - 1}, \tag{6.52}$$

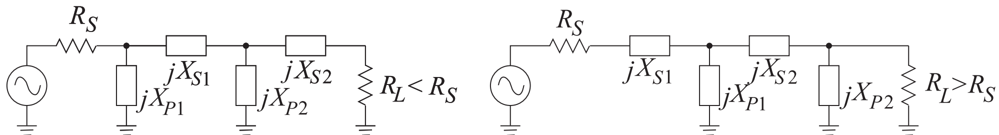
where $Q$ is the desired loaded $Q$ of the network. Each L network is calculated in exactly the same manner as was done for the Pi network matching. That is, $R_S$ is matched to $R_V$ and $R_V$ is matched to $R_L$. Once again there will be four possible designs for the T network, given $R_S$, $R_L$, and $Q$.

### 6.6.6 Broadband (Low $Q$) Matching

L network matching does not allow the circuit $Q$, and hence bandwidth, to be selected. However, Pi network and T network matching allows the circuit $Q$ to be selected independent of the source and load impedances, provided that the chosen $Q$ is larger than that which can be obtained with an L network. Thus the Pi and T networks result in narrower bandwidth designs.

One design solution for broadband matching is to use two (or more) series-connected L sections (see Figure 6-27). Design is still based on the concept of a virtual resistor, $R_V$, placed at the junction of the two L networks (as in Figure 6-28), but now $R_V$ is chosen to be between $R_S$ and $R_L$:

$$R_{\min} \leq R_V \leq R_{\max}, \tag{6.53}$$




(a) (b)

**Figure 6-27:** Broadband matching networks.

**Figure 6-28:** Matching network with two L networks.

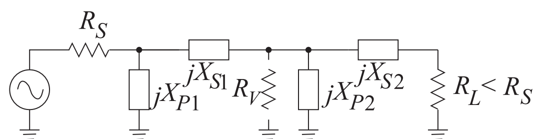
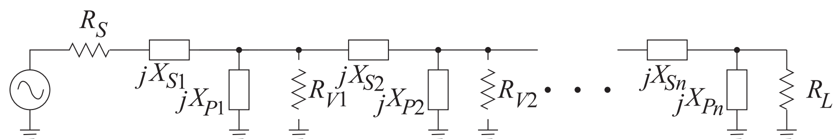



**Figure 6-29:** Cascaded L networks for broadband matching.




where $R_{\min} = \min(R_L, R_S)$ and $R_{\max} = \max(R_L, R_S)$. Then one of the two networks will have

$$Q_1 = \sqrt{\frac{R_V}{R_{\min}} - 1} \quad \text{and the other} \quad Q_2 = \sqrt{\frac{R_{\max}}{R_V} - 1}. \tag{6.54}$$

The maximum bandwidth (minimum $Q$) available is obtained when

$$Q_1 = Q_2 = \sqrt{\frac{R_V}{R_{\min}} - 1} = \sqrt{\frac{R_{\max}}{R_V} - 1}\ . \tag{6.55}$$

That is, the maximum matching bandwidth is obtained when $R_V$ is the geometric mean of $R_S$ and $R_L$:

$$R_V = \sqrt{R_L R_S}. \tag{6.56}$$

Even wider bandwidths can be obtained by cascading more than two L networks, as shown in Figure 6-29. In this circuit

$$R_S < R_{V1} < R_{V2} \ldots < R_{Vn-1} < R_L. \tag{6.57}$$

For optimum bandwidth the ratios should be equal,

$$\frac{R_{V_1}}{R_S} = \frac{R_{V_2}}{R_{V_1}} = \frac{R_{V_3}}{R_{V_2}} = \cdots = \frac{R_L}{R_{V_{n-1}}}, \tag{6.58}$$

and the $Q$ is given by

$$Q = \sqrt{\frac{R_{V_1}}{R_S} - 1} = \sqrt{\frac{R_{V_2}}{R_1} - 1} = \cdots = \sqrt{\frac{R_L}{R_{V_{n-1}}} - 1}. \tag{6.59}$$

If there are $N$ L networks used in the match, the maximum bandwidth will be obtained if the $i$th virtual resistor is

$$R_{Vi} = (R_S R_L)^{i/N}, \quad i = 1, \ldots, (N-1). \tag{6.60}$$

**EXAMPLE 6.9** Two-Section Matching Network Design

Consider matching a 10 Ω source to a 1000 Ω load using two L matching networks and designing for a $Q$ of 3. How many matching sections are required?

**Solution:**

Here the approximate $Q$s achieved with a single L matching network and with an optimum two-section design are compared. For a single L network design

$$Q = \sqrt{\frac{R_L}{R_S} - 1} = 9.95. \tag{6.61}$$

Now consider an optimum two-section design:

$$R_V = \sqrt{R_S R_L};\ Q_2 = \sqrt{\frac{R_L}{R_V} - 1} = \sqrt{\sqrt{\frac{R_L}{R_S}} - 1} = 3. \tag{6.62}$$

Thus the $Q$ is 3 compared to the $Q$ of an L section of 9.95. If the fractional bandwidth is inversely proportional to $Q$, then the bandwidth of the two-section design is 9.95/3 = 3.32 times more than that of the L section.

Now consider how many sections are required to obtain a $Q$ of 2:

$$(1 + Q^2) = \frac{R_{V_1}}{R_S} = \frac{R_{V_2}}{R_{V_1}} = \ldots = \frac{R_L}{R_{V_{n-1}}} \Rightarrow \tag{6.63}$$

$$\left(1 + Q^2\right)^n = \frac{R_L}{R_S} \Rightarrow n \ln\left(1 + Q^2\right) = \ln\frac{R_L}{R_S} \Rightarrow n = \frac{\ln(R_L/R_S)}{\ln\left(1 + Q^2\right)}. \tag{6.64}$$

For $Q = 2$ and $R_L/R_S = 100$, $n = 2.86$, which rounds to $n = 3$, and three sections are required.

## 6.7 Impedance Matching Using Smith Charts

The lumped-element matching networks presented up to now can also be developed using Smith charts which provide a fairly intuitive approach to network design. With experience it will be found that this is the preferred approach to developing designs, as trade-offs can be captured graphically. Smith chart-based design will be presented using examples.

### *6.7.1 Two-Element Matching*

The examples here build on the preceding lumped-element matching network designs now using the Smith chart introduced in Chapter 3. Capacitive and inductive regions on the Smith chart are shown in Figure 6-30. In the design examples presented here, circles of constant resistance or constant conductance are followed and these correspond to varying reactance or susceptance, respectively.

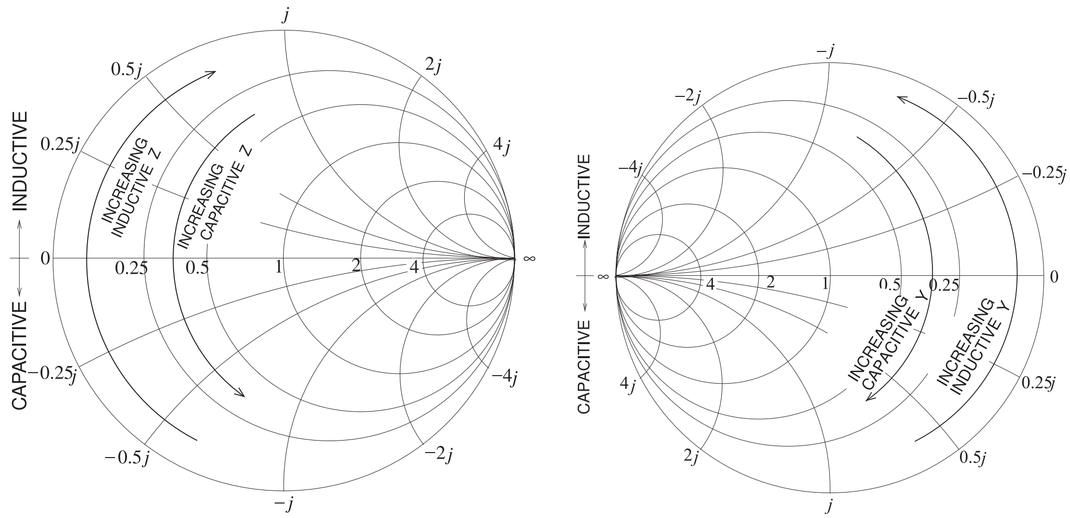



**Figure 6-30:** Inductive and capacitive regions on Smith charts. Increasing capacitive impedance ($Z$) indicates smaller capacitance; increasing inductive admittance ($Y$) indicates smaller inductance.

## EXAMPLE 6.10 Two-Element Matching Network Design Using a Smith Chart

Develop a two-element matching network to match a source with an impedance of $R_S = 25\ \Omega$ to a load $R_L = 200\ \Omega$ (see Figure 6-31).

**Solution:**

The design objective is to present conjugate matched impedances to the source and load. However, since here the source and load impedances are real, the design objective is $Z_1 = R_S$ and $Z_2 = R_L$. The load and source resistances are plotted on the Smith chart in Figure 6-33(a) after choosing a normalization impedance of $Z_0 = 50\ \Omega$ (and so $r_S = R_S/Z_0 = 0.5$ and $r_L = R_L/Z_0 = 4$). The normalized source impedance, $r_S$, is Point A, and the normalized load impedance, $r_L$, is Point C. The matching network must be lossless, which means that the design must follow lines of constant resistance (on the impedance part of the Smith chart) or constant conductance (on the admittance part of the Smith chart). So Points A and C must be on the above circles and the circles must intersect if a design is possible. The design can be viewed as moving back from the source toward the load or moving back from the load toward the source. (The views result in identical designs.) Here the view taken is moving back from the source toward the load.

One possible design is shown in Figure 6-33(a). From Point A, the line of constant resistance is followed to Point B (there is increasing series reactance along this path). From Point B, the locus follows a line of constant conductance to the final point, Point C. There is also an alternative design that follows the path shown in Figure 6-33(b). There are only two designs that have a path from A to B following just two arcs. At this point two designs have been outlined. The next step is assigning element values.

The design shown in Figure 6-33(a) begins with $r_S$ followed by a series reactance, $x_S$, taking the locus from A to B. Then a shunt capacitive susceptance, $b_P$, takes the locus from B to C and $r_L$. At Point A the reactance $x_A = 0$, at Point B the reactance $x_B = 1.323$. This value is read off the Smith chart, requiring that an arc as shown be interpolated between the arcs provided. It should be noted that not all versions of Smith charts include negative signs, as the chart becomes too complicated. Thus the user needs to be aware and add signs where appropriate. The normalized series reactance is

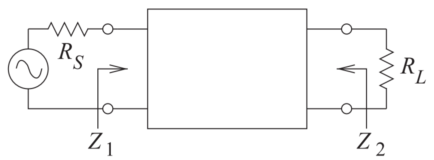
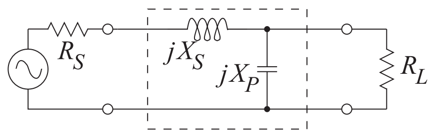
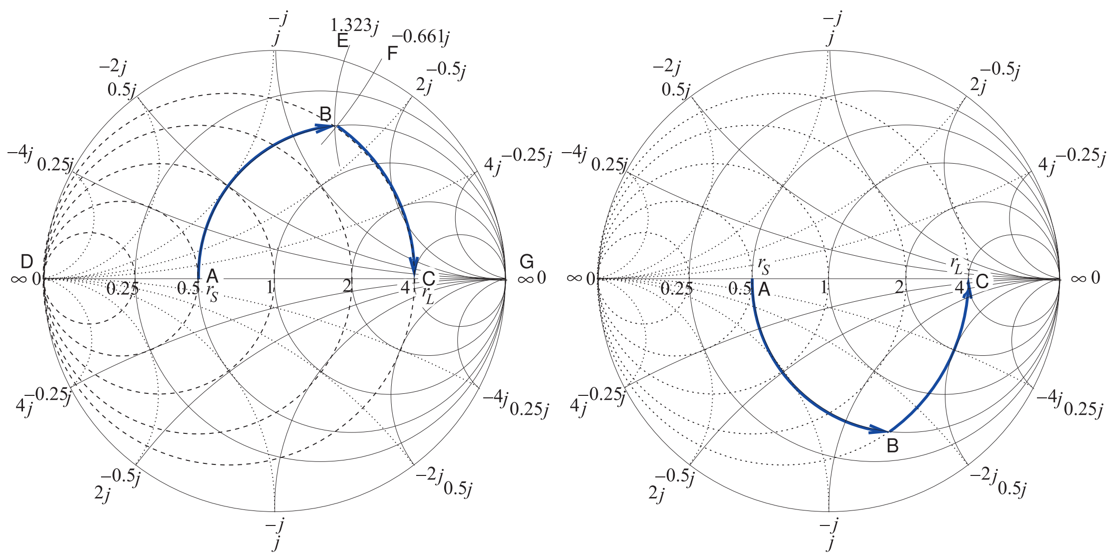



**Figure 6-31:** Design objectives for Example 6.10. $R_S = 25\ \Omega$, $R_L = 200\ \Omega$.




$X_S = 66.1\ \Omega$.  $X_P = -75.6\ \Omega$

**Figure 6-32:** Final design for Example 6.10 using the path shown in Figure 6-33(a).




(a) Design 1 (b) Design 2

**Figure 6-33:** Alternative designs for Example 6.10. The normalization impedance is $50\ \Omega$.

$$x_S = x_B - x_A = 1.323 - 0 = 1.323, \tag{6.65}$$

that is,

$$X_S = x_s Z_0 = 1.323 \times 50 = 66.1\ \Omega. \tag{6.66}$$

A shunt capacitive element takes the locus from Point B to Point C and

$$b_P = b_C - b_B = 0 - (-0.661) = 0.661, \tag{6.67}$$

so

$$B_P = b_P/Z_0 = 0.661/50 = 13.22\ \text{mS} \quad \text{or} \quad X_P = -1/B_P = -75.6\ \Omega. \tag{6.68}$$

The final design is shown in Figure 6-32.

One of the advantages of using the Smith chart is that the design progresses in stages, with the structure of the design developed before actual numerical values are calculated. Of course, it is difficult to extract accurate values from a chart, so designs are regularly roughed out on a Smith chart and refined using CAD tools. Example 6.10 matched a resistive source to a resistive load. The next example considers the matching of complex load and source impedances. In the earlier algorithmic approach to matching network design absorption and resonance were introduced as strategies for dealing with complex terminations. Design was not always straightforward. It will be seen that this complication disappears with a Smith chart-based design, as it is conceptually not much different from the resistive problem of Example 6.10.

## EXAMPLE 6.11 Matching Network Design With Complex Impedances

Develop a two-element matching network to match a source with an impedance of $Z_S = 12.5 + 12.5\jmath\ \Omega$ to a load $Z_L = 50 - 50\jmath\ \Omega$, as shown in Figure 6-34.

**Solution:**

The design objective is to present conjugate matched impedances to the source and load; that is, $Z_1 = Z_S^*$ and $Z_2 = Z_L^*$. The choice here is to design for $Z_1$; that is, elements will be inserted in front of $Z_L$ to produce the impedance $Z_1$. The normalized source and load impedances are plotted in Figure 6-35(a) using a normalization impedance of $Z_0 = 50\ \Omega$, so $z_S = Z_S/Z_0 = 0.25 + 0.25\jmath$ (Point S) and $z_L = Z_L/Z_0 = 1 - \jmath$ (Point C).

The impedance to be synthesized is $z_1 = Z_1/Z_0 = z_S^* = 0.25 - 0.25\jmath$ (Point A). The matching network must be lossless, which means that the lumped-element design must follow lines of constant resistance (on the impedance part of the Smith chart) or constant conductance (on the admittance part of the Smith chart). Points A and C must be on the above circles and the circles must intersect if a design is possible.

The design can be viewed as moving back from the load impedance toward the conjugate of the source impedance. The direction of the impedance locus is important. One possible design is shown in Figure 6-35(a). From Point C the line of constant conductance is followed to Point B (there is increasing positive [i.e., capacitive] shunt susceptance along this path). From Point B the locus follows a line of constant resistance to the final point, Point A.

The design shown in Figure 6-35(a) begins with a shunt susceptance, $b_P$, taking the locus from Point C to Point B and then a series inductive reactance, $x_S$, taking the locus to Point A. At Point C the susceptance $b_C = 0.5$, at Point B the susceptance $b_B = 1.323$. This value is read off the Smith chart, requiring that an arc of constant susceptance, as shown, be interpolated between the constant susceptance arcs provided. The normalized shunt susceptance is

$$b_P = b_B - b_C = 1.323 - 0.5 = 0.823, \tag{6.69}$$

that is,

$$B_P = b_P/Z_0 = 0.823/(50\ \Omega) = 16.5\ \text{mS or } X_P = -1/B_P = -60.8\ \Omega. \tag{6.70}$$

A series reactive element takes the locus from Point B to Point A, so

$$x_S = x_A - x_B = -0.25 - (-0.661) = 0.411, \tag{6.71}$$

so

$$X_S = x_S Z_0 = 0.411 \times 50\ \Omega = 20.6\ \Omega. \tag{6.72}$$

The final design is shown in Figure 6-36.

There are only two designs that have a path from Point C to Point A following just two arcs. In Design 1, shown in Figure 6-35(a), Path CBA is much shorter than Path CHA for Design 2 shown in Figure 6-35(b). The path length is an approximate indication of the total reactance required, and the higher the reactance, the greater the energy storage and hence the narrower the bandwidth of the design. (The actual relative bandwidth depends on the voltage and current levels in the network; the path length criteria, however, is an important rule of thumb.) Thus Design 1 can be expected to have a much higher bandwidth than Design 2. Since designing broader bandwidth is usually an objective, a design requiring a shorter path on a Smith chart is usually preferable.

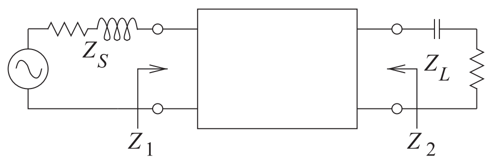
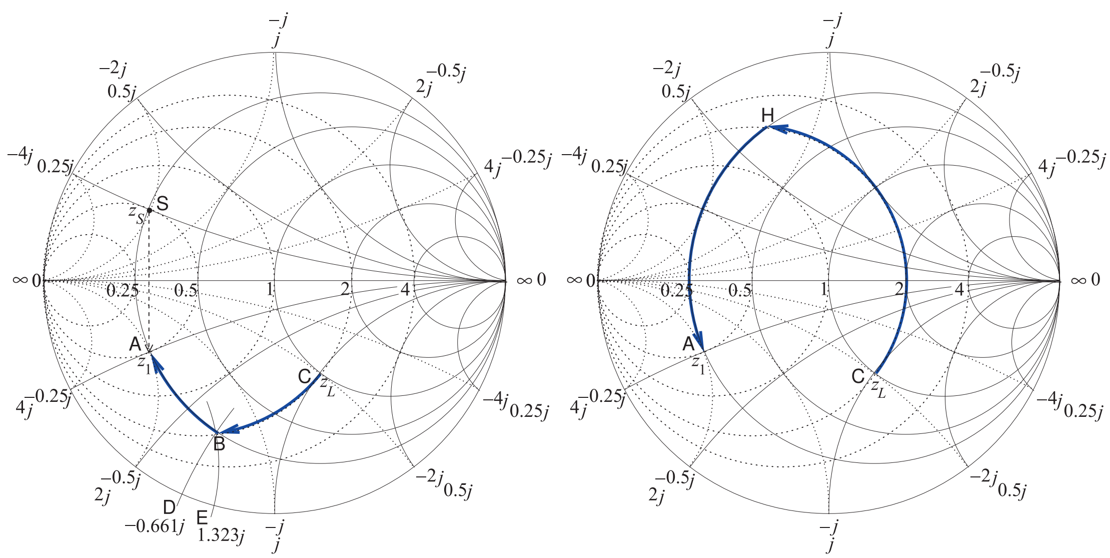



**Figure 6-34:** Design objectives for Example 6.11.




(a) Design 1

(b) Design 2

**Figure 6-35:** Smith chart-based designs used in Example 6.11. (50 Ω normalization used.)

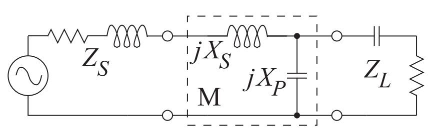



**Figure 6-36:** Final circuit for Design 1 of Example 6.11. $X_S = 20.6\ \Omega$, $X_P = -60.8\ \Omega$.

## 6.8 Distributed Matching

Matching using lumped elements leads to series and shunt lumped elements. The shunt elements can be implemented using shunt transmission lines, as a short length (less than one-quarter wavelength long) of short-circuited transmission line looks like an inductor and a short section of open-circuited transmission line looks like a capacitor. However, in most transmission line technologies it is not possible to realize the series elements as lengths of transmission lines. While it has been shown that a short length of transmission line is inductive, replacing series inductors by a length of transmission line of high characteristic impedance is not the best approach to realizing networks. The solution is to use lengths of transmission line together with shunt elements. If space is not at a premium, this is an optimum solution, as transmission lines have much lower loss than a lumped inductor. The series transmission lines rotate the reflection coefficient on the Smith chart.

As with all matching design, using transmission lines begins with a topology in mind. Several topologies are shown in Figure 6-37. Figure 6-37(a) is the

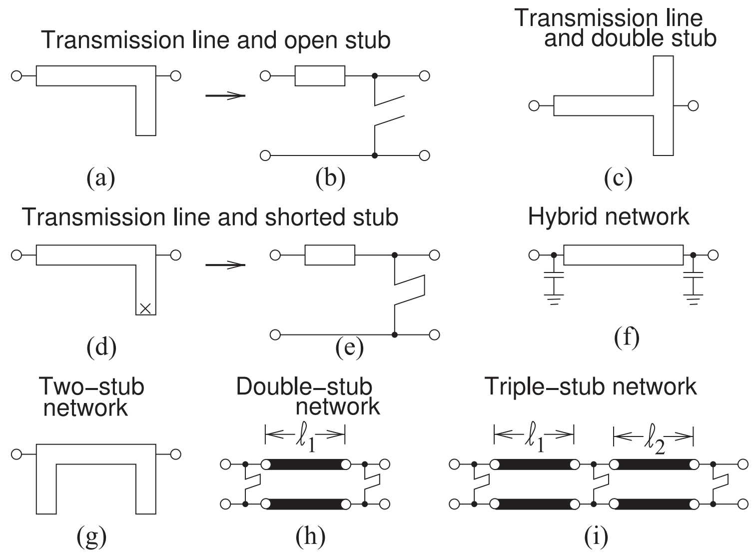
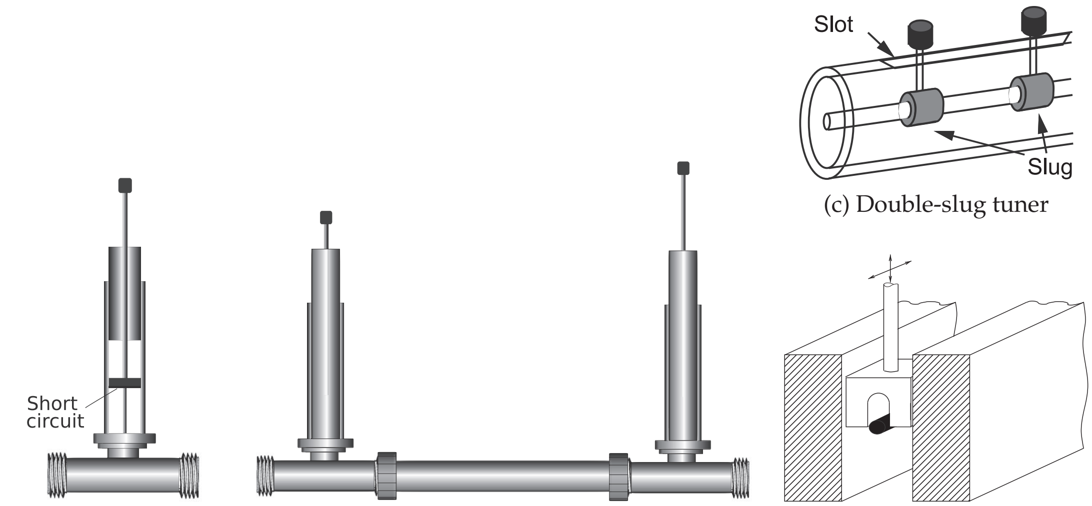



**Figure 6-37:** Matching networks with transmission line elements.

top view of a microstrip matching network with a series transmission line and stub realized as an open-circuited transmission line. Figure 6-37(b) is a shorthand schematic for this circuit. Matching network design then becomes a problem of choosing the lengths and characteristic impedances of the lines. The stub here is used to realize a capacitive shunt element. This network corresponds to two-element matching with a shunt capacitor. The value of the shunt capacitance can be increased using a dual stub, as shown in Figure 6-37(c), where the capacitive input impedances of each stub are in parallel. The dual circuit to that in Figure 6-37(a) is shown in Figure 6-37(d) together with its schematic representation in Figure 6-37(e). This circuit has a short-circuited stub that realizes a shunt inductance.

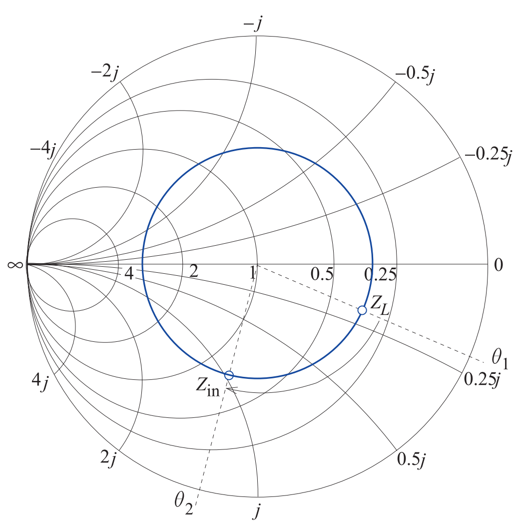
Mixing lumped capacitors with a transmission line element, as shown in Figure 6-37(f), realizes a much more space-efficient network design. There are many variations to stub-based matching network design, including the two-stub design in Figure 6-37(g).

A common situation encountered in the laboratory is the matching of circuits that are in development. Laboratory items available for matching include the **stub tuner**, shown in Figure 6-38(a), and the **double-stub tuner**, shown in Figure 6-38(b). With the double-stub tuner the length of the series transmission line is fixed, but stubs can have variable length using lengths of transmission lines with sliding short circuits. Not all impedances can be matched using a double stub tuner, however. A triple-stub tuner can match all impedances presented to it [2]. The **double-slug tuner** shown in Figure 6-38(c) has dielectric slugs each of which introduces a short section of lower impedance line. The slugs are moved up and down the line and avoid the rapid changes in impedances that occur with the stub tuners and as a result the double-slug tuner provides a broader bandwidth match than does the double stub tuner.. The **slide-screw slug tuner** shown in Figure 6-38(d) can achieve a broadband match. Here a metal slug can be lowered into the slabline changing the impedance of a section of transmission line and mostly affects the magnitude of the reflection coefficient while moving the metal slug along the line mostly affects the phase. This is the type of tuner incorporated in computer-controlled automated tuners.




(a) Single-stub tuner (b) Double-stub tuner (c) Double-slug tuner (d) Slide-screw slug tuner

**Figure 6-38:** Laboratory tuners.



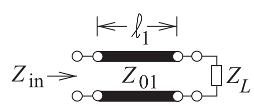




**Figure 6-39:** Rotation of the input impedance of a transmission line on a Smith chart normalized to $Z_{01}$ as the line length increases.

## 6.8.1 Stub Matching

In this section matching using one series transmission line and one stub will be considered. This corresponds to the microstrip circuit topologies shown in Figures 6-37(a and d). First, consider the terminated transmission line shown in Figure 6-39. When the length, $\ell_1$, of the line is zero, the input impedance of the line, $Z_{\text{in}}$, equals $Z_L$. How it changes is best described by considering the input reflection coefficient, $\Gamma_{\text{in}}$, of the line. If the reflection coefficient is normalized to $Z_{01}$, then the magnitude of $\Gamma_{\text{in}}$ and its phase varies as twice the electrical length of the line. This situation is shown on the Smith chart in Figure 6-39, where $Z_L$ is chosen arbitrarily. The input reflection coefficient of

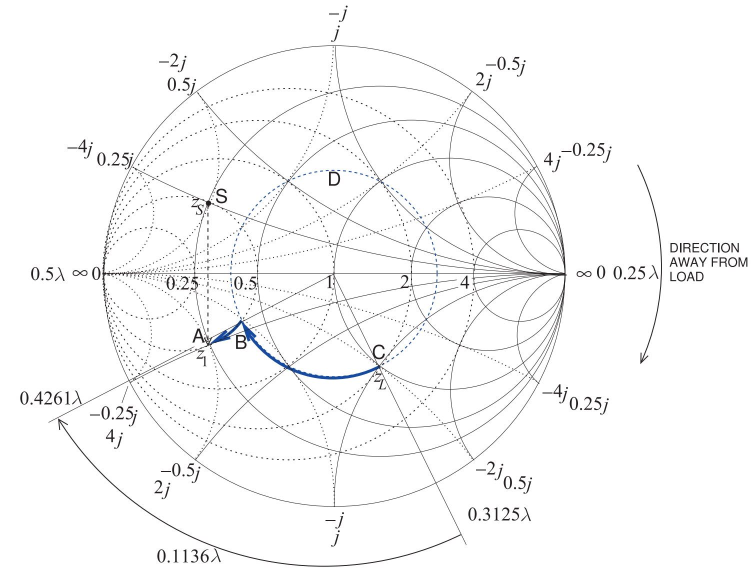



**Figure 6-40:** Design for Example 6.12.

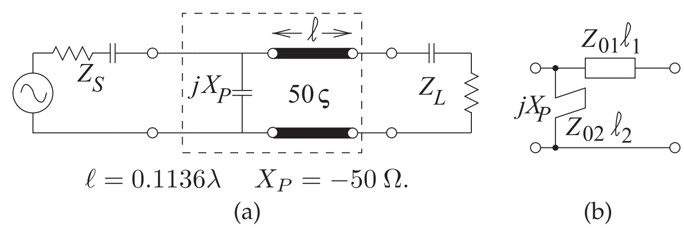



**Figure 6-41:** Single-stub matching network design of Example 6.12: (a) electrical design; and (b) electrical design with a shunt stub.

the line rotates in a clockwise direction as the length of the line increases. One way of remembering this is to consider an open-circuited line. When the line length is zero, $Y_{\text{in}} = 0$ and $\Gamma_{\text{in}} = +1$. A short length of this line is capacitive so that its reflection coefficient will be in the bottom half of the Smith chart. A length of line can be used to rotate the impedance to an appropriate point to follow a line of constant conductance to the desired input impedance.

## EXAMPLE 6.12 Matching Network Design With a Transmission Line and a Single Stub

Design a two-element matching network to match a source with an impedance $Z_S = 12.5 + 12.5\jmath\ \Omega$ to a load $Z_L = 50 - 50\jmath\ \Omega$, as shown in Figure 6-34. This example repeats the design in Example 6.11, but now using a transmission line.

**Solution:**

As in Example 6.11, choose $Z_0 = 50\ \Omega$ and the design path is from $z_L = Z_L/Z_0 = 1 - \jmath$ to $z_s^*$, where $z_s = 0.25 + 0.25\jmath$. One possible design solution is indicated in Figure 6-40. The line length, $\ell$ (taking the locus from Point C to Point B), is

$$\ell = 0.4261\lambda - 0.3125\lambda = 0.1136\lambda,$$

and the normalized shunt susceptance, $b_P$ (taking the locus from Point B to Point C), is

$$b_P = b_A - b_B = 2 - 1 = 1.$$

Thus $X_P = (-1/b_P) \times 50\ \Omega = -50\ \Omega$. The final design is shown in Figure 6-41(a). The stub design of Figure 6-41(b) follows the procedure described in Example 6.5.

### 6.8.2 Hybrid Lumped-Distributed Matching

A lossless matching network can have transmission lines as well as inductors and capacitors. If the system reference or normalization impedance is the characteristic impedance of a transmission line, then the locus of the input impedance (or reflection coefficient) of the line with respect to the length of the line is an arc on a circle centered at the origin of the Smith chart. The direction of the arc is clockwise as the electrical length of the line moves away from the load. So a hybrid matching network is possible that combines a length of transmission line with a lumped element (preferably a capacitor rather than an a inductor as the inductor would have a relatively lower $Q$).

## 6.9 Matching Options Using the Smith Chart

The purpose of this section is to use the Smith chart to present several design options for matching a source to a load, see Figure 6-42. The designs here provide another view of design using the Smith chart.

### 6.9.1 Locating the Design Points

The first design choice to be made is the reference impedance to use. Here $Z_{\mathrm{REF}} = 50\ \Omega$ will be chosen largely because this is in the center of the design space for microstrip lines. Generally the characteristic impedance, $Z_0$, of a microstrip line needs to be between $20\ \Omega$ and $100\ \Omega$. A microstrip line with $Z_0 < 20\ \Omega$ will be wide and there is a possibility of multimoding due to transverse resonance. Also $20\ \Omega$ line is about six times wider than a $50\ \Omega$ line and so takes up a lot of room and there is a good chance that it could be close to other microstrip lines or perhaps the wall of an enclosure. This is based on the rule of thumb (developed in Example 3.4 of [1]) that $Z_0 \propto \sqrt{h/w}$ where $h$ is the substrate thickness and $w$ is the strip width. The thickness is usually fixed, i.e. it is not always a readily changed design choice. If $Z_0 > 100\ \Omega$ the characteristic impedance is getting close to the wave impedance of free space and of the dielectric of the substrate. As such it is likely that field lines are not tightly constrained by the metal of the strip and the fields can more likely radiate. Then radiation loss can be high or coupling to a neighboring microstrip can be high.

The normalized source and load impedances are $z_S = Z_S/Z_{\mathrm{REF}} = [(29.36 - \jmath 12.05)\ \Omega]/(50\ \Omega) = 0.587 - \jmath 0.241$ and $z_L = Z_L/Z_{\mathrm{REF}} = [(132.7 - \jmath 148.8)\ \Omega/(50\ \Omega) = 2.655 - \jmath 2.976 = r_L + \jmath x_L$, respectively. Maximum power transfer requires that the input impedance of the matching network terminated in $Z_L$ be $Z_1 = Z_S^*$, i.e. $z_1 = z_S^* = 0.587 + \jmath 0.241 = r_1 + \jmath x_1$. These impedances are plotted on the normalized Smith chart in Figure 6-43.

The normalized load impedance is Point L. To locate this point the arcs corresponding to the real and imaginary parts of $z_L$ are considered separately. The resistive part of $z_L$ is $r_L = 2.655$ and the resistance labels are

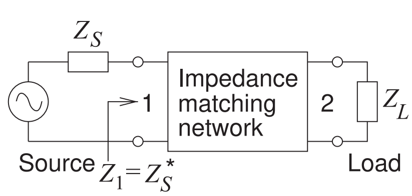



**Figure 6-42:** Matching problem with the matching network between the source and load designed for maxium power transfer. $Z_S = R_S + \jmath X_S = 29.36 - \jmath 12.05$, $Z_1 = R_1 + \jmath X_1 = Z_S^* = R_S - \jmath X_S = 29.36 + \jmath 12.05$, and $Z_L = R_L + \jmath X_L = 32.7 - \jmath 148.8$.

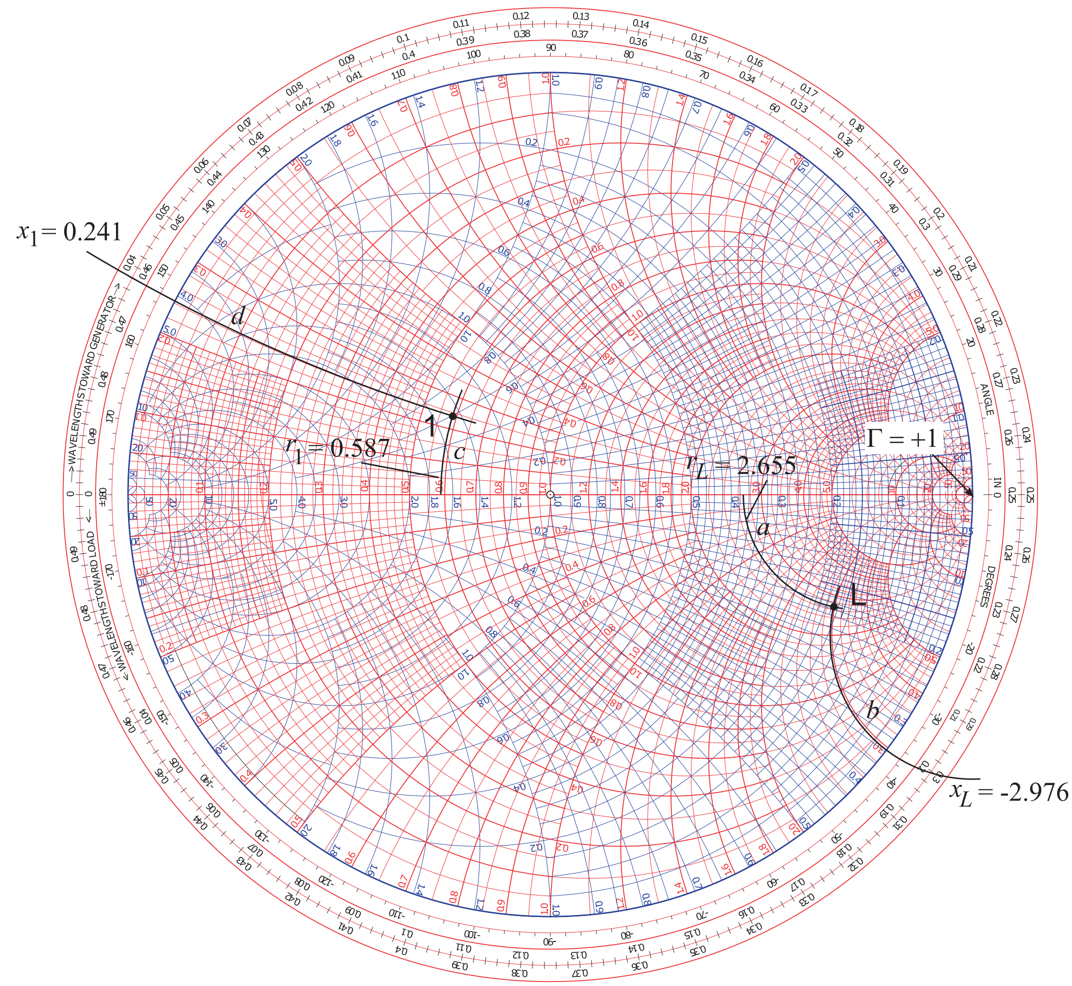



**Figure 6-43:** Locating $Z_L$ at Point L and $Z_1 = Z_S^*$ at Point 1.

located on the horizontal axis (or equator) of the Smith chart. There are two sets of labels, one for normalized resistance, $r$ (which is above the horizontal axis), and one for normalized conductance, $g$ (which is below the horizontal axis). The way to remember which is which is to realize that the infinite impedance point is at the $\Gamma = +1$ (open circuit) location on the right of the graph. At the origin (center) of the Smith chart $r = 1 = g$ and to the right of the center the values of $r$ should be greater than one. The closest $r$ labels to $r_L = 2.655$ are $r = 2.0$ and $r = 3.0$. There are five divisions so the unlabeled curves correspond to $2.2$, $2.4$, $\ldots$. The arc corresponding to $r = 2.655$ must be interpolated and this interpolation is shown as the Path 'a'.

The imaginary part of $z_L$ is $x_L = -2.976$. The labels for the arcs of constant reactance are given adjacent to the unit circle. There are two sets of labels, one for reactance and one for susceptance. To recall which is which, the point of infinite impedance can be used and the required reactance labels should increase towards the $\Gamma = +1$ point. Recall that the Smith chart does not include signs of reactances (there is not enough room) so note must be made that positive reactances are in the top half of the Smith chart and negative reactances are in the bottom half. Since $x_L$ is negative it will be in the bottom half of the Smith chart. The closest labels are $x = 2.0$ (this is actually $-2.0$) and $x = 3.0$ (this is actually $-3.0$) so the arc for $x = -2.976$ is interpolated as the Path 'b'. Point L, i.e. $z_L$, is located at the intersection of Paths 'a' and'b'. The normalized impedance $z_1$ is located similarly at Point 1 by finding the point of intersection of the $r_1 = 0.587$ arc, Path 'c', and the $x_1 = +0.241$ arc, Path 'd'.

### 6.9.2 Design Options

By convention design follows a process of beginning with $z_L$ and adding series and shunt elements in front of it evolving the impedance (or reflection coefficient) until the input impedance is $z_1 = z_S^*$. Three electrical designs are shown in Figure 6-44 and the corresponding lumped-element and microstrip topologies are shown in Figure 6-45. The subscript on the circuit elements correspond to the paths on the Smith chart in Figure 6-44 . The designs will be elaborated in the following subsections.

### 6.9.3 Design 1, Hybrid Design

Design 1 on its own is shown in Figure 6-46. The concept here is to use a transmission line and a shunt to go from the load Point L to the Point 1. The reason why a shunt element is chosen and not a series element is that the shunt element can be implemented as stub line and a series element, i.e. a series stub, cannot be implemented in microstrip. A lumped element limits a design to the low microwave range as losses become prohibitively large especially for inductors. Also if a microstrip line is going to be used anyway then a decision already has been made that there is enough room to implement a transmission-line based design and so the shunt lumped element can reasonably be replaced by a stub.

Design follows trial and error. The first attempt, and the one that works here, is to draw a circle through L centered on the origin at Point O. This circle describes a transmission line whose characteristic impedance is the same as the reference impedance of the Smith chart, here 50 Ω. The next step is to draw a circle of constant conductance through Point 1. The combination path from L to 1 needs to lie on these circles and the intermediate point will be where these circles intersect. One other constraint is that with a transmission line the locus (as the line length increases) of the input reflection coefficient of the line must rotate in the clockwise direction. It is seen that there are two points of intersection and the first of these, at A, is chosen in design. So the electrical design is defined by the directed Paths 'g' and 'h'. Path 'g' defines the properties of the transmission line and Path 'h' defines the properties of the shunt element. As 'h' is directed towards the infinite inductive susceptance point, Path 'h' defines an inductor. The topology of this design is shown in Figure 6-45(a).

The characteristic impedance of the transmission line (defined by Path 'g') is $Z_{0g} = 50\ \Omega$. The electrical length of the line is defined by the angle subtended by the arc 'g'. The electrical length of the line is determined from the outermost circular scale which is labeled 'WAVELENGTHS TOWARDS

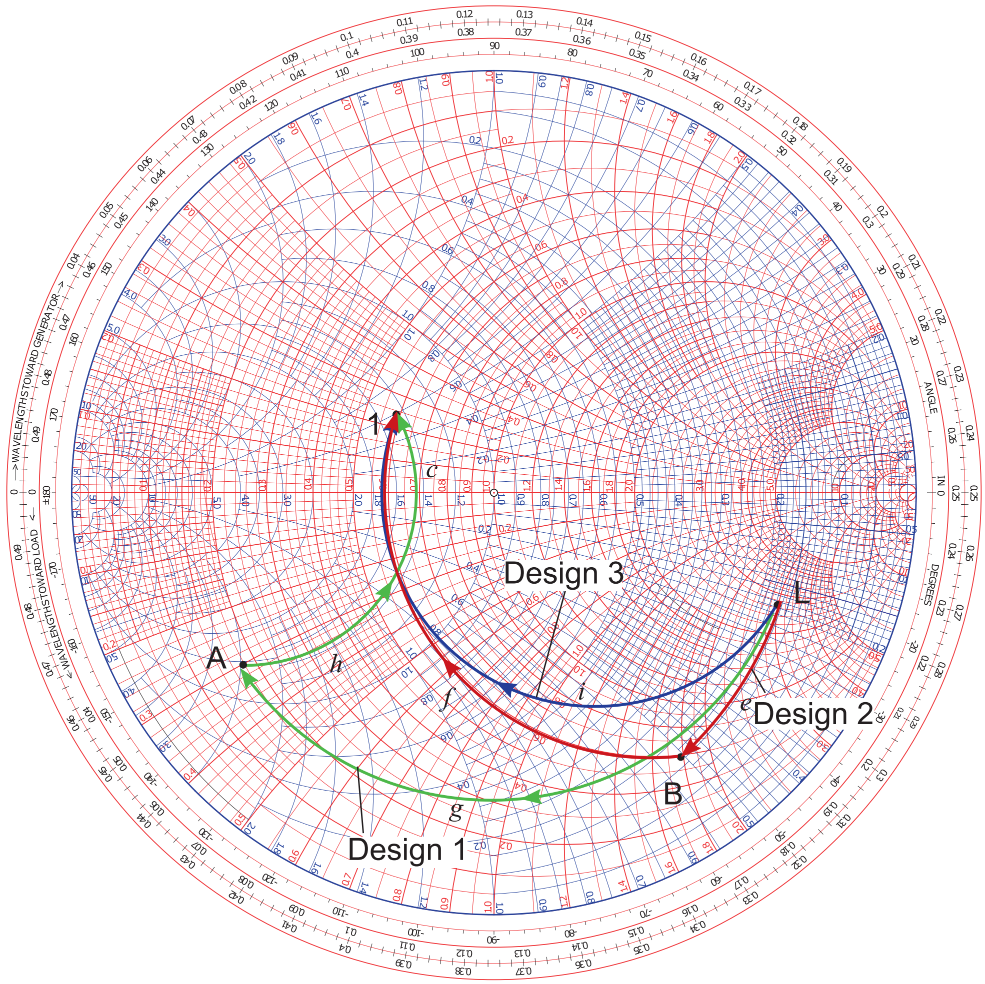
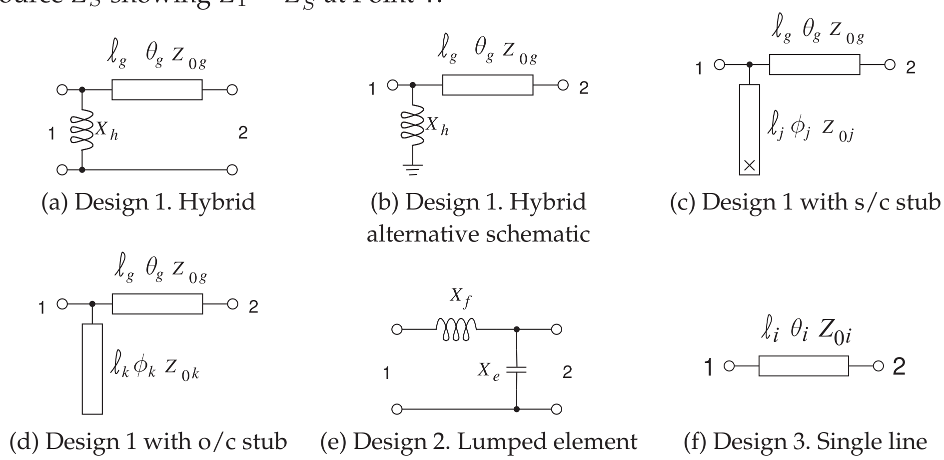



**Figure 6-44:** Three matching network electrical designs matching a load impedance $Z_L$ at Point L to a source $Z_S$ showing $Z_1 = Z_S^*$ at Point 1.




(a) Design 1. Hybrid

(b) Design 1. Hybrid alternative schematic

(c) Design 1 with s/c stub

(d) Design 1 with o/c stub

(e) Design 2. Lumped element

(f) Design 3. Single line

**Figure 6-45:** Matching network topologies using lumped elements and microstrip lines. In the stub layouts X is a via to the ground plane implementing a short circuit (s/c) and an open circuit o/c simply does not show a connection to the microstrip ground plane.

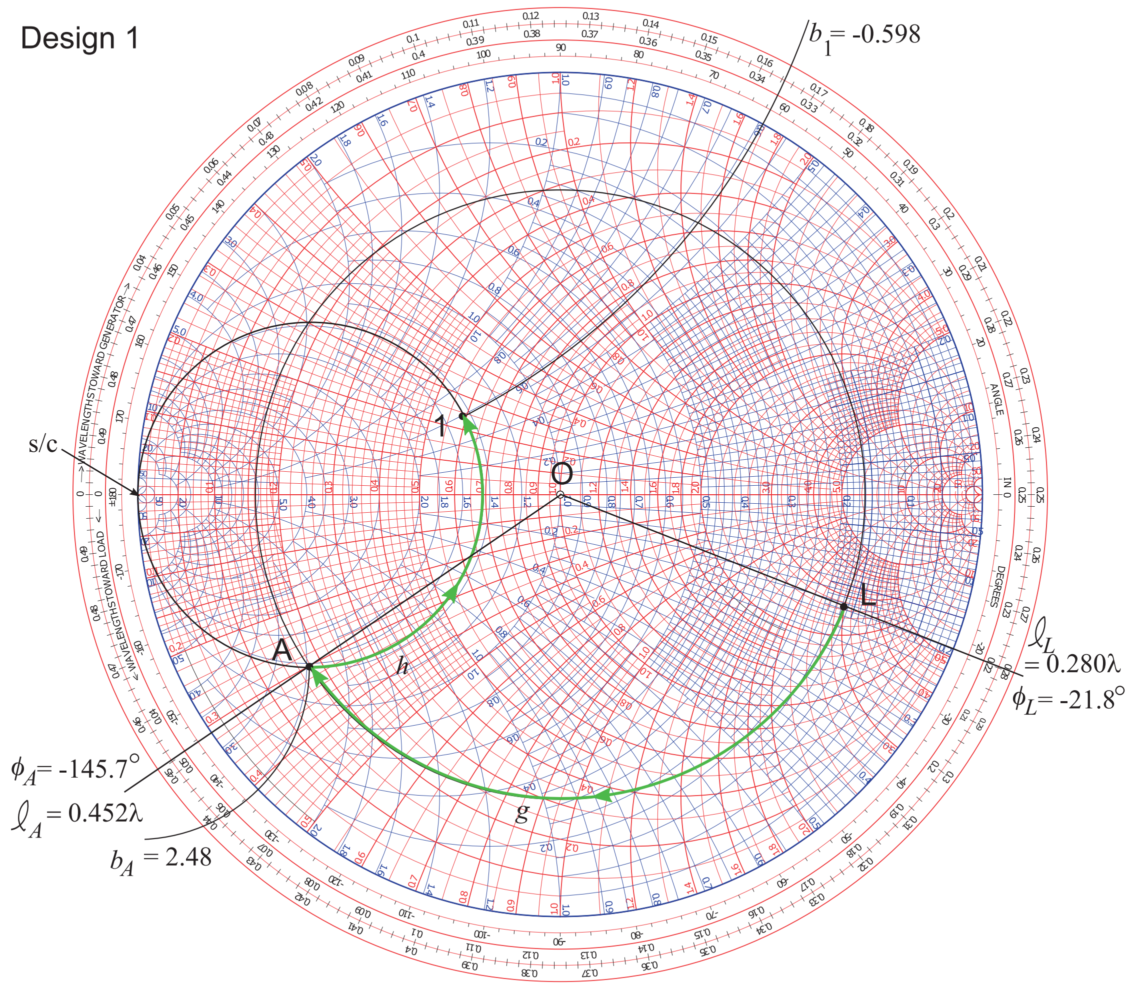



**Figure 6-46:** Design 1. Hybrid design combining a transmission line with a lumped element in shunt. The design is identified by paths 'g' and 'h'.

GENERATOR.' A line drawn from O through L intersecting the scale has a scale reading of $\ell_L = 0.280\lambda$. Then the scale reading at A is similarly found as $\ell_A = 0.452\lambda$ and so the line length is $\ell_g = \ell_A - \ell_L = 0.452\lambda - 0.280\lambda = 0.172\lambda$. Another way of determining the electrical length of the line is from the change in reflection coefficient angle. For L the reflection coefficient angle is $\phi_L = -21.8°$ read from the innermost circular scale. This angle is just the angle from the polar plot. Then the angle at A is read as $\phi_A = -145.7°$. The difference is $|\phi_A - \phi_L| = |-145.7 - (-21.8°)| = 124.9°$. The electrical length of the line is half the change in reflection coefficient angle and so the electrical length of the line is $\theta_g = \frac{1}{2}124.9° = 62.5°$. Now $\lambda$ corresponds to an electrical length of $360°$ so $\theta_g$ corresponds to $62.5/360\lambda = 0.174\lambda$ corresponding to the previously determined length of $0.172\lambda$ which is very good agreement given that these were derived from graphical readings.

Path 'h' defines a shunt inductor and a circle of constant conductance is

followed with only the susceptance changing. The susceptance indicated by Path 'g' is $b_g = b_1 - b_A$. To obtain $b_A$ extend the circle of constant susceptance through A out to the unit circle. The extended circle cross the unit circle between the susceptance labels 2.0 and 3.0. A check is that susceptance is positive in the bottom half of the Smith chart so the signs of the labels do not need to be adjusted. There are two scales adjacent to the unit circle, one for normalized susceptance and one for normalized reactance. Since the intersection is close to the infinite susceptance point at the s/c (short-circuit) so the values that are becoming very large towards s/c are used. Interpolation results in the reading $b_A = 2.48$. A similar process applied to Point 1 results in $b_1 = -0.598$ where the negative sign has been applied to the scale reading since the point of intersection between the arc of constant susceptance passing through Point 1 and the unit circle is in the top half of the Smith chart. Thus $b_h = b_1 - b_A = -0.598 - 2.48 = -3.08$ and so the normalized reactance of the shunt element is $x_h = -1/b_h = 0.325$. The un-normalized reactance of the shunt element is $X_h = x_h Z_{\rm REF} = 16.2\ \Omega$.
The final Design 1 hybrid layout is shown in Figure 6-45(a) with $X_h = 16.2\ \Omega$, $Z_{0g} = 50\ \Omega$, and $\ell_g = 0.172\lambda$. That is all that is needed to define the electrical design, providing the electrical length in degrees, $\theta_g = 62.5°$ is redundant but provided anyway. The transmission line in Figure 6-45(a) is shown as as the top view of the strip of a microstrip line as is commonly done. A more common way of representing this schematic is shown in Figure 6-45(b) where the ground connections at Ports 1 and 2 have been removed and the ground connection of the inductor shown separately.

### 6.9.4 Design 1 with an Open-Circuited Stub

In the previous section Design 1 was left as a hybrid design with a transmission line and a lumped-element inductor. In this section the lumped-element inductor is implemented as an open-circuited stub, see Figure 6-45(d). Recall that the $50\ \Omega$-normalized susceptance of the inductor is $b_h = -3.08$. If the stub is also implemented as a $50\ \Omega$ line then $b_h$ can be used unchanged. Point C in Figure 6-47 corresponds to the normalized admittance $0 - \jmath 3.08$. The unit circle is the zero conductance circle (and is also the zero resistance circle) and the susceptance is read from the scale adjacent to the unit circle again using as reference that susceptances in the top half of the Smith chart need to incorporate a negative sign and the susceptance scale is identified by the susceptance values becoming larger approaching the s/c point. Point C also corresponds to $x_h = -1/b_h = 3.25$ and indeed this is the value read from the normalized reactance scale.
A transmission line needs to be designed to have a normalized input susceptance of $b_h = -3.08$. Choosing an open circuit, o/c, termination the point corresponding to o/c is as identified in the figure. At the o/c point the length scale reads $\ell_{\rm o/c} = 0.250\lambda$. The locus rotates in the clockwise direction up to Point C where the direct electrical reading reading is $\ell_C = 0.050\lambda$. Using this directly to determine the line length $\ell_k = \ell_C - \ell_{\rm o/c} = 0.050\lambda - 0.250\lambda = -0.20\lambda$ which indicates that the stub has a negative length. Clearly an erroneous result. This apparent discrepancy comes about because the length scale resets at the short circuit point where the length scale abruptly goes from $0.5\lambda$ to $0\lambda$. Thus the corrected $\ell_C$ reading needs to have an additional $0.5\lambda$. Thus the corrected value of $\ell_C = (0.5 + 0.050)\lambda = 0.550\lambda$

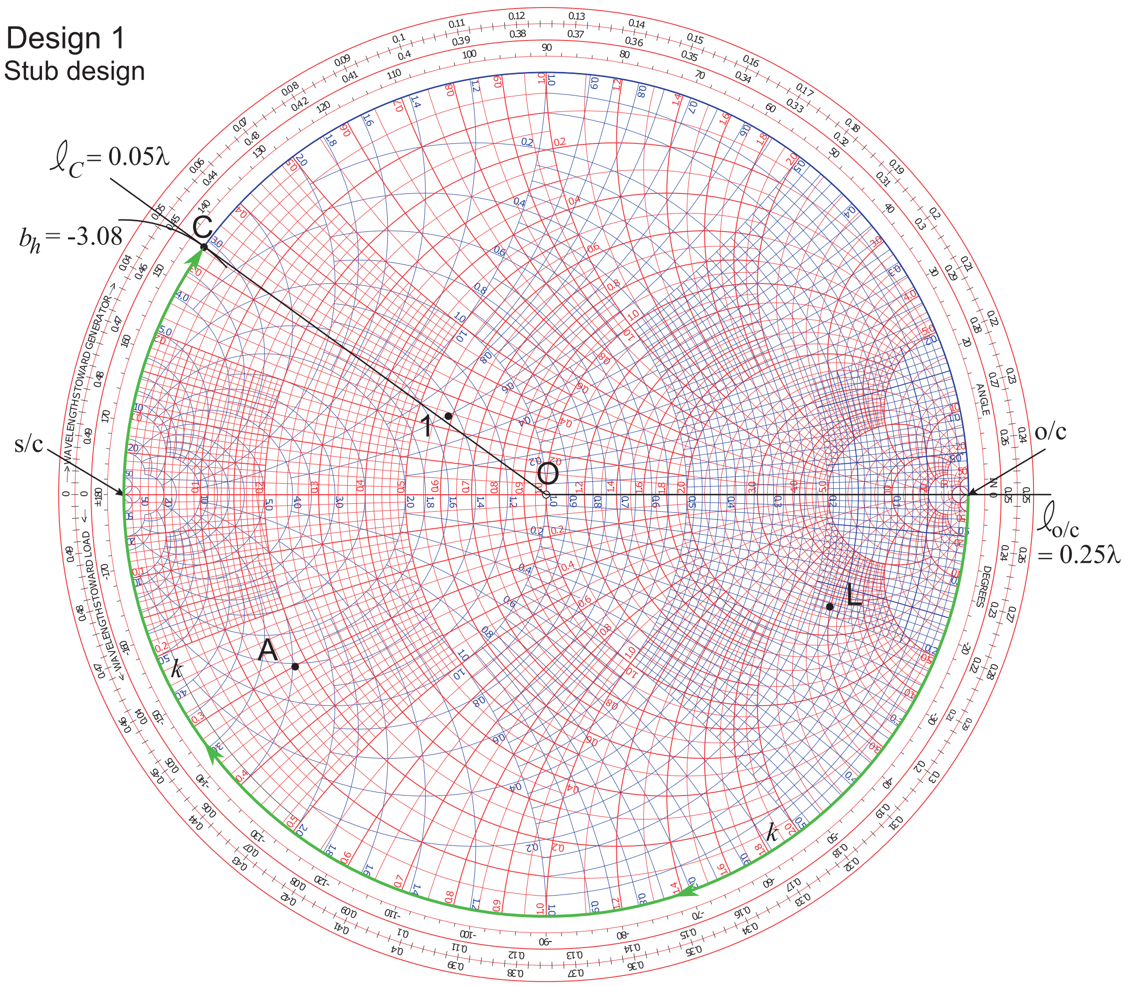



**Figure 6-47:** Design 1. Design of an open-circuit stub having normalized input susceptance $b_h$.

and $\ell_k = \ell_C - \ell_{\text{o/c}} = 0.550\lambda - 0.250\lambda = 0.300\lambda$.
Thus the final design is as shown in Figure 6-45(d) with $Z_{0k} = 50\ \Omega$, and $\ell_g = 0.300\lambda$, $Z_{0g} = 50\ \Omega$, and $\ell_g = 0.172\lambda$.
The stub could also have been implemented as a short-circuit stub as shown in Figure 6-45(c). Now the beginning of the line would be at the s/c point and the line length would be $0.050\lambda$

## 6.9.5 Design 2, Lumped-Element Design

Design 2 is a lumped-element design and the Smith-chart-based electrical design is shown in Figure 6-48 resulting in the schematic shown in Figure 6-45(e). Design proceeds by identifying where circles of constant conductance and constant resistance passing through the Points L and 1 intersect. One solution is shown in Figure 6-48. A circle of constant conductance passes through L and part of a circle of constant resistance passes through 1. If the circle had continued there would have been a second intersection with

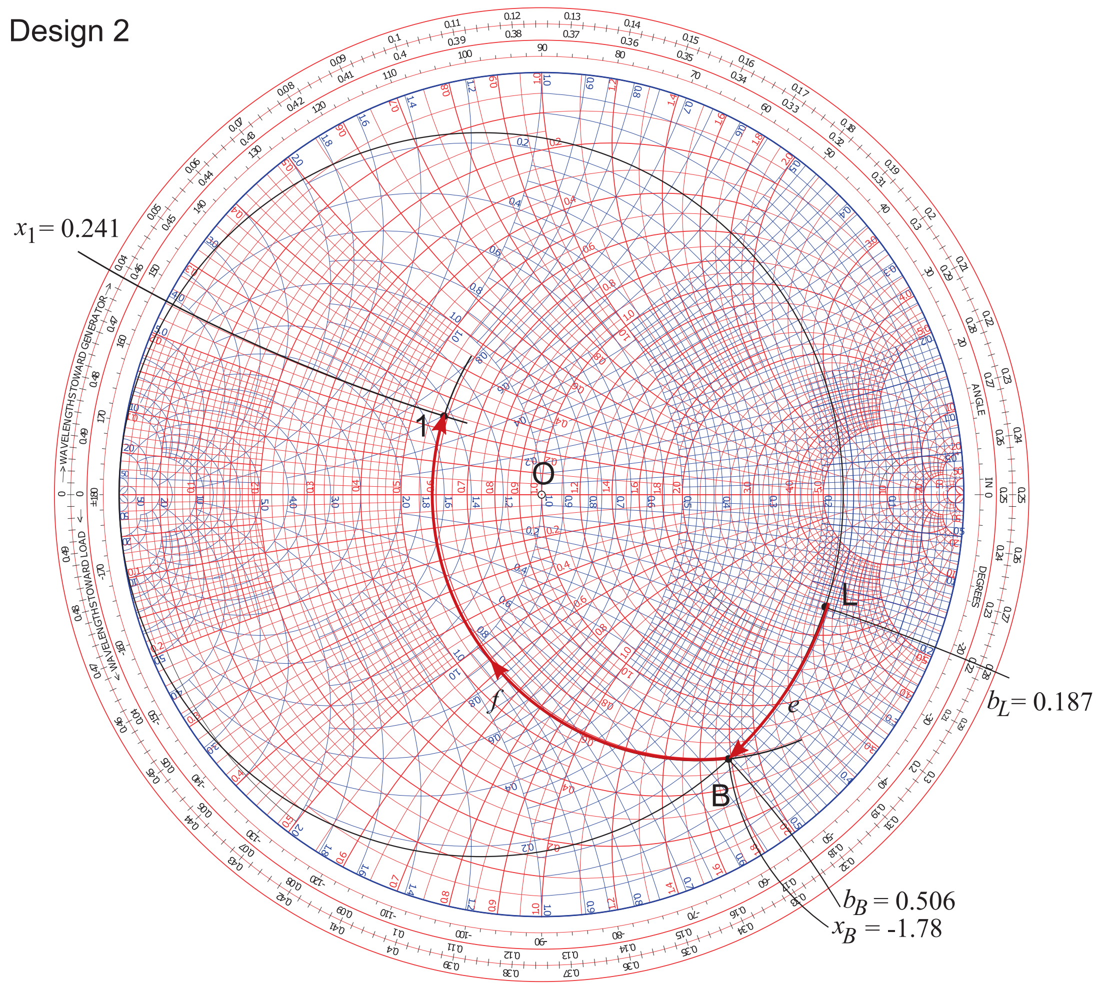



**Figure 6-48:** Design 2.

the circle through L. Both of these intersections mean that there is a shunt element adjacent to the load and series element adjacent to the source. Recall that in a lumped-element design that under no circumstances can the locus of a lumped element pass through the short circuit point (if it is a susceptance) or open-circuit point (if it is a reactance), the susceptance and reactance infinity points respectively.

Returning to the actual design shown in Figure 6-48. The first intersection of the two circles is Point B so that the design is specified by the Paths 'e' and 'f'. Design has largely been completed by identifying these paths and the next stage is determining the circuit elements that correspond to these paths. Path 'e' follows a circle of constant conductance and so indicates a shunt susceptance and the direction of the locus indicates a capacitance. The value of this normalized susceptance is $b_e = b_B - b_L = 0.506 - 0.187 = 0.319$. (Remember to check the signs of the readings since the Smith chart omits signs of the reactances and susceptances.) Path 'f' identifies a series inductor with a reactance $x_f = x_1 - x_B = 0.241 - (-1.78) = 2.02$. The final design is shown in Figure 6-45(e) with $X_e = Z_0/b_e = 50/0.319\ \Omega = 158\ \Omega$ and $X_f = Z_0 x_f = 50 \cdot 2.02\ \Omega = 101\ \Omega$.

### 6.9.6 Design 3, Single Line Matching

Design 3 uses a single transmission line to match the source and load as shown in the schematic of Figure 6-45(f). This design is akin to using a quarter-wave transmission line transformer but with a Smith chart being used the approach can now be used with complex source and load impedances. Recall from Sections 3.5.2 and 4.5 that the locus of a terminated transmission line is a circle on the Smith chart even if the characteristic impedance of the transmission line, $Z_{0i}$ in Design 3, and the reference impedance, $Z_{\mathrm{REF}}$, are not the same. Furthermore the center of the circle will be on the horizontal axis. The Smith-chart-based electrical design of Design 3 is shown in Figure 6-49 where $Z_{\mathrm{REF}} = 50\ \Omega$. There is only one way to draw a circle that passes through two points with the center of the circle constrained to be on the horizontal axis. That circle is shown in Figure 6-49 with Path 'i' on the circle going from Points L to 1 tracing out the locus of the reflection coefficient. This locus must be in the clockwise direction. The center of the circle is at Point D and the center of the circle is (the reflection coefficient normalized to $Z_{\mathrm{REF}}$) is $C_D = 0.240$. (The radius is also given but this is not necessary.) The angle subtended by Path 'i' is twice the electrical length of the line. This angle cannot be directly measured from the scales (although it is possible by placing a chart over the top and and aligning the center of the polar plot with D) and here was read using a protractor. The angle is $\phi_D = 150^\circ$ so that the electrical length of the line is $\theta_i = \frac{1}{2}150^\circ = 75^\circ$, i.e. $\ell_i = 74/360\lambda = 0.208\lambda$. The only parameter not known is the characteristic impedance, $Z_{0i}$, of the line. This must be arrived at iteratively. From Equation (4.40) (after replacing $Z_{01}$, $Z_{02}$ and $C_{Z02}$ by $Z_{0i}$, $Z_{\mathrm{REF}}$ and $C_D$, respectively)

$$Z_{0i} \approx Z_{\mathrm{REF}}(1 + C_D)/(1 - C_D) \tag{6.73}$$

with the approximation improving the closer $Z_{0i}$ is to $Z_{\mathrm{REF}}$. Substituting $Z_{\mathrm{REF}} = 50\ \Omega$ and $C_D = 0.240$, the first iteration of $Z_{0i}$ is

$$^1Z_{0i} = 50 \left(\frac{1 + 0.240}{1 - 0.240}\right)\ \Omega = 82\ \Omega. \tag{6.74}$$

Re-plotting using a new reference impedance of $Z_{\mathrm{REF}} = 82\ \Omega$ yields a new center of 0.07 from which an updated iterate is $^2Z_{0i} = 94\ \Omega$ and then a new center of 0.02 and $^3Z_{0i} = 98\ \Omega$. Continuing to iterate results in an asymptotic value $Z_{0i} = 100\ \Omega$. $Z_{0i}$ can also be estimated from the tabulated values in Table 4-1. Reading the line with radius of 0.5 and center of 0.24 yields $Z_{0i} = 50 \cdot 1.980 = 99\ \Omega$.

The final design is as shown in Figure 6-45(f) with $\ell_i = 0.208\lambda$, $\theta_i = 75^\circ$, and $Z_{0i} = 100\ \Omega$.

As was seen with the quarter-wave transformer design using multiple stages significantly increases bandwidth by matching to intermediate resistances determined as geometric means of the source and load resistance. The corresponding strategy here is to draw a line between the Points 1 and L and, if there are to be two stages, choose the intermediate matching impedance

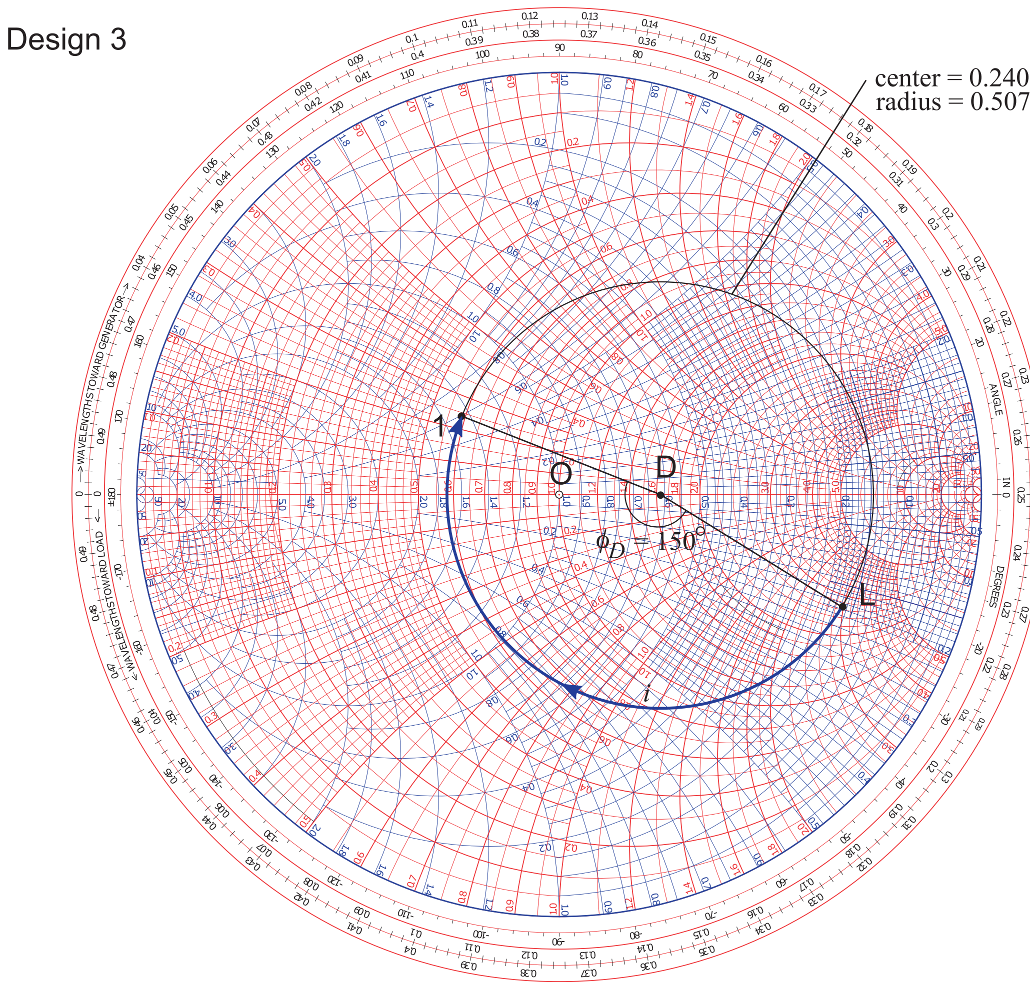



**Figure 6-49:** Design 3.

at the mid-point of the line. If the source and loads are complex then the reactive storage elements in the source and load will set a limit on maximum achievable bandwidth.

## *6.9.7 Summary*

This section presented three quite different designs for a matching network. One of the particular benefits of using the Smith chart is identifying topologies and initial design values. Design can then transfer to a microwave circuit simulator. The Smith chart enables back-of-the-envelope design studies. While with experience it is possible to complete many of these steps with a computer-based Smith chart tool, even experienced designers doodle with a printed Smith chart when exploring design options.

## 6.10 Summary

This chapter presented techniques for impedance matching that achieve maximum power transfer from a source to a load. The simplest matching network uses a series and a shunt element, a two-element matching network, to realize a single-frequency match. This type of impedance matching network uses lumped elements and can be used up to a few gigahertz. Performance is limited by the self-resonant frequency of lumped elements and by their loss, particularly that of inductors. The shunt element can be replaced by a shunt stub, but in most transmission line technologies, including microstrip, the series element cannot be implemented as a stub. Matching networks can also be realized using transmission line segments only, principally shunt stubs and cascaded transmission lines. A tunable double-stub matching network, which uses two stubs separated by a transmission line, is standard equipment in microwave laboratories and facilitates matching of a circuit under development.

The bandwidth of a matching network is set by the maximum allowable reflection coefficient of the terminated network. Two-element matching nearly always results in a narrow match and for typical communications applications often achieves acceptable matching over bandwidths of only 1%–3%. The most significant determinant of the quality of the match that can be achieved is the ratio of the source and load resistances, as well as the reactive energy storage of the source and load. Clearly if the load and source are resistances of the same value, the bandwidth of the matching network is infinite, as it is no more than a wired connection.

An important concept in matching network design is a technique for controlling bandwidth. The concept is based on matching to an intermediate resistance $R_v$. Increased bandwidth is obtained if $R_v$ is the geometric mean of the source and load resistances. This new network consists of two two-element matching networks. If $R_v$ is greater or less than both the source and load resistances, then the bandwidth of the matching network is reduced. Matching network synthesis can also be addressed using filter design techniques, enabling simultaneous control over the quality and bandwidth of the match. It is always a good idea to have no more bandwidth in the system than is needed, as this minimizes the propagation of noise.

A powerful graphical matching tool is the Smith chart on which the load and source can be plotted. The design objective is then to determine the path, subject to constraints, from the load back to an input, which is the complex conjugate of the source, a task that a human is particularly adept at performing. Alternatively the perspective could be flipped and the role of the source and load interchanged.

Design becomes increasingly more difficult as the required bandwidth increases. Many times it is sufficient to have small-to-moderate bandwidths that can be tuned rather than providing one large instantaneous bandwidth. Also many of the evolving wireless systems require multiple functionality, which in turn requires adjustability of a matching network. In some applications the matching network may require adjustment to match a variable load impedance. A good example is dealing with a cell phone antenna where the user may put his or her hand over the antenna and alter the load seen by the RF frontend. Some types of matching network designs are more adjustable than others. Such designs require variable components, so matching design can be a source of competitive advantage.

## 6.11 References

[1] M. Steer, *Microwave and RF Design, Transmission Lines*, 3rd ed. North Carolina State University, 2019.

[2] R. Collins, *Foundations for Microwave Engineering*. McGraw Hill, 1966.

## 6.12 Exercises

1. Consider the design of a magnetic transformer that will match the 3 Ω output resistance of a power amplifier (this is the source) to a 50 Ω load. The secondary of the transformer is on the load side.
   (a) What is the ratio of the number of primary turns to the number of secondary turns for ideal matching?
   (b) If the transformer ratio could be implemented exactly (the ideal situation), what is the reflection coefficient normalized to 3 Ω looking into the primary of the transformer with the 50 Ω load?
   (c) What is the ideal return loss of the loaded transformer (looking into the primary)? Express your answer in dB.
   (d) If there are 100 secondary windings, how many primary windings are there in your design? Note that the number of windings must be an integer. (This practical situation will be considered in the rest of the problem.)
   (e) What is the input resistance of the transformer looking into the primary?
   (f) What is the reflection coefficient normalized to 3 Ω looking into the primary of the transformer with the 50 Ω load?
   (g) What is the actual return loss (in dB) of the loaded transformer (looking into the primary)?
   (h) If the maximum available power from the amplifier is 20 dBm, how much power (in dBm) is reflected at the input of the transformer?
   (i) Thus, how much power (in dBm) is delivered to the load ignoring loss in the transformer?

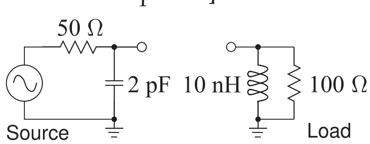
2. Consider the design of a magnetic transformer that will match a 50 Ω output resistance to the 100 Ω load presented by an amplifier. The secondary of the transformer is on the load (amplifier) side.
   (a) What is the ratio of the number of primary turns to the number of secondary turns for ideal matching?
   (b) If the transformer ratio could be implemented exactly (the ideal situation), what is the reflection coefficient normalized to 50 Ω looking into the primary of the transformer with the load?
   (c) What is the ideal return loss of the loaded transformer (looking into the primary)? Express your answer in dB.
   (d) If there are 20 secondary windings, how many primary windings are there in your design? Note that the number of windings must be an integer? (This situation will be considered in the rest of the problem.)
   (e) What is the input resistance of the transformer looking into the primary?
   (f) What is the reflection coefficient normalized to 50 Ω looking into the primary of the loaded transformer?
   (g) What is the actual return loss (in dB) of the loaded transformer (looking into the primary)?
   (h) If the maximum available power from the source is $-10$ dBm, how much power (in dBm) is reflected from the input of the transformer?
   (i) Thus, how much power (in dBm) is delivered to the amplifier ignoring loss in the transformer?

3. Consider the design of an L-matching network centered at 1 GHz that will match the 2 Ω output resistance of a power amplifier (this is the source) to a 50 Ω load. [Parallels Example 6.3 but note the DC blocking requirement below.]
   (a) What is the $Q$ of the matching network?
   (b) The matching network must block DC current. Draw the topology of the matching network.
   (c) What is the reactance of the series element in the matching network?
   (d) What is the reactance of the shunt element in the matching network?
   (e) What is the value of the series element in the matching network?
   (f) What is the value of the shunt element in the matching network?
   (g) Draw and label the final design of your matching network including the source and load resistances.
   (h) Approximately, what is the 3 dB bandwidth

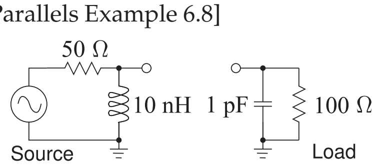
of the matching network?

4. Consider the design of an L-matching network centered at 100 GHz that will match a source with a Thevenin resistance of 50 Ω to the input of an amplifier presenting a load resistance of 100 Ω to the matching network. [Parallels Example 6.4 but note the DC blocking requirement below.]
   (a) What is the $Q$ of the matching network?
   (b) The matching network must block DC current. Draw the topology of the matching network.
   (c) What is the reactance of the series element in the matching network?
   (d) What is the reactance of the shunt element in the matching network?
   (e) What is the value of the series element in the matching network?
   (f) What is the value of the shunt element in the matching network?
   (g) Draw and label the final design of your matching network including the source and load resistance.
   (h) Approximately, what is the 3 dB bandwidth of the matching network?

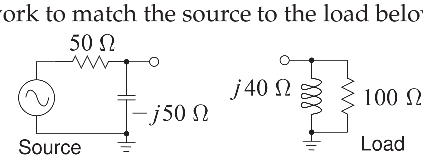
5. Design a Pi network to match the source configuration to the load configuration below. The design frequency is 900 MHz and the desired $Q$ is 10. [Parallels Example 6.8]



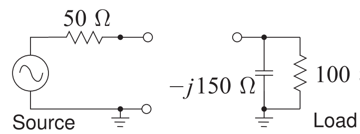

6. Design a Pi network to match the source configuration to the load configuration below. The design frequency is 900 MHz and the desired $Q$ is 10. [Parallels Example 6.8]



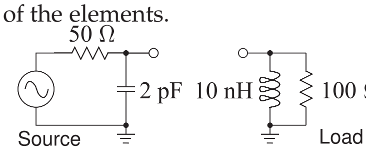

7. Develop the electrical design of an L-matching network to match the source to the load below.




8. Develop the electrical design of an L-matching network to match the source to the load below.




9. Design a lowpass lumped-element matching network to match the source and load shown below. The design frequency is 1 GHz. You must use a Smith Chart and clearly show your working and derivations. You must develop the final values of the elements.

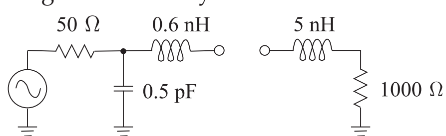



10. Consider the design of an L-matching network centered at 100 GHz that will match a source with a Thevenin resistance of 50 Ω to the input of an amplifier presenting a load resistance of 200 Ω to the matching network. [Parallels Example 6.4 but note the DC blocking requirement below.]
   (a) What is the $Q$ of the matching network?
   (b) The matching network must block DC current. Draw the topology of the matching network.
   (c) What is the reactance of the series element in the matching network?
   (d) What is the reactance of the shunt element in the matching network?
   (e) What is the value of the series element in the matching network?
   (f) What is the value of the shunt element in the matching network?
   (g) Draw and label the final design of your matching network including the source and load resistance.
   (h) Approximately, what is the 3 dB bandwidth of the matching network?

11. Consider the design of the output matching network of a 15 GHz small-signal amplifier. The amplifier consists of an active two-port and input and output matching networks. Port 1 is the input of the active device and Port 2 is its output, and its 50 Ω $S$ parameters are $S_{11} = 0.5\angle 45^\circ$, $S_{12} = 0.1\angle 0^\circ$, $S_{21} = 2\angle 90^\circ$, and $S_{22} = 0.75\angle -45^\circ$.
   (a) If the input of the active device is terminated in 50 Ω, what is the impedance looking into the output of the amplifier?
   (b) Design a two-element lumped-element matching network for maximum power transfer from the output of the transistor

into a 50 Ω load. Develop at least two designs and compare them. (Note that the output of the active device physically appears as a resistance in parallel with a capacitance and this can be used in contrasting your designs.)

12. Develop a two-element matching network for the source/load configuration shown in the figure below. The matching network must pass DC. The center frequency of the matching network is $f$ = 1 GHz. There are a number of design considerations that should be considered before embarking on network synthesis.




(a) Source (b) Load

Consider the following:

(i) The source needs to be collapsed to an equivalent circuit with one resistance and one reactance.

(ii) The reactive elements in the source and the load will be accommodated using absorption or resonance. Absorption is preferred, but not always possible.

(iii) The DC requirements necessitate a lowpass matching network. So there must be a series inductance and a shunt capacitor in the matching network. Since the load resistance is greater than the source resistance, the most likely design has a shunt capacitor on the load side. However, this may change when the transforming properties of the source and load reactances are considered.

(iv) The source reactance should be handled by a series inductor or a shunt capacitor. The input impedance of the source must be considered to determine which.

(v) The load reactance will be resonated out by a shunt capacitor. Looks like absorption will be a possibility at the load.

(a) What is the input impedance of the source? Treat the voltage generator as a short circuit.

(b) What is the reactance of the series element that will resonate the effective input reactance of the source?

(c) What is the input admittance of the load?

(d) What is the shunt reactance required to resonate the load?

(e) What is the resistive matching problem? That is, since the reactances of the load and source have been resonated out of consideration, what are the effective source and load resistances. Derive the required matching network. Keep the element values of the matching network as reactances.

(f) Draw the complete matching network showing source and load elements required for resonance as well as the matching network for the resistive problem. Keep the element values as reactances.

(g) Draw the final matching network combining all resonant and matching elements. Keep the element values as reactances. This is the electrical design of the matching network.

(h) Calculate the inductance and capacitance values of the matching network.

13. Design a two-element matching network to interface a source with a 25 Ω Thevenin equivalent impedance to a load consisting of a capacitor in parallel with a resistor so that the load admittance is $Y_L = 0.02 + \jmath 0.02$ S. Use the absorption method to handle the reactive load.

14. Design a matching network to interface a source with a 25 Ω Thevenin equivalent impedance to a load consisting of a capacitor in parallel with a resistor so that the load admittance is $Y_L = 0.01 + \jmath 0.01$ S.

(a) If the complexity of the matching network is not limited, what is the minimum $Q$ that could possibly be achieved in the complete network consisting of the matching network and the source and load impedances?

(b) Outline the procedure for designing the matching network for maximum bandwidth if only four elements can be used in the network. You do not need to design the network.

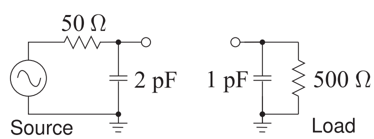
15. Design a lumped-element matching network to match a source, with a Thevenin equivalent impedance of 50 Ω, to a load that consists of a 100 Ω resistor in parallel with a 5 pF capacitor. Design the matching network for maximum bandwidth at 1 GHz using no more than four lumped elements.

(a) How many elements are there in the matching network?

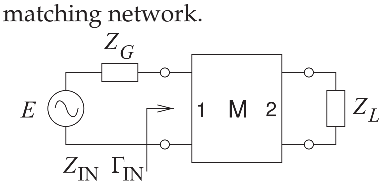
(b) Outline how you will design the matching network.

(c) Design the matching network. You must draw the final design, including the source and load elements, and label each of the lumped elements using reactance values. (That is, do not calculate values of the inductance and capacitances in your design.) (Do not use a Smith chart.)

16. The output of a transistor amplifier is modeled as a current source in parallel with both a 50 Ω resistor and a 1 pF capacitor. This is to be matched to a load consisting of a 25 Ω resistor in series with a 0.02 nH inductor. The task is to design a matching network that will enable DC bias to be applied from the load to the transistor output, thus the matching network must be a lowpass type. The center frequency of the system is 10 GHz and a bandwidth of 50 MHz is required.
    (a) What is the fractional bandwidth of the system?
    (b) What is the $Q$ of the system?
    (c) Indicate the form of the matching network if no more than four reactive elements are to be used; that is, sketch the matching network.
    (d) Complete the design of the amplifier providing numerical element values.
17. Design a Pi network to match the source configuration to the load configuration below. The design frequency is 900 MHz and the desired $Q$ is 10.



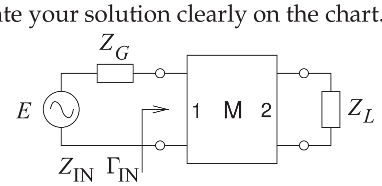

18. Design a passive matching network that will achieve maximum bandwidth matching from a source with an impedance of 2 Ω (typical of the output impedance of a power amplifier) to a load with an impedance of 50 Ω. The matching network can have a maximum of three reactive elements. You need only calculate reactances and not the capacitor and inductor values.
19. Design a passive matching network that will achieve maximum bandwidth matching from a source with an impedance of 20 Ω to a load with an impedance of 125 Ω. The matching network can have a maximum of four reactive elements. You need only calculate reactances and not the capacitor and inductor values.
    (a) Will you use two, three, or four elements in your matching network?
    (b) With a diagram, and perhaps equations, indicate the design procedure.
    (c) Design the matching network. It is sufficient to use reactance values.
20. Design a passive matching network that will achieve maximum bandwidth matching from a source with an impedance of 60 Ω to a load with an impedance of 5 Ω. The matching network can have a maximum of four reactive elements. You need only calculate reactances and not the capacitor and inductor values.
    (a) Will you use two, three, or four elements in your matching network?
    (b) With a diagram and perhaps equations, indicate the design procedure.
    (c) Design the matching network. It is sufficient to use reactance values.
21. Design a T network to match a 50 Ω source to a 1000 Ω load. The desired loaded $Q$ is 15.
22. Repeat Example 6.2 with an inductor in series with the load. Show that the inductance can be adjusted to obtain any positive shunt resistance value.
23. Design a three-lumped-element matching network that interfaces a source with an impedance of 5 Ω to a load with an impedance consisting of a resistor with an impedance of 10 Ω. The network must have a $Q$ of 6.
24. A source with a Thevenin equivalent impedance of 75 Ω must drive a load with an impedance of 5 Ω. A matching network with maximum possible bandwidth between the source and the load must be designed to achieve maximum power transfer. Design the matching network for maximum possible bandwidth using no more than four reactive elements.
    (a) Sketch the schematic of the matching network.
    (b) Describe the design procedure.
    (c) Complete the design of the matching network. Determine the values of the elements if the center frequency is 1 GHz.
25. A two-port matching network is shown below with a generator and a load. The generator impedance is 40 Ω and the load impedance is $Z_L = 50 - \jmath 20$ Ω. Use a Smith chart to design the matching network.




    (a) What is the condition for maximum power transfer from the generator? Express your answer using impedances.
    (b) What is the condition for maximum power transfer from the generator? Express your answer using reflection coefficients.
    (c) What system reference impedance are you going to use to solve the problem?

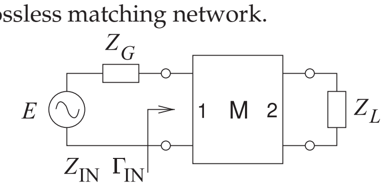
(d) Plot $Z_L$ on the Smith chart and label the point. (Remember to use impedance normalization if required.)
(e) Plot $Z_G$ on the Smith chart and label the point.
(f) Design a matching network using only transmission lines. Show your work on the Smith chart. You must express the lengths of the lines in terms of electrical length (either degrees or wavelengths). Characteristic impedances of the lines are required. (You will therefore have a design that consists of one stub and one other length of transmission line.)

26. A two-port matching network is shown below with a generator and a load. The generator impedance is 60 Ω and the load impedance is $Z_L = 30 + \jmath 30$ Ω. Use a Smith chart to design a lossless matching network. It is important that your solution can be followed, so you must indicate your solution clearly on the chart.




(a) What is the condition for maximum power transfer from the generator? Express your answer using impedances.
(b) What is the condition for maximum power transfer from the generator? Express your answer using reflection coefficients.
(c) What system reference impedance are you going to use to solve the problem?
(d) Plot $Z_L$ on the Smith chart and label the point. (Remember to use impedance normalization if required.)
(e) Plot $Z_G$ on the Smith chart and label the point.
(f) Design a lossless lumped-element matching network showing your design process on the Smith chart. Label critical points on the Smith chart. Draw the matching network and show the reactance values.

27. A two-port matching network is shown below with a generator and a load. The generator impedance is 30 Ω and the load impedance is $Z_L = 90 - \jmath 30$ Ω. Use a Smith chart to design a lossless matching network.




(a) What is the condition for maximum power transfer from the generator? Express your answer using impedances.
(b) What is the condition for maximum power transfer from the generator? Express your answer using reflection coefficients.
(c) What system reference impedance are you going to use to solve the problem?
(d) Plot $Z_L$ on the Smith chart and label the point. (Remember to use impedance normalization if required.)
(e) Plot $Z_G$ on the Smith chart and label the point.
(f) Design a lossless matching network showing your design process on the Smith chart. Label critical points on the Smith chart. Draw the matching network and show the reactance values.

28. Use Smith chart techniques to design a double-stub matching network to match a load with a normalized admittance $y_L = 0.7 - \jmath 5$ to a source with a normalized admittance of 1. The stubs are short-circuited and are separated by a transmission line of length $\lambda/8$. The load is at the position of the first stub. All transmission lines have the system characteristic impedance. Your design should yield the lengths of the two stubs.
(a) Plot the load on a Smith chart. Clearly indicate the load.
(b) Determine the admittances of each of the stubs. Clearly show and describe your design technique so that it can be understood. Label your efforts on a Smith chart and describe the design steps. Note that a description is required and not simply markings on a Smith chart.
(c) Determine the electrical lengths of the stubs (express your answer in terms of wavelengths or degrees).

29. Use a lossless transmission line and a series reactive element to match a source with a Thevenin equivalent impedance of $25 + \jmath 50$ Ω to a load of 100 Ω. (That is, use one transmission line and one series reactance only.)
(a) Draw the matching network with the source and load.
(b) What is the value of the series reactance in the matching network (you can leave this in ohms)?
(c) What is the length and characteristic impedance of the transmission line?

30. Consider a load $Z_L = 100 - \jmath 150$ Ω. Use the Smith chart to design a two-stub matching network that will match the load to a 50 Ω genera-

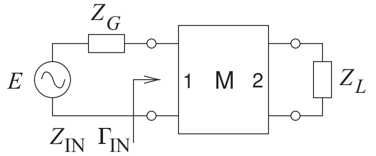
tor. Use 50 Ω transmission lines throughout and assume that the load is immediately next to the first stub. The two stubs are separated by a line with an electrical length of 45°. Both stubs are short-circuited.

(a) Draw the matching stub system.
(b) What is the normalized load impedance?
(c) Briefly indicate the procedure used to design the two-stub matching network. You will need to use stylized Smith charts.
(d) Plot the load on a Smith chart.
(e) What is the admittance of the first stub (Stub 1)?
(f) What is the electrical length of Stub 1? (Note that the stub is short circuited.)
(g) What is the admittance of the second stub near the generator (Stub 2)?
(h) What is the electrical length of Stub 2? (Note that the stub is short circuited.)

31. Consider a load $Z_L$ = 80 + $\jmath$40 Ω. Use the Smith chart to design a matching network consisting of only two transmission lines that will match the load to a generator of 40 Ω.

(a) Draw the matching network with transmission lines. If you use a stub, it should be a short-circuited stub.
(b) Indicate your choice of characteristic impedance for your transmission lines. What is the normalized load impedance? What is the normalized source impedance?
(c) Briefly outline the design procedure you will use. You will need to use Smith chart sketches.
(d) Plot the load and source on a Smith chart.
(e) Complete the design of the matching network, providing the lengths of the transmission lines.

32. A two-port matching network is shown below with a generator and a load. The generator impedance is 40 Ω and the load impedance is $Z_L = 20 - \jmath 50$ Ω. Use a Smith chart to design the matching network.




(a) What is the condition for maximum power transfer from the generator? Express your answer using impedances.
(b) What is the condition for maximum power transfer from the generator? Express your answer using reflection coefficients.
(c) What system reference impedance are you going to use to solve the problem?
(d) Plot $Z_L$ on a Smith chart and label the point. (Remember to use impedance normalization if required.)
(e) Plot $Z_G$ on a Smith chart and label the point.
(f) Design a matching network using only transmission lines and show your work on a Smith chart. You must express the lengths of the lines in terms of electrical length (either degrees or wavelengths long). Characteristic impedances of the lines are required. (You will therefore have a design that consists of one stub and one other length of transmission line.)

33. Use a Smith chart to design a microstrip network to match a load $Z_L = 10 - \jmath 30$ Ω to a source $Z_S = 60 + \jmath 40$ Ω. Use a substrate with permittivity $\varepsilon_r = 10.0$ and thickness 500 μm.

(a) What is the condition for maximum power transfer?
(b) Develop the electrical design of the matching network using the Smith chart using 50 Ω lines only.
(c) Develop the full physical design of the matching network. Draw the microstrip layout and label critical dimensions. That is, you need to find the dimensions of the microstrip circuit.

34. Use a Smith chart to develop the electrical design of a microstrip network to match a load $Z_L = 25$ Ω to a source $Z_S = 250$ Ω. Use only 50 Ω transmission lines. Use one series transmission line and one open-circuited stub. You must use an actual Smith chart and not a sketch of one. Answer parts b, c, and j-q, on a sheet separate from the Smith chart. Make your work easy to follow.

(a) Draw the matching network problem labeling the matching network as M.
(b) What is the condition for maximum power transfer in terms of the source impedance?
(c) What is the condition for maximum power transfer in terms of the source reflection coefficient?
(d) Use a 50 Ω reference impedance and plot the normalized source and load impedances on a Smith chart.
(e) Draw the locus (as the line length increases) of the reflection coefficient looking into a 50 Ω line terminated in the source impedance.
(f) Draw the locus of the reflection coefficient looking into a 50 Ω line terminated in the load impedance.

(g) Draw the locus of a shunt susceptance in parallel with the source (with the shunt susceptance varying in value).
(h) Draw the locus of a shunt susceptance in parallel with the load.
(i) Hence identify two matching network designs on the Smith chart identifying them as Design 1 and Design 2. Make the trajectory of the designs clearly visible including directions and label the critical lengths and susceptances on the Smith chart.
(j) Draw the topology of your Design 1 indicating the source and load ends and labeling critical dimensions?
(k) What is the length of the series transmission line in your Design 1?
(l) What is the normalized input admittance of the stub in your Design 1?
(m) What is the length of the open-circuited stub in your Design 1?
(n) Draw the topology of your Design 2 indicating the source and load ends and labeling critical dimensions?
(o) What is the length of the series transmission line in your Design 2?
(p) What is the normalized input admittance of the stub in your Design 2?
(q) What is the length of the open-circuited stub in your Design 2?

35. Repeat exercise 34 but now with $Z_L = 200\ \Omega$ and $Z_S = 20\ \Omega$).

36. Use a Smith chart to design a microstrip network to match a load $Z_L = 100 - \jmath 100\ \Omega$ to a source $Z_S = 34 - \jmath 40\ \Omega$). Use transmission lines only and do not use short-circuited stubs. Use a reference impedance of $40\ \Omega$.
(a) Draw the matching network problem labeling impedances and the impedance looking into the matching network from the source as $Z_1$.
(b) What is the condition for maximum power transfer in terms of impedances?
(c) What is the condition for maximum power transfer in terms of reflection coefficients?
(d) Identify, i.e. draw, at least two suitable microstrip matching networks.
(e) Develop the electrical design of the matching network using the Smith chart using $40\ \Omega$ lines only. You only need do one design.
(f) Draw the microstrip layout of the matching network identify critical parameters such characteristic impedances and electrical length. Ensure that you identify which is the source side and which is the load side. You do not need to determine the widths of the lies or their physical lengths.

37. Repeat exercise S10.31 but now with $Z_L = 10 - \jmath 40\ \Omega$ and $Z_S = 28 - \jmath 28\ \Omega$).

38. Use a Smith chart to design a two-element lumped-element lossless matching network to interface a source with an admittance $Y_S = 6 - \jmath 12$ mS to a load with admittance $Y_L = 70 - \jmath 50$ mS.

39. Use a Smith chart to design a two-element lumped-element lossless matching network to interface a load $Z_L = 50 + \jmath 50\ \Omega$ to a source $Z_S = 10\ \Omega$.

### 6.12.1 Exercises by Section

†challenging

§6.3 1, 2
§6.4 3, 4
§6.5 5, 6, 7, 8, 9, 10, 11†, 12†, 13†, 14†, 15†, 16†, 17†
§6.6 18†, 19†, 20†, 21†, 22†, 23†, 24†
§6.7 25†, 26†, 27†, 28†, 29†, 30†, 31†, 32†
§6.9 33†, 34, 35, 36, 37, 38, 39

### 6.12.2 Answers to Selected Exercises

11 $43.6 - \jmath 106\ \Omega$
12(g) series: $\jmath 219\ \Omega$, shunt: $-\jmath 225\ \Omega$
16(b) 200
19(c) $Q = 1.22467$
22 $C = 1/(\omega_d^2 L_P)$
25(d) $1.25 - \jmath 0.5$
29(b) $-\jmath 50\ \Omega$
32(f) $40\ \Omega$, 0.085 $\lambda$ long line before load, $40\ \Omega$, 0.076 $\lambda$ long shorted stub

CHAPTER 7

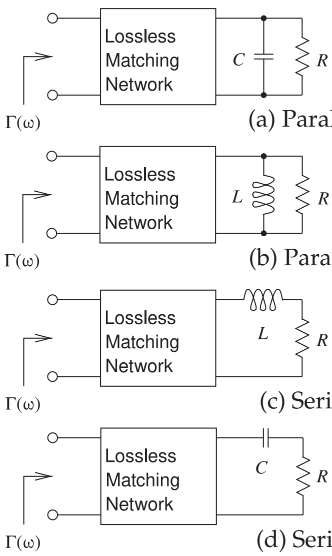
# Broadband Matching



## 7.1 Introduction

This chapter presents techniques for developing broadband matching networks. Sometimes these are akin to cascaded two-port matching networks. The general concept that works most of the time is to gradually step from an initial impedance to a final impedance. Design of broadband matching networks should also be done with the understanding that many RF systems will have multiple matching networks and so the maximum passband transmission loss of an individual matching network needs to be a fraction of a decibel so that when multiple networks are cascaded the insertion loss at the overall band edges will accumulate to equal the desired limit on loss. It is common to use a 1 dB insertion loss threshold to define bandwidth so the individual matching networks may need to have a bandwidth defined by a loss that is much less with insertion loss thresholds as low as 0.1 dB often used.

Bandwidth is limited by energy storage and Section 7.2 introduces the Fano-Bode limits which are theoretical limits of what can be achieved in matching given reactive loads. It is not possible to exceed the Fano-Bode limits on bandwidth. Section 7.3 introduces the constant $Q$ circles plotted on Smith charts to provide a visual guide for the design of broadband matching networks. The next three sections describe three types of broadband matching networks. The first, in Section 7.4, describes the stepped-impedance transmission line transformer which is a generalization of the quarter-wave transformer. Section 7.5 describes tapered transmission-line




$$\int_0^\infty \ln\left[\frac{1}{|\Gamma(\omega)|}\right] d\omega \leq \frac{\pi}{RC}$$

$$\int_0^\infty \frac{1}{\omega^2}\ln\left[\frac{1}{|\Gamma(\omega)|}\right] d\omega \leq \frac{\pi L}{R}$$

$$\int_0^\infty \ln\left[\frac{1}{|\Gamma(\omega)|}\right] d\omega \leq \frac{\pi R}{L}$$

$$\int_0^\infty \frac{1}{\omega^2}\ln\left[\frac{1}{|\Gamma(\omega)|}\right] d\omega \leq \pi RC$$

**Figure 7-1:** Fano-Bode limits for circuits with reactive loads.

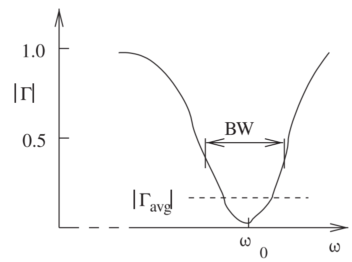



**Figure 7-2:** Response looking into matching network used in defining nonintegral Fano-Bode criteria. The bandwidth of the passband (low Γ, is BW).

transformers which are generalizations of stepped impedance broadband matching networks. An approach to broadband matching to reactive loads is described in Section 7.7.

## 7.2 Fano-Bode Limits

A complex load having energy storage elements limits the bandwidth of the match achieved by a matching network. Theoretical limits addressing the bandwidth and the quality of the match were developed by Fano [1, 2] based on earlier work by Bode [3]. These theoretical limits are known as the Fano-Bode criteria or the Fano-Bode limits. The limits for simple loads are shown in Figure 7-1. More general loads are treated by Fano [1]. The Fano-Bode criteria are used to justify the broad assertion that the more reactive energy stored in a load, the narrower the bandwidth of a match.

The Fano-Bode criteria include the term $1/|\Gamma(\omega)|$, which is the inverse of the magnitude of the reflection coefficient looking into the matching network, as shown in Figure 7-1. A matching network provides matching over a radian bandwidth $\mathrm{BW}$, and outside the matching frequency band the magnitude of the reflection coefficient approaches 1. Introducing $\Gamma_{\mathrm{avg}}$ as the average absolute value of $\Gamma(\omega)$ within the passband, and with $f_0 = \omega_0/(2\pi)$ as the center frequency of the match (see Figure 7-2), then the four Fano-Bode criteria shown in Figure 7-1 can be written as

Parallel RC load:
$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \leq \frac{\pi}{R(\omega_0 C)} \tag{7.1}$$

Parallel RL load:
$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \leq \frac{\pi(\omega_0 L)}{R} \tag{7.2}$$

Series RL load:
$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \leq \frac{\pi R}{(\omega_0 L)} \tag{7.3}$$

Series RC load:
$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \leq \pi R(\omega_0 C). \tag{7.4}$$

In terms of reactance and susceptance these can be written as

Parallel load:

$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \le \frac{\pi G}{|B|} \tag{7.5}$$

Series load:

$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \le \frac{\pi R}{|X|}, \tag{7.6}$$

where $G = 1/R$ is the conductance of the load, $B$ is the load susceptance, and $X$ is the load reactance. $\mathrm{BW}/\omega_0$ is the fractional bandwidth of the matching network. Equations (7.5) and (7.6) indicate that the greater the proportion of energy stored reactively in the load compared to the power dissipated in the load, the smaller the fractional bandwidth ($\mathrm{BW}/\omega_0$) for the same average in-band reflection coefficient $\Gamma_{\mathrm{avg}}$.

Equations (7.5) and (7.6) can be simplified one step further:

$$\frac{\mathrm{BW}}{\omega_0}\ln\left(\frac{1}{\Gamma_{\mathrm{avg}}}\right) \le \frac{\pi}{Q}, \tag{7.7}$$

where $Q$ is that of the load. Several general results can be drawn from Equation (7.7) as follows:

1. If the load stores any reactive energy, so that the $Q$ of the load is nonzero, the in-band reflection coefficient looking into the matching network cannot be zero across the passband.
2. The higher the $Q$ of the load, the narrower the bandwidth of the match for the same average in-band reflection coefficient.
3. The higher the $Q$ of the load, the more difficult it will be to design the matching network to achieve a specified matching bandwidth.
4. A match over all frequencies is only possible if the $Q$ of the load is zero; that is, if the load is resistive. In this case a resistive load could be matched to a resistive source by using a magnetic transformer. Using a matching network with lumped $L$ and $C$ components will result in a match over a finite bandwidth. However, with more than two $L$ and $C$ elements the bandwidth of the match can be increased.
5. Multielement matching networks are required to maximize the matching network bandwidth and minimize the in-band reflection coefficient. The matching network design becomes more difficult as the $Q$ of the load increases.

## 7.3 Constant $Q$ Circles

One strategy for wideband matching is based on the concept of matching to an intermediate resistance that is the geometric mean of the source and load impedances. This concept was introduced in Section 6.6 and can be generalized to intermediate impedances and can be represented on a Smith chart using constant $Q$ as shown in Figure 7-3. If the load and source impedances, $R_L$ and $R_S$, are resistive, then the normalizing resistance ($R_v$) of the Smith chart should (although it is not necessary) be chosen as the geometric mean of the source and load resistance (i.e., $R_v = \sqrt{R_L R_S}$). To maintain a specific circuit $Q$, and hence bandwidth, the locus of impedances in the design must stay within or touch, the corresponding constant $Q$ circle.

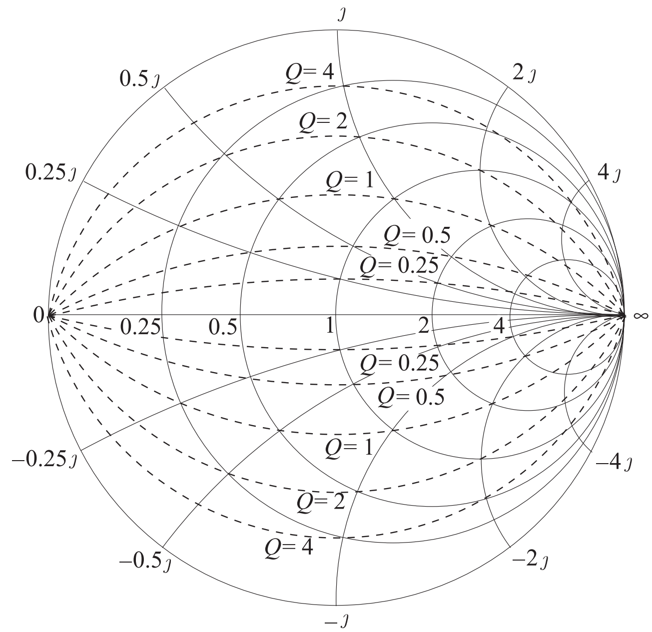



**Figure 7-3:** Impedance Smith chart with constant $Q$ circles.

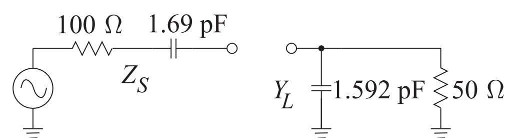



**Figure 7-4:** Matching problem in Example 7.1.

## EXAMPLE 7.1 Broadband Matching Using Constant $Q$ Circles

Design a broadband matching network at 1 GHz for the situation shown in Figure 7-4.

**Solution:**

$Z_S = 100 - \jmath 94.25\ \Omega$, $Y_L = 0.02 + \jmath 0.01$ S, and $Z_L = 40.0 - \jmath 20.0\ \Omega$. Normalizing these to 100 Ω results in $z_S = 1 - \jmath 0.9425$, $y_L = 2 + \jmath 1$, and $z_L = 0.400 - \jmath 0.200$. Also the $Q$ of the source, $Q_S = 0.9425$ and the $Q$ of the load is $Q_L = 0.500$. The matching design puts a network in front of $z_L$ to present an impedance $z_S^*$ to the source. These values are plotted in Figure 7-5. The design objective is to maintain maximum bandwidth and this is done by staying inside the $Q = 0.9425$ circle, where the $Q$ is the larger of the source and load $Q$s.

Design can proceed by moving back from the load toward the source, or from the source to the load. Most commonly the perspective used is moving back from the load. Then the load $z_L$ is transformed to $z_S^*$. Maximum bandwidth is approximately achieved if the matching stages do not exceed the maximum $Q$ of the load or of the source. So the $Q$ of the stages cannot exceed 0.9425. An appropriate matching concept is shown in Figure 7-6. The locus is B→C→D→E→F→G→A. The matching network is therefore of the form shown in Figure 7-7. The rest of the design is extraction of numerical values, but the difficult part of the design has been done.

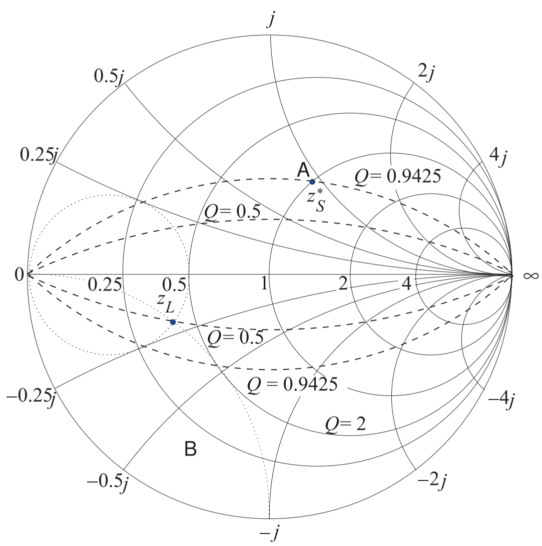



**Figure 7-5:** Impedance Smith chart indicating the matching problem in Example 7.1 with load and source plotted and their constant $Q$ circles. The normalization impedance is 100 Ω.

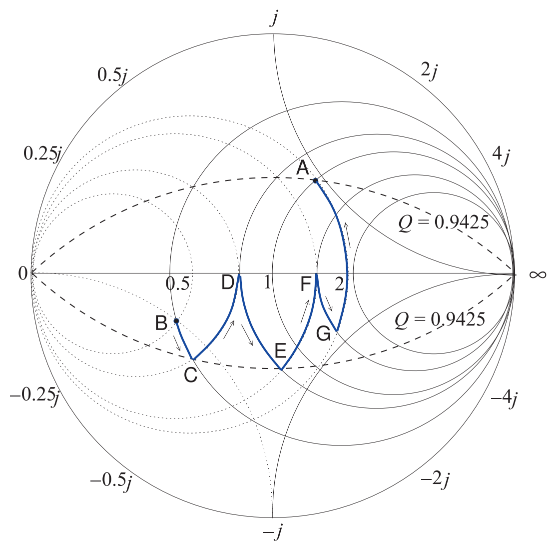



**Figure 7-6:** Outline of matching concept in Example 7.1.

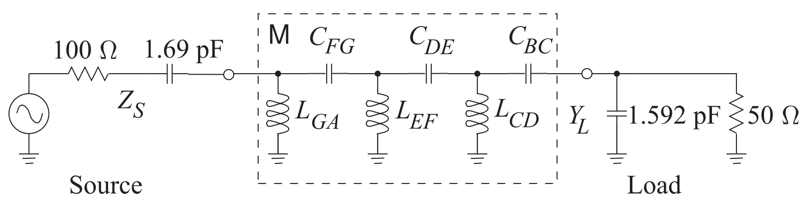



**Figure 7-7:** Matching network M in Example 7.1.

## 7.4 Stepped-Impedance Transmission Line Transformer

The wideband matching techniques described in this section use multiple quarter-wavelength-long transmission line sections with the lines having characteristic impedances which are stepped from the source impedance to the load impedance. They are conceptual extensions of the quarter-wave transformer and differ by how the characteristic impedances of the sections are chosen. The methods strictly are applicable to resistive source and load impedances yet achieve reasonably wideband matches with moderately reactive source and load impedances.

### *7.4.1 Quarter-Wave Transformer using Geometric Means*

Design here uses multiple quarter-wave long transmission lines the characteristic impedances of which are chosen as geometric means of the source and load impedances. The procedure is described in the next example.

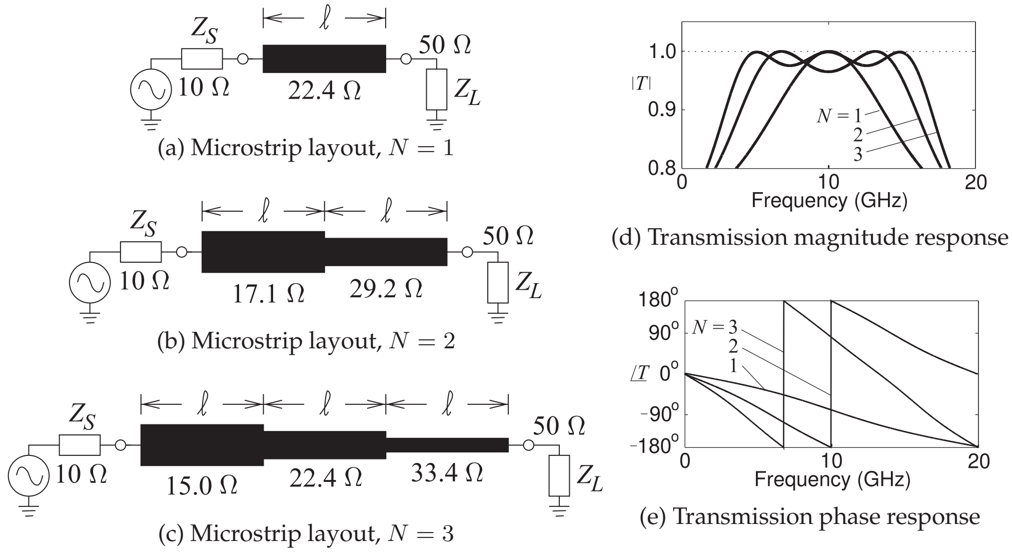



**Figure 7-8:** Transmission line transformer designed in Example 7.2. Each section is of length $\ell = \lambda_g/4 = 2.83$ mm where $\lambda_g$ is the midband wavelength (at 10 GHz).

**EXAMPLE 7.2** Multisection Quarter-Wave Transmission Line Transformer

Design one-, two, and three-section quarter-wave transformers in microstrip to connect a power amplifier with an output impedance of 10 Ω to a 50 Ω cable.

**Solution:**

The parameters are $Z_S = 10\ \Omega$ and $Z_L = 50\ \Omega$. The characteristic impedance of a single, $N = 1$, quarter-wave transformer is $Z_{01} = \sqrt{Z_S Z_L} = 22.36\ \Omega$.

With a two-section, $N = 2$, quarter-wave transformer (using geometric means)

$$Z_{01} = \sqrt[3]{Z_S^2 Z_L} = 17.10\ \Omega \quad Z_{02} = \sqrt[3]{Z_S Z_L^2} = 29.24\ \Omega.$$

With a three-section, $N = 3$, quarter-wave stepped-impedance transformer

$$Z_{01} = \sqrt{Z_S Z_{02}} = 14.95\ \Omega \quad Z_{02} = \sqrt{Z_S Z_L} = 22.36\ \Omega \quad Z_{03} = \sqrt{Z_{02} Z_L} = 33.44\ \Omega. \tag{7.8}$$

The microstrip layouts are shown in Figure 7-8 where each section is a quarter-wavelength long at mid band. The simulated transmission characteristics of the design realized at 10 GHz (on alumina, $\varepsilon_r = 10$, and attenuation of 1.87 dB/m are shown in Figure 7-8(d and e).

### 7.4.2 Design Based on the Theory of Small Reflections

Another design method for choosing the characteristic impedances of cascaded lines is based on the theory of small reflections [4, 5]. The reflection coefficients at each boundary in Figure 7-9 are defined as

$$\Gamma_0 = \frac{Z_{01} - Z_S}{Z_{01} + Z_S} \quad \Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \quad \Gamma_N = \frac{Z_L - Z_{0N}}{Z_L + Z_{0N}}. \tag{7.9}$$

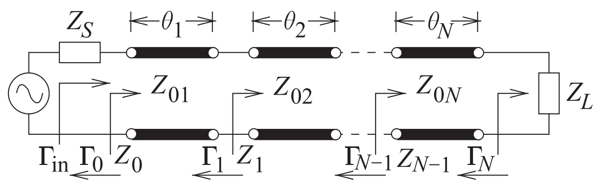



**Figure 7-9:** Stepped-impedance transmission line transformer with the $n$th section having characteristic impedance $Z_{0n}$ and electrical length $\theta_n$. $\Gamma_n$ is the reflection coefficient, and $Z_n$ the impedance, considering only the $(n+1)$th line.

As these are small reflections (since $Z_{0n}$ changes gradually) the theory of small reflections (described in Section 2.6.5 of [6]) can be invoked and so, using Equation ((2.201) of [6]), the total reflection coefficient seen from the source $Z_S$ is

$$\Gamma_{in} \approx \Gamma_0 + e^{-2\jmath\theta_1}\left(\Gamma_1 + e^{-2\jmath\theta_2}\left(\Gamma_2 + \cdots e^{-2\jmath\theta_N}\Gamma_N\right)\right). \tag{7.10}$$

It is now necessary to make a design choice. It has been found that a multisection transformer that provides a good broadband match at both the source and load has symmetrical reflection coefficients, i.e. $\Gamma_n = \Gamma_{N-n}$ [5]. Another design choice is that the electrical lengths of the sections are the same, i.e. $\theta_n = \theta$. Then Equation (7.10) becomes

$$\Gamma_{in} = \Gamma_0 + \Gamma_1 e^{-2\jmath\theta_1} + \Gamma_2 e^{-4\jmath\theta_2} + \cdots \Gamma_N e^{-2\jmath N\theta_N} \tag{7.11}$$

$$= \Gamma_0\left[1 + e^{-2\jmath N\theta}\right] + \Gamma_1\left[e^{-2\jmath\theta} + e^{-2\jmath(N-1)\theta}\right] + \ldots$$

$$= e^{-\jmath N\theta}\left\{\Gamma_0\left[e^{\jmath N\theta} + e^{-\jmath N\theta}\right] + \Gamma_1\left[e^{\jmath(N-2)\theta} + e^{-\jmath(N-2)\theta}\right] + \ldots\right\}$$

$$= 2e^{-\jmath N\theta}\left\{\Gamma_0\cos(N\theta) + \Gamma_1\cos[(N-2)\theta] + \ldots\right\} \tag{7.12}$$

using the trigonometric identity $\cos x = \frac{1}{2}(e^{\jmath x} + e^{-\jmath x})$ (and this is where symmetry is used). The last term in Equation (7.12) is $\frac{1}{2}\Gamma_{N/2}$ if $N$ is even and $\Gamma_{(N-1)/2}\cos\theta$ if $N$ is odd.The design variables here are the reflection coefficients at each line boundary (from which the characteristic impedances of the lines can be found) and the mid-band electrical length $\theta_0$.
The general design approach is to assume a functional form for $\Gamma_{in}(\theta)$ and then to derive the $\Gamma_n$s that result in that functional form. $\Gamma_{in}(\theta)$ will now be used to indicate that $\Gamma_{in}$ is a function of $\theta$ and hence of frequency. Also the mid-band electrical length $\theta_0$ is set to $\pi/2$ corresponding to the sections being a quarter-wavelength long. This may seem arbitrary but it has been shown to be optimum [7] (for maximum bandwidth). The final design step is to derive the characteristic impedances of the line sections. Using Equation (7.9)

$$Z_{0N} = Z_L\left(\frac{1-\Gamma_N}{1+\Gamma_N}\right) \quad \text{and} \quad Z_{0n} = Z_{0(n+1)}\left(\frac{1-\Gamma_n}{1+\Gamma_n}\right). \tag{7.13}$$

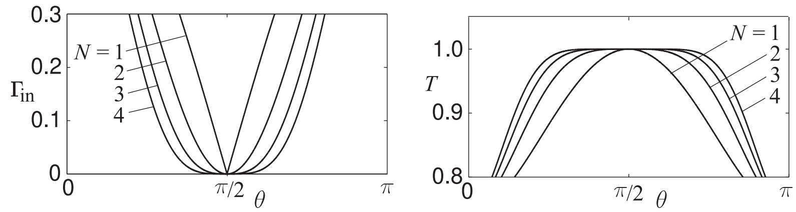
### 7.4.3 Maximally Flat Stepped Impedance Transformer

The design objective here is to set the first $N$ derivatives of $\Gamma_{in}$ at midband to zero. This results in a very smooth response and that is what is desired in some situations. If the following assignment is made

$$|\Gamma_{in}(\theta)| \propto |\cos(\theta)|^N \tag{7.14}$$

then $d^n|\Gamma_{in}(\theta)|/d\theta^n = 0$ at $\theta = \pi/2 = \theta_0$ for $n = 0, 1, \ldots, (N-1)$. An assignment that results in this is the binomial expansion

$$\Gamma_{in}(\theta) = A\left(1 + e^{-2\jmath\theta}\right)^N = \sum_{n=0}^{N}\binom{N}{n}e^{-2\jmath n\theta} \tag{7.15}$$



(a) Input reflection coefficient (b) Transmission coefficient

**Figure 7-10:** Characteristics of order $N = 1, 2, 3$ and 4 maximally flat stepped-impedance transformers with an impedance mismatch $\delta_z = \max(Z_L/Z_S, Z_S/Z_L) = 2$.

where (since $N$ and $n$ are integers)

$$\binom{N}{n} = \frac{N!}{(N-n)!n!} \tag{7.16}$$

is the binomial coefficient. Equating Equations (7.11) and (7.15)

$$\Gamma_n = A\binom{N}{n}. \tag{7.17}$$

To find $A$ consider zero frequency. Then $\theta = 0$ and the transformer has no effect and $\Gamma_{\text{in}}$ is just the mismatch of the source and load impedances and Equation (7.15) becomes

$$\Gamma_{\text{in}}(0) = A2^N = \frac{Z_L - Z_S}{Z_L + Z_S} \quad \text{so that} \quad A = 2^{-N}\left(\frac{Z_L - Z_S}{Z_L + Z_S}\right). \tag{7.18}$$

Thus $$\Gamma_n(\theta) = 2^{-N}\left(\frac{Z_L - Z_S}{Z_L + Z_S}\right)\binom{N}{n} \tag{7.19}$$

and $Z_{0n}$ comes from Equation (7.13) with the design accuracy determined by the approximation of a small discontinuity at each transmission line boundary.

The reflection $\Gamma_{\text{in}}$ and transmission $T$ characteristics of maximally flat stepped-impedance transformers are shown in Figure 7-10 for various orders. As with all lossless two-ports $|T| = \sqrt{1 - |\Gamma_{\text{in}}|^2}$. It can be seen that the bandwidth increases with increasing order and the transmission is remarkably flat.

## EXAMPLE 7.3 Maximally Flat Multisection Transmission Line Transformer

Design a three-section maximally flat stepped-impedance transformer in microstrip to connect a $Z_S = 5\ \Omega$ source to a $Z_L = 50\ \Omega$ load.

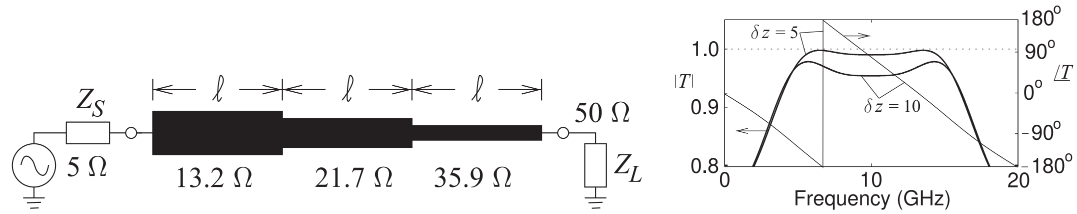
**Solution:**

The design will have three transmission lines of different characteristic impedance and each section will be a quarter-wavelength long at mid band. Now $N = 3$ so from Equation (7.19)

$$\Gamma_n = 2^{-N}\left(\frac{Z_L - Z_S}{Z_L + Z_S}\right)\binom{N}{n} = 2^{-3}\left(\frac{50-5}{50+5}\right)\binom{3}{n} = \frac{45}{8 \cdot 55}\binom{3}{n}. \tag{7.20}$$

$$\Gamma_3 = 0.1023\frac{3!}{0!3!} = \frac{1}{16}\frac{3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1} = 0.1023, \quad \Gamma_2 = 0.3068, \quad \Gamma_1 = 0.3068, \quad \Gamma_0 = 0.1023$$

resulting in (a sanity check is the expectation that $Z_L > Z_{03} > Z_{02} \ldots$)




(a) Microstrip layout, $\delta z = Z_L/Z_S = 10$ (b) Transmission response

**Figure 7-11:** Maximally flat transmission line transformer designed in Example 7.3. At 10 GHz each section is of length $\ell = \lambda_g/4 = 2.83$ mm where $\lambda_g$ is the midband wavelength.

$$Z_{03} = 50\frac{1-0.1023}{1+0.1023}\ \Omega = 40.72\ \Omega, \quad Z_{02} = 21.60\ \Omega, \quad Z_{01} = 11.46\ \Omega, \quad \text{and} \quad Z'_S = 9.333\ \Omega$$

where $Z'_S$ is ideally (the complex conjugate of) the original $Z_S$ but is actually significantly different. This is because the design is based on the theory of small reflections and so multiple reflections at the transmission line boundaries were not considered. As a further investigation, and noting that each section is a quarter-wavelength long at midband, another estimate for $Z_S$, call this $Z''_S$, is found using Equation ((2.132)) of [6],

$$Z_2 = Z_{03}^2/Z_L = 33.16\ \Omega, \quad Z_1 = Z_{02}^2/Z_2 = 14.07\ \Omega, \quad Z''_S = Z_{01}^2/Z_1 = 9.334\ \Omega.$$

The microstrip layout is shown in Figure 7-11(a). Simulation of the design realized at 10 GHz (on alumina, $\varepsilon_r = 10$, a 2.83 mm section length, and a line attenuation of 1.87 dB/m for an overall attenuation of 0.016 dB) results in the transmission characteristics identified by $\delta z = Z_L/Z_S = 10$ in Figure 7-11(b). The final design does not have the ideal maximally flat transmission characteristics and this is due to deficiencies in the small reflection assumption. Final design requires a small amount of optimization as the synthesized design has a maximum in-band insertion loss of 0.4 dB ($T = 0.953$).

Repeating the design with $Z_S = 10\ \Omega$ and $Z_L = 50\ \Omega$ results in $\Gamma_0 = \Gamma_3 = 0.08333$, $\Gamma_1 = \Gamma_2 = 0.2500$, $Z_{01} = 15.23\ \Omega$, $Z_{02} = 25.38\ \Omega$, $Z_{03} = 42.31\ \Omega$, and $Z'_S = Z''_S = 12.89\ \Omega$. The transmission characteristics are identified by $\delta z = Z_L/Z_S = 5$ in Figure 7-11(b). The minimum in-band transmission coefficient is 0.990 for a maximum insertion loss of 0.091 dB. Thus design accuracy improves for a lower impedance transformation ratio.

### 7.4.4 Stepped Impedance Transformer With Chebyshev Response

Expressing the input reflection coefficient $\Gamma_{\text{in}}$ of a stepped-impedance transformer in terms of a Chebyshev polynomial results in a good match in-band that rapidly transitions outside the passband. Compared to the maximally flat transformer a much better match can be obtained for the same number of line sections if the resulting ripples in the passband can be tolerated. With reference to Equation (7.10), the design choice is

$$\begin{aligned}\Gamma_{\text{in}}(\theta) &= \Gamma_0 + \mathrm{e}^{-2\jmath\theta}\left(\Gamma_1 + \mathrm{e}^{-2\jmath\theta}\left(\Gamma_2 + \cdots \mathrm{e}^{-2\jmath\theta}\Gamma_N\right)\right)\\ &= A\mathrm{e}^{-\jmath N\theta}T_N(\cos\theta/\cos\theta_m).\end{aligned} \tag{7.21}$$

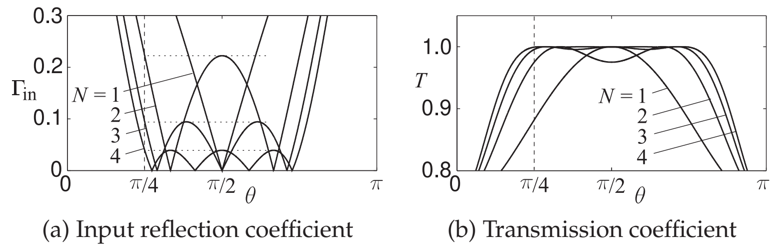



**Figure 7-12:** Characteristics of order $N = 1, 2, 3$ and 4 Chebyshev stepped-impedance transformers with impedance mismatch $\delta_z = \max(Z_L/Z_S, Z_S/Z_L) = 2$ and $\theta_m = \pi/4$. The transmission ripple cannot be seen for $N \geq 3$.

where $T_N$ is the $N$th order Chebyshev polynomial of the first kind (as described in Section 1.A.10 of [6]). In Equation (7.21) $\theta_m$ defines the passband of the transformer as being between $\theta_m \leq \theta \leq (\pi - \theta_m)$ and in the passband $|\Gamma_{\text{in}}(\theta)| \leq \Gamma_m$ and at $\theta = \theta_m$ and $\theta = (\pi - \theta_m)$ (the edges of the passband)

$$|\Gamma_{\text{in}}(\theta_m)| = |\Gamma_{\text{in}}(\pi - \theta_m)| = \Gamma_m = |AT_N(\cos\theta_m/\cos\theta_m)| = |AT_N(1)|. \quad (7.22)$$

$$\text{Since } |T_N(1)| = 1 \qquad A = \Gamma_m \quad (7.23)$$

To proceed another expression is needed for A so that $\Gamma_m$, $N$, and $\theta_m$ (and thus bandwidth) can be related. This is obtained by considering the mismatch at zero frequency, that is when $\theta = 0$:

$$\Gamma_{\text{in}}(0) = \left|\frac{Z_L - Z_S}{Z_L + Z_S}\right| = AT_N(\sec\theta_m). \quad (7.24)$$

(where $\sec\theta_m = 1/\cos\theta_m$) and so

$$A = \left|\frac{Z_L - Z_S}{Z_L + Z_S}\right| \frac{1}{T_N(\sec\theta_m)}. \quad (7.25)$$

Substituting for $A$ in Equation (7.21) and using Equation (7.10) results in

$$\begin{aligned} e^{\jmath N\theta}\Gamma_{\text{in}}(\theta) &= AT_N(\cos\theta/\cos\theta_m) = \left(\frac{Z_L - Z_S}{Z_L + Z_S}\right)\frac{T_N(\cos\theta/\cos\theta_m)}{T_N(\sec\theta_m)} \\ &= 2\left\{\Gamma_0\cos(N\theta) + \Gamma_1\cos[(N-2)\theta] + \ldots\right\}. \end{aligned} \quad (7.26)$$

The expansion of $T_N(\cos\theta/\cos\theta_m)$ is given in Equation ((1.198)) of [6] and has terms in $\cos(m\theta)$ and so design (which requires $\Gamma_n$) proceeds by equating terms in Equation (7.26) having the same $\cos(m\theta)$ for $m = 1, \ldots, N$. This will be illustrated in an example.

The reflection $\Gamma_{\text{in}}$ and transmission $T$ characteristics of Chebyshev stepped-impedance transformers are shown in Figure 7-12 for orders from one to four for $\theta_m = \pi/4$ (this indicates a 100% bandwidth). It is seen that the maximum inband $\Gamma_{\text{in}}$) reduces with increasing order. So with a Chebyshev response there is a trade-off between passband ripple and bandwidth.

An expression for bandwidth of the match will now be developed. Equating Equations (7.23) and (7.25)

$$T_N(\sec\theta_m) = \frac{1}{\Gamma_m}\left|\frac{Z_L - Z_S}{Z_L + Z_S}\right| \quad (7.27)$$

Using the identity

$$T_N(\sec\theta_m) = \cosh\left(N\cosh^{-1}(\sec\theta_m)\right) = \frac{1}{\Gamma_m}\left|\frac{Z_L - Z_S}{Z_L + Z_S}\right| \quad (7.28)$$

**Table 7-1:** Relationship of order $N$, fractional bandwidth, impedance ratio $\delta z = \max(Z_L/Z_S, Z_S/Z_L)$, and reflection coefficient ripple $\Gamma_m$ for a Chebyshev stepped-impedance transformer. A transmission ripple of 0.1 dB has $\Gamma_m = 0.151$. (For example, a two-section ($N = 2$) transformer has a 95.4% bandwidth with $\delta_z = 3.0$.)

| $N =$ | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 4 | 5 | 5 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Gamma_m =$ | 0.05 | 0.1 | 0.151 | 0.05 | 0.1 | 0.151 | 0.05 | 0.1 | 0.151 | 0.05 | 0.1 | 0.151 | 0.05 | 0.1 | 0.151 |
| $\delta z$ | Fractional bandwidth, $\Delta f/f_0$ | | | | | | | | | | | | | | |
| 1.0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| 1.2 | 0.741 | $\infty$ | $\infty$ | 1.275 | $\infty$ | $\infty$ | 1.502 | $\infty$ | $\infty$ | 1.622 | $\infty$ | $\infty$ | 1.696 | $\infty$ | $\infty$ |
| 1.3 | 0.501 | 1.112 | $\infty$ | 1.069 | 1.526 | $\infty$ | 1.346 | 1.680 | $\infty$ | 1.500 | 1.759 | $\infty$ | 1.596 | 1.807 | $\infty$ |
| 1.4 | 0.388 | 0.819 | 1.441 | 0.951 | 1.333 | 1.714 | 1.252 | 1.544 | 1.808 | 1.425 | 1.655 | 1.856 | 1.534 | 1.722 | 1.885 |
| 1.6 | 0.278 | 0.571 | 0.907 | 0.814 | 1.134 | 1.395 | 1.137 | 1.396 | 1.588 | 1.330 | 1.540 | 1.689 | 1.455 | 1.629 | 1.750 |
| 1.8 | 0.224 | 0.455 | 0.708 | 0.735 | 1.024 | 1.250 | 1.066 | 1.310 | 1.483 | 1.271 | 1.471 | 1.607 | 1.405 | 1.573 | 1.684 |
| 2.0 | 0.192 | 0.388 | 0.598 | 0.683 | 0.951 | 1.158 | 1.018 | 1.252 | 1.415 | 1.229 | 1.424 | 1.554 | 1.369 | 1.534 | 1.641 |
| 2.5 | 0.149 | 0.300 | 0.458 | 0.604 | 0.844 | 1.026 | 0.943 | 1.162 | 1.313 | 1.164 | 1.351 | 1.607 | 1.313 | 1.472 | 1.574 |
| 3.0 | 0.128 | 0.256 | 0.390 | 0.561 | 0.784 | **0.954** | 0.900 | 1.110 | 1.254 | 1.125 | 1.307 | 1.426 | 1.280 | 1.436 | 1.535 |
| 4.0 | 0.106 | 0.213 | 0.324 | 0.513 | 0.718 | 0.874 | 0.850 | 1.051 | 1.188 | 1.081 | 1.257 | 1.372 | 1.241 | 1.393 | 1.490 |
| 6.0 | 0.085 | 0.179 | 0.271 | 0.471 | 0.660 | 0.804 | 0.805 | 0.997 | 1.128 | 1.040 | 1.211 | 1.322 | 1.204 | 1.354 | 1.449 |
| 8.0 | 0.082 | 0.164 | 0.249 | 0.452 | 0.634 | 0.772 | 0.784 | 0.971 | 1.100 | 1.020 | 1.189 | 1.299 | 1.187 | 1.335 | 1.429 |
| 10 | 0.078 | 0.156 | 0.236 | 0.441 | 0.618 | 0.754 | 0.772 | 0.957 | 1.083 | 1.009 | 1.176 | 1.285 | 1.176 | 1.323 | 1.417 |

and so
$$\theta_m = \sec^{-1}\left\{\cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_S}{Z_L + Z_S}\right|\right)\right]\right\}. \tag{7.29}$$

The fractional bandwidth can be obtained by noting that $\theta$ and thus $\theta_m$ are proportional to frequency $f$. That is, $f = k\theta$. At the passband center frequency $f_0$, $\theta = \pi/2$ and so $k = 2f_0/\pi$. Thus if $f_m$ is the frequency at the lower band edge, $f_m = k\theta_m = 2f_0\theta_m/\pi$. Then the fractional bandwidth (with the passband defined by when $|\Gamma_{\text{in}}| \leq \Gamma_m$) is

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi}. \tag{7.30}$$

Thus Equations (7.29) and (7.30) relate the fractional bandwidth, the maximum passband reflection coefficient $\Gamma_m$, the impedance mismatch $\delta z = \max(Z_L/Z_S, Z_S/Z_L)$, and the Chebyshev order, $N$. Table 7-1 enables the required transformer order to be selected for a specified bandwidth and impedance mismatch.

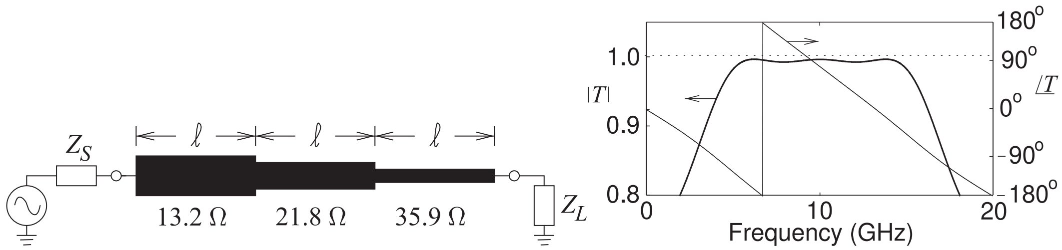
### EXAMPLE 7.4 Chebyshev Multisection Transmission Line Transformer

Design a 100% bandwidth three-section Chebyshev stepped-impedance transformer in microstrip to connect a power amplifier with an output impedance of 10 Ω to a 50 Ω cable.

**Solution:**

The design parameters are $N = 3$, $Z_S = 10\ \Omega$, and $Z_L = 50\ \Omega$. From Equation (7.30) the fractional bandwidth $\Delta f/f_0 = 1 = 2 - (4\theta_m)/\pi$ so that $\theta_m = \pi/4$. Then, with $\sec\theta_m = 1/\cos\theta_m = 1/\cos(\pi/4) = \sqrt{2}$ and $T_3(\sqrt{2}) = 7.071$ (from Equation ((1.193) of [6])), Equation (7.25) yields

$$\Gamma_m = A = \left(\frac{Z_L - Z_S}{Z_L + Z_S}\right)\frac{1}{T_3(\sec\theta_m)} = \left(\frac{50-10}{50+10}\right)\frac{1}{7.071} = 0.09428. \tag{7.31}$$




(a) Microstrip layout, $\ell = \lambda_g/4 = 2.83$ mm (b) Transmission response

**Figure 7-13:** Chebyshev transformer of Example 7.4. ($\lambda_g$ is the midband wavelength.)

Using Equation (7.26) and the Chebyshev expansion for $T_3$ this leads to

$$\begin{aligned} AT_3(\cos\theta/\cos\theta_m) &= A\left\{\sec^3\theta_m[\cos(3\theta) + 3\cos\theta] - 3\sec\theta_m\cos\theta\right\} \\ &= 2\left[\Gamma_0\cos(3\theta) + \Gamma_1\cos(\theta)\right]. \end{aligned} \tag{7.32}$$

Thus after equating like terms in $\cos(m\theta)$ (noting that $\Gamma_n = \Gamma_{(N-n)}$ since symmetry is required)

$$\begin{aligned} \Gamma_0 = \Gamma_3 &= \tfrac{1}{2}A\sec^3\theta_m = \tfrac{1}{2}0.09428\cdot\left(\sqrt{2}\right)^3 = 0.1333 \\ \Gamma_1 = \Gamma_2 &= \tfrac{3}{2}A\left[\sec^3\theta_m - \sec\theta_m\right] = \tfrac{3}{2}0.09428\cdot\left[\left(\sqrt{2}\right)^3 - \sqrt{2}\right] = 0.2000 \end{aligned} \tag{7.33}$$

Then the characteristic impedances of the three line sections are (using Equation (7.13))

$$Z_{01} = 13.19\ \Omega,\quad Z_{02} = 21.78\ \Omega,\quad \text{and}\quad Z_{03} = 35.94\ \Omega \tag{7.34}$$

and each section is a quarter-wavelength long at midband. The microstrip layout is shown in Figure 7-13(a). The simulated transmission characteristics of the design realized at 10 GHz (on alumina, $\varepsilon_r = 10$, section lengths of 2.83 mm, and an attenuation of 1.87 dB/m (for an overall attenuation of 0.016 dB) are shown in Figure 7-13(b). The expected lossless ripple (from $T = \sqrt{1-\Gamma_m^2} = 0.9955 = -0.039$ dB) is 0.039 dB. The simulated minimum insertion loss is 0.024 dB and the maximum in-band insertion loss is 0.057 dB for a passband ripple of 0.033 dB (line loss is known to reduce ripple).

### 7.4.5 Stepped Impedance Transformer Design

The multisection stepped-impedance transformers change the characteristic impedances of the section between the source and load resistances but do so in steps. It is possible to achieve a similar result by continuously tapering the characteristic impedance of the transmission line taper or, equivalently, tapering the width of microstrip line as shown in Figure 7-14.

### 7.4.6 Summary

The multisection impedance transformer design described in this section is based on transmission line sections each a quarter-wavelength long at the center frequency of the match. It is tempting to think that a better result could be obtained by having sections of various lengths. However it has been shown that optimum matching transformer designs are of the quarter-wave

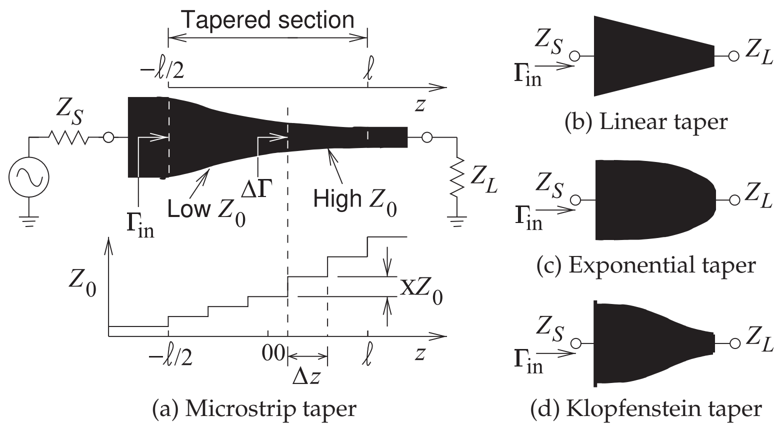



**Figure 7-14:** Tapered impedance transformers with length $\ell$. The width of the microstrip line is approximately inversely proportional to the characteristic impedance of the line. (Here $Z_L > Z_S$ and both $Z_L$ and $Z_S$ are resistive.)

Chebyshev type and only minimum enhancement is possible [7].

## 7.5 Tapered Matching Transformers

Tapered impedance transformers match an impedance $Z_S$ to an impedance $Z_L$ using a transmission line having a characteristic impedance $Z_0$ that gradually and monotonically varies from $Z_S$ to $Z_L$ along the length of the line, see Figure 7-14. This figure references a microstrip line but the key aspect is the gradual change in characteristic impedance that applies to any transmission line. The central design problem is how to choose the function $Z_0(z)$. If the length of the line, $\ell$ here, is not constrained then the Klopfentein taper [8, 9] is regarded as the optimum approach for design of the taper and will be discussed after first reviewing approaches used when the length of the line is constrained. Note that both $Z_L$ and $Z_S$ are resistive.

### *7.5.1 Small Reflection Theory and Tapered Lines*

In this section the theory behind the synthesis of a taper is developed beginning with the theory of small reflections. The reflection at point $z$ on the line for a taper segment of length $\Delta z$ is (refer to Figure 7-14(a))

$$\Delta\Gamma = \frac{(Z_0(z) + \Delta Z) - Z_0(z)}{(Z_0 + \Delta Z) + Z_0(z)} = \frac{\Delta Z}{2Z_0(z) + \Delta Z} \approx \frac{\Delta Z}{2Z_0(z)}, \tag{7.35}$$

where $Z_0(z)$ is the characteristic impedance of the taper at $z$ and $\Delta Z$ is the change of $Z_0(z)$ from one side of the taper segment to the other. In the limit as $\Delta Z \to 0$, $\Delta Z$ is replaced by $dZ$ and $\Delta\Gamma$ is replaced by $d\Gamma$ so that Equation (7.35) becomes (and putting in differential operator form)

$$d\Gamma = \frac{dZ_0}{2Z_0(z)} \quad \to \quad \frac{d\Gamma}{dz} = \frac{\mathrm{d}\Gamma}{\mathrm{d}z} = \frac{1}{2Z_0(z)}\frac{\mathrm{d}Z_0}{\mathrm{d}z}. \tag{7.36}$$

The next step is to refer all of the small reflections, $\Delta\Gamma$, to the beginning of the taper at $z = -\ell/2$. The theory of small reflections is that these small reflections, accounting for the electrical lengths from the start of the taper to each taper segment, can be summed, see Equation (7.10). This is the same as saying that the small reflection from one taper segment is not affected by its own reflection from another taper segment. Noting that the electrical length from the beginning of the taper to a taper segment at $z$ is $\theta = \beta(\ell/2 + z)$, the reflection at the input of the tapered line is found as

$$\Gamma_{\text{in}}(\ell) = \int_{z=-\ell/2}^{z=\ell/2} \mathrm{e}^{-2\jmath(\ell/2+\beta z)} \mathrm{d}\Gamma \tag{7.37}$$

$$= \tfrac{1}{2}\mathrm{e}^{-\jmath\beta\ell} \int_{-\ell/2}^{\ell/2} \frac{\mathrm{e}^{-2\jmath\beta z}}{Z_0(z)} \frac{\mathrm{d}Z_0(z)}{\mathrm{d}z}\,\mathrm{d}z, \tag{7.38}$$

where $\Gamma_{\text{in}}$ is shown explicitly as a function of the length of the taper. So the design problem becomes choosing the characteristic impedance function, $Z_0(z)$ to provide the desired input reflection coefficient $\Gamma_{\text{in}}$. This is difficult to achieve with the form of Equation (7.38). The problem can be simplified by noting that (and introducing $Z_1$ as a normalizing impedance so that the argument of $\ln$ is dimensionless)

$$\frac{\ln\left(Z_0(z)/Z_1\right)}{\mathrm{d}z} = \frac{Z_1}{Z_0(z)} \frac{\mathrm{d}(Z_0(z)/Z_1)}{\mathrm{d}z} = \frac{Z_1}{Z_0(z)} \frac{1}{Z_1} \frac{\mathrm{d}(Z_0(z)/Z_1)}{\mathrm{d}z}$$
$$= \frac{1}{Z_0(z)} \frac{\mathrm{d}(Z_0(z)/Z_1)}{\mathrm{d}z}, \tag{7.39}$$

and so, making it clear that $d\Gamma$ varies along the taper,

$$d\Gamma(z) = \tfrac{1}{2} \frac{\mathrm{d}(Z_0(z)/Z_1)}{\mathrm{d}z}\,dz. \tag{7.40}$$

Thus after assuming the form of $Z_0(z)$, the incremental reflection coefficient $d\Gamma$ is obtained using Equation (7.37). Alternatively (after integrating Equation (7.40)) a form for $\Gamma_{\text{in}}$ can be assumed and $d\Gamma(z)$ determined and then $Z_0(z)$. This will become clearer below when specific tapers are considered.

### 7.5.2 Linear taper

In the linear tapered line design $Z_0(z)$ varies linearly from the source impedance $Z_S$ to $Z_L$:

$$Z_0(z) = Z_S + (Z_L - Z_S)z/\ell. \tag{7.41}$$

This is often approximated in microstrip by a linear taper of the width of the microstrip line as shown in Figure 7-14(b). A simple expression for the input reflection coefficient is not available and so must be found from simulation. This is the simplest taper and the taper performs better the greater its electrical length (i.e. at higher frequencies or longer physical length). The performance of the linear taper is compared to other tapers later.

### 7.5.3 Exponential taper

The exponential taper has an exponential taper of the line's characteristic impedance. Setting

$$Z_0(z) = Z_x \mathrm{e}^{az} \quad \text{with} \quad a = \frac{1}{\ell}\ln\left(Z_L/Z_S\right) \quad \text{and} \quad Z_x = Z_S \mathrm{e}^{-a\ell/2} \tag{7.42}$$

results in the input reflection coefficient, derived using Equation (7.38),

$$\Gamma_{\rm in}(\ell) = \tfrac{1}{2} Z_x \mathrm{e}^{-\jmath\beta\ell} \int_{-\ell/2}^{\ell/2} \mathrm{e}^{-2\jmath\beta z} \frac{\mathrm{d}\ln(e^{az})}{\mathrm{d}z}\,\mathrm{d}z = \tfrac{1}{2}\ln\left(Z_L/Z_S\right)\frac{\sin(\beta\ell)}{\beta\ell}. \quad (7.43)$$

So $\Gamma_{\rm in}$ has a sinc function characteristic with the variations of $\Gamma_{\rm in}$ reducing as the taper becomes longer. The main problem with this taper comes from the abrupt impedance discontinuity at the $Z_L$ end of the taper. This taper will not be considered further as the Klopfenstein taper considered next has much better performance.

### 7.5.4 Klopfenstein taper

The Klopfenstein taper [8, 9] results in a specified reflection coefficient ripple (and thus transmission ripple) above a minimum passband frequency. It is believed to achieve the minimum taper length over a passband defined by the maximum allowable reflection coefficient mismatch, $\Gamma_m$, (and so minimum transmission loss) in the passband. The linear taper and most other tapers used in matching [10, 11] assume the form of a taper's characteristic impedance profile and the broadband reflection and transmission characteristics are whatever results. In contrast the Klopfenstein taper derives the required impedance profile for a source and load impedance mismatch ratio ($Z_L/Z_S$) and $\Gamma_m$.

Klopfenstein [8] showed that the input reflection coefficient of the taper could be expressed as the limiting form of a high-order Chebyshev polynomial. Thus the Klopfenstein taper has the passband ripples that occur with Chebyshev-based multi-section impedance transformers and Chebyshev filters. The maximum magnitude of the reflection coefficient in the passband is determined by the line length. The appropriate characteristic impedance is computed from [8, 9]

$$\ln Z_0(z) = \ln\left(\sqrt{Z_S Z_L}\right) + \ln\left(\sqrt{Z_L/Z_S}\right)\cdot(\cosh A)^{-1} \times \left[A^2\phi\left(2z/\ell, A\right) + U\left(z-\tfrac{1}{2}\ell\right) + U\left(z+\tfrac{1}{2}\ell\right) - 1\right] \quad (7.44)$$

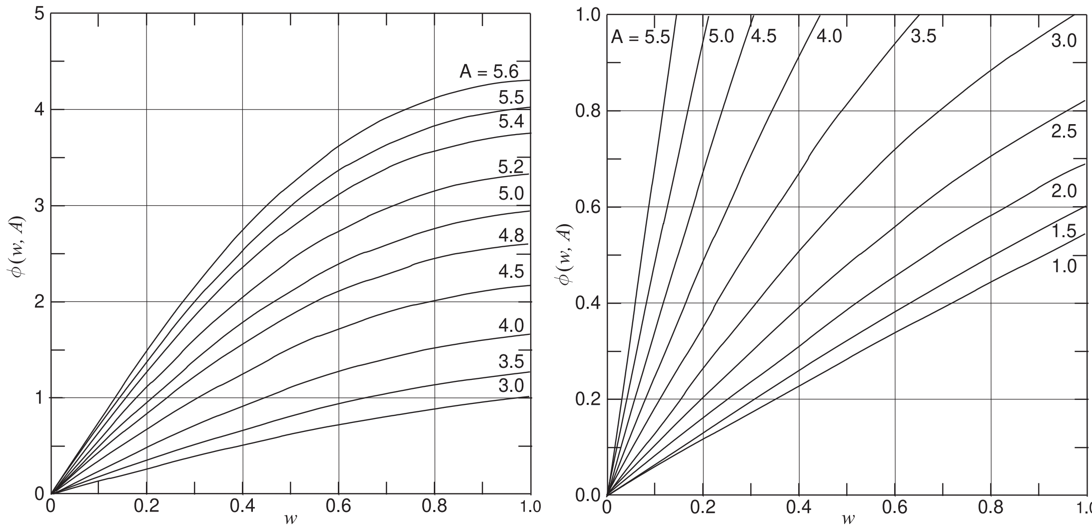
where $U(x)$ is the unit step function so that $U(x) = 0$ for $x < 0$ and $U(x) = 1$ for $x \geq 0$.

The maximum reflection coefficient amplitude $\Gamma_m = \rho_0/\cosh A$ where $\rho_0 = (Z_L - Z_S/(Z_L + Z_S)$ is the reflection coefficient when the load and source are directly connected. Klopfenstein found that this introduced a small error attributed to the limitation of the small reflection assumption. He determined that a better estimate is $\rho_0 = \frac{1}{2}\ln(Z_L/Z_S)$. Thus

$$A = \cosh^{-1}\left[\ln(Z_L/Z_S)/\Gamma_m\right]. \quad (7.45)$$

That is, $\Gamma_m$, the maximum reflection coefficient in the passband, and the mismatch, $Z_L/Z_S$, determines $A$. Substituting Equation (7.45) in Equation (7.44) yields

$$\ln Z_0(z) = \ln\left(\sqrt{Z_S Z_L}\right) + \ln\left(\sqrt{Z_L/Z_S}\right)\Gamma_m\frac{(Z_L+Z_S)}{(Z_L-Z_S)} \times \left[A^2\phi\left(2z/\ell, A\right) + U\left(z-\tfrac{1}{2}\ell\right) + U\left(z+\tfrac{1}{2}\ell\right) - 1\right] \quad (7.46)$$




(a) $\phi(w, A)$ (b) $\phi(w, A)$ detail

**Figure 7-15:** Klopfenstein taper function $\phi(w, A)$ used in designing a taper segment. The taper has length $\ell$, $z$ is the coordinate at the center of a taper segment, the center of the taper is at $z = 0$, and $\phi(-w, A) = -\phi(w, A)$, $w = 2z/\ell$.

and the function $\phi(w, A)$ is computed as [12]

$$\phi(w, A) = \sum_{k=0}^{\infty} a_k b_k \tag{7.47}$$

(summation up to $k = 20$ is sufficient) with the recursion formulas

$$a_0 = 1,\ a_k = \frac{A^2}{4k(k+1)} a_{k-1};\ \ b_0 = \tfrac{1}{2}w,\ b_k = \frac{\frac{1}{2}w(1-w^2)^k + 2kb_{k-1}}{2k+1}. \tag{7.48}$$

The results are shown in Figure 7-15.

It is interesting to derive the characteristic impedance at the center of the taper. At the center of the taper, $z = 0$, $\phi(0, A) = 0$ and Equation (7.44) becomes

$$\ln Z_0(0) = \tfrac{1}{2}\ln(Z_S Z_L), \quad \text{that is,} \quad Z_0(0) = \sqrt{Z_S Z_L}, \tag{7.49}$$

which is the geometric mean of the source and load resistances.

The Klopfenstein taper trades off line length $\ell$, minimum passband frequency $f_{\min}$, and maximum passband reflection coefficient $\Gamma_m$. The passband of the taper is all frequencies above $f_{\min}$. This is a remarkable result with the line length being considerably less than that of a linear taper. The limitation is that with such a short line the reflections along the line cannot always be considered to be small so that it is often necessary to increase the line length slightly above the length derived from this synthesis procedure.

### 7.5.5 Simplified Klopfenstein taper

The simplified form of the Klopfenstein taper is obtained by noting that the acceptable value for the maximum in-band reflection coefficient $\Gamma_m$ will

| $w$ | Exact $Z_0(w)/Z_S$ | Simplified $Z_0(w)/Z_S$ | Error |
|---|---|---|---|
| −1.0 | 1.163 | 1.177 | 1.23% |
| −0.8 | 1.326 | 1.344 | 1.32% |
| −0.6 | 1.577 | 1.597 | 1.26% |
| −0.4 | 1.944 | 1.964 | 1.04% |
| −0.2 | 2.460 | 2.476 | 0.64% |
| 0.0 | 3.162 | 3.162 | 0% |
| 0.2 | 4.065 | 4.039 | 0.64% |
| 0.4 | 5.145 | 5.092 | 1.03% |
| 0.6 | 6.214 | 6.262 | 1.25% |
| 0.8 | 7.539 | 7.440 | 1.31% |
| 1.0 | 8.520 | 8.542 | 1.12% |

**Table 7-2:** Comparison of $Z_0$ of the exact and simplified Klopfenstein tapers (using Equation (7.46) with Equations (7.47) and (7.50) respectively) for a maximum passband transmission loss $T_m = 0.1$ dB and $Z_L/Z_S = 10$ ($\Gamma_m = 0.151$ and $A = 2.72$.) Errors are less for larger $T_m$ and smaller $Z_L/Z_S$:
For $Z_L/Z_S = 20$, $T_m = 0.1$ dB, the max. error is 2.78%.
For $Z_L/Z_S = 20$, $T_m = 0.2$ dB, the max. error is 1.49%.
For $Z_L/Z_S = 10$, $T_m = 0.1$ dB, the max. error is 1.33%.
For $Z_L/Z_S = 10$, $T_m = 0.2$ dB, the max. error is 0.66%.
For $Z_L/Z_S = 5$, $T_m = 0.1$ dB, the max. error is 0.44%.
For $Z_L/Z_S = 5$, $T_m = 0.2$ dB, the max. error is 0.21%.

typically be small and for a maximum transmission loss of between 0.1 dB and 1 dB (corresponding to $\Gamma_m = 0.151$ and $\Gamma_m = 0.454$ respectively) and maximum $Z_L/Z_S = 10$. Then the maximum value of $A$ is 2.72 and this is when the simplified Klopfenstein taper will have the most error. So retaining only the first three terms in Equation (7.47), $\phi(w, A)$ can be approximated as

$$\phi(w, A) = a_0 b_0 + a_1 b_1 + a_2 b_2 \tag{7.50}$$

and used in Equation (7.46). A comparison of the calculated impedances for a relatively high transmission loss of 1 dB (0.1 dB is more typical) and a large impedance mismatch $Z_L/Z_S = 10$ is given in Table 7-2. The maximum error of 1.33% is comparable to the characteristic impedance error of fabricated transmission lines. So for practical purpose the simplified approach can be used to design the Klopfenstein taper.

## EXAMPLE 7.5 Design of a Klopfenstein Taper

Design a microstrip Klopfenstein taper to match a $Z_S = 10\ \Omega$ source to a $Z_L = 50\ \Omega$ load. The maximum transmission ripple is to be 0.1 dB and the minimum passband frequency is 8 GHz. The substrate has a thickness $h = 0.635$ μm, and relative permittivity $\varepsilon_r = 10.0$.

**Solution:**

First determine the maximum reflection coefficient $\Gamma_m$ in the passband. The minimum transmission in the passband is $T = 10^{0.1/20}$ and $\Gamma_m = \sqrt{1 - T^2} = 0.151$. Since $Z_L/Z_S = 5$ it is seen from Table 7-2 that the simplified Klopfenstein taper can be used with a maximum error of 1.33%. Using Equation (7.45) the required electrical length of the taper at 8 GHz is

$$A = \cosh^{-1}\left[\ln(Z_L/Z_S)/\Gamma_m\right] = \cosh^{-1}\left[\ln(50/5)/0.151\right] = 2.720.$$

A taper with ten segments is chosen and in the table below the normalized length $w' = 2z/\ell$ is used to distinguish the parameter from the microstrip width $w$. The microstrip parameters were obtained by interpolating Table 3-3 of [6]. $\bar{Z}_0$ and $\bar{\varepsilon}_e$ are the average characteristic impedance and effective permittivity of a segment, a linear taper, extending from the width on the previous line to that on the same line. The electrical length of each segment is $\beta(\Delta\ell) = A/10 = 0.272$ radians so that the physical length of a segment $\ell = 0.272\lambda_{8\text{GHz}}/(2\pi\sqrt{\varepsilon_e})$ where $\lambda_{8\text{GHz}} = 3.745$ cm is the free-space wavelength at 8 GHz.

| Segment | $w'$ | $\phi(w', A)$ | $Z_0$ (Ω) | $u = h/w$ | $w$ (mm) | $\bar{Z}_0$ (Ω) | $\bar{\varepsilon}_e$ | $\Delta\ell$ ($\mu$m) |
|---|---|---|---|---|---|---|---|---|
| | −1.0 | −0.774 | 11.68 | 8.28 | | | | |
| 1 | −0.8 | −0.661 | 12.84 | 7.38 | | 12.2 | 8.35 | |
| 2 | −0.6 | −0.521 | 14.44 | 6.39 | | 14.1 | 8.19 | |
| 3 | −0.4 | −0.360 | 16.52 | 5.40 | | 16.0 | 8.05 | |
| 4 | −0.2 | −0.184 | 19.16 | 4.45 | | 17.8 | 7.93 | |
| 5 | 0 | 0 | 22.36 | 3.62 | | 20.8 | 7.75 | |
| 6 | 0.2 | 0.184 | 26.10 | 2.90 | | 24.7 | 7.54 | |
| 7 | 0.4 | 0.360 | 30.26 | 2.33 | | 28.7 | 7.35 | |
| 8 | 0.6 | 0.521 | 34.63 | 1.88 | | 32.9 | 7.18 | |
| 9 | 0.8 | 0.661 | 38.93 | 1.54 | | 37.3 | 7.03 | |
| 10 | 1.0 | 0.774 | 42.83 | 1.29 | | 41.4 | 6.90 | |

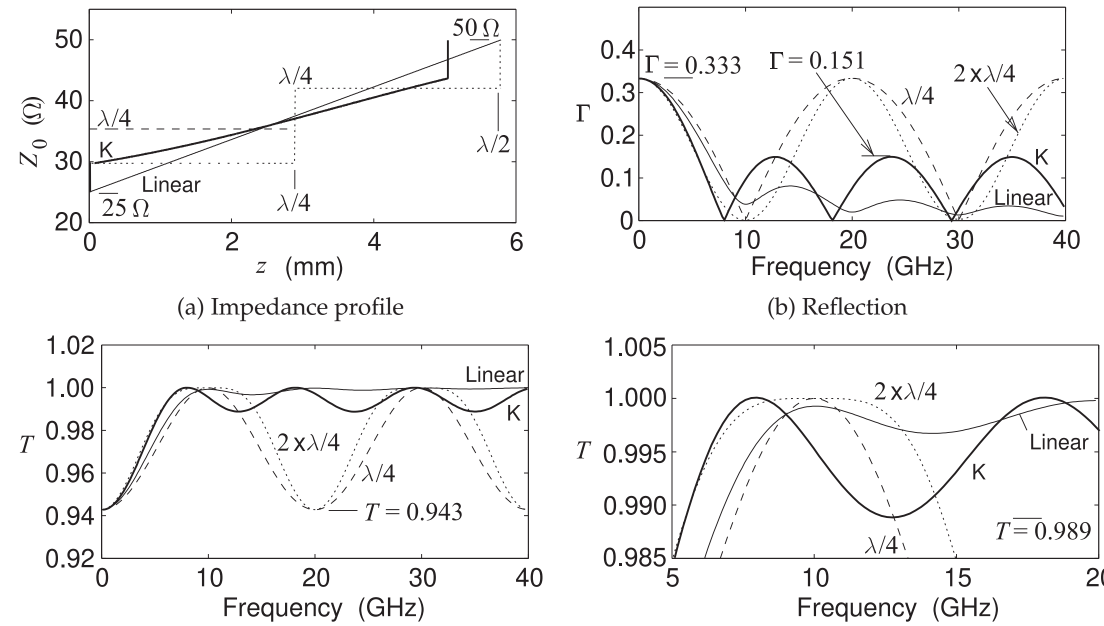
As well at $w = (-1.0 - 1/\infty)$ $Z_0 = 510.03\ \Omega$ and at $w = (1.0 + 1/\infty)$ $Z_0 = 49.81\ \Omega$ matching the source and load impedances respectively and thus there is a step discontinuity at both ends, a characteristic of the Klopfenstein taper.

### 7.5.6 Comparison of Transmission Line Impedance Transformers

In this section the four main impedance transformers are compared: the linear taper, the Klopfenstein taper, the quarter-wave transformer and the two-section quarter-wave transformer. These transformers are lengths of nonuniform transmission line with a characteristic impedance that varies along the length of the line, i.e. $Z_0 = Z_0(z)$ where $z$ is the position along the line of total length $\ell$. The $N$-section quarter-wave transformer has step changes in $Z_0(z)$ at $n\lambda/4$ where $n = 1,\ 2 < \ldots N$ but practically $N = 1$ or 2 is the limit usually considered as much better performance can be obtained with the Klopfenstein taper with a legth typically between $\lambda/4$ and $\lambda/2$.

Figure 7-16 compares the performance of the tapers for $Z_S = 25\ \Omega$ and $Z_L = 50\ \Omega$ but the results are applicable in general for $Z_L/Z_S = 2$. Figure 7-16(a) shows the $Z_0$ profile for the transmission line transformers and where the length of the linear taper has been chosen to provide comparable passband responses defined as where the transmission loss is less than 0.1 dB corresponding to a maximum reflection coefficient $\Gamma_m = 0.151$ and a minimum transmission factor $T = 0.989$. The reflection coefficient response is shown in Figure 7-16(b). First consider the responses of the quarter-wave transformers. Both provide an ideal match at the passband center frequency $f_0 = 10$ GHz and this repeats at odd multiples of $f_0$ as a $3\lambda/4$-long line is electrically identical to a $\lambda/4$-long line. The linear taper, chosen here as $\lambda_m/2$ long where $\lambda_m$ is the guide wavelength at $f_0$, has a reflection coefficient mismatch that reduces as frequency increases as then the line becomes electrically longer.

The Klopfenstein taper for $\Gamma_m = 0.151$ and $Z_L/Z_S = 2$ has $A = 1.1103$ so that the passband of the Klopfenstein taper extends indefinitely above an electrical length of 1.1103 radians which defines the physical length of the line for a chosen minimum passband frequency $f_{\min}$. The design choice here is $f_{\min} = 0.532 f_0$ so that $f_{\min}$ was comparable to that of the two-section quarter-wave transformer. Then the electrical line length required is $0.87\lambda_m/2$. That is, a slightly shorter Klopfenstein taper has the same minimum passband frequency as a two-section quarter-wave transformer.




(a) Impedance profile (b) Reflection

(c) Transmission (d) Transmission detail

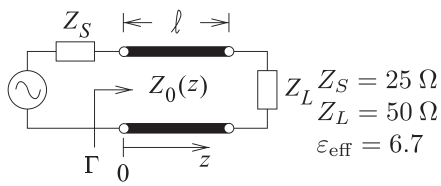
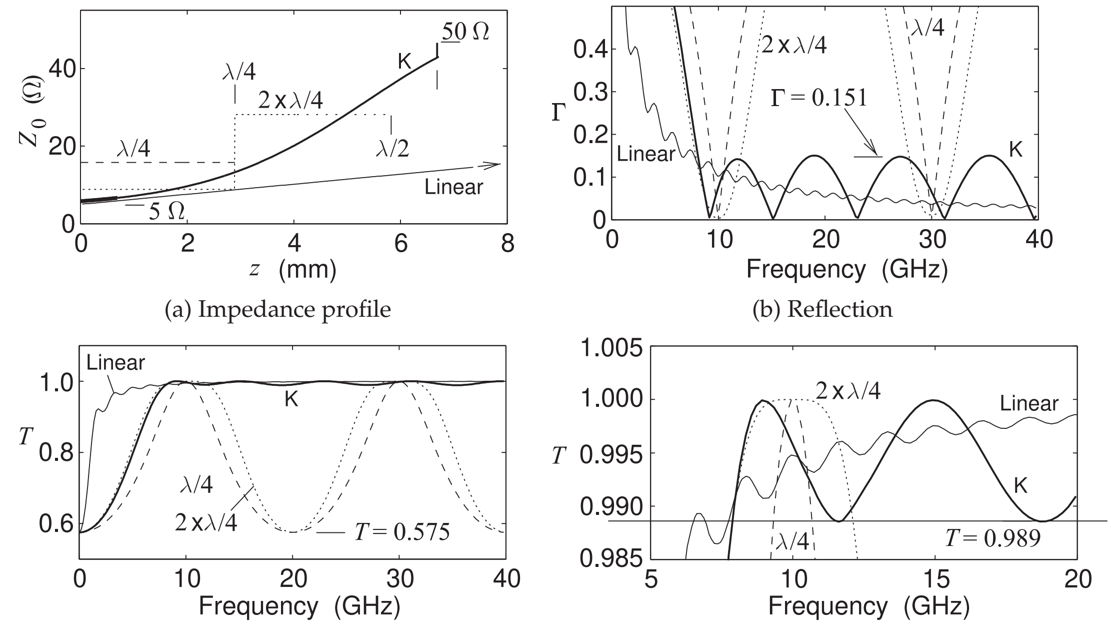



**Figure 7-16:** Characteristics of simulated transmission line transformers. Midband wavelength $\lambda_m = 11.58$ mm at $f_0 = 10$ GHz. Linear: Linear taper, $\ell = \lambda_m/2$. $\lambda/4$: $\lambda/4$ transformer, $\ell = \lambda_m/4$. 2x$\lambda/4$: two-section $\lambda/4$ transformer, $\ell = \lambda_m/2$. K: Klopfenstein taper, $\ell = 3.88\ \text{mm} = 0.335\lambda_m$.

The Klopfenstein taper has the distinct advantage that the passband extends indefinitely above $f_{\min}$ where as one-stage and multi-stage quarter-wave transformers have a finite bandwidth.

The previous paragraph considered matching when $Z_L/Z_S = 2$. A similar comparison is shown in Figure 7-17 for a much higher source and load impedance discontinuity with $Z_S = 5\ \Omega$ and $Z_L = 50\ \Omega$ and the results are applicable in general for $Z_L/Z_S = 10$. Figure 7-17(a) shows the $Z_0$ profile for the transmission line transformers and where the length of the linear taper has been chosen to provide comparable passband responses defined as where the transmission loss is less than 0.1 dB (corresponding to a maximum reflection coefficient magnitude of 0.151). The reflection coefficient response is shown in Figure 7-17(b). First consider the responses of the single- and two-section quarter-wave transformers. Both provide an ideal match at the passband center frequency $f_0 = 10$ GHz and repeating at odd multiples of $f_0$. Again the passband defined by the two-section quarter-wave transformer is used to determine the length of the linear and Klopfenstein tapers resulting in the linear taper being $3\lambda_m$ long and the Klopfenstein taper being $0.595\ \lambda_m$ long, slightly longer than the two-section quarter-wave transformer. The linear taper, chosen here to be $\lambda_m/2$ long where $\lambda_m$ is the guide wavelength at $f_0$, has a reflection coefficient mismatch that reduces as frequency increases




(a) Impedance profile

(b) Reflection

(c) Transmission

(d) Transmission detail

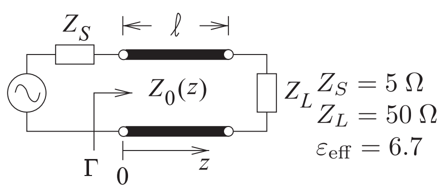



**Figure 7-17:** Characteristics of simulated transmission line transformers. Midband wavelength $\lambda_m = 11.58$ mm at $f_0 = 10$ GHz. Linear: Linear taper, $\ell = \lambda_m/2$. $\lambda/4$: $\lambda/4$ transformer, $\ell = \lambda_m/4$. 2x$\lambda/4$: two-section $\lambda/4$ transformer, $\ell = \lambda_m/2$. K: Klopfenstein taper, $\ell = 3.88$ mm $= 0.335\lambda_m$.

**Table 7-3:** Comparison of passbands of the four transmission line impedance transformers considered in Section 7.5.6 with $\lambda_m$ being the guide wavelength at 10 GHz. The lengths of the tapers were chosen to have the same minimum passband frequency as the two-section quarter-wave transformer.

| $Z_L/Z_S = 2$ | $\ell$ | Passband | Bandwidth |
|---|---|---|---|
| Linear taper | $0.5\lambda_m$ | >6.53 GHz | |
| $\lambda/4$ | $0.25\lambda_m$ | 7.16–11.84 GHz | 50% |
| $2 \times \lambda/4$ | $0.5\lambda_m$ | 5.43–14.57 GHz | 91% |
| Klopfenstein taper | $0.335\lambda_m$ | >5.43 GHz | |
| $Z_L/Z_S = 10$ | | | |
| Linear taper | $3\lambda_m$ | >7.72 GHz | |
| $\lambda/4$ | $0.25\lambda_m$ | 9.32–10.69 GHz | 14% |
| $2 \times \lambda/4$ | $0.5\lambda_m$ | 7.87–12.12 GHz | 42% |
| Klopfenstein taper | $0.595\lambda_m$ | >7.87GHz | |
| exponential taper | $1.68\lambda_m$ | >7.87GHz | |

as the line becomes electrically longer.

The Klopfenstein taper for $\Gamma_m = 0.151$ and $Z_L/Z_S = 10$ has $A = 2.720$ so that the passband of the Klopfenstein taper extends indefinitely above an electrical length of 2.720 radians and so choosing the minimum passband frequency $f_{\rm min}$ determines the physical length of the line.

The 0.1 dB passbands of the transmission line transformers are compared in Table 7-3. The two-section quarter-wave transformer and the Klopfenstein transformer have comparable performance near the center frequency of the design with the choice being made on whether it is more important to have good transmission properties indefinitely above $f_{\min}$ or to provide some frequency selectivity by having a poorer match at the second harmonic frequency of the center match frequency, here $f_0$.

### 7.5.7 Summary

The transmission line transformers considered in this section match resistive source and load impedances. However these impedance transformers provide guidance for design strategies when the source and load include reactances. When the source and load are resistances then the clear choice for a transmission-line-based impedance transformer is the Klopfenstein tape.

With a reactive load the challenge is achieving broadband match since of course the reactance of the load and/or load will vary with frequency and so impose an overall bandwidth constraint. Design of the matching network needs to take into account the frequency characteristic of the load.

Different loads will have different frequency characteristics and hence variations in the type of matching network required. Four basic loads that are commonly encountered in microwave engineering and to which matching is required include the series RC model of the input of a FET transistor with a series reactance that is inversely proportional to frequency; the series LR model encountered in bonding to the input of a device where the inductance comes from a bond wire and which has a series reactance that is proportional to frequency; and a parallel RC load encountered at the output of a transistor with a susceptance that reduces with frequency. These two-element models of sources and loads are simple and other parasitics may need to be included. At microwave frequencies the $Q$ of the impedances encountered with active devices is typically in the range of 0.5 to 3, and source/load resistance mismatch typically ranges from 1.5 to 10. For example, high mismatches are encountered at the output of power amplifiers.

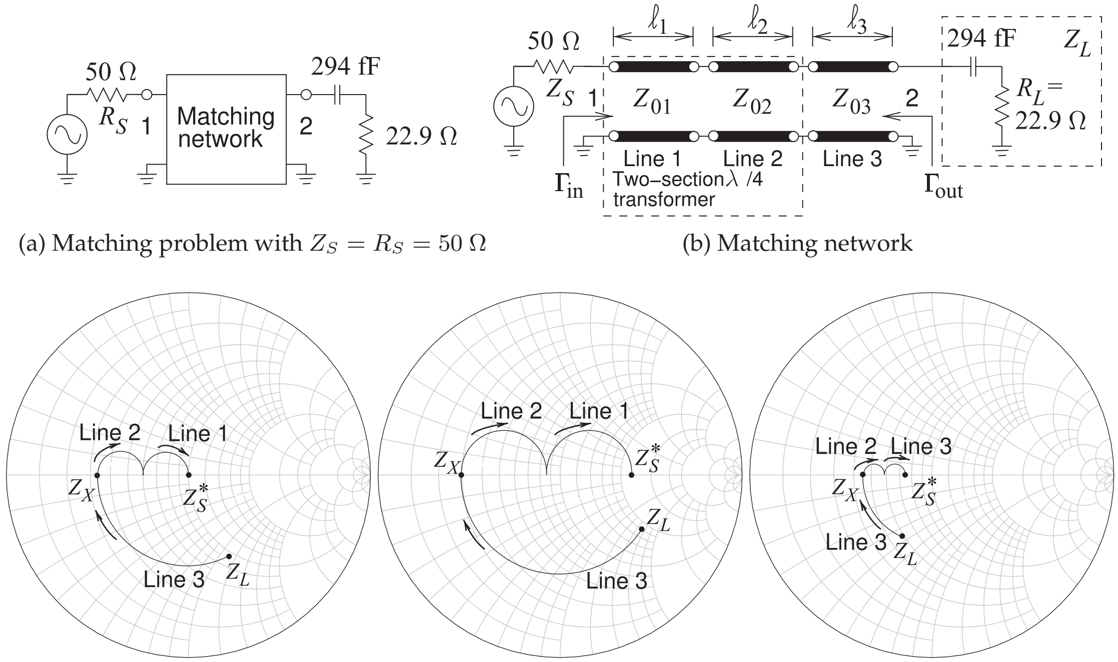
## 7.6 Matching a Series RC load

The matching network design described in this section is appropriate for a real source impedance and series RC load where the load resistance is less than the source resistance. Also a two-section quarter-wave impedance transformer will be considered as this has performance that is representative of what is reasonable to achieve using transmission-line based impedance transformers. Tapers are not considered here as their particular advantage of an indefinite passband when matching resistive source and load impedance disappears when the load (or source) is reactive.

#### Design Examples

Figure 7-18(a) presents the problem of matching to the input of a transistor which is modeled here as a capacitor in series with a resistive load. This is the typical model for the input of a FET.

With a two-section cascaded quarter-wave transformer an appropriate matching network is shown in Figure 7-18(b). This topology is based on the design concepts shown in Figures 7-18(c, d and e) where the single-frequency reflection coefficient loci with respect to increasing line length are shown. The overall concept is that line 3 rotates the load impedance to a resistance $R_X$ and then $R_X$ is matched to $R_S$ using a two-section quarter-wave transformer,




(a) Matching problem with $Z_S = R_S = 50\ \Omega$ (b) Matching network

(c) $Z_{03} = 50\ \Omega$ (d) Low impedance, $Z_{03} \ll 50\ \Omega$ (e) High impedance, $Z_{03} \gg 50\ \Omega$

**Figure 7-18:** Matching problem of the case study of Section 7.6 using a two-section cascaded quarter-wave impedance transformer. The arrows in (c), (d), and (e) indicate increasing line length at 10 GHz. The Smith charts are normalized to $Z_{03}$ and present three design concepts. (The choice of high $Z_{03}$ is appropriate for $R_L < Z_S$ and a capacitor in the load.)

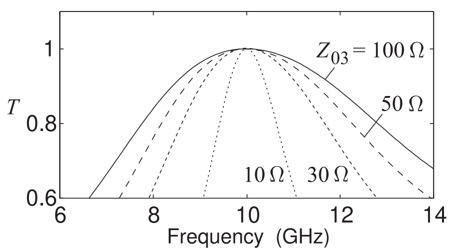
that is, lines 1 and 2.

At 10 GHz the 294 fF capacitor has a reactance of $-54.13\ \Omega$ and with the resistive part of the load the input of the transistor has a $Q$ of 2.36. This is plotted on the $50\ \Omega$ Smith chart in Figure 7-18(c). The design concept is that line 3 rotates the load $Z_L$ to a purely resistive load of $10.0\ \Omega$. Then a $\lambda/4$-long line, line 2, rotates the reflection coefficient to an intermediate impedance, and this is followed by line 1, another $\lambda/4$-long line which takes the input impedance to $50\ \Omega$. Note that the electrical lengths of the lines are given by the angle subtended by the arcs and do not correspond to the drawn lengths of the arcs. Here the electrical length of line 1 is $50.0°$ and that of lines 2 and 3 are $90°$.

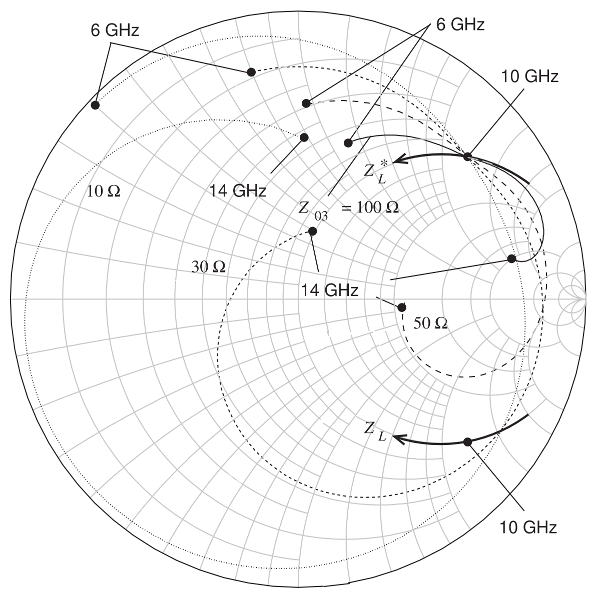
Two alternative design concepts are shown in Figures 7-18(d and e). Figure 7-18(d) describes a low impedance ($Z_{03} \ll 50\ \Omega$) design concept. If $Z_{03} = 20\ \Omega$ then the electrical length of the line is $72.3°$ and the intermediate impedance $Z_X = 2.41\ \Omega$. An alternative is to use a high characteristic impedance for line 3 as outlined in Figure 7-18(e). With $Z_{03} = 100\ \Omega$ the electrical length of line 3 is $29.4°$ and $Z_X = 17.54\ \Omega$. Of these three design concepts the third design concept with the highest $Z_{03}$ is preferable on two accounts. One of these is that the electrical length of line 3 is the smallest, and since the electrical lengths of lines 1 and 2 are fixed (each is $\lambda/8$ long), this results in the lowest overall electrical line length. Secondly the intermediate resistance is closest

| $Z_{03}$ ($\Omega$) | $\beta\ell_3$ | $Z_X$ ($\Omega$) | $Z_{02}$ ($\Omega$) | $Z_{01}$ ($\Omega$) | 0.1 dB Bandwidth (GHz) |
|---|---|---|---|---|---|
| 10 | 81.1° | 0.65 | 1.92 | 18.86 | 9.90–10.18 (0.28) |
| 20 | 72.3° | 2.41 | 5.14 | 23.4 | 9.76–10.25 (0.49) |
| 30 | 64.1° | 4.86 | 8.70 | 27.9 | 9.69–10.39 (0.70 |
| 40 | 56.6° | 7.50 | 9.62 | 31.1 | 9.62–10.46 ( 0.84) |
| 50 | 50.0° | 10.0 | 9.55 | 33.4 | 9.55-10.52 (0.97) |
| 60 | 44.4° | 12.2 | 17.3 | 35.1 | 9.54-10.53 (0.99) |
| 70 | 39.6° | 14.0 | 19.2 | 36.3 | 9.48–10.60 (1.11) |
| 80 | 35.6° | 15.4 | 20.7 | 37.3 | 9.48–10.60 (1.11) |
| 90 | 32.3° | 16.6 | 21.9 | 38.0 | 9.48–10.60 (1.11) |
| 100 | 29.4° | 17.5 | 22.8 | 38.5 | 9.48–10.60 (1.11) |

**Table 7-4:** Matching network performance with different $Z_{03}$ with electrical length $\beta\ell_3$. The minimum and maxium frequencies of the passband are when $T = 0.989$, i.e. when the transmission loss is 0.1 dB.




(a) Transmission




(b) $\Gamma_{\text{out}}$

**Figure 7-19:** Characteristics of the matching network in Figure 7-18(b) for $Z_{03} = 10\ \Omega, 30\ \Omega, 50\ \Omega$, and $100\ \Omega$. The Smith chart in (b) is normalized to $22.9\ \Omega$ and show the frequency loci of $\Gamma_{\text{out}}$ for four matching network designs for $Z_{03} = 10\ \Omega, 30\ \Omega, 50\ \Omega$, and $100\ \Omega$. The Smith chart also shows the frequency locus of $Z_L$ and $Z_L^*$. Note that $\Gamma_{\text{out}}$ is equal to the reflection coefficient of $Z_L^*$ at 10 GHz for all matching networks.

to the complex conjugate of the source impedance, $Z_S^*$, which of course is just $Z_S$ here since the source is real.

Table 7-4 compares several trial designs with varying $Z_{03}$ and it is apparent that the characteristic impedance of $Z_{03}$ needs to be high. A maximum $Z_{03}$ of $100\ \Omega$ is typical of many transmission line technologies and especially of planar transmission lines like microstrip. Note that the maximum bandwidth is 11% which is considerably less than the approximately 90% bandwidth that could be obtained for a two-section quarter-wave transformer if the capacitance in the load was not present (see Table 7-3).

The transmission characteristics of the matching network designed using four values of $Z_{03}$ ranging from $10\ \Omega$ to $100\ \Omega$ are shown in Figure 7-19(a). Examination of the Smith chart plot, Figure 7-19(b), reveals the fundamental problem in broadband matching of reactive loads to a resistive source impedance. First consider the frequency locus of the load $Z_L$ which is seen to have a clockwise rotation on the Smith chart as is typical of non-resonant impedances. Matching requires that the impedance presented by the network at port 2 be the complex conjugate of $Z_L$, i.e. $Z_L^*$. $Z_L^*$ is seen to rotate in the counter-clockwise direction with increasing frequency. The frequency loci of the reflection coefficients, $\Gamma_{\rm out}$, that are presented looking into port 2 of the matching network (in Figure 7-18(a)) for each of the four designs are also plotted in Figure 7-19(b). These all rotate in the clockwise direction so that it is only possible for these matching networks to present the desired impedance $Z_L^*$ at a single frequency, here 10 GHz. The angular length of the $\Gamma_{\rm out}$ loci from 6 GHz to 14 GHz for the four designs differ. The line with $Z_{03} = 100\ \Omega$ has the shortest $\Gamma_{\rm out}$ locus having the shortest total angular length (and hence shortest electrical and physical lengths). The passband responses of the various designs are shown in Figure 7-19(a) and the broadest passband response is obtained when $Z_{03} = 100\ \Omega$ and this impedance is about the largest that could be tolerated for a planar line as otherwise the microstrip characteristic impedance is become close to the $377\ \Omega$ free space and radiation (and this loss) from the microstrip line is starting to be significant.

### 7.6.1 Summary

The lessons learned from the design in this section can be generalized with the results shown in Table 7-5. While these results were obtained for a particular form of the load (a series RC load) they are a broad indication of

**Table 7-5:** Bandwidths achievable for various $R_L/Z_S$ ratios and $Qs$ using the two-section quarter-wave transformer of Figure 7-18(b) and $Z_{03} = 2Z_S$. The percentage bandwidth is calculated as 100 times the difference of the high and low frequencies of the passband divided by the ideal match frequency, $f_0$. The notation >33 indicates that the passband is at all frequencies above $0.33f_0$ as for small impedance discontinuities and low $Q$, the two-section transformer is close to being a linear taper. $Q$ is inversely proportion to frequency. Transmission losses of 0.1 dB, 0.2 dB, 0.5 dB, and 1.0 dB correspond to a maximum $|\Gamma_{\rm in}|$ in the passband of $|\Gamma_m| = 0.151, 0.212$, 0.330, and 0.454 respectively. (The same bandwidths are obtained with inverted values of $R_L/Z_S$ but requires a different design concept for the matching network.)

| 0.1 dB max. transmission loss, % bandwidth | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $Z_L/Z_S$ | $Q$ at $f_0$ | | | | | | | | |
| | 0 | 0.25 | 0.5 | 0.75 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| 0.10 | 50 | 37 | 31 | 27 | 23 | 18 | 14 | 12 | 10 |
| 0.25 | 59 | 47 | 38 | 31 | 26 | 19 | 14 | 11 | 9 |
| 0.50 | 85 | 56 | 45 | 34 | 27 | 19 | 14 | 11 | 8 |
| 0.75 | $\infty$ | 78 | 50 | 36 | 27 | 18 | 13 | 9 | 7 |
| 1.00 | $\infty$ | 116 | 56 | 37 | 27 | 17 | 11 | 8 | 6 |

| 0.5 dB max. transmission loss, % bandwidth | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $Z_L/Z_S$ | $Q$ at $f_0$ | | | | | | | | |
| | 0 | 0.25 | 0.5 | 0.75 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| 0.10 | 66 | 58 | 52 | 46 | 42 | 34 | 29 | 24 | 21 |
| 0.25 | 96 | 77 | 65 | 56 | 49 | 38 | 31 | 25 | 21 |
| 0.50 | 187 | 108 | 82 | 66 | 55 | 40 | 30 | 24 | 19 |
| 0.75 | $\infty$ | >41 | 104 | 75 | 58 | 39 | 28 | 22 | 17 |
| 1.00 | $\infty$ | >33 | 150 | 83 | 60 | 37 | 26 | 19 | 14 |

| 0.2 dB max. transmission loss, % bandwidth | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $Z_L/Z_S$ | $Q$ at $f_0$ | | | | | | | | |
| | 0 | 0.25 | 0.5 | 0.75 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| 0.10 | 51 | 45 | 39 | 35 | 31 | 24 | 20 | 16 | 14 |
| 0.25 | 73 | 59 | 49 | 41 | 34 | 26 | 20 | 16 | 14 |
| 0.50 | 98 | 77 | 58 | 46 | 38 | 26 | 20 | 15 | 12 |
| 0.75 | $\infty$ | 106 | 68 | 50 | 39 | 25 | 18 | 14 | 11 |
| 1.00 | $\infty$ | 181 | 80 | 52 | 38 | 24 | 16 | 12 | 9 |

| 1.0 dB max. transmission loss, % bandwidth | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $Z_L/Z_S$ | $Q$ at $f_0$ | | | | | | | | |
| | 0 | 0.25 | 0.5 | 0.75 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| 0.10 | 82 | 72 | 64 | 58 | 53 | 45 | 38 | 33 | 29 |
| 0.25 | 123 | 98 | 83 | 72 | 63 | 51 | 42 | 34 | 30 |
| 0.50 | $\infty$ | >38 | 114 | 88 | 74 | 55 | 42 | 34 | 27 |
| 0.75 | $\infty$ | >27 | >47 | 172 | 82 | 55 | 40 | 31 | 24 |
| 1.00 | $\infty$ | >24 | >43 | 160 | 88 | 53 | 37 | 27 | 21 |

what can be achieved without exploiting resonance in the matching network. (Resonance is a technique to obtain improved results and is be considered in Section 7.7.) The performance metrics in Table 7-5 are much more realistic than the Fano-Bode limits which could require lossless networks of infinite complexity, and only consider the average passband reflection coefficient rather than the maximum in-band reflection coefficient.

For example, with a factor of 2 difference between $Z_L$ and $Z_S$ and with a $Q$ of 1, the fractional bandwidth that can be readily achieved is 27%, 55%, 38%, and 74% for maximum passband losses of 0.1 dB, 0.2 dB, 0.5 dB, and 1 dB respectively. Since matching networks are not usually used for frequency selectivity (which is more appropriate for a filter), and since there will be multiple matching networks in a design, it is common to use a maximum transmission loss of 0.1 dB as a passband criterion.

### 7.6.2 Matching using Cascaded Transmission Lines and Constant $Q$ Circles

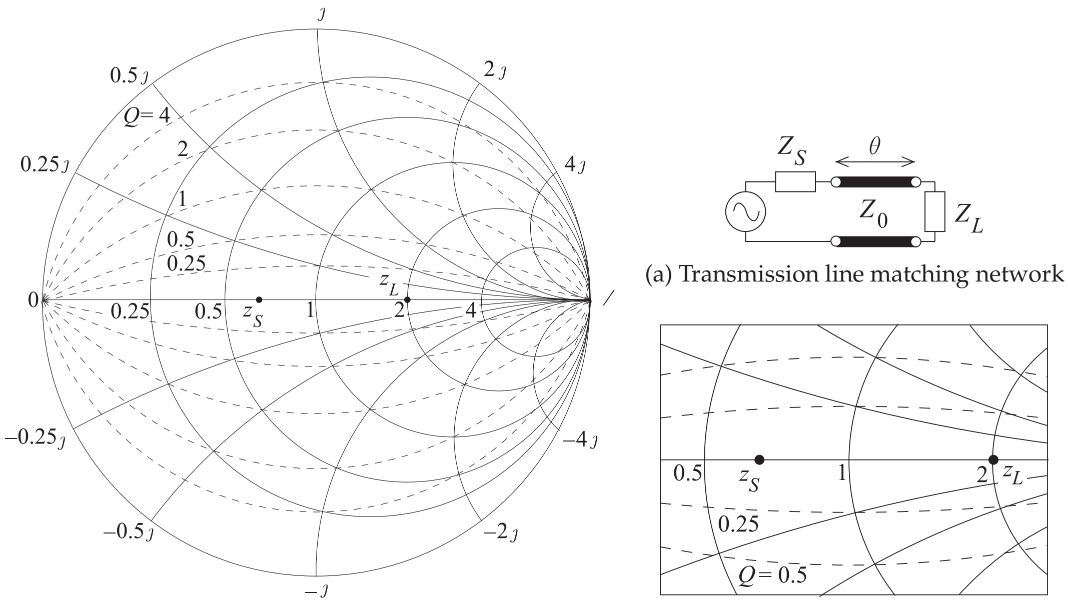
The stepped-impedance transformer designs utilizing quarter-wavelength-long lines can be very long. In this section the design of a stepped-impedance transmission line transformer with shortened lines is considered. Design is conceptually derived from the quarter-wave transformer but design choices are made based on constant $Q$ circles.

First consider the single-line matching problem shown in Figure 7-20(a). The normalized impedances are plotted on a Smith chart with constant $Q$ circles in Figure 7-20(b). A detailed view is given in Figure 7-20(c) and this will be used to describe the design procedure. The normalizing reference impedance $Z_0$ is arbitrary and does not need to be related to the source or load impedance, or to the characteristic impedance of the line.

Figure 7-21(a) shows a quarter-wavelength-long transmission line matching a normalized load $z_L$ to a normalized source impedance $z_S$ with the arrow on the reflection coefficient locus indicating the direction of increasing line length. The arc subtends an angle of $180°$ corresponding to the line having an electrical length of $90°$, i.e. it is $\lambda/4$ long. The maximum $Q$ along the arc is near 0.6 indicating an approximate fractional bandwidth of $1/Q$ or 1.6.

Greater bandwidth of the match can be obtained by using more line sections and matching to an intermediate impedance. This situation is shown in Figure 7-21(b) where there are two quarter-wavelength-long transmission lines each matching to a normalized intermediate resistance $r_v = \sqrt{z_s z_L}$ since both $z_s$ and $z_L$ are resistances. The two arcs in the locus each are part of a circle whose center can be used to determine the characteristic impedance of each of the lines. As an approximation, the center of the circle containing the arc for transmission line $n$ is $C_n$ referenced to the impedance $Z_0$ and then $Z_{0n} \approx Z_0(1 + 2C_{0n})$ (see Section 4.5). (Of course we already know that each of the lines is a quarter-wave transformer so $Z_{01} = Z_0\sqrt{z_s r_v}$ and $Z_{02} = Z_0\sqrt{z_v r_L}$). The maximum $Q$ set by the matching network is now 0.28 so the fractional bandwidth has increased to 1/0.28 = 3.57. This process can be continued indefinitely. With a matching network with three quarter-wave lines, as shown in Figure 7-21(c) the maximum $Q$ is further reduced and the fractional bandwidth increased.

The cascaded quarter-wave lines can reliably be used to obtain wide-bandwidth matching but the overall length of the network becomes quite




(a) Transmission line matching network

(b) Smith chart with constant $Q$ circles

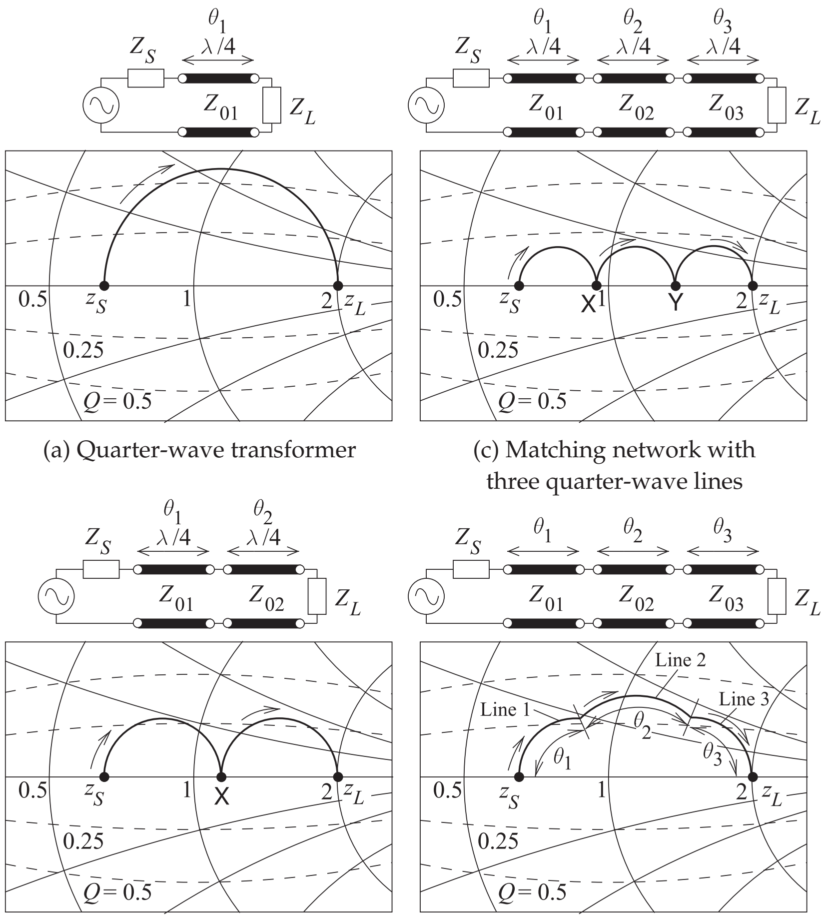
(c) Detailed view of Smith chart

**Figure 7-20:** Transmission line matching a source impedance $Z_S$ to a load impedance $Z_L$.

large. A more compact matching network with shorter overall line length can be obtained using lines that are shorter than $\lambda/4$. Such a network design is shown in Figure 7-21(d) with electrical lengths of each of the three lines being less than that of a quarter-wave line (each is about $\lambda/8$ long). The design problem then becomes one of determining the number of lines (or sections), determining the electrical lengths of the lines, the Θs, and then the characteristic impedance of the lines.

A reasonably good estimate can be obtained using a Smith chart and the constant $Q$ circles. Generally maximum bandwidth is obtained if no one point on the locus sets the maximum circuit $Q$. A better solution is when multiple arcs on the locus all touch the same $Q$ line.

While the illustration used here has resistive load and source impedances the technique can be used with complex source and load impedances. The complicating factor is that if the source and/or load impedances have reactances, then the source and load impedances will vary with frequency. Still resistive matching provides a good initial point in a design and starting from here optimization in a microwave circuit simulator can be used to finalize a design. A good approach is to absorb the impedance variation with frequency into the matching network.




(a) Quarter-wave transformer

(c) Matching network with three quarter-wave lines

(b) Matching network with two quarter-wave lines

(d) Matching network with three lines

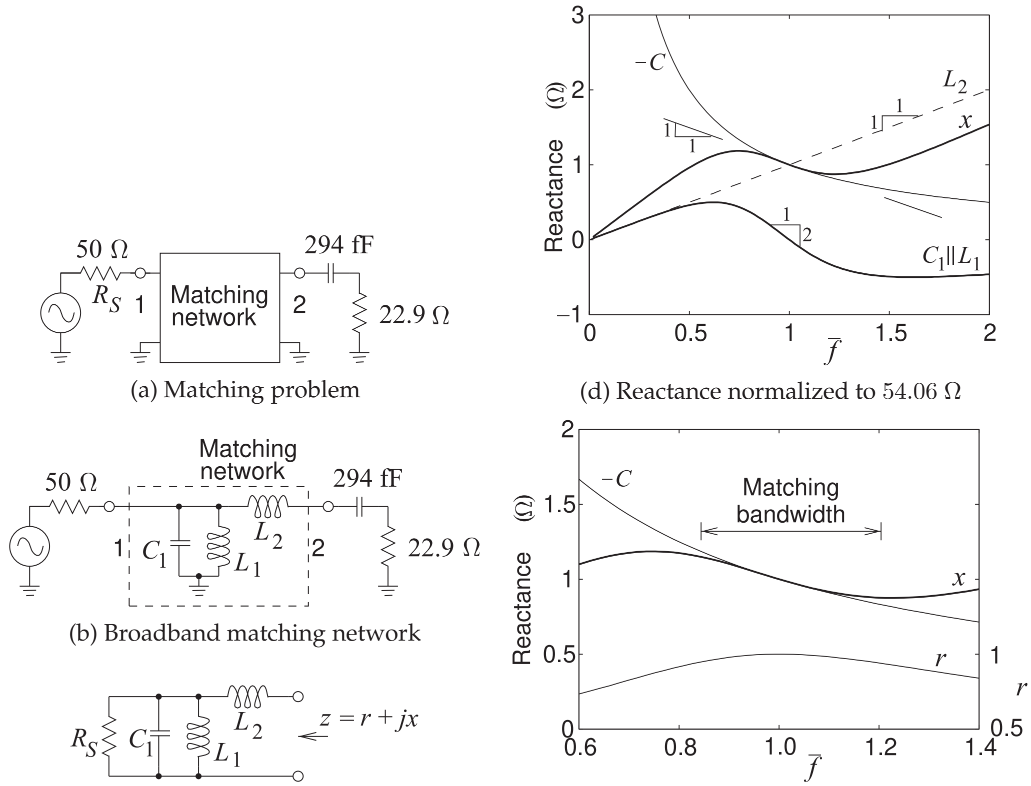
**Figure 7-21:** Broadband cascaded line matching networks. The arrows in the Smith chart locus indicate the direction with increasing line length.

## 7.7 Broadband Matching to Reactive Loads

Previous sections presented methods for broadband matching to resistive loads. Usually these techniques work quite well if the load is moderately reactive but this is not always the case. Inputs and outputs of transistors can have larger reactive parts than resistive parts. Broadband matching to such loads requires customization taking into account the frequency locus of the loads which nearly always rotates in the clockwise direction on a Smith chart so that the locus of the complex conjugate match rotates in the counter-clockwise direction. Circuits that achieve broadband match to these loads exploit resonance and as such have limited bandwidths so that half-octave matching is usually the most that can be achieved.

### *7.7.1 Broadband Matching to a Series RC Load*

Consider matching to the input of a transistor. A transistor such as a FET has an input that can be modeled as a capacitor in series with a resistor as shown in Figure 7-22(a). At 10 GHz the 294 fF capacitor has a reactance of $-54.06\ \Omega$ so that the $Q$ of the load is 2.36. The Fano-Bode limit, see Equation (7.7), indicates that the maximum fractional bandwidth that can be achieved




(a) Matching problem

(b) Broadband matching network

(c) $R_S = 1\ \Omega$, $C_1 = 1$ pF, $L_1 = L_2 = 1$ nH

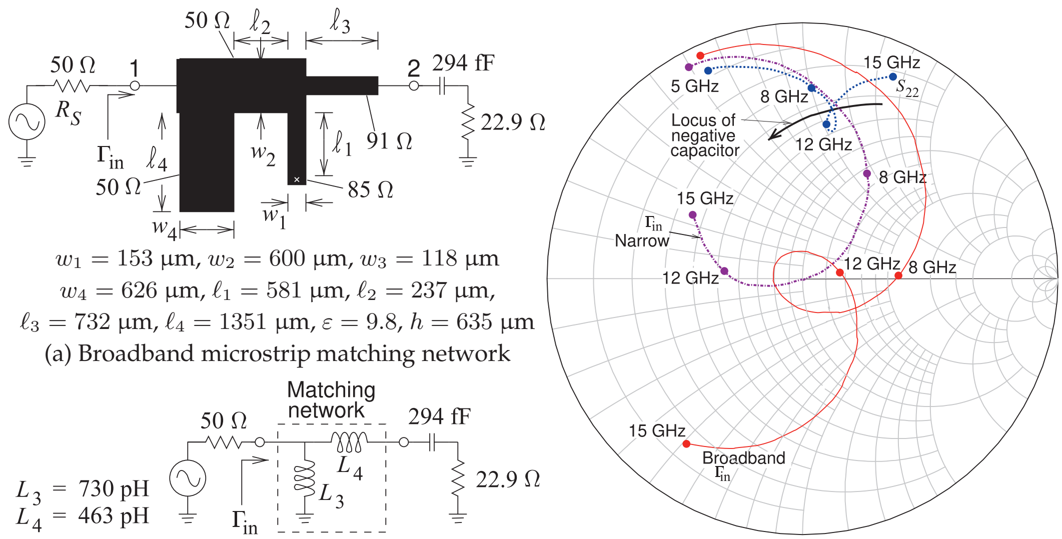
(d) Reactance normalized to $54.06\ \Omega$

(e) Detailed normalized reactance and resistance

**Figure 7-22:** Broadband matching with normalized frequency $\bar{f}$ in radians/s normalized to 10 GHz.

for an average reflection coefficient, $\Gamma_{\rm avg}$, at Port 1 of 0.11 is 60%. This $\Gamma_{\rm avg}$ corresponds to an average transmission loss of 0.05 dB for a maximum transmission loss of approximately 0.1 dB in the bandwidth of the match. (Note that if the load was purely resistive, then $Q = 0$ and it is theoretically possible to achieve infinite bandwidth.)

Matching would be greatly simplified if the matching network presented a negative capacitor to the load. The reactance normalized to $54.06\ \Omega$ versus frequency of the required negative capacitance (of capacitance $-294$ fF) is shown as the curve identified as $-C$ in Figure 7-22(d). At the normalized frequency $\bar{f} = 1$ (frequency normalized to 10 GHz) the slope of this curve is $-1$. A circuit that approximates this over a moderate bandwidth is the broadband matching network shown in Figure 7-22(b). To see how this is achieved, consider the input impedance of the circuit in Figure 7-22(c). The reactance of the parallel LC subcircuit is shown in Figure 7-22(d) as the curve labeled $C_1 \| L_1$. At $\hat{f} = 1$ this reactance has a slope of $-2$ and adding a series inductor, $L_2$, (having the reactance curve $L_2$ in Figure 7-22(d)) results in a reactance $x$, see Figure 7-22(d), which does have a slope of $-1$ at $\hat{f} = 1$. Thus the total reactance, $x$, closely matches the reactance of a negative capacitor over a limited, but still broad, bandwidth. The other part




(a) Broadband microstrip matching network

(b) Narrowband matching network

(c) $\Gamma_{\text{in}}$, and $S_{22}$ of the matching network

**Figure 7-23:** Broadband and narrowband matching networks with $\Gamma_{\text{in}}$ of both networks and $S_{22}$ of the broadband network shown with the the locus of the ideal conjugate match (identified as the locus of negative capacitor). In (a) $\ell_1 = 0.048\ \lambda$, $\ell_2 = 0.020\ \lambda$, $\ell_3 = 0.060\ \lambda$, and $\ell_4 = 0.117\ \lambda$ at 10 GHz. In (b) a $50\ \Omega$ Smith chart is used.

of the matching problem is matching the source and load resistances and with appropriate choice of matching network values the resistance $r$ (ideally $22.9\ \Omega$ normalized to $54.06\ \Omega$) will be approximately constant in the matching region, see Figure 7-22(e).

A microstrip realization centered at 10 GHz of the broadband matching network concept is shown in Figure 7-23(a). At Port 1 is an open-circuited stub with a relatively short electrical length at 10 GHz and so presents the capacitance $C_1$. This is followed by a short section of line that separates the two stubs and provides an extra degree of freedom to be used in matching the source and load resistances. Then follows a short shorted stub that implements $L_1$. This is followed by a short high-impedance line which introduces the series inductance $L_2$ (see Section 2.4.5 of [6]). The performance of the matching network is shown in Figure 7-23(c) where it is compared to that of a narrowband matching network. The narrowband network, shown in Figure 7-23(b), is a conventional two-element matching network designed using the absorption method so that the 294 fF capacitor is absorbed into the network but still requires an additional inductance $L_4$ to compensate for the capacitance. The match of both the broadband and narrowband matching networks is ideal at 10 GHz. The $\Gamma_{\text{in}}$ loci of the two networks are shown on a $50\ \Omega$ Smith chart in Figure 7-23(c).

The range of match for a maximum transmission loss of 0.1 dB is from 9.04 GHz to 11.53 GHz (a 2.49 GHz bandwidth) for the broadband network and 9.47 GHz to 10.62 GHz (a 1.15 GHz bandwidth) for the narrowband network. Using a 0.5 dB bandwidth criterion the bandwidth of the region of match for the broadband microstrip network is 8.13 GHz to 12.95 GHz (a 4.82 GHz bandwidth) and the narrowband network has a passband from 8.54 GHz to 12.34 GHz (a 3.80 GHz bandwidth).

Also plotted in Figure 7-23(c) is $S_{22}$ of the broadband microstrip network and from this the reason why a good match is achieved can be seen. Typically the reflection coefficient locus of simple networks rotates clockwise on the Smith chart with increasing frequency. For a small frequency range near the center match frequency $S_{22}$ has a loop and effectively rotates in the counter-clockwise direction approximating the locus of a negative capacitor. Such a behavior is obtained in the lumped element version of the broadband network by the resonance of $L_1$, $C_1$, and $L_2$. A very good match is therefor possible over a small frequency range. The good match is obtained over about half an octave (of frequency) and this is typically the best that can be achieved when matching to the inputs and outputs of microwave transistors. The microstrip broadband matching network has a finite length and width. Including the widths as well as the lengths of the lines, the broadband matching network has a width and length of $0.11\ \lambda$, considerably less than that of a quarter-wave transformer used to match resistive source and load impedances when they are resistive but not when the load has a large reactance as here.

### 7.7.2 Summary

The broadband matching concept presented in this section is using resonance to present an impedance to a load or source that rotates in the counter-clockwise direction (with respect to frequency) on a Smith chart. This is achievable only over a moderate bandwidth and typically half-octave bandwidths are regarded as the limit of what can be achieved when matching to a load that is more reactive than resistive. There are techniques also that are sometimes able to achieve broader effective matches by incorporating the parasitic reactances of a device to be matched into a distributed transmission line.

## 7.8 Summary

This chapter presented design concepts for realizing matching networks with broad bandwidths. Matching a load and/or a source with a reactance presents a particular problem because reactive loads include energy storage elements and energy storage elements limit bandwidth. Theoretically, if negative capacitances and inductors could be realized then it would be possible to have infinite bandwidth matching of complex loads and/or sources provided that the elements of the matching network are lossless. Since negative capacitors and inductors cannot be realized, there will be a limit on matching bandwidth. The Fano-Body limits indicate the trade-off between the quality of the match in terms of the minimum reflection coefficient that can be achieved and the bandwidth of the match. While ideal magnetic transformers can achieve infinite matching of source and load resistors, actual magnetic transformers have self inductances and thus energy storage and so matching bandwidth is limited even with magnetic transformers.

All matching networks introduce energy storage elements. This includes capacitor, inductor, and transmission line elements, as well as magnetic transformers. Thus design techniques are required that enable design trade-offs of the quality of a match and bandwidth. The constant $Q$ circles on a Smith chart enable the maximum $Q$ and hence fractional bandwidth, which is very approximately $1/Q$, of a matched to be controlled. As long as the locus of the input reflection coefficient of an evolving terminated matching network does not go outside a specific pair of equal-valued constant $Q$ circles the bandwidth is constrained. This concept applies to loads and sources that are resistive only or include reactances. The stepped-impedance and tapered transmission-line matching networks presented enable broadband matching of source and load resistances but the concepts of gradual matching in stages to interim resistances levels can be extrapolated to matching general sources and loads.

The final concept of this chapter describes a topology for matching a reactive load, in particular a load comprising a resistor and a capacitor. This is what the input and output of a transistor looks like. The matching network described in Section 7.7 approximates a negative capacitor over a limited bandwidth and is one of the best such networks known and this matching network topology is commonly used in matching the input and output of transistors. Other broadband network topologies have been invented and these developments are tracked by microwave engineers involved in matching network design.

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## 7.10 Exercises

1. A load is modeled as a 50 Ω resistance in series with a reactance of 50 Ω. This load is to be matched to a source with a Thevenin equivalent resistance of 50 Ω. Use the Fano-Bode criteria to determine the upper limit on the matching network bandwidth when the average in-band reflection coefficient is $-10$ dB.

2. A load is modeled as a 50 Ω resistance in series with a reactance of 50 Ω. This load is to be

matched to a source with a Thevenin equivalent resistance of 50 Ω. Use the Fano-Bode criteria to determine the upper limit on the matching network bandwidth when the average in-band reflection coefficient is $-20$ dB.

3. The output of a transistor amplifier operating at 1 GHz is modeled as a 100 Ω resistor in parallel with a 10 pF capacitor. The amplifier must drive the input of a $\lambda/2$ dipole antenna with an input resistance of 73 Ω. To do this efficiently a matching network is required. Consider that the input resistance of the antenna is independent of frequency, and assume that the matching network is lossless. This is the same as assuming that its bandwidth is much greater than the bandwidth required. If the required fractional bandwidth of the matching network is 5%, and using the Fano-Bode criteria, determine the following:
   (a) The lower limit on the average in-band reflection coefficient of the matching network.
   (b) The upper limit on the average transmission coefficient of the matching network.
4. Design a broadband matching network at 1 GHz to match a source $Z_S = 80+\jmath 50$ Ω to a load with an impedance $Z_L = 60.0+\jmath 20.0$ Ω. Maintain the maximum bandwidth possible with this source and load. [Parallels Example 7.1]
5. Design a broadband matching network at 1 GHz to match a source $Z_S = 45+\jmath 10$ Ω to a load with an impedance $Z_L = 50.0+\jmath 80.0$ Ω. Maintain the maximum bandwidth possible with this source and load. [Parallels Example 7.1]
6. Consider the problem of matching a source with a Thevenin equivalent impedance of 25 Ω to a load of admittance $0.035 + \jmath 0.035$.
   (a) What is the minimum $Q$ that can be achieved for the network, and what is the topology of the matching network that will yield the match with the widest bandwidth?
   (b) Design the matching network with the widest bandwidth possible if the matching network can have at most four elements.
7. Develop the electrical design of a three-section quarter-wave transformer to match a 50 Ω cable to an antenna with a 10 Ω input impedance. [Parallels Example 7.2]
8. Design of a two-section quarter-wave transformer to match a 50 Ω cable to a 75 Ω cable. [Parallels Example 7.2]
9. Develop the electrical design of a two-section quarter-wave transformer to match a 50 Ω cable to a 75 Ω cable. [Parallels Example 7.2]
10. Develop the electrical design of a three-section maximally flat stepped-impedance transformer to match a source $Z_S = 20$ Ω to a load $Z_L = 50$ Ω load. [Parallels Example 7.3]
11. Design a stepped impedance transmission line transformer with two transmission line sections to match a 50 Ω source to a load with an impedance of 25 Ω. Design for a maximally flat response. [Parallels Example 7.3]
12. Design a maximally flat four-section stepped impedance transmission line transformer matching a basestation amplifier with a 2 Ω output impedance to a 50 Ω cable. [Parallels Example 7.3]
13. Develop the electrical design of a 100% bandwidth three-section Chebyshev stepped-impedance transformer in microstrip to connect a power amplifier with an output impedance of 2 Ω to a 50 Ω cable. [Parallels Example 7.4]
14. Design a microstrip Klopfenstein taper to match a $Z_S = 15$ Ω source to a $Z_L = 75$ Ω load. The maximum transmission ripple is to be 0.5 dB and the minimum passband frequency is 60 GHz. Only an electrical design is required but draw the microstrip layout. [Parallels Example 7.5]

### 7.10.1 Exercises by Section

†challenging, ‡very challenging

| | |
|---|---|
| §7.2 1, 2, 3 | §7.4 7, 8, 9, 10, 11, 12, 13 |
| §7.3 4, 5, 6 | §7.5 14 |

### 7.10.2 Answers to Selected Exercises

| | | |
|---|---|---|
| A 41.36 meV | B 662.6 fJ | C 3.25 cm |

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