Record your answer in Table 20.2. Hint: The center of mass of the stick is at its exact center. Use this fact to find $d$ in terms of $L$.

Remove the aluminum stick and replace it so now it is now hanging about the more central hole. Do not change the length of the simple pendulum. Set the two pendula in motion as before.

**Question 3:** What did you observe? Give a reason(s) for what you observed.

**Question 4:** Use Equation 20.4 to determine the mathematical expression for the experimental moment of inertia of the stick about this new axis. Express your answer in terms of the mass $M$ of the stick and the length $L$ of the stick. Draw and label any figures that may be required in the calculation. Record your answer in Table 20.2. Hint: The central hole is located at a distance of $\frac{2}{3}L$ from the far end of the stick (see Figure 20.3b). Use this fact to find $d$ in terms of $L$.

**Question 5:** Use the parallel axis theorem (Equation 20.7) to compute the theoretical moment of inertia of the stick about an axis through its end and again about an axis through the more central hole in the stick. Draw and label any figures that may be required in the calculation. Express your answers in terms of the mass $M$ of the stick and the length $L$ of the stick. The moment of inertia of a stick of mass $M$ and length $L$ about an axis through its center of mass is $I_{\text{cm}} = \frac{1}{12}ML^2$. Record your answers in Table 20.2. Hint: The distance $h$ from the center of mass to the rotation point is the same as the distance $d$ in questions 2 and 4.

**Question 6:** Compare your answers to Question 5 to those in Questions 2 and 4. Treating the result derived from the parallel axis theorem as the accepted values, compute the percent experimental error for each of the two determinations of moment of inertia for the stick. Record your answers in Table 20.2.

| | |
|---|---|
| Experimental Moment of Inertia about the end (kg m$^2$) | |
| Theoretical Moment of Inertia about the end (kg m$^2$) | |
| Percent Experimental Error (%) | |
| | |
| Experimental Moment of Inertia about the central hole (kg m$^2$) | |
| Theoretical Moment of Inertia about the central hole (kg m$^2$) | |
| Percent Experimental Error (%) | |

Table 20.2: Moment of Inertia of Aluminum Stick

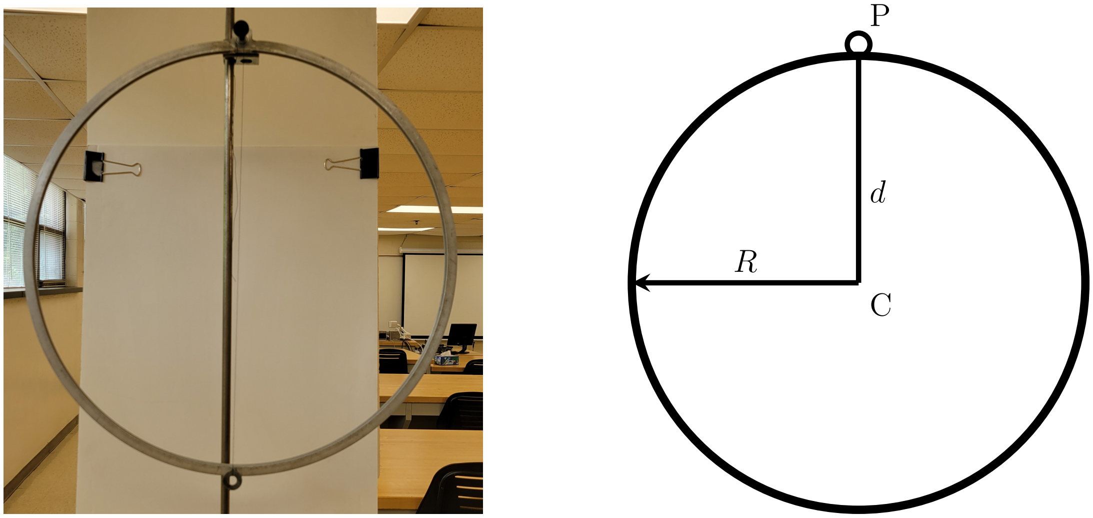
**Question 7:** Compute the radius of gyration of the aluminum stick for each of the axes of rotation considered here. Draw and label any figures that may be required in the calculation. Your answers should be in the form $k = aL$ where $a$ is a decimal number. Record your results in Table 20.3.

| Radius of Gyration About the End (cm) | Radius of Gyration About the Central Hole (cm) |
|---|---|
| | |

Table 20.3: Radius of Gyration of the Aluminum Stick

#### 20.2.1.2 Moment of Inertia of an Aluminum Hoop About an Axis Passing Through a Point on Its Rim

Insert the bolt into the aluminum sleeve then place the aluminum hoop over the aluminum sleeve then clamp the bolt to the stand. Adjust the length of the simple pendulum so that it swings with the same period of oscillation as the hoop (see Figure 20.4a).



(a) Aluminum Hoop with Simple Pendulum (b) Diagram of the Aluminum Hoop

Figure 20.4: Aluminum Hoop

**Question 8:** Having determined the length to which you must adjust the simple pendulum in order too have it oscillate with the same period as the aluminum hoop, determine the ratio of the length of the equivalent simple pendulum $l$ to the radius $R$ of the hoop.

**Question 9:** Use Equation 20.4 to determine the mathematical expression for the experimental moment of inertia of the hoop about an axis through a point on its rim. Express your answer in terms of the mass $M$ of the hoop and the radius $R$ of the hoop. No numerical answer is required. Record your answer in Table 20.4. Hint: The center of mass of the hoop is at its exact center. Use this fact to find $d$ in terms of $R$.

**Question 10:** Use the parallel axis theorem to compute the theoretical moment of inertia of the hoop about an axis passing through a point on the rim of the hoop. Express your answers in terms of the mass $M$ of the hoop and the radius $R$ of the hoop. The moment of inertia of a hoop of mass $M$ and radius $R$ about an axis through its center of mass is $I_{\text{cm}} = MR^2$. Record your answers in Table 20.4. Hint: The distance $h$ from the center of mass to the rotation point is the same as the distance $d$ in question 9.

**Question 11:** Compare your answer to question 10 to that in questions 9. Treating the result derived from the parallel axis theorem as the accepted value, compute the percent experimental error for the determination of moment of inertia for the hoop about an axis through a point on its rim. Record your answers in Table 20.4.

| | |
|---|---|
| Experimental Moment of Inertia (kg m$^2$) | |
| Theoretical Moment of Inertia (kg m$^2$) | |
| Percent Experimental Error (%) | |

Table 20.4: Moment of Inertia of Aluminum Hoop

**Question 12:** Compute the radius of gyration of the hoop. Your answers should be in the form $k = aR$ where $a$ is a decimal number. Record your results in Table 20.5.

| Radius of Gyration of the Hoop Through a Point on Its Rim (cm) |
|---|
| |

Table 20.5: Radius of Gyration of the Hoop

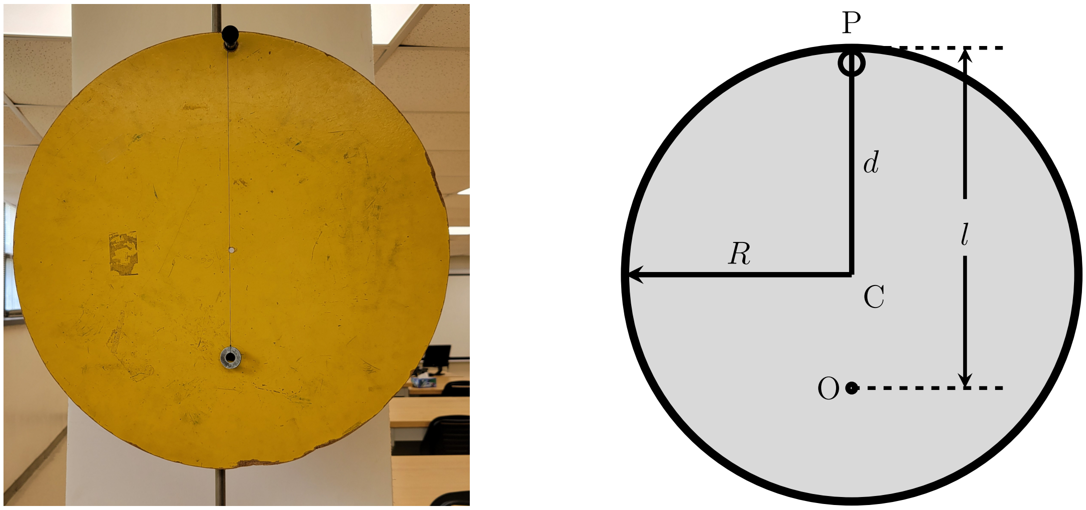
#### 20.2.1.3 Moment of Inertia of an Masonite Disk About an Axis Passing Through a Point on Its Edge

Insert the bolt into the aluminum sleeve then place the masonite disk over the aluminum sleeve then clamp the bolt to the stand. Adjust the length of the simple pendulum so that it swings with the same period of oscillation as the hoop (see Figure 20.5a).

**Question 13:** Having determined the length to which you must adjust the simple pendulum in order too have it oscillate with the same period as the masonite disk, determine the ratio of the length of the equivalent simple pendulum $l$ to the radius $R$ of the disk.

**Question 14:** Use Equation 20.4 to determine the mathematical expression for the experimental moment of inertia of the disk about an axis through a point on its edge. Express your answer in terms of the mass $M$ of the disk and the radius $R$ of the disk. No numerical answer is required. Record your answer in Table 20.6. Hint: The center of mass of the disk is at its exact center. Use this fact to find $d$ in terms of $R$.

**Question 15:** Use the parallel axis theorem to compute the theoretical moment of inertia of the disk about an axis passing through a point on the edge of the disk. Express your answers in



(a) Masonite Disk with Simple Pendulum (b) Diagram of the Masonite Disk

Figure 20.5: Masonite Disk

terms of the mass $M$ of the disk and the radius $R$ of the disk. The moment of inertia of a disk of mass $M$ and radius $R$ about an axis through its center of mass is $I_{\text{cm}} = \frac{1}{2}MR^2$. Record your answers in Table 20.6. Hint: The distance $h$ from the center of mass to the rotation point is the same as the distance $d$ in question 14.

**Question 16:** Compare your answer to question 15 to that in questions 14. Treating the result derived from the parallel axis theorem as the accepted value, compute the percent experimental error for the determination of moment of inertia for the disk about an axis through a point on its edge. Record your answers in Table 20.6.

| | |
|---|---|
| Experimental Moment of Inertia (kg m$^2$) | |
| Theoretical Moment of Inertia (kg m$^2$) | |
| Percent Experimental Error (%) | |

Table 20.6: Moment of Inertia of the Masonite Disk

**Question 17:** Compute the radius of gyration of the disk. Your answers should be in the form $k = aR$ where $a$ is a decimal number. Record your results in Table 20.7.

| Radius of Gyration of the Disk Through a Point on Its Edge (cm) |
|---|
| |

Table 20.7: Radius of Gyration of the Masonite Disk

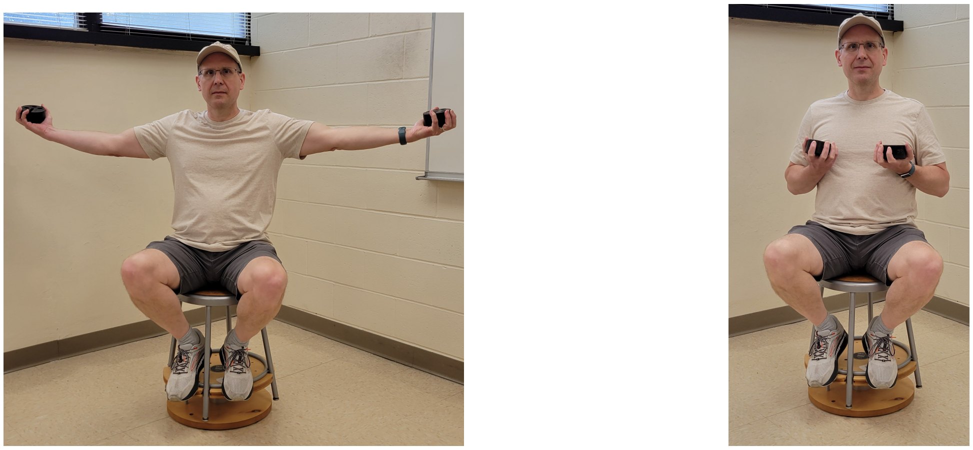
**Question 18:** If the disc were pivoted at the center of oscillation O (see Figure 20.5b) instead of at the point P, what would be the period of oscillation of the disc as compared to its period when pivoted about point P?

**Question 19:** In each of the preceding experiments, would your results be changed if you had used a bob of a different mass on the equivalent simple pendulum? Why?

### 20.2.2 Demonstration of the Theorem of Conservation of Angular Momentum

You will now attempt to demonstrate to yourself that the total angular momentum of a system along a fixed axis is conserved in the absence of an external torque about that axis. You will require four pieces of apparatus: (1) a frictionless rotating stool, (2) a set of two kilogram weights, (3) a rim-loaded bicycle wheel, and (4) a willing student volunteer.

Request that the student sit on the stool with his/her arms extended horizontally with a two kilogram weight in each hand. Have someone start him/her spinning. Assume that friction is negligible and exerts no torque about the vertical axis. Now have the student on the rotating stool bring his/her hands to his/her side (see Figure 20.6).



(a) Individual Spinning with Weights Held Away from Body

(b) Individual Spinning with Weights Held Next to Body

Figure 20.6: Individual Spinning on Rotating Stool

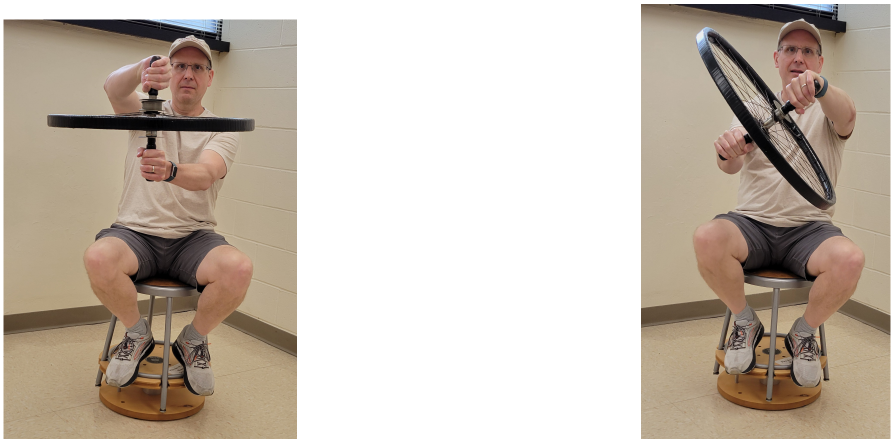
**Question 20:** What did you observe?

The only external forces acting on the student are the force due to gravity and the normal force that the stool exerts on the student. Both of these forces, however, exert no torque on the student about the vertical axis because each acts through the center of mass of the student plus weights, which lies along the vertical axis, and thus each has zero moment arm about the vertical axis of rotation. Hence, the angular momentum of the system of student plus weights along the vertical axis $L_z$ is conserved.

$$L_{z\mathrm{i}} = L_{z\mathrm{f}} \tag{20.11}$$
$$I_\mathrm{i}\omega_\mathrm{i} = I_\mathrm{f}\omega_\mathrm{f}$$

**Question 21:** How does the moment of inertia about the vertical axis of the system of student plus weights when the student has his/her hands down to his/her side compare with the moment of inertia when he/she has his/her arms outstretched? Explain carefully. Apply Equation 20.11 as part of your answer.

Have a student sit on the stool. Have them hold the axle of the weighted rim bicycle wheel with its axis vertical. Wind the cord around the central hub about three revolutions. Pull the cord to set the wheel spinning. During this procedure, the student and the stool should remain at rest. Now ask the student to change the direction of the bicycle wheel axle so that it makes some angle less than 90° with the vertical direction (see Figure 20.7).



(a) Individual at Rest on a Chair with Wheel Spinning Counter-Clockwise

(b) Individual Spinning on a Chair with Wheel Rotated

Figure 20.7: Individual on Stool with Rotating Wheel

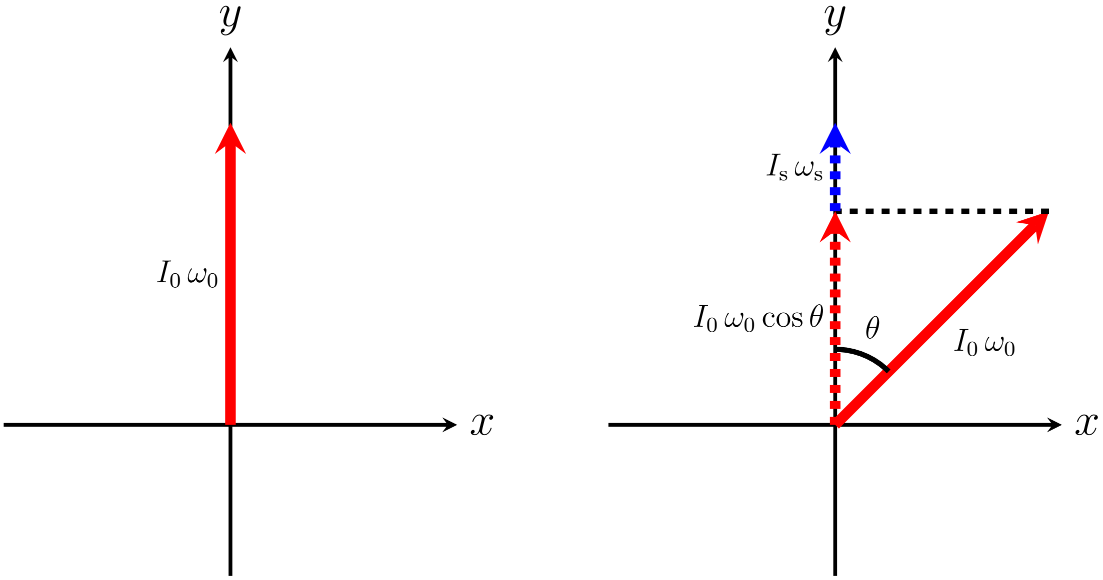


Figure 20.8: Conservation of Angular Momentum Diagram

When the student turns the axis of the wheel through an angle $\theta$ from the vertical direction, he/she must supply a torque. This torque, however, is internal to the system of student, wheel, and rotating stool. Since there is no external torque about the vertical axis acting on the system, the vertical component of the angular momentum of the system must be conserved. However, since the wheel is now spinning about an axis making an angle $\theta$ with the vertical, it contributes a vertical component of angular momentum of $I_0\omega_0 \cos\theta$ to the system, where $I_0$ is the moment of inertia of the wheel about its axis and $\omega_0$ is the angular speed of the wheel about its axis. Hence, the student and stool must supply the additional angular momentum about the vertical axis in order that the total angular momentum along the vertical axis be conserved, i.e., equal the total angular momentum $I_0\omega_0$ when the axle of the wheel was in the initial position. Hence, the student and platform begin to rotate about the vertical direction (see Figure 20.8).

Suppose $I_{\rm s}\omega_{\rm s}$ is the angular momentum of student plus stool about the vertical direction, where $I_{\rm s}$ is the moment of inertia of the student plus stool about the vertical axis and $\omega_{\rm s}$ is the angular speed about this axis. Then

$$I_{\rm s}\omega_{\rm s} + I_0\omega_0 \cos\theta = I_0\omega_0 \tag{20.12}$$

according to the conservation of angular momentum.

**Question 22:** Solve Equation 20.12 for $I_{\rm s}\omega_{\rm s}$ the angular momentum of the student plus stool about the vertical direction. For what angle $\theta$ does Equation 20.12 predict that the angular momentum of student plus stool will have a maximum value? Find the maximum value of $I_{\rm s}\omega_{\rm s}$ in terms of $I_0$ and $\omega_0$. Is this in agreement with what you observed? Hint: Refer to Figure 20.9.

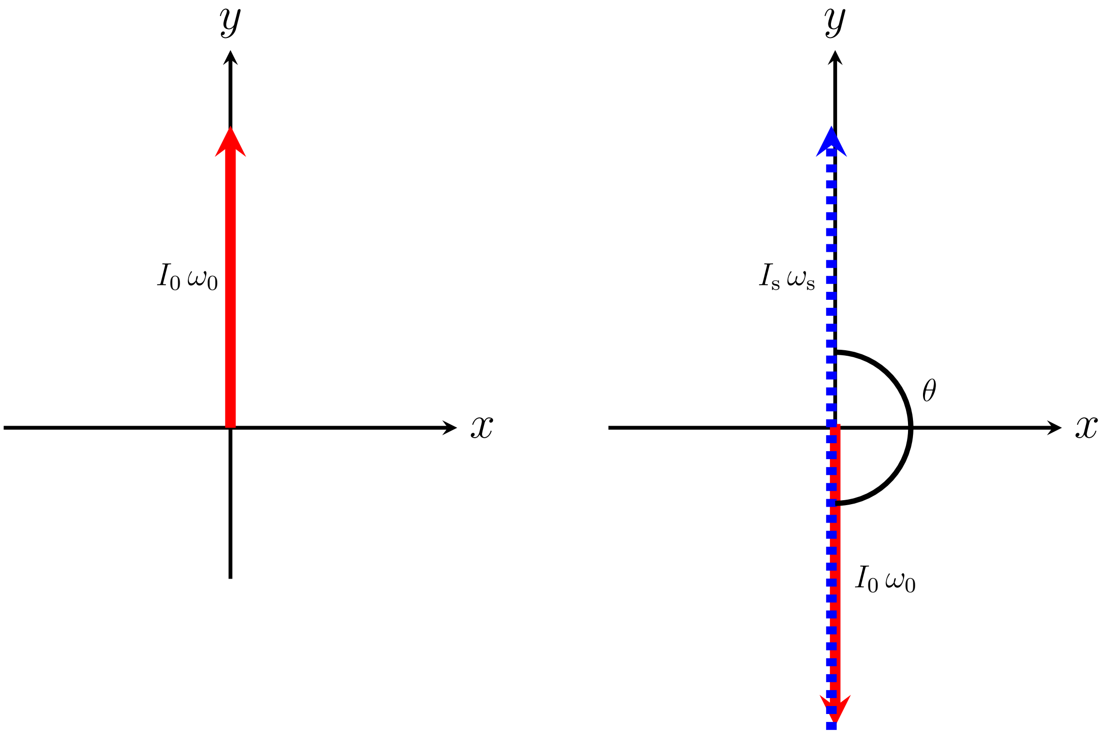


Figure 20.9: Maximum Angular Momentum of Student and Stool

# Part VII

# Simple Harmonic Oscillations

# Lab 21

# Simple Harmonic Motion

An important type of motion frequently encountered in nature is motion that repeats itself at regular intervals. This type of motion is called vibratory, oscillatory, or periodic. One type of periodic motion of a particle is that in which the restoring force or torque acting on the particle is proportional to the particle's displacement from an equilibrium position and is in the opposite direction. Such motion is called simple harmonic motion (SHM). The mathematical description of simple harmonic motion can be used to explain such things as the vibration of molecules and the behavior of electronic circuits. In this experiment you will study SHM as approximated by the motion of a simple pendulum, a mass suspended from a helical spring, and a cart attached to two springs on the linear air track.

## 21.1 Theory

### 21.1.1 The Simple Pendulum

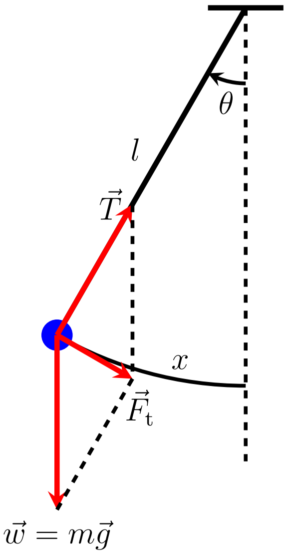


Figure 21.1: The Forces Acting on a Simple Pendulum

The simple pendulum consists of a particle of mass $m$ suspended from a massless inextensible string of length $l$. If this pendulum, approximated by a heavy bob on a thin string, is displaced through a small angle $\theta$ and released, the resulting motion will be oscillatory about the horizontal axis. This oscillatory motion closely approximates SHM, provided $\theta$ is small ($\theta < 15°$). Consider the simple pendulum of length $l$ in Figure 21.1, displaced so the string makes an angle of $\theta$ with the vertical.

The forces acting on the bob are the weight $\vec{w} = m\vec{g}$ and the tension in the string $\vec{T}$. The resultant of these two forces, $\vec{F}_\mathrm{t}$, is tangential to the arc $x$ through which the bob swings. Its magnitude is given by

$$F_\mathrm{t} = mg\sin\theta \tag{21.1}$$

The angular displacement $\theta$ may be expressed in terms of the arc $x$, which is the linear displacement of the pendulum.

$$\theta = \frac{x}{l} \tag{21.2}$$

Combining Equations 21.1 and 21.2 gives

$$F_\mathrm{t} = mg\sin\left(\frac{x}{l}\right) \tag{21.3}$$

Note that the motion is *not* simple harmonic because $F_\mathrm{t} \propto \sin\left(\frac{x}{l}\right)$. That is, the restoring force is not proportional to the displacement; it is proportional to a function of the displacement. However, for small angles $\theta$, $\sin\theta \approx \theta$ (for $\theta < 15°$, $\theta$ differs from $\sin\theta$ by about 0.5%), therefore $\sin\left(\frac{x}{l}\right) \approx \left(\frac{x}{l}\right)$ and

$$F_\mathrm{t} \approx \frac{mg}{l}x \tag{21.4}$$

Equation 21.4 satisfies the definition of SHM that the particle undergoes a restoring force $\vec{F}$ such that

$$F = kx \tag{21.5}$$

where the arbitrary constant $k$ in this case is given by

$$k = \frac{mg}{l} \tag{21.6}$$

It can be shown that the period $T$ of an object undergoing SHM can be expressed as

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{21.7}$$

For the case of the simple pendulum, we may substitute for $k$ in Equation 21.7 the value expressed in Equation 21.6. We then have for the period $T$ of a simple pendulum of length $l$ undergoing a small displacement

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{21.8}$$

### 21.1.2 The Motion of a Mass Suspended from a Helical Spring

Consider a helical spring from is suspended a mass $m$ (see Figure 21.2).

When the mass is pulled down from the equilibrium position through a displacement $\vec{x}$ and released, the net force acting on the mass is the elastic restoring force $\vec{F}$.

$$\vec{F} = -k\vec{x} \tag{21.9}$$

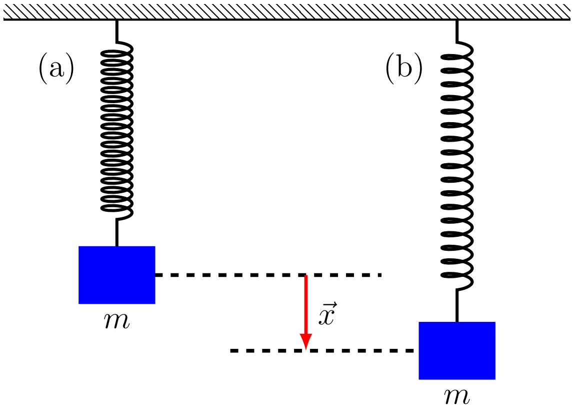


Figure 21.2: (a) A Mass $m$ Suspended from a Helical Spring in Equilibrium Position. (b) The Mass $m$ Moved from Equilibrium Position Through a Displacement $\vec{x}$.

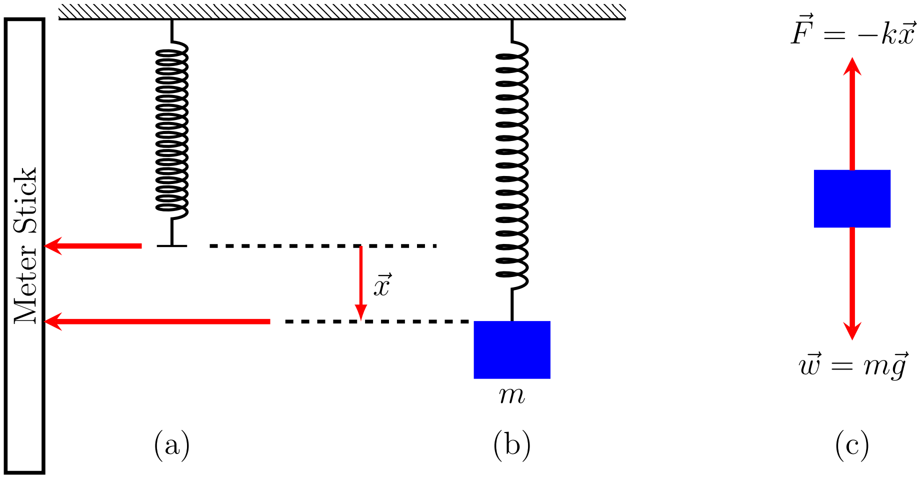


Figure 21.3: (a) A Helical Spring in its Equilibrium Position. (b) Displacement $\vec{x}$ of Spring Caused by the Addition of Mass $m$. (c) Free-Body Diagram of Forces Acting on the Suspended Mass in Figure 21.3 (b)

The negative sign indicates that the direction of force is opposite that of the displacement and $k$ is the spring constant. To experimentally determine the spring constant, $k$, of the spring, hang the spring vertically as shown in Figure 21.3.

Place a meter stick next to the hanging spring and measure the location of the end of the spring to determine its equilibrium position (see Figure 21.3a). By adding various masses, $m$, to the spring and recording the corresponding displacement $\vec{x}$ with the use of the meter stick, the spring constant, $k$, can be determined (see Figure 21.3b). If the spring and suspended mass, $m$, are at rest, then the magnitude of the restoring force, $kx$, must equal the weight, $mg$, of the suspended mass (see Figure 21.3c).

$$kx = mg \tag{21.10}$$

From Equation 21.10, the spring constant can be determined by

$$k = \frac{mg}{x} \tag{21.11}$$

Since there is an unbalanced force $\vec{F}$ acting on the mass for the situation described in Figure

21.2b, the mass will experience an acceleration $\vec{a}$ according to Newton's second law

$$\vec{F} = m\vec{a} \tag{21.12}$$

Combining Equations 21.9 and 21.12 gives us

$$m\vec{a} = -k\vec{x} \tag{21.13}$$

or

$$\vec{a} = -\frac{k}{m}\vec{x} \tag{21.14}$$

The acceleration of the mass is proportional to the displacement and is in the opposite direction. In this discussion we have neglected the mass of the spring whose particles are also undergoing SHM. It can be shown that the equivalent mass of the vibrating system is the mass of the suspended object plus one-third the mass of the spring. The period of a system undergoing SHM is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{21.15}$$

In the case of a mass suspended from a helical spring, $m$ is the equivalent mass of the system (suspended mass plus one-third the mass of the spring), and $k$ is the spring constant.

### 21.1.3 Motion of a Cart Between Two Stretched Springs on an Air Track

Consider an object of mass $m$ resting on a frictionless surface in equilibrium between two stretched springs as shown in Figure 21.4.

If as in Figure 21.4b, the object is moved to the right through a displacement $\vec{x}$ (we will define the positive $x$ axis as horizontal to the right) and released, the unbalanced forces acting on it will be the restoring forces in both springs. The restoring forces of the springs are given by Hooke's law.

$$\vec{F}_1 = -k_1\vec{x} \tag{21.16}$$

and

$$\vec{F}_2 = -k_2\vec{x} \tag{21.17}$$

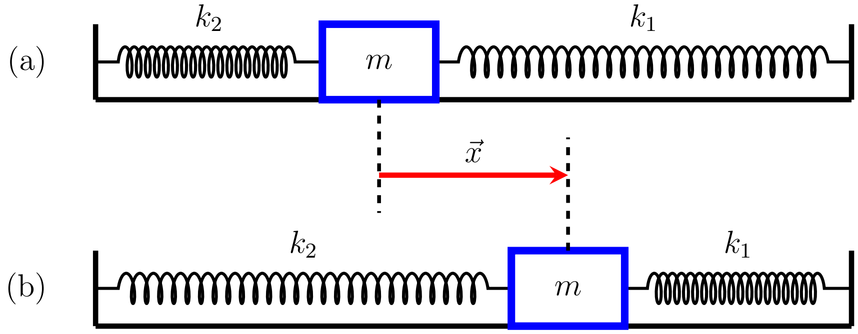


Figure 21.4: (a) Mass $m$ in Equilibrium Between Two Stretched Springs of Spring Constants $k_1$ and $k_2$. (b) Mass $m$ Moved Through a Displacement $\vec{x}$.

The total unbalanced restoring force $\vec{F}_{\mathrm{t}}$ will be

$$\vec{F}_{\mathrm{t}} = \vec{F}_1 + \vec{F}_2 \tag{21.18}$$

or

$$\vec{F}_{\mathrm{t}} = -\left(k_1 + k_2\right)\vec{x} \tag{21.19}$$

Since the total unbalanced restoring force is proportional to the displacement and in the opposite direction, the object will undergo simple harmonic motion.

**Question 1:** Derive an expression for the period of the object shown in Figure 21.4.

## 21.2 Experiment

### 21.2.1 Measuring the Spring Constant

You will be given two springs (a steel spring and a brass spring). Hang the steel spring from a physics stand and measure the position of the bottom of the spring. Add a mass to the spring and note the new position, then compute the displacement (elongation) of the spring (see Figure 21.5). Do this for five different masses. Repeat the procedure for the brass spring. Record all of your results in Table 21.1.

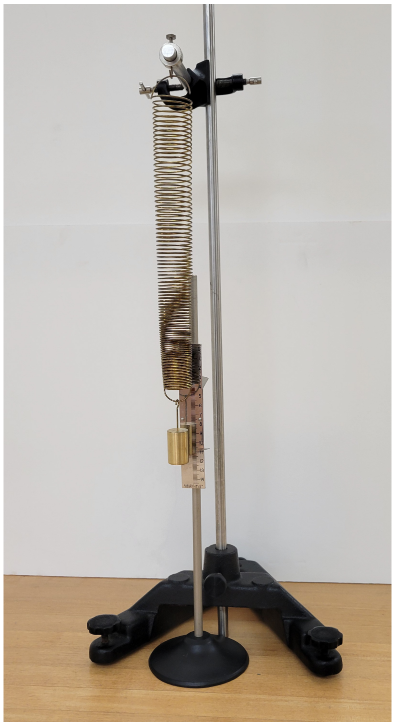


Figure 21.5: Mass Hanging From a Vertical Spring

**Question 2:** Plot your results for each spring. Plot the weight on the $y$-axis and the elongation on the $x$-axis. How do the shapes of your graphs verify Hooke's law? What does the slope of each curve represent? What is the value of the spring constant $k$ for each spring? Record your values in Table 21.2.

| | Mass Suspended From Spring ($m$) (kg) | Weight ($w = mg$) (N) | Elongation of Spring ($\Delta x$) (m) |
|---|---|---|---|
| Spring 1 (steel) | 1. | | |
| | 2. | | |
| | 3. | | |
| | 4. | | |
| | 5. | | |
| Spring 2 (brass) | 1. | | |
| | 2. | | |
| | 3. | | |
| | 4. | | |
| | 5. | | |

Table 21.1: Masses Suspended From Spring

| | Spring Constant ($k$) (N/m) |
|---|---|
| Spring 1 (steel) | |
| Spring 2 (brass) | |

Table 21.2: Spring Constants for Each Spring

### 21.2.2 Determining the Period of Oscillation of a Mass on a String

1.) Measure the mass of the steel spring. From the steel spring, suspend a mass of 0.40 kg. Displace the mass 0.05 m from the equilibrium position and release. Measure the time for ten complete oscillations. Record your values in Table 21.3.

2.) Repeat the same procedure as in (1) above but now displace the mass by 0.10 m and release. Record your values in Table 21.3.

3.) Repeat the same procedure as in (1) above but now suspend a mass of 0.20 kg. Displace the mass 0.05 m from the equilibrium position and release. Record your values in Table 21.3.

4.) Repeat the procedures 1 - 3 from above with the brass spring. Record all of your values in Table 21.3.

**Question 3:** Using the time for 10 oscillations, compute the experimental period for each condition. Show your calculations and record your values in Table 21.3.

**Question 4:** Using Equation 21.15, calculate the theoretical period for each condition. Note that $m$ is the equivalent mass of the system and $k$ is the spring constant found in Table 21.2. Show your calculations and record your values in Table 21.3.

| Mass of Spring ($m_{\text{sp}}$) (kg) | Mass Suspended From Spring ($m_{\text{s}}$) (kg) | Equivalent Mass of System ($m = m_{\text{sp}} + m_{\text{s}}$) (kg) | Disp. ($\Delta x$) (m) | Time for 10 Cycles ($t$) (s) | Exp. Period ($T_{\text{exp}}$) (s) | Theor. Period ($T_{\text{th}}$) (s) | Percent Experimental Error (%) |
|---|---|---|---|---|---|---|---|
| Steel Spring | | | | | | | |
| | 0.40 | | 0.05 | | | | |
| | 0.40 | | 0.10 | | | | |
| | 0.20 | | 0.05 | | | | |
| Brass Spring | | | | | | | |
| | 0.40 | | 0.05 | | | | |
| | 0.40 | | 0.10 | | | | |
| | 0.20 | | 0.05 | | | | |

Table 21.3: Period of Oscillation of Masses on Springs

**Question 5:** Compare the experimental and theoretical periods for each condition by computing the percent experimental errors. Show your calculations and record your values in Table 21.3.

### 21.2.3 Determining the Period of Oscillation of a Cart Connected to Two Springs on An Air Track

Figure 21.6: Cart Connected to Two Springs on an Air Track

1.) Using the linear air track, set up an experiment similar to that shown in Figure 21.6 (Note that the two springs are not identical and have different spring constants). Using the methods in Section 21.2.1, determine the spring constants of the two springs. Record your results in Table 21.4. Determine the mass of the cart and record its value in the table. Displace the cart by 0.05 m, release it and measure the time to complete ten oscillations. Record your results in Table 21.4.

2.) Add two cylindrical masses to the cart (approximatly 0.10 kg) and repeat procedure (1). Record your results in Table 21.4.

**Question 6:** Using the time for 10 oscillations, compute the experimental period for each condition. Show your calculations and record your values in Table 21.4.

**Question 7:** Using the equation you derived in question 1, calculate the theoretical period for each condition. Show your calculations and record your values in Table 21.4.

**Question 8:** Compare the experimental and theoretical periods for each condition by computing the percent experimental errors. Show your calculations and record your values in Table 21.4.

| Spring Constant 1 ($k_1$) (N/m) | Spring Constant 2 ($k_2$) (N/m) | Mass of Cart ($m$) (kg) | Time for 10 Oscillations ($t$) (s) | Exp. Period ($T_{\text{exp}}$) (s) | Theor. Period ($T_{\text{th}}$) (s) | Percent Experimental Error (%) |
|---|---|---|---|---|---|---|
| Cart Alone | | | | | | |
| | | | | | | |
| Cart with Two Cylindrical Masses | | | | | | |
| | | | | | | |

Table 21.4: Period of Cart Connected to Two Springs on Air Track

# Lab 22

# The Physical Pendulum

In this experiment you will determine the period of a physical pendulum, theoretically and experimentally, and compare this period with the observed period of an equivalent simple pendulum.

## 22.1 Theory

The period $T$ of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{22.1}$$

where $l$ is the length of the simple pendulum and $g$ is the acceleration due to gravity.

A physical pendulum is defined as any rigid body hinged about a horizontal axis and allowed to oscillate in its own plane. Its period is given by

$$T = 2\pi\sqrt{\frac{I}{Mgd}} \tag{22.2}$$

where $I$ is the moment of inertia about the axis of oscillation, $M$ is the mass, $g$ is the acceleration due to gravity, and $d$ is the distance from the axis of oscillation to the center of mass. No matter what the period of a physical pendulum, there must be a simple pendulum of some length $l$ such that it has the same period as the physical pendulum. Such a pendulum is called the equivalent simple pendulum.

A uniform rod free to oscillate about a horizontal axis through one end has a moment of inertia given by

$$I = \frac{1}{3}Ml_{\mathrm{r}}^2 \tag{22.3}$$

where $M$ is the mass and $l_{\mathrm{r}}$ is the length of the rod. Substituting Equation 22.3 into Equation 22.2 we have

$$T = 2\pi\sqrt{\frac{\frac{1}{3}Ml_{\mathrm{r}}^2}{Mgd}} \tag{22.4}$$

For a uniform rod hinged through the end, the distance from the axis of oscillation to the center of mass $d$, is one-half the length of the rod $l_{\mathrm{r}}$

$$d = \frac{1}{2}l_{\mathrm{r}} \tag{22.5}$$

Substituting Equation 22.5 into Equation 22.4 we have

$$T = 2\pi\sqrt{\frac{\frac{1}{3}Ml_{\mathrm{r}}^2}{\frac{1}{2}Mgl_{\mathrm{r}}}} \tag{22.6}$$

or

$$T = 2\pi\sqrt{\frac{2l_{\mathrm{r}}}{3g}} \tag{22.7}$$

Comparing Equation 22.7 and Equation 22.1, it is obvious that the length of the equivalent simple pendulum is two-thirds the length of the rod ($l = \frac{2}{3}l_{\mathrm{r}}$).

## 22.2 Experiment

### 22.2.1 Period of Aluminum Rod (Stick)

Determine the mass and length of the aluminum stick. Using $I = \frac{1}{3}Ml_{\mathrm{r}}^2$, calculate its moment of inertia about the end. Record the values in Table 22.1.

| Mass ($M$) (kg) | Length ($l_{\mathrm{r}}$) (m) | Moment of Inertia About End ($I$) (kg m$^2$) |
|---|---|---|
| | | |

Table 22.1: Moment of Inertia About the End of the Aluminum Stick

Insert a bolt into the aluminum sleeve then place the aluminum stick over the aluminum sleeve (make sure the aluminum stick is hanging from the hole at the end of the stick). Then clamp the bolt to a heavy ring stand. Hang a simple pendulum consisting of a washer tied to a string from the bolt. The length of the simple pendulum can be adjusted by making a few turns of the string around the bolt (see Figure 22.1).

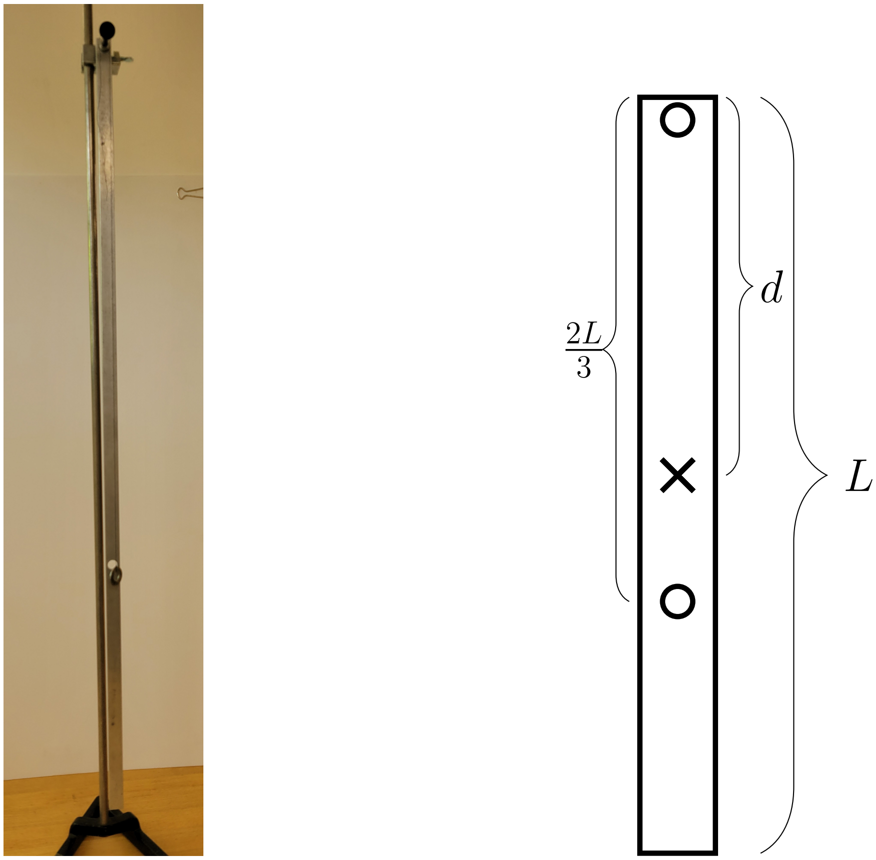
Displace the aluminum stick about 15° and release it. Measure the time for ten oscillations and record your value in Table 22.2.

| Time for 10 Oscillations ($t$) (s) | Experimental Period ($T_{\mathrm{exp}}$) (s) | Theoretical Period ($T_{\mathrm{th}}$) (s) | Percent Experimental Error (%) |
|---|---|---|---|
| | | | |

Table 22.2: Period of Aluminum Stick About End

**Question 1:** Using the time for 10 oscillations, compute the experimental period for the aluminum stick. Show your calculation and record your value in Table 22.2.

**Question 2:** Using Equation 22.7, calculate the theoretical period of the aluminum stick. Show your calculation and record your value in Table 22.2.



(a) Aluminum Stick with Simple Pendulum - Period Experiment

(b) Diagram of the Aluminum Stick - Period Experiment

Figure 22.1: Aluminum Stick - Period Experiment

**Question 3:** Compare the experimental and theoretical period by computing the percent experimental error. Show your calculation and record your value in Table 22.2.

Based on Equation 22.7, construct an equivalent simple pendulum for the aluminum stick (hint: its length should be two-thirds the length of the aluminum stick $l = \frac{2}{3}l_{\mathrm{r}}$). Displace the simple pendulum by 15° and release it. Measure the time for ten oscillations. Record your value in Table 22.3.

**Question 4:** Using the time for 10 oscillations, compute the experimental period for the equivalent simple pendulum for the aluminum stick. Show your calculation and record your value in Table 22.3. How does your value compare to the experimental period of the aluminum stick found in Table 22.2.

| Time for 10 Oscillations ($t$) (s) | Experimental Period ($T_{\mathrm{exp}}$) (s) |
|---|---|
| | |

Table 22.3: Period of Equivalent Simple Pendulum for the Aluminum Stick

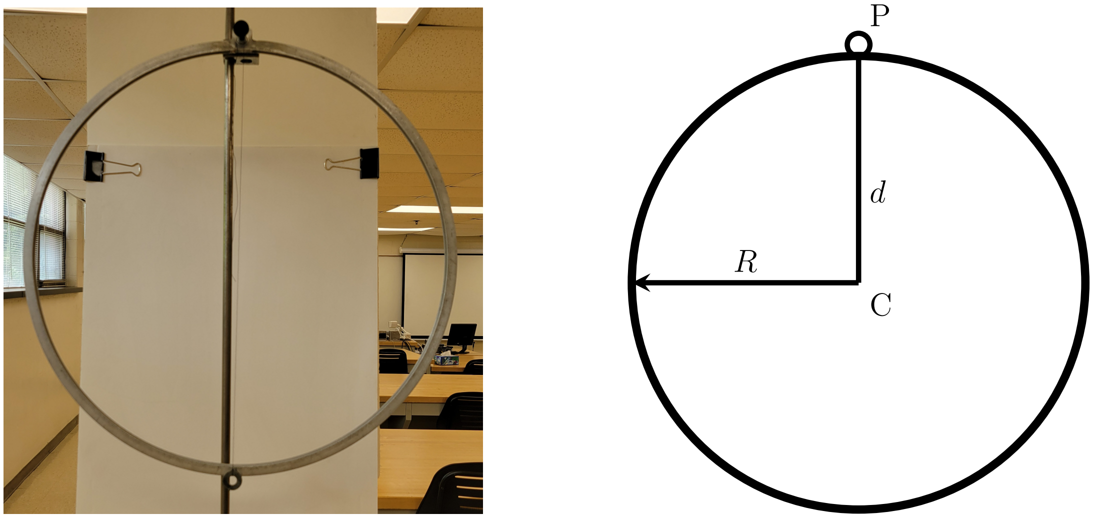
### 22.2.2 Period of Aluminum Hoop

Determine the mass and radius of the aluminum hoop. Using $I = 2MR^2$, calculate its moment of inertia about the edge. Record the values in Table 22.4.

| Mass ($M$) (kg) | Radius ($R$) (m) | Moment of Inertia of Hoop About Edge ($I$) (kg m$^2$) |
|---|---|---|
| | | |

Table 22.4: Moment of Inertia of Aluminum Hoop About a Point on the Edge

Insert the bolt into the aluminum sleeve then place the aluminum hoop over the aluminum sleeve then clamp the bolt to the stand. Hang a simple pendulum consisting of a washer tied to a string from the bolt. The length of the simple pendulum can be adjusted by making a few turns of the string around the bolt (see Figure 22.2).



(a) Aluminum Hoop with Simple Pendulum - Period Experiment

(b) Diagram of the Aluminum Hoop - Period Experiment

Figure 22.2: Aluminum Hoop - Period Experiment

Displace the aluminum hoop about 15° and release it. Measure the time for ten oscillations and record your value in Table 22.5.

| Time for 10 Oscillations ($t$) (s) | Experimental Period ($T_{\text{exp}}$) (s) | Theoretical Period ($T_{\text{th}}$) (s) | Percent Experimental Error (%) |
|---|---|---|---|
| | | | |

Table 22.5: Period of Aluminum Hoop About Its Edge

**Question 5:** Using the time for 10 oscillations, compute the experimental period for the aluminum hoop. Show your calculation and record your value in Table 22.5.

**Question 6:** Starting with Equation 22.2, and using the fact that the moment of inertia of a hoop about its edge is $I = 2MR^2$ and the location of the center of mass of a hoop relative to the edge is $d = R$, derive an equation for the period of the hoop about its edge. Using the equation you derived, calculate the theoretical period of the aluminum stick. Show your calculation and record your value in Table 22.5.

**Question 7:** Compare the experimental and theoretical period by computing the percent experimental error. Show your calculation and record your value in Table 22.5.

Based on the equation that you derived in Question 6, construct an equivalent simple pendulum for the aluminum hoop (hint: its length should be two times the radius of the hoop $l = 2R$). Displace the simple pendulum by 15° and release it. Measure the time for ten oscillations. Record your value in Table 22.6.

**Question 8:** Using the time for 10 oscillations, compute the experimental period for the equivalent simple pendulum for the aluminum hoop. Show your calculation and record your value in Table 22.6. How does your value compare to the experimental period of the aluminum hoop found in Table 22.5.

| Time for 10 Oscillations ($t$) (s) | Experimental Period ($T_{\text{exp}}$) (s) |
|---|---|
| | |

Table 22.6: Period of Equivalent Simple Pendulum for the Aluminum Hoop

### 22.2.3 Period of Masonite Disk

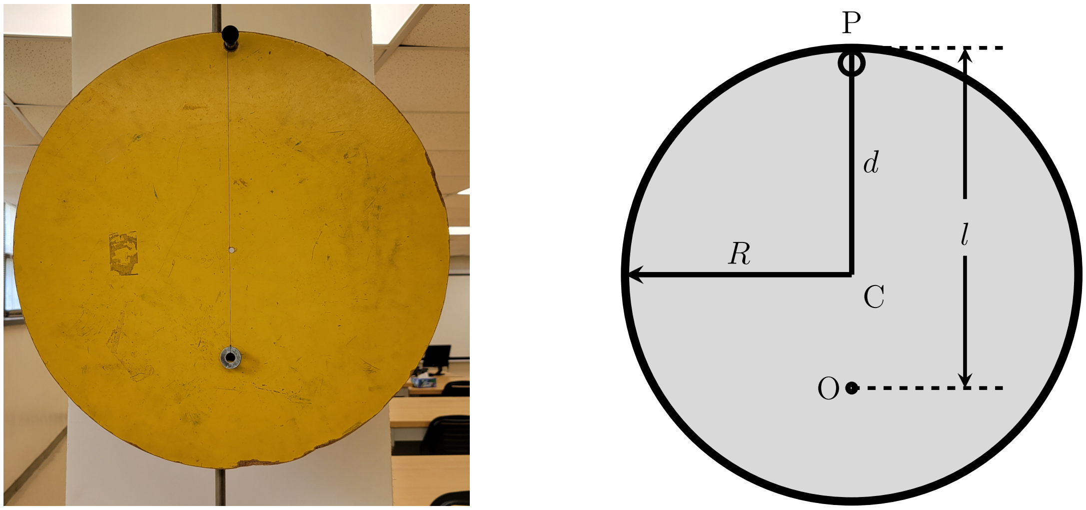
Determine the mass and radius of the masonite disk. Using $I = \frac{3}{2}MR^2$, calculate its moment of inertia about the edge. Record the values in Table 22.7.

| Mass ($M$) (kg) | Radius ($R$) (m) | Moment of Inertia of Disk About Edge ($I$) (kg m$^2$) |
|---|---|---|
| | | |

Table 22.7: Moment of Inertia of Masonite Disk About a Point on the Edge

Insert the bolt into the aluminum sleeve then place the masonite disk over the aluminum sleeve then clamp the bolt to the stand. Hang a simple pendulum consisting of a washer tied to a string from the bolt. The length of the simple pendulum can be adjusted by making a few turns of the string around the bolt (see Figure 22.3).

Displace the masonite disk about 15° and release it. Measure the time for ten oscillations and record your value in Table 22.8.



(a) Masonite Disk with Simple Pendulum - Period Experiment

(b) Diagram of the Masonite Disk - Period Experiment

Figure 22.3: Masonite Disk - Period Experiment

| Time for 10 Oscillations ($t$) (s) | Experimental Period ($T_{\text{exp}}$) (s) | Theoretical Period ($T_{\text{th}}$) (s) | Percent Experimental Error (%) |
|---|---|---|---|
| | | | |

Table 22.8: Period of Masonite Disk About Its Edge

**Question 9:** Using the time for 10 oscillations, compute the experimental period for the masonite disk. Show your calculation and record your value in Table 22.8.

**Question 10:** Starting with Equation 22.2, and using the fact that the moment of inertia of a masonite disk about its edge is $I = \frac{3}{2}MR^2$ and the location of the center of mass of a disk relative to the edge is $d = R$, derive an equation for the period of the disk about its edge. Using the equation you derived, calculate the theoretical period of the masonite disk. Show your calculation and record your value in Table 22.8.

**Question 11:** Compare the experimental and theoretical period by computing the percent experimental error. Show your calculation and record your value in Table 22.8.

Based on the equation that you derived in Question 10, construct an equivalent simple pendulum for the masonite disk (hint: its length should be three-halves times the radius of the disk $l = \frac{3}{2}R$). Displace the simple pendulum by 15° and release it. Measure the time for ten oscillations. Record your value in Table 22.9.

**Question 12:** Using the time for 10 oscillations, compute the experimental period for the equivalent simple pendulum for the masonite disk. Show your calculation and record your value in Table 22.9. How does your value compare to the experimental period of the masonite disk found in Table 22.8.

| Time for 10 Oscillations ($t$) (s) | Experimental Period ($T_{\text{exp}}$) (s) |
|---|---|
| | |

Table 22.9: Period of Equivalent Simple Pendulum for the Masonite Disk

# Part VIII

# Fluid Statics and Dynamics

# Lab 23

# Experimental Studies in Fluid Statics

An interesting and useful consequence of the formation of fluid statics is the fact that a body that is either completely or partially immersed in a fluid (be it liquid or gas) at rest is buoyed up with a force that is equal to the weight of the fluid displaced by the body. The force that buoys the body up acts vertically upward through a point in the body called the center of buoyancy, located at a point that corresponds to the center of gravity of the fluid that would have occupied the volume of the body were the body not in the fluid. These results were deduced by the Greek philosopher Archimedes around 250 B.C. and are summarized by a statement commonly referred to as Archimedes' principle. In this experiment you will investigate the meaning of Archimedes' principle and then use it to determine the specific gravity of various materials. The specific gravity of a material is the ratio of the density of the material to the density of pure water. Thus, specific gravity has nothing to do with gravity; the term is, therefore, misleading and its origin is purely historical.

## 23.1 Theory

Application of Newton's laws of motion and the properties of a fluid at rest readily leads to a simple explanation of Archimedes' principle. Consider the fluid of Figure 23.1 and the arbitrary portion of fluid bounded by the imaginary surface indicated. Assume that the fluid is at rest.

The arrows in Figure 23.1 represent the forces that are exerted by the surrounding fluid at various points on the imaginary surface, bonding the fluid. The force at each point on this surface is normal to the surface at that point. Also, if we consider very small areas $A$ on the surface, then the force on such a small area has a magnitude $pA$, where $p$ is the pressure, which depends only on the vertical depth below the free surface and not on the shape or orientation of the boundary surface. The pressure $p$ increases with depth.

Now, if the fluid inside our imaginary boundary is at rest, then the horizontal component of the resultant of the surface forces must be zero. The vertical component of the resultant of these surface forces, $\vec{B}$, must be equal in magnitude to the weight of the fluid, $\vec{w} = m\vec{g}$, inside the arbitrary surface. The line of action of the force $\vec{B}$ must pass through the center of gravity of the bounded fluid. We call the force $\vec{B}$ the buoyant force and the point at which $\vec{B}$ acts, b, the center of buoyancy.

If we should now replace the fluid inside the arbitrary surface of Figure 23.1 by some solid body having exactly the same shape as our imaginary surface, then the pressure at every point on its surface will be the same as it was before, so that the buoyant force $\vec{B}$ will be unchanged. That is, the fluid will exert an upward force $\vec{B}$ on the body whose magnitude is equal to the weight of the

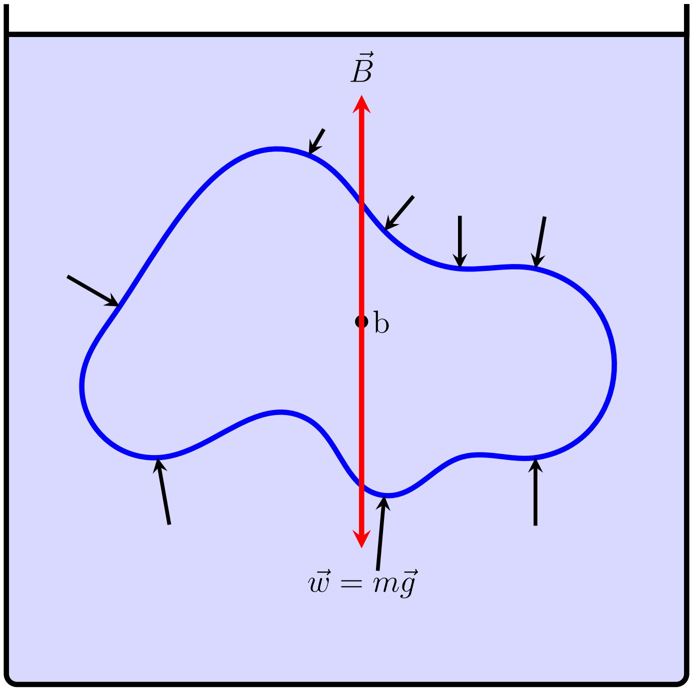


Figure 23.1: Archimedes' Principle. The Buoyant Force $\vec{B}$ is Equal in Magnitude to the Weight $\vec{w}$ of the Displaced Fluid.

fluid originally bounded by the surface. This can be summarized by the equation

$$B = w_{\mathrm{d}} = m_{\mathrm{d}}g = \rho_{\mathrm{d}}V_{\mathrm{d}}g \tag{23.1}$$

where $w_{\mathrm{d}}$ is the weight of the fluid displaced by the object, $m_{\mathrm{d}}$ is the mass of the fluid displaced, $V_{\mathrm{d}}$ is the volume of the fluid displaced, and $\rho_{\mathrm{d}}$ is the density of the fluid displaced. The line of action of $\vec{B}$ will pass through the location of the original center of gravity of the fluid bounded by the surface. Thus, we have a simple explanation of Archimedes' principle.

Note: In general, a submerged body may not be in equilibrium. That is, its weight may be greater or less in magnitude than the buoyant force. Furthermore, if the body is not homogeneous, then its center of gravity will not correspond to the center of buoyancy. If this is the case, then a force couple will be set up by the buoyant force $\vec{B}$ and the weight vector of the body and, therefore, the body may rotate as it rises or falls.

## 23.2 Experiment

You will find on your bench a triple beam balance, a wooden block, an aluminum cylinder, an overflow can, and a catch bucket (see Figure 23.2). We will use these to perform several fluid static experiments involving Archimedes' principle.

### 23.2.1 Experiment With An Object (Wooden Block) Less Dense Than Water

Place the wooden block and empty catch bucket on the balance and measure its mass. Now fill the overflow can until water comes out of the spout. When water has ceased to come from the spout,

Figure 23.2: Fluid Statics Apparatus

place the dry catch bucket under the spout and lower the wooden block carefully into the overflow can until it floats (see Figure 23.3). When the flow of water from the spout of the can has ceased, place the catch bucket with water that it contains on the balance to determine the combined mass of bucket plus water. Record your results in Table 23.1.

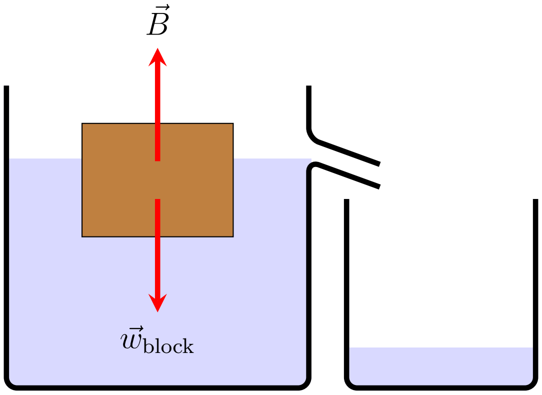


Figure 23.3: Block of Wood Displacing Water

| | |
|---|---|
| Mass of Wooden Block Plus Empty Catch Bucket (g) | |
| Mass of Bucket With Displaced Water (g) | |

Table 23.1: Mass of Wooden Block and Displaced Water

**Question 1:** Compare the combined mass of the wooden block and bucket with the combined mass of bucket plus water displaced by the block. Taking into account the experimental error involved in your procedure, what can you conclude from this comparison? Also, what can you conclude about the relationship between the buoyant force and the weight of the block? Hint: From Figure 23.3 we can see that since the wooden block is in translational equilibrium then summing the forces in the $y$ direction gives

$$\begin{aligned} B - w_{\text{block}} &= 0 \\ B &= w_{\text{block}} \\ m_{\text{d}} g &= m_{\text{block}} g \\ m_{\text{d}} &= m_{\text{block}} \end{aligned}$$

**Question 2:** If a body weighs less than the fluid displaced by it when it is completely submerged, what will, therefore, happen to such a body when we submerge it and then release it? Why?

### 23.2.2 Experiment With An Object (Aluminum Cylinder) More Dense Than Water

Place the aluminum cylynder and empty catch bucket on the balance and measure the mass. Record your value in Table 23.2.

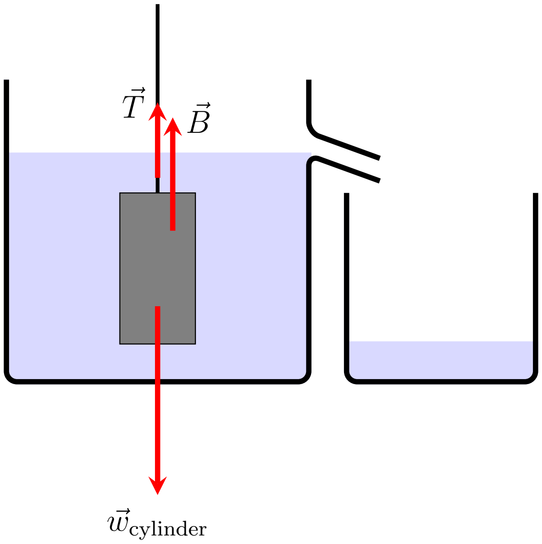


Figure 23.4: Aluminum Cylinder Completely Submerged and Displacing Water

Fill the overflow can as you did before, so that the water is just at the entrance to the spout. Place the balance up on the rod of the physics stand. Suspend the aluminum cylinder from the hook attached underneath the balance (see Figure 23.2). Completely immerse the cylinder into the overflow can, being careful not to allow the cylinder to touch either the sides or the bottom of the container. It is also important that you get rid of air bubbles tending to cling to the surface of the cylinder. Collect the water from the overflow can in the catch bucket (see Figure 23.4). Measure the apparent mass of the cylinder while submerged in the water. Remove the cylinder from the balance and now measure the mass of the catch bucket with the displaced water. Finally, measure the mass of the aluminum cylinder alone in air and record all of your values in Table 23.2.

| | |
|---|---|
| Combined Mass of Aluminum Cylinder And Empty Catch Bucket (g) | |
| Apparent Mass of Aluminum Cylinder in Water (g) | |
| Mass of the Catch Bucket Plus Water (g) | |
| Mass of Aluminum Cylinder in Air (g) | |

Table 23.2: Mass of Aluminum Cylinder and Displaced Water

**Question 3:** Why is it important that the precautions mentioned above be taken? List any possible consequences that could arise as a result of failure to comply with these precautions.

**Question 4:** Determine the apparent mass of the aluminum cylinder when in water plus the mass of the catch bucket with water. Compare this value with the combined mass of the empty bucket and aluminum cylinder when measured in air. Taking into account the experimental error in your procedure, what can you conclude from this comparison? Hint: From Figure 23.4 we can see that since the aluminum cylinder is in translational equilibrium then summing the forces in the $y$ direction gives

$$\begin{aligned}
T + B - w_{\text{cylinder}} &= 0 \\
T + B &= w_{\text{cylinder}} \\
w_{\text{apparent}} + B &= w_{\text{cylinder}} \\
m_{\text{apparent}} g + m_{\text{d}} g &= m_{\text{cylinder}} g \\
m_{\text{apparent}} + m_{\text{d}} &= m_{\text{cylinder}}
\end{aligned}$$

**Question 5:** If the weight of the fluid displaced by a body is less than the weight of the body, what will happen to the body when it is placed in the fluid? Why?

**Question 6:** What is the meaning of (mass of cylinder in air - apparent mass of cylinder water) ×$g$, where $g$ is the acceleration due to gravity?

**Question 7:** Collect all your information from the procedures and comment upon Archimedes' principle. Be original, but concise. Summarize what happens when you have completely immersed into water an object that is less dense than water and an object that is more dense than water.

### 23.2.3 Specific Gravity

The specific gravity of a body is the ratio of the density of the body to the density of pure water

$$\text{S.G.} = \frac{\rho_{\text{b}}}{\rho_{\text{w}}}$$

Using the definition of density, we have

$$\rho_{\text{b}} = \frac{m_{\text{b}}}{V_{\text{b}}}$$

and

$$\rho_{\text{w}} = \frac{m_{\text{w}}}{V_{\text{w}}}$$

Assuming that the body is completely submerged then the volume of the body is equal to the volume of the displaced water $V_{\text{b}} = V_{\text{w}}$. Note that if a fluid other than water is being used, the volume of the submerged body would equal the volume of the displaced new fluid.

Therefore, we have

$$\text{S.G.} = \frac{\rho_{\text{b}}}{\rho_{\text{w}}} = \frac{m_{\text{b}}}{m_{\text{w}}} \tag{23.2}$$

#### 23.2.3.1 Specific Gravity of Aluminum

**Question 8:** Use Equation 23.2 to find an experimental value for the specific gravity of aluminum. You already have compiled all the necessary data. Compare this value to the accepted value of 2.7 and compute your percent experimental error. Record all your values in Table 23.3.

| Experimental Specific Gravity of Aluminum | Accepted Specific Gravity of Aluminum |
|---|---|
| | 2.7 |
| | |
| Percent Experimental error (%) | |

Table 23.3: Specific Gravity of Aluminum

#### 23.2.3.2 Specific Gravity of Ethyl Alcohol

Fill the overflow can with ethyl alcohol. Since we won't be collecting the displaced alcohol, you don't need to fill it all the way up to the spout, just enough so the aluminum cylinder can be completely submerged. Hang the aluminum cylinder from the balance and submerge it into the ethyl alcohol. Measure the apparent mass of the aluminum cylinder while it is submerged in the alcohol. Record your value in Table 23.4. The other values in Table 23.4 can be copied over from Table 23.2.

| | |
|---|---|
| Mass of Aluminum Cylinder in Air (g) | |
| Apparent Mass of Aluminum Cylinder in Water (g) | |
| Apparent Mass of Aluminum Cylinder in Ethyl Alcohol (g) | |

Table 23.4: Mass Values to Determine the Specific Gravity of Ethyl Alcohol

**Question 9:** Determine the specific gravity of ethyl alcohol. Compare this value with the accepted value of 0.79 by computing the percent experimental error. Record your values in Table 23.5. Hint: If the volume of the displaced alcohol and displaced water are the same (equal to the volume of the aluminum cylinder) then

$$\text{S.G.} = \frac{\rho_{\text{alcohol}}}{\rho_{\text{water}}} = \frac{\frac{m_{\text{d,alcohol}}}{V_{\text{d,alcohol}}}}{\frac{m_{\text{d,water}}}{V_{\text{d,water}}}} = \frac{m_{\text{d,alcohol}}}{m_{\text{d,water}}}$$

Since the mass of the displaced fluid is in general equal to the mass of the aluminum cylinder in air minus the apparent mass of the aluminum cylinder in that fluid we have

$$\text{S.G.} = \frac{m_{\text{d,alcohol}}}{m_{\text{d,water}}} = \frac{m_{\text{cylinder,air}} - m_{\text{apparent, alcohol}}}{m_{\text{cylinder,air}} - m_{\text{apparent, water}}}$$

| Experimental Specific Gravity of Ethyl Alcohol | Accepted Specific Gravity of Ethyl Alcohol |
|---|---|
| | 0.79 |
| | |
| Percent Experimental error (%) | |

Table 23.5: Specific Gravity of Ethyl Alcohol

# Lab 24

# Experimental Studies in Fluid Dynamics

## 24.1 Theory

Fluid Dynamics, also called Hydrodynamics, is the study of fluids in motion. Two important questions regarding fluids in motion are:

- How does the velocity vary in a fluid flow?
- How does the pressure vary in a fluid flow?

The first question is answered by the "Continuity Equation". The second question is answered by the Bernoulli Equation. Each of these equations will be considered for an "Ideal Fluid Flow". An Ideal Fluid Flow is a flow that is "steady", "incompressible", "irrotational", and "non-viscous". In this experiment, we will approximate the air flow under study to be an "Ideal Fluid Flow".

A fluid flow is said to be steady at a given point in the fluid flow if and only if every fluid particle passing through that point passes through with the same velocity. Otherwise, the fluid flow is said to be "non-steady". Fluid flows that are "steady" are also said to be "laminar".

A fluid flow is said to be "incompressible" if and only if the mass per unit volume density of the fluid is everywhere the same within the fluid flow. Otherwise, the fluid flow is said to be "compressible".

A fluid flow is said to be "irrotational" if and only if there do not exist any regions within the fluid flow in which fluid particles exhibit rotational motion. Otherwise, the fluid flow is said to be "rotational". A rotational fluid flow can lead to a condition called "turbulence" in which the fluid particles can assume very complex motions, such as within a vortex.

A fluid flow is said to be "non-viscous" if and only if there are no drag forces between the fluid particles or between the fluid particles and the walls of the pipeline in which the fluid is flowing. Otherwise, the fluid flow is said to be "viscous".

The Continuity Equation for an Ideal Fluid Flow is given by

$$Q = Av = \text{ constant} \tag{24.1}$$

Or for two arbitrary points 1 and 2 in the fluid flow

$$A_2 v_2 = A_1 v_1 \tag{24.2}$$

$Q$ is called the "flow rate" and measures the volume of fluid that passes through a region in the pipeline per unit time. In SI units $Q$ is measured in $\text{m}^3/\text{s}$. $A$ is the cross-sectional area of the region in the pipeline where the flow is being measured, and $v$ is the velocity measured in m/s of the flow in that region.

Bernoulli's Equation for an Ideal Fluid Flow is given by:

$$p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 \tag{24.3}$$

where $p_1$ and $p_2$ are the "static" pressures at positions 1 and 2, respectively, $\frac{1}{2}\rho v_1^2$ and $\frac{1}{2}\rho v_2^2$ are the "dynamic" pressures at positions 1 and 2, and $y_1$ and $y_2$ are the elevations of the positions 1 and 2 above a chosen reference level.

From the Continuity Equation, it is easy to conclude that the velocity of the flow is largest where the cross-sectional area of the pipeline is the smallest. From Bernoulli's Equation, it is easy to conclude that the static pressure is the greatest where the velocity of the flow and "dynamic" pressure are the smallest.

## 24.2 Experiment

Figure 24.1 shows the apparatus we will be using in the experiment. The apparatus consists of a high powered fan, a Venturi tube, and a manometer to measure changes in pressure. It is manufactured by Leybold and distributed by Klinger Educational products.

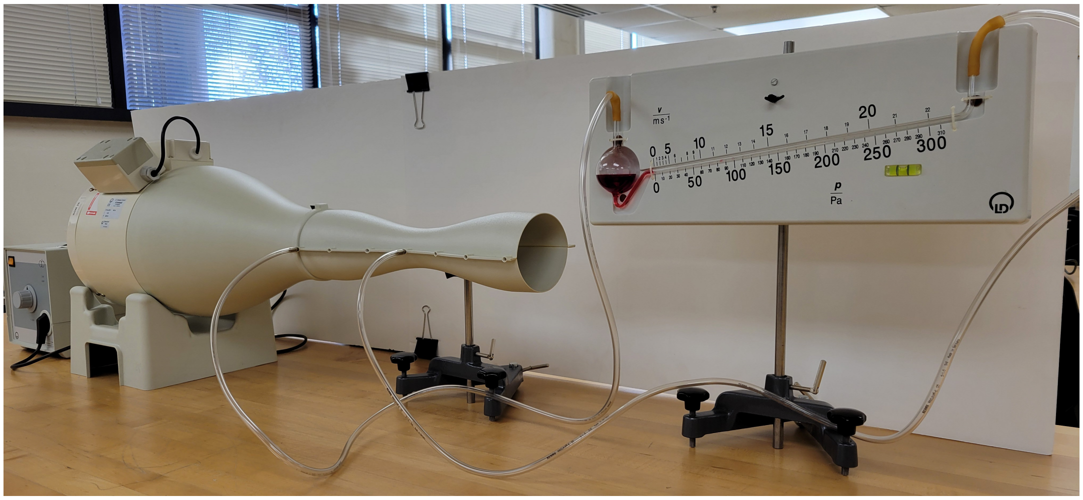


Figure 24.1: Fluid Dynamics Apparatus

According to the manufacturer, the inside diameter of the Venturi tube at position a (smallest diameter) is 50 mm, and the inside diameter of the Venturi tube at position b (largest diameter) is 100 mm. We will need these values in the computations that follow.

$$d_{\mathrm{a}} = 50 \text{ mm} = 0.050 \text{ m}$$
$$d_{\mathrm{b}} = 100 \text{ mm} = 0.100 \text{ m}$$

### 24.2.1 Measurement of the Static Pressure Difference ($p_{\mathbf{b}} - p_{\mathbf{a}}$)

Connect hoses from the holes in the Venturi tube at positions a and b (see Figure 24.2) to the manometer shown in Figure 24.3a. The final set-up should look similar to that in Figure 24.1.

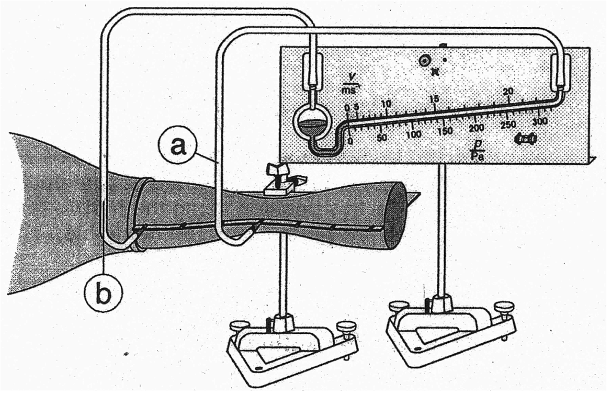
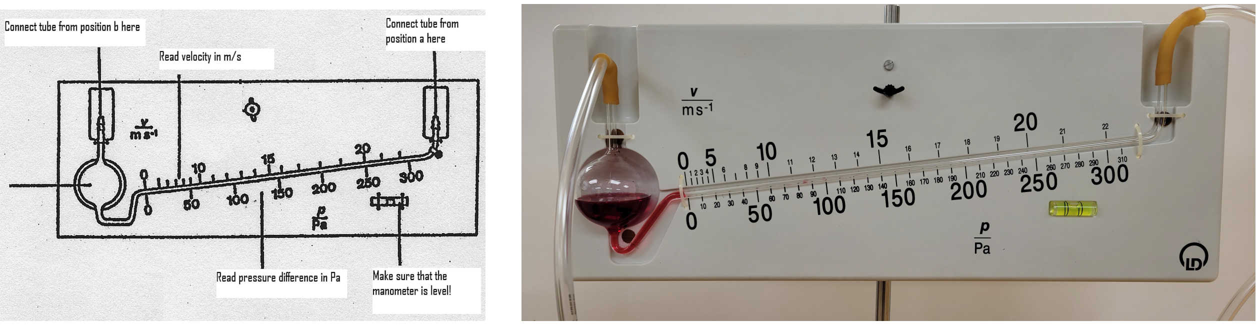


Figure 24.2: Venturi Tube Connections to Manometer - Courtesy of Klinger Educational Products



(a) Manometer Connections - Courtesy of Klinger Educational Products

(b) Fluid Dynamics Apparatus - Close-up of Manometer

Figure 24.3: Connections to Manometer

Measure the pressure difference ($p_{\mathrm{b}} - p_{\mathrm{a}}$) by reading the bottom of the scale on the manometer (see Figure 24.3b). Your reading will be in Pascals (Pa). Record your value in Table 24.1.

If we apply the Continuity Equation (Equation 24.2) to positions a and b in the air flow, we have

$$A_{\mathrm{a}} v_{\mathrm{a}} = A_{\mathrm{b}} v_{\mathrm{b}}$$

from which it follows that

$$v_{\mathrm{b}} = \left(\frac{A_{\mathrm{a}}}{A_{\mathrm{b}}}\right) v_{\mathrm{a}} \tag{24.4}$$

Since the cross-sectional area of the Venturi tube at positions a and b have the shape of a circle we have

$$A_{\mathrm{a}} = \frac{\pi d_{\mathrm{a}}^2}{4}$$

$$A_{\mathrm{b}} = \frac{\pi d_{\mathrm{b}}^2}{4}$$

Inserting these results into Equation 24.4 and simplifying gives the following relationship between $v_{\mathrm{a}}$ and $v_{\mathrm{b}}$.

$$v_{\mathrm{b}} = \left(\frac{d_{\mathrm{a}}}{d_{\mathrm{b}}}\right)^2 v_{\mathrm{a}} \tag{24.5}$$

We can now use this relationship and Bernoulli's Equation to find an equation for $v_{\mathrm{a}}$. Applying Bernoulli's Equation (Equation 24.3) between positions a and b on the Venturi tube gives

$$p_{\mathrm{a}} + \frac{1}{2}\rho v_{\mathrm{a}}^2 + \rho g y_{\mathrm{a}} = p_{\mathrm{b}} + \frac{1}{2}\rho v_{\mathrm{b}}^2 + \rho g y_{\mathrm{b}}$$

Assuming we stay along the central axis of the Venturi tube, $y_{\mathrm{a}}$ and $y_{\mathrm{b}}$ are the same, therefore, the equation simplifies to

$$p_{\mathrm{a}} + \frac{1}{2}\rho v_{\mathrm{a}}^2 = p_{\mathrm{b}} + \frac{1}{2}\rho v_{\mathrm{b}}^2 \tag{24.6}$$

Substituting the results from Equation 24.5 into Equation 24.6 gives

$$p_{\mathrm{a}} + \frac{1}{2}\rho v_{\mathrm{a}}^2 = p_{\mathrm{b}} + \frac{1}{2}\rho \left(\frac{d_{\mathrm{a}}}{d_{\mathrm{b}}}\right)^4 v_{\mathrm{a}}^2$$

Finally, solving algebraically for $v_{\mathrm{a}}$ gives

$$v_{\mathrm{a}} = \sqrt{\frac{2\left(p_{\mathrm{b}} - p_{\mathrm{a}}\right)}{\rho\left[1 - \left(\frac{d_{\mathrm{a}}}{d_{\mathrm{b}}}\right)^4\right]}} \tag{24.7}$$

**Question 1:** Assume that the mass per unit volume density $\rho$ of the air in the air flow is 1.29 kg/m$^3$. Using your measured pressure difference $(p_{\mathrm{b}} - p_{\mathrm{a}})$, the measured diameters at positions a and b, and the density of air, compute the velocity of the air flow at position a using Equation 24.7. Show your calculation and record your value in Table 24.1.

**Question 2:** Using Equation 24.5 and the value of $v_{\mathrm{a}}$ that you computed in Question 1, compute the velocity $v_{\mathrm{b}}$ of the air flow at position b. Show your calculation and record your value in Table 24.1.

| Pressure Difference<br>$(p_{\mathrm{b}} - p_{\mathrm{a}})$ (Pa) | Velocity at Position a<br>$v_{\mathrm{a}}$ (m/s) | Velocity at Position b<br>$v_{\mathrm{b}}$ (m/s) |
|---|---|---|
| | | |

Table 24.1: Velocity of Air Flow at Two Positions in the Venturi Tube

### 24.2.2 Measurement of the Velocity of Air Flow

Next, you will measure the velocity of the air flow near the open end of the Venturi tube. Figure 24.4 provides a picture of the "pressure head" accessory that will be used in this part of the experiment.

Connect the "front end" of the pressure head to the bulb end of the manometer and the "right end" to the opposite end of the manometer. Place the pressure head into the open end of the Venturi tube until the tube on the pressure end with the hole on its side aligns with the central axis of the Venturi tube and is in direct alignment with the hole on the wall of the Venturi tube that is closest to its open end. The axis of the "pressure head" should be perpendicular to the cross

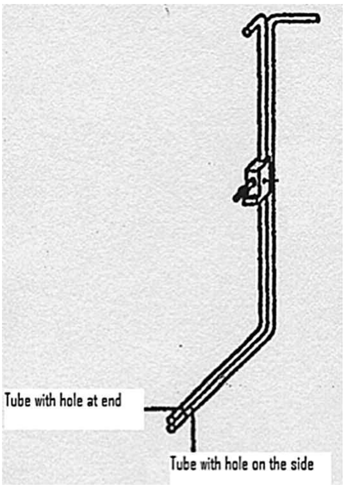


Figure 24.4: Pressure Head - Courtesy of Klinger Educational Products

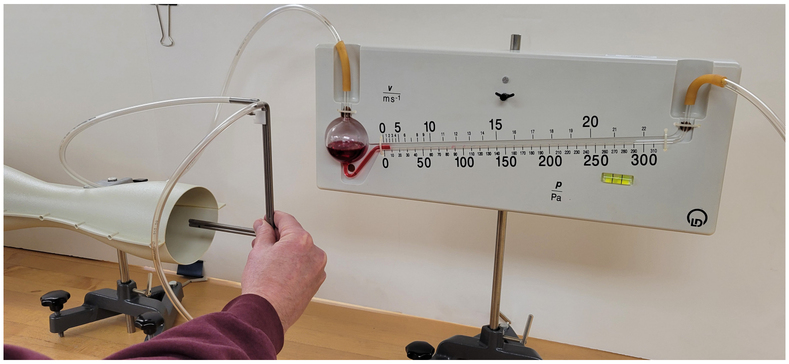


Figure 24.5: Fluid Dynamics Apparatus - Measuring with Pressure Head

sectional area of the Venturi tube (see Figure 24.5). Use the pressure scale on the manometer to measure the dynamic pressure at the end $\frac{1}{2}\rho v_{\mathrm{end}}^2$. Record your measurement in Table 24.2.

**Question 3:** From your result for the dynamics pressure, $\frac{1}{2}\rho v_{\mathrm{end}}^2$, compute $v_{\mathrm{end}}$, the velocity of air flow near the end of the Venturi tube. Show your calculation and record your result in Table 24.2.

**Question 4:** Since the diameter of the tube at the end position is the same as at position b, the velocity, $v_{\mathrm{end}}$ should be equal to $v_{\mathrm{b}}$ that you computed in Question 2. Compare your velocity values by computing a percent experimental error. Assume $v_{\mathrm{b}}$ is the accepted value. Show your calculation and record your result in Table 24.2.

| Dynamic Pressure $\frac{1}{2}\rho v_{\mathrm{end}}^2$ (Pa) | Velocity at End of Venturi Tube ($v_{\mathrm{end}}$) (m/s) | Percent Experimental Error (%) |
|---|---|---|
| | | |

Table 24.2: Velocity of Air Flow at the End of the Venturi Tube

# Part IX

# Thermodynamics

# Lab 25

# The Thermal Expansion of Solids

It is a well-founded experimental fact that changes in the size and state of a material result when we change the temperature of the material (be it solid, liquid, or gas). In this experiment you will investigate changes of sizes of solid materials that occur with changes of temperature but without changes of state. In particular, you will determine an experimental measure of the change in a linear dimension, such as length, of a solid with an increase in temperature. We shall refer to this change as the linear expansion of the solid.

## 25.1 Theory

As a simple model of a crystalline solid, consider the solid to be made up of a very large number ($10^{23}$ per cubic centimeter) of atoms that are held together in some regular array by small springs so that the solid appears to be fashioned in the form of a microscopic bedspring (see Figure 25.1).

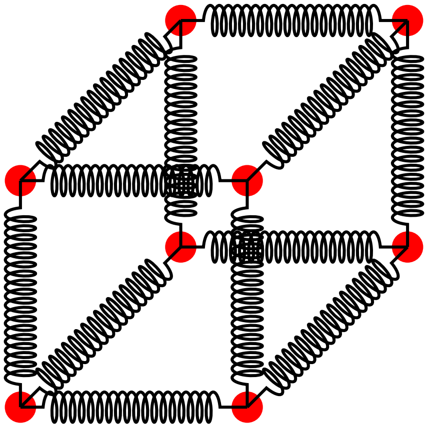


Figure 25.1: "Bedspring" Model of a Solid

In actuality, the atoms are held together by forces that are of electrical origin, but the "bedspring" approximation turns out not to be such a terribly bad model of what really happens.

### 25.1.1 Linear Expansion of a Solid

If the temperature of the solid is increased, then, with the exception of a few materials, the average distance between the atoms of the solid increases. This leads to an expansion of the entire solid body as the temperature is increased. Experiments show that the increase $\Delta L$ in the length $L$ of a solid (see Figure 25.2) when its temperature is increased by an amount $\Delta T$ is given by

$$\Delta L = \alpha L \Delta T \tag{25.1}$$

where $\alpha$, called the coefficient of linear expansion, has a value that is characteristic of the type of solid. In this experiment, you will measure the coefficient of linear expansion for aluminum, copper, and steel. We may rewrite Equation 25.1 as

$$\alpha = \frac{1}{L}\frac{\Delta L}{\Delta T} \tag{25.2}$$

so that $\alpha$ has the meaning of a fractional change in length per degree temperature change. We shall assume that $\alpha$ is a constant for a given material, independent of the temperature. In actuality, however, Equation 25.2 is an approximation. That is, $\alpha$ really depends on the actual temperature and reference temperature to determine $L$. However, its variation with temperature is negligible compared to the accuracy of this experiment and, indeed, to the accuracy with which most engineering measurements need to be made. Experimental values of the coefficient of linear expansion for various metals are given in the standard physics–chemistry handbook.

Note (A): The temperature interval $\Delta T$ that appears both in Equation 25.1 and Equation 25.2 is measured in units of one Celcius degree (1C°) and not in degrees Celcius (°C). That is, one Celcius degree is a temperature interval of one unit on the Celcius scale, whereas one degree Celcius is a specific temperature reading on that scale.

Note (B): The values of coefficients of linear expansion for solid materials aare, in general, very small numbers. Therefore, it is very important to express them in scientific notation.

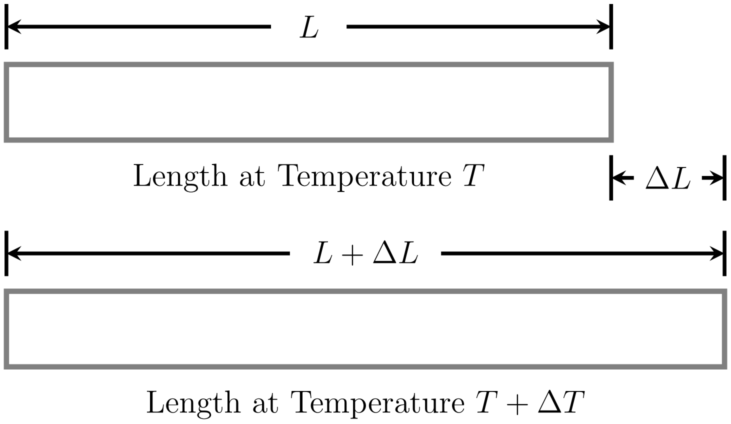


Figure 25.2: Thermal Expansion of a Solid. When the Temperature is Increased by an Amount $\Delta T$, the length increases by an amount $\Delta L = \alpha L \Delta T$

### 25.1.2 Linear Interpolation to Determine Temperature based on Resistance

In this lab, the temperature of the solid is measured using a thermistor. A thermistor enables temperatures to be represented as electrical resistance values. We will need to convert from resistance values in ohms to temperature in °C. As you can see from Figure 25.3, the temperature is a nonlinear function of the resistance. To calculate the temperature given a particular resistance

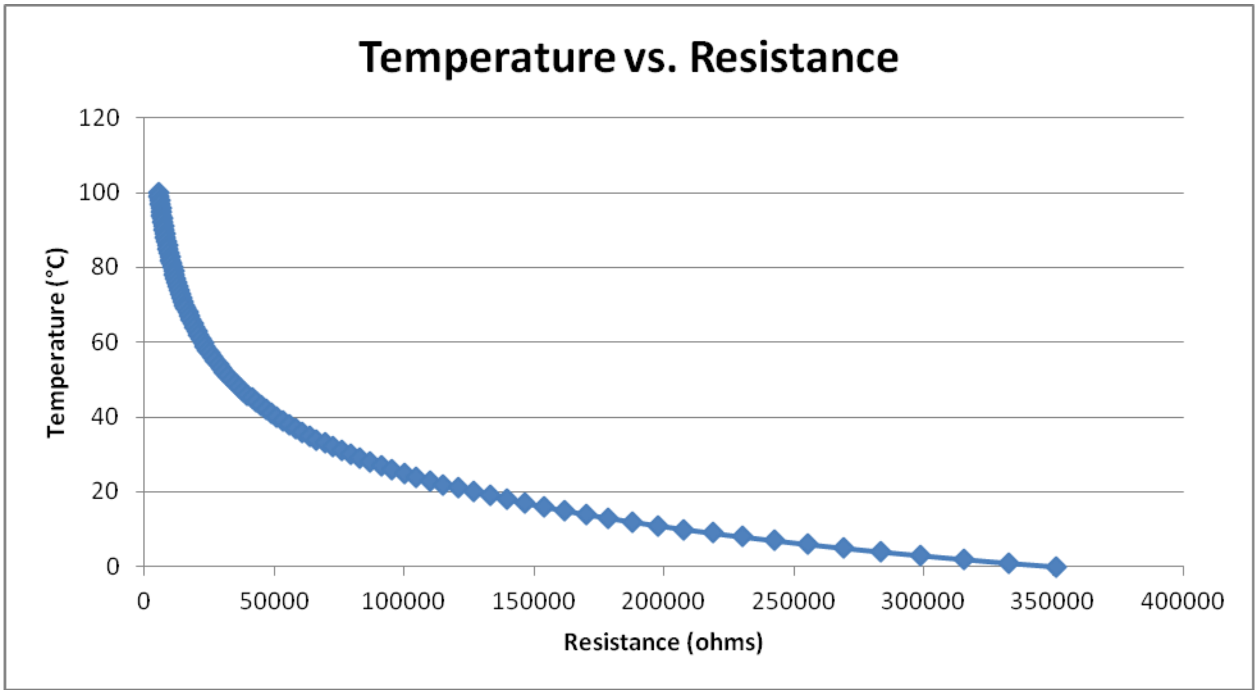


Figure 25.3: Thermistor Temperature versus Resistance Graph

value, we will need to use the table in Figure 25.5 and apply linear interpolation between the two closest resistance/temperature pairs of values.

For example, assume the resistance is measured to be 5800 Ω. What is the corresponding temperature value using linear interpolation?

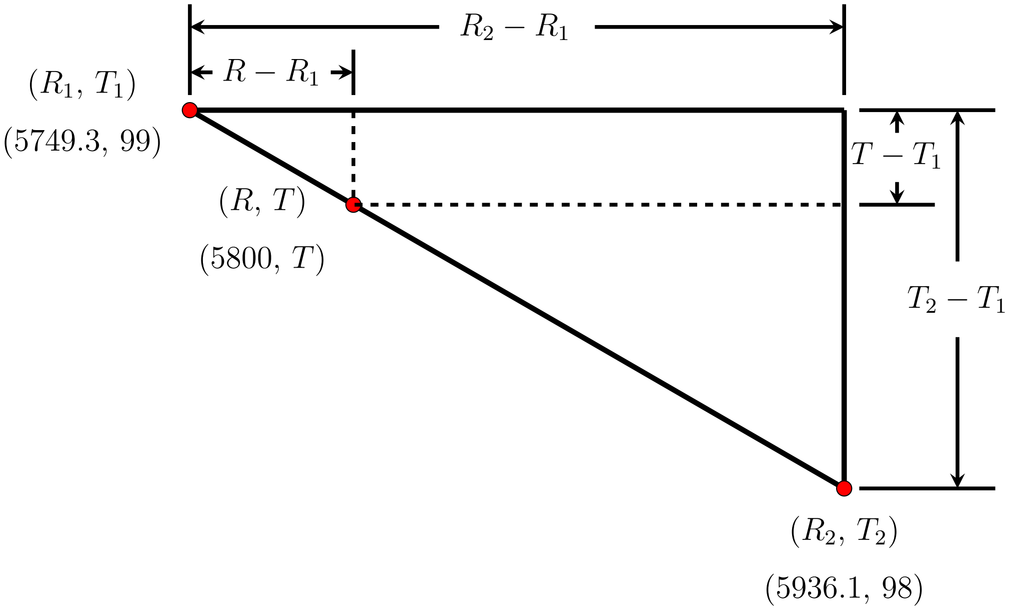
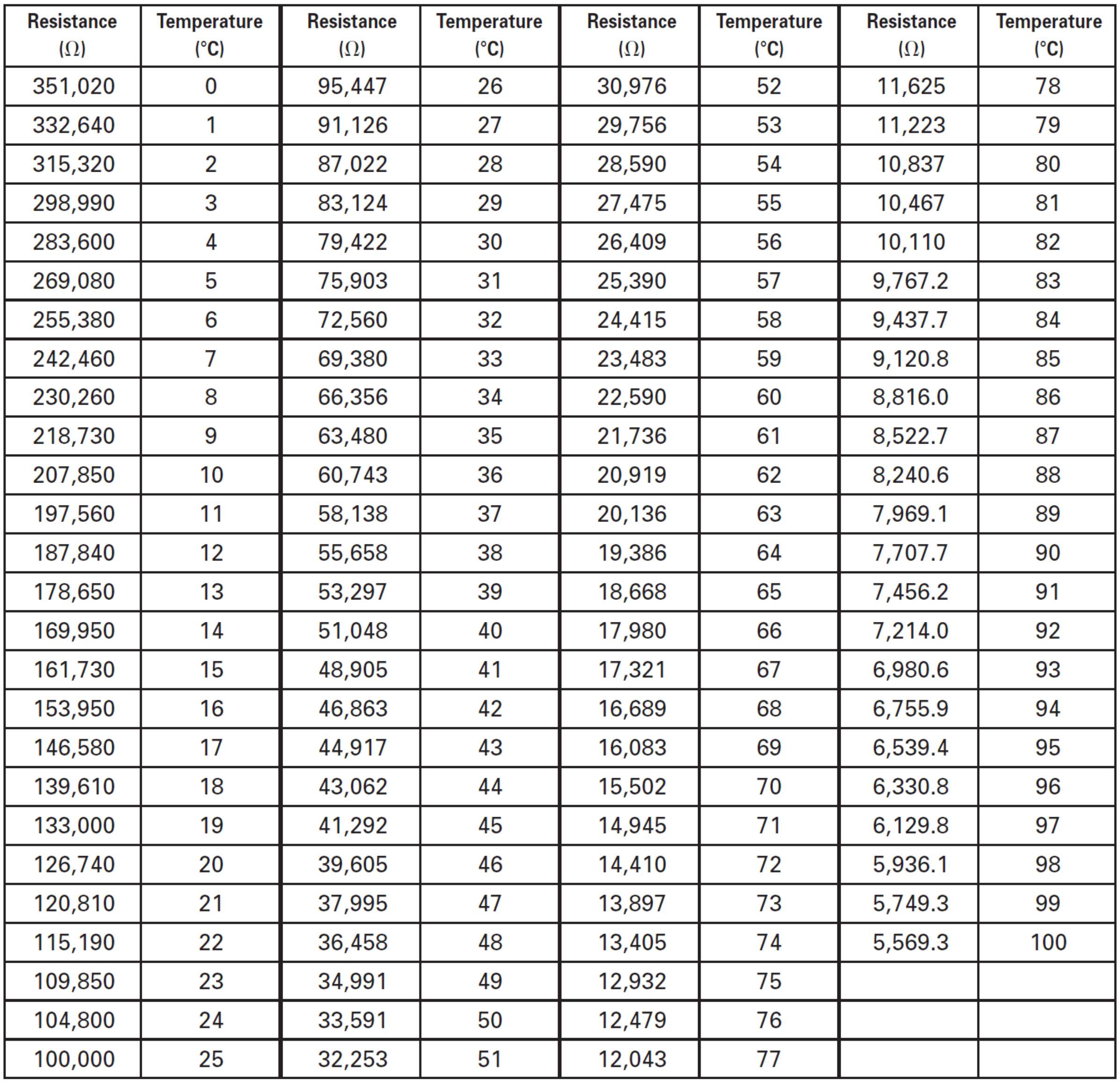


Figure 25.4: Using Linear Interpolation to Find Temperature



| Resistance (Ω) | Temperature (°C) | Resistance (Ω) | Temperature (°C) | Resistance (Ω) | Temperature (°C) | Resistance (Ω) | Temperature (°C) |
|---|---|---|---|---|---|---|---|
| 351,020 | 0 | 95,447 | 26 | 30,976 | 52 | 11,625 | 78 |
| 332,640 | 1 | 91,126 | 27 | 29,756 | 53 | 11,223 | 79 |
| 315,320 | 2 | 87,022 | 28 | 28,590 | 54 | 10,837 | 80 |
| 298,990 | 3 | 83,124 | 29 | 27,475 | 55 | 10,467 | 81 |
| 283,600 | 4 | 79,422 | 30 | 26,409 | 56 | 10,110 | 82 |
| 269,080 | 5 | 75,903 | 31 | 25,390 | 57 | 9,767.2 | 83 |
| 255,380 | 6 | 72,560 | 32 | 24,415 | 58 | 9,437.7 | 84 |
| 242,460 | 7 | 69,380 | 33 | 23,483 | 59 | 9,120.8 | 85 |
| 230,260 | 8 | 66,356 | 34 | 22,590 | 60 | 8,816.0 | 86 |
| 218,730 | 9 | 63,480 | 35 | 21,736 | 61 | 8,522.7 | 87 |
| 207,850 | 10 | 60,743 | 36 | 20,919 | 62 | 8,240.6 | 88 |
| 197,560 | 11 | 58,138 | 37 | 20,136 | 63 | 7,969.1 | 89 |
| 187,840 | 12 | 55,658 | 38 | 19,386 | 64 | 7,707.7 | 90 |
| 178,650 | 13 | 53,297 | 39 | 18,668 | 65 | 7,456.2 | 91 |
| 169,950 | 14 | 51,048 | 40 | 17,980 | 66 | 7,214.0 | 92 |
| 161,730 | 15 | 48,905 | 41 | 17,321 | 67 | 6,980.6 | 93 |
| 153,950 | 16 | 46,863 | 42 | 16,689 | 68 | 6,755.9 | 94 |
| 146,580 | 17 | 44,917 | 43 | 16,083 | 69 | 6,539.4 | 95 |
| 139,610 | 18 | 43,062 | 44 | 15,502 | 70 | 6,330.8 | 96 |
| 133,000 | 19 | 41,292 | 45 | 14,945 | 71 | 6,129.8 | 97 |
| 126,740 | 20 | 39,605 | 46 | 14,410 | 72 | 5,936.1 | 98 |
| 120,810 | 21 | 37,995 | 47 | 13,897 | 73 | 5,749.3 | 99 |
| 115,190 | 22 | 36,458 | 48 | 13,405 | 74 | 5,569.3 | 100 |
| 109,850 | 23 | 34,991 | 49 | 12,932 | 75 | | |
| 104,800 | 24 | 33,591 | 50 | 12,479 | 76 | | |
| 100,000 | 25 | 32,253 | 51 | 12,043 | 77 | | |

Figure 25.5: Thermistor Conversion Table: Temperature versus Resistance

To calculate $T$, let's examine Figure 25.4. We first need to find the equation of the line that passes through $(R_1, T_1)$ and $(R_2, T_2)$. The general equation is

$$T - T_1 = m\,(R - R_1) = \left(\frac{T_2 - T_1}{R_2 - R_1}\right)(R - R_1)$$

where $m = \frac{T_2 - T_1}{R_2 - R_1}$ is the slope of the line.

Solving for $T$ gives

$$T = T_1 + \left(\frac{T_2 - T_1}{R_2 - R_1}\right)(R - R_1)$$

For our specific example, since we want to calculate the temperature $T$ corresponding to $R = 5800\,\Omega$, we have to first find the closest resistance/temperature pair values in the table in Figure 25.5. These are $(R_1, T_1) = (5749.3, 99)$ and $(R_2, T_2) = (5936.1, 98)$. Plugging these values into the linear equation gives

$$T = 99 + \left(\frac{98 - 99}{5936.1 - 5749.3}\right)(5800 - 5749.3) = 98.8°\text{C}$$

Therefore the temperature that corrsponds to a resistance value of $5800\,\Omega$ is 98.8°C rounded to the nearest 0.2°C.

## 25.2 Experiment

You will use the linear expansion apparatus to determine the coefficient of linear expansion of aluminum, copper, and steel in the form of tubes. Equation 25.2 gives the coefficient of linear expansion $\alpha$ in terms of experimentally measurable quantities. In order to apply this equation, you must first measure the length $L$ of the tube at the initial reference temperature $T_{\text{i}}$, and then measure the change in the length of the tube when the tube expands to a final length $L + \Delta L$ at the final temperature $T_{\text{f}}$. From these measurements, the coefficient of linear expansion can be determined. The result is given by

$$\alpha = \frac{1}{L}\left(\frac{\Delta L}{T_{\text{f}} - T_{\text{i}}}\right) \tag{25.3}$$

Equation 25.3 gives us an experimental determination of the coefficient of linear expansion for the test metal. Take $2.3 \times 10^{-5}$ $\text{C}^{\circ -1}$ as the accepted value for aluminum. Take $1.7 \times 10^{-5}$ $\text{C}^{\circ -1}$ as the accepted value for copper. Take $1.2 \times 10^{-5}$ $\text{C}^{\circ -1}$ as the accepted value for steel. The apparatus being used in this experiment is manufactured by PASCO Scientific (see Figure 25.6).

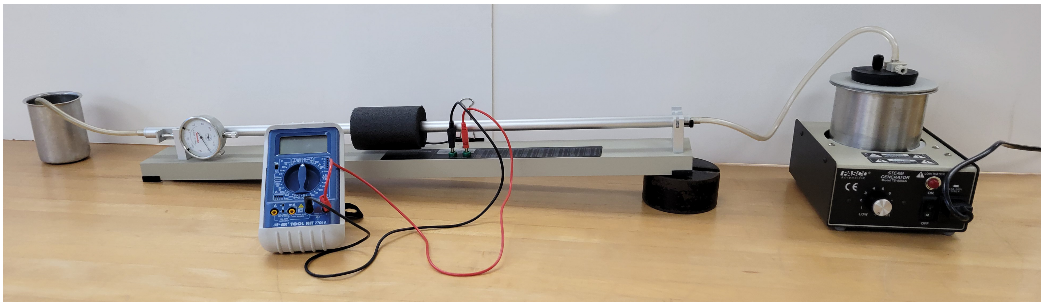


Figure 25.6: Linear Expansion Apparatus

The expansion of the metal can be measured to within 0.01 mm using the built-in gauge. The temperature of the metal can be measured to within 0.5 °C using a thermistor that attaches to the center of the tube. A thermistor, as explained above, is a device whose electrical resistance is very sensitive to its temperature. A piece of foam insulation is provided and is to be wrapped around the thermistor connection in order to avoid heat loss. A steam generator is used to produce steam that is run through the tube. One end of the apparatus should be elevated, as shown in Figure 25.6, so that any water that condenses inside the tube will drain properly at the opposite end.

The temperatures $T_{\text{i}}$ and $T_{\text{f}}$ are actually found from measurements of the electrical resistance offered by the thermistor. These can be computed by making use of the table in Figure 25.5 (provided by PASCO Scientific with the apparatus) and then applying linear interpolation as described in the theory section above.

Measure the length $L$ of the aluminum tube before allowing steam to pass through it. Take this measurement from the inner edge of the stainless steel pin on one end to the inner edge of the angle bracket at the other end. Record your value in Table 25.1. Mount the aluminum tube onto the apparatus base so that the stainless steel pin on the tube fits into the slot on the slotted mounting block and the angle bracket on the tube presses against the spring needle of the gauge. Attach the thermistor to the middle of the tube by securing it with the thumbscrew. Cover the thermistor with the foam insulator. Plug the leads from the multimeter into the connectors labeled "thermistor" and record the initial resistance $R_{\mathrm{i}}$ in Table 25.1. Align the indicator needle on the gauge with the zero point on the gauge's dial. Each division on the dial is .01 mm. Attach the tube from the steam generator to the end of the aluminum tube and allow the steam to pass through the tube. Catch the exhaust steam along with any water droplets at the other end with the small catch bucket. Note the expansion of the tube by checking the gauge and the reading on the multimeter. Once both the gauge and multimeter reading stabilize, record the expansion of the tube, $\Delta L$, and the final resistance, $R_{\mathrm{f}}$, of the thermistor in Table 25.1.

Repeat this procedure for the copper and stainless steel tubes. Be very careful when you remove the previous tube since it will be very hot.

| | Initial Length ($L$) (mm) | Change in Length ($\Delta L$) (mm) | Initial Resistance ($R_{\mathrm{i}}$) ($\Omega$) | Final Resistance ($R_{\mathrm{f}}$) ($\Omega$) | Initial Temp. ($T_{\mathrm{i}}$) (°C) | Final Temp. ($T_{\mathrm{f}}$) (°C) | Change in Temp. ($\Delta T$) (C°) |
|---|---|---|---|---|---|---|---|
| Aluminum | | | | | | | |
| Copper | | | | | | | |
| Steel | | | | | | | |

Table 25.1: Length, Resistance, and Temperature Values for Each Metal Tube

**Question 1:** For each tube, use the conversion table in Figure 25.5 and linear interpolation to covert your initial and final resistance measurements ($R_{\mathrm{i}}$ and $R_{\mathrm{f}}$) into corresponding initial and final temperature measurements ($T_{\mathrm{i}}$ and $T_{\mathrm{f}}$). Using the initial and final temperature values, compute the difference in temperature $\Delta T$. Show all your calculations and record your values in Table 25.1.

**Question 2:** Using your measured values, compute the experimental coefficient of linear expansion $\alpha$ values for each metal using Equation 25.3. Show your calculations and record your values in Table 25.2.

| | Experimental Coefficient of Linear Exansion ($\alpha_{\mathrm{exp}}$) ( $\mathrm{C}^{\mathrm{o}-1}$) | PRAAD for $\alpha$ (%) | Theoretical Coefficient of Linear Exansion ($\alpha_{\mathrm{theory}}$) ( $\mathrm{C}^{\mathrm{o}-1}$) | Percent Experimental Error (%) |
|---|---|---|---|---|
| Aluminum | | | $2.3 \times 10^{-5}$ | |
| Copper | | | $1.7 \times 10^{-5}$ | |
| Steel | | | $1.2 \times 10^{-5}$ | |

Table 25.2: Coefficient of Linear Expansion Values

**Question 3:** Calculate percent relative average absolute deviation (PRAAD) for each determination of coefficient of linear expansion $\alpha$. Show your calculations and record your results in Table 25.2. Hint: PRAAD $\alpha$ = PRAAD $\Delta L$ + PRAAD $L$ + PRAAD $\Delta T$. Since we only performed a single measurement, each PRAAD can be estimated by using the resolution of the corresponding instrument. PRAAD $\Delta L = \frac{0.01 \text{ mm}}{\Delta L \text{ mm}} \times 100\%$, PRAAD $L = \frac{1 \text{ mm}}{L \text{ mm}} \times 100\%$, and PRAAD $\Delta T = \frac{0.2\text{C}^\circ}{\Delta T\text{C}^\circ} \times 100\%$.

**Question 4:** Compare the experimental coefficients of linear expansion with the theoretical coefficients of linear expansion by computing the percent experimental errors. Show all of your calculations and record the values in Table 25.2.

**Question 5:** Does the value of $\alpha$ depend on the unit of length used? Explain.

**Question 6:** When F° are used instead of C° as a unit of temperature change, does the numerical value of $\alpha$ change? If so, how? Hint: $T_{^\circ\text{F}} = \frac{9}{5}T_{^\circ\text{C}} + 32$, therefore $\Delta T_{\text{F}^\circ} = \frac{9}{5}\Delta T_{\text{C}^\circ}$

**Question 7:** List two major causes of error in this experiment and discuss their influence on your experimental determination of the coefficient of linear expansion.

**Question 8:** Using a bi-metallic strip made out of aluminum and copper, design a thermostatic device that will work on the priciple of the linear expansion of metals. Explain how your device works.

**Question 9:** A steel bridge is 200 ft long on a winter day when the temperature is -15°C. Find the length of the bridge on a summer day when the temperature is 35°C.

**Question 10:** Pyrex is a very useful material for cooking. It can withstand very large temperature changes without breaking. Why?

# Lab 26

# The Behavior of Gases

## 26.1 Theory

An ideal gas is a simplified model of a dilute gas in which the molecules are considered to be point particles that move independently and don't interact with one another except when they undergo elastic collisions. In this lab, we will make the approximation that the air in the room is an ideal gas.

Experimentally, an equation relating the absolute pressure, absolute temperature, and volume can be determined for ideal gases. This equation is called the ideal gas law and is given as $PV = nRT$ where $P$ is the absolute pressure, $V$ is the volume, $T$ is the absolute temperature, $n$ is the number of moles, and $R$ is the universal gas constant. This equation is also referred to as the equation of state of an ideal gas. In a typical situation, the number of molecules of gas is kept fixed ($nR$ is constant) and the other quantities are independently varied or held constant. In this case, the ideal gas law is more useful in the following form:

$$\frac{PV}{T} = \text{constant or } \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \tag{26.1}$$

Absolute zero is theoretically the lowest possible attainable temperature. We can get an estimate of the value of absolute zero by plotting the absolute temperature vs. absolute pressure for an ideal gas (keeping a fixed amount of gas and a fixed volume) and extrapolate the graph to the temperature where the pressure is zero. Rearranging the ideal gas law for temperature gives

$$T = \frac{V}{nR}P \tag{26.2}$$

for which you can see that temperature is a linear function of pressure. By plotting several pairs of points ($P$, $T$) we can perform a linear fit to the data and the $y$-intercept of the line will represent the value of absolute zero in Kelvin. Theoretically, the value should be 0 K. This value in the units of degrees Celsius is –273.15 °C.

## 26.2 Experiment

### 26.2.1 The Relationship Between Pressure and Volume at Constant Temperature

We will use the PASCO gas syringes and the GLX dataloggers for the first two parts of the experiment (see Figure 26.1).

Figure 26.1: Ideal Gas Law Apparatus

With the syringe adapters disconnected, compress the syringe to a volume of 21 cc. Fill the syringe with air to a volume of 42 cc. Connect the pressure/temperature sensor to the GLX datalogger at port 1 then connect the syringe adapters to the pressure/temperature sensor. The absolute pressure vs. time graph should appear on the screen. Change the graph to temperature vs. time and set the units to Kelvin. You can do this by pressing the check button twice to select the y-axis label and then select temperature instead of absolute pressure from the menu.

Start the data collection by pressing the blue arrow button. Wait a few seconds, note the initial temperature, and then slowly compress the syringe fully to 21 cc (there is a built-in stopper at 21 cc). Hold the syringe at 21 cc for several seconds (wait until the temperature gets back near the baseline value) then slowly let it expand back to near 42 cc. Press the blue arrow again to stop the data collection. If you need to recollect the data, you need to disconnect the pressure sensor and follow the previous instructions again. Press the blue button to collect additional trials if necessary.

To analyze the graph, go to tools and select "Smart Tool." Move the smart cursor to the beginning of the trial where the temperature is constant and note the exact time. Then move the cursor to a region on the graph where the syringe was fully compressed and the temperature is back to the baseline value. Again, note the exact time. Using the check button, switch the graph to absolute pressure vs. time. Record the absolute pressure at the two times where the temperature was at the baseline value. Record the values in Table 26.1.

| | Time of Measurement (s) | Volume (cc) | Pressure (kPa) |
|---|---|---|---|
| Before Compressing (1) | | 42.0 | |
| After Compressing (2) | | 21.0 | |

Table 26.1: Volume and Pressure Values at a Constant Temperature

Since we held the temperature constant (when we made the measurements) then the ideal gas law simplifies to

$$P_1V_1 = P_2V_2 \text{ or } \frac{P_2}{P_1} = \frac{V_1}{V_2}$$

The ratio of $V_1$ to $V_2$ is $\frac{42.0\text{ cc}}{21.0\text{ cc}} = 2.0$ in this experiment.

**Question 1:** Calculate $\frac{P_2}{P_1}$ and compare it to the accepted value of 2.0 by computing the percent experimental error. Show all of your work and record your values in Table 26.2.

| Calculated $\frac{P_2}{P_1}$ | Accepted $\frac{V_1}{V_2}$ | Percent Experimental Error (%) |
|---|---|---|
| | 2.0 | |

Table 26.2: Ratio of Pressure Values

### 26.2.2 The Ideal Gas Law with Varying Pressure, Volume, and Temperature

In this part of the lab, we will again use the syringe to explore the ideal gas law. However, we will now allow pressure, volume, and temperature to vary.

Disconnect the syringe from the sensor and completely compress it then fill it up with air to 42 cc. Reconnect the syringe to the sensor. Set up the datalogger so the temperature vs. time graph is displayed. Start the data collection, note the initial temperature and wait a few seconds before compressing the syringe. This time, compress the syringe quickly to 21 cc. Hold the syringe at 21 cc for several seconds while the temperature goes back to baseline then you may decompress the syringe back to 42 cc. Finally, stop the data collection.

To analyze the data, begin by moving the smart cursor to the beginning of the trial where the temperature is at a constant baseline level and note the exact time and record the temperature. Next, locate the time of maximum temperature and note the exact time and record the temperature. Finally, switch the graph to absolute pressure vs. time and record the pressure at the two times that were noted (the pressure that corresponded to the baseline temperature and the pressure that corresponded to the maximum temperature). Record all of your values in Table 26.3.

| | Time of Measurement (s) | Volume (cc) | Pressure (kPa) | Temperature (K) |
|---|---|---|---|---|
| Before Compressing (1) | | 42.0 | | |
| After Compressing (2) | | 21.0 | | |

Table 26.3: Volume, Pressure, and Temperature Values

In this part of the lab, pressure, volume, and temperature were all varies. In this case, the ideal gas law is most useful in the follow form:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \text{constant}$$

**Question 2:** Calculate $\frac{P_1V_1}{T_1}$ and $\frac{P_2V_2}{T_2}$ and compare these ratios by computing the absolute percent differences. Show all your calculations and record your results in Table 26.4.

| $\frac{P_1V_1}{T_1}$ $\left(\frac{\text{kPa}\cdot\text{cc}}{\text{K}}\right)$ | $\frac{P_2V_2}{T_2}$ $\left(\frac{\text{kPa}\cdot\text{cc}}{\text{K}}\right)$ | $\frac{\left\lvert\frac{P_1V_1}{T_1}-\frac{P_2V_2}{T_2}\right\rvert}{\frac{P_1V_1}{T_1}} \times 100(\%)$ | $\frac{\left\lvert\frac{P_1V_1}{T_1}-\frac{P_2V_2}{T_2}\right\rvert}{\frac{P_2V_2}{T_2}} \times 100(\%)$ |
|---|---|---|---|
| | | | |

Table 26.4: Ratio of the Product of Pressure and Volume to Temperature

### 26.2.3 Determining the Value of Absolute Zero

In this part of the lab, we will use the absolute zero apparatus (a constant volume gas thermometer) to determine the temperature of absolute zero. Fill the white 3-liter container about half full with hot water from the tap. Connect the absolute zero apparatus to the pressure/temperature sensor (see Figure 26.2).

Figure 26.2: Absolute Zero Apparatus

Go to the home menu then choose the sensors icon. Choose mode (F1) and select manual (verify by hitting OK (F1)). This will allow us to collect data at certain instances by pressing the button with the flag on it (instead of having the data collected over the entire time). In this part of the lab we will use the digits display to display our data. Go to the home menu, then choose digits from the main menu and set it up to display both absolute pressure in (kPa) and temperature in (K).

Submerge the absolute zero apparatus in the hot water and start data collection by pressing the blue arrow. The values will change but will not be recorded until you press the flag button. Wait until the temperature plateaus then press the flag button to collect the data. You will have to verify the data point by pressing “OK” (F1 key). The pressure and temperature values for that instant should now be stored but I would also write them down as a backup.

Add a handful or two of ice, wait until the temperature plateaus again and press the flag button. This will give you pressure and temperature values for that instant. Record these values by hand as well. Do this again for two additional conditions where you add more ice and cool the water down further. When all four pairs of data values have been collected, you can stop the data collection by pressing the blue arrow.

To retrieve the data go to the home menu and choose “table”. The first column should be displaying absolute pressure in (kPa). You want to set up the second column to display temperature in (K). Now you can record your four pairs of values. You should also have these values previously written down as a backup. Record your values in Table 26.5.

| Condition | Absolute Pressure (kPa) | Temperature (K) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

Table 26.5: Absolute Pressure and Temperature Values in a Constant Volume Gas Thermometer

**Question 3:** Plot Temperature (K) versus Absoute Pressure (kPa). What is the shape of the graph? Perform a linear fit to the data. What does the $y$-intercept represent? Hint: Consider the ideal gas law written in the form

$$T = \frac{V}{nR}P = kP$$

**Question 4:** Convert your absolute zero value from Kelvin into degrees Celsius by subtracting 273.15. Compare your experimental value of absolute zero with the accepted value of -273.15°C by computing the percent experimental error. Show your calculations and record all of your values in Table 26.6.

| Experimental Absolute Zero (°C) | Accepted Absolute Zero (°C) | Percent Experimental Error (%) |
|---|---|---|
| | -273.15 | |

Table 26.6: Absolute Zero in Degrees Celsius

# Lab 27

# The Specific Heat of a Solid

The specific heat of a substance is a measure of the thermal behavior of the substance. The theoretical development of the specific heat comes from the field of physics called thermodynamics. Thermodynamics is concerned with a macroscopic description of heat phenomena and investigates those processes in which a system changes from one equilibrium state to another. In this experiment you will measure the specific heat at constant pressure for two different metals.

## 27.1 Theory

You will acquire an understanding of the fundamental nature of heat when the principles of thermodynamics are discussed in your lecture. Here, let us simply define heat as the amount of energy transferred from one body to another because of a temperature difference between the two bodies. For examples that demonstrate this definition of heat, you need only call upon your personal experiences. Note that this definition of heat implies that it is wrong to say that a body contains a certain amount of heat, for heat is energy transferred from one body to another. It is, however, correct to speak of the amount of heat given off or absorbed by a body.

Let us now turn to a definition for the specific heat of a solid. Consider the event of dropping into some cool water a block of iron that has just been taken from a hot furnace (see Figure 27.1).

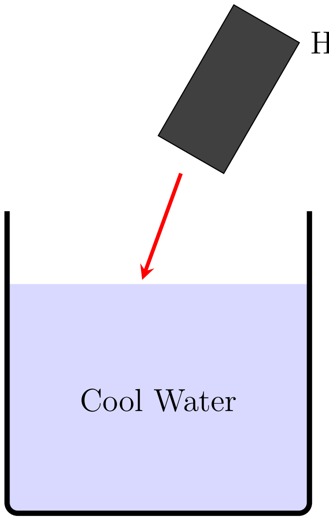


Figure 27.1: Hot Iron Into Cool Water

Suppose that the iron block gives off an amount of heat $Q$ to the surrounding water resulting in a rise in temperature $\Delta T$ of the water. Then, the ratio of the heat supplied to the water to the corresponding temperature rise of the water is called the heat capacity of the water, $C$.

$$C = \frac{Q}{\Delta T} \tag{27.1}$$

Of course, we need not have confined ourselves to iron and water. Equation 27.1 gives then the definition for the heat capacity of a body, i.e., the amount of heat that must be given to it in order to increase its temperature by one degree. The heat capacity is really not a characteristic property of the substance that makes up the body, however, since, in general, the larger the body the more heat must be supplied to raise the temperature of the entire body one degree. Therefore, in order to use Equation 27.1 meaningfully we must have knowledge of the size of the body.

A more useful quantity that is characteristic of the substance that makes up the body may be obtained by simply dividing Equation 27.1 by the mass of the body. This quantity is the specific heat $c$ of the substance that makes up the body.

$$c = \frac{C}{m} = \frac{1}{m}\frac{Q}{\Delta T} \tag{27.2}$$

Note: Energy transferred in the form of heat is measured in units of calories or Btu's. Unlike other units, the unit of heat cannot be attached to something that is easily stored in a bureau of standards. This is because of the definition of heat as being the energy that is transferred from one body to another because of a temperature difference between the two bodies. So, what one does is select some standard body and consider the change in its temperature due to the amount of heat it gains. The standard chosen is water. One then defines:

- 1 calorie = The quantity of heat that must be supplied to one gram of water to raise its temperature from 14.5 °C to 15.5 °C.
- 1 Btu = The quantity of heat that must be supplied to one pound of water to raise its temperature from 63 °F to 64 °F.

The relationship between these units of heat energy and the units of energy with which you are familiar (i.e., the ft-lb and joule) is accomplished by making use of the principle of conservation of energy in experiments in which a measured amount of mechanical energy is completely converted into a measured quantity of heat. The result is that

$$778 \text{ ft-lb} = 1 \text{ Btu}$$
$$4.186 \text{ joules} = 1 \text{ calorie}$$

As a side note, the "calorie" that you refer to so diligently in the course of dieting (the energy content of food) is actually 1,000 calories, as we have defined the calorie.

In order to measure the specific heat of a solid, you will use an instrument called a water calorimeter. In order to see how this instrument gives a determination of the specific heat of a sample, consider the following thought experiment. Figure 27.2 illustrates the experimental setup.

The calorimeter can (cup), which is made of polished aluminum, has a mass $M_{\text{can}}$ and contains an amount of water of mass $M_{\text{w}}$. The water is thoroughly stirred and its temperature is found to be $T_0$. The calorimeter can is assumed to be at the same temperature as the water. The unknown sample x is "boiled" in water and is therefore assumed to be at the temperature of boiling water,

Figure 27.2: Calorimeter Apparatus for Determining the Specific Heat

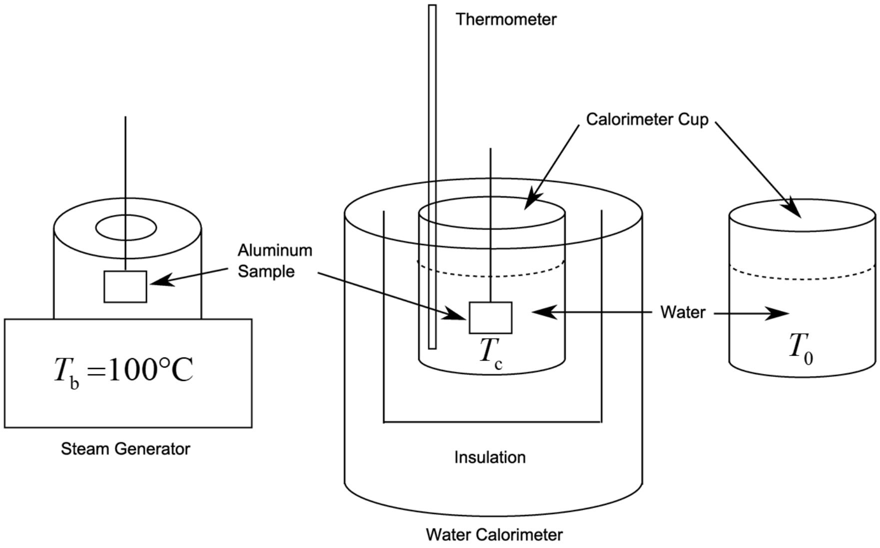


Figure 27.3: Calorimeter Procedure for Determining the Specific Heat

$T_{\mathrm{b}}$. The sample is quickly transferred from the boiling water to the water in the calorimeter can. A cork is immediately inserted into the hole at the top of the calorimeter in order to seal it. The water inside the calorimeter can is again stirred, and a common temperature of sample and water plus can $T_{\mathrm{c}}$ is finally reached (see Figure 27.3).

We can use this data together with the principle of conservation of energy to find the specific heat of the sample. That is,we have that

$$\begin{array}{l}\text{Heat given off}\\\text{by sample } (Q_{\mathrm{x}})\end{array} = \begin{array}{l}\text{Heat absorbed}\\\text{by water } (Q_{\mathrm{w}})\end{array} + \begin{array}{l}\text{Heat absorbed by}\\\text{calorimeter can } (Q_{\mathrm{can}})\end{array} \tag{27.3}$$

However, according to Equation 27.2

$$Q_{\mathrm{x}} = c_{\mathrm{x}} M_{\mathrm{x}} \Delta T_{\mathrm{x}} \tag{27.4}$$

where $c_{\mathrm{x}}$ is the specific heat of the sample, $M_{\mathrm{x}}$ is the mass of the sample, and $\Delta T_{\mathrm{x}}$ is the change in temperature that the sample has experienced because it has given off an amount of heat $Q_{\mathrm{x}}$. Also,

$$Q_{\mathrm{w}} = c_{\mathrm{w}} M_{\mathrm{w}} \Delta T_{\mathrm{w}} \tag{27.5}$$

where $c_{\mathrm{w}}$ is the specific heat of water $= 1 \frac{\mathrm{cal}}{\mathrm{g\,C^{\circ}}}$, $M_{\mathrm{w}}$ is the mass of the water, and $\Delta T_{\mathrm{w}}$ is the change in temperature that the water in the can has experienced because of the heat $Q_{\mathrm{w}}$ absorbed by it. Finally,

$$Q_{\mathrm{can}} = c_{\mathrm{a}} M_{\mathrm{can}} \Delta T_{\mathrm{can}} \tag{27.6}$$

where $c_{\mathrm{a}}$ is the specific heat of aluminum (the calorimeter can is aluminum), $M_{\mathrm{can}}$ is the mass of the aluminum calorimeter can, and $\Delta T_{\mathrm{can}}$ is the change in temperature that the can has experienced because of the heat $Q_{\mathrm{can}}$ absorbed by it.

Inserting Equations 27.4, 27.5, and 27.6 into Equation 27.3 gives

$$c_{\mathrm{x}} M_{\mathrm{x}} \Delta T_{\mathrm{x}} = c_{\mathrm{w}} M_{\mathrm{w}} \Delta T_{\mathrm{w}} + c_{\mathrm{a}} M_{\mathrm{can}} \Delta T_{\mathrm{can}} \tag{27.7}$$

But,

$$\Delta T_{\mathrm{x}} = T_{\mathrm{b}} - T_{\mathrm{c}}$$

and

$$\Delta T_{\mathrm{w}} = \Delta T_{\mathrm{can}} = T_{\mathrm{c}} - T_0$$

Note that because of how Equation 27.3 was written, all of the change in temperature $\Delta T$ values are considered to be positive quantities. Therefore, Equation 27.7 becomes

$$c_{\mathrm{x}} M_{\mathrm{x}} (T_{\mathrm{b}} - T_{\mathrm{c}}) = c_{\mathrm{w}} M_{\mathrm{w}} (T_{\mathrm{c}} - T_0) + c_{\mathrm{a}} M_{\mathrm{can}} (T_{\mathrm{c}} - T_0) \tag{27.8}$$

Solving Equation 27.8 for $c_{\mathrm{x}}$ gives

$$c_{\mathrm{x}} = \frac{c_{\mathrm{w}} M_{\mathrm{w}} (T_{\mathrm{c}} - T_0) + c_{\mathrm{a}} M_{\mathrm{can}} (T_{\mathrm{c}} - T_0)}{M_{\mathrm{x}} (T_{\mathrm{b}} - T_{\mathrm{c}})} \tag{27.9}$$

Now suppose the unknown sample were aluminum. Then

$$c_{\mathrm{x}} = c_{\mathrm{a}} \text{ and } M_{\mathrm{x}} = M_{\mathrm{a}}$$

In this case, Equation 27.8 would become

$$c_{\mathrm{a}} M_{\mathrm{a}} (T_{\mathrm{b}} - T_{\mathrm{c}}) = c_{\mathrm{w}} M_{\mathrm{w}} (T_{\mathrm{c}} - T_0) + c_{\mathrm{a}} M_{\mathrm{can}} (T_{\mathrm{c}} - T_0)$$

or

$$c_{\mathrm{a}} \left[ M_{\mathrm{a}} (T_{\mathrm{b}} - T_{\mathrm{c}}) - M_{\mathrm{can}} (T_{\mathrm{c}} - T_0) \right] = c_{\mathrm{w}} M_{\mathrm{w}} (T_{\mathrm{c}} - T_0)$$

Therefore

$$c_{\mathrm{a}} = \frac{c_{\mathrm{w}} M_{\mathrm{w}} (T_{\mathrm{c}} - T_0)}{M_{\mathrm{a}} (T_{\mathrm{b}} - T_{\mathrm{c}}) - M_{\mathrm{can}} (T_{\mathrm{c}} - T_0)} \tag{27.10}$$

Equation 27.10 then gives a determination of the specific heat of aluminum provided that certain assumptions are made. These assumptions will be left for the student to comment upon at the end of the experiment.

## 27.2 Experiment

Before suggesting a procedure to follow in this experiment, the authors must confess the absence of at least three major refinements that would have to be taken into account in a more accurate experiment to determine the specific heat of a solid. These refinements are manifested in the assumptions that were made in the derivation of Equations 27.9 and 27.10 above. As the student conducts the experiment, they are urged to take careful note of the procedure, for it will offer hints as to what these assumptions may be. The aforementioned refinements have been purposely omitted from the suggested procedure because it is the opinion of the authors that they would serve to obscure the true purpose of the experiment, namely to acquaint the student with some basic thermodynamics and to measure as simply as possible the characteristic property of a solid that we call the specific heat. The price that must be paid for omitting the refinements is probably an induced error of about 3–5%.

The suggested procedure for measuring the specific heat of the metals on your bench is as follows. Pour water into the steam generator and plug it in. While the water in the generator is being heated, determine the mass of the calorimeter can and also the mass of your sample (aluminum or copper) on the electronic balance. Record your data in Table 27.1. Pour tap water into the calorimeter can until it is about half full and determine the mass of the can plus water on the electronic balance. Compute the mass of the water in the can. Record your measurements in Table 27.1. Place the calorimeter can into the calorimeter and stir the water gently with the thermometer. Now place the aluminum sample into the steam generator by means of a string, but do not allow the sample to touch the bottom of the steam generator. When the water in the steam generator has come to a boil and the sample has been in the water for a sufficiently long time (4 or 5 minutes), measure the temperature $T_0$ of the water in the calorimeter can. Assume that the calorimeter can is at the same temperature as the water within it. Now quickly pull the sample from the steam generator and slip it into the calorimeter, being careful not to splash any water. Replace the cork on the top of the calorimeter. Gently stir the water with the thermometer as best you can and take several readings (up to 10) in order to determine the common temperature $T_\mathrm{c}$ reached by the sample, water, and calorimeter can (you can stop taking readings when the temperature plateaus). Record these readings in Table 27.1. Assume that the amount of heat absorbed by the calorimeter jacket is negligible.

Repeat the same procedure for the copper sample.

**Question 1:** Use Equation 27.10 to compute the specific heat of the aluminum sample, $c_\mathrm{a}$. The accepted value for $c_\mathrm{a}$ is 0.217 $\frac{\mathrm{cal}}{\mathrm{g\ C^\circ}}$ at room temperature and at a pressure of one atmosphere. Compare your experimental value with the accepted value by computing the percent experimental error. Show your calculations and record your values in Table 27.1.

**Question 2:** Use Equation 27.9 to compute the specific heat of the copper sample, $c_\mathrm{copper}$. For the value of $c_\mathrm{a}$ in the equation, use the accepted value of 0.217 $\frac{\mathrm{cal}}{\mathrm{g\ C^\circ}}$. The accepted value for $c_\mathrm{copper}$ is 0.093 $\frac{\mathrm{cal}}{\mathrm{g\ C^\circ}}$ at room temperature and at a pressure of one atmosphere. Compare your experimental value with the accepted value by computing the percent experimental error. Show your calculations and record your values in Table 27.1.

**Question 3:** What would be the advantage of starting the experiment with the temperature of the water in the calorimeter can a few degrees lower than room temperature and ending about the same number of degrees above room temperature?

**Question 4:** List what you think may be three major refinements that would have to be taken into account in a more accurate measurement of specific heat using your apparatus and discuss how neglecting these refinements affected your results.

| | Aluminum | Cooper |
|---|---|---|
| Mass of Calorimeter Can (g) | | |
| Mass of Sample (g) | | |
| Mass of Calorimeter Can Plus Water (g) | | |
| Mass of Water in Can (g) | | |
| Temperature of Boiling Water ($T_\mathrm{b}$) (°C) | 100 | 100 |
| Initial Temperature of Calorimeter Can Plus Water ($T_0$) (°C) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 1.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 2.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 3.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 4.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 5.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 6.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 7.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 8.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 9.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Measurement 10.) | | |
| Common Temperature of Can, Water, and sample ($T_\mathrm{c}$) (°C) Final Value | | |
| Experimental Value of Specific Heat ($\frac{\mathrm{cal}}{\mathrm{g\ C^\circ}}$) | | |
| Accepted Value of Specific Heat ($\frac{\mathrm{cal}}{\mathrm{g\ C^\circ}}$) | 0.217 | 0.093 |
| Percent Experimental Error (%) | | |

Table 27.1: Measurements to Determine Specific Heat of Aluminum and Copper

**Question 5:** List any other causes of experimental error that you think are important.

**Question 6:** We are all familiar with aluminum and copper cooking utensils. Based upon the value of the specific heat of copper as compared to that for aluminum, give an argument that would favor a pan of one of these metals over the pan made of the other metal. Hint: Review the definition of heat and that for specific heat.

# Lab 28

# Heat of Fusion and Vaporization

In Experiment 27 you determined the specific heat of various substances; that is, the heat required to raise the temperature of a given mass of the substance a specific amount, $\Delta T$. However, in these experiments the change in temperature $\Delta T$ of the substance was never over a range such that the substance underwent a change in state or phase. We find experimentally when a substance does undergo a phase change, a specific amount of heat is absorbed or given off without resulting in any change in temperature. The heat absorbed by a substance (without temperature change) in changing from the solid to the liquid state is called the heat of fusion of that substance. The quantity of heat absorbed as the substance undergoes a transition from the liquid to the vapor state is called the heat of vaporization. The term heat of transition ($L$) is sometimes used to refer to either the heat of fusion or the heat of vaporization of a substance (see Figure 28.1).

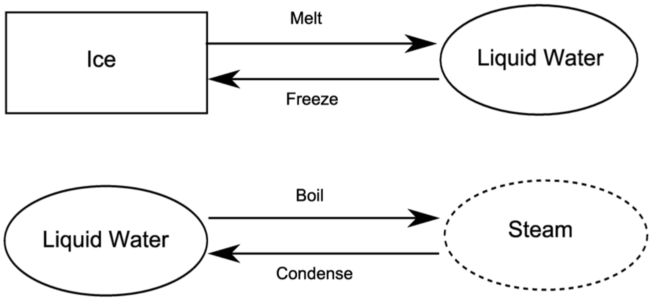


Figure 28.1: Phase Transition Diagram

The object of this experiment is to use the techniques of calorimetry to experimentally determine a value for the heat of fusion of ice and the heat of vaporization of water.

## 28.1 Theory

### 28.1.1 Change in Temperature of 1 g of Ice as Heat is Absorbed

Consider an experiment in which heat is absorbed at a known rate by a piece of ice of mass 1 g and the corresponding change in temperature of the ice is recorded. Assume also that this experiment was carried out over a temperature range of -20°C to 120°C so that during the course of the experiment two phase changes were encountered. At 0°C the ice would change to water and at

100°C the water would change to the vapor state. Plotting the results of this thought experiment would result in a graph similar to Figure 28.2.

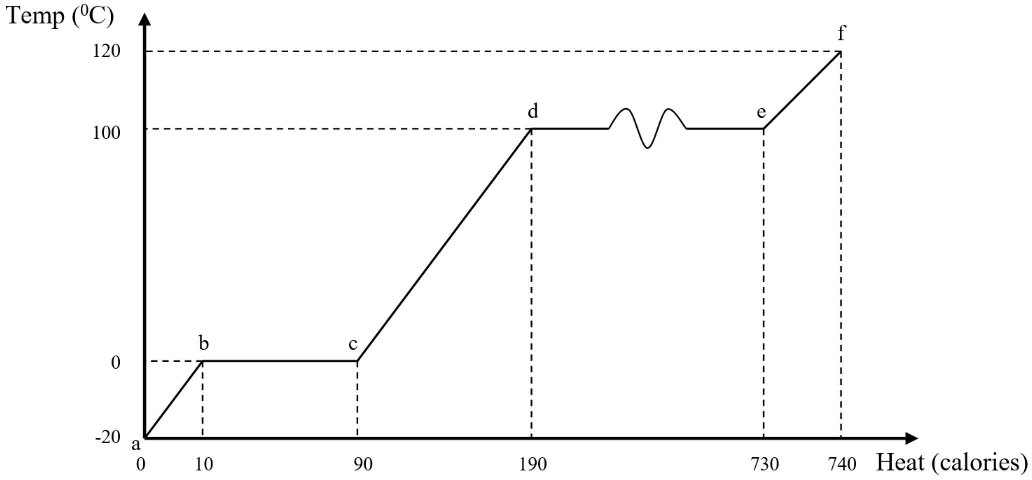


Figure 28.2: Change of Temperature of 1 g of Ice, Water, and Water Vapor Versus Number of Calories of Heat Added

**Question 1:** What do the two plateaus bc and de in Figure 28.2 represent?

**Question 2:** What does the slope of line ab in Figure 28.2 represent? What is its numerical value? What do the slopes of lines cd and ef represent? What are their respective numerical values?

The amount of heat absorbed by the ice at 0°C to convert it completely to water at the same temperature is defined as the heat of fusion of ice. Its numerical value is 80 calories per gram. The amount of heat absorbed by the water at 100°C to convert it completely to water vapor at the same temperature is defined as the heat of vaporization of water. Its numerical value is 539 calories per gram. Listings of the value of the heats of fusion and vaporization of other substances may be found in your text. In our notation we will use the letter $L$ to represent the heat of transformation; that is, the heat absorbed or given off in a change of phase per unit mass. The total heat $Q$ absorbed or given off in a phase transition of mass $m$ may then be given by:

$$Q = mL \tag{28.1}$$

### 28.1.2 Experimental Determination of Heat of Fusion $L_\mathrm{f}$ of Ice

A block of ice of initial temperature 0°C and mass $m_\mathrm{i}$ is immersed in water of mass $m_\mathrm{w}$ contained in an aluminum calorimeter can of mass $m_\mathrm{can}$ and specific heat $c_\mathrm{can}$. The initial temperature of the water and the cup is $T_1$. Allowing the ice to melt completely we find the cup, water, and water resulting from the melted ice have reached a final equilibrium temperature of $T_2$ ($T_1 > T_2 > 0$°C) (see Figure 28.3).

Since we have been very careful not to allow any heat to be given off to the surroundings we may state that the heat lost by the water and cup in cooling down from $T_1$ to $T_2$ is equal to the heat gained by the ice changing phase plus the heat gained by the ice water in raising its temperature

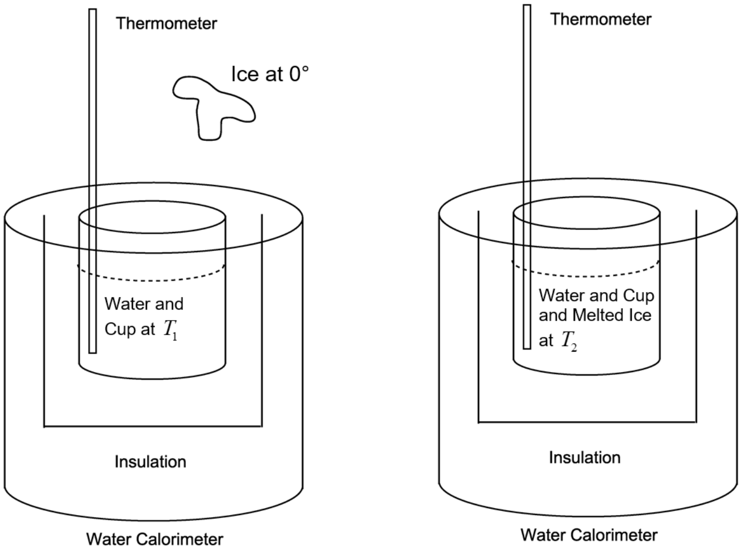


Figure 28.3: Calorimeter Procedure for Determining Latent Heat of Fusion

from 0°C to $T_2$. We may therefore write:

$$m_{\rm ice}L_{\rm f} + m_{\rm ice}c_{\rm w}(T_2 - 0) = m_{\rm w}c_{\rm w}(T_1 - T_2) + m_{\rm can}c_{\rm can}(T_1 - T_2) \tag{28.2}$$

where $m_{\rm ice}L_{\rm f}$ is the heat gained by the ice changing phase, $m_{\rm ice}c_{\rm w}(T_2 - 0)$ is the heat gained by the resulting ice water in going from 0°C to a final temperature of $T_2$, $m_{\rm w}c_{\rm w}(T_1 - T_2)$ is the heat lost by the water in the calorimeter cup, and $m_{\rm can}c_{\rm can}(T_1 - T_2)$ is the heat lost by the can itself.

**Question 3:** Solve Equation 28.2 for the heat of fusion $L_{\rm f}$.

**Question 4:** Suppose the ice had been at a temperature of -10°C initially. Rewrite Equation 28.2 for this case.

**Question 5:** In this experiment it is a good practice to let $T_1$ be about the same amount above room temperature as $T_2$ is below. Why?

### 28.1.3 Experimental Determination of the Heat of Vaporization $L_{\rm v}$ of Water

Begin with a calorimeter can of mass $m_{\rm can}$ and a specific heat $c_{\rm can}$ containing water of mass $m_{\rm w}$. The initial temperature of the water and the cup is $T_1$. Steam of temperature 100°C is allowed to bubble through the water, with a mass of steam $m_{\rm s}$ condensing in the calorimeter. The final temperature of the system (cup, water, and water resulting from condensed steam) is $T_2$ ($100°{\rm C} > T_2 > T_1$) (see Figure 28.4).

Again assuming no heat is lost to the surroundings, we can write:

$$m_{\rm s}L_{\rm v} + m_{\rm s}c_{\rm w}(100 - T_2) = m_{\rm w}c_{\rm w}(T_2 - T_1) + m_{\rm can}c_{\rm can}(T_2 - T_1) \tag{28.3}$$

**Question 6:** Explain what is meant by each of the four groups of terms in Equation 28.3.

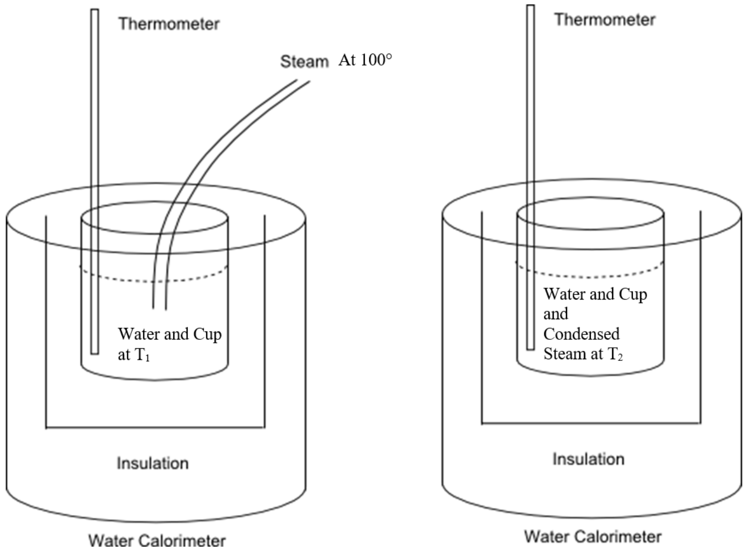


Figure 28.4: Calorimeter Procedure for Determining Latent Heat of Vaporization

**Question 7:** Solve Equation 28.3 for $L_\mathrm{v}$.

## 28.2 Experiment

### 28.2.1 Heat of Fusion of Ice

Determine the mass of the small calorimeter can. Fill it about one-half full of water at a temperature of about 30°C. Determine the mass of the water and the cup, place the cup in the calorimeter jacket, and record the initial temperature $T_1$. Add small pieces of ice and stir gently with the thermometer until the ice has melted. Continue this until a temperature of about 10°C is reached. Make sure all the ice has melted, stir gently, and record the final temperature $T_2$. Determine the mass of the cup, water, and water resulting from the melted ice. Record all of your data in Table 28.1.

**Question 8:** Using your observed values and your answer to Question 3, solve for your experimental value of $L_\mathrm{f}$. Compare your value to the accepted value of 80 cal/g by computing the percent experimental error. Show all of your calculations and record your values in Table 28.1.

**Question 9:** Explain why your results will be more accurate if you dry the ice with a paper towel before putting it in the calorimeter.

| | |
|---|---|
| Mass of Cup and Initial Water (g) | |
| Mass of Cup (g) | |
| Mass of Initial Water (g) | |
| Mass of Cup, Initial Water, and Water From Ice (g) | |
| Mass of Ice (g) | |
| Initial Temperature ($T_1$) (°C) | |
| Final Temperature ($T_2$) (°C) | |
| | |
| Experimental Latent Heat of Fusion ($L_f$) (cal/g) | |
| Accepted Latent Heat of Fusion (cal/g) | 80 |
| Percent Experimental Error (%) | |

Table 28.1: Data to Compute Latent Heat of Fusion

### 28.2.2 Heat of Vaporization of Water

Determine the mass of the aluminum calorimeter can, fill it about one-half full of water (approximately 10°C) and determine the mass of the cup and water. Fill your steam generator about two-thirds full with water and connect it to a water trap using plastic tubing. Plug in your steam generator and when steam is given off continuously run the tube from the water trap into the water in the calorimeter, after determining the initial temperature of the water and calorimeter cup $T_1$ (see Figure 28.5).

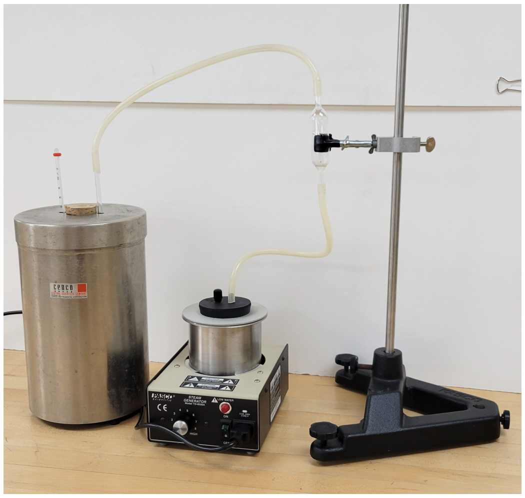


Figure 28.5: Calorimeter Apparatus for Determining Latent Heat of Vaporization

Let the steam bubble through the calorimeter cup until the temperature of the system (aluminum can, water, and condensed steam) is about the same amount above room temperature as your initial temperature was below room temperature. Remove the steam tube from the calorimeter and immediately record your final temperature $T_2$. Determine the final mass of the cup, water, and condensed steam. Record all of your values in Table 28.2.

| | |
|---|---|
| Mass of Cup and Initial Water (g) | |
| Mass of Cup (g) | |
| Mass of Initial Water (g) | |
| Mass of Cup, Initial Water, and Condensed Steam (g) | |
| Mass of Steam (g) | |
| Initial Temperature ($T_1$) (°C) | |
| Final Temperature ($T_2$) (°C) | |
| | |
| Experimental Latent Heat of Vaporization ($L_\mathrm{v}$) (cal/g) | |
| Accepted Latent Heat of Vaporization (cal/g) | 539 |
| Percent Experimental Error (%) | |

Table 28.2: Data to Compute Latent Heat of Vaporization

**Question 10:** Explain how water that condensed in the steam tube and rolled down into the calorimeter cup will affect your results.

**Question 11:** Use your answer to Question 7 to determine your experimental value for $L_\mathrm{v}$. Compare your value to the accepted value of 539 cal/g by computing the percent experimental error. Show all of your calculations and record your values in Table 28.2.

# Part X

# Wave Motion and Sound

# Lab 29

# Measurement of the Speed of Sound in Air

Resonance is the intensification of sound that results when a reflected disturbance unites with a direct disturbance to yield a disturbance of increased amplitude. The phenomenon of resonance is easily produced in the apparatus called the resonance tube. In this experiment, you will use the resonance tube to investigate resonance in an air column that is closed at one end. You will also measure the speed of sound in air. The resonance tube that we will use in this experiment is shown in Figure 29.1.

Figure 29.1: Resonance Tube Apparatus For Measuring the Speed of Sound

An open-ended speaker that can be adjusted to oscillate at a range of frequencies is placed near the open end of the tube. The resonance tube contains a smaller inner tube with a plastic barrier that can be slid in and out, effectively changing the length of the air column in the main resonance tube. When the open-ended speaker vibrates toward the tube from a neutral position, a condensation of air is produced that travels down the tube, strikes the barrier, and is reflected back up the tube. If the reflected condensation reaches the open end of the tube when the speaker has completed one half of a vibration and is in a position to vibrate outward (thus producing a condensation ready to issue forth in the air away from the tube), then the reflected condensation will combine with this direct condensation to produce a condensation of greater amplitude. The listener thus experiences an intensification of sound (resonance).

If one investigates the resonance phenomenon by considering the addition of direct and reflected sound waves, then one obtains the standing wave pattern. That is, the mathematical description of the resonance phenomenon is that of standing waves. The standing wave pattern constructed in the air in the tube consists of a node at the boundary and an antinode near the open end of the tube. The number of closed loops in the standing wave pattern depends on the frequency of the speaker. Figure 29.2 shows a typical standing wave pattern set up in a resonance tube closed at one end. The spatial distance $s$ between successive nodes is one-half a wavelength; hence, the wavelength $\lambda$ of the sound waves propagating in the tube can be determined by

$$s = \frac{\lambda}{2} \tag{29.1}$$
$$\lambda = 2s$$

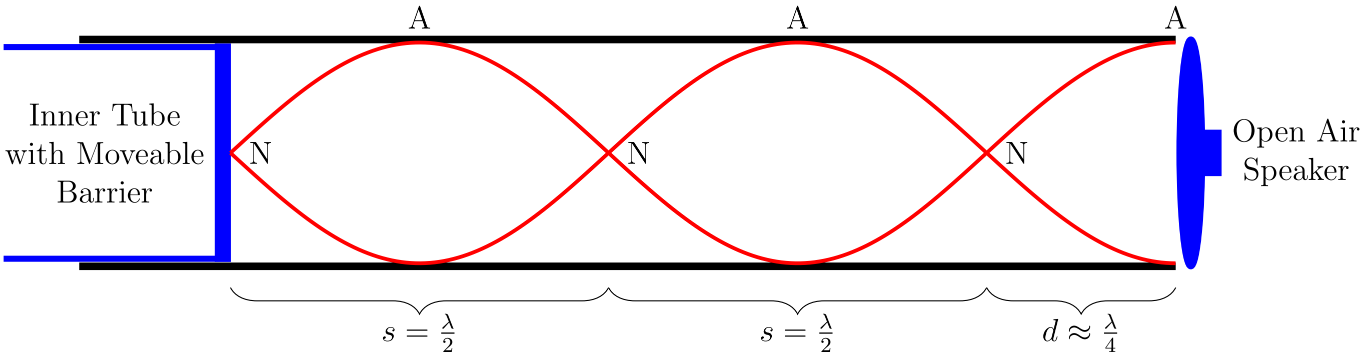


Figure 29.2: Standing Wave Pattern in Resonance Tube

The speed of sound $v$ can be determined if the frequency $f$ of the source and the wavelength $\lambda$ of the sound wave are known. That is,

$$v = f\lambda \tag{29.2}$$

Combining Equations 29.1 and 29.2 we readily find that

$$v = 2fs \tag{29.3}$$

Thus, if one can determine the positions of the boundary in the resonance tube for which resonance occurs, then one has a measure of the speed of sound in the air in the tube.

In reality some of the energy carried by the sound waves escapes at each reflection of a condensation from the boundary and, in addition, is also radiated in the form of spherical waves from the open end of the tube. The air ahead of the open end is therefore in vibration and the result is that the effective length of the air column for which resonance occurs is greater than the actual length of the air column in the tube. All of this means that the open end of the tube is not exactly an antinode, but the antinode is actually located at some small distance ahead of the open end of the tube. Hence, the distance $d$ (see Figure 29.2) of the boundary from the open end of the tube for which resonance is first detected is slightly less than a quarter of a wavelength. If one is to use the measurement of $d$ to determine the speed of sound in the air column, an end correction $\delta$ must first be made. H. von Helmholtz (1821–1894) deduced from theoretical considerations that the end correction $\delta$ that must be added on to the value of $d$ in order that $d+\delta$ be a quarter of a wavelength is given by

$$\delta = \frac{\pi R}{4} \tag{29.4}$$

where $R$ is the radius of the resonance tube. If a second position of the boundary for which resonance is detected can be obtained, then one can determine the value of $\delta$ experimentally.

## 29.1 Experiment

Place the speaker at the open end of the resonance tube. Use the sine wave generator to set the speaker to oscillate at 500 Hz. Make sure that you keep the intensity level on the sine wave generator low so you don't damage your hearing. Move the inner tube out slowly to determine the lengths of the air column at which resonance occurs (you are locating the nodes of the standing wave) (see Figure 29.3). The centimeter scale taped to the inner tube represents the distance of the barrier from the end of the tube. By taking the differences of the successive node positions, you can then determine the node to node distances $s_1$, $s_2$, and $s_3$. You should be able find 4 node locations enabling you to calculate 3 node to node distances when the frequency is 500 Hz. Record the data in Table 29.1.

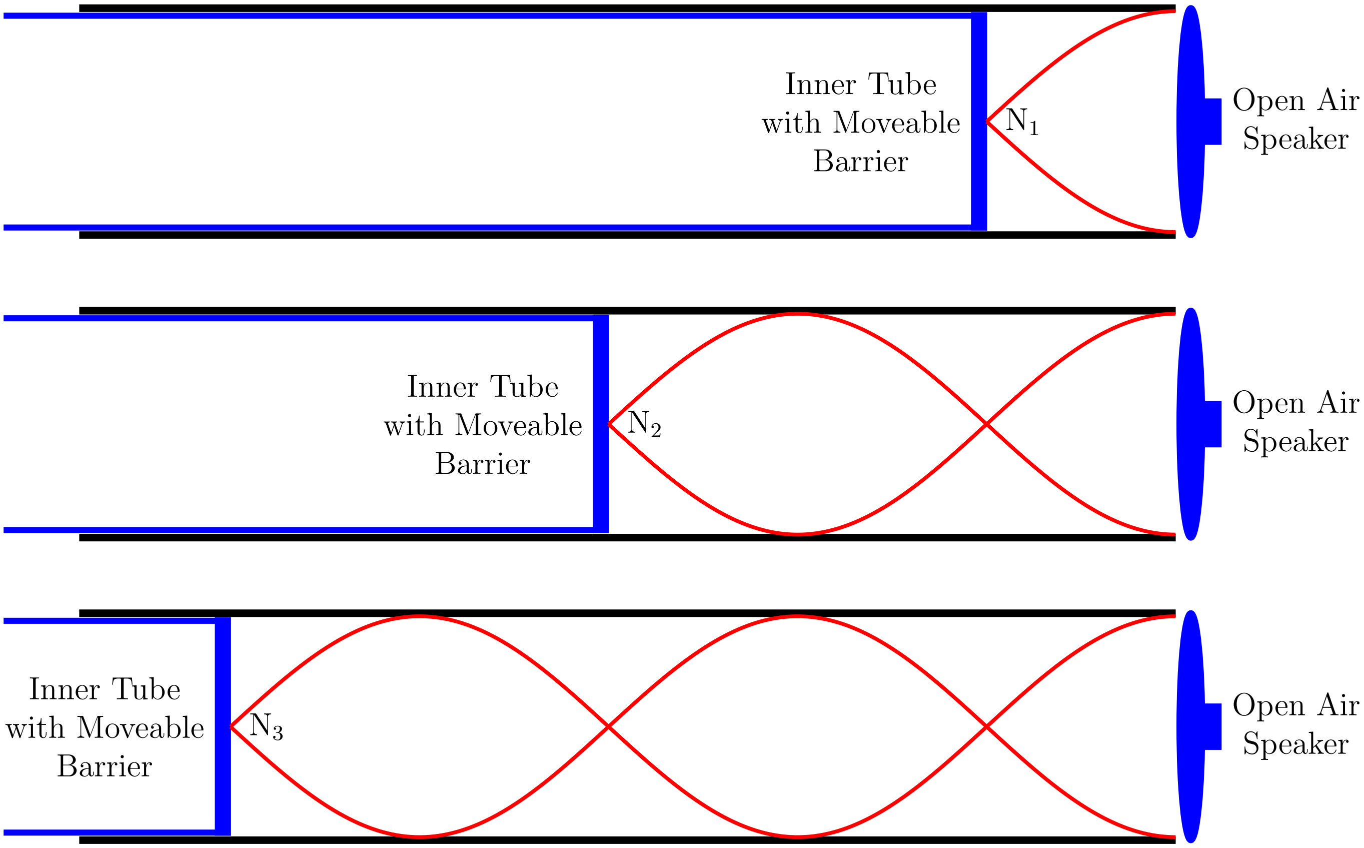


Figure 29.3: Node Positions in Resonance Tube

Change the frequency of the vibrating speaker to 650 Hz. Again determine the lengths of the air column at which resonance occurs and the node to node distances. You should be able find 5 node locations enabling you to calculate 4 node to node distances when the frequency is 650 Hz. Record your data in Table 29.2.

**Question 1:** For each frequency, calculate an average value $s_{\text{ave}}$ for the distance between successive nodes in the standing wave. Record your values in Tables 29.1 and 29.2.

**Question 2:** For each frequency, calculate the corresponding wavelength of the sound waves propagating in the air column. Use Equation 29.1 with $s = s_{\text{ave}}$, $\lambda = 2s_{\text{ave}}$. Record your values in Table 29.3.

| Frequency 500 Hz | | |
|---|---|---|
| | | |
| Node Number | Position of End of Tube at Resonance (m) | Distance Between Nodes (m) |
| 1 | | $s_1 =$ |
| 2 | | $s_2 =$ |
| 3 | | $s_3 =$ |
| 4 | | |
| | | |
| Average Node to Node Distance ($s_{\text{ave}}$) (m) | | |

Table 29.1: Node to Node Distances - Frequency = 500 Hz

| Frequency 650 Hz | | |
|---|---|---|
| | | |
| Node Number | Position of End of Tube at Resonance (m) | Distance Between Nodes (m) |
| 1 | | $s_1 =$ |
| 2 | | $s_2 =$ |
| 3 | | $s_3 =$ |
| 4 | | $s_4 =$ |
| 5 | | |
| | | |
| Average Node to Node Distance ($s_{\text{ave}}$) (m) | | |

Table 29.2: Node to Node Distances - Frequency = 650 Hz

**Question 3:** For each of the values of the wavelength determined in question 2 and the corresponding frequency of the speaker, calculate the experimental speed of sound in air. Use Equation 29.2, $v = f\lambda$. Record your values in Table 29.3.

**Question 4:** Calculate the theoretical speed of sound in air correcting for temperature. Use the following equation:

$$v = v_0\sqrt{1 + \frac{T_c}{273.15}}$$

where $v_0 = 331.36$ m/s and $T_c$ is the room temperature in ° Celsius. Compare the experimental and theoretical speeds of sound by computing the percent experimental error. Record your values in Table 29.3.

| Frequency ($f$) (Hz) | Wavelength ($\lambda$) (m) | Speed Sound ($v$) (m/s) |
|---|---|---|
| 500 | | |
| 650 | | |
| | | |
| Average Experimental Speed of Sound (m/s) | | |
| Theoretical Speed of Sound Corrected for Temperature (m/s) | | |
| Percent Experimental Error (%) | | |

Table 29.3: Speed of Sound in Air

# Lab 30

# Demonstration of Resonance in a Stretched String

## 30.1 Introduction

Whenever a system capable of oscillating or vibrating is acted upon by a driving force whose frequency approaches one of the natural frequencies of oscillation or vibration of the system, the system is set into oscillation or vibration with a relatively large amplitude. This phenomenon is called resonance, and the system is said to resonate with the driving force when the frequency of the driving force is equal to one of the natural frequencies of the system.

Some common examples of resonance are the shimmying of the unaligned front end suspension of an automobile at high speeds, and the way a person's eardrums seem to buzz when certain sounds are heard. Another familiar example of resonance occurs in the tuning of a radio. When you turn the dial of the radio to select a particular station, you are bringing a selected circuit in the radio into resonance with radio waves of a particular frequency being broadcast by the radio station.

In this experiment you will study the phenomenon of resonance by employing a method devised over a century ago by Franz Emil Melde. The apparatus you will use is rather simple and is shown in Figure 30.1. Melde used a tuning fork instead of an vibrating blade.

The string ABC is fastened to the vibrating blade at A and the other end passes over a pulley at B. A suspended weight is attached to the end of the string at C (also see Figure 30.2). The frequency of vibration of the vibrating blade is fixed by the alternating current passing through the vibrator at 60 Hz. When the vibrating blade is vibrating steadily, it forces the string to vibrate with the same frequency. The form of the response made apparent in the string depends ultimately on the frequency of the vibrating blade and the time it takes a wave to travel along the string from the blade at A to the pulley at B.

## 30.2 Theory

The waves that will be generated in this experiment are examples of mechanical waves. They originate in the displacement of some portion of an elastic medium (in this case the string) from an equilibrium position. The elastic properties of the medium (string) result in the propagation of the initial disturbance through the medium (the string). The mechanical waves generated in the string in this experiment are examples of transverse mechanical waves. Transverse mechanical waves are waves whose direction of propagation is perpendicular to the direction along which the particles of

Figure 30.1: Equipment to Generate Standing Waves on a String

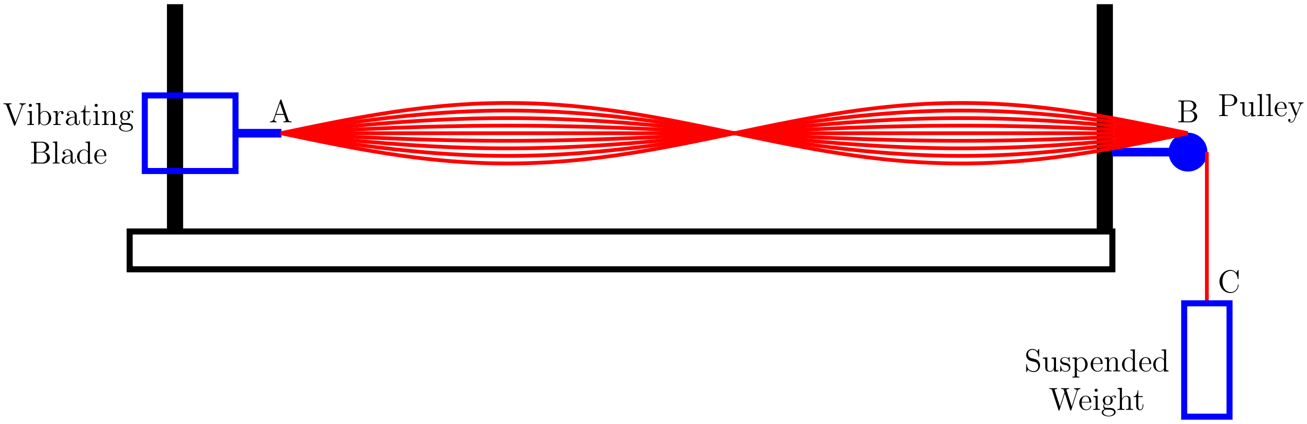


Figure 30.2: Set-up to Generate Standing Waves on a String

the medium vibrate. For example, in the case of the waves you will generate in this experiment, the waves will propagate along the length of the string but the particles of string vibrate at right angles to the direction of propagation of the wave. An example of a transverse wave that needs no material medium for propagation is an electromagnetic wave. Electromagnetic waves, therefore, are not mechanical waves.

Mechanical waves that propagate along a direction that is the same as that along which the particles of the medium vibrate are called longitudinal waves. Examples of longitudinal waves are sound waves. We will explore sound waves in a separate lab.

When the vibrating blade is set into vibration, waves are generated in the string that are transverse in nature and travel from the vibrating blade at A to the pulley at B and back again during any whole number of vibrations of the vibrating blade. The waves traveling from the pulley at B toward the blade at A are those that have been reflected at the pulley. If the pulley is treated as a perfectly rigid boundary, then the waves that are reflected from the pulley are of the same amplitude as those incident on the pulley; that is, no energy has been absorbed by the pulley if the pulley is a perfectly rigid boundary. The waves incident on the pulley and the waves reflected from the pulley interfere and give rise to what is called a standing wave. Figure 30.3 illustrates this interference and the resulting standing wave at intervals of one-quarter of a period of vibration of the vibrating blade.

In the standing wave pattern, each particle of string vibrates along a vertical direction with

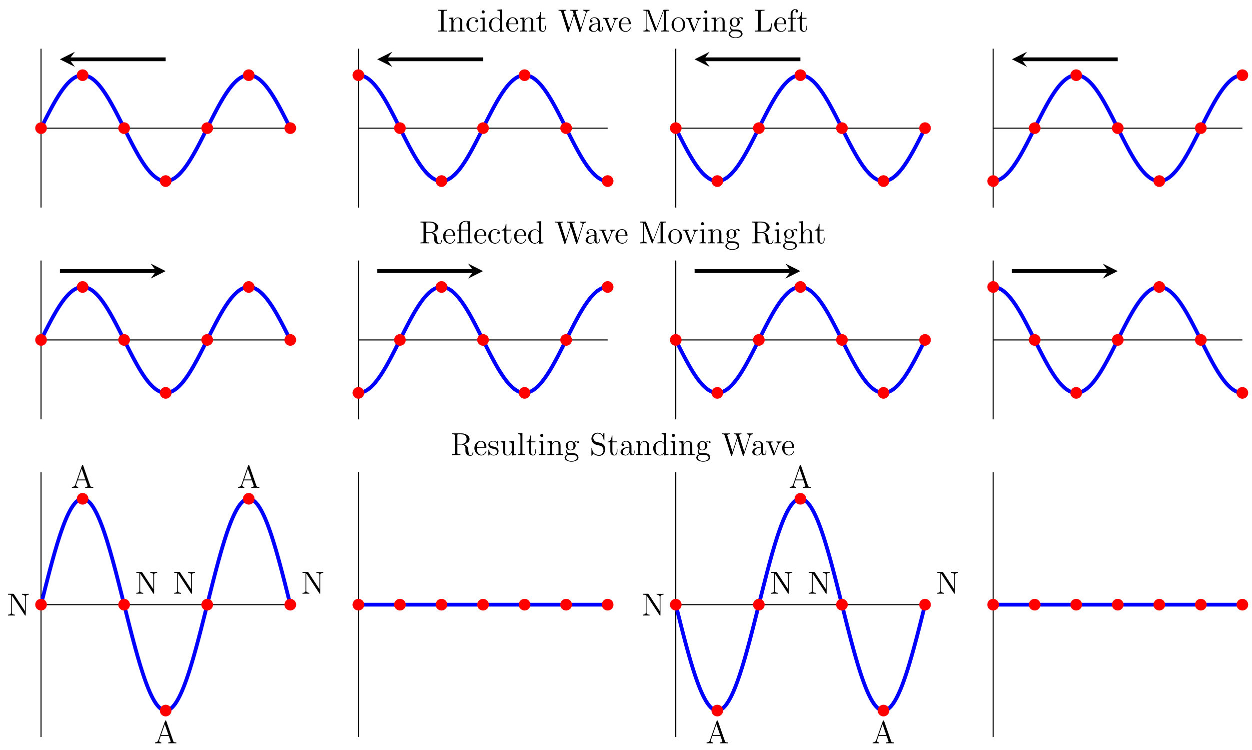


Figure 30.3: Sum of Incident and Reflecting Wave Forms a Standing Wave

the exception of particles at positions labeled N (see Figure 30.3 along the resulting standing wave row). The points where there is no apparent motion of the string are called nodes. The points labeled A are called antinodes. They correspond to positions of maximum displacement of particles of string from their equilibrium position (i.e., horizontal unstretched position of the string). The actual appearance of the standing wave form setup in the string in slow motion would appear as illustrated by Figure 30.3 under resulting standing wave. Energy is not transported along the string to the right or to the left. This is because energy cannot flow past the nodal points N in the string, which are permanently at rest. Thus, the energy in the string remains “standing” and alternates between vibrational kinetic energy of the particles of string and elastic potential energy of position of the particles of string. When the string is set vibrating, you will not be able to discern the various positions of the string as noted by Figure 30.4 because the string will be vibrating too rapidly. What you will observe is an envelope of the motion. The string will look like a blur with no motion apparent and a fixed stationary wave form. The spatial distance between successive nodes (or antinodes) is one-half of a wavelength. The pulley will be very nearly a true node. The vibrating blade is almost a node. The number $n$ of half-wavelengths (i.e., loops) that will appear in the standing wave form will depend upon the tension $T$ in the string, the frequency $f$ of the vibrating blade, the length $L$ of string between pulley and vibrating blade, and the linear density $\mu$, of the string. The tension in the string is determined by the weight hung at C (see Figure 30.2).

If the string has a mass $M$ and a total length $L$, then the linear density is given by

$$\mu = \frac{M}{L} \tag{30.1}$$

For a string of linear density $\mu$ and under tension $T$, the velocity of a transverse wave traveling along the string is given by

$$v = \sqrt{\frac{T}{\mu}} \tag{30.2}$$

If $\lambda$ is the wavelength of the wave and $f$ is the frequency of vibration of the string, then we also have that

$$v = f\lambda \tag{30.3}$$

But if $L$ is the length of the string between the pulley and vibrating blade, then

$$L = n\frac{\lambda}{2}$$

or

$$\lambda = \frac{2L}{n};\ n = 1, 2, 3, \ldots \tag{30.4}$$

where $n$ is the number of loops observed between the pulley and vibrating blade (see Figure 30.4).

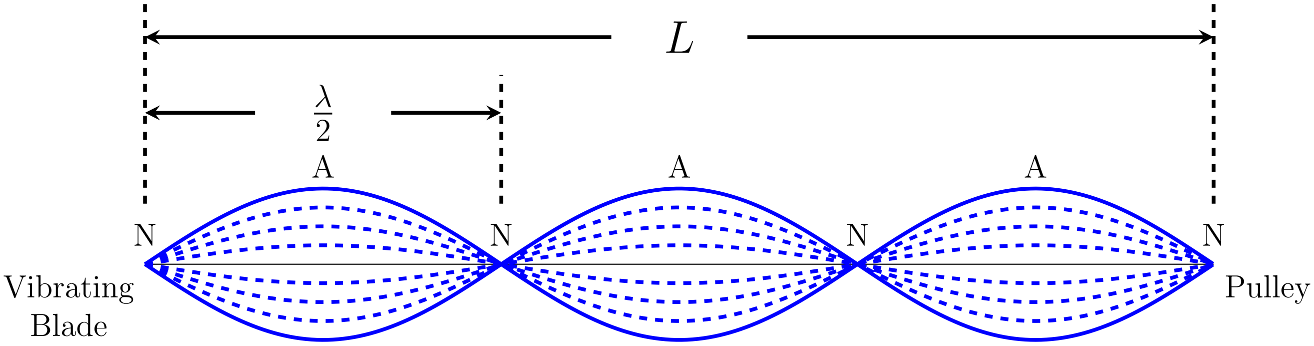


Figure 30.4: Standing Wave With Three Loops

Solving Equation 30.3 for $\lambda$ gives

$$\lambda = \frac{v}{f} \tag{30.5}$$

Inserting Equation 30.2 into Equation 30.5, we get

$$\lambda = \frac{1}{f}\sqrt{\frac{T}{\mu}} \tag{30.6}$$

The frequency of the waves in the string is that of the vibrating blade (i.e. 60 Hz).

Equation 30.6 clearly demonstrates that, for a given string, the wavelength of the waves is directly proportional to the square root of the tension in the string. If, for example, the tension increased fourfold, the wavelength increases twofold. One may solve Equation 30.6 for the tension $T$ which gives

$$T = \lambda^2 f^2 \mu \tag{30.7}$$

Inserting Equation 30.4 into Equation 30.7, we get an equation that gives the tension $T$ required to give $n$ loops (i.e. $n$ half-wavelengths). The result is

$$T = \frac{4\mu f^2 L^2}{n^2} \tag{30.8}$$

## 30.3 Experiment

Measure the mass and length of the string.

**Question 1:** Compute the linear density of the string and record your value in Table 30.1.

| | |
|---|---|
| Length of String Between the Vibrating Blade and Pulley ($L$) (m) | 1 |
| Linear Mass Density of String ($\mu$) (kg/m) | |
| Frequency of Vibrating Blade ($f$) (Hz) | 60 |

Table 30.1: Parameters for Standing Waves on a String

Set up the apparatus as shown in Figures 30.1 and 30.2. Measure the length of the string AB between the pulley and the vibrating blade. Record your value in Table 30.1 (note: we will typically use a default value of $L = 1$ m).

Suspend enough mass on the hanger on the end of the string at C so that exactly one loop appears in the string when the vibrating blade is turned on. If the "standing" wave pattern wiggles about, then the mass must be adjusted as resonance has not been achieved. When the standing wave is *fixed* with one loop, the appropriate mass will have been suspended at C. Record the value of the mass in Table 30.3. Using $w = mg$, compute the weight of the mass (don't forget to include the mass of the hanger). We will consider this to be the experimental tension in the string. Record the value in Table 30.3.

**Question 2:** Use Equation 30.4 to determine the wavelength when there is one loop in the string. Show your calculation and record your value in Table 30.3. Sketch a picture of the loop labeling the nodes and the antinodes. Also indicate the wavelength (or a part of it) on the diagram.

**Question 3:** Use Equation 30.8 to determine the theoretical value of the tension in the string appropriate for one loop. Compare this value with the experimental value of the tension and compute the percent experimental error. Show your calculations and record your values in Table 30.3.

Now suspend enough mass on the hanger on the end of the string at C so that exactly two loops appears in the string when the vibrating blade is turned on. Record the value of the mass in Table 30.3. Using $w = mg$, compute the weight of the mass (don't forget to include the mass of the hanger). We will consider this to be the experimental tension in the string. Record the value in Table 30.3.

**Question 4:** Use Equation 30.4 to determine the wavelength when there are two loops in the string. Show your calculation and record your value in Table 30.3. Sketch a picture of the loop labeling the nodes and the antinodes. Also indicate on your sketch one wavelength.

**Question 5:** Use Equation 30.8 to determine the theoretical value of the tension in the string appropriate for two loops. Compare this value with the experimental value of the tension and compute the percent experimental error. Show your calculations and record your values in Table 30.3.

**Question 6:** Using Equation 30.8, determine an expression for the value of the tension in the string appropriate for $n$ loops, $T_{\mathrm{n}}$, in terms of the value of the tension in the string appropriate for one loop, $T_1$. Use this expression to compute the tension in the string appropriate for two loops. Your value should match what you computed in question 5.

**Question 7:** Derive an equation that gives the wave velocity of the transverse waves that make up the standing wave. Write the wave velocity $v$ in terms of the frequency of the vibrating blade $f$, the length of string between the blade and pulley $L$, and the number of loops $n$ in the standing waveform.

**Question 8:** Compute the wave velocity for the case of one loop and two loops, respectively. What happens to the wave velocity when the number of loops is increased from one to two? Record your values in Table 30.2.

| Number of Loops ($n$) | Wave Velocity ($v$) (m/s) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

Table 30.2: Velocity of Traveling Waves

Now suspend enough mass on the hanger on the end of the string at C so that exactly three loops appears in the string when the vibrating blade is turned on. Record the value of the mass in Table 30.3. Using $w = mg$, compute the weight of the mass (don't forget to include the mass of the hanger). We will consider this to be the experimental tension in the string. Record the value in Table 30.3.

**Question 9:** Use Equation 30.4 to determine the wavelength when there are three loops in the string. Show your calculation and record your value in Table 30.3. Sketch a picture of the loop labeling the nodes and the antinodes. Also indicate on your sketch one wavelength.

**Question 10:** Use your derived equation from question 6 to determine the tension in the string appropriate for three loops. Compare this value with the experimental value of the tension and compute the percent experimental error. Show your calculations and record your values in Table 30.3.

**Question 11:** Compute the wave velocity for the case of three loops. What happens to the wave velocity when the number of loops is increased from one to three? Record your value in Table 30.2.

Now suspend enough mass on the hanger on the end of the string at C so that exactly four loops appears in the string when the vibrating blade is turned on. Record the value of the mass in

Table 30.3. Using $w = mg$, compute the weight of the mass (don't forget to include the mass of the hanger). We will consider this to be the experimental tension in the string. Record the value in Table 30.3.

**Question 12:** Use Equation 30.4 to determine the wavelength when there are four loops in the string. Show your calculation and record your value in Table 30.3. Sketch a picture of the loop labeling the nodes and the antinodes. Also indicate on your sketch one wavelength.

**Question 13:** Use your derived equation from question 6 to determine the tension in the string appropriate for four loops. Compare this value with the experimental value of the tension and compute the percent experimental error. Show your calculations and record your values in Table 30.3.

**Question 14:** Compute the wave velocity for the case of four loops. What happens to the wave velocity when the number of loops is increased from one to four? Record your value in Table 30.2.

| Number of Loops ($n$) | Wavelength ($\lambda$) (m) | Mass ($m$) (kg) | Experimental Tension (weight) (N) | Theoretical Tension ($T$) (N) | Percent Experimental Error (%) |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

Table 30.3: Values for Standing Waves on a String

**Question 15:** Suppose that you were given a string whose linear density was twice that of the string used in this experiment, but the length of the string between pulley and vibrating blade were kept constant. How would the tensions required to produce one, two, three, and four loops, respectively, in this new string compare with the tensions required to produce one, two, three, and four loops, respectively, in the string used in this experiment? Support your answer with an equation. How would the wavelengths in the two strings compare in each case (i.e., for one loop, two loops, three loops, and four loops). Use an equation to support your answer. How would the wave velocities compare in each case? Use an equation to support your answer.

**Question 16:** While the string is resonating, energy is being continually "pumped in" by the vibrator. Why doesn't the amplitude of the standing waveform increase continually without limit? Explain carefully.

**Question 17:** When resonance in the string is not exactly achieved, the waveform wiggles about and the amplitude of the waveform is small. Why? In your explanation, use the analogy of a child being pushed on a swing.

# Part XI

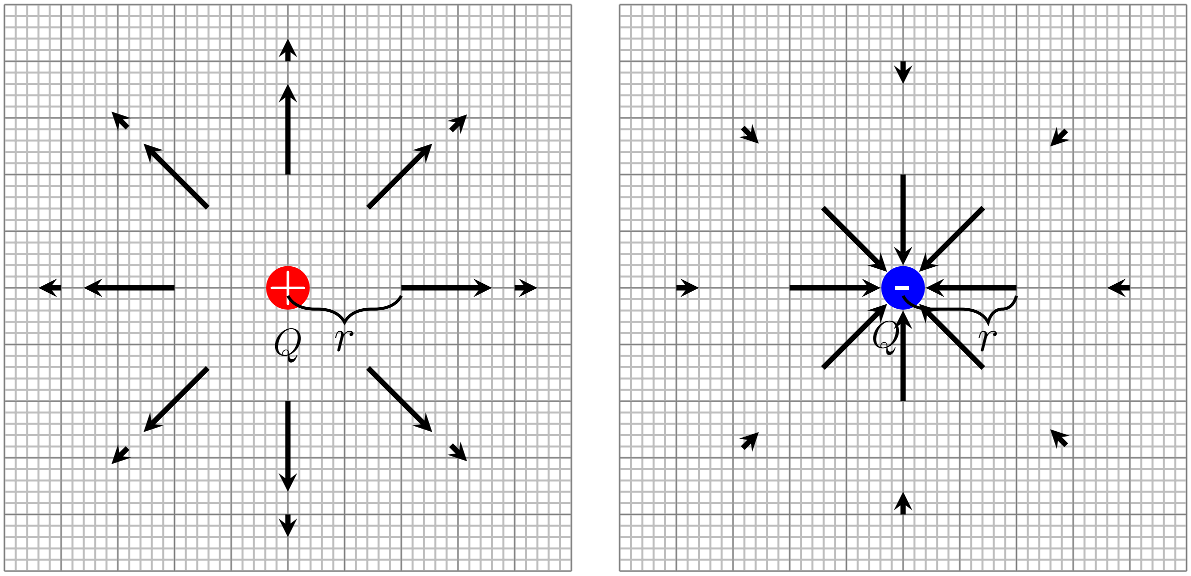
# Basic Electrostatics and Circuit Analysis

# Lab 31

# Basic Exploration of Electric Fields Due to Point Charges

In this lab, we will explore electric fields by using the Charges and Fields simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/en/simulations/charges-and-fields

## 31.1 Theory



(a) Electric Field Due to a Positive Point Charge

(b) Electric Field Due to a Negative Point Charge

Figure 31.1: Electric Field Due to a Point Charge

An electric field $\vec{E}(r)$ is a vector field. For a point charge, the magnitude of the electric field is given as

$$E(r) = \frac{k|Q|}{r^2} \left[\frac{\text{V}}{\text{m}}\right] \text{ or } \left[\frac{\text{N}}{\text{C}}\right] \tag{31.1}$$

where $Q$ is the source charge in Coulombs (C), $k = 9 \times 10^9 \left[\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right]$ is the electrostatic constant, and $r$ is the radial distance in meters (m) from the point charge to the location where you are measuring the electric field. If the charge is positive, the electric field points in the radial direction away from the charge. If the charge is negative, the electric field points in the radial direction towards the charge (see Figures 31.1a and 31.1b).

### 31.1.1 Calculating the Electric Field Due to a Single Point Charge at a Particular Location

As an example of computing the electric field, we will compute the electric field at the point $(4, 0)$ due to a single positive point charge $Q = 5$ nC at $(0, 0)$ (see Figure 31.2).

To calculate the magnitude of the electric field, we use Equation 31.1.

$$E = \frac{k|Q|}{r^2} = \frac{9 \times 10^9 |5 \times 10^{-9}|}{4^2} = 2.81 \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

Since the direction of the electric field is always directed radially outward from a positive point charge, the direction of the field at the location $(4, 0)$ would be in the positive $\hat{\imath}$ direction. Therefore the electric field at position $(4, 0)$ is

$$\vec{E} = \left(2.81\hat{\imath} + 0\hat{\jmath}\right) \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

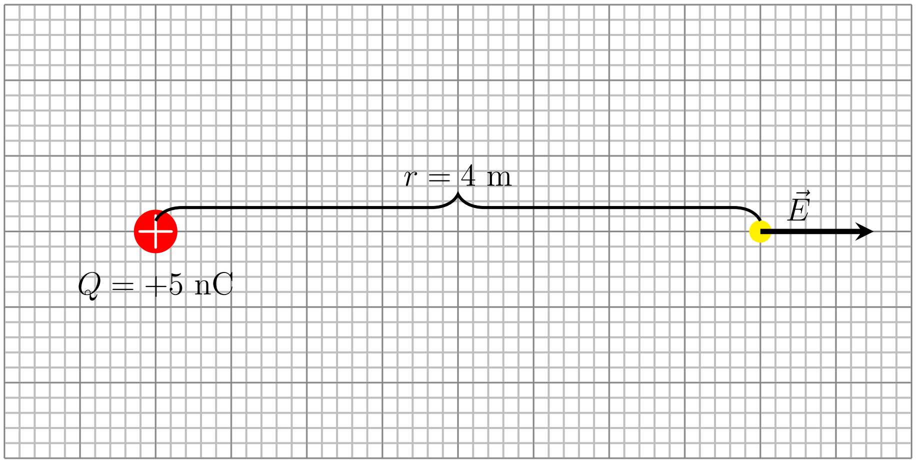


Figure 31.2: Computing the Electric Field Due to a Single Positive Point Charge

### 31.1.2 Calculating the Electric Field Due to Several Point Charges at Different Locations

To determine the electric field due to several point charges, we have to first find the electric field due to each point charge separately, then add them together vectorially.

As an example, consider two point charges $Q_1$ and $Q_2$. $Q_1$ is at the position $(0, 0)$ and has a charge of $Q_1 = +2$ nC. $Q_2$ is at the position $(0, -1)$ and has a charge of $Q_2 = -6$ nC. We will compute the electric field at the point $(4, 0)$ (see Figure 31.3).

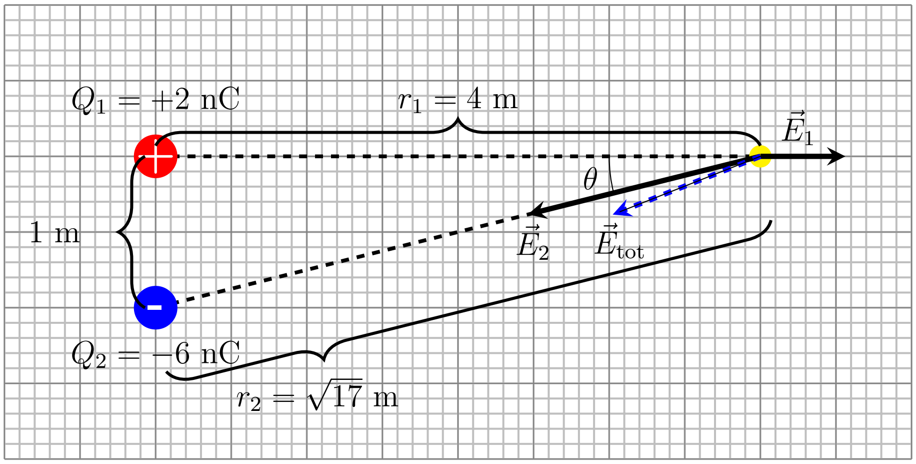


Figure 31.3: Computing the Electric Field Due to a Multiple Point Charges

First, let's find the magnitude of the electric field due to charge 1 and charge 2 separately. Since $Q_1$ is at the position $(0, 0)$ and the location at which we want to measure the electric field is at $(4, 0)$, this means that $r_1 = 4$ m. Knowing that the two charges are 1 m apart, we can use the Pythagorean theorem to find that $r_2 = \sqrt{17}$ m. The magnitude of $\vec{E}_1$ is then

$$E_1 = \frac{k\,|Q_1|}{r_1^2} = \frac{9 \times 10^9\,|2 \times 10^{-9}|}{4^2} = 1.13 \left[\frac{\text{V}}{\text{m}}\right]$$

The magnitude of $\vec{E}_2$ is then

$$E_2 = \frac{k\,|Q_2|}{r_2^2} = \frac{9 \times 10^9\,| - 6 \times 10^{-9}|}{(\sqrt{17})^2} = 3.18 \left[\frac{\text{V}}{\text{m}}\right]$$

The direction of $\vec{E}_1$ is along the positive $x$-axis so written in component form we have

$$\vec{E}_1 = \left(1.13\hat{\imath} + 0\hat{\jmath}\right) \left[\frac{\text{V}}{\text{m}}\right]$$

To determine the direction of $\vec{E}_2$ we need to find the reference angle that $\vec{E}_2$ makes with respect to the $-x$-axis. Using trigonometry gives $\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.0°$. We can then find the components of $\vec{E}_2$ as

$$E_{2x} = -3.18\cos(14.0) = -3.08 \left[\frac{\text{V}}{\text{m}}\right]$$

$$E_{2y} = -3.18\sin(14.0) = -0.77 \left[\frac{\text{V}}{\text{m}}\right]$$

In component form, $\vec{E}_2$ is then given as

$$\vec{E}_2 = \left(-3.08\hat{\imath} - 0.77\hat{\jmath}\right) \left[\frac{\text{V}}{\text{m}}\right]$$

To find the total electric field at $(4,0)$ $\vec{E}_{\text{tot}}$ (shown as the dashed blue vector in Figure 31.3) we just add the vectors together

$$\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 = \left(1.13\hat{\imath} + 0\hat{\jmath}\right) + \left(-3.08\hat{\imath} - 0.77\hat{\jmath}\right) = \left(-1.96\hat{\imath} - 0.77\hat{\jmath}\right)\left[\frac{\text{V}}{\text{m}}\right]$$

The magnitude of $\vec{E}_{\text{tot}}$ is then found by using the Pythagorean theorem

$$E_{\text{tot}} = \sqrt{(-1.96)^2 + (-0.77)^2} = 2.10\left[\frac{\text{V}}{\text{m}}\right]$$

The direction is found by using the arctangent function.

$$\phi = \tan^{-1}\left(\frac{0.77}{1.96}\right) = 21.5^\circ \text{ S of W or } 201.5^\circ$$

## 31.2 Experiment

### 31.2.1 A Single Point Charge

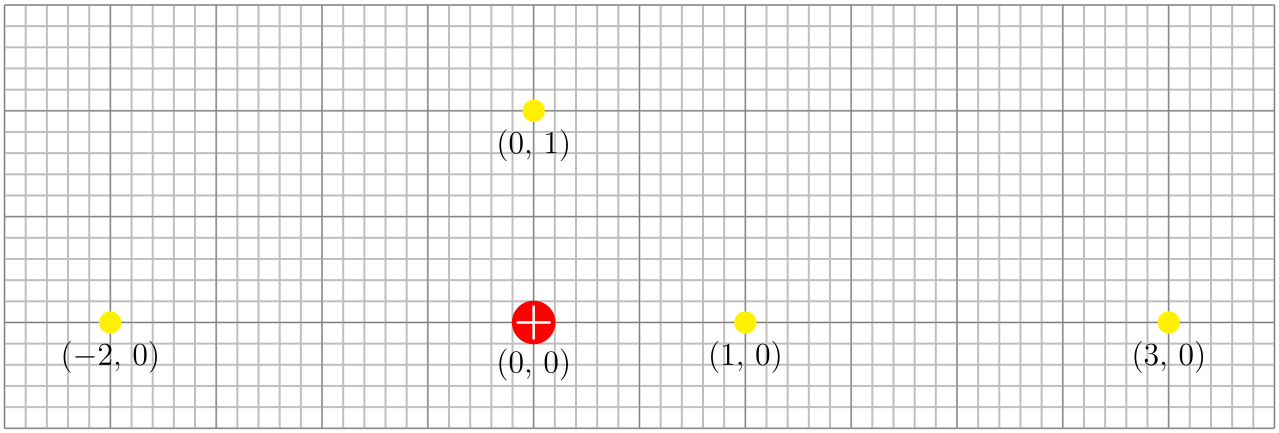


Figure 31.4: Single Point Charge

**Question 1:** A +1 nC charge represented by the red circle is placed at the location $(0,0)$. Calculate the magnitude of the electric field at the following locations (represented by the small yellow circles): $(1,0)$, $(3,0)$, $(0,1)$, and $(-2,0)$, where the ordered pairs represent the $x$- and $y$-coordinates in meters (see Figure 31.4). Fill in the calculated electric field strength in Table 31.1. Show all of your calculations.

**Question 2:** Go to the PhET Charges and Fields Simulation https://phet.colorado.edu/en/simulations/charges-and-fields and set up the configuration of charges and sensors as in Figure 31.4. On the menu of simulation options, you should uncheck the "electric field" but check "values" and "grid" (the simulation screen should look similar to Figure 31.5 except that you will see the values of the electric field at the different locations). Note that two major gridlines in the simulation represent 1 meter. Measure the magnitude of the electric field at each of the sensor locations. Record your measured electric field strength values in Table 31.1.

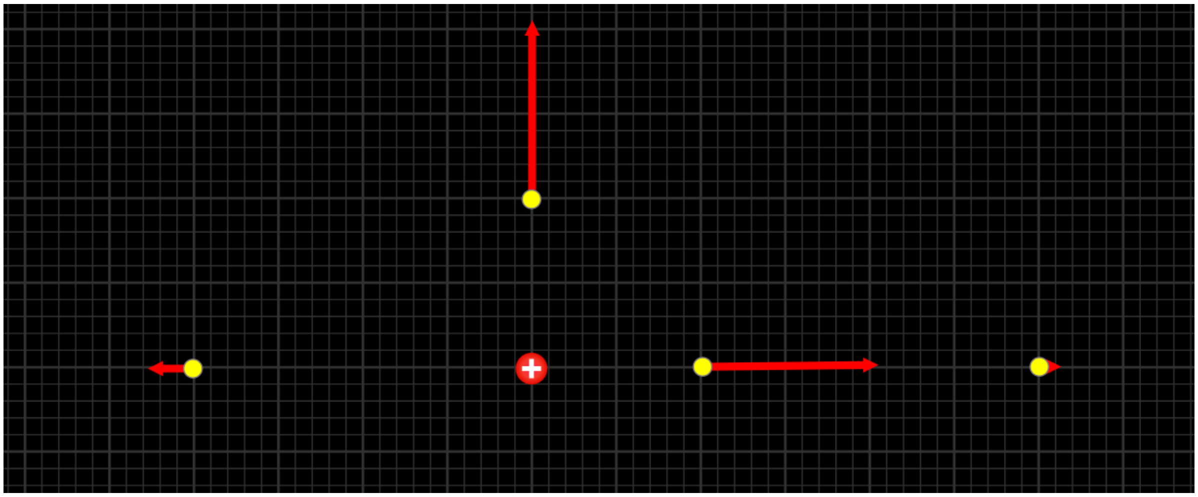


Figure 31.5: PhET Simulation Set-Up for Investigating the Electric Field Strength of a Single Point Charge

| Location $(x, y)$ (m,m) | Calculated Electric Field Strength (V/m) | Measured Electric Field Strength From Simulation (V/m) | Percent Experimental Error (%) |
|---|---|---|---|
| $(1, 0)$ | | | |
| $(3, 0)$ | | | |
| $(0, 1)$ | | | |
| $(-2, 0)$ | | | |

Table 31.1: Electric Field Strengths at Different Locations

**Question 3:** Compare the calculated values to the measured values by computing the percent experimental errors. Consider the calculated values to be the accepted values. Show all of your calculations and record your values in Table 31.1.

**Question 4:** If you double the point charge to +2 nC, predict how the magnitudes of the electric field at the four coordinate locations change. You can simulate a +2 nC charge by putting another +1 nC charge on top of the existing +1 nC charge. Do the results of the simulation match your predictions?

**Question 5:** If the point charge is replaced by -2 nC (replace the two +1 nC charges with two -1 nC charges in the simulation), how do the magnitudes of the electric field at each location compare to the values found in question 4? How do the directions of the electric field at each location compare to what was found in question 4?

### 31.2.2 Superposition of Electric Fields

To calculate the electric field at a particular location when you have a configuration of several point charges, you must find the electric field due to each point charge separately, then add them all together. This is called the principle of supeerposition (to review this refer back to the example in the theory section 31.1.2).

**Question 6:** You are given the following static charge configuration: A +2 nC charge at $(0, 0)$, a -2 nC charge at $(2, 0)$, and a +1 nC charge at $(0, 2)$. Calculate the magnitude and direction of the electric field at $(2, 2)$ (see Figure 31.6). Record your values in Table 31.2. Show all of your calculations.

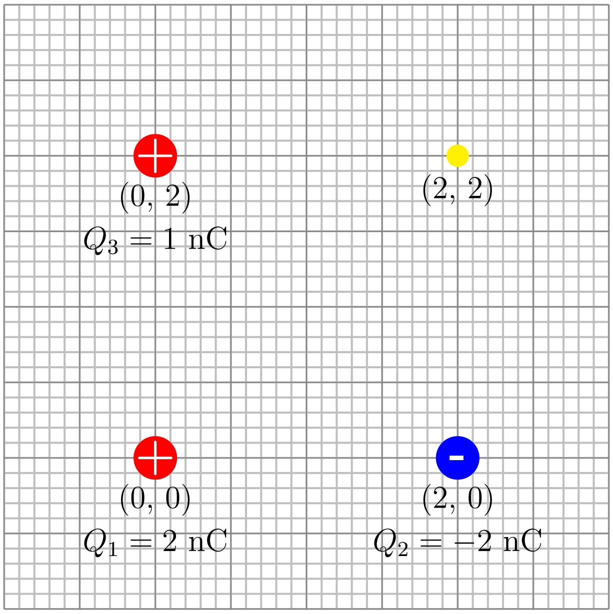


Figure 31.6: Electric Field Due to a Configuration of Several Charges

**Question 7:** Set up the simulation with the same configuration of point charges and measure the magnitude and direction of the electric field at $(2, 2)$ (see Figure 31.7). Compare the calculated values from question 6 and the measured values from the simulation by computing the percent experimental errors. Record your values in Table 31.2. Show all of your calculations.

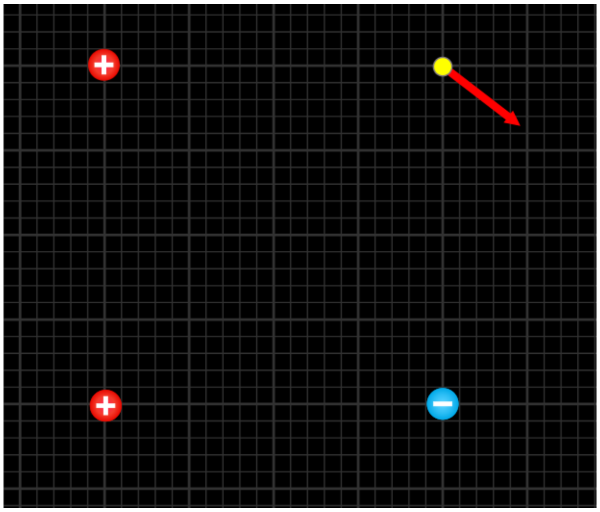


Figure 31.7: PhET Simulation Set-Up for Investigating the Electric Field Strength of Several Point Charges (Configuration 1)

| | Magnitude of Electric Field (V/m) | Direction of Electric Field (°) |
|---|---|---|
| Calculated Values | | |
| Measured Values | | |
| | | |
| Percent Experimental Error (%) | | |

Table 31.2: Magnitude and Direction of Electric Field (Configuration 1)

**Question 8:** You are given the following static charge configuration: A +2 nC charge at $(0, 0)$, a -2 nC charge at $(2, 0)$, and a -2 nC charge at $(1, 2)$. Calculate the magnitude and direction of the electric field at $(2, 2)$ (see Figure 31.8). Record your values in Table 31.3. Show all of your calculations.

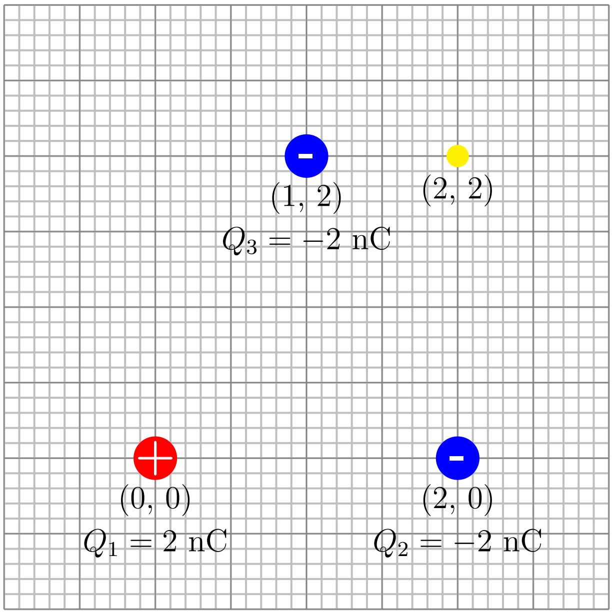


Figure 31.8: Electric Field Due to a Configuration of Several Charges

**Question 9:** Set up the simulation with the same configuration of point charges and measure the magnitude and direction of the electric field at $(2, 2)$ (see Figure 31.9). Compare the calculated values from question 8 and the measured values from the simulation by computing the percent experimental errors. Record your values in Table 31.3. Show all of your calculations.

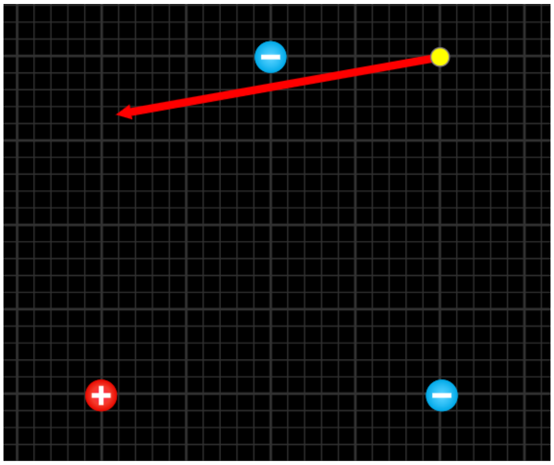


Figure 31.9: PhET Simulation Set-Up for Investigating the Electric Field Strength of Several Point Charges (Configuration 2)

| | Magnitude of Electric Field (V/m) | Direction of Electric Field (°) |
|---|---|---|
| Calculated Values | | |
| Measured Values | | |
| | | |
| Percent Experimental Error (%) | | |

Table 31.3: Magnitude and Direction of Electric Field (Configuration 2)

**Question 10:** Given the configuration of point charges shown below (+3 nC at $(0, 0)$ and +1 nC at $(4, 0)$), there must be a location where the electric field is 0 V/m (see Figure 31.10). Calculate that location and record your value in Table 31.4. Show all your calculations and verify your answer by checking the result with the simulation. Hint: Use symmetry to find the $y$-coordinate of the location where the electric field is 0 V/m. You will need to solve a quadratic equation to find the $x$-coordinate where the electric field is 0 V/m.

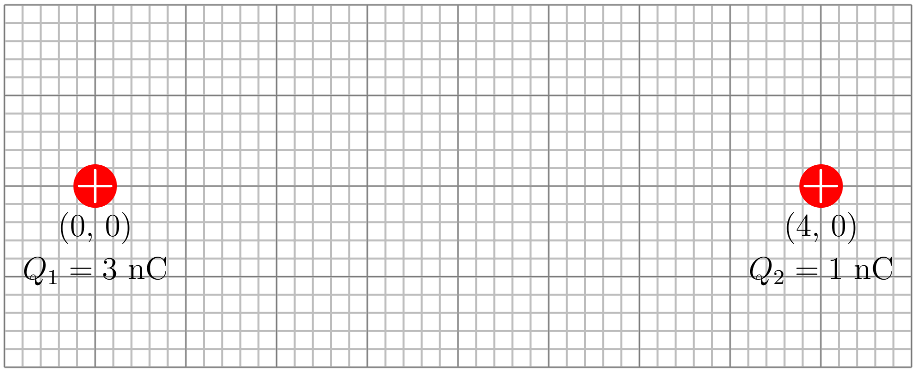


Figure 31.10: Zero Electric Field Due to a Configuration of Two Positive Charges

| | |
|---|---|
| Coordinates Where the Electric Field is 0 V/m | |

Table 31.4: Location Where the Electric Field is Zero

# Lab 32

# Basic Circuit Analysis

The amount of charge per unit time flowing in an electric circuit is defined as the current. Current is measured in amperes. The potential difference or voltage across the conductor is necessary to cause the current. The property of the circuit that opposes the current is called the resistance and is measured in ohms. It was found experimentally that the amount of current in a conductor is directly proportional to the potential difference applied across the conductor and inversely proportional to the resistance of the conductor. This relation called Ohm's law holds for certain conductors if they are held at constant temperatures. Ohm's law may be expressed in the form:

$$I = \frac{V}{R} \tag{32.1}$$

where $I$ is the current measured in amps (A), $V$ is the voltage measured in volts (V), and $R$ is the resistance measured in ohms ($\Omega$). Ohm's law is basic in studying the properties of electric circuits. In this experiment, you will measure values of current, voltage, and resistance in both series and parallel circuits, showing that Ohm's law is obeyed.

## 32.1 Theory

### 32.1.1 Series Circuits

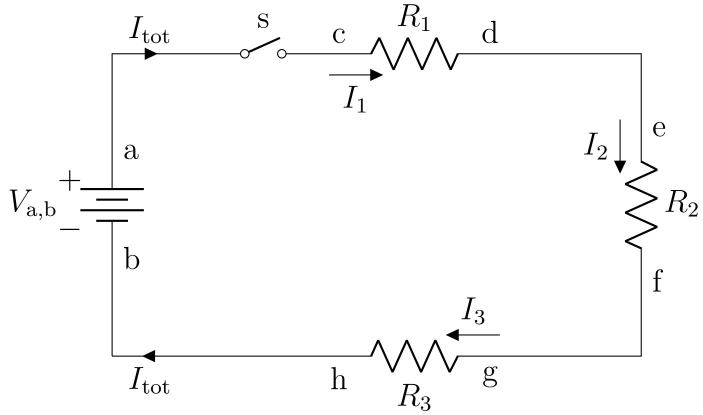


Figure 32.1: A Series Circuit

Consider the circuit shown in Figure 32.1. It consists of a source of potential difference, $V_{\mathrm{a,b}}$ (a battery in this case), connected by a conductor of negligible resistance (a copper wire) to three circuit elements of resistance $R_1$, $R_2$, and $R_3$. When the switch s is closed, there will be a current in the circuit. Note that the total charge must flow through all three circuit elements ($R_1$, $R_2$, and $R_3$). Therefore, the total current is equal to the current passing through each resistor ($I_{\mathrm{tot}} = I_1 = I_2 = I_3$). Such circuit elements are said to be connected in series. For the circuit shown in Figure 32.1, according to Ohm's law:

$$I_{\mathrm{tot}} = \frac{V_{\mathrm{tot}}}{R_{\mathrm{tot}}} \tag{32.2}$$

where $I_{\mathrm{tot}}$ is the total current, $V_{\mathrm{tot}}$ is the total voltage ($V_{\mathrm{tot}} = V_{\mathrm{a,b}}$ in this case) and $R_{\mathrm{tot}}$ is the total resistance. When resistors $R_1, R_2, R_3, \ldots R_n$ are connected in series the total resistance $R_{\mathrm{tot}}$ is given by:

$$R_{\mathrm{tot}} = R_1 + R_2 + \cdots + R_n \tag{32.3}$$

The total potential difference $V_{\mathrm{a,b}}$ across a series circuit is equal to the sum of the potential differences ($IR$ drops) across each resistor. Hence, referring to Figure 32.1:

$$V_{\mathrm{a,b}} = V_{\mathrm{c,d}} + V_{\mathrm{e,f}} + V_{\mathrm{g,h}} \tag{32.4}$$

or

$$V_{\mathrm{a,b}} = I_{\mathrm{tot}}R_1 + I_{\mathrm{tot}}R_2 + I_{\mathrm{tot}}R_3 \tag{32.5}$$

### 32.1.2 Parallel Circuits

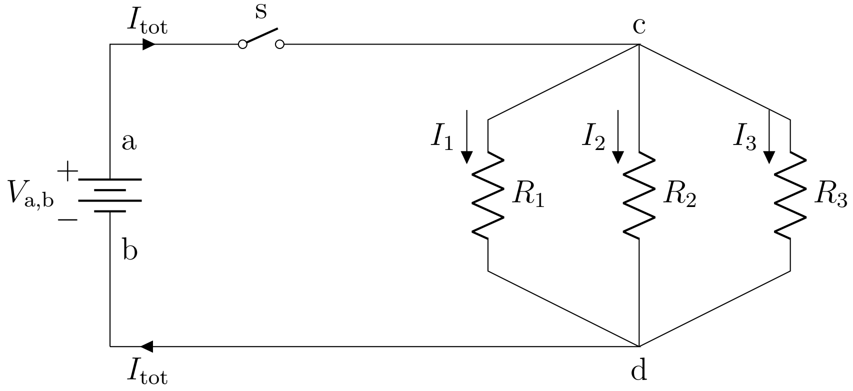


Figure 32.2: A Parallel Circuit

Consider the circuit shown in Figure 32.2. In the parallel circuit, the three resistors $R_1$, $R_2$, and $R_3$ are connected in such a manner that the flow of charge in the circuit will divide at point c with various portions of the charge flowing through $R_1$, $R_2$, and $R_3$. Note that in this case the total current does not pass through each resistor as in the case of the series circuit. In a parallel circuit the voltage across each parallel branch is the same. Hence, in our example of a parallel circuit, the voltage across each resistor is $V_{\mathrm{c,d}}$, which obviously is equal to the voltage drop across the battery $V_{\mathrm{a,b}}$. The total resistance $R_{\mathrm{tot}}$ of $n$ resistors, $R_1, R_2, \ldots, R_n$, connected in parallel is given by:

$$\frac{1}{R_{\mathrm{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \tag{32.6}$$

The total current $I_{\text{tot}}$ in a parallel circuit is calculated from Ohm's law.

$$I_{\text{tot}} = \frac{V_{\text{a,b}}}{R_{\text{tot}}} \tag{32.7}$$

Also, the total current $I_{\text{tot}}$ is the sum of the currents in each parallel branch. From Figure 32.2

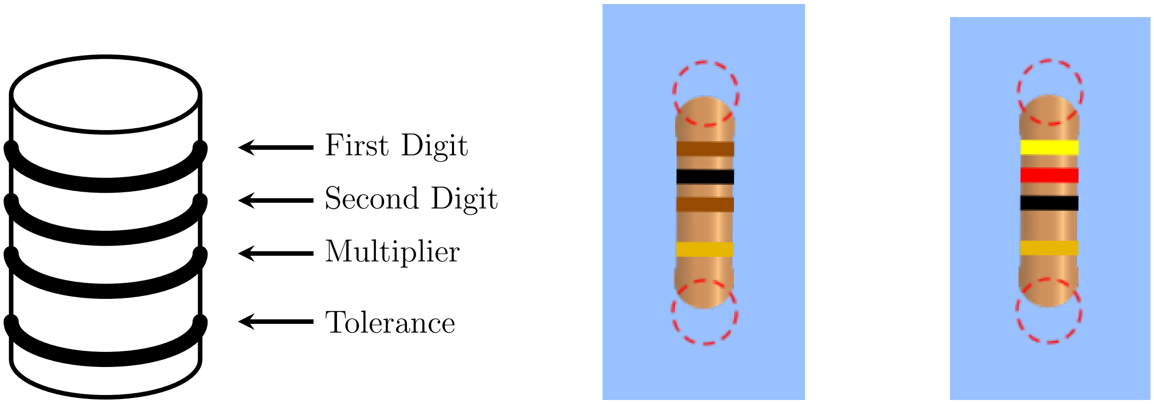
$$I_{\text{tot}} = I_1 + I_2 + I_3 \tag{32.8}$$

### 32.1.3 Reading Values of Resistors

The resistance of each resistor may be determined from the coded colorbands on them. See Table 32.1 and Figure 32.3. For example, the resistor in Figure 32.3b shows bands of colors brown, black, brown, and gold. The first brown band represents the first digit which is a 1. The second black band represents the second digit which is a 0. The third brown band represents the multiplier which is a $10^1 = 10$. Therefore, this resistor has a resistance of $10 \times 10 = 100\,\Omega$. The fourth gold band indicates that the tolerance is $\pm 5\,\%$. In Figure 32.3c, the bands are colored yellow, red, black, and gold. In this case, the resistor has a resistance of $42 \times 1 = 42\,\Omega$.

| Color | Number | Multiplier | Tolerance |
|---|---|---|---|
| Black | 0 | $10^0 = 1$ | |
| Brown | 1 | $10^1 = 10$ | |
| Red | 2 | $10^2 = 100$ | |
| Orange | 3 | $10^3 = 1000$ | |
| Yellow | 4 | $10^4 = 10000$ | |
| Green | 5 | $10^5 = 100000$ | |
| Blue | 6 | $10^6 = 1000000$ | |
| Violet | 7 | $10^7 = 10000000$ | |
| Gray | 8 | $10^8 = 100000000$ | |
| White | 9 | $10^9 = 1000000000$ | |
| Gold | | $10^{-1}$ | $\pm 5\,\%$ |
| Silver | | $10^{-2}$ | $\pm 10\,\%$ |
| No Color | | | $\pm 20\,\%$ |

Table 32.1: Resistor Color Code Values

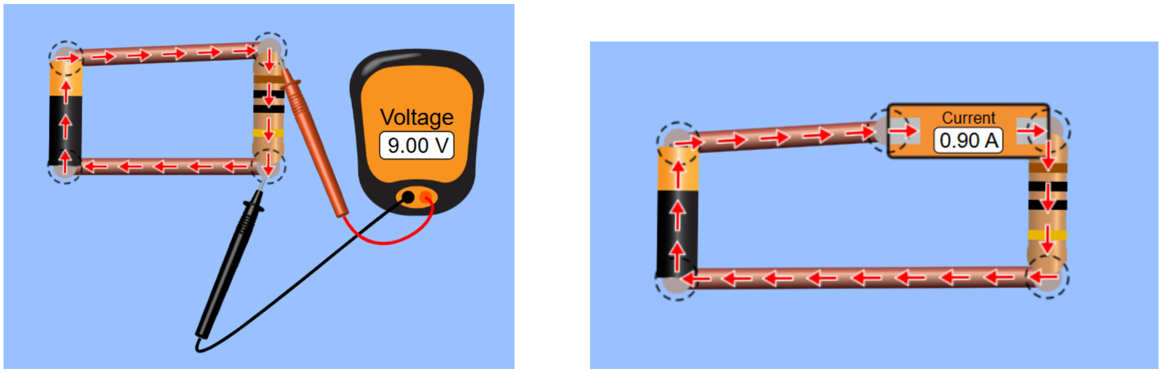


(a) Resistor Color Code (b) 100 Ω Resistor (c) 42 Ω Resistor

Figure 32.3: Resistance Value of Resistors Based on Color Code

## 32.2 Experiment

In this lab we are going to use a PhET circuit simulation and we will also construct physical circuits. In the simulation, circuit measurements will be made with a voltage meter and a current meter. Measurements within the physical circuits will be performed with a volt-ohm-milliammeter (VOM). Remember, voltage readings are always taken in parallel (across the resistor or battery, etc.) and current readings are taken in series (a wire is removed and the ammeter becomes a part of the circuit) (see Figure 32.4).



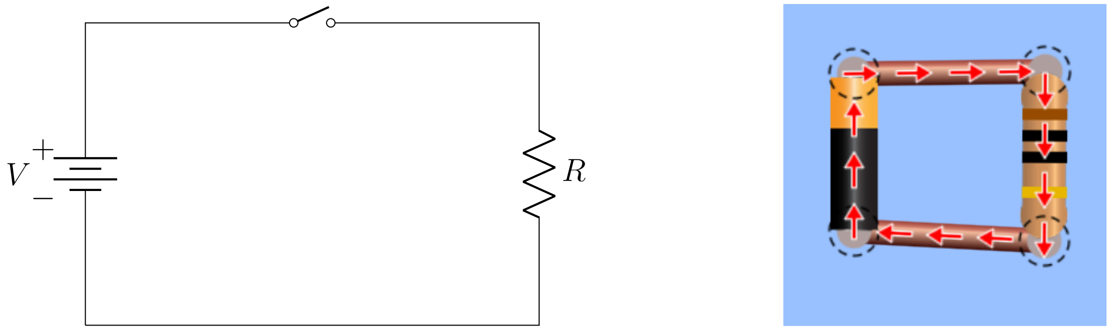
(a) Measuring Voltage (b) Measuring Current

Figure 32.4: Measuring Voltage and Current in a Simple Circuit

### 32.2.1 Simulation Experiment Part 1

The first several parts of this lab involve using the Circuit Construction Kit: DC simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/sims/html/circuit-construction-kit-dc/latest/circuit-construction-kit-dc_all.html

Using the simulation, construct the simple circuit shown in Figure 32.5a. Use a 9 V battery as the voltage source and a 10 ohm resistor (you don't need to include the switch). The completed circuit should look similar to that in Figure 32.5b. Measure the voltage across the resistor and measure the current in the circuit. Set-up the simple circuit again using a 20 ohm resistor and again measure the voltage and current.



(a) Simple Circuit (b) Simple Circuit in Simulation

Figure 32.5: Simple Circuit

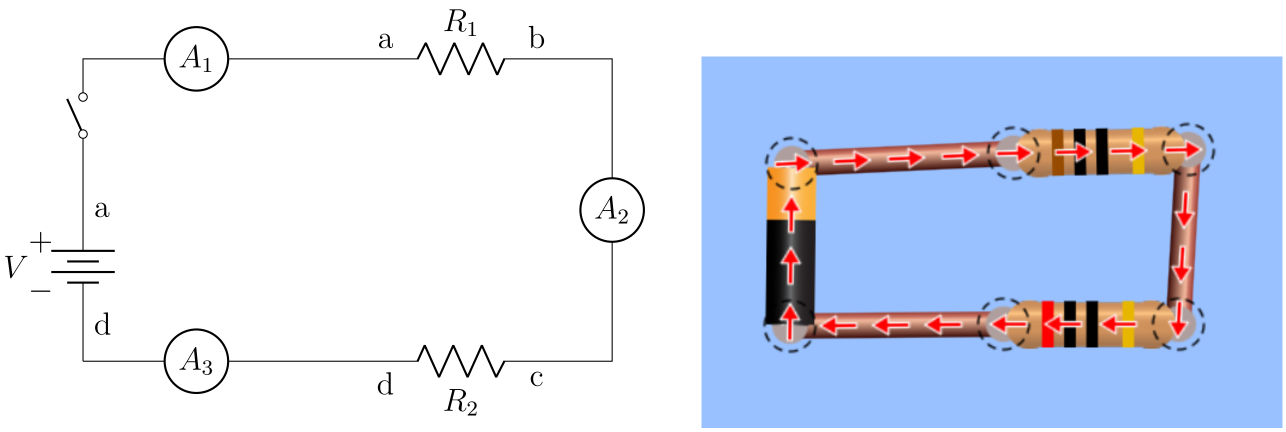
**Question 1:** Calculate the current through the circuit for both circuits using Ohm's law. Compare the measured currents from the simulation to the calculated currents by computing the percent experimental error. Show all of your calculations and record your values in Table 32.2.

| Resistance (Ω) | Voltage Across the Resistor (V) | Measured Current (A) | Calculated Current (A) | Percent Experimental Error (%) |
|---|---|---|---|---|
| $R_1 = 10\,\Omega$ | | | | |
| $R_2 = 20\,\Omega$ | | | | |

Table 32.2: Voltage and Current in a Simple Circuit

### 32.2.2 Simulation Experiment Part 2

Using the simulation, construct the series circuit shown in Figure 32.6a. Use a 9 V battery as the voltage source and two resistors ($R_1 = 10\,\Omega$ and $R_2 = 20\,\Omega$) (you don't need to include the switch). The completed circuit should look similar to that in Figure 32.6b. Measure the voltage across each resistor and across the battery ($\Delta V_{\mathrm{a,b}}$, $\Delta V_{\mathrm{c,d}}$, and $\Delta V_{\mathrm{a,d}}$). Note that $\Delta V_{\mathrm{tot}} = \Delta V_{\mathrm{a,d}}$. Measure the currents at the locations indicated by $A_1$, $A_2$, and $A_3$ in Figure 32.6a. The $A$'s represent the location of the ammeter, therefore, you will be determining the current $I_1$, $I_2$, and $I_3$ at those locations. To measure current, remember that the ammeter must be placed in series with the circuit. Record all of your values in Table 32.3.



(a) Series Circuit (b) Series Circuit in Simulation

Figure 32.6: Series Circuit

| | | | |
|---|---|---|---|
| Current (A) | $I_1 =$ | $I_2 =$ | $I_3 =$ |
| Voltage (V) | $\Delta V_{\text{a,b}} =$ | $\Delta V_{\text{c,d}} =$ | $\Delta V_{\text{a,d}} =$ |
| Resistance ($\Omega$) | $R_1 = 10$ | $R_2 = 20$ | $R_{\text{tot}} =$ |

Table 32.3: Voltage and Current in a Series Circuit

**Question 2:** Calculate $R_{\text{tot}} = R_1 + R_2$ to simplify the series circuit into a simple circuit with an equivalent resistance value of $R_{\text{tot}}$. Record your values in Table 32.3. Use Ohm's law to compute $I_{\text{tot}}$ ($I_{\text{tot}} = \frac{\Delta V_{\text{tot}}}{R_{\text{tot}}}$). You can think of $I_{\text{tot}}$ as the current that travels to and from the battery. Record your value in Table 32.4. Show all of your calculations.

| | |
|---|---|
| Calculated Current ($I_{\text{tot}}$) (A) | |

Table 32.4: Calculated Current Through a Series Circuit

**Question 3:** For a series circuit, several relationships between the voltage, current, and resistance must be true. Confirm that the following relations are true by showing the equations with your measured values:

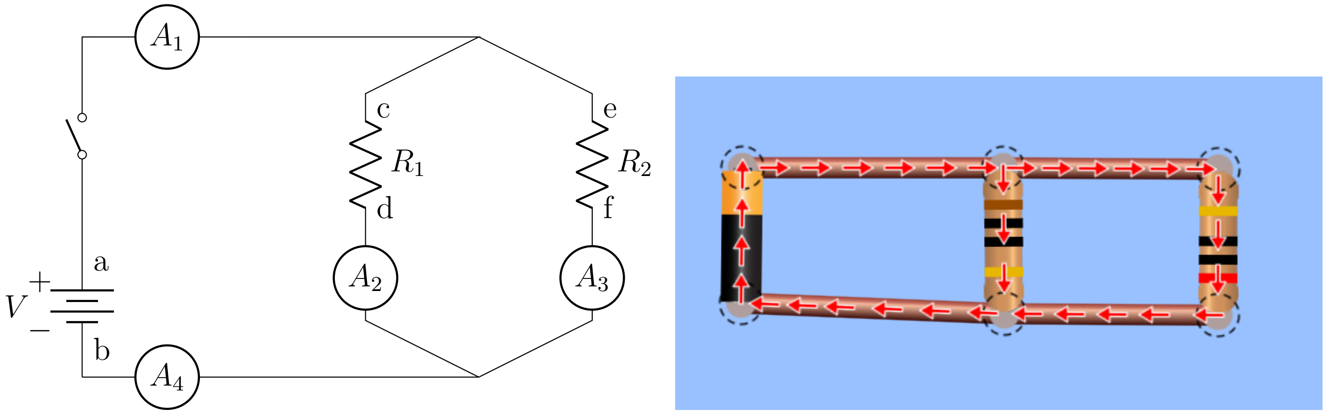
A.) Is $I_1 = I_2 = I_3 = I_{\text{tot}}$?

B.) Does $\Delta V_{\text{a,d}} = \Delta V_{\text{a,b}} + \Delta V_{\text{c,d}}$?

C.) Does $\Delta V_{\text{a,d}} = I_{\text{tot}} R_1 + I_{\text{tot}} R_2$ ?

### 32.2.3 Simulation Experiment Part 3

Using the simulation, construct the parallel circuit shown in Figure 32.7a. Use a 9 V battery as the voltage source and two resistors ($R_1 = 10\,\Omega$ and $R_2 = 20\,\Omega$) (you don't need to include the switch). The completed circuit should look similar to that in Figure 32.7b. Measure the voltage across each resistor and across the battery ($\Delta V_{\mathrm{a,b}}$, $\Delta V_{\mathrm{c,d}}$, and $\Delta V_{\mathrm{e,f}}$). Note that $\Delta V_{\mathrm{tot}} = \Delta V_{\mathrm{a,b}}$. Measure the currents at the locations indicated by $A_1$, $A_2$, $A_3$, and $A_4$ in Figure 32.6a. The $A$'s represent the location of the ammeter, therefore, you will be determining the current $I_1$, $I_2$, $I_3$, and $I_4$ at those locations. Record all of your values in Table 32.5.



(a) Parallel Circuit (b) Parallel Circuit in Simulation

Figure 32.7: Parallel Circuit

| | | | | | |
|---|---|---|---|---|---|
| Current (A) | $I_1 =$ | $I_2 =$ | $I_3 =$ | $I_4 =$ | $I_{\mathrm{tot}} =$ |
| Voltage (V) | $\Delta V_{\mathrm{a,b}} =$ | $\Delta V_{\mathrm{c,d}} =$ | $\Delta V_{\mathrm{e,f}} =$ | | $\Delta V_{\mathrm{tot}} =$ |
| Resistance ($\Omega$) | $R_1 = 10$ | $R_2 = 20$ | | | $R_{\mathrm{tot}} =$ |

Table 32.5: Voltage and Current in a Parallel Circuit

**Question 4:** Compute the total current $I_{\mathrm{tot}}$ in the circuit using Ohm's law $I_{\mathrm{tot}} = \frac{\Delta V_{\mathrm{tot}}}{R_{\mathrm{tot}}}$. You will first need to simplify the parallel circuit into a simple circuit with an equivalent resistance value of $R_{\mathrm{tot}}$. $R_{\mathrm{tot}}$ may be calculated using

$$\frac{1}{R_{\mathrm{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or equivalently } R_{\mathrm{tot}} = \frac{R_1 R_2}{R_1 + R_2}$$

Show all of your calculations and record your values in Table 32.5.

**Question 5:** Compare the computed value of the total current to the measured value of the total current (i.e. compare the computed $I_{\mathrm{tot}}$ to the measured $I_4$) by computing the percent experimental error. Note that we could use either the measured $I_4$ or the measured $I_1$ as our measured value of the total current as they are the same. Also note that we will assume the computed $I_{\mathrm{tot}}$ is our accepted value when computing the percent experimental error. Show all of your calculations and record your results in Table 32.6.

**Question 6:** Calculate the total current in the circuit as the sum of the measured values $I_2$ and $I_3$. Compare this value to $I_4$ by computing the percent experimental error. Assume the sum $I_2 + I_3$ is our accepted value when computing the percent experimental error. Show all of your calculations and record your results in Table 32.6.

| $I_{\text{tot}}$ Computed (A) (accepted) | $I_4$ Measured (A) (experimental) | Percent Experimental Error (%) |
|---|---|---|
| | | |
| | | |
| $I_2 + I_3$ Measured (A) (accepted) | $I_4$ Measured (A) (experimental) | Percent Experimental Error (%) |
| | | |

Table 32.6: Measured and Computed Total Current

### 32.2.4 Simulation Experiment Part 4

Using the simulation, construct the circuit shown in Figure 32.8.

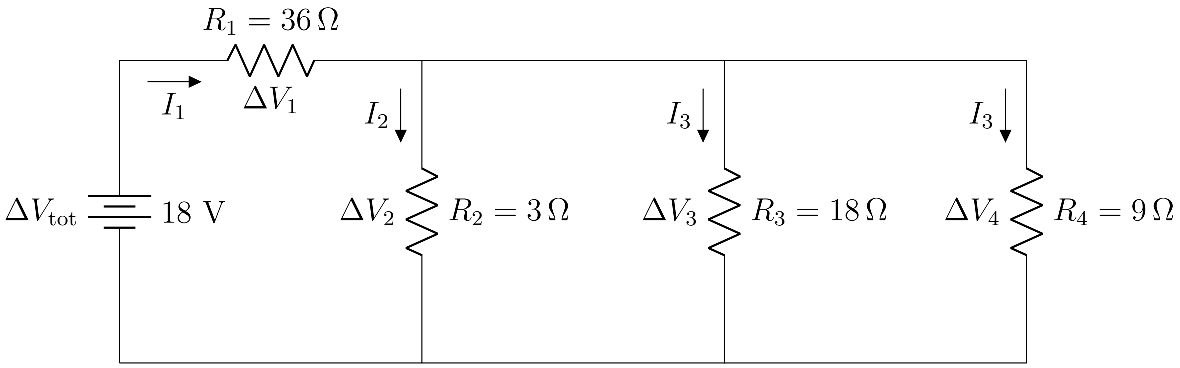


Figure 32.8: Circuit with Resistors in Series and Parallel

| | Calculated Values | Measured Values |
|---|---|---|
| $I_1$ | | |
| $I_2$ | | |
| $I_3$ | | |
| $I_4$ | | |
| $\Delta V_1$ | | |
| $\Delta V_2$ | | |
| $\Delta V_3$ | | |
| $\Delta V_4$ | | |

Table 32.7: Current and Voltage Values

**Question 7:** Calculate $I_1$, $I_2$, $I_3$, $I_4$, $\Delta V_1$, $\Delta V_2$, $\Delta V_3$, and $\Delta V_4$ using the techniques you learned in physics lecture. Now measure $I_1$, $I_2$, $I_3$, $I_4$, $\Delta V_1$, $\Delta V_2$, $\Delta V_3$, and $\Delta V_4$ using the simulation. Show all of your calculations and record all of your values in Table 32.7. Were there differences between the calculated values and measured values? They should be identical within rounding error.

### 32.2.5 Physical Circuit Experiment Part 5

Using the wooden circuit board, construct the series circuit shown in Figure 32.6a. Use a 1.5 V battery as the voltage source and two resistors $R_1 = 10\,\Omega$ (blue) and $R_2 = 20\,\Omega$ (red) (you don't need to include the switch). The completed circuit should look similar to that in Figure 32.9. Measure the voltage across each resistor and across the battery ($\Delta V_{\mathrm{a,b}}$, $\Delta V_{\mathrm{c,d}}$, and $\Delta V_{\mathrm{a,d}}$). Measure the currents at the locations indicated by $A_1$, $A_2$, and $A_3$ in Figure 32.6a. The $A$'s represent the location of the ammeter, therefore, you will be determining the current $I_1$, $I_2$, and $I_3$ at those locations. To measure voltage, remember that the VOM multimeter must be set up to measure voltage then placed in parallel with the resistor or voltage source. To measure current, remember that the VOM multimeter must be set up to measure current and placed in series with the circuit (see Figures 32.10 and 32.11). Record all of your values in Table 32.8.

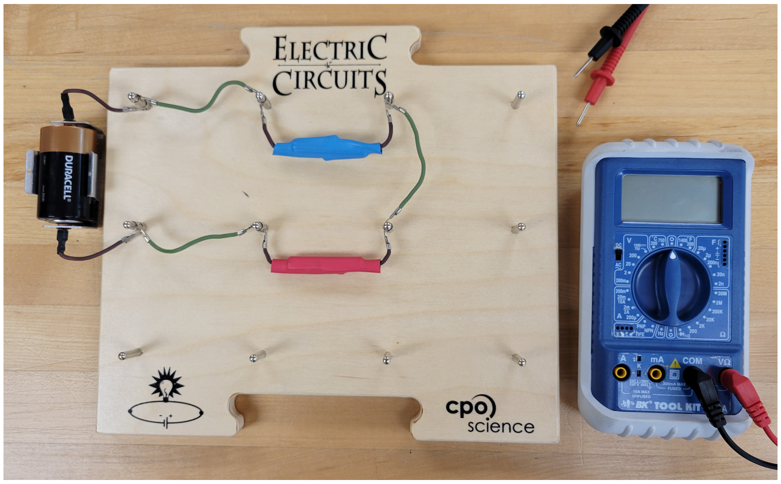


Figure 32.9: Physical Series Circuit

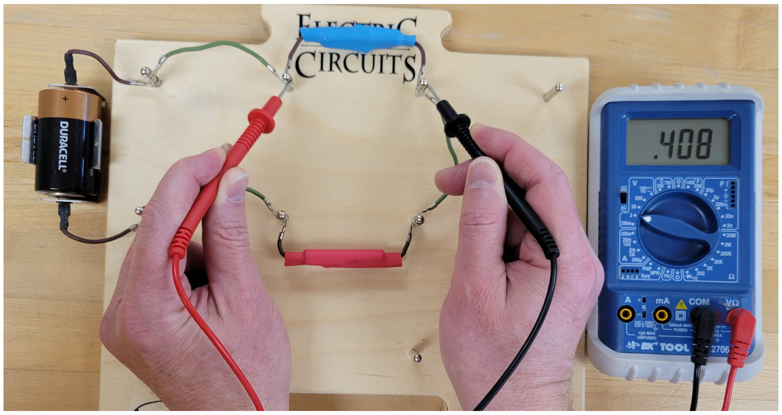


Figure 32.10: Using a Multimeter to Measure Voltage

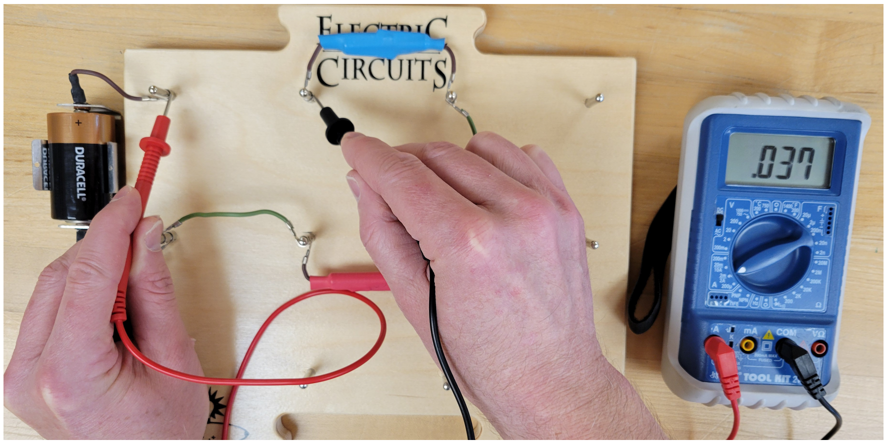


Figure 32.11: Using a Multimeter to Measure Current

| Physical Series Circuit | | | |
|---|---|---|---|
| Current (A) | $I_1 =$ | $I_2 =$ | $I_3 =$ |
| Voltage (V) | $\Delta V_{\mathrm{a,b}} =$ | $\Delta V_{\mathrm{c,d}} =$ | $\Delta V_{\mathrm{a,d}} =$ |
| Resistance ($\Omega$) | $R_1 = 10$ | $R_2 = 20$ | |

Table 32.8: Voltage and Current in a Physical Series Circuit

Construct the exact same series circuit using the PhET simulation and measure the same voltage drops and currents. Record your values in Table 32.9.

| Simulated Series Circuit | | | |
|---|---|---|---|
| Current (A) | $I_1 =$ | $I_2 =$ | $I_3 =$ |
| Voltage (V) | $\Delta V_{\mathrm{a,b}} =$ | $\Delta V_{\mathrm{c,d}} =$ | $\Delta V_{\mathrm{a,d}} =$ |
| Resistance ($\Omega$) | $R_1 = 10$ | $R_2 = 20$ | |

Table 32.9: Voltage and Current in a Simulated Series Circuit

**Question 8:** Compare the values measured in the physical series circuit to those measured in the simulated series circuit. Were there differences? If yes, what are some potential reasons for the differences.

### 32.2.6 Physical Circuit Experiment Part 6

Using the wooden circuit board, construct the parallel circuit shown in Figure 32.7a. Use a 1.5 V battery as the voltage source and two resistors $R_1 = 10\,\Omega$ (blue) and $R_2 = 20\,\Omega$ (red) (you don't need to include the switch). The completed circuit should look similar to that in Figure 32.12. Measure the voltage across each resistor and across the battery ($\Delta V_{\mathrm{a,b}}$, $\Delta V_{\mathrm{c,d}}$, and $\Delta V_{\mathrm{e,f}}$). Measure the currents at the locations indicated by $A_1$, $A_2$, $A_3$, and $A_4$ in Figure 32.7a. The $A$'s represent the location of the ammeter, therefore, you will be determining the current $I_1$, $I_2$, $I_3$, and $I_4$ at those locations. Record all of your values in Table 32.10.

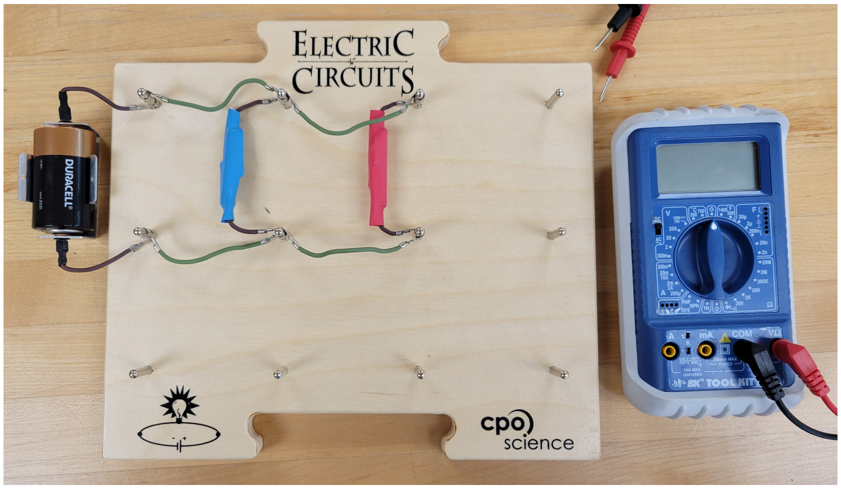


Figure 32.12: Physical Parallel Circuit

| Physical Parallel Circuit | | | | |
|---|---|---|---|---|
| Current (A) | $I_1 =$ | $I_2 =$ | $I_3 =$ | $I_4 =$ |
| Voltage (V) | $\Delta V_{\mathrm{a,b}} =$ | $\Delta V_{\mathrm{c,d}} =$ | $\Delta V_{\mathrm{e,f}} =$ | |
| Resistance (Ω) | $R_1 = 10$ | $R_2 = 20$ | | |

Table 32.10: Voltage and Current in a Physical Parallel Circuit

Construct the exact same parallel circuit using the PhET simulation and measure the same voltage drops and currents. Record your values in Table 32.11.

| Simulated Parallel Circuit | | | | |
|---|---|---|---|---|
| Current (A) | $I_1 =$ | $I_2 =$ | $I_3 =$ | $I_4 =$ |
| Voltage (V) | $\Delta V_{\mathrm{a,b}} =$ | $\Delta V_{\mathrm{c,d}} =$ | $\Delta V_{\mathrm{e,f}} =$ | |
| Resistance (Ω) | $R_1 = 10$ | $R_2 = 20$ | | |

Table 32.11: Voltage and Current in a Simulated Parallel Circuit

**Question 9:** Compare the values measured in the physical parallel circuit to those measured in the simulated parallel circuit. Were there differences? If yes, what are some potential reasons for the differences.

# Part XII

# Geometric Optics

# Lab 33

# Geometric Optics: Reflection and Refraction of Light

The solar eclipse, the reflection of a face from the surface of a placid lake, the rainbow that we admire after a summer thunderstorm, the glitter of a precious gem, the colors in an oil slick, the blue color of the sky, and many other common visual experiences that we have all had at one time or another can be explained by three simple empirical laws: The law of rectilinear propagation, the law of reflection, and the law of refraction; these three laws are the basis of geometrical optics. Geometrical optics is concerned with the description of the motion of light, i.e., the kinematics of light. Geometrical optics, therefore, does not try to answer the question, what is light? It is concerned instead with the behavior of light. Models for the nature of light will be discussed in lecture.

In this experiment, you will be concerned primarily with the study of the law of reflection and the law of refraction.

## 33.1 Theory

### 33.1.1 The Law of the Rectilinear Propagation of Light

The law of rectilinear propagation states that in a homogeneous medium, light travels in straight lines. This law is valid when the dimensions of the apertures through which the light passes and the obstacles it encounters are large compared to the wavelength of the light. A rather dramatic consequence of this law is made evident in the phenomenon of the solar eclipse (see Figure 33.1). The law of rectilinear propagation suggests that we represent a beam of light by a single line, called a ray, parallel to the direction of propagation of the light.

### 33.1.2 The Law of the Reflection

The law of rectilinear propagation describes the motion of light as it travels from one point to another in a homogeneous medium. However, further study is needed to formulate the behavior of light when it encounters various materials. As you are most probably aware, light, upon encountering a material, does not continue in the same straight line but bounces off the surface of the material into a new straight line that changes when the orientation of the surface is changed. The question is, what is the relationship between the angle at which the light is incident to the surface and the

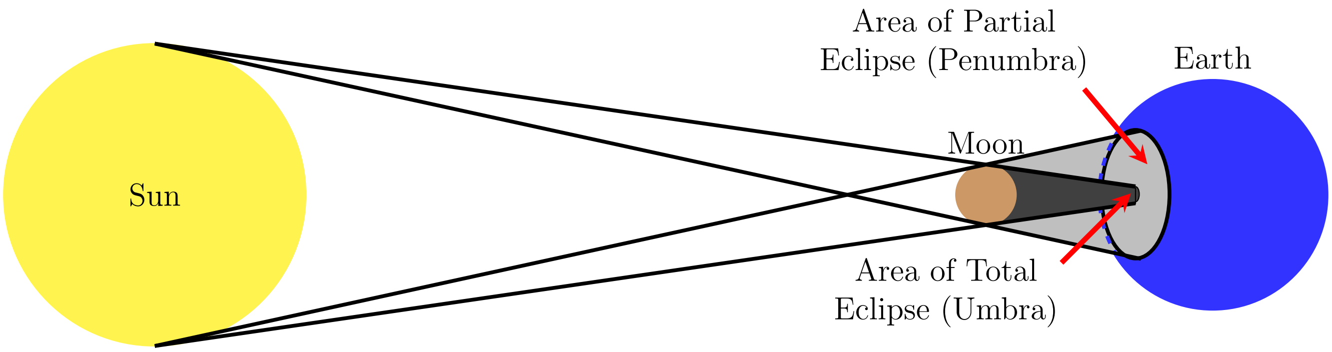


Figure 33.1: Diagram of a Solar Eclipse

angle at which the light is reflected from the surface? Suppose that a light ray is incident to an arbitrarily curved surface at a point P (see Figure 33.2).

We define the angle of incidence $\theta_1$ as the angle that the incident ray to P makes with the normal to the surface at the point P. The angle of reflection $\theta_1'$ is then defined as the angle that the reflected ray makes with the normal to the surface at point P. Thus, the law of reflection is formulated rather simply as

$$\theta_1 = \theta_1' \tag{33.1}$$

$$\text{Angle of Incidence} = \text{Angle of Reflection}$$

This law was known to the Greek mathematician, Euclid, around 300 B.C.

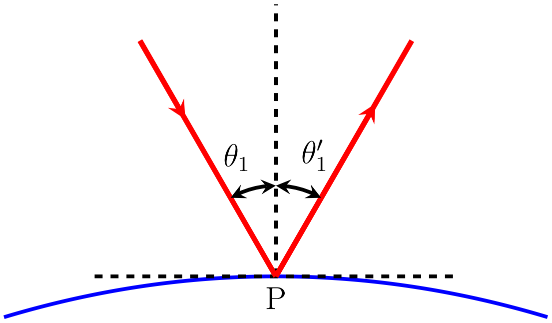


Figure 33.2: Reflection of a Light Ray from an Arbitrarily Curved Surface at the Point P

### 33.1.3 The Law of Refraction

The law of reflection is a relatively simple proposition, but what happens when light goes from one medium into another? What is found is that when light goes from one medium into another (say from air into water), the light does not, in general, continue in a straight line, but instead it bends. Figure 33.3 shows a ray of light incident at an angle $\theta_1$ to the interface between air and water. The angle $\theta_1'$ is the angle of reflection from the interface back into air. The angle $\theta_2$ is the angle of bending, commonly called the angle of refraction, which the light ray makes with the normal to the interface as it travels in the water.

The path of the light ray in water is not parallel to the path of the light ray in air, but in fact is at an inclination to its path in air. When $\theta_1$ is small, it is found that $\theta_2$ is also small. However, when $\theta_1$ is large, the angle $\theta_2$ is also found to become large. The correct relationship between the angles

$\theta_1$ and $\theta_2$ was not successfully discovered until 1621 when Willebrord Snell, a Dutch mathematician, found that if $\theta_1$ is the angle of the light ray in air and $\theta_2$ is the angle of the light ray in water, then $\sin\theta_1$ is equal to a constant multiple of the $\sin\theta_2$. That is,

$$\sin\theta_1 = n_{2,1}\sin\theta_2 \tag{33.2}$$

The constant $n_{2,1}$ is called the index of refraction of medium 2 (in this case water) relative to medium 1 (in this case air). When medium 1 is air and medium 2 is water, $n_{2,1}$ is found to be approximately 1.33. Equation 33.2 is called Snell's law and is the law of refraction.

If one medium is a vacuum, then the index of refraction of the second medium relative to a vacuum is defined by

$$n = \frac{c}{v} \tag{33.3}$$

where $c$ is the speed of light in a vacuum and has the value $3 \times 10^8$ m/s, and $v$ is the speed of light in the second medium.

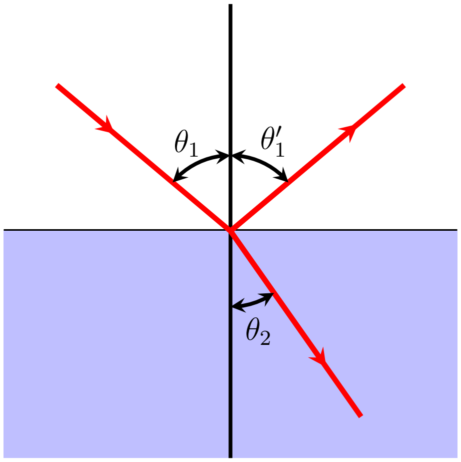


Figure 33.3: Refraction of a Light Ray when it Passes from One Medium into Another

It can also be shown that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \tag{33.4}$$

where $v_1$ is the speed of light in medium 1 and $v_2$ is the speed of light in medium 2. This result follows from a fundamental principle of geometric optics known as Huygen's principle.

Equations 33.3 and 33.4 imply that

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

or

$$n_1\sin\theta_1 = n_2\sin\theta_2 \tag{33.5}$$

where $n_1$ and $n_2$ are the indices of refraction of the two media relative to a vacuum. Equation 33.5 is just a restatement of Equation 33.2, Snell's law, when the index of refraction of each medium is taken relative to a vacuum. The index of refraction of a medium (other than a vacuum) relative to a vacuum is always a number greater than unity. This is due to the fact that light moves slower in a material medium than it does in a vacuum (i.e., free space).

## 33.2 Experiment

For this experiment, we will use a variety of apparatus including a laser ray box along with plane, concave, and convex mirrors, glass prisms, and a water refraction tank. We will also use the Bending Light simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/sims/html/bending-light/latest/bending-light_all.html

### 33.2.1 The Law of Reflection

#### 33.2.1.1 Reflection by a Plane Mirror

Use the laser ray box to produce a single ray of light. Trace the mirror and a line perpendicular (normal) to the mirror on a piece of paper. Reflect the ray of light from the plane mirror by shining the light on the mirror at some incident angle relative to the normal line. Trace the incident ray and reflected ray as shown in Figure 33.4. Measure the angle of incidence and angle of reflection using a protractor. Position the mirror at a new location on the paper, and again draw the mirror and a line normal to it. Reflect the ray of light from the mirror again but choose a new angle of incidence. Again measure the angle of incidence and reflection. Do this for a total of four conditions and perform two sets of measurements per sheet of paper. Record your measurements in Table 33.1.

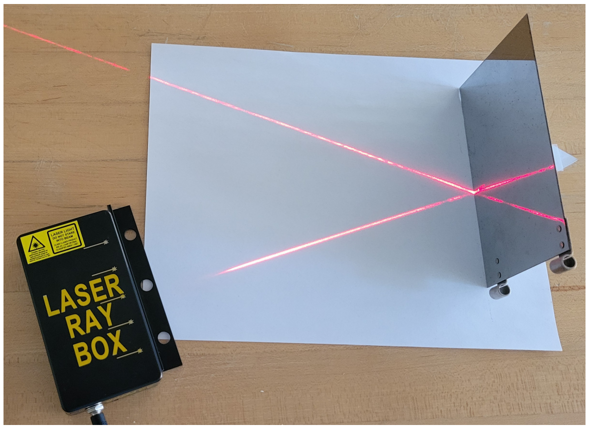


Figure 33.4: Reflection from a Plane Mirror

**Question 1:** Compute the value of $\frac{\theta_1'}{\theta_1}$ for the different rays. What should the value of $\frac{\theta_1'}{\theta_1}$ be in each case? Compute the percent experimental error comparing your computed ratio to the theoretical ratio in each case. Record your values in Table 33.1.

#### 33.2.1.2 Reflection by a Concave Mirror

Place a sheet of paper on the table. Set up the laser ray box so that three parallel rays are emitted. Set the cylindrical mirror perpendicularly on the slits and so that the central ray from the slits strikes the mirror perpendicularly at the center (see Figure 33.5).

Trace the reflecting surface of the mirror on the paper and trace the paths taken by the incident rays and the reflected rays. Label the incident and reflected rays with the appropriate arrows. The reflected rays will intersect at the focal point of the mirror. Locate this point and label it F. The

| Condition | $\theta_1$ (Incident Angle) | $\theta_1'$ (Reflected Angle) | Ratio $\frac{\theta_1'}{\theta_1}$ | Percent Experimental Error (%) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |

Table 33.1: Angles of Incidence and Reflection from a Plane Mirror

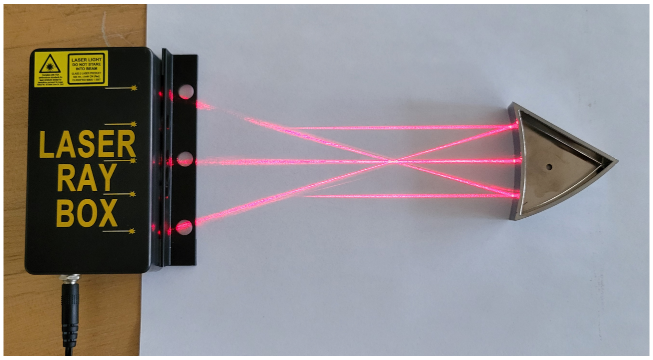


Figure 33.5: Reflection from a Concave Mirror

focal length of the mirror, $f$, is FV, i.e., the distance between the focal point F and the vertex V of the mirror (see Figure 33.6). Measure the experimental focal length $f$ and record the value on your drawing and in Table 33.2.

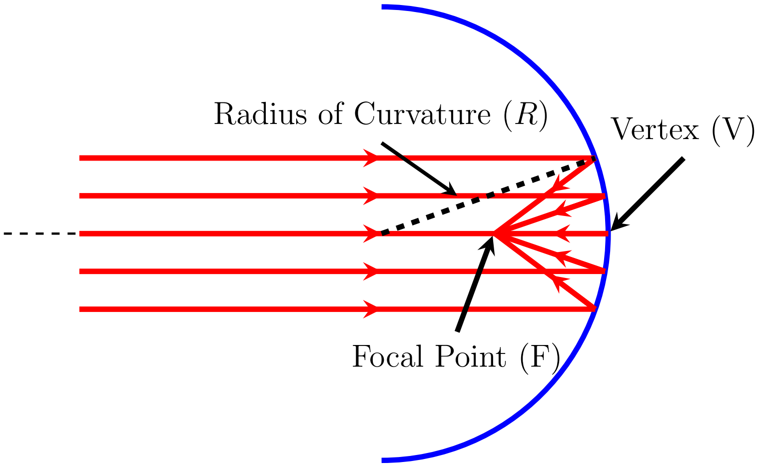


Figure 33.6: Reflection by a Concave Mirror

The focal length of the mirror is related to the object distance $d_o$ and the image distance $d_i$ by the formula

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \tag{33.6}$$

When the object is at infinity, the rays incident on the mirror will be parallel (as was approximately the case in this experiment). Hence, when the object distance $d_o$ is infinite, the image distance $d_i$ is the focal length $f$. For both concave and convex mirrors, if we assume that the rays are near the center of the mirror and the reflection angles are small, then it can be shown that the focal length is one-half the radius of curvature.

$$f = \frac{1}{2}R \tag{33.7}$$

#### 33.2.1.3 Reflection by a Convex Mirror

Place a sheet of plain white paper on the table. Set up the laser ray box so that three parallel rays are emitted. Set the cylindrical mirror perpendicularly on the paper so that its convex side faces the slits and so that the central ray strikes the mirror perpendicularly and reflects back on itself (see Figure 33.7).

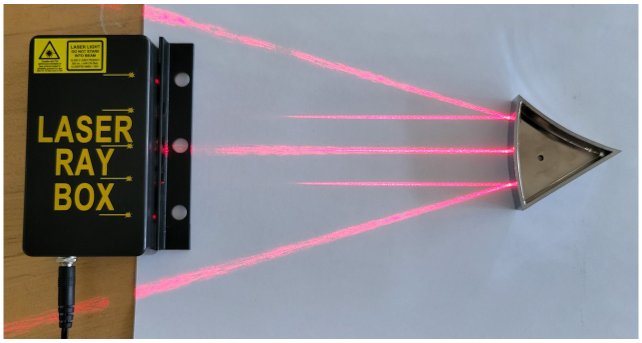


Figure 33.7: Reflection from a Convex Mirror

Trace the reflecting surface of the mirror on the paper and trace the paths taken by the incident and reflected rays. Label the incident and reflected rays with the appropriate arrows. The reflected rays will all appear to come from the focal point of the convex mirror. Extend the reflected rays backward to locate the focal point and label it F. The focal length of the mirror, $f$, is FV, i.e., the distance between the focal point F and the vertex V of the mirror (see Figure 33.8). Measure the experimental focal length $f$ and record the value on your drawing and in Table 33.2.

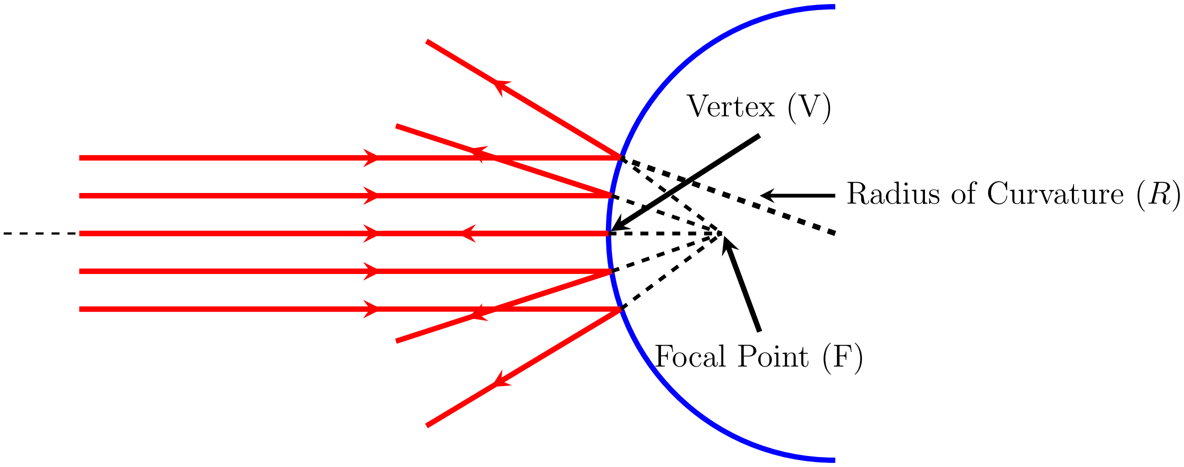


Figure 33.8: Reflection by a Convex Mirror

**Question 2:** Using Equation 33.7 and the values for the radius of curvature $R$ given in Table 33.2, calculate the theoretical focal length for the concave and convex mirrors. Compare the experimental and theoretical focal lengths for the concave and convex mirrors by computing the percent experimental errors. Show all of your work and record your values in Table 33.2.

| Mirror | Experimental Focal Length ($f$) (cm) | Radius of Curvature ($R$) (cm) | Theoretical Focal Length ($f$) (cm) | Percent Experimental Error (%) |
|---|---|---|---|---|
| Concave | | 12.5 | | |
| Convex | | 12.5 | | |

Table 33.2: Focal Lengths of Concave and Convex Mirrors

**Question 3:** What is the difference between a concave mirror and a convex mirror?

## 33.2.2 The Law of Refraction

### 33.2.2.1 Refraction of Light Through a Glass Plate - Simulation

Launch the PhET Bending Light simulation and choose the Prisms option. Choose a rectangular glass prism and rotate it clockwise about 45°. Set up the laser so that the light enters first face of the prism, is refracted as it passes through the prism, then exists the second face. Make sure the "Normal Lines" option is selected. The outgoing ray should emerge parallel to the incoming ray but be displaced vertically (see Figure 33.9).

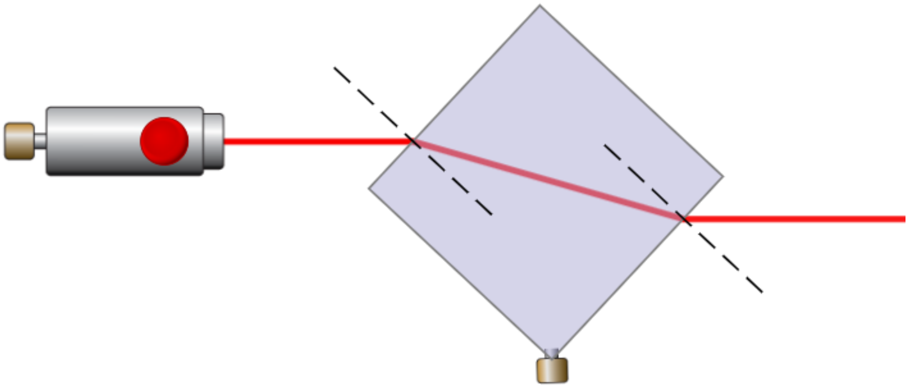


Figure 33.9: Refraction Through a Glass Plate

Using the protractor tool, measure the incident and refraction angles ($i_1$ and $r_1$) at the first face. Also measure the incident and refraction angles ($i_2$ and $r_2$) at the second face (see Figure 33.10). Record your values in Table 33.3.

**Question 4:** Use Snell's law (Equation 33.2) to calculate the index of refraction of glass relative to air $n_{\text{glass,air}}$ using the angles $i_1$ and $r_1$. Record your values in Table 33.3.

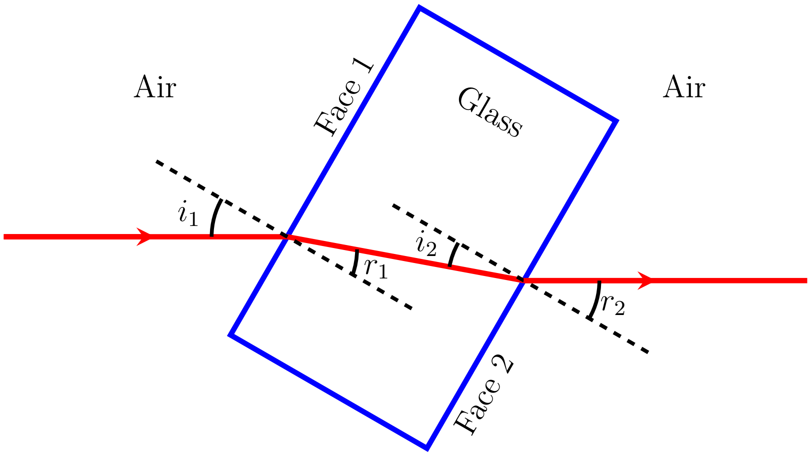


Figure 33.10: Refraction of Light Through Glass

**Question 5:** Use Snell's law (Equation 33.2) to calculate the index of refraction of glass relative to air $n_{\text{glass,air}}$ using the angles $i_2$ and $r_2$. Record your values in Table 33.3.

**Question 6:** How does your answer in Question 4 compare with your answer in Question 5?

| | | | |
|---|---|---|---|
| Angle of Incidence ($i_1$) (°) | | Angle of Incidence ($i_2$) (°) | |
| Angle of Refraction ($r_1$) (°) | | Angle of Refraction ($r_2$) (°) | |
| | | | |
| $n_{\text{glass,air}} = \frac{\sin i_1}{\sin r_1}$ | | $n_{\text{glass,air}} = \frac{\sin r_2}{\sin i_2}$ | |

Table 33.3: Index of Refraction of Glass Relative to Air for a Rectangular Prism

#### 33.2.2.2 Refraction of Light Through a Glass Prism

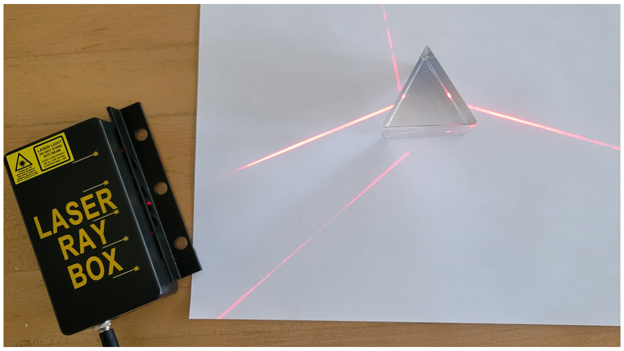


Figure 33.11: Refraction Through a Glass Prism

Place the glass prism at the center of a sheet of plain white paper. Outline the shape of the prism with a sharp pencil. Using the laser ray box, shine a single ray of light through the prism as shown in Figure 33.11.

Trace the incident and refracted light rays and the edges of the prism in order to determine the angles of incidence and refraction at the two faces of the prism. Draw a perpendicular line at each face then measure the two angles of incidence ($i_1$ and $i_2$) and the two angles of refraction ($r_1$ and $r_2$) with a protractor (see Figure 33.12). Also measure the prism angle $A$ with the protractor. Label your values on the drawing and record them in the Table 33.4.

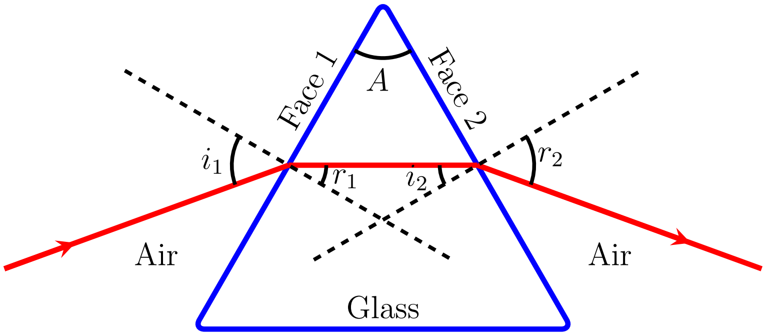


Figure 33.12: Refraction of Light Through A Glass Prism

| | | | |
|---|---|---|---|
| Angle of Incidence ($i_1$) (°) | | Angle of Incidence ($i_2$) (°) | |
| Angle of Refraction ($r_1$) (°) | | Angle of Refraction ($r_2$) (°) | |
| | | | |
| $n_{\text{glass,air}} = \frac{\sin i_1}{\sin r_1}$ | | $n_{\text{glass,air}} = \frac{\sin r_2}{\sin i_2}$ | |
| | | | |
| $A$ Measured (°) | | $A$ Formula (°) | |
| | | Percent Experimental Error for $A$ (%) | |

Table 33.4: Index of Refraction of Glass Relative to Air for a Triangular Prism

**Question 7:** Using Snell's law (Equation 33.2), calculate the index of refraction of glass relative to air $n_{\text{glass,air}}$ using the angles $i_1$ and $r_1$. Record your values in Table 33.4.

**Question 8:** Using Snell's law (Equation 33.2), calculate the index of refraction of glass relative to air $n_{\text{glass,air}}$ using the angles $i_2$ and $r_2$. Record your values in Table 33.4.

**Question 9:** How does your answer in Question 7 compare with your answer in Question 8?

Measure the prism angle $A$ with your protractor, and record the values in Table 33.4.

**Question 10:** Derive the relation between the angles $A$, $r_1$ and $i_2$. Calculate the value of the prism angle $A$ using this derived equation. Compare the calculated value of $A$ with the measured value of $A$ by computing the percent experimental error (use the calculated value of $A$ as the accepted value). Record your results in Table 33.4.

### 33.2.2.3 Refraction of Light Through a Glass Prism - Simulation

Launch the PhET Bending Light simulation and choose the Prisms option. Choose a triangular glass prism and rotate it clockwise about 20°. Set up the laser so that the light enters first face of the prism, is refracted as it passes through the prism (the light that passes through the prism should be approximately parallel to the base of the prism), then exists the second face. Make sure the Normal lines option is selected (see Figure 33.13).

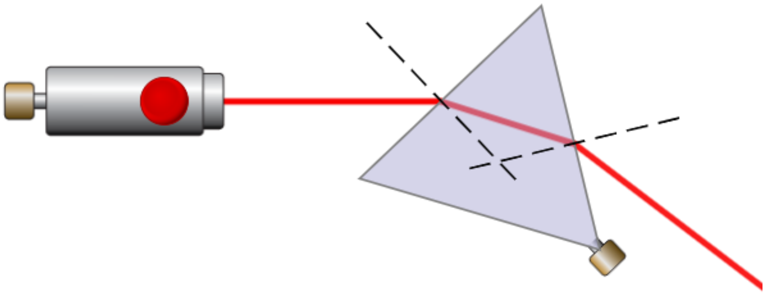


Figure 33.13: Refraction Through a Glass Prism - Simulation

Using the protractor tool, measure the incident and refraction angles ($i_1$ and $r_1$) at the first face. Also measure the incident and refraction angles ($i_2$ and $r_2$) at the second face (see Figure 33.12). Record your values in Table 33.5.

| | | | |
|---|---|---|---|
| Angle of Incidence ($i_1$) (°) | | Angle of Incidence ($i_2$) (°) | |
| Angle of Refraction ($r_1$) (°) | | Angle of Refraction ($r_2$) (°) | |
| | | | |
| $n_{\text{glass,air}} = \frac{\sin i_1}{\sin r_1}$ | | $n_{\text{glass,air}} = \frac{\sin r_2}{\sin i_2}$ | |
| | | | |
| $A$ Measured (°) | | $A$ Formula (°) | |
| | | Percent Experimental Error for $A$ (%) | |

Table 33.5: Index of Refraction of Glass Relative to Air for a Triangular Prism - Simulation

**Question 11:** Using Snell's law (Equation 33.2), calculate the index of refraction of glass relative to air $n_{\text{glass,air}}$ using the angles $i_1$ and $r_1$. Record your values in Table 33.5.

**Question 12:** Using Snell's law (Equation 33.2), calculate the index of refraction of glass relative to air $n_{\text{glass,air}}$ using the angles $i_2$ and $r_2$. Record your values in Table 33.5.

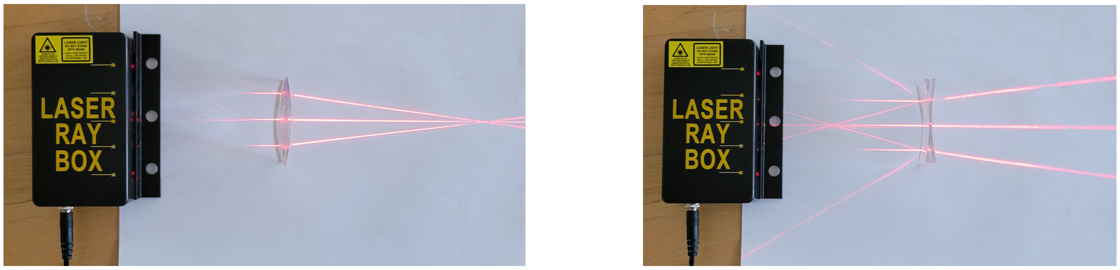
**Question 13:** How does your answer in Question 11 compare with your answer in Question 12?

Measure the prism angle $A$ in the simulation with your protractor, and record the values in Table 33.5.

**Question 14:** Calculate the value of the prism angle $A$ using the equation you derived in Question 10. Compare the calculated value of $A$ with the measured value of $A$ by computing the percent experimental error (use the calculated value of $A$ as the accepted value). Record your results in Table 33.5.

#### 33.2.2.4 Refraction of Light Through Lenses

Place a sheet of plain white paper on the ray table and place one of the two lenses provided on the paper (one is a double convex lens and the other is a double concave lens). Set up the laser ray box so that three parallel rays are emitted and the central ray strikes the lens perpendicularly. Trace the outline of the lens and trace the paths of the rays as they pass through the lens. Using a clean sheet of white paper, repeat the above procedure with the other lens (see Figures 33.14a and 33.14b).



(a) Refraction Through a Double Convex Lens (b) Refraction Through a Double Concave Lens

Figure 33.14: Refraction Through Lenses

**Question 15:** One of the lenses used is known as a converging lens and the other is known as a diverging lens. Label each of the two drawings you have made with the appropriate name for the lens and record the appropriate name in Table 33.6. What are the characteristics of a convergent lens and those of a divergent lens? Indicate on each drawing where you think the focal point of the lens is located and label this point F.

| Type of Lens | Converging or Diverging |
|---|---|
| Double Concave Lens | |
| Double Convex Lens | |

Table 33.6: Converging and Diverging Lenses

#### 33.2.2.5 Refraction of Light Through a Liquid

In this experiment you will use an apparatus, called the refraction apparatus, to measure the index of refraction of water with respect to air (see Figure 33.15).

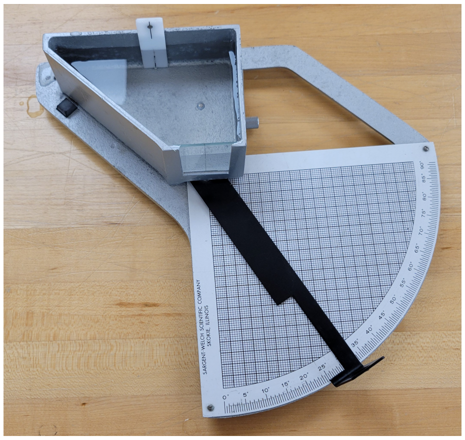


Figure 33.15: Refraction Apparatus

The apparatus consists of an aluminum base, a tank for holding various liquids (water, alcohol, etc.), reference lines, a radius arm, and an arc which is graduated to the nearest 0.5°. The radius arm is provided with a sighting wire. The tank is provided with a fixed index line which is set exactly at the center of the arc in which the index line on the radius arm swings. There is also provided a movable reference slit which should be clipped to the other side of the tank.

In order to use the apparatus to determine the index of refraction of a liquid, fill the tank about half full of the liquid whose index of refraction is to be determined. The beam of light is to be considered as traveling from the movable reference slit on the back of the tank, through the liquid in the tank, and entering the air at the position of the front index line on the glass plate. The angle of incidence $\theta_1$ is obtained by sighting all three index lines through the liquid in the tank. The angle of refraction $\theta_2$ is obtained by sighting all three index lines through the air above the liquid in the tank (see Figure 33.16).

In order to obtain the angle of refraction $\theta_2$, sight the three index lines such that the line on the front glass plate and the reference line on the back plate of the tank are directly behind the sighting line on the movable radius arm when viewed through the air. The angle of refraction is then obtained by reading the degree scale at the position where the beveled edge of the radius arm comes over the degree scale. The angle of incidence $\theta_1$ is obtained from the same scale but by sighting the three lines such that when viewed through the liquid in the tank all three lines line up. In order to obtain an accurate reading, you should sight the lines so that your eye is positioned about one foot from the sighting wire on the movable radius arm.

To obtain an accurate determination of the index of refraction of the liquid, you should take

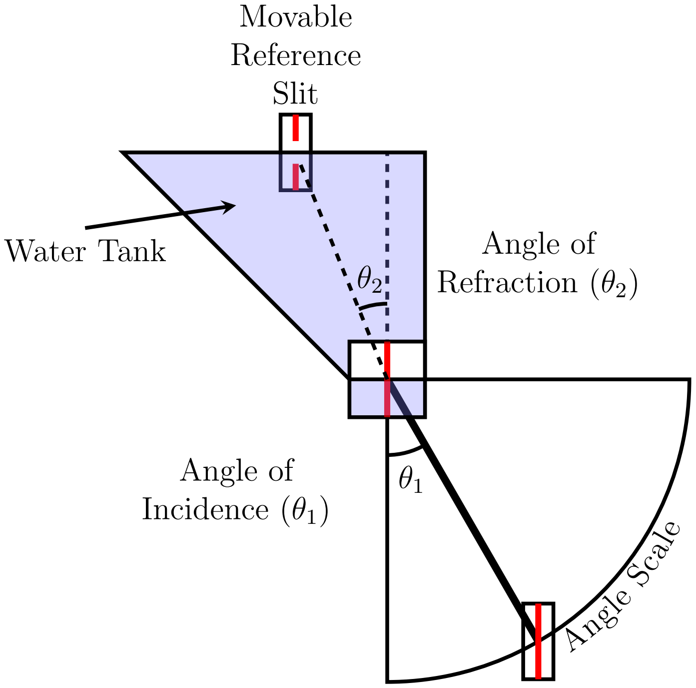


Figure 33.16: Water Tank Refraction Apparatus

about five different readings at small angles as well as at large angles. This can be achieved by starting with the movable reference slit on the back of the tank near the center line, then moving it outward (see Figure 33.16).

| Angle of Incidence ($\theta_1$) (°) | Angle of Refraction ($\theta_2$) (°) | Index of Refraction of Water Relative to Air ($n_{\text{water,air}}$) |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | Average Index of Refraction | |
| | Percent Experimental Error (%) | |

Table 33.7: Index of Refraction of Water Relative to Air

**Question 16:** Fill the tank about half full of water. Measure the angle of incidence and the angle of refraction by placing the movable slit at five different locations. Using Snell's law (Equation 33.2), calculate the index of refraction of water relative to air $n_{\text{glass,air}}$ using the angle of incidence $\theta_1$ and the angle of refraction $\theta_2$ at each location. Compute the average of your results then compare this value with the accepted value for the index of refraction of water relative to air $n_{\text{water,air}} = 1.33$ by computing the percent experimental error. Record all of your values in Table 33.7.

#### 33.2.2.6 Refraction of Light Through a Liquid - Simulation

Launch the PhET Bending Light simulation and choose the Intro option. Make sure that the first medium is air and the second medium is water. Set up the protractor to enble the measurement of the angles of incidence, reflection, and refraction (see Figure 33.17).

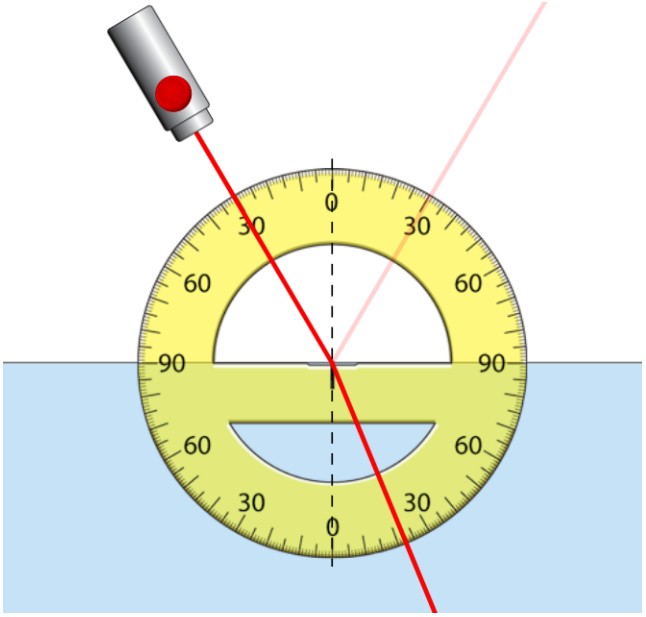


Figure 33.17: Reflection and Refraction - Simulation

**Question 17:** For each angle of incidence given in Table 33.8, use the protractor to measure the angle of reflection. Compute the ratio of the angle of reflection to the angle of incidence for each angle. Find the average ratio then compare it to the accepted value of 1.0 by computing the percent experimental error. Show all calculations and record your results in Table 33.8.

**Question 18:** For each angle of incidence given in Table 33.9, use the protractor to measure the angle of refraction as light passes from the air into the water. Using Snell's law (Equation 33.2), calculate the index of refraction of water relative to air $n_{\text{glass,air}}$. Compute the average index of refraction then compare this value with the accepted value for the index of refraction of water relative to air $n_{\text{water,air}} = 1.33$ by computing the percent experimental error. Show all calculations and record your results in Table 33.9.

| Angle of Incidence ($\theta_1$) (°) | Angle of Reflection ($\theta_2$) (°) | Ratio $\frac{\theta_2}{\theta_1}$ |
|---|---|---|
| 20 | | |
| 30 | | |
| 40 | | |
| 50 | | |
| 60 | | |
| 70 | | |
| | | |
| | Average Ratio | |
| | Percent Experimental Error (%) | |

Table 33.8: Ratio of the Angle of Reflection to the Angle of Incidence

| Angle of Incidence ($\theta_1$) (°) | Angle of Refraction ($\theta_2$) (°) | Index of Refraction of Water Relative to Air ($n_{\text{water,air}}$) |
|---|---|---|
| 20 | | |
| 30 | | |
| 40 | | |
| 50 | | |
| 60 | | |
| 70 | | |
| | | |
| | Average Index of Refraction | |
| | Percent Experimental Error (%) | |

Table 33.9: Index of Refraction of Water Relative to Air - Simulation

# Lab 34

# Dispersion and Focal Length of Lenses

## 34.1 Theory

### 34.1.1 Dispersion of Light Through a Glass Prism

For a discussion of the law of refraction, Snell's law, the student is referred to Experiment 33. Below we discuss the phenomenon of dispersion.

One model that attempts to describe the physical nature of light is a wave model. Its development is based on the theory that light is electromagnetic in nature. That is, the classical electromagnetic theory of light suggests that light waves are propagated by the oscillation of electric and magnetic fields. When the light interacts with matter it is necessary to determine the effects that the electric and magnetic fields have on the atoms. In order to successfully describe a variety of interactions of light with the fundamental particles of matter, however, it is found that a new theory of light is needed, known as quantum-electrodynamics. Here, we will confine our attention to a description of light according to classical electromagnetic theory.

The frequency of the electromagnetic wave that is descriptive of the light, $f$, is determined by the light source and is thus independent of the medium in which the light propagates. If $\lambda$ is the wavelength of the light in a vacuum and $\lambda_n$ is the wavelength of the light in a medium with index of refraction (relative to a vacuum) $n$, then

$$c = f\lambda \tag{34.1}$$

and

$$v = f\lambda_n \tag{34.2}$$

where $c$ is the speed of light in a vacuum and $v$ is the speed of light in the medium with index of refraction $n$. But,

$$n = \frac{c}{v} \tag{34.3}$$

Equations 34.1, 34.2, and 34.3 therefore imply

$$n = \frac{\lambda}{\lambda_n} \tag{34.4}$$

This result clearly shows: (1) that the wavelength of the light changes when the light enters a material transparent medium and (2) that the index of refraction for the medium varies with the wavelength of the light that enters the medium. The fact that the index of refraction of a material medium has a different value for different wavelengths of light is known as the phenomenon of dispersion. This phenomenon was first reported by Newton in his treatise Opticks in 1704. Newton found, as you will in this experiment, that a beam of white light is dispersed into a spectrum of separated colors by a prism (see Figure 34.1). Newton's experiments thus showed that the index of refraction of the material of the prism was dependent upon the wavelength (color) of the light that was refracted. Using Maxwell's equations of classical electromagnetic theory, which relate the electric and magnetic fields to one another, and Newton's second law of motion, one can derive an approximate relationship, which gives the index of refraction as a function of the wavelength of the refracted light,

$$n \approx n_0 + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \frac{D}{\lambda^6} + \cdots \tag{34.5}$$

where $n_0$, $B$, $C$, $D$, $\ldots$ are constants. For purposes of this experiment $n$ is the index of refraction of glass relative to air. Equation 34.5 is known as Cauchy's dispersion formula. It is clear from Cauchy's dispersion formula that $n$ increases when $\lambda$ decreases.

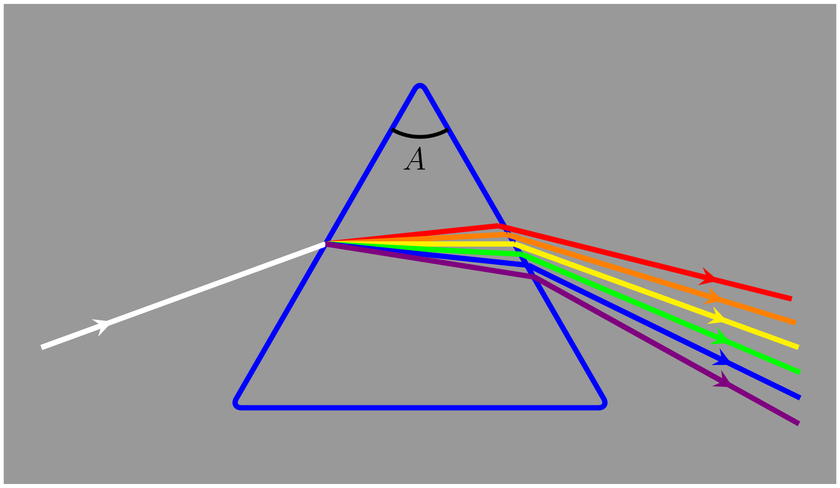


Figure 34.1: Dispersion of White Light

### 34.1.2 Converging and Diverging Lenses

The measurement of the focal length of lenses is of prime importance in the construction of optical instruments. There are two basic types of lenses: converging and diverging lenses. The converging lens is thicker in the middle than at the edges; it is called a double convex lens. The diverging lens is thinner at the center than at the edges; it is called a double concave lens. A parallel beam of light incident on a double convex lens will converge toward a point called the focal point F. The distance between the center of the lens and the focal point is called the focal length $f$ (see Figure 34.2). A parallel beam of light incident upon a double concave lens will diverge after passing through the lens as though it came from a point F′ on the same side of the lens as the light source. Since the light rays do not pass through F′ no image can be formed at this point and it is called a virtual focus (see Figure 34.3). (Note: Consult the diagrams you made in Experiment 33 for an example of parallel beams of light passing through convex and concave lenses.)

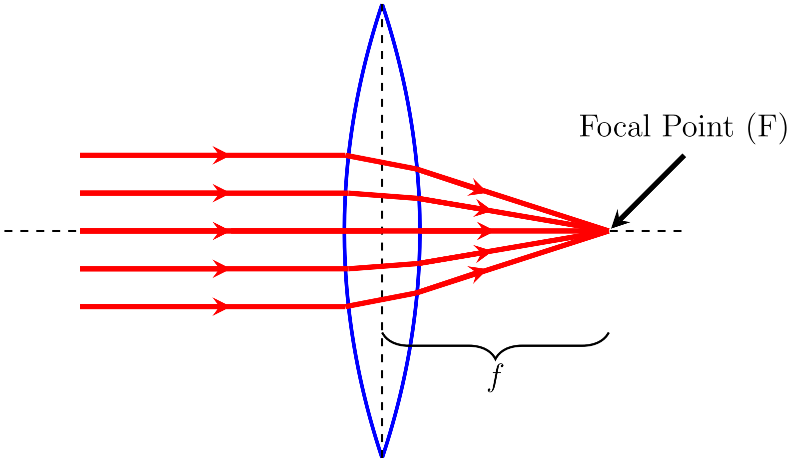


Figure 34.2: Converging Lens

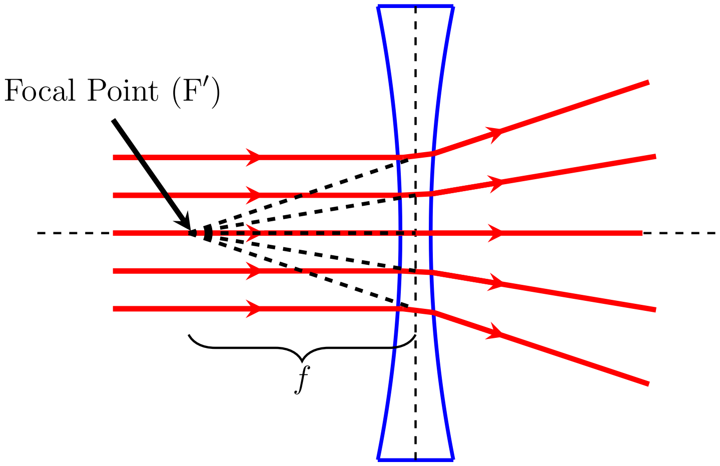


Figure 34.3: Diverging Lens

#### 34.1.2.1 Image Formation by a Double Convex Lens

Figure 34.4 shows a diagram of the formation of an image A'B' of object AB by a double convex lens. The size and position of the image may be determined from a geometric construction. Set the object AB parallel to the lens with one end on the principal axis of the lens. Draw three rays from the top of the object B through the lens. The first ray is drawn parallel to the principal axis and will be bent by the lens through the focal point. The second ray is drawn from the top of the object B through the center C of the lens. Such a ray is unaltered by the thin lens. Finally, the third ray is drawn so that it passes through the near focal point and emerges parallel to the principal axis after passing through the lens. Where the three beams intersect locates the top of the image B'. The bottom of the image A' is on the principal axis. (If the object is between the focal point and the lens, three rays drawn in the above manner will not intersect. The image is a virtual one on the same side of the lens as the object.)

#### 34.1.2.2 Image Formation by a Double Concave Lens

Figure 34.5 shows the construction of the virtual image A'B' formed by a diverging lens of object AB. The virtual image A'B' may be geometrically constructed by drawing three rays from the top of the object B. The first ray, traveling through the center C of the lens, is unaltered as it passes

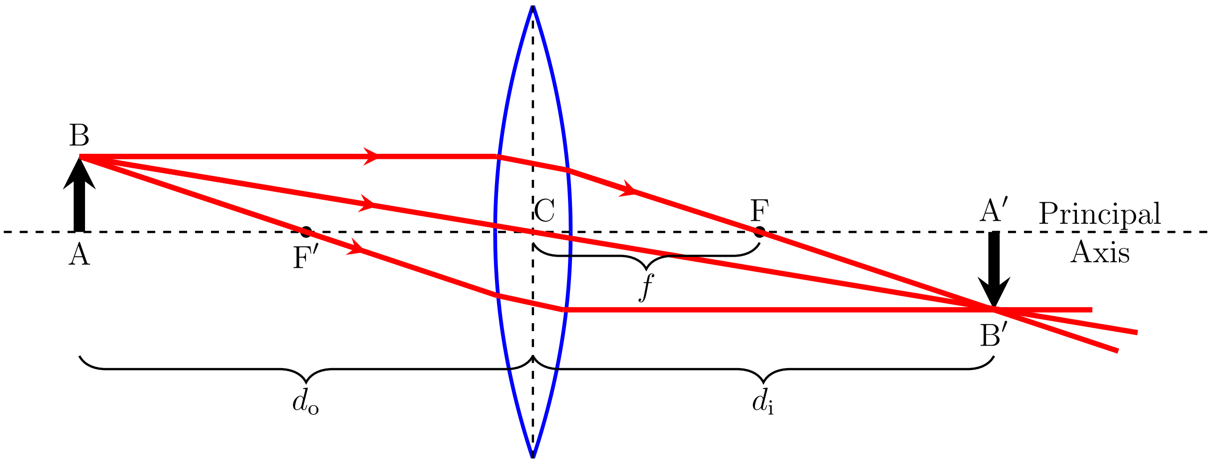


Figure 34.4: Image Formed by a Converging Lens

through the thin lens. The second ray drawn parallel to the principal axis is caused to diverge by the lens in such a manner that when it is projected backward it passes through the focal point F′ on the same side of the lens as the object. The third ray, drawn so that it would pass through the focal point F on the other side of the lens, emerges parallel to the principal axis after passing through the lens. If the three rays are extended backwards, the intersection represents the top of the virtual image B'. The bottom of the virtual image A' is on the principal axis.

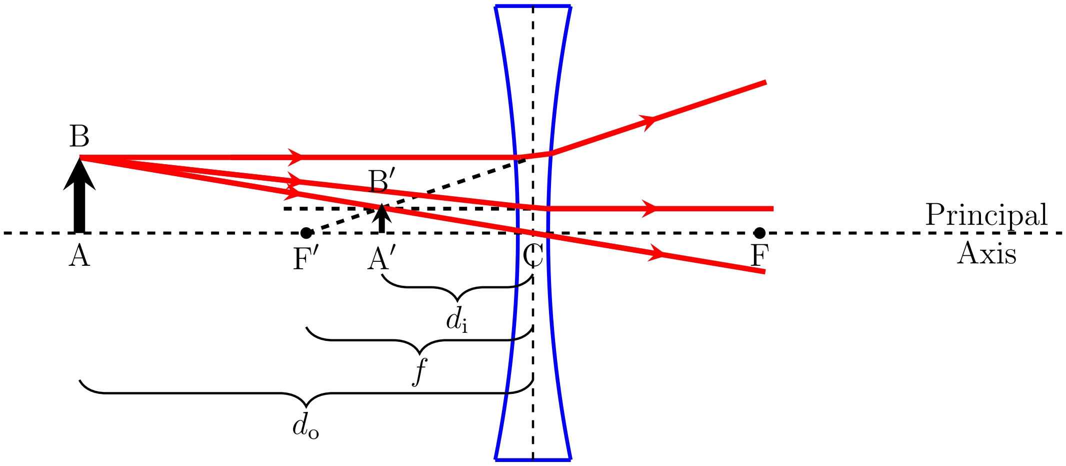


Figure 34.5: Image Formed by a Diverging Lens

### 34.1.2.3 The Thin Lens Equation

It may be shown by geometric proof from Figure 34.4 that

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{34.6}$$

$$\text{or } f = \frac{d_o d_i}{d_o + d_i}$$

where $d_o$ is the object distance, $d_i$ is the image distance, and $f$ is the focal length. In applying this thin lens equation to lens problems, the following sign convention should be followed:

- Numerical values of object distances are positive.
- Numerical values of the focal lengths of double convex (converging) lenses are positive; values of the focal length of double concave (diverging) lenses are negative.
- Numerical values of real image distances are positive; values of virtual image distances are negative.

Note: Only use positive and negative signs when the numerical values are inserted in Equation 34.6.

#### 34.1.2.4 Magnification

The magnification $M$ of a lens is defined as the ratio of the image size to the object size. It can be shown geometrically that:

$$M = -\frac{d_\mathrm{i}}{d_\mathrm{o}} \tag{34.7}$$

where $d_\mathrm{i}$ is the image distance, $d_\mathrm{o}$ is the object distance, and the negative sign indicates an inverted image.

#### 34.1.2.5 Experimental Determination of the Focal Length of a Converging Lens Using an Optical Bench

A method of determining the focal length of a double convex (converging) lens utilizes an optical bench in which an object, lens holder and lens, and a screen are attach to a rigid beam (see Figure 34.6). The beam has a build in tape measure to allow for the measurement of their relative positions. In this method, an illuminated object is placed a distance $d_\mathrm{o}$ from the lens. A screen is then positioned at the correct distance $d_\mathrm{i}$ from the lens such that a sharp image is formed on the screen. The focal length of the lens can then be calculated using Equation 34.6.

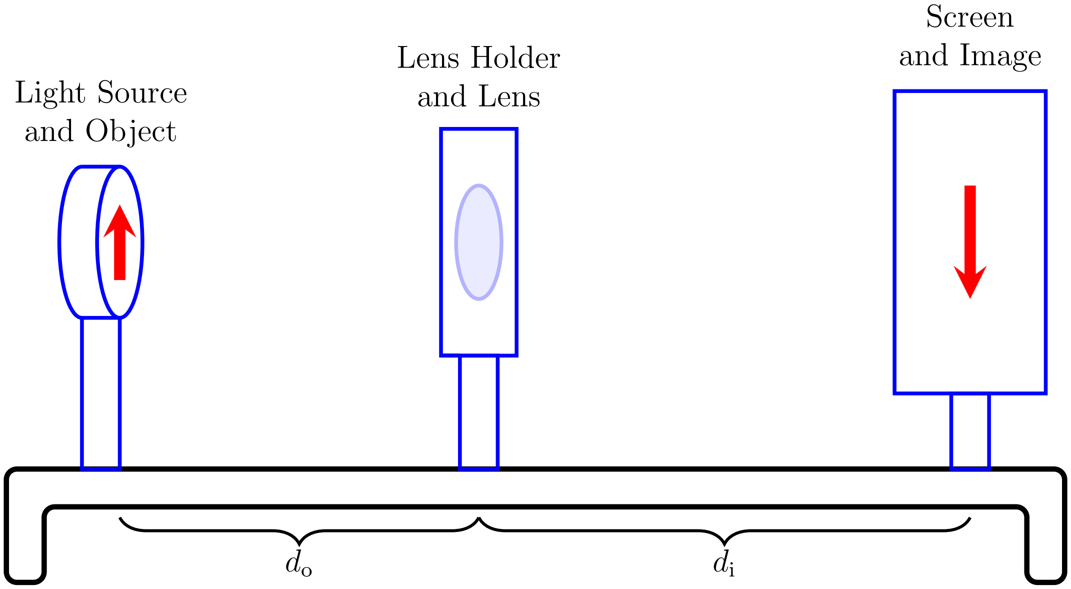


Figure 34.6: Optical Bench Used to Determine the Focal Length of Lenses

#### 34.1.2.6 Lens Combinations

If two thin lenses are placed in contact in such a manner that their principal axes coincide, then the focal length of the combination of lenses is given by:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \tag{34.8}$$

where $f_1$ and $f_2$ are the focal lengths of the two respective lenses. We will use Equation 34.8 to determine the focal length of a double concave (diverging) lens.

## 34.2 Experiment

### 34.2.1 Dispersion of White Light From a Prism

Set up your *light* ray box so that all but one of the slits is covered and a single ray of white light is emitted. Place a sheet of plain white paper on the table and place the prism on the paper so that the path of the ray inside the prism is parallel to the base of the prism (see Figure 34.7). Trace the prism and the ray incident on the prism. Note the ordering of the colors which emerge from the prism. Remove the prism and the paper from the ray table and draw an exaggerated picture of the rays emerging from the prism; label the colors in the order as they appeared. Note: You may want to place another piece of paper further out to more clearly see the dispersion of colors. The effect is subtle and should look similar to the simulation shown in Figure 34.8.

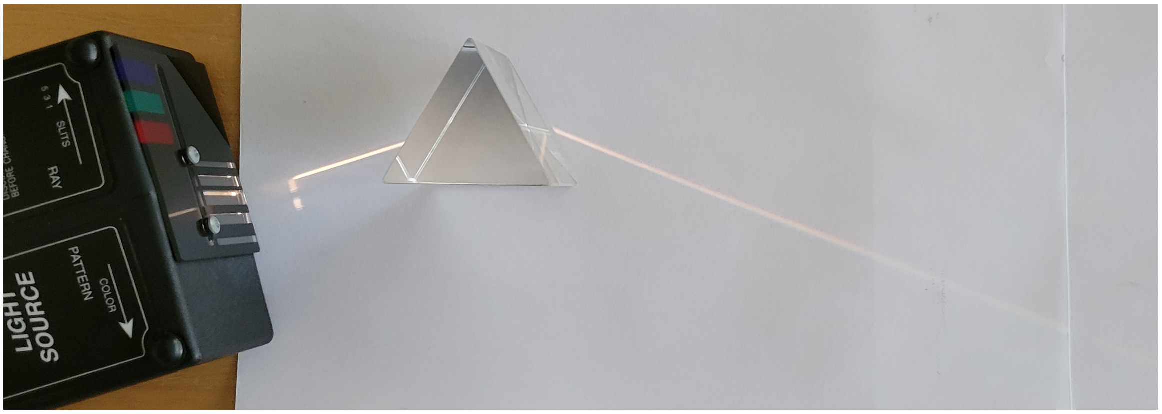


Figure 34.7: Dispersion of White Light Through a Prism

**Question 1:** Based on your understanding of Snell's law and the Cauchy's dispersion formula, which would you say has the greater wavelength, red light or blue light? Support your answer with a logical argument.

**Question 2:** Is the index of refraction of glass relative to air the same for all colors of light? How does what you observed support your answer?

**Question 3:** Sketch a graph indicating the behavior of the index of refraction of glass relative to air as a function of the color of light. Label the $y$-axis $n$ for the index of refraction and label the $x$-axis with the color of the light (violet, red, yellow, green, etc.) in the correct order.

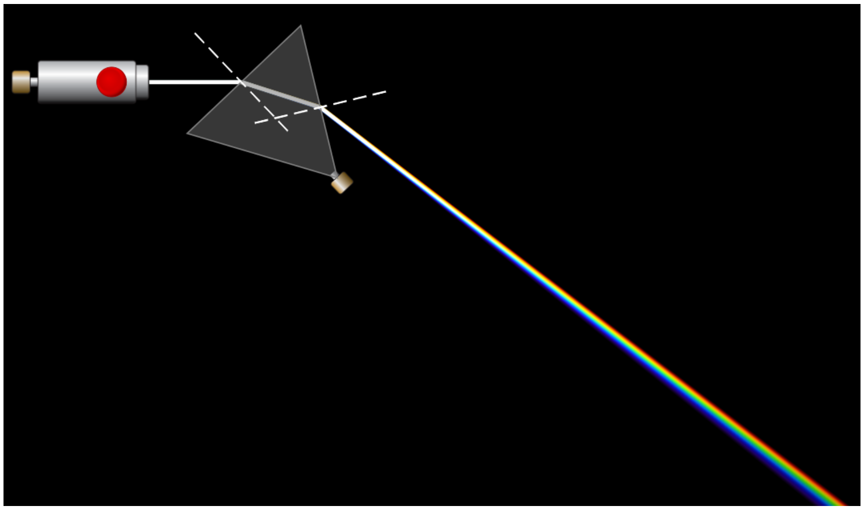


Figure 34.8: Dispersion of White Light - Simulation

### 34.2.2 Differences Between a Continuous and Discrete Spectrum

In this section we will briefly look at the differences between the spectrum generated by the dispersion of white light and the spectrum generated by a helium discharge tube. Set up the prism spectrometer so that a lamp emitting white light is placed in front of the collimator slit. View the dispersion of white light through the telescope. If you fail to see a spectrum of if it is not clear, ask your instructor to adjust the spectrometer (see Figure 34.9).

Now place the helium discharge tube with its accompanying power supply in front of the collimator slit. View the dispersion of the light emitted by the excited helium gas through the telescope (see Figure 34.10).

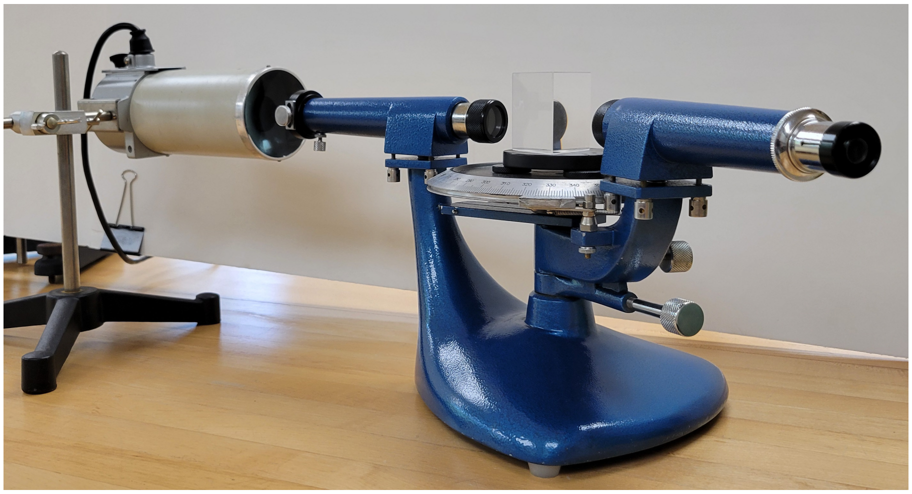


Figure 34.9: Prism Spectrometer with White Light Source

Figure 34.10: Prism Spectrometer with Helium Discharge Tube Source

**Question 4:** What were the differences in the observed spectrums? Can you give an explanation for these differences?

### 34.2.3 Determining the Focal Length and Magnification of a Double Convex Lens

In this part of the experiment, we will use the optical bench to securely hold the illuminated object, the lens holder and lens, and the screen in place (see Figure 34.11). Secure the lens holder at a position of 50 cm, then place one of the double convex lenses in the lens holder. Place the illuminated object (the lighted vertical arrow) a distance $d_{\mathrm{o}} = 20$ cm to the left the double convex lens. Finally, adjust the location of the screen until a sharp image of the arrow is formed on the screen. The distance between the lens and the screen is the image distance $d_{\mathrm{i}}$. Record your value for the location of the image in Table 34.1. Perform the procedure again for the other double convex lens.

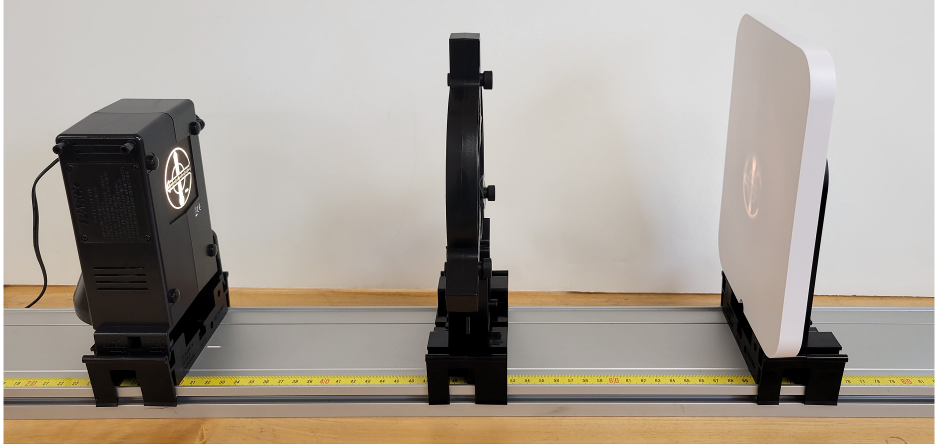


Figure 34.11: Optical Bench Used to Determine the Focal Length of Lenses

| Condition | Location of Object (cm) | Location of Lens (cm) | Location of Image (cm) | Object Distance ($d_o$) (cm) | Image Distance ($d_i$) (cm) |
|---|---|---|---|---|---|
| Lens 1 (10 cm) | 30 | 50 | | 20 | |
| Lens 2 (12.5 cm) | 30 | 50 | | 20 | |

Table 34.1: Image Distance Formed by Double Convex Lenses

**Question 5:** Compute the image distance $d_i$ for both double convex lenses. Record your values in Table 34.1.

**Question 6:** Compute the focal length $f$ for both double convex lenses using the thin lens equation (Equation 34.6). Compare the calculated focal lengths with the accepted focal lengths by computing the percent experimental errors. Show all of your calculations and record your results in Table 34.2.

| Condition | Accepted $f$ (cm) | Calculated $f$ (cm) | Percent Experimental Error (%) |
|---|---|---|---|
| Lens 1 (10 cm) | 10 | | |
| Lens 2 (12.5 cm) | 12.5 | | |

Table 34.2: Focal Lengths of Double Convex Lenses

**Question 7:** Measure the object size (length of object arrow) and the image size (length of image arrow) for each double convex lens. Calculate the experimental magnification for each double convex lens by computing the ratio of the image size to the object size (remember that if the image is inverted the magnification is negative). Calculate the theoretical magnification for each lens using Equation 34.7. Compare the experimental and theoretical magnification values for both lenses by computing the percent experimental errors. Show all of your calculations and record your values in Table 34.3.

| Condition | Object Size (cm) | Image Size (cm) | Magnification (experimental) $=\frac{\text{Image Size}}{\text{Object Size}}$ | Magnification (theoretical) | Percent Experimental Error (%) |
|---|---|---|---|---|---|
| Lens 1 (10 cm) | | | | | |
| Lens 2 (12.5 cm) | | | | | |

Table 34.3: Magnification by Double Convex Lenses

### 34.2.4 Determining the Focal Length of a Double Concave Lens

The determination of the focal length of the double concave (diverging) lens is more difficult because the image is virtual. A screen can't be placed at the virtual image location to obtain a focused image. However, if we combine the diverging lens with the 10 cm converging lens, a real image will form on a screen placed on the other side of the lenses. We can then find the focal length of the combination of lenses using Equation 34.6. Finally, we can compute the focal length of the double concave (diverging) lens by using Equation 34.8 in the following form:

$$\frac{1}{f_{\text{combo}}} = \frac{1}{f_{\text{converging}}} + \frac{1}{f_{\text{diverging}}} \tag{34.9}$$
$$\text{or } f_{\text{diverging}} = \frac{f_{\text{combo}} f_{\text{converging}}}{f_{\text{converging}} - f_{\text{combo}}}$$

Place both the double concave (diverging) lens and the 10 cm double convex (converging) lens in the lens holder. Locate the illuminated object 40 cm away from the lens combination. Adjust the location of the screen such that a sharp image is formed. Measure the location of the image and record the value in Table 34.4.

| Condition | Location of Object (cm) | Location of Lens (cm) | Location of Image (cm) | Object Distance ($d_o$) (cm) | Image Distance ($d_i$) (cm) |
|---|---|---|---|---|---|
| Lens (Combo) | 10 | 50 | | 40 | |

Table 34.4: Image Distance Formed by a Combination of a Double Concave and a Double Convex Lens

**Question 8:** Compute the image distance $d_i$ for the combination of lenses. Record your value in Table 34.4.

**Question 9:** Calculate the focal length $f_{\text{combo}}$ for the combination of the double concave and 10 cm double convex lenses using the thin lens equation (Equation 34.6). Now calculate the focal length for the double concave (diverging) lens using Equation 34.9. Compare the calculated focal length of the diverging lens with the accepted focal length by computing the percent experimental error. Show all of your calculations and record your results in Table 34.5.

| | Accepted $f_{\text{diverging}}$ (cm) | Calculated $f_{\text{diverging}}$ (cm) | Percent Experimental Error (%) |
|---|---|---|---|
| Lens (Double Concave) | -20 | | |

Table 34.5: Focal Length of Double Concave Lens

### 34.2.5 Determining the Focal Length of Lenses Using a Simulation

In this last section, we will calculate the focal length of a double convex (converging) and double concave (diverging) lens using the Geometric Optics simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/en/simulations/geometric-optics

#### 34.2.5.1 Double Convex (Converging) Lens

Start the simulation and choose the "Lens" option. Leave the default settings in place (radius of curvature = 80 cm, index of refraction = 1.5, and diameter = 80 cm, focal points and virtual image remain selected) but select the radio button to show the principal rays. For the object, choose the blue arrow. For the lens, choose the double convex (converging) lens. Set up the object so that it has a height of 20 cm and is 120 cm to the left of the lens (therefore, the object distance $d_o$ = 120 cm). To measure your distances, use the simulated horizontal and vertical rulers. Your setup should look similar to Figure 34.12.

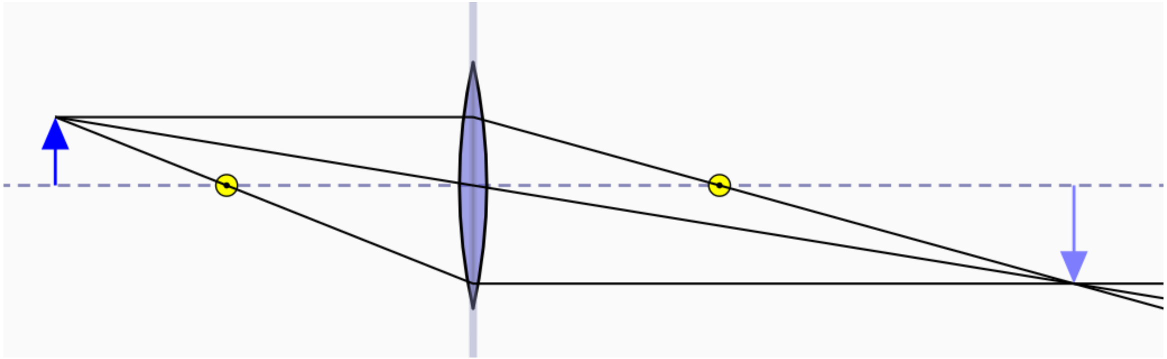


Figure 34.12: Image Formed by Converging Lens

**Question 10:** Measure the image distance $d_i$ and record the value in Table 34.6.

| Condition | Object Distance ($d_o$) (cm) | Image Distance ($d_i$) (cm) |
|---|---|---|
| Lens (Double Convex) | 120 | |

Table 34.6: Image Distance Formed by a Double Convex Lens - Simulation

**Question 11:** Measure the focal length $f_{\text{converging}}$ directly in the simulation. Calculate the focal length using the thin lens equation (Equation 34.6). Compare your calculated value with the measured value by computing the percent experimental error. Show all of your calculations and record your results in Table 34.7.

| | Measured $f_{\text{converging}}$ (cm) | Calculated $f_{\text{converging}}$ (cm) | Percent Experimental Error (%) |
|---|---|---|---|
| Lens (Double Convex) | | | |

Table 34.7: Focal Length of Double Convex Lens - Simulation

**Question 12:** Measure the object size (length of object arrow) and the image size (length of image arrow). Calculate the experimental magnification computing the ratio of the image size to the object size (remember that if the image is inverted the magnification is negative). Calculate the theoretical magnification using Equation 34.7. Compare the experimental and theoretical magnification values by computing the percent experimental error. Show all of your calculations and record your values in Table 34.8.

| | Object Size (cm) | Image Size (cm) | Magnification (experimental) $=\frac{\text{Image Size}}{\text{Object Size}}$ | Magnification (theoretical) | Percent Experimental Error (%) |
|---|---|---|---|---|---|
| Lens (Double Convex) | | | | | |

Table 34.8: Magnification by Double Convex Lens - Simulation

#### 34.2.5.2 Double Concave (Diverging) Lens

Using the same options for the simulation, now choose the double concave (diverging) lens. Set up the object so that it has a height of 40 cm and is 120 cm to the left of the lens (therefore, the object distance $d_o = 120$ cm). To measure your distances, use the simulated horizontal and vertical rulers. Your setup should look similar to Figure 34.13.

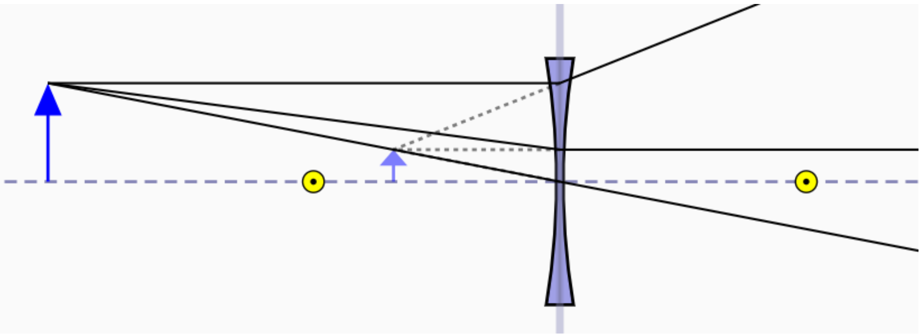


Figure 34.13: Virtual Image Formed by Diverging Lens

**Question 13:** Even though a virtual image is created, since this is a simulation, we can now measure the image distance directly. Measure the image distance $d_i$ and record the value in Table 34.9. Remember that since the image is virtual, the image distance is negative.

| Condition | Object Distance ($d_o$) (cm) | Image Distance ($d_i$) (cm) |
|---|---|---|
| Lens (Double Concave) | 120 | |

Table 34.9: Virtual Image Distance Formed by a Double Concave Lens - Simulation

**Question 14:** Measure the focal length $f_{\text{diverging}}$ directly in the simulation (remember that it will be negative). Calculate the focal length using the thin lens equation. Compare your calculated value with the measured value by computing the percent experimental error. Show all of your calculations and record your results in Table 34.10.

| | Measured $f_{\text{diverging}}$ (cm) | Calculated $f_{\text{diverging}}$ (cm) | Percent Experimental Error (%) |
|---|---|---|---|
| Lens (Double Concave) | | | |

Table 34.10: Focal Length of Double Concave Lens - Simulation

**Question 15:** Measure the object size (length of object arrow) and the image size (length of image arrow). Calculate the experimental magnification computing the ratio of the image size to the object size. Calculate the theoretical magnification using Equation 34.7. Compare the experimental and theoretical magnification values by computing the percent experimental error. Show all of your calculations and record your values in Table 34.11.

| | Object Size (cm) | Image Size (cm) | Magnification (experimental) $=\frac{\text{Image Size}}{\text{Object Size}}$ | Magnification (theoretical) | Percent Experimental Error (%) |
|---|---|---|---|---|---|
| Lens (Double Concave) | | | | | |

Table 34.11: Magnification by Double Concave Lens - Simulation

# Appendices

# Appendix A

# Good Graphing Techniques

In many of the labs throughout this course, you will be plotting your data as a graph. The graphs may be created using graphing software such as Pasco Capstone or Microsoft Excel. However, to be a useful graph it should contain several features. Figure A.1 an example of a poorly drawn graph.

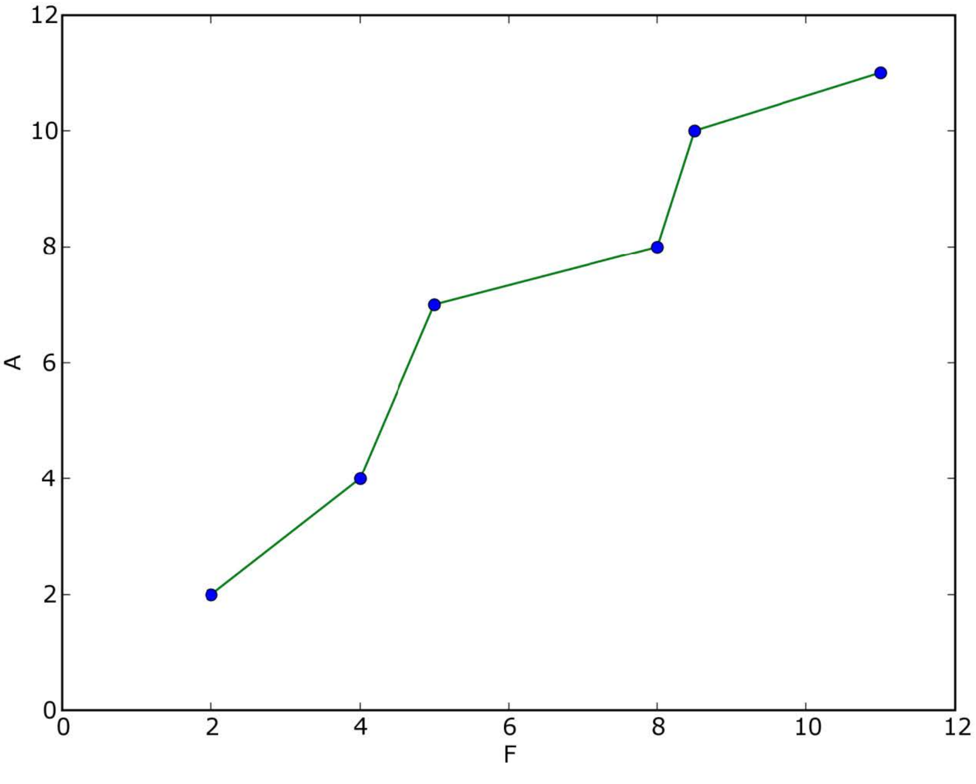


Figure A.1: Poorly Drawn Graph

Notice that although the graph is labeled (with "F" on the $x$-axis and "A" on the $y$-axis), the labels are not clear. We don't easily know what the "F" and "A" indicate. Maybe the "F" indicates "Force" and the "A" indicates "Acceleration". For a useful graph, you want to make sure that you have clear labels. Also, in this graph there are no units. What is the force measured in (N, dynes, etc.), what is the acceleration measured in (m/s$^2$, cm/s$^2$, etc.)? Make sure you include units with the labels. A useful graph should also have a title. In general, the title name can be the $y$ variable (dependent variable) versus the $x$ variable (independent variable). In this example I could use the title ”Acceleration versus Force”. There are a few more subtle points to mention as well. Usually we want to show the relationship between the $y$ variable and the $x$ variable. Connecting the data points with line segment connections is not the best way to do this. A better technique is to fit the data with a curve of best fit. In this example, a linear fit would probably work best. In general, choosing the mathematical function of the fitted curve will depend on the data from the underlying physical process. Lastly, since several trials were usually performed to obtain each data point in the graph, you will typically want to represent the error involved with obtaining each data point. Some researchers do this by including error bars representing the measure of precision (ie. +/- 1 standard deviation, etc.) or to qualitatively show that there was error in obtaining the data point you could circle the data points. In our labs, we won’t focus on indicating the error on a graph but in general it is important.

Figure A.2 below shows a graph that is drawn better and is useful. It has clear labels with units and a title. A curve of best fit (in this case it was a sin function) was drawn through the data points. And to qualitatively indicate the error in the measurement process, each data point was circled.

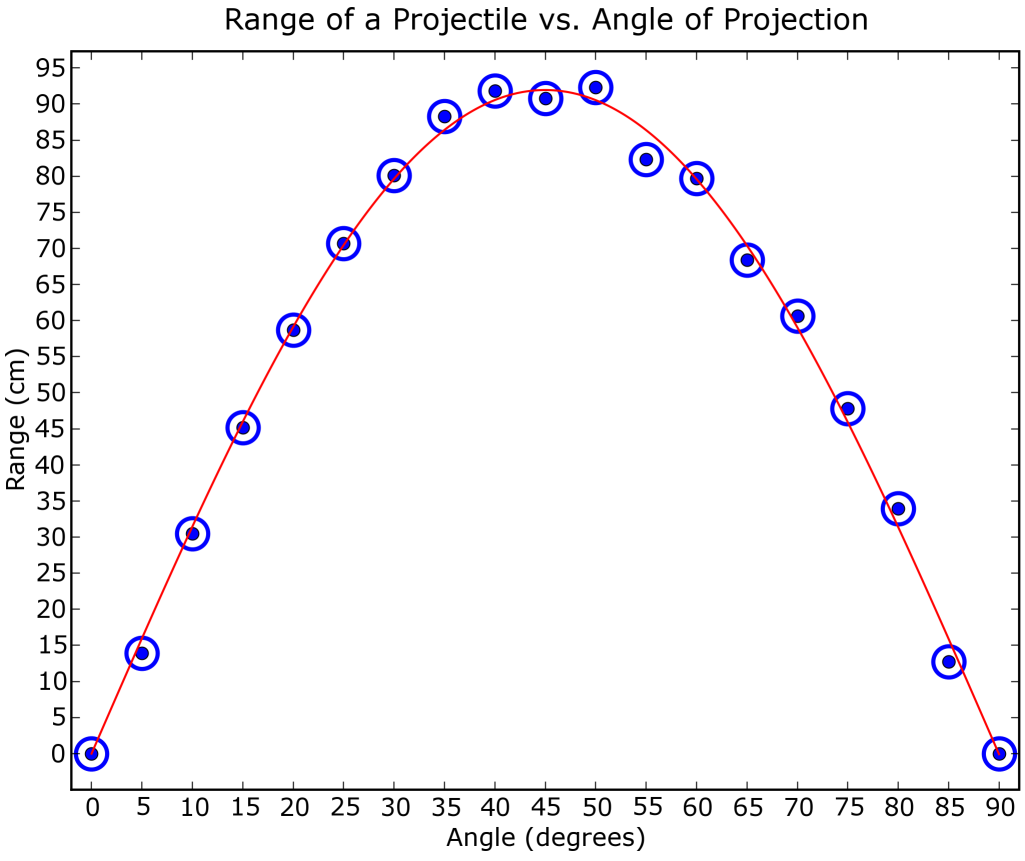


Figure A.2: Well Drawn Graph

# Appendix B

# Measurements of Accuracy and Precision

When performing experiments and making measurements, the data are never exact. There is always some error which may be due to the measurement process, the limitations of the instrument, a bias in the instrument, random variations, etc. To better understand these errors the scientific community uses the terms accuracy and precision.

Accuracy is defined as the degree of closeness of measures of a quantity to the true value of that quantity. In our lab experiments, we will measure accuracy by computing the percent experimental error. Precision is defined as the degree to which repeated measurements show the same results. Precision indicates the variability of the measurements. In our lab experiments we will measure precision by computing the percent relative average absolute deviation (PRAAD). Finally, the resolution of an instrument is defined as the fineness to which a measuring instrument can be read. For example, if a vernier caliper is used to measure length and it can be read to two decimal places, then its resolution is 0.01 cm. This indicates the smallest length that could be measured with the vernier caliper.

To better indicate the difference between accuracy and precision, the archery target diagram is often used (see Figure B.1)

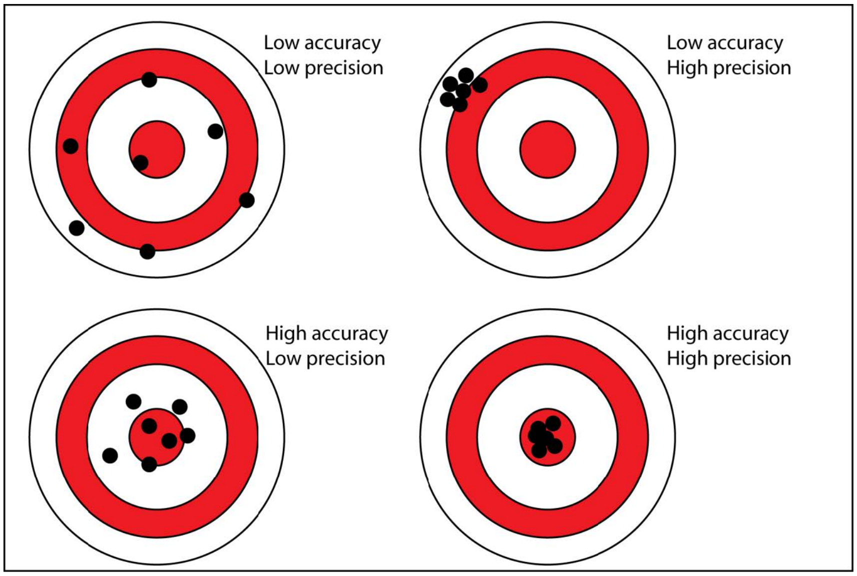


Figure B.1: Difference Between Accuaracy and Precision
Retrieved August 31, 2015, from http://cdn.antarcticglaciers.org/wp-content/uploads
/2013/11/precision_accuracy.png.

Let's analyze each target in the figure. Starting with the target in the top left corner, notice that if your were to average those experimental data values (black points) the average would be outside of the bull's eye (probably a little south of the bull's eye). The center of the bull's eye represents the true value of the variable and since the average experimental value is outside of the bull's eye, it represents low accuracy. The experimental data points are also spread out which means there is a lot of variability in that measure. Therefore, this target also represents low precision.

The target in the top right corner definitely displays low accuracy because the average experimental value is far from the center of the bull's eye. However, the data is tightly clustered with low variability so this represnts high precision.

The target in the bottom left represents high accuracy because the average experimental value is right in the center of the bull's eye and very close to the true value of that variable. However, the data is still spread out which displays some variability representing low or maybe medium precision.

Finally, the target in the bottom right is the ideal case. The average experimental value is right in the center of the bull's eye and the data are tightly clustered with low variability. This target represents high accuracy and high precision.

## B.1 Calculating Percent Relative Average Absolute Deviation (PRAAD)

This example illustrates how to calculate the area of a rectangular wooden board. In particular, emphasis is placed on how to calculate the variability associated with the length, width, and area measures. We will compute the percent relative average absolute deviation (PRAAD) to quantify the variability. Remember that precision is the degree to which repeated measurements show the same results. Therefore, PRAAD is a measure of the precision of the measurements. A lower PRAAD indicates less variability and higher precision.

Table B.1 consists of 10 measurements of the length and width of a rectangular board using a meter stick. Each measurement is recorded to one decimal place so this indicates that the resolution of the meter stick is 0.1 cm. To calculate PRAAD for length, we first need to average the 10 length values and round to the resolution of the instrument. This gives 8.1 cm. Next, we find all of the absolute deviations which is how far each data value is from the average value. The first few absolute deviations would be $|8.2 - 8.1| = 0.1$, $|8.1 - 8.1| = 0.0$, and $|8.0 - 8.1| = 0.1$. Notice that the absolute deviation values are always non-negative. Next, we average the absolute deviations and round to the resolution of the instrument. This gives the average absolute deviation which is 0.1 cm in this case. Finally, to compute PRAAD we take the average absolute deviation divide it by our average length and multiply it by 100 to represent it as a percentage.

$$\text{PRAAD for Length} = \frac{0.1}{8.1} \cdot 100\% = 1.2\%$$

Therefore, the length of the rectangular board can be written as 8.1 cm ± 1.2 % If we perform the same procedure for the width then the average width is 3.2 cm and the PRAAD for width is

$$\text{PRAAD for Width} = \frac{0.1}{3.2} \cdot 100\% = 3.1\%$$

The width of the rectangular board is then 3.2 cm ± 3.1%

Now we may be interested in computing the area of the rectangular board. To do that we simply substitute in our average length and average width values into the formula for the area of a rectangle ($A = l \cdot w$).

$$\text{Area of Board} = l \cdot w = 8.1 \text{ cm} \cdot 3.2 \text{ cm} = 26 \text{ cm}^3$$

Finally, let's compute the PRAAD for the area of the rectangular board. To do this, notice that the formula to compute area involves multiplying the length and width of the board. In general, to calculate the PRAAD of a quantity that is represented by an equation in which the original variables are **multiplied** or **divided**, you must **add** the respective PRAAD values associated with the original variables. Therefore,

$$\text{PRAAD for Area} = \text{PRAAD for Length} + \text{PRAAD for Width} = 1.2\% + 3.1\% = 4.3\%$$

| Length (cm) | Absolute Deviations (cm) | Width (cm) | Absolute Deviations (cm) |
|---|---|---|---|
| 8.2 | 0.1 | 3.3 | 0.1 |
| 8.1 | 0.0 | 3.3 | 0.1 |
| 8.0 | 0.1 | 3.2 | 0.0 |
| 8.2 | 0.1 | 3.2 | 0.0 |
| 8.3 | 0.2 | 3.3 | 0.1 |
| 8.1 | 0.0 | 3.0 | 0.2 |
| 7.9 | 0.2 | 3.3 | 0.1 |
| 8.0 | 0.1 | 3.4 | 0.2 |
| 8.4 | 0.3 | 3.1 | 0.1 |
| 8.2 | 0.1 | 3.3 | 0.1 |
| Average = $\frac{81.4}{10} = 8.1$ | Average = $\frac{1.2}{10} = 0.1$ | Average = $\frac{32.4}{10} = 3.2$ | Average = $\frac{1.0}{10} = 0.1$ |

Table B.1: Length and Width Measurements of a Rectangular Board

Note that the PRAAD can never have a value of 0% because there is always some error associated with the measuring instrument itself. For example, let's assume you perform 10 measurements of the length with the meter stick again and find that most of the 10 measurements have the same value (see Table B.2). In this case, most of the absolute deviation values are 0.0 and if you average them and round to the resolution of the instrument you get 0.0 for the average absolute deviation. Computing PRAAD for length would now give

$$\text{PRAAD for Length} = \frac{0.0}{8.1} \cdot 100\% = 0.0\%$$

However, since we assume PRAAD can't be 0%, how do we resolve this? The solution is to make sure the average absolute deviation is at least the resolution of your instrument (0.1 in this example).

Therefore, the correct PRAAD for length would be

$$\text{PRAAD for Length} = \frac{0.1}{8.1} \cdot 100\% = 1.2\%$$

| Length (cm) | Absolute Deviations (cm) |
|---|---|
| 8.1 | 0.0 |
| 8.1 | 0.0 |
| 8.0 | 0.1 |
| 8.1 | 0.0 |
| 8.1 | 0.0 |
| 8.1 | 0.0 |
| 8.1 | 0.0 |
| 8.0 | 0.1 |
| 8.1 | 0.0 |
| 8.1 | 0.0 |
| Average = $\frac{80.8}{10} = 8.08 = 8.1$ | Average = $\frac{0.2}{10} = 0.02 = 0.0$<br>must use 0.1 |

Table B.2: Length Measurements of a Rectangular Board with Low Variability

## B.2 Calculating Percent Experimental Error

The percent experimental error is a measure of how much a measured value (experimental value) of a physical quantity differs from a predicted (or accepted) value of that quantity. It is a measure of the accuracy of that measurement. For example, assume that you perform an experiment to find the density of a small aluminum cylinder (see Lab 1). In this experiment you find the density of the aluminum cylinder to be 2.6 g/cm$^3$. This is your experimental value. Next, you look up the accepted value for the density of aluminum and find it to be 2.7 g/cm$^3$. This is the accepted or predicted value (sometimes we also use the term ”gold standard value”). To determine how accurate your measurement was you can compute the percent experimental error using the following formula

$$\text{Percent Experimental Error} = \frac{|\text{Experimental Value} - \text{Accepted Value}|}{\text{Accepted Value}} \cdot 100\%$$

For this example we have

$$\text{Percent Experimental Error} = \frac{|2.6 - 2.7|}{2.7} \cdot 100\% = \frac{|-0.1|}{2.7} \cdot 100\% = \frac{0.1}{2.7} \cdot 100\% = 3.7\%$$

Note that 0% is an acceptable value when computing percent experimental error.

# Appendix C

# Practice Lab: Accuracy, Precision, and Graphing

## C.1 Measuring the Volume of a Textbook

A meter stick is used to measure the length ($l$), width ($w$) , and height ($h$) of a physics textbook. Five trials are performed for each measurement. Using the data in the tables (Tables C.1, C.2, and C.3) below, complete the tables by computing the average and PRAAD for the length, width, and height.

| Trial | Length (cm) | Absolute Deviation (cm) |
|---|---|---|
| 1 | 10.7 | |
| 2 | 10.4 | |
| 3 | 10.1 | |
| 4 | 10.1 | |
| 5 | 10.2 | |
| Average | | |
| | | |
| PRAAD for Length (%) | | |

Table C.1: Length Measurements of a Physics Textbook

| Trial | Width (cm) | Absolute Deviation (cm) |
|---|---|---|
| 1 | 8.7 | |
| 2 | 8.5 | |
| 3 | 8.4 | |
| 4 | 8.3 | |
| 5 | 8.5 | |
| Average | | |
| | | |
| PRAAD for Width (%) | | |

Table C.2: Width Measurements of a Physics Textbook

| Trial | Height (cm) | Absolute Deviation (cm) |
|---|---|---|
| 1 | 2.9 | |
| 2 | 2.9 | |
| 3 | 2.9 | |
| 4 | 2.9 | |
| 5 | 2.9 | |
| Average | | |
| | | |
| PRAAD for Height (%) | | |

Table C.3: Height Measurements of a Physics Textbook

The volume of a rectangular solid is given as

$$V = l \cdot w \cdot h$$

Assume the physics textbook has the shape of a rectangular solid and compute the volume of the textbook and the PRAAD for volume. Fill in Table C.4 below.

| | |
|---|---|
| Volume of Textbook ($\text{cm}^3$) | |
| PRAAD for Volume (%) | |

Table C.4: Volume of Physics Textbook

## C.2 Measuring the Volume of a Basketball

The diameter of a basketball is measured 10 times with a caliper that has a resolution of 0.01 cm. Using the data in Table C.5 below, complete the table by computing the average diameter and the PRAAD for diameter.

| Trial | Diameter (cm) | Absolute Deviation (cm) |
|---|---|---|
| 1 | 23.94 | |
| 2 | 23.85 | |
| 3 | 24.01 | |
| 4 | 23.88 | |
| 5 | 24.10 | |
| 1 | 23.92 | |
| 2 | 23.96 | |
| 3 | 23.85 | |
| 4 | 23.89 | |
| 5 | 23.91 | |
| Average | | |
| | | |
| PRAAD for Diameter (%) | | |

Table C.5: Diameter Measurements of a Basketball

The formula for the volume of a sphere is given as

$$V = \frac{4}{3}\pi r^3$$

where $r$ is the radius. Since the diameter of a sphere is twice the radius $d = 2r$ or $r = \frac{d}{2}$, we can rewrite the formula for the volume as a function of the diameter as

$$V = \frac{4}{3}\pi \left(\frac{d}{2}\right)^3 = \frac{1}{6}\pi d^3$$

Assuming the basketball is a perfect sphere, compute the volume of the basketball and the PRAAD for volume. Fill in Table C.6 below. Note that the formula for volume involves multiplying the diameter variable three times $d^3 = d \cdot d \cdot d$. The numbers $\frac{1}{6}$ and $\pi$ are pure numbers and have no error associated with them. Therefore, to compute the PRAAD for volume you must add the PRAAD for diameter three times.

$$\begin{aligned}\text{PRAAD for Volume} &= \text{PRADD for diameter} + \text{PRADD for diameter} + \text{PRADD for diameter}\\ &= 3 \cdot \text{PRADD for diameter}\end{aligned}$$

| | |
|---|---|
| Volume of Basketball ($\text{cm}^3$) | |
| PRAAD for Volume (%) | |

Table C.6: Volume of Basketball

Suppose that the accepted value for the volume of a regulation basketball is 7130 $\text{cm}^3$. Compare your experimental average volume value with the accepted volume value by computing the percent experimental error. Fill in Table C.7 below.

| | |
|---|---|
| Accepted Value for Volume of Basketball ($\text{cm}^3$) | 7130 |
| Experimental Value for Volume of Basketball ($\text{cm}^3$) | |
| | |
| Percent Experimental Error (%) | |

Table C.7: Percent Experimental Error for Volume

## C.3 Graphing Practice

The position and time data presented in Table C.8 were collected for a cart moving at a constant velocity on a horizontal air track.

Use Pasco Capstone, Excel, or your other favorite graphing software and plot a graph of position versus time. Perform a linear fit and record the slope of the graph in Table C.9 below. The slope represents the velocity of the cart in cm/s. Regardless of what software you use, your graph should look similar to that in Figure C.1.

The position and time data presented in Table C.10 were collected for an object that was dropped in the vicinity of the Earth's surface.

Plot a graph of position versus time. The data for this graph were generated based on the function $y = 4.9t^2$, therefore, you should perform a quadratic fit to your data. Your graph should look similar to that in Figure C.2.

| Time (s) | Position (cm) |
|---|---|
| 0.0 | 0.0 |
| 0.1 | 2.9 |
| 0.2 | 6.1 |
| 0.3 | 8.7 |
| 0.4 | 12.3 |
| 0.5 | 14.7 |
| 0.6 | 17.6 |
| 0.7 | 21.0 |
| 0.8 | 23.9 |
| 0.9 | 27.4 |

Table C.8: Position versus Time Data - Constant Velocity

| | |
|---|---|
| Slope of Graph (Velocity of Cart) (cm/s) | |

Table C.9: Slope of Graph (Velocity of Cart)

| Time (s) | Position (cm) |
|---|---|
| 0.0 | 0.00 |
| 0.1 | 0.05 |
| 0.2 | 0.20 |
| 0.3 | 0.44 |
| 0.4 | 0.76 |
| 0.5 | 1.23 |
| 0.6 | 1.80 |
| 0.7 | 2.40 |
| 0.8 | 3.10 |
| 0.9 | 4.02 |

Table C.10: Position versus Time Data - Dropped Object

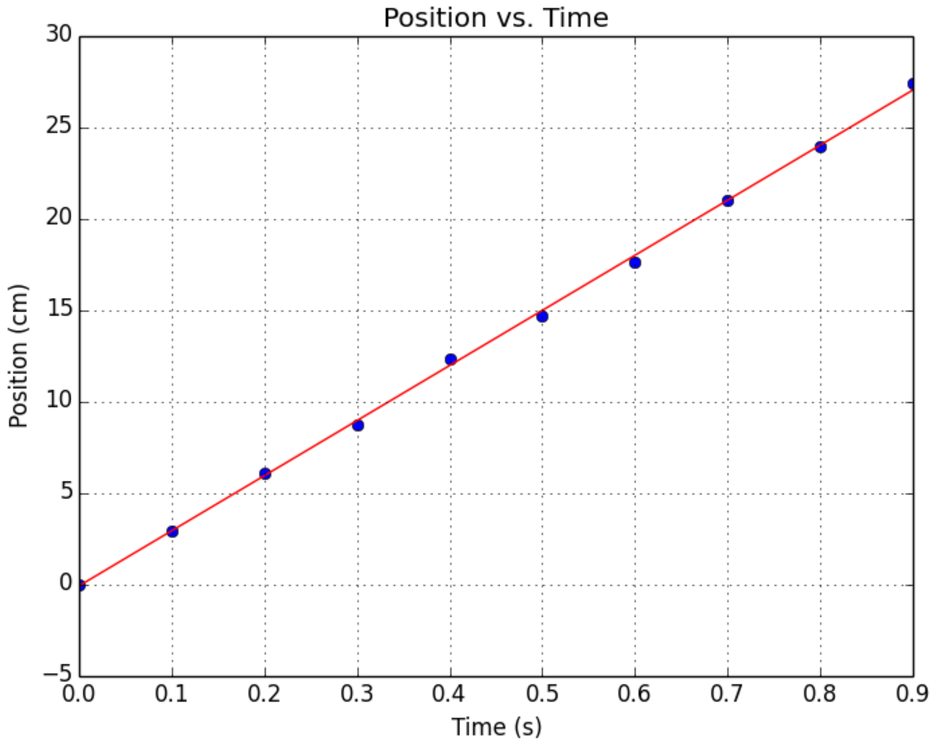


Figure C.1: Position vs. Time - Constant Velocity

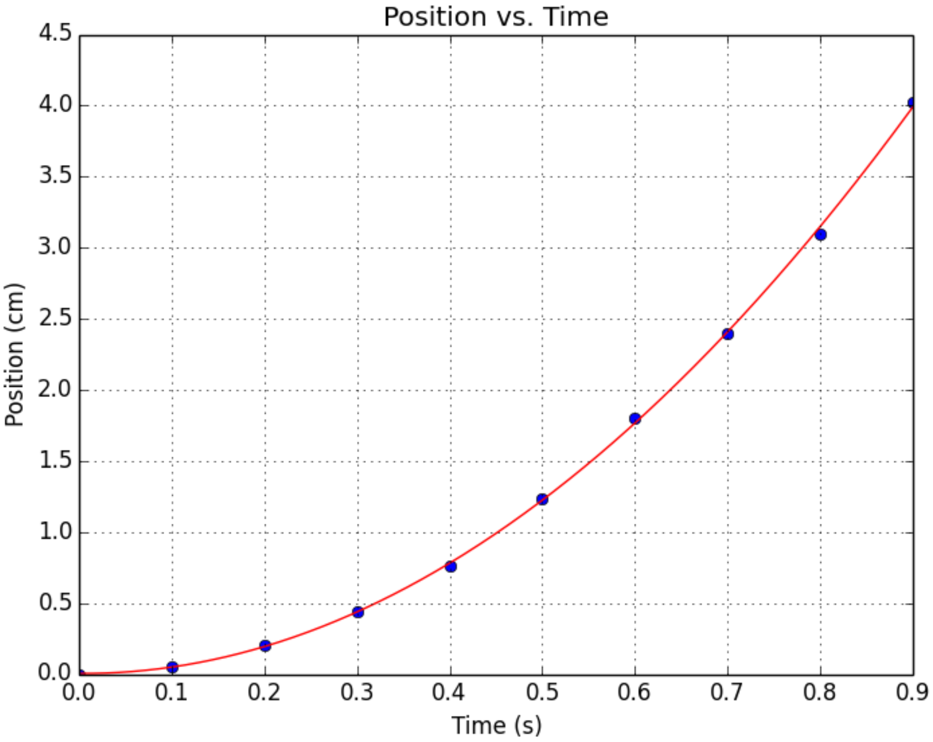


Figure C.2: Position vs. Time - Dropped Object