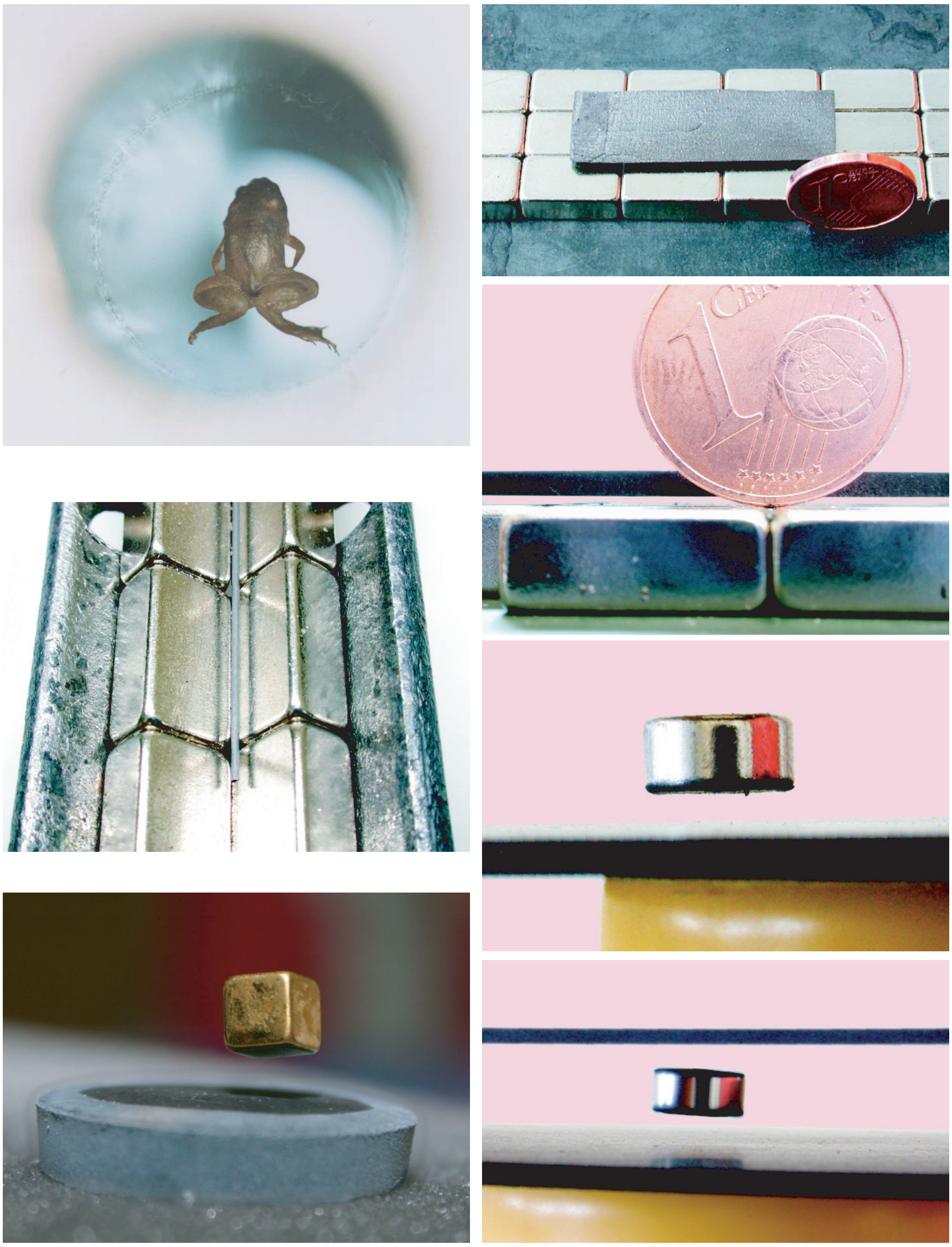

FIGURE 169 Stable diamagnetic levitation. Left: a living frog in a 16 T magnet, a graphite bar over rectangular permanent magnets, and a brass-coloured magnet over a superconducting ring. Right: two levitating graphite plates, one seen from above and another from the side; below, levitation of a 4 mm diameter NdFeB permanent magnet, above a graphite plate and between two graphite plates, near a large ring magnet that is not shown (© Lijnis Nelemans, Peter Nussbaumer, and Joachim Schlichting from Ref. 179).

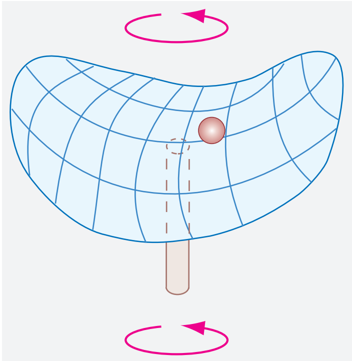

FIGURE 170 Trapping a metal sphere using a variable speed drill and a plastic saddle.

FIGURE 171 Floating 'magic' nowadays available in toy shops, left, with a spinning top and, right, with a spinning magnetic sphere levitating above a large ring magnet (© Kay Kublenz).

Ref. 184 device yourself.

Even free electrons can be levitated, letting them float above the surface of fluid helium. Ref. 185 In the most recent twist of the science of levitation, in 1995 Stephen Haley predicted that the suspension height of small magnetic particles above a superconducting ring should be quantized. However, the prediction has not been verified by experiment yet.

For the sake of completeness we mention that nuclear forces cannot be used for levitation in everyday life, as their range is limited to a few femtometres. However, we will Vol. V, page 209 see later that the surface matter of the Sun is prevented from falling into the centre by these interactions; we could thus say that it is indeed levitated by nuclear interactions.

Does gravity make charges radiate?

We learned in the section on general relativity that gravitation has the same effects as acceleration. This means that a charge kept fixed at a certain height is equivalent to a charge accelerated by $9.8\,\mathrm{m/s^2}$, which would imply that it radiates electromagnetically, since all accelerated charges radiate. However, the world around us is full of charges at fixed heights, and there is no such radiation. How is this possible?

Ref. 171 The question has been a pet topic for many years. Generally speaking, the concept of radiation is not observer invariant: If one observer detects radiation, a second one does not necessarily do so as well. The exact way a radiation field changes from one observer to the other depends on the type of relative motion and on the field itself.

A detailed exploration of the problem shows that for a uniformly accelerated charge, an observer undergoing the same acceleration only detects an electrostatic field. In contrast, an inertial observer detects a radiation field. Since gravity is (to a high precision) equivalent to uniform acceleration, we get a simple result: gravity does not make electrical charges radiate for an observer at rest with respect to the charge – as is indeed observed. The results holds true also in the quantum theoretical description.

Ref. 172

Matter, levitation and electromagnetic effects

The levitation used by magicians mostly falls into another class. When David Copperfield, a magician performing for young girls at the end of the twentieth century, 'flies' during his performances, he does so by being suspended on thin fishing lines that are rendered invisible by clever lighting arrangements. (How could one check this?) In fact, if we want to be precise, we should count fishing lines, plastic bags, as well as every table and chair as levitation devices. (Tabloid journalists would even call them 'anti-gravity' devices.) Contrary to our impression, a hanging or lying object is not really in contact with the suspension, if we look at the critical points with a microscope. The proof about lack of contact will arise in the quantum part of our walk.*

Challenge 217 s

But if a lying object is not in contact with its support, why don't we fall through a table or through the floor? We started the study of mechanics by stating that a key property of matter its *solidity*, i.e., the impossibility of having more than one body at the same place at the same time. But what is the origin of solidity? Solidity is due to electricity inside matter. Again, we will be discover the details only in the forthcoming, quantum part of our adventure, but we can already collect the first clues at this point.

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Not only solidity is due to electricity. Many other – in fact all – experiments show that matter is constituted of charged particles. Indeed, matter can be moved and influenced by electromagnetic fields in many ways. Over the years, material scientists have produced a long list of such effects, all of which are based on the existence of charged constituents in matter. An overview is given in Table 17. Can you find or imagine a new effect? For example, can electric charge change the colour of objects?

Ref. 186
Challenge 219 r

TABLE 17 Selected matter properties related to electromagnetism, showing among other things the role it plays in the constitution of matter; at the same time a short overview of atomic, solid state, fluid and business physics.

Property	Example	Definition
thermal radiation, heat radiation or incandescence	every object	temperature-dependent radiation emitted by any macroscopic amount of matter
emissivity	all bodies	ability to emit thermal light
Interactions with charges and currents (transport-related effects)		

Challenge 218 ny

* The issue is far from simple: which one of the levitation methods described above is used by tables or chairs?

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
electrification	separating metals from insulators	spontaneous charging
triboelectricity	glass rubbed on cat fur	charging through rubbing
barometer light	mercury slipping along glass	gas discharge due to triboelectricity Ref. 187
insulation	air	no current flow below critical voltage drop
semiconductivity	diamond, silicon or gallium arsenide	current flows only when material is impure ('doped')
conductivity	copper, metals	current flows easily
superconductivity	niobium below 9 K	current flows indefinitely
ionization	fire flames	current flows easily
localization (weak, Anderson)	disordered solids	resistance of disordered solids
resistivity, Joule effect	graphite, W	heating due to current flow
thermoelectric effects at contacts: Seebeck effect, Peltier effect	ZnSb, PbTe, PbSe, BiSeTe, Bi_2Te_3, etc.	current flow due to temperature difference, cooling due to current flow
thermoelectric effect in the bulk: Thomson effect	Fe, Bi, Co, Sb, Cu, Ag, etc.	cooling due to temperature gradients
acousto-electric effect	CdS	sound generation by currents, and vice versa
magnetoresistance (several different effects)	permalloy, perovskites, metal multilayers	electrical resistance changes with applied magnetic field Ref. 188
recombination	smoke detector	charge carriers combine to make neutral atoms or molecules
annihilation	positron tomography	particle and antiparticle, e.g. electron and positron, disappear into photons
Penning effect	H, Ne, Ar	neutral metastable excited atoms ionize other atoms through collisions
Richardson effect, thermal emission	BaO_2, W, Mo, used in tv and electron microscopes	emission of electrons from hot metals
skin effect	Cu, all conductors	high current density on exterior of wire at high frequency
pinch effect	InSb, plasmas	high current density on interior of wire
Josephson effect	Nb-Oxide-Nb	tunnel current flows through insulator between two superconductors
Sasaki–Shibuya effect	n-Ge, n-Si	anisotropy of conductivity due to applied electric field
switchable magnetism	InAs:Mn	voltage switchable magnetization Ref. 189

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
Hall effect	silicon and other semiconductors; used for magnetic field measurements	voltage perpendicular to current flow in applied magnetic field
Ettingshausen–Nernst effect	Bi	appearance of electric field in materials with temperature gradients in magnetic fields
optogalvanic effect	plasmas	change of discharge current due to light irradiation
Interactions with magnetic fields		
ferromagnetism	Fe, Ni, Co, Gd	spontaneous magnetization; material strongly attracted by magnetic fields
paramagnetism	Fe, Al, Mg, Mn, Cr	induced magnetization parallel to applied field; attracted by magnetic fields
diamagnetism	water, Au, graphite, NaCl	induced magnetization opposed to applied field; repelled by magnetic fields
magnetostriction (and the related Joule effect, Villari effect, Wiedemann effect, Matteucci effect, Barret effect and Nagaoka-Honda effect)	CeB_6, $CePd_2Al_3$, TbDyFe	change of shape or volume by applied magnetic field
magnetoelastic effect	Fe, Ni	change of magnetization by tension or pressure
acousto-magnetic effect	metal alloys, anti-theft stickers	excitation of mechanical oscillations through magnetic field
spin valve effect	metal multilayers	electrical resistance depends on spin direction of electrons with respect to applied magnetic field
Zeeman effect	atoms, e.g., Cd	change of emission frequency with magnetic field
optical orientation	paramagnetic gases	circularly polarized light and magnetic field align atomic spins due to Zeeman effect
Hanle effect	Hg, paramagnetic gases	change of polarization of fluorescence with magnetic field
Paschen–Back effect, Back–Goudsmit effect,	atomic gases	change of emission frequency in strong magnetic fields
magneto-optical activity or Faraday effect or Faraday rotation	flint glass	polarization angle is rotated with magnetic field; different refraction index for right and left circularly polarized light, as in magneto-optic (MO) recording

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
magnetic circular dichroism	gases	different absorption for right- and left-circularly polarized light; essentially the same as the previous one
Majorana effect	colloids	specific magneto-optic effect
photoelectromagnetic effect	InSb	current flow due to light irradiation of semiconductor in a magnetic field
inverse Faraday effect	GdFeCo	switch of magnetisation by a femtosecond laser pulse
Voigt effect	vapours	birefringence induced by applied magnetic field
Cotton–Mouton effect	liquids	birefringence induced by applied magnetic field
Shubnikov–de Haas effect	Bi	periodic change of resistance with applied magnetic field
thermomagnetic effects: Ettingshausen effect, Righi–Leduc effect, Nernst effect, magneto–Seebeck effect	BiSb alloys	relation between temperature, applied fields and electric current
photonic Hall effect	CeF_3	transverse light intensity depends on the applied magnetic field Ref. 190
magnetocaloric effect	gadolinium, GdSiGe alloys	material cools when magnetic field is switched off Ref. 191
cyclotron resonance	semiconductors, metals	selective absorption of radio waves in magnetic fields
magnetoacoustic effect	semiconductors, metals	selective absorption of sound waves in magnetic fields
magnetic resonance (many types)	most materials, used for imaging in medicine for structure determination of molecules	selective absorption of radio waves in magnetic fields; includes NMR, EPR, etc.
magnetorheologic effect	liquids, used in advanced car suspensions	change of viscosity with applied magnetic fields
Meissner effect	type 1 superconductors, used for levitation	expulsion of magnetic field from superconductors
Interactions with electric fields		
polarizability	all matter	polarization changes with applied electric field
ionization, field emission, Schottky effect	all matter, tv	charges are extracted at high fields

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
paraelectricity	$BaTiO_3$	applied field leads to polarization in same direction
dielectricity	deionized water, insulators	in opposite direction
ferroelectricity	$BaTiO_3$	spontaneous polarization below critical temperature
piezoelectricity	the quartz lighter used in the kitchen, human bones, $LiNbO_3$	polarization appears with tension, stress, or pressure
electrostriction	platinum sponges in acids	shape change with applied voltage Ref. 192
pyroelectricity	$CsNO_3$, tourmaline, crystals with polar axes; used for infrared detection	change of temperature produces charge separation
electro-osmosis or electrokinetic effect	many ionic liquids	liquid moves under applied electric field Ref. 193
electrowetting	salt solutions on gold	wetting of surface depends on applied voltage
electrolytic activity	sulphuric acid	charge transport through liquid
liquid crystal effect	watch displays	molecules turn with applied electric field
electro-optical activity: Pockels effect, Kerr effect	crystalline solids ($LiNbO_3$), liquids (e.g. oil)	electric field rotates light polarization, i.e., produces birefringence
Freederichsz effect, Schadt–Helfrichs effect	nematic liquid crystals	electrically induced birefringence
Stark effect	hydrogen, mercury	colour change of emitted light in electric field
field ionization	helium near tungsten tips in field ion microscope	ionization of gas atoms in strong electric fields
Zener effect	Si	energy-free transfer of electrons into conduction band at high fields
field evaporation	W	evaporation under strong applied electric fields
Linear interactions with light		
absorption	coal, graphite	transformation of light into heat or other energy forms (which ones?) Challenge 220 s
blackness	coal, graphite	complete absorption in visible range
colour	ruby	absorption depending on light frequency
metallic shine	metal, doped crystals	ability to act as ‘good’ mirror

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
chromatic dispersion	all materials	phase speed of light depends on wavelength
photostriction	PbLaZrTi	light induced piezoelectricity
photography	AgBr, AgI	light precipitates metallic silver
photoelectricity, photoeffect	Cs	current flows into vacuum due to light irradiation
internal photoelectric effect	Si p–n junctions, solar cells	voltage generation and current flow due to light irradiation
photon drag effect	p-Ge	current induced by photon momentum
transparency	glass, quartz, diamond	low reflection, low absorption, low scattering
reflectivity	metals	light bounces on surface
polarization	elongated silver nanoparticles in glass	light transmission depending on polarization angle
optical activity	sugar dissolved in water, quartz	rotation of polarization
birefringence, linear dichroism	calcite, cornea, thin polymer sheets	refraction index depends on linear polarization direction, light beams are split into two beams
circular dichroism	aminoacids, andalusite	absorption depends on circular polarization
optically induced anisotropy, Weigert effect	AgCl	optically induced birefringence and dichroism
Compton effect	momentum measurements	change of wavelength of X-rays and gamma radiation colliding with electrons
electrochromicity	wolframates	colour change with applied electric field
scattering	gases, liquids	light changes direction
Mie scattering	dust in gases	light changes direction
Raleigh scattering	sky	light changes direction, sky is blue
Raman effect or Smekal–Raman effect	molecular gases	scattered light changes frequency
switchable mirror	LaH	voltage controlled change from reflection to transparency Ref. 194
radiometer effect	bi-coloured windmills	irradiation turns mill (see page 122)
luminous pressure	*idem*	irradiation turns mill directly
solar sail effect	future satellites	motion due to solar wind
acousto-optic effect	TeO_2, $LiNbO_3$	diffraction of light by sound in transparent materials
photorefractive materials	$Bi_{12}SiO_{20}$, $LiNbO_3$, GaAs, InP	light irradiation changes refractive index
Auger effect	Auger electron spectroscopy	electron emission due to atomic reorganization after ionization by X-rays

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
Bragg reflection	crystal structure determination	X-ray diffraction by atomic planes
Mößbauer effect	^{57}Fe, used for spectroscopy	recoil-free resonant absorption of gamma radiation
pair creation	Pb	transformation of a photon in a charged particle–antiparticle pair
photoconductivity	Se, CdS	change of resistivity with light irradiation
optoacoustic effect, photoacoustic effect	gases, solids	creation of sound due to absorption of pulsed light; used for imaging of animal and human tissue
Light emission		
luminescence: general term for opposite of incandescence	GaAs, tv	light emission by cold matter
fluorescence	CaF_2, X-ray production, light tubes, cathode ray tubes, television tubes, dyes, coloured polymers, doped crystals	light emission during and after light absorption or other energy input
phosphorescence	$TbCl_3$, crystals doped with heavy metals	light emission due to light, electrical or chemical energy input, continuing *long after* stimulation
semiconductor luminescence	light-emitting diodes (LEDs), pointer lasers	emission of light due to electron hole recombination at p-n junctions
electroluminescence	ZnS powder	emission of light due to alternating electrical field
photoluminescence	ZnS : Cu, $SrAlO_4$: Eu, Dy, hyamine	light emission triggered by UV light, used in safety signs
chemoluminescence	H_2O_2, phenyl oxalate ester, dye solutions	chemically excited cold light emission, used in light sticks for divers and fun
bioluminescence	glow-worm, deep sea fish	cold light emission in animals, special type of chemoluminescence
triboluminescence	sugar	light emission during friction or crushing, not practical for lighting
thermoluminescence	quartz, feldspar, metastable ion dopants in crystals	light emission during heating, often shows irradiation memory, used e.g. for archaeological dating of pottery Ref. 195
sonoluminescence	air in water	light emission during cavitation
gravitoluminescence	does not exist; Challenge 221 s why?	

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
bremsstrahlung	X-ray generation	radiation emission through fast deceleration of electrons
Čerenkov effect	water, polymer particle detectors	light emission in a medium due to particles, e.g. emitted by radioactive processes, moving faster than the speed of light in that medium
transition radiation	any material	light emission due to fast particles moving from one medium to a second with different refractive index
Non-linear interactions with light		
laser activity, superradiation	beer, ruby, He–Ne, etc.	emission of stimulated radiation
quantum cascade laser	semiconductor multilayers	emission of stimulated infrared radiation through intersubband transitions
second, third n-th harmonic generation	$LiNbO_3$, KH_2PO_4	light partially transformed to double, threefold, n-fold frequency
phase conjugating mirror activity	gaseous CS_2, solid $Bi_{12}SiO_{20}$	reflection of light with locally opposite phase
additional optical nonlinear effects: parametric amplification, frequency mixing, saturable absorption, n-th harmonic generation, optical Kerr effect, Raman amplification, stimulated Brillouin scattering, etc.		
Interactions with vacuum		
Casimir effect	metals	attraction of uncharged, conducting bodies
General mechanical and thermal material properties		
solidity, impenetrability	floors, columns, ropes, buckets	at most one object per place at a given time
plasticity	metals	permanent deformation under stress
elasticity	solids	reversible deformation under stress
ferroelasticity	Ni-Ti alloys	spontaneous strain
viscosity	liquids, solids	deformation under stress due to component motion
heat capacity and heat conductivity	silver, marble, air	ability to store and to transport disordered atomic motion
Any other everyday material property	every material	

All matter properties given in the list can be influenced by electromagnetic fields or directly depend on them. This shows in detail:

▷ The nature of all everyday material properties is electromagnetic.

In other words, electric charges and their interactions are an essential and fundamental part of the structure of objects. The table shows so many different electromagnetic properties that the motion of charges inside each material must be complex indeed. Most effects are the topic of solid state physics,* fluid physics or plasma physics.

Solid state physics is by far the most important part of physics, when measured by the impact it has on society. Almost all its effects have applications in technical products, and give employment to many people. Can you name a product or business application

Challenge 222 e for any randomly chosen effect from the table?

In our mountain ascent however, we look at only one example from the above list: thermal radiation, the emission of light by hot bodies.

All bodies emit radiation

Earnshaw's theorem about the impossibility of a stable equilibrium for charged particles at rest implies that the charges inside matter must be *moving*. For any charged particle in motion, Maxwell's equations for the electromagnetic field show that it radiates energy by emitting electromagnetic waves. In short, we predict that all matter must radiate electromagnetic energy.

Interestingly, we know from experience that this is indeed the case. Hot bodies light up depending on their temperature; the working of light bulbs thus proves that metals are made of charged particles. *Incandescence*, as it is called, requires charges. Actually, *every* body emits radiation, even at room temperature. This radiation is called *thermal radiation*; at room temperature it lies in the infrared. Its intensity is rather weak in everyday

Ref. 196 life; it is given by the general expression

$$I(T) = fT^4 \frac{2\pi^5 k^4}{15c^2h^3} \quad \text{or} \quad I(T) = f\sigma T^4 \quad \text{with} \quad \sigma = 56.7\,\mathrm{nW/K^4m^2}\,, \qquad (85)$$

where f is a material-, shape- and temperature-dependent factor, with a value between zero and one, and is called the *emissivity*. The constant σ is called the *Stefan–Boltzmann black body radiation constant* or *black body radiation constant*. A body whose emissivity is given by the ideal case $f = 1$ is called a *black body*, because at room temperature such a body also has an ideal absorption coefficient and thus appears black. (Can you see why?)

Challenge 223 s The heat radiation such a body emits is called *black body radiation*. In the expression, h is Planck's constant; h is nature's quantum of action. The emission of thermal radiation is thus a quantum effect.

Ref. 197 By the way, which object radiates more energy: a human body or an average piece of

Challenge 224 s the Sun of the same mass? Guess first!

Challenges and curiosities about electromagnetic effects

The vertical electric field of the atmosphere, about 200 V/m, was already mentioned a few times. Incredibly, certain spiders use this field to fly! Can you imagine how

Challenge 225 e this might work? After you tried, watch the video at https://www.youtube.com/watch?

* Probably the best and surely the most entertaining introductory English language book on the topic is the one by Neil Ashcroft & David Mermin, *Solid State Physics*, Holt Rinehart & Winston, 1976.

v=JrS0igctMi0 to learn about the details.

* *

‘Inside a conductor there is no electric field.’ This statement is often found. In fact the truth is not that simple. Indeed, a *static* field or a *static* charge on the metal surface of a body does not influence fields and charges inside it. A closed metal surface thus forms a shield against an electric field. Can you give an explanation? In fact, a tight metal layer is not required to get the effect; a cage is sufficient. One speaks of a *Faraday cage*.

Challenge 226 s

The detailed mechanism allows you to answer the following question: do Faraday cages for gravity exist? Why?

For *moving* external fields or charges, the issue is more complex. Fields due to accelerated external charges – radiation fields – decay exponentially through a shield. Fields due to external charges moving at constant speed are strongly reduced, but do not disappear. The reduction depends on the thickness and the resistivity of the metal enclosure used. For sheet metal, the field suppression is very high; it is not necessarily high for metal sprayed plastic. Plastic shields will not necessarily protect a device from a close lightning stroke.

Ref. 198

In practice, there is no danger if lightning hits an aeroplane or a car, as long they are made of metal. (There is one film on the internet of a car hit by lightning; the driver does not even notice.) However, if your car is hit by lightning in dry weather, you should wait a few minutes before getting out of it. Can you imagine why?

Faraday cages also work the other way round. (Slowly) changing electric fields that are inside a Faraday cage are not felt outside. For this reason, radios, mobile phones and computers are surrounded by boxes made of metal or metal-sprayed plastics. The metal keeps the so-called *electromagnetic smog* to a minimum.

There are thus three reasons to surround electric appliances by a grounded shield: to protect the appliance from outside fields, to protect people and other machines from electromagnetic smog, and to protect people against the mains voltage accidentally being fed into the box (for example, when the insulation fails). In high precision experiments, these three functions can be realized by three separate cages.

For purely magnetic fields, the situation is more complex. It is quite difficult to shield the inside of a machine from outside magnetic fields. How would you do it? In practice one often uses layers of so-called *mu-metal*; can you guess what this material does?

Challenge 227 s

* *

Not only electric fields are dangerous. Also time-varying electromagnetic fields can be. In 1997, in beautiful calm weather, a Dutch hot air balloon approached the powerful radio transmitter in Hilversum. After travelling for a few minutes near to the antenna, the gondola suddenly detached from the balloon, killing all the passengers inside.

An investigation team reconstructed the facts a few weeks later. In modern gas balloons the gondola is suspended by high quality nylon ropes. To avoid damage by lightning and in order to avoid electrostatic charging problems all these nylon ropes contain thin metal wires which form a large equipotential surface around the whole balloon. Unfortunately, in the face of the radio transmitter, these thin metal wires absorbed radio energy from the transmitter, became red hot, and melted the nylon wires. It was the first time that this had ever been observed.

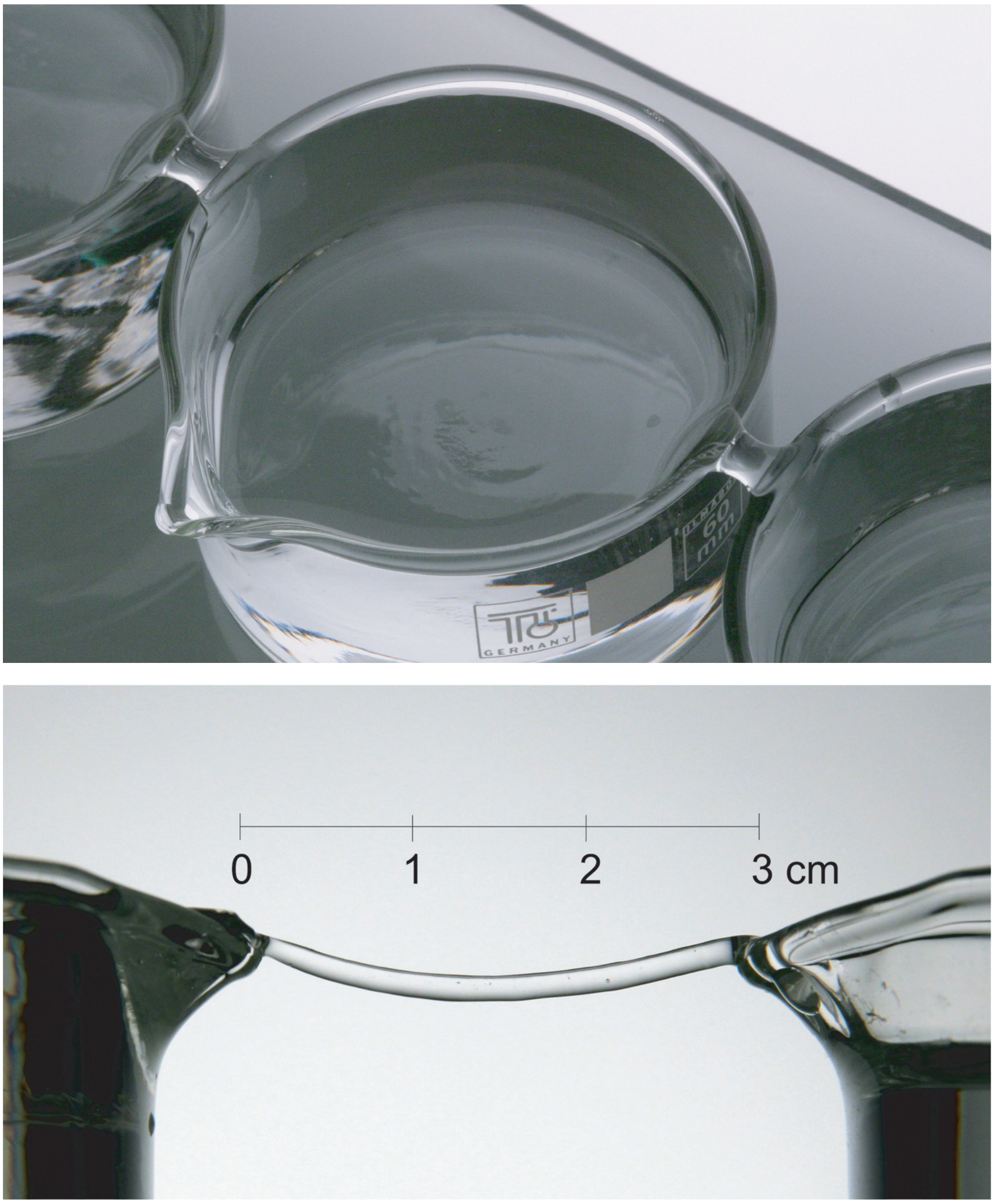

FIGURE 172 Floating water bridges sustained by high voltage between water containers and an example of the length that can be achieved in this way (© Elmar Fuchs).

* *

Ref. 200 Some researchers are trying to detect tooth decay with the help of electric currents, using the observation that healthy teeth are bad conductors, in contrast to teeth with decay. How would you make use of this effect in this case? (By the way, it might be that the totally unrelated techniques of imaging with terahertz waves or with optical coherence tomography could yield similar results.) Challenge 228 ny

* *

Something interesting occurs when high voltage, about 25 kV, is applied to two touching glasses filled with purified water and the glasses are then pulled apart. A floating water bridge appears. Examples are shown in Figure 172. The high voltage – it is dangerous, thus do not do this at home – makes the water flow from one glass to the other in a tube hanging in the air. For a complete introduction to this electrohydrodynamic effect see the beautiful website ecfuchs.com/?page=waterbridge. Ref. 199

* *

Ref. 201 Human bone is piezoelectric: it produces electric signals when stressed. When we move and grow, the electric signals are used by the body to reinforce the bones in the regions that are in need. The piezoelectricity of the bones thus controls and guides their growth. This connection is also used to make fractured bones heal more rapidly: by applying *pulsed* magnetic fields to a broken bone, the healing is stimulated and accelerated. (Static magnetic fields obviously do not work for this aim.) Also teeth are piezoelectric, and the effect plays also a role in their growth.

* *

In shops, one can buy piezoelectric devices – similar to a gas lighter – that are applied to mosquito bites and are said to reduce itching and even swelling. (Some product names are 'zanza click' and 'skeeter click') Can these claims be true? Challenge 229 e

* *

A team of camera men in the middle of the Sahara were using battery-driven electrical equipment to make sound recordings. Whenever the microphone cable was a few tens of metres long, they also heard a 50 Hz power supply noise, even though the next power supply was hundreds of kilometres away. An investigation revealed that the high voltage lines in Europe lose a considerable amount of power by irradiation; these 50 Hz waves are reflected by the ionosphere around the Earth and thus can disturb recording in the middle of the desert. Can you estimate whether this observation implies that living directly near a high voltage line is dangerous? Challenge 230 s

* *

When solar plasma storms are seen on the Sun, astronomers first phone the electricity company. They know that about 24 to 48 hours later, the charged particles ejected by the storms will arrive on Earth, making the magnetic field on the surface fluctuate. Since power grids often have closed loops of several thousands of kilometres, additional electric currents are induced, which can make transformers in the grid overheat and then switch off. Other transformers then have to take over the additional power, which can lead to their overheating, etc. On several occasions in the past, millions of people have been left without electrical power due to solar storms. Today, the electricity companies avoid the problems by disconnecting the various grid sections, by avoiding large loops, by reducing the supply voltage to avoid saturation of the transformers and by disallowing load transfer from failed circuits to others.

* *

Ref. 202 If the electric field is described as a sum of components of different frequencies, its so-called *Fourier components*, the amplitudes are given by

$$\hat{\boldsymbol{E}}(k,t) = \frac{1}{(2\pi)^3/2} \int \boldsymbol{E}(x,t) \mathrm{e}^{-i\boldsymbol{k}\boldsymbol{x}} \, \mathrm{d}^3 x \qquad (86)$$

and similarly for the magnetic field. It then turns out that a Lorentz invariant quantity N, describing the energy per circular frequency ω, can be defined:

$$N = \frac{1}{8\pi} \int \frac{|\boldsymbol{E}(k,t)|^2 + |\boldsymbol{B}(k,t)|^2}{c|\boldsymbol{k}|} \, \mathrm{d}^3 k \; . \qquad (87)$$

Challenge 231 s Can you guess what N is physically? (Hint: think about quantum theory.)

* *

Page 48 Faraday discovered, as told above, how to change magnetism into electricity, knowing that electricity could be transformed into magnetism. The issue is subtle. Faraday's law is not the dual of Ampère's, as that would imply the use of magnetic monopoles; neither is it the reciprocal, as that would imply the displacement current. But he was looking for a link and he found a way to relate the two observations – in a novel way, as it turned out.

Faraday also discovered how to transform electricity into light and into chemistry. He then tried to change gravitation into electricity. But he was not successful. Why not? Challenge 232 s

* *

At high altitudes (60 km to 1000 km) above the Earth, gases are partly or completely ionized; no atom is neutral. One speaks of the *ionosphere*, as space is full of positive ions and free electrons. Even though both charges appear in exactly the same number, a satellite moving through the ionosphere acquires a negative charge. Why? How does the charging stop? Vol. I, page 366 Challenge 233 s

* *

A capacitor of capacity C is charged with a voltage U. The stored electrostatic energy is $E = CU^2/2$. The capacitor is then detached from the power supply and branched on to an empty capacitor of the same capacity. After a while, the voltage obviously drops to $U/2$. However, the stored energy now is $C(U/2)^2$, which is half the original value. Where did the energy go? Challenge 234 s

* *

Challenge 235 s How can you give somebody an electric shock using a 4.5 V battery and some wire?

* *

An old puzzle about electricity results from the equivalence of mass and energy. It is known from experiments that the size d of electrons is surely smaller than 10^{-22} m. This Ref. 203

Challenge 236 e means that the electric field surrounding it has an energy content E given by at least

$$E_{\text{nergy}} = \frac{1}{2}\varepsilon_0 \int E^2_{\text{lectric field}}\, \mathrm{d}V = \frac{1}{2}\varepsilon_0 \int_d^\infty \left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\right)^2 4\pi r^2 \mathrm{d}r$$
$$= \frac{q^2}{8\pi\varepsilon_o}\frac{1}{d} > 1.2\,\mu\mathrm{J}\,. \qquad (88)$$

On the other hand, the *mass* of an electron, usually given as $511\,\mathrm{keV}/c^2$, corresponds to an energy of only 82 fJ, ten million times *less* than the value just calculated. In other words, classical electrodynamics has considerable difficulty describing electrons.

In fact, a consistent description of charged point particles within classical electrodynamics is impossible. Ref. 204 This topic receives only a rare – but then often passionate – interest nowadays, because the puzzle is solved in a different way in the quantum parts of our adventure.

* *

Even though the golden days of materials science are over, the various electromagnetic properties of matter and their applications in devices do not seem to be completely explored yet. About once a year a new effect is discovered that merits inclusion in the list Page 231 of electromagnetic matter properties of Table 17. Among others, some newer semiconductor technologies will still have an impact on electronics, such as the recent introduction of low cost light detecting integrated circuits built in CMOS (complementary metal oxide silicon) technology.

* *

The building of light sources of high quality has been a challenge for many centuries and remains one for the future. Light sources that are intense, tunable and with large coherence length or sources that emit extreme wavelengths are central to many research pursuits. As one example of many, the first X-ray lasers have recently been built; however, they are several hundred metres in size and use modified particle accelerators. The construction of compact X-ray lasers is still many years off – if it is possible at all.

* *

In many materials, left and right circularly polarized light is absorbed differently. The effect, called *circular dichroism*, was discovered by Aimé Cotton in 1896. Since circular dichroism appears in optically active chiral molecules, the measurement of circular dichroism spectra is a simple and important method for the structure determination of biological molecules.

* *

Effects of atmospheric electricity are also observed around waterfalls. Various studies have shown that large waterfalls produce negatively charged water droplets in the air around them. It even seems that inhaling these droplets is healthy, especially for people with asthma.

* *

But maybe the biggest challenge imaginable in classical electrodynamics is to decode the currents inside the brain. Will it be possible to read our thoughts with an apparatus placed outside the head?

Challenge 237 r One could start with a simpler challenge: Would it be possible to distinguish the thought 'yes' from the thought 'no' by measuring electrical or magnetic fields around the head? In other words, is simple mind-reading possible? The answer is yes, as the feat has already been achieved. Even more, using brain imaging, it is already possible to distinguish between simple concepts that a person has in mind. Ref. 205

Page 94 As we have seen above, partial mind-reading is also possible already for motion-related tasks, including some video games.

In fact, it is now possible to use a cap with electrical contacts and use passwords that you simply think about to secure computer systems. The advantage of such a password is that it is hard to steal. (Is this system secure?) Challenge 238 s

The twenty-first century will surely bring many new results also for the mind reading of cognitive tasks. The team first performing such a feat will become instantly famous.

Chapter 6

SUMMARY AND LIMITS OF CLASSICAL ELECTRODYNAMICS

All of classical electrodynamics can be summarized in three principles. Every adventurer should know them, because they will help us later on, when we approach the top of Motion Mountain, the goal of our adventure. We will discover that we can reach the top only if we express things as simply as possible. The three principles of classical electrodynamics are:

▷ *Definition:* Electric charges exert forces on other charges; for charges at rest, the force falls off as the inverse square of the distance. Equivalently, charges are surrounded by an *electromagnetic field*.

▷ *Conservation:* Electric charges are conserved.

▷ *Invaraince of c:* Charges move more slowly than light. Equivalently, all charged particles have mass.

Ref. 39 From these three principles we can deduce all of electrodynamics. In particular, we can deduce the following *basic statements*:

— The electromagnetic field is a physical observable, as shown e.g. by compass needles.
— The sources of the electromagnetic field are the (moving) charges, as shown by amber, lodestone or mobile phones.
— The electromagnetic field changes the motion of electrically charged objects via the Lorentz expression as shown, for example, by electric motors.
— The electromagnetic field can exist in empty space and moves in it as a wave, as shown, for example, by the light from the stars.
— The electromagnetic field behaves like a continuous quantity and is described by Maxwell's evolution equations, as shown, for example, by radio, the internet and electric toothbrushes.

More precisely, the motion of the electric field **E** and the magnetic field **B** is described by the Lagrangian density

$$\mathcal{L} = \frac{\varepsilon_0}{2} E^2 - \frac{1}{2\mu_0} B^2 . \qquad (89)$$

Like for any motion described by a Lagrangian, the motion of the electromagnetic field is reversible, continuous, conserved and deterministic. However, there is quite some fun in the offing; even though this description is correct in everyday life, during the rest of our mountain ascent we will find that the last basic statement must be wrong: fields do not always follow Maxwell's equations. A simple example shows this.

At a temperature of zero kelvin, when matter does not radiate thermally, we have the paradoxical situation that the charges inside matter cannot be moving, since no emitted radiation is observed, but they cannot be at rest either, due to Earnshaw's theorem. In short, the simple existence of matter – with its charged constituents – shows that classical electrodynamics is wrong.

In fact, the overview of the numerous material properties and electromagnetic effects given in Table 17 makes the same point even more strongly; classical electrodynamics can describe many of the effects listed, *but it cannot explain the origin and numerical values of any of them*. Even though few of the effects will be studied in our walk – they are not essential for our adventure – the general concepts necessary for their description will be the topic of the upcoming part of this mountain ascent, that on quantum physics.

Page 231

In fact, classical electrodynamics fails in *two* domains.

Space is curved, not flat

First of all, classical electrodynamics fails in regions with *extremely strong fields*. When electromagnetic fields are extremely strong, their energy density will *curve* space-time. Classical electrodynamics, which assumes flat space-time, is not valid in such situations.

The failure of classical electrodynamics is most evident in the most extreme case of all: when the fields are extremely strong, they will lead to the formation of black holes. The existence of black holes, together with the discreteness of charge, imply *maximum* electric and magnetic field values. These upper limits were already mentioned in Table 3, which lists various electric field values found in nature, and in Table 8, which lists possible magnetic field values. Can you deduce the values of these so-called *Planck fields*? The precise argument that limits electric and magnetic fields values in nature is not simple; and there are still many physicists who – mistakenly – deny the limits.

Page 26
Page 37
Challenge 239 s

The interplay between curvature of space and electrodynamics has many aspects. For example, the maximum force in nature limits the maximum charge that a black hole can carry. Can you find the relation? As another example, it seems that magnetic fields effectively increase the stiffness of empty space, i.e., they increase the difficulty to bend empty space. Not all interactions between gravity and electrodynamics have been studied up to now; more examples should appear in the future.

Vol. II, page 107
Challenge 240 ny
Ref. 206

In summary, classical electrodynamics does not work for extremely high field values, when general relativity plays a role.

Charge values are discrete, not continuous

Classical electrodynamics fails to describe nature correctly also for *extremely weak fields*. This happens also in flat space-time and is due to a reason already mentioned a number of times: *electric charges are discrete*. Electric charges do not vary continuously, but change in fixed steps. Not only does nature show a smallest value of entropy – as we found in our exploration of heat, – and smallest amounts of matter; nature also shows a smallest charge.

Vol. I, page 399
Vol. I, page 400

▷ Electric charge values are quantized.

In metals, the quantization of charge is noticeable in the flow of electrons. In electrolytes, i.e., electrically conducting liquids, the quantization of charge appears in the flow of charged atoms, usually called *ions*. All batteries have electrolytes inside; also water is an electrolyte, though a poorly conducting one. In plasmas, like fire or fluorescent lamps, both ions and electrons move and show the discreteness of charge. Also in all known types of particle radiation – from the electron beams inside cathode ray tubes in televisions, the channel rays formed in special low-pressure glass tubes, the cosmic radiation hitting us all the time, up to the omnipresent radioactivity – charges are quantized.

In all known experiments, the *same* smallest value e for electric charge has been found. The most precise result is

$$e = 0.160\,217\,656\,5(35)\ \mathrm{aC}\ , \tag{90}$$

around a sixth of an attocoulomb. All observed electric charges in nature are multiples of this so-called *elementary charge*.

In short, like all flows in nature, also the flow of electricity is due to a flow of discrete particles. In fact, the nature of the charged particles differs from situation to situation: they may be electrons, ions, muons or various other kinds of particles. However, the charge steps are always exactly the same. In fact, at this point of our adventure, the equality of the elementary charge for all matter particles is unexplained. We will discover the reason at the very end of our adventure.

Above all, a smallest charge change has a simple implication:

▷ Classical electrodynamics is *wrong*.

Classical electrodynamics is just a good approximation for *medium-sized* field values. Indeed, a smallest charge implies that no infinitely small test charges exist. But such Page 25 infinitely small test charges are necessary to *define* electric and magnetic fields. For a *finite* test charge, the disturbance of the field introduced by the test charge itself makes a precise field measurement – and thus a precise field definition – impossible. As a consequence, the values of electric and magnetic field measured with finite test charges are always somewhat fuzzy. This fuzziness is most apparent for low field values. For example, for low intensities of light, experiments detect *photons*, discrete light particles. All low light intensities are time-averages of low photon numbers; they are not continuous fields.

The lower limit on charge magnitude also implies that there is no fully correct way of defining an instantaneous electric current in classical electrodynamics. Indeed, the Vol. IV, page 171 position and the momentum of a charge are always somewhat fuzzy, as we will find out.

In summary,

▷ Maxwell's evolution equations are only *approximate*.

Classical electrodynamics does not work for extremely *low* field values, when quantum effects play a role, and does not work for extremely *high* field values, when gravity plays a role. We will explore these two extreme cases in the remaining legs of our adventure, those on quantum theory and those on unification. Only some effects of the discreteness of charge can be treated in classical physics; a few instructive examples follow.

How fast do charges move?

In a vacuum, such as inside a colour television tube or inside an electron microscope, charged particles accelerated by a voltage of 30 kV move with a third of the speed of light. At higher voltage, the speed is even higher. In modern particle accelerators charges move so rapidly that their speed is indistinguishable from that of light for all practical purposes.

Challenge 241 s

Inside a metal, electric signals move with speeds of the order of the speed of light. The precise value depends on the capacity and impedance of the cable and is usually in the range $0.3c$ to $0.5c$. This high speed is due to the ability of metals to easily take in arriving charges and to let others depart. The ability for rapid reaction is due to the high mobility of the charges inside metals, which in turn is due to the properties of metallic bonds and to the small mass and size of the involved charges, the electrons.

The high signal speed in metals appears to contradict another determination. The *drift speed* v of the electrons in a metal wire, i.e., the average speed of the charges, obviously obeys

$$v = \frac{I}{Ane} , \tag{91}$$

where I is the current, A the cross-section of the wire, e the charge of a single electron and n the number density of electrons. The electron density in copper is $8.5 \cdot 10^{28}\,\mathrm{m}^{-3}$. Using a typical current of 0.5 A and a typical cross-section of a square millimetre, we get a drift speed of 0.37 μm/s. In other words, electrons move a thousand times slower than ketchup inside its bottle. Worse, if a room lamp used direct current instead of alternate current, the electrons would take several days to get from the switch to the bulb! Nevertheless, the lamp goes on or off almost immediately after the switch is activated. Similarly, the electrons from an email transported with direct current would arrive much later than a paper letter sent at the same time; nevertheless, the email arrives quickly. Why?

Water pipes show a similar effect. A long hose provides water almost in the same instant as the tap is opened, even if the water takes a long time to arrive from the tap to the end of the hose. The speed with which the water reacts, the signal speed, is given by the speed for pressure waves, or sound waves, in water. For water hoses, the signal speed is much higher than the speed of the water flow, much higher than the speed of the molecules.

Also everyday life provides us with a similar effect. Imagine a long queue of cars (representing electrons) waiting in front of a red traffic light. In an ideal world, all drivers look at the light. As soon as the light turns green, everybody starts driving. Even though the driving speed might be only 10 m/s, the speed of traffic flow onset was that of light. It is this latter speed which is the signal speed. The signal speed is much higher than the speed of the cars.

In short, inside a metal, the electrons move slowly; the speed of electrical signals is not given by the electron speed, but by the speed of electron density waves, which in turn is due to the electromagnetic field. In fact, a typical house has only an alternating current supply. In this typical case, the electrons inside the copper wires only vibrate back and forwards by a tiny distance, as you might want to check.

Challenge 242 e

Inside liquids, charges move with a different speed from that inside metals, and their charge to mass ratio is also different. We all know this from direct experience. Our *nerves* work by using electric signals and take a few milliseconds to respond to a stimulus, even though they are (only) metres long. A similar speed is observed inside batteries. In all these systems, moving charge is transported by ions. *Ions* are charged atoms. Ions, like atoms, are large, composed and heavy entities, in contrast to the tiny and light electrons. As a result, ions move much more slowly than electrons do. Our limited reaction time is a consequence of ion motion.

In still other matter systems, charges move both as electrons and as ions. Examples are neon lamps, fire, plasmas and the Sun. This leads us to ask:

What motion occurs inside atoms?

Inside atoms, electrons behave strangely. We tend to imagine that electrons orbit the nucleus (as we will see later) at a rather high speed, as the orbital radius is so small. However, it turns out that in most atoms many electrons do not orbit the nucleus at all: many electrons have no orbital angular momentum around the nucleus. How can this be?

Worse, some electrons do have orbital momentum. But if these electrons were orbiting the atomic nucleus like planets orbit the Sun, they would move under constant acceleration. Thus they would emit electromagnetic radiation until they would fall into the nucleus. But this is not the case: atoms are stable! How can this be?

And why are all atoms of the same size anyway? Atom size should depend on the angular momentum of the electrons inside it. But what determines the orbital momentum of electrons around the nucleus?

We will discover soon that in nature there is a smallest angular momentum value. This value fixes the size of atoms. And we will discover that moving electrons, in contrast to everyday objects, are *not* described by trajectories in space, thus allowing atoms to be stable. The strange story of atoms and their structure will be told in the quantum legs of our adventure, in the volumes following this one. Vol. IV, page 181

Challenges and curiosities about charge discreteness

Challenge 243 s How would you show experimentally that electrical charge comes in smallest chunks?

* *

The discreteness of charge implies that we can estimate the size of atoms by observing galvanic deposition of metals. How? Challenge 244 ny

* *

Classical electrodynamics implies that *point-like* charges cannot exist. Can you explain the argument? Then, can you answer whether the reasoning applies also to nature? Challenge 245 s

* *

Cosmic radiation consists of charged particles hitting the Earth. (We will discuss this in more detail later.) Astrophysicists explain that these particles are accelerated by the magnetic fields around the Galaxy. However, the expression of the Lorentz acceleration shows that magnetic fields can only change the direction of the velocity of a charge, not Vol. V, page 162 Ref. 207

Challenge 246 ny its magnitude. How can nature get acceleration nevertheless?

* *

What would be the potential of the Earth in volt if we could take far away all the electrons of a drop of water? Challenge 247 s

* *

When a voltage is applied to a resistor, how long does it take until the end value of the current, given by Ohm's 'law', is reached? The first to answer this question was Paul Drude* in the years around 1900. He reasoned that when the current is switched on, the speed v of an electron increases as $v = (eE/m)t$, where E is the electrical field, e the charge and m the mass of the electron. Drude's model assumes that the increase of electron speed stops when the electron hits an atom, loses its energy and begins to be accelerated again. Drude deduced that the average time τ up to the collision is related to the specific resistance by

$$\rho = \frac{E}{j} = \frac{E}{env} = \frac{2m}{\tau e^2 n} , \qquad (92)$$

with n being the electron number density. The right side does not depend on E any more; it is a constant. Drude had thus explained *Ohm's 'law'* $U = RI$ (or $E = j\rho$) from material properties, by assuming that resistance is due to moving electrons that continuously collide and speed up again. Inserting numbers for copper ($n = 8.5 \cdot 10^{28}\,/\mathrm{m}^{-3}$ and $\rho = 0.16 \cdot 10^{-7}\,\Omega\mathrm{m}$), we get a time $\tau = 51\,\mathrm{ps}$. This time is so short that the switch-on process can usually be neglected.

* *

Does it make sense to write Maxwell's equations *in vacuum*? Both electrical and magnetic fields require charges in order to be measured. But in vacuum there are no charges at all. And fields are defined by using infinitesimally small test charges. But, as we mentioned already, infinitesimally small charges do not exist. In fact, only quantum theory solves this issue. Are you able to imagine how? Challenge 248 d

* *

We have seen that in cases of fields of medium values, classical electrodynamics is a good approximation, despite charge discreteness. One useful system makes use of discrete charge but can nevertheless be described in many of its aspects with classical electrodynamics. It merits a separate discussion: our brain.

* Paul Karl Ludwig Drude (b. 1863 Braunschweig, d. 1906 Berlin), physicist, predicted with his electron gas model of metals – that the ratio between the thermal conductivity and the electric conductivity at a given temperature should be the same for all metals; this is roughly correct. Drude also conceived ellipsometry and introduced c as the symbol for the speed of light.

Chapter 7

THE STORY OF THE BRAIN

> Alles was überhaupt gedacht werden kann,
> kann klar gedacht werden.**
>
> Ludwig Wittgenstein, *Tractatus*, 4.116

In our quest for increased precision in the description of all motion around us, it is time to take a break, sit down and look back. In our walk so far, which has led us to investigate mechanics, general relativity and electrodynamics, we used several concepts without defining them. Examples are 'information', 'memory', 'measurement', 'set', 'number', 'infinity', 'existence', 'universe' and 'explanation'. Each of these is a common and important term. In this intermezzo, we take a look at these concepts and try to give some simple, but sufficiently precise definitions, keeping them as provocative and entertaining as possible. For example, can you explain to your parents what a concept is?

Challenge 249 e

We need to study the definitions of concepts in order to get to the top of Motion Mountain, i.e., to the full description of motion. In the past, many have lost their way because of lack of clear concepts. In order to avoid these difficulties, physics has a special guiding role. All sciences share one result: *every type of change observed in nature is a form of motion*. In this sense, but in this sense *only*, physics, focusing on motion itself, forms the basis for all the other sciences. In other words, the search for the famed 'theory of everything' is an arrogant and misleading expression for the search for a *final theory of motion*. Even though the knowledge of motion is basic, its precise description does not imply a description of 'everything': just try to solve a marriage problem using the Schrödinger equation to note the difference.

Given the basic importance of motion, it is necessary that in physics all statements on observations be as *precise* as possible. For this reason, many thinkers have investigated physical statements with particular care, using all criteria imaginable. *Physics is precise prattle by curious people about moving things.* What does precision mean? The meaning appears once we ask: which abilities does such prattle require? You might want to fill in the list yourself before reading on.

Challenge 250 e

The abilities necessary for talking are a topic of research even today. The way that the human species acquired the ability to chat about motion is studied by evolutionary biologists. Child psychologists study how the ability develops in a single human being.

** 'Everything that can be thought at all can be thought clearly.' This and other quotes of Ludwig Wittgenstein are from the equally short and famous *Tractatus logico-philosophicus*, written in 1918, first published in 1921; it has now been translated into many other languages.

FIGURE 173 Ludwig Wittgenstein (1889–1951).

Physiologists, neurologists and computer scientists are concerned with the way the brain and the senses make this possible; linguists focus on the properties of the language we use, while logicians, mathematicians and philosophers of science study the general properties of correct statements about nature. All these fields investigate tools that are essential for the development of physics, the understanding of motion and the specification of the undefined concepts listed above. The fields structure the following exploration.

Evolution

> A hen is only an egg's way of making another egg.
>
> Samuel Butler, *Life and Habit.*

The evolution of the human species is the result of a long story that has been told in many excellent books. A summarizing table on the history of the universe that includes evolution was given in the exploration of general relativity. The almost incredible chain of events that has lead to one's own existence includes the formation of nuclei, atoms, galaxies, stars, the planets, the Moon, the atmosphere, the oceans, the first cells, the water animals, the land animals, the mammals, the hominids, the humans, the ancestors, the family and finally, oneself.

Ref. 208 Vol. II, page 230 Ref. 209

The way that the atoms we are made of moved during this sequence, being blown through space, being collected on Earth, becoming organized to form organic matter and then people, is one of the most awe-inspiring examples of motion. Remembering and meditating about this cosmic sequence of motion every now and then can be an enriching experience.

Challenge 251 e

In particular, without biological evolution, we would not be able to talk about motion at all; only moving bodies can study moving bodies. Without evolution, we would have no muscles, no senses, no nerves and no brains. And without a brain, we would not be able to think or talk. Evolution was also the fount of childhood and curiosity. Indeed, in the present and the next chapters we will discover that most concepts of classical physics are already introduced by every little child, in the experiences it has while growing up.

Children, laws and physics

> Physicists also have a shared reality. Other than that, there isn't really a lot of difference between being a physicist and being a schizophrenic.
>
> Richard Bandler

Ref. 210

During childhood, everybody is a physicist. When we follow our own memories backwards in time as far as we can, we reach a certain stage, situated before birth, which forms the starting point of human experience. In that magic moment, we sensed somehow that apart from ourselves, there is something else. The first observation we make about the world, during the time in the womb, is thus the recognition that we can distinguish two parts: ourselves and the rest of the world. This distinction is an example – perhaps the first – of a large number of 'laws' of nature that we stumble upon in our lifetime. Being a physicist started back then. And it continued. By discovering more and more distinctions we bring structure in the chaos of experience. We quickly find out that the world is made of *related parts*, such as mama, papa, milk, earth, toys, etc. We divide the parts in objects and images.

Ref. 211

Later, when we learn to speak, we enjoy using more difficult words and we call the surroundings the *environment*. Depending on the context, we call the whole formed by oneself and the environment together the (physical) *world*, the (physical) *universe*, *nature*, or the *cosmos*. These concepts are not distinguished from each other in this walk;* they are all taken to designate the sum of all parts and their relations. They are simply taken here to designate the whole.

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The discovery of the first distinction in nature starts a chain of similar discoveries that continue throughout our life. We extract the numerous distinctions that are possible in the environment, in our own body and in the various types of interactions between them. The ability to distinguish is the central ability that allows us to change our view from that of the world as *chaos*, i.e., as a big mess, to that of the world as a *system*, i.e., a structured set, in which parts are related in specific ways. (If you like precision, you may ponder whether the two choices of 'chaos' and 'system' are the only possible ones.)

Challenge 252 s

In particular, the observation of the differences between oneself and the environment goes hand in hand with the recognition that not only are we not independent of the environment, but we are firmly tied to it in various inescapable ways: we can fall, get hurt, feel warm, cold, etc. Such relations are called *interactions*. Interactions express the observation that even though the parts of nature can be distinguished, they cannot be isolated. In other words,

Page 322

▷ Interactions describe the difference between the whole and the sum of its parts.

No part can be defined without its relation to its environment. (Do you agree?)

Challenge 253 e

Interactions are not arbitrary; just take touch, smell or sight as examples. They differ in reach, strength and consequences. We call the characteristic aspects of interactions *patterns of nature*, or *properties of nature*, or *rules of nature* or, equivalently, with their historical but unfortunate name, *'laws' of nature*. The term 'law' stresses their general validity; unfortunately, it also implies design, aim, coercion and punishment for infringement. However, no design, aim or coercion is implied in the properties of nature, nor is

* The differences in their usage can be deduced from their linguistic origins. 'World' is derived from old Germanic 'wer' – person – and 'ald' – old – and originally means 'lifetime'. 'Universe' is from the Latin, and designates the one – 'unum' – which one sees turning – 'vertere', and refers to the starred sky at night which turns around the polar star. 'Nature' comes also from the Latin, and means 'what is born'. 'Cosmos' is from Greek κόσμος and originally means 'order'.

infringement possible. The ambiguous term 'law of nature' was made popular by René Descartes (b. 1596 La Haye en Touraine, d. 1650 Stockholm) and has been adopted enthusiastically because it gave weight to the laws of the state – which were far from perfect at that time – and to those of other organizations – which rarely are. The expression is an anthropomorphism coined by an authoritarian world view, suggesting that nature is 'governed'. We will therefore use the term as rarely as possible in our walk and it will, if we do, be always between 'ironical' parentheses. Nature cannot be forced in any way. The 'laws' of nature are not obligations for nature or its parts, they are obligations only for physicists and all other people: the patterns of nature oblige us to use certain descriptions and to discard others. Whenever one says that 'laws govern nature' one is talking nonsense (or asking for money); the correct expression is *rules describe nature*.

During childhood we learn to distinguish between interactions with the environment, or *perceptions*: some are shared with others and called *observations*, others are uniquely personal and are called *sensations*.* A still stricter criterion of 'sharedness' is used to divide the world into 'reality' and 'imagination' (or 'dreams'). Our walk will show – at the very end – that this distinction is not essential, provided that we stay faithful to the quest for ever increasing precision: we will find, surprisingly, that the description of motion that we are looking for does not depend on whether the world is 'real' or
Vol. VI, page 431
'imagined', 'personal' or 'public'. The fundamental principles of motion in reality and in dreams are the same. Nevertheless, these same principles allow us to distinguish the two.

Humans enjoy their ability to distinguish parts, which in other contexts they also call *details*, *aspects* or *entities*, and enjoy their ability to associate them or to observe the *re-*
Ref. 213
lations between them. Humans call this activity *classification*. Colours, shapes, objects, mother, places, people and ideas are some of the entities that humans discover first.

Our anatomy provides a handy tool to make efficient use of these discoveries: *memory*. It stores a large amount of input that is called *experience* afterwards. Memory is a tool used by both young and old children to organize their world and to achieve a certain security in the chaos of life.

Memorized classifications are called *concepts*. Jean Piaget was the first researcher to describe the influence of the environment on the concepts that every child forms. Step by step, children learn that objects are localized in space, that space has three dimensions, that objects fall, that collisions produce noise, etc. In particular, Piaget showed that space and time are not a priori concepts, but result from the interactions of every child with its environment.**

* A child that is unable to make this distinction among perceptions – and who is thus unable to lie – almost
Ref. 212
surely develops or already suffers from *autism*, as recent psychological research has shown.

** An overview of the origin of developmental psychology is given by J. H. Flavell, *The Developmental Psychology of Jean Piaget*, 1963. This work summarizes the observations by the Jean Piaget (b. 1896 Neuchâtel, d. 1980 Geneva), the central figure in the field. He was one of the first researchers to look at child development in the same way that a physicist looks at nature: carefully observing, taking notes, making experiments, extracting hypotheses, testing them, deducing theories. His astonishingly numerous publications, based on his extensive observations, cover almost all stages of child development. His central contribution is the detailed description of the stages of development of the cognitive abilities of humans. He showed that all cognitive abilities of children, the formation of basic concepts, their way of thinking, their ability to talk, etc., result from the continuous interaction between the child and the environment.

In particular, Piaget described the way in which children first learn that they are different from the ex-

Around the time that a child goes to school, it starts to understand the idea of *permanence of substances*, e.g. liquids, and the concept of *contrary*. Only at that stage does its subjective experience becomes *objective*, with abstract comprehension. Still later, the child's description of the world stops to be animistic: before this step, the Sun, a brook or a cloud are *alive*. In short, only after puberty does a human become ready for physics, the science of motion.

Ref. 215

Even though everyone has been a physicist in their youth, most people stop at *Galilean* physics, where matter is approximated to be continuous and space to be flat. In the present adventure we go much further, by using all the possibilities of a toy with which nature provides us: the brain.

> "Experience is the name everyone gives to their mistakes."
>
> Oscar Wilde, *Lady Windermere's Fan*.

Polymer electronics

TABLE 18 Some aspects of the human brain.

Aspect	Details	Computer equivalent
Hardware		
Ultrashort term memory	5 to 9 concepts	cache
Hippocampus	novelty detector, spatial memory, learning	RAM and Flash memory
Amygdala	emotions, learning	priority scheduler of operating system
Ventral striatum, dopamine and opioid provider	rewards system	priority scheduler of operating system
Suprachiasmatic nucleus	day-night control	sleep controller

ternal environment, and how they then learn about the physical properties of the world. Of his many books related to physical concepts, two especially related to the topic of this walk are J. Piaget, *Les notions de mouvement et de vitesse chez l'enfant*, Presses Universitaires de France, 1972 and *Le developpement de la notion de temps chez l'enfant*, Presses Universitaires de France, 1981, this last book being born from a suggestion by Albert Einstein. These texts should be part of the reading of every physicist and science philosopher interested in these questions.

Piaget also describes how in children the mathematical and verbal intelligence derives from sensomotorial, practical intelligence, which itself stems from habits and acquired associations to construct new concepts. Practical intelligence requires the system of reflexes provided by the anatomical and morphological structure of our organism. Thus his work shows in detail that our faculty for mathematical description of the world is based, albeit indirectly, on the physical interaction of our organism with the world.

Some of his opinions on the importance of language in development are now being revised, notably through the rediscovery of the work of Lev Vigotsky, who argues that all higher mental abilities, emotions, recollective memory, rational thought, voluntary attention and self-awareness, are not innate, but learned. This learning takes place through language and culture, and in particular through the process of talking to oneself.

Ref. 214

At www.piaget.org you can find the website maintained by the Jean Piaget Society.

TABLE 18 (Continued) Some aspects of the human brain.

Aspect	Details	Computer equivalent
Neurons in human brain	$86(8) \cdot 10^9$	storage and access electronics
Neurons in cortex	women *c.* $19(2) \cdot 10^9$, men $22(2) \cdot 10^9$	hard disk and processor
Glial cells in brain	about as many as neurons	power supply, structure
Neuron number decay	women: $e^{3.05-0.00145 \cdot \mathrm{age}/a} \cdot 10^9$, men: $e^{3.2-0.00145 \cdot \mathrm{age}/a} \cdot 10^9$	hard disk scratching
Pulses exchanged between both brain halves	$4 \cdot 10^9/s$	internal bus speed
Synapses per neuron	10^4	
Total synapse connections	*c.* $2 \cdot 10^{14}$	memory cells
Input pathways from the eye	*c.* $2 \cdot 10^6$	camera wire
Input pathways from the ear	*c.* $2 \cdot 3000$	microphone line, inclination sensor line
Input pathways from skin, mouth, and nose	*c.* $0.5 \cdot 10^6$	sensor interfaces
Input signal capacity (total, 300 pulses/s per pathway)	*c.* 100 MB/s	input bandwidth
Output pathways (muscles, organs)	*c.* $1.5 \cdot 10^6$	actuator and motor interfaces
Output signal capacity (total, 300 pulses/s per pathway)	*c.* 50 MB/s	output bandwidth
Non-serious – probably too low – estimate of the processing capacity	10 PFlop	several dozens of supercomputers
Typical mass (Einstein's brain)	1.230 kg; varies between 0.7 and 2.0 kg	0.001 to 5000 kg
Power consumption (average)	1600 to 2200 kJ/d or 18 to 25 W (with 750 ml/min blood supply)	1 W to 20 kW
Lifetime	130 years	2 years or more
Size	0.14 m 0.17 m0.09 m	from a few cm^3 to $1\ m^3$
Software and processing		
Learning	changing synapse strength through long-term potentiation	activate, classify, store
Deep sleep and learning storage	structured writing from hippocampus to cortex	clean-up and back-up to hard disk

TABLE 18 (Continued) Some aspects of the human brain.

Aspect	Details	Computer equivalent
REM (rapid eye movement, or dream) sleep	offline processing	data compression in batch process

The brain is an electrical device. This was definitely shown in 1924, when the neurologist Hans Berger (b. 1873 Neuses, d. 1941 Jena) recorded and named the first *electroencephalogram*. A modern electroencephalogram is shown in Figure 176.* In more detail, the brain is a flexible, metal-free, short-lived, sensitive, unreliable, electronic polymer device. Incidentally, all these properties are shared by all electronic polymer devices, whether alive or not. Higher reliability is the main reason that commercial electronics is usually silicon-based instead of polymer-based.

Ref. 216 The polymer electronics that forms the brain is organized like a computer. Some details of its organization are shown in Table 18, Figure 174 and Figure 175. Though the functional blocks of a brain and of a computer are astonishingly similar, the specific mechanisms they use are usually completely different.

Ref. 217 Like a computer, a brain consists of numerous parts *dedicated* to specific tasks and of a *general* computing part, the grey matter. The division between dedicated and specific is almost fifty–fifty. Also the computing power in a modern computer is divided in this way; for example, graphics cards are often as powerful as the central processing unit.

In a generation or two, this section could be entitled 'how to build a brain'. Unfortunately, there is not enough knowledge yet to realize this aim. Maybe you can help in this pursuit?

Why a brain?

> "Denken ist bereits Plastik.**"
>
> Ref. 218 Joseph Beuys.

The brain exists to control the motion of an organism. The more complex the motions of an organism are, the larger its brain is. Living beings that do not move around, such as trees or dandelions, do not have a brain. Living beings that stop moving around, such as sea squirts (*Ascidiae* or *Ascidiacea*) digest their own brain when they attach themselves to a rock in the sea.

The brain – together with some parts of the central nervous system – controls motion by processing the input provided by the various senses and sending the results of the processing to the various muscles in the body. Numerous observations show that sense input is processed, i.e., classified, stored and retrieved in the brain. Notably, lesions of

* In the electric signals generated by the brain one distinguishes, *irregular signals* during data processing, *beta waves*, mainly during attention, with a frequency between 14 and 30 Hz, *alpha waves*, during relaxation, with a frequency between 8 and 13 Hz, *theta waves*, during early sleep and during rapid eye movement (REM) sleep, with a frequency between 3 and 7 Hz, and *delta waves*, during deep sleep, with a frequency between 0.5 and 2 Hz.
** 'Thinking is already sculpture.' Joseph Beuys (b. 1920 Krefeld, d. 1986 Düsseldorf) was a famous sculptor.

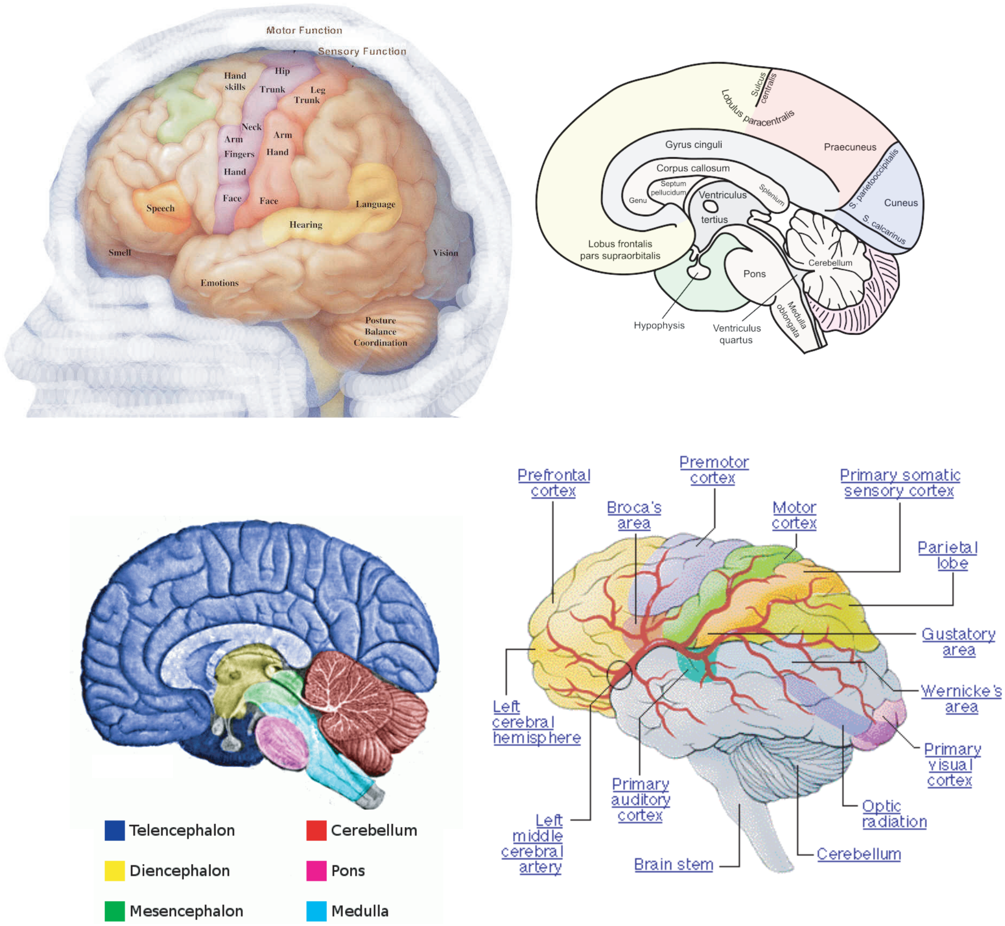

FIGURE 174 Sections and side view of the human brain, all in false colours (images WikiCommons).

Ref. 219 the brain can lead to the loss of part or all of these functions. Among the important consequences of these basic abilities of the brain are thought and language. All brain
Ref. 220 abilities result from the construction, i.e., from the 'hardware' of the brain.

Systems with the ability to deduce classifications from the input they receive are called
Ref. 221 *classifiers*, and are said to be able to *learn*. Our brain shares this property with many complex systems; the brain of many animals, but also certain computer algorithms, such as the so-called 'neural networks', are examples of classifiers. Classifiers are studied in sev-
Ref. 222 eral fields, from biology to neurology, mathematics and computer science. All classifiers have the double ability to discriminate and to associate; and both abilities are fundamental to thinking.

Machine classifiers have a lot in common with the brain. As an example, following
Ref. 223 an important recent hypothesis in evolutionary biology, the necessity to cool the brain in an effective way is one reason for the upright, bipedal walk of humans. The brain, which uses around a quarter of all energy burned in the human body, needs a powerful cooling system to work well. In this, brains resemble modern computers, which usually have powerful fans or even water cooling systems built into them. It turns out that the

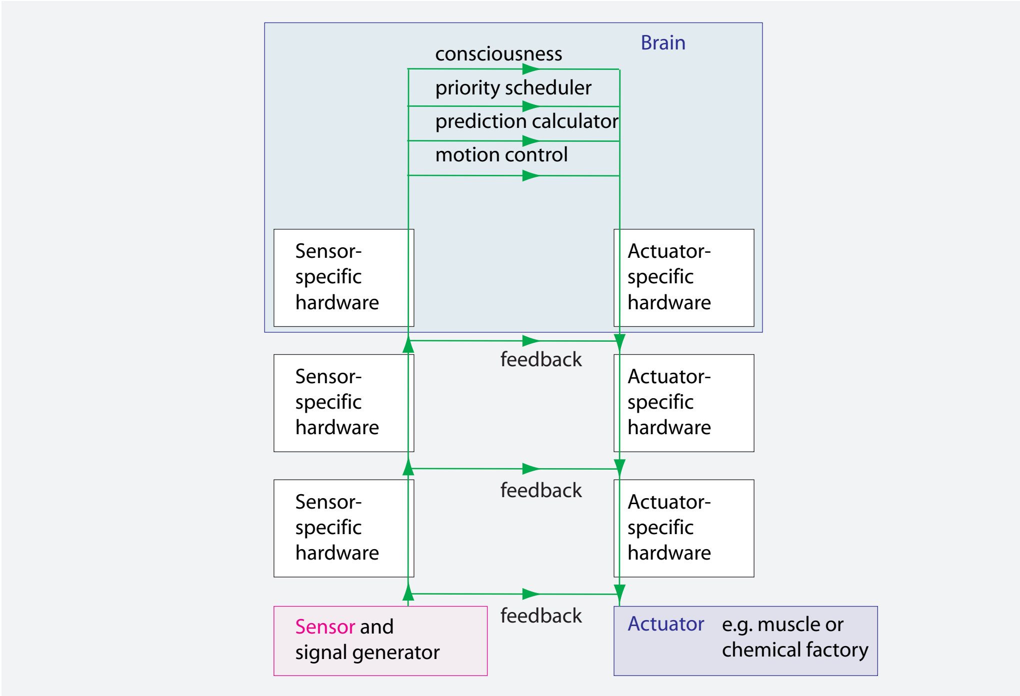

FIGURE 175 The general structure of the nervous system, with some typical feedback loops it contains and an example of its sensor-specific hardware.

human species has the most powerful cooling system of all mammals. An upright posture allowed the air to cool the body most effectively in the tropical environment where humans evolved. For even better cooling, humans have also no body hair, except on their head, where it protects the brain from direct heating by the Sun. The upright posture in turn allowed humans to take breath independently of their steps, a feat that many animals cannot perform. This ability increased the cooling again, and in turn allowed humans to develop speech. Speech in turn developed the brain further.

All classifiers are built from smallest *classifying units*, sometimes large numbers of them. Usually, the smallest units can classify input into only *two* different groups. The larger the number of these units, often called 'neurons' by analogy to the brain, the more sophisticated classifications can be produced by the classifier. Classifiers thus work by applying more or less sophisticated combinations of 'same' and 'different'. The distinction by a child of red and blue objects is such a classification; the distinction of compact and non-compact gauge symmetry groups in quantum theory is a more elaborate classification, but relies on the same fundamental ability.

Ref. 224

Neurons and networks

In the brain, the classifying units are the *neurons*. Neurons are specialised cells that process, produce and transport electrical signals. In the brain, the main classifying units are the *multipolar* neurons. Like every classifying unit, they have input and output channels. And like every classifying unit, such neurons produce just two different output signals:

FIGURE 176 A modern electroencephalogram, taken at a number of positions at the head. The measured voltages are around 0.1 mV (© Wikimedia).

an electrical spike of fixed height and duration or no spike at all.

In all classifiers, the smallest classifying units *interact* with each other. Usually, but not always, these interactions are channelled via connections. For neurons, the connections are the dendrites and the axions. The full configuration of classifiers thus forms a *network*. In these connections, signals are exchanged, via moving objects, such as electrons or photons. Thus we arrive at the conclusion that the ability of the brain to classify the physical world – for example to distinguish moving objects interacting with each other – arises because the brain itself consists of moving objects interacting with each other. Humans would not have become such a successful animal species without our built-in powerful classifier. And only the motion inside our brain allows us to talk about motion in general.

Numerous researchers are identifying the parts of the brain used when different intellectual tasks are performed. Such experiments are possible using magnetic resonance imaging and similar imaging techniques. Other researchers are studying how thought processes can be modelled from the brain structure. Modern neurology is still making regular progress. In particular, neurologists have destroyed the belief that thinking is *more* than a physical process. This false belief results from various personal fears, as you might want to test by introspection. The fears and the belief will disappear as time goes by. How would you argue that thought is just a physical process?

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Challenge 254 s

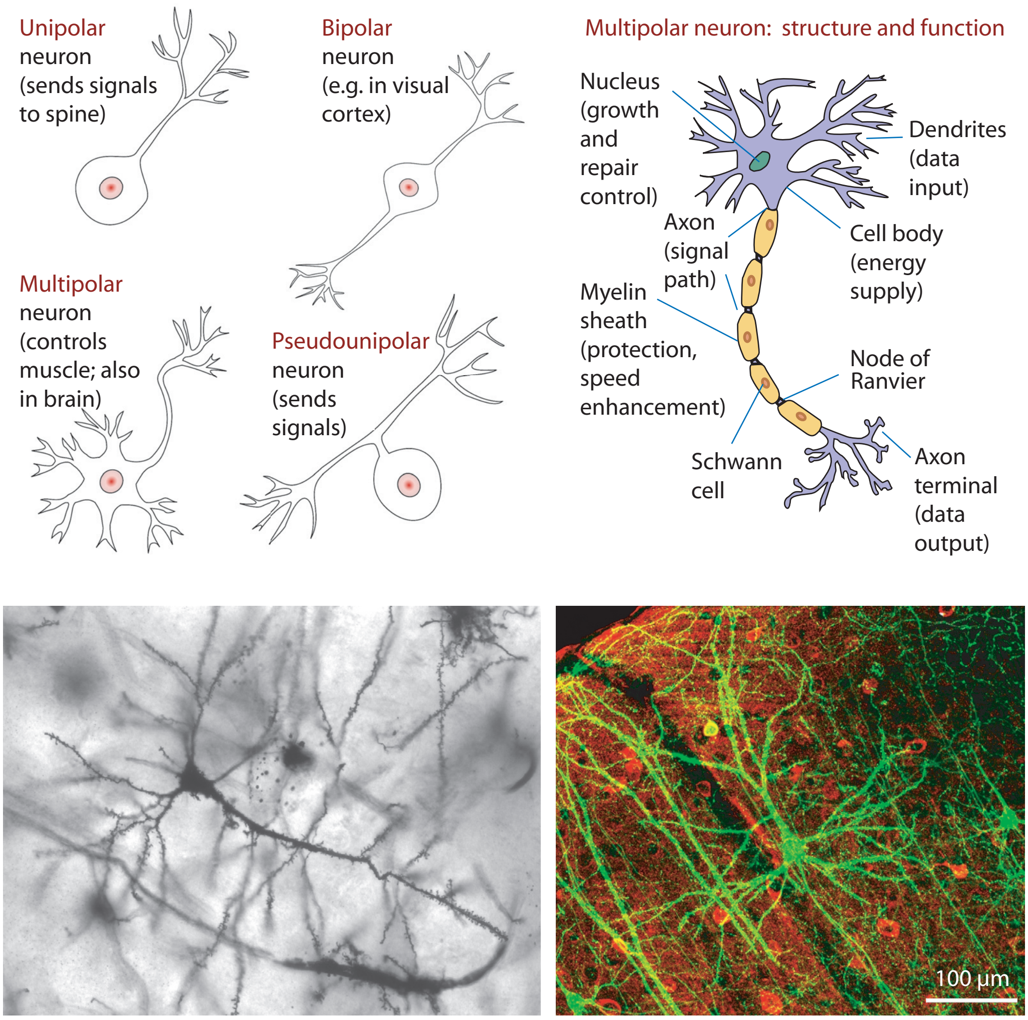

FIGURE 177 An overview of the four main types of neurons, a schematic diagram of a multipolar neuron, and two photographs of stained brain tissue (© Wikimedia, Wikimedia, MethoxyRoxy, Wei-Chung Allen Lee & al.)

Evolution developed the brain, with all its capabilities, as a tool that helps every person to find her way through the challenges that life poses. The human brain is so large because of two reasons: the sensory input is vast, and the processing is complex. More concretely, the brain is so large in order to process what we see. The amount of information provided by the eyes to the brain is indeed huge.

What is information?

> “These thoughts did not come in any verbal formulation. I rarely think in words at all. A thought comes, and I may try to express it in words afterward.”
>
> Albert Einstein

Ref. 225

We started our adventure by stating that studying physics means to talk about motion. To *talk* is to transmit information. Can information be measured? Can we measure the progress of physics in this way? Is the universe made of information? To answer these questions, we start with the definition.

▷ Information is the result of classification.

A classification is the answer to one or to several yes–no questions. Such yes–no questions are the simplest classifications possible; they provide the basic *units* of classification, from which all others can be built. Therefore,

▷ Information is measured by counting the implied yes–no questions, the number of *bits*, leading to it.

Examples of information values are given in Table 19.

Are you able to say how many bits are necessary to define the place where you live? Obviously, the number of bits depends on the set of questions with which we start; that could be the names of all streets in a city, the set of all coordinates on the surface of the Earth, the names of all galaxies in the universe, the set of all letter combinations in the address. What is the most efficient method you can think of? A variation of the combination method is used in computers. For example, the story of the present adventure required about nine thousand million bits of information. But since the amount of information in a story depends on the set of questions with which we start, it is impossible to define a precise measure for information in this way.

Challenge 255 s

TABLE 19 Some measures of information.

Kind of information	Amount
Words spoken on an average day by a man	*c.* 5000
Words spoken on an average day by a woman	*c.* 7000
Bits processed by the ears	1 to 10 Mbit/s
Light sensitive cells per retina (120 million rods and 6 million cones)	$126 \cdot 10^{6}$
Bits processed by the eyes	1 to 10 Gbit/s
Words spoken during a lifetime (2/3 time awake, 30 words per minute)	$3 \cdot 10^{8}$
Words heard and read during a lifetime	10^{9}
Letters (base pairs) in haploid human DNA	$3 \cdot 10^{9}$
Pulses exchanged between both brain halves every second	$4 \cdot 10^{9}$
Bits in a compact disc	$6.1 \cdot 10^{9}$
Neurons in the human brain	$86(8) \cdot 10^{9}$

TABLE 19 (Continued) Some measures of information.

Kind of information	Amount
Printed words available in (different) books around the world ($c.$ $100 \cdot 10^6$ books consisting of 50 000 words)	$c.$ $5 \cdot 10^{12}$
Memory bits in the human brain	$> 10^{16}$
Image pixels seen in a lifetime: $3 \cdot 10^9$ s (lifetime) $\cdot 2/3$ (awake) $\cdot 10^6$ (connections to the brain) /15 ms (eye speed) Ref. 258	10^{17}
Bits of information processed in a lifetime (the above times $c.$ 32)	10^{19}

The only way to measure information precisely is to take the largest possible set of questions that can be asked about a system, and to compare it with what is known about the system. In this case, the amount of unknown information is called entropy, a concept that we have already encountered. Vol. I, page 395 With this concept you should able to deduce yourself, for example, whether it is really possible to measure the advance of physics. Challenge 256 s

Since classification or categorization is an activity of the brain and other, similar classifiers, information as defined here is a concept that applies to the result of activities by people and by other classifiers. In short, information is produced when talking about the universe.

Information is the result of classification. This implies that the universe itself is *not the same* as information. There is a growing number of publications based on the opposite of this view; however, this is a conceptual short circuit. Any transmission of information implies an interaction; physically speaking, this means that any information needs *energy* for transmission and *matter* for storage. Without either of these, there is no information. In other words, the universe, with its matter and energy, has to exist *before* transmission of information is possible. Saying that the universe *is made of* information, or that it *is* information, is as meaningful and as correct as saying that the universe is made of toothpaste.

The aim of physics is to give a *complete* classification of all types and examples of motion, in other words, to know everything about motion. Is this possible? Or are you able to find an argument against this endeavour? Challenge 257 s

What is memory?

> “The brain is my second favorite organ.”
> Woody Allen

Memory is the collection of records of perceptions. The production of such records is the essential aspect of observation. Records can be stored in human memory, i.e., in the brain, or in machine memory, as in computers, or in object memory, such as notes on paper. Without memory, there is no science, no life – since life is based on the records inside the DNA – and especially, no fun, as proven by the sad life of those who lose their memory. Ref. 219

Many animals and people have a memory, because a memory helps to move in a way that maximises reproduction and survival. Memory is found in all mammals, but also in insects and snails. The well-known sea snail *Aplysia californica* has memory – it shows

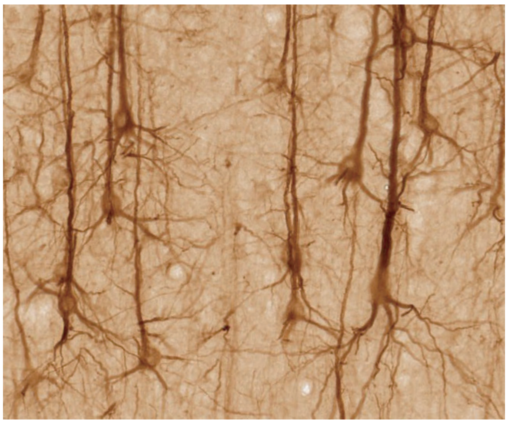

FIGURE 178 Photograph of stained pyramidal neurons in the cerebral cortex of the human cortex, showing their interconnections (© Medlat.

conditioning, like Pawlow's dogs – even though it has only 20 000 neurons. Experiments confirm that individual *long-time* memory is stored in the strength of neuron connections, the *synapses*. This statement was made already in 1949 by the Canadian psychologist Donald Hebb. In that year Hebb specified the physical embodiment of the observations of the psychologists Sigmund Freud and William James from the 1890s, who had already deduced that memory is about the strengthening and weakening of connections inside the brain. In short, observations and learning, everything we call *memories*, are recorded in the synapses.*

Obviously every record is an object. But under which conditions does an object qualify as a record? A signature can be the record of the agreement on a commercial transaction. A single small dot of ink is not a record, because it could have appeared by mistake, for example by an accidental blot. In contrast, it is improbable that ink should fall on paper exactly in the shape of a signature. (The simple signatures of physicians are obviously exceptions.) Simply speaking, a *record* is any object, which, in order to be copied, has to be forged. More precisely, a record is an object or a situation that cannot arise nor disappear by mistake or by chance. Our personal memories, be they images or voices, have the same property; we can usually trust them, because they are so detailed that they cannot have arisen by chance or by uncontrolled processes in our brain.

Can we estimate the probability for a record to appear or disappear by chance? Yes, we can. Every record is made of a characteristic number N of small entities, for example the

* The brain has various modes of learning, i.e., of storing in long-time memory, that depend on its hardware. Long-time learning always relies on changing synapse strengths or on growing new synapses. In a traumatic event, the brain learns within a few seconds to avoid similar situations for the rest of its life. In contrast, learning at school can take many months for a simple idea. In fact everybody can influence the ease and speed of learning; by mentally attaching images, voices, emotions, fantasies or memories to a topic or situation, one can speed up learning and reduce learning effort considerably.

Ref. 226 Research has shown that in the amygdala, where emotions and memory are combined, the enzyme calcineurin and the gene regulator Zif268 are important for traumatic memory: low calcineurin levels lead to increased expression of the gene regulator and to longer-lasting traumatic memory, high levels reduce the traumatic effect.

For usual long-time learning, the CPEB proteins, in particular Orb2A and Orb2B, seem to decide which synapses increase in strength.

number of the possible ink dots on paper, the number of iron crystals in a cassette tape, the electrons in a bit of computer memory, the silver iodide grains in a photographic negative, etc. The chance disturbances in any memory are due to internal fluctuations, also called *noise*. Noise makes the record unreadable; it can be dirt on a signature, thermal magnetization changes in iron crystals, electromagnetic noise inside a solid state memory, etc. Noise is found in all classifiers, since it is inherent in all interactions and thus in all information processing.

It is a general property that internal fluctuations due to noise decrease when the size, i.e., the number of components of the record is increased. In fact, the probability p_{mis}

Challenge 258 ny

for a misreading or miswriting of a record changes as

$$p_{\text{mis}} \sim 1/\sqrt{N} \,, \tag{93}$$

where N is the number of particles or subsystems used for storing it. This relation appears because, for large numbers, the so-called *normal distribution* is a good approximation of almost any process. In particular, the width of the normal distribution, which determines the probability of record errors, grows less rapidly than its integral when the number of entities is increased; for large numbers, such statements become more and more precise.

We conclude that any good record must be made from a *large* number of entities. The larger the number, the less sensitive the memory is to fluctuations. Now, a system of large size with small fluctuations is called a *(physical) bath*. Only baths make memories possible. In other words, every record contains a bath. We conclude that any *observation* of a system is the interaction of that system with a bath. This connection will be used several times in the following, in particular in quantum theory. When a record is produced by a machine, the 'observation' is usually called a *(generalized) measurement*. Are you able

Challenge 259 s

to specify the bath in the case of a person looking at a landscape?

From the preceding discussion we can deduce a powerful conclusion: since we have such a good memory at our disposition, we can deduce that we are made of many small parts. And since records exist, the world must also be made of a large number of small parts. No microscope of any kind is needed to confirm the existence of molecules or similar small entities; such a tool is only needed to determine the *sizes* of these particles. Their existence can be deduced simply from the observation that we have memory. (Of course, another argument proving that matter is made of small parts is the ubiquity of

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noise.)

A second conclusion was popularized in the late 1920s by Leo Szilard. Writing a memory does not necessarily produce entropy; it is possible to store information into a memory without increasing entropy. However, entropy is produced in every case that the memory is *erased*. It turns out that the (minimum) entropy created by erasing one bit is

Challenge 260 e

given by

$$S_{\text{per erased bit}} = k \ln 2 \,, \tag{94}$$

and the number $\ln 2 \approx 0.69$ is the natural logarithm of 2. Erasing thus on the one hand reduces the disorder of the data – the local entropy–, but on the other hand increases the total entropy. As is well known, energy is needed to reduce the entropy of a local system. In short, any system that erases memory requires energy. For example, a logical AND gate effectively erases one bit per operation. Logical thinking thus requires energy.

It is also known that *dreaming* is connected with the erasing and reorganization of information. Could that be the reason that, when we are very tired, without any energy left, we do not dream as much as usual? In dreams, the brain reorganizes the experiences made in the past. Dreams tell us what keeps our unconscious busy. Every person must decide by herself what to do with dreams that we recall. In short, dreams have no meaning – we give them meaning. In any case, dreams are one of the brain's ways to use memory efficiently.

Challenge 261 s

Entropy is thus necessarily created when we forget. This is evident when we remind ourselves that forgetting is similar to the deterioration of an ancient manuscript. Entropy increases when the manuscript is not readable any more, since the process is irreversible and dissipative.* Another way to see this is to recognize that to clear a memory, e.g. a magnetic tape, we have to put energy into it, and thus increase its entropy. Conversely, *writing* into a memory can often *reduce* entropy; we remember that signals, the entities that write memories, carry negative entropy. For example, the writing of magnetic tapes usually reduces their entropy.

Ref. 227

The capacity of the brain

> Computers are boring. They can give only answers.
>
> (Wrongly) attributed to Pablo Picasso

The human brain is built in such a way that its fluctuations cannot destroy its contents. The brain is well protected by the skull for exactly this reason. In addition, the brain literally grows connections, called *synapses*, between its various *neurons*, which are the cells doing the signal processing. The neuron is the basic processing element of the brain, performing the basic classification. It can only do two things: to fire and not to fire. (It is possible that the time at which a neuron fires also carries information; this question is not yet settled.) The neuron fires depending on its input, which comes via the synapses from hundreds of other neurons. A neuron is thus an element that can distinguish the inputs it receives into two cases: those leading to firing and those that do not. Neurons are thus classifiers of the simplest type, able only to distinguish between two situations.

Ref. 230, Ref. 231

Every time we store something in our long term memory, such as a phone number, the connection strength of existing synapses is changed or new synapses are grown. The connections between the neurons are much stronger than the fluctuations in the brain. Only strong disturbances, such as a blocked blood vessel or a brain lesion, can destroy neurons and lead to loss of memory.

* As Wojciech Zurek clearly explains, the entropy created inside the memory is the main reason that even Maxwell's demon cannot reduce the entropy of two volumes of gases by opening a door between them in such a way that fast molecules accumulate on one side and slow molecules accumulate on the other. (Maxwell had introduced the 'demon' in 1871, to clarify the limits posed by nature to the gods.) This is just another way to rephrase the old result of Leo Szilard, who showed that the measurements by the demon create more entropy than they can save. And every measurement apparatus contains a memory.

Ref. 228, Ref. 229

To play being Maxwell's demon, look for one of the many computer game implementations around the internet.

As a whole, the brain provides an extremely efficient memory. Despite intense efforts, engineers have not yet been able to build a memory with the capacity of the brain in the same volume. Let us estimated this memory capacity. By multiplying the number of neurons, about 10^{11},* by the average number of synapses per neuron, about 100, and also by the estimated average number of bits stored in every synapse, about 10**, we arrive at a conservative estimate for the storage capacity of the brain of about

$$M_{\text{rewritable}} \approx 10^{14}\,\text{bit} \approx 10^{4}\,\text{GB} . \tag{95}$$

(One *byte*, abbreviated B, is the usual name for eight bits of information.) Note that evolution has managed to put as many neurons in the brain as there are stars in the galaxy, and that if we add all the dendrite lengths, we get a total length of about 10^{11} m, which corresponds to the distance to from the Earth to the Sun. Our brain truly is *astronomically* complex.

However, this standard estimate of 10^{14} bits is not really correct! It assumes that the only component storing information in the brain is the synapse strength. Therefore it only measures the *erasable* storage capacity of the brain. In fact, information is also stored in the structure of the brain, i.e., in the exact configuration in which every cell is connected to other cells. Most of this structure is fixed at the age of about two years, but it continues to develop at a lower level for the rest of human life. Assuming that for each of the N cells with n connections there are $f\,n$ connection possibilities, this

Challenge 262 e

write once capacity of the brain can be estimated as roughly $N\sqrt{fn}\,fn\log fn$ bits. For $N = 10^{11}$, $n = 10^{2}$, $f = 6$, this gives

$$M_{\text{writeonce}} \approx 10^{16}\,\text{bit} \approx 10^{6}\,\text{GB} . \tag{96}$$

Ref. 232

Structural brain changes are measurable. Recent measurements confirmed that bilingual persons, especially early bilinguals, have a higher density of grey mass in the small parietal cortex on the left hemisphere of the brain. This is a region mainly concerned with language processing. The brain thus makes also use of structural changes for optimized storage and processing. Structure changes are also known for other populations, such as autistics, homophiles and hyperactive children. Intense and prolonged experiences during pregnancy or childhood seem to induce such structural developments.

Sometimes it is claimed that people use only between 5 % or 10 % of their brain capacity. This myth, which goes back to the nineteenth century, would imply that it is possible to measure the actual data stored in the brain and compare it with its available maximum. Alternatively, the myth implies that the processing capacity can be measured and compared with an available maximum capacity. The myth also implies that nature would develop and maintain an organ with 90 % overcapacity, wasting all the energy and material to build, repair and maintain it. The myth is wrong. At present, the storage capacity and the processing capacity of a brain cannot be measured, but only estimated.

* The number of neurons seems to be constant, and fixed at birth. The growth of interconnections is highest between age one and three, when it is said to reach up to 10^{7} new connections per second.

** This is an average. Some types of synapses in the brain, in the hippocampus, are known to store only one bit.

The large storage capacity of the brain also shows that human memory is filled by the environment and is not inborn: one human ovule plus one sperm have a mass of about 1 mg, which corresponds to about $3 \cdot 10^{16}$ atoms. Obviously, fluctuations make it impossible to store 10^{16} bits in these systems. In fact, nature stores only about $6 \cdot 10^9$ DNA base pairs or $12 \cdot 10^9$ bits in the genes of a fecundated ovule, using $3 \cdot 10^6$ atoms per bit. In contrast, a typical brain has a mass of 1.5 to 2 kg and contains about 5 to $7 \cdot 10^{25}$ atoms, which makes it as efficient a memory as an ovule. The difference between the number of bits in human DNA and those in the brain nicely shows that almost all information stored in the brain is taken from the environment; it cannot be of genetic origin, even allowing for smart decompression of stored information.

In total, all the tricks used by nature result in the most powerful classifier yet known.* Are there any limits to the brain's capacity to memorize and to classify? With the tools that humans have developed to expand the possibilities of the brain, such as paper, writing and printing to support memory, and the numerous tools available to simplify and to abbreviate classifications explored by mathematicians, brain classification is only limited by the time spent practising it. Without tools, there are strict limits, of course. The two-millimetre thick cerebral cortex of humans has a surface of about four sheets of A4 paper, a chimpanzee's yields one sheet, and a monkey's is the size of a postcard. It is estimated that the total intellectually accessible memory is of the order of

Ref. 233

$$M_{\text{intellectual}} \approx 1\ \text{GB}\ , \tag{97}$$

though with a large experimental error.

The brain is also unparalleled in its processing capacity. This is most clearly demonstrated by the most important consequence deriving from memory and classification: thought and language. Indeed, the many types of thinking or language we use, such as comparing, distinguishing, remembering, recognizing, connecting, describing, deducing, explaining, imagining, etc., all describe different ways to classify memories or perceptions. In the end, every type of thinking or talking directly or indirectly classifies observations. But how far are computers from achieving this! The first attempt, in 1966, was a programming joke by Joseph Weizenbaum: the famous chatterbot program Eliza (try it at www.manifestation.com/neurotoys/eliza.php3) is a parody of a psychoanalyst. Even today, over 40 years later, conversation with a computer program, such as Friendbot (found at www.friendbot.co.uk), is still a disappointing experience. The huge capacity of the brain is the main reason for this disappointment.

Incidentally, even though the brains of sperm whales and of elephants can be five to six times as heavy as those of humans, the number of neurons and connections, and thus the capacity, is lower than for humans. Snails, ants, small fish have neuron numbers of the order of 10 000; the well-studied nematode *Caenorhabditis elegans* has only 302, though other animals have even fewer.

* Also the power consumption of the brain is important: even though it contains only about 2 % of the body's mass, it uses 25 % of the energy taken in by food. The brain is the reason that humans like eating fruit.

Curiosities about the brain and memory

Teachers and learners should all be brain experts. The brain learns best when it has an *aim*. Without an aim, both the lecture preparation and the lecture performance will lose most of its possible effects. How many teachers state the aim of their class at its beginning? How many learners formulate learning aims?

The brain also learns best when it is *motivated*. Different students need different motivations: potential applications, curiosity, competition, activation of already acquired knowledge, impressing the opposite sex, or exploring the unknown. And students need motivations on different levels of difficulty. Which teacher provides this mix?

Finally, brains in students and learners have different ways to *create concepts*: using words, sounds, images, emotions, body sensations, etc. Which teacher addresses them all in his lessons?

* *

The brain plays strange games on the people that carry it. For example, the brain is always on the search for something to do. Many habits and many addictions grow in this way.

* *

The brain often commands the person that carry it. Modern research has shown that school pupils can be distinguished into five separate groups. Ref. 234

1. Smart students
2. Uninterested students
3. Students who overestimate themselves (often, but not always, boys)
4. Students who underestimate themselves (often, but not always, girls)
5. Struggling/weak students

This has to be kept in mind when teaching classes. To which group do/did you belong?

* *

Many cognitive activities of the brain are located in specific regions of the *cerebral cortex*, also called *grey matter* (see Figure 174). It is known that all grey matter is built of a large number of parallel, but largely independent structures, the so-called *neocortical columns*; they are similar to microprocessors. Each neocortical column has input and outputs, but works independently of the others; it is about 2 mm in height, 0.5 mm in diameter, and contains about 1000 neurons of various types. (See neurolex.org/wiki/Category:Neuron for a list.) The human cortex contains several millions of these columns, arranged in six layers. At present, researchers are able to simulate *one* neocortical column with *one* supercomputer. For more details, see the bluebrain.epfl.ch website. In short, your brain corresponds to several million supercomputers. Take good care of it.

* *

A beautiful atlas of the brain can be found at bigbrain.loris.ca. On this website, researchers from across the world collect the best images of the brain that modern research provides. Ref. 235

* *

The brain has many interesting sides. The technique of *neurofeedback* is an example. A few electrodes are attached to the skin of the head, and a feedback loop is created with the help of a visualization on a screen. Such a visualisation helps to put oneself into high-theta state – corresponding to deep relaxation –, or into the SMR state – corresponding to rest and concentration –, or into alpha-dominated states – corresponding to relaxation with closed eyes. Learning to switch rapidly between these states is helping athletes, surgeons, dancers, musicians, singers and children with attention deficit syndrome. After a few sessions, the effects stay for over a year. For attention deficit syndrome, the results
Ref. 236 are as good as with medication.

* *

One interesting side of the human brain is the wide range of *passions* it produces. For example, there are people whose passion drives them to dedicate all their life to singing. There are people whose life-long passion is to invent languages; John Ronald Tolkien is the most famous example. There are other people whose passion is to help murderers to find peace of mind. Some people dedicate their life to raising handicapped children unwanted by their parents. Other people dedicate their life to implementing rapid solutions for infrastructure problems – water, gas and electricity supplies – in cities under war. The examples one can find are fascinating.

* *

Many functions in the brain are not performed by the programmable part of the brain, the cortex, but by specialized hardware. The list of known specialized hardware parts of the brain is still growing, as discoveries are still being made.

Ref. 237 Figure 174 only shows the basic regions. Researchers have discovered dedicated neurons that control the walking process in each leg, dedicated neurons – the so-called *mirror neurons* – that re-enact what people we see are feeling or acting, and dedicated neurons from the eye to the brain that control the day–night cycle. These recent discoveries complement the older ones that there is specialized hardware for every sense in the neural system, from touch to smell to proprioception. In short, many basic functions of the neural system are wired in, and many advanced functions are as well. The full list of wired-in systems is not known yet. For example, only future research will help us to understand how much of our subconscious is due to hardware, and how much is due to the software in the cortex.

* *

Cats are smart animals, and everybody who interacts with them knows how elaborate their behaviour and the spectrum of their activities is. All this is organized by a brain of the size of a walnut, with about 300 million neurons.

Interestingly, every human has roughly the same number of neurons that are found in a cat's brain in a place outside the brain: the belly. This group of neurons is called the enteric nervous system. This large collection of neurons, over 100 millions of them, controls the behaviour of the gut cells – in particular, the first layer of gut cells that comes in contact with food – and controls the production of many enzymes and neurotransmitters, which in turn influence our mood. 95 % of the serotonin produced in the body are produced by the intestine. It may well turn out that treating depression requires treating

Vol. V, page 54 the intestine first. The enteric nervous system also determines whether vomiting is necessary or not, it triggers constipation and diarrhea, influences stress levels, regulates our immune system and controls numerous other processes. In short, the enteric nervous Ref. 238 system is the anatomical basis for our 'gut feelings' and probably for our general well-being. Research on these topics is still ongoing.

* *

We learn better if we *recall* what we learned. Experiments show that remembering strengthens synapses, and thus strengthens our memory. We learn better if we know the *reasons* for the things we are learning. Experiments show that causality strengthens synapses.

* *

We learn while sleeping. The brain stores most things we experience during the day in a region called the *hippocampus*. During deep sleep, i.e., in the sleep time without dreams, the brain selects which of those experiences need to be stored in its long-time memory, the *neocortex*. The selection is based on the emotions attached to the memory, especially excitement, fear or anger. But also the expectation of a reward – such as a present or the possibility to make good impression when asked about the topic – is extremely effective in transferring content into the neocortex, as research by Jan Born has shown. If this rule is followed, sleeping just after learning, and in particular, deep sleep, is the best way to study. The most effective way to learn a language, to learn a new topic, or to memorize a presentation is to sleep just after study or training.

Deep sleep helps learning. Deep sleep can be promoted in many ways. Effort, sport, and even electric stimulation increases deep sleep. The pharmaceutic industry is now trying to develop sleeping pills that increase deep sleep. Alcohol, most sleeping pills, television, the internet and traumatic events decrease deep sleep. Jan Born states that most probably, sleep exists in order to enable us to learn; no other explanation for the loss of consciousness during deep sleep is convincing.

How do we sleep? When we are awake, all sense input is sent to the *thalamus*, which filters it and sends it to the neocortex. During sleep the neocortex effectively switches off large parts of the thalamus, so that almost no sense input arrives to the neocortex. Modelling these processes even allows understanding how sleep starts and to reproduce Ref. 239 the brain waves seen during the beginning of sleep.

* *

Many modern research results on animal and human brains can be found at the Brain Map website, available at www.brain-map.org.

* *

Brains and computers differ markedly in the way they work. Brains are analog, computers are digital. How exactly do computers work? The general answer is: computers are a smart and organized collection of electrical switches. To make matters as easy as possible, the calculation engine inside a computer – the so-called *central processing unit*, the heart of the computer – calculates using binary numbers. The 'on' and 'off' states of a switch are associated to the digits '1' and '0'. Can you devise a simple collection of switches that allows adding two binary numbers of one digit? Of many digits? And to multiply two numbers? Try it – it is an interesting exercise.

Computers are called *digital* because they are based on switches. Indeed, all integrated circuits inside a pocket calculator or inside a laptop are just collections of electrical switches; modern specimen can contain several millions of them, each switch with a specific function.

* *

During pregnancy, the brain of the embryo grows at a rate of 250 000 neurons per minute. The rate shows how fascinating a process life is.

* *

The signal communication between the brain and the arms differs from the signal communication between the brain and the legs. When the brain sends a command for some arm or leg movement to the spine, the spine then in turn sends it to the arms or to the legs. For the arms (and hands) – but *not* for the legs – the spine sends a copy of the command it is sending there back to the brain. This feedback seems to allow the brain to specify its next motion command more precisely. Thus the body and the brain are hard-wired for the fine motor skills that help us to use our fingers and hands as precisely as possible. The importance of the fine motor skills was already known to the ancient Greeks; Anaxagoras said that humans are the most clever living beings because they have hands.

* *

Epilepsy is a group of brain disorders that affects a large percentage of the human population. Epilepsy is an electric malfunction of the brain. It leads to regular electrical oscillations inside the brain, during which the person loses awareness or even gets fits. Epilepsy is also one of the reasons for autistic behaviour. Epilepsy can be triggered by genetic defects, by injuries and by other causes. Research in epilepsy is a vast field.

Many genetic types of epilepsy are due to mutations in genes that code ion channels. When ion channels do not work properly, the concentration of cations such as sodium does not behave properly, leading to the electric malfunctions. Research into the origin of epilepsy has shown that some genetic mutations are not inherited from the parents, but are *de novo*: they appear only in the child.

* *

Researchers have tentatively linked gene defects to the propensity to forget things in everyday life. However, one can question whether an error in the DRD2 gene is really the cause for forgetting where the car keys are.

* *

Many switches has three states; one could call them '-1', '0', '1'. Thus, building computers based on 'trits' instead of 'bits' is a realistic option. Why are there no 27-trit computers?

Challenge 263 s

* *

Challenge 264 e

Does water have memory? Certain people make a living from this statement. However, the molecules in water have an average speed of 590 m/s at room temperature. The liquid state, together with this high speed, prevents the formation of stable aggregates beyond a length scale of a handful of molecules. Experiments trying to look for memory effects are all negative, apart those from crooks. Water has no memory.

Ref. 240

* *

The neurotransmitters that influence moods are still a topic of intense research. Such research has shown that a specific peptide called *hypocretin* or also *orexin* leads to high alertness, to increases appetite and above all to good mood. Whether this really is the 'happiness hormone', as is sometimes claimed, still has to be tested.

* *

In 2015, for the first time, lymphatic vessels have been discovered in the brain: both an entry and an exit of lymphatic liquid exist. This astonishing discovery by Antoine Louveau and his collaborators might well change the way researchers tackle Alzheimer's disease, autism, multiple sclerosis and many other neuro-immune diseases.

Ref. 241

* *

In 2015, 'speed cells' have been discovered inside brains. These cells are part of the navigation system of the brain, and fire with a frequency that is proportional to the proper speed of the organism. These cells thus have the same role as the speedometer in a car. Completing the understanding the navigation system of the brain is one of the present challenges of the neurosciences.

Ref. 242

* *

A fascinating aspect of the brain is the way it controls motion. Many research groups are exploring how arm, leg and body motion is learned, structured, stored and controlled in the brain. The brain controls motion by constructing its commands to muscles from a small set of *motion primitives*.

Other research groups are exploring how the brain coordinates the motion of lips, tongue, jaws and larynx during speech. Also in this case it seems that speech motion is controlled as a sequence of learned motion primitives; these primitives seem similar or at least closely related to syllables.

* *

The aim to read someone's thoughts is still distant. But reading someone's emotions is already possible with magnetic resonance machines. A glimpse of the approach is shown in Figure 179. By properly weighing the neural activity of certain areas of the brain it became possible to distinguish anger, disgust, envy, fear, happiness, lust, pride, sadness and shame – though with an accuracy of the order of 85 %.

Vol. V, page 162

Ref. 243

* *

Can one influence the brain without chemicals or touch, i.e., without the subject noticing? It seems so. Recent research on monkeys has shown that focussed ultrasound can influence the decisions of the brain . The effect is small and new. Future will tell if this

Ref. 244

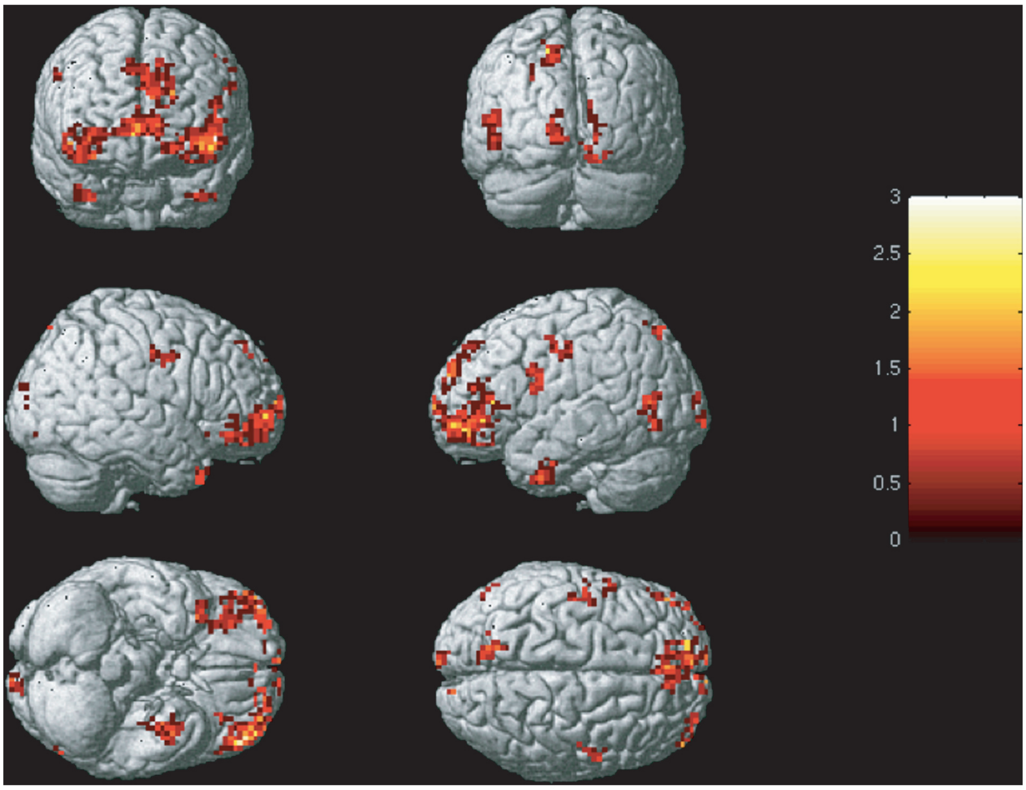

FIGURE 179 Brain scans with functional magnetic resonance superimposed on a brain picture allow to determine the emotion felt, if the shown locations are properly weighted (© Karim S. Kassan).

stays so.

* *

Recent research has show that by far the best way to keep the brain healthy is regular movement. For example, regular movement is more effective than any known medicine in preventing both arteriosclerosis and Alzheimer's disease.

Chapter 8

LANGUAGE AND CONCEPTS

> Reserve your right to think, for even to think wrongly is better than not to think at all.
>
> Hypatia of Alexandria

Language possibly is the most wonderful gift of human nature. We have all learned it from somebody who cared about us. Nevertheless, the origins of language are hidden in the distant past of humanity. Language is produced and transmitted from one brain to another. Because we have repeatedly stated that physics is talking about motion, we have to explore language also in our adventure. Physics is a precise language specialized for the description of motion. We will find out in our walk that this specific definition of language is not restrictive, because everything in the world is in motion. Thus, physics is a precise language for everything. And our quest for precision demands that we explore the meaning, use and limits of language.

What is language?

> Ein Satz kann nur sagen, *wie* ein Ding ist, nicht *was* es ist.**
>
> Ludwig Wittgenstein, *Tractatus*, 3.221

Using the ability to produce sounds and to put ink on paper, people attach certain *symbols*,*** also called *words* or *terms* in this context, to the many partitions they specify with the help of their thinking. Such a categorization is then said to define a *concept* or *notion*, and is set in *italic typeface* in this text. *A standard set of concepts forms a language.*****

Ref. 245 In other words, we have:

** 'A proposition can only say *how* a thing is, not *what* it is.'

*** A symbol is a type of *sign*, i.e., an entity associated by some convention to the object it refers. Following Charles Peirce (b. 1839 Cambridge, d. 1914 Milford) – see www.peirce.org – the most original philosopher born in the United States, a symbol differs from an *icon* (or *image*) and from an *index*, which are also attached to objects by convention, in that it does not resemble the object, as does an icon, and in that it has no contact with the object, as is the case for an index.

**** The recognition that language is based on a partition of ideas, using the various differences between them to distinguish them from each other, goes back to Ferdinand de Saussure (b. 1857 Geneva, d. 1913 Vufflens), who is regarded as the founder of linguistics. His textbook *Cours de linguistique générale*, Editions Payot, 1985, has been the reference work of the field for over half a century. Note that Saussure, in contrast to Peirce, prefers the term 'sign' to 'symbol', and that his definition of the term 'sign' includes also the object to which it refers.

TABLE 20 Language basics.

Aspect	Value
Human phonemes	*c.* 70
English phonemes	44
German phonemes	40
Italian phonemes	30
Words of the English language (more than all other languages, with the possible exception of German)	*c.* 350 000
Number of languages on Earth in the year 2000	*c.* 6000

▷ A *(human) language* is a standard way of symbolic interaction between people.

There are human languages based on facial expressions, on gestures, on spoken words, on whistles, on written words, and more. The use of *spoken* language is considerably younger than the human species; it seems that it appeared only about one hundred thousand years ago. *Written* language is even younger, namely only about six thousand years old. The set of concepts used in language, the *vocabulary*, is still expanding. For humans, the understanding of language begins soon after birth (perhaps even before), the active use begins at around a year of age, the ability to read can start as early as two, and personal vocabulary continues to grow as long as curiosity is alive.

Evolutionary biologists like to stress a further point that is necessary for the definition of language:

▷ Only a symbolic interaction system that allows communication about what is *not present* – not here and not now – forms a *language*.

Pointing at an object or a place is a sign that is understood and used by many animals. But the ability to use signs to talk about the past, about the future and about a different location is required to transform a symbol collection into a language. It is not clear whether an animal species with this ability exists; the great apes are obvious candidates.

Physics being a lazy way to chat about motion, it needs language as an essential tool. Of the many aspects of language, from literature to poetry, from jokes to military orders, from expressions of encouragement, dreams, love and emotions, physics uses only a small and rather special segment. This segment is defined by the inherent restriction to talk about motion. Since motion is an observation, i.e., an interaction with the environment that several people experience in the same way, this choice puts a number of restrictions on the contents – the vocabulary – and on the form – the grammar – of such discussions.

For example, from the definition that observations are shared by others, we get the requirement:

▷ Statements describing observations must be translatable into all languages.

But when can a statement be translated? On this question two extreme points of view are possible: the first maintains that *all* statements can be translated, since it follows from the properties of human languages that each of them can express every possible statement. In this view, we can say:

▷ Only sign systems that allow expressing the complete spectrum of human messages form a *human language*.

This definition of language distinguishes human spoken and sign language from animal languages, such as the signs used by apes, birds or honey bees, and also from computer languages, such as Pascal or C. With this meaning of language, all statements can be translated by definition.

It is more challenging for a discussion to follow the opposing view, namely that precise translation is possible only for those statements which use terms, word types and grammatical structures found in *all* languages. Linguistic research has invested considerable effort in the distillation of phonological, grammatical and semantic *universals*, as they are called, from the 6000 or so languages thought to exist today.*

Language components and their hierarchy

> Jedes Wort ist ein Vorurteil.
>
> Friedrich Nietzsche**

The investigations into the *phonological* aspect of language showed for example that every human language has at least two consonants and two vowels and that the number of phonemes in the world is limited. However, such studies do not provide any material for the discussion of translation.***

Studies of the *grammatical* (or *syntactic*) aspect showed that all languages use smallest elements, called 'words', which they group into sentences. They all have pronouns for the first and second person, 'I' and 'you', and always contain nouns and verbs. All languages use *subjects* and *predicates* or, as one usually says, the three entities *subject*, *verb* and
Challenge 265 e
object, though not always in this order. Just check the languages you know.

Exploring the *semantic* aspect of language, the long list of lexical universals, i.e., words that appear in all languages, such as 'mother' or 'Sun', has recently been given a struc-

* A professional database by the linguist Merritt Ruhlen with 5700 languages and many details on each language can be found at ehl.santafe.edu/intro1.htm. A long but unprofessional list with 6 900 languages (and with 39 000 language and dialect names) can be found on the website www.ethnologue.com. Beware, it is edited by a fringe religious group that aims to increase the number of languages as much as possible.

It is estimated that 15 000 ± 5 000 languages have existed in the past.

Nevertheless, in today's world, and surely in the sciences, it is often sufficient to know one's own language plus English. Since English is the language with the largest number of words, learning it well is a
Ref. 246
greater challenge than learning most other languages.

** 'Every word is a prejudice.' Friedrich Nietzsche (b. 1844 Röcken, d. 1900 Weimar) was an influential philologist and philosopher.

*** Phonological studies also explore topics such as the observation that in many languages the word for 'little' contains an 'i' (or high pitched 'e') sound: petit, piccolo, klein, tiny, pequeño, chiisai; exceptions are: small, parvus. Other researchers have shown that languages in regions that are warm and have many trees have more vowels and fewer consonants.

TABLE 21 The universal semantic primitives, following Anna Wierzbicka.

I, you, someone, something, people	[substantives]
this, the same, one, two, all, much/many	[determiners and quantifiers]
know, want, think, feel, say	[mental predicates]
do, happen	[agent, patient]
good, bad	[evaluative]
big, small	[descriptors]
very	[intensifier]
can, if (would)	[modality, irrealis]
because	[causation]
no (not)	[negation]
when, where, after (before), under (above)	[time and place]
kind of, part of	[taxonomy, partonomy]
like	[hedge/prototype]

ture. The linguist Anna Wierzbicka performed a search for the building blocks from which all concepts can be built. She looked for the definition of every concept with the help of simpler ones, and continued doing so until a fundamental level was reached that cannot be further reduced. The set of concepts that are left over are the semantic primitives. By repeating this exercise in many languages, Wierzbicka found that the list is the same in all cases. She thus had discovered *universal semantic primitives*. In 1992, the list contained the terms given in Table 21.

Following the life-long research of Anna Wierzbicka and her research school, all these concepts exist in all languages of the world studied so far.* They have defined the meaning of each primitive in detail, performed consistency checks and eliminated alternative approaches. They have checked this list in languages from all language groups, in languages from all continents, thus showing that the result is valid everywhere. In every language all other concepts can be defined with the help of the semantic primitives. Ref. 247

Simply stated, learning to speak means learning these basic terms, learning how to combine them and learning the names of these composites. The definition of language given above, namely as a means of communication that allows one to express everything one wants to say, also about other places and other times, can thus be refined:

▷ Only a set of concepts that includes the universal semantic primitives forms a *human language*.

For physicists – who aim to talk in as few words as possible – the list of semantic primitives has three facets. First, the approach is similar to physics' own aim: the idea of primitives gives a structured summary of *everything that can be said*, just as the atomic

* It is easy to imagine that this research steps on the toes of many people. A list that maintains that we only have about thirty basic concepts in our heads is taken to be offensive by many small minds. In addition, a list that maintains that 'true', 'creation', 'life', 'mother' and 'god', are composite will elicit intense reactions, despite the correctness of the statements. Also the terms 'light' and 'motion' are missing. Indeed, some of these terms were added in later version of the list.

FIGURE 180 One goal of physics is to describe all of nature like a smurf: using only a single concept. (© Peyo 2016, licensed through I.M.P.S., Brussels, www.smurf.com)

elements structure all material objects that can be touched. Second, the list of primitives can be divided into *two groups*: one group contains all terms describing motion (do, happen, when, where, feel, small, etc. – probably a term from the semantic field around light or colour should be added) and the other group contains all terms necessary to talk about abstract sets and relations (this, all, kind of, no, if, etc.). Even for linguistics, aspects of motion and logical concepts are the basic entities of human experience and human thinking. To bring the issue to a point,

Challenge 266 e

▷ The semantic primitives contain the basic elements of physics and the basic elements of mathematics.

All humans are thus both physicists and mathematicians.

The third facet about Wierzbicka's list of semantic primitives is that for a physicist it is *too long*. The division of the list into two groups directly suggests shorter lists; we just have to ask physicists and mathematicians for concise summaries of their respective fields. To appreciate this aim, try to define what 'if' means, or what 'no' or an 'opposite' is – and explore your own ways of reducing the list.

Challenge 267 d

Reducing the list of primitives is also one of our aims in this adventure. Indeed, we will explore the mathematical group of primitives in this chapter. The physical group will occupy us in the rest of our adventure. However, a shorter list of primitives is not sufficient:

▷ Physics' (and our) goal is to arrive at a description of nature consisting of only *one* basic concept.

Physicists want to speak like *smurfs:* using just a single term. Reaching this goal is not simple, though. On the one hand, we need to check whether the set of classical physical concepts that we have discovered so far is *complete*. For example, can classical physical concepts describe *all* observations – with precision? The volume on quantum physics is devoted to this question. On the other hand, we need to *reduce* the list. This task is not straightforward; we have already discovered that physics is based on a circular definition: in Galilean physics, space and time are defined using matter, and matter is defined using space and time. We will need quite some effort to overcome this obstacle. The final part of this text tells the precise story on how to reduce the list. After various adventures we will indeed discover a basic concept on which all other concepts can be based.

Vol. IV, page 15
Vol. I, page 438
Vol. VI, page 148

We can summarize all the above-mentioned results of linguistics in the following way. If we construct a statement consisting of nouns, verbs and a few other concepts built from the semantic primitives, we are sure that it can be translated into all languages. This might explain why physics textbooks are often so *boring*: the authors are often too afraid to depart from this basic scheme of telling things. On the other hand, research has shown that such simple and straightforward statements are not restrictive. Exaggerating somewhat: *With a few nouns and verbs we can say* everything *that can be said.*

> Every word was once a poem.
>
> Ralph Waldo Emerson*

Is mathematics a language?

> Die Sätze der Mathematik sind Gleichungen, also Scheinsätze. Der Satz der Mathematik drückt keinen Gedanken aus.**
>
> Ludwig Wittgenstein, *Tractatus*, 6.2, 6.21

There is a group of people that has taken the strict view on translation and on precision to the extreme. They build all concepts from an even smaller set of primitives, namely only two: 'set' and 'relation', and explore the various possible combinations of these two concepts, studying their various classifications. Step by step, this radical group, commonly called *mathematicians*, came to define with full precision concepts such as numbers, points, curves, equations, symmetry groups and more. The construction of these concepts is summarized partly in the following and partly in the next volume of this adventure.

Vol. IV, page 223

Mathematics is a *vocabulary* that helps us to talk with precision. Mathematics can be seen as the exploration of *all* possible concepts that can be constructed from the two fundamental bricks 'set' and 'relation' (or some alternative, but equivalent pair).

▷ *Mathematics* is the science of symbolic necessities.

Rephrased again, mathematics is the exploration of all possible types of classifications. Or, less humorously: mathematics is the exploration of tautologies. These aspects explain the usefulness of mathematics in all situations where complex, yet precise classifications of observations are necessary, such as in physics.

* Ralph Waldo Emerson (b. 1803 Boston, d. 1882 Concord) was an influential essayist and philosopher.
** 'The propositions of mathematics are equations, and therefore pseudo-propositions. A proposition of mathematics does not express a thought.'

However, mathematics cannot express everything that humans want to communicate, such as wishes, ideas or feelings. Just try to express the fun of swimming using mathematics. Indeed, mathematics is the science of symbolic *necessities*; thus mathematics is not a language, nor does it contain one. Mathematical concepts, being based on *abstract* sets and relations, do not pertain to nature. Despite its beauty, mathematics does not allow us to talk about nature or the observation of motion. Mathematics does not tell *what* to say about nature; it does tell us *how* to say it.

In short, the precision of mathematics, in particular, its axiomatic structure, has an unwanted consequence: no mathematical concept talks about nature or about observations.*

▷ Mathematics is not a language.

Therefore, the study of motion needs other, more useful concepts.

In his famous 1900 lecture in Paris, the important mathematician David Hilbert** gave a list of 23 great challenges facing mathematics. The sixth of Hilbert's problems was to find a mathematical treatment of the axioms of physics. Our adventure so far has shown that physics started with a *circular definition* that has not yet been eliminated after Vol. I, page 438 2500 years of investigations: space-time is defined with the help of objects and objects are defined with the help of space and time. Being based on a circular definition, physics is thus *not* modelled after mathematics, even if many physicists and mathematicians, Ref. 262 including Hilbert, would like it to be so. Physicists must live with logical problems and must walk on unsure ground in order to achieve progress. In fact, they have done so for 2500 years.

In summary, mathematics is not a language; physics is. Mathematics is an axiomatic system. Physics is not. If physics were an axiomatic system, it would not contain circular definitions; in that case, it would cease to be a language and would cease to describe nature.

---

* Insofar as we can say that mathematics is based on the concepts of 'set' and 'relation', which are based on experience, we can say that mathematics explores a section of reality, and that its concepts are *derived* from experience. This and similar views of mathematics are called *platonism*. More concretely, platonism is the view that the concepts of mathematics exist *independently* of people, and that they are discovered, and not created, by mathematicians.

In fact, since mathematics makes use of the brain, which is a physical system, actually *mathematics is applied physics*.

However, we will discover that the concept of 'set' does *not* apply to nature; this changes the discussion Vol. VI, page 106 in completely.

** David Hilbert (b. 1862 Königsberg, d. 1943 Göttingen) was professor of mathematics in Göttingen and the greatest mathematician of his time. He was a central figure to many parts of mathematics, and also played an important role both in the birth of general relativity and of quantum theory. His textbooks are still in print. His famous personal credo was: 'Wir müssen wissen, wir werden wissen.' (We must know, we will know.) His famous Paris lecture is published e.g. in *Die Hilbertschen Probleme*, Akademische Verlagsgesellschaft Geest & Portig, 1983. The lecture galvanized all of mathematics. (Despite efforts and promises of similar fame, *nobody* in the world had a similar overview of mathematics that allowed him or her to repeat the feat in the year 2000.) In his last decade he suffered the persecution of the Nazi regime; the persecution eliminated Göttingen from the list of important science universities, without recovering its place up to this day.

▷ Physics is a language because it is not an axiomatic system.

Vol. VI, page 110 We will return to this central issue in the last part of our adventure. We noted that the concepts needed for the precise description of motion must be physical, as mathematical concepts are not sufficient.

> Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und sofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit.*
>
> Albert Einstein

What is a concept?

> Concepts are merely the results, rendered permanent by language, of a previous process of comparison.
>
> William Hamilton

What properties must a useful concept have? For example, what is 'passion' and what is a 'cotton bud'? Obviously, a useful concept implies a list of its parts, its aspects and their internal relations, as well as their relation to the exterior world. Therfore thinkers in all fields, from mathematics to physics, even from philosophy to politics, agree that the definition is:

▷ A *concept* has
1. explicit and fixed *content*,
2. explicit and fixed *limits*,
3. explicit and fixed *domain of application*.

The inability to state these properties or to keep them fixed is often the easiest way to distinguish *crackpots* from more reliable thinkers. Unclearly defined terms, which thus do not qualify as concepts, regularly appear in myths, e.g. 'dragon' or 'sphinx', or in ideologies, e.g. 'worker', 'soul' or 'paradigm'. Even physics is not immune. For example, we will discover later that neither 'universe' nor 'creation' are concepts. Are you able to argue the case? Challenge 268 s

But the three defining properties of any concept are interesting in their own right. Explicit content means that concepts are built one onto another. In particular, the most fundamental concepts appear to be those that have no parts and no external relations, but only internal ones. Can you think of one? Challenge 269 s

The requirements of explicit limits and explicit contents also imply that all concepts describing nature are *sets* or *relations* or both – since sets and relations obey the requirements for concepts.** Since mathematics is based on the concepts of 'set' and of 'relation', we follow directly that mathematics can provide the *form* for any concept, especially whenever high precision is required, as in the study of motion. Obviously, the

* 'In so far as mathematical statements describe reality, they are not certain, and as far as they are certain, they are not a description of reality.'

** We see that every physical concept is an example of a (mathematical) *category*, i.e., a combination of objects and mappings/relations. For more details about categories, with a precise definition of the term, see page 290.

content of the description is only provided by the study of nature itself; only then do concepts become useful.

Physics is the precise description of motion. In physics, the search for sufficiently precise concepts can be seen as the single theme structuring the long history of the field. Regularly, new concepts have been proposed, explored in all their properties, and tested. Finally, concepts are rejected or adopted, in the same way that children reject or adopt a new toy. Children do this unconsciously, scientists do it consciously, using language.* For this reason, physical concepts, and thus all concepts, are universally intelligible.

Note that the concept '*concept*' itself is not definable independently of experience; a concept is something that helps us to act and react to the world in which we live. Moreover, concepts do not live in a world separate from the physical one: every concept requires memory from its user, since the user has to remember the way in which it was formed; therefore every concept needs a material support for its use and application. Thus all thinking and all science is fundamentally based on experience.

In conclusion, all concepts are based on the idea that nature is made of related parts. This idea leads to complementing couples such as 'noun–verb' in linguistics, 'set–relation' or 'definition–theorem' in mathematics, and 'aspect of nature–pattern of nature' in physics. These couples constantly guide human thinking, from childhood onwards, as developmental psychology can testify. We now explore some specific concepts of importance in our adventure.

What are sets? What are relations?

> Alles, was wir sehen, könnte auch anders sein. Alles, was wir überhaupt beschreiben können, könnte auch anders sein. Es gibt keine Ordnung der Dinge a priori.**
>
> Ludwig Wittgenstein, *Tractatus*, 5.634

Defining sets and defining relations are the two fundamental acts of our thinking. This can be seen most clearly in any book about mathematics; such a book is usually divided into paragraphs labelled 'definition' on the one hand, and 'theorem', 'lemma' and 'corollary' on the other hand. The first type of paragraph introduces some set, and the other types of paragraphs express relations, i.e., connections between these sets or their elements. *Mathematics* is thus the exploration of the possible symbolic concepts and their relations. As we said above, mathematics is the science of symbolic necessities.

Sets and relations are tools of classification; that is why they are also the tools of any bureaucrat. (See Figure 181.) This class of humans is characterized by heavy use of paper clips, files, metal closets, archives – which all define various types of sets – and by the extensive use of numbers, such as reference numbers, customer numbers, passport

* Concepts formed unconsciously in our early youth are the most difficult to define precisely, i.e., with language. Some who were unable to define them, such as the influential philosopher Immanuel Kant (b. 1724 Königsberg, d. 1804 Königsberg), used to call them 'a priori' concepts (such as 'space' and 'time') to contrast them with the more clearly defined 'a posteriori' concepts. Today, this distinction has been shown to be unfounded both by the study of child psychology (see the footnote on page 256) and by physics itself, so that these qualifiers are therefore not used in our walk.

** 'Everything we see could also be otherwise. Everything we describe at all could also be otherwise. There is no order of things a priori.'

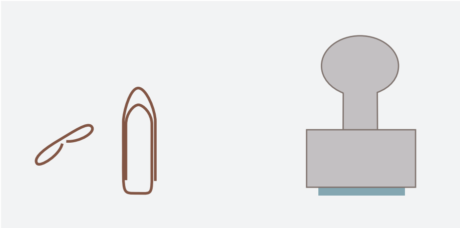

FIGURE 181 Devices for the definition of sets (left) and of relations (right).

TABLE 22 The defining properties of a set – the ZFC axioms.

The axioms of Zermelo–Fraenkel–C set theory
– Two sets are equal if and only if they have the same elements. (Axiom of extensionality)
– The empty set is a set. (Axiom of the null set)
– If x and y are sets, then the unordered pair $\{x, y\}$ is a set. (Axiom of unordered pairs)
– If x is a set of sets, the union of all its members is a set. (Union or sum set axiom)
– The entity $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\{\emptyset\}\}\}, ...\}$ is a set [a] – in other words, infinite collections, such as the natural numbers, are sets. (Axiom of infinity)
– An entity defined by all elements having a given property is a set, provided this property is reasonable; some important technicalities defining 'reasonable' are necessary. (Axiom of separation)
– If the domain of a function is a set, so is its range. (Axiom of replacement)
– The entity y of all subsets of x is also a set, called the power set. (Axiom of the power set)
– A set is not an element of itself – plus some technicalities. (Axiom of regularity)
– The product of a family of non-empty sets is non-empty. Equivalently, picking elements from a list of sets allows one to construct a new set – plus technicalities. (Axiom of choice, C)

a. The more common formulation (though equivalent to the above) is this: The entity $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, ...\}$ is a set.

numbers, account numbers, law article numbers – which define various types of relations between the items, i.e., between the elements of the sets.

In short, mathematics, at its worst, is the bureaucracy of physics. Mathematics, at its best, leads our thoughts to new knowledge.

Both the concepts of set and of relation express, in different ways, the fact that nature can be *described*, i.e., that it can be *classified* into parts that form a whole. The act of grouping together aspects of experience, i.e., the act of classifying them, is expressed in formal language by saying that a set is defined. In other words, a *set* is a collection of *elements* of our thinking. Every set distinguishes the elements from each other and from the set itself. This definition of 'set' is called the *naive* definition. For physics, the definition is sufficient, but you won't find many who will admit this. In fact, mathematicians have refined the definition of the concept 'set' several times, because the naive definition does not work well for infinite sets. A famous example is the story about sets which do not contain themselves. Obviously, any set is of two sorts: either it contains itself or it does not. If we take the set of all sets that do *not* contain themselves, to which sort does it belong?

Challenge 270 s

To avoid problems with the concept of 'set', mathematics requires a precise definition.

The first such definition was given by the mathematicians Ernst Zermelo (b. 1871 Berlin, d. 1951 Freiburg i.B.) and Adolf/Abraham Fraenkel (b. 1891 München, d. 1965 Jerusalem). Later, the so-called *axiom of choice* was added, in order to make it possible to manipulate a wider class of infinite sets. The result of these efforts is called the *ZFC set definition* and is given in Table 22.* From this basic definition we can construct all mathematical concepts used in physics. From a practical point of view, it is sufficient to keep in mind that for the whole of physics, the naive definition of a set is equivalent to the precise ZFC definition, actually even to the simpler ZF definition. Subtleties appear only for some special types of infinite sets, but these are not used in physics. In short, from the basic, naive set definition we can construct all concepts used in physics.

Ref. 249 The naive set definition is far from boring. To satisfy two people when dividing a cake, we follow the rule: I cut, you choose. The method has two properties: it is *just*, as everybody thinks that they have the share that they deserve, and it is *fully satisfying*, as everybody has the feeling that they have at least as much as the other. What rule is needed for three people? And for four? Challenge 272 d

Apart from defining sets, every child and every brain creates links between the different aspects of experience. For example, when it hears a voice, it automatically makes the connection that a human is present. In formal language, connections of this type are called *relations*. Relations connect and differentiate elements along other lines than sets: the two form a complementing couple. Defining a set unifies many objects and at the same time divides them into two: those belonging to the set and those that do not; defining a (binary) relation unifies elements two by two and divides them into many, namely into the many couples it defines.

Sets and relations are closely interrelated concepts. Indeed, one can define (mathematical) relations with the help of sets. A *(binary) relation* between two sets X and Y is a subset of the product set, where the *product set* or *Cartesian product* $X \times Y$ is the set of all ordered pairs (x, y) with $x \in X$ and $y \in Y$. An *ordered pair* (x, y) can easily be defined with the help of sets. Can you find out how? Challenge 273 s For example, in the case of the relation 'is wife of', the set X is the set of all women and the set Y that of all men; the relation is given by the list all the appropriate ordered pairs, which is much smaller than the product set, i.e., the set of all possible woman–man combinations.

It should be noted that the definition of relation just given is not really complete, since every construction of the concept 'set' already contains certain relations, such as the relation 'is element of.' It does not seem to be possible to reduce either one of the concepts 'set' or 'relation' completely to the other one. This situation is reflected in the physical cases of sets and relations, such as space (as a set of points) and distance, which also

* A global overview of axiomatic set theory is given by Paul J. Cohen & Reuben Hersch, *Non-Cantorian set theory*, Scientific American **217**, pp. 104–116, 1967. Those were the times when Scientific American was a quality magazine.

For a good introduction to the axiom of choice, see the www.math.vanderbilt.edu/~schectex/ccc/choice.html website.

Ref. 248 Other types of entities, more general than standard sets, obeying other properties, can also be defined, and are also subject of (comparatively little) mathematical research. To find an example, see the section on cardinals later on. Page 289 Such more general entities are called classes whenever they contain at least one set. Can you give an example? Challenge 271 s In the final part of our mountain ascent we will meet physical concepts that are described neither by sets nor by classes, containing no set at all. That is where the real fun starts.

seem impossible to separate completely from each other. In other words, even though mathematics does not pertain to nature, its two basic concepts, sets and relations, are taken from nature. In addition, the two concepts, like those of space-time and particles, are each defined with the other.

Infinity – and its properties

Mathematicians soon discovered that the concept of 'set' is only useful if one can also call collections such as $\{0, 1, 2, 3...\}$, i.e., of the number 0 and all its successors, a 'set'. To achieve this, one property in the Zermelo–Fraenkel list defining the term 'set' – given in Table 22 – explicitly specifies that this infinite collection can be called a set. (In fact, also the axiom of replacement states that sets may be infinite.) Infinity is thus put into mathematics and added to the tools of our thinking right at the very beginning, in the definition of the term 'set'. When describing nature, with or without mathematics, we should never forget this fact. A few additional points about infinity should be of general knowledge to any expert on motion.

A set is infinite if there is a function from it into itself that is *injective* (i.e., different elements map to different results) but not *onto* (i.e., some elements do not appear as images of the map); e.g. the map $n \mapsto 2n$ shows that the set of integers is infinite. Infinity also can be checked in another way: a set is infinite if it remains so also after removing one element, even repeatedly. We just need to remember that the empty set is *finite*.

Only *sets* can be infinite. And sets have parts, namely their elements. When a thing or a concept is called 'infinite' one can *always* ask and specify what its parts are: for space the parts are the points, for time the instants, for the set of integers the integers, etc. An indivisible or a finitely divisible entity cannot be called infinite.*

There are *many types* of infinities, all of different sizes.** This important result was discovered by the important mathematician Georg Cantor (b. 1845 Saint Petersburg, d. 1918 Halle an der Saale). He showed that from the countable set of natural numbers one can construct other infinite sets which are not countable. He did this by showing that the *power set* $P(\omega)$, namely the set of all subsets, of a countably infinite set is infinite, but *not* countably infinite. Sloppily speaking, the power set is 'more infinite' than the original set. The real numbers $\mathbb{R}$, to be defined shortly, are an example of an uncountably infinite set; there are many more of them than there are natural numbers. (Can you show this?) However, *any* type of infinite set contains at least one subset which is countably infinite.

Challenge 274 s

Even for an infinite set we can define size as the number of its elements. Cantor called this concept of size the *cardinality* of a set. The cardinality of a finite set is simply given by the number of its elements. The cardinality of a power set is 2 exponentiated by the cardinality of the set. The cardinality of the set of integers is called $\aleph_0$, pronounced 'aleph zero', after the first letter of the Hebrew alphabet. The smallest *uncountable* cardinal is called $\aleph_1$. The next cardinal is called $\aleph_2$ etc. A whole branch of mathematics is con-

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* Therefore, most gods, being concepts and thus sets, are either finite or, in the case where they are infinite, they are divisible. It seems that only polytheistic and pantheistic world views are not disturbed by this conclusion.

** In fact, there is such a huge number of types of infinities that none of these infinities itself actually describes this number. Technically speaking, there are as many infinities as there are ordinals.

cerned with the manipulation of these infinite 'numbers'; addition, multiplication, exponentiation are easily defined. For some of them, even logarithms and other functions make sense.

Ref. 250

The cardinals defined using power sets, including $\aleph_n$, $\aleph_\omega$ and $\aleph_{\aleph_\aleph}$, are called *accessible*, because since Cantor, people have defined even larger types of infinities, called *inaccessible*. These numbers (inaccessible cardinals, measurable cardinals, supercompact cardinals, etc.) need additional set axioms, extending the ZFC system. Like the ordinals and the cardinals, they form examples of what are called *transfinite* numbers.

The real numbers have the cardinality of the power set of the integers, namely $2^{\aleph_0}$. Can you show this? The result leads to the famous question: Is $\aleph_1 = 2^{\aleph_0}$ or not? The statement that this be so is called the *continuum hypothesis* and was unproven for several generations. The surprising answer came in 1963: the usual definition of the concept of set is not specific enough to fix the answer. By specifying the concept of set in more detail, with additional axioms – remember that axioms are defining properties – you can make the continuum hypothesis come out either right or wrong, as you prefer.

Challenge 275 s

Ref. 251

Another result of research into transfinites is important: for every definition of a type of infinite cardinal, it seems to be possible to find a larger one. In everyday life, the idea of infinity is often used to stop discussions about size: 'My big brother is stronger than yours.' 'But mine is infinitely stronger than yours!' Mathematics has shown that questions on size do continue afterwards: 'The strength of my brother is the power set of that of yours!' Rucker reports that mathematicians conjecture that there is no possible nor any conceivable end to these discussions.

Ref. 250

For physicists, a simple question appears directly. Do infinite quantities exist in nature? Or better, is it necessary to use infinite quantities to describe nature? You might want to clarify your own opinion on the issue. The question will be settled during the rest of our adventure.

Challenge 276 e

Functions and structures

Which relations are useful to describe patterns in nature? A typical example is 'larger stones are heavier'. Such a relation is of a specific type: it relates one specific value of an observable 'volume' to one specific value of the observable 'weight'. Such a one-to-one relation is called a *(mathematical) function* or *mapping*. Functions are the most specific types of relations; thus they convey a maximum of information. In the same way as numbers are used for observables, functions allow easy and precise communication of relations between observations. All physical rules and 'laws' are therefore expressed with the help of functions and, since physical 'laws' are about measurements, functions of numbers are their main building blocks.

A *function* f, or *mapping*, is a thus binary relation, i.e., a set $f = \{(x, y)\}$ of ordered pairs, where for every value of the first element x, called the *argument*, there is only *one* pair (x, y). The second element y is called the *value* of the function at the argument x. The set X of all arguments x is called the *domain of definition* and the set Y of all second arguments y is called the *range* of the function. Instead of $f = \{(x, y)\}$ one writes

$$f : X \to Y \quad \text{and} \quad f : x \mapsto y \quad \text{or} \quad y = f(x) , \tag{98}$$

where the type of arrow – with initial bar or not – shows whether we are speaking about sets or about elements.

We note that it is also possible to use the couple 'set' and 'mapping' to define all mathematical concepts; in this case a relation is defined with the help of mappings. A modern school of mathematical thought formalized this approach by the use of (mathematical) *categories*, a concept that includes both sets and mappings on an equal footing in its definition.*

To think and talk more clearly about nature, we need to define more specialized concepts than sets, relations and functions, because these basic terms are too general. The most important concepts derived from them are operations, algebraic structures and numbers.

A *(binary) operation* is a function that maps the Cartesian product of two copies of a set X into itself. In other words, an operation w takes an ordered couple of arguments $x \in X$ and assigns to it a value $y \in X$:

$$w : X \times X \to X \quad \text{and} \quad w : (x, x) \mapsto y \,. \tag{99}$$

Challenge 277 s Is division of numbers an operation in the sense just defined?

Now we are ready to define the first of three basic concepts of mathematics. An *algebraic structure*, also called an *algebraic system*, is (in the most restricted sense) a set together with certain operations. The most important algebraic structures appearing in physics are groups, vector spaces, and algebras. Vol. IV, page 235

In addition to algebraic structures, mathematics is based on *order structures* and on *topological structures*. Order structures are building blocks of numbers and necessary to define comparisons of any sort. Topological structures are built, via subsets, on the concept of neighbourhood. They are necessary to define continuity, limits, dimensionality, topological spaces and manifolds. Vol. V, page 363

Obviously, most mathematical structures are combinations of various examples of these three basic structure types. For example, the *system* of real numbers is given by the *set* of real numbers with the *operations* of addition and multiplication, the *order relation* 'is larger than' and a *continuity* property. They are thus built by combining an algebraic structure, an order structure and a topological structure. Let us delve a bit into the details. Ref. 253

Numbers

> Which numbers are multiplied by six when their last digit is taken away and transferred to the front?

Challenge 278 s

* A (mathematical) *category* is defined as a collection of objects and a collection of 'morphisms', or mappings. Morphisms can be composed; the composition is associative and there is an identity morphism. More details can be found in the literature. Ref. 252

Note that every category contains a set; since it is unclear whether nature contains sets, as we will discuss below,, it is questionable whether categories will be useful in the unification of physics, despite their intense and abstract charm. Page 328

Numbers are the oldest mathematical concept and are found in all cultures. The notion of number, in Greek ἀριθμός, has been changed several times. Each time the aim was to include wider classes of objects, but always retaining the general idea that numbers are entities that can be added, subtracted, multiplied and divided.

The modern way to write numbers, as e.g. in $12\,345\,679 \cdot 54 = 666\,666\,666$, is essential for science.* It can be argued that the lack of a good system for writing down and for calculating with numbers delayed the progress of science by several centuries. By the way, the same delay can be claimed for the lack of affordable mass reproduction of written texts.

The simplest numbers, 0, 1, 2, 3, 4, ..., are usually seen as being taken directly from experience. However, they can also be constructed from the notions of 'relation' and 'set'.

Challenge 279 s

One of the many possible ways to do this (can you find another?) is by identifying a natural number with the set of its predecessors. With the relation 'successor of', abbreviated S, this definition can be written as

$$0 := \emptyset \quad , \quad 1 := S\,0 = \{0\} = \{\emptyset\} \quad ,$$
$$2 := S\,1 = \{0,1\} = \{\emptyset, \{\emptyset\}\} \quad \text{and} \quad n+1 := S\,n = \{0, ..., n\} \,. \tag{100}$$

This set, together with the binary operations 'addition' and 'multiplication,' constitutes the algebraic system $N = (N, +, \cdot, 1)$ of the *natural numbers*. For all number systems the algebraic system and the set are often sloppily designated by the same symbol. The algebraic system N is what mathematician call a semi-ring. (Some authors prefer not to count the number zero as a natural number.) Natural numbers are fairly useful.

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TABLE 23 Some large numbers.

Number	Example in nature
Around us	
1	number of angels that can be in one place at the same time, following Thomas Aquinas Ref. 255
8	number of times a newspaper can be folded in alternate perpendicular directions
12	largest number of times a paper strip has been folded in the same direction Ref. 256
20	number of digits in precision measurements that will probably never be achieved
21, 34, 55, 89	petals of common types of daisy and sunflower Ref. 257
57	faces of a diamond with brilliant cut
2000 to 6000	stars visible in the night sky
15 000	average number of objects in a European household
10^5	leaves of a tree (10 m beech)
6 to $7 \cdot 10^9$	humans in the year 2000

* However, there is no need for written numbers for doing mathematics, as shown in the interesting book by Marcia Ascher, *Ethnomathematics – A Multicultural View of Mathematical Ideas*, Brooks/Cole, 1991.

Number	Example in nature
10^{17}	ants in the world
c. 10^{20}	number of snowflakes falling on the Earth per year
c. 10^{24}	grains of sand in the Sahara desert
10^{22}	stars in the universe
$10^{25\pm1}$	cells on Earth
$1.1 \cdot 10^{50}$	atoms making up the Earth ($6370^3\,\mathrm{km}^3 \cdot 4 \cdot 3.14/3 \cdot 5500\,\mathrm{kg/m}^3 \cdot 30\,\mathrm{mol/kg} \cdot 6 \cdot 10^{23}\,/\mathrm{mol}$)
10^{81}	atoms in the visible universe
10^{90}	photons in the visible universe
10^{169}	number of atoms fitting in the visible universe
10^{244}	number of space-time points inside the visible universe
Information	
51	record number of languages spoken by one person
c. 5000	words spoken on an average day by a man
c. 7000	words spoken on an average day by a woman
c. 2 000 000	number of scientists on Earth around the year 2000
$3 \cdot 10^8$	words spoken during a lifetime (2/3 time awake, 30 words per minute)
10^9	words heard and read during a lifetime
$4 \cdot 10^9$	pulses exchanged between both brain halves every second
$3 \cdot 10^{12}$	number of trees on Earth
10^{17}	image pixels seen in a lifetime ($3 \cdot 10^9\,\mathrm{s} \cdot (1/15\,\mathrm{ms}) \cdot 2/3$ (awake) $\cdot 10^6$ (nerves to the brain) Ref. 258
10^{19}	bits of information processed in a lifetime (the above times 32)
c. $5 \cdot 10^{12}$	printed words available in (different) books around the world (*c.* $100 \cdot 10^6$ books consisting of 50 000 words)
$2^{10} \cdot 3^7 \cdot 8! \cdot 12!$	
$= 4.3 \cdot 10^{19}$	possible positions of the $3 \times 3 \times 3$ Rubik's Cube Ref. 259
$5.8 \cdot 10^{78}$	possible positions of the $4 \times 4 \times 4$ Rubik-like cube
$5.6 \cdot 10^{117}$	possible positions of the $5 \times 5 \times 5$ Rubik-like cube
c. 10^{200}	possible games of chess
c. 10^{800}	possible games of go
c. 10^{10^7}	possible states in a personal computer
Parts of us	
600	numbers of muscles in the human body, of which about half are in the face
150 000 ± 50 000	hairs on a healthy head
900 000	neurons in the brain of a grasshopper
$126 \cdot 10^6$	light sensitive cells per retina (120 million rods and 6 million cones)
$86(8) \cdot 10^9$	neurons in the human brain
$500 \cdot 10^6$	blinks of the eye during a lifetime (about once every four seconds when awake)
$300 \cdot 10^6$	breaths taken during human life

Number	Example in nature
$3 \cdot 10^9$	heart beats during a human life
$3 \cdot 10^9$	letters (base pairs) in haploid human DNA
$10^{15\pm1}$	cells in the human body
$10^{16\pm1}$	bacteria carried in the human body

The system of *integers* $Z = (..., -2, -1, 0, 1, 2, ..., +, \cdot, 0, 1)$ is the minimal ring that is an extension of the natural numbers. The system of *rational numbers* $Q = (Q, +, \cdot, 0, 1)$ is the minimal field that is an extension of the ring of the integers. (The terms 'ring' and 'field' are defined in all details in the next volume.) The system of *real numbers* $R = (R, +, \cdot, 0, 1, >)$ is the minimal extension of the rationals that is continuous and totally ordered. (For the definition of continuity, see volume IV, page 224, and volume V, page 364.) Equivalently, the reals are the minimal extension of the rationals forming a complete, totally strictly-Archimedean ordered field. This is the historical construction – or definition – of the integer, rational and real numbers from the natural numbers. However, it is not the only one construction possible. The most beautiful definition of all these types of numbers is the one discovered in 1969 by John Conway, and popularized by him, Donald Knuth and Martin Kruskal.

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Ref. 260

▷ A *number* is a sequence of bits.

The two bits are usually called 'up' and 'down'. Examples of numbers and the way to write them are given in Figure 182.

The empty sequence is the number zero. A finite sequence of n ups is the integer number n, and a finite sequence of n downs is the integer $-n$. Finite sequences of mixed ups and downs give the *dyadic rational numbers*. Examples are 1, 2, 3, −7, 19/4, 37/256, etc. They all have denominators with a power of 2. The other *rational numbers* are those that end in an infinitely repeating string of ups and downs, such as the *reals*, the *infinitesimals* and simple infinite numbers. Longer countably infinite series give even more crazy numbers.

The complete class of numbers that is defined by a sequence of bits is called the class of *surreal* numbers.*

There is a second way to write surreal numbers. The first is the just mentioned sequence of bits. But in order to define addition and multiplication, another notation is usually used, deduced from Figure 182. A surreal α is defined as the earliest number of all those between two series of earlier surreals, the left and the right series:

$$\alpha = \{a, b, c, ...|A, B, C, ...\} \quad \text{with} \quad a, b, c, < \alpha < A, B, C\,. \tag{101}$$

* The surreal numbers do *not* form a set since they contain all *ordinal numbers*, which themselves do not form a set, even though they of course *contain* sets. In short, ordinals and surreals are classes which are *larger* than sets.

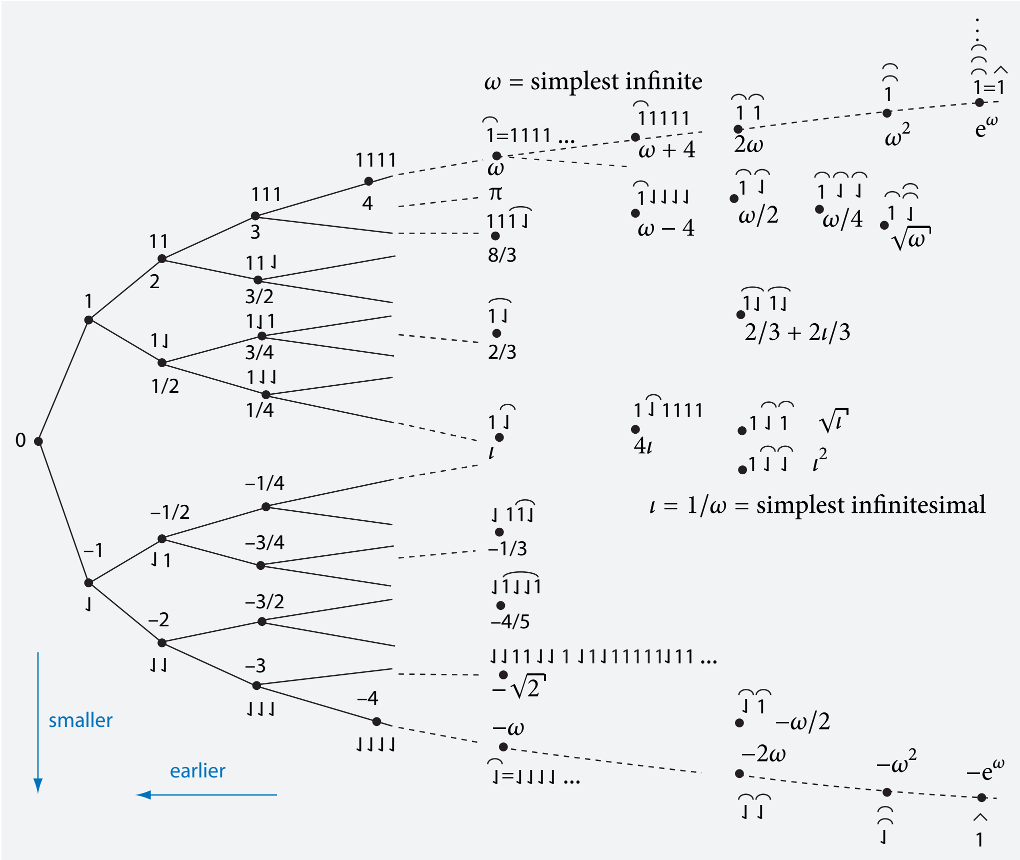

FIGURE 182 The surreal numbers in conventional and in bit notation.

For example, we have

$$\{0|\} = 1\ ,\ \{0,1\,|\} = 2\ ,\ \{|0\} = -1\ ,\ \{|-1,0\} = -2\ ,\ \{0\,|1\} = 1/2\ ,$$
$$\{0\,|1/2,1/4\} = 1\ ,\ \{0,1,3/2,25/16\,|\,41/16,13/8,7/4,2\} = 1 + 37/64\ , \qquad (102)$$

showing that the finite surreals are the *dyadic numbers* $m/2^n$ (n and m being integers). Given two surreals $\alpha = \{...,a,...|...,A,...\}$ with $a < \alpha < A$ and $\beta = \{...,b,...|...,B,...\}$ with $b < \beta < B$, addition is defined recursively, using earlier, already defined numbers, as

$$\alpha + \beta = \{...,a+\beta,...,\alpha+b,...|...,A+\beta,...,\alpha+B,...\}\ . \qquad (103)$$

This definition is used simply because it gives the same results as usual addition for integers and reals. Can you confirm this? By the way, addition is not always commutative. Are you able to find the exceptions, and to find the definition for subtraction?

Challenge 280 s Multiplication is also defined recursively, namely by the expression

$$\begin{aligned}\alpha\beta =&\{..., a\beta + \alpha b - ab, ..., A\beta + \alpha B - AB, ...|\\ &..., a\beta + \alpha B - aB, ..., A\beta + \alpha b - Ab, ...\} .\end{aligned} \qquad (104)$$

These definitions allow us to write $\iota = 1/\omega$, and to talk about numbers such as $\sqrt{\omega}$, the square root of infinity, about $\omega+4$, $\omega-1$, 2ω, e^{ω} and about other strange numbers shown Ref. 260 in Figure 182. However, the surreal numbers are not commonly used. More common is one of their subsets.

The *real numbers* are those surreals whose decimal expansion is not larger than infinity and in addition, equate numbers such as 0.999999... and 1.000000..., as well as all similar cases. In other words, the surreals distinguish the number 0.999999... from the number 1, whereas the reals do not. Indeed, between these two surreal numbers there Challenge 281 s are infinitely many other surreals. Can you name a few?

Reals are more useful for describing nature than surreals, first because they form a *set* – which the surreals do not – and secondly because they allow the definition of *integration*. Other numbers defined with the help of reals, e.g. the complex numbers $\mathbb{C}$, the quaternions $\mathbb{H}$ and a few more elaborate number systems, are presented in the next Vol. IV, page 223 volume.

To conclude, in physics it is usual to call *numbers* the elements of any set that is a semi-ring (e.g. $\mathbb{N}$), a ring (e.g. $\mathbb{Z}$) or a field ($\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$). Since numbers allow us to compare magnitudes and thus to *measure*, these numbers play a central role in the description of observations.

Ref. 261

> “A series of equal balls is packed in such a way that the area of needed wrapping paper is minimal. For small numbers of balls the linear package, with all balls in one row, is the most efficient. For which number of balls is the linear package no longer a minimum?”

Challenge 282 s

Is mathematics always useful?

> “Die Forderung der Möglichkeit der einfachen Zeichen ist die Forderung der Bestimmtheit des Sinnes.*”
>
> Ludwig Wittgenstein, *Tractatus*, 3.23

Numbers, as well as most other mathematical concepts, were developed precisely with the aim of describing nature.

▷ Numbers and mathematical concepts were developed right from the start to provide as succinct a description as possible.

This property is one consequence of mathematics being the science of *symbolic* necessities. Mathematical concepts are *tools* that help our thinking. This is the reason that

* ‘The requirement that simple signs be possible is the requirement that sense be determinate.’

mathematics is used in physics, the science of motion.

Several well-known physicists have repeatedly asked *why* mathematics is so useful.
Ref. 263 For example, Niels Bohr is quoted as having said: 'We do not know why the language of mathematics has been so effective in formulating those laws in their most succinct form.' Eugene Wigner wrote an often cited paper entitled 'The unreasonable effective-
Ref. 264 ness of mathematics.' At the start of science, many centuries earlier, Pythagoras and his contemporaries were so overwhelmed by the usefulness of numbers in describing nature that Pythagoras was able to organize a sect based on this connection. The members of the inner circle of this sect were called 'learned people,' in Greek '*mathematicians*', from the Greek μάθημα 'teaching'. This sect title then became the name of the modern profession. But wondering about the effectiveness of mathematics is akin to wondering about the effectiveness of carpenter tools.

Perhaps we are being too dismissive. Perhaps the mentioned thinkers mainly wanted to express their feeling of wonder when experiencing that language works, that thinking and our brain works, and that life and nature are so beautiful. This would put the accent nearer to the well-known statement by Albert Einstein: 'The most incomprehensible fact about the universe is that it is comprehensible.' Comprehension is another word for description, i.e., for classification. Obviously, any separable system is comprehensible, and there is nothing strange about it. But is the universe separable?

The basic assumption we made at our start was the separability of nature and the universe. This is the central idea that Pythagoras' sect expressed in their core belief

▷ Everything in nature is numbers.

As long as the universe is described as being made of particles and vacuum, the belief is correct. But Pythagoras and his sect were wrong. Like for so many beliefs, observation will show the opposite. Numbers are indeed useful in everyday life; but numbers are not at the basis of nature. We reach this conclusion in the last part of our adventure. Indeed, the assumption that observations in nature can be separated is an approximation. In short:

▷ Counting is always an approximation.

Even the counting of apples is an approximation; it is valid only for low energies and low curvatures. Physics is built on sand. We need a better foundation. In fact, mathematics is not useful for achieving a unified description of nature. With the better foundation, the quoted 'incomprehensibility' of nature then becomes the amazement at the precision of the counting approximation. This experience will be the high point of our adventure.

“Die Physik ist für Physiker viel zu schwer.*
David Hilbert”

* 'Physics is much too difficult for physicists.'

Curiosities and fun challenges about mathematics

Challenge 283 s What is the largest number that can be written with four digits of 2 and no other sign? And with four 4s?

* *

Pythagorean triplets are integers that obey $a^2 + b^2 = c^2$. Give at least ten examples. Then show the following three properties: at least one number in a triplet is a multiple of 3; at least one number in a triplet is a multiple of 4; at least one number in a triplet is a multiple of 5. Challenge 284 e

* *

Here is how to multiply numbers between 5 and 10 using your hands, without multiplication table. Take 8 and 7 as an example. In each hand, extend as many finger as the excess above 5. In the example that implies to extend 3 fingers in one hand and 2 in the other. The *sum* of the extended fingers gives the tens, the *product* of the remaining bent fingers the units. In the example 5 tens and 6 units, thus 56.

* *

Challenge 285 e How many zeroes are there at the end of 1000! ?

* *

A mother is 21 years older than her child, and in 6 years the child will be 5 times younger than the mother. Where is the father? This is the *young mother puzzle*. Challenge 286 s

* *

The number $1/n$, when written in decimal notation, has a periodic sequence of digits. The period is at most $n - 1$ digits long, as for $1/7 = 0.142857\,142857\,1428....$ Which other numbers $1/n$ have periods of length $n - 1$? Challenge 287 d

* *

Felix Klein was a famous professor of mathematics at Göttingen University. There were two types of mathematicians in his department: those who did research on whatever they wanted and those for which Klein provided the topic of research. To which type did Klein belong? Challenge 288 s

Obviously, this is a variation of another famous puzzle. A barber shaves all those people who do not shave themselves. Does the barber shave himself? Challenge 289 s

* *

Everybody knows what a *magic square* is: a square array of numbers, in the simplest case from 1 to 9, that are distributed in such a way that the sum of all rows, columns (and possibly all diagonals) give the same sum. Can you write down the simplest $3 \times 3 \times 3$ *magic cube*? Challenge 290 s

* *

In the history of recreational mathematics, several people have independently found the

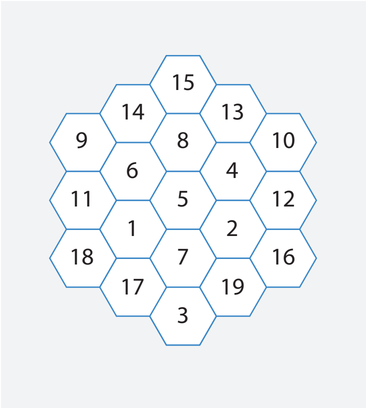

FIGURE 183 The only magic hexagon starting with the number 1 (up to reflections and rotations).

well-known magic hexagon shown in Figure 183. The discoverer was, in 1887, Ernst von Hasselberg. The hexagon is called magic because all lines add up to the same number, 38. Hasselberg also proved the almost incredible result that no other magic hexagon exists. Can you confirm this?

Challenge 291 d

* *

The digits 0 to 9 are found on keyboards in two different ways. Calculators and keyboards have the 7 at the top left, whereas telephones and automatic teller machines have the digit 1 at the top left. The two standards, respectively by the International Standards Organization (ISO) and by the International Telecommunication Union (ITU, formerly CCITT), evolved separately and have never managed to merge.

Ref. 265

* *

Can you devise a machine that counts the hair on the head of a person?

Challenge 292 e

* *

Leonhard Euler in his notebooks sometimes wrote down equations like

$$1 + 2^2 + 2^4 + 2^6 + 2^8 + \ldots = -\frac{1}{3} \,. \tag{105}$$

Can this make sense?

Challenge 293 d

* *

In many flowers, numbers from the *Fibonacci series* 1, 1, 2, 3, 5, 8, 13, 21 etc., appear. Figure 190 gives a few examples. It is often suggested that this is a result of some deep sense of beauty in nature. This is *not* the case, as Figure 184 shows. Mark a spot on a surface, and put washers around it in by hand in a spiral manner; you will find the same spirals that you find in many flowers, and thus, at their border, the same Fibonacci numbers. This argument by Donald Simanek shows that there is nothing deep, complicated or even mysterious in the appearance of Fibonacci numbers in plants. For an opposite point of

Vol. I, page 246
Ref. 257

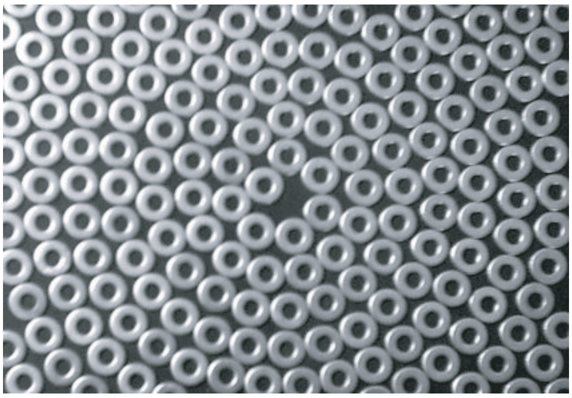

FIGURE 184 Fibonacci numbers and spirals from washers (© Donald Simanek).

view, see reference Ref. 257 and many publications about the patterns in sunflowers.

* *

Prime numbers are a favourite playground for mathematicians. A famous result on all prime numbers p_i states

$$\prod_{i=1}^{\infty}(1-\frac{1}{p_i^2})=\frac{6}{\pi^2} \tag{106}$$

Challenge 294 s Can you imagine how this result is proven?

* *

Digits owe their name to the latin word ‘digitum’ or finger. In times when writing on paper was expensive, it was already possible to count up to 9999 using the two hands, with a system developed by Beda Venerabilis and popularized, for example, by Luca Pacioli.
Challenge 295 e Can you develop a similar system?

CHAPTER 9

OBSERVATIONS, LIES AND PATTERNS OF NATURE

> Die Grenzen meiner Sprache bedeuten die Grenzen meiner Welt.**
>
> Ludwig Wittgenstein, *Tractatus*, 5.6

> Der Satz ist ein Bild der Wirklichkeit. Der Satz ist ein Modell der Wirklichkeit, so wie wir sie uns denken.***
>
> Ludwig Wittgenstein, *Tractatus*, 4.01

Ref. 266

In contrast to mathematics, physics does aim at being a language. But it is ambitious: it aims to express *everything*, with *complete* precision, and, in particular, all examples and possibilities of change. All observations are about change. Now, physics is the study of motion. But because all change is due to motion, we can also call physics the study of change.

Physics is the language of change. Like any language, physics consists of concepts and sentences. In order to be able to express everything, it must aim to use few words for a lot of facts.**** Physicists are essentially *lazy* people: they try to minimize the effort in everything they do. The concepts in use today have been optimized by the combined effort of many people to be as practical, i.e., as powerful as possible. A concept is called *powerful* when it allows one to express in a compact way a large amount of information, meaning that it can rapidly convey a large number of details about observations.

General statements about many examples of motion are called *rules* or *patterns*. In the past, it was often said that 'laws govern nature', using an old and inappropriate ideology. A physical 'law' is only a way of saying as much as possible with as few words as possible. When saying 'laws govern nature' we actually mean to say 'being lazy, we describe observations with patterns'. Laws are the epitome of laziness. Formulating laws is pure sloth. In fact, the correct expression is

** 'The limits of my language are the limits of my world.'

*** 'A proposition is a picture of reality. A proposition is a model of reality as we imagine it.'

**** A particular, specific observation, i.e., a specific example of input shared by others, is called a *fact*, or in other contexts, an *event*. A striking and regularly observed fact is called a *phenomenon*, and a general observation made in many different situations is called a *(physical) principle*. (Often, when a concept is introduced that is used with other meaning in other fields, in this walk it is preceded by the qualifier 'physical' or 'mathematical' in parentheses.) Actions performed towards the aim of collecting observations are called *experiments*. The concept of experiment became established in the sixteenth century; in the evolution of a child, it can best be compared to that activity that has the same aim of collecting experiences: *play*.

▷ Patterns describe nature.

Physicists have written about the laziness necessary to find patterns in much detail. In order to become a master of laziness, we need to distinguish lazy patterns from those which are not, such as lies, beliefs, and other statements that are not about observations or motion at all. We do this below. Page 304

The quest for laziness is the origin, among others, of the use of numbers in physics. Observables are often best described with the help of numbers, because numbers allow easy and precise communication and classification. Length, velocity, angles, temperature, voltage or field strength are of this type. The notion of 'number', used in every measurement, is constructed, often unconsciously, from the notions of 'set' and 'relation', as shown above. Apart from the notion of number, other concepts are regularly defined to allow fast and compact communication of the 'laws' of nature; all are 'abbreviation tools.' In this sense, the statement 'the level of the Kac–Moody algebra of the Lagrangian of the heterotic superstring model is equal to one' contains precise information, explainable to everybody; however, it would take dozens of pages to express it using only the terms 'set' and 'relation.' In short, the *precision* common in physics results from its *quest for laziness*.

> Es ist besser, daß die Leute nicht wissen, wie Gesetze und Wurst zustande kommen. Sonst könnten sie nachts nicht ruhig schlafen.*
>
> Bismarck, Otto von

Are physical concepts discovered or created?

> Das logische Bild der Tatsachen ist der Gedanke.**
>
> Ludwig Wittgenstein, *Tractatus*, 3

The title question is often rephrased as: are physical concepts free of beliefs, taste or personal choices? The question has been discussed so much that it even appears in Hollywood films. We give a short summary that can help you to distinguish honest from dishonest teachers.

If concepts were created, instead of discovered, would imply that there is free choice between many alternative possibilities. The chosen alternative for the definition of a concept would then be due to the beliefs or tastes used. But in physics (in obvious contrast to other, more ideological fields of enquiry), we know that different physical descriptions of observations are either equivalent or, in the opposite case, imprecise or even wrong. A physical description of observations is thus essentially unique: any choices of concepts are only apparent. *There is no real freedom in the definition of physical concepts.* In this property, physics is in strong contrast to artistic activity.

If two different physical concepts can be used to describe the same aspect of observations, they must be equivalent, even if the relation that leads to the equivalence is not immediately clear. In fact, the requirement that people with different standpoints and

* 'It is better that people do not know how laws and sausages are made. Otherwise they would not sleep well at night.' Otto von Bismarck (b. 1815 Schönhausen, d. 1898 Friedrichsruh) was Prussian Chancellor.
** 'A logical picture of facts is a thought.'

observing the same event deduce equivalent descriptions lies at the very basis of physics. It expresses the requirement that observations are observer-independent. In short, the strong requirement of viewpoint independence makes the free choice of concepts a logical impossibility.

The conclusion that concepts describing observations are discovered rather than created is also reached independently in the field of linguistics by the above-mentioned research on semantic primitives,* in the field of psychology by the observations on the formation of the concepts in the development of young children, and in the field of ethology by the observations of animal development, especially in the case of mammals. In all three fields detailed observations have been made of how the interactions between an individual and its environment lead to concepts, of which the most basic ones, such as space, time, object or interaction, are common across the sexes, cultures, races and across many animal species populating the world. Curiosity and the way that nature works leads to the same concepts for all people and even for all animals. The world offers only one possibility, without room for imagination. Imagining that physical concepts can be created at your leisure is a mistaken belief – or a useful exercise – but never successful.

Physical concepts are classifications of observations. The activity of classification itself follows the patterns of nature; it is a mechanical process that machines can also perform. This means that any distinction, i.e., any statement that A is different from B, is a theory-free statement. No belief system is necessary to distinguish different entities in nature. Cats and pigs can also do so. Physicists *can* be replaced by animals, even by machines. Our mountain ascent will repeatedly confirm this point.

As already mentioned, the most popular physical concepts allow us to describe observations as succinctly and as accurately as possible. They are formed with the aim of having the largest possible amount of understanding with the smallest possible amount of effort. Both Occam's razor – the requirement not to introduce unnecessary concepts – and the drive for unification automatically reduce the number and the type of concepts used in physics. In other words, the progress of physical science was and is based on a programme that reduces the possible choice of concepts as drastically as possible.

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In summary, we found that physical concepts are the same for everybody and are free of beliefs and personal choices: they are, first of, all *boring*: they are correct and accurate. Moreover, as they could stem from machines instead of people, concepts are *born of laziness*: they are as efficient as possible. These human analogies – not meant to be taken too seriously – confirm that physical concepts are *not* created; they are discovered. If a teacher tells you the opposite, he is lying. Unfortunately, there are many liars of this kind.

Having handled the case of physical concepts, let us now turn to physical statements. The situation is somewhat similar: physical statements must be *correct*, *boring*, *lazy* and *arrogant*. Let us see why.

* Anna Wierzbicka concludes that her research clearly indicates that semantic primitives are *discovered*, in particular that they are deduced from the fundamentals of human experience, and not invented.

Ref. 247

TABLE 24 The 'scientific method'.

Normal description **Curiosity**	Lobbyist description **Scientific method**
1. look around a lot	1. interact with the world
2. don't believe anything told	2. forget unproven statements
3. choose something interesting and explore it yourself	3. observe and measure
4. make up your own mind and describe precisely what you saw	4. use reason, build hypothesis
5. check if you can also describe similar situations in the same way	5. analyse hypothesis
6. increase the precision of observation until the checks either fail or are complete	6. perform experiments to check hypothesis
7. depending on the case, continue with step 4 or 1	7. ask authority for more money

> Wo der Glaube anfängt, hört die Wissenschaft auf.*
>
> Ernst Haeckel, *Natürliche Schöpfungsgeschichte*, 1879.

How do we find physical concepts, patterns and rules?

> Grau, theurer Freund, ist alle Theorie,
> Und grün des Lebens goldner Baum.**
>
> J.W. v. Goethe, *Faust*.

> Physics is usually presented as an objective science, but I notice that physics changes and the world stays the same, so there must be something subjective about physics.
>
> Richard Bandler

Progressing through the exploration of motion reflects a young child's attitude towards life: a child is driven by curiosity. The progress follows the simple programme on the left of Table 24. Adult scientists do the same, except that they use more fashionable terms, given on the right of the table. Adults also have specialized professions to make money from their curiosity. The experts of step 7, who request more money, are variously called *lobbyists* or *fund raisers*; instead of calling this program *curiosity*, they call it the *scientific method.*

Physics being the talk about motion,*** and motion being a vast topic, a lot can be explored and told. The experts of step 6, those who check hypotheses, are called *exper-*

* 'Where belief starts, science ends.'
** 'Grey, dear friend, is all theory, and green the golden tree of life.' Johann Wolfgang von Goethe (b. 1749 Frankfurt am Main, d. 1832 Weimar), the influential German poet.
*** Several sciences have the term 'talk' as part of their name, namely all those whose name finishes in '-logy', such as e.g. biology. The ending stems from ancient Greek and is deduced from λήγηιν meaning 'to say, to talk'. Physics as the science of motion could thus be called 'kinesiology' from κίνησις, meaning 'motion'; but for historical reasons this term has a different meaning, namely the study of human muscular

imental physicists or simply *experimentalists*, a term derived from the Latin 'experiri', meaning 'to try out'. Most of them are part of the category 'graduate students'. The experts of steps 5 and 4, those who build and analyse hypotheses, are called *theoretical physicists* or simply *theoreticians*.* This is a rather modern term; the first professors of theoretical physics were appointed around the start of the twentieth century. The term 'theory' is derived from the Greek θεωρία meaning 'observation, contemplation'. Finally, there are the people who focus on steps 1 to 4, and who induce others to work on steps 5 and 6; they are called *geniuses*. The geniuses are those people who introduce the concepts that best help to describe nature.

Obviously an important point is hidden in step 6: how do all these people know whether their checks fail? How do they know if a concept or statement applies to nature? How do they recognize truth?

> All professions are conspiracies against laymen.
>
> George Bernard Shaw

What is a lie?

> Get your facts straight, and then you can distort them at your leisure.
>
> Mark Twain

In most countries, every person must know what 'truth' is, since in a law court for example, telling an untruth can lead to a prison sentence. And the courts are full of experts in lie detection. **

In court, a *lie* is a statement that knowingly contrasts with observations.*** The truth of a statement is thus checked by observation. The check itself is sometimes called the *proof* of the statement. For law courts, and for physics, we thus have

▷ ***Truth*** is the correspondence with facts.

▷ ***Facts*** are observations shared with other people or machines.

Therefore, also in science, we have

▷ A ***lie*** is a statement in contrast with facts.

activity – and also, unfortunately, a lot of esoteric nonsense. The term 'physics' is either derived from the Greek φύσικη (τέχνη is understood) meaning '(the art of) nature', or from the title of Aristotle' works τά φυσικά meaning 'natural things'. Both expressions are derived from φύσις, meaning 'nature'.

* If you like theoretical physics, have a look at the refreshingly candid web page by Nobel Prize winner Gerard 't Hooft with the title *How to become a good theoretical physicist*. It can be found at www.phys.uu.nl/~thooft/theorist.html.

** Some scholars have spent most of their research career on lies and lying. A well-known example is Paul Ekman, whose fascinating website at www.paulekman.com tells how to spot lies from the behaviour of the person telling it.

*** Statements not yet checked with observations are variously called *speculations*, *conjectures*, *hypotheses*, or – wrongly – simply *theses*. Statements that are in correspondence with observations are called *correct* or *true*; statements that contrast with observations are called *wrong* or *false*.

Except in court, lies are fun statements, because we can draw *any imaginable conclusion* from them. A well-known discussion between two Cambridge professors early in the twentieth century makes the point. McTaggart asked: 'If $2 + 2 = 5$, how can you prove that I am the pope?' Godfrey Hardy: 'If $2 + 2 = 5$, then $4 = 5$; subtract 3; then $1 = 2$; but McTaggart and the pope are two; therefore McTaggart and the pope are one.' As noted long ago, *ex falso quodlibet*; from what is wrong, anything imaginable can be deduced. Therefore, in our mountain ascent we need to build on previously deduced results and our trip could not be completed if we had a false statement somewhere in our chain of arguments.

Nevertheless, lying is such an important activity that one should learn to perform it well – in order to learn to discover it in others. The art of lying has three stages: the animal stage, the child stage and the adult stage. Many animals have been shown to deceive their kin. Children start lying just before their third birthday, by hiding experiences. Psychological research has even shown that children who lack the ability to lie cannot complete their personal development towards a healthy human being.

Ref. 212

Adults are habitual liars. Many adults cheat on taxes. Others lie to cover up their wrongdoings. The worst examples of liars are those violent contemporaries – often politicians or intellectuals – who claim that truth "does not exist". If you ever lie in court, you better do it well; indeed, experience shows that you might get away with many criminal activities.

What is a good lie?

> The pure truth is always a lie.
>
> Bert Hellinger

Since a lie is a statement in contrast with facts – or shared observations – a *good* lie is a lie whose contrast with facts is hard to discover. In contrast, a *successful* lie is a statement that lets you earn money. We do not explore this type of lie here; we will just explore the art of good lies.

The first way of lying is to put an emphasis on the sharedness only. Populists and polemics do this regularly. ('Every foreigner is a danger for the values of our country.') Since almost any imaginable opinion, however weird, is held by some group – and thus shared – one can always claim it as true.* Unfortunately, it is no secret that ideas also get shared because they are fashionable, imposed or opposed to somebody who is generally disliked. Often a sibling in a family has this role – remember Cassandra.** For a good lie we thus need more than sharedness, more than *intersubjectivity* alone.

A good lie should be, like a true statement, really independent of the listener and the observer and, in particular, independent of their age, their sex, their education, their

* The work of the sociologist Gabriel Tarde (b. 1843 Sarlat, d. 1903 Paris), especially his concepts of *imitation* and group mind, already connects to this fact.

** The implications of birth order on creativity in science and on acceptance of new ideas has been studied in the fascinating book by Frank J. Sulloway, *Born to Rebel – Birth Order, Family Dynamics and Creative Lives*, Panthon Books, 1996. This exceptional book tells the result of a life-long study correlating the personal situations in the families of thousands of people and their receptivity to about twenty revolutions in the recent history. The book also includes a test in which the reader can deduce their own propensity to rebel, on a scale from 0 to 100 %. Darwin scores 96 % on this scale.

civilization or the group to which they belong. For example, it is especially hard – but not impossible – to lie with mathematics. The reason is that the basic concepts of mathematics, be they 'set', 'relation' or 'number', are taken from observation and are intersubjective, so that statements about them are easily checked. Therefore, good lies avoid mathematics.*

Thirdly, a 'good' lie should avoid statements about observations and use *interpretations* instead. For example, some people like to talk about other universes, which implies talking about fantasies, not about observations. However, a really good lie has to avoid to make statements which are meaningless; the most destructive comment that can be made about a statement is the one used by the great physicist Wolfgang Pauli: 'That is not even wrong.'

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Fourthly, a good lie avoids talking about observations, but focuses on imagination. Only truth needs to be *empirical*; *speculative* statements differ from truth by not caring about observations. If you want to lie 'well' even with empirical statements, you need to pay attention. There are two types of empirical statements: *specific* statements and *universal* statements. For example, 'On the 2nd of June 1960 I saw a green swan swimming on the northern shore of the lake of Varese' is specific, whereas 'All ravens are black' is universal, since it contains the term 'all'. There is a well-known difference between the two, which is important for lying well:

▷ Specific statements cannot be falsified, they are only verifiable.

▷ Universal statements cannot be verified, they are only falsifiable.

Ref. 267

Let us explore the reason.

Universal statements, such as 'the speed of light is constant', cannot be tested for *all* possible cases. (Note that if they could, they would not be universal statements, but just a list of specific ones.) However, they can be reversed by a counter-example. Another example of the universal type is: 'Apples fall upwards.' Since it is falsified by an observation conducted by Newton several centuries ago, or by everyday experience, it qualifies as an (easily detectable) lie. In general therefore, lying by stating the opposite of a theory is usually unsuccessful. If somebody insists on doing so, the lie becomes a *superstition*, a *belief*, a *prejudice* or a *doctrine*. These are the low points in the art of lying. A famous case of insistence on a lie is that of the colleagues of Galileo, who are said to have refused to look through his telescope to be convinced that Jupiter has moons, an observation that would have shaken their belief that everything turns around the Earth. Obviously these astronomers were amateurs in the art of lying. A good universal lie is one whose counter-example is not so easily spotted.

There should be no insistence on lies in physics. Unfortunately, classical physics is full of lies. We will dispel them during the rest of our walk.

Lying by giving specific instead of universal statements is much easier. ('I can't remember.') Even a specific statement such as 'yesterday the Moon was green, cubic and

* In mathematics, 'true' is usually specified as 'deducible' or 'provable'; this is in fact a special case of the usual definition of truth, namely 'correspondence with facts', if one remembers that mathematics studies the properties of classifications.

smelled of cheese' can never be completely falsified: there is no way to show with absolute certainty that this is wrong. The only thing that we can do is to check whether the statement is compatible with other observations, such as whether the different shape affected the tides as expected, whether the smell can be found in air collected that day, etc. A *good* specific lie is thus not in evident contrast with other observations.*

Incidentally, universal and specific statements are connected: the *opposite* of a universal statement is always a specific statement, and vice versa. For example, the opposite of the general statement 'apples fall upwards', namely 'some apples fall downwards', is specific. Similarly, the specific statement 'the Moon is made of green cheese' is in opposition to the universal statement 'the Moon is solid for millions of years and has almost no smell or atmosphere.'

In other words, law courts and philosophers disagree. Law courts have no problem with calling theories true, and specific statements lies. Many philosophers avoid this. For example, the statement 'ill-tempered gaseous vertebrates do not exist' is a statement of the universal type. If a universal statement is in agreement with observations, and if it is falsifiable, law courts call it *true*. The opposite, namely the statement: 'ill-tempered gaseous vertebrates do exist', is of the *specific* type, since it means 'Person X has observed an ill-tempered gaseous vertebrate in some place Y at some time Z'. To verify this, we need a record of the event. If such a record, for example a photographs or testimony does not exist, and if the statement *can* be falsified by other observations, law courts call the specific statement a *lie*. Even though these are the rules for everyday life and for the law, there is no agreement among philosophers and scientists that this is acceptable. Why? Intellectuals are a careful lot, because many of them have lost their lives as a result of exposing lies too openly.

In short, specific lies, like all specific statements, can never be falsified with certainty. This is what makes them so popular. Children learn specific lies first. ('I haven't eaten the jam.') General lies, like all general statements, can always be corroborated by examples. This is the reason for the success of ideologies. But the criteria for recognizing lies, even general lies, have become so commonplace that beliefs and lies try to keep up with them. It became fashionable to use expressions such as 'scientific fact' – there are no non-scientific facts –, or 'scientifically proven' – observations cannot be proven otherwise – and similar empty phrases. These are not 'good' lies; whenever we encounter a sentence beginning with 'science says ...' or 'science and religion do ...' we just need to

* It is often difficult or tedious to verify statements concerning the past, and the difficulty increases with the distance in time. That is why people can insist on the occurrence of events which are supposed to be exceptions to the patterns of nature ('miracles'). Since the advent of rapid means of communication these checks are becoming increasingly easy, and no miracles are left over. This can be seen in Lourdes in France, where even though today the number of visitors is much higher than in the past, no miracles have been seen in decades. (In fact there is one exception that has with several witnesses. In 1998, a man in a wheelchair was pushed into the holy water. When he came out again, miraculously, his wheelchair had new tires.)

In fact, all modern so-called 'miracles' are kept alive only by consciously eschewing checks, such as the supposed yearly liquefaction of blood in Napoli, the milk supposedly drunk by statues in temples, the supposed healers in television evangelism, etc. Most miracles only remain because many organizations make money out of the difficulty of falsifying specific statements. For example, when the British princess Diana died in a car crash in 1997, even though the events were investigated in extreme detail, the scandal press could go on almost without end about the 'mysteries' of the accident.

Ref. 268

replace 'science' by 'knowledge' or 'experience' to check whether such a sentence are to be taken seriously or not.*

Lies differ from true statements in their emotional aspect. Specific statements are usually boring and fragile, whereas specific lies are often sensational and violent. In contrast, general statements are often daring and fragile whereas general lies are usually boring and violent. The truth is fragile. True statements require the author to stick his neck out to criticism. Researchers know that if one doesn't stick the neck out, it can't be an observation or a theory. (A *theory* is another name for one or several connected, not yet falsified universal statements about observations.)** Telling the truth does make vulnerable. For this reason, theories are often *daring*, *arrogant* or *provoking*; at the same time they have to be *fragile* and *vulnerable*. For many men, theories thus resemble what they think about women. Darwin's *The origin of species* illustrates the stark contrast between the numerous boring and solid facts that Darwin collected and the daring theory that he deduced. Boredom of facts is a sign of truth.

In contrast, the witch-hunters propagating 'creationism' or so-called 'intelligent design' are examples of liars. The specific lies they propagate, such as 'the world was created in October 4004 BCE', are sensational, whereas the general lies they propagate, such as 'there have not been big changes in the past', are boring. This is in full contrast with common sense. Moreover, lies, in contrast to true statements, make people violent. The worse the lie, the more violent the people. This connection can be observed regularly in the news. In other words, 'creationism' and 'intelligent design' are not only lies, they are bad lies. A 'good' *general lie*, like a good physical theory, seems crazy and seems vulnerable, such as 'people have free will'. A 'good' *specific lie* is boring, such as 'this looks like bread, but for the next ten minutes it is not'. Good lies do not induce violence. Feelings can thus be a criterion to judge the quality of lies, if we pay careful attention to the type of statement. A number of common lies are discussed later in this chapter.

An important aspect of any 'good' lie is to make as few *public* statements as possible, so that critics can check as little as possible. (For anybody sending corrections of mistakes in this text, I provide a small reward.) To detect lies, public scrutiny is important, though not always reliable. Sometimes, even scientists make statements which are not based on observations. However, a 'good' lie is always well prepared and told on purpose; accidental lies are frowned upon by experts. Examples of good lies in science are 'aether', 'UFOs', 'creation science', or 'cold fusion'. Sometimes it took many decades to

* To clarify the vocabulary usage of this text: *religion* is spirituality plus a varying degree of beliefs and power abuse. The mixture depends on each person's history, background and environment. *Spirituality* is the open participation in the whole of nature. Most, maybe all, people with a passion for physics are spiritual. Most are not religious.

** In other words, a set of not yet falsified patterns of observations on the same topic is called a *(physical) theory*. The term 'theory' will always be used in this sense in this walk, i.e., with the meaning 'set of correct general statements'. This use results from its Greek origin: 'theoria' means 'observation'; its original meaning, 'passionate and emphatic contemplation', summarizes the whole of physics in a single word. ('Theory', like 'theatre', is formed from the root θέ, meaning 'the act of contemplating'.) Sometimes, however, the term 'theory' is used – being confused with 'hypothesis' – with the meaning of 'conjecture', as in 'your theory is wrong', sometimes with the meaning of 'model', as in 'Chern–Simons' theory and sometimes with the meaning of 'standard procedure', as in 'perturbation theory'. These incorrect uses are avoided here. To bring the issue to a point: the *theory of evolution* is not a conjecture, but a set of correct statements based on observation.

detect the lies in these domains.

To sum up, the central points of the art of lying without being caught are two: do not divulge details, and allow some select group to earn money with your lies. Be *vague*. All the methods used to verify a statement ask for details, for *precision*. For any statement, its degree of precision allows one to gauge the degree to which the author is sticking his neck out. The more precision that is demanded, the weaker a statement becomes, and the more likely a fault will be found, if there is one. This is the main reason that we chose an increase in precision as a guide for our mountain ascent: we want to avoid lies completely. (And, besides, we do not look for money in this trip either.) By the way, the same method is used in criminal trials. To discover the truth, investigators typically ask all the witnesses a large number of questions, allowing as many *details* as possible to come to light. When sufficient details are collected, and the precision is high enough, the situation becomes clear. Telling 'good' lies is much more difficult than telling the truth; it requires an excellent imagination.

> Truth is an abyss.
>
> Democritus

> To teach superstitions as truth is a most terrible thing.
>
> Hypatia of Alexandria (*c.* 355–415)

> [Absolute truth:] It is what scientists say it is when they come to the end of their labors.
>
> Charles Peirce

Ref. 269

Is this statement true? – A bit about nonsense

> There are three types of people: those who believe in Father Christmas, those who do not believe in Father Christmas, and those who are Father Christmas.
>
> Anonymous

> Truth is a rhetorical concept.
>
> Paul Feyerabend

Feyerabend's statement is nonsense. Everybody should be able to spot nonsense. Here is how to do so.

Not all statements can be categorized as true or false. There is a third option: statements can simply make *no sense*. There are even such statements in mathematics, where they are called *undecidable*. Indeed,

$$\triangleright \text{ `\textit{Undecidable}' \textit{means} `\textit{nonsense}'} . \qquad (107)$$

An example is the continuum hypothesis. This hypothesis is undecidable because it makes a statement that depends on the precise meaning of the term 'set'. In standard mathematical usage the term 'set' is not defined with sufficient precision: therefore a truth value cannot be assigned to the continuum hypothesis. In short, statements can be undecidable, i.e., can be nonsensical, because the concepts contained in them are not sharply defined.

Statements can also be undecidable for other reasons. Phrases such as 'This statement is not true' illustrate the situation. The phrase is undecidable because it references to itself. Kurt Gödel* has even devised a general way of constructing such undecidable statements in the domain of logic and mathematics. The different variations of these *self-referential* statements, especially popular both in the field of logic and computer science, have captured a large public.** Similarly undecidable statements can be constructed with
Ref. 270 terms such as 'calculable', 'provable' and 'deducible'.

In fact, self-referential statements are undecidable because they are meaningless. If the usual definition of 'true', namely corresponding to facts, is substituted into the sentence 'This statement is not true', we quickly see that it has no meaningful content. A famous meaningless sentence was constructed by the linguist Noam Chomsky:

Colorless green ideas sleep furiously. (108)

Ref. 220 It is often used as an example for the language processing properties of the brain, but nobody sensible elevates it to the status of a paradox and writes philosophical discussions about it. To do that with the title of this section is a similar waste of energy.

The main reason for the popular success of self-reference is the difficulty in perceiving the lack of meaning.*** A good example is the statement:

This statement is false or you are an angel. (109)

Challenge 297 s We can actually deduce from it that 'you are an angel.' Can you see how? If you want, you can change the second half and get even more interesting statements. Such examples show that statements referring to themselves have to be ignored. In short, whenever you meet somebody who tries to use the self-referential construction by Kurt Gödel to deduce another statement, take a step back, or better, a few more. Self-reference, especially the type defined by Gödel, is a hard but common path – especially amongst wannabe-intellectuals – to think, tell and write *nonsense*.

▷ Self-reference is a form of nonsense.

* Kurt Gödel (b. 1906 Brünn, d. 1978 Princeton), famous logician.

** A general introduction is given in the beautiful books by Raymond Smullyan: *Satan, Cantor and Infinity and Other Mind-boggling Puzzles*, Knopf, 1992; *What is the Name of This Book? The Riddle of Dracula and Other Logical Puzzles*, Touchstone, 1986, and *The Lady or the Tiger? And Other Puzzles*, Times Books, 1982. Also definitions can have no content, such as David Hilbert's 'smallest number that has not been mentioned this century' or 'the smallest sequence of numbers that is described by more signs than this sentence'.

*** A well-known victim of this difficulty is Paulus of Tarsus. The paradox of the Cretan poet *Epimenedes* (6th century BCE) who said 'All Cretans lie' is too difficult for the notoriously humour-impaired Paulus, who in his letter to Titus (chapter 1, verses 12 and 13, in the christian bible) calls Epimenedes a 'prophet', adds some racist comments, and states that this 'testimony' is true. But wait! There is a final twist to this
Ref. 271 story. The statement 'All Cretans lie' is *not* a paradox at all; a truth value can actually be ascribed to it,
Challenge 296 s because the statement is not really self-referential. Can you confirm this? The only *genuine* paradox is 'I am lying', to which it is indeed impossible to ascribe a truth value.

Nothing useful can be deduced from nonsense. Well, not entirely; it does help to meet a psychiatrist on a regular basis.*

In physics, in the other natural sciences and in legal trials self-referential statements are not used. Therefore there are no problems.** In fact, the work of logicians confirms, often rather spectacularly, that there is no way to extend the term 'truth' beyond the definition of 'correspondence with facts.'

> Ein Satz kann unmöglich von sich selbst aussagen, daß er wahr ist.***
>
> Ludwig Wittgenstein, *Tractatus*, 4.442

Curiosities and fun challenges about lies and nonsense

> A man is his own easiest dupe, for what he wishes to be true he generally believes to be true.
>
> Demosthenes, 349 BCE.

Ref. 272

> Quator vero sunt maxima comprehendendae veritatis offendicula, quae omnem quemcumque sapientem impediunt, et vix aliquem permittunt ad verum titulum sapientiae pervenire: videlicet fragilis et indignae auctoritatis exemplum, consuetudinis diurnitatis, vulgi sensus imperiti, et propriae ignorantiae occultatio cum ostentatione sapientiae apparentis.****
>
> Roger Bacon, *Opus majus*, 1267.

> Es ist ja nicht alles, was ich den Bürgern sage, gelogen.*****
>
> Konrad Adenauer, 1962, West German Chancellor.

Some lies are entertaining and funny – and are better called jokes –, some are signs of psychic disturbance, and some are made with criminal intent. Some statements are not lies, but simply nonsense. Have fun distinguishing them.

* *

During a church sermon, a man fell asleep. He dreamt about the French revolution: he was being brought to the guillotine. At that moment, his wife noticed that he was sleeping. In the same moment in which the man dreamt that the knife was hitting him, his wife gave him a tap on his neck with her fan. The shock instantly killed the man. – Is this story true or false?

Challenge 299 e

* Also Gödel had therapy.

** Why are circular definitions, like those at the basis of Galilean physics, not self-referential?

Challenge 298 s

*** 'It is quite impossible for a proposition to state that it itself is true.'

**** 'There are four stumbling blocks to truth and knowledge: weak and unworthy authority, custom, popular prejudice, and the concealment of ignorance with apparent knowledge.'

***** 'Indeed, not everything that I tell the people is a lie.'

* *

A well-known bad lie: 'Yesterday I drowned.'

* *

Starting in the 1990s, so-called *crop circles* are regularly produced by people walking with stilts, a piece of wood and some rope into fields of crops. Nevertheless, many pretended and even more believed that these circles were made by extraterrestrial beings. Can you provide some reasons why this is impossible?

Challenge 300 s

* *

Often one hears or reads statements like: 'mind (or spirit or soul) is stronger than matter.' Beware of anybody who says this; he wants something from you. Can you show that such statements are all and always wrong?

Challenge 301 e

* *

In certain countries, two lies were particularly frequent in the early twenty-first century. The first: global warming does not exist. The second: global warming is not due to human causes. Are these good or bad lies?

Challenge 302 s

* *

Sometimes it is heard that a person whose skin is completely covered with finest metal powder will die, due to the impossibility of the skin to breathe. Can you show from you own observation that this is wrong?

Challenge 303 s

* *

A famous mixture of hoax and belief premises that the Earth was created about six thousand years ago. (Some believers even use this lie as justification for violence against non-believers.) Can you explain why the number is wrong?

Challenge 304 s

* *

A famous provocation: the world has been created last Saturday. Can you decide whether this is wrong?

Challenge 305 s

* *

Hundreds of hoaxes are found on the www.museumofhoaxes.com website. It gives an excellent introduction into the art of lying; of course it exposes only those who have been caught. Enjoy the science stories, especially those about archaeology. Several other sites with similar content can be found on the internet.

* *

In the 1990s, many so-called 'healers' in the Philippines earned large amounts of money by suggesting patients that they were able to extract objects from their bodies without operating. Why is this not possible? (For more information on health lies, see the www.quackwatch.com website.)

Challenge 306 e

* *

Challenge 307 s Is homoeopathy a lie?

* *

'Amber helps against tooth ache.' 'A marriage partner should have the correct blood group/zodiacal sign.' 'Opening an umbrella inside a house brings bad luck.' 'The number 8 brings good luck.' These are common statements of nonsense around the world.

* *

Since the 1980s, certain persons have claimed that it is possible to acquire knowledge simply from somebody 1000 km away, without any communication between the two people. However, the assumed 'morphogenetic fields' realizing this feat cannot exist. Why not? Challenge 308 e

* *

It is claimed that a Fire Brigade building in a city in the US hosts a light bulb that has been burning without interruption since 1901 (at least this was the case in 2005). Can this be true? Challenge 309 s Hundreds of such stories, often called 'urban legends,' can be found on the www.snopes.com website. However, some of the stories are not urban legends, but true, as the site shows.

* *

A common lie in science and business is the promise of free energy. False proofs of this lie often use electromagnetism. On the other hand, electromagnetism is based on relativity, and relativity is often sufficient to show that the claims are false. Train yourself to do it whenever the occasion arises.

For people who want funding for free energy devices the answer should always be the same as that given in the Middle Age to the alchemists seeking funding for making gold: if you were right, you would earn the money yourself.

* *

'This statement has been translated from French into English.' Is the statement true, false or neither?

* *

Aeroplanes have no seat row 13. Many tall hotels have no floor 13. What is the lie behind this habit? Challenge 310 s What is the truth behind it? Once the author asked a singer in Napoli to sing 'Fenesta ca lucive', a beautiful song performed by Enrico Caruso and many others since. The singer refused, explaining that the local public would run out in rage, and that the owners of the place would be forced to clean the whole place with salt, to get rid of bad luck. Many superstitions are found across the world.

* *

For about a thousand years, certain people pretend that they have been stigmatized, i.e., that they have 'miraculously' suffered wounds that are similar to those of Jesus's crucifixion. How can one prove by a one-second observation that all of these people, without exception, produced the wounds by themselves? Challenge 311 s

* *

‘In the middle age and in antiquity, people believed in the flat Earth.’ This is a famous lie that is rarely questioned. The historian Reinhard Krüger has shown that the lie is most of all due to the writers Thomas Paine (1794) and Washington Irving (1928). Fact is that since Aristotle, everybody believed in a spherical Earth.

* *

Challenge 312 s Is the term ‘multiverse’, a claimed opposite to ‘universe’, a lie or a belief?

* *

The following is not a lie. A good way to suppress curiosity in children is used in many environments: let the child watch television whenever it wants. Do it for a few weeks and you will not recognize the child any more. Do it for a few years, and its curiosity will not come back at all. The internet and smartphones have the same effect.

* *

Challenge 313 e How would you show that ‘Earth rays’ are a lie?

* *

How would you show that the statement ‘the laws of nature could change any time’ is a lie? Challenge 314 s

* *

Challenge 315 e ‘I can generate energy from the vacuum.’ Show that this is a lie.

* *

‘Not everything that exists can be measured.’ ‘There are things that cannot be measured.’ Show that these frequent statements are lies – without exception. Challenge 316 e

* *

‘Not everything is known.’ This statement is quite interesting: modern physics indeed claims the opposite in several domains. For example, all forms of energy are known; so are all forms of moving entities. In short, even though this statement is correct – indeed, not everything is known, especially in medicine – it is often used by liars. Be careful when you hear it; if the statement is made without evidence, it is made by a crook.

* *

Here is a lie that uses mathematics, from a journalist: ‘Your university exams treat women applicants worse than men; your statistics show that only 41 % of all female, but 57 % of all male applicants are admitted.’ The university is small and has only two faculties; so it checks its numbers.

Faculty 1 admitted 60 % of all males (60 of 100 applicants) and 65 % of all applicant females (13 of 20 applicants). Faculty 2 admitted 30 % of all males (3 of 10 applicants) and 32 % of all females (16 of 50 applicants).

In total, the university thus admitted 63 of 110 male applicants (or 57 %) and 29 of 70 female applicants (or 41 %). In other words, even though in each faculty the percentage of admitted females was *higher*, the *total* admission percentage for females was *lower*. Why? In fact, this is a true story; in this version, the numbers are simplified, to make the situation as clear as possible. But a large university once got in trouble with journalists in this way, despite preferring women in each of its departments. Some journalists are excellent liars.

Challenge 317 e

* *

Many lies consist of just one concept, sometimes just a single word. Examples are 'laser sword', 'aether', 'transubstantiation' or 'spaceship'. Long time ago, every word was a poem – nowadays, many words are lies. In fact, *science fiction* is a common source of lies.

Page 282

* *

Another domain in which lies are common is the food industry. It is now possible to buy artificial eggs, artificial tomato, or artificial shrimps. But also usual products are not immune. Many products contain cysteine; for decades, cysteine was extracted from human hair. In Europe, most food products also do not tell the country of origin or the content of genetic engineering. Most Bavarian pretzels are made in China, for example.

* *

A famous lie: genetically engineered crops are good for the food supply. In fact, they increase the use of pesticides, have reduced fertility, cost more and increased food problems. Biofuel for cars has produced the same disastrous effects.

* *

'X is the oldest science.' Such statements, with X variously taken to be metallurgy, astronomy, geography, mathematics or some other field, are regularly heard. Obviously, all are both lies and nonsense.

* *

Physicists have helped to reveal that many common statements are lies. Examples are: "astrology holds" – "creation did occur" – "perpetua mobilia are possible" – "vacuum is an energy source" – "lightning is thrown by Zeus" – "certain actions bring bad luck" – "energy speeds faster than light exist" – "telepathy is possible" – "more than three spatial dimensions exist" – "there are things that cannot be measured" – "miracles contradict the laws or rules of nature" – "exceptions to the rules of nature exist" – "quantum theory implies many worlds" – "there are no measurement limits" – "infinite quantities exist in nature" – "supersymmetry is valid" – "particles are membranes" – "a multiverse exists" – "mind is stronger than matter". Other lies and many funny prejudices and superstitions are mentioned throughout our adventure.

* *

The British Broadcasting Corporation is famous for its April 1st pranks. One of the best ever is its documentary on flying penguins. Simply search on the internet for the beautiful film showing how a species of penguins takes off and flies.

Challenge 318 e

Observations and their collection

> Knowledge is a sophisticated statement of ignorance.
>
> Attributed to Karl Popper

The collection of a large number of true statements about a type of observations, i.e., of a large number of facts, is called *knowledge*. Where the domain of observations is sufficiently extended, one speaks of a *science*. A *scientist* is thus somebody who collects knowledge.* We found above that an observation is classified input in the memory of several people. Since there is motion everywhere around us, describing all these observations is a mammoth task. As for every large task, to a large extent the use of appropriate tools determines the degree of success that can be achieved. These tools, in physics and in all other sciences, fall in three groups: tools for the collection of observations, tools to communicate observations and tools to communicate relations between observations. The latter group has been already discussed in the section on language and on mathematics. We just touch on the other two.

Ref. 273

Did instruments collect enough observations?

> Measure what is measurable; make measurable what is not.
>
> Often attributed, though incorrectly, to Galileo.

Ref. 278

Physics is an experimental science; it rests on the *collection* of observations. To realize this task effectively, all sorts of *instruments*, i.e., tools that facilitate observations, have been developed and built. Microscopes, telescopes, oscilloscopes, as well as thermometers, hygrometers, manometers, pyrometers, spectrometers amongst others are familiar examples. The precision of many of these tools is being continuously improved even today; their production is a sizeable part of modern industrial activity, examples being electrical measuring apparatus and diagnostic tools for medicine, chemistry and biology. Instruments can be as small as a tip of a few tungsten atoms to produce an electron beam of a few volts, and as large as 27 km in circumference, producing an proton beam with more than 13 TV effective accelerating voltage at CERN in Geneva. Instruments have been built that contain and measure the coldest known matter in the universe. Other instruments can measure length variations of far less than a proton diameter over kilometre long distances. Instruments have been put deep inside the Earth, on the Moon, on several planets, and have been sent outside the solar system.

Every day, better, cheaper and more precise measurement instruments are being developed and invented. Despite the interest of these activities, in this walk, instruments are only described in passing; many good textbooks on the topic are available. Also most observations collected by instruments are not mentioned in our adventure; they are only

Ref. 275, Ref. 276

* The term 'scientist' is a misnomer peculiar to the English language. Properly speaking, a 'scientist' is a follower of *scientism*, an extremist philosophical school that tried to resolve all problems through science. For this reason, some religious sects have the term in their name. Since the English language did not have a shorter term to designate 'scientific persons', as they used to be called, the term 'scientist' started to appear in the United States, from the eighteenth century onwards. Nowadays the term is used in all English-speaking countries – but not outside them, fortunately.

summarized or cited. The most important measurement results in physics are recorded
Ref. 277 in standard publications, such as the Landolt–Börnstein series and the physics journals.
Vol. I, page 469 Appendix C gives a general overview of reliable information sources.

Will there be significant new future observations in the domain of the foundations of motion? At present, *in this specific domain*, even though the number of physicists and publications is at an all-time high, the number of new experimental discoveries has been steadily diminishing for many years and is now fairly small. The sophistication and investment necessary to obtain new results has become extremely high. In many cases, measuring instruments have reached the limits of technology, of budgets or even those of nature, as CERN shows. The number of new experiments that produce results showing no deviation from theoretical predictions is increasing steadily. The number of historical papers that try to enliven stalled or even dull fields of enquiry are increasing. Claims of new effects and discoveries which turn out to be due to measurement errors, self-deceit or even fraud have become so frequent that scepticism to new results has become a common response.

Most importantly, no difference between observations and the present fundamental theories of motion – general relativity and quantum field theory – are known, as we will discover in the next two volumes. Although in many domains of science, including physics, discoveries are still expected, new observations on the foundations of motion are only a remote possibility.

In short, the task of collecting observations on the foundations of motion – though not on other topics of physics – seems to be fairly *complete*. Indeed, the vast majority of observations described in this adventure were obtained before the end of the twentieth century. We are not too early with our walk.

> Every generation is inclined to define 'the end of physics' as coincident with the end of their scientific contributions.
>
> Julian Schwinger*

Are all physical observables known?

> Scientists have odious manners, except when you prop up their theory; then you can borrow money from them.
>
> Mark Twain

The most practical way to *communicate* observations was developed a long time ago: by measurements. A measurement allows effective communication of an observation to other times and places. This is not always as trivial as it sounds; for example, in the Middle Ages people were unable to compare with precision the 'coldness' of the winters of two different years! The invention of the thermometer provided a reliable solution

* Julian Seymour Schwinger (b. 1918 New York City, d. 1994 Los Angeles), child prodigy and physicist, was famous for his clear thinking and his excellent lectures. He worked on waveguides and synchrotron radiation, made contributions to nuclear physics and developed quantum electrodynamics. For the latter he received the 1965 Nobel Prize in Physics together with Tomonaga and Feynman. He was a thesis advisor
Ref. 274 to many famous physicists and wrote several excellent and influential textbooks. Nevertheless, at the end of his life, he became strangely interested in a hoax turned sour: cold fusion.

to this requirement. A *measurement* is thus the classification of an observation into a standard set of observations. To put it simply:

▷ A measurement is a *comparison with a standard*.

This definition of a measurement is precise and practical, and has therefore been universally adopted. For example, when the length of a house is measured, this aspect of the house is classified into a certain set of standard lengths, namely the set of lengths defined by multiples of a unit. A *unit* is the abstract name of the standard for a certain observable. Numbers and units allow the most precise and most effective communication of measurement results.

For all measurable quantities, practical standard units and measurement methods have been defined; the main ones are listed and defined in Appendix A. All units are derived from a few fundamental ones; this is ultimately due to our limited number of senses: length, time and mass are related to sight, hearing and touch. Our limited number of senses is, in turn, due to the small number of observables of nature. Animals and machines share the same fundamental senses.

We call *observables* the different measurable aspects of a system. Most observables, such as size, speed, position, etc. can be described by numbers, and in this case they are *quantities*, i.e., multiples of some standard unit. Observables are usually abbreviated by *(mathematical) symbols*, usually letters from some alphabet. For example, the symbol c commonly specifies the velocity of light. For most observables, standard symbols have been defined by international bodies.* The symbols for the observables that describe the state of an object are also called *variables*. Variables on which other observables depend are often called *parameters*. (Remember: a parameter is a variable constant.) For example, the speed of light is a constant, the position a variable, and the temperature is often a parameter, on which the length of an object, for example, can depend. Note that not all observables are quantities; in particular, parities are not multiples of any unit.

Physical observables are tools to communicate observations. Is it possible to talk about observations at all? Yes, as we do it every day. But it is many a philosopher's hobby to pretend otherwise. Thy discuss whether there actually is an example for an 'Elementarsatz' – an atomic fact – mentioned by Wittgenstein in his *Tractatus*. Physicists have at least one that fits: *Differences exist*. It is a simple statement; in the final part of our walk, it will play a central role.

Today, all physical observables are known. The task of defining tools for the communication of observations can thus be considered *complete*. This is a simple and strong statement. It shows that the understanding of the fundamentals of motion is near completion.

Indeed, the BIPM, the Bureau International des Poids et Mesures, has stopped adding new units. The last unit, the katal, was introduced in 1999 as an abrreviation of or mol/s.

* All mathematical symbols used in this walk, together with the alphabets from which they are taken, are listed in Appendix A on notation. They follow international standards whenever they are defined. The standard symbols of the physical quantities, as defined by the International Standards Organization (ISO), the International Union of Pure and Applied Physics (IUPAP) and the International Union of Pure and Applied Chemistry (IUPAC), can be found for example in the *bible*, i.e., the *CRC Handbook of Chemistry and Physics*, CRC Press, 1992.

The full list of physical units is presented in Appendix A.

No new observables are expected to be found. In the past, the importance of a physicist could be ranked by the number of observables he or she had discovered. Discovering obervables had always been less common than discovering new patterns, or 'laws' of nature. Even a great scientist such as Einstein, who discovered several pattern of nature, only introduced one new observable, namely the metric tensor for the description of gravity. Following this criterion – as well as several others – Maxwell might be the most important physicist, having introduced several material dependent observables. For Schrödinger, the wave function describing electron motion could be counted as an observable (even though it is a quantity necessary to calculate measurement results, and not itself an observable). Incidentally, the introduction of *any* term that is taken up by others is a rare event; 'gas', 'entropy' or 'kinetic energy' are such examples. Usually, observables were developed by many people cooperating together. Indeed, almost no observables bear people's names, whereas many 'laws' do.

Given that the list of observables necessary to describe nature is complete, does this mean that all the patterns or rules of nature are known? Not necessarily; in the history of physics, observables were usually defined and measured long *before* the precise rules connecting them were found. For example, all observables used in the description of motion itself – such as time, position and its derivatives, momentum, energy and all the thermodynamic quantities – were defined before or during the nineteenth century, whereas the most precise versions of the patterns or 'laws' of nature connecting them, *special relativity* and *non-equilibrium thermodynamics*, have been found only in the twentieth century. The same is true for all observables connected to electromagnetic interaction. The correct patterns of nature, *quantum electrodynamics*, was discovered long after the corresponding observables. The observables that were discovered last were the fields of the strong and the weak nuclear interactions. Also, in this case, the patterns of nature were formulated much later.

In summary, all observables about the fundamentals of motion have been discovered. We are, at this moment of history, in a fortunate situation: we can talk with precision about all motion observed in nature. The last part of our adventure will explore the tiny possibility for errors or loopholes in this statement.

Do observations take time?

An observation is an interaction with some part of nature leading to the production of a record, such as a memory in the brain, data on a tape, ink on paper, or any other fixed pattern applied to a support. The necessary irreversible interaction process is often called *writing* the record. Obviously, writing takes a certain amount of time; zero interaction time would give no record at all. Therefore any recording device, including our brain, always records some *time average* of the observation, however short it may be.

In summary, what we call a fixed image, be it a mental image or a photograph, is always the time average of a moving situation. Without time averaging, we would have no fixed memories. On the other hand, any time averaging introduces a blur that hides certain details; and in our quest for precision, at a certain moment, these details are bound to become important. The discovery of these details will begin in the upcoming part of the walk, the volume that explores quantum theory.

In the final part of our mountain ascent we will discover that there is a shortest possible averaging time. Observations of that short duration show so many details that even the distinction between particles and empty space is lost. In contrast, our concepts of everyday life appear only after relatively long time averages. The search for an average-free description of nature is one of the big challenges of our adventure.

Is induction a problem in physics?

> Nur *gesetzmäßige* Zusammenhänge sind *denkbar*.*
> Ludwig Wittgenstein, *Tractatus*, 6.361

> There is a tradition of opposition between adherents of induction and of deduction. In my view it would be just as sensible for the two ends of a worm to quarrel.
> Alfred North Whitehead

Induction is the usual term used for the act of making, from a small and finite number of experiments, general conclusions about the outcome of *all* possible experiments performed in other places, or at other times. In a sense, it is the technical term for sticking out one's neck, which is necessary in every scientific statement. Universal statements, including the so-called 'laws' and patterns of nature, rely on induction. Induction has been a major topic of discussion for science commentators. Frequently one finds the remark that knowledge in general, and physics in particular, relies on induction for its statements. According to some, induction is a type of hidden belief that underlies all sciences but at the same time contrasts with them.

To avoid wasting energy, we make only a few remarks. The first can be deduced from a simple experiment. Try to convince a critic of induction to put their hand into a fire. Nobody who honestly calls induction a belief should conclude from a few unfortunate experiences in the past that such an act would also be dangerous in the future... In short, somehow induction works.

A second point is that physical universal statements are always openly stated; they are never hidden. The refusal to put one's hand into a fire is a consequence of the invariance of observations under time and space translations. Indeed, general statements of this type form the very basis of physics. However, no physical statement is a belief only because it is universal; it always remains open to experimental checks. Physical induction is not a hidden method of argumentation, it is an explicit part of experimental statements.
Vol. I, page 281
In fact, the complete list of 'inductive' statements used in physics is well known: we gave it in the first part of our adventure. These statements are so important that they have been given a special name: they are called *symmetries*. The list of all known symmetries of nature is the list of all inductive statements used in physics.

Perhaps the best argument for the use of induction is that there is no way to avoid it when one is thinking. There is no way to think, to talk or to remember without using concepts, i.e., without assuming that most objects or entities or processes have the same properties over time. There is also no way to communicate with others without assuming that the observations made from the other's viewpoint are similar to one's own. There

* 'Only connexions that are *subject to law* are *thinkable*.'

is no way to think without symmetry and induction. Indeed, the concepts related to symmetry and induction, such as space and time, belong to the fundamental concepts of language. In fact, the only sentences which do not use induction, the sentences of logic, do not have any content (*Tractatus*, 6.11). Without induction, we cannot classify observations at all! Evolution has given us memory and a brain because induction works. To criticize induction is not to criticize natural sciences, it is to criticize the use of thought in general. We should never take too seriously people who themselves do what they criticize in others; sporadically pointing out the ridicule of this endeavour is just the right amount of attention they deserve.

Page 280
Ref. 273
Challenge 319 s

The topic could be concluded here, were it not for some interesting developments in modern physics that put two additional nails in the coffin of arguments against induction. First, in physics whenever we make statements about all experiments, all times or all velocities, such statements are actually about a *finite number* of cases. We know – today more than ever – that infinities, both in size and in number, do not occur in nature. The infinite number of cases appearing in statements in classical physics and in quantum mechanics are apparent, not real, and due to human simplifications and approximations. Statements that a certain experiment gives the same result 'everywhere' or that a given equation is correct for 'all times', always encompass only a *finite* number of examples. A great deal of otherwise often instinctive repulsion to such statements is avoided in this way. In the sciences, as well as in this book, 'all' *never* means an infinite number of cases.

Secondly, it is well known that extrapolating from a few cases to many is false when the few cases are independent of each other. However, this conclusion is correct if the cases are interdependent. From the observation that somebody found a penny on the street on two subsequent months, cannot follow that he will find one the coming month. Induction is only correct if we know that all cases have similar behaviour, e.g. because they follow from the same origin. For example, if a neighbour with a hole in his pocket carries his salary across that street once a month, and the hole always opens at that point because of the beginning of stairs, then the conclusion would be correct.

The results of modern physics encountered in the final part of our walk show that all situations in nature are indeed interdependent, and thus we will prove that what is called 'induction' is in fact a logically correct conclusion.

> In the progress of physics, the exception often turned out to be the general case.

THE QUEST FOR PRECISION AND ITS IMPLICATIONS

> Der Zweck der Philosophie ist die logische Klärung der Gedanken.*
> Ludwig Wittgenstein, *Tractatus*, 4.112

To talk well about motion means to talk precisely. Precision requires avoiding three common mistakes in the description of nature.

* 'The object of philosophy is the logical clarification of thoughts.'

First, concepts must be *consistent*. Concepts should never have a contradiction built into their definition. For example, any phenomenon occurring in nature evidently is a 'natural' phenomenon; therefore, to talk about either 'supernatural' phenomena or 'unnatural' phenomena is a mistake that nobody interested in motion should let go unchallenged; such terms contain a logical contradiction. Naturally, *all* observations are natural. Incidentally, there is a reward of more than a million dollars for anybody proving the opposite. In over twenty years, despite many attempts, nobody has yet been able to collect it.

Ref. 279

Second, concepts must be *fixed*. Concepts should not have unclear or constantly changing definitions. Their content and their limits must be kept constant and explicit. The opposite of this is often encountered in crackpots or populist politicians; it distinguishes them from more reliable thinkers. Physicists also fall into the trap; for example, there is, of course, only one *single* (physical) universe, as even the name says. To talk about more than one universe is an increasingly frequent error.

Ref. 280

Third, concepts must be *used as defined*. Concepts should not be used outside their domain of application. It is easy to succumb to the temptation to transfer results from physics to philosophy without checking the content. An example is the question: 'Why do particles follow the laws of nature?' The flaw in the question is due to a misunderstanding of the term 'laws of nature' and to a confusion with the laws of the state.

If nature were governed by 'laws', they could be changed by parliament.

We must remember that 'laws of nature' simply means 'pattern', 'property' or 'description of behaviour'. Then we can rephrase the question correctly as 'Why do particles behave in the way we describe their behaviour?' and we can recognize its senselessness.

In the course of our walk, we will often be tempted by these three mistakes. A few such situations follow, with the ways of avoiding them.

> Consistency is the last refuge of the unimaginative.
>
> Oscar Wilde

What are interactions? – No emergence

> The whole is always more than the sum of its parts.
>
> Aristotle, *Metaphysica*, 10f–1045a.

In the physical description of nature, the whole is always *more* than the sum of its parts. Actually, the difference has a special name:

▷ The difference between a whole and the sum of its parts is called the *interaction between the parts.*

For example, the energy of the whole minus the sum of the energies of its parts is called the energy of interaction. The study of interactions is the main topic of physics. In other words, physics is concerned *primarily* with the difference between the parts and the whole. This is contrary to what is often suggested by bad journalists or other sloppy thinkers.

Note that the term 'interaction' is based on the general observation that anything that affects anything else is, in turn, affected by it:

▷ Interactions are reciprocal.

For example, if one body changes the momentum of another, then the second changes the momentum of the first by the same (negative) amount. The reciprocity of interactions is a result of conservation 'laws'. The reciprocity is also the reason that somebody who uses the term 'interaction' is considered a heretic by monotheistic religions, as theologians regularly point out. These belief experts regularly stress that such a reciprocity implicitly denies the immutability of the deity. (Are they correct?)

Ref. 281
Challenge 320 s

The simple definition of interaction given above sounds elementary, but it leads to surprising conclusions. Take the atomic idea of Democritus in its modern form: nature is made of vacuum and of particles. The first consequence is the *paradox of incomplete description*: experiments show that there are interactions between vacuum and particles. However, interactions are differences between parts and the whole, in this case between vacuum and particles on the one hand, and the whole on the other. We thus have deduced that nature is not made of vacuum and particles alone.

The second consequence is the *paradox of overcomplete description*. It starts from the result that is deduced later on:

Vol. IV, page 198

▷ Experiments show that interactions happen through exchange of particles.

However, we have counted particles already as basic building blocks of nature. Does this mean that the description of nature by vacuum and particles is an overdescription, counting things twice? We will resolve both paradoxes in the last part of our mountain ascent.

Challenge 321 s
Vol. VI, page 85

The application of the definition of interaction also settles the frequently heard question of whether in nature there are 'emergent' properties, i.e., properties of systems that cannot be deduced from the properties of their parts and interactions. By the definition of interaction, there are no emergent properties. 'Emergent' properties can only appear if interactions are approximated or neglected. The idea of 'emergent' properties is a product of minds with restricted horizons, unable to see or admit the richness of consequences that general principles can produce. In defending the idea of emergence, one belittles the importance of interactions, working, in a seemingly innocuous, maybe unconscious, but in fact sneaky way, against the use of reason in the study of nature. 'Emergence' is a superstition.

Vol. I, page 426
Ref. 282

What is existence?

> "You know what I like most? Rhetorical questions."

Ref. 283 Assume a friend tells you 'I have seen a *grampus* today!' You would naturally ask what it looks like. What answer do we expect? We expect something like 'It's an animal with a certain number of heads similar to a X, attached to a body like a Y, with wings like a Z, it make noises like a U and it felt like a V' – the letters denoting some other animal or object. Generally speaking, in the case of an object, this scene from Darwin's voyage to South America shows that in order to talk to each other, we first need certain basic, common concepts ('animal', 'head', 'wing', etc.).* In addition, for the definition of a new entity we need a characterization of its parts ('size', 'colour'), of the way these parts relate to each other, and of the way that the whole interacts with the outside world ('feel', 'sound'). In other words, for an object to exist, we must be able to give a list of relations with the outside world.

▷ An object *exists* if we can interact with it.

Challenge 322 s Is observation sufficient to determine existence?

For an abstract concept, such as 'time' or 'superstring', the definition of existence has to be refined only marginally:

▷ *(Physical) existence* is the effectiveness to describe interactions accurately.

This definition applies to trees, time, virtual particles, imaginary numbers, entropy and so on. It is thus pointless to discuss whether a physical concept 'exists' or whether it is 'only' an abstraction used as a tool for descriptions of observations. The two possibilities coincide. The point of dispute can only be whether the description provided by a concept is or is not *precise*.

For mathematical concepts, existence has a somewhat different meaning: a mathematical concept is said to exist if it has no built-in contradictions. This is a much weaker requirement than physical existence. It is thus incorrect to deduce physical existence from mathematical existence. This is a frequent error; from Pythagoras' times onwards it was often stated that since mathematical concepts exist, they must therefore also exist in nature. Historically, this error occurred in the statements that planet orbits 'must' be circles, that planet shapes 'must' be spheres or that physical space 'must' be Euclidean. Today this is still happening with the statements that space and time 'must' be continuous and that nature 'must' be described by sets. In all these cases, the reasoning is wrong. In fact, the continuous attempts to deduce physical existence from mathematical existence hide that the opposite is correct: a short reflection shows that mathematical existence is a special case of physical existence. Challenge 323 s

We note that there is also a different type of existence, namely *psychological existence*. A concept can be said to exist psychologically if it describes human internal experience. Thus a concept can exist psychologically even if it does not exist physically. It is easy to find examples from the religions or from systems that describe inner experiences. Challenge 324 s Also myths, legends and comic strips define concepts that only exist psychologically, not

* By the way, a grampus was the old name for what is called an 'orca' today.

physically. In our walk, whenever we talk about existence, we mean physical existence only.

Do things exist?

> Wer Wissenschaft und Kunst besitzt,
> Hat auch Religion;
> Wer jene beiden nicht besitzt,
> Der habe Religion.*
>
> Johann Wolfgang von Goethe, *Zahme Xenien, IX*

Using the above definition of existence, the question becomes either trivial or imprecise. It is trivial in the sense that things necessarily exist if they describe observations, since they were defined that way. But perhaps the questioner meant to ask: Does reality exist independently of the observer?

Using the above, this question can be rephrased: 'Do the things we observe exist independently of observation?' After thousands of years of extensive discussion by professional philosophers, logicians, sophists and amateurs the answer is the same: it is 'Yes', because the world did not change after great-grandmother died. The disappearance of observers does not seem to change the universe. These experimental findings can be corroborated by inserting the definition of 'existence' into the question, which then becomes: 'Do the things we observe interact with other aspects of nature when they do not interact with people?' The answer is evident. Several popular books on quantum mechanics fantasize about the importance of the 'mind' of observers – whatever this term may mean; they provide pretty examples of authors who see themselves as irreplaceable, seemingly having lost the ability to see themselves as part of a larger entity.

Of course there are other opinions about the existence of things. The most famous is that of the unmarried George Berkeley (b. 1685 Kilkennys, d. 1753 Oxford) who rightly understood that thoughts based on observation alone, if spread, would undermine the basis of the religious organization of which he was one of the top managers. To counteract this tendency, in 1710 he published *A Treatise Concerning the Principles of Human Knowledge*, a book denying the existence of the material world. This reactionary book became widely known in like-minded circles (it was a time when few books were written) even though it is based on a fundamentally flawed idea: it assumes that the concept of 'existence' and that of 'world' can be defined independently. (You may be curious to try the feat.)

Challenge 325 e

Berkeley had two aims when he wrote his book. First, he tried to deny the capacity of people to arrive at judgements on nature or on any other matter *from their own experience*. Second, he also tried to deny the *ontological reach* of science, i.e., the conclusions one can draw from experience on the questions about human existence. (Later, a university was not ashamed to use his name.) Even though Berkeley is generally despised nowadays, he actually achieved his main aim: he was the originator of the statement that science and religion do not contradict, but *complement* each other. By religion, Berkeley did not mean either morality or spirituality; every scientist is a friend of both of these. By religion, Berkeley meant that the standard set of beliefs for which he stood is above

* He who possesses science and art, also has religion; he who does not possess the two, better have religion.

the deductions of reason. The widely cited statement about the compatibility of science and religion, itself a belief, is still held dearly by many even to this day.

Another mistake is to ask *why* things exist. The question makes no sense. It is a waste of time due to bizarre beliefs. When searching for the origin of motion, *all* beliefs stand in the way. Carrying beliefs is like carrying oversized baggage: doing so prevents us from reaching the goal of our adventure.

Does the void exist?

> Teacher: 'What is found between the nucleus and the electrons?'
> Student: 'Nothing, only air.'

In philosophical discussions '*void*' is usually defined as 'non-existence'. It then becomes a game of words to ask for a yes or no answer to the question 'Does the void exist?' The expression 'the existence of non-existence' is either a contradiction of terms or is at least unclearly defined; the topic would not seem to be of great interest. However, similar questions do appear in physics, and a physicist should be prepared to notice the difference of this from the previous one. Does a vacuum exist? Does empty space exist? Or is the world 'full' everywhere, as the more conservative biologist Aristotle maintained? In the past, people have been killed for giving an answer that was unacceptable to authorities.

It is not obvious, but it is nevertheless important, that the modern physical concepts of 'vacuum' and 'empty space' are not the same as the philosophical concept of 'void'. 'Vacuum' is not defined as 'non-existence'; on the contrary, it is defined as the absence of matter and radiation. Vacuum is an entity with specific observable properties, such as its number of dimensions, its electromagnetic constants, its curvature, its vanishing mass, its interaction with matter through curvature and through its influence on decay, etc. A table of the properties of a physical vacuum is given on page 137. Historically, it took a long time to clarify the distinction between a physical vacuum and a philosophical void. People confused the two concepts and debated the existence of the vacuum for more than two thousand years. The first to state that it existed, with the courage to try to look through the logical contradiction at the underlying physical reality, were Leucippus and Democritus, the most daring thinkers of antiquity. Their speculations in turn elicited the reactionary response of Aristotle, who rejected the concept of vacuum. Aristotle and his disciples propagated the belief about nature's *horror of the vacuum*.

The discussion changed completely in the seventeenth century, when the first experimental method to realize a vacuum was devised by Torricelli.* Using mercury in a glass
Challenge 326 s
tube, he produced the first laboratory vacuum. Can you guess how? Arguments against the existence of the vacuum again appeared around 1900, when it was argued that light needed 'aether' for its propagation, using almost the same arguments that had been used two hundred years earlier, but in different words. However, experiments failed to detect any of the supposed properties of this unclearly defined concept. Experiments in the field of general relativity showed that a vacuum can move – though in a completely different

* Evangelista Torricelli (b. 1608 Faenza, d. 1647 Florence), physicist, pupil and successor to Galileo. The (non-SI) pressure unit 'torr' is named after him.

way from the way in which the aether was expected to move – that the vacuum can be bent, but it then tends to return to its shape. Then, in the late twentieth century, quantum field theory again argued against the existence of a true vacuum and in favour of a space full of virtual particle–antiparticle pairs. The issue culminated in the discussions around Vol. VI, page 58 the cosmological constant.

In short, the vacuum exists. The question 'Does the void exist?' is settled conclusively Vol. VI, page 87 only in the last part of this walk, in a rather surprising way.

> Natura abhorret vacuum.
>
> Antiquity

Is nature infinite?

> It is certain and evident to our senses, that in the world some things are in motion. Now whatever is moved is moved by another... If that by which it is moved be itself moved, then this also needs to be to be moved by another, and that by another again. But this cannot go on to infinity, because then there would be no first mover and consequently, no other mover, seeing that subsequent movers move only inasmuch as they are moved by the first mover, as the staff moves only because it is moved by the hand. Therefore it is necessary to arrive at a first mover, moved by no other; and this everyone understands to be god.
>
> Thomas Aquinas (b. *c.* 1225 Aquino, d. 1274 Fossanova) *Summa Theologiae*, I, q. 2.

Most of the modern discussions about set theory centre on ways to defining the term 'set' for various types of infinite collections. For the description of motion this leads to two questions: Is the universe infinite? Is it a set? We begin with the first one. Illuminating the question from various viewpoints, we will quickly discover that it is both simple and imprecise.

Do we need infinite quantities to describe nature? Certainly, in classical and quantum physics we do, e.g. in the case of space-time. Is this necessary? We can say already a few things.

Any set can be finite in one aspect and infinite in another. For example, it is possible to proceed along a finite mathematical distance in an infinite amount of time. It is also possible to travel along any distance whatsoever in a given amount of mathematical time, making infinite speed an option, even if relativity is taken into account, as was explained Vol. II, page 48 earlier.

Despite the use of infinities, scientists are still limited. We saw above that many types Page 288 of infinities exist. However, no infinity larger than the cardinality of the real numbers plays a role in physics. No space of functions or phase space in classical physics and no Ref. 284 Hilbert space in quantum theory has higher cardinality. Despite the ability of mathematicians to define much larger kinds of infinities, the description of nature does not need them. Even the most elaborate descriptions of motion use only the infinity of the real numbers.

Challenge 327 s But is it possible at all to say *of nature* or of one of its aspects that it is indeed *infinite*? Can such a statement be compatible with observations? No. It is evident that every statement that claims that something in nature is infinite is a belief, and is not backed by observations. We shall patiently eliminate this belief in the following.

Ref. 269 The possibility of introducing false infinities make any discussion on whether humanity is near the 'end of science' rather difficult. The amount of knowledge and the time required to discover it are unrelated. Depending on the speed with which one advances through it, the end of science can be near or unreachable. In practice, scientists have thus the power to *make* science infinite or not, e.g. by reducing the speed of progress. As scientists need funding for their work, one can guess the stand that they usually take.

In short, the universe cannot be proven to be infinite. But can it be *finite*? At first sight, this would be the only possibility left. (It is not, as we shall see.) But even though many have tried to describe the universe as finite in all its aspects, no one has yet been successful. In order to understand the problems that they encountered, we continue with the other question mentioned above:

Is the universe a set?

Ref. 285 A simple observation leads us to question whether the universe is a set. For 2500 years it has been said that the universe is made of vacuum and particles. This implies that the universe is made of a certain *number* of particles. Perhaps the only person to have taken this conclusion to the limit was the astrophysicist Arthur Eddington (b. 1882 Kendal, d. 1944 Cambridge), who wrote:

Ref. 286
> I believe there are 15,747,724,136,275,002,577,605,653,961,181,555,468,044, 717,914,527,116,709,366,231,425,076,185,631,031,296 protons in the universe and the same number of electrons.

Eddington was ridiculed over and over again for this statement and for his beliefs that lead up to it. His arguments were indeed based on his personal preferences for certain pet numbers. However, we should not laugh too loudly. In fact, for 2500 years almost all scientists have thought along the same line, the only difference being that they have left the precise number unspecified! In fact, *any other number* put into the above sentence would be equally ridiculous. Avoiding specifying it is just a coward's way of avoiding looking at this foggy aspect of the particle description of nature.

Is there a particle number at all in nature? If you smiled at Eddington's statement, or if you shook your head over it, it may mean that you instinctively believe that nature is not a set. Is this so? Whenever we define the universe as the totality of events, or as the totality of all space-time points and objects, we imply that space-time points can be distinguished, that objects can be distinguished and that both can be distinguished from each other. We thus assume that nature is separable and a set. But is this correct? The question is important. The ability to distinguish space-time points and particles from each other is often called *locality*. Thus the universe is separable or is a set if and only if our description of it is local.* And in everyday life, locality is observed without exception.

* In quantum mechanics also other, more detailed definitions of locality are used. We will mention them

In daily life we also observe that nature is separable and a whole at the same time. It is a 'many that can be thought as one': in daily life nature is a set. Indeed, the basic characteristic of nature is its diversity. In the world around us we observe changes and differences; we observe that nature is separable. Furthermore, all aspects of nature belong together: there are relations between these aspects, often called 'laws,' stating that the different aspects of nature form a whole, usually called the universe.

In other words, the possibility of describing observations with the help of 'laws' follows from our experience of the separability of nature. The more precisely the separability is specified, the more precisely the 'laws' can be formulated. Indeed, if nature were not separable or were not a unity, we could not explain why stones fall downwards. Thus we are led to speculate that we should be able to deduce all 'laws' from the observation that nature is separable.

In addition, only the separability allows us to describe nature at all. A description is a classification, that is, a mapping between certain aspects of nature and certain concepts. All concepts are sets and relations. Since the universe is separable, it can be described with the help of sets and relations. Both are separable entities with distinguishable parts. A precise description is commonly called an understanding. In short, the universe is comprehensible only because it is separable.

Moreover, only the separability of the universe makes our brain such a good instrument. The brain is built from a large number of connected components, and only the brain's separability allows it to function. In other words, thinking is only possible because nature is separable.

Finally, only the separability of the universe allows us to distinguish reference frames, and thus to define all symmetries at the basis of physical descriptions. And in the same way that separability is thus necessary for *covariant* descriptions, the unity of nature is necessary for *invariant* descriptions. In other words, the so-called 'laws' of nature are based on the experience that nature is both separable and unifiable – that it is a set.

These arguments seem overwhelmingly to prove that the universe is a set. However, these arguments apply only to everyday experience, everyday dimensions and everyday energies. Is nature a set also *outside* the domains of daily life? Are objects different at all energies, even when they are looked at with the highest precision possible?

We have three open issues left: the issue of the number of particles in the universe; the circular definition of space, time and matter; and the issue as to whether describing nature as made of particles and void is an overdescription, an underdescription, or neither. These three issues make us doubt whether objects are countable at all energies.
Vol. VI, page 106
We will discover in the final part of our mountain ascent that indeed, objects in nature cannot be counted at high energy. The consequences will be extensive and fascinating. As an example, try to answer the following: if the universe is not a set, what does that
Challenge 328 s
mean for space and time?

Vol. IV, page 153
in the quantum part of this text. The issue mentioned here is a different, more fundamental one, and not connected with that of quantum theory.

Does the universe exist?

> Each progressive spirit is opposed by a thousand men appointed to guard the past.
>
> Maurice Maeterlink

Following the definition above, existence of a concept means its usefulness to describe interactions. Now, there are two common definitions of the concept of 'universe'. The first is the totality of all matter, energy, space and time. But this usage results in a strange consequence: since nothing can interact with this totality, we cannot claim that the universe exists.

So let us take the second, more restricted view, namely that the universe is only the totality of all matter and energy. But also in this case it is impossible to interact with the
Challenge 329 s
universe. Can you give a few arguments to support this?

In short, we arrive at the conclusion that the universe does not exist. We will indeed
Vol. VI, page 111
confirm this result in more detail later on in our walk. In particular, since the universe does not exist, it does not make sense to even try to answer *why* it exists. The best answer
Ref. 220
might be: because of furiously sleeping, colourless green ideas.

What is creation?

> (Gigni) De nihilo nihilum, in nihilum nil posse reverti.*
>
> Ref. 287 Persius, *Satira*, III, v. 83-84.

> Anaxagoras, discovering the ancient theory that nothing comes from nothing, decided to abolish the concept of creation and introduced in its place that of discrimination; he did not hesitate to state, in effect, that all things are mixed to the others and that discrimination produces their growth.
>
> Ref. 288 *Anonymous fragment*, Middle Ages.

The term 'creation' is often heard when talking about nature. It is used in various contexts with different meanings.

One speaks of creation as the characterization of human actions, such as observed in an artist painting or a secretary typing. Obviously, this is one type of change. In the classification of change introduced at the beginning of our walk, the changes cited are movements of objects, such as the electrons in the brain, the molecules in the muscles, the material of the paint, or the electrons inside the computer. This type of creation is thus a special case of motion.

One also speaks of creation in the biological or social sense, such as in 'the creation of life', or 'creation of a business', or 'the creation of civilization'. These events are forms of growth or of self-organization; again, they are special cases of motion.

Physicists often say that a lamp 'creates' light or that a stone falling into a pond 'creates' water ripples. Similarly, they talk of 'pair creation' of matter and antimatter. It
Vol. IV, page 192
was one of the important discoveries of physics that all these processes are special types

* Nothing (can appear) from nothing, nothing can disappear into nothing.

Vol. V, page 113 of motion, namely excitation of fields.

In popular writing on cosmology, 'creation' is also a term commonly applied, or better misapplied, to the *big bang*. However, the expansion of the universe is a pure example of motion, and contrary to a frequent misunderstanding, the description of the big bang contains only processes that fall into one of the previous three categories, as shown in Vol. II, page 248 the relevant chapter in general relativity. The big bang is not an example of creation. Quantum cosmology provides more reasons that show why the naive term 'creation' is Vol. II, page 248 not applicable to the big bang. First, it turns out that the big bang was not an event. Second, it was not a beginning. Third, it did not provide a *choice* from a large set of possibilities. The big bang does not have any properties attributed to the term 'creation'.

In summary, we conclude that in all cases, *creation is a type of motion*. (The same applies to the notions of 'disappearance' and 'annihilation'.) No other type of creation is observed in nature. In particular, the naive sense of 'creation', namely 'appearance from nothing' – *ex nihilo* in Latin – is never observed in nature. All observed types of 'creation' require space, time, forces, energy and matter for their realization. Creation requires something to exist already, in order to take place. In addition, precise exploration shows that no physical process and no example of motion has a beginning. Our walk will show us that nature does not allow us to pinpoint beginnings. This property alone is sufficient to show that 'creation' is not a concept applicable to what happens in nature. Worse still, creation is applied only to physical systems; we will discover that nature is not a system and worse, that systems do not exist at all.

The opposite of creation is *conservation*. The central statements of physics are conservation theorems: for energy, mass, linear momentum, angular momentum, charge, etc. In fact, every conservation 'law' is a detailed and accurate rejection of the concept of creation. The ancient Greek idea of atoms already contains this rejection. Atomists stated that there is no creation and no disappearance, but only motion of atoms. Every transformation of matter is a motion of atoms. In other words, the idea of the atom was a direct consequence of the negation of creation. It took humanity over 2000 years before Vol. I, page 335 it stopped locking people in jail for talking about atoms, as had happened to Galileo.

However, there is one exception in which the naive concept of creation does apply: it describes what magicians do on stage. When a magician makes a rabbit appear from nowhere, we indeed experience 'creation' from nothing. At its best such magic is a form of entertainment, at its worst, a misuse of gullibility. The idea that the universe results from either of these two does not seem appealing; on second thought though, maybe looking at the universe as the ultimate entertainment could open up a fresh and more productive approach to life.

Voltaire (b. 1694 Paris, d. 1778 Paris) popularized an argument against creation often used in the past: we do not know whether creation has taken place or not. Today the situation is different: we *do* know that it has *not* taken place, because creation is a type of motion and, as we will see in the concluding part of our mountain ascent, motion did not exist near the big bang.

Have you ever heard the expression 'creation of the laws of nature'? It is one of the most common examples of disinformation. First of all, this expression confuses the 'laws' with nature itself. A description is not the same as the thing itself; everybody knows that giving their beloved a description of a rose is different from giving an actual rose. Second, the expression implies that nature is the way it is because it is somehow 'forced' to follow the 'laws' – a rather childish and, what is more, incorrect view. And third, the expression assumes that it is possible to 'create' descriptions of nature. But a 'law' is a description, and a description by definition cannot be created: so the expression makes no sense at all. The expression 'creation of the laws of nature' is the epitome of confused thinking.

It may well be that calling a great artist 'creative' or 'divine', as was common during the Renaissance, is not blasphemous, but simply an encouragement to the gods to try to do as well. In fact, whenever one uses the term 'creation' to mean anything other than some form of motion, one is discarding both observations and human reason. 'Creation' is one of the last pseudo-concepts of our modern time; no expert on motion should forget this. It is impossible to complete our adventure without getting rid of 'creation'. This is not easy. We will encounter several attempts to bring back creation, among them in the study of entropy, in the study of biological evolution and in quantum theory.

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> Every act of creation is first of all an act of destruction.
>
> Pablo Picasso

Is nature designed?

> In the beginning the universe was created. This has made a lot of people very angry and has been widely regarded as a bad move.
>
> Douglas Adams, *The Restaurant at the End of the Universe.*

The tendency to infer the intentional creation of an object from its simple existence is widespread. Some people jump to this conclusion every time they see a beautiful landscape. This habit stems from the triple prejudice that a beautiful scene implies a complex description, in turn implying complex building instructions, and therefore pointing to an underlying *design*.

This chain of thought contains several mistakes. First, in general, beauty is not a consequence of complexity. Usually it is the opposite: indeed, the study of chaos and of self-organization demonstrates how beautifully complex shapes and patterns can be generated with extremely simple descriptions.

Ref. 289
Vol. I, page 415

True, for most human artefacts, complex descriptions indeed imply complex building processes; a personal computer is a good example of a complex object with a complex production process. But in nature, this connection does not apply. We have seen above that even the amount of information needed to construct a human body is about a million times smaller than the information stored in the brain alone. Similar results have been found for plant architecture and for many other examples of patterns in nature. The simple descriptions behind the apparent complexities of nature have been and are still being uncovered by the study of self-organization, chaos, turbulence and fractal shapes. In nature, complex structures derive from *simple* processes. Beware of anyone who says that nature has 'infinite' or 'high complexity': first of all, complexity is not a measurable entity, despite many attempts to quantify it. In addition, all known complex system can be described by (relatively) few parameters and simple equations. Finally, nothing in nature is infinite.

The second mistake in the argument for design is to link a description with an 'instruction', and maybe even to imagine that some unknown 'intelligence' is somehow pulling the strings of the world's stage. The study of nature has consistently shown that there is no hidden intelligence and no instruction behind the processes of nature. An instruction is a list of orders to an executioner. But there are no orders in nature, and no executioners. There are no 'laws' of nature, only descriptions of processes. Nobody is building a tree; the tree is an outcome of the motion of molecules making it up. The genes in the tree do contain information; but no molecule is given any instructions. What seem to be instructions to us are just natural movements of molecules and energy, described by the same patterns taking place in non-living systems. The whole idea of instruction – like that of 'law' of nature – is an ideology, born from an analogy with monarchy or even tyranny, and a typical anthropomorphism.

The third mistake in the argument for design is the suggestion that a complex description for a system implies an underlying design. This is not correct. A complex description only implies that the system has a long evolution behind it. The correct deduction is: something of large complexity exists; therefore it has *grown*, i.e., it has been transformed through input of (moderate) energy over time. This deduction applies to flowers, mountains, stars, life, people, watches, books, personal computers and works of art; in fact it applies to all objects in the universe. The complexity of our environment thus points out the considerable age of our environment and reminds us of the shortness of our own life.

The lack of basic complexity and the lack of instructions in nature confirm a simple result: there is not a single observation in nature that implies or requires design or creation. On the other hand, the variety and intensity of nature's phenomena fills us with deep awe. The wild beauty of nature shows us how *small* a part of nature we actually
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are, both in space and in time.* We shall explore this experience in detail. And we shall find that remaining open to nature's phenomena in all their overwhelming intensity is central to the rest of our adventure.

> There is a separation between state and church, but not yet between state and science.
>
> Paul Feyerabend

What is a description?

> In theory, there is no difference between theory and practice. In practice, there is.

Following standard vocabulary usage, a description of an observation is a list of the de-
Page 324
tails. The above example of the grampus showed this clearly. In other words, a description of an observation is the act of categorizing it, i.e., of comparing, by identifying or distinguishing, the observation with all the other observations already made.

▷ A description is a classification.

In short, *to describe means to see as an element of a larger set.*

* The search for a 'sense' in life or in nature is a complicated, and necessary, way to try to face the smallness of human existence.

A description can be compared to the 'you are here' sign on a city tourist map. Out of a set of possible positions, the 'you are here' sign gives the actual one. Similarly, a description highlights the given situation in comparison with all other possibilities. For example, the formula $a = GM/r^2$ is a description of the observations relating motion to gravity, because it classifies the observed accelerations a according to distance to the central body r and to its mass M; indeed such a description sees each specific case as an example of a general pattern. The habit of generalizing is one reason for the often disturbing dismissiveness of scientists: when they observe something, their professional training usually makes them classify it as a special case of a known phenomenon and thus keeps them from being surprised or from being exited about it.

A description is thus the opposite of a *metaphor*; the latter is an analogy relating an observation with another *special* case; a description relates an observation with a *general* case, such as a physical theory.

> Felix qui potuit rerum cognoscere causas,
> atque metus omnis et inexorabile fatum
> subjecit pedibus strepitumque acherontis avari.
> Vergilius*

Reason, purpose and explanation

> Der ganzen modernen Weltanschauung liegt
> die Täuschung zugrunde, daß die sogenannten
> Naturgesetze die Erklärungen der
> Naturerscheinungen seien.**
> Ludwig Wittgenstein, *Tractatus*, 6.371

Compare the following two types of questions and answers:

1. Why are the leaves of most trees green? Because they absorb red and blue light. Why do they absorb those colours? Because they contain chlorophyll. Why is chlorophyll green? Because all chlorophyll types contain magnesium between four pyrrole groups, and this chemical combination gives the green colour, as a result of its quantum mechanical energy levels. Why do plants contain chlorophyll? Because this is what land plants can synthesize. Why only this? Because all land plants originally evolved from the green algae, who are only able to synthesize this compound, and not the compounds found in the blue or in the red algae, which are also found in the sea.
2. Why do children climb trees, and why do some people climb mountains? Because of the sensations they experience during their activity: the feelings of achievement, the symbolic act to go upwards, the wish to get a wider view of the world are part of this type of adventure.

* 'Happy he who can know the causes of things and who, free of all fears, can lay the inexorable fate and the noise of Acheron to his feet.' *Georgica*, book II, verses 490 ss. Publius Vergilius Maro (b. 70 Mantua, d. 19 bce Brindisi), the great Roman poet, is author of the Aeneid. Acheron was the river crossed by those who had just died and were on their way to the Hades.

** 'The whole modern conception of the world is founded on the illusion that the so-called laws of nature are the explanations of natural phenomena.'

The two types of 'why'-questions show the general difference between reasons and purposes (although the details of these two terms are not defined in the same way by everybody). A *purpose* or *intention* is a classification applied to the actions of humans or animals; strictly speaking, it specifies the quest for a feeling, namely for achieving some type of satisfaction after completion of the action. On the other hand, a *reason* is a specific relation of a fact with the rest of the universe, usually its past. What we call a reason always rests outside the observation itself, whereas a purpose is always internal to it.

Reasons and purposes are the two possibilities of explanations, i.e., the two possible answers to questions starting with 'why'. Usually, physics is not concerned with purpose or with people's feeling, mainly because its original aim, to talk about motion with precision, does not seem to be achievable in this domain. Therefore, *physical* explanations of facts are never purposes, but are always reasons.

▷ A *physical explanation* of an observation is always the description of its relation with the rest of nature.

Ref. 290

We note that purposes are *not* put aside because they pertain to the future, but because they are inadmissible anthropomorphisms. In fact, for deterministic systems, we can equally say that the future is actually a *reason* for the present and the past, a fact often forgotten.

A question starting with 'why' is thus accessible to physical investigation as long as it asks for a reason and not for a purpose. In particular, questions such as 'why do stones fall downwards and not upwards?' or 'why do electrons have that value of mass, and why do they have mass at all?' or 'why does space have three dimensions and not thirty-six?' can be answered, as these ask for the connection between specific observations and more general ones. Of course, not all demands for explanation have been answered yet, and there are still problems to be solved. Our present trail only leads from a few answers to some of the more fundamental questions about motion.

The most general quest for an explanation derives from the question: why is the universe the way it is? The topic is covered in our mountain ascent using two usual approaches.

Unification and demarcation

> “Tout sujet est un; et, quelque vaste qu'il soit, il peut être renfermé dans un seul discours.* ”
> Buffon, *Discours sur le style*.

Studying the properties of motion, constantly paying attention to increase the accuracy of description, we find that explanations are generally of two types:**

1. 'It is like all such cases; also this one is described by …' The situation is recognized as a *special case* of a general behaviour.
2. 'If the situation were different, we would have a conclusion in contrast with observa-

* Every subject is one and, however vast it is, it can be comprised in a single discourse.

Challenge 330 s

** Are these the only possible ones?

tions.' The situation is recognized as the *only possible case*.*

In other words, the first procedure to find explanations is to formulate patterns, rules or 'laws' that describe larger and larger numbers of observations, and compare the observation with them. This endeavour is called the *unification* of physics – by those who like it; those who don't like it, call it 'reductionism'. For example, the same rule describes the flight of a tennis ball, the motion of the tides at the sea shore, the timing of ice ages, and the time at which the planet Venus ceases to be the evening star and starts to be the morning star. These processes are all consequences of universal gravitation. Similarly, it is not evident that the same rule describes the origin of the colour of the eyes, the formation of lightning, the digestion of food and the working of the brain. These processes are described by quantum electrodynamics.

Unification has its most impressive successes when it predicts an observation that has not been made before. A famous example is the existence of antimatter, predicted by Dirac when he investigated the solutions of an equation that describes the precise behaviour of common matter.

The second procedure in the search for explanations is to eliminate all other imaginable alternatives in favour of the actually correct one. This endeavour has no commonly accepted name: it could be called the *demarcation* of the 'laws' of physics – by those who like it; others call it 'anthropocentrism', or simply 'arrogance'.

When we discover that light travels in such a way that it takes the shortest possible time to its destination, when we describe motion by a principle of least action, or when we discover that trees are branched in such a way that they achieve the largest effect with the smallest effort, we are using a demarcation viewpoint.

In summary, unification, answering 'why' questions, and demarcation, answering 'why not' questions, are typical for the progress throughout the history of physics. We can say that the dual aspects of unification and demarcation form the composing and the opposing traits of physics. They stand for the desire to *know everything*.

However, neither demarcation nor unification can explain the universe as a whole. Can you see why? In fact, apart from unification and demarcation, there is a third possibility that merges the two and allows one to say more about the universe. Can you find it? Our walk will automatically lead to it later.

Challenge 331 s

Challenge 332 s

Pigs, apes and the anthropic principle

> “Das wichtigste Hilfsmittel des Wissenschaftlers ist der Papierkorb.**”
>
> Several authors

* These two cases have not to be confused with similar sentences that *seem* to be explanations, but that aren't:

— 'It is like the case of ...' A similarity with another *single* case is *not* an explanation.
— 'If it were different, it would contradict the idea that ...' A contradiction with an *idea* or with a theory is *not* an explanation.

** 'The most important instrument of a scientist is the waste paper basket.'

The wish to achieve demarcation of the patterns of nature is most interesting when we follow the consequences of different possible rules of nature until we find them in contradiction with the most striking observation: our own human existence. In this special case the program of demarcation is often called the *anthropic principle* – from the Greek ἄνθρωπος, meaning 'man'. Is it really possible to deduce all the properties of nature from our own existence?

For example, if the Sun–Earth distance were different from what it is, the resulting temperature change on the Earth would have made impossible the appearance of life, which needs liquid water. Similarly, our brain would not work if the Moon did not circle the Earth. It is also well-known that if there were fewer large planets in the solar system, the evolution of humans would have been impossible. The large planets divert large numbers of comets, preventing them from hitting the Earth. The spectacular collision of comet Shoemaker–Levy-9 with Jupiter, the astronomical event of July 1994, was an example of this diversion of a comet.*

Also the anthropic principle has its most impressive successes when it predicts unknown observations. The most famous example stems from the study of stars. Carbon atoms, like all other atoms except most hydrogen, helium or lithium atoms, are formed in stars through fusion. While studying the mechanisms of fusion in 1953, the well-known astrophysicist Fred Hoyle** found that carbon nuclei could not be formed from the alpha particles present inside stars at reasonable temperatures, unless they had an excited state with an increased cross-section. From the fact of our existence, which is based on carbon, Hoyle thus predicted the existence of a previously unknown excited state of the
Ref. 291 carbon nucleus. And, indeed, the excited state was found a few months later by Willy Fowler.***

Ref. 292 In the *serious* form, we have:

▷ The anthropic principle is the quest to deduce the complete description of nature from the experimental fact of human existence.

The anthropic principle is thus not a principle. It is better called the *anthropic quest* or the *anthropic conjecture*.

Unfortunately, in the popular literature the anthropic principle is often changed from a quest to a *perverted* form, a melting pot of absurd metaphysical ideas in which everybody mixes up their favourite beliefs. Most frequently, the experimental observation of our own existence has been perverted to reintroduce the idea of 'design', i.e., that the universe has been constructed with the aim of producing humans. Often it is even suggested that the anthropic principle is an *explanation* of the rules of nature – a gross

* For a collection of pictures of this event, see e.g. the garbo.uwasa.fi/pc/gifslevy.html website.
** Fred Hoyle (b. 1915 Bingley, d. 2001 Bournemouth), important astronomer and astrophysicist, was the first and maybe only physicist who ever made a specific prediction – namely the existence of an excited state of the carbon nucleus – from the simple fact that humans exist. A permanent maverick, he coined the term 'big bang' even though he did not accept the evidence for it, and proposed another model, the 'steady state'. His most important and well-known research was on the formation of atoms inside stars. He also propagated the belief that life was brought to Earth from extraterrestrial microbes.
*** William A. Fowler (b. 1911 Pittsbrugh, d. 1995 Pasadena) shared the 1983 Nobel Prize in Physics with Subramanyan Chandrasekhar for this and related discoveries.

example of disinformation.

How can we distinguish between the serious and the perverted form? We start with an observation. We would get exactly the same rules and patterns of nature if we used the existence of pigs or monkeys as a starting point. In other words, if we would reach *different* conclusions by using the *porcine principle* or the *simian principle*, we are using the perverted form of the anthropic principle, otherwise we are using the serious form. (The carbon-12 story is thus an example of the serious form.) This test is effective because there is no known pattern or 'law' of nature that is particular to humans but unnecessary for apes or pigs.*

It might even be that one day a computer will start to talk about the 'computer principle'. That would be another example of the perverted form.

> Er wunderte sich, daß den Katzen genau an den Stellen Löcher in den Pelz geschnitten wären, wo sie Augen hätten.**
>
> Georg Christoph Lichtenberg

Do we need cause and effect in explanations?

> There are in nature neither rewards nor punishments – there are only consequences.
>
> Robert Ingersoll

> The world owes you nothing. It was there first.
>
> Mark Twain

> No matter how cruel and nasty and evil you may be, every time you take a breath you make a flower happy.
>
> Mort Sahl

Historically, the two terms 'cause' and 'effect' have played an important role in philosophical discussions. The terms were attached to certain processes or observations. In
Ref. 293 particular, during the birth of modern mechanics, it was important to point out that every effect has a cause, in order to distinguish precise thought from thought based on beliefs, such as 'miracles', 'divine surprises' or 'evolution from nothing'. It was equally essential to stress that effects are different from causes; this distinction avoids pseudo-explanations such as the famous example by Molière where the doctor explains to his patient in elaborate terms that sleeping pills work because they contain a 'dormitive virtue'.

But in physics, the concepts of cause and effect are not used at all. That miracles do not appear is expressed every time we use symmetries, conservation theorems or rules of nature. The observation that cause and effect differ from each other is inherent in any

* Though apes do not seem to be good physicists, as described in the text by D. J. Povinelli, *Folk Physics for Apes: the Chimpanzee's Theory of How the World Works*, Oxford University Press, 2000.

** 'He was amazed that cats had holes cut into their fur precisely in those places where they had eyes.' Georg Christoph Lichtenberg (b. 1742 Ober-Ramstadt, d. 1799 Göttingen), physicist and intellectual, professor in Göttingen, still famous today for his extremely numerous and witty aphorisms and satires. Among others of his time, Lichtenberg made fun of all those who maintained that the universe was made exactly to the measure of man, a frequently encountered idea in the foggy world of the anthropic principle.

evolution equation. Moreover, the concepts of cause and effect are not clearly defined; for example, it is especially difficult to define what is meant by one cause or one effect as opposed to several of them. Both terms are also impossible to quantify and to measure. In other words, useful as 'cause' and 'effect' may be in personal and everyday life, they are not necessary in physics. In the exploration of motion, cause and effect play no role.

> Ἀγαθον καὶ ξαξόν · ἓν καὶ ταὐτό.*
>
> Heraclitus

> Wenn ein Arzt hinter dem Sarg seines Patienten geht, so folgt manchmal tatsächlich die Ursache der Wirkung.**
>
> Robert Koch

Is consciousness required?

> Variatio delectat.***
>
> Cicero

Ref. 294

A lot of mediocre discussions are going on about consciousness, and we will skip them here. What is consciousness? Most simply and concretely, consciousness means the possession of a small part of oneself that is watching what the rest of oneself is perceiving, feeling, thinking and doing. In short, consciousness is the ability to observe oneself, and in particular one's inner mechanisms and motivations.

▷ *Consciousness* is the ability of introspection.

The definition of consciousness explains why it is so difficult to grasp precisely. Indeed, observing our own consciousness would mean to observe the part inside us that observes the rest of ourselves. This seems an almost impossible task, independently of whether consciousness is a hardware or a software aspect of our brain. This impossibility is the basis of the fascination and the mystery of our consciousness and of our human nature.

The definition of consciousness tells us that it is *not* a prerequisite for studying motion. Indeed, animals, plants or machines are also able to observe motion, because they contain sensors, i.e., measurement devices. For the same reason, consciousness is *not* necessary to observe quantum mechanical motion, though measurement is. On the other hand, the exploration of motion and the exploration of oneself have a lot in common: the need to observe carefully, to overcome preconceptions, to overcome fear and the pleasure of doing so.

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For the time being, we have put enough emphasis on the precision of concepts. Talking about motion is something to be deeply enjoyed. Let us see why.

> Precision and clarity obey the indeterminacy relation: their product is constant.
>
> Niels Bohr

* 'Good and bad – one and the same.'
** 'When a physician walks behind the coffin of his patient, indeed the cause sometimes follows the effect.'
*** 'Change pleases.' Marcus Tullius Cicero (b. 106 Arpinum, d. 43 BCE Formiae), important lawyer, orator and politician at the end of the Roman republic.

Curiosity

> Precision is the child of curiosity.

Like the history of every person, also the history of mankind charts a long struggle to avoid the pitfalls of accepting the statements of authorities as truth, without checking the facts. Indeed, whenever curiosity leads us to formulate a question, there are always two general ways to proceed. One is to check the facts personally, the other is to ask somebody. However, the last way is dangerous: it means to give up a part of oneself. Healthy people, children whose curiosity is still alive, as well as scientists, choose the first way. After all, science is due to adult curiosity.

Curiosity, also called the *exploratory drive*, plays strange games with people. Starting with the original experience of the world as a big 'soup' of interacting parts, curiosity can drive one to find *all* the parts and *all* the interactions. It drives not only people. It has been observed that when rats show curious behaviour, certain brain cells in the hypothalamus get active and secrete hormones that produce positive feelings and emotions. If a rat has the possibility, via some implanted electrodes, to excite these same cells by pressing a switch, it does so voluntarily: rats get *addicted* to the feelings connected
Ref. 295 with curiosity. Like rats, humans are curious because they enjoy it. They do so in at least four ways: because they are artists, because they are fond of pleasure, because they are adventurers and because they are dreamers. Let us see how.

Originally, curiosity stems from the desire to interact in a positive way with the environment. Young children provide good examples: curiosity is a natural ingredient of their life, in the same way that it is for other mammals and a few bird species; incidentally, the same taxonomic distribution is found for play behaviour. In short, all animals that play are curious, and vice versa. Curiosity provides the basis for learning, for creativity and thus for every human activity that leaves a legacy, such as art or science. The sculptor and art theoretician Joseph Beuys had as his own guiding principle that *every* creative
Ref. 296 act is a form of art. Humans, and especially children, enjoy curiosity because they feel its importance for creativity, and for growth in general.

Curiosity regularly leads one to exclaim: 'Oh!', an experience that leads to the second reason to be curious: relishing feelings of wonder and surprise. Epicurus (Epikuros) (b. 341 Samos, d. 271 bce Athens) maintained that this experience, θαυμάζειν, is the origin of philosophy. These feelings, which nowadays are variously called religious, spiritual, numinous, etc., are the same as those to which rats can become addicted. Among these feelings, Rudolf Otto has introduced the now classical distinction into the fascinating and the frightening. He named the corresponding experiences 'mysterium fascinans' and 'mysterium tremendum'.* Within these distinctions, physicists, scientists, children and connoisseurs take a clear stand: they choose the fascinans as the starting point for their actions and for their approach to the world. Such feelings of fascination induce some children who look at the night sky to dream about becoming astronomers, some

* This distinction is the basis of Rudolf Otto, *Das Heilige – Über das Irrationale in der Idee des Göttlichen und sein Verhältnis zum Rationalen*, Beck 1991. This is a new edition of the epoch-making work originally published at the beginning of the twentieth century. Rudolf Otto (b. 1869 Peine, d. 1937 Marburg) was one of the most important theologians of his time.

who look through a microscope to become biologists or physicists, and so on. (It could also be that genetics plays a role in this pleasure of novelty seeking.) Ref. 297

Perhaps the most beautiful moments in the study of physics are those appearing after new observations have shaken our previously held thinking habits, have forced us to give up a previously held conviction, and have engendered the feeling of being lost. When, in this moment of crisis, we finally discover a more adequate and precise description of the observations, which provide a better insight into the world, we are struck by a feeling usually called illumination. Anyone who has kept alive the memory and the taste for these magic moments knows that in these situations one is pervaded by a feeling of union between oneself and the world.* The pleasure of these moments, the adventures of the change of thought structures connected with them, and the joy of insight following them provides the drive for many scientists. Little talk and lots of pleasure is their common denominator. In this spirit, the important physicist Victor Weisskopf (b. 1908 Vienna, d. 2002 Newton) liked to say jokingly: 'There are two things that make life worth living: Mozart and quantum mechanics.'

The choice of moving away from the tremendum towards the fascinans stems from an innate desire, most obvious in children, to reduce uncertainty and fear. This drive is the father of all adventures. It has a well-known parallel in ancient Greece, where the first men studying observations, such as Epicurus, stated explicitly that their aim was to free people from unnecessary fear by deepening knowledge and transforming people from frightened passive victims into fascinated, active and responsible beings. Those ancient thinkers started to popularize the idea that, like the common events in our life, the rarer events also follow rules. For example, Epicurus underlined that lightning is a natural phenomenon caused by interactions between clouds, and stressed that it was a natural process, i.e., a process that followed rules, in the same way as the falling of a stone or any other familiar process of everyday life.

Investigating the phenomena around them, philosophers and later scientists succeeded in freeing people from most of their fears caused by uncertainty and a lack of knowledge about nature. This liberation played an important role in the history of human culture and still pervades in the personal history of many scientists. The aim to arrive at stable, rock-bottom truths has inspired (but also hindered) many of them; Albert Einstein is a well-known example for this, discovering relativity, helping to start up but then denying quantum mechanics.

Interestingly, in the experience and in the development of every human being, curiosity, and therefore the sciences, appears *before* magic and superstition. Magic needs deceit to be effective, and superstition needs indoctrination; curiosity doesn't need either. Conflicts of curiosity with superstitions, ideologies, authorities or the rest of society are thus preprogrammed.

Curiosity is the exploration of limits. For every limit, there are two possibilities: the limit can turn out to be real or apparent. If the limit is real, the most productive attitude is that of acceptance. Approaching the limit then gives strength. If the limit is only apparent

* Several researchers have studied the situations leading to these magic moments in more detail, notably the physician and physicist Hermann von Helmholtz (b. 1821 Potsdam, d. 1894 Charlottenburg) and the mathematician Henri Poincaré (b. 1854 Nancy, d. 1912 Paris). They distinguish four stages in the conception of an idea at the basis of such a magic moment: saturation, incubation, illumination and verification. Ref. 298

and in fact non-existent, the most productive attitude is to re-evaluate the mistaken view, extract the positive role it performed, and then cross the limit. Distinguishing between real and apparent limits is only possible when the limit is investigated with great care, openness and unintentionality. Most of all, exploring limits need courage.

> Das gelüftete Geheimnis rächt sich.*
>
> Bert Hellinger

Courage

> Il est dangereux d'avoir raison dans des choses où des hommes accrédités ont tort.**
>
> Voltaire

> Manche suchen Sicherheit, wo Mut gefragt ist, und suchen Freiheit, wo das Richtige keine Wahl läßt.***
>
> Bert Hellinger

Most of the material in this chapter is necessary to complete our adventure. But we need
Ref. 301 more. Like any enterprise, curiosity also requires courage, and complete curiosity, as aimed for in our quest, requires complete courage. In fact, it is easy to get discouraged on this journey. The quest is often dismissed by others as useless, uninteresting, childish, confusing, damaging, crazy or even evil and deserving punishment. For example, between the death of Socrates in 399 BCE and Paul-Henri Thiry, Baron d'Holbach, in 1770, no book was published with the statement 'gods do not exist', because of the threats to the life of anyone who dared to make the point. Even today, this type of attitude still abounds, as the newspapers show.

Curiosity and scientific activity are implicitly opposed to any idea, person or organization that tries to avoid the comparison of statements with observations. These 'avoiders' demand to live with superstitions and beliefs. But superstitions and beliefs produce unnecessary fear. And fear is the basis of all unjust authorities. One gets into a vicious circle: avoiding comparison with observation produces fear – fear keeps unjust authority in place – unjust authority avoids comparison with observation – etc.

Curiosity and science are fundamentally opposed to unjust authority, a connection that made life difficult for people such as Anaxagoras in ancient Greece, Hypatia in the christian Roman Empire, Galileo Galilei in the former Papal States, Antoine Lavoisier in revolutionary France and Albert Einstein (and many others) in Nazi Germany. In the second half of the twentieth century, victims were Robert Oppenheimer, Melba Phillips and Chandler Davis in the United States, and Andrei Sakharov in the Soviet Union. Each of them tell a horrible but instructive story, as have, more recently, Fang Lizhi, Xu Liangying, Liu Gang and Wang Juntao in China, Kim Song-Man in South Corea, Otanazar Aripov in Uzbekistan, Ramadan al-Hadi al-Hush in Libya, Bo Bo Htun in Burma, Sami Kilani and Salman Salman in Palestine, Abdus Salam in Pakistan, as well as many hundreds of others. In many authoritarian societies the antagonism between curiosity and

* 'The unveiled secret takes revenge.'

Ref. 299 ** 'It is dangerous to be right in matters where established men are wrong.'

*** 'Some look for security where courage is required and look for freedom where the right way doesn't
Ref. 300 leave any choice.'

injustice has hindered or even completely suppressed the development of physics, other natural sciences and engineering, with extremely negative economic, social and cultural consequences.

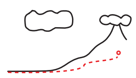
When embarking on the adventure to understand motion, we need to be conscious of what we are doing. In fact, we can avoid external obstacles or at least largely reduce them by keeping the project to ourselves. Other difficulties still remain, this time of personal nature. Curiosity often leads us to face painful discoveries. Many have tried to embark on the adventure of motion with some hidden or explicit intention, usually of an ideological nature, and then have got entangled before reaching the end. Some have not been prepared to accept the humility required for such an endeavour. Others were not prepared for the openness and honesty required, which can shatter deeply held beliefs. Still others were not ready to turn towards the unclear, the dark and the unknown, confronting them at every occasion.

On the other hand, the dangers of curiosity are worth it. By taking curiosity as a maxim, facing disinformation and fear with all our courage, we achieve freedom from all beliefs. In exchange, we come to savour some among the fullest pleasures and the deepest satisfaction that life has to offer.

In summary: we continue our hike. At this point, the trail towards the top of Motion Mountain is leading us towards the next adventure: discovering the origin of sizes, shapes and colours in nature.

> “And the gods said to man: ‘Take what you want, and pay the price.’ ”
>
> Popular saying

> “It is difficult to make a man miserable while he feels he is worthy of himself. ”
>
> Abraham Lincoln



Chapter 10

CLASSICAL PHYSICS IN A NUTSHELL

Classical electrodynamics, with mechanics, thermodynamics and relativity, completes our walk through classical physics. In the structure of physics, classical physics encompasses four of the eight points that make up all of physics, the science of motion. As a whole, classical physics describes the motion of everyday Page 8 bodies, the motion of heat, the motion of extremely fast objects, the motion of empty space, and the motion of light and electric charge. By completing classical physics, we have covered the first half of our adventure. Let us summarize what we have found out about motion so far – and what we did not.

What can move?

In nature, four entities can move: *objects*, *radiation*, *space-time* and *horizons*. In all cases, their motion happens in such a way as to minimize change. Change is also called (physical) action. In short, all motion in nature minimizes action.

In all cases of motion, we distinguish the permanent or *intrinsic properties* from the varying *state*. We learned to distinguish and to characterize the possible intrinsic properties and the possible states of each moving entity.

About *objects*, we found that in everyday life, all sufficiently small objects or particles are described completely by their *mass* and their electric *charge*. There is no magnetic charge. Mass and electric charge are thus the only localized intrinsic properties of classical, everyday objects. Both mass and electric charge are defined by the accelerations they produce around them. Both quantities are conserved; thus they can be added (with certain precautions). Mass, in contrast to charge, is always positive, i.e., always attractive. Mass describes the interaction of objects in collisions and in gravitation, charge the interaction with electromagnetic fields.

All *varying* aspects of objects, i.e., their *state*, can be described using *momentum* and *position*, as well as *angular momentum* and *orientation*. These four quantities can vary continuously in amount and direction. Therefore the set of all possible states forms a space, the so-called *phase space*. The state of extended, shape-changing objects is given by the states of all its constituent particles. These particles make up all objects by interacting electromagnetically.

The Lagrangian determines the action, or total change, of any kind of motion. Action, or change, is independent of the observer; the state is not. The states found by different observers are related: the relations are called the 'laws' or properties of motion. For different times they are called *evolution equations*, for different places and orientations they are called *transformation relations*, and for different gauges they are called *gauge transformations*. Motion of each everyday objects is fully described by the principle of least action: motion minimizes action.

Radiation also moves. Everyday types of radiation, such as light, radio waves and their related forms, are travelling electromagnetic waves. They are described by same equations that describe the interaction of charged or magnetic objects. Electromagnetic fields have no mass; their speed in vacuum is the maximum possible energy speed in nature and is the same for all observers. The motion of radiation describes the motion of images. The *intrinsic properties* of radiation are its dispersion relation and its energy–angular momentum relation. The *state* of radiation is described by its electromagnetic field strength, its phase, its polarization and its coupling to matter. The motion of electromagnetic fields and waves minimizes action and change.

Space-time is also able to move, by changing its curvature. The state of space-time is given by the metric, which describes distances and curvature, and thus the local warpedness. The warpedness can oscillate and propagate, so that empty space can move like a wave. Also the motion of space-time minimizes change. The principle of least action is valid. The intrinsic properties of space-time are the number of dimensions, its metric signature and its topology. Experiments show that space-time has 3+1 dimensions, its metric signature is $+ + +-$ and the topology of space-time is simple.

Horizons can be seen as limit cases of either space-time or matter-radiation. They share the same intrinsic and state properties. The dark night sky, the boundary of the universe, is the most important example of a horizon. Other examples are the boundaries of black holes. The universe, both its space-time and its matter content, shows maximum age and distance values. The history of the universe is long, about three times as long as the history of the Earth. On large scales, all matter in the universe moves away from all other matter: the universe, and its horizon, is expanding.

Properties of classical motion

Around us, we observe motion for objects, radiation, space-time and horizons. In our exploration of classical physics, we distilled six specific properties of all classical – or everyday – motion.

1. Everyday motion is *continuous*. Continuous motion allows defining space and time. All energy moves in the way space-time dictates it, and space moves the way energy dictates it. This relation describes the motion of the stars, of thrown stones, of light beams and of the tides. Rest and free fall are the same, and gravity is curved space-time. Mass breaks conformal symmetry and thus distinguishes space from time.

   The continuity of motion is limited: The (local) speed of energy, mass and charge is bound from above by a universal constant c, and (local) energy change per time is bound from above by a universal constant $c^5/4G$. The speed value c is realized for the motion of massless particles. It also relates space to time. The power value $c^5/4G$ is realized by horizons. Horizons are found around black holes and at the border of the universe. The maximum power value also relates space-time curvature to energy flow and thus describes the elasticity of space-time.

   The continuity of motion is limited in a second way: No two objects can be at the same spot at the same time. This is the first statement that humans encounter about electromagnetism. The statement is due to the repulsion of charges of the same sign found in matter. More detailed investigation shows that electric charges accelerate other charges, that charges are necessary to define length and time intervals, and that charges are the source of electromagnetic fields. Also light is such a field. Light travels at the maximum possible velocity c. In contrast to objects, light and electromagnetic fields can interpenetrate.

2. Everyday motion *conserves* mass, electric charge, energy, linear momentum and angular momentum. For these quantities, *nothing appears out of nothing.* Conservation applies to all kinds of motion: to linear motion, to rotational motion, and to motion of matter, radiation, space-time and horizons. Energy and momentum are similar to continuous substances: they are never destroyed, never created, but always redistributed. Not even heat, growth, transformations, biological evolution or friction are exceptions to conservation.
3. Everyday motion is *relative*: motion depends on the observer. Not even the firm floor below our feet contradicts relativity.
4. Everyday motion is *reversible*: everyday motion can occur backwards. Not even friction, the breaking of objects or death are exceptions to reversibility.
5. Everyday motion is *mirror-invariant*: everyday motion can occur in a mirror-reversed way. In short, we found that the classical motion of objects, radiation and space-time is right–left symmetric. Human-made objects, such as writing, are no exceptions to mirror-invariance.
6. Everyday motion is *lazy*: motion happens in a way that minimizes change, i.e., physical action. In Galilean physics and electrodynamics, action is the time average of the difference between kinetic and potential energy. In general relativity, action takes into account the curvature and elasticity of space-time. The principle of least action – or cosmic laziness – hold for all cases.

In short, our exploration of classical physics showed us:

▷ Motion is *lazy*: it is *predictable* and *limited*.

In other terms, nature follows *patterns* and *rules*. Motion is deterministic. There are *no* surprises in nature. Nature *cannot* do whatever it likes to do.

We will discover later that some rare examples of non-everyday motion violate reversibility and mirror-invariance in a subtle way. The subtle violations disappear if the terms are properly extended in their meaning. Also mass conservation is violated separately, but becomes, in relativity, part of energy conservation. In short, the general statements about motion, suitably corrected, remain valid across all of nature.

Above all, we saw that *motion minimizes action*. Also this deep result remains valid throughout our adventure. In other terms, the universe has no freedom to determine what occurs inside it.

Challenge 333 e

After completing the classical parts of this adventure, you might think that you know classical physics well. If you do so, read the excellent collection by Friedrich Herrmann, *Historical Burdens on Physics*, available for free download, at www.physikdidaktik.uni-karlsruhe.de/index_en.html. If the topics presented there –

all simple to understand – are clear to you – even if you disagree – you have become a real expert on classical physics.

The future of planet Earth

Maybe nature shows no surprises, but it still provides many adventures. On the 2nd of March 2009, a small asteroid 'almost' hit the Earth. It passed at a distance of only 63 500 km from our planet. On impact, it would have destroyed a region the size of London. Such events occur regularly.* Several other adventures can be predicted by classical physics; they are listed in Table 25. Several items are problems facing humanity in the distant future, but some, such as volcanic eruptions or asteroid impacts, could happen at any time. All are research topics. Ref. 302

TABLE 25 Examples of disastrous motion of possible future importance.

Critical situation	Years from now
Giant tsunami from volcanic eruption at Canary islands	*c.* 10-200
End of fundamental physics, with a definite proof that nature is simple	*c.* 20 (around year 2030)
Major nuclear material accident or weapon use	unknown
Explosion of volcano in Greenland, Italy or elsewhere, leading to long darkening of sky	unknown
Explosion of Yellowstone or other giant volcano leading to year-long volcanic winter	0 to 100 000
Earth's mantle instability leading to massive volcanic activity	unknown
Mini ice age due to collapse of gulf stream	unknown
Ozone shield reduction	*c.* 100
Rising ocean levels due to greenhouse warming	> 100
Several magnetic north and south poles appear, allowing solar storms to disturb radio and telecommunications, to interrupt electricity supplies, to increase animal mutations and to disorient migrating animals such as wales, birds and tortoises	*c.* 800
Our interstellar gas cloud detaches from the solar system, changing the size of the heliosphere, and thus expose us more to aurorae and solar magnetic fields	*c.* 3 000
Reversal of Earth's magnetic field, implying a time with almost no magnetic field, with increased cosmic radiation levels and thus more skin cancers and miscarriages	unknown
Atmospheric oxygen depletion due to forest reduction and exaggerated fuel consumption	> 1000
Upcoming ice age	*c.* 15 000
Possible collision with interstellar gas cloud assumed to be crossed by the Earth every 60 million years, maybe causing mass extinctions	*c.* 50 000

* The web pages around www.minorplanetcenter.net/iau/lists/Closest.html provide more information on such close encounters.

TABLE 25 (Continued) Examples of disastrous motion of possible future importance.

Critical situation	Years from now
Possible genetic degeneration of homo sapiens due to Y chromosome reduction	*c.* 200 000
Africa collides with Europe, transforming the Mediterranean into a lake that starts evaporating	around $3 \cdot 10^6$
Gamma-ray burst from within our own galaxy, causing radiation damage to many living beings	between 0 and $5 \cdot 10^6$
Asteroid hitting the Earth, generating tsunamis, storms, darkening sunlight, etc.	between 0 and $50 \cdot 10^6$
Neighbouring star approaching, starting comet shower through destabilization of Oort cloud and thus risk for life on Earth	$> 10^6$
American continent collides with Asia	$> 100 \cdot 10^6$
Molecular cloud engulfs the solar system	unknown
Instability of solar system	$> 100 \cdot 10^6$
Low atmospheric CO_2 content stops photosynthesis	$> 100 \cdot 10^6$
Collision of Milky Way with star cluster or other galaxy	$> 150 \cdot 10^6$
Sun ages and gets hotter, evaporating seas	$> 250 \cdot 10^6$
Ocean level increase due to Earth rotation slowing/stopping (if not evaporated before)	$> 10^9$
Temperature rise/fall (depending on location) due to Earth rotation stop	$> 10^9$
Sun runs out of fuel, becomes red giant, engulfs Earth	$5.0 \cdot 10^9$
Sun stops burning, becomes white dwarf	$5.2 \cdot 10^9$
Earth core solidifies, removing magnetic field and thus Earth's cosmic radiation shield	$10.0 \cdot 10^9$
Nearby nova (e.g. Betelgeuse) bathes Earth in annihilation radiation	unknown
Nearby supernova (e.g. Eta Carinae) blasts over solar system	unknown
Galaxy centre destabilizes rest of galaxy	unknown
Universe recollapses – if ever (see page 136, volume II)	$> 20 \cdot 10^9$
Matter decays into radiation – if ever (see Appendix B in vol. V)	$> 10^{33}$
Problems with naked singularities	only in science fiction
Vacuum becomes unstable	only in science fiction
End of applied physics	never

Despite the fascination of the predictions – all made in the year 2000 – we leave aside these literally tremendous issues and continue on our adventure.

> “I'm an old man and I've known many troubles. Most of them never happened.”
>
> Anonymous wisdom

The essence of classical physics – the infinitely small and the lack of surprises

In the first three parts of our walk, on classical physics, we found that motion minimizes change. Every type of motion around us confirms that nature is lazy. The 'laziness' of the classical description of nature is based on an important statement.

▷ Classical physics is the description of motion using the concept of the *infinitely small.*

All concepts used so far, be they for space, time or other observables, assume that the infinitely small exists. Special relativity, despite the speed limit, still allows infinitely small velocities; general relativity, despite its black hole limits, still allows infinitely small force and power values. Similarly, in the description of electrodynamics and gravitation, both integrals and derivatives are abbreviations of mathematical processes that use and assume infinitely small distances and time intervals. In other words, the classical description of nature introduces and is based on the infinitely small in the description of motion.

Using the infinitely small as a research tool, the classical description of motion discovers that energy, momentum, angular momentum and electric charge are *conserved*. They are conserved also for infinitely small dimensions or time intervals. The detailed exploration of conservation at infinitely small scale has led us to a strong conclusion:

▷ Motion has no surprises. Motion is deterministic, predictable and limited. Motion has no choices and no options.

Experiments confirm all these properties. Experiments thus imply

▷ Nature provides no miracles.

In this statement, a 'miracle' is a term used for a process *against* the rules of nature. Some people argue that infinity is the necessary ingredient needed to perform miracles. Classical physics shows the opposite: *the existence of the infinitely small prevents miracles.*

Laziness, conservation and the lack of surprises also imply that motion and nature are not described by concepts such as 'punishment' or 'reward' or 'fairness'. This is also the case for disasters, catastrophes, luck or happy occurrences. Laziness, conservation and the lack of surprises also imply that motion and nature are *not designed* and have *no aim*. Various people claim the opposite; they are mistaken.

Classical physics implies the absence of surprises. As reassuring as this result may be, it leaves us with a doubt. Both special and general relativity have eliminated the existence of the infinitely large. There is no infinitely large force, power, size, age or speed. Why should the infinitely small exist, but the infinitely large not? And if the infinitely small is also eliminated, can miracles occur again? In fact, there are still more open questions about motion.

Summary: Why have we not yet reached the top of the mountain?

> The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote... Our future discoveries must be looked for in the sixth place of decimals.
>
> Albert Michelson, 1894.*

We might think that we know nature now, as did Albert Michelson at the end of the nineteenth century. He claimed that electrodynamics and Galilean physics implied that the major laws of physics were well known. The statement is often quoted as an example of flawed predictions, since it reflects an incredible mental closure to the world around him. Not only did Michelson overlook the need for understanding the darkness of the sky and general relativity.

Michelson – in contrast to many physicists of his time – had also overlooked three contradictions between electrodynamics and nature for which he had no excuse. First of all, we found above that clocks and metre bars are necessarily made of matter and necessarily based on electromagnetism. Page 250 But as we saw, classical electrodynamics does not explain the stability and properties of matter and atoms. Matter is made of small particles, but the relation between these particles, electricity and the smallest charges is not clear.

▷ We do not understand matter.

If we do not understand matter, we do not yet fully understand space and time, because we defined space and time using measurement devices made of matter.

Secondly, Michelson knew that the origin of not a single colour observed in nature is described by classical electrodynamics.

▷ We do not understand colour.

Classical electrodynamics can only explain colour differences and colour changes, but it cannot describe absolute colour values.

Worse, Michelson overlooked a third limitation of the classical description of nature:

▷ We do not understand life.

The abilities of living beings – growing, seeing, hearing, feeling, thinking, being healthy or sick, reproducing and dying – are all unexplained by classical physics. In fact, all these abilities *contradict* classical physics. Nevertheless, Michelson received the Nobel Prize in Physics many years after his speech.

* From his address at the dedication ceremony for the Ryerson Physical Laboratory at the University of Chicago. Michelson (b. 1852 Strelno, d. 1931 Pasadena) was an important and influential physicist; he was awarded the Nobel Prize in Physics in 1907 for his experiments on the invariance of the speed of light.

At the end of the nineteenth century, the progress in technology due to the use of electricity, chemistry and vacuum technology allowed better and better machines and apparatuses to be built. All were built with classical physics in mind. In the years between 1890 and 1920, these classical machines completely destroyed the foundations of classical physics. Experiments with these apparatuses showed that matter is made of atoms of finite and constant size, that electrical charge comes in smallest amounts, that there is a smallest entropy value, a smallest angular momentum value and a smallest action value in nature, and that both matter particles and light behave randomly.

In short, precise experiments show that in nature, the existence of the infinitely small is wrong in many cases: many observables come in *quanta*. Like an old empire, classical physics collapsed. Classical physics does *not* describe nature correctly at small scales. *Quantum physics* is needed.

In summary, understanding light, matter and its interactions, including life itself, is the aim of the upcoming parts of our adventure. And to understand life we need to understand the size, shape, colour and material properties of all things – including atoms. This understanding takes place at small scales. More specifically, in order to understand matter, colour and life, we need to study *particles*. A lot is still left to explore. And this exploration will lead us from wonder to wonder.

Appendix A

UNITS, MEASUREMENTS AND CONSTANTS

Measurements are comparisons with standards. Standards are based on *units*. Many different systems of units have been used throughout the world. Most of these standards confer power to the organization in charge of them. Such power can be misused; this is the case today, for example in the computer industry, and was so in the distant past. The solution is the same in both cases: organize an independent and global standard. For measurement units, this happened in the eighteenth century: in order to avoid misuse by authoritarian institutions, to eliminate problems with differing, changing and irreproducible standards, and – this is not a joke – to simplify tax collection and to make it more just, a group of scientists, politicians and economists agreed on a set of units. It is called the *Système International d'Unités*, abbreviated *SI*, and is defined by an international treaty, the 'Convention du Mètre'. The units are maintained by an international organization, the 'Conférence Générale des Poids et Mesures', and its daughter organizations, the 'Commission Internationale des Poids et Mesures' and the 'Bureau International des Poids et Mesures' (BIPM). All originated in the times just before the French revolution. Ref. 303

SI units

All SI units are built from seven *base units*. Their simplest definitions, translated from French into English, are the following ones, together with the dates of their formulation and a few comments:

- 'The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.' (1967) The 2019 definition is equivalent, but much less clear.*
- 'The *metre* is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.' (1983) The 2019 definition is equivalent, but much less clear.*
- 'The *kilogram*, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,15 \cdot 10^{-34}$ when expressed in the unit $J \cdot s$, which is equal to $kg \cdot m^2 \cdot s^{-1}$.' (2019)*
- 'The *ampere*, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,634 \cdot 10^{-19}$ when expressed in the unit C, which is equal to $A \cdot s$.' (2019)* This definition is equivalent to: One ampere is $6.241\,509\,074... \cdot 10^{18}$ elementary charges per second.
- 'The *kelvin*, symbol K, is the SI unit of thermodynamic temperature. It is defined

by taking the fixed numerical value of the Boltzmann constant k to be $1.380649 \cdot 10^{-23}$ when expressed in the unit $\mathrm{J \cdot K^{-1}}$.' (2019)*

▪ 'The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \cdot 10^{23}$ elementary entities.' (2019)*

▪ 'The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ hertz and has a radiant intensity in that direction of (1/683) watt per steradian.' (1979) The 2019 definition is equivalent, but much less clear.*

We note that both time and length units are defined as certain properties of a standard example of motion, namely light. In other words, also the Conférence Générale des Poids et Mesures makes the point that the observation of motion is a *prerequisite* for the definition and construction of time and space. *Motion is the fundament of every observation and of all measurement.* By the way, the use of light in the definitions had been proposed already in 1827 by Jacques Babinet.**

From these basic units, all other units are defined by multiplication and division. Thus, all SI units have the following properties:

▪ SI units form a system with *state-of-the-art precision*: all units are defined with a precision that is higher than the precision of commonly used measurements. Moreover, the precision of the definitions is regularly being improved. The present relative uncertainty of the definition of the second is around 10^{-14}, for the metre about 10^{-10}, for the kilogram about 10^{-9}, for the ampere 10^{-7}, for the mole less than 10^{-6}, for the kelvin 10^{-6} and for the candela 10^{-3}.

▪ SI units form an *absolute* system: all units are defined in such a way that they can be reproduced in every suitably equipped laboratory, independently, and with high precision. This avoids as much as possible any error or misuse by the standard-setting organization. In fact, the SI units are as now as near as possible to Planck's natural units, which are presented below. In practice, the SI is now an international standard defining the numerical values of the seven constants $\Delta\nu_{\mathrm{Cs}}$, c, $\hbar$, e, k, N_{A} and K_{cd}. After over 200 years of discussions, the CGPM has little left to do.

▪ SI units form a *practical* system: the base units are quantities of everyday magnitude. Frequently used units have standard names and abbreviations. The complete list includes the seven base units just given, the supplementary units, the derived units and the admitted units.

The *supplementary* SI units are two: the unit for (plane) angle, defined as the ratio of arc length to radius, is the *radian* (rad). For solid angle, defined as the ratio of the subtended area to the square of the radius, the unit is the *steradian* (sr).

The *derived* units with special names, in their official English spelling, i.e., without capital letters and accents, are:

---

* The symbols of the seven units are s, m, kg, A, K, mol and cd. The full offical definitions are found at www.bipm.org. For more details about the levels of the caesium atom, consult a book on atomic physics. The Celsius scale of temperature θ is defined as: $\theta/°\mathrm{C} = T/\mathrm{K} - 273.15$; note the small difference with the number appearing in the definition of the kelvin. In the definition of the candela, the frequency of the light corresponds to 555.5 nm, i.e., green colour, around the wavelength to which the eye is most sensitive.

Ref. 304

** Jacques Babinet (1794–1874), French physicist who published important work in optics.

Name	Abbreviation
hertz	Hz = 1/s
pascal	Pa = N/m^2 = kg/m s^2
watt	W = kg m^2/s^3
volt	V = kg m^2/As^3
ohm	Ω = V/A = kg m^2/A^{2}s^3
weber	Wb = Vs = kg m^2/As^2
henry	H = Vs/A = kg m^2/A^{2}s^2
lumen	lm = cd sr
becquerel	Bq = 1/s
sievert	Sv = J/kg = m^2/s^2

Name	Abbreviation
newton	N = kg m/s^2
joule	J = Nm = kg m^2/s^2
coulomb	C = As
farad	F = As/V = A^{2}s^4/kg m^2
siemens	S = 1/Ω
tesla	T = Wb/m^2 = kg/As^2 = kg/Cs
degree Celsius	°C (see definition of kelvin)
lux	lx = lm/m^2 = cd sr/m^2
gray	Gy = J/kg = m^2/s^2
katal	kat = mol/s

We note that in all definitions of units, the kilogram only appears to the powers of 1, 0 and −1. Can you try to formulate the reason? Challenge 334 s

The *admitted* non-SI units are *minute*, *hour*, *day* (for time), *degree* $1° = \pi/180$ rad, *minute* $1' = \pi/10\,800$ rad, *second* $1'' = \pi/648\,000$ rad (for angles), *litre*, and *tonne*. All other units are to be avoided.

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called *prefixes*:*

Power	Name	
10^1	deca	da
10^2	hecto	h
10^3	kilo	k
10^6	Mega	M
10^9	Giga	G
10^{12}	Tera	T
10^{15}	Peta	P

Power	Name	
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Power	Name	
10^{18}	Exa	E
10^{21}	Zetta	Z
10^{24}	Yotta	Y
unofficial:		
10^{27}	Xenta	X
10^{30}	Wekta	W
10^{33}	Vendekta	V
10^{36}	Udekta	U

Power	Name	
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y
Ref. 305		
10^{-27}	xenno	x
10^{-30}	weko	w
10^{-33}	vendeko	v
10^{-36}	udeko	u

▪ SI units form a *complete* system: they cover in a systematic way the full set of observables of physics. Moreover, they fix the units of measurement for all other sciences as well.

* Some of these names are invented (yocto to sound similar to Latin *octo* 'eight', zepto to sound similar to Latin *septem*, yotta and zetta to resemble them, exa and peta to sound like the Greek words ἑξάκις and πεντάκις for 'six times' and 'five times', the unofficial ones to sound similar to the Greek words for nine, ten, eleven and twelve); some are from Danish/Norwegian (atto from *atten* 'eighteen', femto from *femten* 'fifteen'); some are from Latin (from *mille* 'thousand', from *centum* 'hundred', from *decem* 'ten', from *nanus* 'dwarf'); some are from Italian (from *piccolo* 'small'); some are Greek (micro is from μικρός 'small', deca/deka from δέκα 'ten', hecto from ἑκατόν 'hundred', kilo from χίλιοι 'thousand', mega from μέγας 'large', giga from γίγας 'giant', tera from τέρας 'monster').

Translate: I was caught in such a traffic jam that I needed a microcentury for a picoparsec and that my car's fuel consumption was two tenths of a square millimetre. Challenge 335 e

▪ SI units form a *universal* system: they can be used in trade, in industry, in commerce, at home, in education and in research. They could even be used by extraterrestrial civilizations, if they existed.

▪ SI units form a *self-consistent* system: the product or quotient of two SI units is also an SI unit. This means that in principle, the same abbreviation, e.g. 'SI', could be used for every unit.

The SI units are not the only possible set that could fulfil all these requirements, but they are the only existing system that does so.*

The meaning of measurement

Challenge 336 e

Every measurement is a comparison with a standard. Therefore, any measurement requires *matter* to realize the standard (even for a speed standard), and *radiation* to achieve the comparison. The concept of measurement thus assumes that matter and radiation exist and can be clearly separated from each other.

Every measurement is a comparison. Measuring thus implies that space and time exist, and that they differ from each other.

Every measurement produces a measurement result. Therefore, every measurement implies the *storage* of the result. The process of measurement thus implies that the situation before and after the measurement can be distinguished. In other terms, every measurement is an *irreversible* process.

Every measurement is a process. Thus every measurement takes a certain amount of time and a certain amount of space.

All these properties of measurements are simple but important. Beware of anybody who denies them.

Precision and accuracy of measurements

Measurements are the basis of physics. Every measurement has an *error*. Errors are due to lack of precision or to lack of accuracy. *Precision* means how well a result is reproduced when the measurement is repeated; *accuracy* is the degree to which a measurement corresponds to the actual value.

Lack of precision is due to accidental or *random errors*; they are best measured by the *standard deviation*, usually abbreviated σ; it is defined through

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 , \tag{110}$$

* Apart from international units, there are also *provincial* units. Most provincial units still in use are of Roman origin. The mile comes from *milia passum*, which used to be one thousand (double) strides of about 1480 mm each; today a nautical mile, once defined as minute of arc on the Earth's surface, is defined as exactly 1852 m. The inch comes from *uncia/onzia* (a twelfth – now of a foot). The pound (from *pondere* 'to weigh') is used as a translation of *libra* – balance – which is the origin of its abbreviation lb. Even the habit of counting in dozens instead of tens is Roman in origin. These and all other similarly funny units – like the system in which all units start with 'f', and which uses furlong/fortnight as its unit of velocity – are now officially defined as multiples of SI units.

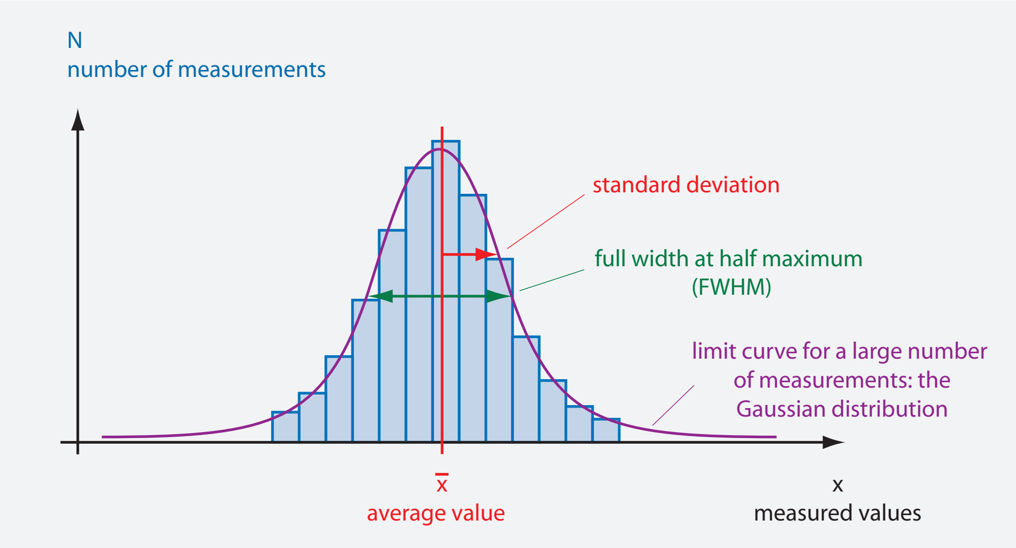


FIGURE 185 A precision experiment and its measurement distribution. The precision is high if the width of the distribution is narrow; the accuracy is high if the centre of the distribution agrees with the actual value.

where $\bar{x}$ is the average of the measurements x_i. (Can you imagine why $n-1$ is used in the formula instead of n?) Challenge 337 s

For most experiments, the distribution of measurement values tends towards a normal distribution, also called *Gaussian distribution*, whenever the number of measurements is increased. The distribution, shown in Figure 185, is described by the expression

$$N(x) \approx \mathrm{e}^{-\frac{(x-\bar{x})^2}{2\sigma^2}} . \tag{111}$$

The square σ^2 of the standard deviation is also called the *variance*. For a Gaussian distribution of measurement values, 2.35σ is the full width at half maximum. Challenge 338 e

Lack of accuracy is due to *systematic errors*; usually these can only be estimated. This estimate is often added to the random errors to produce a *total experimental error*, sometimes also called *total uncertainty*. The *relative* error or uncertainty is the ratio between the error and the measured value. Ref. 306

For example, a professional measurement will give a result such as 0.312(6) m. The number between the parentheses is the standard deviation σ, in units of the last digits. As above, a Gaussian distribution for the measurement results is assumed. Therefore, a value of 0.312(6) m implies that the actual value is expected to lie Challenge 339 e

— within 1σ with 68.3 % probability, thus in this example within 0.312 ± 0.006 m;
— within 2σ with 95.4 % probability, thus in this example within 0.312 ± 0.012 m;
— within 3σ with 99.73 % probability, thus in this example within 0.312 ± 0.018 m;
— within 4σ with 99.9937 % probability, thus in this example within 0.312 ± 0.024 m;
— within 5σ with 99.999 943 % probability, thus in this example within 0.312 ± 0.030 m;
— within 6σ with 99.999 999 80 % probability, thus within 0.312 ± 0.036 m;

— within 7σ with 99.999 999 999 74 % probability, thus within 0.312 ± 0.041 m.

Challenge 340 s (Do the latter numbers make sense?)

Note that standard deviations have one digit; you must be a world expert to use two, and a fool to use more. If no standard deviation is given, a (1) is assumed. As a result, among professionals, 1 km and 1000 m are *not* the same length!

What happens to the errors when two measured values A and B are added or subtracted? If the all measurements are independent – or uncorrelated – the standard deviation of the sum *and* that of difference is given by $\sigma = \sqrt{\sigma_A^2 + \sigma_B^2}$. For both the product or ratio of two measured and uncorrelated values C and D, the result is $\rho = \sqrt{\rho_C^2 + \rho_D^2}$, where the ρ terms are the *relative* standard deviations.

Challenge 341 s Assume you measure that an object moves 1 m in 3 s: what is the measured speed value?

Limits to precision

What are the limits to accuracy and precision? There is no way, even in principle, to measure a length x to a *precision* higher than about 61 digits, because in nature, the ratio between the largest and the smallest measurable length is $\Delta x/x > l_{\mathrm{Pl}}/d_{\mathrm{horizon}} = 10^{-61}$.

Challenge 342 e (Is this ratio valid also for force or for volume?) In the final volume of our text, studies

Vol. VI, page 94 of clocks and metre bars strengthen this theoretical limit.

But it is not difficult to deduce more stringent practical limits. No imaginable machine can measure quantities with a higher precision than measuring the diameter of the Earth within the smallest length ever measured, about 10^{-19} m; that is about 26 digits of precision. Using a more realistic limit of a 1000 m sized machine implies a limit of 22 digits. If, as predicted above, time measurements really achieve 17 digits of precision, then they are nearing the practical limit, because apart from size, there is an additional practical restriction: cost. Indeed, an additional digit in measurement precision often means an additional digit in equipment cost.

Physical constants

In physics, general observations are deduced from more fundamental ones. As a consequence, many measurements can be deduced from more fundamental ones. The most fundamental measurements are those of the physical constants.

The following tables give the world's best values of the most important physical constants and particle properties – in SI units and in a few other common units – as published in the standard references. The values are the world averages of the best measurements made up to the present. As usual, experimental errors, including both random and estimated systematic errors, are expressed by giving the standard deviation in the last digits. In fact, behind each of the numbers in the following tables there is a long story which is worth telling, but for which there is not enough room here.

Ref. 307, Ref. 308

In principle, *all* quantitative properties of matter can be calculated with quantum theory – more precisely, equations of the standard model of particle – and a set of *basic* physical constants that are given in the next table. For example, the colour, density and elastic properties of any material can be predicted, in principle, in this way.

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TABLE 27 Basic physical constants.

Quantity	Symbol	Value in SI units	Uncert.[a]
Constants that define the SI measurement units			
Vacuum speed of light[c]	c	299 792 458 m/s	0
Original Planck constant[c]	h	$6.626\,070\,15 \cdot 10^{-34}$ Js	0
Reduced Planck constant, quantum of action	$\hbar$	$1.054\,571\,817 \ldots \cdot 10^{-34}$ Js	0
Positron charge[c]	e	0.160 217 6634 aC	0
Boltzmann constant[c]	k	$1.380\,649 \cdot 10^{-23}$ J/K	0
Avogadro's number	N_{A}	$6.022\,140\,76 \cdot 10^{23}$ 1/mol	0
Constant that *should* define the SI measurement units			
Gravitational constant	G	$6.674\,30(15) \cdot 10^{-11}\ \mathrm{Nm^2/kg^2}$	$2.2 \cdot 10^{-5}$
Other fundamental constants			
Number of space-time dimensions		3 + 1	0[b]
Fine-structure constant[d] or	$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$	1/137.035 999 084(21)	$1.5 \cdot 10^{-10}$
e.m. coupling constant	$= g_{\mathrm{em}}(m_{\mathrm{e}}^2 c^2)$	= 0.007 297 352 5693(11)	$1.5 \cdot 10^{-10}$
Fermi coupling constant[d] or	$G_{\mathrm{F}}/(\hbar c)^3$	$1.166\,3787(6) \cdot 10^{-5}\ \mathrm{GeV^{-2}}$	$5.1 \cdot 10^{-7}$
weak coupling constant	$\alpha_{\mathrm{w}}(M_{\mathrm{Z}}) = g_{\mathrm{w}}^2/4\pi$	1/30.1(3)	$1 \cdot 10^{-2}$
Strong coupling constant[d]	$\alpha_{\mathrm{s}}(M_{\mathrm{Z}}) = g_{\mathrm{s}}^2/4\pi$	0.1179(10)	$8.5 \cdot 10^{-3}$
Weak mixing angle	$\sin^2 \theta_{\mathrm{W}}(\overline{MS})$	0.231 22(4)	$1.7 \cdot 10^{-4}$
	$\sin^2 \theta_{\mathrm{W}}$ (on shell) $= 1 - (m_{\mathrm{W}}/m_{\mathrm{Z}})^2$	0.222 90(30)	$1.3 \cdot 10^{-3}$
CKM quark mixing matrix	$\lvert V \rvert$	$\begin{pmatrix} 0.97383(24) & 0.2272(10) & 0.00396(9) \\ 0.2271(10) & 0.97296(24) & 0.04221(80) \\ 0.00814(64) & 0.04161(78) & 0.999100(34) \end{pmatrix}$	
Jarlskog invariant	J	$3.08(18) \cdot 10^{-5}$	
PMNS neutrino mixing m.	$\lvert P \rvert$	$\begin{pmatrix} 0.82(2) & 0.55(4) & 0.150(7) \\ 0.37(13) & 0.57(11) & 0.71(7) \\ 0.41(13) & 0.59(10) & 0.69(7) \end{pmatrix}$	
Electron mass	m_{e}	$9.109\,383\,7015(28) \cdot 10^{-31}$ kg	$3.0 \cdot 10^{-10}$
		$5.485\,799\,090\,65(16) \cdot 10^{-4}$ u	$2.9 \cdot 10^{-11}$
		0.510 998 950 00(15) MeV	$3.0 \cdot 10^{-10}$
Muon mass	m_{μ}	$1.883\,531\,627(42) \cdot 10^{-28}$ kg	$2.2 \cdot 10^{-8}$
		105.658 3755(23) MeV	$2.2 \cdot 10^{-8}$
Tau mass	m_{τ}	$1.776\,82(12)\ \mathrm{GeV}/c^2$	$6.8 \cdot 10^{-5}$
El. neutrino mass	$m_{\nu_{\mathrm{e}}}$	$< 2\ \mathrm{eV}/c^2$	
Muon neutrino mass	$m_{\nu_{\mu}}$	$< 2\ \mathrm{eV}/c^2$	
Tau neutrino mass	$m_{\nu_{\tau}}$	$< 2\ \mathrm{eV}/c^2$	
Up quark mass	u	$21.6(+0.49/-0.26)\ \mathrm{MeV}/c^2$	
Down quark mass	d	$4.67(+0.48/-0.17)\ \mathrm{MeV}/c^2$	

TABLE 27 (Continued) Basic physical constants.

Quantity	Symbol	Value in SI units	Uncert.[a]
Strange quark mass	s	$93(+11/-5)\,\mathrm{MeV}/c^2$	
Charm quark mass	c	$1.27(2)\,\mathrm{GeV}/c^2$	
Bottom quark mass	b	$4.18(3)\,\mathrm{GeV}/c^2$	
Top quark mass	t	$172.9(0.4)\,\mathrm{GeV}/c^2$	
Photon mass	γ	$< 2 \cdot 10^{-54}\,\mathrm{kg}$	
W boson mass	$W^{\pm}$	$80.379(12)\,\mathrm{GeV}/c^2$	
Z boson mass	Z^0	$1\,91.1876(21)\,\mathrm{GeV}/c^2$	
Higgs mass	H	$125.10(14)\,\mathrm{GeV}/c^2$	
Gluon mass	$g_{1...8}$	$c.\ 0\,\mathrm{MeV}/c^2$	

a. Uncertainty: standard deviation of measurement errors.
b. Measured from to 10^{-19} m to 10^{26} m.
c. Defining constant.
d. All coupling constants depend on the 4-momentum transfer, as explained in the section on renormalization. Page 131 *Fine-structure constant* is the traditional name for the electromagnetic coupling constant g_{em} in the case of a 4-momentum transfer of $Q^2 = m_e^2 c^2$, which is the smallest one possible. At higher momentum transfers it has larger values, e.g., $g_{\mathrm{em}}(Q^2 = M_W^2 c^2) \approx 1/128$. In contrast, the strong coupling constant has lover values at higher momentum transfers; e.g., $\alpha_s(34\,\mathrm{GeV}) = 0.14(2)$.

Why do all these basic constants have the values they have? For any basic constant *with a dimension*, such as the quantum of action $\hbar$, the numerical value has only historical meaning. It is $1.054 \cdot 10^{-34}$ Js because of the SI definition of the joule and the second. The question why the value of a *dimensional* constant is not larger or smaller therefore always requires one to understand the origin of some *dimensionless* number giving the ratio between the constant and the corresponding *natural unit* that is defined with c, G, k, N_A and $\hbar$. Details and values for the natural units are given in the dedicated section. Vol. IV, page 208

In other words, understanding the sizes of atoms, people, trees and stars, the duration of molecular and atomic processes, or the mass of nuclei and mountains, implies understanding the ratios between these values and the corresponding natural units. The key to understanding nature is thus the understanding of all measurement ratios, and thus of all dimensionless constants. This quest, including the understanding of the fine-structure constant α itself, is completed only in the final volume of our adventure.

The basic constants yield the following useful high-precision observations.

TABLE 28 Derived physical constants.

Quantity	Symbol	Value in SI units	Uncert.
Vacuum permeability	μ_0	$1.256\,637\,062\,12(19)\,\mu\mathrm{H/m}$	$1.5 \cdot 10^{-10}$
Vacuum permittivity	$\varepsilon_0 = 1/\mu_0 c^2$	$8.854\,187\,8128(13)\,\mathrm{pF/m}$	$1.5 \cdot 10^{-10}$
Vacuum impedance	$Z_0 = \sqrt{\mu_0/\varepsilon_0}$	$376.730\,313\,668(57)\,\Omega$	$1.5 \cdot 10^{-10}$
Loschmidt's number	N_L	$2.686\,780\,111... \cdot 10^{25}\,1/\mathrm{m}^3$	0
at 273.15 K and 101 325 Pa			

TABLE 28 (Continued) Derived physical constants.

Quantity	Symbol	Value in SI units	Uncert.
Faraday's constant	$F = N_{A}e$	96 485.332 12... C/mol	0
Universal gas constant	$R = N_{A}k$	8.314 462 618... J/(mol K)	0
Molar volume of an ideal gas at 273.15 K and 101 325 Pa	$V = RT/p$	22.413 969 54... l/mol	0
Rydberg constant [a]	$R_{\infty} = m_{e}c\alpha^{2}/2h$	10 973 731.568 160(21) m^{-1}	$1.9 \cdot 10^{-12}$
Conductance quantum	$G_{0} = 2e^{2}/h$	77.480 917 29... µS	0
Magnetic flux quantum	$\varphi_{0} = h/2e$	2.067 833 848... fWb	0
Josephson frequency ratio	$2e/h$	483.597 8484... THz/V	0
Von Klitzing constant	$h/e^{2} = \mu_{0}c/2\alpha$	25 812.807 45... Ω	0
Bohr magneton	$\mu_{B} = e\hbar/2m_{e}$	9.274 010 0783(28) yJ/T	$3.0 \cdot 10^{-10}$
Classical electron radius	$r_{e} = e^{2}/4\pi\varepsilon_{0}m_{e}c^{2}$	2.817 940 3262(13) fm	$4.5 \cdot 10^{-10}$
Compton wavelength	$\lambda_{C} = h/m_{e}c$	2.426 310 238 67(73) pm	$3.0 \cdot 10^{-10}$
of the electron	$\lambda\!\!\!^{-}_{c} = \hbar/m_{e}c = r_{e}/\alpha$	0.386 159 267 96(12) pm	$3.0 \cdot 10^{-10}$
Bohr radius [a]	$a_{\infty} = r_{e}/\alpha^{2}$	52.917 721 0903(80) pm	$1.5 \cdot 10^{-10}$
Quantum of circulation	$h/2m_{e}$	3.636 947 5516(11) cm^{2}/s	$3.0 \cdot 10^{-10}$
Specific positron charge	e/m_{e}	175.882 001 076(55) GC/kg	$3.0 \cdot 10^{-10}$
Cyclotron frequency of the electron	$f_{c}/B = e/2\pi m_{e}$	27.992 489 872(9) GHz/T	$3.0 \cdot 10^{-10}$
Electron magnetic moment	μ_{e}	−9.284 764 7043(28) yJ/T	$3.0 \cdot 10^{-10}$
	μ_{e}/μ_{B}	−1.001 159 652 181 28(18)	$1.7 \cdot 10^{-13}$
	μ_{e}/μ_{N}	$-1\,838.281\,971\,88(11) \cdot 10^{3}$	$6.0 \cdot 10^{-11}$
Electron g-factor	g_{e}	−2.002 319 304 362 56(35)	$1.7 \cdot 10^{-13}$
Muon–electron mass ratio	m_{μ}/m_{e}	206.768 2830(46)	$2.2 \cdot 10^{-8}$
Muon magnetic moment	μ_{μ}	$-4.490\,448\,30(10) \cdot 10^{-26}$ J/T	$2.2 \cdot 10^{-8}$
Muon g-factor	g_{μ}	−2.002 331 8418(13)	$6.3 \cdot 10^{-10}$
Atomic mass unit	$1\,u = m_{^{12}C}/12$	$1.660\,539\,066\,60(50) \cdot 10^{-27}$ kg	$3.0 \cdot 10^{-10}$
Proton mass	m_{p}	$1.672\,621\,923\,69(51) \cdot 10^{-27}$ kg	$3.1 \cdot 10^{-10}$
		1.007 276 466 621(53) u	$5.3 \cdot 10^{-11}$
		938.272 088 16(29) MeV	$3.1 \cdot 10^{-10}$
Proton–electron mass ratio	m_{p}/m_{e}	1 836.152 673 43(11)	$6.0 \cdot 10^{-11}$
Specific proton charge	e/m_{p}	$9.578\,833\,1560(29) \cdot 10^{7}$ C/kg	$3.1 \cdot 10^{-10}$
Proton Compton wavelength	$\lambda_{C,p} = h/m_{p}c$	1.321 409 855 39(40) fm	$3.1 \cdot 10^{-10}$
Nuclear magneton	$\mu_{N} = e\hbar/2m_{p}$	$5.050\,783\,7461(15) \cdot 10^{-27}$ J/T	$3.1 \cdot 10^{-10}$
Proton magnetic moment	μ_{p}	$1.410\,606\,797\,36(60) \cdot 10^{-26}$ J/T	$4.2 \cdot 10^{-10}$
	μ_{p}/μ_{B}	$1.521\,032\,202\,30(46) \cdot 10^{-3}$	$3.0 \cdot 10^{-10}$
	μ_{p}/μ_{N}	2.792 847 344 63(82)	$2.9 \cdot 10^{-10}$
Proton gyromagnetic ratio	$\gamma_{p} = 2\mu_{p}/h$	42.577 478 518(18) MHz/T	$4.2 \cdot 10^{-10}$
Proton g factor	g_{p}	5.585 694 6893(16)	$2.9 \cdot 10^{-10}$
Neutron mass	m_{n}	$1.674\,927\,498\,04(95) \cdot 10^{-27}$ kg	$5.7 \cdot 10^{-10}$

TABLE 28 (Continued) Derived physical constants.

Quantity	Symbol	Value in SI units	Uncert.
		1.008 664 915 95(43) u	$4.8 \cdot 10^{-10}$
		939.565 420 52(54) MeV	$5.7 \cdot 10^{-10}$
Neutron–electron mass ratio	m_n/m_e	1 838.683 661 73(89)	$4.8 \cdot 10^{-10}$
Neutron–proton mass ratio	m_n/m_p	1.001 378 419 31(49)	$4.9 \cdot 10^{-10}$
Neutron Compton wavelength	$\lambda_{C,n} = h/m_n c$	1.319 590 905 81(75) fm	$5.7 \cdot 10^{-10}$
Neutron magnetic moment	μ_n	$-0.966\,236\,51(23) \cdot 10^{-26}$ J/T	$2.4 \cdot 10^{-7}$
	μ_n/μ_B	$-1.041\,875\,63(25) \cdot 10^{-3}$	$2.4 \cdot 10^{-7}$
	μ_n/μ_N	−1.913 042 73(45)	$2.4 \cdot 10^{-7}$
Stefan–Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	56.703 744 19... nW/m^{2}K^4	0
Wien's displacement constant	$b = \lambda_{max} T$	2.897 771 955... mmK	0
		58.789 257 57... GHz/K	0
Electron volt	eV	0.160 217 6634... aJ	0
Bits to entropy conversion const.	$k \ln 2$	10^{23} bit = 0.956 994... J/K	0
TNT energy content		3.7 to 4.0 MJ/kg	$4 \cdot 10^{-2}$

a. For infinite mass of the nucleus.

Some useful properties of our local environment are given in the following table.

TABLE 29 Astronomical constants.

Quantity	Symbol	Value
Tropical year 1900 [a]	a	31 556 925.974 7 s
Tropical year 1994	a	31 556 925.2 s
Mean sidereal day	d	$23^h 56' 4.090\,53''$
Average distance Earth–Sun [b]		149 597 870.691(30) km
Astronomical unit [b]	AU	149 597 870 691 m
Light year, based on Julian year [b]	al	9.460 730 472 5808 Pm
Parsec	pc	30.856 775 806 Pm = 3.261 634 al
Earth's mass	$M_♁$	$5.973(1) \cdot 10^{24}$ kg
Geocentric gravitational constant	GM	$3.986\,004\,418(8) \cdot 10^{14}$ m^3/s^2
Earth's gravitational length	$l_♁ = 2GM/c^2$	8.870 056 078(16) mm
Earth's equatorial radius [c]	$R_{♁eq}$	6378.1366(1) km
Earth's polar radius [c]	$R_{♁p}$	6356.752(1) km
Equator–pole distance [c]		10 001.966 km (average)
Earth's flattening [c]	$e_♁$	1/298.25642(1)
Earth's av. density	$\rho_♁$	5.5 Mg/m^3
Earth's age	$T_♁$	4.50(4) Ga = 142(2) Ps
Earth's normal gravity	g	9.806 65 m/s^2
Earth's standard atmospher. pressure	p_0	101 325 Pa

TABLE 29 (Continued) Astronomical constants.

Quantity	Symbol	Value
Moon's radius	$R_{☾\text{v}}$	1738 km in direction of Earth
Moon's radius	$R_{☾\text{h}}$	1737.4 km in other two directions
Moon's mass	$M_☾$	$7.35 \cdot 10^{22}$ kg
Moon's mean distance [d]	$d_☾$	384 401 km
Moon's distance at perigee [d]		typically 363 Mm, historical minimum 359 861 km
Moon's distance at apogee [d]		typically 404 Mm, historical maximum 406 720 km
Moon's angular size [e]		average 0.5181° = 31.08′, minimum 0.49°, maximum 0.55°
Moon's average density	$\rho_☾$	3.3 Mg/m^3
Moon's surface gravity	$g_☾$	1.62 m/s^2
Moon's atmospheric pressure	$p_☾$	from 10^{-10} Pa (night) to 10^{-7} Pa (day)
Jupiter's mass	$M_♃$	$1.90 \cdot 10^{27}$ kg
Jupiter's radius, equatorial	$R_♃$	71.398 Mm
Jupiter's radius, polar	$R_♃$	67.1(1) Mm
Jupiter's average distance from Sun	$D_♃$	778 412 020 km
Jupiter's surface gravity	$g_♃$	24.9 m/s^2
Jupiter's atmospheric pressure	$p_♃$	from 20 kPa to 200 kPa
Sun's mass	$M_\odot$	$1.988\,43(3) \cdot 10^{30}$ kg
Sun's gravitational length	$2GM_\odot/c^2$	2.953 250 08(5) km
Heliocentric gravitational constant	$GM_\odot$	$132.712\,440\,018(8) \cdot 10^{18}$ m^3/s^2
Sun's luminosity	$L_\odot$	384.6 YW
Solar equatorial radius	$R_\odot$	695.98(7) Mm
Sun's angular size		0.53° average; minimum on fourth of July (aphelion) 1888″, maximum on fourth of January (perihelion) 1952″
Sun's average density	$\rho_\odot$	1.4 Mg/m^3
Sun's average distance	AU	149 597 870.691(30) km
Sun's age	$T_\odot$	4.6 Ga
Solar velocity around centre of galaxy	$v_{\odot\text{g}}$	220(20) km/s
Solar velocity against cosmic background	$v_{\odot\text{b}}$	370.6(5) km/s
Sun's surface gravity	$g_\odot$	274 m/s^2
Sun's lower photospheric pressure	$p_\odot$	15 kPa
Distance to Milky Way's centre		8.0(5) kpc = 26.1(1.6) kal
Milky Way's age		13.6 Ga
Milky Way's size		*c.* 10^{21} m or 100 kal
Milky Way's mass		10^{12} solar masses, *c.* $2 \cdot 10^{42}$ kg

TABLE 29 (Continued) Astronomical constants.

Quantity	Symbol	Value
Most distant galaxy cluster known	SXDF-XCLJ 0218-0510	$9.6 \cdot 10^9$ al

a. Defining constant, from vernal equinox to vernal equinox; it was once used to define the second. (Remember: π seconds is about a nanocentury.) The value for 1990 is about 0.7 s less, corresponding to a slowdown of roughly 0.2 ms/a. (Watch out: why?) There is even an empirical formula for the change of the length of the year over time.
Challenge 343 s
Ref. 309

b. The truly amazing precision in the average distance Earth–Sun of only 30 m results from time averages of signals sent from Viking orbiters and Mars landers taken over a period of over twenty years. Note that the International Astronomical Union distinguishes the average distance Earth–Sun from the *astronomical unit* itself; the latter is defined as a fixed and exact length. Also the *light year* is a unit defined as an exact number by the IAU. For more details, see www.iau.org/public/measuring.

c. The shape of the Earth is described most precisely with the World Geodetic System. The last edition dates from 1984. For an extensive presentation of its background and its details, see the www.wgs84.com website. The International Geodesic Union refined the data in 2000. The radii and the flattening given here are those for the 'mean tide system'. They differ from those of the 'zero tide system' and other systems by about 0.7 m. The details constitute a science in itself.

d. Measured centre to centre. To find the precise position of the Moon at a given date, see the www.fourmilab.ch/earthview/moon_ap_per.html page. For the planets, see the page www.fourmilab.ch/solar/solar.html and the other pages on the same site.

e. Angles are defined as follows: 1 degree = $1^{\circ} = \pi/180$ rad, 1 (first) minute = $1' = 1°/60$, 1 second (minute) = $1'' = 1'/60$. The ancient units 'third minute' and 'fourth minute', each 1/60th of the preceding, are not in use any more. ('Minute' originally means 'very small', as it still does in modern English.)

Some properties of nature at large are listed in the following table. (If you want a challenge, can you determine whether any property of the universe itself is listed?)
Challenge 344 s

TABLE 30 Cosmological constants.

Quantity	Symbol	Value
Cosmological constant	Λ	*c.* $1 \cdot 10^{-52}\ \mathrm{m}^{-2}$
Age of the universe [a]	t_0	$4.333(53) \cdot 10^{17}$ s = $13.8(0.1) \cdot 10^9$ a
(determined from space-time, via expansion, using general relativity)		
Age of the universe [a]	t_0	over $3.5(4) \cdot 10^{17}$ s = $11.5(1.5) \cdot 10^9$ a
(determined from matter, via galaxies and stars, using quantum theory)		
Hubble parameter [a]	H_0	$2.3(2) \cdot 10^{-18}\ \mathrm{s}^{-1} = 0.73(4) \cdot 10^{-10}\ \mathrm{a}^{-1}$
	$= h_0 \cdot 100\ \mathrm{km/s\,Mpc} = h_0 \cdot 1.0227 \cdot 10^{-10}\ \mathrm{a}^{-1}$	
Reduced Hubble parameter [a]	h_0	0.71(4)
Deceleration parameter [a]	$q_0 = -(\ddot{a}/a)_0/H_0^2$	−0.66(10)
Universe's horizon distance [a]	$d_0 = 3ct_0$	$40.0(6) \cdot 10^{26}$ m = 13.0(2) Gpc
Universe's topology		trivial up to 10^{26} m

TABLE 30 (Continued) Cosmological constants.

Quantity	Symbol	Value
Number of space dimensions		3, for distances up to 10^{26} m
Critical density	$\rho_c = 3H_0^2/8\pi G$	$h_0^2 \cdot 1.878\,82(24) \cdot 10^{-26}\,\mathrm{kg/m^3}$
of the universe		$= 0.95(12) \cdot 10^{-26}\,\mathrm{kg/m^3}$
(Total) density parameter [a]	$\Omega_0 = \rho_0/\rho_c$	1.02(2)
Baryon density parameter [a]	$\Omega_{B0} = \rho_{B0}/\rho_c$	0.044(4)
Cold dark matter density parameter [a]	$\Omega_{CDM0} = \rho_{CDM0}/\rho_c$	0.23(4)
Neutrino density parameter [a]	$\Omega_{\nu 0} = \rho_{\nu 0}/\rho_c$	0.001 to 0.05
Dark energy density parameter [a]	$\Omega_{X0} = \rho_{X0}/\rho_c$	0.73(4)
Dark energy state parameter	$w = p_X/\rho_X$	−1.0(2)
Baryon mass	m_b	$1.67 \cdot 10^{-27}$ kg
Baryon number density		$0.25(1)\,/\mathrm{m^3}$
Luminous matter density		$3.8(2) \cdot 10^{-28}\,\mathrm{kg/m^3}$
Stars in the universe	n_s	$10^{22\pm1}$
Baryons in the universe	n_b	$10^{81\pm1}$
Microwave background temperature [b]	T_0	2.725(1) K
Photons in the universe	n_γ	10^{89}
Photon energy density	$\rho_\gamma = \pi^2 k^4/15T_0^4$	$4.6 \cdot 10^{-31}\,\mathrm{kg/m^3}$
Photon number density		$410.89\,/\mathrm{cm^3}$ or $400\,/\mathrm{cm^3}(T_0/2.7\,\mathrm{K})^3$
Density perturbation amplitude	$\sqrt{S}$	$5.6(1.5) \cdot 10^{-6}$
Gravity wave amplitude	$\sqrt{T}$	$< 0.71\sqrt{S}$
Mass fluctuations on 8 Mpc	σ_8	0.84(4)
Scalar index	n	0.93(3)
Running of scalar index	$\mathrm{d}n/\mathrm{d}\ln k$	−0.03(2)
Planck length	$l_{Pl} = \sqrt{\hbar G/c^3}$	$1.62 \cdot 10^{-35}$ m
Planck time	$t_{Pl} = \sqrt{\hbar G/c^5}$	$5.39 \cdot 10^{-44}$ s
Planck mass	$m_{Pl} = \sqrt{\hbar c/G}$	21.8 µg
Instants in history [a]	t_0/t_{Pl}	$8.7(2.8) \cdot 10^{60}$
Space-time points	$N_0 = (R_0/l_{Pl})^3 \cdot$	$10^{244\pm1}$
inside the horizon [a]	(t_0/t_{Pl})	
Mass inside horizon	M	$10^{54\pm1}$ kg

a. The index 0 indicates present-day values.
b. The radiation originated when the universe was 380 000 years old and had a temperature of about 3000 K; the fluctuations ΔT_0 which led to galaxy formation are today about $16 \pm 4\,\mu\mathrm{K} = 6(2) \cdot 10^{-6}\,T_0$.

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Useful numbers

Ref. 310

π	$3.14159\,26535\,89793\,23846\,26433\,83279\,50288\,41971\,69399\,37510_5$
e	$2.71828\,18284\,59045\,23536\,02874\,71352\,66249\,77572\,47093\,69995_9$
γ	$0.57721\,56649\,01532\,86060\,65120\,90082\,40243\,10421\,59335\,93992_3$
ln 2	$0.69314\,71805\,59945\,30941\,72321\,21458\,17656\,80755\,00134\,36025_5$
ln 10	$2.30258\,50929\,94045\,68401\,79914\,54684\,36420\,76011\,01488\,62877_2$
$\sqrt{10}$	$3.16227\,76601\,68379\,33199\,88935\,44432\,71853\,37195\,55139\,32521_6$

CHALLENGE HINTS AND SOLUTIONS

Challenge 1, page 10: Do not hesitate to be demanding and strict. The next edition of the text will benefit from it.

Challenge 3, page 17: The electric field distorts the flame towards and against the comb. A photograph of the effect is shown in Figure 187. A video of a similar effect in stronger fields is found at www.youtube.com/watch?v=a7_8Gc_Llr8.

Challenge 4, page 20: The water drops have to detach from the flow *inside* the metal counter-electrodes. There is always a tiny charge somewhere on the metal structures (due to cosmic rays, rubbing, previous charging, etc.). In Figure 186, this initial charge is the positive charge drawn on the lower left and upper right metal structure. When the water droplets form, they get a charge that is opposite to that of the metal surrounding the region where they form. The negative droplets fall into the other metal structure. Through the negative charge accumulating there, the positive charge in the first structure increases. When the charge on the metal structure increases, the charge separation in the droplets is more efficient. In other words, water droplet formation inside the metal structures *amplifies* any initial charge. After a while, the charge value and the associated voltage are so high that it leads to a loud bang (if everything is dry, including the air.) Then the process starts again. In fact, a vaguely similar charge separation mechanism leads to cloud charging and to lightning. If you want to build a Kelvin generator at home, have a look at the de.wikipedia.org/wiki/Kelvin-Generator web page, or search for other internet sites on the topic.

To avoid the sparks in the fuel tanks of its cars, Opel simply earthed the metal tube at the tank inlet; they had forgotten to ensure electric contact between the tube and the rest of the car.

The explosion of fuel can also occur if you pour fuel into your car from a metal container. Several times, fuel thieves were 'punished' by an explosion triggered by static electricity when they tried to pour stolen fuel into their own car. You can see on every airport how to avoid the problem: before even attaching the fuel tube to an aeroplane, the worker attaches a conducting cable to connect the truck (or the tank) to the aeroplane.

Challenge 5, page 21: We look at the two sparks through a rapidly rotating mirror. In this way, small timing differences lead to position differences of the two sparks. In the 19th century, the speed values measured in this way varied between 6000 km/s and way over 100 000 km/s, because the speed depends on the effective capacitance and inductance of wire and set-up. Only if these effects can be neglected is the measured speed the same as that of light in vacuum, namely around
Page 32 300 000 km/s. In modern cables, the speed is typically around a third of this value.

Challenge 6, page 22: A lot of noise appeared while the metal pendulum banged wildly between the two fixed bells.

Challenge 8, page 26: No.

Challenge 9, page 26: The field at a distance of 1 m from an electron is 1.4 nV/m.

Challenge 10, page 27: The inverse square law is a simple geometrical effect: anything flowing out homogeneously from a sphere diminishes with the square of the distance.

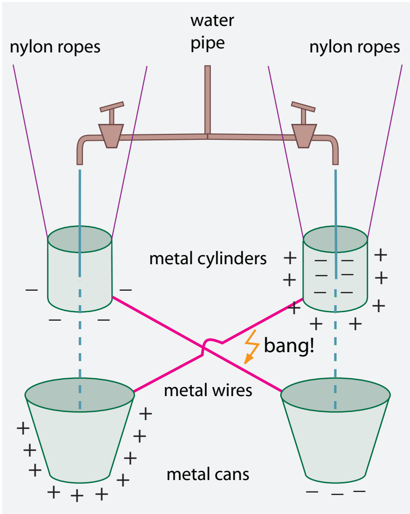

FIGURE 186 The key process in the Kelvin generator: charge separation during droplet formation.

FIGURE 187 The way a flame reacts to a rubbed comb (© Shubham Das and Rakesh Kumar).

Challenge 11, page 28: One gets $F = \alpha\hbar c N_A^2/4R^2 = 3 \cdot 10^{12}$ N, an enormous force, corresponding to the weight of 300 million tons. It shows the enormous forces that keep matter together. Obviously, there is no imaginable way to keep 1 g of positive charge together in a box, as the repulsive forces among the charges would be even larger.

Challenge 13, page 28: To show the full equivalence of Coulomb's and Gauss's 'laws', first show that it holds for a single point charge. Then expand the result for more than one point charge. That gives Gauss's 'law' in integral form, as given just before this challenge.

To deduce the integral form of Gauss's 'law' for a single point charge, we have to integrate over the closed surface. The essential point here is to note that the integration can be carried out for an inverse square dependence only. This dependence allows transforming the scalar product between the local field and the area element into a normal product between the charge and the solid angle Ω:

$$\boldsymbol{E}\,\mathrm{d}\boldsymbol{A} = \frac{q\mathrm{d}A\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{q\mathrm{d}\Omega}{4\pi\varepsilon_0} \,. \tag{112}$$

In case that the surface is closed the integration is then straightforward.

To deduce the differential form of (the static) Gauss's 'law', namely

$$\nabla \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \,, \tag{113}$$

we make use of the definition of the charge density ρ and of the purely mathematical relation

$$\oint_{\text{closed surface}} \boldsymbol{E}\,\mathrm{d}\boldsymbol{A} = \int_{\text{enclosed volume}} \nabla \boldsymbol{E}\,\mathrm{d}V \,. \tag{114}$$

This mathematical relation, valid for any vector field $\boldsymbol{E}$, is called *Gauss's theorem*. It simply states that the flux is the volume integral of the divergence.

To deduce the full form of Gauss's law, including the time-derivative of the magnetic field, we need to include relativistic effects by changing viewpoint to a moving observer.

Challenge 15, page 29: Uncharged bodies can attract each other if they are made of charged constituents neutralizing each other, and if the charges are constrained in their mobility. The charge fluctuations then lead to attraction. Most molecules interact among each other in this way; such forces are also at the basis of surface tension in liquids and thus of droplet formation.

Challenge 16, page 30: No; batteries only separate charges and pump them around.

Challenge 18, page 31: The ratio q/m of electrons and that of the free charges inside metals is not exactly the same.

Challenge 20, page 34: Find out a way to test the issue, perform the experiment, and publish it!

Challenge 21, page 41: If you can, publish the result. Researchers have tried to put people on the ocean during clouded days, have tried experiments in dark rooms, but nothing has been found so far. Also the experiences of people in magnetic resonance imaging equipment is inconclusive so far.

Challenge 23, page 47: No.

Challenge 25, page 47: The correct version of Ampère's 'law' is

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{j} \tag{115}$$

whereas the expression mentioned in the text misses the term $\frac{\partial \boldsymbol{E}}{\partial t}$.

For another way to state the difference between the correct and the wrong version of Ampère's 'law', see Richard P. Feynman, Robert B. Leighton & Matthew Sands, *The Feynman Lectures on Physics*, volume II, Addison Wesley, p. 21-1, 1977. They can be read online for free at www.feynmanlectures.info.

Challenge 26, page 48: Only boosts with relativistic speeds mix magnetic and electric fields to an appreciable amount.

Challenge 28, page 50: The dual field $*\boldsymbol{F}$ is defined on page 78.

Challenge 29, page 50: Scalar products of four vectors are always, by construction, Lorentz invariant quantities.

Challenge 30, page 51: X-rays production needs high concentration of energy; such levels are impossible in biological systems.

Challenge 31, page 51: Electric waves of low frequency are produced in nervous systems, and in brains in particular. As mentioned above, various fish communicate via time-varying electric
Ref. 16 dipole fields. But no communication via *radio* waves has ever been found. In fact, there is little hope that such systems exist. Why? (Hint: ponder the involved frequencies, their generation, and the physical properties of water and air.)

Challenge 34, page 54: Almost all neutral particles are made of charged ones. So the speed limit holds for them as well. There is only one exception: neutrinos. For them, this argument is not valid. However, even neutrinos have charged *virtual* particles around them, so that the maximum speed also applies to them.

Page 54 **Challenge 35**, page 55: As explained earlier on, for an observer who flies along the wire, the entrance and exit events for charges at the two ends events do not occur simultaneously any more; the wire is charged for a moving observer. Thus there is a magnetic field around a wire for *any* moving observer.

Challenge 36, page 57: The illumination of the sun changes the ionization in the upper atmosphere and provokes convection in the ionosphere. The tides move the ions in the ocean and in
Page 64 the atmosphere. These currents lead to magnetic fields which can be seen in sensitive compass needles.

Challenge 37, page 57: If you find such an effect and are able to demonstrate it, publish it in a didactic journal.

Challenge 38, page 57: Usually, the cables of high voltage lines are too warm to be comfortable.

Challenge 39, page 57: Move them to form a T shape.

Challenge 40, page 57: Hint: a shining bulb is hot.

Challenge 41, page 57: For three and more switches, one uses inverters; an inverter is a switch with two inputs and two outputs which in one position, connects first and second input to first and second output respectively, and in the other position connects the first input to the second output and vice versa. (There are other possibilities, though; wires can be saved using electromagnetic relay switches.)

Challenge 43, page 58: They behave differently: a full alkaline battery contains a gel that dampens the rebound. An empty battery has no gel and bounces.

Challenge 44, page 58: Blond children tend to have the thinnest hair, thus giving the greatest effect. Dry weather is needed to avoid that the moisture in the air discharges the head thus preventing the hair to raise at all.

Challenge 45, page 58: Wireless electrical power transport via waves is possible; however, the systems so far are usually large, expensive and are dangerous for human health.

The idea to collect solar power in deep space and then beam it to the Earth as microwaves has often been aired. Finances and dangers have blocked it so far.

The inductive systems that are found more and more in recent years do not use waves for transport, even though they are wireless. The development towards higher power transfers, such as needed for charging electric cars, will surely continue for many years to come.

Challenge 47, page 60: Glue two mirrors together at a right angle. Or watch yourself on TV using a video camera.

Challenge 48, page 60: This is again an example of combined triboluminescence and triboelectricity. See also the websites scienceworld.wolfram.com/physics/Triboluminescence.html and www.geocities.com/RainForest/9911/tribo.htm.

Challenge 51, page 63: Pepper is lighter than salt, and thus reacts to the spoon before the salt does.

Challenge 52, page 64: For a wavelength of 546.1 nm (standard green), that is a bit over 18 wavelengths.

Challenge 53, page 66: The angular size of the Sun is too large; diffraction plays no role here.

Challenge 54, page 66: Just use a high speed camera.

Challenge 55, page 67: The current flows perpendicularly to the magnetic field and is thus deflected. It pulls the whole magnet with it.

Challenge 56, page 67: The most simple equivalent to a coil is a rotating mass being put into rotation by the flowing water. A transformer would then be made of two such masses connected through their axis.

Challenge 57, page 67: Light makes seven turns of the Earth in one second.

Challenge 61, page 69: There are no permanent magnets in nature that fit in a floor and that are strong enough to achieve a floating height of 50 to 80 cm. (Note that in one image the floating height is so large that the legs of the woman do not touch the floor.) And anybody who has tried this with an electromagnet knows that such a device would be larger than a complete room.

Looking carefully at the images, you will also note that they are not photographs: there are errors with the shadow and with the reflected image of the woman. And most of all, nobody would cut half the bed out of an image with a woman on the bed. Finally, nobody has ever seen the floating bed shown in the images.

Challenge 63, page 70: The mathematics required to find the solution is fascinating. Explore it!

Challenge 64, page 71: The charged layer has the effect that almost only ions of one charge pass the channels. As a result, charges are separated on the two sides of the liquid, and a current is generated.

Challenge 65, page 71: The attraction at low distances is due to the 'image force', the attraction of a charge to any conducting surface. Measuring the distance d from the centre of the sphere, the repulsion of the point charge starts for values $d > 1.618R$.

Challenge 66, page 72: Leakage currents change the picture. The long term voltage ratio is given by the leakage resistance ratio $V_1/V_2 = R_1/R_2$, as can be easily verified in experiments.

Challenge 67, page 72: The wire parallel to the high voltage line forms a capacitor. The voltage difference that appears is sufficient to trigger the neon lamp.

Challenge 68, page 72: The water disrupts the small discharge sparks, called *aigrettes*. When a new one appears, it makes a small noise. Then, with the arrival of new water, they are disrupted again, and the process repeats. Aigrettes are a form of corona discharge; the also lead to power losses and to radio interference.

Challenge 69, page 72: See above, in the section on invariants.

Challenge 72, page 74: The model does not work in three dimensions. An attempt to correct this is F. De Flaviis, M. Noro & N. G. Alexopoulos, *Diaz-Fitzgerald time domain (D-FTD) method applied to dielectric and lossy materials*, preprint available online at www.researchgate.net.

Challenge 73, page 75: Search on the web, for example on the pages blog.biodiversitylibrary.org/2012/06/narwhal-oceans-one-toothed-wonder.html or narwhalslefttooth.blogspot.de/2011/05/narwhal-tusk-debate.html.

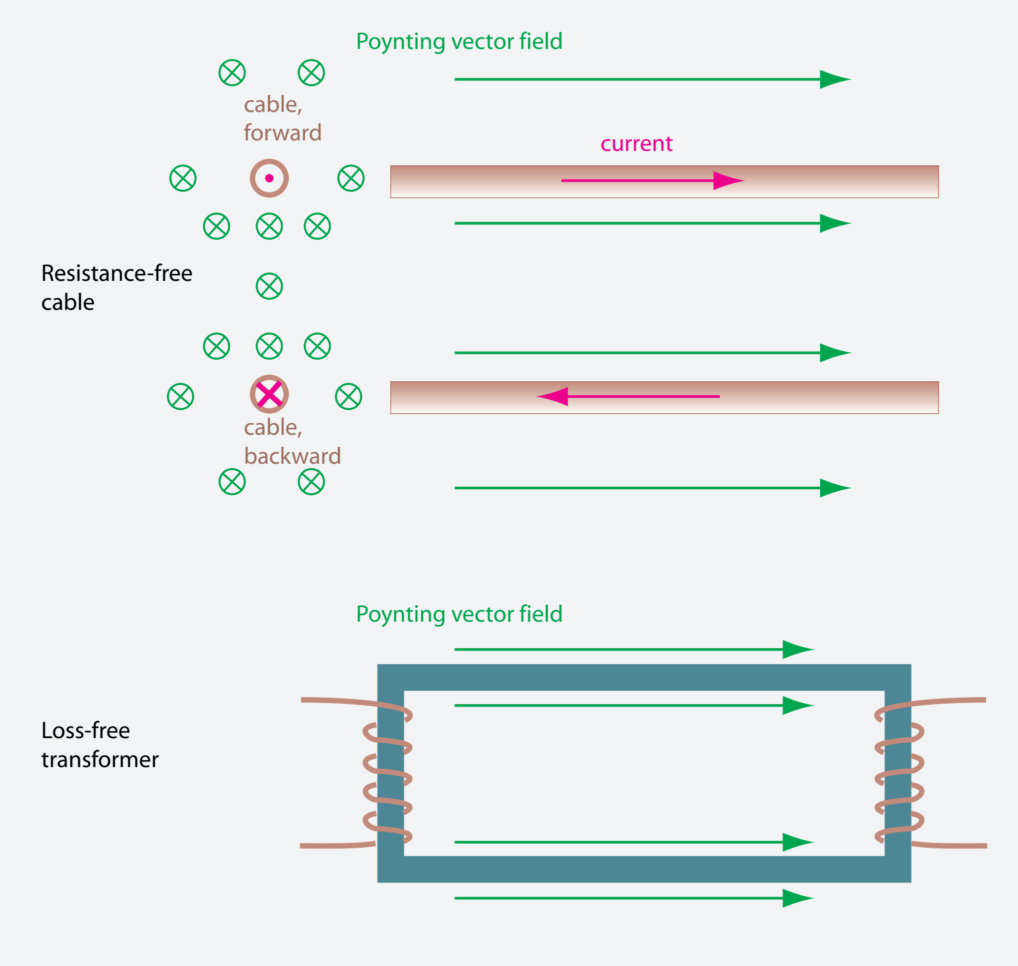

FIGURE 188 The Poynting vector field for a cable without electrical resistance and the situation a long transformer without losses.

Challenge 78, page 81: Some momentum, usually a very small part, is carried away by the electromagnetic field. Given that the electromagnetic momentum is given by the vector potential, are you able to check whether everything comes out right?

Challenge 79, page 82: Field lines and equipotential surfaces are always orthogonal to each other. Thus a field line cannot cross an equipotential surface twice.

Challenge 91, page 89: See Figure 188. If the cable is resistance-free, most of the energy flows just outside the two conductors and parallel to them. If the cable does have resistance, the Poynting vectors point slightly towards the conductors. For the case of a transformer, which can be deduced from the case of the cable via the analogy sketched in the picture, see the beautiful paper by F. Herrmann & G. B. Schmid, *The Poynting vector field and the energy flow within a transformer*, American Journal of Physics 54, pp. 528–531, 1986.

Challenge 90, page 89: The argument is the same as for the increase in entropy: reverse processes are possible, but the probability is so low that they do not appear in practice. The extremely low probability is due to the fluctuations induced by the environment.

Challenge 93, page 90: Just draw a current through a coil with its magnetic field, then draw the mirror image of the current and redraw the magnetic field.

Challenge 94, page 90: Other asymmetries in nature include the helicity of the DNA molecules making up the chromosomes and many other molecules in living systems, the right hand preference of most humans, the asymmetry of fish species which usually stay flat on the bottom of the seas.

Challenge 95, page 91: Explaining the difference of left and right is not possible at all using gravitational or electromagnetic systems or effects. The only way is to use the weak nuclear interaction, as shown in the chapter on the nucleus.

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Challenge 96, page 91: The Lagrangian does not change if one of the three coordinates is changed by its negative value.

Challenge 97, page 91: The image flips up: a 90 degree rotation turns the image by 180 degrees.

Challenge 98, page 92: Imagine $\boldsymbol{E}$ and $\boldsymbol{B}$ as the unit vectors of two axes in complex space. Then any rotation of these axes is also a generalized duality symmetry.

Challenge 100, page 95: The angular momentum was put into the system when it was formed. If we bring a point charge from infinity along a straight line to its final position close to a magnetic dipole, the magnetic force acting on the charge is not directed along the line of motion. It therefore creates a non-vanishing torque about the origin. See J. M. Aguirregabiria & A. Hernandez, *The Feynman paradox revisited*, European Journal of Physics 2, pp. 168–170, 1981.

Challenge 101, page 95: Show that even though the radial magnetic field of a spherical wave is vanishing by definition, Maxwell's equations would require it to be different from zero. Since electromagnetic waves are transversal, it is also sufficient to show that it is impossible to comb a hairy sphere without having a (double) vortex or two simple vortices. Despite these statements, quantum theory changes the picture somewhat: the emission probability of a photon from an excited atom in a degenerate state is spherically symmetric exactly.

Challenge 102, page 95: If the conservation of linear and angular momentum are taken into account, there is no ambiguity of the Poynting vector. See, for example, W. H. Furry, *Examples of momentum distributions in the electromagnetic field and in matter*, American Journal of Physics 37, pp. 621–636, 1969.

Challenge 103, page 96: The emitted radiation is strongly suppressed because the size of the dipole (the plug) is much smaller than the wavelength of the field.

Challenge 105, page 96: No. Neither electromagnetic motors nor coils have been found in any living system. Muscles, the most powerful actuators in biology, are mainly made of large numbers of electrostatic motors. The fundamental reason for this difference is the low efficiency of microscopic *electromagnetic* motors, which contrasts with the high efficiency of microscopic *electrostatic* motors. At macroscopic sizes, the efficiency advantages switches.

Challenge 107, page 102: In every case of interference, the energy is redistributed into other directions. This is the general rule; sometimes it is quite tricky to discover this other direction.

Challenge 108, page 102: The author regularly sees about 7 lines; assuming that the distance is around 20 μm, this makes about 3 μm per line. The wavelength must be smaller than this value and the frequency thus larger than 100 THz. The actual values for various colours are given in the table of the electromagnetic spectrum.

Challenge 110, page 104: The distance l between the lines of an interference pattern is given by $l = \lambda d/s$, where d is the distance to the screen, and s is the source separation.

To learn more about interference and the conditions for its appearance, explore the concept of Fresnel number. For example, the Fresnel number allows to distinguish the 'far field' from the

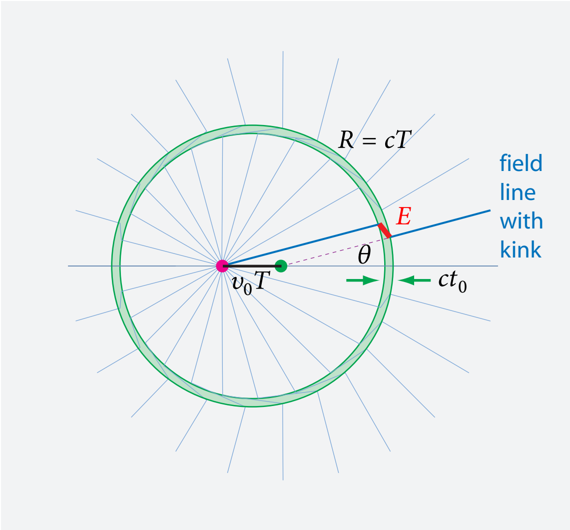


FIGURE 189 Calculating the transverse field E of a briefly accelerated charge.

'near field', two situations that occur in many wave phenomena.

Challenge 111, page 105: He noted that when a prism produces a rainbow, a thermometer placed in the region after the colour red shows a temperature rise.

Challenge 114, page 112: Birefringence appears when the refraction depends on polarization. Only two linear independent polarizations are possible, thus there is no trirefringence in nature. This holds true also for crystals which have three different indices of refraction in three directions!

Challenge 115, page 113: Light reflected form a water surface is partly polarized. Mirages are not.

Challenge 116, page 116: Figure 189 shows electrical field lines. We assume that the charge moves at a initial velocity v_0 that is small compared to c and that it decelerates to zero velocity during a time t_0. After a time T has elapsed, the radiation pulse has travelled a distance $R = cT$, where $T \gg t_0$. The figure shows that at a given kink, drawn in red, the ratio of the transverse field E_t and of the radial field E_r is given by the steepness of the of the kink. (Why?) Geometry then leads to

$$\frac{E_\mathrm{t}}{E_\mathrm{r}} = \frac{v_0 T \sin\theta}{c t_0} = \frac{aR\sin\theta}{c^2} \;. \tag{116}$$

Inserting Coulomb's expression for the radial field we get

$$E_\mathrm{t} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{a\sin\theta}{R} \;. \tag{117}$$

The magnitude of the transversal field thus decreases with $1/R$. In addition, the field depends on the angle θ; this is clearly visible both in Figure 189 and in Figure 69 on page 117. In other words, transmitter antennas have a preferred direction of power emission, namely perpendicularly to the direction of acceleration.

Challenge 117, page 119: Such an observer would experience a wavy but static field, which cannot exist, as the equations for the electromagnetic field show.

Challenge 118, page 119: You would never die. Could you reach the end of the universe?

Challenge 121, page 121: A surface of $1\,\mathrm{m}^2$ perpendicular to the light receives about 1 kW of radiation. It generates the same pressure as the weight of about 0.3 mg of matter. That generates 3 μPa for black surfaces, and the double for mirrors.

Challenge 123, page 122: The shine side gets twice the momentum transfer as the black side, and thus should be pushed backwards.

Challenge 126, page 124: Rotation of light can be understood in two ways: rotating the intensity pattern around the direction of propagation, or rotating the polarization pattern around the propagation. Both are possible: a Dove prism rotates intensity (and polarization) and a half-wave waveplate just rotates polarization, for a fixed wavelength. Both aspects can also be rotated with the mirror arrangements explained above. Page 139

Challenge 129, page 125: The interference patterns change when colours are changed. Rainbows also appear because different colours are due to different frequencies.

Challenge 132, page 126: Ternary and quaternary rainbows form a bow around the Sun. To see them, typically one has to be behind a building or tree that covers the direct view to the Sun. In 2011, there were only a handful of photographs of a ternary rainbow and only a single photograph of a quaternary rainbow, world-wide.

Challenge 133, page 126: The full rainbow is round like a circle. You can produce one with a garden hose, if you keep the hose in your hand while you stand on a chair, with your back to the evening Sun. (Well, one small part is missing; can you imagine which part?) The circle is due to the spherical shape of droplets. If the droplets were of different shape, *and* if they were all aligned, the rainbow would have a different shape than a simple circle.

Challenge 136, page 133: Take a film of a distant supernova explosion, or better, an optical or gamma-ray burst, and check whether the outburst occurs at the same time for each colour separately. This has been done extensively, and no differences have been detected within experimental errors.

Challenge 138, page 136: The first part of the forerunner is a feature with the shortest possible effective wavelength; thus it is given by taking the limit for infinite frequency.

Challenge 139, page 136: The light is pulsed; thus it is the energy velocity.

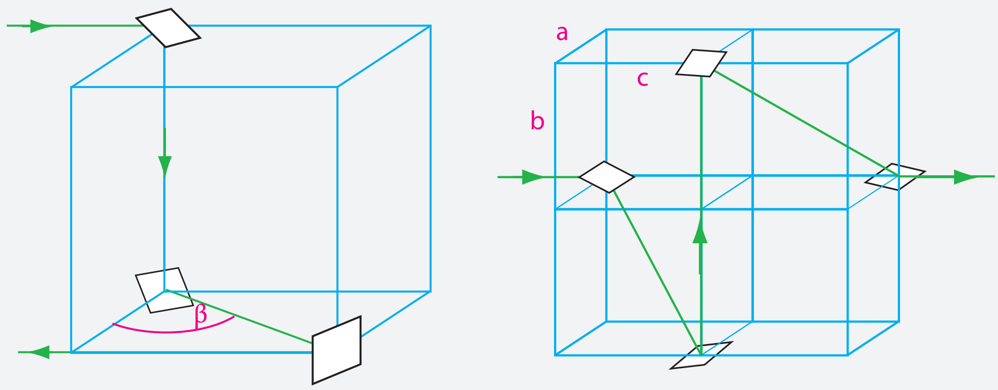
Challenge 140, page 136: Inside matter, the energy is transferred to atoms, then back to light, then to the next atoms, etc. That takes time and slows down the propagation.

Challenge 142, page 138: For single photons, permeability, permittivity and the wave impedance are not well-defined. Conformal invariance, dimensionality and topology are not are not valid at the tiny Planck scales. Near black holes, if quantum effects are taken into account, there is friction on moving bodies. Quantum field theory shows that vacuum contains and consists of virtual particle–antiparticle pairs. Cosmology shows that the vacuum has non-zero energy content, and the same is suggested by quantum field theory. General relativity shows that curved vacuum can move, and so does quantum gravity. In summary, one can say that vacuum has all the properties that were once ascribed to the aether, but in a way that differs fundamentally from what was discussed by its proponents.

Challenge 143, page 138: Almost no light passes; the intensity of the little light that is transmitted depends exponentially on the ratio between wavelength and hole diameter. One also says that after the hole there is an evanescent wave.

Challenge 144, page 138: The energy density is $1\,\mathrm{kW/m^2}/c = 3.3\,\mathrm{\mu J/m^3}$. Assuming sinusoidal waves, the (root mean square) electric field is $\sqrt{3.3\,\mathrm{\mu J/m^3}/\varepsilon_0} = 610\,\mathrm{V/m}$ – quite a high value. The (root mean square) magnetic field is $610\,\mathrm{V/m}/c = 2.1\,\mathrm{\mu T}$ – a rather low value.

Challenge 145, page 138: Any example of light has only one colour.



Input and output beams are antiparallel, β determines the polarization rotation.

Input and output beams are parallel, the length ratios a : b : c determine the polarization rotation.

FIGURE 190 Two mirror arrangements that rotate the polarization of a light beam by a predetermined angle.

Challenge 147, page 139: Describing light as a substance helps in understanding light beams. On the other hand, light is quite a special substance: it has no everyday permanence – it can be absorbed – and it has no mass.

Challenge 148, page 139: Too much light is wasted, the wind shields are too expensive, and there is no reason to do something if nobody else does.

Challenge 150, page 142: Three mirrors is the minimum. Two such mirror arrangements are shown in Figure 190. There is also a three-mirror arrangement with parallel input and output
Ref. 98 beams; can you find it? The ideas behind these arrangements are well explained in the papers by Enrique Galvez and his collaborators.

Challenge 151, page 143: In the left interferometer, light exits in direction B, in the right one, in direction A. The problem can also be generalized to arbitrary interferometer shapes. The way to solve it in this case is the use of *Berry's phase*. If you are interested, explore this interesting concept with the help of your favourite library.

Challenge 152, page 149: The average temperature of the Earth is thus 287 K. The energy from the Sun is proportional to the fourth power of the temperature. The energy is spread (roughly) over half the Earth's surface. The same energy, at the Sun's surface, comes from a much smaller surface, given by the same angle as the Earth subtends there. We thus have $E \sim 2\pi R^2_{\text{Earth}} T^4_{\text{Earth}} = T^4_{\text{Sun}} R^2_{\text{Earth}} \alpha^2$, where α is half the angle subtended by the Sun. As a result, the temperature of the Sun is estimated to be $T_{\text{Sun}} = (T^4_{\text{Earth}}/\alpha^2)^{0.25} = 4\,\text{kK}$.

Challenge 156, page 150: Because the maximum of a spectrum in wavelengths and in frequencies is not the same, thus does cannot and does *not* follow $c = f\lambda$.

Challenge 159, page 150: At high temperature, all bodies approach black bodies. The colour is more important than other colour effects. The oven and the objects have the same temperature. Thus they cannot be distinguished from each other. To do so nevertheless, illuminate the scene with powerful light and then take a picture with small sensitivity. Thus one always needs bright light to take pictures of what happens inside fires.

Challenge 160, page 151: Achieving a higher temperature would allow to break the second principle of thermodynamics. To explore this question further, read in textbooks about the so-called *Kirchhoff laws*.

Challenge 161, page 152: The effective temperature of laser light can also be described as *higher than infinite*; this allows also to heat targets to extremely high temperatures.

Challenge 164, page 157: For small mirrors or lenses, like those used in microscopes, mass production is easier for lenses. In contrast, large mirrors are much easier and cheaper to fabricate, mount and use than large lenses, because mirrors use less glass, are lighter, and allow changing their shape with actuators.

Challenge 165, page 157: Syrup shows an even more beautiful effect in the following setting. Take a long transparent tube closed at one end and fill it with syrup. Shine a red helium–neon laser into the tube from the bottom. Then introduce a linear polarizer into the beam: the light seen in the tube will form a spiral. By rotating the polarizer you can make the spiral advance or retract. This effect, called the *optical activity* of sugar, is due to the ability of sugar to rotate light polarization and to a special property of plants: they make only one of the two mirror forms of sugar.

Challenge 167, page 158: The relation, the so-called 'law' of refraction is

$$\frac{c_1}{c_2} = \frac{\sin\alpha_1}{\sin\alpha_2} . \tag{118}$$

The particular speed ratio between vacuum (or air, which is almost the same) and a material gives the *index of refraction* n of that material:

$$n = \frac{c_1}{c_0} = \frac{\sin\alpha_1}{\sin\alpha_0} \tag{119}$$

Many incorrectly call the 'law' of refraction 'Snell's law', or 'Descartes' law' even though many others found it before them (and even though the family name is 'Snel').

Challenge 168, page 163: The thin lens formula is

$$\frac{1}{d_{\mathrm{o}}} + \frac{1}{d_{\mathrm{i}}} = \frac{1}{f} . \tag{120}$$

It is valid for diverging and converging lenses, as long as their own thickness is negligible. The strength of a lens can thus be measured with the quantity $1/f$. The unit $1\,\mathrm{m}^{-1}$ is called a *diopter*; it is used especially for reading glasses. Converging lenses have positive, diverging lenses negative values.

However, the thin lens formula is only an approximation, and is never used in lens design. It is a relic of old textbooks. Modern lens designers always use Gaussian optics for calculations. See, for example, Francis A. Jenkins & Harvey E. White, *Fundamentals of Optics*, McGraw-Hill, 1957.

Challenge 170, page 164: A light microscope is basically made of two converging lenses. One lens – or lens system – produces an enlarged real image and the second one produces an enlarged virtual image of the previous real image. Figure 191 also shows that microscopes always turn images upside down. Due to the wavelength of light, light microscopes have a maximum resolution of about 1 µm. Note that the *magnification* of microscopes is unlimited; what is limited is their *resolution*. This is exactly the same behaviour shown by digital images. The *resolution* is simply the size of the smallest possible pixel that makes sense.

The microscope seems to have been invented by Girolamo Fracastro in 1538. The first viable microscopes were built in the Netherlands around 1590. Progress in microscopes was so slow

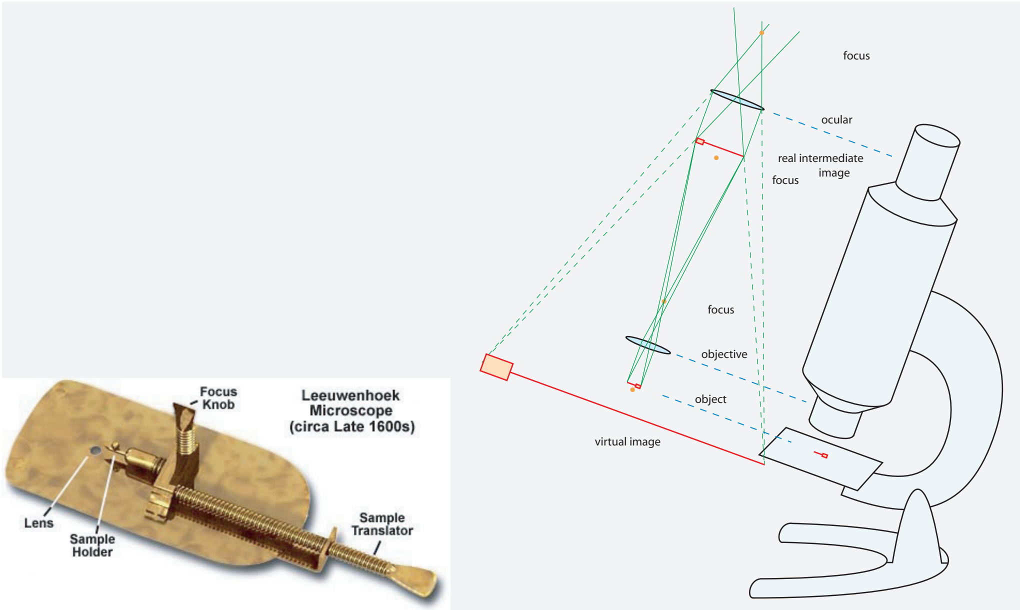


FIGURE 191 One lens made the oldest commercial microscope, from 1680 (length *c.* 8 cm, to be held close to the eye), but two converging lenses make a modern microscope (photo WikiCommons).

because glass and lens production was extremely difficult at those times, especially for small lenses. Therefore, David Brewster proposed in 1819 to build a microscope using the lens of a fish eye; when this idea was realized with the lens of an eel, it resulted in a microscope with astonishing performance. To learn more about microscopes, read the beautiful text by Elizabeth M. Slater & Henry S. Slater, *Light and Electron Microscopy*, Cambridge University Press, 1993, or explore dedicated websites, such as www.mikroskopie-muenchen.de or micro.magnet.fsu.edu/primer/techniques.

Challenge 172, page 167: The dispersion at the lens leads to different apparent image positions, as shown in Figure 192. For more details on the dispersion in the human eye and the ways of using it to create three-dimensional effects, see the article by C. Ucke & R. Wolf, *Durch Farbe in die dritte Dimension*, Physik in unserer Zeit **30**, pp. 50–53, 1999.

Challenge 173, page 170: The 1 mm beam would return 1000 times as wide as the 1 m beam. A perfect 1 m-wide beam of green light would be 209 m wide on the Moon; can you deduce this result from the (important) formula that involves distance, wavelength, initial diameter and final diameter? Try to guess this beautiful formula first, and then deduce it. In reality, the values are a few times larger than the theoretical minimum thus calculated. See the www.csr.utexas.edu/mlrs and ilrs.gsfc.nasa.gov websites.

Challenge 174, page 170: It is often said that evolution tuned the number of cones in the eye to the maximum theoretical resolution possible with open pupil and its aberrations; the experts on the subject however maintain that there are somewhat larger numbers of cones, as a reservoir.

Challenge 175, page 170: The answer should lie between one or two dozen kilometres, assuming ideal atmospheric circumstances.

Challenge 178, page 180: In fact, there is no way that a hologram of a person can walk around and frighten a real person. A hologram is always transparent; one can always see the background through the hologram. A hologram thus always gives an impression similar to what moving

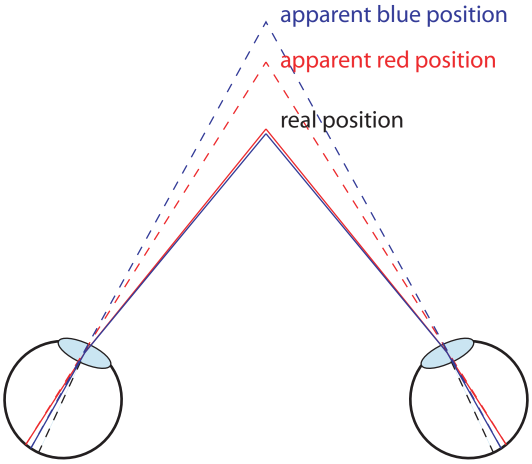


FIGURE 192 The relation between the colour depth effect and the lens dispersion of the human eye.

pictures usually show as ghosts. If the background is black, shine with a torch onto it to find out.

Challenge 179, page 180: The small wavelength of light probably prevents achieving this dream. For a true holographic display, the pixels need to be smaller than the wavelength of light and must be able to reproduce phase information. Thus the next question is: how much of the dream can be realized? If you find a solution, you will become rich and famous.

Challenge 182, page 187: There is a *blind spot* in the eye; that is a region in which images are not perceived. The brain than assumes that the image at that place is the same than at its borders. If a spot falls exactly inside it, it disappears.

Challenge 183, page 187: The mechanism that compensates the missing blue receptors in the fovea does not work so rapidly: you will see a spot due to the fovea.

Challenge 185, page 189: The eye and brain surely do not switch the up and the down direction at a certain age. Besides, where does the idea come from that babies see upside-down?

Challenge 186, page 196: The eye and vision system subtract patterns that are constant in time.

Challenge 187, page 200: Not really; a Cat's-eye uses two reflections at the sides of a cube. A living cat's eye has a large number of reflections. The end effect is the same though: light returns back to the direction it came from.

Challenge 190, page 205: Use diffraction; watch the pattern on a wall a few metres behind the hair.

Challenge 192, page 205: At 10 pc=32.6 al, the Sun would have apparent magnitude 4.7. At 20 pc=65.2 al, it would appear 4 times fainter, thus about 1.5 magnitudes more, therefore with an apparent visual magnitude of about 6.2. This is near the limit magnitude of the eye. The actual limiting magnitude of the eye is neither constant nor universal, so the distance of 50 light years is not a sharp limit. The limiting magnitude, – like the night vision, or *scotopic sensitvity*

– depends on the partial pressure of oxygen in the atmosphere the observer is breathing, on the clarity of the air, on the zenith distance, and, above all, on the degree of dark adaptation. An eye exposed to the full brightness of the night sky in a very dark location far from light pollution is still not completely dark-adapted. You can easily see 7th-magnitude stars by blocking off most of the sky and just looking at a little patch of it. Some observers, under ideal conditions, have reliably reported seeing stars near 8th magnitude.

Challenge 193, page 205: The green surface seen at a low high angle is larger than when seen vertically, where the soil is also seen; the soil is covered by the green grass in low angle observation.

Challenge 194, page 207: It is indeed true. Modern telescopes have a large surface collecting light (up to $50\,\mathrm{m}^2$) and have extremely sensitive detectors. The number of photons emitted by a match lit on the moon into the direction of a large telescope (how many?) is sufficient to trigger the detector.

Challenge 195, page 208: Of course not, as the group velocity is not limited by special relativity. The energy velocity is limited, but is not changed in this experiments.

Challenge 196, page 210: He bought clothes for his mother and for himself whose colours were inappropriate.

Challenge 198, page 210: The Prussian explorer Alexander von Humboldt extensively checked this myth in the nineteenth century. He visited many mine pits and asked countless mine workers in Mexico, Peru and Siberia about their experiences. He also asked numerous chimney-sweeps. Neither him nor anybody else has ever seen the stars during the day.

Challenge 199, page 210: Watch the Sun with closed eyes, and remember the shade of red you see. Go into a closed room, turn a light bulb on, and watch it with closed eyes. Choose the distance from the bulb that yields the same shade of red. Then deduce the power of the Sun from the power of the light bulb and the inverse square dependence.

Challenge 200, page 211: If you unroll a roll of adhesive tape, in addition to light, also X-rays are emitted. This is an example of triboluminescence. See the experiment live in the film at www.youtube.com/watch?v=J3i8oRi0WNc.

Challenge 209, page 221: The human body is slightly conducting and changes the shape of the field and thus effectively short circuits it. Usually, the field cannot be used to generate energy, as the currents involved are much too small. (Lightning bolts are a different story, of course. They are due – very indirectly – to the field of the Earth, but they are too irregular to be used consistently. Franklin's lightning rod is such an example.) The fair weather field cannot be used as a power source because its internal resistance is $3\,\mathrm{G\Omega/m}$.

Challenge 210, page 222: The field at the surface of a sphere of radius r is given by $E = Q/4\pi\varepsilon_0 r^2$. Inserting $E = 200\,\mathrm{V/m}$, we get $Q = 0.9\,\mathrm{MC}$.

Challenge 211, page 225: If you find a method that is different from the known estimates, publish it.

Challenge 217, page 231: All the illusions of the flying act look as if the magician is hanging on lines, as observed by many, including myself. (Photographic flashes are forbidden, a shimmery background is set up to render the observation of the lines difficult, no ring is ever actually pulled over the magician, the aquarium in which he floats is kept open to let the fishing lines pass through, always the same partner is 'randomly' chosen from the public, etc.) Information from eyewitnesses who have actually seen the fishing lines used by David Copperfield explains the reasons for these set-ups. The usenet news group alt.magic.secrets, in particular Tilman Hausherr, was central in clearing up this issue in all its details, including the name of the company that made the suspension mechanism.

Challenge 219, page 231: Any new one is worth a publication.

Challenge 220, page 235: Sound energy is also possible, as is mechanical work.

Challenge 221, page 237: Space-time deformation is not related to electricity; at least at everyday energies. Near Planck energies, this might be different, but nothing has been predicted yet.

Challenge 223, page 239: Ideal absorption is blackness (though it can be redness or whiteness at higher temperatures).

Challenge 224, page 239: Indeed, the Sun emits about $4 \cdot 10^{26}$ W from its mass of $2 \cdot 10^{30}$ kg, about 0.2 mW/kg. The adult human body (at rest) emits about 100 W (you can check this in bed at night), thus about 1.2 W/kg. This is about 6000 times more than the Sun. The reason: only the very centre of the Sun actually emits energy. If that energy amount is then divided by the full mass, including all the mass that does not emit energy at all, one gets a small average value. By the way, any candle or, better, any laser pointer emits even more light per mass, for similar reasons.

Challenge 226, page 240: The charges on a metal box rearrange so that the field inside remains vanishing. This makes cars and aeroplanes safe against lightning. Of course, if the outside field varies so quickly that the rearrangement cannot follow, fields *can* enter the Faraday cage. (By the way, also fields with long wavelengths penetrate metals; specialized remote controls for opening security doors regularly use frequencies of 25 kHz to achieve this.) However, one should wait a bit before stepping out of a car after lightning has hit, as the car is on rubber wheels with low conduction; waiting gives the charge time to flow into the ground.

For gravity and solid cages, mass rearrangement is not possible, so that there is no gravity shield.

Challenge 227, page 240: Mu-metal is a nickel-iron alloy, often containing traces of other metals, that has a high relative permeability μ_r in the range of 50 000 to 140 000; it is astonishingly ductile. The high permeability value effectively concentrates the magnetic fields inside the alloy and thus leads applied magnetic fields through the mu-metal and around the enclosed volume, which therefore is shielded as a result.

Challenge 230, page 242: This is a touchy topic. It is not clear whether 50 Hz fields are dangerous to humans. There is a high probability that they are not; but the issue is not settled.

Challenge 231, page 243: The number of photons times the quantum of action $\hbar$.

Challenge 232, page 243: First, Faraday could have found a superficial link using the mentioned tube experiment. But he was looking for a possible deep connection. However, gravitation and electricity are not at all connected, as one is due to mass, the other due to charge. Much after Faraday, people discovered that gravity also includes gravitomagnetism, i.e., measurable effects due to moving masses – but still no relation to electromagnetism. A distant connection between gravitation and electricity will only appear in the last part of this adventure.

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Challenge 233, page 243: The charging stops because a negatively charged satellite repels electrons and thus stops any electron collecting mechanism. Electrons are captured more frequently than ions because it is easier for them than for ions to have an inelastic collision with the satellite, due to their larger speed at a given temperature.

Challenge 234, page 243: Any loss mechanism will explain the loss of energy, such as electrical resistance or electromagnetic radiation. After a fraction of a second, the energy will be lost. This little problem is often discussed on the internet.

Challenge 235, page 243: Use the wire as shown in Figure 193. *If* the oscillation is properly tuned in frequency, and *if* the contact detaches properly at the tip, and *if* you touch the two contacts with a strong grip, you will get a stronger shock than you can stand.

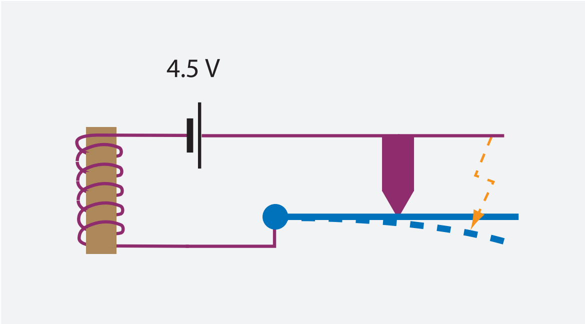

FIGURE 193 How to get electrical shocks from a 4.5 V pocket battery.

Challenge 237, page 245: This should be possible in the near future; but both the experiment, which will probably measure brain magnetic field details, and the precise check of its seriousness will not be simple.

Challenge 238, page 245: No, the system is not secure. In any system, the security is given by the weakest spot. And in any password system, the weakest spots are the transport of the raw data – in this case the signals from the electric cap to the computer – and the password checking system. Both are as vulnerable as any other password system. (If you want to learn about security, read the writings of Bruce Schneier, most of which are available on the internet.)

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Challenge 239, page 247: The maximum electric and magnetic field values are those that exert the maximum possible force $c^4/4G$ on an elementary charge e. Table 3 gives the maximum electric field value and Table 8 the maximum magnetic field value.

Challenge 241, page 249: See challenge 29.

Challenge 243, page 250: One can measure many tony charges and show that they are always multiples of the same unit. This method was used by Millikan in his famous experiment. One can also measure current fluctuations, and show that they follow from shot noise, i.e., from the flow of discrete charges of the same value.

Challenge 245, page 250: Point-like charges would imply an infinite energy density. This is impossible. Whether this argument applies only to classical electrodynamics or to nature is a heated debate. The majority opinion is that quantum theory allows point-like charges, because quantum particles never are at rest, so that an infinite energy density is effectively avoided.

Challenge 247, page 251: Earth's potential would be $U = -q/(4\pi\varepsilon_0 R) = 60\,\mathrm{MV}$, where the number of electrons in water must be taken into account.

Challenge 248, page 251: There is always a measurement error when measuring field values, even when measuring a 'vanishing' electromagnetic field. In addition, quantum theory leads to arbitrary small charge density values through the probability density due to wave functions.

Challenge 252, page 255: The issue is: is the 'universe' a concept? In the last part of this adventure we will show that it is not.

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Challenge 254, page 262: When thinking, physical energy, momentum and angular momentum are conserved, and thermodynamic entropy is not destroyed. Any experiment showing anything different would point to unknown processes. However, there is no evidence for such processes.

Challenge 255, page 264: The best method cannot be much shorter than what is needed to describe 1 in 6000 million, or 33 bits. The Dutch and UK post code systems (including the letters NL or UK) are not far from this value and thus can claim to be very efficient.

Challenge 256, page 265: For complex systems, when the unknowns are numerous, the advance is thus simply given by the increase in answers. For the universe as a whole, the number of open issues is quite low, as shown later on; in this topic there has not been much advance in the past years. But the advance is clearly measurable in this case as well.

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Challenge 257, page 265: Is it possible to use the term 'complete' when describing nature? Yes, it is. In fact, humanity is not far from a complete description of motion. For a clear-cut survey, see the last volume of our adventure.

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Challenge 259, page 267: There are many baths in series: thermal baths in each light-sensitive cell of the eyes, thermal baths inside the nerves towards the brain and thermal baths inside brain cells.

Challenge 261, page 268: Yes.

Challenge 263, page 274: Chips based on trits would have to be redesigned from scratch. This would be a waste of resources and of previous work.

Challenge 267, page 281: Reducing the list of semantic primitives is not hard; you will easily find ways to do so, both for the mathematical and for the physical concepts. But is the list of semantic primitives really *complete?* For the mathematical primitives this is the case. But physicists might claim that the properties of objects, of space-time and of interactions form their smallest list possible. However, this list of properties is longer than the one found by linguists! One reason is that physicists have found 'physical primitives' that do not appear in everyday life. The other reason is that physicists have not achieved unification. In a sense, the aim of physicists is limited, at present, by the list of unexplained questions about nature. That list is given later on; it forms the starting point of the last part of this adventure.

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By the way, it can be argued that the list of primitives indeed is already complete, as it allows to talk about everything. This implies that the average person already has, without knowing it, a *theory of everything*. Therefore, physicists just need to catch up with the average person...

Challenge 268, page 284: Neither has a defined content, clearly stated limits or a domain of application.

Challenge 269, page 284: Impossible! That would not be a concept, as it has no content. The solution to the issue must be and will be different. The last part of this walk will propose one.

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Challenge 270, page 286: To neither. This paradox shows that such a 'set of all sets' does not exist.

Challenge 271, page 287: The most famous is the class of all sets that do not contain themselves. This is not a set, but a class.

Challenge 272, page 287: Dividing cakes is difficult. A simple method that solves many – but not all – problems among N persons P1 ... PN is the following:

— P1 cuts the cake into N pieces.
— P2 to PN choose a piece.
— P1 keeps the last part.
— P2 ... PN assemble their parts back into one.
— Then P2 ... PN repeat the algorithm for one person less.

The problem is much more complex if the reassembly is not allowed. A *just* method (in finite many steps) for 3 people, using nine steps, was published in 1944 by Steinhaus, and a *fully satisfactory* method in the 1960s by John Conway. A fully satisfactory method for four persons was found only in 1995; it has 20 steps.

Challenge 273, page 287: $(x, y) := \{x, \{x, y\}\}$.

Challenge 274, page 288: Hint: show that any countable list of reals misses at least one number. This was proven for the first time by Cantor. His way was to write the list in decimal expansion and then find a number that is surely not in the list. Second hint: his world-famous trick is called the *diagonal argument*.

Challenge 275, page 289: Hint: all reals are limits of series of rationals.

Challenge 277, page 290: Yes, but only provided division by zero is not allowed, and numbers are restricted to the rationals and reals.

Challenge 278, page 290: There are infinitely many of these so-called *parasitic numbers*. The smallest is already large: 1016949152542372881355932203389830508474576271186440677966. If the number 6 is changed in the puzzle, one finds that the smallest solution for 1 is 1, for 4 is 102564, for 5 is 142857, for 8 is 1012658227848, for 2 is 105263157894736842, for 7 is 1014492753623188405797, for 3 is 1034482758620689655172413793, and for 9 is 10112359550561797752808988764044943820224719. The smallest solution for the number 6 is by far the largest of this list.

Challenge 279, page 291: One way was given above: $0 := \emptyset$, $1 := \{\emptyset\}$, $2 := \{\{\emptyset\}\}$ etc.

Challenge 280, page 294: Subtraction is easy. Addition is not commutative only for cases when infinite numbers are involved: $\omega + 2 \neq 2 + \omega$.

Challenge 281, page 295: Examples are $1 - \varepsilon$ or $1 - 4\varepsilon^2 - 3\varepsilon^3$.

Challenge 282, page 295: The answer is 57; the cited reference gives the details.

Challenge 283, page 297: $2^{2^{2^2}}$ and $4^{4^{4^4}}$.

Challenge 286, page 297: The child is minus 0.75 years old, or minus 9 months old; the father is thus very near the mother.

Challenge 287, page 297: This is not an easy question. The first non-trivial numbers are 7, 23, 47, 59, 167 and 179. See Robert Matthews, *Maximally periodic reciprocals*, Bulletin of the Institute of Mathematics and its Applications **28**, pp. 147–148, 1992. Matthews shows that a number n for which $1/n$ generates the maximum of $n - 1$ decimal digits in the decimal expansion is a special sort of prime number that can be deduced from the so-called *Sophie Germain primes* S; one must have $n = 2S + 1$, where both S and $2S + 1$ must be prime and where S mod 20 must be 3, 9, or 11.

Thus the first numbers n are 7, 23, 47, 59, 167 and 179, corresponding to values for S of 3, 11, 23, 29, 83 and 89. In 1992, the largest known S that meets the criteria was

$$S = (39051 \cdot 2^{6002}) - 1 , \qquad (121)$$

a 1812-digit long Sophie Germain prime number that is 3 mod 20. It was discovered by Wilfred Keller. This Sophie Germain prime leads to a prime n with a decimal expansion that is around 10^{1812} digits long before it starts repeating itself. Read your favourite book on number theory to find out more. Interestingly, the solution to this challenge is also connected to that of challenge 278. Can you find out more?

Challenge 288, page 297: Klein did not belong to either group. As a result, some of his nastier students concluded that he was not a mathematician at all.

Challenge 289, page 297: A barber cannot belong to either group; the definition of the barber is thus contradictory and has to be rejected.

Challenge 290, page 297: See the members.shaw.ca/hdhcubes/cube_basics.htm web page for more information on magic cubes.

Challenge 293, page 298: Such an expression is derived with the intermediate result $(1 - 2^2)^{-1}$. The handling of divergent series seems absurd, but mathematicians know how to give the expression a defined content. (See Godfrey H. Hardy, *Divergent Series*, Oxford University Press, 1949.) Physicists often use similar expressions without thinking about them, in quantum field theory.

Challenge 291, page 298: Try to find another magic hexagon and then prove the uniqueness of the known one.

Challenge 294, page 299: The result is related to Riemann's zeta function. For an introduction, see en.wikipedia.org/wiki/Prime_number.

Challenge 296, page 310: 'All Cretans lie' is *false*, since the opposite, namely 'some Cretans say the truth' is true in the case given. The trap is that the opposite of the original sentence is usually, but *falsely*, assumed to be 'all Cretans say the truth'.

Challenge 297, page 310: The statement cannot be false, due to the first half and the 'or' construction. Since it is true, the second half must be true and thus you are an angel.

Challenge 298, page 311: The terms 'circular' and 'self-referential' describe two different concepts.

Challenge 300, page 312: Extraterrestrials cannot be at the origin of crop circles because, like Father Christmas or ghosts, they do not exist on Earth.

Challenge 302, page 312: This can be debated; in any case it is definitely known that both statements are lies indeed, as shown in detail later on..

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Challenge 303, page 312: If this false statement were true, swimmers or divers would also die, as their skin cannot breathe either.

Challenge 304, page 312: It is equally correct to claim that the Earth was created a hundred ago, and that our environment and our memories were created in our brain to make us believe that the Earth is older. It is hard to disprove such nonsense, but it is possible. See also the next challenge.

Challenge 305, page 312: It is surprisingly hard to disprove such nonsense, if well thought through. The reason for the particular date (or for any other date) is not obvious. Neither is obvious what is meant by the term 'creation'.

Challenge 307, page 313: No. As many experiments demonstrate, homoeopathy is a set of *numerous* lies. For example, the internet provides films of people swallowing – without any harm – hundreds of homoeopathic pills at a time that are labelled as 'extremely dangerous when overdosed'. By the way, many of the homoeopathy lies have been generated by a single person. As always, the most sucessful lies are those that allow a select group of people to earn a lot of money.

Challenge 309, page 313: The light bulb story seems to be correct. The bulb is very weak, so that the wire is not evaporating.

Challenge 310, page 313: The origin might be the number of people present in the last supper in the New Testament; or the forgotten 13th sign of the Zodiac. There is no truth in this superstition. In fact, *every superstition is a lie.* However, beware of people who are jealous of those who do not care about superstitions, and who get violent as a reaction.

Challenge 311, page 313: Without exception so far, all those who pretend to have been stigmatized have wounds in the *palms* of their hands. However, in crucifixion, the nails are driven through the *wrist*, because nails driven through the palms cannot carry the weight of a human body: the palms would tear open.

Challenge 312, page 314: The term 'multiverse' is both a superstition and a lie. Above of all, it is nonsense. It is akin to attempting to produce a plural for the word 'everything'.

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Challenge 314, page 314: In which frame of reference? How? Beware of anybody making that statement: he is a crook.

Challenge 319, page 321: Only induction allows us to make use of similarities and thus to define concepts.

Challenge 320, page 323: This depends on the definition (of the concept) of deity used. Pantheism does not have the issue, for example.

Vol. VI, page 106 **Challenge 321**, page 323: Yes, as we shall find out.

Challenge 322, page 324: Yes, as observation implies interaction.

Challenge 323, page 324: Lack of internal contradictions means that a concept is valid as a thinking tool; as we use our thoughts to describe nature, mathematical existence is a specialized version of physical existence, as thinking is itself a natural process. Indeed, mathematical concepts are also useful for the description of the working of computers and the like.

Another way to make the point is to stress that all mathematical concepts are built from sets and relations, or some suitable generalizations of them. These basic building blocks are taken from our physical environment. Sometimes the idea is expressed differently; many mathematicians have acknowledged that certain mathematical concepts, such as natural numbers, are taken directly from experience.

Challenge 324, page 324: Examples are Achilles, Odysseus, Mickey Mouse, the gods of polytheism and spirits.

Challenge 326, page 326: Torricelli made vacuum in a U-shaped glass tube, using mercury, the same liquid metal used in thermometers. Can you imagine how? A more difficult question: where did he get mercury from?

Challenge 327, page 328: Stating that something is infinite are not beliefs if the statement is falsifiable. An example is the statement 'There are infinitely many mosquitoes.' Such a statement is just wrong. Other statements are not falsifiable, such as 'The universe continue without limit behind the horizon.' Such a statement is a belief. Both cases of statements on infinities are not facts.

Challenge 328, page 329: They are not sets either and thus not collections of points.

Challenge 329, page 330: There is still no possibility to interact with all matter and energy, as this includes oneself.

Challenge 330, page 335: No. There is only a generalization encompassing the two.

Challenge 331, page 336: An explanation of the universe is not possible, as the term explanation require the possibility to talk about systems outside the one under consideration. The universe is not part of a larger set.

Challenge 332, page 336: Both can in fact be seen as two sides of the same argument: There is no other choice; there is only one possibility. Equivalently, the rest of nature shows that observations have to be the way they are, because everything depends on everything.

Challenge 334, page 354: Mass is a measure of the amount of energy. The 'square of mass' makes no sense.

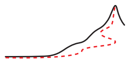
Challenge 337, page 356: The formula with $n - 1$ is a better fit. Why?

Challenge 340, page 357: No! They are much too precise to make sense. They are only given as an illustration for the behaviour of the Gaussian distribution. Real measurement distributions are not Gaussian to the precision implied in these numbers.

Challenge 341, page 357: About 0.3 m/s. It is *not* 0.33 m/s, it is *not* 0.333 m/s and it is *not* any longer strings of threes!

Challenge 343, page 363: The slowdown goes *quadratically* with time, because every new slowdown adds to the old one!

Challenge 344, page 363: No, only properties of parts of the universe are listed. The universe itself has no properties, as shown in the final volume. Vol. VI, page 112



BIBLIOGRAPHY

> [...] moi, qui trouve toujours tous les livres trop longs, et surtout les miens [...]
>
> Voltaire, *Lettre à M. Cideville.**

1 Julian Schwinger, L. L. DeRaad, K. A. Milton & W. Y. Tsai, *Classical Electrodynamics*, Perseus, 1998. An excellent text on the topic by one of its greatest masters.
See also the beautiful problem book by André Butoli & Jean-Marc Lévy-Leblond, *La physique en questions – électricité et magnétisme*, Vuibert, 1999. Cited on pages 16 and 82.

2 A pretty book about the history of magnetism and the excitement it generates is James D. Livingston, *Driving Force – the Natural Magic of Magnets*, Harvard University Press, 1996. Cited on page 17.

3 R. Edwards, *Filling station fires spark cars' recall*, New Scientist, pp. 4–5, 4 March 1995. Cited on page 19.

4 S. Desmet, F. Orban & F. Grandjean, *On the Kelvin electrostatic generator*, European Journal of Physics **10**, pp. 118–122, 1989. You can also find construction plans for it in various places on the internet. Cited on page 20.

5 F. Steinle, *Exploratives Experimentieren – Georges Dufay und die Entdeckung der zwei Elektrizitäten*, Physik Journal **3**, pp. 47–52, 2004. Cited on page 21.

6 For an etching of Franklin's original ringing rod, see E. P. Krider, *Benjamin Franklin and lightning rods*, Physics Today **59**, pp. 42–48, 2006. Cited on page 22.

7 W. Rueckner, *An improved demonstration of charge conservation*, American Journal of Physics **75**, pp. 861–863, 2007. Cited on page 23.

8 For more details on various electromagnetic units, see the standard text by J. D. Jackson, *Classical Electrodynamics*, 3rd edition, Wiley, 1998. Cited on pages 27 and 390.

9 See the old but beautiful papers by Richard C. Tolman & T. Dale Stewart, *The electromotive force produced by the acceleration of metals*, Physical Review **8**, pp. 97–116, 1916, Richard C. Tolman & T. Dale Stewart, *The mass of the electric carrier in copper, silver and aluminium*, Physical Review **9**, pp. 164–167, 1917, and the later but much more precise experiment by C. F. Kettering & G. G. Scott, *Inertia of the carrier of electricity in copper and aluminum*, Physical Review **66**, pp. 257–267, 1944. (Obviously the American language dropped the 'i' from aluminium during that period.) The first of these papers is also a review of the preceding attempts, and explains the experiment in detail. The last paper shows what had to be taken into consideration to achieve sufficient precision. Cited on page 30.

* '[...] me, who always finds all books too long, first of all my own [...]'.

10 This effect has first been measured by S. J. Barnett, *A new electron-inertia effect and the determination of m/e for the free electron in copper*, Philosophical Magazine **12**, p. 349, 1931. Cited on page 30.

11 See for example C. Schiller, A. A. Koomans, T.L. van Rooy, C. Schönenberger & H. B. Elswijk, *Decapitation of tungsten field emitter tips during sputter sharpening*, Surface Science Letters **339**, pp. L925–L930, 1996. Cited on page 30.

12 L. I. Schiff & M. V. Barnhill, *Gravitational-induced electric field near a metal*, Physical Review **151**, pp. 1067–1071, 1966. F. C. Witteborn & W. M. Fairbank, *Experimental comparison of the gravitational force on freely falling electrons and metallic electrons*, Physical Review Letters **19**, pp. 1049–1052, 1967. Cited on page 31.

13 J. Lepak & M. Crescimanno, *Speed of light measurement using ping*, electronic preprint available at arxiv.org/abs/physics/0201053. Cited on page 32.

14 This story was printed on its fron page by the *Wall Street Journal* on 15 December 2006 under the title *Firms seek edge through speed as computer trading expands*. Cited on page 32.

15 J. D. Pettigrew, *Electroreception in monotremes*, Journal of Experimental Biology **202**, pp. 1447–1454, 1999. Cited on page 34.

16 For an excellent review article on the fascinating field of electric fish, see C. D. Hopkins, *Electrical Perception and Communication*, Encyclopedia of Neuroscience **3**, pp. 813–831, 2009. Hopkin's research laboratory can be found at www.nbb.cornell.edu. Cited on pages 34 and 369.

17 On the search for magnetic monopoles, see the website of the Particle Data Group, the world's reference, at pdg.web.cern.ch. See also H. Jeon & M. Longo, *Search for magnetic monopoles trapped in matter*, Physical Review Letters **75**, pp. 1443–1447, 1995. See also A. S. Goldhaber & W. P. Trower, *Resource letter MM-1: magnetic monopoles*, American Journal of Physics **58**, pp. 429–439, 1990. Cited on page 36.

18 Pierre de Maricourt, *Tractatus de magnete*, 1269. Cited on page 37.

19 R. Wiltschko & W. Wiltschko, *Magnetic Orientation in Animals*, Springer, 1995. Cited on page 40.

20 M. Lauwers & al., *An iron-rich organelle in the cuticular plate of avian hair cells*, Current biology **23**, pp. 924–929, 2013. This paper presets the newest candidate for the location of the basic magnetic sensor in birds. Cited on page 41.

21 I. A. Solov'yov, K. Schulten & W. Greiner, *Nur dem Schnabel nach?*, Physik Journal **9**, pp. 23–28, 2010. Cited on page 41.

22 The ratio of angular L to magnetic M moment is

$$\frac{L}{M} = \frac{2m}{e} \cdot \frac{1}{g} , \qquad (122)$$

where e is the electron charge and m its mass. Both L and M are measurable. The first measurements were published with a g-value of 1, most probably because the authors expected the value. In later experiments, de Haas found other values. Measurements by other researchers gave values nearer to 2 than to 1, an observation that was only understood with the discovery of spin. The original publications are A. Einstein & W. J. de Haas, *Proefondervinderlijk bewijs voor het bestaan der moleculaire stroomen van Ampère*, Konninklijke Akademie der Wetenschappen te Amsterdam, Verslagen **23**, p. 1449, 1915, and A. Einstein & W. J. de Haas, *Experimental proof of the existence of Ampère's molecular currents*, Konninklijke Akademie der Wetenschappen te Amsterdam, Proceedings **18**, p. 696, 1916. Cited on page 44.

23 S. J. Barnett, *Magnetization by rotation*, Physical Review **6**, pp. 171–172, 1915, and S. J. Barnett, *Magnetization by rotation*, Physical Review **6**, pp. 239–270, 1915. Cited on page 45.

24 See J. D. Jackson, *Classical Electrodynamics*, 3rd edition, Wiley, 1998, or also R. F. Harrington, *Time Harmonic Electromagnetic Fields*, McGraw–Hill, 1961. Cited on pages 49 and 82.

25 The best available book on the brain is the one by Eric R. Kandel, James H. Schwartz & Thomas M. Jessell, *Principles of Neural Science*, fifth edition, McGraw-Hill, 2000. The suhep.phy.syr.edu/courses/modules/MM/brain/brain.html website gives an introduction into brain physiology. Cited on page 49.

26 N. Salingaros, *Invariants of the electromagnetic field and electromagnetic waves*, American Journal of Physics **53**, pp. 361–363, 1985. Cited on page 50.

27 A. L. Hodgkin & A. F. Huxley, *A quantitative description of membrane current and its application to conduction and excitation in nerve*, Journal of Physiology **117**, pp. 500–544, 1952. This famous paper of theoretical biology earned the authors the Nobel Prize in Medicine in 1963. Cited on page 51.

28 See the excellent overview article by T. Heimburg, *Die Physik von Nerven*, Physik Journal **8**, pp. 33–39, 2009. See also S. S. L. Andersen, A. D. Jackson & T. Heimburg, *Towards a thermodynamic theory of nerve pule propagation*, Progress in Neurobiology **88**, pp. 104–113, 2009, the website membranes.nbi.dk, and the text Thomas Heimburg, *Thermal Biophysics of Membranes*, Wiley-VCH, 2007. Cited on page 52.

29 A.C. de la Torre, *$v \leqslant c$ in 1820?*, European Journal of Physics **20**, pp. L23–L24, March 1999. Cited on page 53.

30 See U. Fantz & A. Lotter, *Blitze zum Anfassen*, Physik in unserer Zeit **33**, pp. 16–19, 2002. More information is available on www.physik.uni-augsburg.de/epp. Cited on page 61.

31 R. H. Tyler, S. Maus & H. Lühr, *Magnetic signal due to ocean tidal flow identified in satellite observations*, Science **299**, pp. 239–241, 2003. The films derived from the data can be found on the www.tu-braunschweig.de/institute/geophysik/spp/publikationen.html website. Cited on page 63.

32 H. Montgomery, *Unipolar induction: a neglected topic in the teaching of electromagnetism*, European Journal of Physics **20**, pp. 271–280, 1999. Cited on page 66.

33 On the geodynamo status, see the articles G. A. Glatzmaier & P. H. Roberts, *Rotation and magnetism of Earth's inner core*, Science **274**, pp. 1887–1891, 1996, and P. H. Roberts & G. A. Glatzmaier, *Geodynamo theory and simulations*, Reviews of Modern Physics 72, pp. 1081–1123, 2000. An older article is R. Jeanloz & B. Romanowicz, *Geophysical dynamics at the center of the Earth*, Physics Today pp. 22–27, August 1997. Cited on pages 67 and 224.

34 A. Yazdani, D. M. Eigler & N. D. Lang, *Off-resonance conduction through atomic wires*, Science **272**, pp. 1921–1924, 28 June 1996. For aluminium, gold, lead, niobium, as well as the influence of chemical properties, see Elke Scheer, *The signature of chemical valence in the electric conduction through a single-atom contact*, Nature **394**, pp. 154–157, 9 July 1998. Cited on page 70.

35 J. Yang, F. Lu, L. W. Kostiuk & D. Y. Kwok, *Electrokinetic microchannel battery by means of electrokinetic and microfluidic phenomena*, Journal of Micromechanics and Microengineering **13**, pp. 963–970, 2003. Cited on page 70.

36 See L. Kowalski, *A myth about capacitors in series*, The Physics Teacher **26**, pp. 286–287, 1988, and A. P. French, *Are the textbook writers wrong about capacitors?*, The Physics Teacher **31**, pp. 156–159, 1993. Cited on page 71.

37 A discussion of a different electrical indeterminacy relation, between current and charge, can be found in Y-Q. Li & B. Chen, *Quantum theory for mesoscopic electronic circuits and its applications*, preprint at arxiv.org/abs/cond-mat/9907171. Cited on page 73.

38 A sober but optimistic evaluation, free of the cheap optimism of tabloid journalism, is R. W. Keyes, *Miniaturization of electronics and its limits*, IBM Jounal of Research and Development **32**, pp. 84–88, 1988. In its last figure, it predicted that the lower limit kT for the energy dissipated by a logical operation would be reached around 2015. Cited on page 74.

39 J. A. Heras, *Can Maxwell's equations be obtained from the continuity equation?*, American Journal of Physics **75**, pp. 652–657, 2007, preprint at arxiv.org/abs/0812.4785. The point is made even more clearly in J. A. Heras, *How to obtain the covariant form of Maxwell's equations from the continuity equation*, European Journal of Physics **30**, pp. 845–854, 2009, and in J. A. Heras, *An axiomatic approach to Maxwell's equations*, European Journal of Physics **37**, p. 055204, 2016, preprint at arxiv.org/abs/1608.00659. See also L. Burns, *Maxwell's equations are universal for locally conserved quantities*, Advances in Applied Clifford Algebras **29**, p. 62, 2019, preprint at arxiv.org/abs/1906.02675. Cited on pages 75, 80, 92, and 246.

40 A similar summary is the basis of Friedrich W. Hehl & Yuri N. Obukov, *Foundations of Classical Electrodynamics – Charge, Flux and Metric*, Birkhäuser 2003. Cited on page 75.

41 On the non-existence of closed magnetic field lines in the general case, see J. Slepian, *Lines of force in electric and magnetic fields*, American Journal of Physics **19**, pp. 87–90, 1951, M. Lieberherr, *The magnetic field lines of a helical coil are not simple loops*, American Journal of Physics **78**, pp. 1117–1119, 2010, F. Herrmann & R. von Baltz, *Altlasten der Physik (128): Geschlossene magnetische Feldlinien*, Praxis der Naturwissenschaften: Physik in der Schule **60**, pp. 48–49, 2011. Cited on page 79.

42 Oleg D. Jefimenko, *A relativistic paradox seemingly violating conservation of momentum law in electromagnetic systems*, European Journal of Physics **20**, pp. 39–44, 1999. Cited on page 81.

43 H. Van Dam & E. P. Wigner, *Classical relativistic mechanics of interacting point particles*, Physical Review **136B**, pp. 1576–1582, 1965. Cited on page 81.

44 Mark D. Semon & John R. Taylor, *Thoughts on the magnetic vector potential*, American Journal of Physics **64**, pp. 1361–1369, 1996. Cited on pages 83 and 85.

45 Jean Sivardière, *Simple derivation of magnetic vector potentials*, European Journal of Physics **14**, pp. 251–254, 1993. Cited on page 83.

46 T. T. Wu & C. N. Yang, 1975, *Concept of nonintegrable phase factors and global formulation of gauge fields*, Physical Review D **12**, pp. 3845–3857, Cited on page 86.

47 See reference Ref. 8 or A. M. Stewart, *Angular momentum of the electromagnetic field: the plane wave paradox explained*, European Journal of Physics **26**, pp. 635–641, 2005. Cited on page 89.

48 An electrodynamics text completely written with (mathematical) forms is Kurt Meetz & Walter L. Engl, *Elektromagnetische Felder – mathematische und physikalische Grundlagen*, Springer, 1980. Cited on page 87.

49 See for example the discussion by M. C. Corballis & I. L. Beale, *On telling left from right*, Scientific American **224**, pp. 96–104, March 1971. Cited on page 91.

50 In 1977, Claus Montonen and David Olive showed that quantum theory allows duality transformations even *with* the inclusion of matter, if specific types of magnetic monopoles, the so-called *dyons*, exist. The fundamental paper is D. Olive & C. Montonen, *Magnetic monopoles as gauge particles*, Physics Letters **72B**, pp. 117–120, 1977. Many other papers built on this one; however, no experimental support for the approach has ever appeared. Cited on page 92.

51 Wolfgang Rindler, *Essential Relativity – Special, General, and Cosmological*, revised 2nd edition, Springer Verlag, 1977, page 247. There is also the beautiful paper by M. Le Bellac & J.-M. Lévy-Leblond, *Galilean electrodynamics*, Nuovo Cimento B **14**, p. 217, 1973, that explains the possibilities but also the problems appearing when trying to define the theory non-relativistically. Cited on page 93.

52 L.-C. Tu, J. Luo & G. T. Gilles, *The mass of the photon*, Reports on Progress of Physics **68**, pp. 77–130, 2005. Cited on page 93.

53 The system for typing by thought alone is described in many papers, such as B. Blankertz, F. Losch, M. Krauledat, G. Dornhege, G. Curio & K.-R. Müller, *The Berlin Brain-Computer Interface: accurate performance from first session in BCI-naïve subjects*, IEEE Transactions on biomedial engineering **55**, pp. 2452–2462, 2008. See the website www.bbci.de for more information. Cited on page 94.

54 See, for example, the paper by I. Martinovic, D. Davies, M. Frank, D. Perito, T. Ros & D. Song, *On the feasibility of side-channel attacks with brain-computer interfaces*, presented at USENIX Security, 2012, found at www.usenix.org/conference/usenixsecurity12. Cited on page 95.

55 D. Singleton, *Electromagnetic angular momentum and quantum mechanics*, American Journal of Physics **66**, pp. 697–701, 1998, Cited on page 95.

56 The magnetic pole strength is discussed in the textbooks by J.C. Maxwell, A. Sommerfeld, J.D. Jackson and others. Cited on page 96.

57 C. Hoyos, N. Sircar & J. Sonnenschein, *New knotted solutions of Maxwell's equations*, J. Phys. A: Math. Theor. **48**, p. 255204, 2015, preprint at arxiv.org/abs/1502.01382. The paper also provides a short review of recent research. Cited on page 96.

58 For a captivating account on the history of the ideas on light, see David Park, *The Fire Within the Eye: a Historical Essay on the Nature and Meaning of Light*, Princeton University Press, 1997. For an example of the complex history of optics, see the famous text by Alhazen or Ibn al-Haytham, *Book of Optics* 1021. However, no Arabic website allows reading the text, and the Arabic Wikipedia articles on the topic are much shorter than the French or English ones. Indeed, like most ancient Middle-East thinkers, Alhazen (b. *c.* 965 Basra, d. 1039 Cairo) is better known in Europe than in his home region. A Latin translation of the Book of Optics can be read at the imgbase-scd-ulp.u-strasbg.fr/displayimage.php?album=44&pos=0 website of the Université de Strasbourg. Around the year 1000, Alhazen performed many experiments on refraction of light, as did Ptolemy almost nine hundred years before him. The measurement results of Ptolemy are still known. But neither researcher found the Snell–Descartes expression for refraction. Alhazen even knew the sine function; despite this knowledge, he did not find the expression. For more details, see E. Kirchner, *Wie ontdekte de wet van Snellius?*, Nederlands Tijdschrift voor Natuurkunde **81**, pp. 198–201, 2015. Cited on page 97.

59 See the text by Raymond L. Lee & Alistair B. Fraser, *The Rainbow Bridge: Rainbows in Art, Myth, and Science*, Pennsylvania State University Press, 2000. A chapter can be found at the www.usna.edu/Users/oceano/raylee/RainbowBridge/Chapter_8.html website. Cited on page 103.

60 For a detailed explanation of supernumerary rainbows, see www.atoptics.co.uk/fz696.htm. For a beautiful picture collection, see www.flickr.com/groups/supernumeraryrainbows/. An excellent article on rainbows and on the effects of drop shapes, with beautiful photographs, graphics and drawings, is I. Sadeghi, A. Munoz, P. Laven, W. Jarosz, F. Seron, D. Gutierrez & H. W. Jensen, *Physically-based simulation of rainbows*, ACM Transactions on Graphics **31**, pp. 1–6, 2011. They show, among others, how flattened water drops yield flattened bows. Cited on pages 103 and 131.

61 The beautiful slit experiment was published by E. A. Montie, E. C. Cosman, G. W. 't Hooft, M.B. van der Mark & C. W. J. Beenakker, *Observation of the optical analogue of quantized conductance of a point contact*, Nature **350**, pp. 594–595, 18 April 1991, and in the longer version E. A. Montie, E. C. Cosman, G. W. 't Hooft, M.B. van der Mark & C. W. J. Beenakker, *Observation of the optical analogue of the quantised conductance of a point contact*, Physica B **175**, pp. 149–152, 1991. The result was also publicized in numerous other scientific magazines. Cited on page 103.

62 A recent measurement of the frequency of light is presented in Th. Udem, A. Huber, B. Gross, J. Reichert, M. Prevedelli, M. Weitz & T. W. Hausch, *Phase-coherent measurement of the hydrogen 1S–2S transition frequency with an optical frequency interval divider chain*, Physical Review Letters **79**, pp. 2646–2649, 1997. Another is C. Schwob, L. Jozefowski, B. de Beauvoir, L. Hilico, F. Nez, L. Julien, F. Biraben, O. Acef & A. Clairon, *Optical frequency measurement of the 2S-12D transitions in hydrogen and deuterium: Rydberg constant and Lamb shift determinations*, Physical Review Letters **82**, pp. 4960–4963, 21 June 1999. Cited on page 105.

63 The discoverors of such a method, the frequency comb, Theodor Hänsch and John Hall were awarded, together with Roy Glauber, the 2005 Nobel Prize in Physics. See John L. Hall & Theodor W. Hänsch, *History of optical comb development*, in Jun Ye & Steven T. Cundiff, editors, *Femtosecond Optical Frequency Comb: Principle, Operation, and Applications*, Springer, 2004. Cited on page 105.

64 M. Burresi, D. van Osten, T. Kampfrath, H. Schoenmaker, R. Heideman, A. Leinse & L. Kuipers, *Probing the magnetic field of light at optical frequencies*, Science Express October 2009. Cited on page 106.

65 K. L. Kelly, *Color designations for colored lights*, Journal of the Optical Society of America **33**, pp. 627–632, 1943. Cited on page 109.

66 About the polarization pattern and its use by insects, see K. Pfeiffer & U. Homberg, *Coding of azimuthal directions via time-compensated combination of celestial compass cues*, Current Biology **17**, pp. 960–965, 2007. Cited on pages 112 and 418.

67 The best introduction to mirages are the web pages by Andrew Young at aty.sdsu.edu/mirages/mirintro.html. See also the pages aty.sdsu.edu/bibliog/alphindex.html and aty.sdsu.edu/bibliog/toc.html. He explains the many types that exist: inferior mirages, superior mirages, fata morganas, mock mirages, Wegener-type mirages, Nachspiegelung, and also gives many references, clearly distinguishing which ones give correct and which one give incorrect explanations. He also simulates mirages, as explained on the page aty.sdsu.edu/mirages/mirsims/mirsimintro.html. There is no modern review article on the topic yet. See also A. T. Young, G. W. Kattawar & P. Parviainen, *Sunset science I – the mock mirage*, Applied Optics **36**, pp. 2689–2700, 1997. For a further aspect of mirages, see G. Horváth, J. Gál & R. Wehner, *Why are water-seeking insects not attracted by mirages? The polarization pattern of mirages*, Naturwissenschaften **83**, pp. 300–303, 1997. Cited on page 113.

68 W. K. Haidinger, *Über das direkte Erkennen des polarisierten Lichts*, Poggendorf's Annalen **63**, pp. 29–39, 1844, W. K. Haidinger, *Beobachtung des Lichtpolarisationsbüschels in geradlinig polarisiertem Lichte*, Poggendorf's Annalen **68**, pp. 73–87, 1846, W. K. Haidinger, *Dauer des Eindrucks der Polarisationsbüschel auf der Netzhaut*, Poggendorf's Annalen **93**, pp. 318–320, 1854. Cited on page 114.

69 See the chapter on polarization brushes in Marcel G. J. Minnaert, *Light and Colour in the Outdoors*, Springer, 1993, or the original book series, Marcel G. J. Minnaert, *De natuurkunde van 't vrije veld*, Thieme & Cie, 1937. For more details, see G. P. Mission, *Form and behaviour of Haidinger's brushes*, Ophthalmology and Physiological Optics **137**, pp. 392–396, 1993, or J. Grebe-Ellis, *Zum Haidinger-Büschel*, 2002, at didaktik.physik.hu-berlin.de/forschung/optik/download/veroeffentlichungn/haidinger.pdf. On the birefringence of the eye, see L. Bour, *Een eigenaardige speling der natuur*, Nederlands tijdschrift voor natuurkunde **67**, pp. 362–364, December 2001. In particular, a photograph of the eye using linear polarized illumination and taken through an analyser shows a black *cross* inside the pupil. Cited on page 114.

70 T. W. Cronin & J. Marshall, *Patterns and properties of polarized light in air and water*, Philosophical Transactions of the Royal Society B **366**, pp. 619–626, 2011, available free online at rstb.royalsocietypublishing.org. Cited on page 115.

71 Edward M. Purcell, *Electricity and Magnetism – Berkeley Physics Course Volume 2*, McGraw–Hill, 1984. Cited on page 116.

72 This was the book series in twenty volumes by Aaron Bernstein, *Naturwissenschaftliche Volksbücher*, Duncker, 1873-1874. The young Einstein read them, between 1892 and 1894, with 'breathless attention', as he wrote later on. They can still be read in many libraries. Cited on page 119.

73 On the ways to levitate and manipulate small glass beads with lasers, see the article D. McGloin, *Optical tweezers: 20 years on*, Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences **364**, pp. 3521–3537, 2006. The photographs shown on page 120 are from T. Li, S. Kheifets, D. Medellin & M. G. Raizen, *Measurement of the instantaneous velocity of a Brownian particle*, Science **328**, pp. 1673–1675, 2010, and T. Li, S. Kheifets & M. G. Raizen, *Millikelvin cooling of an optically trapped microsphere in vacuum*, Nature Physics 7, pp. 527–530, 2011. Cited on page 120.

74 The first correct explanation of the light mill was given by Osborne Reynolds, *On certain dimensional properties of matter in the gaseous state*, Royal Society Philosophical Transactions Part 2, 1879. The best discussion is the one given on the web by Phil Gibbs, in the frequently asked question list of the usenet news group sci.physics; it is available at the www.desy.de/user/projects/Physics/General/LightMill/light-mill.html website. A film of a rotating radiometer is found in commons.wikimedia.org. Cited on page 122.

75 P. Lebedew, *Untersuchungen über die Druckkräfte des Lichtes*, Annalen der Physik **6**, pp. 307–458, 1901. Lebedew confirmed Kepler's result that light pressure is the basis for the change of direction of the tails of comet when they circle around the Sun. Cited on page 122.

76 P. Galajda & P. Ormos, Applied Physics Letters **78**, p. 249, 2001. Cited on page 122.

77 A short overview is given by Miles Padgett & Les Allen, *Optical tweezers and spanners*, Physics World pp. 35–38, September 1997. The original papers by Ashkin's group are A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm & S. Chu, *Observation of a gradient force optical trap for dielectric particles*, Optics Letters **11**, p. 288, 1986, and A. Askin,

J. M. Dziedzic & T. Yamane, *Optical trapping and manipulation of single cells using infrared laser beams*, Nature **330**, p. 769, 1987. A pedagogical explanation on optical spanners, together with a way to build one, can be found in D. N. Moothoo, J. Arlt, R. S. Conroy, F. Akerboom, A. Voit & K. Dholakia, *Beth's experiment using optical tweezers*, American Journal of Physics **69**, pp. 271–276, 2001, and in S. P. Smith, S. R. Bhalotra, A. L. Brody, B. L. Brown, E. K. Boyda & M. Prentiss, *Inexpensive optical tweezers for undergraduate laboratories*, American Journal of Physics **67**, pp. 26–35, 1999. Cited on pages 122 and 123.

78 R. A. Beth, *Mechanical detection and measurement of the angular momentum of light*, Physical Review **50**, p. 115, 1936. For modern measurements, see N. B. Simpson, K. Dholakia, L. Allen & M. J. Padgett, *Mechanical equivalence of spin and orbital angular momentum of light: an optical spanner*, Optics Letters **22**, pp. 52–54, 1997, and M. E. J. Friese, T. A. Nieminen, N. R. Heckenberg & H. Rubinsztein-Dunlop, *Optical torque controlled by elliptical polarization*, Optics Letters **23**, pp. 1–3, 1998. See also J. H. Poynting, *The wave motion of a revolving shaft, and a suggestion as to the angular momentum in a beam of circularly polarised light*, Proceedings of the Royal Society London A **82**, pp. 560–567, 1908. Cited on page 124.

79 The photographs are from P. H. Jones, F. Palmisano, F. Bonaccorso, P. G. Gucciardi, G. Calogero, A. C. Ferrari & O. M. Marago, *Rotation detection in light-driven nanorotors*, ACS Nano **3**, pp. 3077–3084, 2009. Cited on pages 124 and 418.

80 A. Valenzuela, G. Haerendel, H. Föppl, F. Melzner, H. Neuss, E. Rieger, J. Stöcker, O. Bauer, H. Höfner & J. Loidl, *The AMPTE artificial comet experiments*, Nature **320**, pp. 700–703, 1986. Cited on page 124.

81 See the Latin text by Dietrich von Freiberg, *De iride et radialibus impressionibus*, *c.* 1315. Cited on page 126.

82 J. Walker, *Multiple rainbows from single drops of water and other liquids*, American Journal of Physics **44**, pp. 421–433, 1976, and his *How to create and observe a dozen rainbows in a single drop of water*, Scientific American **237**, pp. 138–144, 1977. See also K. Sassen, *Angular scattering and rainbow formation in pendant drops*, Journal of the Optical Society of America **69**, pp. 1083–1089, 1979. A beautiful paper with the formulae of the angles of all rainbows is E. Willerding, *Zur Theorie von Regenbögen, Glorien und Halos*, 2003, preprint on the internet. It also provides sources for programs that allow to simulate rainbows on a personal computer. Cited on page 126.

83 There are also other ways to see the green ray, for longer times, namely when a mirage appears at sunset. An explanation with colour photograph is contained in M. Vollmer, *Gespiegelt in besonderen Düften ...– Oasen, Seeungeheuer und weitere Spielereien der Fata Morgana*, Physikalische Blätter **54**, pp. 903–909, 1998. Cited on page 127.

84 The resulting colouring of the Sun's rim is shown clearly on Andrew Young's web page mintaka.sdsu.edu/GF/explain/simulations/std/rims.html. His website mintaka.sdsu.edu/GF offers the best explanation of the green flash, including the various types that exist (explained at mintaka.sdsu.edu/GF/papers/Zenit/glance.html), how to observe it, and the numerous physical effects involved. Detailed simulations and extensive material is available. See also his paper A. T. Young, *Sunset science – III. Visual adaptation and green flashes*, Journal of the Optical Society of America A **17**, pp. 2129–2139, 2000. Cited on pages 126 and 127.

85 See the wonderful website by Les Cowley on atmospheric optics, www.atoptics.co.uk. Or the book David K. Lynch & William Livingston, *Color and Light in Nature*, second edition, Cambridge University Press, 2001. They updated and expanded the fascination for colours in nature – such as, for example, the halos around the Moon and the Sun, or the colour of shadows – that was started by the beautiful and classic book already mentioned earlier on: Marcel G. J. Minnaert, *Light and Colour in the Outdoors*, Springer, 1993, an updated version based on the wonderful original book series Marcel G. J. Minnaert, *De natuurkunde van 't vrije veld*, Thieme & Cie, 1937. Cited on page 127.

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86 About the colour of the ozone layer seen at dawn and the colour of the sky in general, see G. Hoeppe, *Die blaue Stunde des Ozons*, Sterne und Weltraum pp. 632–639, August 2001, and also his extensive book Götz Hoeppe, *Blau: Die Farbe des Himmels*, Spektrum Akademischer Verlag, 1999, also available in English as the extended revision Götz Hoeppe, *Why the Sky is Blue: Discovering the Color of Life*, Princeton University Press, 2007, This beautiful text also tells why bacteria were essential to produce the colour of the sky. Cited on page 128.

87 The beautiful *RGB Color Atlas* from 2011 by Tauba Auerbach is presented on her astonishing website at taubaauerbach.com/view.php?id=286&alt=698. The books were produced together with Daniel E. Kelm. In fact, they produced three such books, with spines in different directions, as shown on the website. Cited on page 129.

88 This famous discovery is by Brent Berlin & Paul Kay, *Basic Color Terms: Their Universality and Evolution*, University of California Press, 1969. The status of their decades-long *world colour survey* is summarized on www1.icsi.berkeley.edu/wcs. Of course there are also ongoing studies to find possible exceptions; but the basic structure is solid, as shown in the conference proceedings C. L. Hardin & Luisa Maffi, *Colour Categories in Thought and Language*, Cambridge University Press, 1997. Cited on page 130.

89 For a thorough discussion of the various velocities connected to wave trains, see the classic text by Louis Brillouin, *Wave Propagation and Group Velocity*, Academic Press, New York, 1960. It expands in detail the theme discussed by Arnold Sommerfeld, *Über die Fortpflanzung des Lichtes in dispergierenden Medien*, Annalen der Physik, 4th series, **44**, pp. 177–202, 1914. See also Arnold Sommerfeld, *Optik*, Dietrichssche Verlagsbuchandlung, Wiesbaden 1950, section 22. An English translation Arnold Sommerfeld, *Lectures on Theoretical Physics: Optics*, 1954, is also available. Cited on pages 133 and 135.

90 Changing the group velocity in fibres is now even possible on demand, as shown by M. González-Herráez, K. -Y. Song & L. Thévenaz, *Optically controlled slow and fast light in optical fibers using stimulated Brillouin scattering*, Applied Physics Letters **87**, p. 081113, 2005. They demonstrate group velocities from 0.24c to plus infinity and beyond, to negative values.

Another experiment was carried out by S. Chu & S. Wong, *Linear pulse propagation in an absorbing medium*, Physical Review Letters **48**, pp. 738–741, 1982. See also S. Chu & D. Styer, *Answer to question #52. Group velocity and energy propagation*, American Journal of Physics **66**, pp. 659–661, 1998. Another example was described in 1993 by the group of Raymond Chiao for the case of certain nonlinear materials in R. Chiao, P. G. Kwait & A. M. Steinberg, *Faster than light?*, Scientific American **269**, p. 52, August 1993, and R. Y. Chiao, A. E. Kozhekin & G. Kurizki, *Tachyonlike excitations in inverted two-level media*, Physical Review Letters 77, pp. 1254–1257, 1996. On still another experimental set-up using anomalous dispersion in caesium gas, see L. J. Wang, A. Kuzmich & A. Dogarin, *Gain-assisted superluminal light propagation*, Nature **406**, pp. 277–279, 20 July 2000. Cited on page 135.

91 G. Nimtz, A. Enders & H. Spieker, Journal de Physique I (Paris) **4**, p. 565, 1994. Unfortunately, Nimtz himself seems to believe that he transported energy or signals faster than light; he is aided by the often badly prepared critics of his quite sophisticated experiments. See A. Enders & G. Nimtz, Physikalische Blätter **49**, p. 1119, Dezember 1993, and the weak replies in Physikalische Blätter **50**, p. 313, April 1994. See also A. M. Steinberg, Journal de Physique I (Paris) **4**, p. 1813, 1994, A. M. Steinberg, P. G. Kwiat & R. Y. Chiao, Physical Review Letters **71**, pp. 708–711, 1993, and A. Ranfagni, P. Fabeni, G. P. Pazzi & D. Mugnai, Physical Review E **48**, p. 1453, 1993. Cited on page 136.

92 Y. P. Terletskii, *Paradoxes in the Theory of Relativity*, Plenum Press, 1968. Cited on page 135.

93 See the excellent explanation by Kirk T. McDonald, *Negative group velocity*, American Journal of Physics **69**, pp. 607–614, 2001. Cited on page 135.

94 A summary of all evidence about the motion of the aether is given by R. S. Shankland, S. W. McCuskey, F. C. Leone & G. Kuerti, *New analysis of the interferometer observations of Dayton C. Miller*, Review of Modern Physics **27**, pp. 167–178, 1955. An older text is H. Witte, Annalen der Physik **26**, p. 235, 1908. Cited on page 137.

95 The history of the concept of vacuum can be found in the book by E. Grant, *Much Ado About Nothing*, Cambridge University Press, 1981, and in the extensive reference text by Edmund T. Whittaker, *A History of the Theories of Aether and Electricity*, Volume 1: *The Classical Theories*, Volume 2: *The Modern Theories*, Tomash Publishers, American Institute of Physics 1951, 1987.

The various *aether models* – gears, tubes, vortices – proposed in the nineteenth century were dropped for various reasons. Since many models used to explain electric and magnetic fields as motion of some entities, it was concluded that the speed of light would depend on electric or magnetic fields. One type of field was usually described by linear motion of the entities, the other by rotatory or twisting motion; both assignments are possible. As a consequence, aether must be a somewhat strange fluid that flows perfectly, but that resists rotation of volume elements, as McCullogh deduced in 1839. However, experiments show that the speed of light in vacuum does not depend on electromagnetic field intensity. Vortices were dropped because real world vortices were found out to be unstable. All models received their final blow when they failed to meet the requirements of special relativity. Cited on pages 137 and 138.

96 M. von Laue, *Zur Thermodynamik der Interferenzerscheinungen*, Annalen der Physik **20**, pp. 365–378, 1906. Cited on page 138.

97 See, for example, the review by L. C. Tu, J. Luo & G. T. Gillies, *The mass of the photon*, Reports on Progress in Physics **68**, pp. 77–130, 2005. Cited on page 139.

98 To learn about the geometric phase in optics, see E. J. Galvez & P. M. Koch, *Use of four mirrors to rotate linear polarization but preserve input-ouput collinearity II*, Journal of the Optical Society of America **14**, pp. 3410–3414, 1999, E. J. Galvez & C. D. Holmes, *Geometric physe of optical rotators*, Journal of the Optical Society of America **16**, pp. 1981–1985, 1999, as well as various other papers by Enrique Galvez. See also the paper by R. Bhandari, *Geometric phase in interference experiments*, Current Science **67**, pp. 224–-230, 1994. Cited on pages 140 and 375.

99 A useful collection of historical papers is Frank Wilczek & Alfred Shapere, eds., *Geometric Phases in Physics*, World Scientific, 1989. See also the vivid paper M. Berry, *Pancharatnam, virtuoso of the Poincaré sphere: an appreciation*, Current Science **67**, pp. 220–223, 1994. Cited on page 142.

100 Dénes Száz & Gábor Horváth, *Success of sky-polarimetric Viking navigation: revealing the chance Viking sailors could reach Greenland from Norway*, Royal Society Open Science 5, p. 172187, 2018. Cited on page 142.

101 Stephen G. Lipson, David S. Tannhauser & Henry S. Lipson, *Optical Physics*, Cambridge University Press, 1995. Cited on page 143.

102 The original paper is J. F. Nye & M. V. Berry, *Dislocations in wave trains*, Proceedings of the Royal Society A **336**, pp. 165–190, 1974. A new summary is M. V. Berry, *Exploring the colours of dark light*, New Journal of Physics **4**, pp. 74.1–74.14, 2002, free online at www.njp.org. Cited on page 143.

103 O. Arteaga, E. Garcia-Caurel & R. Ossikovski, *Stern-Gerlach experiment with light: separating photons by spin with the method of A. Fresnel*, Optics Express **27**, pp. 4758–4768, 2019. Cited on page 143.

104 M. Arrayás & J. L. Trueba, *Electromagnetic torus knots*, preprint at arxiv.org/abs/1106.1122. Cited on page 144.

105 There are many good introductions to optics in every library. A good introduction that explains the fundamental concepts step by step is the relevant chapter in the physics book by Eric Mazur, available on the internet; one day it will be published by Prentice Hall. Cited on page 145.

106 A good overview of the invention and the life of Frits Zernike is given by Menno van Dijk, *Ken uw klassieken: hoe Frits Zernike fasecontrast ontdekte*, Nederlands tijdschrift voor natuurkunde **71**, pp. 194–196, June 2005. Cited on page 145.

107 See its www.cie.co.at/cie website. Cited on page 148.

108 P. D. Jones, M. New, D. E. Parker, S. Martin & I. G. Rigor, *Surface air temperature and its changes over the past 150 years*, Reviews of Geophysics **37**, pp. 173–199, May 1999. Cited on page 149.

109 He recalls this episode from 1933 in M. Planck, *Mein Besuch bei Adolf Hitler*, Physikalische Blätter p. 143, 1947. Cited on page 149.

110 Pictures of objects in a red hot oven and at room temperature are also shown in C. H. Bennett, *Demons, engines and the second law*, Scientific American **255**, pp. 108–117, November 1987. Cited on page 150.

111 If you want to read more on the topic, have a look at the classic text by Warren J. Smith, *Modern Optical Engineering : the Design of Optical Systems*, 3rd edition, McGraw-Hill, 2000. The main historic reference is R. Clausius, *Über die Concentration von Wärme und Lichtstrahlen und die Gränzen ihrer Wirkung*, Poggendorff's Annalen der Physik **121**, pp. 1–44, 1864. Cited on pages 151 and 214.

112 Measured values and ranges for physical quantities are collected in Horst Völz & Peter Ackermann, *Die Welt in Zahlen*, Spektrum Akademischer Verlag, 1996. Cited on page 154.

113 See, for example, K. Codling & L. J. Frasinski, *Coulomb explosion of simple molecules in intense laser fields*, Contemporary Physics **35**, pp. 243–255, 1994. Cited on page 154.

114 The standard reference on the propagation of light is Max Born & Emil Wolf, *Principles of Optics – Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, Pergamon Press, 6th edition, 1998. Cited on page 157.

115 E. D. Palik, *Handbook of optical constants of solids*, Academic Publishing, 1998. Cited on page 160.

116 More mirage photographs, even mirage films, can be found on www.polarimage.fi/mirages/mirages.htm and virtual.finland.fi/netcomm/news/showarticle.asp?intNWSAID=25722. Cited on page 161.

117 E. J. J. Kirchner, *De uitvinding van het telescoop in 1608: gewoon twee lenzen*, Nederlands tijdschrift voor natuurkunde **74**, pp. 356–361, 2008. Cited on page 164.

118 A fascinating overview about what people have achieved in this domain up to now is given in the classic reference text by Rolf Riehker, *Fernrohre und ihre Meister*, VEB Verlag Technik, second edition, 1990. See also by Peter Manly, *Unusual Telescopes*, Cambridge University Press, 1991, and Henry C. King, *The History of the Telescope*, Dover, 2003. Cited on page 164.

119 An introduction to the topic of the 22° halo, the 46° halo, Sun dogs, and the many other arcs and bows that can be seen around the Sun, see the beautifully illustrated paper by R. Greenler, *Lichterscheinungen, Eiskristalle und Himmelsarchäologie*, Physikalische Blätter **54**, pp. 133–139, 1998, or the book Robert Greenler, *Rainbows, Halos, and Glories*, Cambridge University Press, 1980. Cited on page 166.

120 J. Aizenberg, V. C. Sundar, A. D. Yablon, J. C. Weaver & G. Chen, *Biological glass fibers: Correlation between optical and structural properties*, Proceedings of the National Academy of Sciences **101**, pp. 3358–3363, 2004, also available online for free at www.pnas.org. Cited on page 167.

121 K. Franz & al., *Müller cells are living optical fibers in the vertebrate retina*, Proceedings of the National Academy of Sciences **104**, pp. 8287–8292, 2007. Cited on page 168.

122 A complete list of data and arguments showing that the hair of polar bears have no fibre function is found on the pages it.stlawu.edu/~koon/mar-ref.html and it.stlawu.edu/~koon/polar.html. Cited on page 168.

123 The prediction of negative refraction is due to V. G. Veselago, *The electrodynamics of substances with simultaneously negative values of ε and μ*, Soviet Physics Uspekhi **10**, p. 509, 1968. (The original paper in Russian is from 1967.) The explanation with different refraction directions was published by P. M. Valanju, R. M. Walser & A. P. Valanju, *Wave refraction in negative-index media: always positive and very inhomogeneous*, Physical Review Letters **88**, p. 187401, 8 May 2002. Also Fermat's principle is corrected, as explained in V. G. Veselago, *About the wording of Fermat's principle for light propagation in media with negative refraction index*, arxiv.org/abs/cond-mat/0203451. Cited on page 168.

124 The first example of material system with a negative refraction index were presented by David Smith and his team. R. A. Schelby, D. R. Smith & S. Schultz, *Experimental verification of a negative index of refraction*, Science **292**, p. 77-79, 2001. More recent examples are A. A. Houck, J. B. Brock & I. L. Chuang, *Experimental observations of a left-handed material that obeys Snell's law*, Physical Review Letters **90**, p. 137401, 2003, C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbah & M. Tanielian, *Experimental verification and simulation of negative index of refraction using Snell's law*, Physical Review Letters **90**, p. 107401, 2003. S. Foteinopoulou, E. N. Economou & C. M. Soukoulis, *Refraction in media with a negative refractive index*, Physical Review Letters **90**, p. 107402, 2003. Cited on page 168.

125 S. A. Ramakrishna, *Physics of negative refractive index materials*, Reorts on Progress of Physics **68**, pp. 449–521, 2005. Cited on pages 168 and 169.

126 J. Pendry, *Negegative refraction makes a perfect lens*, Physical Review Letters **85**, p. 3966, 2000. See also J. B. Pendry, D. Schurig & D. R. Smith, *Controlling electromagnetic fields*, Science **312**, pp. 1780–1782, 2006, and D. Schurig, J. J. Mock, B. J. Justice,

S. A. Cummer, J. B. Pendry, A. F. Starr & D. R. Smith, *Metamaterial electromagnetic cloak at microwave frequencies*, Science **314**, pp. 977–980, 2006. Cited on page 169.

127 On metamaterials, see A. Lai, C. Caloz & T. Itoh, *Composite rightleft-handed transmission metamaterials*, IEEE Microwave Magazine **5**, pp. 34–50, September 2004. Cited on page 169.

128 M. Zedler & P. Russer, *Investigation on the Dispersion Relation of a 3D LC-based Metamaterial with an Omnidirectional Left-Handed Frequency Band*, 2006 International Microwave Symposium Digest, San Francisco pp. 1477–1479, 2006. M. Zedler, C. Caloz & P. Russer, *A 3D Isotropic left-handed metamaterial based on the rotated transmission line matrix (TLM) scheme*, IEEE Transactions on Microwave Theory and Techniques **55**, pp. 2930–2941, 2007. Cited on page 169.

129 Read Grimaldi's text online at fermi.imss.fi.it/rd/bdv?/bdviewer/bid=000000300682. Cited on page 170.

130 James E. Faller & E. Joseph Wampler, *The lunar laser reflector*, Scientific American pp. 38–49, March 1970. Cited on page 170.

131 Neil Armstrong of Apollo 11, Jim Lovell of Apollo 8 and Apollo 13, and Jim Irwin of Apollo 15 extensively searched for it and then made negative statements, as told in Science News p. 423, 24 & 31 December 1994. From the space shuttle however, which circles only a few hundred kilometres above the Earth, the wall *can* be seen when the Sun is low enough such that the wall appears wider through its own shadow, as explained in Science News **149**, p. 301, 1996. Cited on page 171.

132 S. W. Hell, *Strategy for far-field optical imaging and writing without diffraction limit*, Physics Letters A **326**, pp. 140–145, 2004, see also V. Westphal & S. W. Hell, *Nanoscale resolution in the focal plane of an optical microscope*, Physical Review Letters **94**, p. 143903, 2005, and V. Westphal, J. Seeger, T. Salditt & S. W. Hell, *Stimulated emission depletion microscopy on lithographic microstructures*, Journal of Physics B **38**, pp. S695–S705, 2005. Cited on page 173.

133 M. Shih, M. Segev & G. Salamo, *Three-dimensional spiraling of interacting spatial solitons*, Physical Review Letters **78**, pp. 2551–2554, 1997. See also the more readable paper by M. Segev & G. Stegeman, *Self-trapping of optical beams: spatial solitons*, Physics Today **51**, pp. 42–48, August 1998. Cited on page 174.

134 On Talbot-Lau imaging with X-rays, see for example the paper A. Momose & al., *X-ray phase imaging: from synchrotron to hospital*, Philosophical Transanactions of the Royal Society A **372**, p. 20130023, 2014, free to read at rsta.royalsocietypublishing.org. Cited on page 175.

135 See the wonderful summary by Frank Schaeffel, *Processing of information in the human visual system*, pp. 1–33, in Alexander Hornberg, editor, *Handbook of Machine Vision*, Wiley-VCH, 2006. Cited on page 187.

136 W. H. Ehrenstein & B. Lingelbach, *Das Hermann–Gitter*, Physik in unserer Zeit **6**, pp. 263–268, 2002. The journal also shows a colour variation of these lattices. Cited on page 187.

137 To enjoy many other flowers under ultraviolet illumination, go to the extensive collection at www.naturfotograf.com/index2. Cited on page 190.

138 For an example of such research, see S. A. Baccus, B. P. Olveczky, M. Manu & M. Meister, *A retinal circuit that computes object motion*, Journal of Neuroscience **28**, pp. 6807–6817, 2008. For an older review, see M. Meister & M. J. Berry, *The neural code of the retina*, Neuron **22**, pp. 435–450, 1999. Cited on page 192.

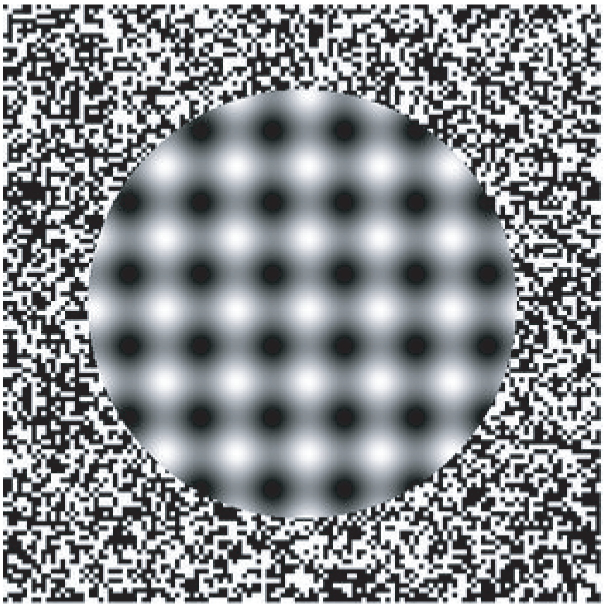

FIGURE 194 The Ouchi illusion of motion.

139 See for example, the summary by D. M. Berson, *Strange vision: ganglion cells as circadian photoreceptors*, Trends in Neurosciences **26**, pp. 314–320, 2003. Cited on page 192.

140 This amazing story is from the wonderful blog watchingtheworldwakeup.blogspot.de/2008/11/mountain-biking-moonlight-color-vision.html – a blog that shows what passion for nature is. Cited on page 194.

141 The eye sensitivity myth is debunked in detail by B. H. Soffer & D. K. Lynch, *Some paradoxes, errors, and resolutions concerning the spectral optimization of human vision*, American Journal of Physics **67**, pp. 946–953, 1999. Cited on page 194.

142 A. Brückner, J. Duparré, F. Wippermann, R. Leitel, P. Dannberg & A. Bräuer, *Ultra-compact close-up microoptical imaging system*, **Proceedings of the SPIE**, 7786, p. 77860A, 2010. Cited on page 195.

143 David R. Williams, *Supernormal Vision*, Science News **152**, pp. 312–313, 15 November 1997. See also aria.cvs.rochester.edu/team/williams_d/ as well as the photographs at roorda.vision.berkeley.edu/ of the interior of living human eyes. Their last publication is A. Roorda, A. Metha, P. Lennie & D. R. Williams, *Packing arrangement of the three cone classes in the primate retina*, Vision Research **41**, pp. 1291–1306, 2001. Cited on page 196.

144 D. Hillmann, H. Spahr, C. Pfäffle, H. Sudkamp, G. Franke & G. Hüttmann, *In vivo optical imaging of physiological responses to photostimulation in human photoreceptors*, Proceedings of the National Academy of Sciences (USA) **113**, pp. 13138–13143, 2016. Cited on page 196.

145 See, for example, the beautiful book by Simon Ings, *Das Auge - Meisterstück der Evolution*, Hoffmann & Campe, 2008. On the limitations of the eye, see Thomas Ditzinger, *Illusionen des Sehens: Eine Reise in die Welt der visuellen Wahrnehmung*, Südwest, 1998, which includes the fascinating *Ouchi illusion* shown in Figure 194. Cited on page 199.

146 This happened to Giovanni Bellini (b. *c.* 1430 Venice, d. 1516 Venice) the great Renaissance painter, who even put this experience into writing, thus producing one of the greatest 'gaffes' ever. If you take a photograph of the effect with a remotely controlled camera, you can prove that your camera is holy as well. Cited on page 200.

147 S. R. Wilk, *How retroreflectors really work*, Optics & Photonics News, pp. 6–7, December 1993. Cited on page 200.

148 G. G. P. van Gorkum, *Introduction to Zeus displays*, Philips Journal of Research **50**, pp. 269–280, 1996. See also N. Lambert, E. A. Montie, T. S. Baller, G. G. P. van Gorkum, B. H. Hendriks, P. H. Trompenaars & S. T. de Zwart, *Transport and extraction in Zeus displays*, Philips Journal of Research **50**, pp. 295–305, 1996. Cited on page 202.

149 Among the many papers on pit vipers, see the excellent summary by B. Schwarzschild, *Neural-network model may explain the surprisingly good infrared vision of snakes*, Physics Today pp. 18–20, September 2006; it is based on the fascinating results by A. B. Sichert, P. Friedel & J. L. van Hemmen, *Snake's perspective on heat: reconstruction of input using an imperfect detection system*, Physical Review Letters **97**, p. 068105, 2006. Cited on page 202.

150 J. Cybulski, J. Clements & M. Prakash, *Foldscope: Origami-based paper microscope*, preprint at arxiv.org/abs/1403.1211. Cited on page 205.

151 For an explanation, see S. Y. van der Werf, G. P. Können & W. H. Lehn, *Novaya Zemlya effect and sunsets*, Applied Optics **42**, pp. 367–378, 2003. Cited on page 205.

152 E. W. Streed, A. Jechow, B. G. Norton & D. Kielpinski, *Absorption imaging of a single atom*, Nature Communications, **3**, p. 933, 2012, preprint at arxiv.org/abs/1201.5280. Cited on page 205.

153 This problem was suggested by Vladimir Surdin. Cited on page 205.

154 For deviations from the geometric 'law' of reflection see M. Merano, A. Aiello, M. P. van Exter & J. P. Woerdman, *Observing angular deviations in the specular reflection of a light beam*, Nature Photonics **3**, pp. 337 – 340, 2009. See also M. Merano, A. Aiello, G. W. 't Hooft, M. P. van Exter, E. R. Eliel & J. P. Woerdman, *Observation of Goos-Hänchen shifts in metallic reflection*, Optics Express **15**, pp. 15928–15934, 2007. This beautiful research field is in need of a good review article. For a measurement of the time delay in total reflection, around 28 fs, see D. Chauvat & al., *Timing the total reflection of light*, Physics Letters A **336**, pp. 271–273, 2005. Cited on page 207.

155 Such a claim was implicitly made by D. Mugnai, A. Ranfagni & R. Ruggieri, *Observation of superluminal behaviors in wave propagation*, Physical Review Letters **84**, p. 4830, 2000. An excellent explanation and rebuttal was given by W. A. Rodrigues, D. S. Thober & A. L. Xavier, *Causal explanation for observed superluminal behavior of microwave propagation in free space*, preprint at arxiv.org/abs/physics/0012032. Cited on page 208.

156 If you want to see more on how the world looks for the different types of colour blind, have a look at the webexhibits.org/causesofcolor/2.html or the www.vischeck.com/examples web pages. Cited on page 210.

157 H. Kobayashi & S. Kohshima, *Unique morphology of the human eye*, Nature **387**, pp. 767–768, 1997. They explored 88 primate species. Cited on page 210.

158 A. N. Heard-Booth & E. C. Kirk, *The influence of maximum running speed on eye size: a test of Leuckart's law in mammals*, The Anatomical Record **295**, pp. 1053–1062, 2012. Cited on page 216.

159 Most of the world's experts in lightning are Russian. Two good books are Vladimir A. Rakov & Martin A. Uman, *Lightning: Physics and Effects*, Cambridge University Press, 2003, and Eduard M. Bazelyon & Yuri P. Raizer, *Lightning Physics and Lightning Protection*, Institute of Physics Publishing, 2000. For a simple introduction, see also the lightning section of the webiste www.nrcan-rncan.gc.ca. Cited on page 218.

160 On the life-long passion that drove Luke Howard, see the book by Richard Hamblyn, *The Invention of Clouds*, Macmillan 2001. Cited on page 218.

161 See J. Latham, *The electrification of thunderstorms*, Quartely Journal of the Royal Meteorological Society **107**, pp. 277–289, 1981. For a more recent and wider review, see Earle R. Williams, *The tripole structure of thunderstorms*, Journal of Geophysical Research **94**, pp. 13151–13167, 1989. See also the book by the National Research Council Staff, *The Earth's Electrical Environment*, Studies in Geophysics, National Academy Press, 1986. Cited on page 218.

162 The exploration of how charges are separated in clouds is a research field in its own. See, for example, the overview and literature list at enviromom.us/lightning/lightningformation.html. The precise atomic scale mechanism is not fully settled. There are two main reasons: experiments are difficult, and electrification is not fully understood in most known material systems, including the well-known process of rubbing glass rods with a fur. Cited on page 218.

163 A. V. Gurevich & K. P. Zybin, *Runaway breakdown and the mysteries of lightning*, Physics Today **58**, pp. 37–43, May 2005. Cited on page 218.

164 To learn more about atmospheric currents, you may want to have a look at the popularizing review of the US work by E. A. Bering, A. A. Few & J. R. Benbrook, *The global electric circuit*, Physics Today **51**, pp. 24–30, October 1998, or the more technical overview by E. Bering, Reviews of Geophysics (supplement) **33**, p. 845, 1995. Cited on page 221.

165 The use of Schumann resonances in the Earth–ionosphere capacitor for this research field is explained in K. Schlegel & M. Füllerkrug, *Weltweite Ortung von Blitzen*, Physik in unserer Zeit **33**, pp. 257–261, 2002. Cited on page 222.

166 J. R. Dwyer, M. A. Uman, H. K. Rassoul, M. Al-Dayeh, E. L. Caraway, J. Jerauld, V. A. Rakov, D. M. Jordan, K. J. Rambo, V. Corbin & B. Wright, *Energetic radiation produced by rocket-triggered lightning*, Science **299**, pp. 694–697, 2003. Cited on page 221.

167 J. R. Dwyer, *A fundamental limit on electric fields in air*, Geophysical Research Letters **30**, p. 2055, 2003. Cited on page 221.

168 B. M. Smirnov, *Physics of ball lightning*, Physics Reports **224**, pp. 151–236, 1993. See also D. Finkelstein & J. Rubinstein, *Ball lightning*, Physical Review **135**, pp. 390–396, 1964. For more folklore on the topic, just search the world wide web. Cited on page 222.

169 G. D. Shabanov, *The optical properties of long-lived luminous formations*, Technical Physics Letters **28**, pp. 164–166, 2002, A. I. Egorov & S. I. Stepanov, *Long-lived plasmoids produced in humid air as analogues of ball lightning*, Technical Physics **47**, pp. 1584–1586, 2002, A. E. Egorov, S. I. Stepanov & G. D. Shabanov, Physics Uspekhi *Laboratory demonstration of ball lightning*, **47**, pp. 99–101, 2004, and G. D. Shabanov & B. Yu. Sokolovskii, *Macroscopic separation of charges in a pulsed electric discharge*, Plasma Physics Reports **31**, pp. 512–518, 2005. (All these are English translations of earlier Russian papers.) See the websites biod.pnpi.spb.ru/pages_ru/Stepanov/index.html stealthtank.narod.ru, balllightning.narod.ru/hvewd.html and www.ipp.mpg.de/ippcms/eng/presse/pi/05_06_pi.html, for more details and more spectacular films. Cited on page 222.

170 G. Silva Paiva, A. C. Pavão, E. Alpes de Vasconcelos, O. Mendes & E. F. da Silva, *Production of ball-lightning-like luminous balls by electrical discharges in silicon*, Physics Review Letters **98**, p. 048501, 2007. Cited on page 223.

171 For a recent summary, see S. Parrott, arxiv.org/abs/gr-qc/9711027. See also T. A. Abbott & D. J. Griffiths, *Acceleration without radiation*, American Journal of Physics **53**, pp. 1203–1211, 1985. See also A. Kovetz & G. E. Tauber, *Radiation from an accelerated charge and the principle of equivalence*, American Journal of Physics **37**, pp. 382–385, 1969. Cited on page 230.

172 C. de Almeida & A. Saa, *The radiation of a uniformly accelerated charge is beyond the horizon: a simple derivation*, American Journal of Physics **74**, pp. 154–158, 2006. Cited on page 231.

173 A summary on these well-known simulations is G. A. Glatzmaier, *Geodynamo simulations - how realistic are they?*, Ann. Rev. Earth Planet. Sci. **30**, pp. 237–257, 2002. A central experimental confirmation is J. Zhang, X. D. Song, Y. C. Li, P. G. Richards, X. L. Sun & F. Waldhauser, *Inner core differential motion confirmed by earthquake doublet waveform doublets*, Science **309**, pp. 1357–1360, 2005. Cited on page 226.

174 An excellent review is E. H. Brandt, *Levitation in Physics*, Science **243**, pp. 349–355, 1989. Cited on pages 226 and 228.

175 See the article by R. Tuckermann, S. Bauerecker & B. Neidhart, *Levitation in Ultraschallfeldern – Schwebende Tröpfchen*, Physik in unserer Zeit **32**, pp. 69–75, February 2001. Liquid drops up to 1 g have been levitated in this way. Cited on page 226.

176 F. C. Moon & P. Z. Chang, *Superconducting Levitation – Applications to Bearings and Magnetic Transportation*, Wiley & Sons, 1994. Cited on pages 227 and 228.

177 W. T. Scott, *Who was Earnshaw?*, American Journal of Physics **27**, pp. 418–419, 1959. Cited on page 227.

178 The trick is to show that div $\boldsymbol{E} = 0$, curl $\boldsymbol{E} = 0$, thus $E\nabla^2 E = 0$ and, from this, $\nabla^2 E^2 \geqslant 0$; there are thus no local electric field maxima in the absence of free charges. The same proof works for the magnetic field. However, bodies with dielectric constants *lower* than their environment *can* be levitated in static electric fields. An example is gas bubbles in liquids, as shown by T. B. Jones & G. W. Bliss, *Bubble dielectrophoresis*, Journal of Applied Physics **48**, pp. 1412–1417, 1977. Cited on page 227.

179 B. Scharlau, V. Nordmeier & H. J. Schlichting, *Magnetische Levitation*, in Deutsche Physikalische Gesellschaft, (editor) *Didaktik der Physik*, Lehmanns, 2003. Cited on pages 228 and 229.

180 See A. K. Geim, M. D. Simon, M. I. Boamfa & L. O. Heflinger, *Magnet levitation at your fingertips*, Nature **400**, pp. 323–324, 1999. Cited on page 228.

181 The first photographs of a single *ion* were in W. Neuhauser, M. Hohenstatt, P. E. Toschek & H. Dehmelt, *Localized visible* Ba^+ *mono-ion oscillator*, Physical Review A **22**, pp. 1137–1140, 1980. See also D. J. Wineland & W. M. Itano, Physics Letters A **82**, p. 75, 1981, as well as F. Dietrich & H. Walter, Physical Review Letters **58**, p. 203, 1987.

For single *atoms*, see photographs in Z. Hu & H. J. Kimble, Optics Letters **1**, p. 1888, 1994, F. Ruschewitz, D. Bettermann, J. L. Peng & W. Ertmer, Europhysics Letters **34**, p. 651, 1996, D. Haubrich, H. Schadwinkel, F. Strauch, B. Ueberholz, R. Wynands & D. Meschede, Europhysics Letters **34**, p. 663, 1996. Cited on page 228.

182 See for example Mark Buchanan, *And God said...let there be levitating strawberries, flying frogs and humans that hover over Seattle*, New Scientist pp. 42–43, 26 July 1997, or

C. Wu, *Floating frogs*, Science News **152**, pp. 632–363, 6 December 1997, and C. Wu, *Molecular magnetism takes off*, Physics World April 1997, page 28. The experiments by Andre Geim, Jan Kees Maan, Humberto Carmona and Peter Main were made public by P. Rodgers, Physics World **10**, p. 28, 1997. Some of the results can be found in M. V. Berry & A. K. Geim, *Of flying frogs and levitrons*, European Journal of Physics **18**, pp. 307–313, 1997. See also their www.ru.nl/hfml/research/levitation/ website. Cited on page 228.

183 The well-known toy allows levitation without the use of any energy source and is called the 'Levitron'. It was *not* invented by Bill Hones of Fascination Toys & Gifts in Seattle, as the www.levitron.com website explains. The toy is discussed by Ron Edge, *Levitation using only permanent magnets*, Physics Teacher **33**, p. 252, April 1995. It is also discussed in M. V. Berry, *The Levitron™: an adiabatic trap for spins*, Proceedings of the Royal Society A **452**, pp. 1207–1220, 1996, (of Berry's phase fame) as well as by M. D. Simon, L. O. Heflinger & S. L. Ridgeway, *Spin stabilized magnetic levitation*, American Journal of Physics **65**, pp. 286–92, 1997, and by T. B. Jones, M. Washizu & R. Gans, *Simple theory for the Levitron*, Journal of Applied Physics **82**, pp. 883–889, 1997. Cited on page 228.

184 The drill trick and the building of a Levitron are described in the beautiful lecture script by Josef Zweck, *Physik im Alltag*, Skript zur Vorlesung im WS 1999/2000 der Universität Regensburg. Cited on page 230.

185 The prediction about quantized levitation is by Stephen B. Haley, *Length quantization in levitation of magnetic microparticles by a mesoscopic superconducting ring*, Physical Review Letters **74**, pp. 3261–3264, 1995. The topic is discussed in more detail in Stephen B. Haley, *Magnetic levitation, suspension, and superconductivity: macroscopic and mesoscopic*, Physical Review B **53**, p. 3506, 1996, reversed in order with Stephen B. Haley, *Quantized levitation of superconducting multiple-ring systems*, Physical Review B **53**, p. 3497, 1996, as well as Stephen B. Haley, *Quantized levitation by multiply-connected superconductors*, LT-21 Proceedings, in Czechoslovak Journal of Physics **46**, p. 2331, 1996. In 1998, there was not yet an experimental confirmation (Stephen Haley, private communication). Cited on page 230.

186 Detailed descriptions of many of these effects can be found in the excellent overview edited by Manfred von Ardenne, Gerhard Musiol & Siegfried Reball, *Effekte der Physik und ihre Anwendungen*, Harri Deutsch, 2004. Cited on page 231.

187 R. Buddakian, K. Weninger, R. A. Hiller & Seth J. Putterman, *Picosecond discharges and stick–slip friction at a moving meniscus of mercury in glass*, Nature **391**, pp. 266–268, 15 January 1998. See also Science News **153**, p. 53, 24 January 1998. Cited on page 232.

188 Henk Swagten & Reinder Coehoorn, *Magnetische tunneljuncties*, Nederlands tijdschrift voor natuurkunde **64**, pp. 279–283, November 1998. Cited on page 232.

189 H. Ohno, D. Chiba, F. Matsukura, T. Omiya, E. Abe, T. Dietl, Y. Ohno & K. Ohtani, *Electric-field control of ferromagnetism*, Nature **408**, pp. 944–946, 21-28 December 2000. Cited on page 232.

190 This effect was discovered by G. Rikken, B. van Tiggelen & A. Sparenberg, *Lichtverstrooiing in een magneetveld*, Nederlands tijdschrift voor natuurkunde **63**, pp. 67–70, maart 1998. Cited on page 234.

191 Vitalij Pecharsky & Karl A. Gschneidner, *Giant magnetocaloric effect in Gd5(Si2Ge2)*, Physical Review Letters **78**, pp. 4494–4497, 1995, and, from the same authors, *Tunable magnetic regenerator alloys with a giant magnetocaloric effect for magnetic* *refrigeration from ~20 to ~2990 K*, Applied Physics Letters **70**, p. 3299, 1997. Cited on page 234.

192 J. Weissmüller, R. N. Viswanath, D. Kramer, P. Zimmer, R. Würschum & H. Gleiter, *Charge-induced reversible strain in a metal*, Science **300**, pp. 312–315, 11 April 2003. Cited on page 235.

193 A. Ajdari, *Electro-osmosis on inhomogeneously charged surfaces*, Physical Review Letters **75**, pp. 755–758, 1995. Cited on page 235.

194 This effect was discovered by J. N. Huiberts, R. Griessen, J. H. Rector, R. J. Wijngarden, J. P. Dekker, D. G. de Groot & N. J. Koeman, *Yttrium and lanthanum hydride films with switchable optical properties*, Nature **380**, pp. 231–234, 1996. A good introduction is R. Griessen, *Schaltbare Spiegel aus Metallhydriden*, Physikalische Blätter **53**, pp. 1207–1209, 1997. Cited on page 236.

195 M. J. Aitken, *Thermoluminescence Dating*, Academic Press, 1985. The precision of the method is far worse that C14 dating, however, as shown by H. Huppertz, *Thermolumineszenzdatierung: eine methodologische Analyse aufgrund gesicherter Befunde*, Peter Lang Verlag, 2000. Cited on page 237.

196 See any book on thermostatics, such as Linda Reichl, *A Modern Course in Statistical Physics*, Wiley, 2nd edition, 1998. Cited on page 239.

197 The Sun emits about $4 \cdot 10^{26}$ W from its mass of $2 \cdot 10^{30}$ kg, about 0.2 mW/kg; a person with an average mass of 75 kg emits about 100 W (you can check this in bed at night), i.e., about 500 times more. Cited on page 239.

198 See for example, J. M. Aguirregabiria, A. Hernandez & M. Rivas, *Velocity fields inside a conducting sphere near a slowly moving charge*, American Journal of Physics **62**, pp. 462–466, 1994. Cited on page 240.

199 This example of electrohydrodynamics was discovered in the 1890s and was explored in detail since. Numerous videos about the phenomenon can be found on the internet, including on the beautiful page at ecfuchs.com/?page=waterbridge, which also includes a literature list. Recent papers are E. C. Fuchs, M. Sammer, A. D. Wexler, P. Kuntke & J. Woisetschläger, *A floating water bridge produces water with excess charge*, Journal of Physics D: Applied Physics **49**, p. 125502, 2016, and A. G. Marín & D. Lohse, *Building water bridges in air: electrohydrodynamics of the floating water bridge*, preprint at arxiv.org/abs/1010.4019. Cited on page 242.

200 Philip Cohen, *Open wide, this won't hurt a bit*, New Scientist p. 5, 3 February 1996. Cited on page 241.

201 For a reference list on bone piezoelectricity, see the website silver.neep.wisc.edu/~lakes/BoneElectr.html. Cited on page 242.

202 J. E. Avron, E. Berg, D. Goldsmith & A. Gordon, *Is the number of photons a classical invariant?*, European Journal of Physics **20**, pp. 153–159, 1999. Cited on page 243.

203 This is deduced from the $g - 2$ measurements, as explained in his Nobel Prize talk by Hans Dehmelt, *Experiments with an isolated subatomic particle at rest*, Reviews of Modern Physics **62**, pp. 525–530, 1990, and in Hans Dehmelt, *Is the electron a composite particle?*, Hyperfine Interactions **81**, pp. 1–3, 1993. Cited on page 243.

204 A good and short introduction is the paper F. Rohrlich, *The self-force and radiation reaction*, American Journal of Physics **68**, pp. 1109–1112, 2000. Cited on page 244.

205 Distinguishing between the thought 'yes' and 'no' is already possible with a simple electroencephalogram. For a video demonstration of the differentaiation of concepts using brain imaging techniques, see www.youtube.com/watch?v=JVLu5_hvr8s. Cited on page 245.

206 C. G. Tsagas, *Magnetic tension and the geometry of the universe*, Physical Review Letters **86**, pp. 5421–5424, 2001. An overview of the topic is C. G. Tsagas, *Geometrical aspects of cosmic magnetic fields*, arxiv.org/abs/gr-qc/0112077. Cited on page 247.

207 A. D. Erlykin & A. W. Wolfendale, *The origin of cosmic rays*, European Journal of Physics **20**, pp. 409–418, 1999, Cited on page 250.

208 See for example the beautiful textbook Stephen C. Stearns & Rolf F. Hoekstra, *Evolution: An Introduction*, Oxford University Press, 2000. For a fascinating story of evolution for non-specialists, see Richard Fortey, *Life – An Unauthorized Biography*, Harper Collins, 1997, or also Menno Schilthuizen, *Frogs, Flies & Dandelions – the Making of Species*, Oxford University Press, 2001. See also Stephen J. Gould, *The Panda's thumb*, W.W. Norton & Co., 1980, one of the several interesting and informative books on evolutionary biology by the best writer in the field. An informative overview over the results of evolution, with the many-branched family tree that it produced, is given on the phylogeny.arizona.edu/tree website. About the results of evolution for human beings, see the informative text by K. Kusch & S. Kusch, *Der Mensch in Zahlen*, Spektrum Akademischer Verlag, 2nd edn., 2000. The epochal work by Charles Darwin, *On the Origin of Species*, can be found on the web, e.g. on on the darwin-online.org.uk websites. Cited on page 254.

209 A simple description is Malcolm Ross Macdonald, *The Origin of Johnny*, Jonathan Cape, 1976. See also Bas Haring, *Kaas en de evolutietheorie*, Houtekiet, 2001. Cited on page 254.

210 Richard Bandler, *Using Your Brain for a Change*, Real People Press, p. 18, 1985. Cited on page 254.

211 There is disagreement among experts about the precise timing of this experience. Some say that only birth itself is that moment. However, there are several standard methods to recall memories of early life, even of the time before birth. One is by Norbert J. Mayer, *Der Kainkomplex – neue Wege der systemischen Familientherapie*, Integral Verlag, 1998. Cited on page 255.

212 Sanjida O'Connell, *Mindreading – How We Learn to Love and Lie*, Arrow, 1998. This interesting book describes the importance of *lying* in the development of a human being, and explains the troubles of those people who cannot read other minds and thus cannot lie, such as autists. Cited on pages 256 and 305.

213 The approach to describe observations as related parts is called *structuralism*; the starting point for this movement was de Saussure's *Cours de linguistique générale* (see the footnote on page 277). A number of thinkers have tried to use the same approach in philosophy, mythology and literature theory, though with little success. An overview of the (modest) success of structuralism in linguistics and its failure in other fields is given by L. Jackson, *The Poverty of Structuralism: Literature and Structuralist Theory*, Longman, 1991. The author argues that when one reduces systems to interactions, one neglects the specific content and properties of the elements of the system, and this approach prevents a full understanding of the system under discussion. Cited on page 256.

214 For a view of the mental abilities different from that of Piaget (described on page 256), a presently much discussed author is the Soviet experimental psychologist Lev Vigotsky, whose path-breaking ideas and complicated life are described, e.g., in Lev Vigotsky, *Mind in Society*, Harvard University Press, 1978, or in René van der Veer & Jaan Valsiner, *Understanding Vigotsky: a Quest for Synthesis*, Blackwell Publishers, 1994. More extensive material can be found in the extensive work by René van der Veer & Jaan Valsinger, *The Vigotsky Reader*, Blackwell, 1994. Cited on page 257.

215 A somewhat unconventional source for more details is the beautiful text by Bruno Bettelheim, *The Uses of Enchantment: the Meaning and Importance of Fairy Tales*, Knopf, 1976. Cited on page 257.

216 A simple introduction is Manfred Spitzer, *Lernen – Gehirnforschung und Schule des Lebens*, Elsevier, 2007. Cited on page 259.

217 See the beautiful textbook by Martin Trepel, *Neuroanatomie: Struktur und Funktion*, Urban & Fischer, 5th edition, 2012. It also shows the parts of the brain dedicated to motion planing and control. Cited on page 259.

218 Quoted in V. Harlen, R. Rappmann & P. Schata, *Soziale Plastik – Materialien zu Joseph Beuys*, Achberger Verlag, 1984, p. 61. Cited on page 259.

219 The problems appearing when one loses the ability to classify or to memorise are told in the beautiful book by the neurologist Oliver Sacks, *The Man Who Mistook His Wife for a Hat*, Picador, 1985, which collects many case studies he encountered in his work. More astonishing cases are collected in his equally impressive text *An Anthropologist on Mars*, Picador, 1995.

See also the beautiful text Donald D. Hoffman, *Visual Intelligence – How We Create What We See*, W.W. Norton & Co., 1998, and the www.cogsci.uci.edu/~ddhoff website associated to it. Cited on pages 260 and 265.

220 For a passionate introduction to the connections between language and the brain from a Chomskian perspective, see the bestselling book by Steven Pinker, *The Language Instinct – How the Mind Creates Language*, Harper Perennial, 1994. The green idea sentence is discussed in a chapter of the book. Cited on pages 260, 310, and 330.

221 An introduction to neurology is Joseph Ledoux, *Synaptic Self: How Our Brains Become Who We Are*, Viking Press, 2002. Cited on page 260.

222 Another good introduction into the study of classifiers is James A. Anderson, *An Introduction to Neural Networks*, MIT Press, 1995. An introduction to computer science is given in J. Glenn Brookshear, *Computer Science, An Overview*, 6th edition, Addison Wesley, 2000, or in Rick Decker & Stuart Hirshfield, *The Analytical Engine: An Introduction to Computer Science Using the Internet*, Brooks/Cole Publishers, 1998. Cited on page 260.

223 An overview of the status of the research into the origin of bipedalism is given by B. Wood, *Four legs good, two legs better*, Nature **363**, pp. 587–588, 17 June 1983. Cited on page 260.

224 A good introduction to neural nets is J. Hertz, A. Krogh & R. Palmer, *Introduction to the Theory of Neural Computation*, Addison Wesley, 1991. Cited on page 261.

225 Quoted from H. Eves, *Mathematical Circles Squared*, Prindle, Weber and Schmidt, 1972. Cited on page 264.

226 K. Baumgärtel, D. Genoux, H. Welzl, R. Y. Tweedie-Cullen, K. Koshibu, M. Livingstone-Zatchej, C. Mamie & I. M. Mansuy, *Control of the establishment of aversive memory by calcineurin and Zif268*, Nature Neuroscience **11**, pp. 572–578, 2008. Cited on page 266.

227 More about the connection between entropy and computers can be found in the classic paper by R. Landauer, *Irreversibility and heat generation in the computing process*, IBM Journal of Research and Development **5**, pp. 183–191, 1961, and in C. H. Bennett & R. Landauer, *The fundamental physical limits of computation*, Scientific American 253, pp. 48–56, 1985. Cited on page 268.

228 W. H. Zurek, *Thermodynamic cost of computation, algorithmic complexity and the information metric*, Nature **341**, pp. 119–124, 14 August 1989. Cited on page 268.

229 L. Szilard, *Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen*, Zeitschrift für Physik 53, p. 840, 1929. This classic paper can also be found in English translation in the collected works by Leo Szilard. Cited on page 268.

230 J. J. Hopfield, Nature **376**, pp. 33–36, 1995. This paper by one of the fathers of the field presents one possibility by which the *timing* of nerve signals, instead of the usually assumed firing frequency, could also carry information. Cited on page 268.

231 The details of the properties of the firing patterns of neurons are nicely described in the article by M. Mahowald & R. Douglas, *A silicon neuron*, Nature **354**, pp. 515–518, 19/26 December 1991, in which they show how to simulate a neuron's electrical behaviour using a silicon circuit. Cited on page 268.

232 A. Mechelli, J. T. Crinion, U. Noppeney, J. O'Doberty, J. Ashburner, R. S. Frackowiak & C. J. Price, *Neurolinguistics: structural plasticity in the bilingual brain*, Nature **431**, p. 757, 2004. Cited on page 269.

233 The discussion whether the brain is or is not superior to a computer is nicely summarised by G. Vollmer, *Algorithmen, Gehirne, Computer – Was sie können und was sie nicht können, Teil I und Teil II*, Naturwissenschaften **78**, p. 481, 1991, and **78**, pp. 533–542, 1991. Cited on page 270.

234 T. Seidel, *The role of student characteristics in studying micro teaching-learning environments*, Learning Environments Research **9**, pp. 253–257, 2006. Cited on page 271.

235 For an introduction, see K. Amunts & al., *BigBrain: an ultrahigh-resolution 3d human brain model*, Science **340**, pp. 1472–1475, 2013. Cited on page 271.

236 The results with children are due to Niels Birbaumer, those for stage performers to Boris Kleber, both at the Universität Tübingen. More information is found on www.dgbfb.de and on applied-neuroscienc.org. Cited on page 272.

237 J. T. Choi & A. J. Bastian, *Adaptation reveals independent control networks for human walking*, Nature Neuroscience **10**, pp. 1055–1062, 2007. Cited on page 272.

238 An entertaining introduction into the importance of the intestine and the enteric nervous system is Giulia Enders, *Darm mit Charme*, Ullstein, 2014. It is well worth reading and contains many interesting references. Cited on page 273.

239 On this aspect of sleep research, see J. Mayer, H. G. Schuster, J. Ch. Claussen & M. Mölle, *Corticothalamic projections control synchronization in locally coupled bistable thalamic oscillators*, Physical Review Letters **99**, p. 068102, 2007. Cited on page 273.

240 The most famous discussion on the topic is the one summarized by J. Maddox, J. Randi & W. W. Stewart, *"High-dilution" experiments a delusion*, Nature **334**, pp. 287–290, 1988. In particular, it was shown that two researchers on the team were paid by a company with interest in spreading the delusion.

The clear animations at www.physik.fu-berlin.de/en/einrichtungen/ag/ag-netz/movies/water_dynamics/ visualize the structure of liquid water and the motion of its molecules. Cited on page 275.

241 A. Louveau, I. Smirnov, T. J. Keyes, J. D. Eccles, S. J. Rouhani, J. D. Peske, N. C. Derecki, D. Castle, J. W. Mandell, K. S. Lee, T. H. Harris & J. Kipnis, *Structural and functional features of central nervous system lymphatic vessels*, Nature **523**, pp. 337–341, 2015. Cited on page 275.

242 E. Kropff, J. E. Carmichael, M. -B. Moser & E. I. Moser, *Speed cells in the medial entorhinal cortex*, Nature **523**, pp. 419–424, 2015. Cited on page 275.

243 K. S. Kassam, A. R. Markey, V. L. Cherkassky, G. Loewenstein & M. A. Just, *Identifying emotions on the basis of neural activation*, PLoS One **8**, p. e66032, 2013, freely available at www.plosone.org. Cited on page 275.

244 A pretty study is J. Kubanek, J. Brown, P. Ye, K. Butts Pauly, T. Moore & W. Newsome, *Remote, brain region–specific control of choice behavior with ultrasonic waves*, Science Advances **6**, p. eaaz4193, 2020. Cited on page 275.

245 For slightly different definitions and a wealth of other interesting information about language, see the beautiful book by David Crystal, *The Cambridge Encyclopedia of Language*, Cambridge University Press, 1987. Cited on page 277.

246 However, the language with the largest available *dictionary* is Dutch, with the 40 volumes of the *Wordenboek der Nederlandsche Taal*, which appeared between 1864 and 1998. It has almost 400 000 entries. Cited on page 279.

247 The list and the remark on discovery on concepts is due to a personal communication from Anna Wierzbicka. A longer list is published in her book *Semantics, Primes and Universals*, Oxford University Press, 1996. Cited on pages 280 and 302.

248 W. S. Hatcher, *Foundations of Mathematics*, W.B. Saunders Co., 1968. There is also the article by P. J. Cohen & R. Hersch, *Non-Cantorian set theory*, Scientific American **217**, pp. 104–116, 1967. Cohen was the mathematician who in 1963 proved that the negation of the continuum hypothesis could be added to the axioms of set theory and still produce a consistent theory; he calls such sets *non-Cantorian*. Cited on page 287.

249 See the beautiful article by I. Stewart, *Fair shares for all*, New Scientist, pp. 42–46, 17 June 1995. Cited on page 287.

250 Many results on infinity are summarized in the excellent and delightful paperback by Rudy Rucker, *Infinity and the Mind – the Science and Philosophy of the Infinite*, Bantam, 1983. Cited on page 289.

251 The proof of the independence of the continuum hypothesis came in two parts. First, Kurt Gödel proved in 1940 that an axiom can be consistently added to ZFC set theory so that the continuum hypothesis is correct. Then, in 1963, Paul Cohen proved that an axiom can be consistently added to ZFC set theory so that the continuum hypothesis is false. Cited on page 289.

252 The strange world of category theory, sometimes called the abstraction of all abstractions, is presented in F. William Lawvere & Stephen H. Schanuel, *Conceptual Mathematics: a First Introduction to Categories*, Cambridge University Press, 1997. Cited on page 290.

253 This general division of mathematics is nicely explained in the text by Pierre Basieux, *Die Architektur der Mathematik – Denken in Strukturen*, Rororo, 2000. Cited on page 290.

254 Umberto Pelizzari, *L'homme et la mer*, Flammarion, 1994. No citations.

255 The issue is treated in Thomas Aquinas, *Summa Theologica*, in question 52 of the first part. The complete text, several thousand pages, can be found on the www.newadvent.org website. We come back to it in the part on quantum theory, in the section on the Pauli exclusion principle. It seems that the whole question goes back to Peter (the) Lombard, *Liber Sententiarum c.* 1150. Cited on page 291.

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256 B. C. Gallivan, *How to fold paper in half twelve times: an "impossible challenge" solved and explained*, Histrical Society of Pomona Valley, 2002, also found at www.osb.net/Pomona/12times.htm. See also www.sciencenews.org/20040124/mathtrek.asp. Cited on page 291.

257 I. Stewart, *Daisy, daisy, give me your answer, do*, Scientific American, pp. 76–79, January 1995. This pedagogical article explains how the growth of plants usually leads to flowers whose number of petals is from the Fibonacci series 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, etc. (The figure on page 246 gives a few examples.) Deviations from this ideal case are also explained. The original work are two articles by S. Douady & Y. Couder, *La physique des spirales végétales*, La Recherche **24**, pp. 26–36, 1993, and *Phyllotaxis as a self-organized growth process*, in *Growth Patterns in Physical Sciences and Biology*, edited by J.M. Garcia-Ruiz & al., Plenum Press, 1993. Despite this and many other publications on the Fibonacci series, the argument on page 298 shows that most of these papers are based on sand. Cited on pages 291, 298, and 299.

258 H. Davson, *The Eye*, Academic Press, 1962. Cited on pages 265 and 292.

259 See the akbar.marlboro.edu/~mahoney/cube/NxN.txt website. Cited on page 292.

260 An introduction to the surreal numbers is given by the article by Polly Shulman, *Infinity plus one, and other surreal numbers*, Discover, pp. 99–105, December 1995. There is also the text by D. Knuth, *Surreal Numbers: How two ex-Students Turned on to Pure Mathematics and Found Total Happiness*, Addison Wesley, 1974, or www-cs-faculty.stanford.edu/~knuth/sn.html. The usually quoted references on the topic include John H. Conway, *On Numbers and Games*, Academic Press, 1976, E. R. Berlekamp, J. H. Conway & R. K. Guy, *Winning Ways for Your Mathematical Plays, Volume I: Games in General*, Academic Press, 1982, and H. Gonshor, *An Introduction to Surreal Numbers*, Cambridge University Press, 1986. Cited on pages 293 and 295.

261 This beautiful problem is discussed by Ian Stewart, *A bundling fool beats the wrap*, Scientific American, pp. 109–111, June 1993. In four dimensions, the answer is known to lie somewhere between 50 000 and 100 000, whereas the five-dimensional answer is conjectured to be 'never'. Cited on page 295.

262 Alfred Tarski, *Introduction to Modern Logic*, Dover, 1946. See also the famous children's book by the mathematician and photographer Lewis Carroll, *Alice in Wonderland*. Cited on page 283.

263 A. Pais, *Niels Bohr's Times: in Physics, Philosophy, and Polity*, Oxford University Press, 1991, page 176. Cited on page 296.

264 Eugene Wigner, *Symmetries and Reflections*, Indiana University Press, 1962. Cited on page 296.

265 Göran Wikell, *The layout of digits on pushbutton telephones – a review of the literature*, Tele 34, pp. 34–42, 1982. Cited on page 298.

266 A clear overview of philosophy of science, often called *epistemology*, without unnecessary detail, is given by Robert Blanché, *L'Epistémologie*, Presses Universitaires de France, 1972. Cited on page 300.

267 About the different aspects of falsifiability of general statements it is commonplace to cite the work by the epitemologist Karl Popper (b. 1902 Vienna, d. 1994 London), especially his long and boring book *Logik der Forschung*, first published in 1934. The reason for this boredom is that Popper's work is simply a warming-up of Pierre Duhem's ideas. Cited on page 306.

268 For a good way of making blood that liquefies, see L. Garlaschelli, F. Ramaccini & S. Della Scala, *Working bloody miracles*, Nature 353, p. 507, 1991. The *Grand dictionnaire universel du XIXe siècle*, by Pierre Larousse, also contains a recipe; it was again shown to the public in the 1980s by Henri Broch. A wonderful and classic text is Harry Houdini, *Miracle Mongers and their Methods*, Prometheus Books, Buffalo, 1981.

The original, written in 1920, by the world famous magician named 'The Great Houdini', is also available on the etext.lib.virginia.edu/toc/modeng/public/HouMirM.html website. The milk drinking Indian statues were common around the world in 1994 and 1995. About healers, see James Randi, *Flim-flam!*, Prometheus Books, Buffalo, New York, 1987, and the exquisite book by Hans Conrad Zander, *Warum ich Jesus nicht leiden kann*, Rowohlt, 1994. Cited on page 307.

269 John Horgan, *The End of Science – Facing the Limits of Knowledge in the Twilight of the Scientific Age*, Broadway Books, 1997, pp. 31–32, and chapter 2, note 2. Cited on pages 309 and 328.

270 For an opinion completely contrary to the one described here, see the book by Gregory J. Chaitin, *The Limits of Mathematics*, Springer Verlag, 1997, which can also be found on the author's website at www.cs.auckland.ac.nz/CDMTCS/chaitin/lm.html, along with his other works. Chaitin has devoted most of his life to the questions discussed in the section, especially on computability. Cited on page 310.

271 See the book by J. Barwise & J. Etchemendy, *The Liar*, Oxford University Press, New York, 1987. Cited on page 310.

272 Demosthenes, *Third Olynthiac*, section 19. Cited on page 311.

273 This definition (statement 4.11) and many other statements about science are in the beautiful and rightly famous text by Ludwig Wittgenstein, *Tractatus logico-philosophicus*, Edition Suhrkamp, 1963. It gives a condensed summary of the basis of science, thought and language in a collection of highly structured and numbered sentences. Cited on pages 316 and 321.

274 See M. Dresden, *The Klopsteg memorial lecture*, American Journal of Physics **66**, pp. 468–482, 1998. Cited on page 317.

275 Well-known books are e.g. Friedrich Kohlrausch, *Praktische Physik*, Teubner, 24. Auflage, 1996. Cited on page 316.

276 Results are described e.g. in L. Bergmann & C. Schäfer, *Lehrbuch der Experimentalphysik*, Band I, II, III und IV, W. de Gruyter. Cited on page 316.

277 Landolt-Börnstein, edited by K.-H. Hellwege & O. Madelung, *Zahlenwerte und Funktionen aus Naturwissenschaften und Technik*, Neue Serie, Springer Verlag, Berlin, 1984. This series of more than one hundred volumes contains all important observations in the domain of physics. Cited on page 317.

278 The origin of this incorrect attribution is the book by Gerhard Szczesny, *Brecht, Leben des Galilei – Dichtung und Wirklichkeit*, Ullstein, Berlin 1966, p. 6. The statement has never been made by Galilei; this issue has been discussed at length in specialist circles, e.g. by F. Kleinert, *"Messen was meßbar ist" - Über ein angebliches Galilei-Zitat*, Berichte zur Wissenschaftgeschichte **11**, p. 221, 1988, or on the internet by Peter Jaencke. Cited on page 316.

279 The strange and sometimes dangerous consequences of beliefs can be found e.g. in Martin Gardner, *Fads and Fallacies*, Dover, 1957, and in James Randi, *Faith Healers*, Prometheus Books, 1989. The million dollar prize for showing any paranormal or supernormal effect is available from his www.randi.org website. Cited on page 322.

280 See the nice collection of cranks on the www.crank.net website. Cited on page 322.

281 It is interesting to observe that most modern theologians, in the age of the internet, avoid to repeat these old and incorrect beliefs and to put them online. Cited on page 323.

282 The opposite view on the emergence of properties is strongly defended in the book by Robert Laughlin, *A Different Universe: Reinventing Physics from the Botton Down* Basic Books, 2005, or by P. JENSEN, *Particle physics and our everyday world*, Physics Today pp. 58–59, July 1998. Their convictions are worth being pondered. Cited on page 323.

283 See page 133 of the bibliography by JOHN BOWLBY, *Charles Darwin*, Pimlico, 1991. Cited on page 324.

284 ROGER PENROSE, *The Road to Reality: A Complete Guide to the Laws of the Universe*, Jonathan Cape, 2004, page 378. Cited on page 327.

285 A beautiful introduction to Greek philosophy is EDUARD ZELLER, *Outlines of the History of Greek Philosophy*, Dover, 1980, a reprint of a book published in 1928. Among others, it gives a clear exposition of the philosophy of Democritus and the other presocratics. Cited on page 328.

286 The famous quote is found at the beginning of chapter XI, 'The Physical Universe', in ARTHUR EDDINGTON, *The Philosophy of Physical Science*, Cambridge, 1939. Cited on page 328.

287 GIUSEPPE FUMAGALLI, *Chi l'ha detto?*, Hoepli, Milano, 1983. Cited on page 330.

288 See JEAN-PAUL DUMONT, *Les écoles présocratiques*, Folio Essais, Gallimard, p. 653, 1991. Cited on page 330.

289 For a beautiful text on fractals, see HEINZ-OTTO PEITGEN, HARTMUT JÜRGENS & DIETMAR SAUPE, *Fractals for the Classroom*, Springer Verlag, 1992, pp. 232–245. It is also available in several other languages. Cited on page 332.

290 As has been pointed out by René Descartes. Cited on page 335.

291 The famous carbon ^{12}C resonance was found by Willy Fowler, as described in E. MARGARET BURBRIDGE, G.R. BURBRIDGE, W.A. FOWLER & F. HOYLE, *Synthesis of the elements in stars*, Reviews of Modern Physics **29**, pp. 547–560, 1957. Cited on page 337.

292 An extensive overview of the topic is given in the thick book by JOHN D. BARROW & FRANK J. TIPLER, *The Anthropic Cosmological Principle*, Oxford University Press, 1986. The term itself is due to Brandon Carter, who coined it in 1973 and presented it in a symposium devoted to the 500th anniversary of Nicolaus Copernicus. For more literature, see YURI I. BALASHOV, *Resource Letter AP-1: the anthropic principle*, American Journal of Physics **59**, pp. 1069–1076, 1991. Cited on page 337.

293 VOLTAIRE, *Candide ou l'optimisme*, 1759. See also the footnote on page 255 in volume I. The book is so good that it was still being seized by the US customs in 1930, and the US post office refused to transport it as late as 1944. For more details, search for 'banned books online' on the world-wide web. Cited on page 338.

294 The number of books on consciousness is large and the contents not always interesting, and often not based on fact, as shown by KARL R. POPPER & JOHN ECCLES, *The Self and its Brain – an Argument for Interactionism*, Rutledge, 1993. Cited on page 339.

295 See e.g. the *Encyclopedia Britannica*, Macropaedia, in the entry on animal behaviour. Cited on page 340.

296 A straight and informative introduction to the work and ideas of Joseph Beuys (in German) is by RENATE GEORGI, *Joseph Beuys*, RAAbits Kunst, Raabe Fachverlag, September 1996. Cited on page 340.

297 Two studies, one by R.P. EBSTEIN & al., *Dopamine D4 receptor (D4DR) exon III polymorphism associated with human personality trait of novelty seeking*, Nature Genetics **12**, pp. 78–80, January 1996, and another study by J. BENJAMIN & al., *Population and familial association between the D4 dopamine receptor gene and measures of novelty seeking*, Nature Genetics **12**, pp. 81–84, January 1996, found that people with a special form of the

D4 dopamine receptor gene, or D4DR, are more prone to novelty seeking than people with the usual form. The D4DR gene regulates the formation of dopamine receptors, a chemical messenger in the brain that has been a candidate for some time for a substance involved in the propensity for novelty seeking. Cited on page 341.

298 See Jacques Hadamard, *The Mathematician's Mind – The Psychology of Invention in the Mathematical Field*, Princeton Science Library, 1996. For a modern perspective, see Pierre de Gennes, *Fragile Objects: Soft Matter, Hard Science, and the Thrill of Discovery*, Springer, 1996. where de Gennes criticizes certain educational systems that put too much stress on mathematics, thus destroying creativity. Cited on page 341.

299 Voltaire writes this in his *Catalogue pour la plupart des écrivains français qui ont paru dans Le Siècle de Louis XIV, pour servir à l'histoire littéraire de ce temps* (1752). Cited on page 342.

300 This is from the beautiful booklet by Bert Hellinger, *Verdichtetes*, Carl-Auer Systeme Verlag, 1996. Cited on page 342.

301 For example, one needs the courage to face envy. About this topic see the classic text by Helmut Schoeck, *Der Neid*, 1966, published in English as *Envy: A Theory of Social Behavior*, 1969. It is the standard work in the field. Cited on page 342.

302 Bill McGuire, *A Guide to the End of the World: Everything You Never Wanted to Know*, Oxford University Press, 2002. On past disasters, see introduction by Tony Hallam, *Catastrophes and Lesser Calamities – the Causes of Mass Extinctions*, Oxford University Press, 2004. Cited on page 347.

303 *Le Système International d'Unités*, Bureau International des Poids et Mesures, Pavillon de Breteuil, Parc de Saint Cloud, 92310 Sèvres, France. All new developments concerning SI units are published in the journal *Metrologia*, edited by the same body. Showing the slow pace of an old institution, the BIPM launched a website only in 1998; it is now reachable at www.bipm.fr. See also the www.utc.fr/~tthomass/Themes/Unites/index.html website; this includes the biographies of people who gave their names to various units. The site of its British equivalent, www.npl.co.uk/npl/reference, is much better; it provides many details as well as the English-language version of the SI unit definitions. Cited on page 352.

304 The bible in the field of time measurement is the two-volume work by J. Vanier & C. Audoin, *The Quantum Physics of Atomic Frequency Standards*, Adam Hilge, 1989. A popular account is Tony Jones, *Splitting the Second*, Institute of Physics Publishing, 2000.

The site opdaf1.obspm.fr/www/lexique.html gives a glossary of terms used in the field. For precision *length* measurements, the tools of choice are special lasers, such as mode-locked lasers and frequency combs. There is a huge literature on these topics. Equally large is the literature on precision *electric current* measurements; there is a race going on for the best way to do this: counting charges or measuring magnetic forces. The issue is still open. On *mass* and atomic mass measurements, see Volume II, page 71. On high-precision *temperature* measurements, see Volume I, page 548. Cited on page 353.

305 The unofficial SI prefixes were first proposed in the 1990s by Jeff K. Aronson of the University of Oxford, and might come into general usage in the future. See New Scientist **144**, p. 81, 3 December 1994. Other, less serious proposals also exist. Cited on page 354.

306 The various concepts are even the topic of a separate international standard, ISO 5725, with the title *Accuracy and precision of measurement methods and results*. A good introduction is John R. Taylor, *An Introduction to Error Analysis: the Study of Uncertainties in Physical Measurements*, 2nd edition, University Science Books, Sausalito, 1997. Cited on page 356.

307 P. J. Mohr & B. N. Taylor, *CODATA recommended values of the fundamental physical constants: 1998*, Reviews of Modern Physics **59**, p. 351, 2000. This is the set of constants resulting from an international adjustment and recommended for international use by the

Committee on Data for Science and Technology (CODATA), a body in the International Council of Scientific Unions, which brings together the International Union of Pure and Applied Physics (IUPAP), the International Union of Pure and Applied Chemistry (IUPAC) and other organizations. The website of IUPAC is www.iupac.org. Cited on page 357.

308 Some of the stories can be found in the text by N. W. Wise, *The Values of Precision*, Princeton University Press, 1994. The field of high-precision measurements, from which the results on these pages stem, is a world on its own. A beautiful introduction to it is J. D. Fairbanks, B. S. Deaver, C. W. Everitt & P. F. Michaelson, eds., *Near Zero: Frontiers of Physics*, Freeman, 1988. Cited on page 357.

309 For details see the well-known astronomical reference, P. Kenneth Seidelmann, *Explanatory Supplement to the Astronomical Almanac*, 1992. Cited on page 363.

310 See the corresponding reference in the first volume. Cited on page 365.

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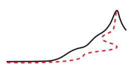

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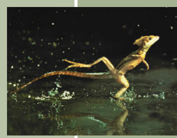

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