

examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in [Figure 11.23](#).

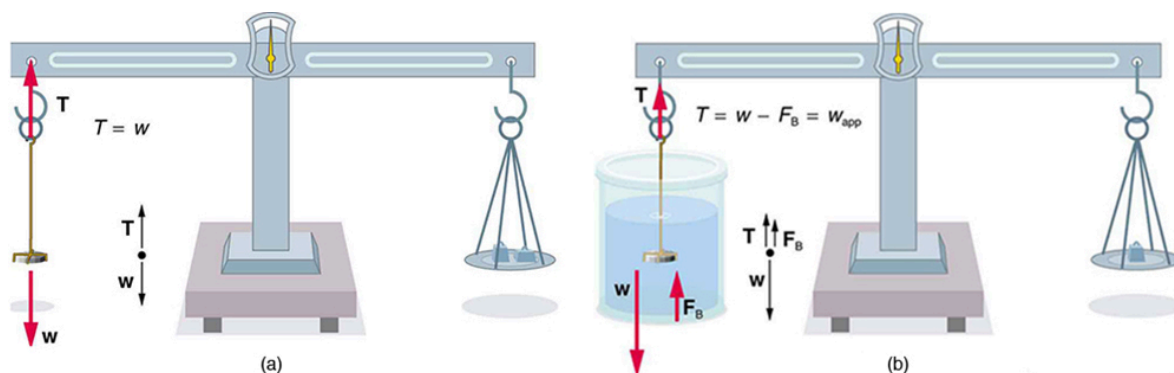


FIGURE 11.23 (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\text{apparent weight loss} = \text{weight of fluid displaced} \quad 11.44$$

or

$$\text{apparent mass loss} = \text{mass of fluid displaced.} \quad 11.45$$

The next example illustrates the use of this technique.



EXAMPLE 11.10

Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in [Figure 11.23](#), its apparent mass is 7.800 g. Calculate its density, given that water has a density of 1.000 g/cm^3 and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V .

Solution

The volume of water is $V_w = \frac{m_w}{\rho_w}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_w = 8.630 \text{ g} - 7.800 \text{ g} = 0.830 \text{ g}$. Thus the volume of water is $V_w = \frac{0.830 \text{ g}}{1.000 \text{ g/cm}^3} = 0.830 \text{ cm}^3$. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_c = \frac{m_c}{V_c} = \frac{8.630 \text{ g}}{0.830 \text{ cm}^3} = 10.4 \text{ g/cm}^3. \quad 11.46$$

Discussion

You can see from [Table 11.1](#) that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand cohesive and adhesive forces.
- Define surface tension.
- Understand capillary action.

Cohesion and Adhesion in Liquids

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See [Figure 11.24](#).) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called **cohesive forces**. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called **adhesive forces**. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

Cohesive Forces

Attractive forces between molecules of the same type are called cohesive forces.

Adhesive Forces

Attractive forces between molecules of different types are called adhesive forces.

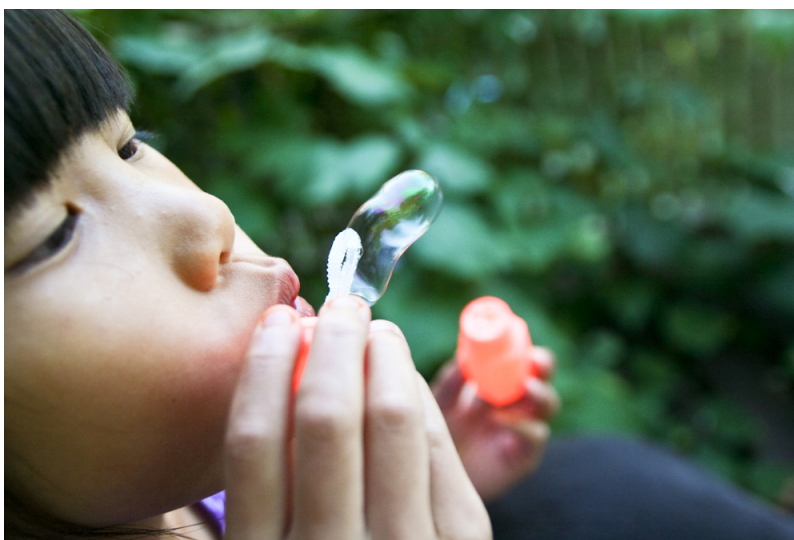


FIGURE 11.24 The soap bubbles in this photograph are caused by cohesive forces among molecules in liquids. (credit: Steve Ford Elliott)

Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension**. Molecules on the surface are pulled inward by cohesive forces, reducing the surface area. Molecules inside the liquid experience zero net force, since they have neighbors on all sides.

Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.

Making Connections: Surface Tension

Forces between atoms and molecules underlie the macroscopic effect called surface tension. These attractive forces pull the molecules closer together and tend to minimize the surface area. This is another example of a submicroscopic explanation for a macroscopic phenomenon.

The model of a liquid surface acting like a stretched elastic sheet can effectively explain surface tension effects. For example, some insects can walk on water (as opposed to floating in it) as we would walk on a trampoline—they dent the surface as shown in [Figure 11.25\(a\)](#). [Figure 11.25\(b\)](#) shows another example, where a needle rests on a water surface. The iron needle cannot, and does not, float, because its density is greater than that of water. Rather, its weight is supported by forces in the stretched surface that try to make the surface smaller or flatter. If the needle were placed point down on the surface, its weight acting on a smaller area would break the surface, and it would sink.

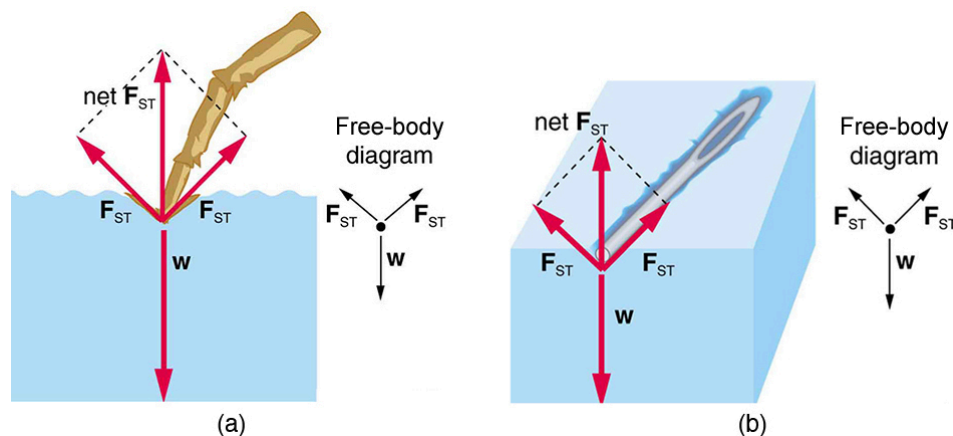


FIGURE 11.25 Surface tension supporting the weight of an insect and an iron needle, both of which rest on the surface without penetrating it. They are not floating; rather, they are supported by the surface of the liquid. (a) An insect leg dents the water surface. F_{ST} is a restoring force (surface tension) parallel to the surface. (b) An iron needle similarly dents a water surface until the restoring force (surface tension) grows to equal its weight.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid. Surface tension γ is defined to be the force F per unit length L exerted by a stretched liquid membrane:

$$\gamma = \frac{F}{L}. \quad 11.47$$

Table 11.3 lists values of γ for some liquids. For the insect of Figure 11.25(a), its weight w is supported by the upward components of the surface tension force: $w = \gamma L \sin \theta$, where L is the circumference of the insect's foot in contact with the water. Figure 11.26 shows one way to measure surface tension. The liquid film exerts a force on the movable wire in an attempt to reduce its surface area. The magnitude of this force depends on the surface tension of the liquid and can be measured accurately.

Surface tension is the reason why liquids form bubbles and droplets. The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside. It can be shown that the gauge pressure P inside a spherical bubble is given by

$$P = \frac{4\gamma}{r}, \quad 11.48$$

where r is the radius of the bubble. Thus the pressure inside a bubble is greatest when the bubble is the smallest. Another bit of evidence for this is illustrated in Figure 11.27. When air is allowed to flow between two balloons of unequal size, the smaller balloon tends to collapse, filling the larger balloon.

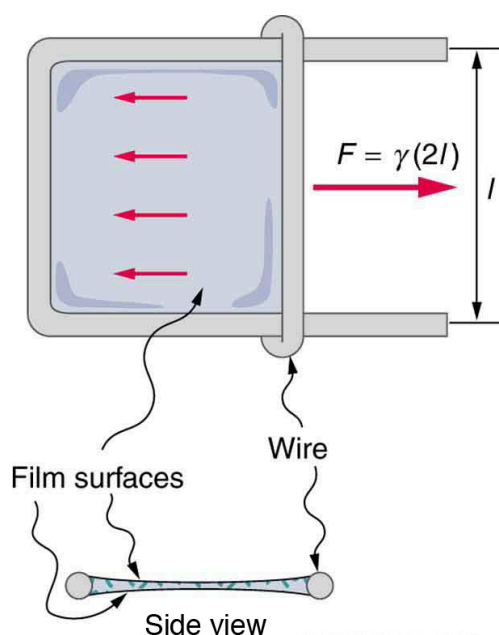


FIGURE 11.26 Sliding wire device used for measuring surface tension; the device exerts a force to reduce the film's surface area. The force needed to hold the wire in place is $F = \gamma L = \gamma(2l)$, since there are two liquid surfaces attached to the wire. This force remains nearly constant as the film is stretched, until the film approaches its breaking point.

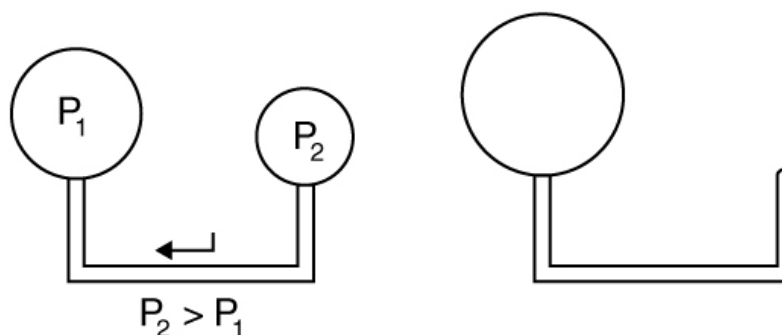


FIGURE 11.27 With the valve closed, two balloons of different sizes are attached to each end of a tube. Upon opening the valve, the smaller balloon decreases in size with the air moving to fill the larger balloon. The pressure in a spherical balloon is inversely proportional to its radius, so that the smaller balloon has a greater internal pressure than the larger balloon, resulting in this flow.

Liquid	Surface tension γ (N/m)
Water at 0°C	0.0756
Water at 20°C	0.0728
Water at 100°C	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465

TABLE 11.3 Surface Tension of Some Liquids¹

¹ At 20°C unless otherwise stated.

Liquid	Surface tension γ (N/m)
Olive oil	0.032
Tissue fluids (typical)	0.050
Blood, whole at 37°C	0.058
Blood plasma at 37°C	0.073
Gold at 1070°C	1.000
Oxygen at –193°C	0.0157
Helium at –269°C	0.00012

TABLE 11.3 Surface Tension of Some Liquids¹



EXAMPLE 11.11

Surface Tension: Pressure Inside a Bubble

Calculate the gauge pressure inside a soap bubble 2.00×10^{-4} m in radius using the surface tension for soapy water in [Table 11.3](#). Convert this pressure to mm Hg.

Strategy

The radius is given and the surface tension can be found in [Table 11.3](#), and so P can be found directly from the equation $P = \frac{4\gamma}{r}$.

Solution

Substituting r and γ into the equation $P = \frac{4\gamma}{r}$, we obtain

$$P = \frac{4\gamma}{r} = \frac{4(0.037 \text{ N/m})}{2.00 \times 10^{-4} \text{ m}} = 740 \text{ N/m}^2 = 740 \text{ Pa.} \quad 11.49$$

We use a conversion factor to get this into units of mm Hg:

$$P = (740 \text{ N/m}^2) \frac{1.00 \text{ mm Hg}}{133 \text{ N/m}^2} = 5.56 \text{ mm Hg.} \quad 11.50$$

Discussion

Note that if a hole were to be made in the bubble, the air would be forced out, the bubble would decrease in radius, and the gauge pressure would reduce to zero, and the absolute pressure inside would *decrease* to atmospheric pressure (760 mm Hg).

Our lungs contain hundreds of millions of mucus-lined sacs called *alveoli*, which are very similar in size, and about 0.1 mm in diameter. (See [Figure 11.28](#).) You can exhale without muscle action by allowing surface tension to contract these sacs. Medical patients whose breathing is aided by a positive pressure respirator have air blown into the lungs, but are generally allowed to exhale on their own. Even if there is paralysis, surface tension in the alveoli will expel air from the lungs. Since pressure increases as the radii of the alveoli decrease, an occasional deep cleansing breath is needed to fully reinflate the alveoli. Respirators are programmed to do this and we find it natural, as do our companion dogs and cats, to take a cleansing breath before settling into a nap.

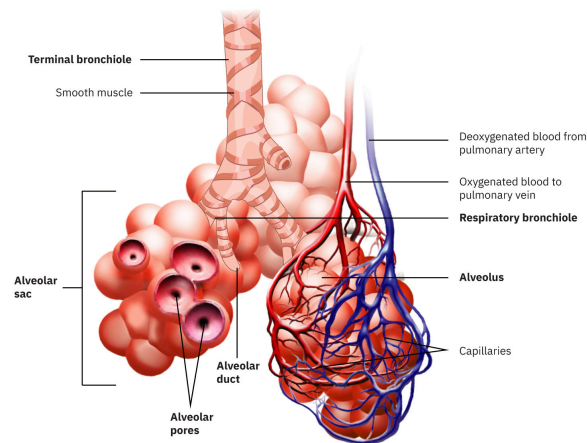


FIGURE 11.28 Bronchial tubes in the lungs branch into ever-smaller structures, finally ending in alveoli. The alveoli act like tiny bubbles. The surface tension of their mucous lining aids in exhalation and can prevent inhalation if too great.

The tension in the walls of the alveoli results from the membrane tissue and a liquid on the walls of the alveoli containing a long lipoprotein that acts as a surfactant (a surface-tension reducing substance). The need for the surfactant results from the tendency of small alveoli to collapse and the air to fill into the larger alveoli making them even larger (as demonstrated in [Figure 11.27](#)). During inhalation, the lipoprotein molecules are pulled apart and the wall tension increases as the radius increases (increased surface tension). During exhalation, the molecules slide back together and the surface tension decreases, helping to prevent a collapse of the alveoli. The surfactant therefore serves to change the wall tension so that small alveoli don't collapse and large alveoli are prevented from expanding too much. This tension change is a unique property of these surfactants, and is not shared by detergents (which simply lower surface tension). (See [Figure 11.29](#).)

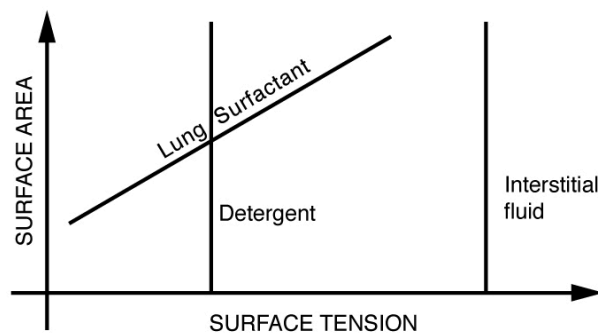


FIGURE 11.29 Surface tension as a function of surface area. The surface tension for lung surfactant decreases with decreasing area. This ensures that small alveoli don't collapse and large alveoli are not able to over expand.

If water gets into the lungs, the surface tension is too great and you cannot inhale. This is a severe problem in resuscitating drowning victims. A similar problem occurs in newborn infants who are born without this surfactant—their lungs are very difficult to inflate. This condition is known as *hyaline membrane disease* and is a leading cause of death for infants, particularly in premature births. Some success has been achieved in treating hyaline membrane disease by spraying a surfactant into the infant's breathing passages. Emphysema produces the opposite problem with alveoli. Alveolar walls of emphysema victims deteriorate, and the sacs combine to form larger sacs. Because pressure produced by surface tension decreases with increasing radius, these larger sacs produce smaller pressure, reducing the ability of emphysema victims to exhale. A common test for emphysema is to measure the pressure and volume of air that can be exhaled.

Making Connections: Take-Home Investigation

- (1) Try floating a sewing needle on water. In order for this activity to work, the needle needs to be very clean as even the oil from your fingers can be sufficient to affect the surface properties of the needle.
- (2) Place the bristles of a paint brush into water. Pull the brush out and notice that for a short while, the bristles will stick

together. The surface tension of the water surrounding the bristles is sufficient to hold the bristles together. As the bristles dry out, the surface tension effect dissipates. (3) Place a loop of thread on the surface of still water in such a way that all of the thread is in contact with the water. Note the shape of the loop. Now place a drop of detergent into the middle of the loop. What happens to the shape of the loop? Why? (4) Sprinkle pepper onto the surface of water. Add a drop of detergent. What happens? Why? (5) Float two matches parallel to each other and add a drop of detergent between them. What happens? Note: For each new experiment, the water needs to be replaced and the bowl washed to free it of any residual detergent.

Adhesion and Capillary Action

Why is it that water beads up on a waxed car but does not on bare paint? The answer is that the adhesive forces between water and wax are much smaller than those between water and paint. Competition between the forces of adhesion and cohesion are important in the macroscopic behavior of liquids. An important factor in studying the roles of these two forces is the angle θ between the tangent to the liquid surface and the surface. (See [Figure 11.30](#).) The **contact angle** θ is directly related to the relative strength of the cohesive and adhesive forces. The larger the strength of the cohesive force relative to the adhesive force, the larger θ is, and the more the liquid tends to form a droplet. The smaller θ is, the smaller the relative strength, so that the adhesive force is able to flatten the drop. [Table 11.4](#) lists contact angles for several combinations of liquids and solids.

Contact Angle

The angle θ between the tangent to the liquid surface and the surface is called the contact angle.

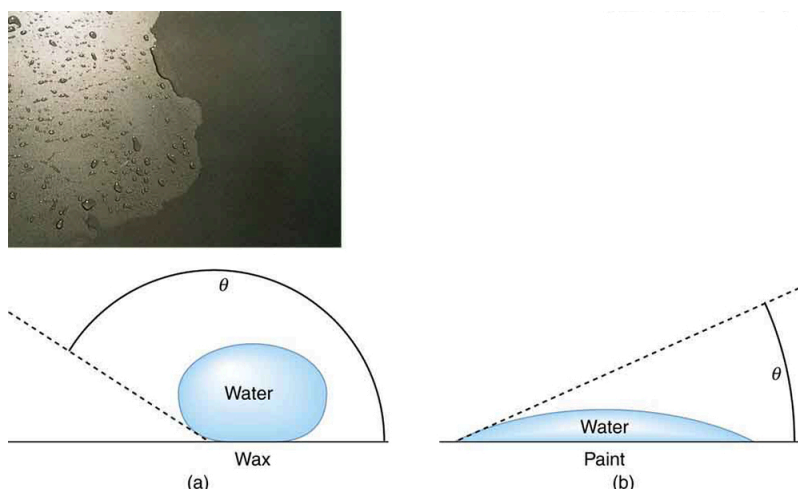


FIGURE 11.30 In the photograph, water beads on the waxed car paint and flattens on the unwaxed paint. (a) Water forms beads on the waxed surface because the cohesive forces responsible for surface tension are larger than the adhesive forces, which tend to flatten the drop. (b) Water beads on bare paint are flattened considerably because the adhesive forces between water and paint are strong, overcoming surface tension. The contact angle θ is directly related to the relative strengths of the cohesive and adhesive forces. The larger θ is, the larger the ratio of cohesive to adhesive forces. (credit: P. P. Urone)

One important phenomenon related to the relative strength of cohesive and adhesive forces is **capillary action**—the tendency of a fluid to be raised or suppressed in a narrow tube, or *capillary tube*. This action causes blood to be drawn into a small-diameter tube when the tube touches a drop.

Capillary Action

The tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube, is called capillary action.

If a capillary tube is placed vertically into a liquid, as shown in [Figure 11.31](#), capillary action will raise or suppress the liquid inside the tube depending on the combination of substances. The actual effect depends on the relative strength of the cohesive and adhesive forces and, thus, the contact angle θ given in the table. If θ is less than 90° ,

then the fluid will be raised; if θ is greater than 90° , it will be suppressed. Mercury, for example, has a very large surface tension and a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop. The curved surface of a fluid in a tube is called a **meniscus**. The tendency of surface tension is always to reduce the surface area. Surface tension thus flattens the curved liquid surface in a capillary tube. This results in a downward force in mercury and an upward force in water, as seen in [Figure 11.31](#).

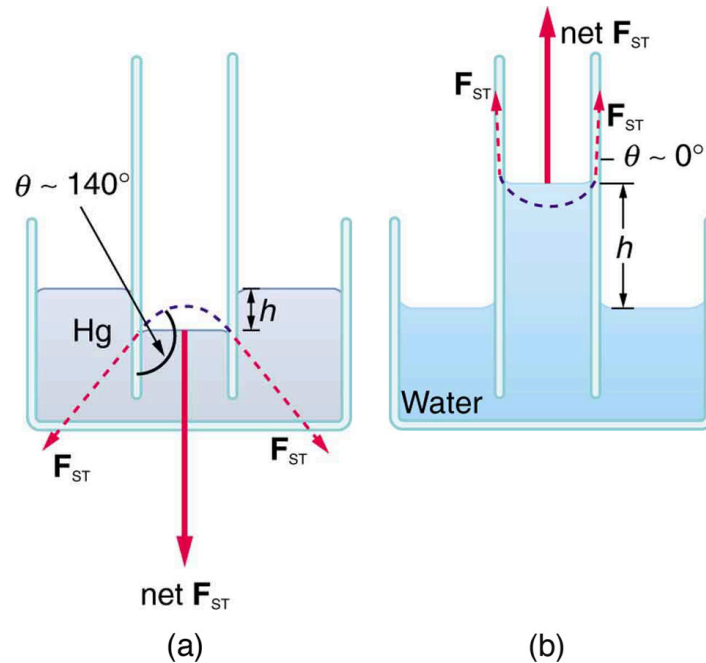


FIGURE 11.31 (a) Mercury is suppressed in a glass tube because its contact angle is greater than 90° . Surface tension exerts a downward force as it flattens the mercury, suppressing it in the tube. The dashed line shows the shape the mercury surface would have without the flattening effect of surface tension. (b) Water is raised in a glass tube because its contact angle is nearly 0° . Surface tension therefore exerts an upward force when it flattens the surface to reduce its area.

Interface	Contact angle θ
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°
Kerosene–glass	26°

TABLE 11.4 Contact Angles of Some Substances

Capillary action can move liquids horizontally over very large distances, but the height to which it can raise or suppress a liquid in a tube is limited by its weight. It can be shown that this height h is given by

$$h = \frac{2\gamma \cos \theta}{\rho g r}. \quad 11.51$$

If we look at the different factors in this expression, we might see how it makes good sense. The height is directly

proportional to the surface tension γ , which is its direct cause. Furthermore, the height is inversely proportional to tube radius—the smaller the radius r , the higher the fluid can be raised, since a smaller tube holds less mass. The height is also inversely proportional to fluid density ρ , since a larger density means a greater mass in the same volume. (See [Figure 11.32](#).)

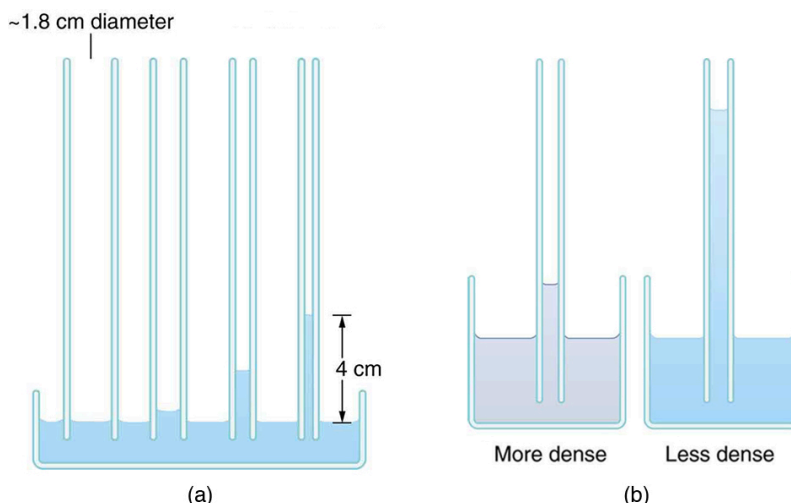


FIGURE 11.32 (a) Capillary action depends on the radius of a tube. The smaller the tube, the greater the height reached. The height is negligible for large-radius tubes. (b) A denser fluid in the same tube rises to a smaller height, all other factors being the same.



EXAMPLE 11.12

Calculating Radius of a Capillary Tube: Capillary Action: Tree Sap

Can capillary action be solely responsible for sap rising in trees? To answer this question, calculate the radius of a capillary tube that would raise sap 100 m to the top of a giant redwood, assuming that sap's density is 1050 kg/m^3 , its contact angle is zero, and its surface tension is the same as that of water at 20.0°C .

Strategy

The height to which a liquid will rise as a result of capillary action is given by $h = \frac{2\gamma \cos \theta}{\rho g r}$, and every quantity is known except for r .

Solution

Solving for r and substituting known values produces

$$\begin{aligned} r &= \frac{2\gamma \cos \theta}{\rho g h} = \frac{2(0.0728 \text{ N/m})\cos(0^\circ)}{(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(100 \text{ m})} \\ &= 1.41 \times 10^{-7} \text{ m.} \end{aligned} \quad 11.52$$

Discussion

This result is unreasonable. Sap in trees moves through the *xylem*, which forms tubes with radii as small as $2.5 \times 10^{-5} \text{ m}$. This value is about 180 times as large as the radius found necessary here to raise sap 100 m. This means that capillary action alone cannot be solely responsible for sap getting to the tops of trees.

How *does* sap get to the tops of tall trees? (Recall that a column of water can only rise to a height of 10 m when there is a vacuum at the top—see [Example 11.5](#).) The question has not been completely resolved, but it appears that it is pulled up like a chain held together by cohesive forces. As each molecule of sap enters a leaf and evaporates (a process called transpiration), the entire chain is pulled up a notch. So a negative pressure created by water evaporation must be present to pull the sap up through the xylem vessels. In most situations, *fluids can push but can exert only negligible pull*, because the cohesive forces seem to be too small to hold the molecules tightly together. But in this case, the cohesive force of water molecules provides a very strong pull. [Figure 11.33](#) shows one

device for studying negative pressure. Some experiments have demonstrated that negative pressures sufficient to pull sap to the tops of the tallest trees *can* be achieved.

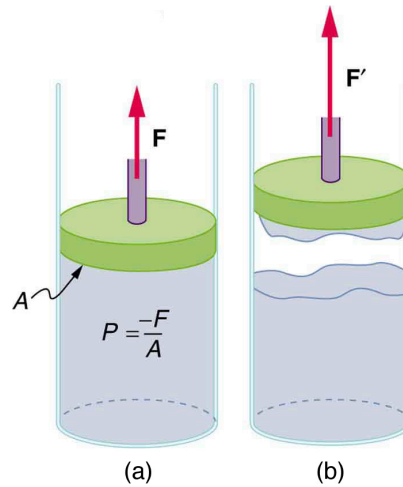


FIGURE 11.33 (a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$. (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

11.9 Pressures in the Body

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the concept of pressure the in human body.
- Explain systolic and diastolic blood pressures.
- Describe pressures in the eye, lungs, spinal column, bladder, and skeletal system.

Pressure in the Body

Next to taking a person's temperature and weight, measuring blood pressure is the most common of all medical examinations. Control of high blood pressure is largely responsible for the significant decreases in heart attack and stroke fatalities achieved in the last three decades. The pressures in various parts of the body can be measured and often provide valuable medical indicators. In this section, we consider a few examples together with some of the physics that accompanies them.

[Table 11.5](#) lists some of the measured pressures in mm Hg, the units most commonly quoted.

Body system	Gauge pressure in mm Hg
Blood pressures in large arteries (resting)	
<i>Maximum (systolic)</i>	100–140
<i>Minimum (diastolic)</i>	60–90
Blood pressure in large veins	4–15
Eye	12–24
Brain and spinal fluid (lying down)	5–12
Bladder	

TABLE 11.5 Typical Pressures in Humans

Body system	Gauge pressure in mm Hg
<i>While filling</i>	0–25
<i>When full</i>	100–150
Chest cavity between lungs and ribs	–8 to –4
Inside lungs	–2 to +3
Digestive tract	
<i>Esophagus</i>	–2
<i>Stomach</i>	0–20
<i>Intestines</i>	10–20
Middle ear	<1

TABLE 11.5 Typical Pressures in Humans

Blood Pressure

Common arterial blood pressure measurements typically produce values of 120 mm Hg and 80 mm Hg, respectively, for systolic and diastolic pressures. Both pressures have health implications. When systolic pressure is chronically high, the risk of stroke and heart attack is increased. If, however, it is too low, fainting is a problem.

Systolic pressure increases dramatically during exercise to increase blood flow and returns to normal afterward. This change produces no ill effects and, in fact, may be beneficial to the tone of the circulatory system. **Diastolic pressure** can be an indicator of fluid balance. When low, it may indicate that a person is hemorrhaging internally and needs a transfusion. Conversely, high diastolic pressure indicates a ballooning of the blood vessels, which may be due to the transfusion of too much fluid into the circulatory system. High diastolic pressure is also an indication that blood vessels are not dilating properly to pass blood through. This can seriously strain the heart in its attempt to pump blood.

Blood leaves the heart at about 120 mm Hg but its pressure continues to decrease (to almost 0) as it goes from the aorta to smaller arteries to small veins (see [Figure 11.34](#)). The pressure differences in the circulation system are caused by blood flow through the system as well as the position of the person. For a person standing up, the pressure in the feet will be larger than at the heart due to the weight of the blood ($P = h\rho g$). If we assume that the distance between the heart and the feet of a person in an upright position is 1.4 m, then the increase in pressure in the feet relative to that in the heart (for a static column of blood) is given by

$$\Delta P = \Delta h\rho g = (1.4 \text{ m})(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2) = 1.4 \times 10^4 \text{ Pa} = 108 \text{ mm Hg.} \quad 11.53$$

Increase in Pressure in the Feet of a Person

$$\Delta P = \Delta h\rho g = (1.4 \text{ m})(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2) = 1.4 \times 10^4 \text{ Pa} = 108 \text{ mm Hg.} \quad 11.54$$

Standing a long time can lead to an accumulation of blood in the legs and swelling. This is the reason why soldiers who are required to stand still for long periods of time have been known to faint. Elastic bandages around the calf can help prevent this accumulation and can also help provide increased pressure to enable the veins to send blood back up to the heart. For similar reasons, doctors recommend tight stockings for long-haul flights.

Blood pressure may also be measured in the major veins, the heart chambers, arteries to the brain, and the lungs.

But these pressures are usually only monitored during surgery or for patients in intensive care since the measurements are invasive. To obtain these pressure measurements, qualified health care workers thread thin tubes, called catheters, into appropriate locations to transmit pressures to external measuring devices.

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body ([Figure 11.34](#)). Right-heart failure, for example, results in a rise in the pressure in the vena cavae and a drop in pressure in the arteries to the lungs. Left-heart failure results in a rise in the pressure entering the left side of the heart and a drop in aortal pressure. Implications of these and other pressures on flow in the circulatory system will be discussed in more detail in [Fluid Dynamics and Its Biological and Medical Applications](#).

Two Pumps of the Heart

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body.

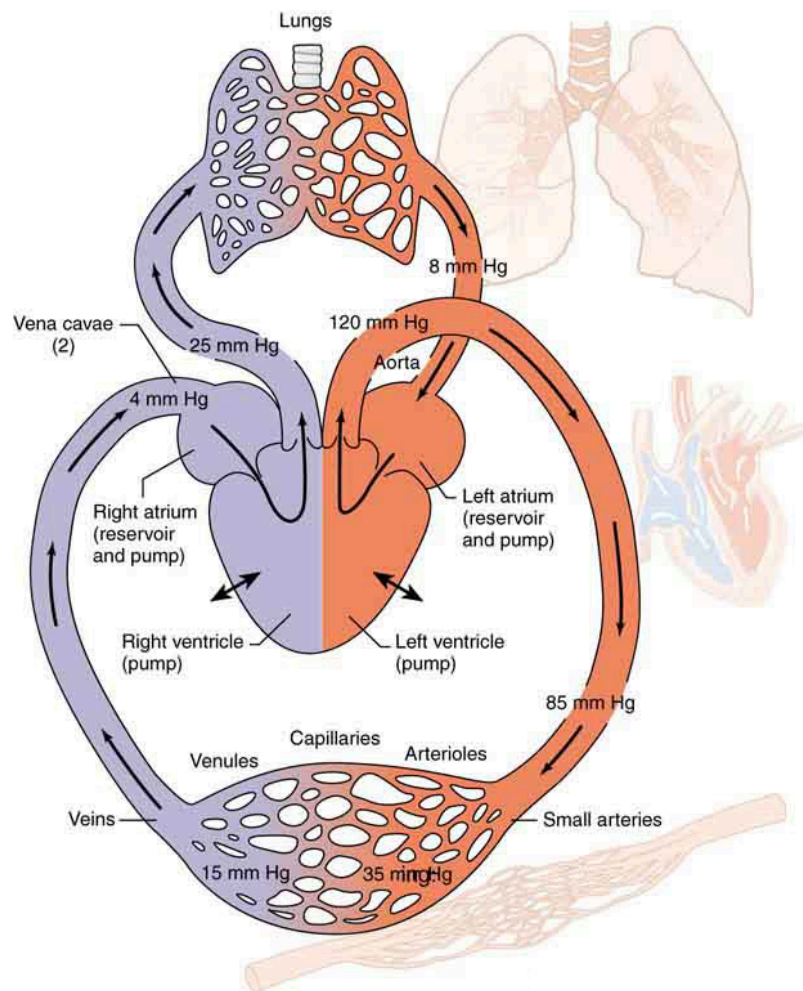


FIGURE 11.34 Schematic of the circulatory system showing typical pressures. The two pumps in the heart increase pressure and that pressure is reduced as the blood flows through the body. Long-term deviations from these pressures have medical implications discussed in some detail in the [Fluid Dynamics and Its Biological and Medical Applications](#). Only aortal or arterial blood pressure can be measured noninvasively.

Pressure in the Eye

The shape of the eye is maintained by fluid pressure, called **intraocular pressure**, which is normally in the range of 12.0 to 24.0 mm Hg. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called **glaucoma**. The net pressure can become as great as 85.0 mm Hg, an abnormally large pressure

that can permanently damage the optic nerve. To get an idea of the force involved, suppose the back of the eye has an area of 6.0 cm^2 , and the net pressure is 85.0 mm Hg . Force is given by $F = PA$. To get F in newtons, we convert the area to m^2 ($1 \text{ m}^2 = 10^4 \text{ cm}^2$). Then we calculate as follows:

$$F = h\rho gA = (85.0 \times 10^{-3} \text{ m})(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.0 \times 10^{-4} \text{ m}^2) = 6.8 \text{ N.} \quad 11.55$$

Eye Pressure

The shape of the eye is maintained by fluid pressure, called intraocular pressure. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The force is calculated as

$$F = h\rho gA = (85.0 \times 10^{-3} \text{ m})(13.6 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.0 \times 10^{-4} \text{ m}^2) = 6.8 \text{ N.} \quad 11.56$$

This force is the weight of about a 680-g mass. A mass of 680 g resting on the eye (imagine 1.5 lb resting on your eye) would be sufficient to cause it damage. (A normal force here would be the weight of about 120 g, less than one-quarter of our initial value.)

People over 40 years of age are at greatest risk of developing glaucoma and should have their intraocular pressure tested routinely. Most measurements involve exerting a force on the (anesthetized) eye over some area (a pressure) and observing the eye's response. A noncontact approach uses a puff of air and a measurement is made of the force needed to indent the eye (Figure 11.35). If the intraocular pressure is high, the eye will deform less and rebound more vigorously than normal. Excessive intraocular pressures can be detected reliably and sometimes controlled effectively.

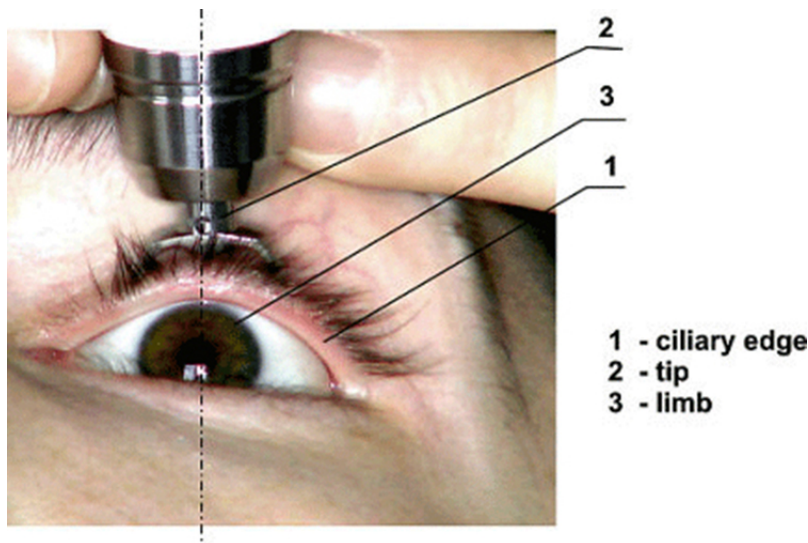


FIGURE 11.35 The intraocular eye pressure can be read with a tonometer. (credit: DevelopAll at the Wikipedia Project.)



EXAMPLE 11.13

Calculating Gauge Pressure and Depth: Damage to the Eardrum

Suppose a 3.00-N force can rupture an eardrum. (a) If the eardrum has an area of 1.00 cm^2 , calculate the maximum tolerable gauge pressure on the eardrum in newtons per meter squared and convert it to millimeters of mercury. (b) At what depth in freshwater would this person's eardrum rupture, assuming the gauge pressure in the middle ear is zero?

Strategy for (a)

The pressure can be found directly from its definition since we know the force and area. We are looking for the gauge pressure.

Solution for (a)

$$P_g = F/A = 3.00 \text{ N}/(1.00 \times 10^{-4} \text{ m}^2) = 3.00 \times 10^4 \text{ N/m}^2. \quad 11.57$$

We now need to convert this to units of mm Hg:

$$P_g = 3.0 \times 10^4 \text{ N/m}^2 \left(\frac{1.0 \text{ mm Hg}}{133 \text{ N/m}^2} \right) = 226 \text{ mm Hg}. \quad 11.58$$

Strategy for (b)

Here we will use the fact that the water pressure varies linearly with depth h below the surface.

Solution for (b)

$P = h\rho g$ and therefore $h = P/\rho g$. Using the value above for P , we have

$$h = \frac{3.0 \times 10^4 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.06 \text{ m}. \quad 11.59$$

Discussion

Similarly, increased pressure exerted upon the eardrum from the middle ear can arise when an infection causes a fluid buildup.

Pressure Associated with the Lungs

The pressure inside the lungs increases and decreases with each breath. The pressure drops to below atmospheric pressure (negative gauge pressure) when you inhale, causing air to flow into the lungs. It increases above atmospheric pressure (positive gauge pressure) when you exhale, forcing air out.

Lung pressure is controlled by several mechanisms. Muscle action in the diaphragm and rib cage is necessary for inhalation; this muscle action increases the volume of the lungs thereby reducing the pressure within them [Figure 11.36](#). Surface tension in the alveoli creates a positive pressure opposing inhalation. (See [Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action](#).) You can exhale without muscle action by letting surface tension in the alveoli create its own positive pressure. Muscle action can add to this positive pressure to produce forced exhalation, such as when you blow up a balloon, blow out a candle, or cough.

The lungs, in fact, would collapse due to the surface tension in the alveoli, if they were not attached to the inside of the chest wall by liquid adhesion. The gauge pressure in the liquid attaching the lungs to the inside of the chest wall is thus negative, ranging from -4 to -8 mm Hg during exhalation and inhalation, respectively. If air is allowed to enter the chest cavity, it breaks the attachment, and one or both lungs may collapse. Suction is applied to the chest cavity of surgery patients and trauma victims to reestablish negative pressure and inflate the lungs.

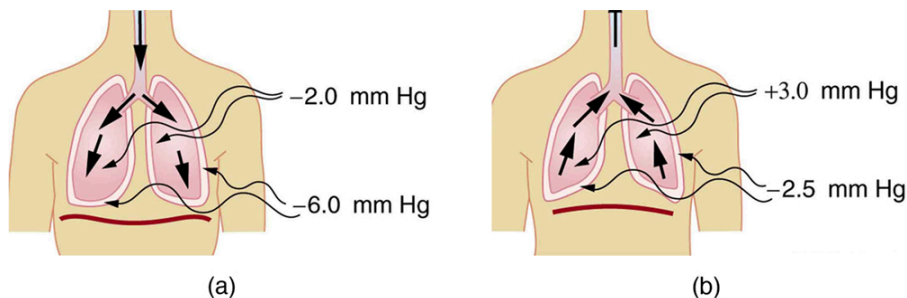


FIGURE 11.36 (a) During inhalation, muscles expand the chest, and the diaphragm moves downward, reducing pressure inside the lungs to less than atmospheric (negative gauge pressure). Pressure between the lungs and chest wall is even lower to overcome the positive pressure created by surface tension in the lungs. (b) During gentle exhalation, the muscles simply relax and surface tension in the alveoli creates a positive pressure inside the lungs, forcing air out. Pressure between the chest wall and lungs remains negative to keep them attached to the chest wall, but it is less negative than during inhalation.

Other Pressures in the Body

Spinal Column and Skull

Normally, there is a 5- to 12-mm Hg pressure in the fluid surrounding the brain and filling the spinal column. This cerebrospinal fluid serves many purposes, one of which is to supply flotation to the brain. The buoyant force supplied by the fluid nearly equals the weight of the brain, since their densities are nearly equal. If there is a loss of fluid, the brain rests on the inside of the skull, causing severe headaches, constricted blood flow, and serious damage. Spinal fluid pressure is measured by means of a needle inserted between vertebrae that transmits the pressure to a suitable measuring device.

Bladder Pressure

This bodily pressure is one of which we are often aware. In fact, there is a relationship between our awareness of this pressure and a subsequent increase in it. Bladder pressure climbs steadily from zero to about 25 mm Hg as the bladder fills to its normal capacity of 500 cm³. This pressure triggers the **micturition reflex**, which stimulates the feeling of needing to urinate. What is more, it also causes muscles around the bladder to contract, raising the pressure to over 100 mm Hg, accentuating the sensation. Coughing, straining, tensing in cold weather, wearing tight clothes, and experiencing simple nervous tension all can increase bladder pressure and trigger this reflex. So can the weight of a pregnant woman's fetus, especially if it is kicking vigorously or pushing down with its head! Bladder pressure can be measured by a catheter or by inserting a needle through the bladder wall and transmitting the pressure to an appropriate measuring device. One hazard of high bladder pressure (sometimes created by an obstruction), is that such pressure can force urine back into the kidneys, causing potentially severe damage.

Pressures in the Skeletal System

These pressures are the largest in the body, due both to the high values of initial force, and the small areas to which this force is applied, such as in the joints.. For example, when a person lifts an object improperly, a force of 5000 N may be created between vertebrae in the spine, and this may be applied to an area as small as 10 cm². The pressure created is $P = F/A = (5000 \text{ N}) / (10^{-3} \text{ m}^2) = 5.0 \times 10^6 \text{ N/m}^2$ or about 50 atm! This pressure can damage both the spinal discs (the cartilage between vertebrae), as well as the bony vertebrae themselves. Even under normal circumstances, forces between vertebrae in the spine are large enough to create pressures of several atmospheres. Most causes of excessive pressure in the skeletal system can be avoided by lifting properly and avoiding extreme physical activity. (See [Forces and Torques in Muscles and Joints](#).)

There are many other interesting and medically significant pressures in the body. For example, pressure caused by various muscle actions drives food and waste through the digestive system. Stomach pressure behaves much like bladder pressure and is tied to the sensation of hunger. Pressure in the relaxed esophagus is normally negative because pressure in the chest cavity is normally negative. Positive pressure in the stomach may thus force acid into the esophagus, causing "heartburn." Pressure in the middle ear can result in significant force on the eardrum if it differs greatly from atmospheric pressure, such as while scuba diving. The decrease in external pressure is also noticeable during plane flights (due to a decrease in the weight of air above relative to that at the Earth's surface). The Eustachian tubes connect the middle ear to the throat and allow us to equalize pressure in the middle ear to avoid an imbalance of force on the eardrum.

Many pressures in the human body are associated with the flow of fluids. Fluid flow will be discussed in detail in the [Fluid Dynamics and Its Biological and Medical Applications](#).

Glossary

absolute pressure the sum of gauge pressure and atmospheric pressure

adhesive forces the attractive forces between molecules of different types

Archimedes' principle the buoyant force on an object equals the weight of the fluid it displaces

buoyant force the net upward force on any object in any fluid

capillary action the tendency of a fluid to be raised or lowered in a narrow tube

cohesive forces the attractive forces between molecules of the same type

contact angle the angle θ between the tangent to the liquid surface and the surface

density the mass per unit volume of a substance or object

diastolic pressure minimum arterial blood pressure; indicator for the fluid balance

diastolic pressure the minimum blood pressure in the artery

fluids liquids and gases; a fluid is a state of matter that yields to shearing forces

gauge pressure the pressure relative to atmospheric

pressure

glaucoma condition caused by the buildup of fluid pressure in the eye

intraocular pressure fluid pressure in the eye

micturition reflex stimulates the feeling of needing to urinate, triggered by bladder pressure

Pascal's Principle a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

pressure the force per unit area perpendicular to the force, over which the force acts

pressure the weight of the fluid divided by the area supporting it

specific gravity the ratio of the density of an object to a fluid (usually water)

surface tension the cohesive forces between molecules which cause the surface of a liquid to contract to the smallest possible surface area

systolic pressure maximum arterial blood pressure; indicator for the blood flow

systolic pressure the maximum blood pressure in the artery

Section Summary

11.1 What Is a Fluid?

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

11.2 Density

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as $\rho = \frac{m}{V}$.
- The SI unit of density is kg/m^3 .

11.3 Pressure

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as $P = \frac{F}{A}$.
- The SI unit of pressure is pascal and $1 \text{ Pa} = 1 \text{ N/m}^2$.

11.4 Variation of Pressure with Depth in a Fluid

- Pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of

the container):

$$P = \frac{mg}{A}.$$

- Pressure due to the weight of a liquid is given by $P = h\rho g$, where P is the pressure, h is the height of the liquid, ρ is the density of the liquid, and g is the acceleration due to gravity.

11.5 Pascal's Principle

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Aneroid gauge measures pressure using a bellows-and-spring arrangement connected to the pointer of a calibrated scale.
- Open-tube manometers have U-shaped tubes and

one end is always open. It is used to measure pressure.

- A mercury barometer is a device that measures atmospheric pressure.

11.7 Archimedes' Principle

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- Attractive forces between molecules of the same type are called cohesive forces.

Conceptual Questions

11.1 What Is a Fluid?

1. What physical characteristic distinguishes a fluid from a solid?
2. Which of the following substances are fluids at room temperature: air, mercury, water, glass?
3. Why are gases easier to compress than liquids and solids?
4. How do gases differ from liquids?

11.2 Density

5. Approximately how does the density of air vary with altitude?
6. Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

- Attractive forces between molecules of different types are called adhesive forces.
- Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
- Capillary action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube which is due to the relative strength of cohesive and adhesive forces.

11.9 Pressures in the Body

- Measuring blood pressure is among the most common of all medical examinations.
- The pressures in various parts of the body can be measured and often provide valuable medical indicators.
- The shape of the eye is maintained by fluid pressure, called intraocular pressure.
- When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma.
- Some of the other pressures in the body are spinal and skull pressures, bladder pressure, pressures in the skeletal system.

7. [Figure 11.37](#) shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.



FIGURE 11.37

11.3 Pressure

8. How is pressure related to the sharpness of a knife and its ability to cut?
9. Why does a dull hypodermic needle hurt more than a sharp one?
10. The outward force on one end of an air tank was calculated in [Example 11.2](#). How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

11. Why is force exerted by static fluids always perpendicular to a surface?
12. In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?
13. How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?
14. Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.
15. How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?
20. Why is it difficult to swim under water in the Great Salt Lake?
21. Is there a net force on a dam due to atmospheric pressure? Explain your answer.
22. Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?
23. You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

11.5 Pascal's Principle

24. Suppose the pedal cylinder in a hydraulic system is at a greater height than the wheel cylinder. Explain how this will affect the force produced at the wheel cylinder.

11.4 Variation of Pressure with Depth in a Fluid

16. Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?
17. Why does atmospheric pressure decrease more rapidly than linearly with altitude?
18. What are two reasons why mercury rather than water is used in barometers?
19. [Figure 11.38](#) shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.

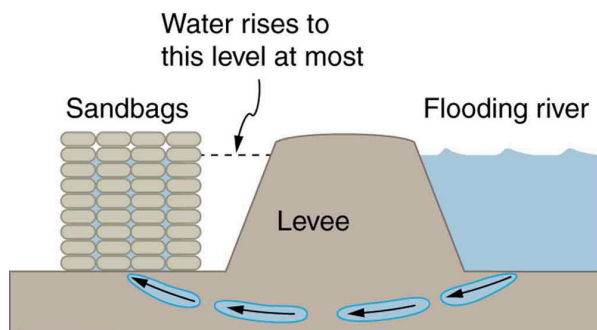


FIGURE 11.38 Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.

11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

25. Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.
26. [Figure 11.15](#) shows how a common measurement of arterial blood pressure is made. Is there any effect on the measured pressure if the manometer is lowered? What is the effect of raising the arm above the shoulder? What is the effect of placing the cuff on the upper leg with the person standing? Explain your answers in terms of pressure created by the weight of a fluid.
27. Considering the magnitude of typical arterial blood pressures, why are mercury rather than water manometers used for these measurements?

11.7 Archimedes' Principle

28. More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.
29. Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.
30. Will the same ship float higher in salt water than in freshwater? Explain your answer.

- 31.** Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- 32.** The density of oil is less than that of water, yet a loaded oil tanker sits lower in the water than an empty one. Why?
- 33.** Is surface tension due to cohesive or adhesive forces, or both?
- 34.** Is capillary action due to cohesive or adhesive forces, or both?
- 35.** Birds such as ducks, geese, and swans have greater densities than water, yet they are able to sit on its surface. Explain this ability, noting that water does not wet their feathers and that they cannot sit on soapy water.
- 36.** Water beads up on an oily sunbather, but not on her neighbor, whose skin is not oiled. Explain in terms of cohesive and adhesive forces.
- 37.** Could capillary action be used to move fluids in a “weightless” environment, such as in an orbiting space probe?
- 38.** What effect does capillary action have on the reading of a manometer with uniform diameter? Explain your answer.
- 39.** Pressure between the inside chest wall and the outside of the lungs normally remains negative. Explain how pressure inside the lungs can become positive (to cause exhalation) without muscle action.

Problems & Exercises

11.2 Density

- 1.** Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?
- 2.** Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?
- 3.** (a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body’s volume and density.
- 4.** A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0 cm³ of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes’ principle.)
- 5.** Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.
- 6.** (a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.
- 7.** A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?
- 8.** A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?
- 9.** What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.
- 10.** There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10³ kg/m³. The nucleus of an atom has a radius about 10^{−5} that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is 7 × 10⁸ m)?

11.3 Pressure

11. As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of 1.50 cm^2 and the woman's mass is 55.0 kg . Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)
12. The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2 ?
13. Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $3.00 \times 10^9 \text{ N/m}^2$? (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

11.4 Variation of Pressure with Depth in a Fluid

14. What depth of mercury creates a pressure of 1.00 atm ?
15. The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.
16. Verify that the SI unit of $h\rho g$ is N/m^2 .
17. Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $3.00 \times 10^5 \text{ N/m}^2$?
18. The aqueous humor in a person's eye is exerting a force of 0.300 N on the 1.10-cm^2 area of the cornea. (a) What pressure is this in mm Hg ? (b) Is this value within the normal range for pressures in the eye?
19. How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?
20. What pressure is exerted on the bottom of a 0.500-m -wide by 0.900-m -long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?
21. Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is 50.0 cm^2 and he exerts a force of 800 N on it. Express the pressure in N/m^2 and compare it with the $1.00 \times 10^6 \text{ Pa}$ pressures sometimes encountered in the skeletal system.
22. The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of 15.0 cm^2 . What force does it exert to accomplish this?
23. Show that the total force on a rectangular dam due to the water behind it increases with the *square* of the water depth. In particular, show that this force is given by $F = \rho g h^2 L/2$, where ρ is the density of water, h is its depth at the dam, and L is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See Figure 11.39.)

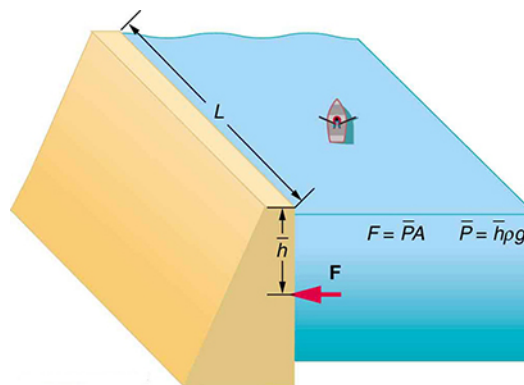


FIGURE 11.39

11.5 Pascal's Principle

24. How much pressure is transmitted in the hydraulic system considered in Example 11.6? Express your answer in pascals and in atmospheres.
25. What force must be exerted on the pedal cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the wheel cylinder? The pedal cylinder has a 2.00-cm diameter and the wheel has a 24.0-cm diameter.
26. A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

- 27.** A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the wheel cylinder to the area of the pedal cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.
- 28.** (a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?
- 33.** Suppose you measure a standing person's blood pressure by placing the cuff on his leg 0.500 m below the heart. Calculate the pressure you would observe (in units of mm Hg) if the pressure at the heart were 120 over 80 mm Hg. Assume that there is no loss of pressure due to resistance in the circulatory system (a reasonable assumption, since major arteries are large).
- 34.** A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.
- 35.** Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground. The bicycle plus rider has a mass of 80.0 kg, and the gauge pressure in the tires is 3.50×10^5 Pa.

11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

- 29.** Find the gauge and absolute pressures in the balloon and peanut jar shown in [Figure 11.14](#), assuming the manometer connected to the balloon uses water whereas the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar, taking $h = 0.0500$ m for each.
- 30.** (a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid ($P = h\rho g$) rather than a conversion factor. (b) Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.
- 31.** How tall must a water-filled manometer be to measure blood pressures as high as 300 mm Hg?
- 32.** Pressure cookers have been around for more than 300 years, although their use has strongly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is 25.0 cm in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid.

11.7 Archimedes' Principle

- 36.** What fraction of ice is submerged when it floats in freshwater, given the density of water at 0°C is very close to 1000 kg/m^3 ?
- 37.** Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?
- 38.** Find the density of a fluid in which a hydrometer having a density of 0.750 g/mL floats with 92.0% of its volume submerged.
- 39.** If your body has a density of 995 kg/m^3 , what fraction of you will be submerged when floating gently in: (a) freshwater? (b) salt water, which has a density of 1027 kg/m^3 ?
- 40.** Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0 g and its apparent mass when submerged is 3.60 g (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?
- 41.** A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

- 42.** Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in [Table 11.1](#) (c) Calculate the fluid's density and identify it.
- 43.** In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?
- 44.** Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is 1015 kg/m^3 ?
- 45.** (a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? You can neglect the volume of the rubber.
- 46.** (a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?
- 47.** A certain man has a mass of 80 kg and a density of 955 kg/m^3 (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?
- 48.** A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?
- 49.** What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?
- 50.** Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?
- 51.** A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The weight of the mattress is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?
- 52.** Referring to [Figure 11.19](#), prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas A . Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.
- 53.** (a) A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- 54.** What is the pressure inside an alveolus having a radius of $2.50 \times 10^{-4} \text{ m}$ if the surface tension of the fluid-lined wall is the same as for soapy water? You may assume the pressure is the same as that created by a spherical bubble.
- 55.** (a) The pressure inside an alveolus with a 2.00×10^{-4} -m radius is $1.40 \times 10^3 \text{ Pa}$, due to its fluid-lined walls. Assuming the alveolus acts like a spherical bubble, what is the surface tension of the fluid? (b) Identify the likely fluid. (You may need to extrapolate between values in [Table 11.3](#).)
- 56.** What is the gauge pressure in millimeters of mercury inside a soap bubble 0.100 m in diameter?
- 57.** Calculate the force on the slide wire in [Figure 11.26](#) if it is 3.50 cm long and the fluid is ethyl alcohol.

58. [Figure 11.32](#)(a) shows the effect of tube radius on the height to which capillary action can raise a fluid. (a) Calculate the height h for water in a glass tube with a radius of 0.900 cm—a rather large tube like the one on the left. (b) What is the radius of the glass tube on the right if it raises water to 4.00 cm?
59. We stated in [Example 11.12](#) that a xylem tube is of radius 2.50×10^{-5} m. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is 1050 kg/m^3 , its contact angle is zero, and its surface tension is the same as that of water at 20.0°C .
60. What fluid is in the device shown in [Figure 11.26](#) if the force is 3.16×10^{-3} N and the length of the wire is 2.50 cm? Calculate the surface tension γ and find a likely match from [Table 11.3](#).
61. If the gauge pressure inside a rubber balloon with a 10.0-cm radius is 1.50 cm of water, what is the effective surface tension of the balloon?
62. Calculate the gauge pressures inside 2.00-cm-radius bubbles of water, alcohol, and soapy water. Which liquid forms the most stable bubbles, neglecting any effects of evaporation?
63. Suppose water is raised by capillary action to a height of 5.00 cm in a glass tube. (a) To what height will it be raised in a paraffin tube of the same radius? (b) In a silver tube of the same radius?
64. Calculate the contact angle θ for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids?
65. When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?
66. Calculate the ratio of the heights to which water and mercury are raised by capillary action in the same glass tube.
67. What is the ratio of heights to which ethyl alcohol and water are raised by capillary action in the same glass tube?

11.9 Pressures in the Body

68. During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of 60.0 mm Hg between the lungs and chest wall. What force in newtons does this pressure create on the 600 cm^2 surface area of the diaphragm?
69. You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of 500 N with your tooth on an area of 1.00 mm^2 ?
70. One way to force air into an unconscious person's lungs is to squeeze on a balloon appropriately connected to the subject. What force must you exert on the balloon with your hands to create a gauge pressure of 4.00 cm water, assuming you squeeze on an effective area of 50.0 cm^2 ?
71. Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve -3.00 cm water pressure with your lungs 60.0 cm below the surface?
72. Gauge pressure in the fluid surrounding an infant's brain may rise as high as 85.0 mm Hg (5 to 12 mm Hg is normal), creating an outward force large enough to make the skull grow abnormally large. (a) Calculate this outward force in newtons on each side of an infant's skull if the effective area of each side is 70.0 cm^2 . (b) What is the net force acting on the skull?
73. A full-term fetus typically has a mass of 3.50 kg. (a) What pressure does the weight of such a fetus create if it rests on the mother's bladder, supported on an area of 90.0 cm^2 ? (b) Convert this pressure to millimeters of mercury and determine if it alone is great enough to trigger the micturition reflex (it will add to any pressure already existing in the bladder).
74. If the pressure in the esophagus is -2.00 mm Hg while that in the stomach is $+20.0$ mm Hg, to what height could stomach fluid rise in the esophagus, assuming a density of 1.10 g/mL ? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)

75. Pressure in the spinal fluid is measured as shown in Figure 11.40. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.

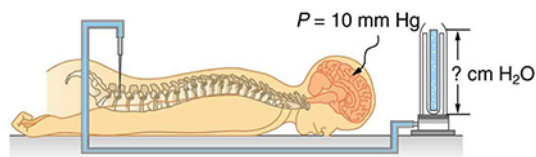


FIGURE 11.40 A water manometer used to measure pressure in the spinal fluid. The height of the fluid in the manometer is measured relative to the spinal column, and the manometer is open to the atmosphere. The measured pressure will be considerably greater if the person sits up.

76. Calculate the maximum force in newtons exerted by the blood on an aneurysm, or ballooning, in a major artery, given the maximum blood pressure for this person is 150 mm Hg and the effective area of the aneurysm is 20.0 cm^2 . Note that this force is great enough to cause further enlargement and subsequently greater force on the ever-thinner vessel wall.
77. During heavy lifting, a disk between spinal vertebrae is subjected to a 5000-N compressional force. (a) What pressure is created, assuming that the disk has a uniform circular cross section 2.00 cm in radius? (b) What deformation is produced if the disk is 0.800 cm thick and has a Young's modulus of $1.5 \times 10^9 \text{ N/m}^2$?
78. When a person sits erect, increasing the vertical position of their brain by 36.0 cm, the heart must continue to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 100 mL of blood raised 36.0 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (c) Discuss how the gain in gravitational potential energy and the decrease in pressure are related.
79. (a) How high will water rise in a glass capillary tube with a 0.500-mm radius? (b) How much gravitational potential energy does the water gain? (c) Discuss possible sources of this energy.

80. A negative pressure of 25.0 atm can sometimes be achieved with the device in Figure 11.41 before the water separates. (a) To what height could such a negative gauge pressure raise water? (b) How much would a steel wire of the same diameter and length as this capillary stretch if suspended from above?

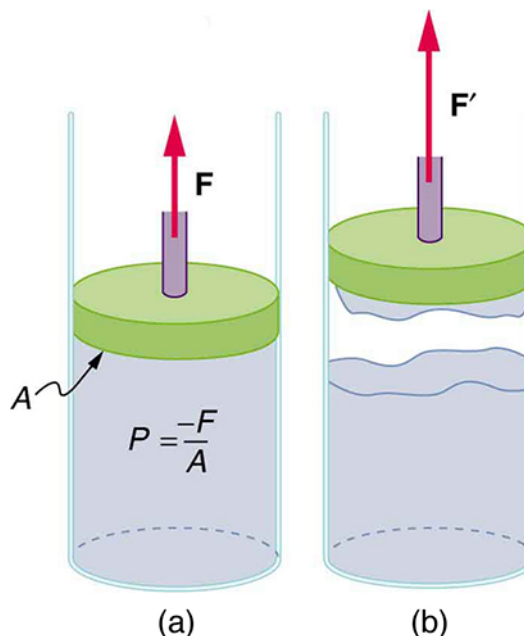


FIGURE 11.41 (a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$. (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

81. Suppose you hit a steel nail with a 0.500-kg hammer, initially moving at 15.0 m/s and brought to rest in 2.80 mm. (a) What average force is exerted on the nail? (b) How much is the nail compressed if it is 2.50 mm in diameter and 6.00-cm long? (c) What pressure is created on the 1.00-mm-diameter tip of the nail?
82. Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is 11.0 km and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?

- 83.** The hydraulic system of a backhoe is used to lift a load as shown in [Figure 11.42](#). (a) Calculate the force F the secondary cylinder must exert to support the 400-kg load and the 150-kg brace and shovel. (b) What is the pressure in the hydraulic fluid if the secondary cylinder is 2.50 cm in diameter? (c) What force would you have to exert on a lever with a mechanical advantage of 5.00 acting on a primary cylinder 0.800 cm in diameter to create this pressure?

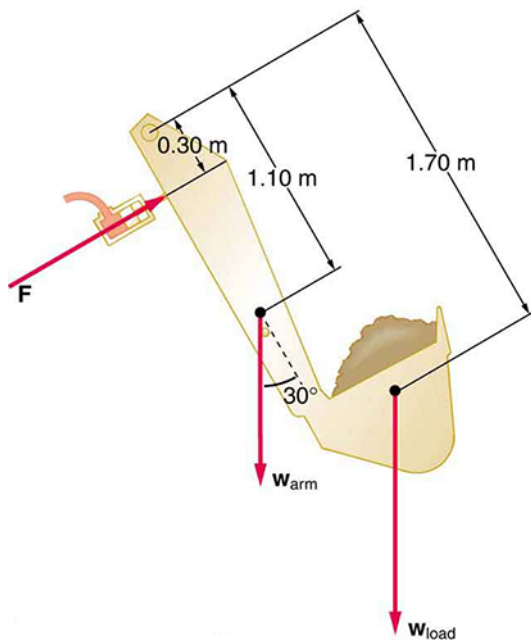


FIGURE 11.42 Hydraulic and mechanical lever systems are used in heavy machinery such as this back hoe.

- 84.** Some miners wish to remove water from a mine shaft. A pipe is lowered to the water 90 m below, and a negative pressure is applied to raise the water. (a) Calculate the pressure needed to raise the water. (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise?
- 85.** You are pumping up a bicycle tire with a hand pump, the piston of which has a 2.00-cm radius. (a) What force in newtons must you exert to create a pressure of 6.90×10^5 Pa (b) What is unreasonable about this (a) result? (c) Which premises are unreasonable or inconsistent?
- 86.** Consider a group of people trying to stay afloat after their boat strikes a log in a lake. Construct a problem in which you calculate the number of people that can cling to the log and keep their heads out of the water. Among the variables to be considered are the size and density of the log, and what is needed to keep a person's head and arms above water without swimming or treading water.
- 87.** The alveoli in emphysema victims are damaged and effectively form larger sacs. Construct a problem in which you calculate the loss of pressure due to surface tension in the alveoli because of their larger average diameters. (Part of the lung's ability to expel air results from pressure created by surface tension in the alveoli.) Among the things to consider are the normal surface tension of the fluid lining the alveoli, the average alveolar radius in normal individuals and its average in emphysema sufferers.

CHAPTER 12

Fluid Dynamics and Its Biological and Medical Applications



FIGURE 12.1 Many fluids are flowing in this scene. Water from the hose and smoke from the fire are visible flows. Less visible are the flow of air and the flow of fluids on the ground and within the people fighting the fire. (credit: Andrew Magill, Flickr)

CHAPTER OUTLINE

12.1 Flow Rate and Its Relation to Velocity

12.2 Bernoulli's Equation

12.3 The Most General Applications of Bernoulli's Equation

12.4 Viscosity and Laminar Flow; Poiseuille's Law

12.5 The Onset of Turbulence

12.6 Motion of an Object in a Viscous Fluid

12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

INTRODUCTION TO FLUID DYNAMICS AND ITS BIOLOGICAL AND MEDICAL APPLICATIONS We have dealt with many situations in which fluids are static. But by their very definition, fluids flow. Examples come easily—a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? The physics of fluids in motion—**fluid dynamics**—allows us to answer these and many other questions.

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12.1 Flow Rate and Its Relation to Velocity

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in [Figure 12.2](#). In symbols, this can be written as

$$Q = \frac{V}{t}, \quad 12.1$$

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s , but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters (10^{-3} m^3 or 10^3 cm^3). In this text we shall use whatever metric units are most convenient for a given situation.

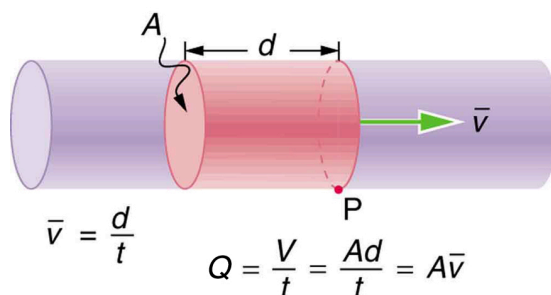


FIGURE 12.2 Flow rate is the volume of fluid per unit time flowing past a point through the area A . Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t . The volume of the cylinder is Ad and the average velocity is $\bar{v} = d/t$ so that the flow rate is $Q = Ad/t = A\bar{v}$.



EXAMPLE 12.1

Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V/t$ for volume gives

$$V = Qt. \quad 12.2$$

Substituting known values yields

$$\begin{aligned} V &= \left(\frac{5.00 \text{ L}}{1 \text{ min}} \right) (75 \text{ y}) \left(\frac{1 \text{ m}^3}{10^3 \text{ L}} \right) \left(5.26 \times 10^5 \frac{\text{min}}{\text{y}} \right) \\ &= 2.0 \times 10^5 \text{ m}^3. \end{aligned} \quad 12.3$$

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity $\bar{v}$ is

$$Q = A\bar{v}, \quad 12.4$$

where A is the cross-sectional area and $\bar{v}$ is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. [Figure 12.2](#) illustrates how this relationship is obtained. The shaded cylinder has a volume

$$V = Ad, \quad 12.5$$

which flows past the point P in a time t . Dividing both sides of this relationship by t gives

$$\frac{V}{t} = \frac{Ad}{t}. \quad 12.6$$

We note that $Q = V/t$ and the average speed is $\bar{v} = d/t$. Thus the equation becomes $Q = A\bar{v}$.

[Figure 12.3](#) shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

$$\left. \begin{aligned} Q_1 &= Q_2 \\ A_1 \bar{v}_1 &= A_2 \bar{v}_2 \end{aligned} \right\}. \quad 12.7$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.

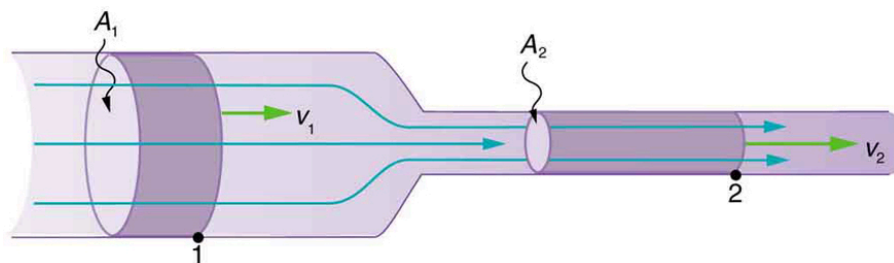


FIGURE 12.3 When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.



EXAMPLE 12.2

Calculating Fluid Speed: Speed Increases When a Tube Narrows

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = A\bar{v}$ for v_1 and note that the cross-sectional area is $A = \pi r^2$, yielding

$$\bar{v}_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}. \quad 12.8$$

Substituting known values and making appropriate unit conversions yields

$$\bar{v}_1 = \frac{(0.500 \text{ L/s})(10^{-3} \text{ m}^3/\text{L})}{\pi(9.00 \times 10^{-3} \text{ m})^2} = 1.96 \text{ m/s}. \quad 12.9$$

Solution for (b)

We could repeat this calculation to find the speed in the nozzle $\bar{v}_2$, but we will use the equation of continuity to give a somewhat different insight. Using the equation which states

$$A_1 \bar{v}_1 = A_2 \bar{v}_2, \quad 12.10$$

solving for $\bar{v}_2$ and substituting πr^2 for the cross-sectional area yields

$$\bar{v}_2 = \frac{A_1}{A_2} \bar{v}_1 = \frac{\pi r_1^2}{\pi r_2^2} \bar{v}_1 = \frac{r_1^2}{r_2^2} \bar{v}_1. \quad 12.11$$

Substituting known values,

$$\bar{v}_2 = \frac{(0.900 \text{ cm})^2}{(0.250 \text{ cm})^2} 1.96 \text{ m/s} = 25.5 \text{ m/s}. \quad 12.12$$

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

$$n_1 A_1 \bar{v}_1 = n_2 A_2 \bar{v}_2, \quad 12.13$$

where n_1 and n_2 are the number of branches in each of the sections along the tube.

**EXAMPLE 12.3****Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System**

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm.

(b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is

5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0 μm , calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = A\bar{v}$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = A\bar{v}$ or $\bar{v} = \frac{Q}{\pi r^2}$ for a cylindrical vessel.

Substituting the known values (converted to units of meters and seconds) gives

$$\bar{v} = \frac{(5.0 \text{ L/min})(10^{-3} \text{ m}^3/\text{L})(1 \text{ min}/60 \text{ s})}{\pi(0.010 \text{ m})^2} = 0.27 \text{ m/s.} \quad 12.14$$

Solution for (b)

Using $n_1 A_1 \bar{v}_1 = n_2 A_2 \bar{v}_2$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \frac{n_1 A_1 \bar{v}_1}{A_2 \bar{v}_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

$$n_2 = \frac{(1)(\pi)(10 \times 10^{-3} \text{ m})^2(0.27 \text{ m/s})}{(\pi)(4.0 \times 10^{-6} \text{ m})^2(0.33 \times 10^{-3} \text{ m/s})} = 5.0 \times 10^9 \text{ capillaries.} \quad 12.15$$

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3 , or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

12.2 Bernoulli's Equation

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the terms in Bernoulli's equation.
- Explain how Bernoulli's equation is related to conservation of energy.
- Explain how to derive Bernoulli's principle from Bernoulli's equation.
- Calculate with Bernoulli's principle.
- List some applications of Bernoulli's principle.

When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem,

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \quad 12.16$$

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the *pressure will drop in a rapidly-moving fluid*, whether or not the fluid is confined to a tube.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a

disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See [Figure 12.4](#).) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.

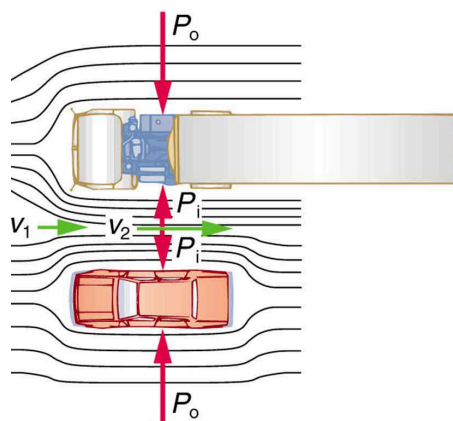


FIGURE 12.4 An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (P_1 is less than P_0). Greater pressure on the outside pushes the car and truck together.

Making Connections: Take-Home Investigation with a Sheet of Paper

Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

Bernoulli's Equation

The relationship between pressure and velocity in fluids is described quantitatively by **Bernoulli's equation**, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an incompressible, frictionless fluid, the following sum is constant:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}, \quad 12.17$$

where P is the absolute pressure, ρ is the fluid density, v is the velocity of the fluid, h is the height above some reference point, and g is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2. \quad 12.18$$

Bernoulli's equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with m replaced by ρ . In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting $\rho = m/V$ into it and gathering terms:

$$\frac{1}{2}\rho v^2 = \frac{\frac{1}{2}mv^2}{V} = \frac{\text{KE}}{V}. \quad 12.19$$

So $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find

$$\rho gh = \frac{mgh}{V} = \frac{PE_g}{V}, \quad 12.20$$

so ρgh is the gravitational potential energy per unit volume. Note that pressure P has units of energy per unit volume, too. Since $P = F/A$, its units are N/m^2 . If we multiply these by m/m , we obtain $\text{N} \cdot \text{m/m}^3 = \text{J/m}^3$, or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

Making Connections: Conservation of Energy

Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results in changes in the fluid's KE and PE_g per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning.

Bernoulli's Equation for Static Fluids

Let us first consider the very simple situation where the fluid is static—that is, $v_1 = v_2 = 0$. Bernoulli's equation in that case is

$$P_1 + \rho gh_1 = P_2 + \rho gh_2. \quad 12.21$$

We can further simplify the equation by taking $h_2 = 0$ (we can always choose some height to be zero, just as we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get

$$P_2 = P_1 + \rho gh_1. \quad 12.22$$

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by h_1 , and consequently, P_2 is greater than P_1 by an amount ρgh_1 . In the very simplest case, P_1 is zero at the top of the fluid, and we get the familiar relationship $P = \rho gh$. (Recall that $P = \rho gh$ and $\Delta PE_g = mgh$.) Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is ρgh . Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter.

Bernoulli's Principle—Bernoulli's Equation at Constant Depth

Another important situation is one in which the fluid moves but its depth is constant—that is, $h_1 = h_2$. Under that condition, Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2. \quad 12.23$$

Situations in which fluid flows at a constant depth are so important that this equation is often called **Bernoulli's principle**. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if v_2 is greater than v_1 in the equation, then P_2 must be less than P_1 for the equality to hold.



EXAMPLE 12.4

Calculating Pressure: Pressure Drops as a Fluid Speeds Up

In [Example 12.2](#), we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01 \times 10^5 \text{ N/m}^2$ (atmospheric, as it must be) and assuming level, frictionless flow.

Strategy

Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find P_1 .

Solution

Solving Bernoulli's principle for P_1 yields

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2). \quad 12.24$$

Substituting known values,

$$\begin{aligned} P_1 &= 1.01 \times 10^5 \text{ N/m}^2 \\ &\quad + \frac{1}{2}(10^3 \text{ kg/m}^3)[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2] \\ &= 4.24 \times 10^5 \text{ N/m}^2. \end{aligned} \quad 12.25$$

Discussion

This absolute pressure in the hose is greater than in the nozzle, as expected since v is greater in the nozzle. The pressure P_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

Applications of Bernoulli's Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle.

Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in [Figure 12.5](#).

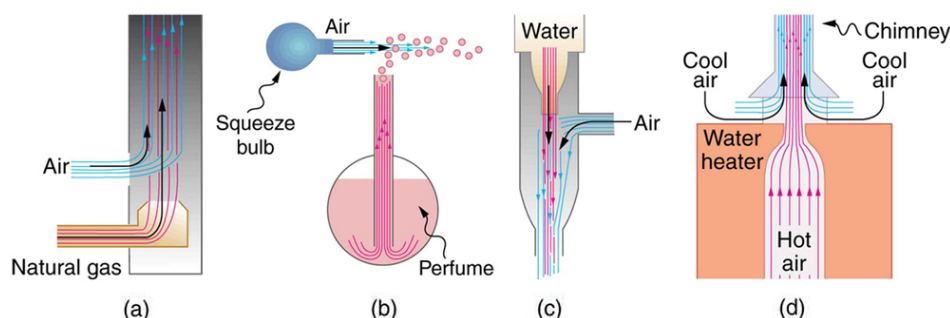


FIGURE 12.5 Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

Wings and Sails

The airplane wing is a beautiful example of Bernoulli's principle in action. [Figure 12.6\(a\)](#) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton's third law.) Sails also have the characteristic shape of a wing. (See [Figure 12.6\(b\)](#).) The pressure on the front side of the sail, P_{front} , is lower than the pressure on the back of the sail, P_{back} .

This results in a forward force and even allows you to sail into the wind.

Making Connections: Take-Home Investigation with Two Strips of Paper

For a good illustration of Bernoulli's principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?

Velocity measurement

Figure 12.7 shows two devices that measure fluid velocity based on Bernoulli's principle. The manometer in Figure 12.7(a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ($v_1 = 0$) in front of it, while fluid passing the other tube has velocity v_2 . This means that Bernoulli's principle as stated in $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ becomes

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2. \quad 12.26$$

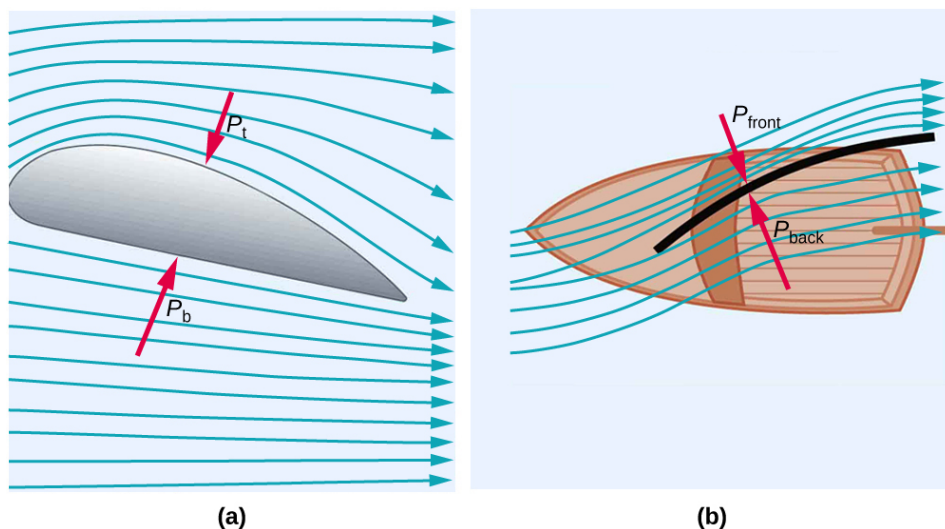


FIGURE 12.6 (a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust.

Thus pressure P_2 over the second opening is reduced by $\frac{1}{2}\rho v_2^2$, and so the fluid in the manometer rises by h on the side connected to the second opening, where

$$h \propto \frac{1}{2}\rho v_2^2. \quad 12.27$$

(Recall that the symbol $\propto$ means “proportional to.”) Solving for v_2 , we see that

$$v_2 \propto \sqrt{h}. \quad 12.28$$

Figure 12.7(b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft.

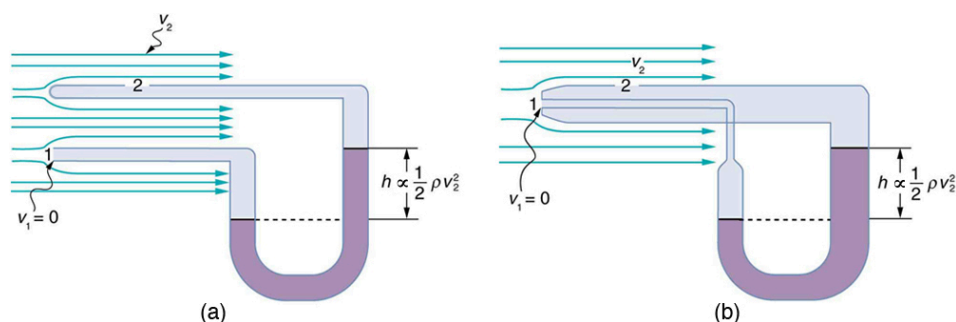


FIGURE 12.7 Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, and so h is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

12.3 The Most General Applications of Bernoulli's Equation

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate using Torricelli's theorem.
- Calculate power in fluid flow.

Torricelli's Theorem

[Figure 12.8](#) shows water gushing from a large tube through a dam. What is its speed as it emerges? Interestingly, if resistance is negligible, the speed is just what it would be if the water fell a distance h from the surface of the reservoir; the water's speed is independent of the size of the opening. Let us check this out. Bernoulli's equation must be used since the depth is not constant. We consider water flowing from the surface (point 1) to the tube's outlet (point 2). Bernoulli's equation as stated in previously is

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2. \quad 12.29$$

Both P_1 and P_2 equal atmospheric pressure (P_1 is atmospheric pressure because it is the pressure at the top of the reservoir. P_2 must be atmospheric pressure, since the emerging water is surrounded by the atmosphere and cannot have a pressure different from atmospheric pressure.) and subtract out of the equation, leaving

$$\frac{1}{2}\rho v_1^2 + \rho gh_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2. \quad 12.30$$

Solving this equation for v_2^2 , noting that the density ρ cancels (because the fluid is incompressible), yields

$$v_2^2 = v_1^2 + 2g(h_1 - h_2). \quad 12.31$$

We let $h = h_1 - h_2$; the equation then becomes

$$v_2^2 = v_1^2 + 2gh \quad 12.32$$

where h is the height dropped by the water. This is simply a kinematic equation for any object falling a distance h with negligible resistance. In fluids, this last equation is called *Torricelli's theorem*. Note that the result is independent of the velocity's direction, just as we found when applying conservation of energy to falling objects.

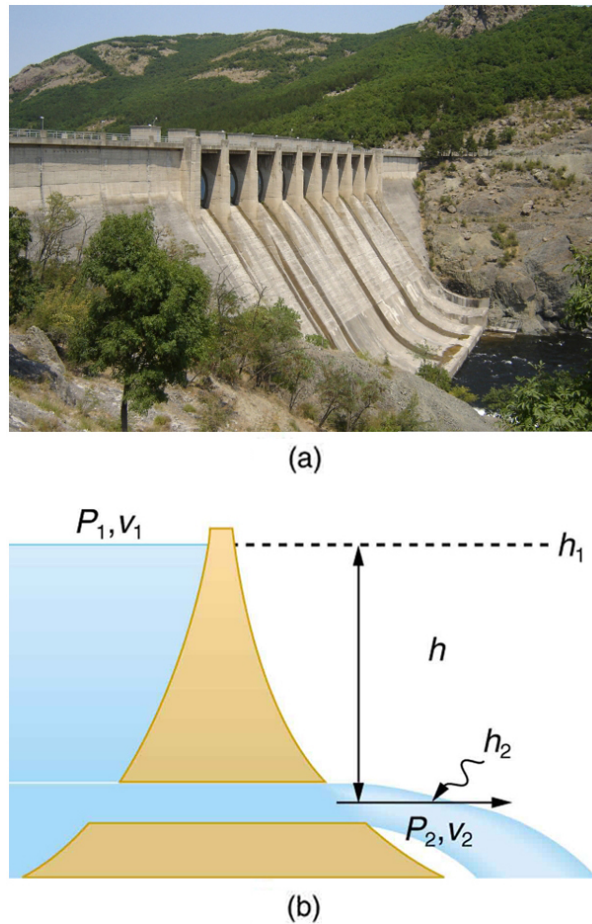


FIGURE 12.8 (a) Water gushes from the base of the Studen Kladenetz dam in Bulgaria. (credit: Kiril Kapustin; <http://www.ImagesFromBulgaria.com>) (b) In the absence of significant resistance, water flows from the reservoir with the same speed it would have if it fell the distance h without friction. This is an example of Torricelli's theorem.

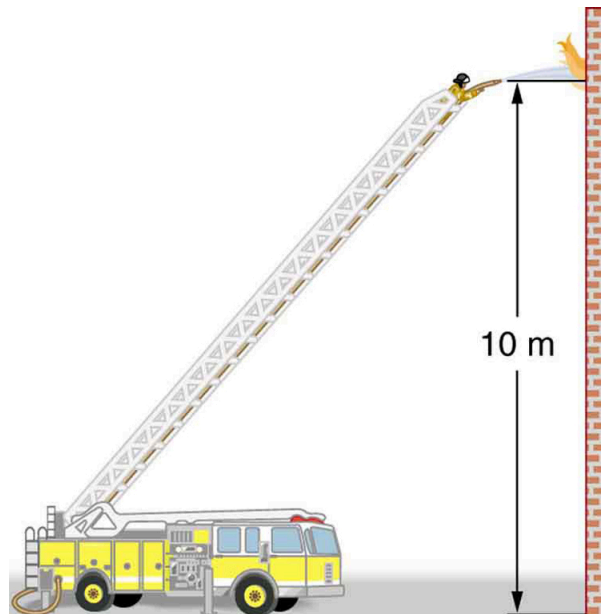


FIGURE 12.9 Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle, and speed increases in the nozzle. In spite of its lowered pressure, the water can exert a large force on anything it strikes, by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air.

All preceding applications of Bernoulli's equation involved simplifying conditions, such as constant height or constant pressure. The next example is a more general application of Bernoulli's equation in which pressure,

velocity, and height all change. (See [Figure 12.9](#).)



EXAMPLE 12.5

Calculating Pressure: A Fire Hose Nozzle

Fire hoses used in major structure fires have inside diameters of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62 \times 10^6 \text{ N/m}^2$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Assuming negligible resistance, what is the pressure in the nozzle?

Strategy

Here we must use Bernoulli's equation to solve for the pressure, since depth is not constant.

Solution

Bernoulli's equation states

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2, \quad 12.33$$

where the subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds v_1 and v_2 . Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3} \text{ m}^3/\text{s}}{\pi(3.20 \times 10^{-2} \text{ m})^2} = 12.434 \text{ m/s}. \quad 12.34$$

Similarly, we find

$$v_2 = 56.588 \text{ m/s}. \quad 12.35$$

(This rather large speed is helpful in reaching the fire.) Now, taking h_1 to be zero, we solve Bernoulli's equation for P_2 :

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho gh_2. \quad 12.36$$

Substituting known values yields

$$\begin{aligned} P_2 &= \\ 1.62 \times 10^6 \text{ N/m}^2 + \frac{1}{2}(1000 \text{ kg/m}^3)[(12.434 \text{ m/s})^2 - (56.588 \text{ m/s})^2] - (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(10.0 \text{ m}) \\ &\approx 1800 \text{ N/m}^2. \end{aligned} \quad 12.37$$

Discussion

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus the nozzle pressure is very close to atmospheric pressure, as it must because the water exits into the atmosphere without changes in its conditions.

Power in Fluid Flow

Power is the *rate* at which work is done or energy in any form is used or supplied. To see the relationship of power to fluid flow, consider Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}. \quad 12.38$$

All three terms have units of energy per unit volume, as discussed in the previous section. Now, considering units, if we multiply energy per unit volume by flow rate (volume per unit time), we get units of power. That is, $(E/V)(V/t) = E/t$. This means that if we multiply Bernoulli's equation by flow rate Q , we get power. In equation form, this is

$$\left(P + \frac{1}{2}\rho v^2 + \rho gh\right)Q = \text{power.} \quad 12.39$$

Each term has a clear physical meaning. For example, PQ is the power supplied to a fluid, perhaps by a pump, to give it its pressure P . Similarly, $\frac{1}{2}\rho v^2 Q$ is the power supplied to a fluid to give it its kinetic energy. And ρghQ is the power going to gravitational potential energy.

Making Connections: Power

Power is defined as the rate of energy transferred, or E/t . Fluid flow involves several types of power. Each type of power is identified with a specific type of energy being expended or changed in form.



EXAMPLE 12.6

Calculating Power in a Moving Fluid

Suppose the fire hose in the previous example is fed by a pump that receives water through a hose with a 6.40-cm diameter coming from a hydrant with a pressure of $0.700 \times 10^6 \text{ N/m}^2$. What power does the pump supply to the water?

Strategy

Here we must consider energy forms as well as how they relate to fluid flow. Since the input and output hoses have the same diameters and are at the same height, the pump does not change the speed of the water nor its height, and so the water's kinetic energy and gravitational potential energy are unchanged. That means the pump only supplies power to increase water pressure by $0.92 \times 10^6 \text{ N/m}^2$ (from $0.700 \times 10^6 \text{ N/m}^2$ to $1.62 \times 10^6 \text{ N/m}^2$).

Solution

As discussed above, the power associated with pressure is

$$\begin{aligned} \text{power} &= PQ \\ &= (0.920 \times 10^6 \text{ N/m}^2)(40.0 \times 10^{-3} \text{ m}^3/\text{s}) \dots \\ &= 3.68 \times 10^4 \text{ W} = 36.8 \text{ kW} \end{aligned} \quad 12.40$$

Discussion

Such a substantial amount of power requires a large pump, such as is found on some fire trucks. (This kilowatt value converts to about 50 hp.) The pump in this example increases only the water's pressure. If a pump—such as the heart—directly increases velocity and height as well as pressure, we would have to calculate all three terms to find the power it supplies.

12.4 Viscosity and Laminar Flow; Poiseuille's Law

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define laminar flow and turbulent flow.
- Explain what viscosity is.
- Calculate flow and resistance with Poiseuille's law.
- Explain how pressure drops due to resistance.

Laminar Flow and Viscosity

When you pour yourself a glass of juice, the liquid flows freely and quickly. But when you pour syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas

syrup has high viscosity. In the previous sections we have considered ideal fluids with little or no viscosity. In this section, we will investigate what factors, including viscosity, affect the rate of fluid flow.

The precise definition of viscosity is based on *laminar*, or nonturbulent, flow. Before we can define viscosity, then, we need to define laminar flow and turbulent flow. [Figure 12.10](#) shows both types of flow. **Laminar** flow is characterized by the smooth flow of the fluid in layers that do not mix. Turbulent flow, or **turbulence**, is characterized by eddies and swirls that mix layers of fluid together.



FIGURE 12.10 Smoke rises smoothly for a while and then begins to form swirls and eddies. The smooth flow is called laminar flow, whereas the swirls and eddies typify turbulent flow. If you watch the smoke (being careful not to breathe on it), you will notice that it rises more rapidly when flowing smoothly than after it becomes turbulent, implying that turbulence poses more resistance to flow. (credit: Creativity103)

[Figure 12.11](#) shows schematically how laminar and turbulent flow differ. Layers flow without mixing when flow is laminar. When there is turbulence, the layers mix, and there are significant velocities in directions other than the overall direction of flow. The lines that are shown in many illustrations are the paths followed by small volumes of fluids. These are called *streamlines*. Streamlines are smooth and continuous when flow is laminar, but break up and mix when flow is turbulent. Turbulence has two main causes. First, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow. Second, high speeds cause turbulence. The drag both between adjacent layers of fluid and between the fluid and its surroundings forms swirls and eddies, if the speed is great enough. We shall concentrate on laminar flow for the remainder of this section, leaving certain aspects of turbulence for later sections.

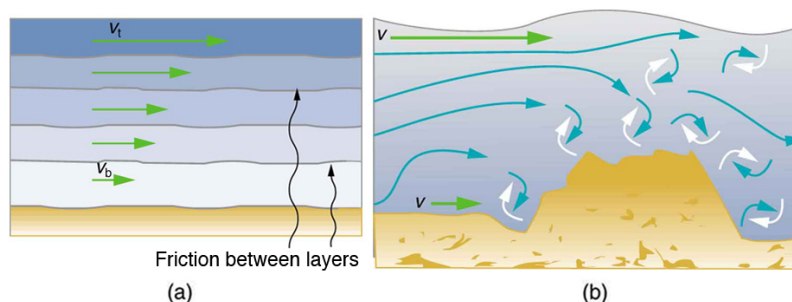


FIGURE 12.11 (a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. (b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow.

Making Connections: Take-Home Experiment: Go Down to the River

Try dropping simultaneously two sticks into a flowing river, one near the edge of the river and one near the middle. Which one travels faster? Why?

Figure 12.12 shows how viscosity is measured for a fluid. Two parallel plates have the specific fluid between them. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, and so the top layer moves at v while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from v to 0 as shown. Care is taken to insure that the flow is laminar; that is, the layers do not mix. The motion in Figure 12.12 is like a continuous shearing motion. Fluids have zero shear strength, but the *rate* at which they are sheared is related to the same geometrical factors A and L as is shear deformation for solids.

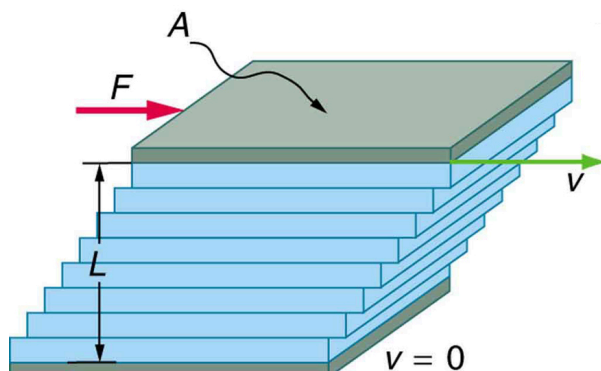


FIGURE 12.12 The graphic shows laminar flow of fluid between two plates of area A . The bottom plate is fixed. When the top plate is pushed to the right, it drags the fluid along with it.

A force F is required to keep the top plate in Figure 12.12 moving at a constant velocity v , and experiments have shown that this force depends on four factors. First, F is directly proportional to v (until the speed is so high that turbulence occurs—then a much larger force is needed, and it has a more complicated dependence on v). Second, F is proportional to the area A of the plate. This relationship seems reasonable, since A is directly proportional to the amount of fluid being moved. Third, F is inversely proportional to the distance between the plates L . This relationship is also reasonable; L is like a lever arm, and the greater the lever arm, the less force that is needed. Fourth, F is directly proportional to the *coefficient of viscosity*, η . The greater the viscosity, the greater the force required. These dependencies are combined into the equation

$$F = \eta \frac{vA}{L}, \quad 12.41$$

which gives us a working definition of fluid **viscosity** η . Solving for η gives

$$\eta = \frac{FL}{vA}, \quad 12.42$$

which defines viscosity in terms of how it is measured. The SI unit of viscosity is $\text{N} \cdot \text{m}/[(\text{m/s})\text{m}^2] = (\text{N}/\text{m}^2)\text{s}$ or $\text{Pa} \cdot \text{s}$. Table 12.1 lists the coefficients of viscosity for various fluids.

Viscosity varies from one fluid to another by several orders of magnitude. As you might expect, the viscosities of gases are much less than those of liquids, and these viscosities are often temperature dependent. The viscosity of blood can be reduced by aspirin consumption, allowing it to flow more easily around the body. (When used over the long term in low doses, aspirin can help prevent heart attacks, and reduce the risk of blood clotting.)

Laminar Flow Confined to Tubes—Poiseuille's Law

What causes flow? The answer, not surprisingly, is pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate Q is in the direction from high to low pressure. The greater the pressure differential between two points, the greater the flow rate. This relationship can be stated as

$$Q = \frac{P_2 - P_1}{R}, \quad 12.43$$

where P_1 and P_2 are the pressures at two points, such as at either end of a tube, and R is the resistance to flow. The resistance R includes everything, except pressure, that affects flow rate. For example, R is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of R . Turbulence greatly increases R , whereas increasing the diameter of a tube decreases R .

If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. Comparing frictionless flow in a tube to viscous flow, as in [Figure 12.13](#), we see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries. We can see the effect of viscosity in a Bunsen burner flame, even though the viscosity of natural gas is small.

The resistance R to laminar flow of an incompressible fluid having viscosity η through a horizontal tube of uniform radius r and length l , such as the one in [Figure 12.14](#), is given by

$$R = \frac{8\eta l}{\pi r^4}. \quad 12.44$$

This equation is called **Poiseuille's law for resistance** after the French scientist J. L. Poiseuille (1799–1869), who derived it in an attempt to understand the flow of blood, an often turbulent fluid.

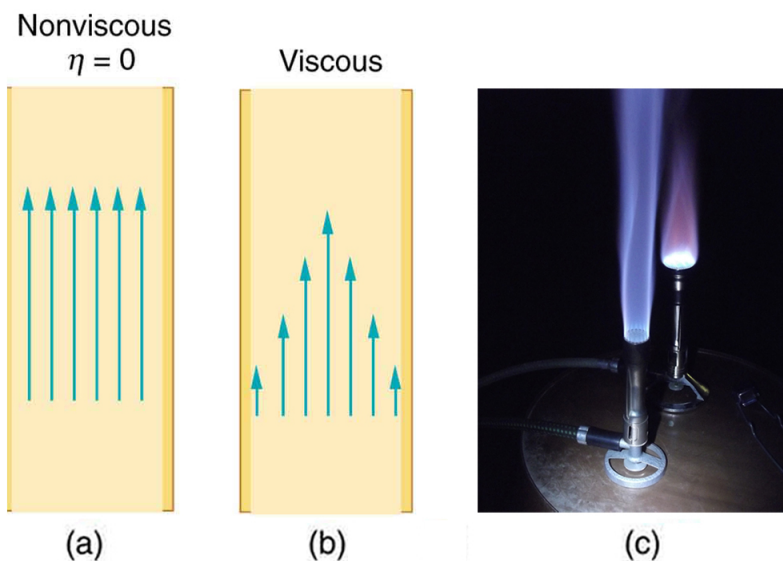


FIGURE 12.13 (a) If fluid flow in a tube has negligible resistance, the speed is the same all across the tube. (b) When a viscous fluid flows through a tube, its speed at the walls is zero, increasing steadily to its maximum at the center of the tube. (c) The shape of the Bunsen burner flame is due to the velocity profile across the tube. (credit: Jason Woodhead)

Let us examine Poiseuille's expression for R to see if it makes good intuitive sense. We see that resistance is directly proportional to both fluid viscosity η and the length l of a tube. After all, both of these directly affect the amount of friction encountered—the greater either is, the greater the resistance and the smaller the flow. The radius r of a tube affects the resistance, which again makes sense, because the greater the radius, the greater the flow (all other factors remaining the same). But it is surprising that r is raised to the *fourth* power in Poiseuille's law. This exponent means that any change in the radius of a tube has a very large effect on resistance. For example, doubling the radius of a tube decreases resistance by a factor of $2^4 = 16$.

Taken together, $Q = \frac{P_2 - P_1}{R}$ and $R = \frac{8\eta l}{\pi r^4}$ give the following expression for flow rate:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}. \quad 12.45$$

This equation describes laminar flow through a tube. It is sometimes called Poiseuille's law for laminar flow, or simply **Poiseuille's law**.



EXAMPLE 12.7

Using Flow Rate: Plaque Deposits Reduce Blood Flow

Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs?

Strategy

Assuming laminar flow, Poiseuille's law states that

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}. \quad 12.46$$

We need to compare the artery radius before and after the flow rate reduction.

Solution

With a constant pressure difference assumed and the same length and viscosity, along the artery we have

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}. \quad 12.47$$

So, given that $Q_2 = 0.5Q_1$, we find that $r_2^4 = 0.5r_1^4$.

Therefore, $r_2 = (0.5)^{0.25}r_1 = 0.841r_1$, a decrease in the artery radius of 16%.

Discussion

This decrease in radius is surprisingly small for this situation. To restore the blood flow in spite of this buildup would require an increase in the pressure difference ($P_2 - P_1$) of a factor of two, with subsequent strain on the heart.

Fluid	Temperature (°C)	Viscosity η (mPa·s)
Gases		
Air	0	0.0171
	20	0.0181
	40	0.0190
	100	0.0218
Ammonia	20	0.00974
Carbon dioxide	20	0.0147
Helium	20	0.0196
Hydrogen	0	0.0090
Mercury	20	0.0450
Oxygen	20	0.0203

TABLE 12.1 Coefficients of Viscosity of Various Fluids

Fluid	Temperature (°C)	Viscosity η (mPa·s)
Steam	100	0.0130
Liquids		
Water	0	1.792
	20	1.002
	37	0.6947
	40	0.653
	100	0.282
Whole blood ¹	20	3.015
	37	2.084
Blood plasma ²	20	1.810
	37	1.257
Ethyl alcohol	20	1.20
Methanol	20	0.584
Oil (heavy machine)	20	660
Oil (motor, SAE 10)	30	200
Oil (olive)	20	138
Glycerin	20	1500
Honey	20	2000–10000
Maple Syrup	20	2000–3000
Milk	20	3.0
Oil (Corn)	20	65

TABLE 12.1 Coefficients of Viscosity of Various Fluids

The circulatory system provides many examples of Poiseuille's law in action—with blood flow regulated by changes in vessel size and blood pressure. Blood vessels are not rigid but elastic. Adjustments to blood flow are primarily made by varying the size of the vessels, since the resistance is so sensitive to the radius. During vigorous exercise, blood vessels are selectively dilated to important muscles and organs and blood pressure increases. This creates both greater overall blood flow and increased flow to specific areas. Conversely, decreases in vessel radii, perhaps

1 The ratios of the viscosities of blood to water are nearly constant between 0°C and 37°C.

2 See note on Whole Blood.

from plaques in the arteries, can greatly reduce blood flow. If a vessel's radius is reduced by only 5% (to 0.95 of its original value), the flow rate is reduced to about $(0.95)^4 = 0.81$ of its original value. A 19% decrease in flow is caused by a 5% decrease in radius. The body may compensate by increasing blood pressure by 19%, but this presents hazards to the heart and any vessel that has weakened walls. Another example comes from automobile engine oil. If you have a car with an oil pressure gauge, you may notice that oil pressure is high when the engine is cold. Motor oil has greater viscosity when cold than when warm, and so pressure must be greater to pump the same amount of cold oil.

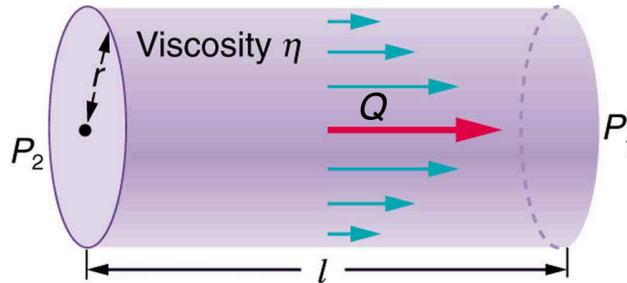


FIGURE 12.14 Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity η through a tube of length l and radius r . The direction of flow is from greater to lower pressure. Flow rate Q is directly proportional to the pressure difference $P_2 - P_1$, and inversely proportional to the length l of the tube and viscosity η of the fluid. Flow rate increases with r^4 , the fourth power of the radius.

EXAMPLE 12.8

What Pressure Produces This Flow Rate?

An intravenous (IV) system is supplying saline solution to a patient at the rate of $0.120 \text{ cm}^3/\text{s}$ through a needle of radius 0.150 mm and length 2.50 cm . What pressure is needed at the entrance of the needle to cause this flow, assuming the viscosity of the saline solution to be the same as that of water? The gauge pressure of the blood in the patient's vein is 8.00 mm Hg . (Assume that the temperature is 20°C .)

Strategy

Assuming laminar flow, Poiseuille's law applies. This is given by

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}, \quad 12.48$$

where P_2 is the pressure at the entrance of the needle and P_1 is the pressure in the vein. The only unknown is P_2 .

Solution

Solving for P_2 yields

$$P_2 = \frac{8\eta l}{\pi r^4} Q + P_1. \quad 12.49$$

P_1 is given as 8.00 mm Hg , which converts to $1.066 \times 10^3 \text{ N/m}^2$. Substituting this and the other known values yields

$$\begin{aligned} P_2 &= \left[\frac{8(1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2)(2.50 \times 10^{-2} \text{ m})}{\pi(0.150 \times 10^{-3} \text{ m})^4} \right] (1.20 \times 10^{-7} \text{ m}^3/\text{s}) + 1.066 \times 10^3 \text{ N/m}^2 \\ &= 1.62 \times 10^4 \text{ N/m}^2. \end{aligned} \quad 12.50$$

Discussion

This pressure could be supplied by an IV bottle with the surface of the saline solution 1.61 m above the entrance to the needle (this is left for you to solve in this chapter's Problems and Exercises), assuming that there is negligible pressure drop in the tubing leading to the needle.

Flow and Resistance as Causes of Pressure Drops

You may have noticed that water pressure in your home might be lower than normal on hot summer days when there is more use. This pressure drop occurs in the water main before it reaches your home. Let us consider flow through the water main as illustrated in [Figure 12.15](#). We can understand why the pressure P_1 to the home drops during times of heavy use by rearranging

$$Q = \frac{P_2 - P_1}{R} \quad 12.51$$

to

$$P_2 - P_1 = RQ, \quad 12.52$$

where, in this case, P_2 is the pressure at the water works and R is the resistance of the water main. During times of heavy use, the flow rate Q is large. This means that $P_2 - P_1$ must also be large. Thus P_1 must decrease. It is correct to think of flow and resistance as causing the pressure to drop from P_2 to P_1 . $P_2 - P_1 = RQ$ is valid for both laminar and turbulent flows.

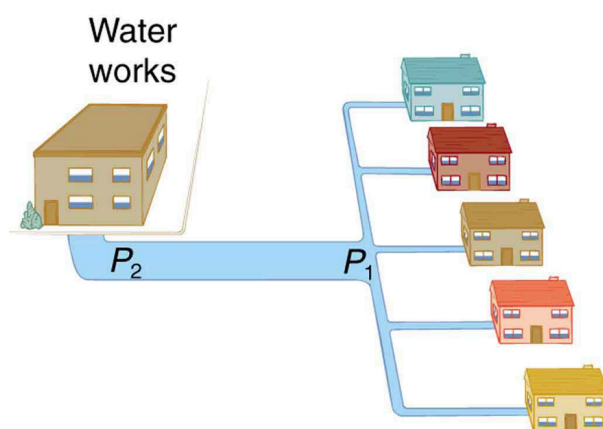


FIGURE 12.15 During times of heavy use, there is a significant pressure drop in a water main, and P_1 supplied to users is significantly less than P_2 created at the water works. If the flow is very small, then the pressure drop is negligible, and $P_2 \approx P_1$.

We can use $P_2 - P_1 = RQ$ to analyze pressure drops occurring in more complex systems in which the tube radius is not the same everywhere. Resistance will be much greater in narrow places, such as an obstructed coronary artery. For a given flow rate Q , the pressure drop will be greatest where the tube is most narrow. This is how water faucets control flow. Additionally, R is greatly increased by turbulence, and a constriction that creates turbulence greatly reduces the pressure downstream. Plaque in an artery reduces pressure and hence flow, both by its resistance and by the turbulence it creates.

[Figure 12.16](#) is a schematic of the human circulatory system, showing average blood pressures in its major parts for an adult at rest. Pressure created by the heart's two pumps, the right and left ventricles, is reduced by the resistance of the blood vessels as the blood flows through them. The left ventricle increases arterial blood pressure that drives the flow of blood through all parts of the body except the lungs. The right ventricle receives the lower pressure blood from two major veins and pumps it through the lungs for gas exchange with atmospheric gases – the disposal of carbon dioxide from the blood and the replenishment of oxygen. Only one major organ is shown schematically, with typical branching of arteries to ever smaller vessels, the smallest of which are the capillaries, and rejoining of small veins into larger ones. Similar branching takes place in a variety of organs in the body, and the circulatory system has considerable flexibility in flow regulation to these organs by the dilation and constriction of the arteries leading to them and the capillaries within them. The sensitivity of flow to tube radius makes this flexibility possible over a large range of flow rates.

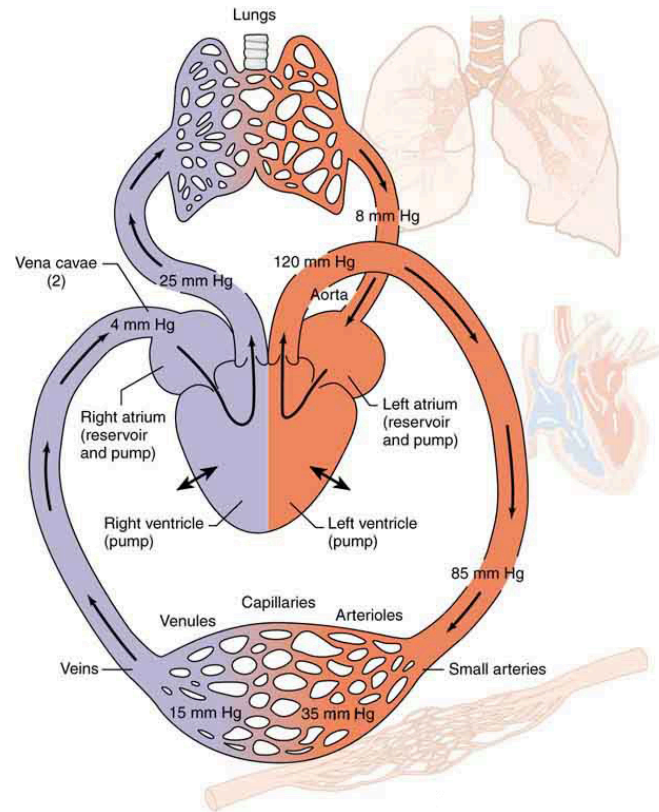


FIGURE 12.16 Schematic of the circulatory system. Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters.

Each branching of larger vessels into smaller vessels increases the total cross-sectional area of the tubes through which the blood flows. For example, an artery with a cross section of 1 cm^2 may branch into 20 smaller arteries, each with cross sections of 0.5 cm^2 , with a total of 10 cm^2 . In that manner, the resistance of the branchings is reduced so that pressure is not entirely lost. Moreover, because $Q = A\bar{v}$ and A increases through branching, the average velocity of the blood in the smaller vessels is reduced. The blood velocity in the aorta (diameter = 1 cm) is about 25 cm/s , while in the capillaries ($20 \mu\text{m}$ in diameter) the velocity is about 1 mm/s . This reduced velocity allows the blood to exchange substances with the cells in the capillaries and alveoli in particular.

12.5 The Onset of Turbulence

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate Reynolds number.
- Use the Reynolds number for a system to determine whether it is laminar or turbulent.

Sometimes we can predict if flow will be laminar or turbulent. We know that flow in a very smooth tube or around a smooth, streamlined object will be laminar at low velocity. We also know that at high velocity, even flow in a smooth tube or around a smooth object will experience turbulence. In between, it is more difficult to predict. In fact, at intermediate velocities, flow may oscillate back and forth indefinitely between laminar and turbulent.

An occlusion, or narrowing, of an artery, such as shown in [Figure 12.17](#), is likely to cause turbulence because of the irregularity of the blockage, as well as the complexity of blood as a fluid. Turbulence in the circulatory system is noisy and can sometimes be detected with a stethoscope, such as when measuring diastolic pressure in the upper arm's partially collapsed brachial artery. These turbulent sounds, at the onset of blood flow when the cuff pressure becomes sufficiently small, are called *Korotkoff sounds*. Aneurysms, or ballooning of arteries, create significant turbulence and can sometimes be detected with a stethoscope. Heart murmurs, consistent with their name, are sounds produced by turbulent flow around damaged and insufficiently closed heart valves. Ultrasound can also be

used to detect turbulence as a medical indicator in a process analogous to Doppler-shift radar used to detect storms.

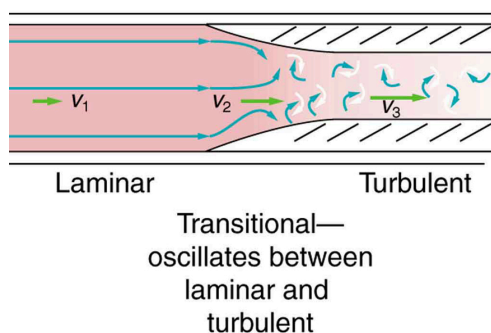


FIGURE 12.17 Flow is laminar in the large part of this blood vessel and turbulent in the part narrowed by plaque, where velocity is high. In the transition region, the flow can oscillate chaotically between laminar and turbulent flow.

An indicator called the **Reynolds number** N_R can reveal whether flow is laminar or turbulent. For flow in a tube of uniform diameter, the Reynolds number is defined as

$$N_R = \frac{2\rho v r}{\eta} (\text{flow in tube}), \quad 12.53$$

where ρ is the fluid density, v its speed, η its viscosity, and r the tube radius. The Reynolds number is a unitless quantity. Experiments have revealed that N_R is related to the onset of turbulence. For N_R below about 2000, flow is laminar. For N_R above about 3000, flow is turbulent. For values of N_R between about 2000 and 3000, flow is unstable—that is, it can be laminar, but small obstructions and surface roughness can make it turbulent, and it may oscillate randomly between being laminar and turbulent. The blood flow through most of the body is a quiet, laminar flow. The exception is in the aorta, where the speed of the blood flow rises above a critical value of 35 m/s and becomes turbulent.



EXAMPLE 12.9

Is This Flow Laminar or Turbulent?

Calculate the Reynolds number for flow in the needle considered in [Example 12.8](#) to verify the assumption that the flow is laminar. Assume that the density of the saline solution is 1025 kg/m^3 .

Strategy

We have all of the information needed, except the fluid speed v , which can be calculated from $\bar{v} = Q/A = 1.70 \text{ m/s}$ (verification of this is in this chapter's Problems and Exercises).

Solution

Entering the known values into $N_R = \frac{2\rho v r}{\eta}$ gives

$$\begin{aligned} N_R &= \frac{2\rho v r}{\eta} \\ &= \frac{2(1025 \text{ kg/m}^3)(1.70 \text{ m/s})(0.150 \times 10^{-3} \text{ m})}{1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2} \\ &= 523. \end{aligned} \quad 12.54$$

Discussion

Since N_R is well below 2000, the flow should indeed be laminar.

Take-Home Experiment: Inhalation

Under the conditions of normal activity, an adult inhales about 1 L of air during each inhalation. With the aid of a watch, determine the time for one of your own inhalations by timing several breaths and dividing the total length by the number of breaths. Calculate the average flow rate Q of air traveling through the trachea during each inhalation.

The topic of chaos has become quite popular over the last few decades. A system is defined to be *chaotic* when its behavior is so sensitive to some factor that it is extremely difficult to predict. The field of *chaos* is the study of chaotic behavior. A good example of chaotic behavior is the flow of a fluid with a Reynolds number between 2000 and 3000. Whether or not the flow is turbulent is difficult, but not impossible, to predict—the difficulty lies in the extremely sensitive dependence on factors like roughness and obstructions on the nature of the flow. A tiny variation in one factor has an exaggerated (or nonlinear) effect on the flow. Phenomena as disparate as turbulence, the orbit of Pluto, and the onset of irregular heartbeats are chaotic and can be analyzed with similar techniques.

12.6 Motion of an Object in a Viscous Fluid

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate the Reynolds number for an object moving through a fluid.
- Explain whether the Reynolds number indicates laminar or turbulent flow.
- Describe the conditions under which an object has a terminal speed.

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number N'_R , defined for an object moving in a fluid to be

$$N'_R = \frac{\rho v L}{\eta} (\text{object in fluid}), \quad 12.55$$

where L is a characteristic length of the object (a sphere's diameter, for example), ρ the fluid density, η its viscosity, and v the object's speed in the fluid. If N'_R is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for N'_R between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an N'_R between 10 and 10^6 , the flow may be either laminar or turbulent and may oscillate between the two. For N'_R greater than about 10^6 , the flow is entirely turbulent, even at the surface of the object. (See [Figure 12.18](#).) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.



EXAMPLE 12.10

Does a Ball Have a Turbulent Wake?

Calculate the Reynolds number N'_R for a ball with a 7.40-cm diameter thrown at 40.0 m/s.

Strategy

We can use $N'_R = \frac{\rho v L}{\eta}$ to calculate N'_R , since all values in it are either given or can be found in tables of density and viscosity.

Solution

Substituting values into the equation for N'_R yields

$$\begin{aligned}
 N'_R &= \frac{\rho v L}{\eta} = \frac{(1.29 \text{ kg/m}^3)(40.0 \text{ m/s})(0.0740 \text{ m})}{1.81 \times 10^{-5} \text{ Pa}\cdot\text{s}} \\
 &= 2.11 \times 10^5.
 \end{aligned}
 \tag{12.56}$$

Discussion

This value is sufficiently high to imply a turbulent wake. Most large objects, such as airplanes and sailboats, create significant turbulence as they move. As noted before, the Bernoulli principle gives only qualitatively-correct results in such situations.

One of the consequences of viscosity is a resistance force called **viscous drag** F_V that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow (N'_R less than about one) viscous drag is proportional to speed, whereas for N'_R between about 10 and 10^6 , viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For N'_R greater than 10^6 , drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere, F_V is proportional to fluid viscosity η , the object's characteristic size L , and its speed v . All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law $F_S = 6\pi r\eta v$. For the special case of a small sphere of radius R moving slowly in a fluid of viscosity η , the drag force F_S is given by

$$F_S = 6\pi R\eta v. \tag{12.57}$$

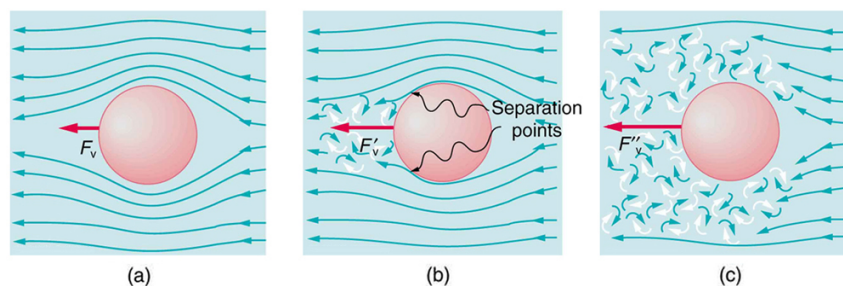


FIGURE 12.18 (a) Motion of this sphere to the right is equivalent to fluid flow to the left. Here the flow is laminar with N'_R less than 1. There is a force, called viscous drag F_V , to the left on the ball due to the fluid's viscosity. (b) At a higher speed, the flow becomes partially turbulent, creating a wake starting where the flow lines separate from the surface. Pressure in the wake is less than in front of the sphere, because fluid speed is less, creating a net force to the left F'_V that is significantly greater than for laminar flow. Here N'_R is greater than 10. (c) At much higher speeds, where N'_R is greater than 10^6 , flow becomes turbulent everywhere on the surface and behind the sphere. Drag increases dramatically.

An interesting consequence of the increase in F_V with speed is that an object falling through a fluid will not continue to accelerate indefinitely (as it would if we neglect air resistance, for example). Instead, viscous drag increases, slowing acceleration, until a critical speed, called the **terminal speed**, is reached and the acceleration of the object becomes zero. Once this happens, the object continues to fall at constant speed (the terminal speed). This is the case for particles of sand falling in the ocean, cells falling in a centrifuge, and sky divers falling through the air. [Figure 12.19](#) shows some of the factors that affect terminal speed. There is a viscous drag on the object that depends on the viscosity of the fluid and the size of the object. But there is also a buoyant force that depends on the density of the object relative to the fluid. Terminal speed will be greatest for low-viscosity fluids and objects with high densities and small sizes. Thus a skydiver falls more slowly with outspread limbs than when they are in a pike position—head first with hands at their side and legs together.

Take-Home Experiment: Don't Lose Your Marbles

By measuring the terminal speed of a slowly moving sphere in a viscous fluid, one can find the viscosity of that fluid (at that temperature). It can be difficult to find small ball bearings around the house, but a small marble will do. Gather two or three fluids (syrup, motor oil, honey, olive oil, etc.) and a thick, tall clear glass or vase. Drop the marble into the center of the fluid and time its fall (after letting it drop a little to reach its terminal speed).

Compare your values for the terminal speed and see if they are inversely proportional to the viscosities as listed in [Table 12.1](#). Does it make a difference if the marble is dropped near the side of the glass?

Knowledge of terminal speed is useful for estimating sedimentation rates of small particles. We know from watching mud settle out of dirty water that sedimentation is usually a slow process. Centrifuges are used to speed sedimentation by creating accelerated frames in which gravitational acceleration is replaced by centripetal acceleration, which can be much greater, increasing the terminal speed.

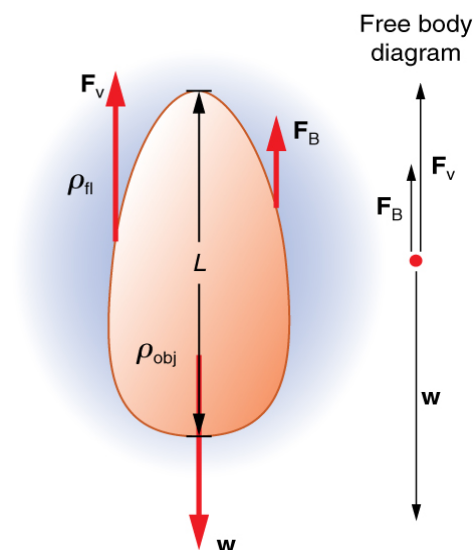


FIGURE 12.19 There are three forces acting on an object falling through a viscous fluid: its weight w , the viscous drag F_v , and the buoyant force F_B .

12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define diffusion, osmosis, dialysis, and active transport.
- Calculate diffusion rates.

Diffusion

There is something fishy about the ice cube from your freezer—how did it pick up those food odors? How does soaking a sprained ankle in Epsom salt reduce swelling? The answer to these questions are related to atomic and molecular transport phenomena—another mode of fluid motion. Atoms and molecules are in constant motion at any temperature. In fluids they move about randomly even in the absence of macroscopic flow. This motion is called a random walk and is illustrated in [Figure 12.20](#). **Diffusion** is the movement of substances due to random thermal molecular motion. Fluids, like fish fumes or odors entering ice cubes, can even diffuse through solids.

Diffusion is a slow process over macroscopic distances. The densities of common materials are great enough that molecules cannot travel very far before having a collision that can scatter them in any direction, including straight backward. It can be shown that the average distance x_{rms} that a molecule travels is proportional to the square root of time:

$$x_{\text{rms}} = \sqrt{2Dt}, \quad 12.58$$

where x_{rms} stands for the **root-mean-square distance** and is the statistical average for the process. The quantity D is the diffusion constant for the particular molecule in a specific medium. [Table 12.2](#) lists representative values of D for various substances, in units of m^2/s .

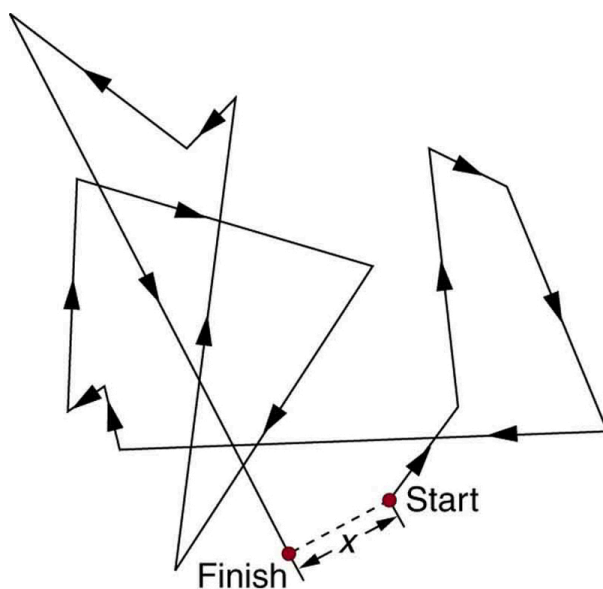


FIGURE 12.20 The random thermal motion of a molecule in a fluid in time t . This type of motion is called a random walk.

Diffusing molecule	Medium	D (m ² /s)
Hydrogen (H ₂)	Air	6.4×10^{-5}
Oxygen (O ₂)	Air	1.8×10^{-5}
Oxygen (O ₂)	Water	1.0×10^{-9}
Glucose (C ₆ H ₁₂ O ₆)	Water	6.7×10^{-10}
Hemoglobin	Water	6.9×10^{-11}
DNA	Water	1.3×10^{-12}

TABLE 12.2 Diffusion Constants for Various Molecules³

Note that D gets progressively smaller for more massive molecules. This decrease is because the average molecular speed at a given temperature is inversely proportional to molecular mass. Thus the more massive molecules diffuse more slowly. Another interesting point is that D for oxygen in air is much greater than D for oxygen in water. In water, an oxygen molecule makes many more collisions in its random walk and is slowed considerably. In water, an oxygen molecule moves only about $40 \mu\text{m}$ in 1 s. (Each molecule actually collides about 10^{10} times per second!). Finally, note that diffusion constants increase with temperature, because average molecular speed increases with temperature. This is because the average kinetic energy of molecules, $\frac{1}{2}mv^2$, is proportional to absolute temperature.



EXAMPLE 12.11

Calculating Diffusion: How Long Does Glucose Diffusion Take?

Calculate the average time it takes a glucose molecule to move 1.0 cm in water.

³ At 20°C and 1 atm

Strategy

We can use $x_{\text{rms}} = \sqrt{2Dt}$, the expression for the average distance moved in time t , and solve it for t . All other quantities are known.

Solution

Solving for t and substituting known values yields

$$\begin{aligned} t &= \frac{x_{\text{rms}}^2}{2D} = \frac{(0.010 \text{ m})^2}{2(6.7 \times 10^{-10} \text{ m}^2/\text{s})} \\ &= 7.5 \times 10^4 \text{ s} = 21 \text{ h.} \end{aligned} \quad 12.59$$

Discussion

This is a remarkably long time for glucose to move a mere centimeter! For this reason, we stir sugar into water rather than waiting for it to diffuse.

Because diffusion is typically very slow, its most important effects occur over small distances. For example, the cornea of the eye gets most of its oxygen by diffusion through the thin tear layer covering it.

The Rate and Direction of Diffusion

If you very carefully place a drop of food coloring in a still glass of water, it will slowly diffuse into the colorless surroundings until its concentration is the same everywhere. This type of diffusion is called free diffusion, because there are no barriers inhibiting it. Let us examine its direction and rate. Molecular motion is random in direction, and so simple chance dictates that more molecules will move out of a region of high concentration than into it. The net rate of diffusion is higher initially than after the process is partially completed. (See [Figure 12.21](#).)

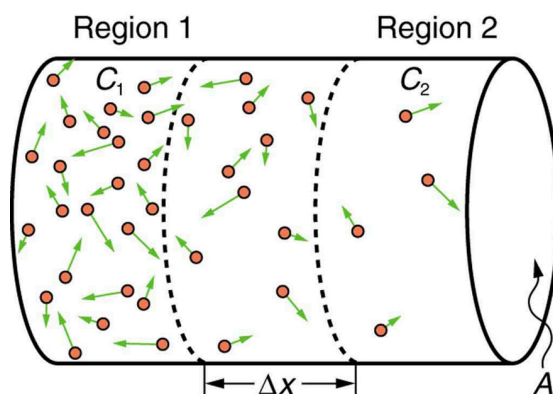


FIGURE 12.21 Diffusion proceeds from a region of higher concentration to a lower one. The net rate of movement is proportional to the difference in concentration.

The net rate of diffusion is proportional to the concentration difference. Many more molecules will leave a region of high concentration than will enter it from a region of low concentration. In fact, if the concentrations were the same, there would be *no* net movement. The net rate of diffusion is also proportional to the diffusion constant D , which is determined experimentally. The farther a molecule can diffuse in a given time, the more likely it is to leave the region of high concentration. Many of the factors that affect the rate are hidden in the diffusion constant D . For example, temperature and cohesive and adhesive forces all affect values of D .

Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger, they needed quicker methods of transportation than net diffusion, because of the larger distances involved in the transport, leading to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Osmosis and Dialysis—Diffusion across Membranes

Some of the most interesting examples of diffusion occur through barriers that affect the rates of diffusion. For example, when you soak a swollen ankle in Epsom salt, water diffuses through your skin. Many substances regularly move through cell membranes; oxygen moves in, carbon dioxide moves out, nutrients go in, and wastes go out, for example. Because membranes are thin structures (typically 6.5×10^{-9} to 10×10^{-9} m across) diffusion rates through them can be high. Diffusion through membranes is an important method of transport.

Membranes are generally selectively permeable, or **semipermeable**. (See [Figure 12.22](#).) One type of semipermeable membrane has small pores that allow only small molecules to pass through. In other types of membranes, the molecules may actually dissolve in the membrane or react with molecules in the membrane while moving across. Membrane function, in fact, is the subject of much current research, involving not only physiology but also chemistry and physics.

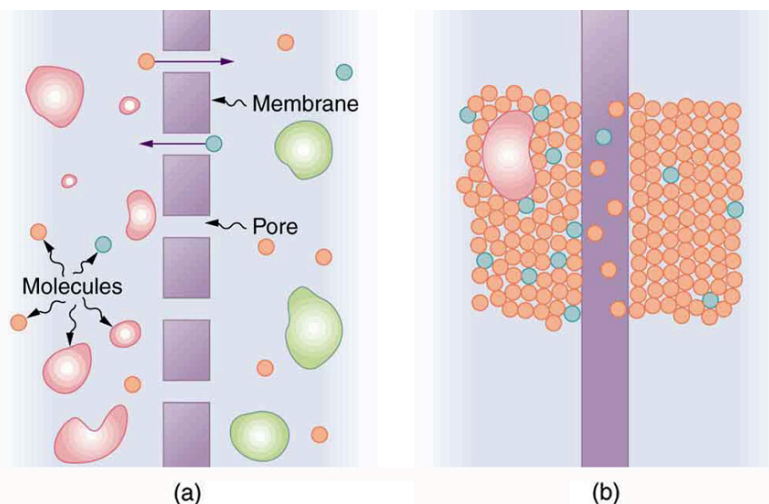


FIGURE 12.22 (a) A semipermeable membrane with small pores that allow only small molecules to pass through. (b) Certain molecules dissolve in this membrane and diffuse across it.

Osmosis is the transport of water through a semipermeable membrane from a region of high concentration to a region of low concentration. Osmosis is driven by the imbalance in water concentration. For example, water is more concentrated in your body than in Epsom salt. When you soak a swollen ankle in Epsom salt, the water moves out of your body into the lower-concentration region in the salt. Similarly, **dialysis** is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood.

Osmosis can create a substantial pressure. Consider what happens if osmosis continues for some time, as illustrated in [Figure 12.23](#). Water moves by osmosis from the left into the region on the right, where it is less concentrated, causing the solution on the right to rise. This movement will continue until the pressure ρgh created by the extra height of fluid on the right is large enough to stop further osmosis. This pressure is called a **back pressure**. The back pressure ρgh that stops osmosis is also called the **relative osmotic pressure** if neither solution is pure water, and it is called the **osmotic pressure** if one solution is pure water. Osmotic pressure can be large, depending on the size of the concentration difference. For example, if pure water and sea water are separated by a semipermeable membrane that passes no salt, osmotic pressure will be 25.9 atm. This value means that water will diffuse through the membrane until the salt water surface rises 268 m above the pure-water surface! One example of pressure created by osmosis is turgor in plants (many wilt when too dry). Turgor describes the condition of a plant in which the fluid in a cell exerts a pressure against the cell wall. This pressure gives the plant support. Dialysis can similarly cause substantial pressures.

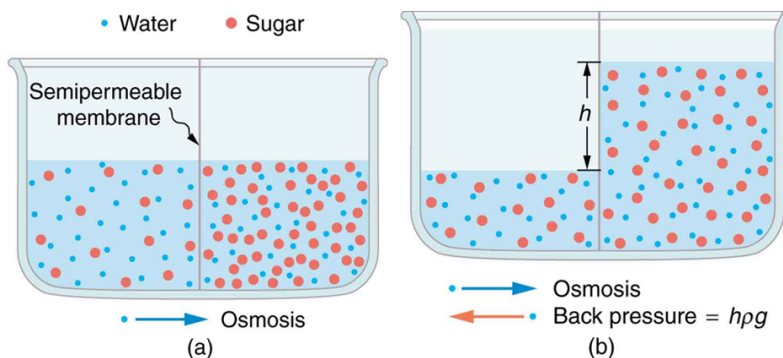


FIGURE 12.23 (a) Two sugar-water solutions of different concentrations, separated by a semipermeable membrane that passes water but not sugar. Osmosis will be to the right, since water is less concentrated there. (b) The fluid level rises until the back pressure ρgh equals the relative osmotic pressure; then, the net transfer of water is zero.

Reverse osmosis and **reverse dialysis** (also called filtration) are processes that occur when back pressure is sufficient to reverse the normal direction of substances through membranes. Back pressure can be created naturally as on the right side of [Figure 12.23](#). (A piston can also create this pressure.) Reverse osmosis can be used to desalinate water by simply forcing it through a membrane that will not pass salt. Similarly, reverse dialysis can be used to filter out any substance that a given membrane will not pass.

One further example of the movement of substances through membranes deserves mention. We sometimes find that substances pass in the direction opposite to what we expect. Cypress tree roots, for example, extract pure water from salt water, although osmosis would move it in the opposite direction. This is not reverse osmosis, because there is no back pressure to cause it. What is happening is called **active transport**, a process in which a living membrane expends energy to move substances across it. Many living membranes move water and other substances by active transport. The kidneys, for example, not only use osmosis and dialysis—they also employ significant active transport to move substances into and out of blood. In fact, it is estimated that at least 25% of the body's energy is expended on active transport of substances at the cellular level. The study of active transport carries us into the realms of microbiology, biophysics, and biochemistry and it is a fascinating application of the laws of nature to living structures.

Glossary

active transport the process in which a living membrane expends energy to move substances across

Bernoulli's equation the equation resulting from applying conservation of energy to an incompressible frictionless fluid: $P + 1/2\rho v^2 + \rho gh = \text{constant}$, through the fluid

Bernoulli's principle Bernoulli's equation applied at constant depth: $P_1 + 1/2\rho v_1^2 = P_2 + 1/2\rho v_2^2$

dialysis the transport of any molecule other than water through a semipermeable membrane from a region of high concentration to one of low concentration

diffusion the movement of substances due to random thermal molecular motion

flow rate abbreviated Q , it is the volume V that flows past a particular point during a time t , or $Q = V/t$

fluid dynamics the physics of fluids in motion

laminar a type of fluid flow in which layers do not mix

liter a unit of volume, equal to 10^{-3} m^3

osmosis the transport of water through a semipermeable membrane from a region of high concentration to one of low concentration

osmotic pressure the back pressure which stops the osmotic process if one solution is pure water

Poiseuille's law the rate of laminar flow of an incompressible fluid in a tube: $Q = (P_2 - P_1)\pi r^4/8\eta l$

Poiseuille's law for resistance the resistance to laminar flow of an incompressible fluid in a tube: $R = 8\eta l/\pi r^4$

relative osmotic pressure the back pressure which stops the osmotic process if neither solution is pure water

reverse dialysis the process that occurs when back pressure is sufficient to reverse the normal direction of dialysis through membranes

reverse osmosis the process that occurs when back pressure is sufficient to reverse the normal direction of osmosis through membranes

Reynolds number a dimensionless parameter that can reveal whether a particular flow is laminar or turbulent

semipermeable a type of membrane that allows only certain small molecules to pass through

terminal speed the speed at which the viscous drag of an object falling in a viscous fluid is equal to the other forces acting on the object (such as gravity), so that the acceleration of the object is zero

turbulence fluid flow in which layers mix together via eddies and swirls

viscosity the friction in a fluid, defined in terms of the friction between layers

viscous drag a resistance force exerted on a moving object, with a nontrivial dependence on velocity

Section Summary

12.1 Flow Rate and Its Relation to Velocity

- Flow rate Q is defined to be the volume V flowing past a point in time t , or $Q = \frac{V}{t}$ where V is volume and t is time.
- The SI unit of volume is m^3 .
- Another common unit is the liter (L), which is 10^{-3} m^3 .
- Flow rate and velocity are related by $Q = A\bar{v}$ where A is the cross-sectional area of the flow and $\bar{v}$ is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

$$\left. \begin{aligned} Q_1 &= Q_2 \\ A_1 \bar{v}_1 &= A_2 \bar{v}_2 \\ n_1 A_1 \bar{v}_1 &= n_2 A_2 \bar{v}_2 \end{aligned} \right\}.$$

12.2 Bernoulli's Equation

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2.$$

- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height h) subtract out, yielding $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$.
- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

12.3 The Most General Applications of Bernoulli's Equation

- Power in fluid flow is given by the equation $(P_1 + \frac{1}{2}\rho v^2 + \rho gh)Q = \text{power}$, where the first term is power associated with pressure, the second is power associated with velocity, and the third is power associated with height.

12.4 Viscosity and Laminar Flow: Poiseuille's Law

- Laminar flow is characterized by smooth flow of

the fluid in layers that do not mix.

- Turbulence is characterized by eddies and swirls that mix layers of fluid together.
- Fluid viscosity η is due to friction within a fluid. Representative values are given in [Table 12.1](#). Viscosity has units of $(\text{N/m}^2)\cdot\text{s}$ or $\text{Pa}\cdot\text{s}$.
- Flow is proportional to pressure difference and inversely proportional to resistance:

$$Q = \frac{P_2 - P_1}{R}.$$

- For laminar flow in a tube, Poiseuille's law for resistance states that

$$R = \frac{8\eta l}{\pi r^4}.$$

- Poiseuille's law for flow in a tube is

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

- The pressure drop caused by flow and resistance is given by

$$P_2 - P_1 = RQ.$$

12.5 The Onset of Turbulence

- The Reynolds number N_R can reveal whether flow is laminar or turbulent. It is

$$N_R = \frac{2\rho v r}{\eta}.$$

- For N_R below about 2000, flow is laminar. For N_R above about 3000, flow is turbulent. For values of N_R between 2000 and 3000, it may be either or both.

Conceptual Questions

12.1 Flow Rate and Its Relation to Velocity

1. What is the difference between flow rate and fluid velocity? How are they related?
2. Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)
3. Identify some substances that are incompressible and some that are not.

12.2 Bernoulli's Equation

4. You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.

12.6 Motion of an Object in a Viscous Fluid

- When an object moves in a fluid, there is a different form of the Reynolds number $N'_R = \frac{\rho v L}{\eta}$ (object in fluid), which indicates whether flow is laminar or turbulent.
- For N'_R less than about one, flow is laminar.
- For N'_R greater than 10^6 , flow is entirely turbulent.

12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

- Diffusion is the movement of substances due to random thermal molecular motion.
- The average distance x_{rms} a molecule travels by diffusion in a given amount of time is given by $x_{\text{rms}} = \sqrt{2Dt}$, where D is the diffusion constant, representative values of which are found in [Table 12.2](#).
- Osmosis is the transport of water through a semipermeable membrane from a region of high concentration to a region of low concentration.
- Dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference.
- Both processes can be reversed by back pressure.
- Active transport is a process in which a living membrane expends energy to move substances across it.

5. Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.
6. Look back to [Figure 12.4](#). Answer the following two questions. Why is P_0 less than atmospheric? Why is P_0 greater than P_1 ?
7. Give an example of entrainment not mentioned in the text.
8. Many entrainment devices have a constriction, called a Venturi, such as shown in [Figure 12.24](#). How does this bolster entrainment?

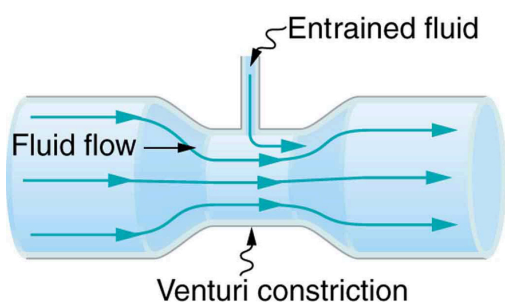


FIGURE 12.24 A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators.

9. Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.
10. Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.
11. Why is it preferable for airplanes to take off into the wind rather than with the wind?
12. Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.
13. Why does a sailboat need a keel?
14. It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.
15. Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

16. A perfume bottle or atomizer sprays a fluid that is in the bottle. ([Figure 12.25](#).) How does the fluid rise up in the vertical tube in the bottle?



FIGURE 12.25 Atomizer: perfume bottle with tube to carry perfume up through the bottle. (credit: Antonia Foy, Flickr)

17. If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

[12.3 The Most General Applications of Bernoulli's Equation](#)

18. Based on Bernoulli's equation, what are three forms of energy in a fluid? (Note that these forms are conservative, unlike heat transfer and other dissipative forms not included in Bernoulli's equation.)
19. Water that has emerged from a hose into the atmosphere has a gauge pressure of zero. Why? When you put your hand in front of the emerging stream you feel a force, yet the water's gauge pressure is zero. Explain where the force comes from in terms of energy.

20. The old rubber boot shown in [Figure 12.26](#) has two leaks. To what maximum height can the water squirt from Leak 1? How does the velocity of water emerging from Leak 2 differ from that of leak 1? Explain your responses in terms of energy.



FIGURE 12.26 Water emerges from two leaks in an old boot.

21. Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

12.4 Viscosity and Laminar Flow: Poiseuille's Law

22. Explain why the viscosity of a liquid decreases with temperature—that is, how might increased temperature reduce the effects of cohesive forces in a liquid? Also explain why the viscosity of a gas increases with temperature—that is, how does increased gas temperature create more collisions between atoms and molecules?
23. When paddling a canoe upstream, it is wisest to travel as near to the shore as possible. When canoeing downstream, it may be best to stay near the middle. Explain why.
24. Why does flow decrease in your shower when someone flushes the toilet?

25. Plumbing usually includes air-filled tubes near water faucets, as shown in [Figure 12.27](#). Explain why they are needed and how they work.

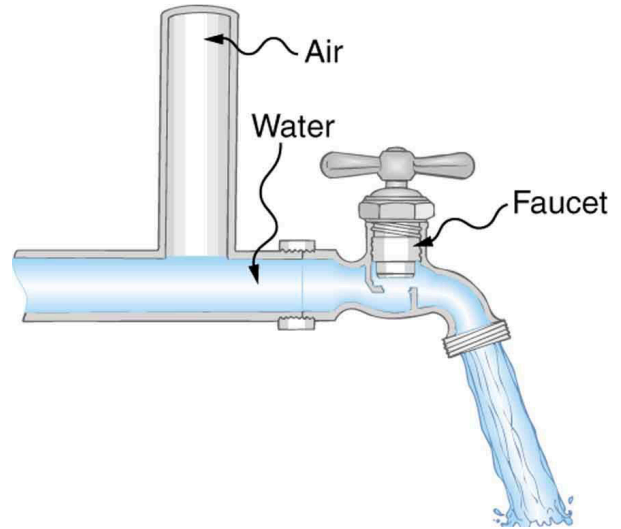


FIGURE 12.27 The vertical tube near the water tap remains full of air and serves a useful purpose.

12.5 The Onset of Turbulence

26. Doppler ultrasound can be used to measure the speed of blood in the body. If there is a partial constriction of an artery, where would you expect blood speed to be greatest, at or nearby the constriction? What are the two distinct causes of higher resistance in the constriction?
27. Sink drains often have a device such as that shown in [Figure 12.28](#) to help speed the flow of water. How does this work?

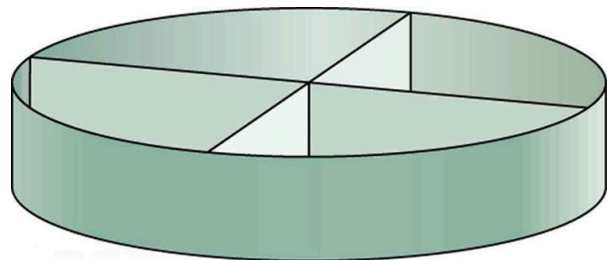


FIGURE 12.28 You will find devices such as this in many drains. They significantly increase flow rate.

28. Some ceiling fans have decorative wicker reeds on their blades. Discuss whether these fans are as quiet and efficient as those with smooth blades.

12.6 Motion of an Object in a Viscous Fluid

29. What direction will a helium balloon move inside a car that is slowing down—toward the front or back? Explain your answer.

30. Will identical raindrops fall more rapidly in 5°C air or 25°C air, neglecting any differences in air density? Explain your answer.
31. If you took two marbles of different sizes, what would you expect to observe about the relative magnitudes of their terminal velocities?

Problems & Exercises

12.1 Flow Rate and Its Relation to Velocity

1. What is the average flow rate in cm^3/s of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?
2. The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to cm^3/s . (b) What is this rate in m^3/s ?
3. Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.
4. Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.
5. The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see Figure 12.29). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

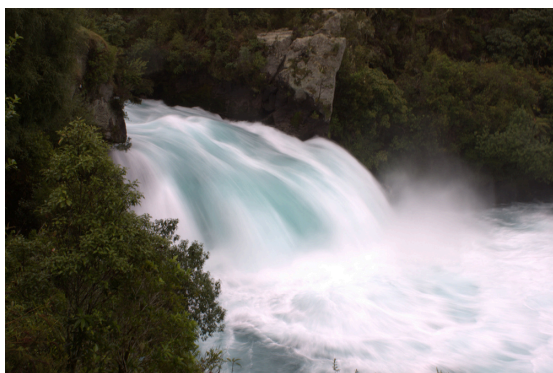


FIGURE 12.29 The Huka Falls in Taupo, New Zealand, demonstrate flow rate. (credit: RaviGogna, Flickr)

6. A major artery with a cross-sectional area of 1.00 cm^2 branches into 18 smaller arteries, each with an average cross-sectional area of 0.400 cm^2 . By what factor is the average velocity of the blood reduced when it passes into these branches?

12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

32. Why would you expect the rate of diffusion to increase with temperature? Can you give an example, such as the fact that you can dissolve sugar more rapidly in hot water?
 33. How are osmosis and dialysis similar? How do they differ?
7. (a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is 10.0 cm^2 , what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0\text{ }\mu\text{m}$?
 8. The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8\text{ }\mu\text{m}$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.
 9. (a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at $5000\text{ m}^3/\text{s}$, into it?
 10. The flow rate of blood through a $2.00 \times 10^{-6}\text{-m}$ -radius capillary is $3.80 \times 10^{-9}\text{ cm}^3/\text{s}$. (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0\text{ cm}^3/\text{s}$? (The large number obtained is an overestimate, but it is still reasonable.)
 11. (a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?
 12. The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

- 13.** Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?
- 14.** Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)
- 15.** Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in cm^3/s ? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.
- 16. Unreasonable Results**
A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches $100,000 \text{ m}^3/\text{s}$. (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?
- 21.** Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of 220 m^2 ? Typical air density in Boulder is 1.14 kg/m^3 , and the corresponding atmospheric pressure is $8.89 \times 10^4 \text{ N/m}^2$. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)
- 22.** (a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be 1.29 kg/m^3 . (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.
- 23.** (a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)
- 24.** (a) Using Bernoulli's equation, show that the measured fluid speed v for a pitot tube, like the one in [Figure 12.7\(b\)](#), is given by $v = \left(\frac{2\rho'gh}{\rho} \right)^{1/2}$, where h is the height of the manometer fluid, ρ' is the density of the manometer fluid, ρ is the density of the moving fluid, and g is the acceleration due to gravity. (Note that v is indeed proportional to the square root of h , as stated in the text.) (b) Calculate v for moving air if a mercury manometer's h is 0.200 m.

12.2 Bernoulli's Equation

- 17.** Verify that pressure has units of energy per unit volume.
- 18.** Suppose you have a wind speed gauge like the pitot tube shown in [Example 12.2\(b\)](#). By what factor must wind speed increase to double the value of h in the manometer? Is this independent of the moving fluid and the fluid in the manometer?
- 19.** If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?
- 20.** Calculate the maximum height to which water could be squirted with the hose in [Example 12.2](#) example if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate.

12.3 The Most General Applications of Bernoulli's Equation

- 25.** Hoover Dam on the Colorado River is the highest dam in the United States at 221 m, with an output of 1300 MW. The dam generates electricity with water taken from a depth of 150 m and an average flow rate of $650 \text{ m}^3/\text{s}$. (a) Calculate the power in this flow. (b) What is the ratio of this power to the facility's average of 680 MW?

- 26.** A frequently quoted rule of thumb in aircraft design is that wings should produce about 1000 N of lift per square meter of wing. (The fact that a wing has a top and bottom surface does not double its area.) (a) At takeoff, an aircraft travels at 60.0 m/s, so that the air speed relative to the bottom of the wing is 60.0 m/s. Given the sea level density of air to be 1.29 kg/m^3 , how fast must it move over the upper surface to create the ideal lift? (b) How fast must air move over the upper surface at a cruising speed of 245 m/s and at an altitude where air density is one-fourth that at sea level? (Note that this is not all of the aircraft's lift—some comes from the body of the plane, some from engine thrust, and so on. Furthermore, Bernoulli's principle gives an approximate answer because flow over the wing creates turbulence.)
- 27.** The left ventricle of a resting adult's heart pumps blood at a flow rate of $83.0 \text{ cm}^3/\text{s}$, increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.
- 28.** A sump pump (used to drain water from the basement of houses built below the water table) is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of $3.00 \times 10^5 \text{ N/m}^2$. (a) The water enters a hose with a 3.00-cm inside diameter and rises 2.50 m above the pump. What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.
- 31.** A glucose solution being administered with an IV has a flow rate of $4.00 \text{ cm}^3/\text{min}$. What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.
- 32.** The pressure drop along a length of artery is 100 Pa, the radius is 10 mm, and the flow is laminar. The average speed of the blood is 15 mm/s. (a) What is the net force on the blood in this section of artery? (b) What is the power expended maintaining the flow?
- 33.** A small artery has a length of $1.1 \times 10^{-3} \text{ m}$ and a radius of $2.5 \times 10^{-5} \text{ m}$. If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is 37°C .)
- 34.** Fluid originally flows through a tube at a rate of $100 \text{ cm}^3/\text{s}$. To illustrate the sensitivity of flow rate to various factors, calculate the new flow rate for the following changes with all other factors remaining the same as in the original conditions. (a) Pressure difference increases by a factor of 1.50. (b) A new fluid with 3.00 times greater viscosity is substituted. (c) The tube is replaced by one having 4.00 times the length. (d) Another tube is used with a radius 0.100 times the original. (e) Yet another tube is substituted with a radius 0.100 times the original and half the length, and the pressure difference is increased by a factor of 1.50.
- 35.** The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.
- 36.** Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?
- 37.** (a) Suppose a blood vessel's radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase? (b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?

12.4 Viscosity and Laminar Flow: Poiseuille's Law

- 29.** (a) Calculate the retarding force due to the viscosity of the air layer between a cart and a level air track given the following information—air temperature is 20°C , the cart is moving at 0.400 m/s, its surface area is $2.50 \times 10^{-2} \text{ m}^2$, and the thickness of the air layer is $6.00 \times 10^{-5} \text{ m}$. (b) What is the ratio of this force to the weight of the 0.300-kg cart?
- 30.** What force is needed to pull one microscope slide over another at a speed of 1.00 cm/s, if there is a 0.500-mm-thick layer of 20°C water between them and the contact area is 8.00 cm^2 ?

38. A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law, $F_s = 6\pi r\eta v$. Show that the terminal speed is given by
- $$v = \frac{2R^2g}{9\eta}(\rho_s - \rho_l),$$
- where R is the radius of the sphere, ρ_s is its density, and ρ_l is the density of the fluid and η the coefficient of viscosity.
39. Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.
40. A skydiver will reach a terminal velocity when the air drag equals their weight. For a skydiver with high speed and a large body, turbulence is a factor. The drag force then is approximately proportional to the square of the velocity. Taking the drag force to be $F_D = \frac{1}{2}\rho A v^2$ and setting this equal to the person's weight, find the terminal speed for a person falling "spread eagle." Find both a formula and a number for v_t , with assumptions as to size.
41. A layer of oil 1.50 mm thick is placed between two microscope slides. Researchers find that a force of 5.50×10^{-4} N is required to glide one over the other at a speed of 1.00 cm/s when their contact area is 6.00 cm^2 . What is the oil's viscosity? What type of oil might it be?
42. (a) Verify that a 19.0% decrease in laminar flow through a tube is caused by a 5.00% decrease in radius, assuming that all other factors remain constant, as stated in the text. (b) What increase in flow is obtained from a 5.00% increase in radius, again assuming all other factors remain constant?
43. [Example 12.8](#) dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of $1.62 \times 10^4 \text{ N/m}^2$ is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)
44. When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?
45. During a marathon race, a runner's blood flow increases to 10.0 times her resting rate. Her blood's viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?
46. Water supplied to a house by a water main has a pressure of $3.00 \times 10^5 \text{ N/m}^2$ early on a summer day when neighborhood use is low. This pressure produces a flow of 20.0 L/min through a garden hose. Later in the day, pressure at the exit of the water main and entrance to the house drops, and a flow of only 8.00 L/min is obtained through the same hose. (a) What pressure is now being supplied to the house, assuming resistance is constant? (b) By what factor did the flow rate in the water main increase in order to cause this decrease in delivered pressure? The pressure at the entrance of the water main is $5.00 \times 10^5 \text{ N/m}^2$, and the original flow rate was 200 L/min. (c) How many more users are there, assuming each would consume 20.0 L/min in the morning?
47. An oil gusher shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. Neglecting air resistance but not the resistance of the pipe, and assuming laminar flow, calculate the gauge pressure at the entrance of the 50.0-m-long vertical pipe. Take the density of the oil to be 900 kg/m^3 and its viscosity to be $1.00 (\text{N/m}^2) \cdot \text{s}$ (or $1.00 \text{ Pa} \cdot \text{s}$). Note that you must take into account the pressure due to the 50.0-m column of oil in the pipe.
48. Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is $8.00 \times 10^6 \text{ N/m}^2$. (a) Calculate the resistance of the hose. (b) What is the viscosity of the concrete, assuming the flow is laminar? (c) How much power is being supplied, assuming the point of use is at the same level as the pump? You may neglect the power supplied to increase the concrete's velocity.
49. **Construct Your Own Problem**
Consider a coronary artery constricted by arteriosclerosis. Construct a problem in which you calculate the amount by which the diameter of the artery is decreased, based on an assessment of the decrease in flow rate.

50. Consider a river that spreads out in a delta region on its way to the sea. Construct a problem in which you calculate the average speed at which water moves in the delta region, based on the speed at which it was moving up river. Among the things to consider are the size and flow rate of the river before it spreads out and its size once it has spread out. You can construct the problem for the river spreading out into one large river or into multiple smaller rivers.

12.5 The Onset of Turbulence

51. Verify that the flow of oil is laminar (barely) for an oil gusher that shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. The vertical pipe is 50 m long. Take the density of the oil to be 900 kg/m^3 and its viscosity to be $1.00 \text{ (N/m}^2\text{)} \cdot \text{s}$ (or $1.00 \text{ Pa} \cdot \text{s}$).
52. Show that the Reynolds number N_R is unitless by substituting units for all the quantities in its definition and cancelling.
53. Calculate the Reynolds numbers for the flow of water through (a) a nozzle with a radius of 0.250 cm and (b) a garden hose with a radius of 0.900 cm, when the nozzle is attached to the hose. The flow rate through hose and nozzle is 0.500 L/s. Can the flow in either possibly be laminar?
54. A fire hose has an inside diameter of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62 \times 10^6 \text{ N/m}^2$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Calculate the Reynolds numbers for flow in the fire hose and nozzle to show that the flow in each must be turbulent.
55. Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is $8.00 \times 10^6 \text{ N/m}^2$. Verify that the flow of concrete is laminar taking concrete's viscosity to be $48.0 \text{ (N/m}^2\text{)} \cdot \text{s}$, and given its density is 2300 kg/m^3 .
56. At what flow rate might turbulence begin to develop in a water main with a 0.200-m diameter? Assume a 20° C temperature.
57. What is the greatest average speed of blood flow at 37° C in an artery of radius 2.00 mm if the flow is to remain laminar? What is the corresponding flow rate? Take the density of blood to be 1025 kg/m^3 .
58. In [Take-Home Experiment: Inhalation](#), we measured the average flow rate Q of air traveling through the trachea during each inhalation. Now calculate the average air speed in meters per second through your trachea during each inhalation. The radius of the trachea in adult humans is approximately 10^{-2} m . From the data above, calculate the Reynolds number for the air flow in the trachea during inhalation. Do you expect the air flow to be laminar or turbulent?
59. Gasoline is piped underground from refineries to major users. The flow rate is $3.00 \times 10^{-2} \text{ m}^3/\text{s}$ (about 500 gal/min), the viscosity of gasoline is $1.00 \times 10^{-3} \text{ (N/m}^2\text{)} \cdot \text{s}$, and its density is 680 kg/m^3 . (a) What minimum diameter must the pipe have if the Reynolds number is to be less than 2000? (b) What pressure difference must be maintained along each kilometer of the pipe to maintain this flow rate?
60. Assuming that blood is an ideal fluid, calculate the critical flow rate at which turbulence is a certainty in the aorta. Take the diameter of the aorta to be 2.50 cm. (Turbulence will actually occur at lower average flow rates, because blood is not an ideal fluid. Furthermore, since blood flow pulses, turbulence may occur during only the high-velocity part of each heartbeat.)
61. **Unreasonable Results**
A fairly large garden hose has an internal radius of 0.600 cm and a length of 23.0 m. The nozzleless horizontal hose is attached to a faucet, and it delivers 50.0 L/s. (a) What water pressure is supplied by the faucet? (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise? (d) What is the Reynolds number for the given flow? (Take the viscosity of water as $1.005 \times 10^{-3} \text{ (N/m}^2\text{)} \cdot \text{s}$.)

12.7 Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

62. You can smell perfume very shortly after opening the bottle. To show that it is not reaching your nose by diffusion, calculate the average distance a perfume molecule moves in one second in air, given its diffusion constant D to be $1.00 \times 10^{-6} \text{ m}^2/\text{s}$.
63. What is the ratio of the average distances that oxygen will diffuse in a given time in air and water? Why is this distance less in water (equivalently, why is D less in water)?

- 64.** Oxygen reaches the veinless cornea of the eye by diffusing through its tear layer, which is 0.500-mm thick. How long does it take the average oxygen molecule to do this?
- 65.** (a) Find the average time required for an oxygen molecule to diffuse through a 0.200-mm-thick tear layer on the cornea. (b) How much time is required to diffuse 0.500 cm^3 of oxygen to the cornea if its surface area is 1.00 cm^2 ?
- 66.** Suppose hydrogen and oxygen are diffusing through air. A small amount of each is released simultaneously. How much time passes before the hydrogen is 1.00 s ahead of the oxygen? Such differences in arrival times are used as an analytical tool in gas chromatography.

CHAPTER 13

Temperature, Kinetic Theory, and the Gas Laws



FIGURE 13.1 Molecules dissolved in surfaces undergo a process of outgassing, or release into the space around them. In a car, this leads to the famous "new car smell." In precision instruments like telescopes, this leads to major distortions due to contamination. NASA engineer Nithin Abraham created a coating to absorb these contaminants. Here she is pictured securing a coated panel beneath the Johnson Space Center's Chamber A, where it was used during testing of the James Webb Space Telescope. (credit: NASA, Chris Gunn/Flickr)

CHAPTER OUTLINE

13.1 Temperature

13.2 Thermal Expansion of Solids and Liquids

13.3 The Ideal Gas Law

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

13.5 Phase Changes

13.6 Humidity, Evaporation, and Boiling

INTRODUCTION TO TEMPERATURE, KINETIC THEORY, AND THE GAS LAWS Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.



FIGURE 13.2 In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)

13.1 Temperature

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

Misconception Alert: Human Perception vs. Reality

On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in [Conduction](#)). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip ([Figure 13.3](#)). Other properties used to measure temperature include electrical resistance and color, as shown in [Figure 13.4](#), and the emission of infrared radiation, as shown in [Figure 13.5](#).

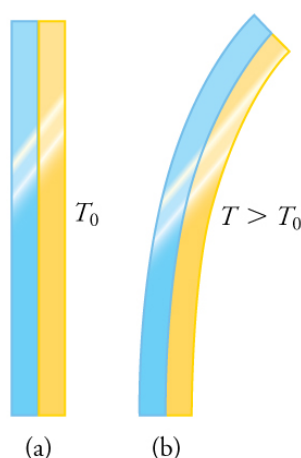


FIGURE 13.3 The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.



FIGURE 13.4 Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95°F, all six squares are black. When the plastic thermometer is exposed to temperature that increases to 95°F, the first liquid crystal square changes color. When the temperature increases above 96.8°F the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)



FIGURE 13.5 Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at 0°C and the boiling point at 100°C. Its unit is the **degree Celsius** (°C). On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at 32°F and the boiling point is at 212°F. The unit of temperature

on this scale is the **degree Fahrenheit**(°F). Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

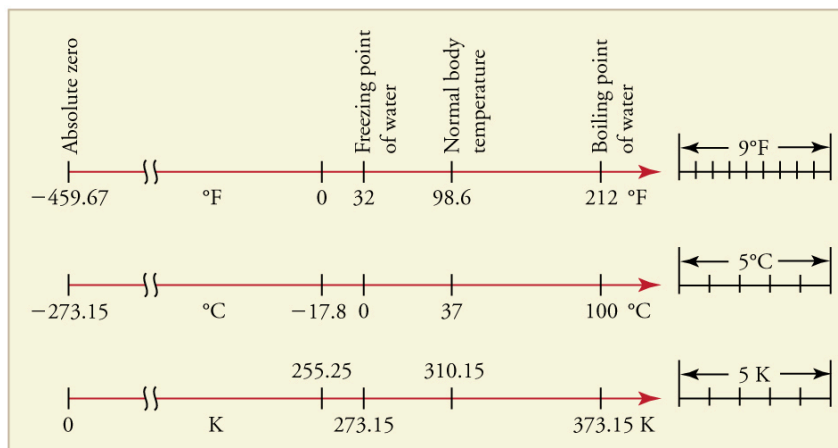


FIGURE 13.6 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in [Figure 13.6](#). Temperatures on these scales can be converted using the equations in [Table 13.1](#).

To convert from . . .	Use this equation . . .	Also written as . . .
Celsius to Fahrenheit	$T(^{\circ}\text{F}) = \frac{9}{5}T(^{\circ}\text{C}) + 32$	$T_{\text{F}} = \frac{9}{5}T_{\text{C}} + 32$
Fahrenheit to Celsius	$T(^{\circ}\text{C}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32)$	$T_{\text{C}} = \frac{5}{9}(T_{\text{F}} - 32)$
Celsius to Kelvin	$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$	$T_{\text{K}} = T_{\text{C}} + 273.15$
Kelvin to Celsius	$T(^{\circ}\text{C}) = T(\text{K}) - 273.15$	$T_{\text{C}} = T_{\text{K}} - 273.15$
Fahrenheit to Kelvin	$T(\text{K}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32) + 273.15$	$T_{\text{K}} = \frac{5}{9}(T_{\text{F}} - 32) + 273.15$
Kelvin to Fahrenheit	$T(^{\circ}\text{F}) = \frac{9}{5}(T(\text{K}) - 273.15) + 32$	$T_{\text{F}} = \frac{9}{5}(T_{\text{K}} - 273.15) + 32$

TABLE 13.1 Temperature Conversions

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

EXAMPLE 13.1

Converting between Temperature Scales: Room Temperature

“Room temperature” is generally defined to be 25°C. (a) What is room temperature in °F? (b) What is it in K?

Strategy

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from °C to °F, use the equation

$$T_{\text{°F}} = \frac{9}{5}T_{\text{°C}} + 32. \quad 13.1$$

2. Plug the known value into the equation and solve:

$$T_{\text{°F}} = \frac{9}{5}25^{\circ}\text{C} + 32 = 77^{\circ}\text{F}. \quad 13.2$$

Solution for (b)

1. Choose the right equation. To convert from °C to K, use the equation

$$T_{\text{K}} = T_{\text{°C}} + 273.15. \quad 13.3$$

2. Plug the known value into the equation and solve:

$$T_{\text{K}} = 25^{\circ}\text{C} + 273.15 = 298 \text{ K}. \quad 13.4$$

EXAMPLE 13.2

Converting between Temperature Scales: the Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is 0°R and the boiling temperature is 80°R. If “room temperature” is 25°C on the Celsius scale, what is it on the Reaumur scale?

Strategy

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is 80°R. On the Celsius scale it is 100°C. Therefore 100°C = 80°R. Both scales start at 0° for freezing, so we can derive a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other:

$$T_{\text{°R}} = \frac{0.8^{\circ}\text{R}}{1^{\circ}\text{C}} \times T_{\text{°C}}. \quad 13.5$$

2. Plug the known value into the equation and solve:

$$T_{\text{°R}} = \frac{0.8^{\circ}\text{R}}{1^{\circ}\text{C}} \times 25^{\circ}\text{C} = 20^{\circ}\text{R}. \quad 13.6$$

Temperature Ranges in the Universe

[Figure 13.8](#) shows the wide range of temperatures found in the universe. Human beings have been known to survive

with body temperatures within a small range, from 24°C to 44°C (75°F to 111°F). The average normal body temperature is usually given as 37.0°C (98.6°F), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see [Figure 13.7](#)).

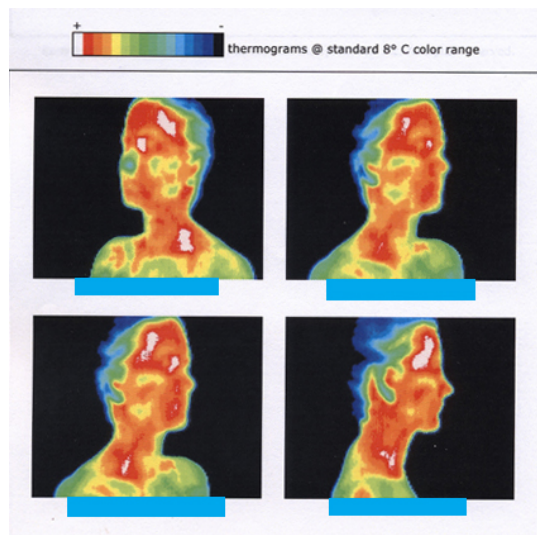


FIGURE 13.7 This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments: 4.5×10^{-10} K at the Massachusetts Institute of Technology (USA), and 1.0×10^{-10} K at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K (−89°C), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

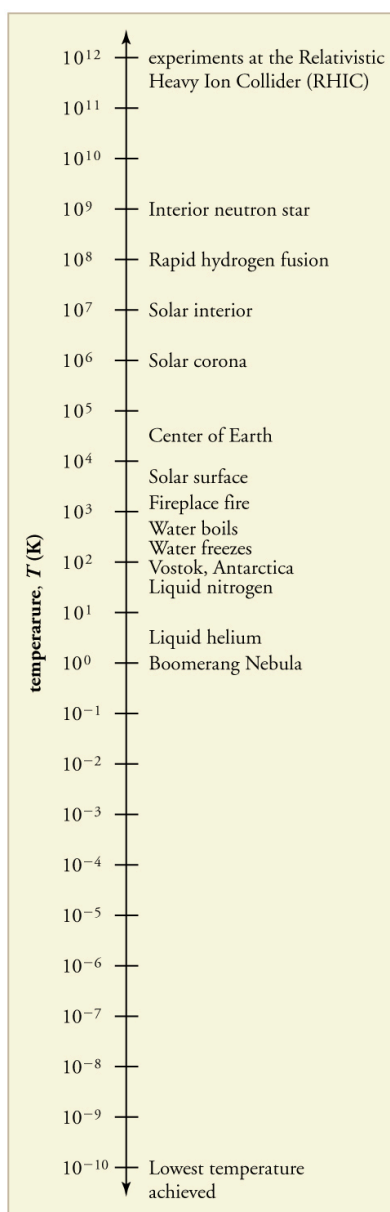


FIGURE 13.8 Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. [Figure 13.9](#) shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, -273.15°C . This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is -273.15°C or 0 K .

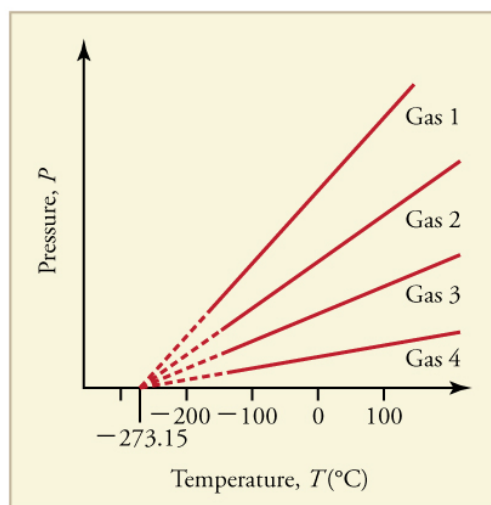


FIGURE 13.9 Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

The Zeroth Law of Thermodynamics

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws (discussed in [Thermodynamics](#)). Suppose, for example, a cold metal block and a hot metal block are both placed on a metal plate at room temperature. Eventually the cold block and the plate will be in thermal equilibrium. In addition, the hot block and the plate will be in thermal equilibrium. By the zeroth law, we can conclude that the cold block and the hot block are also in thermal equilibrium.

✓ CHECK YOUR UNDERSTANDING

Does the temperature of a body depend on its size?

Solution

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

13.2 Thermal Expansion of Solids and Liquids

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define and describe thermal expansion.
- Calculate the linear expansion of an object given its initial length, change in temperature, and coefficient of linear expansion.
- Calculate the volume expansion of an object given its initial volume, change in temperature, and coefficient of volume expansion.
- Calculate thermal stress on an object given its original volume, temperature change, volume change, and bulk modulus.



FIGURE 13.10 Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, the change in size or volume of a given mass with temperature. Hot air rises because its volume increases, which causes the hot air's density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans, and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? As is discussed in [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#), an increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

Linear Thermal Expansion—Thermal Expansion in One Dimension

The change in length ΔL is proportional to length L . The dependence of thermal expansion on temperature, substance, and length is summarized in the equation

$$\Delta L = \alpha L \Delta T,$$

13.7

where ΔL is the change in length L , ΔT is the change in temperature, and α is the **coefficient of linear expansion**, which varies slightly with temperature.

Table 13.2 lists representative values of the coefficient of linear expansion, which may have units of $1/^{\circ}\text{C}$ or $1/\text{K}$. Because the size of a kelvin and a degree Celsius are the same, both α and ΔT can be expressed in units of kelvins or degrees Celsius. The equation $\Delta L = \alpha L \Delta T$ is accurate for small changes in temperature and can be used for large changes in temperature if an average value of α is used.

Material	Coefficient of linear expansion $\alpha(1/^{\circ}\text{C})$	Coefficient of volume expansion $\beta(1/^{\circ}\text{C})$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	51×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or Steel	12×10^{-6}	35×10^{-6}
Invar (Nickel-iron alloy)	0.9×10^{-6}	2.7×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Silver	18×10^{-6}	54×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete, Brick	$\sim 12 \times 10^{-6}$	$\sim 36 \times 10^{-6}$
Marble (average)	7×10^{-6}	2.1×10^{-5}
Liquids		
Ether		1650×10^{-6}
Ethyl alcohol		1100×10^{-6}
Petrol		950×10^{-6}

TABLE 13.2 Thermal Expansion Coefficients at 20°C ¹

¹ Values for liquids and gases are approximate.

Material	Coefficient of linear expansion $\alpha(1/^{\circ}\text{C})$	Coefficient of volume expansion $\beta(1/^{\circ}\text{C})$
Glycerin		500×10^{-6}
Mercury		180×10^{-6}
Water		210×10^{-6}
Gases		
Air and most other gases at atmospheric pressure		3400×10^{-6}

TABLE 13.2 Thermal Expansion Coefficients at 20°C ¹**EXAMPLE 13.3****Calculating Linear Thermal Expansion: The Golden Gate Bridge**

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from -15°C to 40°C . What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

Strategy

Use the equation for linear thermal expansion $\Delta L = \alpha L \Delta T$ to calculate the change in length, ΔL . Use the coefficient of linear expansion, α , for steel from [Table 13.2](#), and note that the change in temperature, ΔT , is 55°C .

Solution

Plug all of the known values into the equation to solve for ΔL .

$$\Delta L = \alpha L \Delta T = \left(\frac{12 \times 10^{-6}}{^{\circ}\text{C}} \right) (1275 \text{ m})(55^{\circ}\text{C}) = 0.84 \text{ m.} \quad 13.8$$

Discussion

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

Thermal Expansion in Two and Three Dimensions

Objects expand in all dimensions, as illustrated in [Figure 13.11](#). That is, their areas and volumes, as well as their lengths, increase with temperature. Holes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the plug was still in place. The plug would get bigger, and so the hole must get bigger too. (Think of the ring of neighboring atoms or molecules on the wall of the hole as pushing each other farther apart as temperature increases. Obviously, the ring of neighbors must get slightly larger, so the hole gets slightly larger).

Thermal Expansion in Two Dimensions

For small temperature changes, the change in area ΔA is given by

$$\Delta A = 2\alpha A \Delta T, \quad 13.9$$

where ΔA is the change in area A , ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.

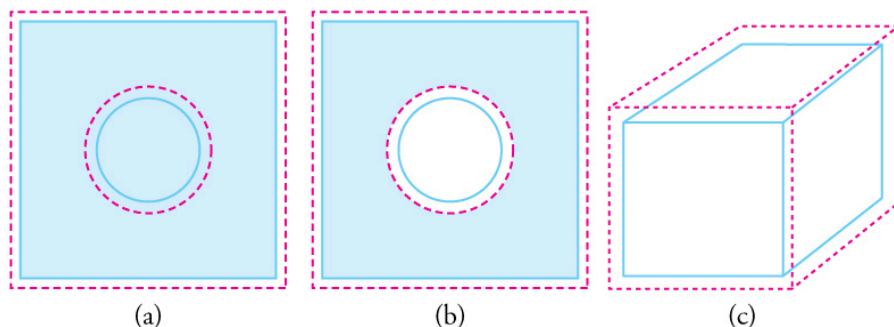


FIGURE 13.11 In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.

Thermal Expansion in Three Dimensions

The change in volume ΔV is very nearly $\Delta V = 3\alpha V \Delta T$. This equation is usually written as

$$\Delta V = \beta V \Delta T, \quad 13.10$$

where β is the **coefficient of volume expansion** and $\beta \approx 3\alpha$. Note that the values of β in [Table 13.2](#) are almost exactly equal to 3α .

In general, objects will expand with increasing temperature. Water is the most important exception to this rule. Water expands with increasing temperature (its density *decreases*) when it is at temperatures greater than 4°C (40°F). However, it expands with *decreasing* temperature when it is between $+4^\circ\text{C}$ and 0°C (40°F to 32°F). Water is densest at $+4^\circ\text{C}$. (See [Figure 13.12](#).) Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to 4°C it is denser than the remaining water and thus will sink to the bottom. This “turnover” results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of 4°C . If the temperature in the surface layer drops below 4°C , the water is less dense than the water below, and thus stays near the top. As a result, the pond surface can completely freeze over. The ice on top of liquid water provides an insulating layer from winter’s harsh exterior air temperatures. Fish and other aquatic life can survive in 4°C water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.

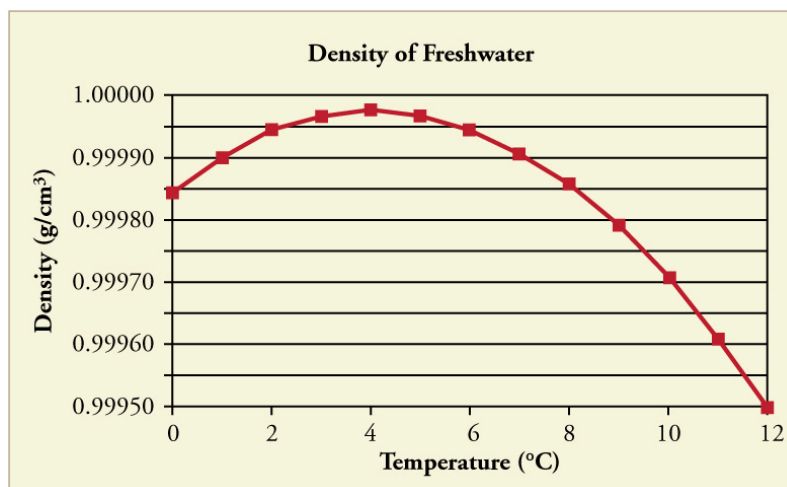


FIGURE 13.12 The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at $+4^\circ\text{C}$ is only 0.0075% greater than the density at 2°C , and 0.012% greater than that at 0°C .

Making Connections: Real-World Connections—Filling the Tank

Differences in the thermal expansion of materials can lead to interesting effects at the gas station. One example is the dripping of gasoline from a freshly filled tank on a hot day. Gasoline starts out at the temperature of the ground under the gas station, which is cooler than the air temperature above. The gasoline cools the steel tank when it is filled. Both gasoline and steel tank expand as they warm to air temperature, but gasoline expands much more than steel, and so it may overflow.

This difference in expansion can also cause problems when interpreting the gasoline gauge. The actual amount (mass) of gasoline left in the tank when the gauge hits “empty” is a lot less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the “add fuel” light goes on, but because the gasoline has expanded, there is less mass. If you are used to getting another 40 miles on “empty” in the winter, beware—you will probably run out much more quickly in the summer.



FIGURE 13.13 Because the gas expands more than the gas tank with increasing temperature, you can't drive as many miles on “empty” in the summer as you can in the winter. (credit: Hector Alejandro, Flickr)



EXAMPLE 13.4

Calculating Thermal Expansion: Gas vs. Gas Tank

Suppose your 60.0-L (15.9-gal) steel gasoline tank is full of gas, so both the tank and the gasoline have a temperature of 15.0°C. How much gasoline has spilled by the time they warm to 35.0°C?

Strategy

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. (The gasoline tank can be treated as solid steel.) We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank.

Solution

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

$$\Delta V_s = \beta_s V_s \Delta T. \quad 13.11$$

2. The increase in volume of the gasoline is given by this equation:

$$\Delta V_{\text{gas}} = \beta_{\text{gas}} V_{\text{gas}} \Delta T. \quad 13.12$$

3. Find the difference in volume to determine the amount spilled as

$$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_s. \quad 13.13$$

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

$$\begin{aligned}
 V_{\text{spill}} &= (\beta_{\text{gas}} - \beta_s) V \Delta T \\
 &= [(950 - 35) \times 10^{-6} / ^\circ\text{C}] (60.0 \text{ L}) (20.0^\circ\text{C}) \\
 &= 1.10 \text{ L.}
 \end{aligned}
 \tag{13.14}$$

Discussion

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel expand quickly. The rate of change in thermal properties is discussed in [Heat and Heat Transfer Methods](#).

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist being compressed with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.

Thermal Stress

Thermal stress is created by thermal expansion or contraction (see [Elasticity: Stress and Strain](#) for a discussion of stress and strain). Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.



EXAMPLE 13.5

Calculating Thermal Stress: Gas Pressure

What pressure would be created in the gasoline tank considered in [Example 13.4](#), if the gasoline increases in temperature from 15.0°C to 35.0°C without being allowed to expand? Assume that the bulk modulus B for gasoline is $1.00 \times 10^9 \text{ N/m}^2$. (For more on bulk modulus, see [Elasticity: Stress and Strain](#).)

Strategy

To solve this problem, we must use the following equation, which relates a change in volume ΔV to pressure:

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0, \tag{13.15}$$

where F/A is pressure, V_0 is the original volume, and B is the bulk modulus of the material involved. We will use the amount spilled in [Example 13.4](#) as the change in volume, ΔV .

Solution

1. Rearrange the equation for calculating pressure:

$$P = \frac{F}{A} = \frac{\Delta V}{V_0} B. \tag{13.16}$$

2. Insert the known values. The bulk modulus for gasoline is $B = 1.00 \times 10^9 \text{ N/m}^2$. In the previous example, the change in volume $\Delta V = 1.10 \text{ L}$ is the amount that would spill. Here, $V_0 = 60.0 \text{ L}$ is the original volume of the gasoline. Substituting these values into the equation, we obtain

$$P = \frac{1.10 \text{ L}}{60.0 \text{ L}} (1.00 \times 10^9 \text{ Pa}) = 1.83 \times 10^7 \text{ Pa.} \tag{13.17}$$

Discussion

This pressure is about 2500 lb/in^2 , much more than a gasoline tank can handle.

Forces and pressures created by thermal stress are typically as great as that in the example above. Railroad tracks

and roadways can buckle on hot days if they lack sufficient expansion joints. (See [Figure 13.14](#).) Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuate the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.



FIGURE 13.14 Thermal stress contributes to the formation of potholes. (credit: Editor5807, Wikimedia Commons)

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion, and are closer to those of teeth.

✓ CHECK YOUR UNDERSTANDING

Two blocks, A and B, are made of the same material. Block A has dimensions $l \times w \times h = L \times 2L \times L$ and Block B has dimensions $2L \times 2L \times 2L$. If the temperature changes, what is (a) the change in the volume of the two blocks, (b) the change in the cross-sectional area $l \times w$, and (c) the change in the height h of the two blocks?

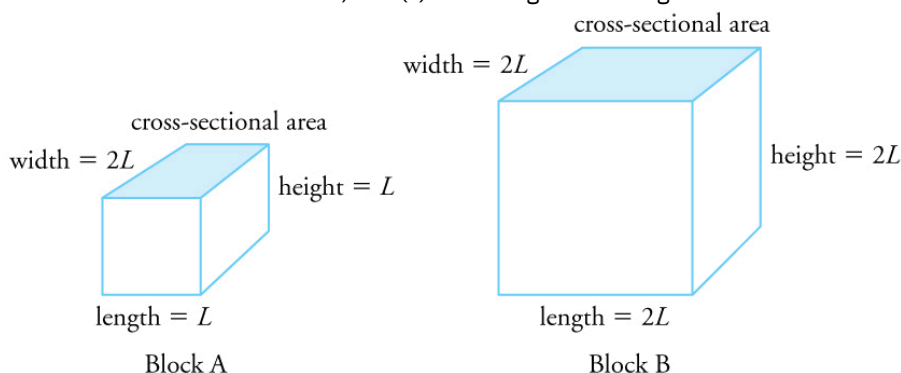


FIGURE 13.15

Solution

(a) The change in volume is proportional to the original volume. Block A has a volume of $L \times 2L \times L = 2L^3$. Block B has a volume of $2L \times 2L \times 2L = 8L^3$, which is 4 times that of Block A. Thus the change in volume of Block B should be 4 times the change in volume of Block A.

(b) The change in area is proportional to the area. The cross-sectional area of Block A is $L \times 2L = 2L^2$, while that of Block B is $2L \times 2L = 4L^2$. Because cross-sectional area of Block B is twice that of Block A, the change in the cross-sectional area of Block B is twice that of Block A.

(c) The change in height is proportional to the original height. Because the original height of Block B is twice that of A, the change in the height of Block B is twice that of Block A.

13.3 The Ideal Gas Law

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State the ideal gas law in terms of molecules and in terms of moles.
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules or moles in a given volume.
- Use Avogadro's number to convert between number of molecules and number of moles.



FIGURE 13.16 The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen, N_2 , and oxygen, O_2 , are composed of two or more atoms. We will primarily use the term “molecule” in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. We can see evidence of this in [Table 13.2](#), where you will note that gases have the *largest* coefficients of volume expansion. The large coefficients mean that gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β . This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in [Figure 13.17](#). Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the

forces between them.

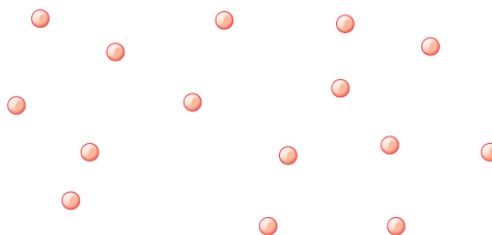


FIGURE 13.17 Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See [Figure 13.18](#).)

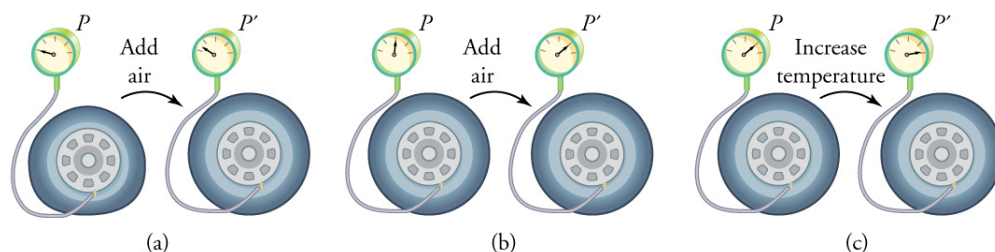


FIGURE 13.18 (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an **ideal gas**, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the **ideal gas law**.

Ideal Gas Law

The **ideal gas law** states that

$$PV = NkT, \quad 13.18$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

$$k = 1.38 \times 10^{-23} \text{ J/K}. \quad 13.19$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V . The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $PV = NkT$ predicts that the pressure should increase in proportion to the number N of

atoms and molecules.



EXAMPLE 13.6

Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of 7.00×10^5 Pa (a gauge pressure of just under 90.0 lb/in²) at a temperature of 18.0°C. What is the pressure after its temperature has risen to 35.0°C? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00 \times 10^5$ Pa, the initial temperature $T_0 = 18.0^\circ\text{C}$, and the final temperature $T_f = 35.0^\circ\text{C}$. We must find the final pressure P_f . How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0V_0 = NkT_0$ and $P_fV_f = NkT_f$. If we divide P_fV_f by P_0V_0 we can come up with an equation that allows us to solve for P_f .

$$\frac{P_f V_f}{P_0 V_0} = \frac{N_f k T_f}{N_0 k T_0} \quad 13.20$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0 , and k , which is a constant. Therefore,

$$\frac{P_f}{P_0} = \frac{T_f}{T_0}. \quad 13.21$$

We can then rearrange this to solve for P_f :

$$P_f = P_0 \frac{T_f}{T_0}, \quad 13.22$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

Solution

1. Convert temperatures from Celsius to Kelvin.

$$\begin{aligned} T_0 &= (18.0 + 273)\text{K} = 291\text{ K} \\ T_f &= (35.0 + 273)\text{K} = 308\text{ K} \end{aligned} \quad 13.23$$

2. Substitute the known values into the equation.

$$P_f = P_0 \frac{T_f}{T_0} = 7.00 \times 10^5 \text{ Pa} \left(\frac{308\text{ K}}{291\text{ K}} \right) = 7.41 \times 10^5 \text{ Pa} \quad 13.24$$

Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

Making Connections: Take-Home Experiment—Refrigerating a Balloon

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?



EXAMPLE 13.7

Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be 0°C and atmospheric pressure.

Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law $PV = NkT$, to find N .

Solution

1. Identify the knowns.

$$\begin{aligned} T &= 0^\circ\text{C} = 273 \text{ K} \\ P &= 1.01 \times 10^5 \text{ Pa} \\ V &= 1.00 \text{ m}^3 \\ k &= 1.38 \times 10^{-23} \text{ J/K} \end{aligned} \quad 13.25$$

2. Identify the unknown: number of molecules, N .

3. Rearrange the ideal gas law to solve for N .

$$\begin{aligned} PV &= NkT \\ N &= \frac{PV}{kT} \end{aligned} \quad 13.26$$

4. Substitute the known values into the equation and solve for N .

$$N = \frac{PV}{kT} = \frac{(1.01 \times 10^5 \text{ Pa})(1.00 \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 2.68 \times 10^{25} \text{ molecules} \quad 13.27$$

Discussion

This number is undeniably large, considering that a gas is mostly empty space. N is huge, even in small volumes. For example, 1 cm^3 of a gas at STP has 2.68×10^{19} molecules in it. Once again, note that N is the same for all types or mixtures of gases.

Moles and Avogadro's Number

It is sometimes convenient to work with a unit other than molecules when measuring the amount of substance. A **mole** (abbreviated mol) is defined to be the amount of a substance that contains as many atoms or molecules as there are atoms in exactly 12 grams (0.012 kg) of carbon-12. The actual number of atoms or molecules in one mole is called **Avogadro's number** (N_A), in recognition of Italian scientist Amedeo Avogadro (1776–1856). He developed the concept of the mole, based on the hypothesis that equal volumes of gas, at the same pressure and temperature, contain equal numbers of molecules. That is, the number is independent of the type of gas. This hypothesis has been confirmed, and the value of Avogadro's number is

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}. \quad 13.28$$

Avogadro's Number

One mole always contains 6.02×10^{23} particles (atoms or molecules), independent of the element or substance. A mole of any substance has a mass in grams equal to its molecular mass, which can be calculated

from the atomic masses given in the periodic table of elements.

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

13.29

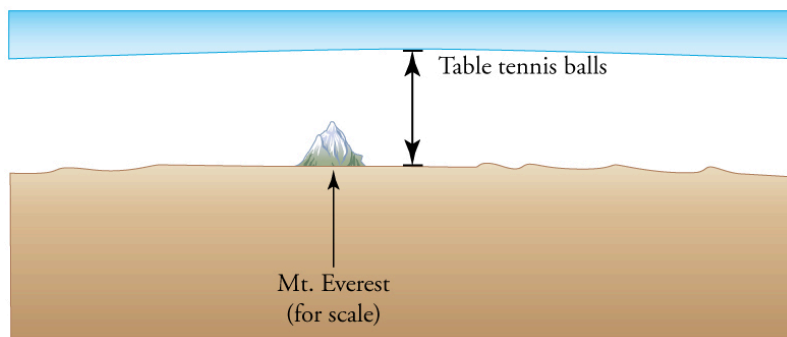


FIGURE 13.19 How big is a mole? On a macroscopic level, one mole of table tennis balls would cover the Earth to a depth of about 40 km.

✓ CHECK YOUR UNDERSTANDING

The active ingredient in a Tylenol pill is 325 mg of acetaminophen ($C_8H_9NO_2$). Find the number of active molecules of acetaminophen in a single pill.

Solution

We first need to calculate the molar mass (the mass of one mole) of acetaminophen. To do this, we need to multiply the number of atoms of each element by the element's atomic mass.

$$\begin{aligned} & (8 \text{ moles of carbon})(12 \text{ grams/mole}) + (9 \text{ moles hydrogen})(1 \text{ gram/mole}) \\ & + (1 \text{ mole nitrogen})(14 \text{ grams/mole}) + (2 \text{ moles oxygen})(16 \text{ grams/mole}) = 151 \text{ g} \end{aligned} \quad 13.30$$

Then we need to calculate the number of moles in 325 mg.

$$\left(\frac{325 \text{ mg}}{151 \text{ grams/mole}} \right) \left(\frac{1 \text{ gram}}{1000 \text{ mg}} \right) = 2.15 \times 10^{-3} \text{ moles} \quad 13.31$$

Then use Avogadro's number to calculate the number of molecules.

$$N = (2.15 \times 10^{-3} \text{ moles}) (6.02 \times 10^{23} \text{ molecules/mole}) = 1.30 \times 10^{21} \text{ molecules} \quad 13.32$$

EXAMPLE 13.8

Calculating Moles per Cubic Meter and Liters per Mole

Calculate: (a) the number of moles in 1.00 m^3 of gas at STP, and (b) the number of liters of gas per mole.

Strategy and Solution

(a) We are asked to find the number of moles per cubic meter, and we know from [Example 13.7](#) that the number of molecules per cubic meter at STP is 2.68×10^{25} . The number of moles can be found by dividing the number of molecules by Avogadro's number. We let n stand for the number of moles,

$$n \text{ mol/m}^3 = \frac{N \text{ molecules/m}^3}{6.02 \times 10^{23} \text{ molecules/mol}} = \frac{2.68 \times 10^{25} \text{ molecules/m}^3}{6.02 \times 10^{23} \text{ molecules/mol}} = 44.5 \text{ mol/m}^3. \quad 13.33$$

(b) Using the value obtained for the number of moles in a cubic meter, and converting cubic meters to liters, we obtain

$$\frac{(10^3 \text{ L/m}^3)}{44.5 \text{ mol/m}^3} = 22.5 \text{ L/mol.} \quad 13.34$$

Discussion

This value is very close to the accepted value of 22.4 L/mol. The slight difference is due to rounding errors caused by using three-digit input. Again this number is the same for all gases. In other words, it is independent of the gas.

The (average) molar weight of air (approximately 80% N_2 and 20% O_2) is $M = 28.8$ g. Thus the mass of one cubic meter of air is 1.28 kg. If a living room has dimensions $5\text{ m} \times 5\text{ m} \times 3\text{ m}$, the mass of air inside the room is 96 kg, which is the typical mass of a human.

CHECK YOUR UNDERSTANDING

The density of air at standard conditions ($P = 1$ atm and $T = 20^\circ\text{C}$) is 1.28 kg/m^3 . At what pressure is the density 0.64 kg/m^3 if the temperature and number of molecules are kept constant?

Solution

The best way to approach this question is to think about what is happening. If the density drops to half its original value and no molecules are lost, then the volume must double. If we look at the equation $PV = NkT$, we see that when the temperature is constant, the pressure is inversely proportional to volume. Therefore, if the volume doubles, the pressure must drop to half its original value, and $P_f = 0.50$ atm.

The Ideal Gas Law Restated Using Moles

A very common expression of the ideal gas law uses the number of moles, n , rather than the number of atoms and molecules, N . We start from the ideal gas law,

$$PV = NkT, \quad 13.35$$

and multiply and divide the equation by Avogadro's number N_A . This gives

$$PV = \frac{N}{N_A} N_A kT. \quad 13.36$$

Note that $n = N/N_A$ is the number of moles. We define the universal gas constant $R = N_A k$, and obtain the ideal gas law in terms of moles.

Ideal Gas Law (in terms of moles)

The ideal gas law (in terms of moles) is

$$PV = nRT. \quad 13.37$$

The numerical value of R in SI units is

$$R = N_A k = (6.02 \times 10^{23} \text{ mol}^{-1}) (1.38 \times 10^{-23} \text{ J/K}) = 8.31 \text{ J/mol} \cdot \text{K}. \quad 13.38$$

In other units,

$$\begin{aligned} R &= 1.99 \text{ cal/mol} \cdot \text{K} \\ R &= 0.0821 \text{ L} \cdot \text{atm/mol} \cdot \text{K}. \end{aligned} \quad 13.39$$

You can use whichever value of R is most convenient for a particular problem.



EXAMPLE 13.9

Calculating Number of Moles: Gas in a Bike Tire

How many moles of gas are in a bike tire with a volume of $2.00 \times 10^{-3} \text{ m}^3$ (2.00 L), a pressure of 7.00×10^5 Pa (a gauge pressure of just under 90.0 lb/in^2), and at a temperature of 18.0°C ?

Strategy

Identify the knowns and unknowns, and choose an equation to solve for the unknown. In this case, we solve the ideal gas law, $PV = nRT$, for the number of moles n .

Solution

1. Identify the knowns.

$$\begin{aligned} P &= 7.00 \times 10^5 \text{ Pa} \\ V &= 2.00 \times 10^{-3} \text{ m}^3 \\ T &= 18.0^\circ\text{C} = 291 \text{ K} \\ R &= 8.31 \text{ J/mol} \cdot \text{K} \end{aligned} \quad 13.40$$

2. Rearrange the equation to solve for n and substitute known values.

$$\begin{aligned} n &= \frac{PV}{RT} = \frac{(7.00 \times 10^5 \text{ Pa})(2.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(291 \text{ K})} \\ &= 0.579 \text{ mol} \end{aligned} \quad 13.41$$

Discussion

The most convenient choice for R in this case is $8.31 \text{ J/mol} \cdot \text{K}$, because our known quantities are in SI units. The pressure and temperature are obtained from the initial conditions in [Example 13.6](#), but we would get the same answer if we used the final values.

The ideal gas law can be considered to be another manifestation of the law of conservation of energy (see [Conservation of Energy](#)). Work done on a gas results in an increase in its energy, increasing pressure and/or temperature, or decreasing volume. This increased energy can also be viewed as increased internal kinetic energy, given the gas's atoms and molecules.

The Ideal Gas Law and Energy

Let us now examine the role of energy in the behavior of gases. When you inflate a bike tire by hand, you do work by repeatedly exerting a force through a distance. This energy goes into increasing the pressure of air inside the tire and increasing the temperature of the pump and the air.

The ideal gas law is closely related to energy: the units on both sides are joules. The right-hand side of the ideal gas law in $PV = NkT$ is NkT . This term is roughly the amount of translational kinetic energy of N atoms or molecules at an absolute temperature T , as we shall see formally in [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#). The left-hand side of the ideal gas law is PV , which also has the units of joules. We know from our study of fluids that pressure is one type of potential energy per unit volume, so pressure multiplied by volume is energy. The important point is that there is energy in a gas related to both its pressure and its volume. The energy can be changed when the gas is doing work as it expands—something we explore in [Heat and Heat Transfer Methods](#)—similar to what occurs in gasoline or steam engines and turbines.

Problem-Solving Strategy: The Ideal Gas Law

Step 1 Examine the situation to determine that an ideal gas is involved. Most gases are nearly ideal.

Step 2 Make a list of what quantities are given, or can be inferred from the problem as stated (identify the known quantities). Convert known values into proper SI units (K for temperature, Pa for pressure, m^3 for volume, molecules for N , and moles for n).

Step 3 Identify exactly what needs to be determined in the problem (identify the unknown quantities). A written list is useful.

Step 4 Determine whether the number of molecules or the number of moles is known, in order to decide which form of the ideal gas law to use. The first form is $PV = NkT$ and involves N , the number of atoms or molecules. The second form is $PV = nRT$ and involves n , the number of moles.

Step 5 Solve the ideal gas law for the quantity to be determined (the unknown quantity). You may need to take a ratio of final states to initial states to eliminate the unknown quantities that are kept fixed.

Step 6 Substitute the known quantities, along with their units, into the appropriate equation, and obtain numerical solutions complete with units. Be certain to use absolute temperature and absolute pressure.

Step 7 Check the answer to see if it is reasonable: Does it make sense?

CHECK YOUR UNDERSTANDING

Liquids and solids have densities about 1000 times greater than gases. Explain how this implies that the distances between atoms and molecules in gases are about 10 times greater than the size of their atoms and molecules.

Solution

Atoms and molecules are close together in solids and liquids. In gases they are separated by empty space. Thus gases have lower densities than liquids and solids. Density is mass per unit volume, and volume is related to the size of a body (such as a sphere) cubed. So if the distance between atoms and molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000.

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Express the ideal gas law in terms of molecular mass and velocity.
- Define thermal energy.
- Calculate the kinetic energy of a gas molecule, given its temperature.
- Describe the relationship between the temperature of a gas and the kinetic energy of atoms and molecules.
- Describe the distribution of speeds of molecules in a gas.

We have developed macroscopic definitions of pressure and temperature. Pressure is the force divided by the area on which the force is exerted, and temperature is measured with a thermometer. We gain a better understanding of pressure and temperature from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.

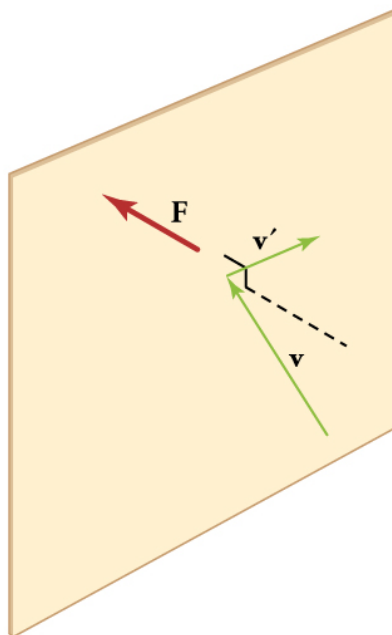


FIGURE 13.20 When a molecule collides with a rigid wall, the component of its momentum perpendicular to the wall is reversed. A force is thus exerted on the wall, creating pressure.

Figure 13.20 shows an elastic collision of a gas molecule with the wall of a container, so that it exerts a force on the wall (by Newton's third law). Because a huge number of molecules will collide with the wall in a short time, we observe an average force per unit area. These collisions are the source of pressure in a gas. As the number of molecules increases, the number of collisions and thus the pressure increase. Similarly, the gas pressure is higher if the average velocity of molecules is higher. The actual relationship is derived in the [Things Great and Small](#) feature below. The following relationship is found:

$$PV = \frac{1}{3}Nm\bar{v}^2, \quad 13.42$$

where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and $\bar{v}^2$ is the average of the molecular speed squared.

What can we learn from this atomic and molecular version of the ideal gas law? We can derive a relationship between temperature and the average translational kinetic energy of molecules in a gas. Recall the previous expression of the ideal gas law:

$$PV = NkT. \quad 13.43$$

Equating the right-hand side of this equation with the right-hand side of $PV = \frac{1}{3}Nm\bar{v}^2$ gives

$$\frac{1}{3}Nm\bar{v}^2 = NkT. \quad 13.44$$

Making Connections: Things Great and Small—Atomic and Molecular Origin of Pressure in a Gas

Figure 13.21 shows a box filled with a gas. We know from our previous discussions that putting more gas into the box produces greater pressure, and that increasing the temperature of the gas also produces a greater pressure. But why should increasing the temperature of the gas increase the pressure in the box? A look at the atomic and molecular scale gives us some answers, and an alternative expression for the ideal gas law.

The figure shows an expanded view of an elastic collision of a gas molecule with the wall of a container. Calculating the average force exerted by such molecules will lead us to the ideal gas law, and to the connection between temperature and molecular kinetic energy. We assume that a molecule is small compared with the

separation of molecules in the gas, and that its interaction with other molecules can be ignored. We also assume the wall is rigid and that the molecule's direction changes, but that its speed remains constant (and hence its kinetic energy and the magnitude of its momentum remain constant as well). This assumption is not always valid, but the same result is obtained with a more detailed description of the molecule's exchange of energy and momentum with the wall.

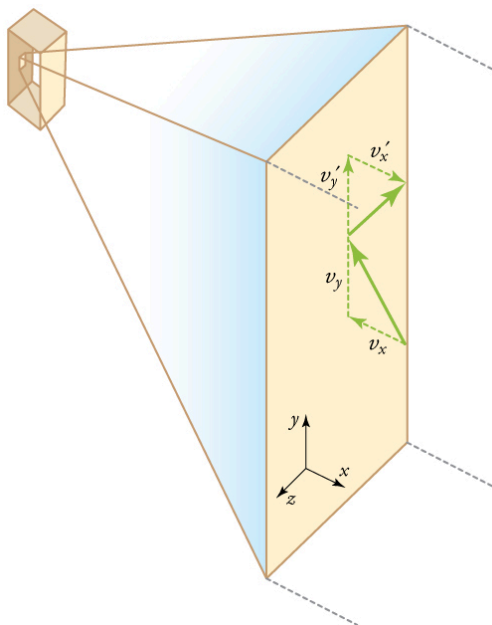


FIGURE 13.21 Gas in a box exerts an outward pressure on its walls. A molecule colliding with a rigid wall has the direction of its velocity and momentum in the x -direction reversed. This direction is perpendicular to the wall. The components of its velocity momentum in the y - and z -directions are not changed, which means there is no force parallel to the wall.

If the molecule's velocity changes in the x -direction, its momentum changes from $-mv_x$ to $+mv_x$. Thus, its change in momentum is $\Delta p = +mv_x - (-mv_x) = 2mv_x$. The force exerted on the molecule is given by

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{\Delta t}. \quad 13.45$$

There is no force between the wall and the molecule until the molecule hits the wall. During the short time of the collision, the force between the molecule and wall is relatively large. We are looking for an average force; we take Δt to be the average time between collisions of the molecule with this wall. It is the time it would take the molecule to go across the box and back (a distance $2l$) at a speed of v_x . Thus $\Delta t = 2l/v_x$, and the expression for the force becomes

$$F = \frac{2mv_x}{2l/v_x} = \frac{mv_x^2}{l}. \quad 13.46$$

This force is due to *one* molecule. We multiply by the number of molecules N and use their average squared velocity to find the force

$$F = N \frac{m\bar{v}_x^2}{l}, \quad 13.47$$

where the bar over a quantity means its average value. We would like to have the force in terms of the speed v , rather than the x -component of the velocity. We note that the total velocity squared is the sum of the squares of its components, so that

$$\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 + \bar{v}_z^2. \quad 13.48$$

Because the velocities are random, their average components in all directions are the same:

$$\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2. \quad 13.49$$

Thus,

$$\bar{v}^2 = 3\bar{v}_x^2, \quad 13.50$$

or

$$\bar{v}_x^2 = \frac{1}{3}\bar{v}^2. \quad 13.51$$

Substituting $\frac{1}{3}\bar{v}^2$ into the expression for F gives

$$F = N \frac{m\bar{v}^2}{3l}. \quad 13.52$$

The pressure is F/A , so that we obtain

$$P = \frac{F}{A} = N \frac{m\bar{v}^2}{3Al} = \frac{1}{3} \frac{Nm\bar{v}^2}{V}, \quad 13.53$$

where we used $V = Al$ for the volume. This gives the important result.

$$PV = \frac{1}{3} Nm\bar{v}^2 \quad 13.54$$

This equation is another expression of the ideal gas law.

We can get the average kinetic energy of a molecule, $\frac{1}{2}mv^2$, from the right-hand side of the equation by canceling N and multiplying by $3/2$. This calculation produces the result that the average kinetic energy of a molecule is directly related to absolute temperature.

$$\overline{\text{KE}} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT \quad 13.55$$

The average translational kinetic energy of a molecule, $\overline{\text{KE}}$, is called **thermal energy**. The equation $\overline{\text{KE}} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$ is a molecular interpretation of temperature, and it has been found to be valid for gases and reasonably accurate in liquids and solids. It is another definition of temperature based on an expression of the molecular energy.

It is sometimes useful to rearrange $\overline{\text{KE}} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT$, and solve for the average speed of molecules in a gas in terms of temperature,

$$\sqrt{\bar{v}^2} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \quad 13.56$$

where v_{rms} stands for root-mean-square (rms) speed.



EXAMPLE 13.10

Calculating Kinetic Energy and Speed of a Gas Molecule

(a) What is the average kinetic energy of a gas molecule at 20.0°C (room temperature)? (b) Find the rms speed of a nitrogen molecule (N_2) at this temperature.

Strategy for (a)

The known in the equation for the average kinetic energy is the temperature.

$$\overline{\text{KE}} = \frac{1}{2}m\bar{v}^2 = \frac{3}{2}kT \quad 13.57$$

Before substituting values into this equation, we must convert the given temperature to kelvins. This conversion gives $T = (20.0 + 273) \text{ K} = 293 \text{ K}$.

Solution for (a)

The temperature alone is sufficient to find the average translational kinetic energy. Substituting the temperature into the translational kinetic energy equation gives

$$\overline{\text{KE}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 6.07 \times 10^{-21} \text{ J.} \quad 13.58$$

Strategy for (b)

Finding the rms speed of a nitrogen molecule involves a straightforward calculation using the equation

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \quad 13.59$$

but we must first find the mass of a nitrogen molecule. Using the molecular mass of nitrogen N_2 from the periodic table,

$$m = \frac{2(14.0067) \times 10^{-3} \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 4.65 \times 10^{-26} \text{ kg.} \quad 13.60$$

Solution for (b)

Substituting this mass and the value for k into the equation for v_{rms} yields

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{4.65 \times 10^{-26} \text{ kg}}} = 511 \text{ m/s.} \quad 13.61$$

Discussion

Note that the average kinetic energy of the molecule is independent of the type of molecule. The average translational kinetic energy depends only on absolute temperature. The kinetic energy is very small compared to macroscopic energies, so that we do not feel when an air molecule is hitting our skin. The rms velocity of the nitrogen molecule is surprisingly large. These large molecular velocities do not yield macroscopic movement of air, since the molecules move in all directions with equal likelihood. The *mean free path* (the distance a molecule can move on average between collisions) of molecules in air is very small, and so the molecules move rapidly but do not get very far in a second. The high value for rms speed is reflected in the speed of sound, however, which is about 340 m/s at room temperature. The faster the rms speed of air molecules, the faster that sound vibrations can be transferred through the air. The speed of sound increases with temperature and is greater in gases with small molecular masses, such as helium. (See [Figure 13.22](#).)

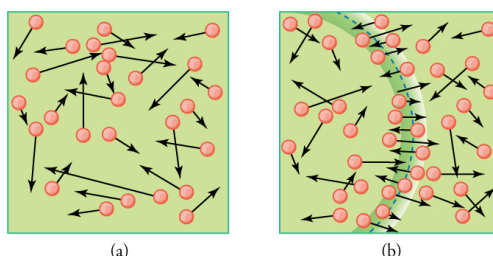


FIGURE 13.22 (a) There are many molecules moving so fast in an ordinary gas that they collide a billion times every second. (b) Individual molecules do not move very far in a small amount of time, but disturbances like sound waves are transmitted at speeds related to the molecular speeds.

Making Connections: Historical Note—Kinetic Theory of Gases

The kinetic theory of gases was developed by Daniel Bernoulli (1700–1782), who is best known in physics for his work on fluid flow (hydrodynamics). Bernoulli's work predates the atomistic view of matter established by Dalton.

Distribution of Molecular Speeds

The motion of molecules in a gas is random in magnitude and direction for individual molecules, but a gas of many molecules has a predictable distribution of molecular speeds. This distribution is called the *Maxwell-Boltzmann distribution*, after its originators, who calculated it based on kinetic theory, and has since been confirmed experimentally. (See [Figure 13.23](#).) The distribution has a long tail, because a few molecules may go several times the rms speed. The most probable speed v_p is less than the rms speed v_{rms} . [Figure 13.24](#) shows that the curve is shifted to higher speeds at higher temperatures, with a broader range of speeds.

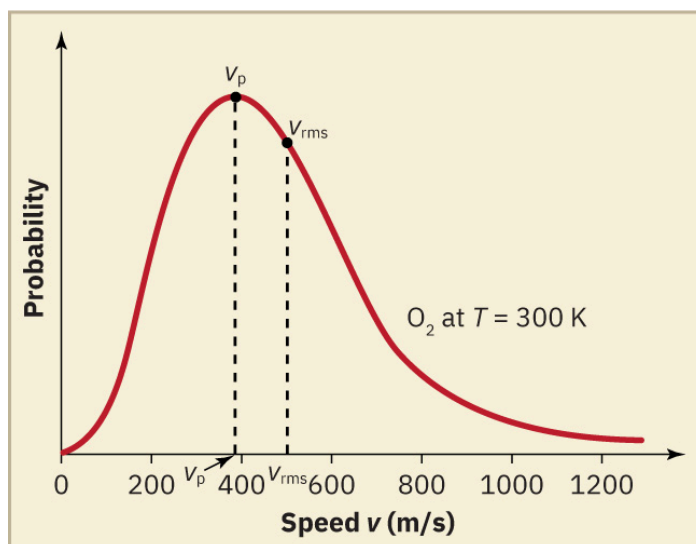


FIGURE 13.23 The Maxwell-Boltzmann distribution of molecular speeds in an ideal gas. The most likely speed v_p is less than the rms speed v_{rms} . Although very high speeds are possible, only a tiny fraction of the molecules have speeds that are an order of magnitude greater than v_{rms} .

The distribution of thermal speeds depends strongly on temperature. As temperature increases, the speeds are shifted to higher values and the distribution is broadened.

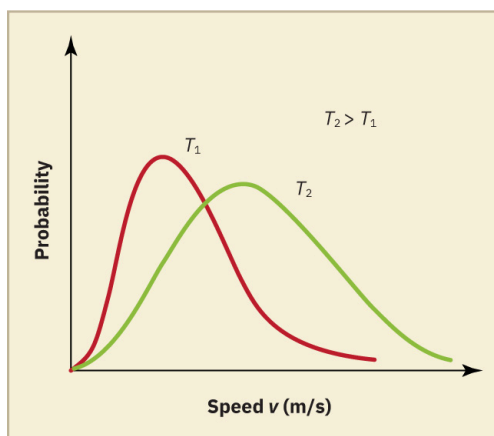


FIGURE 13.24 The Maxwell-Boltzmann distribution is shifted to higher speeds and is broadened at higher temperatures.

What is the implication of the change in distribution with temperature shown in [Figure 13.24](#) for humans? All other things being equal, if a person has a fever, they are likely to lose more water molecules, particularly from linings

along moist cavities such as the lungs and mouth, creating a dry sensation in the mouth.



EXAMPLE 13.11

Calculating Temperature: Escape Velocity of Helium Atoms

In order to escape Earth's gravity, an object near the top of the atmosphere (at an altitude of 100 km) must travel away from Earth at 11.1 km/s. This speed is called the *escape velocity*. At what temperature would helium atoms have an rms speed equal to the escape velocity?

Strategy

Identify the knowns and unknowns and determine which equations to use to solve the problem.

Solution

1. Identify the knowns: v is the escape velocity, 11.1 km/s.
2. Identify the unknowns: We need to solve for temperature, T . We also need to solve for the mass m of the helium atom.
3. Determine which equations are needed.

- To solve for mass m of the helium atom, we can use information from the periodic table:

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}} \quad 13.62$$

- To solve for temperature T , we can rearrange either

$$\overline{\text{KE}} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \quad 13.63$$

or

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad 13.64$$

to yield

$$T = \frac{m \overline{v^2}}{3k}, \quad 13.65$$

where k is the Boltzmann constant and m is the mass of a helium atom.

4. Plug the known values into the equations and solve for the unknowns.

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}} = \frac{4.0026 \times 10^{-3} \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}} = 6.65 \times 10^{-27} \text{ kg} \quad 13.66$$

$$T = \frac{(6.65 \times 10^{-27} \text{ kg})(11.1 \times 10^3 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 1.98 \times 10^4 \text{ K} \quad 13.67$$

Discussion

This temperature is much higher than atmospheric temperature, which is approximately 250 K (−25°C or −10°F) at high altitude. Very few helium atoms are left in the atmosphere, but there were many when the atmosphere was formed. The reason for the loss of helium atoms is that there are a small number of helium atoms with speeds higher than Earth's escape velocity even at normal temperatures. The speed of a helium atom changes from one instant to the next, so that at any instant, there is a small, but nonzero chance that the speed is greater than the escape speed and the molecule escapes from Earth's gravitational pull. Heavier molecules, such as oxygen, nitrogen, and water (very little of which reach a very high altitude), have smaller rms speeds, and so it is much less likely that any of them will have speeds greater than the escape velocity. In fact, so few have speeds above the

escape velocity that billions of years are required to lose significant amounts of the atmosphere. [Figure 13.25](#) shows the impact of a lack of an atmosphere on the Moon. Because the gravitational pull of the Moon is much weaker, it has lost almost its entire atmosphere. The comparison between Earth and the Moon is discussed in this chapter's Problems and Exercises.



FIGURE 13.25 This photograph of Apollo 17 Commander Eugene Cernan driving the lunar rover on the Moon in 1972 looks as though it was taken at night with a large spotlight. In fact, the light is coming from the Sun. Because the acceleration due to gravity on the Moon is so low (about $1/6$ that of Earth), the Moon's escape velocity is much smaller. As a result, gas molecules escape very easily from the Moon, leaving it with virtually no atmosphere. Even during the daytime, the sky is black because there is no gas to scatter sunlight. (credit: Harrison H. Schmitt/NASA)

CHECK YOUR UNDERSTANDING

If you consider a very small object such as a grain of pollen, in a gas, then the number of atoms and molecules striking its surface would also be relatively small. Would the grain of pollen experience any fluctuations in pressure due to statistical fluctuations in the number of gas atoms and molecules striking it in a given amount of time?

Solution

Yes. Such fluctuations actually occur for a body of any size in a gas, but since the numbers of atoms and molecules are immense for macroscopic bodies, the fluctuations are a tiny percentage of the number of collisions, and the averages spoken of in this section vary imperceptibly. Roughly speaking the fluctuations are proportional to the inverse square root of the number of collisions, so for small bodies they can become significant. This was actually observed in the 19th century for pollen grains in water, and is known as the Brownian effect.

PHET EXPLORATIONS

Gas Properties

Pump gas molecules into a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/13-4-kinetic-theory-atomic-and-molecular-explanation-of-pressure-and-temperature\)](https://openstax.org/books/college-physics-2e/pages/13-4-kinetic-theory-atomic-and-molecular-explanation-of-pressure-and-temperature)

13.5 Phase Changes

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Interpret a phase diagram.
- State Dalton's law.
- Identify and describe the triple point of a gas from its phase diagram.
- Describe the state of equilibrium between a liquid and a gas, a liquid and a solid, and a gas and a solid.

Up to now, we have considered the behavior of ideal gases. Real gases are like ideal gases at high temperatures. At lower temperatures, however, the interactions between the molecules and their volumes cannot be ignored. The molecules are very close (condensation occurs) and there is a dramatic decrease in volume, as seen in [Figure 13.26](#). The substance changes from a gas to a liquid. When a liquid is cooled to even lower temperatures, it becomes a solid. The volume never reaches zero because of the finite volume of the molecules.

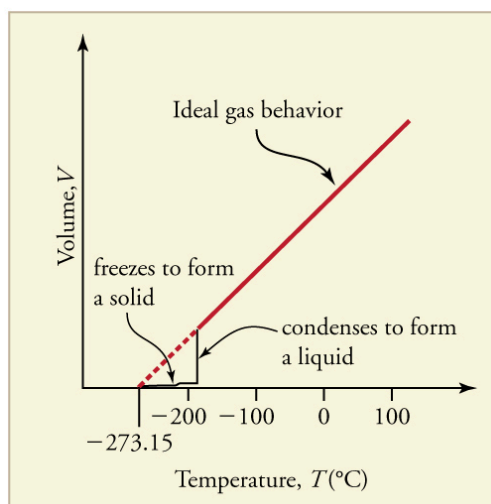


FIGURE 13.26 A sketch of volume versus temperature for a real gas at constant pressure. The linear (straight line) part of the graph represents ideal gas behavior—volume and temperature are directly and positively related and the line extrapolates to zero volume at -273.15°C , or absolute zero. When the gas becomes a liquid, however, the volume actually decreases precipitously at the liquefaction point. The volume decreases slightly once the substance is solid, but it never becomes zero.

High pressure may also cause a gas to change phase to a liquid. Carbon dioxide, for example, is a gas at room temperature and atmospheric pressure, but becomes a liquid under sufficiently high pressure. If the pressure is reduced, the temperature drops and the liquid carbon dioxide solidifies into a snow-like substance at the temperature -78°C . Solid CO_2 is called “dry ice.” Another example of a gas that can be in a liquid phase is liquid nitrogen (LN_2). LN_2 is made by liquefaction of atmospheric air (through compression and cooling). It boils at 77 K (-196°C) at atmospheric pressure. LN_2 is useful as a refrigerant and allows for the preservation of blood, sperm, and other biological materials. It is also used to reduce noise in electronic sensors and equipment, and to help cool down their current-carrying wires. In dermatology, LN_2 is used to freeze and painlessly remove warts and other growths from the skin.

PV Diagrams

We can examine aspects of the behavior of a substance by plotting a graph of pressure versus volume, called a **PV diagram**. When the substance behaves like an ideal gas, the ideal gas law describes the relationship between its pressure and volume. That is,

$$PV = NkT \text{ (ideal gas).} \quad 13.68$$

Now, assuming the number of molecules and the temperature are fixed,

$$PV = \text{constant (ideal gas, constant temperature).} \quad 13.69$$

For example, the volume of the gas will decrease as the pressure increases. If you plot the relationship $PV = \text{constant}$ on a PV diagram, you find a hyperbola. [Figure 13.27](#) shows a graph of pressure versus volume. The

hyperbolas represent ideal-gas behavior at various fixed temperatures, and are called *isotherms*. At lower temperatures, the curves begin to look less like hyperbolas—the gas is not behaving ideally and may even contain liquid. There is a **critical point**—that is, a **critical temperature**—above which liquid cannot exist. At sufficiently high pressure above the critical point, the gas will have the density of a liquid but will not condense. Carbon dioxide, for example, cannot be liquefied at a temperature above 31.0°C. **Critical pressure** is the minimum pressure needed for liquid to exist at the critical temperature. [Table 13.3](#) lists representative critical temperatures and pressures.

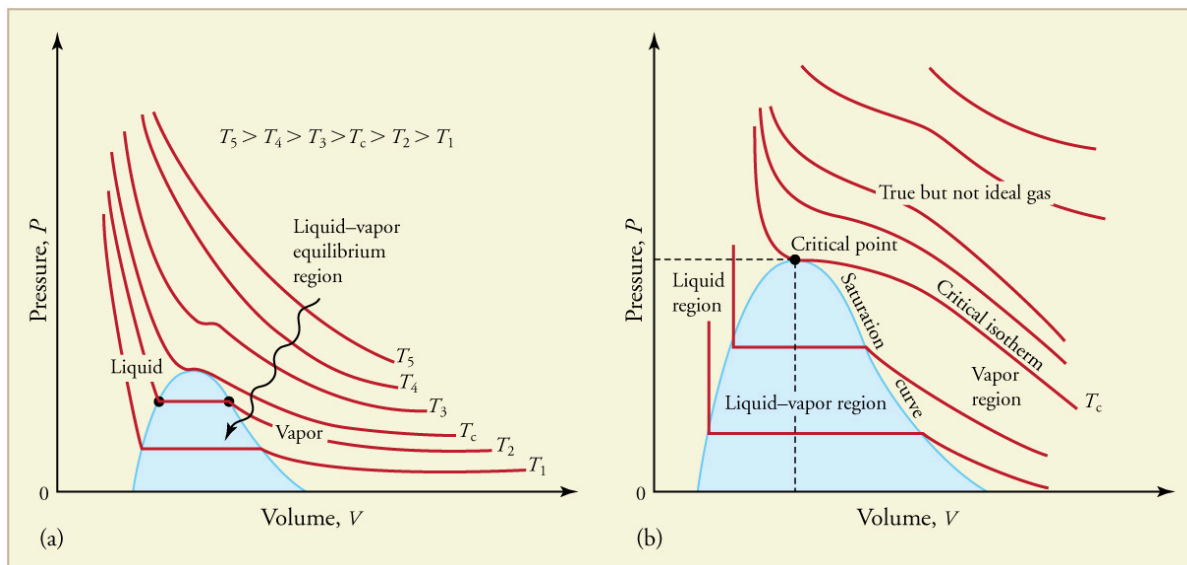


FIGURE 13.27 *PV* diagrams. (a) Each curve (isotherm) represents the relationship between *P* and *V* at a fixed temperature; the upper curves are at higher temperatures. The lower curves are not hyperbolas, because the gas is no longer an ideal gas. (b) An expanded portion of the *PV* diagram for low temperatures, where the phase can change from a gas to a liquid. The term “vapor” refers to the gas phase when it exists at a temperature below the boiling temperature.

Substance	Critical temperature		Critical pressure	
	K	°C	Pa	atm
Water	647.4	374.3	22.12×10^6	219.0
Sulfur dioxide	430.7	157.6	7.88×10^6	78.0
Ammonia	405.5	132.4	11.28×10^6	111.7
Carbon dioxide	304.2	31.1	7.39×10^6	73.2
Oxygen	154.8	−118.4	5.08×10^6	50.3
Nitrogen	126.2	−146.9	3.39×10^6	33.6
Hydrogen	33.3	−239.9	1.30×10^6	12.9
Helium	5.3	−267.9	0.229×10^6	2.27

TABLE 13.3 Critical Temperatures and Pressures

Phase Diagrams

The plots of pressure versus temperatures provide considerable insight into thermal properties of substances. There are well-defined regions on these graphs that correspond to various phases of matter, so *PT* graphs are called

phase diagrams. Figure 13.28 shows the phase diagram for water. Using the graph, if you know the pressure and temperature you can determine the phase of water. The solid lines—boundaries between phases—indicate temperatures and pressures at which the phases coexist (that is, they exist together in ratios, depending on pressure and temperature). For example, the boiling point of water is 100°C at 1.00 atm . As the pressure increases, the boiling temperature rises steadily to 374°C at a pressure of 218 atm . A pressure cooker (or even a covered pot) will cook food faster because the water can exist as a liquid at temperatures greater than 100°C without all boiling away. The curve ends at a point called the *critical point*, because at higher temperatures the liquid phase does not exist at any pressure. The critical point occurs at the critical temperature, as you can see for water from Table 13.3. The critical temperature for oxygen is -118°C , so oxygen cannot be liquefied above this temperature.

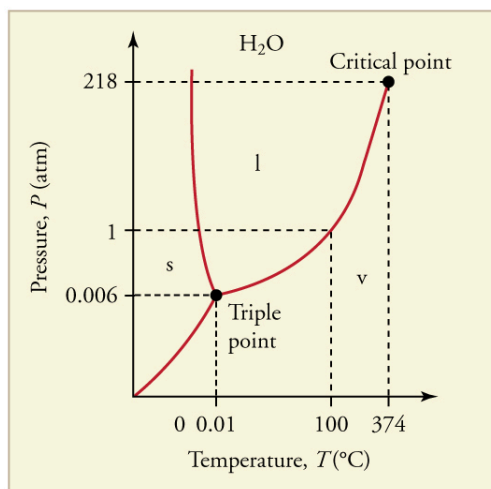


FIGURE 13.28 The phase diagram (PT graph) for water. Note that the axes are nonlinear and the graph is not to scale. This graph is simplified—there are several other exotic phases of ice at higher pressures.

Similarly, the curve between the solid and liquid regions in Figure 13.28 gives the melting temperature at various pressures. For example, the melting point is 0°C at 1.00 atm , as expected. Note that, at a fixed temperature, you can change the phase from solid (ice) to liquid (water) by increasing the pressure. Ice melts from pressure in the hands of a snowball maker. From the phase diagram, we can also say that the melting temperature of ice falls with increased pressure. When a car is driven over snow, the increased pressure from the tires melts the snowflakes; afterwards the water refreezes and forms an ice layer.

At sufficiently low pressures there is no liquid phase, but the substance can exist as either gas or solid. For water, there is no liquid phase at pressures below 0.00600 atm . The phase change from solid to gas is called **sublimation**. It accounts for large losses of snow pack that never make it into a river, the routine automatic defrosting of a freezer, and the freeze-drying process applied to many foods. Carbon dioxide, on the other hand, sublimates at standard atmospheric pressure of 1 atm . (The solid form of CO_2 is known as dry ice because it does not melt. Instead, it moves directly from the solid to the gas state.)

All three curves on the phase diagram meet at a single point, the **triple point**, where all three phases exist in equilibrium. For water, the triple point occurs at 273.16 K (0.01°C), and is a more accurate calibration temperature than the melting point of water at 1.00 atm , or 273.15 K (0.0°C). See Table 13.4 for the triple point values of other substances.

Equilibrium

Liquid and gas phases are in equilibrium at the boiling temperature. (See Figure 13.29.) If a substance is in a closed container at the boiling point, then the liquid is boiling and the gas is condensing at the same rate without net change in their relative amount. Molecules in the liquid escape as a gas at the same rate at which gas molecules stick to the liquid, or form droplets and become part of the liquid phase. The combination of temperature and pressure has to be “just right”; if the temperature and pressure are increased, equilibrium is maintained by the same increase of boiling and condensation rates.

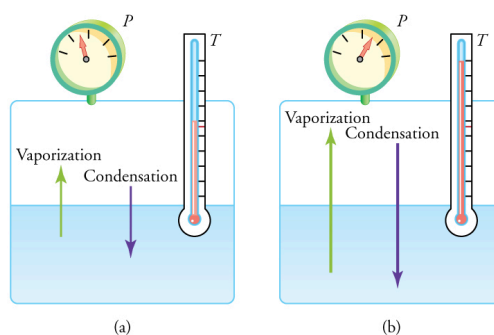


FIGURE 13.29 Equilibrium between liquid and gas at two different boiling points inside a closed container. (a) The rates of boiling and condensation are equal at this combination of temperature and pressure, so the liquid and gas phases are in equilibrium. (b) At a higher temperature, the boiling rate is faster and the rates at which molecules leave the liquid and enter the gas are also faster. Because there are more molecules in the gas, the gas pressure is higher and the rate at which gas molecules condense and enter the liquid is faster. As a result the gas and liquid are in equilibrium at this higher temperature.

Substance	Temperature		Pressure	
	K	°C	Pa	atm
Water	273.16	0.01	6.10×10^2	0.00600
Carbon dioxide	216.55	-56.60	5.16×10^5	5.11
Sulfur dioxide	197.68	-75.47	1.67×10^3	0.0167
Ammonia	195.40	-77.75	6.06×10^3	0.0600
Nitrogen	63.18	-210.0	1.25×10^4	0.124
Oxygen	54.36	-218.8	1.52×10^2	0.00151
Hydrogen	13.84	-259.3	7.04×10^3	0.0697

TABLE 13.4 Triple Point Temperatures and Pressures

One example of equilibrium between liquid and gas is that of water and steam at 100°C and 1.00 atm. This temperature is the boiling point at that pressure, so they should exist in equilibrium. Why does an open pot of water at 100°C boil completely away? The gas surrounding an open pot is not pure water: it is mixed with air. If pure water and steam are in a closed container at 100°C and 1.00 atm, they would coexist—but with air over the pot, there are fewer water molecules to condense, and water boils. What about water at 20.0°C and 1.00 atm? This temperature and pressure correspond to the liquid region, yet an open glass of water at this temperature will completely evaporate. Again, the gas around it is air and not pure water vapor, so that the reduced evaporation rate is greater than the condensation rate of water from dry air. If the glass is sealed, then the liquid phase remains. We call the gas phase a **vapor** when it exists, as it does for water at 20.0°C, at a temperature below the boiling temperature.

✓ CHECK YOUR UNDERSTANDING

Explain why a cup of water (or soda) with ice cubes stays at 0°C, even on a hot summer day.

Solution

The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

Vapor Pressure, Partial Pressure, and Dalton's Law

Vapor pressure is defined as the pressure at which a gas coexists with its solid or liquid phase. Vapor pressure is created by faster molecules that break away from the liquid or solid and enter the gas phase. The vapor pressure of a substance depends on both the substance and its temperature—an increase in temperature increases the vapor pressure.

Partial pressure is defined as the pressure a gas would create if it occupied the total volume available. In a mixture of gases, *the total pressure is the sum of partial pressures of the component gases*, assuming ideal gas behavior and no chemical reactions between the components. This law is known as **Dalton's law of partial pressures**, after the English scientist John Dalton (1766–1844), who proposed it. Dalton's law is based on kinetic theory, where each gas creates its pressure by molecular collisions, independent of other gases present. It is consistent with the fact that pressures add according to [Pascal's Principle](#). Thus water evaporates and ice sublimates when their vapor pressures exceed the partial pressure of water vapor in the surrounding mixture of gases. If their vapor pressures are less than the partial pressure of water vapor in the surrounding gas, liquid droplets or ice crystals (frost) form.

CHECK YOUR UNDERSTANDING

Is energy transfer involved in a phase change? If so, will energy have to be supplied to change phase from solid to liquid and liquid to gas? What about gas to liquid and liquid to solid? Why do they spray the orange trees with water in Florida when the temperatures are near or just below freezing?

Solution

Yes, energy transfer is involved in a phase change. We know that atoms and molecules in solids and liquids are bound to each other because we know that force is required to separate them. So in a phase change from solid to liquid and liquid to gas, a force must be exerted, perhaps by collision, to separate atoms and molecules. Force exerted through a distance is work, and energy is needed to do work to go from solid to liquid and liquid to gas. This is intuitively consistent with the need for energy to melt ice or boil water. The converse is also true. Going from gas to liquid or liquid to solid involves atoms and molecules pushing together, doing work and releasing energy.



PHET EXPLORATIONS

States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/13-5-phase-changes\)](https://openstax.org/books/college-physics-2e/pages/13-5-phase-changes)



13.6 Humidity, Evaporation, and Boiling

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the relationship between vapor pressure of water and the capacity of air to hold water vapor.
- Explain the relationship between relative humidity and partial pressure of water vapor in the air.
- Calculate vapor density using vapor pressure.
- Calculate humidity and dew point.

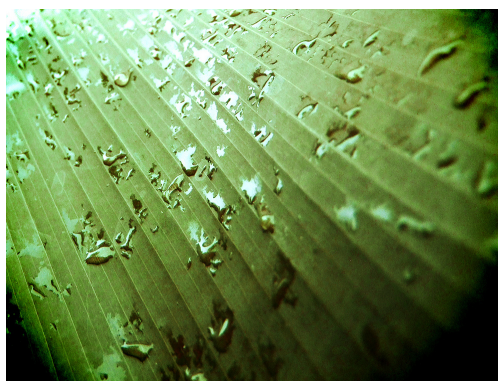


FIGURE 13.30 Dew drops like these, on a banana leaf photographed just after sunrise, form when the air temperature drops to or below the dew point. At the dew point, the rate at which water molecules join together is greater than the rate at which they separate, and some of the water condenses to form droplets. (credit: Aaron Escobar, Flickr)

The expression “it’s not the heat, it’s the humidity” makes a valid point. We keep cool in hot weather by evaporating sweat from our skin and water from our breathing passages. Because evaporation is inhibited by high humidity, we feel hotter at a given temperature when the humidity is high. Low humidity, on the other hand, can cause discomfort from excessive drying of mucous membranes and can lead to an increased risk of respiratory infections.

When we say humidity, we really mean **relative humidity**. Relative humidity tells us how much water vapor is in the air compared with the maximum possible. At its maximum, denoted as **saturation**, the relative humidity is 100%, and evaporation is inhibited. The amount of water vapor in the air depends on temperature. For example, relative humidity rises in the evening, as air temperature declines, sometimes reaching the **dew point**. At the dew point temperature, relative humidity is 100%, and fog may result from the condensation of water droplets if they are small enough to stay in suspension. Conversely, if you wish to dry something (perhaps your hair), it is more effective to blow hot air over it rather than cold air, because, among other things, the increase in temperature increases the energy of the molecules, so the rate of evaporation increases.

The amount of water vapor in the air depends on the vapor pressure of water. The liquid and solid phases are continuously giving off vapor because some of the molecules have high enough speeds to enter the gas phase; see [Figure 13.31\(a\)](#). If a lid is placed over the container, as in [Figure 13.31\(b\)](#), evaporation continues, increasing the pressure, until sufficient vapor has built up for condensation to balance evaporation. Then equilibrium has been achieved, and the vapor pressure is equal to the partial pressure of water in the container. Vapor pressure increases with temperature because molecular speeds are higher as temperature increases. [Table 13.5](#) gives representative values of water vapor pressure over a range of temperatures.

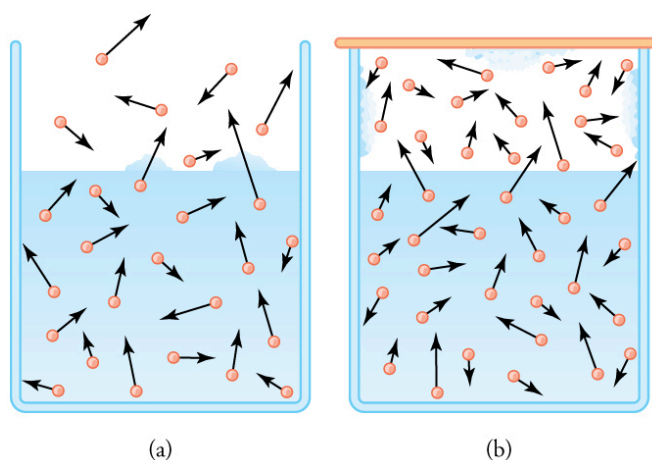


FIGURE 13.31 (a) Because of the distribution of speeds and kinetic energies, some water molecules can break away to the vapor phase even at temperatures below the ordinary boiling point. (b) If the container is sealed, evaporation will continue until there is enough vapor density for the condensation rate to equal the evaporation rate. This vapor density and the partial pressure it creates are the saturation values. They increase with temperature and are independent of the presence of other gases, such as air. They depend only on the vapor pressure of water.

Relative humidity is related to the partial pressure of water vapor in the air. At 100% humidity, the partial pressure is

equal to the vapor pressure, and no more water can enter the vapor phase. If the partial pressure is less than the vapor pressure, then evaporation will take place, as humidity is less than 100%. If the partial pressure is greater than the vapor pressure, condensation takes place. In everyday language, people sometimes refer to the capacity of air to “hold” water vapor, but this is not actually what happens. The water vapor is not held by the air. The amount of water in air is determined by the vapor pressure of water and has nothing to do with the properties of air.

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m ³)
-50	4.0	0.039
-20	1.04×10^2	0.89
-10	2.60×10^2	2.36
0	6.10×10^2	4.84
5	8.68×10^2	6.80
10	1.19×10^3	9.40
15	1.69×10^3	12.8
20	2.33×10^3	17.2
25	3.17×10^3	23.0
30	4.24×10^3	30.4
37	6.31×10^3	44.0
40	7.34×10^3	51.1
50	1.23×10^4	82.4
60	1.99×10^4	130
70	3.12×10^4	197
80	4.73×10^4	294
90	7.01×10^4	418
95	8.59×10^4	505
100	1.01×10^5	598
120	1.99×10^5	1095
150	4.76×10^5	2430

TABLE 13.5 Saturation Vapor Density of Water

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m ³)
200	1.55×10^6	7090
220	2.32×10^6	10,200

TABLE 13.5 Saturation Vapor Density of Water

**EXAMPLE 13.12****Calculating Density Using Vapor Pressure**

Table 13.5 gives the vapor pressure of water at 20.0°C as 2.33×10^3 Pa. Use the ideal gas law to calculate the density of water vapor in g/m³ that would create a partial pressure equal to this vapor pressure. Compare the result with the saturation vapor density given in the table.

Strategy

To solve this problem, we need to break it down into a two steps. The partial pressure follows the ideal gas law,

$$PV = nRT, \quad 13.70$$

where n is the number of moles. If we solve this equation for n/V to calculate the number of moles per cubic meter, we can then convert this quantity to grams per cubic meter as requested. To do this, we need to use the molecular mass of water, which is given in the periodic table.

Solution

1. Identify the knowns and convert them to the proper units:

- temperature $T = 20^\circ\text{C} = 293 \text{ K}$
- vapor pressure P of water at 20°C is $2.33 \times 10^3 \text{ Pa}$
- molecular mass of water is 18.0 g/mol

2. Solve the ideal gas law for n/V .

$$\frac{n}{V} = \frac{P}{RT} \quad 13.71$$

3. Substitute known values into the equation and solve for n/V .

$$\frac{n}{V} = \frac{P}{RT} = \frac{2.33 \times 10^3 \text{ Pa}}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.957 \text{ mol/m}^3 \quad 13.72$$

4. Convert the density in moles per cubic meter to grams per cubic meter.

$$\rho = \left(0.957 \frac{\text{mol}}{\text{m}^3}\right) \left(\frac{18.0 \text{ g}}{\text{mol}}\right) = 17.2 \text{ g/m}^3 \quad 13.73$$

Discussion

The density is obtained by assuming a pressure equal to the vapor pressure of water at 20.0°C. The density found is identical to the value in Table 13.5, which means that a vapor density of 17.2 g/m^3 at 20.0°C creates a partial pressure of $2.33 \times 10^3 \text{ Pa}$, equal to the vapor pressure of water at that temperature. If the partial pressure is equal to the vapor pressure, then the liquid and vapor phases are in equilibrium, and the relative humidity is 100%. Thus, there can be no more than 17.2 g of water vapor per m^3 at 20.0°C , so that this value is the saturation vapor density at that temperature. This example illustrates how water vapor behaves like an ideal gas: the pressure and density are consistent with the ideal gas law (assuming the density in the table is correct). The saturation vapor densities listed in Table 13.5 are the maximum amounts of water vapor that air can hold at various temperatures.

Percent Relative Humidity

We define **percent relative humidity** as the ratio of vapor density to saturation vapor density, or

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100 \quad 13.74$$

We can use this and the data in [Table 13.5](#) to do a variety of interesting calculations, keeping in mind that relative humidity is based on the comparison of the partial pressure of water vapor in air and ice.



EXAMPLE 13.13

Calculating Humidity and Dew Point

(a) Calculate the percent relative humidity on a day when the temperature is 25.0°C and the air contains 9.40 g of water vapor per m³. (b) At what temperature will this air reach 100% relative humidity (the saturation density)? This temperature is the dew point. (c) What is the humidity when the air temperature is 25.0°C and the dew point is –10.0°C?

Strategy and Solution

(a) Percent relative humidity is defined as the ratio of vapor density to saturation vapor density.

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100 \quad 13.75$$

The first is given to be 9.40 g/m³, and the second is found in [Table 13.5](#) to be 23.0 g/m³. Thus,

$$\text{percent relative humidity} = \frac{9.40 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 40.9\% \quad 13.76$$

(b) The air contains 9.40 g/m³ of water vapor. The relative humidity will be 100% at a temperature where 9.40 g/m³ is the saturation density. Inspection of [Table 13.5](#) reveals this to be the case at 10.0°C, where the relative humidity will be 100%. That temperature is called the dew point for air with this concentration of water vapor.

(c) Here, the dew point temperature is given to be –10.0°C. Using [Table 13.5](#), we see that the vapor density is 2.36 g/m³, because this value is the saturation vapor density at –10.0°C. The saturation vapor density at 25.0°C is seen to be 23.0 g/m³. Thus, the relative humidity at 25.0°C is

$$\text{percent relative humidity} = \frac{2.36 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 10.3\% \quad 13.77$$

Discussion

The importance of dew point is that air temperature cannot drop below 10.0°C in part (b), or –10.0°C in part (c), without water vapor condensing out of the air. If condensation occurs, considerable transfer of heat occurs (discussed in [Heat and Heat Transfer Methods](#)), which prevents the temperature from further dropping. When dew points are below 0°C, freezing temperatures are a greater possibility, which explains why farmers keep track of the dew point. Low humidity in deserts means low dew-point temperatures. Thus condensation is unlikely. If the temperature drops, vapor does not condense in liquid drops. Because no heat is released into the air, the air temperature drops more rapidly compared to air with higher humidity. Likewise, at high temperatures, liquid droplets do not evaporate, so that no heat is removed from the gas to the liquid phase. This explains the large range of temperature in arid regions.

Why does water boil at 100°C? You will note from [Table 13.5](#) that the vapor pressure of water at 100°C is 1.01×10^5 Pa, or 1.00 atm. Thus, it can evaporate without limit at this temperature and pressure. But why does it

form bubbles when it boils? This is because water ordinarily contains significant amounts of dissolved air and other impurities, which are observed as small bubbles of air in a glass of water. If a bubble starts out at the bottom of the container at 20°C , it contains water vapor (about 2.30%). The pressure inside the bubble is fixed at 1.00 atm (we ignore the slight pressure exerted by the water around it). As the temperature rises, the amount of air in the bubble stays the same, but the water vapor increases; the bubble expands to keep the pressure at 1.00 atm. At 100°C , water vapor enters the bubble continuously since the partial pressure of water is equal to 1.00 atm in equilibrium. It cannot reach this pressure, however, since the bubble also contains air and total pressure is 1.00 atm. The bubble grows in size and thereby increases the buoyant force. The bubble breaks away and rises rapidly to the surface—we call this boiling! (See [Figure 13.32](#).)

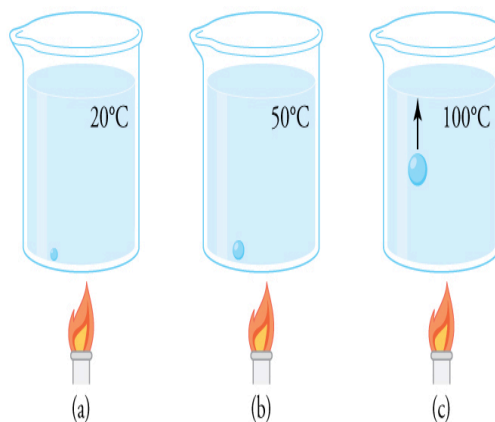


FIGURE 13.32 (a) An air bubble in water starts out saturated with water vapor at 20°C . (b) As the temperature rises, water vapor enters the bubble because its vapor pressure increases. The bubble expands to keep its pressure at 1.00 atm. (c) At 100°C , water vapor enters the bubble continuously because water's vapor pressure exceeds its partial pressure in the bubble, which must be less than 1.00 atm. The bubble grows and rises to the surface.

✓ CHECK YOUR UNDERSTANDING

Freeze drying is a process in which substances, such as foods, are dried by placing them in a vacuum chamber and lowering the atmospheric pressure around them. How does the lowered atmospheric pressure speed the drying process, and why does it cause the temperature of the food to drop?

Solution

Decreased the atmospheric pressure results in decreased partial pressure of water, hence a lower humidity. So evaporation of water from food, for example, will be enhanced. The molecules of water most likely to break away from the food will be those with the greatest velocities. Those remaining thus have a lower average velocity and a lower temperature. This can (and does) result in the freezing and drying of the food; hence the process is aptly named freeze drying.

PHET EXPLORATIONS

States of Matter

Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.

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Glossary

absolute zero the lowest possible temperature; the temperature at which all molecular motion ceases

Avogadro's number N_A , the number of molecules or atoms in one mole of a substance;
 $N_A = 6.02 \times 10^{23}$ particles/mole

Boltzmann constant k , a physical constant that relates energy to temperature;
 $k = 1.38 \times 10^{-23}$ J/K

Celsius scale temperature scale in which the freezing point of water is 0°C and the boiling point of water is 100°C

coefficient of linear expansion α , the change in length, per unit length, per 1°C change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

coefficient of volume expansion β , the change in volume, per unit volume, per 1°C change in temperature

critical point the temperature above which a liquid cannot exist

critical pressure the minimum pressure needed for a liquid to exist at the critical temperature

critical temperature the temperature above which a liquid cannot exist

Dalton's law of partial pressures the physical law that states that the total pressure of a gas is the sum of partial pressures of the component gases

degree Celsius unit on the Celsius temperature scale

degree Fahrenheit unit on the Fahrenheit temperature scale

dew point the temperature at which relative humidity is 100%; the temperature at which water starts to condense out of the air

Fahrenheit scale temperature scale in which the freezing point of water is 32°F and the boiling point of water is 212°F

ideal gas law the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Kelvin scale temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

mole the quantity of a substance whose mass (in grams) is equal to its molecular mass

partial pressure the pressure a gas would create if it occupied the total volume of space available

percent relative humidity the ratio of vapor density to saturation vapor density

phase diagram a graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the three phases of the substance occur

PV diagram a graph of pressure vs. volume

relative humidity the amount of water in the air relative to the maximum amount the air can hold

saturation the condition of 100% relative humidity

sublimation the phase change from solid to gas

temperature the quantity measured by a thermometer

thermal energy $\overline{KE}$, the average translational kinetic energy of a molecule

thermal equilibrium the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

thermal expansion the change in size or volume of an object with change in temperature

thermal stress stress caused by thermal expansion or contraction

triple point the pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas

vapor a gas at a temperature below the boiling temperature

vapor pressure the pressure at which a gas coexists with its solid or liquid phase

zeroth law of thermodynamics law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

Section Summary

13.1 Temperature

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.

- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T_{\text{F}} = \frac{9}{5}T_{\text{C}} + 32$$

$$T_{\text{C}} = \frac{5}{9}(T_{\text{F}} - 32)$$

$$T_{\text{K}} = T_{\text{C}} + 273.15$$

$$T_{\text{C}} = T_{\text{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

13.2 Thermal Expansion of Solids and Liquids

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.
- Linear thermal expansion is $\Delta L = \alpha L \Delta T$, where ΔL is the change in length L , ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.
- The change in area due to thermal expansion is $\Delta A = 2\alpha A \Delta T$, where ΔA is the change in area.
- The change in volume due to thermal expansion is $\Delta V = \beta V \Delta T$, where β is the coefficient of volume expansion and $\beta \approx 3\alpha$. Thermal stress is created when thermal expansion is constrained.

13.3 The Ideal Gas Law

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas: $PV = NkT$, where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant $k = 1.38 \times 10^{-23} \text{ J/K}$.
- A mole is the number of atoms in a 12-g sample of carbon-12.
- The number of molecules in a mole is called Avogadro's number N_{A} ,

$$N_{\text{A}} = 6.02 \times 10^{23} \text{ mol}^{-1}.$$

- A mole of any substance has a mass in grams equal to its molecular weight, which can be determined from the periodic table of elements.
- The ideal gas law can also be written and solved in terms of the number of moles of gas: $PV = nRT$, where n is number of moles and R is the universal gas constant, $R = 8.31 \text{ J/mol} \cdot \text{K}$.
- The ideal gas law is generally valid at temperatures well above the boiling temperature.

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

- Kinetic theory is the atomistic description of gases as well as liquids and solids.
- Kinetic theory models the properties of matter in terms of continuous random motion of atoms and molecules.
- The ideal gas law can also be expressed as $PV = \frac{1}{3} N m \overline{v^2}$, where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and $\overline{v^2}$ is the average of the molecular speed squared.
- Thermal energy is defined to be the average translational kinetic energy $\overline{\text{KE}}$ of an atom or molecule.
- The temperature of gases is proportional to the average translational kinetic energy of atoms and molecules.
$$\overline{\text{KE}} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$
 or
$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}.$$
- The motion of individual molecules in a gas is random in magnitude and direction. However, a gas of many molecules has a predictable distribution of molecular speeds, known as the *Maxwell-Boltzmann distribution*.

13.5 Phase Changes

- Most substances have three distinct phases: gas, liquid, and solid.
- Phase changes among the various phases of matter depend on temperature and pressure.
- The existence of the three phases with respect to pressure and temperature can be described in a phase diagram.

- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures. These are described as a line on a phase diagram.
- The three phases coexist at a single pressure and temperature. This is known as the triple point and is described by a single point on a phase diagram.
- A gas at a temperature below its boiling point is called a vapor.
- Vapor pressure is the pressure at which a gas coexists with its solid or liquid phase.
- Partial pressure is the pressure a gas would create if it existed alone.
- Dalton's law states that the total pressure is the

sum of the partial pressures of all of the gases present.

13.6 Humidity, Evaporation, and Boiling

- Relative humidity is the fraction of water vapor in a gas compared to the saturation value.
- The saturation vapor density can be determined from the vapor pressure for a given temperature.
- Percent relative humidity is defined to be

$$\frac{\text{vapor density}}{\text{saturation vapor density}} \times 100.$$
- The dew point is the temperature at which air reaches 100% relative humidity.

Conceptual Questions

13.1 Temperature

1. What does it mean to say that two systems are in thermal equilibrium?
2. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.
3. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.
4. If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

13.2 Thermal Expansion of Solids and Liquids

5. Thermal stresses caused by uneven cooling can easily break glass cookware. Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.
6. Water expands significantly when it freezes: a volume increase of about 9% occurs. As a result of this expansion and because of the formation and growth of crystals as water freezes, anywhere from 10% to 30% of biological cells are burst when animal or plant material is frozen. Discuss the implications of this cell damage for the prospect of preserving human bodies by freezing so that they can be thawed at some future date when it is hoped that all diseases are curable.

7. One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.
8. Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.
9. Liquids and solids expand with increasing temperature, because the kinetic energy of a body's atoms and molecules increases. Explain why some materials *shrink* with increasing temperature.

13.3 The Ideal Gas Law

10. Find out the human population of Earth. Is there a mole of people inhabiting Earth? If the average mass of a person is 60 kg, calculate the mass of a mole of people. How does the mass of a mole of people compare with the mass of Earth?
11. Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?
12. A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

13. How is momentum related to the pressure exerted by a gas? Explain on the atomic and molecular level, considering the behavior of atoms and molecules.

13.5 Phase Changes

14. A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?
15. Why does condensation form most rapidly on the coldest object in a room—for example, on a glass of ice water?
16. What is the vapor pressure of solid carbon dioxide (dry ice) at -78.5°C ?

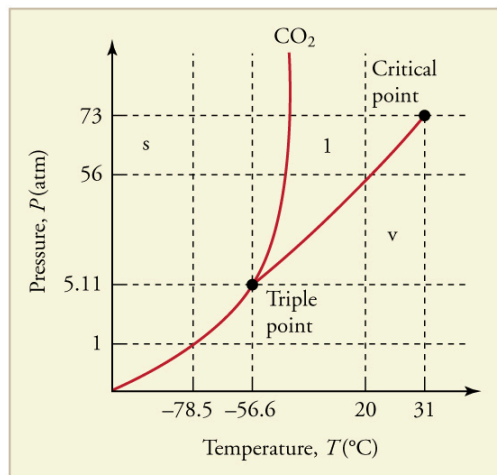


FIGURE 13.33 The phase diagram for carbon dioxide. The axes are nonlinear, and the graph is not to scale. Dry ice is solid carbon dioxide and has a sublimation temperature of -78.5°C .

17. Can carbon dioxide be liquefied at room temperature (20°C)? If so, how? If not, why not? (See [Figure 13.33](#).)
18. Oxygen cannot be liquefied at room temperature by placing it under a large enough pressure to force its molecules together. Explain why this is.
19. What is the distinction between gas and vapor?

13.6 Humidity, Evaporation, and Boiling

20. Because humidity depends only on water's vapor pressure and temperature, are the saturation vapor densities listed in [Table 13.5](#) valid in an atmosphere of helium at a pressure of $1.01 \times 10^5 \text{ N/m}^2$, rather than air? Are those values affected by altitude on Earth?
21. Why does a beaker of 40.0°C water placed in a vacuum chamber start to boil as the chamber is evacuated (air is pumped out of the chamber)? At what pressure does the boiling begin? Would food cook any faster in such a beaker?
22. Why does rubbing alcohol evaporate much more rapidly than water at STP (standard temperature and pressure)?

Problems & Exercises

13.1 Temperature

1. What is the Fahrenheit temperature of a person with a 39.0°C fever?
2. Frost damage to most plants occurs at temperatures of 28.0°F or lower. What is this temperature on the Kelvin scale?
3. To conserve energy, room temperatures are kept at 68.0°F in the winter and 78.0°F in the summer. What are these temperatures on the Celsius scale?
4. A tungsten light bulb filament may operate at 2900 K . What is its Fahrenheit temperature? What is this on the Celsius scale?
5. The surface temperature of the Sun is about 5750 K . What is this temperature on the Fahrenheit scale?
6. One of the hottest temperatures ever recorded on the surface of Earth was 134°F in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

7. (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0°F decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.
8. (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

13.2 Thermal Expansion of Solids and Liquids

9. The height of the Washington Monument is measured to be 170 m on a day when the temperature is 35.0°C . What will its height be on a day when the temperature falls to -10.0°C ? Although the monument is made of limestone, assume that its thermal coefficient of expansion is the same as marble's.

10. How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by 15°C ? Its original height is 321 m and you can assume it is made of steel.
11. What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from 37.0°C to 40.0°C , assuming the mercury is unconstrained?
12. How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature 35.0°C greater than when they were laid? Their original length is 10.0 m.
13. You are looking to purchase a small piece of land in Hong Kong. The price is “only” \$60,000 per square meter! The land title says the dimensions are 20 m $\times$ 30 m. By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature was 20°C above normal?
14. Global warming will produce rising sea levels partly due to melting ice caps but also due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of 1.00°C . Note that this calculation is only approximate because ocean warming is not uniform with depth.
15. Show that 60.0 L of gasoline originally at 15.0°C will expand to 61.1 L when it warms to 35.0°C , as claimed in [Example 13.4](#).
16. (a) Suppose a meter stick made of steel and one made of invar (an alloy of iron and nickel) are the same length at 0°C . What is their difference in length at 22.0°C ? (b) Repeat the calculation for two 30.0-m-long surveyor’s tapes.
17. (a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of 5.00°C , how much will overflow when its temperature reaches 22.0°C ? (b) How much less water would overflow under the same conditions?
18. Most automobiles have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at 10.0°C . What volume of radiator fluid will overflow when the radiator and fluid reach their 95.0°C operating temperature, given that the fluid’s volume coefficient of expansion is $\beta = 400 \times 10^{-6}/^{\circ}\text{C}$? Note that this coefficient is approximate, because most car radiators have operating temperatures of greater than 95.0°C .
19. A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the 350 cm^3 of coffee is in a 7.00-cm-diameter cup and decreases in temperature from 95.0°C to 45.0°C . (Most of the drop in level is actually due to escaping bubbles of air.)
20. (a) The density of water at 0°C is very nearly 1000 kg/m^3 (it is actually 999.84 kg/m^3), whereas the density of ice at 0°C is 917 kg/m^3 . Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.) (b) What are the implications of this result for biological cells that are frozen?
21. Show that $\beta \approx 3\alpha$, by calculating the change in volume ΔV of a cube with sides of length L .

13.3 The Ideal Gas Law

22. The gauge pressure in your car tires is $2.50 \times 10^5\text{ N/m}^2$ at a temperature of 35.0°C when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to -40.0°C ?
23. Convert an absolute pressure of $7.00 \times 10^5\text{ N/m}^2$ to gauge pressure in lb/in^2 . (This value was stated to be just less than 90.0 lb/in^2 in [Example 13.9](#). Is it?)
24. Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of 20.0°C . (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is 60.0°C (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

- 25.** To test a balloon, it is placed in a lab and filled with helium. The temperature of the helium is 10.0°C and the pressure is 1.00 atmosphere. The pressure in the lab is maintained. Assume the membrane of the balloon provides a negligible inward pressure, so it is not considered significant. (a) What is the pressure inside the balloon if the helium is replaced with helium that is at -50.0°C ? and the balloon is filled until it has a volume of 20.0 times its original volume? (b) What is the gauge pressure? (Assume the pressure in the lab remains at 1.00 atmosphere during the experiment.)
- 26.** Confirm that the units of nRT are those of energy for each value of R : (a) $8.31 \text{ J/mol} \cdot \text{K}$, (b) $1.99 \text{ cal/mol} \cdot \text{K}$, and (c) $0.0821 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$.
- 27.** In the text, it was shown that $N/V = 2.68 \times 10^{25} \text{ m}^{-3}$ for gas at STP. (a) Show that this quantity is equivalent to $N/V = 2.68 \times 10^{19} \text{ cm}^{-3}$, as stated. (b) About how many atoms are there in one μm^3 (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?
- 28.** Calculate the number of moles in the 2.00-L volume of air in the lungs of the average person. Note that the air is at 37.0°C (body temperature).
- 29.** An airplane passenger has 100 cm^3 of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50 \times 10^4 \text{ N/m}^2$?
- 30.** (a) What is the volume (in km^3) of Avogadro's number of sand grains if each grain is a cube and has sides that are 1.0 mm long? (b) How many kilometers of beaches in length would this cover if the beach averages 100 m in width and 10.0 m in depth? Neglect air spaces between grains.
- 31.** An expensive vacuum system can achieve a pressure as low as $1.00 \times 10^{-7} \text{ N/m}^2$ at 20°C . How many atoms are there in a cubic centimeter at this pressure and temperature?
- 32.** The number density of gas atoms at a certain location in the space above our planet is about $1.00 \times 10^{11} \text{ m}^{-3}$, and the pressure is $2.75 \times 10^{-10} \text{ N/m}^2$ in this space. What is the temperature there?
- 33.** A bicycle tire has a pressure of $7.00 \times 10^5 \text{ N/m}^2$ at a temperature of 18.0°C and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of 100 cm^3 at atmospheric pressure? Assume tire temperature and volume remain constant.
- 34.** A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40 \times 10^7 \text{ N/m}^2$ and a temperature of 25.0°C . Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature (-78.5°C) to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?
- 35.** Find the number of moles in 2.00 L of gas at 35.0°C and under $7.41 \times 10^7 \text{ N/m}^2$ of pressure.
- 36.** Calculate the depth to which Avogadro's number of table tennis balls would cover Earth. Each ball has a diameter of 3.75 cm. Assume the space between balls adds an extra 25.0% to their volume and assume they are not crushed by their own weight.
- 37.** (a) What is the gauge pressure in a 25.0°C car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and 25.0°C ? Assume the temperature returns to 25.0°C and the volume remains constant.
- 38.** (a) In the deep space between galaxies, the density of atoms is as low as 10^6 atoms/m^3 , and the temperature is a frigid 2.7 K. What is the pressure? (b) What volume (in m^3) is occupied by 1 mol of gas? (c) If this volume is a cube, what is the length of its sides in kilometers?

13.4 Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

- 39.** Some incandescent light bulbs are filled with argon gas. What is v_{rms} for argon atoms near the filament, assuming their temperature is 2500 K?
- 40.** Average atomic and molecular speeds (v_{rms}) are large, even at low temperatures. What is v_{rms} for helium atoms at 5.00 K, just one degree above helium's liquefaction temperature?
- 41.** (a) What is the average kinetic energy in joules of hydrogen atoms on the 5500°C surface of the Sun? (b) What is the average kinetic energy of helium atoms in a region of the solar corona where the temperature is $6.00 \times 10^5 \text{ K}$?

42. The escape velocity of any object from Earth is 11.2 km/s. (a) Express this speed in m/s and km/h. (b) At what temperature would oxygen molecules (molecular mass is equal to 32.0 g/mol) have an average velocity v_{rms} equal to Earth's escape velocity of 11.1 km/s?
43. The escape velocity from the Moon is much smaller than from Earth and is only 2.38 km/s. At what temperature would hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity v_{rms} equal to the Moon's escape velocity?
44. Nuclear fusion, the energy source of the Sun, hydrogen bombs, and fusion reactors, occurs much more readily when the average kinetic energy of the atoms is high—that is, at high temperatures. Suppose you want the atoms in your fusion experiment to have average kinetic energies of 6.40×10^{-14} J. What temperature is needed?
45. Suppose that the average velocity (v_{rms}) of carbon dioxide molecules (molecular mass is equal to 44.0 g/mol) in a flame is found to be 1.05×10^5 m/s. What temperature does this represent?
46. Hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity v_{rms} equal to 193 m/s. What is the temperature?
47. Much of the gas near the Sun is atomic hydrogen. Its temperature would have to be 1.5×10^7 K for the average velocity v_{rms} to equal the escape velocity from the Sun. What is that velocity?
48. There are two important isotopes of uranium— ^{235}U and ^{238}U ; these isotopes are nearly identical chemically but have different atomic masses. Only ^{235}U is very useful in nuclear reactors. One of the techniques for separating them (gas diffusion) is based on the different average velocities v_{rms} of uranium hexafluoride gas, UF_6 . (a) The molecular masses for $^{235}\text{U UF}_6$ and $^{238}\text{U UF}_6$ are 349.0 g/mol and 352.0 g/mol, respectively. What is the ratio of their average velocities? (b) At what temperature would their average velocities differ by 1.00 m/s? (c) Do your answers in this problem imply that this technique may be difficult?
50. (a) What is the vapor pressure of water at 20.0°C? (b) What percentage of atmospheric pressure does this correspond to? (c) What percent of 20.0°C air is water vapor if it has 100% relative humidity? (The density of dry air at 20.0°C is 1.20 kg/m^3 .)
51. Pressure cookers increase cooking speed by raising the boiling temperature of water above its value at atmospheric pressure. (a) What pressure is necessary to raise the boiling point to 120.0°C? (b) What gauge pressure does this correspond to?
52. (a) At what temperature does water boil at an altitude of 1500 m (about 5000 ft) on a day when atmospheric pressure is $8.59 \times 10^4 \text{ N/m}^2$? (b) What about at an altitude of 3000 m (about 10,000 ft) when atmospheric pressure is $7.00 \times 10^4 \text{ N/m}^2$?
53. What is the atmospheric pressure on top of Mt. Everest on a day when water boils there at a temperature of 70.0°C?
54. At a spot in the high Andes, water boils at 80.0°C, greatly reducing the cooking speed of potatoes, for example. What is atmospheric pressure at this location?
55. What is the relative humidity on a 25.0°C day when the air contains 18.0 g/m^3 of water vapor?
56. What is the density of water vapor in g/m^3 on a hot dry day in the desert when the temperature is 40.0°C and the relative humidity is 6.00%?
57. A deep-sea diver should breathe a gas mixture that has the same oxygen partial pressure as at sea level, where dry air contains 20.9% oxygen and has a total pressure of $1.01 \times 10^5 \text{ N/m}^2$. (a) What is the partial pressure of oxygen at sea level? (b) If the diver breathes a gas mixture at a pressure of $2.00 \times 10^6 \text{ N/m}^2$, what percent oxygen should it be to have the same oxygen partial pressure as at sea level?
58. The vapor pressure of water at 40.0°C is $7.34 \times 10^3 \text{ N/m}^2$. Using the ideal gas law, calculate the density of water vapor in g/m^3 that creates a partial pressure equal to this vapor pressure. The result should be the same as the saturation vapor density at that temperature (51.1 g/m^3).
59. Air in human lungs has a temperature of 37.0°C and a saturation vapor density of 44.0 g/m^3 . (a) If 2.00 L of air is exhaled and very dry air inhaled, what is the maximum loss of water vapor by the person? (b) Calculate the partial pressure of water vapor having this density, and compare it with the vapor pressure of $6.31 \times 10^3 \text{ N/m}^2$.

13.6 Humidity, Evaporation, and Boiling

49. Dry air is 78.1% nitrogen. What is the partial pressure of nitrogen when the atmospheric pressure is $1.01 \times 10^5 \text{ N/m}^2$?

- 60.** If the relative humidity is 90.0% on a muggy summer morning when the temperature is 20.0°C , what will it be later in the day when the temperature is 30.0°C , assuming the water vapor density remains constant?
- 61.** Late on an autumn day, the relative humidity is 45.0% and the temperature is 20.0°C . What will the relative humidity be that evening when the temperature has dropped to 10.0°C , assuming constant water vapor density?
- 62.** Atmospheric pressure atop Mt. Everest is $3.30 \times 10^4 \text{ N/m}^2$. (a) What is the partial pressure of oxygen there if it is 20.9% of the air? (b) What percent oxygen should a mountain climber breathe so that its partial pressure is the same as at sea level, where atmospheric pressure is $1.01 \times 10^5 \text{ N/m}^2$? (c) One of the most severe problems for those climbing very high mountains is the extreme drying of breathing passages. Why does this drying occur?
- 63.** What is the dew point (the temperature at which 100% relative humidity would occur) on a day when relative humidity is 39.0% at a temperature of 20.0°C ?
- 64.** On a certain day, the temperature is 25.0°C and the relative humidity is 90.0%. How many grams of water must condense out of each cubic meter of air if the temperature falls to 15.0°C ? Such a drop in temperature can, thus, produce heavy dew or fog.
- 65. Integrated Concepts**
The boiling point of water increases with depth because pressure increases with depth. At what depth will fresh water have a boiling point of 150°C , if the surface of the water is at sea level?
- 66. Integrated Concepts**
(a) At what depth in fresh water is the critical pressure of water reached, given that the surface is at sea level? (b) At what temperature will this water boil? (c) Is a significantly higher temperature needed to boil water at a greater depth?
- 67. Integrated Concepts**
To get an idea of the small effect that temperature has on Archimedes' principle, calculate the fraction of a copper block's weight that is supported by the buoyant force in 0°C water and compare this fraction with the fraction supported in 95.0°C water.
- 68. Integrated Concepts**
If you want to cook in water at 150°C , you need a pressure cooker that can withstand the necessary pressure. (a) What pressure is required for the boiling point of water to be this high? (b) If the lid of the pressure cooker is a disk 25.0 cm in diameter, what force must it be able to withstand at this pressure?
- 69. Unreasonable Results**
(a) How many moles per cubic meter of an ideal gas are there at a pressure of $1.00 \times 10^{14} \text{ N/m}^2$ and at 0°C ? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?
- 70. Unreasonable Results**
(a) An automobile mechanic claims that an aluminum rod fits loosely into its hole on an aluminum engine block because the engine is hot and the rod is cold. If the hole is 10.0% bigger in diameter than the 22.0°C rod, at what temperature will the rod be the same size as the hole? (b) What is unreasonable about this temperature? (c) Which premise is responsible?
- 71. Unreasonable Results**
The temperature inside a supernova explosion is said to be $2.00 \times 10^{13} \text{ K}$. (a) What would the average velocity v_{rms} of hydrogen atoms be? (b) What is unreasonable about this velocity? (c) Which premise or assumption is responsible?
- 72. Unreasonable Results**
Suppose the relative humidity is 80% on a day when the temperature is 30.0°C . (a) What will the relative humidity be if the air cools to 25.0°C and the vapor density remains constant? (b) What is unreasonable about this result? (c) Which premise is responsible?

CHAPTER 14

Heat and Heat Transfer Methods



FIGURE 14.1 Eunice Newton Foote was the first to determine the relationship between carbon dioxide, water vapor, and the potential for global heating. She designed and conducted a number of experiments to uncover the ability of different gases to trap heat, describing what would later be referred to as greenhouse gases. (credit: Carlyn Iverson, NOAA Climate.gov)

CHAPTER OUTLINE

14.1 Heat

14.2 Temperature Change and Heat Capacity

14.3 Phase Change and Latent Heat

14.4 Heat Transfer Methods

14.5 Conduction

14.6 Convection

14.7 Radiation

INTRODUCTION TO HEAT AND HEAT TRANSFER METHODS Energy can exist in many forms and heat is one of the most intriguing. Heat is often hidden, as it only exists when in transit, and is transferred by a number of distinctly different methods. Heat transfer touches every aspect of our lives and helps us understand how the universe functions. It explains the chill we feel on a clear breezy night, or why Earth's core has yet to cool. This chapter defines and explores heat transfer, its effects, and the methods by which heat is transferred. These topics are fundamental, as well as practical, and will often be referred to in the chapters ahead.

14.1 Heat

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define heat as transfer of energy.

In [Work, Energy, and Energy Resources](#), we defined work as force times distance and learned that work done on an object changes its kinetic energy. We also saw in [Temperature, Kinetic Theory, and the Gas Laws](#) that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain

internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

As noted in [Temperature, Kinetic Theory, and the Gas Laws](#), heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00°C—specifically, between 14.5°C and 15.5°C, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00°C. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called “big calorie”) are actually kilocalories (1 kilocalorie = 1000 calories), a fact not easily determined from package labeling.

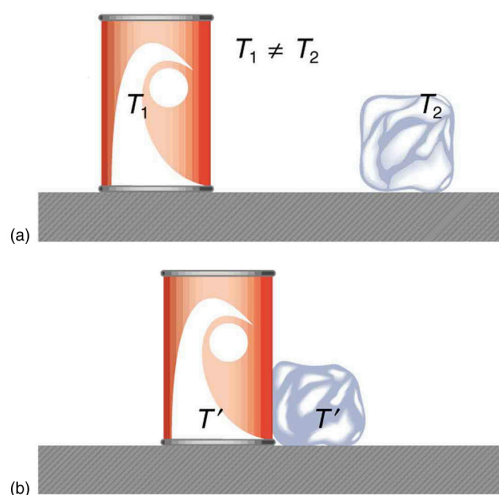


FIGURE 14.2 In figure (a) the soft drink and the ice have different temperatures, T_1 and T_2 , and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T' , achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—the work needed to produce the same effects as heat transfer. In terms of the units used for these two terms, the best modern value for this equivalence is

$$1.000 \text{ kcal} = 4186 \text{ J.} \quad 14.1$$

We consider this equation as the conversion between two different units of energy.

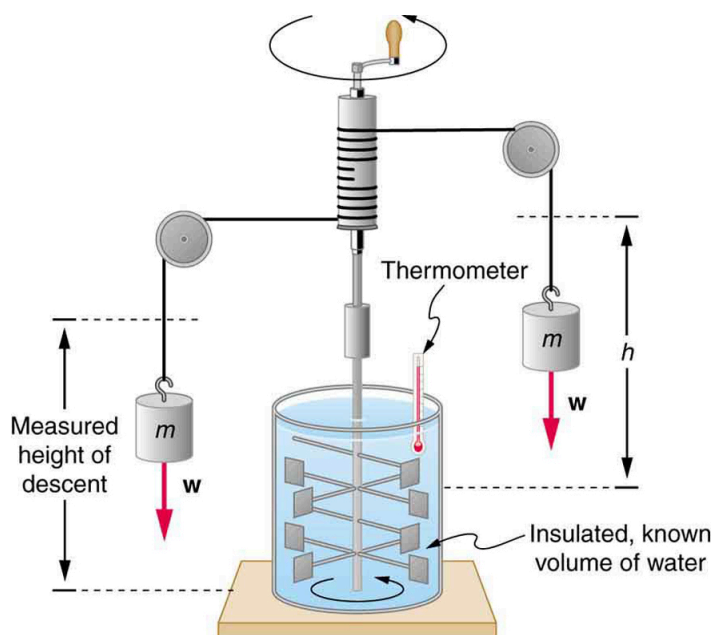


FIGURE 14.3 Schematic depiction of Joule's experiment that established the equivalence of heat and work.

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

✓ CHECK YOUR UNDERSTANDING

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Solution

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

14.2 Temperature Change and Heat Capacity

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

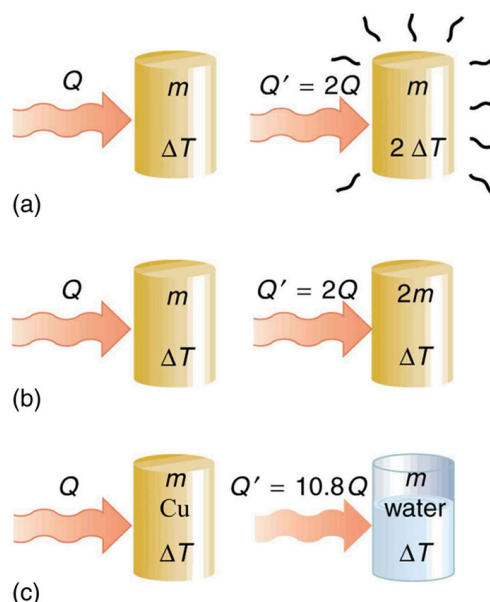


FIGURE 14.4 The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m , you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

Heat Transfer and Temperature Change

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = mc\Delta T, \quad 14.2$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for **specific heat** and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00°C. The specific heat c is a property of the substance; its SI unit is J/(kg · K) or J/(kg · °C). Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then the unit of specific heat is kcal/(kg · °C).

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. [Table 14.1](#) lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.



EXAMPLE 14.1

Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0°C to 80.0°C. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in [Table 14.1](#).

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

$$\Delta T = T_f - T_i = 60.0^\circ\text{C}. \quad 14.3$$

2. Calculate the mass of water. Because the density of water is 1000 kg/m³, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_w = 0.250$ kg.

3. Calculate the heat transferred to the water. Use the specific heat of water in [Table 14.1](#):

$$Q_w = m_w c_w \Delta T = (0.250 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(60.0^\circ\text{C}) = 62.8 \text{ kJ}. \quad 14.4$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in [Table 14.1](#):

$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500 \text{ kg})(900 \text{ J/kg}^\circ\text{C})(60.0^\circ\text{C}) = 27.0 \times 10^4 \text{ J} = 27.0 \text{ kJ}. \quad 14.5$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$Q_{\text{Total}} = Q_w + Q_{Al} = 62.8 \text{ kJ} + 27.0 \text{ kJ} = 89.8 \text{ kJ}. \quad 14.6$$

Thus, the amount of heat going into heating the pan is

$$\frac{27.0 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 30.1\%, \quad 14.7$$

and the amount going into heating the water is

$$\frac{62.8 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 69.9\%. \quad 14.8$$

Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

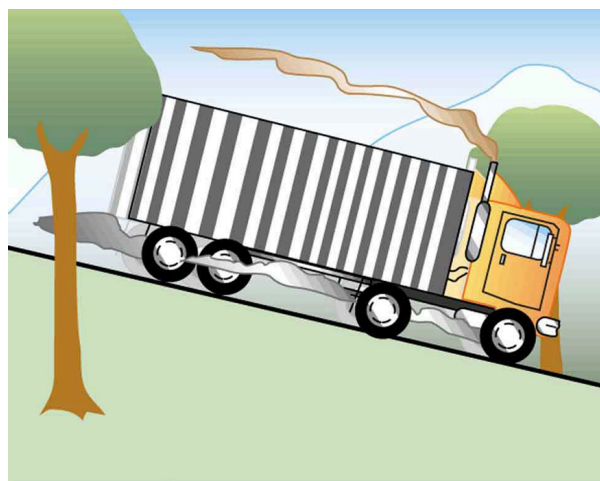


FIGURE 14.5 The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.



EXAMPLE 14.2

Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $800 \text{ J/(kg}^\circ\text{C)}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill

$$Mgh = (10,000 \text{ kg})(9.80 \text{ m/s}^2)(75.0 \text{ m}) = 7.35 \times 10^6 \text{ J.} \quad 14.9$$

2. Calculate the temperature from the heat transferred using $Q = Mgh$ and

$$\Delta T = \frac{Q}{mc}, \quad 14.10$$

where m is the mass of the brake material. Insert the values $m = 100 \text{ kg}$ and $c = 800 \text{ J/(kg}^\circ\text{C)}$ to find

$$\Delta T = \frac{(7.35 \times 10^6 \text{ J})}{(100 \text{ kg})(800 \text{ J/(kg}^\circ\text{C)})} = 92^\circ\text{C.} \quad 14.11$$

Discussion

This same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Substances	Specific heat (c)	
	J(kg°C)	kcal(kg°C) ¹
Solids		
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases ²	$C_v(C_p)$	$C_v(C_p)$
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)

¹ These values are identical in units of cal/g·°C.

² c_v at constant volume and at 20.0°C, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_p at a constant

Substances	Specific heat (c)	
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

TABLE 14.1 Specific Heats³ of Various Substances

Note that [Example 14.2](#) is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.



EXAMPLE 14.3

Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of 20.0°C water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of 150°C. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $|Q_{\text{hot}}| = Q_{\text{cold}}$.

Solution

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\text{hot}} = m_{\text{Al}} c_{\text{Al}} (T_f - 150^\circ\text{C}). \quad 14.12$$

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

$$Q_{\text{cold}} = m_{\text{W}} c_{\text{W}} (T_f - 20.0^\circ\text{C}). \quad 14.13$$

3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -Q_{\text{hot}}, \\ m_{\text{W}} c_{\text{W}} (T_f - 20.0^\circ\text{C}) &= -m_{\text{Al}} c_{\text{Al}} (T_f - 150^\circ\text{C}). \end{aligned} \quad 14.14$$

4. Bring all terms involving T_f on the left hand side and all other terms on the right hand side. Solve for T_f ,

$$T_f = \frac{m_{\text{Al}} c_{\text{Al}} (150^\circ\text{C}) + m_{\text{W}} c_{\text{W}} (20.0^\circ\text{C})}{m_{\text{Al}} c_{\text{Al}} + m_{\text{W}} c_{\text{W}}}, \quad 14.15$$

and insert the numerical values:

$$\begin{aligned} T_f &= \frac{(0.500 \text{ kg})(900 \text{ J/(kg}\cdot^\circ\text{C)})(150^\circ\text{C}) + (0.250 \text{ kg})(4186 \text{ J/(kg}\cdot^\circ\text{C)})(20.0^\circ\text{C})}{(0.500 \text{ kg})(900 \text{ J/(kg}\cdot^\circ\text{C)}) + (0.250 \text{ kg})(4186 \text{ J/(kg}\cdot^\circ\text{C)})} \\ &= \frac{88430 \text{ J}}{1496.5 \text{ J/}^\circ\text{C}} \\ &= 59.1^\circ\text{C}. \end{aligned} \quad 14.16$$

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to 20.0°C than 150°C ? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

Take-Home Experiment: Temperature Change of Land and Water

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

CHECK YOUR UNDERSTANDING

If 25 kJ is necessary to raise the temperature of a block from 25°C to 30°C , how much heat is necessary to heat the block from 45°C to 50°C ?

Solution

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

14.3 Phase Change and Latent Heat

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Examine heat transfer.
- Calculate final temperature from heat transfer.

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.



FIGURE 14.6 Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at 0°C stays at 0°C until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together [Figure 14.7](#).

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$$Q = mL_f \text{ (melting/freezing),} \quad 14.17$$

$$Q = mL_v \text{ (vaporization/condensation),} \quad 14.18$$

where the latent heat of fusion, L_f , and latent heat of vaporization, L_v , are material constants that are determined experimentally. See [Table 14.2](#).

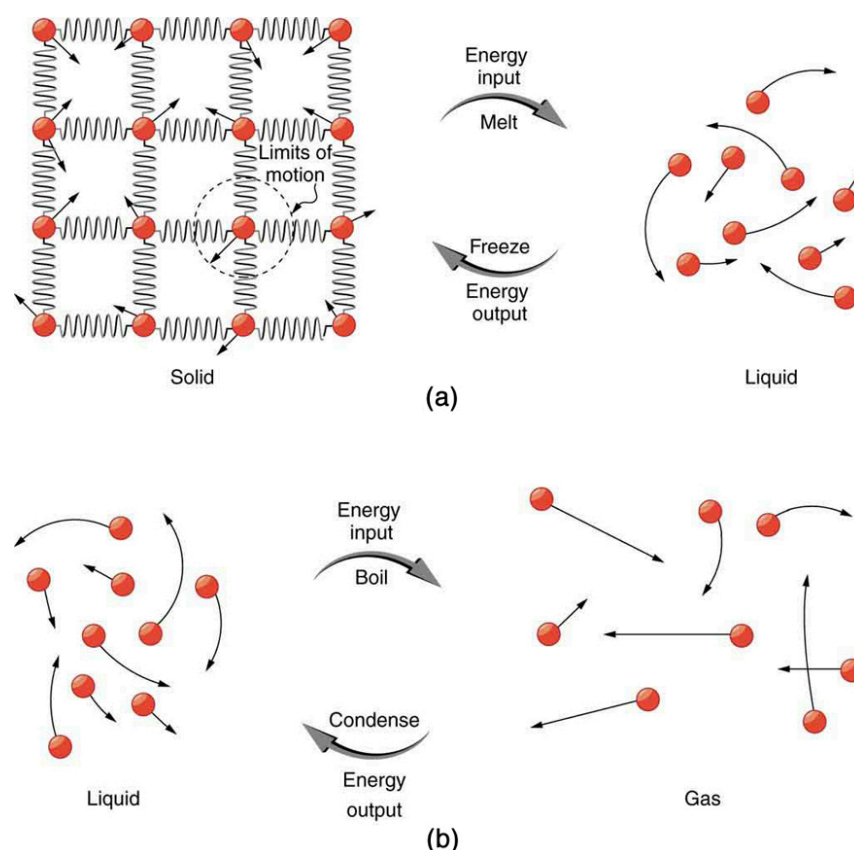


FIGURE 14.7 (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is measured in units of J/kg. Both L_f and L_v depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_f and L_v are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. [Table 14.2](#) lists representative values of L_f and L_v , together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at 0°C to produce a kilogram of water at 0°C . Using the equation for a change in temperature and the value for water from [Table 14.2](#), we find that

$Q = mL_f = (1.0 \text{ kg})(334 \text{ kJ/kg}) = 334 \text{ kJ}$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from 0°C to 79.8°C . Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point (100°C at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Substance	Melting point ($^\circ\text{C}$)	L_f		Boiling point ($^\circ\text{C}$)	L_v	
		kJ/kg	kcal/kg		kJ/kg	kcal/kg
Helium	-269.7	5.23	1.25	-268.9	20.9	4.99
Hydrogen	-259.3	58.6	14.0	-252.9	452	108

TABLE 14.2 Heats of Fusion and Vaporization ⁶

⁶ Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).

	L_f				L_v	
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2430 ⁴	580 ⁵
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

TABLE 14.2 Heats of Fusion and Vaporization ⁶

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above 35.0°C, which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at −20°C (Figure 14.8). The temperature of the ice rises linearly, absorbing heat at a constant rate of 0.50 cal/g · °C until it reaches 0°C. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at 0°C during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of 1.00 cal/g · °C. At 100°C, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of 0.482 cal/g · °C.

⁴ At 37.0°C (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg

⁵ At 37.0°C (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg

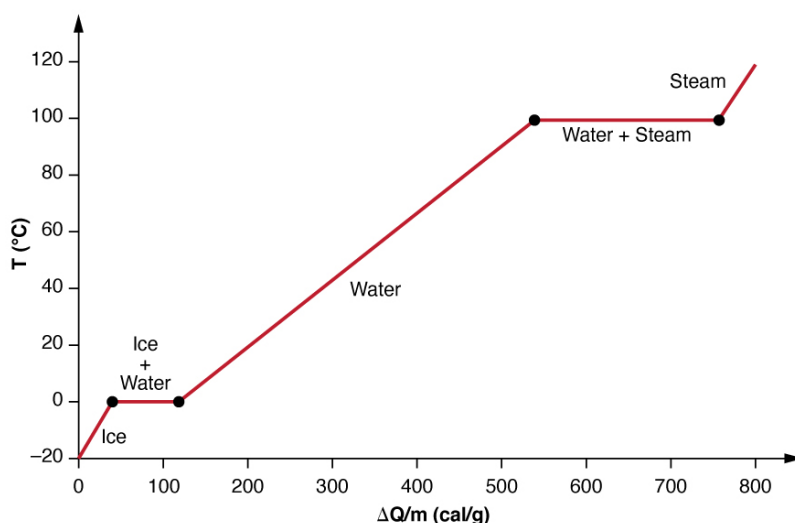


FIGURE 14.8 A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at 0°C and 100°C reflect the large latent heat of melting and vaporization, respectively.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below 100°C is less than that at 100°C, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at 100°C. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.



EXAMPLE 14.4

Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at 20°C with mass $m_{\text{soda}} = 0.25$ kg. The ice is at 0°C and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of 0°C. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at 0°C, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}. \quad 14.19$$

The heat transferred to the ice is $Q_{\text{ice}} = m_{\text{ice}}L_f + m_{\text{ice}}c_W(T_f - 0^\circ\text{C})$. The heat given off by the soda is $Q_{\text{soda}} = m_{\text{soda}}c_W(T_f - 20^\circ\text{C})$. Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

$$m_{\text{ice}}L_f + m_{\text{ice}}c_W(T_f - 0^\circ\text{C}) = -m_{\text{soda}}c_W(T_f - 20^\circ\text{C}). \quad 14.20$$

Bring all terms involving T_f on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_f :

$$T_f = \frac{m_{\text{soda}}c_W(20^\circ\text{C}) - m_{\text{ice}}L_f}{(m_{\text{soda}} + m_{\text{ice}})c_W}. \quad 14.21$$

Solution

1. Identify the known quantities. The mass of ice is $m_{\text{ice}} = 3 \times 6.0$ g = 0.018 kg and the mass of soda is $m_{\text{soda}} = 0.25$ kg.

2. Calculate the terms in the numerator:

$$m_{\text{soda}} c_W (20^\circ\text{C}) = (0.25 \text{ kg})(4186 \text{ J/(kg}\cdot^\circ\text{C)})(20^\circ\text{C}) = 20,930 \text{ J} \quad 14.22$$

and

$$m_{\text{ice}} L_f = (0.018 \text{ kg})(334,000 \text{ J/kg}) = 6012 \text{ J}. \quad 14.23$$

3. Calculate the denominator:

$$(m_{\text{soda}} + m_{\text{ice}}) c_W = (0.25 \text{ kg} + 0.018 \text{ kg})(4186 \text{ J/(kg}\cdot^\circ\text{C)}) = 1122 \text{ J/}^\circ\text{C}. \quad 14.24$$

4. Calculate the final temperature:

$$T_f = \frac{20,930 \text{ J} - 6012 \text{ J}}{1122 \text{ J/}^\circ\text{C}} = 13^\circ\text{C}. \quad 14.25$$

Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is -6°C . However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_v$.



FIGURE 14.9 Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The rate at which water molecules join together exceeds the rate at which they separate, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

Real-World Application

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point (0°C). Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.



FIGURE 14.10 The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below 0°C . Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of “smoke” from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.



(a)



(b)

FIGURE 14.11 Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = mL_s$, where L_s is the **heat of sublimation**, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_s is analogous to L_f and L_v , and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as

that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Problem-Solving Strategies for the Effects of Heat Transfer

1. *Examine the situation to determine that there is a change in the temperature or phase. Is there heat transfer into or out of the system?* When the presence or absence of a phase change is not obvious, you may wish to first solve the problem as if there were no phase changes, and examine the temperature change obtained. If it is sufficient to take you past a boiling or melting point, you should then go back and do the problem in steps—temperature change, phase change, subsequent temperature change, and so on.
2. *Identify and list all objects that change temperature and phase.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or what can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* If there is a temperature change, the transferred heat depends on the specific heat (see [Table 14.1](#)) whereas, for a phase change, the transferred heat depends on the latent heat. See [Table 14.2](#).
6. *Substitute the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* You will need to do this in steps if there is more than one stage to the process (such as a temperature change followed by a phase change).
7. *Check the answer to see if it is reasonable: Does it make sense?* As an example, be certain that the temperature change does not also cause a phase change that you have not taken into account.

CHECK YOUR UNDERSTANDING

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Solution

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0°C. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

14.4 Heat Transfer Methods

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A

less obvious example is thermal radiation from the human body.

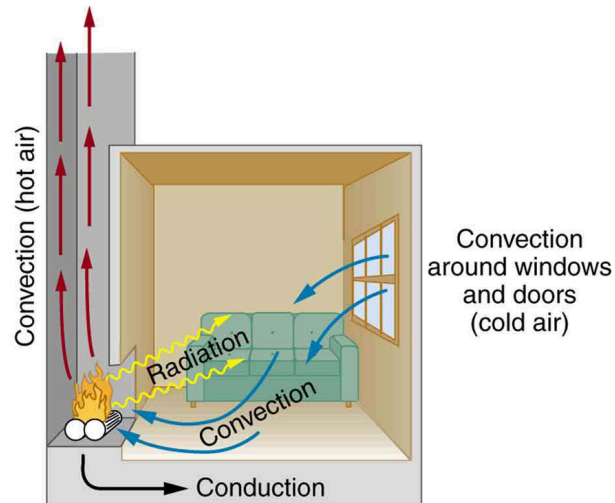


FIGURE 14.12 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

We examine these methods in some detail in the three following modules. Each method has unique and interesting characteristics, but all three do have one thing in common: they transfer heat solely because of a temperature difference [Figure 14.12](#).

✓ CHECK YOUR UNDERSTANDING

Name an example from daily life (different from the text) for each mechanism of heat transfer.

Solution

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista “steams” cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

14.5 Conduction

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Calculate thermal conductivity.
- Observe conduction of heat in collisions.
- Study thermal conductivities of common substances.



FIGURE 14.13 Insulation is used to limit the conduction of heat from the inside to the outside (in winters) and from the outside to the inside (in summers). (credit: Giles Douglas)

Your feet feel cold as you walk barefoot across the living room carpet in your cold house and then step onto the kitchen tile floor. This result is intriguing, since the carpet and tile floor are both at the same temperature. The different sensation you feel is explained by the different rates of heat transfer: the heat loss during the same time interval is greater for skin in contact with the tiles than with the carpet, so the temperature drop is greater on the tiles.

Some materials conduct thermal energy faster than others. In general, good conductors of electricity (metals like copper, aluminum, gold, and silver) are also good heat conductors, whereas insulators of electricity (wood, plastic, and rubber) are poor heat conductors. [Figure 14.14](#) shows molecules in two bodies at different temperatures. The (average) kinetic energy of a molecule in the hot body is higher than in the colder body. If two molecules collide, an energy transfer from the molecule with greater kinetic energy to the molecule with less kinetic energy occurs. The cumulative effect from all collisions results in a net flux of heat from the hot body to the colder body. The heat flux thus depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water. Conversely, if the temperatures are the same, the net heat transfer rate falls to zero, and equilibrium is achieved. Owing to the fact that the number of collisions increases with increasing area, heat conduction depends on the cross-sectional area. If you touch a cold wall with your palm, your hand cools faster than if you just touch it with your fingertip.

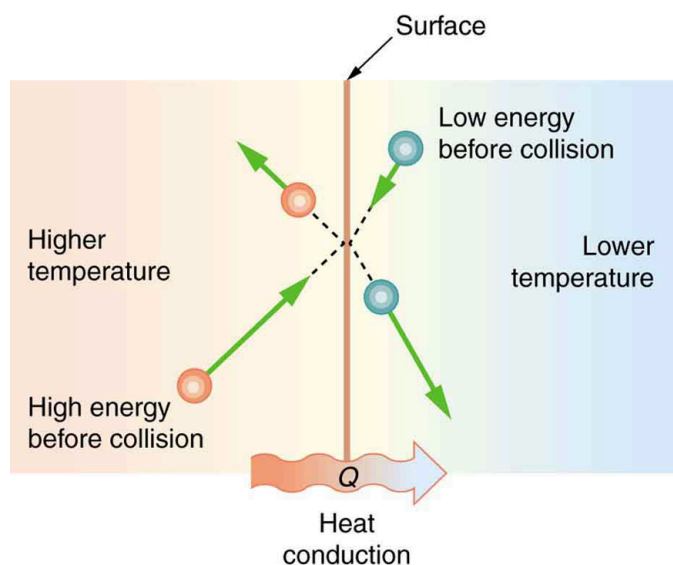


FIGURE 14.14 The molecules in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a molecule in the lower temperature region (right side) has low energy before collision, but its energy increases after colliding with the contact surface. In contrast, a molecule in the higher temperature region (left side) has high energy before collision, but its energy decreases after colliding

with the contact surface.

A third factor in the mechanism of conduction is the thickness of the material through which heat transfers. The figure below shows a slab of material with different temperatures on either side. Suppose that T_2 is greater than T_1 , so that heat is transferred from left to right. Heat transfer from the left side to the right side is accomplished by a series of molecular collisions. The thicker the material, the more time it takes to transfer the same amount of heat. This model explains why thick clothing is warmer than thin clothing in winters, and why Arctic mammals protect themselves with thick blubber.

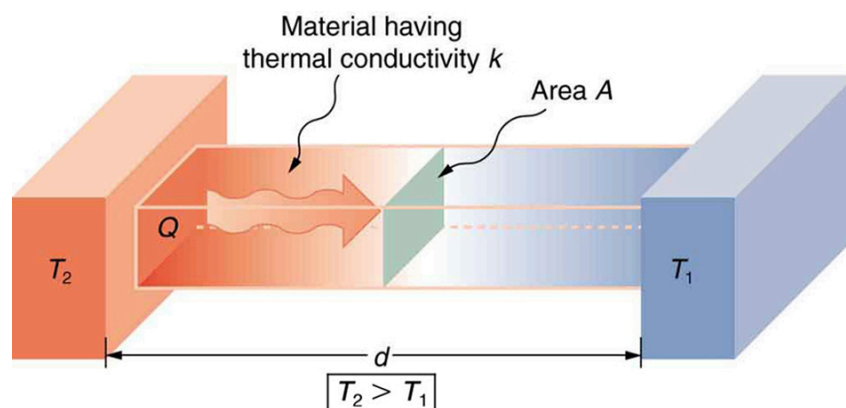


FIGURE 14.15 Heat conduction occurs through any material, represented here by a rectangular bar, whether window glass or walrus blubber. The temperature of the material is T_2 on the left and T_1 on the right, where T_2 is greater than T_1 . The rate of heat transfer by conduction is directly proportional to the surface area A , the temperature difference $T_2 - T_1$, and the substance's conductivity k . The rate of heat transfer is inversely proportional to the thickness d .

Lastly, the heat transfer rate depends on the material properties described by the coefficient of thermal conductivity. All four factors are included in a simple equation that was deduced from and is confirmed by experiments. The **rate of conductive heat transfer** through a slab of material, such as the one in [Figure 14.15](#), is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}, \quad 14.26$$

where Q/t is the rate of heat transfer in watts or kilocalories per second, k is the **thermal conductivity** of the material, A and d are its surface area and thickness, as shown in [Figure 14.15](#), and $(T_2 - T_1)$ is the temperature difference across the slab. [Table 14.3](#) gives representative values of thermal conductivity.



EXAMPLE 14.5

Calculating Heat Transfer Through Conduction: Conduction Rate Through an Ice Box

A Styrofoam ice box has a total area of 0.950 m^2 and walls with an average thickness of 2.50 cm . The box contains ice, water, and canned beverages at 0°C . The inside of the box is kept cold by melting ice. How much ice melts in one day if the ice box is kept in the trunk of a car at 35.0°C ?

Strategy

This question involves both heat for a phase change (melting of ice) and the transfer of heat by conduction. To find the amount of ice melted, we must find the net heat transferred. This value can be obtained by calculating the rate of heat transfer by conduction and multiplying by time.

Solution

1. Identify the knowns.

$$\begin{aligned}
 A &= 0.950 \text{ m}^2; \\
 d &= 2.50 \text{ cm} = 0.0250 \text{ m}; \\
 T_1 &= 0^\circ\text{C}; \\
 T_2 &= 35.0^\circ\text{C}, t = 1 \text{ day} = 24 \text{ hours} = 86,400 \text{ s}.
 \end{aligned}
 \tag{14.27}$$

2. Identify the unknowns. We need to solve for the mass of the ice, m . We will also need to solve for the net heat transferred to melt the ice, Q .

3. Determine which equations to use. The rate of heat transfer by conduction is given by

$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}.$$
14.28

4. The heat is used to melt the ice: $Q = mL_f$.

5. Insert the known values:

$$\frac{Q}{t} = \frac{(0.010 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C})(0.950 \text{ m}^2)(35.0^\circ\text{C} - 0^\circ\text{C})}{0.0250 \text{ m}} = 13.3 \text{ J/s}.$$
14.29

6. Multiply the rate of heat transfer by the time (1 day = 86,400 s):

$$Q = (Q/t)t = (13.3 \text{ J/s})(86,400 \text{ s}) = 1.15 \times 10^6 \text{ J}.$$
14.30

7. Set this equal to the heat transferred to melt the ice: $Q = mL_f$. Solve for the mass m :

$$m = \frac{Q}{L_f} = \frac{1.15 \times 10^6 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 3.44 \text{ kg}.$$
14.31

Discussion

The result of 3.44 kg, or about 7.6 lbs, seems about right, based on experience. You might expect to use about a 4 kg (7–10 lb) bag of ice per day. A little extra ice is required if you add any warm food or beverages.

Inspecting the conductivities in [Table 14.3](#) shows that Styrofoam is a very poor conductor and thus a good insulator. Other good insulators include fiberglass, wool, and goose-down feathers. Like Styrofoam, these all incorporate many small pockets of air, taking advantage of air's poor thermal conductivity.

Substance	Thermal conductivity k (J/s · m · °C)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84

TABLE 14.3 Thermal Conductivities of Common Substances⁷

⁷ At temperatures near 0°C.

Substance	Thermal conductivity k (J/s • m • °C)
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

TABLE 14.3 Thermal Conductivities of Common Substances⁷

A combination of material and thickness is often manipulated to develop good insulators—the smaller the conductivity k and the larger the thickness d , the better. The ratio of d/k will thus be large for a good insulator. The ratio d/k is called the **R factor**. The rate of conductive heat transfer is inversely proportional to R . The larger the value of R , the better the insulation. R factors are most commonly quoted for household insulation, refrigerators, and the like—unfortunately, it is still in non-metric units of $\text{ft}^2 \cdot ^\circ\text{F} \cdot \text{h/Btu}$, although the unit usually goes unstated (1 British thermal unit [Btu] is the amount of energy needed to change the temperature of 1.0 lb of water by 1.0 °F). A couple of representative values are an R factor of 11 for 3.5-in-thick fiberglass batts (pieces) of insulation and an R factor of 19 for 6.5-in-thick fiberglass batts. Walls are usually insulated with 3.5-in batts, while ceilings are usually insulated with 6.5-in batts. In cold climates, thicker batts may be used in ceilings and walls.

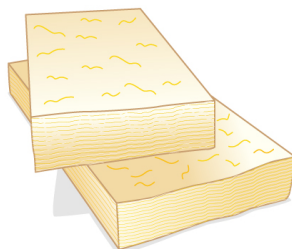


FIGURE 14.16 The fiberglass batt is used for insulation of walls and ceilings to prevent heat transfer between the inside of the building and the outside environment.

Note that in [Table 14.3](#), the best thermal conductors—silver, copper, gold, and aluminum—are also the best electrical conductors, again related to the density of free electrons in them. Cooking utensils are typically made from good conductors.



EXAMPLE 14.6

Calculating the Temperature Difference Maintained by a Heat Transfer: Conduction Through an Aluminum Pan

Water is boiling in an aluminum pan placed on an electrical element on a stovetop. The sauce pan has a bottom that is 0.800 cm thick and 14.0 cm in diameter. The boiling water is evaporating at the rate of 1.00 g/s. What is the temperature difference across (through) the bottom of the pan?

Strategy

Conduction through the aluminum is the primary method of heat transfer here, and so we use the equation for the rate of heat transfer and solve for the temperature difference.

$$T_2 - T_1 = \frac{Q}{t} \left(\frac{d}{kA} \right). \quad 14.32$$

Solution

1. Identify the knowns and convert them to the SI units.

The thickness of the pan, $d = 0.800 \text{ cm} = 8.0 \times 10^{-3} \text{ m}$, the area of the pan, $A = \pi(0.14/2)^2 \text{ m}^2 = 1.54 \times 10^{-2} \text{ m}^2$, and the thermal conductivity, $k = 220 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C}$.

2. Calculate the necessary heat of vaporization of 1 g of water:

$$Q = mL_v = (1.00 \times 10^{-3} \text{ kg}) (2256 \times 10^3 \text{ J/kg}) = 2256 \text{ J}. \quad 14.33$$

3. Calculate the rate of heat transfer given that 1 g of water melts in one second:

$$Q/t = 2256 \text{ J/s or } 2.26 \text{ kW}. \quad 14.34$$

4. Insert the knowns into the equation and solve for the temperature difference:

$$T_2 - T_1 = \frac{Q}{t} \left(\frac{d}{kA} \right) = (2256 \text{ J/s}) \frac{8.00 \times 10^{-3} \text{ m}}{(220 \text{ J/s} \cdot \text{m} \cdot ^\circ\text{C})(1.54 \times 10^{-2} \text{ m}^2)} = 5.33^\circ\text{C}. \quad 14.35$$

Discussion

The value for the heat transfer $Q/t = 2.26 \text{ kW}$ or 2256 J/s is typical for an electric stove. This value gives a remarkably small temperature difference between the stove and the pan. Consider that the stove burner is red hot while the inside of the pan is nearly 100°C because of its contact with boiling water. This contact effectively cools the bottom of the pan in spite of its proximity to the very hot stove burner. Aluminum is such a good conductor that it only takes this small temperature difference to produce a heat transfer of 2.26 kW into the pan.

Conduction is caused by the random motion of atoms and molecules. As such, it is an ineffective mechanism for heat transport over macroscopic distances and short time distances. Take, for example, the temperature on the Earth, which would be unbearably cold during the night and extremely hot during the day if heat transport in the atmosphere was to be only through conduction. In another example, car engines would overheat unless there was a more efficient way to remove excess heat from the pistons.



CHECK YOUR UNDERSTANDING

How does the rate of heat transfer by conduction change when all spatial dimensions are doubled?

Solution

Because area is the product of two spatial dimensions, it increases by a factor of four when each dimension is doubled ($A_{\text{final}} = (2d)^2 = 4d^2 = 4A_{\text{initial}}$). The distance, however, simply doubles. Because the temperature difference and the coefficient of thermal conductivity are independent of the spatial dimensions, the rate of heat transfer by conduction increases by a factor of four divided by two, or two:

$$\left(\frac{Q}{t}\right)_{\text{final}} = \frac{kA_{\text{final}}(T_2 - T_1)}{d_{\text{final}}} = \frac{k(4A_{\text{initial}})(T_2 - T_1)}{2d_{\text{initial}}} = 2 \frac{kA_{\text{initial}}(T_2 - T_1)}{d_{\text{initial}}} = 2 \left(\frac{Q}{t}\right)_{\text{initial}} \quad . \quad 14.36$$

14.6 Convection

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the method of heat transfer by convection.

Convection is driven by large-scale flow of matter. In the case of Earth, the atmospheric circulation is caused by the flow of hot air from the tropics to the poles, and the flow of cold air from the poles toward the tropics. (Note that Earth's rotation causes the observed easterly flow of air in the northern hemisphere). Car engines are kept cool by the flow of water in the cooling system, with the water pump maintaining a flow of cool water to the pistons. The circulatory system is used the body: when the body overheats, the blood vessels in the skin expand (dilate), which increases the blood flow to the skin where it can be cooled by sweating. These vessels become smaller when it is cold outside and larger when it is hot (so more fluid flows, and more energy is transferred).

The body also loses a significant fraction of its heat through the breathing process.

While convection is usually more complicated than conduction, we can describe convection and do some straightforward, realistic calculations of its effects. Natural convection is driven by buoyant forces: hot air rises because density decreases as temperature increases. The house in [Figure 14.17](#) is kept warm in this manner, as is the pot of water on the stove in [Figure 14.18](#). Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another. Both are examples of natural convection.

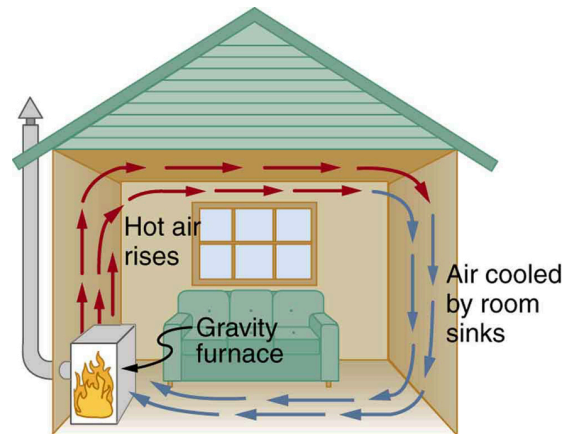


FIGURE 14.17 Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system using natural convection, like this one, can be quite efficient in uniformly heating a home.

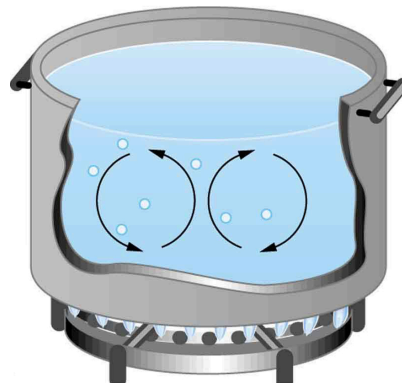


FIGURE 14.18 Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process keeps repeating.

Take-Home Experiment: Convection Rolls in a Heated Pan

Take two small pots of water and use an eye dropper to place a drop of food coloring near the bottom of each. Leave one on a bench top and heat the other over a stovetop. Watch how the color spreads and how long it takes the color to reach the top. Watch how convective loops form.



EXAMPLE 14.7

Calculating Heat Transfer by Convection: Convection of Air Through the Walls of a House

Most houses are not airtight: air goes in and out around doors and windows, through cracks and crevices, following wiring to switches and outlets, and so on. The air in a typical house is completely replaced in less than an hour. Suppose that a moderately-sized house has inside dimensions $12.0\text{ m} \times 18.0\text{ m} \times 3.00\text{ m}$ high, and that all air is replaced in 30.0 min. Calculate the heat transfer per unit time in watts needed to warm the incoming cold air by 10.0°C , thus replacing the heat transferred by convection alone.

Strategy

Heat is used to raise the temperature of air so that $Q = mc\Delta T$. The rate of heat transfer is then Q/t , where t is the time for air turnover. We are given that ΔT is 10.0°C , but we must still find values for the mass of air and its specific heat before we can calculate Q . The specific heat of air is a weighted average of the specific heats of nitrogen and oxygen, which gives $c = c_p \cong 1000\text{ J/kg}\cdot^\circ\text{C}$ from [Table 14.1](#) (note that the specific heat at constant pressure must be used for this process).

Solution

1. Determine the mass of air from its density and the given volume of the house. The density is given from the density ρ and the volume

$$m = \rho V = (1.29\text{ kg/m}^3)(12.0\text{ m} \times 18.0\text{ m} \times 3.00\text{ m}) = 836\text{ kg.} \quad 14.37$$

2. Calculate the heat transferred from the change in air temperature: $Q = mc\Delta T$ so that

$$Q = (836\text{ kg})(1000\text{ J/kg}\cdot^\circ\text{C})(10.0^\circ\text{C}) = 8.36 \times 10^6\text{ J.} \quad 14.38$$

3. Calculate the heat transfer from the heat Q and the turnover time t . Since air is turned over in $t = 0.500\text{ h} = 1800\text{ s}$, the heat transferred per unit time is

$$\frac{Q}{t} = \frac{8.36 \times 10^6\text{ J}}{1800\text{ s}} = 4.64\text{ kW.} \quad 14.39$$

Discussion

This rate of heat transfer is equal to the power consumed by about forty-six 100-W light bulbs. Newly constructed homes are designed for a turnover time of 2 hours or more, rather than 30 minutes for the house of this example. Weather stripping, caulking, and improved window seals are commonly employed. More extreme measures are sometimes taken in very cold (or hot) climates to achieve a tight standard of more than 6 hours for one air turnover. Still longer turnover times are unhealthy, because a minimum amount of fresh air is necessary to supply oxygen for breathing and to dilute household pollutants. The term used for the process by which outside air leaks into the house from cracks around windows, doors, and the foundation is called “air infiltration.”

A cold wind is much more chilling than still cold air, because convection combines with conduction in the body to increase the rate at which energy is transferred away from the body. The table below gives approximate wind-chill factors, which are the temperatures of still air that produce the same rate of cooling as air of a given temperature and speed. Wind-chill factors are a dramatic reminder of convection’s ability to transfer heat faster than conduction. For example, a 15.0 m/s wind at 0°C has the chilling equivalent of still air at about -18°C .

Moving air temperature	Wind speed (m/s)				
(°C)	2	5	10	15	20
5	3	−1	−8	−10	−12
2	0	−7	−12	−16	−18
0	−2	−9	−15	−18	−20
−5	−7	−15	−22	−26	−29
−10	−12	−21	−29	−34	−36
−20	−23	−34	−44	−50	−52
−40	−44	−59	−73	−82	−84

TABLE 14.4 Wind-Chill Factors

Although air can transfer heat rapidly by convection, it is a poor conductor and thus a good insulator. The amount of available space for airflow determines whether air acts as an insulator or conductor. The space between the inside and outside walls of a house, for example, is about 9 cm (3.5 in) —large enough for convection to work effectively. The addition of wall insulation prevents airflow, so heat loss (or gain) is decreased. Similarly, the gap between the two panes of a double-paned window is about 1 cm, which prevents convection and takes advantage of air's low conductivity to prevent greater loss. Fur, fiber, and fiberglass also take advantage of the low conductivity of air by trapping it in spaces too small to support convection, as shown in the figure. Fur and feathers are lightweight and thus ideal for the protection of animals.

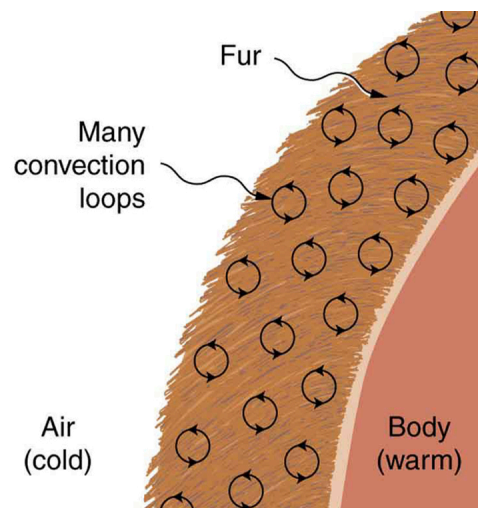


FIGURE 14.19 Fur is filled with air, breaking it up into many small pockets. Convection is very slow here, because the loops are so small. The low conductivity of air makes fur a very good lightweight insulator.

Some interesting phenomena happen *when convection is accompanied by a phase change*. It allows us to cool off by sweating, even if the temperature of the surrounding air exceeds body temperature. Heat from the skin is required for sweat to evaporate from the skin, but without air flow, the air becomes saturated and evaporation stops. Air flow caused by convection replaces the saturated air by dry air and evaporation continues.

**EXAMPLE 14.8****Calculate the Flow of Mass during Convection: Sweat-Heat Transfer away from the Body**

The average person produces heat at the rate of about 120 W when at rest. At what rate must water evaporate from the body to get rid of all this energy? (This evaporation might occur when a person is sitting in the shade and surrounding temperatures are the same as skin temperature, eliminating heat transfer by other methods.)

Strategy

Energy is needed for a phase change ($Q = mL_v$). Thus, the energy loss per unit time is

$$\frac{Q}{t} = \frac{mL_v}{t} = 120 \text{ W} = 120 \text{ J/s.} \quad 14.40$$

We divide both sides of the equation by L_v to find that the mass evaporated per unit time is

$$\frac{m}{t} = \frac{120 \text{ J/s}}{L_v}. \quad 14.41$$

Solution

(1) Insert the value of the latent heat from [Table 14.2](#), $L_v = 2430 \text{ kJ/kg} = 2430 \text{ J/g}$. This yields

$$\frac{m}{t} = \frac{120 \text{ J/s}}{2430 \text{ J/g}} = 0.0494 \text{ g/s} = 2.96 \text{ g/min.} \quad 14.42$$

Discussion

Evaporating about 3 g/min seems reasonable. This would be about 180 g (about 7 oz) per hour. If the air is very dry, the sweat may evaporate without even being noticed. A significant amount of evaporation also takes place in the lungs and breathing passages.

Another important example of the combination of phase change and convection occurs when water evaporates from the oceans. Heat is removed from the ocean when water evaporates. If the water vapor condenses in liquid droplets as clouds form, heat is released in the atmosphere. Thus, there is an overall transfer of heat from the ocean to the atmosphere. This process is the driving power behind thunderheads, those great cumulus clouds that rise as much as 20.0 km into the stratosphere. Water vapor carried in by convection condenses, releasing tremendous amounts of energy. This energy causes the air to expand and rise, where it is colder. More condensation occurs in these colder regions, which in turn drives the cloud even higher. Such a mechanism is called positive feedback, since the process reinforces and accelerates itself. These systems sometimes produce violent storms, with lightning and hail, and constitute the mechanism driving hurricanes.



FIGURE 14.20 Cumulus clouds are caused by water vapor that rises because of convection. The rise of clouds is driven by a positive feedback mechanism. (credit: Mike Love)



FIGURE 14.21 Convection accompanied by a phase change releases the energy needed to drive this thunderhead into the stratosphere. (credit: Gerardo García Moretti)



FIGURE 14.22 The phase change that occurs when this iceberg melts involves tremendous heat transfer. (credit: Dominic Alves)

The movement of icebergs is another example of convection accompanied by a phase change. Suppose an iceberg drifts from Greenland into warmer Atlantic waters. Heat is removed from the warm ocean water when the ice melts and heat is released to the land mass when the iceberg forms on Greenland.

CHECK YOUR UNDERSTANDING

Explain why using a fan in the summer feels refreshing!

Solution

Using a fan increases the flow of air: warm air near your body is replaced by cooler air from elsewhere. Convection increases the rate of heat transfer so that moving air “feels” cooler than still air.

14.7 Radiation

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss heat transfer by radiation.
- Explain the power of different materials.

You can feel the heat transfer from a fire and from the Sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. The space between the Earth and the Sun is largely empty, without any possibility of heat transfer by convection or conduction. In these examples, heat is transferred by radiation. That is, the hot body emits electromagnetic waves that are absorbed by our skin: no medium is required for electromagnetic waves to propagate. Different names are used for electromagnetic waves of different wavelengths: radio waves, microwaves, infrared **radiation**, visible light, ultraviolet radiation, X-rays, and gamma rays.



FIGURE 14.23 Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light, although dramatic, transfers relatively little thermal energy. Convection transfers energy away from the observers as hot air rises, while conduction is negligibly slow here. Skin is very sensitive to infrared radiation, so that you can sense the presence of a fire without looking at it directly. (credit: Daniel X. O'Neil)

The energy of electromagnetic radiation depends on the wavelength (color) and varies over a wide range: a smaller wavelength (or higher frequency) corresponds to a higher energy. Because more heat is radiated at higher temperatures, a temperature change is accompanied by a color change. Take, for example, an electrical element on a stove, which glows from red to orange, while the higher-temperature steel in a blast furnace glows from yellow to white. The radiation you feel is mostly infrared, which corresponds to a lower temperature than that of the electrical element and the steel. The radiated energy depends on its intensity, which is represented in the figure below by the height of the distribution.

[Electromagnetic Waves](#) explains more about the electromagnetic spectrum and [Introduction to Quantum Physics](#) discusses how the decrease in wavelength corresponds to an increase in energy.

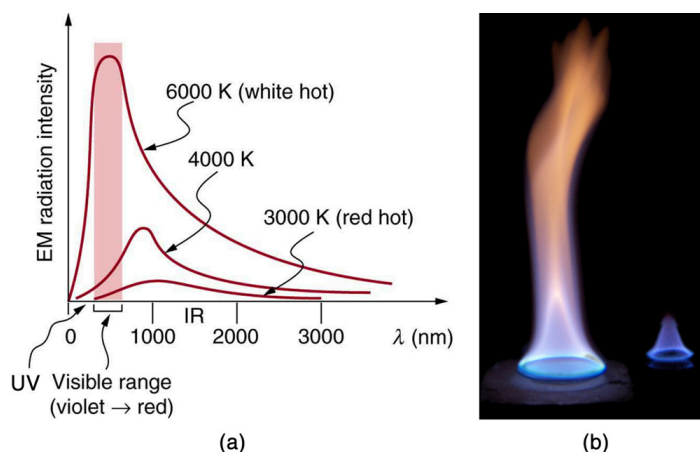


FIGURE 14.24 (a) A graph of the spectra of electromagnetic waves emitted from an ideal radiator at three different temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shaded portion denotes the visible part of the spectrum. It is apparent that the shift toward the ultraviolet with temperature makes the visible appearance shift from red to white to blue as temperature increases. (b) Note the variations in color corresponding to variations in flame temperature. (credit: Tuohirulla)

All objects absorb and emit electromagnetic radiation. The rate of heat transfer by radiation is largely determined by the color of the object. Black is the most effective, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than adjacent gray sidewalk on a summer day, because black absorbs better than gray. The reverse is also true—black radiates better than gray. Thus, on a clear summer night, the asphalt will be colder than the gray sidewalk, because black radiates the energy more rapidly than gray. An *ideal radiator* is the same color as an *ideal absorber*, and captures all the radiation that falls on it. In contrast, white is a poor absorber and is also a poor radiator. A white object reflects all radiation, like a mirror. (A perfect, polished white surface is mirror-like in appearance, and a crushed mirror looks white.)

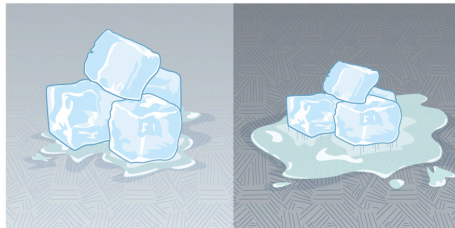


FIGURE 14.25 This illustration shows that the darker pavement is hotter than the lighter pavement (much more of the ice on the right has melted), although both have been in the sunlight for the same time. The thermal conductivities of the pavements are the same.

Gray objects have a uniform ability to absorb all parts of the electromagnetic spectrum. Colored objects behave in similar but more complex ways, which gives them a particular color in the visible range and may make them special in other ranges of the nonvisible spectrum. Take, for example, the strong absorption of infrared radiation by the skin, which allows us to be very sensitive to it.

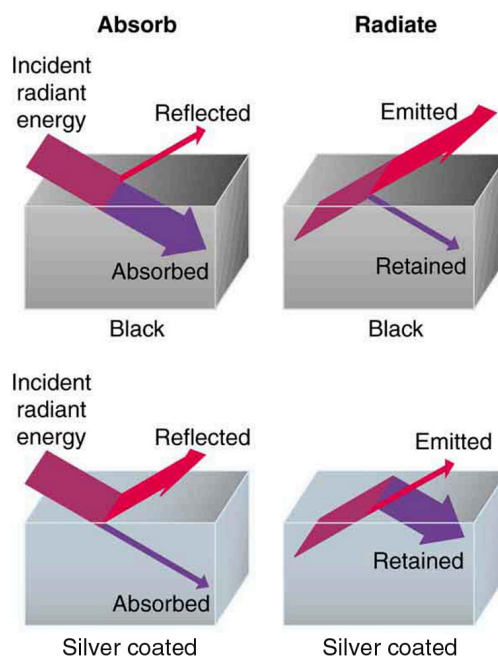


FIGURE 14.26 A black object is a good absorber and a good radiator, while a white (or silver) object is a poor absorber and a poor radiator. It is as if radiation from the inside is reflected back into the silver object, whereas radiation from the inside of the black object is “absorbed” when it hits the surface and finds itself on the outside and is strongly emitted.

The rate of heat transfer by emitted radiation is determined by the **Stefan-Boltzmann law of radiation**:

$$\frac{Q}{t} = \sigma e A T^4, \quad 14.43$$

where $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$ is the Stefan-Boltzmann constant, A is the surface area of the object, and T is its absolute temperature in kelvin. The symbol e stands for the **emissivity** of the object, which is a measure of how well it radiates. An ideal jet-black (or black body) radiator has $e = 1$, whereas a perfect reflector has $e = 0$. Real objects fall between these two values. Take, for example, tungsten light bulb filaments which have an e of about 0.5, and carbon black (a material used in printer toner), which has the (greatest known) emissivity of about 0.99.

The radiation rate is directly proportional to the *fourth power* of the absolute temperature—a remarkably strong temperature dependence. Furthermore, the radiated heat is proportional to the surface area of the object. If you knock apart the coals of a fire, there is a noticeable increase in radiation due to an increase in radiating surface area.



FIGURE 14.27 A thermograph of part of a building shows temperature variations, indicating where heat transfer to the outside is most severe. Windows are a major region of heat transfer to the outside of homes. (credit: U.S. Army)

Skin is a remarkably good absorber and emitter of infrared radiation, having an emissivity of 0.97 in the infrared

spectrum. Thus, we are all nearly (jet) black in the infrared, in spite of the obvious variations in skin color. This high infrared emissivity is why we can so easily feel radiation on our skin. It is also the basis for the use of night scopes used by law enforcement and the military to detect human beings. Even small temperature variations can be detected because of the T^4 dependence. Images, called *thermographs*, can be used medically to detect regions of abnormally high temperature in the body, perhaps indicative of disease. Similar techniques can be used to detect heat leaks in homes [Figure 14.27](#), optimize performance of blast furnaces, improve comfort levels in work environments, and even remotely map the Earth's temperature profile.

All objects emit and absorb radiation. The *net* rate of heat transfer by radiation (absorption minus emission) is related to both the temperature of the object and the temperature of its surroundings. Assuming that an object with a temperature T_1 is surrounded by an environment with uniform temperature T_2 , the **net rate of heat transfer by radiation** is

$$\frac{Q_{\text{net}}}{t} = \sigma e A (T_2^4 - T_1^4), \quad 14.44$$

where e is the emissivity of the object alone. In other words, it does not matter whether the surroundings are white, gray, or black; the balance of radiation into and out of the object depends on how well it emits and absorbs radiation. When $T_2 > T_1$, the quantity Q_{net}/t is positive; that is, the net heat transfer is from hot to cold.

Take-Home Experiment: Temperature in the Sun

Place a thermometer out in the sunshine and shield it from direct sunlight using an aluminum foil. What is the reading? Now remove the shield, and note what the thermometer reads. Take a handkerchief soaked in nail polish remover, wrap it around the thermometer and place it in the sunshine. What does the thermometer read?



EXAMPLE 14.9

Calculate the Net Heat Transfer of a Person: Heat Transfer by Radiation

What is the rate of heat transfer by radiation, with an unclothed person standing in a dark room whose ambient temperature is 22.0°C . The person has a normal skin temperature of 33.0°C and a surface area of 1.50 m^2 . The emissivity of skin is 0.97 in the infrared, where the radiation takes place.

Strategy

We can solve this by using the equation for the rate of radiative heat transfer.

Solution

Insert the temperatures values $T_2 = 295 \text{ K}$ and $T_1 = 306 \text{ K}$, so that

$$\begin{aligned} \frac{Q}{t} &= \sigma e A (T_2^4 - T_1^4) & 14.45 \\ &= (5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4)(0.97)(1.50 \text{ m}^2) [(295 \text{ K})^4 - (306 \text{ K})^4] & 14.46 \\ &= -99 \text{ J/s} = -99 \text{ W}. & 14.47 \end{aligned}$$

Discussion

This value is a significant rate of heat transfer to the environment (note the minus sign), considering that a person at rest may produce energy at the rate of 125 W and that conduction and convection will also be transferring energy to the environment. Indeed, we would probably expect this person to feel cold. Clothing significantly reduces heat transfer to the environment by many methods, because clothing slows down both conduction and convection, and has a lower emissivity (especially if it is white) than skin.

The Earth receives almost all its energy from radiation of the Sun and reflects some of it back into outer space. Because the Sun is hotter than the Earth, the net energy flux is from the Sun to the Earth. However, the rate of

energy transfer is less than the equation for the radiative heat transfer would predict because the Sun does not fill the sky. The average emissivity (e) of the Earth is about 0.65, but the calculation of this value is complicated by the fact that the highly reflective cloud coverage varies greatly from day to day. There is a negative feedback (one in which a change produces an effect that opposes that change) between clouds and heat transfer; greater temperatures evaporate more water to form more clouds, which reflect more radiation back into space, reducing the temperature. The often mentioned **greenhouse effect** is directly related to the variation of the Earth's emissivity with radiation type (see the figure given below). The greenhouse effect is a natural phenomenon responsible for providing temperatures suitable for life on Earth. The Earth's relatively constant temperature is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by carbon dioxide (CO_2) and water (H_2O) in the atmosphere and then re-radiated back to the Earth or into outer space. Re-radiation back to the Earth maintains its surface temperature about 40°C higher than it would be if there was no atmosphere, similar to the way glass increases temperatures in a greenhouse.

The greenhouse effect and its causes were first predicted by Eunice Newton Foote after she designed and conducted experiments on heating of different gases. After filling flasks with carbon dioxide, hydrogen, and regular air, then also modifying moisture, she placed them in the sun and carefully measured their heating and, especially, their heat retention. She discovered that the CO_2 flask gained the most temperature and held it the longest. After subsequent research, her paper "Circumstances affecting the Heat of the Sun's Rays" included conclusions that an atmosphere consisting of more carbon dioxide would be hotter resulting from the gas trapping radiation.

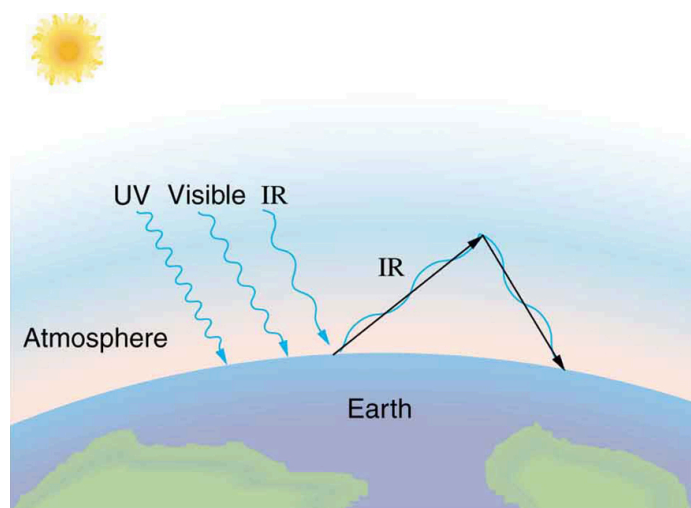


FIGURE 14.28 The greenhouse effect is a name given to the trapping of energy in the Earth's atmosphere by a process similar to that used in greenhouses. The atmosphere, like window glass, is transparent to incoming visible radiation and most of the Sun's infrared. These wavelengths are absorbed by the Earth and re-emitted as infrared. Since Earth's temperature is much lower than that of the Sun, the infrared radiated by the Earth has a much longer wavelength. The atmosphere, like glass, traps these longer infrared rays, keeping the Earth warmer than it would otherwise be. The amount of trapping depends on concentrations of trace gases like carbon dioxide, and a change in the concentration of these gases is believed to affect the Earth's surface temperature.

The greenhouse effect is also central to the discussion of global warming due to emission of carbon dioxide and methane (and other so-called greenhouse gases) into the Earth's atmosphere from industrial production and farming. Changes in global climate could lead to more intense storms, precipitation changes (affecting agriculture), reduction in rain forest biodiversity, and rising sea levels.

Heating and cooling are often significant contributors to energy use in individual homes. Mária Telkes, a Hungarian-born American scientist, was among the foremost developers of solar energy applications in industrial and community use. After inventing a widely deployed solar seawater distiller used on World War II life rafts, she partnered with architect Eleanor Raymond to design the first modern home to be completely heated by solar power. Air warmed on rooftop collectors transferred heat to salts, which stored the heat for later use. Telkes later worked with the Department of Energy to develop the first solar-electrically powered home. Current research efforts into developing environmentally friendly homes quite often focus on reducing conventional heating and cooling through better building materials, strategically positioning windows to optimize radiation gain from the Sun, and opening spaces to allow convection. It is possible to build a zero-energy house that allows for comfortable living in most

parts of the United States with hot and humid summers and cold winters.

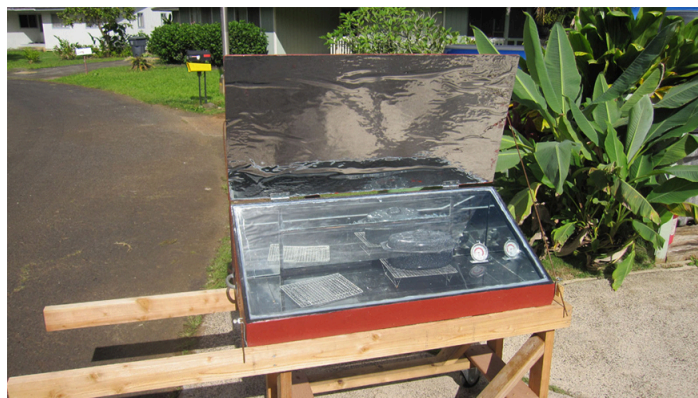


FIGURE 14.29 This simple but effective solar cooker uses the greenhouse effect and reflective material to trap and retain solar energy. Made of inexpensive, durable materials, it saves money and labor, and is of particular economic value in countries with lower energy availability. Mária Telkes, a pioneering innovator in solar applications, developed some of the earliest models of these cookers in the 1950s. (credit: E.B. Kauai)

Conversely, dark space is very cold, about 3 K (− 454°F), so that the Earth radiates energy into the dark sky. Owing to the fact that clouds have lower emissivity than either oceans or land masses, they reflect some of the radiation back to the surface, greatly reducing heat transfer into dark space, just as they greatly reduce heat transfer into the atmosphere during the day. The rate of heat transfer from soil and grasses can be so rapid that frost may occur on clear summer evenings, even in warm latitudes.

✓ CHECK YOUR UNDERSTANDING

What is the change in the rate of the radiated heat by a body at the temperature $T_1 = 20^\circ\text{C}$ compared to when the body is at the temperature $T_2 = 40^\circ\text{C}$?

Solution

The radiated heat is proportional to the fourth power of the *absolute temperature*. Because $T_1 = 293\text{ K}$ and $T_2 = 313\text{ K}$, the rate of heat transfer increases by about 30 percent of the original rate.

Career Connection: Energy Conservation Consultation

The cost of energy is generally believed to remain very high for the foreseeable future. Thus, passive control of heat loss in both commercial and domestic housing will become increasingly important. Energy consultants measure and analyze the flow of energy into and out of houses and ensure that a healthy exchange of air is maintained inside the house. The job prospects for an energy consultant are strong.

Problem-Solving Strategies for the Methods of Heat Transfer

1. *Examine the situation to determine what type of heat transfer is involved.*
2. *Identify the type(s) of heat transfer—conduction, convection, or radiation.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is very useful.*
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. For conduction, equation $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$ is appropriate. [Table 14.3](#) lists thermal conductivities. For convection, determine the amount of matter moved and use equation $Q = mc\Delta T$, to calculate the heat transfer involved in the temperature change of the fluid. If a phase change accompanies convection, equation $Q = mL_f$ or $Q = mL_v$ is appropriate to find the heat transfer involved in the phase change. [Table 14.2](#) lists information relevant to phase change. For radiation, equation $\frac{Q_{\text{net}}}{t} = \sigma eA(T_2^4 - T_1^4)$ gives the

net heat transfer rate.

7. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.*
8. *Check the answer to see if it is reasonable. Does it make sense?*

Glossary

conduction heat transfer through stationary matter by physical contact

convection heat transfer by the macroscopic movement of fluid

emissivity measure of how well an object radiates

greenhouse effect warming of the Earth that is due to gases such as carbon dioxide and methane that absorb infrared radiation from the Earth's surface and reradiate it in all directions, thus sending a fraction of it back toward the surface of the Earth

heat the spontaneous transfer of energy due to a temperature difference

heat of sublimation the energy required to change a substance from the solid phase to the vapor phase

kilocalorie 1 kilocalorie = 1000 calories

latent heat coefficient a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

mechanical equivalent of heat the work needed to produce the same effects as heat transfer

net rate of heat transfer by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma e A (T_2^4 - T_1^4)$$

R factor the ratio of thickness to the conductivity of a material

radiation energy transferred by electromagnetic waves directly as a result of a temperature difference

radiation heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

rate of conductive heat transfer rate of heat transfer from one material to another

specific heat the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 °C

Stefan-Boltzmann law of radiation $\frac{Q}{t} = \sigma e A T^4$, where σ is the Stefan-Boltzmann constant, A is the surface area of the object, T is the absolute temperature, and e is the emissivity

sublimation the transition from the solid phase to the vapor phase

thermal conductivity the property of a material's ability to conduct heat

Section Summary

14.1 Heat

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5°C and 15.5°C.
- The mechanical equivalent of this heat transfer is 1.00 kcal = 4186 J.

14.2 Temperature Change and Heat Capacity

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

14.3 Phase Change and Latent Heat

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as “phases.”

- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:
 $Q = mL$,
where L is the latent heat coefficient.

14.4 Heat Transfer Methods

- Heat is transferred by three different methods: conduction, convection, and radiation.

14.5 Conduction

- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- The rate of heat transfer Q/t (energy per unit time) is proportional to the temperature difference $T_2 - T_1$ and the contact area A and inversely proportional to the distance d between the objects:
$$\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$$

14.6 Convection

- Convection is heat transfer by the macroscopic

movement of mass. Convection can be natural or forced and generally transfers thermal energy faster than conduction. [Table 14.4](#) gives wind-chill factors, indicating that moving air has the same chilling effect of much colder stationary air. *Convection that occurs along with a phase change* can transfer energy from cold regions to warm ones.

14.7 Radiation

- Radiation is the rate of heat transfer through the emission or absorption of electromagnetic waves.
- The rate of heat transfer depends on the surface area and the fourth power of the absolute temperature:

Conceptual Questions

14.1 Heat

1. How is heat transfer related to temperature?
2. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?
3. When heat transfers into a system, is the energy stored as heat? Explain briefly.

14.2 Temperature Change and Heat Capacity

4. What three factors affect the heat transfer that is necessary to change an object's temperature?
5. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v . How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

14.3 Phase Change and Latent Heat

6. Heat transfer can cause temperature and phase changes. What else can cause these changes?
7. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below 0°C , in the vicinity of large bodies of water?
8. What is the temperature of ice right after it is formed by freezing water?
9. If you place 0°C ice into 0°C water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?
10. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

$$\frac{Q}{t} = \sigma e A T^4,$$

where $\sigma = 5.67 \times 10^{-8} \text{ J/s} \cdot \text{m}^2 \cdot \text{K}^4$ is the Stefan-Boltzmann constant and e is the emissivity of the body. For a black body, $e = 1$ whereas a shiny white or perfect reflector has $e = 0$, with real objects having values of e between 1 and 0. The net rate of heat transfer by radiation is

$$\frac{Q_{\text{net}}}{t} = \sigma e A (T_2^4 - T_1^4)$$

where T_1 is the temperature of an object surrounded by an environment with uniform temperature T_2 and e is the emissivity of the object.

11. In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about 35°C (95°F). In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.
12. In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.
13. Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.
14. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.
15. When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.
16. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from [Table 14.2](#).

14.4 Heat Transfer Methods

17. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?
18. When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a 40.0°C hot tub?
19. Figure 14.30 shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.

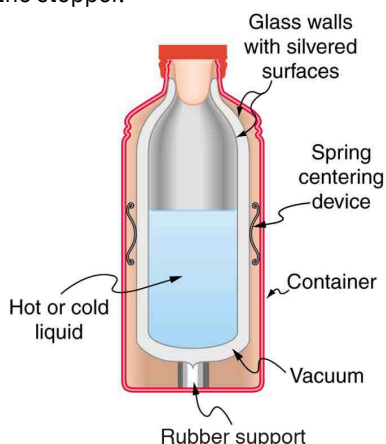


FIGURE 14.30 The construction of a thermos bottle is designed to inhibit all methods of heat transfer.

14.5 Conduction

20. Some electric stoves have a flat ceramic surface with heating elements hidden beneath. A pot placed over a heating element will be heated, while it is safe to touch the surface only a few centimeters away. Why is ceramic, with a conductivity less than that of a metal but greater than that of a good insulator, an ideal choice for the stove top?

21. Loose-fitting white clothing covering most of the body is ideal for desert dwellers, both in the hot Sun and during cold evenings. Explain how such clothing is advantageous during both day and night.



FIGURE 14.31 A jellabiya is worn by many men in Egypt. (credit: Zerida)

14.6 Convection

22. One way to make a fireplace more energy efficient is to have an external air supply for the combustion of its fuel. Another is to have room air circulate around the outside of the fire box and back into the room. Detail the methods of heat transfer involved in each.
23. On cold, clear nights horses will sleep under the cover of large trees. How does this help them keep warm?

14.7 Radiation

24. When watching a daytime circus in a large, dark-colored tent, you sense significant heat transfer from the tent. Explain why this occurs.
25. Satellites designed to observe the radiation from cold (3 K) dark space have sensors that are shaded from the Sun, Earth, and Moon and that are cooled to very low temperatures. Why must the sensors be at low temperature?
26. Why are cloudy nights generally warmer than clear ones?

- 27.** Why are thermometers that are used in weather stations shielded from the sunshine? What does a thermometer measure if it is shielded from the sunshine and also if it is not?

Problems & Exercises

14.2 Temperature Change and Heat Capacity

- On a hot day, the temperature of an 80,000-L swimming pool increases by 1.50°C . What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.
- Show that $1 \text{ cal/g} \cdot ^{\circ}\text{C} = 1 \text{ kcal/kg} \cdot ^{\circ}\text{C}$.
- To sterilize a 50.0-g glass baby bottle, we must raise its temperature from 22.0°C to 95.0°C . How much heat transfer is required?
- The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at 20.0°C : (a) water; (b) concrete; (c) steel; and (d) mercury.
- Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.
- A 0.250-kg block of a pure material is heated from 20.0°C to 65.0°C by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.
- Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?
- (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a 54.9°C temperature increase? (b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.
- On average, would Earth be warmer or cooler without the atmosphere? Explain your answer.
- Following vigorous exercise, the body temperature of an 80.0-kg person is 40.0°C . At what rate in watts must the person transfer thermal energy to reduce the body temperature to 37.0°C in 30.0 min, assuming the body continues to produce energy at the rate of 150 W?
(1 watt = 1 joule/second or $1 \text{ W} = 1 \text{ J/s}$).
- Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails (1 watt = 1 joule/second or $1 \text{ W} = 1 \text{ J/s}$ and $1 \text{ MW} = 1 \text{ megawatt}$). (a) Calculate the rate of temperature increase in degrees Celsius per second ($^{\circ}\text{C/s}$) if the mass of the reactor core is $1.60 \times 10^5 \text{ kg}$ and it has an average specific heat of $0.3349 \text{ kJ/kg} \cdot ^{\circ}\text{C}$. (b) How long would it take to obtain a temperature increase of 2000°C , which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the $5 \times 10^5 \text{ -kg}$ steel containment vessel would also begin to heat up.)

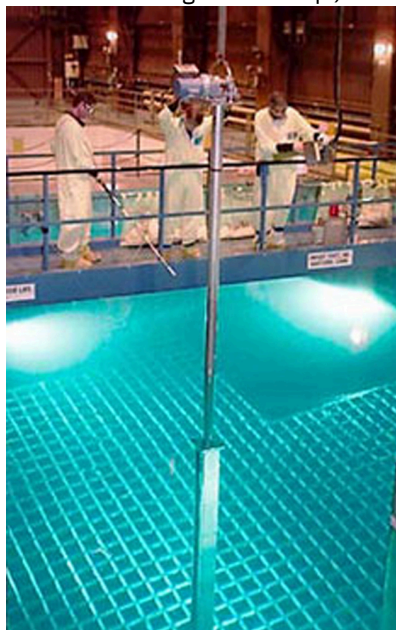


FIGURE 14.32 Radioactive spent-fuel pool at a nuclear power plant. Spent fuel stays hot for a long time. (credit: U.S. Department

of Energy)

14.3 Phase Change and Latent Heat

11. How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at 0°C if their heat of fusion is the same as that of water?
12. A bag containing 0°C ice is much more effective in absorbing energy than one containing the same amount of 0°C water.
 - a. How much heat transfer is necessary to raise the temperature of 0.800 kg of water from 0°C to 30.0°C ?
 - b. How much heat transfer is required to first melt 0.800 kg of 0°C ice and then raise its temperature?
 - c. Explain how your answer supports the contention that the ice is more effective.
13. (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from 30.0°C to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W
 $1 \text{ watt} = 1 \text{ joule/second}$ ($1 \text{ W} = 1 \text{ J/s}$)?
14. The formation of condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of condensation forms on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs.
15. On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at 0°C and completely melts to 0°C water in exactly one day
 $1 \text{ watt} = 1 \text{ joule/second}$ ($1 \text{ W} = 1 \text{ J/s}$)?
16. On a certain dry sunny day, a swimming pool's temperature would rise by 1.50°C if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?
17. (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from -20.0°C to 130°C , including the energy needed for phase changes?
(b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer?
(c) Make a graph of temperature versus time for this process.
18. In 1986, a gargantuan iceberg broke away from the Ross Ice Shelf in Antarctica. It was approximately a rectangle 160 km long, 40.0 km wide, and 250 m thick.
 - (a) What is the mass of this iceberg, given that the density of ice is 917 kg/m^3 ?
 - (b) How much heat transfer (in joules) is needed to melt it?
 - (c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of 100 W/m^2 , 12.00 h per day?
19. How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from 95.0°C to 45.0°C ? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)
20. (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases $2.80 \times 10^7 \text{ J}$ of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water has its temperature raised from 20.0°C to 100°C , it boils, and the resulting steam is raised to 300°C . (b) Discuss additional complications caused by the fact that crude oil has a smaller density than water.
21. The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.
22. To help prevent frost damage, 4.00 kg of 0°C water is sprayed onto a fruit tree.
 - (a) How much heat transfer occurs as the water freezes?
 - (b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35 \text{ kJ/kg} \cdot ^{\circ}\text{C}$, and assume that no phase change occurs.
23. A 0.250-kg aluminum bowl holding 0.800 kg of soup at 25.0°C is placed in a freezer. What is the final temperature if 377 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water? Explicitly show how you follow the steps in [Problem-Solving Strategies for the Effects of Heat Transfer](#).

24. A 0.0500-kg ice cube at -30.0°C is placed in 0.400 kg of 35.0°C water in a very well-insulated container. What is the final temperature?
25. If you pour 0.0100 kg of 20.0°C water onto a 1.20-kg block of ice (which is initially at -15.0°C), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.
26. Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of 500°C rock must be placed in 4.00 kg of 15.0°C water to bring its temperature to 100°C , if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings and take the average specific heat of the rocks to be that of granite.
27. What would be the final temperature of the pan and water in [Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan](#) if 0.260 kg of water was placed in the pan and 0.0100 kg of the water evaporated immediately, leaving the remainder to come to a common temperature with the pan?
28. In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of 808 kg/m^3 .
 - (a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to 3.00°C . (Use c_p and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies.
 - (b) What is this heat transfer rate in kilowatt-hours?
 - (c) Compare the amount of cooling obtained from melting an identical mass of 0°C ice with that from evaporating the liquid nitrogen.
29. Some gun fanciers make their own bullets, which involves melting and casting the lead slugs. How much heat transfer is needed to raise the temperature and melt 0.500 kg of lead, starting from 25.0°C ?

14.5 Conduction

30. (a) Calculate the rate of heat conduction through house walls that are 13.0 cm thick and that have an average thermal conductivity twice that of glass wool. Assume there are no windows or doors. The surface area of the walls is 120 m^2 and their inside surface is at 18.0°C , while their outside surface is at 5.00°C . (b) How many 1-kW room heaters would be needed to balance the heat transfer due to conduction?
31. The rate of heat conduction out of a window on a winter day is rapid enough to chill the air next to it. To see just how rapidly the windows transfer heat by conduction, calculate the rate of conduction in watts through a 3.00-m^2 window that is 0.635 cm thick ($1/4$ in) if the temperatures of the inner and outer surfaces are 5.00°C and -10.0°C , respectively. This rapid rate will not be maintained—the inner surface will cool, and even result in frost formation.
32. Calculate the rate of heat conduction out of the human body, assuming that the core internal temperature is 37.0°C , the skin temperature is 34.0°C , the thickness of the tissues between averages 1.00 cm, and the surface area is 1.40 m^2 .
33. Suppose you stand with one foot on ceramic flooring and one foot on a wool carpet, making contact over an area of 80.0 cm^2 with each foot. Both the ceramic and the carpet are 2.00 cm thick and are 10.0°C on their bottom sides. At what rate must heat transfer occur from each foot to keep the top of the ceramic and carpet at 33.0°C ? For ceramic, use the thermal conductivity for glass.
34. A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?

35. (a) A firewalker runs across a bed of hot coals without sustaining burns. Calculate the heat transferred by conduction into the sole of one foot of a firewalker given that the bottom of the foot is a 3.00-mm-thick callus with a conductivity at the low end of the range for wood and its density is 300 kg/m^3 . The area of contact is 25.0 cm^2 , the temperature of the coals is 700°C , and the time in contact is 1.00 s.
- (b) What temperature increase is produced in the 25.0 cm^3 of tissue affected?
- (c) What effect do you think this will have on the tissue, keeping in mind that a callus is made of dead cells?
36. (a) What is the rate of heat conduction through the 3.00-cm-thick fur of a large animal having a 1.40-m^2 surface area? Assume that the animal's skin temperature is 32.0°C , that the air temperature is -5.00°C , and that fur has the same thermal conductivity as air. (b) What food intake will the animal need in one day to replace this heat transfer?
37. A walrus transfers energy by conduction through its blubber at the rate of 150 W when immersed in -1.00°C water. The walrus's internal core temperature is 37.0°C , and it has a surface area of 2.00 m^2 . What is the average thickness of its blubber, which has the conductivity of fatty tissues without blood?



FIGURE 14.33 Walrus on ice. (credit: Captain Budd Christman, NOAA Corps)

38. Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of 10.0 m^2 and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of 2.00 m^2 , assuming the same temperature difference across each.
39. Suppose a person is covered head to foot by wool clothing with average thickness of 2.00 cm and is transferring energy by conduction through the clothing at the rate of 50.0 W. What is the temperature difference across the clothing, given the surface area is 1.40 m^2 ?
40. Some stove tops are smooth ceramic for easy cleaning. If the ceramic is 0.600 cm thick and heat conduction occurs through the same area and at the same rate as computed in [Example 14.6](#), what is the temperature difference across it? Ceramic has the same thermal conductivity as glass and brick.
41. One easy way to reduce heating (and cooling) costs is to add extra insulation in the attic of a house. Suppose the house already had 15 cm of fiberglass insulation in the attic and in all the exterior surfaces. If you added an extra 8.0 cm of fiberglass to the attic, then by what percentage would the heating cost of the house drop? Take the single story house to be of dimensions 10 m by 15 m by 3.0 m. Ignore air infiltration and heat loss through windows and doors.
42. (a) Calculate the rate of heat conduction through a double-paned window that has a 1.50-m^2 area and is made of two panes of 0.800-cm-thick glass separated by a 1.00-cm air gap. The inside surface temperature is 15.0°C , while that on the outside is -10.0°C . (Hint: There are identical temperature drops across the two glass panes. First find these and then the temperature drop across the air gap. This problem ignores the increased heat transfer in the air gap due to convection.)
- (b) Calculate the rate of heat conduction through a 1.60-cm-thick window of the same area and with the same temperatures. Compare your answer with that for part (a).
43. Many decisions are made on the basis of the payback period: the time it will take through savings to equal the capital cost of an investment. Acceptable payback times depend upon the business or philosophy one has. (For some industries, a payback period is as small as two years.) Suppose you wish to install the extra insulation in [Exercise 9.41](#). If energy cost \$1.00 per million joules and the insulation was \$4.00 per square meter, then calculate the simple payback time. Take the average ΔT for the 120 day heating season to be 15.0°C .

44. For the human body, what is the rate of heat transfer by conduction through the body's tissue with the following conditions: the tissue thickness is 3.00 cm, the change in temperature is 2.00°C , and the skin area is 1.50 m^2 . How does this compare with the average heat transfer rate to the body resulting from an energy intake of about 2400 kcal per day? (No exercise is included.)

14.6 Convection

45. At what wind speed does -10°C air cause the same chill factor as still air at -29°C ?
46. At what temperature does still air cause the same chill factor as -5°C air moving at 15 m/s?
47. The "steam" above a freshly made cup of instant coffee is really water vapor droplets condensing after evaporating from the hot coffee. What is the final temperature of 250 g of hot coffee initially at 90.0°C if 2.00 g evaporates from it? The coffee is in a Styrofoam cup, so other methods of heat transfer can be neglected.
48. (a) How many kilograms of water must evaporate from a 60.0-kg woman to lower her body temperature by 0.750°C ?
(b) Is this a reasonable amount of water to evaporate in the form of perspiration, assuming the relative humidity of the surrounding air is low?
49. On a hot dry day, evaporation from a lake has just enough heat transfer to balance the 1.00 kW/m^2 of incoming heat from the Sun. What mass of water evaporates in 1.00 h from each square meter? Explicitly show how you follow the steps in the [Problem-Solving Strategies for the Effects of Heat Transfer](#).
50. One winter day, the climate control system of a large university classroom building malfunctions. As a result, 500 m^3 of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by 10.0°C (that is, to bring the air to room temperature)?
51. The Kilauea volcano in Hawaii is the world's most active, disgorging about $5 \times 10^5\text{ m}^3$ of 1200°C lava per day. What is the rate of heat transfer out of Earth by convection if this lava has a density of 2700 kg/m^3 and eventually cools to 30°C ? Assume that the specific heat of lava is the same as that of granite.



FIGURE 14.34 Lava flow on Kilauea volcano in Hawaii. (credit: J. P. Eaton, U.S. Geological Survey)

52. During heavy exercise, the body pumps 2.00 L of blood per minute to the surface, where it is cooled by 2.00°C . What is the rate of heat transfer from this forced convection alone, assuming blood has the same specific heat as water and its density is 1050 kg/m^3 ?
53. A person inhales and exhales 2.00 L of 37.0°C air, evaporating $4.00 \times 10^{-2}\text{ g}$ of water from the lungs and breathing passages with each breath.
(a) How much heat transfer occurs due to evaporation in each breath?
(b) What is the rate of heat transfer in watts if the person is breathing at a moderate rate of 18.0 breaths per minute?
(c) If the inhaled air had a temperature of 20.0°C , what is the rate of heat transfer for warming the air?
(d) Discuss the total rate of heat transfer as it relates to typical metabolic rates. Will this breathing be a major form of heat transfer for this person?
54. A glass coffee pot has a circular bottom with a 9.00-cm diameter in contact with a heating element that keeps the coffee warm with a continuous heat transfer rate of 50.0 W
(a) What is the temperature of the bottom of the pot, if it is 3.00 mm thick and the inside temperature is 60.0°C ?
(b) If the temperature of the coffee remains constant and all of the heat transfer is removed by evaporation, how many grams per minute evaporate? Take the heat of vaporization to be 2340 kJ/kg .

14.7 Radiation

55. At what net rate does heat radiate from a 275-m^2 black roof on a night when the roof's temperature is 30.0°C and the surrounding temperature is 15.0°C ? The emissivity of the roof is 0.900.
56. (a) Cherry-red embers in a fireplace are at 850°C and have an exposed area of 0.200 m^2 and an emissivity of 0.980. The surrounding room has a temperature of 18.0°C . If 50% of the radiant energy enters the room, what is the net rate of radiant heat transfer in kilowatts? (b) Does your answer support the contention that most of the heat transfer into a room by a fireplace comes from infrared radiation?
57. Radiation makes it impossible to stand close to a hot lava flow. Calculate the rate of heat transfer by radiation from 1.00 m^2 of 1200°C fresh lava into 30.0°C surroundings, assuming lava's emissivity is 1.00.
58. (a) Calculate the rate of heat transfer by radiation from a car radiator at 110°C into a 50.0°C environment, if the radiator has an emissivity of 0.750 and a 1.20-m^2 surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of 200 hp (1.5 kW) and the efficiency of automobile engines as 25%.
59. Find the net rate of heat transfer by radiation from a skier standing in the shade, given the following. She is completely clothed in white (head to foot, including a ski mask), the clothes have an emissivity of 0.200 and a surface temperature of 10.0°C , the surroundings are at -15.0°C , and her surface area is 1.60 m^2 .
60. Suppose you walk into a sauna that has an ambient temperature of 50.0°C . (a) Calculate the rate of heat transfer to you by radiation given your skin temperature is 37.0°C , the emissivity of skin is 0.98, and the surface area of your body is 1.50 m^2 . (b) If all other forms of heat transfer are balanced (the net heat transfer is zero), at what rate will your body temperature increase if your mass is 75.0 kg ?

61. Thermography is a technique for measuring radiant heat and detecting variations in surface temperatures that may be medically, environmentally, or militarily meaningful. (a) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of 34.0°C compared with that at 33.0°C , such as on a person's skin? (b) What is the percent increase in the rate of heat transfer by radiation from a given area at a temperature of 34.0°C compared with that at 20.0°C , such as for warm and cool automobile hoods?

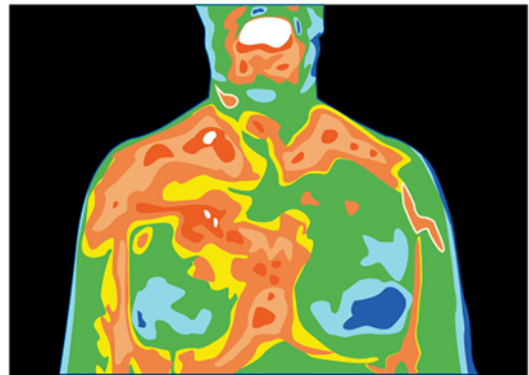


FIGURE 14.35 Artist's rendition of a thermograph of a patient's upper body, showing the distribution of heat represented by different colors.

62. The Sun radiates like a perfect black body with an emissivity of exactly 1. (a) Calculate the surface temperature of the Sun, given that it is a sphere with a $7.00 \times 10^8\text{-m}$ radius that radiates $3.80 \times 10^{26}\text{ W}$ into 3-K space. (b) How much power does the Sun radiate per square meter of its surface? (c) How much power in watts per square meter is that value at the distance of Earth, $1.50 \times 10^{11}\text{ m}$ away? (This number is called the solar constant.)
63. A large body of lava from a volcano has stopped flowing and is slowly cooling. The interior of the lava is at 1200°C , its surface is at 450°C , and the surroundings are at 27.0°C . (a) Calculate the rate at which energy is transferred by radiation from 1.00 m^2 of surface lava into the surroundings, assuming the emissivity is 1.00. (b) Suppose heat conduction to the surface occurs at the same rate. What is the thickness of the lava between the 450°C surface and the 1200°C interior, assuming that the lava's conductivity is the same as that of brick?

- 64.** Calculate the temperature the entire sky would have to be in order to transfer energy by radiation at 1000 W/m^2 —about the rate at which the Sun radiates when it is directly overhead on a clear day. This value is the effective temperature of the sky, a kind of average that takes account of the fact that the Sun occupies only a small part of the sky but is much hotter than the rest. Assume that the body receiving the energy has a temperature of 27.0°C .
- 65.** (a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is 34.0°C and has an emissivity of 0.970. The exposed area of skin is 0.400 m^2 . He receives radiation at the rate of 20.0 W —half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at 34.0°C . (b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.
- 66. Integrated Concepts**
One 30.0°C day the relative humidity is 75.0%, and that evening the temperature drops to 20.0°C , well below the dew point. (a) How many grams of water condense from each cubic meter of air? (b) How much heat transfer occurs by this condensation? (c) What temperature increase could this cause in dry air?
- 67. Integrated Concepts**
Large meteors sometimes strike the Earth, converting most of their kinetic energy into thermal energy. (a) What is the kinetic energy of a 10^9 kg meteor moving at 25.0 km/s ? (b) If this meteor lands in a deep ocean and 80% of its kinetic energy goes into heating water, how many kilograms of water could it raise by 5.0°C ? (c) Discuss how the energy of the meteor is more likely to be deposited in the ocean and the likely effects of that energy.
- 68. Integrated Concepts**
Frozen waste from airplane toilets has sometimes been accidentally ejected at high altitude. Ordinarily it breaks up and disperses over a large area, but sometimes it holds together and strikes the ground. Calculate the mass of 0°C ice that can be melted by the conversion of kinetic and gravitational potential energy when a 20.0 kg piece of frozen waste is released at 12.0 km altitude while moving at 250 m/s and strikes the ground at 100 m/s (since less than 20.0 kg melts, a significant mess results).
- 69. Integrated Concepts**
(a) A large electrical power facility produces 1600 MW of “waste heat,” which is dissipated to the environment in cooling towers by warming air flowing through the towers by 5.00°C . What is the necessary flow rate of air in m^3/s ? (b) Is your result consistent with the large cooling towers used by many large electrical power plants?
- 70. Integrated Concepts**
(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is 20.0%, how long will it take for your body temperature to rise 1.00°C if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?
- 71. Integrated Concepts**
A 76.0-kg person suffering from hypothermia comes indoors and shivers vigorously. How long does it take the heat transfer to increase the person's body temperature by 2.00°C if all other forms of heat transfer are balanced?
- 72. Integrated Concepts**
In certain large geographic regions, the underlying rock is hot. Wells can be drilled and water circulated through the rock for heat transfer for the generation of electricity. (a) Calculate the heat transfer that can be extracted by cooling 1.00 km^3 of granite by 100°C . (b) How long will this take if heat is transferred at a rate of 300 MW , assuming no heat transfers back into the 1.00 km of rock by its surroundings?

73. Integrated Concepts

Heat transfers from your lungs and breathing passages by evaporating water. (a) Calculate the maximum number of grams of water that can be evaporated when you inhale 1.50 L of 37°C air with an original relative humidity of 40.0%. (Assume that body temperature is also 37°C .) (b) How many joules of energy are required to evaporate this amount? (c) What is the rate of heat transfer in watts from this method, if you breathe at a normal resting rate of 10.0 breaths per minute?

74. Integrated Concepts

(a) What is the temperature increase of water falling 55.0 m over Niagara Falls? (b) What fraction must evaporate to keep the temperature constant?

75. Integrated Concepts

Hot air rises because it has expanded. It then displaces a greater volume of cold air, which increases the buoyant force on it. (a) Calculate the ratio of the buoyant force to the weight of 50.0°C air surrounded by 20.0°C air. (b) What energy is needed to cause 1.00 m^3 of air to go from 20.0°C to 50.0°C ? (c) What gravitational potential energy is gained by this volume of air if it rises 1.00 m? Will this cause a significant cooling of the air?

76. Unreasonable Results

(a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with 95.0% of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

77. Unreasonable Results

An Arctic inventor surrounded by ice thinks it would be much less mechanically complex to cool a car engine by melting ice on it than by having a water-cooled system with a radiator, water pump, antifreeze, and so on. (a) If 80.0% of the energy in 1.00 gal of gasoline is converted into “waste heat” in a car engine, how many kilograms of 0°C ice could it melt? (b) Is this a reasonable amount of ice to carry around to cool the engine for 1.00 gal of gasoline consumption? (c) What premises or assumptions are unreasonable?

78. Unreasonable Results

(a) Calculate the rate of heat transfer by conduction through a window with an area of 1.00 m^2 that is 0.750 cm thick, if its inner surface is at 22.0°C and its outer surface is at 35.0°C . (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

79. Unreasonable Results

A meteorite 1.20 cm in diameter is so hot immediately after penetrating the atmosphere that it radiates 20.0 kW of power. (a) What is its temperature, if the surroundings are at 20.0°C and it has an emissivity of 0.800? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

80. Construct Your Own Problem

Consider a new model of commercial airplane having its brakes tested as a part of the initial flight permission procedure. The airplane is brought to takeoff speed and then stopped with the brakes alone. Construct a problem in which you calculate the temperature increase of the brakes during this process. You may assume most of the kinetic energy of the airplane is converted to thermal energy in the brakes and surrounding materials, and that little escapes. Note that the brakes are expected to become so hot in this procedure that they ignite and, in order to pass the test, the airplane must be able to withstand the fire for some time without a general conflagration.

81. Construct Your Own Problem

Consider a person outdoors on a cold night. Construct a problem in which you calculate the rate of heat transfer from the person by all three heat transfer methods. Make the initial circumstances such that at rest the person will have a net heat transfer and then decide how much physical activity of a chosen type is necessary to balance the rate of heat transfer. Among the things to consider are the size of the person, type of clothing, initial metabolic rate, sky conditions, amount of water evaporated, and volume of air breathed. Of course, there are many other factors to consider and your instructor may wish to guide you in the assumptions made as well as the detail of analysis and method of presenting your results.

