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Approaching human vision perception to designing visual graph in data visualization

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In a data visualization system, data are converted to information and/or knowledge by human vision perception of graph representing these data. This research studies the characteristics of human vision perception and the relationships between data variables of dataset to constitute cube models representing dataset as a graph on the cube. The research proposes the process designing multidimensional graph considered as structural graph on cube model to represent multivariate data. Structural graph is designed in the light of the relationships between data variables and human vision perception. After mapping structural graph onto computer screen, the graph is integrated retinal variables to improve objectively visual features and respond subjectively human perception of vision.

KEYWORDS

visualization, vision perception, retinal variable, multidimensional graph, visual graph

1 | INTRODUCTION

Two approaches to converting data to information and knowledge in systems of data analysis and knowledge discovery are mathematical model and visualization (Figure 1). Approaching mathematical model, the system of knowledge discovery in data applies algorithms to answer analytical questions but doesn't need the contribution of user. Meanwhile, the approach of visualization keeps human within the system as a component to answer analytical questions by his vision perception. In visualization system, human views graphs representing data to perceive the significance of data. In other words, visualization system refers to a mapping of data onto information and knowledge through human vision perception.

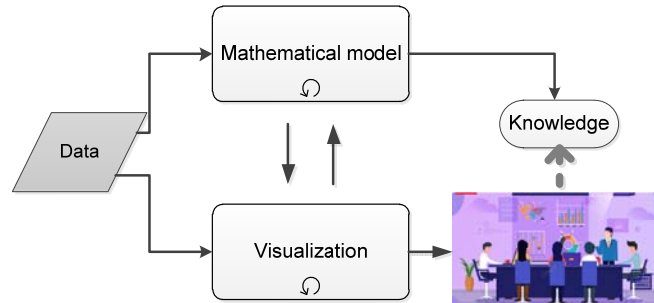


FIGURE 1 Approaches to converting data to knowledge

This research studies human perception of vision and data characteristics to propose an approach to designing visual graph representing data in visualization system composed of two components, visual technique and vision perception. In that, the component of visual technique converts data to visual graph for human perception of vision. The effectiveness of a visualization system is estimated by insights extracted from dataset. Technically, visualization system has to provide user with the best visual graph representing data so that user perceives a lot of significance. In other words, the graph representing data is not only compatible with the characteristics of data but also suitable for human perception of vision.

This article is structured as follows. In the next section, human vision organs along with laws of vision perception of shape and space are studied to cognize how human views and understands the significance of images and graphs. In the third section, dataset is considered as a structure of data variables, where the relationships of data variables are analyzed to result in models of cubes representing multivariate data. The fourth section approaches the technique to structuring multidimensional graph and rendering it on planar screen. In the fifth section, the process for integrating retinal variables [1] into graph to increase visual features is proposed. The sixth section resumes the results of the research.

2 HUMAN VISION PERCEPTION

Human cognizes real world by perceiving and analyzing data through five sense organs each of which collects data by a sensor. Ears collect acoustic data, noses collect smell data, tongue collects taste data, skin collects touch data, and eyes collect image data. In that, vision organs collecting data by eyes is the most broad channel that assists human in cognizing and understanding real world [2]. In other words, vision organs are the most important channel collecting and understanding data shaped images or patterns.

Vision perception is a progress created by human vision organs to cognize and understand what he sees. When human looks at some thing, light rays carrying data of the thing transmit from it to his vision organs to detect information. From the thing, the light rays pass eyes to come retina and fovea, then they are decoded to information and/or knowledge at brain. The characteristics of vision perception result in human capacity of

understanding what he views. In other words, it is necessary to research the features of vision perception to know how to constitute best images or patterns representing data in visualization system.

2.1 | Human Vision Organs

Human vision organs perceive an entity in real world by analyzing data carried in light rays emitting from the entity, where the entity may be a light source or a reflector of some light source. Physically, light is electromagnetic radiation propagating in space as electromagnetic wave, each light ray from a point of entity to eyes carries the features of the point. Generally, light ray emitting from entity is electromagnetic wave carrying data of the entity. In other words, data of elements of entity are modulated in carrier wave and transmitted to human eyes.

After passing pupil, the carrier wave focuses on retina and fovea to detect data and convert them into electrochemical signal to be transmitted to brain for processing and demodulating. At retina, data of black and white elements are detected by receptors of rods, data of colored elements are detected by receptors of cones. At fovea with an extremely dense combination of cones, data are detected in detail when human vision focuses on some part of entity. Data are detected at fovea more detail than at retina.

In human organs of vision perception, retina with rods and cones perceives the features of physical entity in shape and color, while fovea with a lot of cones perceives the features of physical entity in color at levels of more details [2]. At retina, data are demodulated in parallel to fast detect prominent features in the whole. At the fovea, data are demodulated in serial to detect features of entities with more details [2, 3]. Just after being detected from carrier at retina, data are fast processed in parallel for perceiving the whole, including the relationships among elements. After that, data detected at fovea are processed seriously each separate element for perceiving in detail.

At brain, the electrochemical signals of data are translated into significance to store at three levels, iconic memory, short-term memory, and long term-memory (Figure 2). Iconic memory stores provisionally data coming from fovea and retina to process according to two ways, moving continuously to short-term memory or discarding due to nonessential features. Short-term memory makes senses of data and swops old data from long-term memory, where data which are considered as to be worthwhile are moved to long-term memory to store and retrieve in future. Though long-term memory has limited capacity and has to store a lot of data, it is always available for working due to its frequent rearrangement [2].

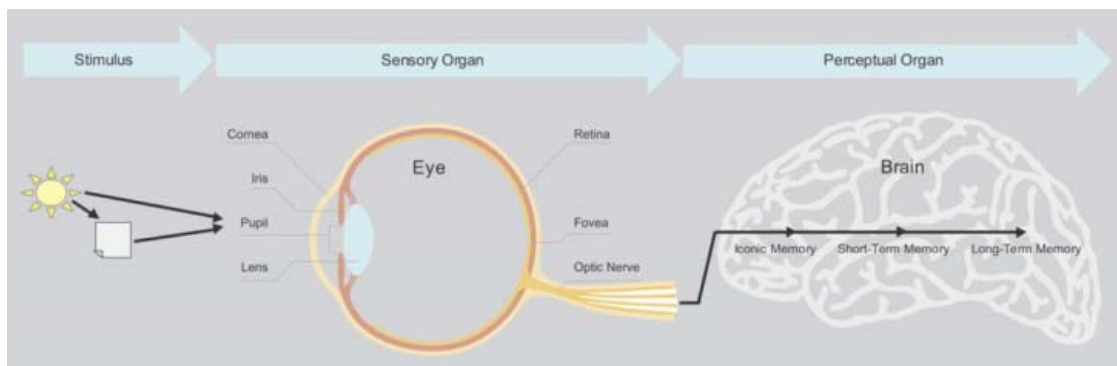


FIGURE 2 Human vision organs (Source [2])

2.2 | Characteristics of Vision Perception

2.2.1 | Laws of Shape Perception

The characteristics of human vision perception are systemized in gestalt laws, where the word “gestalt” is German language, which is understood to be shape, form [4], or configuration [5]. The analysis of human characteristics in perceiving objects or patterns results in the constitution of gestalt laws. The gestalt laws indicate various ways that human vision organs cognize and understand what his eyes look at. According to the laws of shape perception, human vision organs perceive patterns sharing an attribute characteristic or an interval of value to group them into the same cluster.

While researching human perception of shape, Max Wertheimer (1880-1943), Kurt Koffka (1886-1941), Wolfgang Kohler (1887-1967) found out that human perceives the whole before perceiving its components and human can not understand the whole by summing its components [4, 6-9]. In other words, the whole is not equal to the sum of its separate components. While viewing and thinking to understand a set of elements, human can perceive it with many gestalt laws each of which is differently applied. The basic gestalt laws comprise [2, 6, 9-11]:

- **The Figure-Ground Law.** [3] The figure-ground law refers to human perception of objects on background [6]. A pattern is perceived to be figure-ground segregation when contours of objects are clear. A pattern is perceived to be figure-ground ambiguity when contours of objects are fuzzy.
- **The Similarity Law.** The similarity law refers to the features of similarity such as shape, or color, or size, or orientation, etc. [6]. Two patterns are perceived to be similar when they have the same feature of similarity. The perceptually similar patterns are considered as to be in the same group. A set of patterns can be divided into different subsets by different features of similarity, where the subsets can intersect with one another.
- **The Proximity Law.** The proximity law refers to the spaced nearness of patterns [6]. Two spatially near patterns are considered as to be more related than two farther patterns. Two patterns are perceived to be proximity when they are spatially near though they can be different from each other in features.
- **The Symmetry Law.** The symmetry law refers to human perception while looking at elements similar in size, shape, of which relative position of corresponding parts is perceived around a virtual center shaped point or line. The tendency of human vision perception is to integrate symmetric elements into an entity. Color is not utilized to define the symmetry of elements, i.e. the symmetric elements can be in different colors and the elements only similar in color are not symmetric.
- **The Continuation Law.** The continuation law refers to separate patterns which can be perceptually connected with each other to become an entity [6]. The continuation law refers to human perception of the continuation of a separated plane, a dotted line, a discontinuous curve, etc. which is drawn disconnectedly, or a sequence of characters on a hide line or on a hide curve.
- **The Closure Law.** The closure law refers to incomplete pattern, i.e. a pattern exists some gaps in forms [6]. Human vision perception has tendency to fill gaps of a pattern to understand it while looking at. For unmoving elements, the closure law is applied together with the continuation law to strengthen perception. The closure law refers to human perception of closed elements which flicker in convenient frequency as the movement of an element.
- **The Parallelism Law.** The parallelism law refers to patterns parallel to one another. The parallel patterns are considered as to be more related than others. Accordingly, the parallel lines are perceived as to be in the same group.
- **The Common Fate Law.** The common fate law, also called the synchrony law, refers to objects of the same activity. Human vision perception considers objects moving in the same direction as to be more related with each other than with objects which are stationary or move in different directions. Objects which change simultaneously an action are perceived as a group. By common fate law, human vision can perceive bicycle racers moving together in tour, but it is difficult to perceive which team a racer belongs to. The common fate law refers to the capacity of human vision organs perceiving characters or patterns on a signboard by their on/off state of lights rendering them.
- **The Common Region Law.** The common region law refers to the grouping of objects within a bordered region. Human vision organs perceive patterns in the same closed region as to be more related with each

other than elements outside the boundary.

- **The Focal Point Law.** The focal point law refers to dominant elements attracting attention. When looking at a pattern applied well focal point law, people are attracted to look at dominant elements before looking at other parts of the pattern. For a pattern, people view and think the emphasis of dominant elements before spreading their attention for other elements to understand the pattern.
- **The Past Experience Law.** The past experience law refers to the experience of an individual, a group of people, or humanity. Elements are perceived as to be more related with one another than others if they had relationships belonging to a group in the past according to experience. Elements are perceived as to be not related if they had not any relation or had little relations which were cognized in the past.

2.2.2 | Laws of Space Perception

The laws of space perception refer to the characteristics of human vision perception concerning distance [6].

Human vision organs perceive the difference of distance from eye to elements of an entity in real world.

Mathematically, the laws of space perception analyze human vision perception of entities in three-dimension space and perspective patterns rendered on plane.

- **The Vertical Dominance.** The law of vertical dominance refers to the perceptual relation between vertical and horizontal [6]. For two equal bars, the vertical bar is perceived as to be longer than the horizontal. Placing two bars of the same size at the same position, one is vertical and one horizontal, human perceives the dominance of the vertical bar, the vertical bar looks longer than the horizontal.
- **The Part and Totality.** The part and totality law refers to the relation between a part and related parts [6]. For a part in a totality, human vision organs not only perceive the individual part but also are under the influence of other related parts in totality.
- **The Surface.** The law of surface refers to human vision perception on surface [6]. Human vision organs perceive surface of heterogeneous constitution more easily than surface of homogenous constitution.
- **The Overlap.** The overlap law refers to the perceptual relation between overlap and distance of two entities [6]. For two objects of A and B, if the object of A overlaps a part of the object of B, the object of B is perceived as to be more distant than the object of A.
- **The Size.** The size law refers to the perceptual relation between size and distance of two entities [6]. For two entities of different size, the bigger entity is perceived as to be closer than the smaller.
- **The Clearness.** The clearness refers to the perceptual relation between clearness and distance of two entities [6]. For two entities of different clarities, the clearer entity is perceived as to be closer than the dimmer.
- **The Perspective.** The perspective law refers to depth of an entity or the distance from eye to two elements, one is near and one farther [6]. An entity in real world is perceived by three dimensions, horizontal, vertical, and depth. In other words, human vision organs can perceive an entity of three dimensions in real space, perspective pictures and images in mathematical space of three dimensions on plane.
- **The Movement Parallax.** The movement parallax law refers to perceptual relative movement of two entities, one is near and one is very far [6]. When moving and simultaneously looking at two entities, one is near and one is very far, human vision organs perceive the far entity moving on the same direction and the near entity moving on the opposing direction. For example, a person on moving car perceives a tree located far moving on the same direction but perceives trees on roadside moving on the opposite.

2.2.3 | Visual Features

The effectiveness of a pattern representing data is assessed by human's rate perceiving its significance. The analysis of characteristics of human vision organs in perceiving patterns results in the constitution of gestalt laws to enable designers to structure patterns representing visually data so that human is able to perceive much significance of data. The effective rate in perceiving a pattern, e.g. a graph representing data, which is formed by applying gestalt laws is objectively estimated by the following visual features [12].

- **The Association.** The associative feature refers to a pattern which enables human to perceive its similar elements in order to group them into a cluster. In other words, a pattern is considered as to be associative if its elements of the same characteristic are easily perceived by human vision organs according to gestalt laws of similarity, proximity, symmetry, continuation, closure, parallelism, common fate, common region, past experience.
- **The Selection.** The selective feature refers to a pattern which enables human to perceive dominant elements in the pattern and to distinguish clusters of elements in the pattern, of which characteristics are not similar. In other words, a pattern is considered as to be selective if it is easy to perceive dominant elements according to the focal point law or discriminate between different clusters of elements according to gestalt laws of similarity, proximity, symmetry, continuation, closure, parallelism, common fate, common region, past experience.
- **The Order.** The ordinal feature refers to a pattern which enables human to perceive the order of elements. In other words, a pattern is considered as to be ordinal if human vision organs easily perceive the order of elements, e.g. from small to big, or from low to high, or from below to above, or from left to right, or from east to west, and so on.
- **The Quantity.** The quantitative feature refers to a pattern which enables human to perceive the size of elements, the interval or the ratio of size of two elements. In other words, a pattern is considered as to be quantitative if human vision organs easily cognize the different sizes of elements and perceive the interval or the ratio of sizes of elements.
- **The Value Range.** The value range refers to the capacity of screen to display all values of variables. i.e. the screen of limited size and resolution is able to display all values of variables. A pattern is completely perceived if all its elements are displayed on screen, the value range of variables is smaller than the size of screen.

3 CUBE APPROACH TO REPRESENTING MULTIVARIATE DATA

The representation of multivariate data is a mapping of dataset onto computer screen. The laws of space perception consider that human vision organs can perceive the vertical, the horizontal, and the depth of entities in real world. This law regards human vision perceiving perspective pictures or images of three mathematical dimensions drawn on plane as objects in real world. Mathematically, the maximal number of dimensions rendered on plane that human vision organs can perceive is three dimensions shaped as a cube. The problem to be solved is that the graph representing multivariate data must meet the gestalt law of "The whole is greater than the sum of components" and the law of perspective, i.e. all data variables of dataset have to be represented as a graph on multidimensional coordinate system and rendered as a cube of three dimensions. Accordingly, the mapping has to refer to the characteristics of the dataset and human vision perception. In other words, cube representing multivariate data, called multidimensional cube, has to be designed to respond to the characteristics of data and meet the capacity of human vision perception.

3.1 | Structure of Dataset

Mathematically, a dataset is constituted by data variables and the relationships between the variables, where each variable is a collection of values and considered as a component of the dataset [13]. According to functional view, each data variable may be independent from or dependent on the others, where independent variable is utilized as reference variable, dependent variable is function of reference variable. Each value of a dependent variable may associate with one or more values of a reference variable, i.e. there are no two or more values of a dependent variable associating with the same value of a reference variable.

According to GIS (Geographic Information Science), data of object or field are composed of variables depicting attributes, location, and time [14, 15]. Time variable, briefly Time, is a set of temporal values each of which is a time point or a time interval, called a time unit. Time is the naturally independent variable of which each value is associated by one and only one value of each attribute and/or associated by one and only one spatial position of a moving or flying object. Location variable, briefly Space, is considered as an independent variable to indicate the spatial position of stationary object or as a reference variable of attributes changing at locations, as a variable dependent on Time to indicate spatial positions of moving or flying objects, where the spatial position is a function of Time. Attributes are dependent variables, each value of an attribute associates with one or more value of Time or Space.

3.2 | Relations between Variables in Dataset

While studying elementary questions in data analytics, Peuquet considered the relations between questions of What, When, Where and proposed the triad of What-When-Where (3W-triad) [16] (Figure 3a). According to 3W-triad, a question is answered if the factors of two other questions are given. The question of What is answered if the factors of When and Where are given, the question of When is answered if the factors of What and Where are given, the question of Where is answered if the factors of What and When are given. Mathematically, these relations can be depicted as follows: $\text{What} \leftarrow \text{When} + \text{Where}$, $\text{When} \leftarrow \text{What} + \text{Where}$, $\text{Where} \leftarrow \text{What} + \text{When}$.

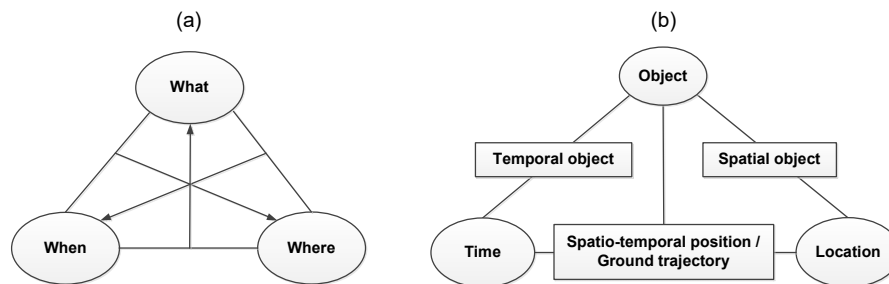


FIGURE 3 (a) The 3W-triad [16]; (b) The OTL-triad [17]

Andrienko et al. converted the 3W-triad by Peuquet to triad of Object-Time-Location (OTL-triad) to depict the relations between object and time, object and location [17] (Figure 3b). An object existing during a determined time interval at undetermined spatial position is considered as a temporal object or an event [18]. An object existing at a determined spatial position during undetermined interval is considered as a spatial object [18]. An object of which spatial and temporal positions are determined is considered as spatio-temporal object [18]. A spatio-temporal object of which spatial position changes over time, i.e. spatial position is a function of time position, is considered as a moving object [18], also called mover.

Both triad models by Peuquet and by Andrienko did not mention the attributes of object as well as their relations with time and/or location. Hong et al. proposed OATL-quad by adding the collection of attributes to OTL-triad [18]. In that, the relations between the collection of object and the value collection of an attribute is a many-one mapping, e.g. an object has only one color but a color may be applied for many objects. The OATL-quad represents the relations between object together with attributes and Time, 2D-Space (collection of locations on ground). OATL-quad represents the association of Attributes with Time and Attributes with 2D-Space, as well as the relations constituting temporal object, spatial object, spatio-temporal object, and moving object.

Generally, Hong et al. consider that the dataset depicting entities, events, or phenomena is structured by data variables such as Object, Attributes, Time, 2D-Space, 3D-Space [18]. The variable of object is the collection

of the names of objects, the variables of attributes are the collections of values of the attributes, the variable of time is the collection of time units each of which is a time point or an time interval, the variable of 2D-Space or 2D-Location is the collection of the positions on ground , and the variable 3D-Space or 3D-Location is the collection of positions in elevated space [18]. In that, the variable of time is independent, the variables of 2D- and 3D-Space may be independent or dependent, the variables of object and attributes are dependent on Time or 2D- and 3D-Space. The relations of the dependent variables of object and attributes to the independent variables of time and space in OATL-Pentad (Figure 4) are classified into three classes of reference, the class of Time reference, the class of 2D-Space reference, and the class of 3D-Space reference.

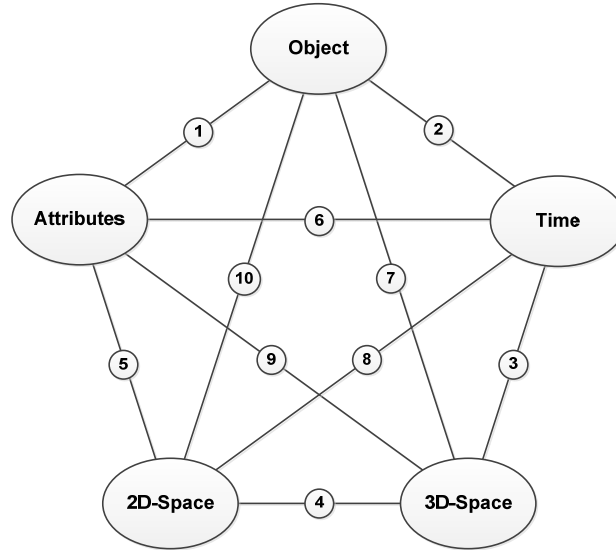


FIGURE 4 The star (OATL-Pentad) representing the relations among data variables in dataset [18], where let O = Object, A = Attributes, T = Time, L^2 = 2D-Space, L^3 = 3D-Space.

3.2.1 | Class of Time Reference

TABLE 1 The class of time reference

Relation combination	Functional relation	Type of objects
6	$T \rightarrow A_1 \times A_2 \times \dots$	Attributes refer to Time
2	$T \rightarrow O$	Object refers to Time
1, 2, 6	$T \rightarrow O \times A_1 \times A_2 \times \dots$	Object together with their attributes refer to Time

The characteristics of time reference of variables in dataset are presented in the table 1. The relation of 6 associates the values of attributes with time units, i.e. each Attribute is a mathematical function of Time. The relation of 2 associates each object with time units, where the variable of object is a collection of temporal objects or events which exist at indicated time units [17], i.e. Object is mathematical function of Time. The relation combination of 1, 2, 6 associates the combination of object and the values of attributes with time units to represent the variation of the attributes of object over time, i.e. each data tuple including object and its attributes associates with a time unit (Figure 4).

3.2.2 | Class of 2D-Space Reference

Considering the relations of an object belonging to the object variable and its attributes belonging to the attribute variable, if the object is indicated its location in 2D-Space at a time unit when it exists, the object is an spatio-

temporal object on ground, if the location in 2D-Space of the spatio-temporal object refers to time units, i.e. the location of the object changes over time, the spatio-temporal object becomes moving object (Figure 4). For spatio-temporal object, the relation of 10 associates each object (element) of the object variable with a location belonging to 2D-Space to indicate the position of the object on ground; the relation combination of 2, 10 associates the object with a time unit and its position on ground to represent the existence of spatio-temporal object of which existent time unit at a location on ground is indicated; the relation combination of 1, 2, 6, 10 associates the object and its attributes with time unit and location on ground. For moving object, the relation combination of 2, 10, 8 associates the object and its ground position with a time unit, where the relation of 8 indicates its ground position changing over time [17]; the relation combination of 1, 2, 6, 10, 8 associates the object along with its attributes and its position on ground with a time unit, where the relation of 8 indicates its ground position changing over time to represent the movement of the object on ground and the variation of its attributes over time; the relation combination of 1, 2, 5, 10, 8 associates the object along with its attributes with a location in 2D-Space, where its ground position changes over time to indicate the movement of the object on ground and the variation of its attributes at ground positions. Mathematically, the relations referring to 2D-Space are presented on the table 2.

TABLE 2 The class of 2D-Space reference

Relation combination	Functional relation	Type of objects
10	$L^2 \rightarrow O$	Spatial objects in 2D-Space (on ground)
2, 10	$L^2 \times T \rightarrow O$	Spatio-temporal objects on ground
1, 2, 6, 10	$L^2 \times T \rightarrow O \times A_1 \times A_2 \times ..$	Spatio-temporal objects along with their attributes on ground
2, 10, 8	$T \rightarrow L^2 \rightarrow O$	Moving objects on ground
1, 2, 5, 10, 8	$T \rightarrow L^2 \rightarrow O \times A_1 \times A_2 \times ..$	Moving objects along with its attributes on ground
1, 2, 6, 10, 8	$T \rightarrow L^2 \times O \times A_1 \times A_2 \times ..$	Moving objects along with its attributes on ground

3.2.3 | Class of 3D-Space Reference

Considering the relations of an object belonging to the object variable and its attributes belonging to the attribute variable, if the object is indicated its location in 3D-Space at time unit when it exists, the object is spatio-temporal object in elevated space, if the location in 3D-Space of the spatio-temporal object refers to time units, i.e. the location of the spatio-temporal object changes over time, the object becomes flying object (Table 3). For a spatio-temporal object in elevated space, the relation of 7 associates the object with a location in 3D-Space to indicate its position in elevated space; the relation combination of 7, 4, 10 associates the object with a location in 3D-Space to indicate its positions in elevated space and its projection on ground, where the relation of 4 associates its position with its projection on ground; the relation combination of 2, 7 associates the object with its location in elevated space and its existent time unit to represent it as an spatio-temporal object. For flying object, the relation combination of 2, 3, 7 associates the object and its positions in 3D-Space with corresponding time units, where the relation of 3 indicates the change of position in 3D-Space over time units to represent flying trajectory of flying object; the relation combination of 2, 3, 7, 4, 8 associates the object and its positions in elevated space with corresponding time units to represent the flight of the object and its projection on ground, where the relation of 4 associates its position in elevated space with its projection on ground and the relation

combination of 4, 8 represents its ground trajectory; the relation combinations of 2, 3, 7, 1, 6 and 2, 3, 7, 4, 8, 1, 6 associate the object along with its position in elevated space and the values of its attributes with corresponding time unit; the relation combination of 2, 3, 7, 4, 8, 1, 9 represents the flight of the object and associates its attributes with elevated locations.

TABLE 3 The relationships referring to 3D-Locations.

Relation combination	Functional relationship	Type of objects
7	$L^3 \rightarrow O$	Spatial objects in 3D-Space (in elevated space)
7, 4, 10	$L^3 \rightarrow O$ and $L^3 \rightarrow L^2$	Spatial objects in elevated space
2, 7	$L^3 \times T \rightarrow O$	Spatio-temporal objects in elevated space
2, 3, 7	$T \rightarrow L^3 \rightarrow O$ $T \rightarrow L^3 \times O$	Flying objects
2, 3, 7, 4, 8	$T \rightarrow L^3 \times O$ and $L^3 \rightarrow L^2$	Flying objects and the projections of their trajectories on ground
2, 3, 7, 1, 6 and 2, 3, 7, 4, 8, 1, 6	$T \rightarrow L^3 \times O \times A_1 \times A_2 \times ..$ and $L^3 \rightarrow L^2$	Flying objects with attributes changing over time and the projections of their trajectories on ground
2, 3, 7, 4, 8, 1, 9	$T \rightarrow L^3 \rightarrow O \times A_1 \times A_2 \times ..$ $T \rightarrow L^3 \times O \times A_1 \times A_2 \times ..$ $L^3 \rightarrow L^2$	Flying objects with attributes changing at locations and the projections of their trajectories on ground

3.3 | Multidimensional Cubes Representing Multivariate Data

The characteristics of perspective perception of human vision organs result in the structure shaped cube to represent multivariate data. The taxonomy of the relations in dataset results in the categorize of multidimensional cube representing multivariate data. Multidimensional graph representing multivariate data is rendered on multidimensional coordinates shaped as a cube on plane. Non-space multidimensional cube results from the relations referring to Time, 2D-Space multidimensional cube results from the relations referring to 2D-Space, 3D-Space multidimensional cube results from the relations referring to 3D-Space [18].

- **Non-space multidimensional cube [18].** Non-space multidimensional cube utilizes parallel axes on the same plane of parallel coordinates to represent the values of attributes, each axis represents the values of an attribute, where the origins of the attribute axes join with the scalar axis perpendicular to attribute axes. The time axes perpendicular to the plane of attribute axes are synchronized time units and join with attribute axes at the scalar axis. The combination of attribute axes, time axes, and scalar axis forms a shaped cube which is called a non-space multidimensional cube representing non-space multivariable data. Each attribute axis combines with a time axis to constitute attribute plane representing the relation of 6 in the figure 4, the variation of the attribute over time. The relation of 1 is rendered on attribute planes, each attribute variable relates to one or more objects of the object variable.
- **2D-space multidimensional cube [18].** 2D-Space multidimensional cube is constituted by joining a space-time cube [18, 19] to a non-space multidimensional cube based on shared axis of time. For spatio-temporal objects, the relation of 10 and the relation combination of 2, 10 are represented by the space-time cube to indicate the ground and space-time positions of objects; the relation combination 1, 2, 6, 10 is represented by the integration of two cubes, the space-time cube indicates the ground positions of objects over time and the non-space multidimensional cube represents the variation of attributes of object over time. For moving objects, the non-space multidimensional cube represents the variation of attributes of the objects over time, the space-time cube indicates the spatial positions of the objects changing over time, the relation of 8 traces trajectories of moving objects on space-time cube [18]. For moving objects, the relation combination of 2,

- 10, 8 is represented on the space-time cube as ground trajectories; the relation combination of 1, 2, 6, 10, 8 is represented on both cubes as ground trajectories and the variation of attributes of objects over time; the relation combination of 1, 2, 5, 10, 8 is represented on the both cubes, where the variation of attributes at ground locations is represented by mapping ground locations onto time axis [18].
- **3D-space multidimensional cube [18].** 3D-Space multidimensional cube is constituted by joining a 3D-Space cube to 2D-Space multidimensional cube based on 2 axes indicating ground locations [18]. For spatio-temporal objects in elevated space [18], the relation of 7 is represented by 2 axes indicating ground locations; the relation combination of 7, 4, 10 is represented by 3D-Space cube; the relation combination 2, 7 is represented by the 3D-Space cube and the space-time cube [18, 19], where the relation of 7 is indicated by three axes of the 3D-Space cube and the relation of 2 is indicated by the time axis of the space-time cube. For flying objects, the relation combinations of 2, 3, 7 and 2, 3, 7, 4, 8 are represented by 3D-Space cube and the space-time cube, where the spatial trajectories of objects are indicated by three axes of 3D-Space cube and the time axis of space-time cube, the ground trajectories of objects are indicated by two axes indicating ground locations; the relation combinations of 2, 3, 7, 1, 6 and 2, 3, 7, 4, 8, 1, 6 are represented by flying trajectories on 3D-Space cube and the time axis of the space-time cube, the attributes changing over time are represented on the non-space multidimensional cube [18] with the time axis shared with the space-time cube [18, 19].

4 | TECHNICAL APPROACH TO STRUCTURING VISUAL GRAPH REPRESENTING DATA

The two-stage visualization system for knowledge discovery in data applies principle “the whole is more than the sum of its parts” [6, 7, 9] of the gestalt laws to design the stage of visual technique. In that, all data of dataset are represented by only one graph, the relations between data variables may be rendered more explicitly to enable people to perceive significance of dataset. The effectiveness of a data visualization system is higher when human extracts much information and/or discovers much knowledge in dataset by viewing graph representing dataset. In other words, graphs designed according to gestalt laws enable human to perceive well implicit significance.

4.1 | Two-component Approach to Converting Data to Knowledge

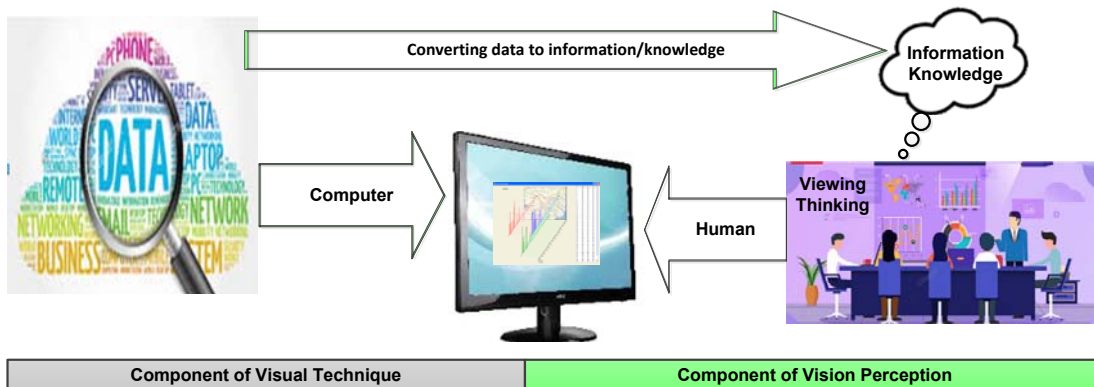


FIGURE 5 Two-component approach to converting data to information/knowledge [12].

Nowadays, while technical processes and systems are researched to reduce man power or exclude human role, the visualization system for discovering knowledge in data keeps human within the system as a component of the

system [18]. As a vision-based system for discovering knowledge from data, a visualization system is considered as a system for the collaboration between human and computer to convert data to knowledge. The system is composed of two components, visual technique and vision perception (Figure 5) [12]. Computer is designed to convert dataset to graph which human can view to think and discover significance implicit in data. Technically, computer along with its soft tools is designed to represent dataset as a visual graph on screen, human together with his available knowledge and experience views the graph to perceive significance of dataset by his vision organs.

4.2 | The Component of Visual Technique

The component of visual technique converts data to structural graph suitable for human perception of vision. The approach to converting dataset to visual graph rendered on planar screen is carried out on two stages, structure and rendering (Figure 6). At structure stage, the dataset of multivariate data is represented as a structural graph of multi-dimensions so that the graph preserves the features of the dataset. At rendering stage, structural multidimensional graph is mapped onto 2-dimensional screen as a planar multidimensional graph so that human vision organs can perceive the features of the set of multidimensional data. In other words, the design at the structure stage is affected more by the characteristics of dataset, the design at the rendering stage is affected more by the characteristics of human capacity of vision. A visualization system is effective for knowledge discovery when the planar multidimensional graph displaying on screen not only represents completely the features of dataset but also enables user to well perceive the significance of dataset.

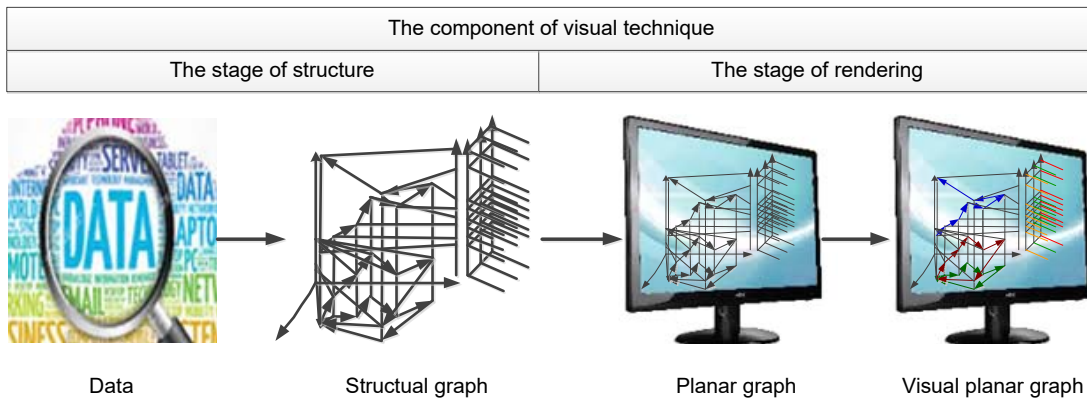


FIGURE 6 The component of visual technique composed of structure stage and rendering stage

4.2.1 | The Stage of Structure

At the stage of structure, dataset is converted to structural graph. Applying laws of shape and space perception, structural graph is demanded to not only render adequately the variables and relations in dataset, but also display as a cube appropriate to human vision perception. The conversion of data to structural graph is carried out according to the following process (Figure 7).

- **Step 1: Structuring data variables.** The values of dataset are grouped into subsets of the dataset by the attributes which the values depict. Each subset is called a data variable of which the characteristic is defined as the mathematical characteristic of its values such as nominal, ordinal, intervallic, or rational characteristic [20-22].
- **Step 2. Surveying the mathematical relations between variables.** According to the view of mathematical function, each data variable is considered as to be independent from or dependent on the others. For an independent variable, the value of the variable varies independently, e.g. variable of time. For a dependent variable, the change of value is mathematically dependent on other variable which is called reference variable or referrer. In other words, a dependent variable is a mathematical function of reference variable, e.g. an attribute of entity is a function of time.
- **Step 3: Grouping data variables.** Data variables are grouped into variable combinations of two or three

variables by their functional relations. Each variable combination contains a variable shared with another variable combination.

- **Step 4. Representing variable combinations.** Each variable combination is represented as a graph of two or three dimensions, where each dimension of the graph represents a data variable, each mark of the graph represents a value of data variable or a tuple of values.
- **Step 5. Designing structural graph.** The graphs of two and three dimensions are joined with one another by sharing reference dimensions to make a structural graph. Graphs of 2-and 3-dimensions are arranged so that the structural graph not only shapes cube but also enables user to easily perceive the significance of dataset. The characteristics of relations between variables in dataset and user's demands indicate the style of structural graph and multidimensional cube.

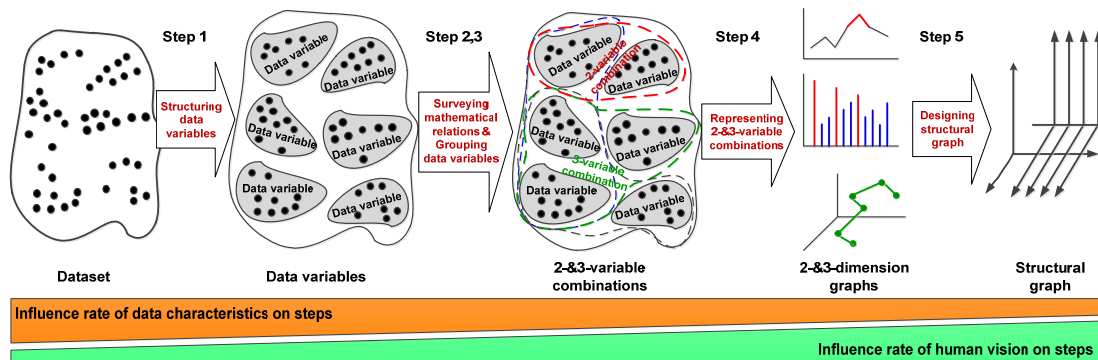


FIGURE 7 The steps converting dataset to structural graph of the structure stage

The process mapping dataset onto structural graph is affected by the characteristics of dataset and user's demands. From step 1 to 5, the impact of data characteristics on the process reduces gradually, meanwhile the impact of user's demands increases gradually. At the step 1, data are grouped into variables by data characteristics. At the step 5, the joining of 2-and 3-dimensional graphs to make structural graph of multi-dimensions is impacted by user. In some cases, user need re-arrange 2-and 3-dimensional graphs to obtain a the best structural graph for extracting information and discovering knowledge.

4.2.2 | The Stage of Rendering

At the stage of rendering, structural graph is mapped onto planar graph of higher visual features to display on screen. The stage of rendering comprises two functions, mapping structural graph onto planar screen and integrating retinal variables into planar graph to improve visual features of the graph [1] (Figure 6).

- **The function for mapping structural graph onto planar screen.** Structural graph of multi-dimensions is converted to planar graph on screen by indicating the positions of dimensions and marks of structural graph on two dimensions of planar screen. Mathematically, the elements of structural graph are indicated on the 2-dimension coordinates of planar screen as marks of planar graph. The challenge to be solved is that the limitation of size and resolution of screen reduces visual features.
- **The function for integrating retinal variables.** The effectiveness of planar graph in extracting information and discovering knowledge is estimated by the visual features of planar graph for human perception of vision [1]. The integration of retinal variables into graph is an effective approach to increase visual features of graph. The problem for integrating retinal variables is discussed in the next section.

4.3 | The Component of Vision Perception

In the component of vision perception, people play an important role in extracting information and discovering knowledge. Human collaborates with computer by interacting on planar graph to increase visual features of graph so that it is suitable for the demands of human vision perception and the problem of data analysis. Tools of graphic software enable human to affect on planar graph in order to answer analytical questions. The flexibility in displaying planar graph results in much profundity in answering questions for analyzing data by viewing and thinking. Due to human position in the component of vision perception, user's available knowledge and experience impact on the results of extracting information and discovering knowledge from dataset.

5 | INTEGRATING RETINAL VARIABLES INTO GRAPH [1]

The effectiveness of a graph representing a dataset is objectively evaluated in the light of visual features. The features of association and selection enable people to perceive marks of the same cluster and discriminate marks of different clusters, respectively. The features of order and value range enable people to perceive the order and quantity of marks, respectively. The integration of retinal variables into graph [1] results in the increase of the visual features and enables people to perceive well the characteristics of marks representing data. The integration of retinal variables into graph [1] is created according to the process as follows.

5.1 | Step 1. Grouping Marks of Graph into Mark Clusters [1]

Human need cognize the similarity or the difference of objects to understand real world. In a data visualization system, human vision organs have to perceive the marks representing values of the same variable or representing values of the same characteristic as well as detect the marks representing values of different characteristics. Accordingly, it is necessary to group the marks representing values of the same characteristic into mark clusters to integrate retinal variables according to laws of shape perception. The results are to increase visual features of graph and assist human capacity of vision perception in data analysis [1].

Mathematically, the marks of structural graph G' are classified as mark clusters, called $G_m | m=1,2,..$, by qualitative and quantitative characteristics of data values which the marks represent. Applying the approach of grouping by quality, marks representing values of the same nominal or ordinal characteristic are grouped into mark clusters of name or order. Applying the approach of grouping by quantity, marks representing the same intervallic or rational value are grouped into mark clusters of interval or ratio.

5.1.1 | Approach to Grouping Qualitative Marks

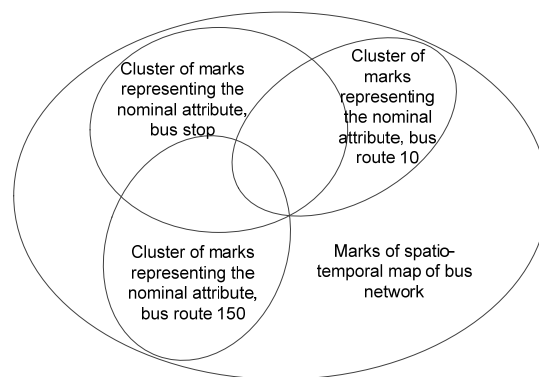


FIGURE 8 The Venn diagram depicting the relations between clusters of marks representing nominal attributes, e.g. the nominal attribute of bus stop, the nominal attribute of bus route 10, the nominal of bus route 150 in a spatio-temporal map of bus network [23, 24].

Marks representing values of the same attribute are grouped into a mark cluster. The mark clusters render the

association of the marks in the same cluster and the discrimination between marks of different mark clusters to respond associative and selective features in data visualization. A mark having many qualitative characteristics may belong to several different mark clusters, e.g. the mark which represents a bus stop used by two bus routes titled 10 and 150 belongs to the intersection of clusters, the mark cluster of bus stop called G_1 , the mark cluster of bus route titled 10 called G_2 , and the mark cluster of bus route titled 150 called G_3 (Figure 8). Technically, the graph G' is the union of mark clusters G_m , i.e. $G' = \bigcup_m G_m \mid m=1,2,..$.

5.1.2 | Approach to Grouping Quantitative Marks

Marks representing quantitative values are grouped into mark clusters to associate marks representing the same interval or rational value and differentiate between marks representing different interval or rational values. The value range of an attribute may be divided into intervals from zero to global maximum or from local minimum to local maximum. Mathematically, the division of value range is a mapping of real value onto natural value by a transfer function (Figure 9). Due to the demands in extracting information from graph, the transfer function is applied for different value ranges. The shape of transfer function is designed to meet the demands of problem of data analysis.

- **Dividing value range from zero or global minimum to global maximum.** In this case, the value range from zero or global minimum to global maximum of an attribute is divided into value segments. Marks representing values of the same segment are grouped into a cluster to differentiate from the values of different segments. The shape of transfer function indicates segments as scales which are attentive.

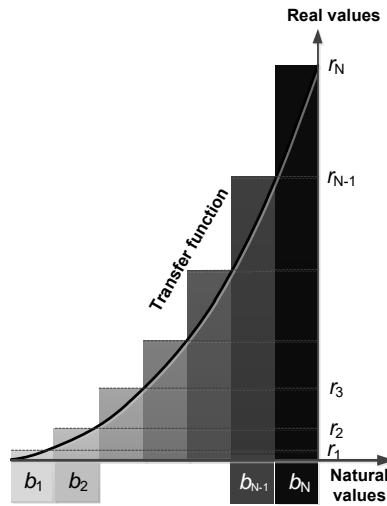


FIGURE 9 The transfer function converting real values of attributes to natural values as scales

- **Dividing value range from local minimum to local maximum.** In this case, each maximum or minimum mark representing a value of local maximum or local minimum, respectively, associates with its reference value. Each maximum or minimum mark and some behaviors are grouped into a cluster, called maximum or minimum cluster, other marks are grouped into different clusters. Technically, differential algorithm is applied to indicate maximum or minimum marks associating with their reference values. Each maximum (or minimum) mark belongs to both maximum (or minimum) cluster and qualitative mark

cluster. In other words, each maximum (or minimum) cluster is a subset of a qualitative cluster and does not intersect other maximum (or minimum) cluster (Figure 10).

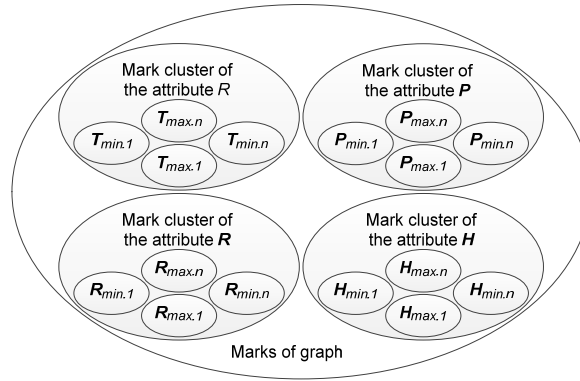


FIGURE 10 The Venn diagram depicting the relations between clusters of attributes (qualitative marks) and clusters of maximum and minimum marks.

5.2 | Step 2. Constituting Generated Retinal Variables [1]

The basic retinal variables are composed of Shape (S), Size (Z), Brightness (B), Symbol (L), Color (C), and Direction (D) [25-27], where the number of elements of each variable is indefinite. A generated retinal variable is constituted from a basic retinal variable as a subset of a basic retinal variable or from a defined Cartesian product of two or more basic retinal variables as a subset of this product [1]. The number of necessary elements of a generated retinal variable is determined by the number of mark clusters [1]. Each generated retinal variable $R^S, R^Z, R^B, R^L, R^C, R^D$ is constituted from a basic retinal variable S, Z, B, L, C, D , respectively, and is a subset of the basic retinal variable $R^S \subset S, R^Z \subset Z, R^B \subset B, R^L \subset L, R^C \subset C, R^D \subset D$, respectively. Each generated retinal variable $R^{SB}, R^{ZL}, R^{BC}, R^{LD}, R^{CS}, R^{DL}$ is constituted from the product of two basic retinal variables SB, ZL, BC, LD, CS, DL , respectively, and is a subset of the basic retinal variable $R^{SB} \subset SB, R^{ZL} \subset ZL, R^{BC} \subset BC, R^{LD} \subset LD, R^{CS} \subset CS, R^{DL} \subset DL$, respectively, and so on. For improving visual features of complex multidimensional graphs representing multivariate data, the approach of product of basic retinal variables increases the number of generated retinal variables to meet the demand of the number of mark clusters divided from graph marks.

5.3 | Step 3. Integrating Retinal Variables into Graph [1]

The integration of retinal variables into graph is an approach to improving visual features, where marks of the same attribute or the same value segment are associated with one another and differentiated from other attributes or segments, respectively [1]. Marks belonging to the intersection of mark clusters are integrated by retinal elements generated from Cartesian product of two or more basic retinal variables so that the marks render adequately different characteristics of the mark clusters to which they belong, e.g. a bus stop belonging to two different bus routes [23, 24] need be integrated generated retinal variable so that it may be perceived to be a bus stop belonging to both bus routes. Mathematically, the integration of retinal variables into graph is definable Cartesian products of generated retinal variables and mark clusters to convert low-visual graph G' to high-visual graph G [1].

5.3.1 | Clusters of Marks Representing Qualitative Variables

Let G' be low-visual graph; G be high-visual graph; $R = \{R^S, R^Z, R^B, R^L, R^C, R^D, R^{SB}, R^{ZL}, \dots\}$ be the set of generated retinal variables; $G^k | k = 1, 2, \dots$ be the graph G' formed by combining the clusters $G_m | m = 1, 2, \dots$

according to the type $k | k = 1, 2, \dots$; $G_m^k | k = 1, 2, \dots; m = 1, 2, \dots$ be the cluster $m | m = 1, 2, \dots$ of the type $k | k = 1, 2, \dots$.

For clusters of marks representing qualitative variables, the mapping of G' onto G is mathematically defined as follows.

$$G = R \times G' = \{R^S, R^Z, \dots\} \times \{G^1, G^2, \dots\}$$

$$G = R \times G' = \{R^S \times G^1, R^Z \times G^2, \dots\} = (R^S \times G^1) \cup (R^Z \times G^2) \cup (R^B \times G^3) \cup \dots$$

The products may be generally defined as follows.

$$R^S \times G^1 = \{s_1, s_2, \dots\} \times \{G_1^1, G_2^1, \dots\} = \{s_1 G_1^1, s_2 G_2^1, \dots\}$$

$$R^Z \times G^2 = \{z_1, z_2, \dots\} \times \{G_1^2, G_2^2, \dots\} = \{z_1 G_1^2, z_2 G_2^2, \dots\}$$

$$R^B \times G^3 = \{b_1, b_2, \dots\} \times \{G_1^3, G_2^3, \dots\} = \{b_1 G_1^3, b_2 G_2^3, \dots\}$$

$$R^L \times G^4 = \{l_1, l_2, \dots\} \times \{G_1^4, G_2^4, \dots\} = \{l_1 G_1^4, l_2 G_2^4, \dots\}$$

$$R^C \times G^5 = \{c_1, c_2, \dots\} \times \{G_1^5, G_2^5, \dots\} = \{c_1 G_1^5, c_2 G_2^5, \dots\}$$

$$R^D \times G^6 = \{d_1, d_2, \dots\} \times \{G_1^6, G_2^6, \dots\} = \{d_1 G_1^6, d_2 G_2^6, \dots\}$$

and so on ...

5.3.2 | Clusters of Marks Representing Quantitative Variables

Each mark belonging to mark cluster of quantitative variables represents simultaneously its quantitative value and attribute. Accordingly, each quantitative mark has to be integrated by a product of two elements of two retinal variables, one indicates attribute and the other indicates quantitative value. For example, let

$G_{m,l} | m = 1, 2, \dots; l = 1, 2, \dots$ be the clusters of marks representing the value scales $l | l = 1, 2, \dots$ of the attributes

$m | m = 1, 2, \dots$; $R^C = \{c_1, c_2, \dots, c_m, \dots\}$ be the generated retinal variable of the basic retinal variable of color C , i.e.

$R^C \subset C$; $R^B = \{b_1, b_2, \dots, b_l, \dots\}$ be the generated retinal variable of the basic retinal variable of brightness B , i.e.

$R^B \subset B$. The product of two generated retinal variables R^C and R^B is $R^{BC} = R^B \times R^C = \{b_1 c_1, b_2 c_1, \dots, b_1 c_2, b_2 c_2, \dots\}$.

The integration of the retinal variables of color and brightness into the clusters $G_{m,l} | m = 1, 2, \dots; l = 1, 2, \dots$ is the following product:

$$R^{BC} \times G_{m,l} = \{b_1 c_1, b_2 c_1, \dots, b_m c_1, \dots\} \times \{G_{1,1}, G_{1,2}, \dots, G_{2,1}, G_{2,2}, \dots\} = \{b_1 c_1 G_{1,1}, b_2 c_1 G_{1,2}, \dots, b_1 c_2 G_{2,1}, b_2 c_2 G_{2,2}, \dots\}$$

6 | CONCLUSION

The research proposed a model of visualization system composed of two components to map a dataset onto information and knowledge. The system keeps human in the visualization system as one of two components, visual technique and vision perception. In the visualization system, user directly extracts information and discovers knowledge from data by the approach to viewing and thinking. In the visualization system, data are converted to visual graph on computer screen to be perceived insights by human. In other words, the visual graph

is demanded to satisfy data characteristics and meet simultaneously the requests of human vision organs.

The research studies the characteristics of human vision perception, which are analyzed as laws of shape perception and laws of space perception. The laws of shape perception depict that human vision organs cognize a pattern from the whole to details and that human vision organs can detect similar or different marks. The perspective law of space perception results in the structure of cube representing multivariate data, which is called multidimensional cube. The laws of shape perception enable designer to apply the approach to grouping marks to integrate retinal variables into graph marks.

Dataset is mathematically structured by five data collections including object, attributes, time, 2D-space, 3D-space, which are considered as data variables. The relations between variables are functionally analyzed as independent and dependent variables, where independent variables are utilized as reference variables or referrers and dependent variables refer to reference variables. The relations between variables are divided into three groups by three reference variables, Time, 2D-Space, 3D-Space. The taxonomy of data variables results in the constitution of three kinds of multidimensional cube representing a dataset of multivariate data, non-space cube, 2D-space cube, and 3D-space cube [18].

The effectiveness of visual graph in extracting information and discovering knowledge is estimated subjectively by human vision perception and objectively by visual features. For applying the approach to integrating of retinal variables into graph to increase visual features, this study divides graph marks into clusters by qualitative and quantitative characteristics [1]. The study proposes to constitute generated retinal variables from one basic retinal variable or from defined Cartesian products of basic retinal variables to integrate them into mark clusters. After integrating retinal variables into graph representing multivariate data, the visual graph is objectively estimated by visual features and interacted by user to subjectively improve human perception of vision.

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Secondary Sources

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