Angle (°)	Speed of Projection (ft/s)			
	25	30	35	40
0	10.64	11.94	13.25	14.56
5	12.51	14.71	17.1	19.7
10	14.67	17.97	21.73	25.96
15	16.92	21.4	26.61	32.55
20	19.1	24.72	31.29	38.85
30	22.65	30.1	38.88	49
34	23.62	31.58	40.97	51.79
35	23.81	31.87	41.39	52.36
36	23.98	32.14	41.77	52.87
37	24.13	32.38	42.11	53.32
38	24.27	32.59	42.4	53.73
39	24.37	32.76	42.66	54.07
40	24.46	32.9	42.87	54.36
41	24.53	33.01	43.03	54.58
42	24.57	33.09	43.15	54.75
43	24.59	33.14	43.23	54.87
44	24.59	33.15	43.26	54.92
45	24.57	33.13	43.25	54.91
46	24.52	33.08	43.19	54.84
47	24.46	33	43.08	54.72
48	24.36	32.88	42.93	54.53
49	24.25	32.73	42.74	54.29
50	24.11	32.54	42.5	53.98

Table 9.17: Horizontal Distance of the Long Jump for Different Launch Speeds and Launch Angles

Part IV

Dynamics and Equilibrium

Lab 10

The Laws of Motion

In this experiment, you will attempt to experimentally verify one of the most important discoveries in classical mechanics, Newton's Second Law of Motion. In addition, you will investigate frictional forces and their affects on the motion of a body.

10.1 Theory

10.1.1 Newton's Second Law of Motion

When a resultant external force acts upon a body, the body experiences a change in its motion that is in the same direction as the resultant external force and is directly proportional to the resultant external force. When the mass of a body does not change during its motion, this statement is formulated by the following vector equation:

$$\vec{F}_{\text{net}} = m\vec{a} \tag{10.1}$$

where $\vec{F}_{\text{net}}$ is the resultant external force, m is the mass, and $\vec{a}$ is the acceleration of the body. In this case, the acceleration is an accurate measure of the change in motion of the body. In those cases in which the mass of the body changes during its motion, the change in the motion must be described in terms of the change in the "momentum" of the body, where momentum is defined as the product of the mass of the body with its velocity. Momentum and its conservation are considered in later experiments.

10.1.2 Frictional Forces

When we try to start the motion of an object across some surface, we discover that the surface offers resistance to our attempt to initiate the motion of the object. The surface exerts a force, called the force of static friction, which opposes the attempt to initiate the motion of the object. When an object is sliding across a surface, we discover again that the surface offers resistance to the sliding by exerting a force, called the force of kinetic friction.

Force of Static Friction ($\vec{f}_{\mathbf{s}}$): The force of static friction is defined as the frictional force that resists the attempt to initiate the motion of one surface relative to another. As you will find in this experiment, the force of static friction ranges in magnitude from zero to some maximum value that we will call $\vec{f}_{\text{s,max}}$.

$$0 \leq f_{\text{s}} \leq f_{\text{s,max}} \tag{10.2}$$

The force of static friction assumes its maximum value, $\vec{f}_{\mathrm{s,max}}$, when the body is on the verge of motion. What is found experimentally is that the magnitude of $\vec{f}_{\mathrm{s,max}}$ is directly proportional to a force, called the normal force, $\vec{N}$, defined below. The proportionality constant is called the coefficient of static friction and is represented by μ_{s}. The value of the coefficient of static friction appears to be intrinsic to the two surfaces involved.

$$f_{\mathrm{s,max}} = \mu_{\mathrm{s}} N \tag{10.3}$$

Normal Force ($\vec{N}$): The normal force is defined as the force that a surface exerts perpendicularly outward on a body placed upon it.

Force of Kinetic Friction ($\vec{f}_{\mathrm{k}}$): The force of kinetic friction is defined as the frictional force that resists the sliding of one surface across another. For surfaces belonging to rigid bodies, the magnitude of the force of kinetic friction does not appear to change with the speed of the body. What is found experimentally is that the magnitude of $\vec{f}_{\mathrm{k}}$ is directly proportional to the normal force, $\vec{N}$, defined above. The proportionality constant is called the coefficient of kinetic friction and is represented by μ_{k}. The value of the coefficient of kinetic friction appears to be intrinsic to the two surfaces involved.

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N \tag{10.4}$$

For surfaces belonging to rigid bodies it is generally found that $\mu_{\mathrm{k}} < \mu_{\mathrm{s}}$.

10.2 Experiment

10.2.1 Experimental Verification of Newton's Second Law of Motion

Today's technology makes it possible to measure not only times very accurately but also various quantities of motion. In this experiment you will actually be able to measure the linear acceleration of a cart as it accelerates along a level air track by using a remarkable little timer, the PASCO Smart Timer®, manufactured by PASCO Scientific. It is capable of a variety of timing modes that include direct measurements of not only time but also of speed and acceleration. One such mode of operation allows us to measure the linear acceleration of a cart down the air track in terms of the rotation of a pulley at the end of the track (see Figure 10.1).

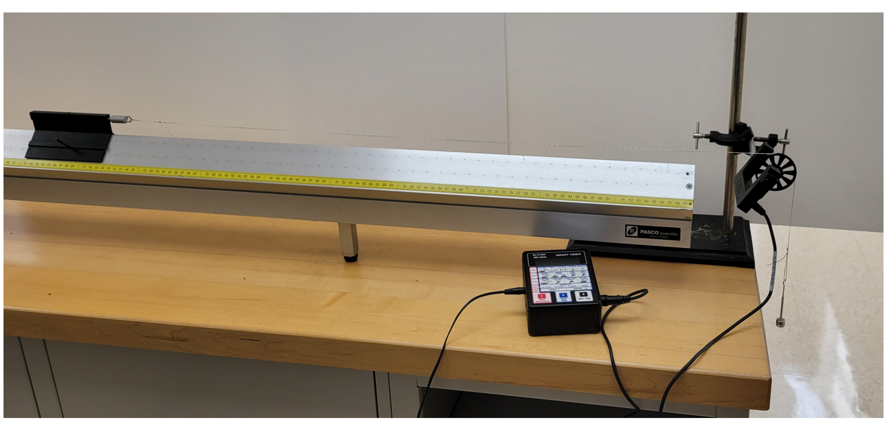


Figure 10.1: Cart Attached to a Suspended Mass Accelerating on an Air Track

Figure 10.2 indicates all of the forces acting on the system of the cart and suspended mass. When the mass m_2 is suspended from the pulley, both the cart and the suspended mass accelerate

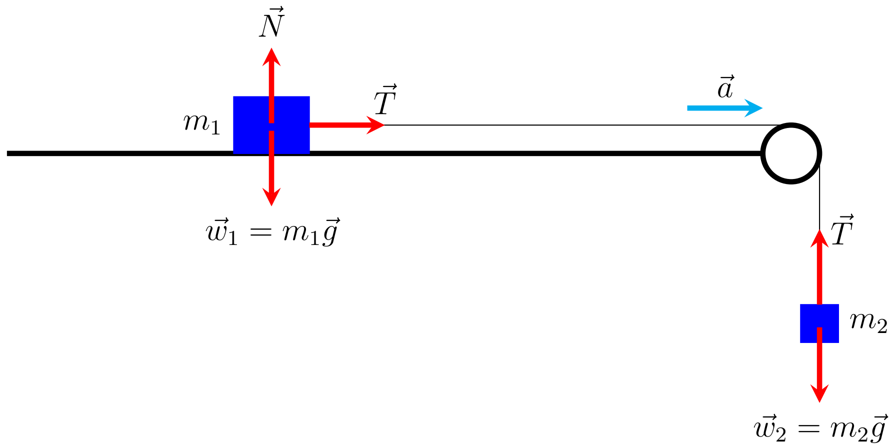


Figure 10.2: Forces acting on the Cart and Suspended Mass

with a common acceleration a. In the absence of friction, the tension, $\vec{T}$, in the cord is the resultant force acting on the cart. Therefore, applying Newton's Second Law in the x direction $\sum F_x = m_1 a_x$ gives

$$T = m_1 a \tag{10.5}$$

For the suspended mass m_2, there are two forces, an upward tension $\vec{T}$ in the cord and a downward force of gravity $m_2\vec{g}$. Since the suspended mass accelerates downward, the resultant force on the suspended mass has a magnitude of $m_2 g - T$. Therefore, if we consider the downward direction to be the positive y direction and apply Newton's Second Law in the y direction $\sum F_y = m_2 a_y$, this gives

$$m_2 g - T = m_2 a \tag{10.6}$$

Adding Equations 10.5 and 10.6 together and solving for the acceleration a, we get

$$a = \frac{m_2 g}{m_1 + m_2} \tag{10.7}$$

Equation 10.7 gives the acceleration as predicted by Newton's Second Law of motion!

When a photogate is connected to the Smart Timer and adjusted so that the spokes of the pulley block the photogate beam as the pulley turns, the timer is capable of determining the linear acceleration of the cord, and thus the cart, as it travels down the air track!

- First make sure that the air track is as level as possible by adjusting the leveling screws on the track. Measure the mass of the cart alone and record its value in Table 10.1. This is the mass m_1. Attach the pulley to the end of the track. Attach a length of string to the cart sufficient in length so that when it passes over the pulley the cart is able to glide about 50 cm before reaching the end of the air track. The other end of the string is to be attached to the special mass hanger that is supplied. The string should be short enough so that the cart will reach the end of the air track before the mass hanger reaches the floor.

- Pull the cart back so that the mass hanger reaches the pulley. Add the small metal mass to the hanger and release it from rest. It should take a couple of seconds for the cart to complete a run down the length of the track. Measure the total mass m_2 (hanger plus small metal mass) that is suspended from the string. Record its value in Table 10.1. Having completed a couple of trial runs, you are now ready to actually make some measurements of the acceleration of the cart.

- Pull the cart back until the mass hanger reaches the pulley. With the Smart Timer in "Acceleration: Linear Pulley" mode, release the cart and measure its acceleration. Do this for a total of 5 trials and record your data in Table 10.2.
- Increase the mass of the cart by using the special cylindrical masses that are provided. Measure the new mass of the cart and record your measurement in Table 10.1. Repeat the procedure to measure the acceleration for the second condition with this increased mass of the cart. Record your values in Table 10.2.

<table>
<tr><td>Condition</td><td colspan="2">Cart Mass (m_1) (g)</td><td colspan="2">Suspended Mass (m_2) (g)</td></tr>
<tr><td>1</td><td>Mass of Cart Alone</td><td></td><td>Hanger with Small Metal Mass</td><td></td></tr>
<tr><td>2</td><td>Mass of Cart With Two Cylindrical Masses</td><td></td><td>Hanger with Small Metal Mass</td><td></td></tr>
</table>

Table 10.1: Mass of Cart and Suspended Mass

<table>
<tr><td></td><td colspan="6">Acceleration of Cart (cm/s^2)</td><td></td></tr>
<tr><td>Condition</td><td>a_1</td><td>a_2</td><td>a_3</td><td>a_4</td><td>a_5</td><td>$\overline{a}$</td><td>PRAAD (%)</td></tr>
<tr><td>1</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Absolute Deviation</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Absolute Deviation</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Table 10.2: Acceleration of System of Cart and Suspended Mass

Question 1: From your data, calculate the average acceleration and the PRAAD for each condition. Record your values in Table 10.2.

Question 2: Calculate the predicted value of the acceleration using Equation 10.7 for each condition. Record your values in Table 10.3.

Question 3: For each of the two conditions, compare the value of the acceleration predicted by Newton's Second Law with the measured acceleration by computing the percent experimental error. Record your values in Table 10.3.

Condition	Predicted Acceleration (cm/s^2)	Experimental Acceleration (cm/s^2)	Percent Experimental Error (%)
1 (Cart Alone)			
2 (Cart with Two Masses)			

Table 10.3: Comparison of Predicted and Experimental Acceleration

Question 4: Based upon the results that you have obtained in this experiment, how does the acceleration of the cart vary with its mass, given that the suspended mass is maintained? Hint: In addition to your data, you may also want to look at Equation 10.7 to answer this question.

Question 5: Based upon the results that you have obtained in this experiment, how does the net force (tension) on the cart need to change with the mass of the cart in order to impart the same acceleration? Hint: In addition to your data, you may also want to look at Equation 10.5 to answer this question.

We will now measure the acceleration of the system again but instead of varying the mass of the cart, we will vary the mass of the suspended mass. Leave the two cylindrical masses on the cart for all 4 conditions. For each condition the suspended mass will be the mass hanger plus the following:

- Condition 1: 1 small metal mass
- Condition 2: 1 small metal mass + 1 large plastic mass
- Condition 3: 1 large metal mass + 1 large plastic mass
- Condition 4: 1 small metal mass + 1 large metal mass + 1 large plastic mass

For each condition, measure the mass of the suspended mass and measure the acceleration of the cart for five trials. Record your values in Table 10.4.

Question 6: Compute the average acceleration over the five trials for each condition. Using Equation 10.6 we can solve for the tension which gives

$$T = m_2 (g - a) \tag{10.8}$$

Using your suspended mass (m_2) and your average acceleration ($\overline{a}$) values, compute the tension (T) for each condition. Note that since we are using cgs units, the tension is computed in units of dynes. Finally, plot tension versus acceleration and perform a linear fit. According to Equation 10.5, the slope should equal the mass of the cart with the two cylindrical masses. Record this slope value as the predicted mass of the cart in Table 10.5. Compare this predicted value with the measured mass of the cart (found in Table 10.1) by computing the percent experimental error.

		Acceleration of Cart (cm/s^2)						
Condition	Suspended Mass (m_2) (g)	a_1	a_2	a_3	a_4	a_5	$\overline{a}$	Tension $T = m_2(g - \overline{a})$ (dynes)
1								
2								
3								
4								

Table 10.4: Acceleration of System of Cart and Varying Suspended Mass

Measured Mass of Cart with Two Cylindrical Masses (g)	Predicted Mass of Cart with Two Cylindrical Masses (g)	Percent Experimental Error (%)

Table 10.5: Comparison of Mass of Cart with Two Cylindrical Masses

10.2.2 Calculating Acceleration Using the PhET Simulation

For this part of the lab, we will calculate acceleration by using the Forces and Motion: Basics motion simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/en/simulation/forces-and-motion-basics

Once you have navigated to the simulation web page, press the play button on the simulation and choose the acceleration simulation option. At the top right of the screen indicate the information displayed by checking off forces, sum of forces, values, masses, and speed. Do not check off acceleration. Place the child on top of the crate (the combination of the child and the crate is your object for this part of the simulation). Set the force applied by the person to be 300 N acting towards the right. This can be done by using the course and fine adjustment buttons (two or one arrow on the button, respectively) or by using the slider (see Figure 10.3).

Question 7: Use Newton's Second Law $\vec{F}_{\text{net}} = m\vec{a}$ to calculate the acceleration as the person applies the force and the speed of the object is increasing. When the person stops pushing (after reaching the top speed of 40 m/s), recalculate the acceleration. Hint: In this case, the acceleration should be negative because the net force is just the force of kinetic friction which is opposing the motion. Run the experiment again with the acceleration button also checked. Now record the measured acceleration from the simulation as the crate and child are increasing speed and when they are slowing back down. Show all of your calculations and record your results in Table 10.6. Were your calculated values equal to the measured values?

Question 8: Perform the same experiment again, but now use two crates (one on top of the

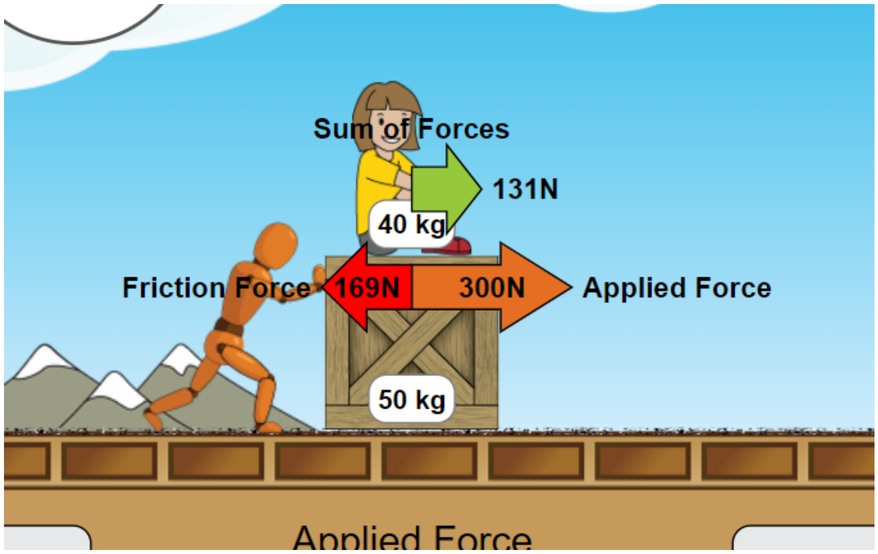


Figure 10.3: PhET Simulation for Measuring Acceleration

other) instead of a crate and the child. Keep the force applied by the person to be 300 N to the right. Calculate the acceleration as the speed of the two crates is increasing and after the two crates reach top speed and the person stops pushing. Run the simulation again with the acceleration button check to determine the experimental acceleration values. Show all of your calculations and record your results in Table 10.6. Were your calculated values equal to the measured values?

Object	Applied Force	Calculated Acceleration (m/s^2)	Measured Acceleration from Simulation (m/s^2)
Crate and Child	Person Pushing (Crate Speeding Up)		
Crate and Child	Person Not Pushing (Crate Slowing Down)		
Two Crates	Person Pushing (Crate Speeding Up)		
Two Crates	Person Not Pushing (Crate Slowing Down)		

Table 10.6: Acceleration Values from PhET Simulation

10.2.3 Alternate Experiment on Newton's Second Law of Motion

This alternate experiment verifies Newton's Second Law of Motion by taking independent measurements of the force on an air puck and its subsequent acceleration down an incline. In order to minimize the effects of friction, we will use a puck that glides over a "cushion" of air. Incline the PASCO Dynamics System track by about 5°. To give the puck a smooth gliding surface, place the plastic-covered cardboard on the track. Two photogates connected to a Smart timer should also be set up so they are about 0.5 m apart and are positioned over the center of the cardboard gliding surface as shown in Figure 10.4.

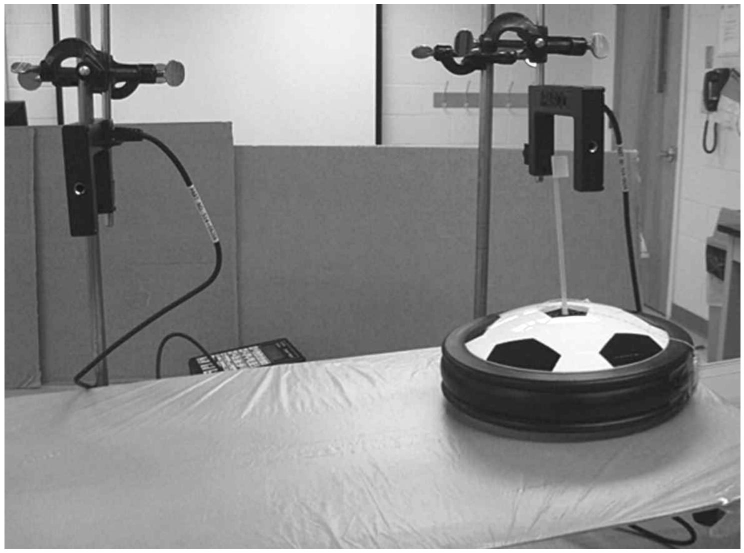


Figure 10.4: Measuring Acceleration of Air Puck

Position the leading edge of the 1 cm flag on the puck such that it is just about ready to intercept the photogate eye of the photogate at position 1 (top photogate) shown in Figure 10.4. Adjust the second photogate at a position 2 (bottom photogate) such that the 1 cm flag will occlude the eye of this photogate when it passes through. With the Smart timer in two-gate mode, measure the time, Δt_{12} ,that it takes the puck to start from rest at position 1 and arrive at position 2. Record your value in Table 10.7. With the Smart timer in stopwatch mode, determine the time, Δt , for the 1 cm flag to pass through the photogate at position 2. Record your value in Table 10.7.

Time from Position 1 to Position 2 Δt_{12} (s)	Time for 1 cm Flag to Pass Through Photogate 2 Δt (s)	Velocity at Position 1 v_1 (cm/s)	Velocity at Position 2 $v_2 = \frac{\Delta x}{\Delta t} = \frac{1 \text{ cm}}{\Delta t}$ (cm/s)	Acceleration $a = \frac{v_2 - v_1}{\Delta t_{12}}$ (cm/s^2)
		0		

Table 10.7: Acceleration Values of Air Puck

Question 9: Using $v_2 = \frac{\Delta x}{\Delta t} = \frac{1 \text{ cm}}{\Delta t}$, calculate the velocity of the puck at photogate 2. Also compute the acceleration $a = \frac{v_2 - v_1}{\Delta t_{12}}$ of the puck as it glides down the track. Show your calculations and record your results in Table 10.7.

In order to complete the verification of Newton's Second Law, you will need to measure the force on the puck and the mass of the puck.

Attach a piece of string to the bumper of the puck and tie the other end to the spring scale that is provided. Align the scale so that it and the string are parallel to the incline. See Figure 10.5. Make sure to calibrate the spring scale so that the indicator is on zero before proceeding. With the air supply of the puck turned on, measure the force, in Newtons, that is required in order to keep the puck stationary and record its value in Table 10.8. This force is the force that is offsetting the component of the force of gravity down the incline, which is the net force acting on the puck when it slides down the incline! Note: As a substitution for the spring scale, you may also use the GLX datalogger with a force sensor to measure the force.

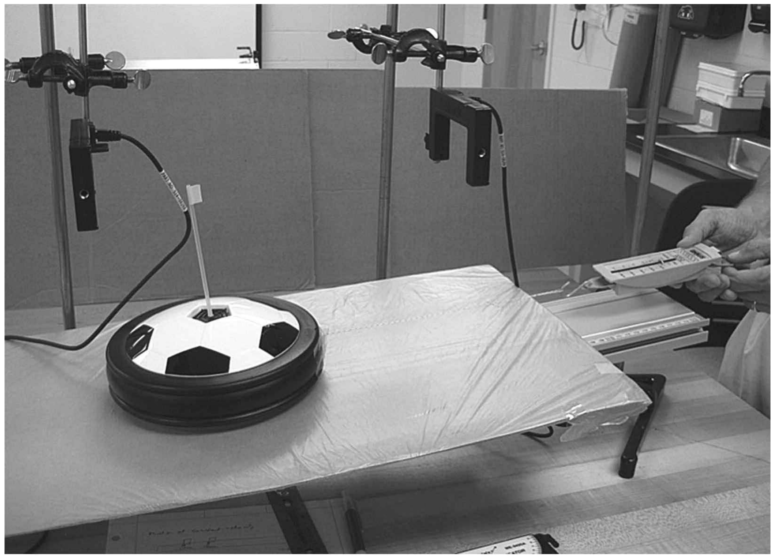


Figure 10.5: Measuring Force on Air Puck

Net Force F (N)	Acceleration a (m/s^2)	Measured Mass of Puck (kg)	Computed Mass of Puck (kg)

Table 10.8: Determination of the Mass of the Air Puck

Question 10: Convert your determination of the acceleration of the puck you recorded in Table 10.7 to units of m/s^2 and record your answer in Table 10.8.

Question 11: Measure the mass of the puck using the electronic scale and record the value in Table 10.8. Finally, using Newton's Second Law, compute the mass of the puck using your values of the net force and acceleration. Record your answer in Table 10.8.

Question 12: Compare the measured mass of the puck with the computed mass of the puck. Compute the percent difference in the values using the following equation:

$$\% \text{ Difference} = \frac{|m_{\text{computed}} - m_{\text{measured}}|}{m_{\text{computed}}} \times 100\%$$

Were you successful in verifying Newton's Second Law of motion?

Question 13: A cart has a mass, M, and is found to accelerate at a rate A when acted upon be a net force, F. What size net force, in terms of F, would be required to give a cart of mass $2M$ the same acceleration, A, as the cart of mass M?

Question 14: A cart of mass M is found to accelerate at a rate A when acted upon by a net force, F. If the same net force, F, were applied to a cart of mass $2M$, what would be its acceleration in terms of the acceleration, A, of the cart of mass, M?

10.2.4 Experimental Study of Frictional Forces

10.2.4.1 Study of Static and Kinetic Frictional Forces

Your station is equipped with a wooden ramp along which a cart can be dragged, a PASCO scientific GLX® data logger, and a force sensor. With this equipment, you will be able to measure both the coefficient of static friction and the coefficient of kinetic friction between two surfaces.

Add two square masses to the cart. Configure the GLX data logger as described in the supplementary instructions. Attach the GLX force sensor to the string. With no force applied to the string, press the zero (tare) button on the force sensor. Press the blue arrow to begin the data collection. Slowly pull on the sensor until the cart just starts to move (see Figure 10.6) and then continue pulling the cart at a constant speed for a distance of about a half meter (see Figure 10.7). Press the blue arrow again to end the data collection. Refer to the supplementary instructions to find the maximal static frictional force $f_{\mathrm{s,max}}$ and kinetic frictional force f_{k} from the force vs. time graph displayed on the data logger. Record the mass of the cart with the two square masses, the normal force, $f_{\mathrm{s,max}}$, f_{k} in the appropriate tables given below (Table 10.9 and Table 10.10). Perform 3 more conditions in which an additional square mass is added to the cart for each new condition. Remember that in the case of a cart on a horizontal surface with no other vertical forces acting on it besides the gravitational force (weight) and the normal force, the magnitude of the normal force is equal to the magnitude of the weight ($N = w = mg$).

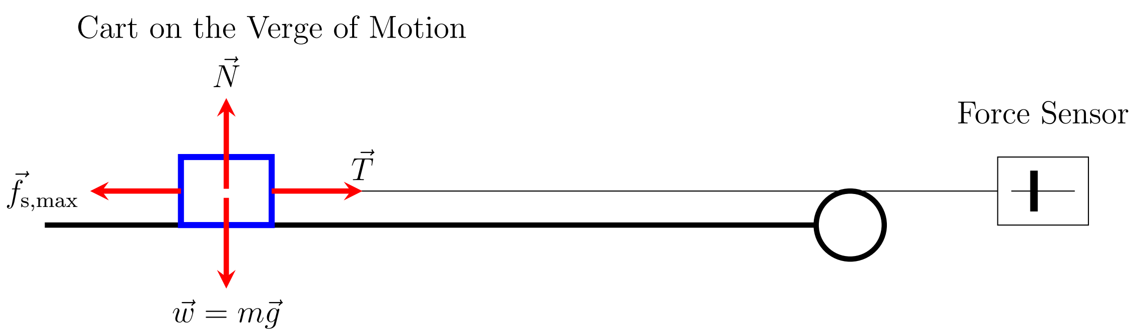


Figure 10.6: Measuring the Maximum Static Frictional Force

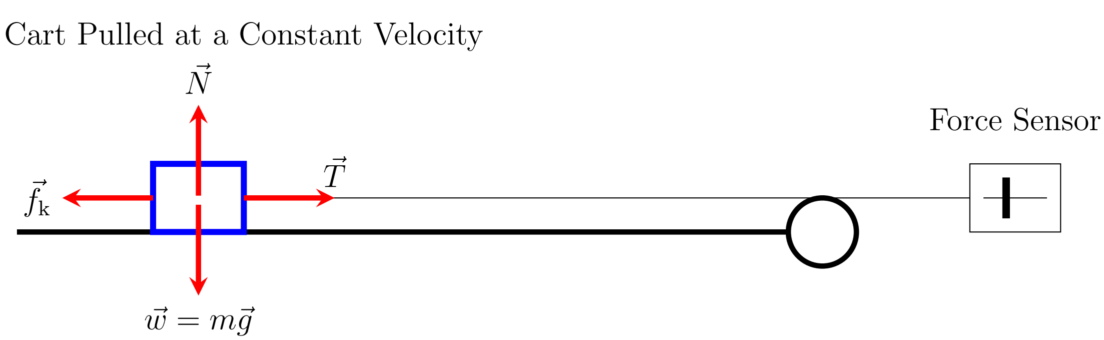


Figure 10.7: Measuring the Kinetic Frictional Force

Condition	Mass of Cart (kg)	Normal Force (N)	$f_{s,max}$ (N)
Cart with 2 Added Masses			
Cart with 3 Added Masses			
Cart with 4 Added Masses			
Cart with 5 Added Masses			
μ_s			

Table 10.9: Measurement of Maximal Static Frictional Force Values

Condition	Mass of Cart (kg)	Normal Force (N)	f_k (N)
Cart with 2 Added Masses			
Cart with 3 Added Masses			
Cart with 4 Added Masses			
Cart with 5 Added Masses			
μ_k			

Table 10.10: Measurement of Kinetic Frictional Force Values

Question 15: Use the data in Tables 10.9 and 10.10 to plot the maximal static frictional force versus normal force and kinetic frictional force versus normal force. Perform a linear fit to each of the graphs and determine the slopes. The slope of the maximal static frictional force versus normal force graph represents μ_s and the slope of the kinetic frictional force versus normal force graph represents μ_k. Record these values in Tables 10.9 and 10.10.

10.2.4.2 Frictional Force on an Inclined Plane

Figure 10.8 shows a diagram for the cart on an incline moving up the incline at a constant velocity. Connected to the cart is a string which passes over a pulley and is attached to a suspended mass. In this part of the experiment, we will use Newton's Second Law and our knowledge of frictional forces to calculate the tension in the string.

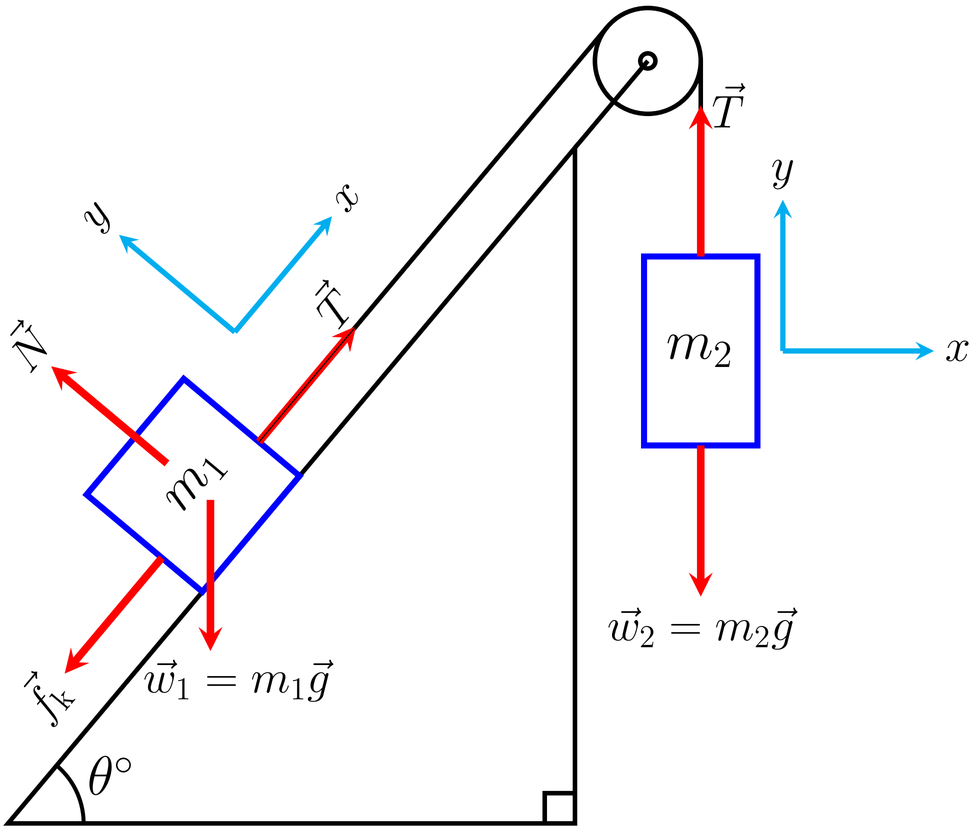


Figure 10.8: Measuring Tension as the Cart Travels Up the Incline at a Constant Speed

Figure 10.9a shows a free body diagram for the cart on the ramp. Since the cart is moving up the incline at a constant velocity, $\sum F_x = 0$. Therefore,

$$T_x + N_x + w_{1x} + f_{kx} = 0$$
$$T + 0 - w_1 \sin\theta - f_k = 0$$

Solving for T gives

$$T = f_k + m_1 g \sin\theta \tag{10.9}$$

Since the cart stays on the inclined ramp, $\sum F_y = 0$. Therefore,

$$T_y + N_y + f_{ky} + w_{1y} = 0$$
$$0 + N + 0 - w_1 \cos\theta = 0$$
$$N = w_1 \cos\theta = m_1 g \cos\theta \tag{10.10}$$

Using

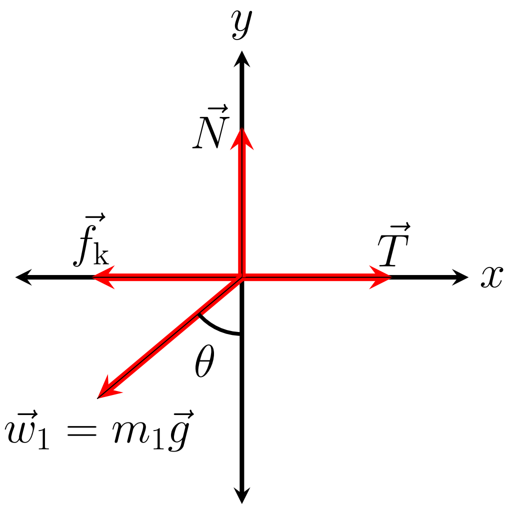
$$f_k = \mu_k N$$

we get

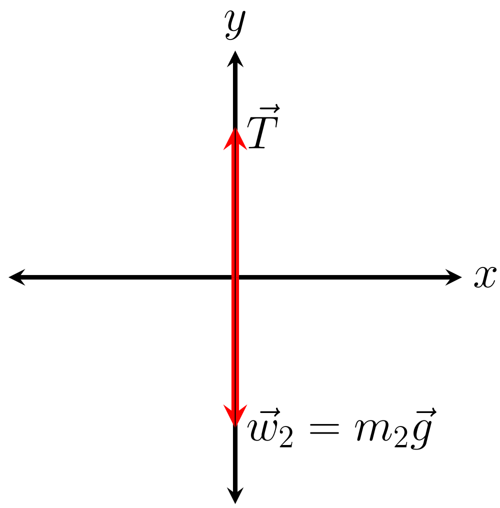
$$f_k = \mu_k m_1 g \cos\theta \tag{10.11}$$

Inserting Equation 10.11 into Equation 10.9 we get

$$T = m_1 g\,(\mu_k \cos\theta + \sin\theta) \tag{10.12}$$



(a) Free-body Diagram for Cart on Ramp



(b) Free-body Diagram for Suspended Mass

Figure 10.9: Free-Body Diagrams for Cart and Suspended Mass

Equation 10.12 gives the theoretical tension pulling the cart up the incline at a constant velocity as a function of the mass of the cart, the angle of the incline, and the coefficient of kinetic friction. Because the cosine function decreases whereas the sine function increases as the angle of inclination increases from 0° toward 90°, we expect that the tension pulling the cart up the incline should peak for some optimum angle θ.

Add just a single square mass to the cart and determine the combined mass. By raising the ramp up the physics stand, you can increase the angle of inclination. You can measure the angle of inclination by using the protractor provided. For each angle, determine the suspended mass m_2 needed to make the cart move up the incline at a constant velocity. Give the cart a slight tap to get it moving, if necessary. Since the suspended mass, m_2 , descends at a constant velocity, you can get an experimental determination of the tension from the following equation (see the free-body diagram in Figure 10.9b).

$$\begin{aligned}\sum F_y &= 0\\ T_y + w_{2y} &= 0\\ T - m_2 g &= 0\\ T &= m_2 g \qquad (10.13)\end{aligned}$$

This equation follows because as the cart moves up the incline at a constant velocity, the suspended mass must move downward at a constant velocity. For each angle, compute the experimental tension T_{exp} given the suspended mass m_2. The theoretical tension T_{theory} can be computed for each angle from Equation 10.12. Record all results in Table 10.11.

Question 16: Graph experimental tension versus angle. Perform a sine fit through the data. Using the coordinate tool, determine the maximum value of T_{exp} and the angle (called the optimal angle) at which it occurred.

Question 17: Graph theoretical tension versus angle. Perform a sine fit through the data. Using the coordinate tool, determine the maximum value of T_{theory} and the angle at which it occurred.

Angle θ (°)	Mass of Cart m_1 (kg)	Suspended Mass m_2 (kg)	Experimental Tension $T_{\text{exp}} = m_2 g$ (N)	Theoretical Tension T_{theory} (N)
20				
30				
40				
50				
60				
65				
70				
75				
80				
85				

Table 10.11: Experimental and Theoretical Tension in the String

Question 18: Compare the maximum experimental and theoretical tensions by computing the percent experimental error. Also compare the experimental and theoretical optimal angles by computing the percent experimental error. Record your results in Table 10.12.

	Experimental Values	Theoretical Values	Percent Experimental Error (%)
Maximum Tension (N)			
Optimal Angle (%)			

Table 10.12: Comparison of Experimental and Theoretical Values for Maximum Tension and Optimal Angle

10.2.5 Calculating Coefficients of Friction Using the PhET Simulation

For this part of the lab, we will calculate the coefficient of static friction and the coefficient of kinetic friction by using the Forces and Motion: Basics motion simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/en/simulation/forces-and-motion-basics

Once you have navigated to the simulation web page, press the play button on the simulation and choose the friction simulation option. At the top right of the screen indicate the information displayed by checking off forces, sum of forces, values, masses, and speed. The single crate will be the object of interest in this simulation. Slowly increase the applied force directed towards the right that the individual exerts on the crate (you can start at 100 N then slowly increase by 1 N). You will see that the static frictional force that opposes the motion increases as well (see Figure 10.10). Find the maximal static frictional force $f_{s,max}$ (you can assume it will be 1 N less than the applied force needed to get the crate moving). Using this information and the fact that the normal force acting on the crate is simply the weight of the crate $N = W = mg$, calculate the coefficient of static friction μ_s.

Now increase the applied force beyond the value needed to get the crate moving. Now that the object is moving, observe that the frictional force (which has decreased and is constant) is the kinetic frictional force f_k. Using this information, calculate the coefficient of kinetic friction μ_k.

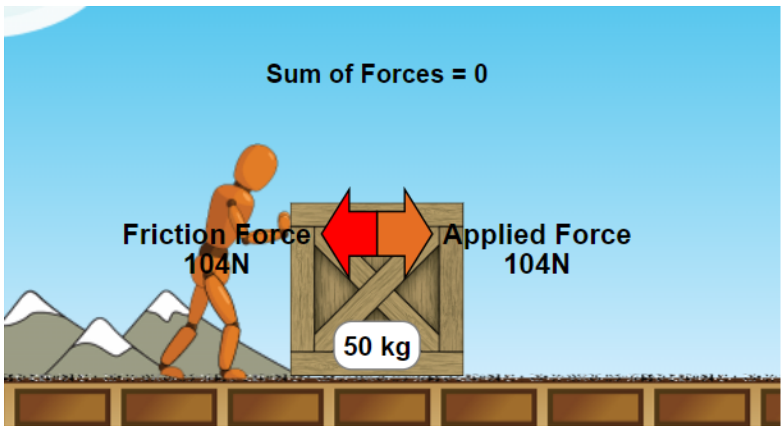


Figure 10.10: PhET Simulation for Calculating Coefficients of Friction

Question 19: Record your values for the coefficients of static and kinetic friction in Table 10.13. Make sure you show all of your calculations.

Coefficients of Friction Values from the Simulation	
Coefficient of Static Friction μ_s	
Coefficient of Kinetic Friction μ_k	

Table 10.13: Coefficients of Friction from the PhET Simulation

Lab 11

Equilibrium of a Rigid Body

There are two conditions necessary for a rigid body to be in equilibrium. First, the vector sum of all the forces acting on the body must equal zero. If this condition is satisfied, there will be no straight line, or linear, acceleration. The second condition of equilibrium of a rigid body is that the clockwise moments of force must equal the counterclockwise moments of force. Another way of stating this is that the vector sum of all the external torques acting on a body must equal zero. If this condition is satisfied, there will be no rotational, or angular, acceleration. It must be emphasized that a body in equilibrium is not necessarily at rest; it may be moving with a constant velocity or rotating with constant angular velocity. In subsequent experiments you will show that the result of a body not being in equilibrium is that it experiences a changing velocity, or an acceleration.

11.1 Theory

11.1.1 Torque

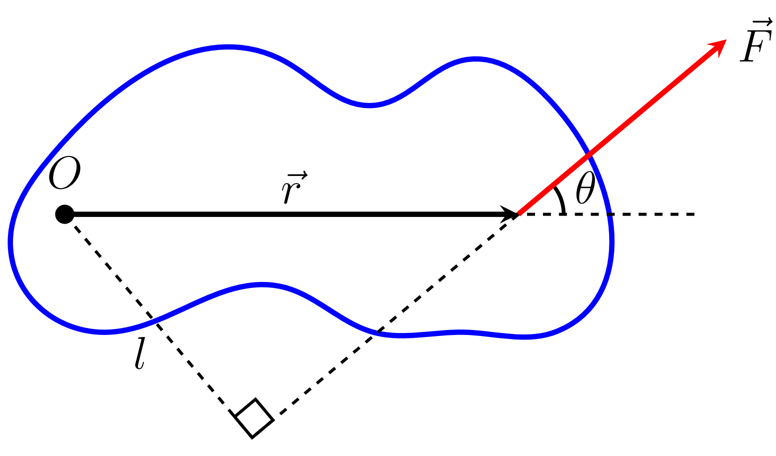


Figure 11.1: A Force $\vec{F}$ Acting on a Rigid Body

Consider a force $\vec{F}$ acting on a rigid object that is free to rotate about point O (see Figure 11.1). Force $\vec{F}$ will tend to cause the body to rotate about O. It should be apparent from your experience that the tendency of $\vec{F}$ to cause rotation will be greater as the perpendicular distance of the line of application of the force from the axis of rotation increases. It therefore becomes useful to define the magnitude of the torque $\vec{\tau}$ as the product of the magnitude of the force $\vec{F}$ and the perpendicular distance l from the axis of rotation to the line of action of the force.

$$\tau = Fl \tag{11.1}$$

The units of torque are force times distance, or the Newton-meter (Nm). Inspection of Figure 11.1 shows that l may be expressed as $r \sin\theta$. Therefore,

$$\tau = Fr\sin\theta \tag{11.2}$$

Torque may therefore be considered a vector quantity represented by the vector product

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{11.3}$$

where $\vec{\tau}$ is the torque vector, which acts perpendicular to the plane of $\vec{r}$ and $\vec{F}$, $\vec{F}$ is the applied force, and $\vec{r}$ is the position vector from the axis of rotation to the point of application of $\vec{F}$. The direction of $\vec{\tau}$ is defined as the direction of a right-handed screw rotated about O in the same sense as the rotation produced by $\vec{F}$ (see Figure 11.2).

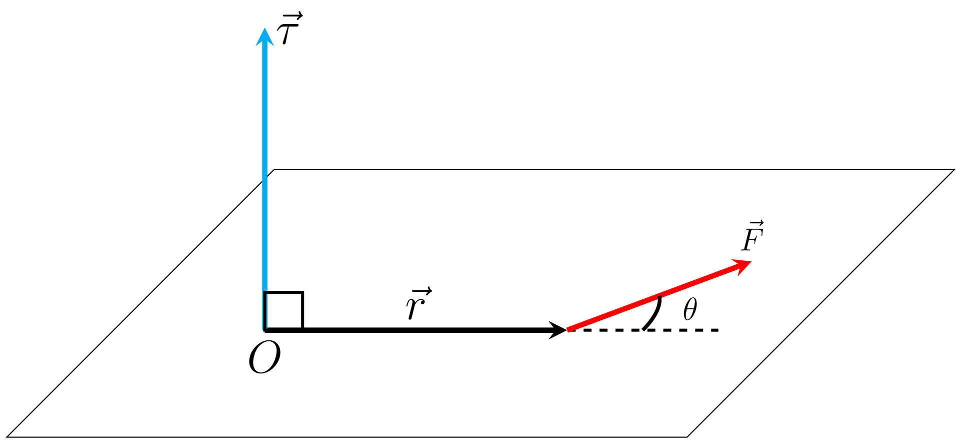


Figure 11.2: Vector Diagram of $\vec{\tau} = \vec{r} \times \vec{F}$

11.1.2 Center of Mass

The explanation of the motion of complex systems becomes simplified if we make use of a concept known as the center of mass. The center of mass of any system behaves as though it were a particle of mass equal to the total mass of the system and acted upon by a net external force equal to that acting on the system. Consider a system of total mass M such that

$$M = m_1 + m_2 + m_3 + \cdots + m_n = \sum_{i=1}^{n} m_i \tag{11.4}$$

where m_1, m_2, …, m_n are the masses of the n particles 1,2, …, n that make up the system. Suppose that these n particles are located at positions $\vec{r}_1$, $\vec{r}_2$, …, $\vec{r}_n$: that is, particle m_1 is located at position $\vec{r}_1$, particle m_2 is located at position $\vec{r}_2$, etc. The position of the center of mass is then given by

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{M} \tag{11.5}$$

Another point, called the center of gravity, also exists. It represents the center of the weight distribution of a system. That is, it represents the point at which the entire weight of the system can be considered to act. For relatively small systems such as those considered in experiments in this book, there is no significant difference in the location of the center of gravity and the center of mass. Therefore, for our purposes, we can consider the weight of a system to act at the center of mass of the system.

Question 1: Does the center of mass of an object always have to be located within the matter of that object? For example, does the center of mass of a tire have to lie within the rubber of the tire? Explain.

11.1.3 Rigid Body Equilibrium

A rigid body is one in which the relative distances between the constituent particles remains fixed. A rigid body is said to be in equilibrium if, (1) the linear acceleration of the center of mass equals zero, and (2) the angular acceleration about any axis equals zero. If the linear acceleration of the center of mass equals zero, the object is said to be in translational equilibrium. If the angular acceleration about any axis equals zero, the object is said to be in rotational equilibrium. In order to be in translational equilibrium the vector sum of all forces acting on the object must also equal zero. That is,

$$\sum_{i=1}^{n} \vec{F}_i = 0 \tag{11.6}$$

This is the first condition for equilibrium. For the object to be in rotational equilibrium the vector sum of all the torques about any axis must equal zero. That is,

$$\sum_{i=1}^{n} \vec{\tau}_i = 0 \tag{11.7}$$

This is the second condition for equilibrium.

11.2 Experiment

11.2.1 Finding the Center of Mass of the Meter Stick

Locate the center of mass of your meter stick by adjusting the position of the meter stick relative to the axis of rotation. This can be done by sliding the meter stick within the clamp located at the axis of rotation. When the center of mass is located at the axis of rotation, the meter stick will be in rotational equilibrium. Record the position of the center of mass in Table 11.1.

Position of the Center of Mass (m)	

Table 11.1: Position of the Center of Mass of the Meter Stick

11.2.2 Verifying the Second Condition for Equilibrium

Place the center of mass of the meter stick at the axis of rotation then suspend a 200 g mass (make sure the total mass is 200 g, the clamp has a mass of 10 g and the blue hanger has a mass of 5 g so just add an additional 185 g) 20 cm to the left of the center of mass. Balance the meter stick by suspending a mass of 100 g at some location to the right of the center of mass of the meter stick (see Figures 11.3 and 11.4).

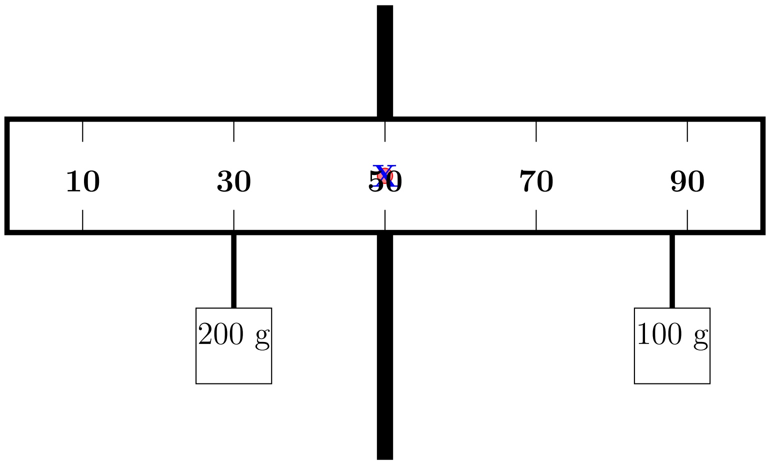


Figure 11.3: Diagram of Set-up to Verify the Second Condition for Equilibrium

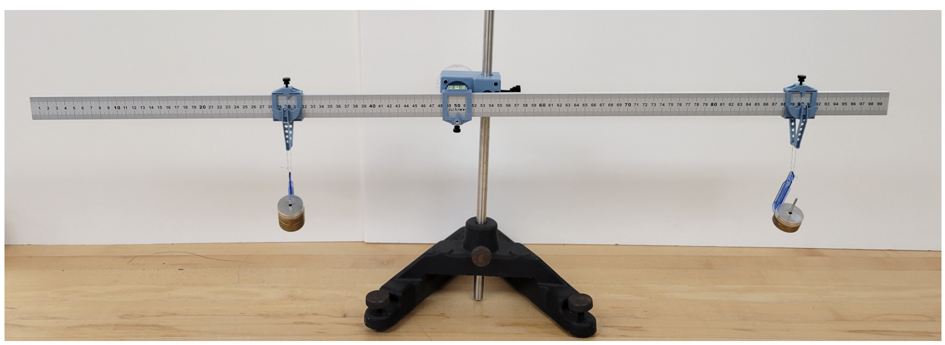


Figure 11.4: Set-up to Verify the Second Condition for Equilibrium

Question 2: Use the location found above to calculate the moment arm for the 100 g mass (l_2) and record the value in Table 11.2. Make sure you convert to meters.

Moment Arm for the 100 g Mass (l_2) (m)	

Table 11.2: Moment Arm for the 100 g Mass

Question 3: Using all of your information, calculate the sum of the torques about the axis of rotation. Show the calculation and record your value in Table 11.3.

Experimental Sum of Torques (Nm)	

Table 11.3: Experimental Sum of Torques about Axis of Rotation

Question 4: Assuming there was no error in placing the masses, reading the locations on the meter stick, etc., what should the answer to question 3 be? Record your answer in Table 11.4.

Theoretical Sum of Torques (Nm)	

Table 11.4: Theoretical Sum of Torques about Axis of Rotation

Now perform the same procedure but suspend 50 g 30 cm to the left of the center of mass and balance the meter stick by placing 100 g at some location to the right of the center of mass.

Question 5: Use the location found above to calculate the moment arm for the 100 g mass (l_2) and record the value in Table 11.5. Make sure you convert to meters.

Moment Arm for the 100 g Mass (l_2) (m)	

Table 11.5: Moment Arm for the 100 g Mass

Question 6: Using all of your information, calculate the sum of the torques about the axis of rotation. Show the calculation and record your value in Table 11.6.

Experimental Sum of Torques (Nm)	

Table 11.6: Experimental Sum of Torques about Axis of Rotation

Question 7: Assuming there was no error in placing the masses, reading the locations on the meter stick, etc., what should the answer to question 6 be? Record your answer in Table 11.7.

Theoretical Sum of Torques (Nm)	

Table 11.7: Theoretical Sum of Torques about Axis of Rotation

11.2.3 Predicting the Mass of the Meter Stick

Assume all of the weight of the meter stick to be concentrated at its center of mass. Place the meter stick so that the center of mass is 20 cm to the right of axis of rotation (for example, if your center of mass is exactly at 50 cm, the meter stick should be placed so that the axis of rotation is at 30 cm). Balance the meter stick by suspending a mass of 200 g to the left of the axis of rotation (see Figure 11.5).

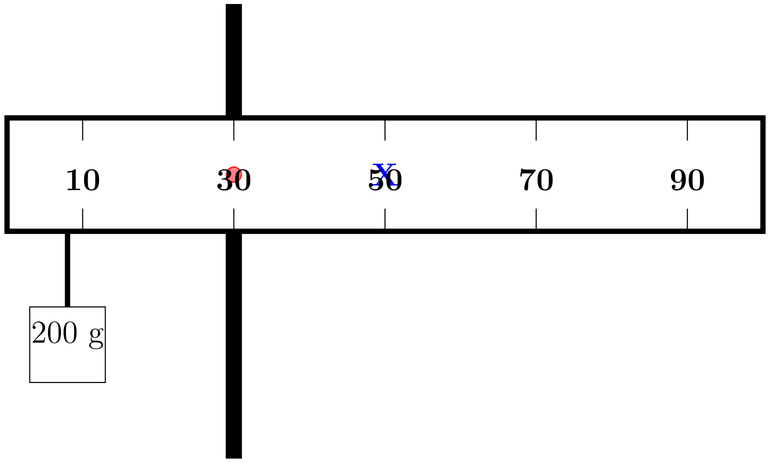


Figure 11.5: Set-up to Predict the Mass of the Meter Stick

Question 8: Use the location found above to calculate the moment arm for the 200 g mass (l) and record the value in Table 11.8. Make sure you convert to meters.

Moment Arm for the 200 g Mass (l) (m)	

Table 11.8: Moment Arm for the 200 g Mass

Question 9: Calculate the sum of the torques about the axis of rotation and set them equal to 0. Using this information, solve for the predicted mass of the meter stick (m_{pred}). Also, determine the experimental mass of the meter stick using the electronic scale and then compare the two values by computing the percent experimental error. Record your results in Table 11.9. Show all of your calculations.

Experimental Mass of Meter Stick (from scale) m_{exp} (kg)	Predicted Mass of Meter Stick m_{pred} (kg)	Percent Experimental Error (%)

Table 11.9: Experimental and Predicted Mass of the Meter Stick

11.2.4 Using the Second Condition for Equilibrium to Compute Forces

Set up the apparatus shown in Figure 11.6. Place a 300 g mass at 30 cm, a 200 g mass at 70 cm and two spring scales at 10 cm and 90 cm, respectively. Don't forget that the meter stick itself has a weight which acts at the center of mass.

Question 10: Calculate the theoretical values for the readings of spring scale A and B. Compare the calculated values with the measured values by computing the percent experimental errors. Show all calculations and record your results in Table 11.10.

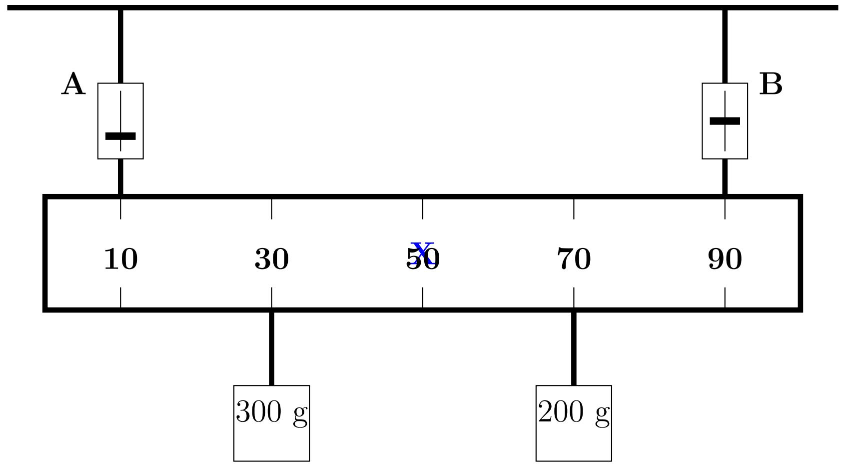


Figure 11.6: Set-up to Compute Tensions in Spring Scales

	Experimental Values	Theoretical Values	Percent Experimental Error (%)
Tension in Spring Scale A (N)			
Tension in Spring Scale B (N)			

Table 11.10: Comparison of Experimental and Theoretical Values for the Tensions in the Spring Scales

Set up the apparatus shown in Figures 11.7 and 11.8. Place the meter stick so that the axis of rotation is at 2 cm. Place a 100 g mass at 30 cm, a 50 g mass at 60 cm, and a 10 g mass (this is just the mass of the clamp) at 90 g. Don't forget that the meter stick itself has a weight which acts at the center of mass. Attach a force sensor with string horizontally from the stand to a position of 90 cm on the meter stick.

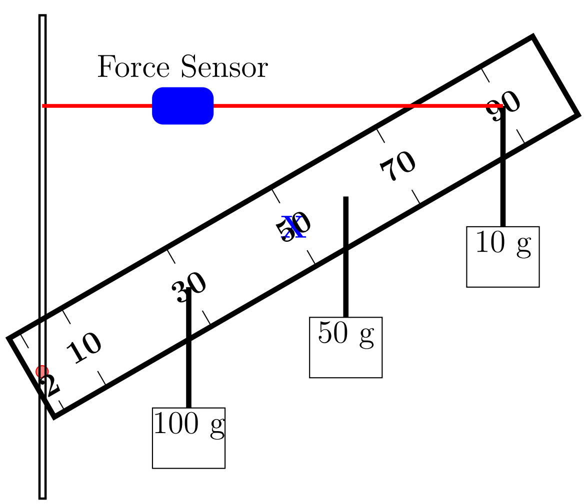


Figure 11.7: Diagram to Compute Tension Recorded by Force Sensor

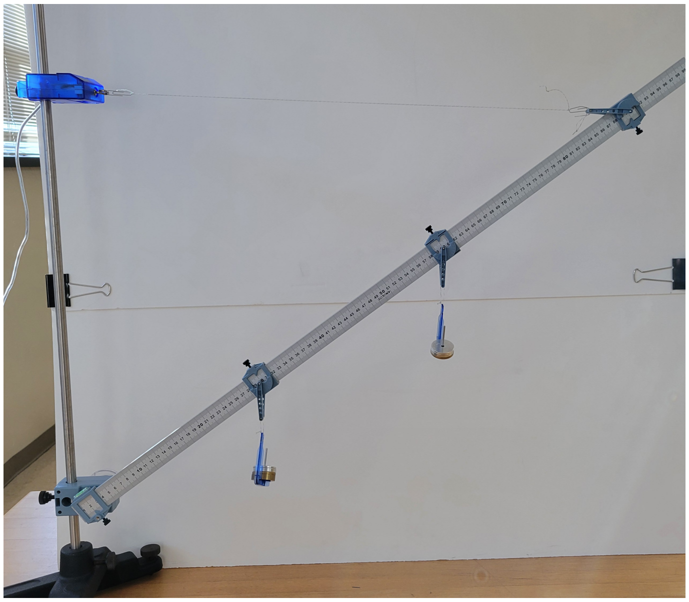


Figure 11.8: Set-up to Compute Tension Recorded by Force Sensor

Question 11: Calculate the theoretical tension in the string. Compare the calculated value with the experimental (measured) value from the force sensor by computing the percent experimental error. Show all calculations and record your results in Table 11.11.

	Experimental Value	Theoretical Value	Percent Experimental Error (%)
Tension in String (N)			

Table 11.11: Comparison of Experimental and Theoretical Values for the Tension in the String

Set up the apparatus shown in Figures 11.9 and 11.10. The wooden beam is horizontal and has a mass of 249.7 g and the hanger has a mass of 300 g. The center of mass of the wooden beam is assumed to be at 30 cm and the hanger is placed at 60 cm. One end of a string is attached to the beam at 40 cm and the other end is attached to the stand at 46 cm above the axis of rotation.

Question 12: Calculate the theoretical tension in the spring scale. Compare the theoretical value with the experimental (measured) value from the spring scale by computing the percent experimental error. Show all calculations and record your results in Table 11.12.

	Experimental Value	Theoretical Value	Percent Experimental Error (%)
Tension in Spring Scale (N)			

Table 11.12: Comparison of Experimental and Theoretical Values for the Tension in the Spring Scale

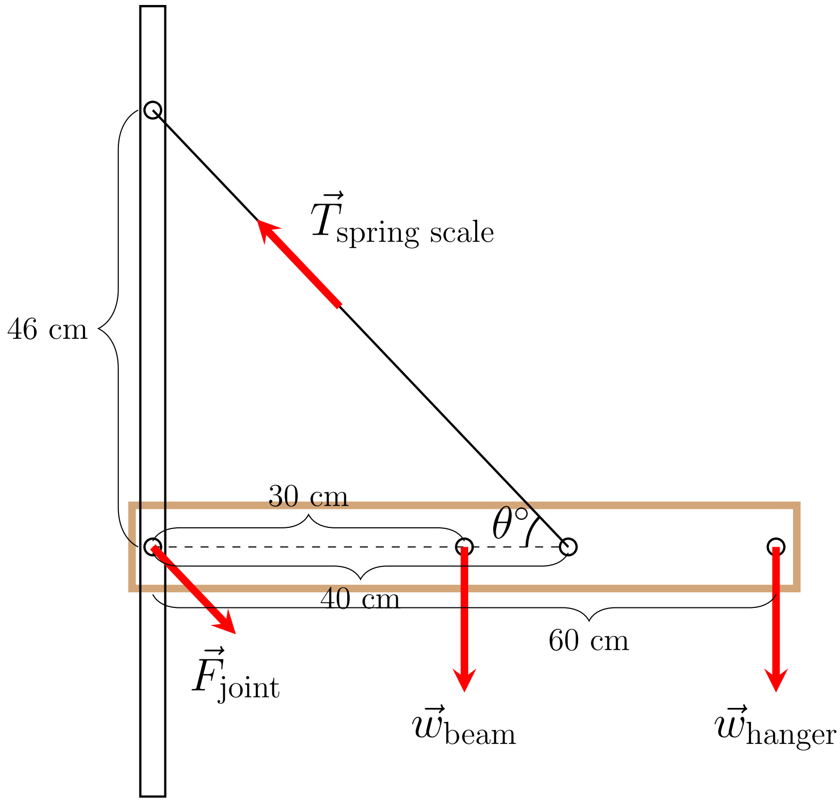


Figure 11.9: Diagram of Wooden Beam Set-up

Figure 11.10: Wooden Beam Set-up

Question 13: Compute the x component of the joint force $F_{\text{joint},x}$ and the y component of the joint force $F_{\text{joint},y}$.

11.2.5 Rotational Equilibrium Using the PhET Simulation

For this part of the lab, we will solve rotational equilibrium problems by using the Balancing Act simulation created by the PhET group at the University of Colorado Boulder (Simulation by PhET Interactive Simulations, University of Colorado Boulder, licensed under CC-BY-4.0 (https://phet.colorado.edu)). Use the following link to access the simulation.
https://phet.colorado.edu/en/simulation/balancing-act

Once you have navigated to the simulation web page, press the play button on the simulation and choose the Balance Lab option. In the upper right hand menu, check off show mass labels, forces from objects, and level. Also make sure that the ruler is also turned on. On the left side of the beam, place a 15 kg brick at 1 meter and 20 kg brick at 0.5 meters. Use the second condition for rotational equilibrium to calculate where a 20 kg brick would have to be placed on the right hand side in order for the beam to be in rotational equilibrium (see Figure 11.11). Note that the beam will not balance with the location of the 20 kg mass shown in the figure. You can check this by pressing the button at the bottom which will remove the pillars on both sides of the beam.

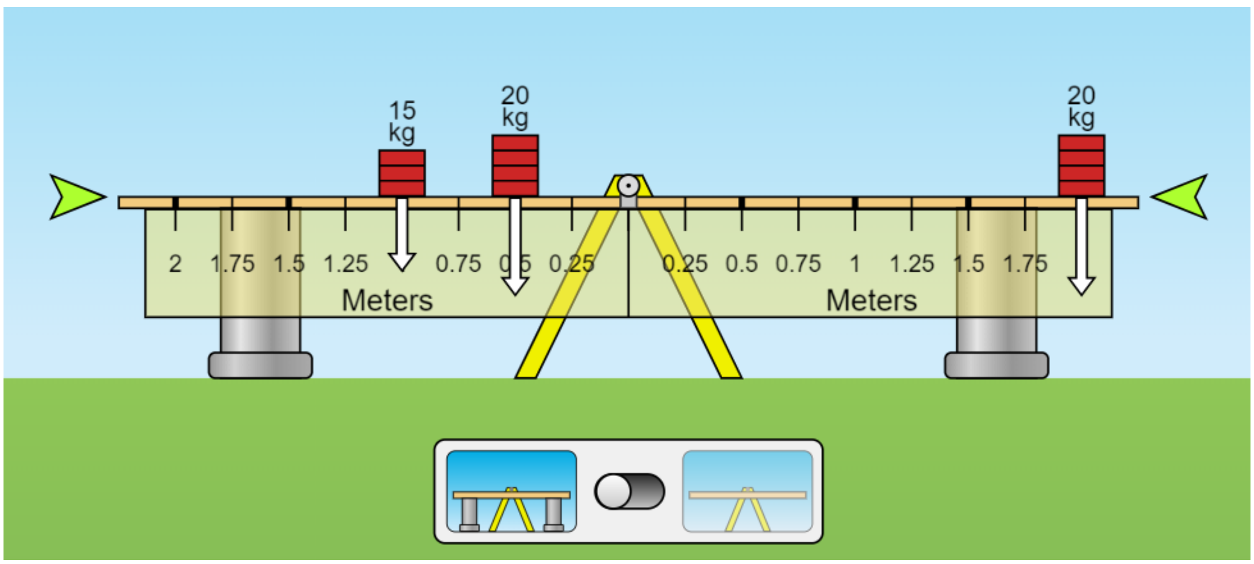


Figure 11.11: PhET Balance Simulation Set-up to Determine Location of 20 Kg Mass

Question 14: Show the calculation for the correct location of the 20 kg mass. Record your result in Table 11.13. Place the 20 kg mass at the location you calculated in the simulation to verify your result.

Location of 20 kg Mass to Achieve Rotational Equilibrium (m)	

Table 11.13: Location of 20 kg Mass from the PhET Simulation

On the left side of the beam, now place a 5 kg brick at 1 meter and 20 kg brick at 0.5 meters. On the right hand side, place package A (with unknown mass) at 0.75 m (the packages with unknown masses can be found by scrolling through the screens of mass objects). You will notice that when you click the button to remove the pillars, the beam is in rotational equilibrium. Use the second condition for rotational equilibrium to calculate the mass of package A (see Figure 11.12).

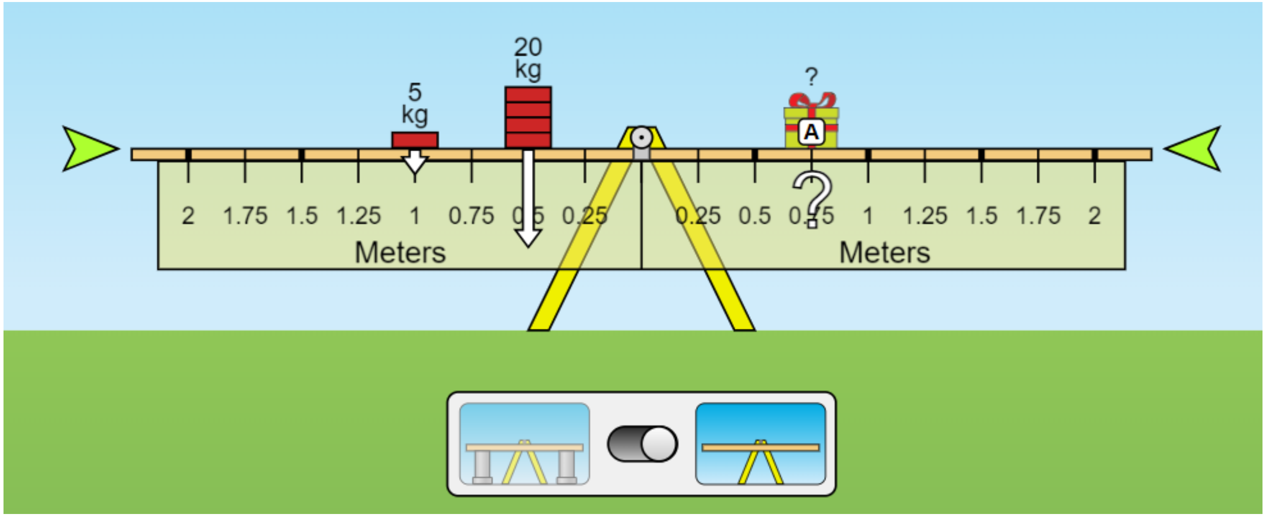


Figure 11.12: PhET Balance Simulation Set-up to Determine Mass of Package A

Question 15: Show the calculation for the mass of package A. Record your result in Table 11.14.

Mass of Package A (kg)	

Table 11.14: Mass of Package A from the PhET Simulation

Question 16: Use the simulation to determine the masses of packages B, C, and D. This may be done by trial and error. Record your results in Table 11.15.

Mass of Package B (kg)	
Mass of Package C (kg)	
Mass of Package D (kg)	

Table 11.15: Mass of Packages B, C, and D from the PhET Simulation

Lab 12

Applications of Rotational Equilibrium

This lab will focus on using the second condition for rotational equilibrium to solve problems in human biomechanics. We will first use a mechanical model of the human arm to measure the biceps force while the model is performing an isometric contraction. We will then use anthropometric data (from cadaver studies) to estimate our own biceps force while performing a similar isometric contraction. Finally, we will determine the location of the center of mass of the human body using a balance board and a scale.

12.1 Mechanical Model of the Human Arm

In this part of the experiment, we will use the Pasco human arm model (see Figure 12.1). The human arm model is a mechanical model that consists of a metal upper arm, a metal forearm and hand, and a rope that attaches from the shoulder to the forearm. A force sensor is connected to the rope and can measure the tension in the rope.

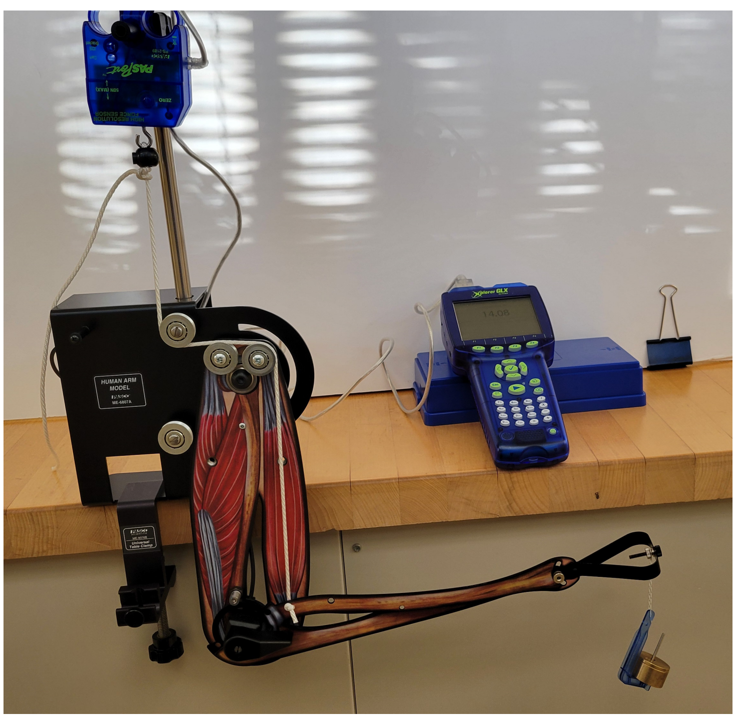


Figure 12.1: Human Arm Model

Note that the mechanical model is a functional, not an anatomical, model of the human arm. It is a simple model that is designed to emulate the basic functions of the arm. Therefore, there are many features of the real human arm that are not present in the mechanical model.

Question 1: Compare your arm with the mechanical model. What are some similarities and some differences? List at least two similarities and two differences in Table 12.1.

List Some Similarities	
List Some Differences	

Table 12.1: Similarities and Differences Between the Model Arm and a Human Arm

12.1.1 The Effects of Different Loads in the Hand on Biceps Force

In this part of the lab, we would like to use the mechanical model to investigate how the biceps force changes as we place different loads in the hand. To do this, we first want to derive an equation to compute the biceps force.

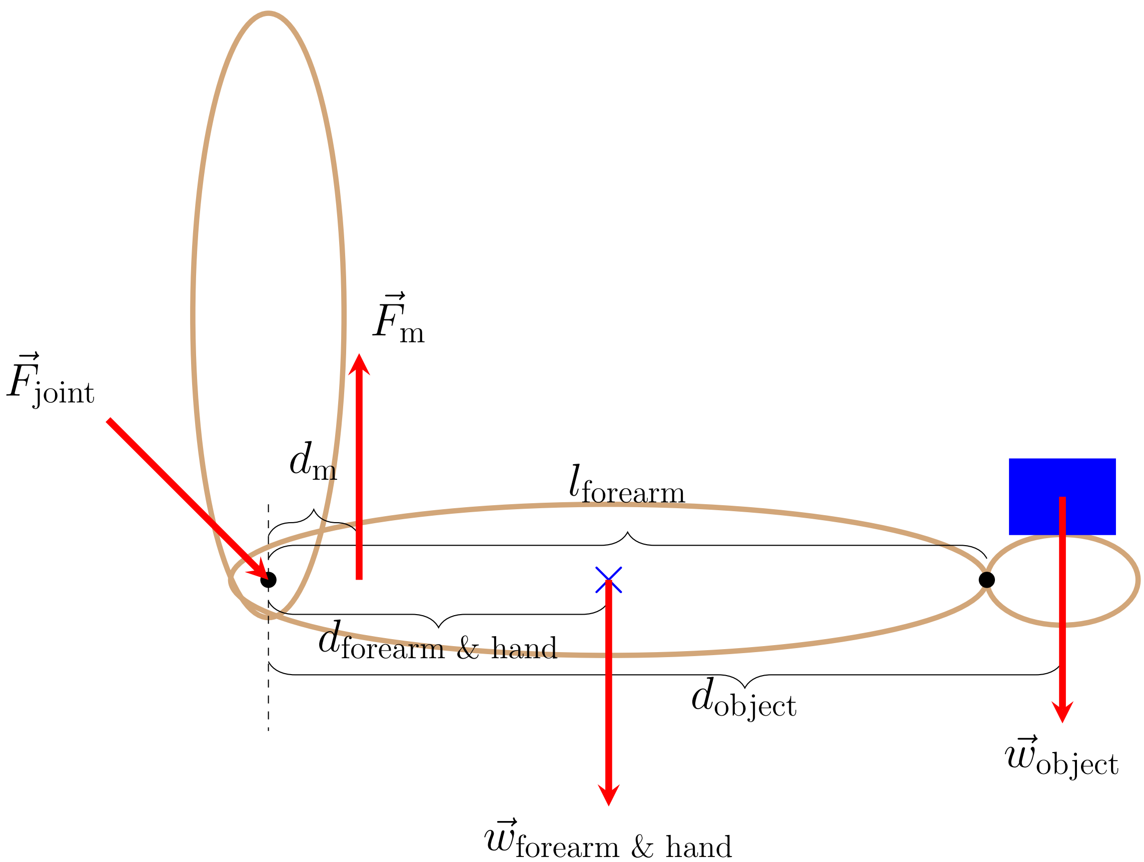


Figure 12.2: Diagram of the Human Arm

As shown in Figure 12.2, there are four forces acting on the forearm and hand. These are the following: the joint force $\vec{F}_{\text{joint}}$ which acts at the elbow joint, the muscle force $\vec{F}_{\text{m}}$ which is attached a distance d_{m} from the elbow joint, the weight of the forearm and hand $\vec{w}_{\text{forearm \& hand}}$ which acts at the center of mass of the forearm and hand which is a distance $d_{\text{forearm \& hand}}$ from the elbow joint, and the weight of the object $\vec{w}_{\text{object}}$ which acts at the center of the object which is a distance d_{object} from the elbow joint.

Assuming that forearm and hand are static (an isometric contraction is being performed), then the sum of the torques due to all of the forces about an axis of rotation must equal zero. Since we want to solve for the muscle force, and don't know the joint force, we will place the axis of rotation at the elbow joint. If we also assume that the upper arm is vertical and the forearm and hand are at a 90° angle with respect to the upper arm, then the distances d_{m}, $d_{\text{forearm \& hand}}$, and d_{object} are the moment arms for the muscle force, the weight of the forearm and hand, and the weight of the object, respectively.

Summing the torques about the elbow joint, and using the convention that forces which tend to rotate the forearm and hand counterclockwise about the elbow joint result in positive torques, gives

$$\sum \tau = 0$$

$$\tau_{\vec{F}_{\text{m}}} + \tau_{\vec{w}_{\text{forearm \& hand}}} + \tau_{\vec{w}_{\text{object}}} + \tau_{\vec{F}_{\text{joint}}} = 0$$

$$F_{\text{m}} d_{\text{m}} - w_{\text{forearm \& hand}} d_{\text{forearm \& hand}} - w_{\text{object}} d_{\text{object}} + 0 = 0$$

Finally, solving for the muscle force we have

$$F_{\text{m}} = \frac{w_{\text{forearm \& hand}} d_{\text{forearm \& hand}} + w_{\text{object}} d_{\text{object}}}{d_{\text{m}}} \tag{12.1}$$

Because we are interested in determining the effect that varying the weight of the object has on the muscle force, let's also write the equation in a slightly different way which will emphasize that this is in fact a linear relationship.

$$F_{\text{m}} = \frac{d_{\text{object}}}{d_{\text{m}}} w_{\text{object}} + \frac{d_{\text{forearm \& hand}}}{d_{\text{m}}} w_{\text{forearm \& hand}} \tag{12.2}$$

Question 2: Assume for a moment that your arm is the mechanical model arm. If you are holding a weight in your hand and are performing an isometric contraction, what muscles are you using to perform the contraction? As you know, there are several elbow flexors in your real arm, why do you think we are modeling them as one single muscle in our model (represented by the tension in the rope).

Question 3: If you were to increase the weight of the load in your hand (or in the mechanical model's hand), how do you think it will affect the biceps force? Predict whether you think the biceps force will increase, decrease, or stay the same.

Now let's model this situation with the Pasco human arm model. Set up the human arm model so that the upper arm is hanging vertically and the forearm is at a 90° angle to the upper arm. Attach a 25 g (0.245 N) weight to the hand (you can do this by hanging a 25 g mass from the stainless steel bolt at the hand). Attach the force sensor to the rod and tie the cord to the force sensor so that the forearm remains at 90° as shown in Figure 12.1. Plug the force sensor into the

GLX device and make sure you have the correct settings. Your lab instructor will give you directions on how to set up the GLX device to measure the force. You are now ready to record the biceps force.

Measure the biceps force for each mass of the load that is placed in the hand. Record your data in Table 12.2.

Mass of the Load In the Hand (g)	Weight of the Load In the Hand (N)	Biceps Force (N)
0		
25		
45		
65		
85		
105		
125		
145		
165		
185		

Table 12.2: Biceps Force in the Mechanical Model

Question 4: For each mass, calculate the weight of the load in Newtons. Use $w = mg$ where m is the mass in kilograms (for example, to calculate the weight of the 25 g mass you would compute $0.025 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 0.245 \text{ N}$).

Question 5: Plot the biceps force versus the weight of the load in the hand. Perform a linear fit to the data and record the values of the slope and y-intercept in Table 12.3.

slope		y-intercept	

Table 12.3: Slope and y-intercept

Question 6: From the shape of your graph, explain the qualitative relationship between the biceps force and the weight of the load in the hand. Was this what you predicted in question 3?

Question 7: What does the value of the slope represent? What does the value of the y-intercept represent? Hint: Refer to Equation 12.2 to help you answer this question.

Question 8: Notice that the biceps force is greater than zero when there is no load in the hand. Why is this the case? Explain.

Question 9: Assume the mass of the model forearm is 100 g = 0.1 kg. Draw a free body diagram (similar to Figure 12.2) showing all the forces acting on the forearm and hand. Take the moment arm of the center of mass of the forearm and hand to be 0.14 m. Take the moment arm of the attachment of the rope (biceps muscle) to be 0.045 m. Finally, take the moment arm of the load in the hand to be 0.35 m. Assume that there is a 185 g = 0.185 kg load in the hand. Using all of this information along with Equation 12.1, compute the biceps force. Note: Remember that the mass is not a force. You must compute the forces (which are the weights) by using $w = mg$ where m is the mass in kg and $g = 9.8$ m/s^2. Show your calculations.

Question 10: Compare this computed value to the experimental value of the biceps force when holding a 185 g mass in the hand (for the experimental value, use the value in Table 12.2). Compute the percent experimental error and record your results in Table 12.4.

Computed Biceps Force (N)	Experimental Biceps Force (N)	Percent Experimental Error (%)

Table 12.4: Comparison of Computed and Experimental Biceps Force with a 185 g Load in the Hand

12.2 Magnitude of the Biceps Force in the Human Arm

We will now estimate the biceps force in your own arm while performing an isometric contraction. To do this, we have to make several assumptions. First, we assume that there is only one elbow flexor which we are calling the biceps muscle. In fact, we known that there are several elbow flexors in the human arm but because we will have only one torque equation, we can have only one unknown variable if we want to solve for it directly. The more realistic case would involve performing some optimizations and is beyond the scope of this experiment. To determine values such as the weight of the forearm and hand, the location of the center of mass of the forearm and hand, etc., we will use cadaver data. Although the average cadaver data for the values of these variables will differ from your actual individual values, we will at least get a reasonable estimate of these values.

Here is the procedure that should be used.

- Weigh the individual in pounds (w_{p} lb) and then convert to Newtons (1 lb = 4.45 N)
- Holding the forearm at 90°, measure the length of the forearm (l_{forearm}) from the elbow joint to the wrist joint. Measure the length in centimeters then convert to meters.
- Calculate the weight of the forearm and hand in Newtons using $w_{\text{forearm \& hand}} = 0.022 \cdot w_{\text{p}}$.
- Calculate the weight of the 2 kg object in Newtons using $w_{\text{object}} = 2\text{ kg} \cdot 9.8\text{ m/s}^2 = 19.6\text{ N}$.
- Find the moment arm of the forearm and hand in meters using $d_{\text{forearm \& hand}} = 0.682 \cdot l_{\text{forearm}}$.
- Find the moment arm of the object (d_{object}) by measuring the distance from the elbow joint to the center of the object while performing the isometric contraction. Measure the length in centimeters then convert to meters.

- Assume the the moment arm for the biceps muscle is $d_{\text{m}} = 4.5$ cm $= 0.045$ m. This is the distance of the attachment of the tendon from the elbow joint and we are making the simplification that it is the same for all individuals.

Question 11: Using all of the information from above, compute the biceps force F_{m} using Equation 12.1. Record all of your results in Table 12.5.

Weight of the Individual (w_{p}) (N)	
Length of Forearm (l_{forearm}) (m)	
Weight of Forearm and Hand ($w_{\text{forearm \& hand}}$) (N)	
Weight of Object (w_{object}) (N)	19.6
Moment Arm of Forearm and Hand ($d_{\text{forearm \& hand}}$) (m)	
Moment Arm of Object (d_{object}) (m)	
Moment Arm of Biceps Muscle (d_{m}) (m)	0.045
Force of Biceps Muscle (F_{m}) (N)	
Ratio of Biceps Force to Weight of Object $\frac{F_{\text{m}}}{w_{\text{object}}}$	

Table 12.5: Biceps Force of an Individual Performing an Isometric Contraction

Question 12: Compare the magnitude of the biceps force to the magnitude of the weight of the object. How many times as large is the biceps force compared to the weight of the object. To answer this question, just compute the ratio of the biceps force to the weight of the object $\left(\frac{F_{\text{m}}}{w_{\text{object}}}\right)$. Does this value seem reasonable?

12.3 Determination of the Location of the Center of Mass in the Human Body

In this part of the lab, we will use the second condition for equilibrium to locate the center of mass in the human body. To do this, we will use the balance board method as shown in Figure 12.3.

12.3.1 Terminology

The following terminology will be used in this experiment.

- Fundamental Standing Position - The person stands in an erect position looking ahead with feet slightly spread apart. The arms are rotated so that the palms of the hands face the thighs.
- Fundamental Lying Position - Same as above except that the person lies down on their back.
- Sagittal plane - An imaginary plane that vertically divides the body into right and left parts.

Figure 12.3: Determination of the Center of Mass of the Human Body

- Frontal plane - An imaginary plane that vertically divides the body into front and back parts.
- Transverse plan - An imaginary plane that horizontally divides the body into upper and lower parts.
- Center of Mass - A point at which the mass of a body may be imagined to be concentrated. The location of the center of mass may be located either within the body or outside the body, depending upon posture. For example, a diver in a pike position may have their center of mass located outside the body.

12.3.2 Theory

12.3.2.1 Transverse Plane

Ultimately, we want to find the three-dimensional location of the center of mass. That is the location where the sagittal plane, the frontal plane, and the transverse plane would pass through the center of mass. In the following derivation, we will find an equation that gives the location where the transverse plane passes through the center of mass. This is also the vertical distance of the center of mass relative to the individual's feet.

Let's define the following variables (see Figure 12.4).

$$\begin{aligned}
\vec{F} &= \text{force exerted by the scale on the board (scale reading)} \\
\vec{w}_{\text{p}} &= \text{weight of the person} \\
\vec{w}_{\text{b}} &= \text{weight of the board} \\
\vec{F}' &= \text{force exerted by the right block of the board} \\
l &= \text{length of the board} \\
z &= \text{distance of center of mass from right knife edge}
\end{aligned}$$

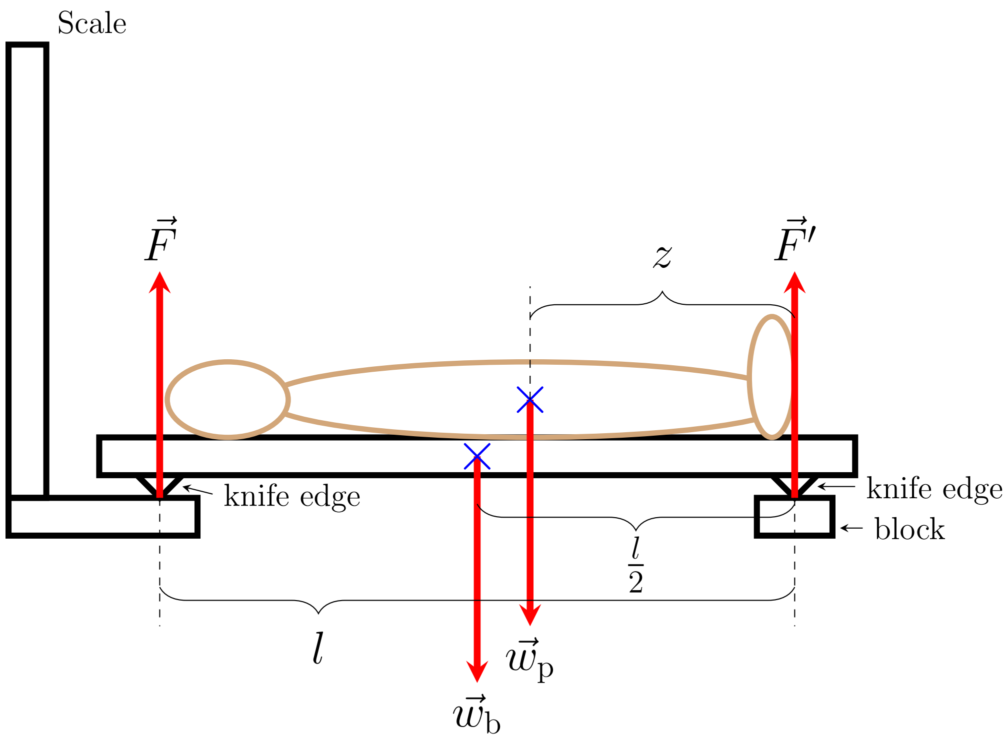


Figure 12.4: Diagram to Determine the Center of Mass of the Human Body

Because the person and the board are static when the person is lying still on the board, the system is in rotational equilibrium. Therefore, the sum of the torques due to all the forces about any point must equal zero. We can't measure F' so let's choose the axis of rotation to be at the right knife edge. Summing the torques (and applying the convention that torques due to forces that tend to rotate the object counterclockwise about the axis of rotation are positive and torques due to forces that tend to rotate the object clockwise about the axis of rotation are negative) gives

$$\sum \tau = 0$$

$$\tau_{\vec{F}} + \tau_{\vec{w}_\mathrm{p}} + \tau_{\vec{w}_\mathrm{b}} + \tau_{\vec{F}'} = 0$$

$$-Fl + w_\mathrm{p} z + w_\mathrm{b} \frac{l}{2} + 0 = 0$$

Solving for z gives

$$z = \frac{\left(F - \frac{w_\mathrm{b}}{2}\right) l}{w_\mathrm{p}} \tag{12.3}$$

Notice that if the individual is standing in the fundamental position, z represents the distance relative to the floor where the transverse plane passes through the center of mass.

12.3.2.2 Sagittal Plane

To determine the location where the sagittal plane passes through the center of mass, the individual must stand on the board such that the sagittal plane is perpendicular to the length of the board (see Figure 12.5). The distance from the right knife edge to the sagittal plane x is then calculated by an equation similar to Equation 12.3.

$$x = \frac{\left(F - \frac{w_{\mathrm{b}}}{2}\right) l}{w_{\mathrm{p}}} \tag{12.4}$$

where F is the scale reading when standing in this position. The values for the other variables are exactly the same.

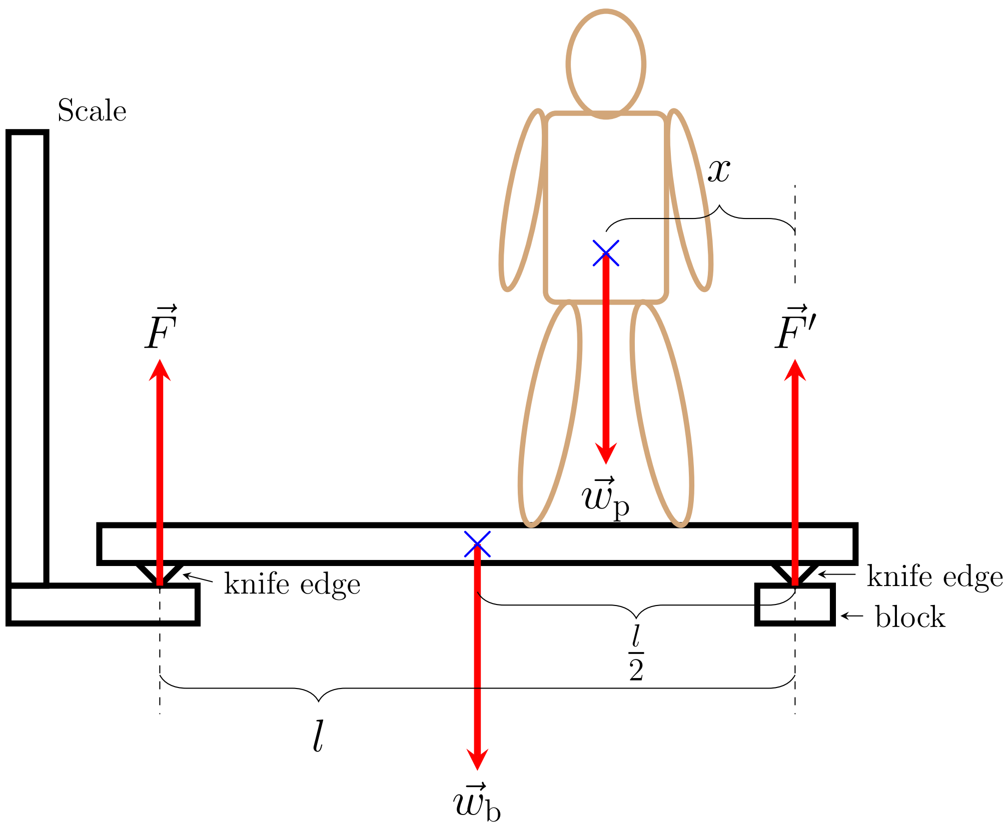


Figure 12.5: Diagram to Determine the Center of Mass of the Human Body - Sagittal Plane

12.3.2.3 Frontal Plane

To determine the location where the frontal plane passes through the center of mass, the individual must stand on the board such that they are facing the scale (or facing away from the scale). In this position, their frontal plane is perpendicular to the length of the board (see Figure 12.6). The distance from the right knife edge to the frontal plane y is again calculated by an equation similar to Equation 12.3.

$$y = \frac{\left(F - \frac{w_{\mathrm{b}}}{2}\right) l}{w_{\mathrm{p}}} \tag{12.5}$$

where F is the scale reading when standing in this position. The values for the other variables are exactly the same.

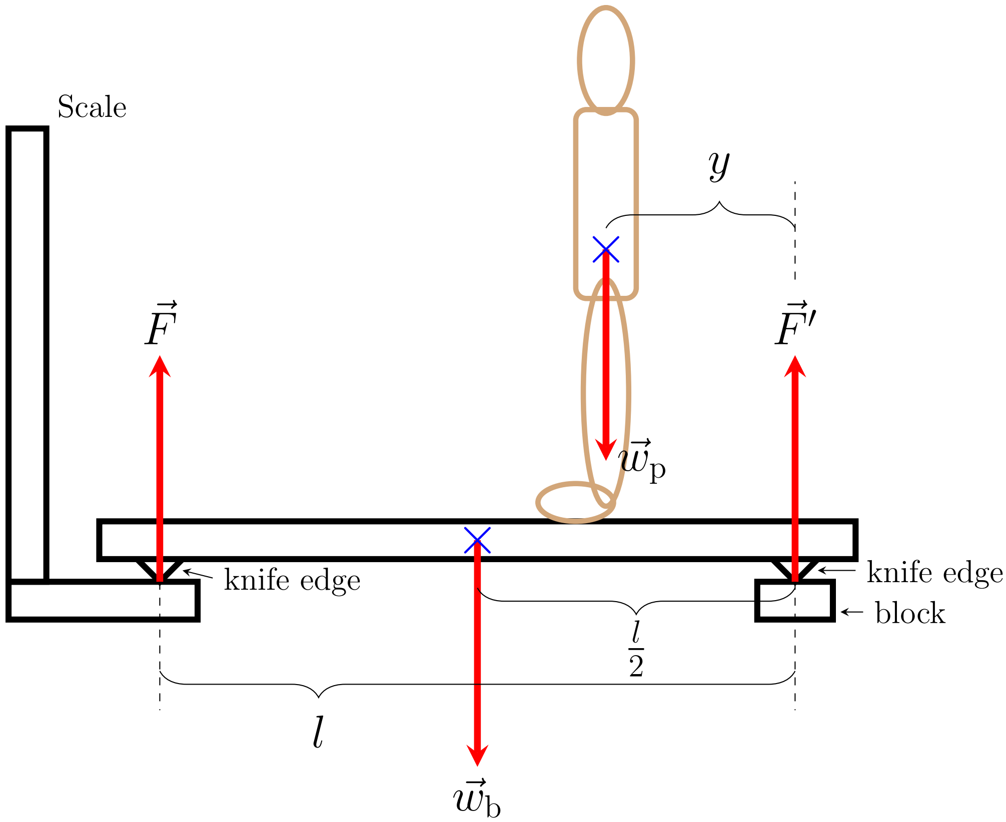


Figure 12.6: Diagram to Determine the Center of Mass of the Human Body - Frontal Plane

12.3.3 Procedure

The following procedure will be used to locate the center of mass.

- Ask for a male and a female volunteer from the class.
- Measure their weight (w_p) in pounds.
- Measure their height (h) in inches.
- While they are lying down on their back with their feet at the right knife edge, measure the force F on the scale (see Figure 12.4).
- Calculate the location z of where the transverse plane passes through the center of mass.
- Tape a large piece of white paper to the board and have the individual stand on the paper while facing perpendicular to the length of the board (see Figure 12.5).
- Trace their feet on the paper, then measure the force F on the scale.
- Calculate the location x of where the sagittal plane passes through the center of mass.
- Measure a distance x from the right knife edge and draw a line on the paper to represent the projection of the sagittal plane.
- Carefully rotate the paper by 90° so that the subject will be facing the scale when they stand in their tracing (see Figure 12.6).

- Measure the force F on the scale when standing in this position.
- Calculate the location y of where the frontal plane passes through the center of mass.
- Measure a distance y from the right knife edge and draw a line on the paper to represent the projection of the frontal plane.
- Record all your values in Table 12.6.

Male Subject		Female Subject	
Weight (w_{p}) (lb)		Weight (w_{p}) (lb)	
Height (h) (in)		Height (h) (in)	
Force of Scale (Transverse) (z) (lb)		Force of Scale (Transverse) (z) (lb)	
Force of Scale (Sagittal) (z) (lb)		Force of Scale (Sagittal) (z) (lb)	
Force of Scale (Frontal) (z) (lb)		Force of Scale (Frontal) (z) (lb)	
Weight of Board (w_{b}) (lb)	32	Weight of Board (w_{b}) (lb)	32
Length of Board (l) (in)	79	Length of Board (l) (in)	79
Center of Mass Transverse (z) (in)		Center of Mass Transverse (z) (in)	
Center of Mass Sagittal (x) (in)		Center of Mass Sagittal (x) (in)	
Center of Mass Frontal (y) (in)		Center of Mass Frontal (y) (in)	
Center of Mass Transverse (z) As a Percentage of Height (%)		Center of Mass Transverse (z) As a Percentage of Height (%)	

Table 12.6: Center of Mass Data

Question 13: Describe in general the location of the center of mass. When you projected the sagittal and frontal planes onto the paper, did your result look similar to Figure 12.7? Now imagine that the paper is shifted upward to where the transverse plane passed through the center of mass. This should give you an idea of the three dimensional location of the center of mass.

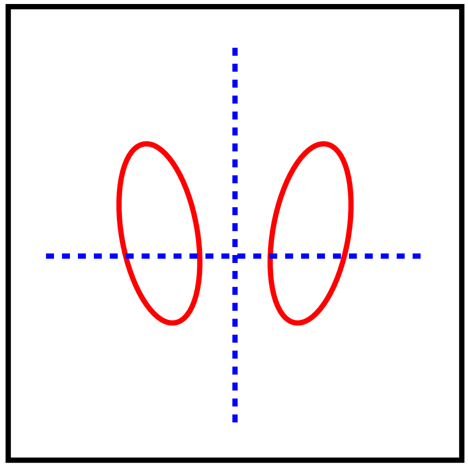


Figure 12.7: Center of Mass Projection of the Sagittal and Frontal Planes

Question 14: We would now like to compare the location where the transverse plane passed through the center of mass between genders. However, we can't compare the z values directly as they may be different due to the differences in subject heights. We can make a direct comparison, however, if we represent z as a percentage of their height. Compute z as a percentage of height and record your results in Table 12.6. Were there differences between genders? Explain.

Part V

Work, Energy, Momentum, and Conservation Laws

Lab 13

An Experimental Test of the Work-Energy Theorem

In this experiment you will test the work-energy theorem by investigating the motion of a cart as it travels down a frictionless incline. In particular, you will apply the work-energy theorem to the problem of determining the speed of the cart at some final position $\vec{r}_\text{f}$ along the incline when at some initial position $\vec{r}_\text{i}$ the cart was at rest.

13.1 Theory

13.1.1 Review of the Work-Energy Theorem

The work done on a body by a *constant* force $\vec{F}$ is given by

$$W = Fd\cos\theta \tag{13.1}$$

where F is the magnitude of the constant force $\vec{F}$, d is the magnitude of the displacement vector $\vec{d}$ that locates the body's final position $\vec{r}_\text{f}$ (point b) relative to its initial position $\vec{r}_\text{i}$ (point a), and θ is the angle between $\vec{F}$ and $\vec{d}$ (see Figure 13.1). Thus, if $\vec{F}$ and $\vec{d}$ are known, then the work done by $\vec{F}$ can be computed. (Note that the magnitude of the constant force $\vec{F}$ that appears in the definition of work done by a constant force, Equation 13.1, need not be the resultant force acting on the body. There may, in fact, be several forces acting on the body; we may compute the work done by any one of any linear combination of these forces, if we wish, using the definition for work.) When the force $\vec{F}$ varies either in magnitude or direction or both, the definition for work done by the force $\vec{F}$ must be generalized to an integral representation.

$$W = \int_a^b \vec{F} \cdot d\vec{r} \tag{13.2}$$

where $\vec{F}$ is the force and $d\vec{r}$ is an infinitesimal displacement along the path. For further details, the student is invited to refer to the literature. The net work done on a body by the resultant force acting on the body in displacing the body from some initial position $\vec{r}_\text{i}$ (i) to some final position $\vec{r}_\text{f}$ (f) is equal to the change in the kinetic energy $K = \frac{1}{2}mv^2$ of the body in its motion from i to f. This is a statement of the work-energy theorem.

$$W_\text{net} = K_\text{f} - K_\text{i} = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \tag{13.3}$$

The theorem follows readily upon application of Newton's second law to the definition of work. The proof of the theorem will be given elsewhere (see, for example, your physics textbook). If the resultant force acting on the body is constant and its magnitude and direction are known, then Equation 13.1 may be employed to compute the work done by the resultant force. Also, knowledge of the work done by the resultant force together with the value of the speed $v_{\rm i}$ of the body at some initial position i, facilitates the use of the work-energy theorem Equation 13.3 to determine the speed $v_{\rm f}$ of the body at some final position f.

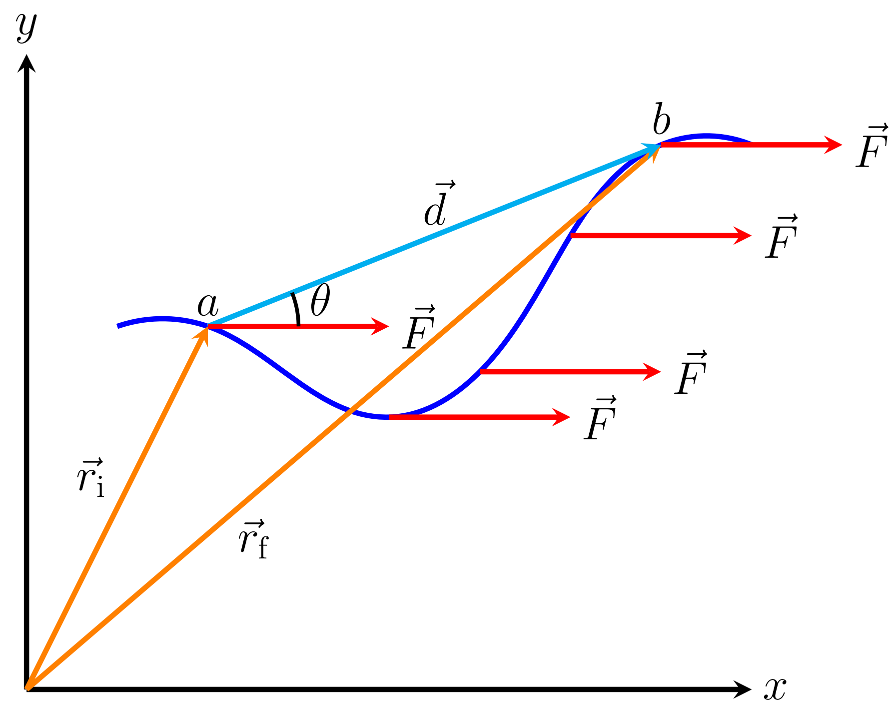


Figure 13.1: Work Done on a Body by a Constant Force

13.1.2 An Application of the Work-Energy Theorem - A Cart Sliding Down a Frictionless Incline

13.1.2.1 Finding the Final Velocity Using the Work-Energy Theorem

Figure 13.2 shows the forces that act on a cart sliding without friction down an incline. As you can see, there are two forces: (1) the force due to gravity $\vec{w}$ (the cart's weight) and (2) the force that the incline exerts on the cart $\vec{N}$ (the normal force). The resultant force $\vec{R}$ acting on the cart is the sum of these two forces.

$$\vec{R} = \vec{w} + \vec{N} \tag{13.4}$$

We have conveniently chosen the positive x direction to be down the incline. The positive y direction is then chosen to be outward and perpendicular to the incline. The x componet of $\vec{R}$ is given by

$$R_x = w_x + N_x = w\sin\theta + 0 = w\sin\theta \tag{13.5}$$

The y component of $\vec{R}$ is given by

$$R_y = w_y + N_y = -w\cos\theta + N = 0 \text{ (the cart is constrained to remain on the incline)} \tag{13.6}$$

Therefore, $\vec{R}$ is directed down the incline (in the $+x$ direction) and has a magnitude of

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(w\sin\theta)^2 + 0^2} = w\sin\theta \tag{13.7}$$

Notice that since θ is kept constant, then $\vec{R}$ is a constant force. Therefore, the definition of work for a constant force (Equation 13.1) may be used to compute the work done by $\vec{R}$. Also notice that $\vec{R}$ is in the same direction as the displacement $\vec{d}$. The net work done by $\vec{R}$ while undergoing a displacement $\vec{d}$ is then given by

$$W_{\text{net}} = (w\sin\theta)d\cos 0^\circ = (w\sin\theta)d \tag{13.8}$$

Inserting Equation 13.8 into the work-energy theorem (Equation 13.3), we find

$$(w\sin\theta)d = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2 \tag{13.9}$$

That is, the amount of work done by the resultant force $\vec{F}$, Equation 13.8, goes into a change in the kinetic energy of the cart that is just its kinetic energy at position f minus its kinetic energy at position i. If the cart is released from rest, then $v_{\text{i}} = 0$ and Equation 13.9 yields

$$(w\sin\theta)d = (mg\sin\theta)d = \frac{1}{2}mv_{\text{f}}^2 \tag{13.10}$$

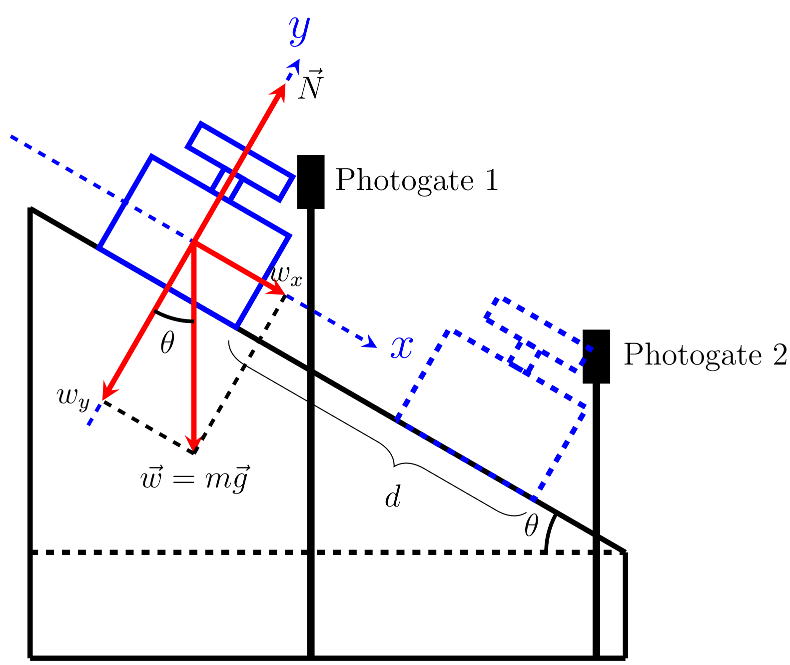


Figure 13.2: Diagram to Use Work-Energy Theorem to Compute the Final Velocity of the Cart

Equation 13.10 says that the amount of kinetic energy that the cart has at a particular position f (when the cart has started from rest at position i) is exactly equal to the amount of work done by the resultant force $\vec{R}$ in accelerating the cart from rest at position i to position f. Equation 13.10 may now be solved for v_{f} to give

$$v_{\text{f}} = \sqrt{2gd\sin\theta} \tag{13.11}$$

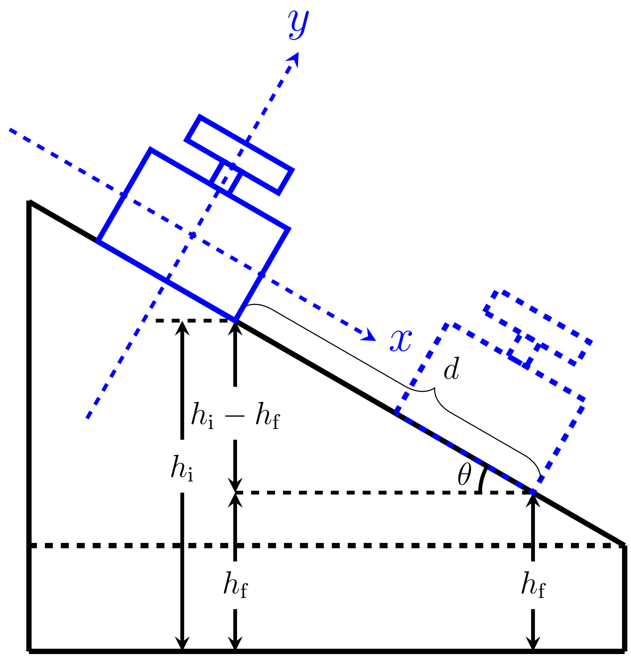


Figure 13.3: Diagram to Compute $\sin\theta$

Please note that from basic trigonometry (see Figure 13.3) we have

$$\sin\theta = \frac{h_\mathrm{i} - h_\mathrm{f}}{d} \tag{13.12}$$

Equation 13.11 then becomes

$$v_\mathrm{f} = \sqrt{2g(h_\mathrm{i} - h_\mathrm{f})} \tag{13.13}$$

Equation 13.13 gives the prediction of v_f from the work-energy theorem. In this experiment, you will measure v_f experimentally and compare your answer with the prediction from Equation 13.13.

13.1.2.2 Finding the Final Velocity Using Kinematics

We can also compute the final velocity of the cart v_f using kinematics. Since the resultant force $\vec{R}$ is constant and $\vec{R} = m\vec{a}$, $\vec{a}$ will also be constant. Applying Newton's Second Law to the x direction gives

$$mg\sin\theta = ma_x \tag{13.14}$$

or

$$a_x = g\sin\theta \tag{13.15}$$

The magnitude of the displacement vector d is given in terms of the average velocity $\overline{v}$ of the cart from position i to position f and the time t it takes the cart to go from position i to position f by

$$d = \overline{v}t \text{ or } \overline{v} = \frac{d}{t} \tag{13.16}$$

Now, because a is constant, we may also write the average velocity as

$$\overline{v} = \frac{v_\mathrm{i} + v_\mathrm{f}}{2} \tag{13.17}$$

However, the cart starts at rest ($v_\mathrm{i} = 0$), therefore from Equation 13.17 we have

$$v_\mathrm{f} = 2\overline{v} \tag{13.18}$$

Using Equations 13.16 and 13.18 then gives

$$v_\mathrm{f} = \frac{2d}{t} \tag{13.19}$$

13.2 The Experiment

We will create a frictionless incline by placing a 2 kg mass under the single leg of the air track. Choose a convenient displacement d (say $d = 100$ cm) for the cart to travel down the air track. Place the cart at the initial position (position i) then move photogate 1 so that the flag is just about to trigger the photogate. Now place the cart a distance d (position f) down the inclined air track and move photogate 2 so that the flag is just about to trigger the photogate. Start the cart at rest at position i and using the Smart Timer in two-gate mode, record the time it takes the cart to travel the distance d (see Figure 13.4). Measure the time for 5 trials and record your results in Table 13.1.

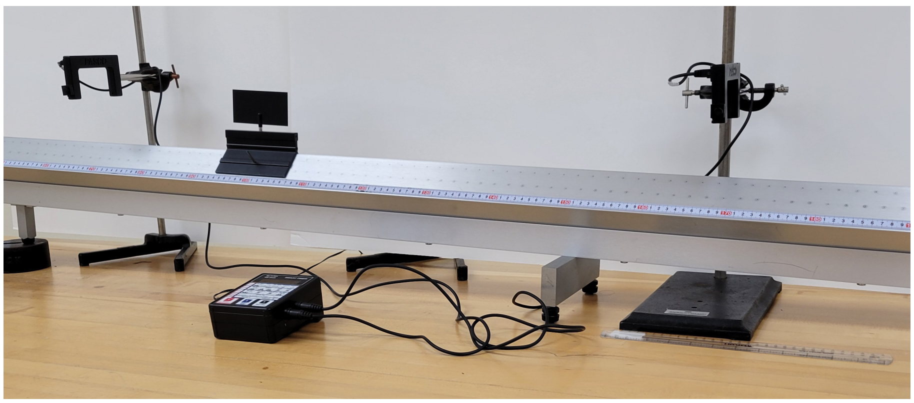


Figure 13.4: Set-Up to Use the Work-Energy Theorem to Compute the Final Velocity of the Cart

Trial	1	2	3	4	5	Average
Time (t) (s)						
Absolute Deviations (s)						
PRAAD (%)						

Table 13.1: Time for Cart to Travel a Distance d Down the Inclined Air Track

Question 1: Calculate the average time and the PRAAD for time. Use Equation 13.19 to compute the experimental final velocity $v_\mathrm{f,exp}$. Record your results in Tables 13.1 and 13.3.

Measure the height of the air track above the lab bench at the initial position i (h_i) and at the final position f (h_f). Record your values in Table 13.2.

Initial Height (h_i) (m)		Final Height (h_f) (m)	

Table 13.2: Height Measurements Needed to Compute $\sin\theta$

Question 2: Use Equation 13.13 to compute the final velocity predicted by the work-energy theorem. Record your value in Table 13.3.

Question 3: Compare the experimental and predicted final velocities by computing the percent experimental error. Record your value in Table 13.3.

Final Predicted Velocity $v_{\mathrm{f,pred}}$ (m/s)	Final Experimental Velocity $v_{\mathrm{f,exp}}$ (m/s)	Percent Experimental Error (%)

Table 13.3: Comparison of Predicted and Experimental Final Velocities

Question 4: How much work is done by the normal force $\vec{N}$ in displacing the cart from the initial position (i) to the final position (f)? Explain.

Question 5: Measure the mass of the cart. Using $W_\mathrm{net} = (w\sin\theta)d = mg\sin\theta d = mg\,(h_\mathrm{i} - h_\mathrm{f})$, compute the net work done by the resultant force $\vec{R}$ in displacing the cart from the initial position (i) to the final position (f). Record your values in Table 13.4.

Mass of Cart (m) (kg)	
Net Work Done by Resultant Force (W_net) (J)	

Table 13.4: Work Done on Cart by Resultant Force

Question 6: How much kinetic energy K_i does the cart have at the initial position (i)? Explain. Record your value in Table 13.5.

Question 7: Compute the kinetic energy of the cart at the final position (f) using $K_\mathrm{f} = \frac{1}{2}mv_{\mathrm{f,exp}}^2$. Note that $v_{\mathrm{f,exp}}$ was computed in question 1. Record your value in Table 13.5.

Question 8: Using your results from questions 6 and 7, compute the change in kinetic energy of the cart in going from the initial position to the final position. Record your value in Table 13.5. How does this value compare to the value computed for the net work done in question 5? Explain.

Question 9: What advantages, if any, does the use of the work-energy theorem offer? Explain.

Question 10: The net work done by the resultant force acting on a body is always equal to the change in the kinetic energy of the body (this is a statement of the work-energy theorem). If

Initial Kinetic Energy (K_{i}) (J)	
Final Kinetic Energy (K_{f}) (J)	
Change in Kinetic Energy (ΔK) (J)	

Table 13.5: Kinetic Energy of the Cart

you have multiple forces acting on a body, can it happen that the work done by one of the forces alone will be greater than the change in kinetic energy? If so, give examples. Hint: Remember that the net work done on an object can be computed as the sum of the work done by each of the individual forces acting on the object. Also note that the work done can be positive (transfer of energy from the environment into the system) or negative (transfer of energy from the system to the environment).

Lab 14

Alternate Experiment on the Work-Energy Theorem

This alternate experiment uses the PASCO Dynamics System along with the PASCO Smart Timer to verify the Work-Energy Theorem and subsequently verify the Law of Conservation of Mechanical Energy.

14.1 Verifying the Work-Energy Theorem

An extremely important theorem in physics is the work-energy theorem. This theorem states that the net work that is performed by the net force acting on a body is equivalent to the change in the kinetic energy of the body.

$$W_{\text{net}} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \tag{14.1}$$

In particular, the net work done by a constant net force $\vec{F}_{\text{net}}$ which acts in the same direction as the displacement $\Delta\vec{x}$ is given by

$$W_{\text{net}} = F_{\text{net}}\Delta x \cos\theta = F_{\text{net}}\Delta x \tag{14.2}$$

where $\theta = 0°$ is the angle between the net force vector $\vec{F}_{\text{net}}$ and the displacement vector $\Delta\vec{x}$.

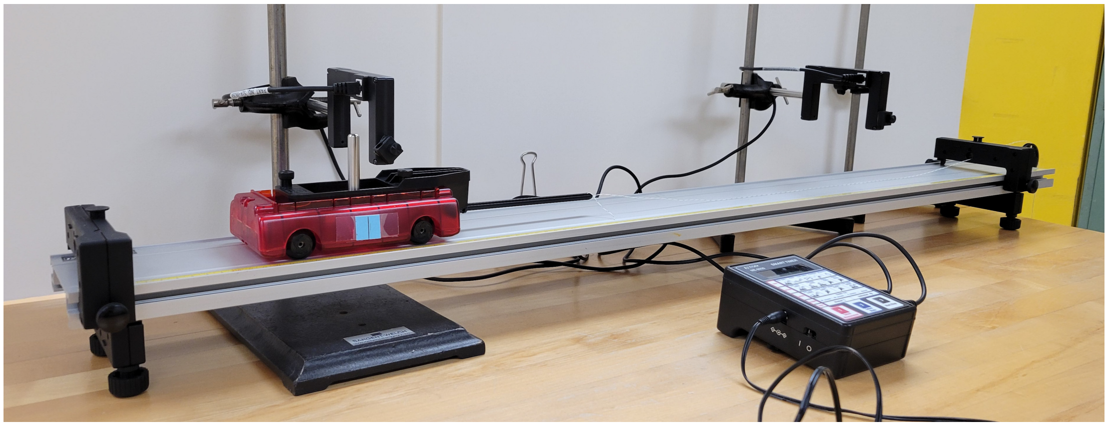


Figure 14.1: Set-Up of Dynamics System to Verify the Work-Energy System

In this activity, you will verify the work-energy theorem by taking measurements using the Pasco Dynamics System. Set up the dynamics track so that it is level and is equipped with a pulley placed at its end. Attach the cart launcher assembly to the car, but do not install the spring (see Figure 14.1). Attach a length of string to the shaft of the cart launcher and run it through the hole of an end stop and over the pulley at the end of the track as shown in Figure 14.2. The string should be long enough so that when a 0.2 Newton weight is attached at its end and allowed to drop, the car will travel the length of the track to the end stop. If necessary, adjust the pulley such that the string is parallel to the track as shown in Figure 14.2.

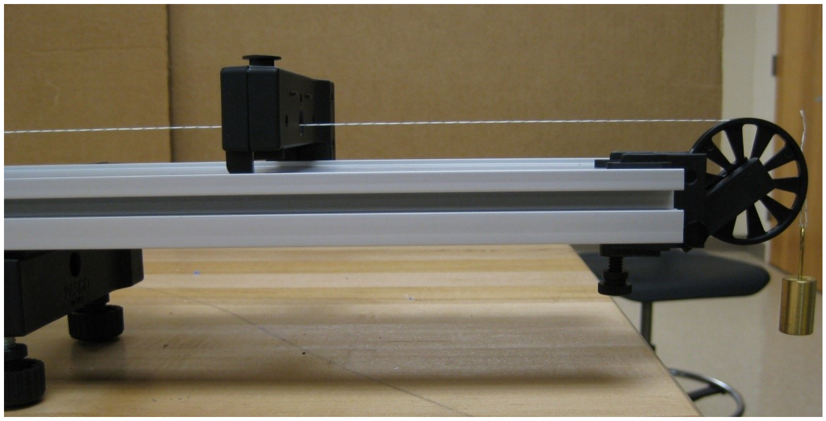


Figure 14.2: Pulley Set-Up for Dynamics System

Position one photogate along the track, as shown in Figures 14.1 and 14.3, such that the 0.935 cm flag (the "flag" is the short steel post) on the car is just about to occlude the photogate eye when the blue marker with the vertical line on the side of the car is at the 20 cm position on the track. Position x_1 is then 20 cm. Position the second photogate along the track, as shown in Figures 14.1 and 14.3, such that the 0.935 cm flag on the car is just about to occlude the photogate eye when the marker on the side of the car is at the 80 cm position on the track. Position x_2 is then 80 cm. The displacement, Δx, of the car as its position changes from x_1 to x_2 is then

$$\Delta x = x_2 - x_1 = 80 \text{ cm} - 20 \text{ cm} = 60 \text{ cm} = 0.60 \text{ m}$$

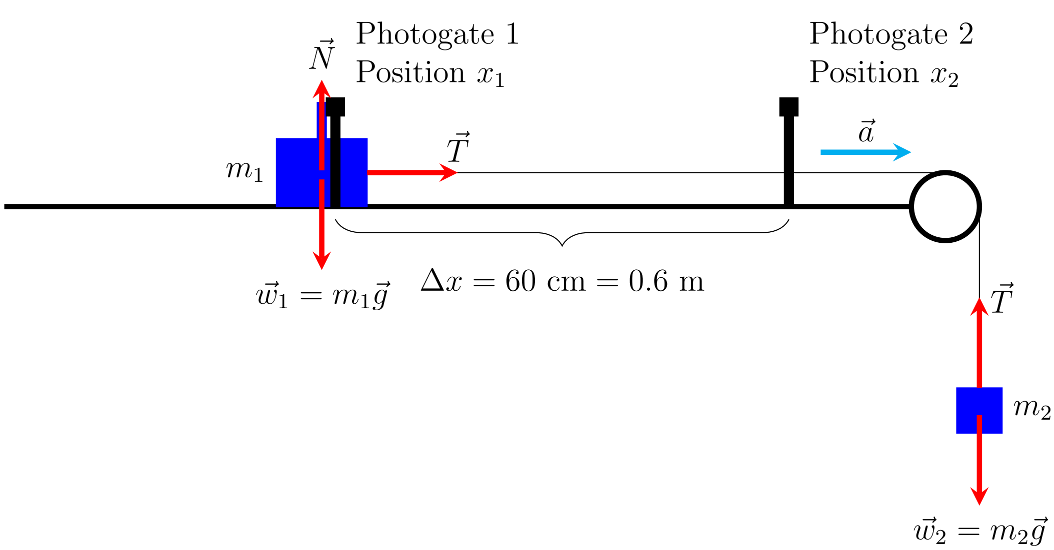


Figure 14.3: Diagram of Dynamics System to Verify the Work-Energy Theorem

Place the Smart Timer in two-gate mode to measure time. Place the car at the position x_1 such that the flag is just about to occlude the photogate eye. Attach the 0.2 Newton weight to the string that passes over the pulley and release the car from rest, measuring the time Δt_{12} for the car to move from position x_1 to position x_2. Perform 5 trials and record your values in Table 14.1. Next, unplug the photogate at position x_1 and put the Smart Timer in stopwatch mode so that you can measure the time Δt for the flag to pass through the photogate at position x_2 (Note: you must remove the connections for both photogates from the Smart Timer then plug the second photogate back into port 1). With the 0.2 Newton weight suspended from the end of the string that passes over the pulley, release the car from rest at position x_1 and measure the time Δt for the flag (steel post) to pass through the photogate at position x_2. Perform 5 trials and record your measurements in Table 14.1.

x_1 (m)	x_2 (m)	$\Delta x = x_2 - x_1$ (m)	Δt_{12} (s)	v_1 (m/s)	Δt (s)	v_2 (m/s)	a (m/s^2)
0.20	0.80	0.60		0			
0.20	0.80	0.60		0			
0.20	0.80	0.60		0			
0.20	0.80	0.60		0			
0.20	0.80	0.60		0			
Average Values		0.60		0			

Table 14.1: Kinematic Values for the Pasco Dynamics Car

Question 1: Compute the velocity v_2 of the car at the second photogate for each trial using

$$v_2 = \frac{d}{\Delta t} = \frac{0.00935}{\Delta t}$$

then compute the average v_2. Record all your results in Table 14.1.

Question 2: Compute the acceleration a of the car as it moved from position x_1 to position x_2 using

$$a = \frac{v_2 - v_1}{\Delta t_{12}}$$

then compute the average acceleration. Record all your results in Table 14.1.

Measure the mass of the car on the electronic balance and record your value in Table 14.2.

Question 3: Copy your average acceleration value from Table 14.1 to Table 14.2. Using Newton's Second Law of Motion, compute the net force acting on the car (the net force is the tension in the string). Record your values in Table 14.2.

Average Acceleration (a) (m/s^2)	Mass of Cart (m) (kg)	Net Force (F_{net}) (N)

Table 14.2: Net Force Acting on Car

Question 4: From the value of the net force on the car F_{net} and the value of the displacement $\Delta x = 0.60$ m, compute the net work done on the car using

$$W_{\text{net}} = F_{\text{net}}\Delta x$$

Record your value in Table 14.3.

Question 5: Compute the change in kinetic energy of the car as it moves from the initial position x_1 to the final position x_2 using

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

How does your answer from question 4 compare to the change in kinetic energy of the car? Compare the two values by computing the percent difference

$$\text{Percent Difference} = \left|\frac{W_{\text{net}} - \Delta K}{W_{\text{net}}}\right| \times 100\%$$

Record your values in Table 14.3.

Work Done by Net Force (W_{net}) (J)	Change in Kinetic Energy (ΔK) (J)	Percent Difference (%)

Table 14.3: Comparison of Net Work Done and Change in Kinetic Energy

14.2 Verification of the Law of Conservation of Mechanical Energy

Using the Work-Energy Theorem, one can derive the Law of Conservation of Mechanical Energy. This law states that when non-conservative forces do no work, the mechanical energy of a system is conserved. Non-conservative forces are forces such as frictional forces, which do work that is path dependent. Conservative forces, such as the force of gravity, do work that is path independent.

$$\text{ME} = K + U_{\text{g}} \tag{14.3}$$

The Mechanical energy (ME) of a body is the sum of the kinetic energy (K) of the body and its gravitational potential energy (U_{g}). Gravitational potential energy results from the work done by the force of gravity. The formulas for kinetic energy K and gravitational potential energy are given as

$$K = \frac{1}{2}mv^2 \tag{14.4}$$

$$U_g = mgy \tag{14.5}$$

where v is the speed of the body and y is the elevation of the center of gravity of the body above a chosen horizontal reference level. g is the gravitational field strength, which has a value of 9.8 N/kg. Note that g is also the acceleration due to gravity, which is 9.8 m/s^2.

In this activity, you will attempt to verify the law of conservation of mechanical energy. You will use the PASCO Dynamics System. Set up the track so that it is inclined at approximately a 5° angle as shown in Figure 14.4.

Figure 14.4: Set-Up of Dynamics System to Verify the Conservation of Mechanical Energy

Attach the cart launcher assembly to the car. The spring that comes with this assembly is to be slipped over the shaft such that the flared end is facing away from the car and the coil at the other end is run through the small hole in the shaft that is next to the car. The spring should then be rotated clockwise so it is locked in place. Adjust the end-stops at the launch site so that they are separated by approximately 7-8 cm. With the car positioned such that the flared end of the spring rests against the end-stop and the spring is *not* compressed, locate the position 1, x_1, of the marker on the side of the car on the track. Record your value in Table 14.4. Position a photogate at this position such that the leading edge of the flag (metal post) is just about to occlude the photogate eye.

Next we need to determine the location along the track to which the car rises to when it is momentarily at rest. Compress the spring by pushing the shaft through the holes in each of the end stops until you see the small pinhole near the end of the shaft. Pass the release pin with the attached string through this small pinhole. When you pull up on the string with a smooth jerk, this will release the car. Practice doing this to get your technique down. You will need to conduct several trials of launching the car in order to accurately determine the location where the car is momentarily at rest (position 2 x_2). Once you have determined this position, record the value in Table 14.4.

Now measure the heights y_1 and y_2 of the bottom of the marker on the side of the car above the laboratory bench when the marker on the side of the car is at position x_1 and position x_2, respectively. Record these values in units of meters in Table 14.4. Using the electronic balance, measure the mass of the car (with the spring attached) and record the value in Table 14.4.

Next, you will determine the velocity of the car at position x_1 by using the Smart Timer in stopwatch mode. Note that only the photgate at position x_1 should be connected to the Smart Timer. Measure the time Δt_1 it takes the flag to pass through the photogate. Peform 5 trials and record your results in Table 14.5.

x_1 (cm)	x_2 (cm)	y_1 (m)	y_2 (m)	Mass (kg)

Table 14.4: Height of Car at the Initial and Final Positions

Time for Flag to Pass Through Photogate (Δt_1) (s)	Velocity at Initial Position (v_1) (m/s)	Velocity at Final Position (v_2) (m/s)
		0
		0
		0
		0
		0
Average Velocity (m/s)		0

Table 14.5: Velocity of Car at Initial and Final Positions

Question 6: Compute the velocity of the car at the initial position ($v_1 = \frac{d}{\Delta t_1} = \frac{0.00935}{\Delta t_1}$) for each trial then calculate the average velocity. Note that the final velocity (v_2) of the car at the final position is 0 m/s since the car is momentarily at rest at that position. Record your results in Table 14.5.

Question 7: Compute the mechanical energy of the car at position 1 using

$$\text{ME}_1 = \frac{1}{2}mv_1^2 + mgy_1$$

where v_1 is the average velocity at position 1. Record your value in Table 14.6.

Question 8: Compute the mechanical energy of the car at position 2 using

$$\text{ME}_2 = \frac{1}{2}mv_2^2 + mgy_2$$

Note that since the velocity at position 2 is $v_2 = 0$ (m/s), the car has no kinetic energy at this instant. Therefore, the mechanical energy is entirely gravitational potential energy. Record your value in Table 14.6.

Question 9: Compare the mechanical energy at position 1 with the mechanical energy at position 2. Compute the percent difference using

$$\text{Percent Difference} = \left| \frac{\text{ME}_2 - \text{ME}_1}{\text{ME}_1} \right| \times 100\%$$

Record your value in Table 14.6.

Mechanical Energy at the Initial Position (ME_1) (J)	Mechanical Energy at the Final Position (ME_2) (J)	Percent Difference (%)

Table 14.6: Mechanical Energy at Initial and Final Positions

Question 10: Were you able to successfully verify the law of conservation of mechanical energy? If not, can you account for any difference between the mechanical energy at position 1 and the mechanical energy at position 2?

Lab 15

An Experimental Demonstration of the Laws of Conservation of Momentum and Energy - Ballistic Pendulum

In this experiment you will investigate two great theorems of physics. Since they appear, in fact, to be valid in applications to all physical phenomena investigated to date, physicists have come to refer to these great theorems as universal laws of nature. These great theorems have thus become known as the law of conservation of momentum and the law of conservation of energy, respectively. Any particular phenomenon of nature that would violate these two laws would certainly create much chaos as well as generate much excitement within the scientific community.

15.1 Theory

15.1.1 Momentum and Its Conservation

Momentum is a *vector* physical quantity used in the discussion of the motion of a particle or system of particles and is defined as the product of the mass m of the particle and the velocity $\vec{v}$ of the particle, i.e.,

$$\vec{p} = m\vec{v} \tag{15.1}$$

Suppose we have a system S consisting of N particles. Now, these N particles are allowed to interact among themselves in various ways. For example, the particles may collide with one another or blow apart or even stick together. When the particles do come in contact with each other, they exert forces on one another. These forces are forces between the particles of the system S and are therefore forces that are internal to the system S. We shall refer to these forces as internal forces. Forces that are due to agents that are external to the system S will be referred to as external forces. An example of an external force would be the gravitational pull of the earth. Note: If there is a net external force acting on a system, then the system will experience an acceleration that is directly proportional to that force.

Now suppose that we look at the system S when it is in some initial state i and again when it is in a final state f. Remember that what we call initial or final is necessarily our own choice since we are the observers of the system. Suppose that when the system S is in the state i there are N particles in the system with masses m_1, m_2, m_3, $\cdots$, m_{N}, respectively, and velocities $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\cdots$, $\vec{v}_{\mathrm{N}}$, respectively. Then the total momentum of the system in the initial state i, call it $\vec{P}_{\mathrm{i}}$ is just the sum of the momenta of the N particles in the system in the sate i. Namely,

$$\vec{P}_{\rm i} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_{\rm N}\vec{v}_{\rm N} \tag{15.2}$$

Now suppose that in the transition of the system S from the state i to the state f something happened to the system so that (1) there are now M particles in the system, where M may or may not be equal to N, (2) the masses of the particles are now $m_1^{'}$, $m_2^{'}$, $m_3^{'}$, $\cdots$, $m_{\rm M}^{'}$, and (3) the velocities of the particles are $\vec{v}_1^{'}$, $\vec{v}_2^{'}$, $\vec{v}_3^{'}$, $\cdots$, $\vec{v}_{\rm M}^{'}$. Let the total momentum of the system of particles in the state f be called $\vec{P}_{\rm f}$. Then $\vec{P}_{\rm f}$ is just the sum of the momenta of the M particles in the system in the state f. Namely,

$$\vec{P}_{\rm f} = m_1^{'}\vec{v}_1^{'} + m_2^{'}\vec{v}_2^{'} + \cdots + m_{\rm M}^{'}\vec{v}_{\rm M}^{'} \tag{15.3}$$

The theorem or law of conservation of momentum is that, in the absence of any external force, the total momentum of a system S remains constant. That is,

$$\vec{P}_{\rm f} = \vec{P}_{\rm i} \tag{15.4}$$

a beautiful, concise statement that appears to be a governing law in nature. Total momentum is then said to be conserved both in magnitude and direction.

15.1.2 Energy and Its Conservation

What is energy? Physicists don't really know. What they do know about energy is the various forms that it can assume; for example, mechanical energy, gravitational potential energy, energy of motion (which we call kinetic energy), chemical energy, electrical energy, nuclear and atomic energy, mass energy (yes, mass energy!), etc. There is one very important thing that physicists have learned about energy, i.e., total energy is conserved. That is, if the total energy of all the various forms of energy of the system S in the state i is $E_{\rm i}$ and the total energy of the system S in the state f is $E_{\rm f}$ then

$$E_{\rm f} = E_{\rm i} \tag{15.5}$$

Note that energy is a scalar quantity. It has a magnitude but no direction.

15.2 The Experiment

15.2.1 An Application of the Laws of Conservation of Momentum and Energy to Ballistics

The ballistic pendulum is an excellent device to use to illustrate the laws of conservation of momentum and energy. Utilizing the laws of conservation of momentum and energy you will determine the initial speed of a ball as it leaves the spring gun of the ballistic pendulum apparatus. But first you will determine the initial speed of the ball as you did in the experiment on the motion of a projectile. You will then compare the results for the initial speed of the ball as determined by these two methods.

15.2.1.1 Determination of the Initial Speed of the Ball by Measuring Its Motion

Place the entire ballistic pendulum apparatus on a small box so the ball can be launched horizontally onto the table. Now place the pendulum in the position shown in Figure 15.1, so that it will not interfere with the free flight of the ball. Place a piece of carbon paper on top of a piece of white paper and tape it to the table at approximately the position the ball will land when launched horizontally.

Figure 15.1: Ballistic Pendulum Position to Launch Ball Horizontally

Measure the height (h) of the launcher by measuring the distance from the table to the bottom of the ball while it is in the launcher. Launch the ball and measure the horizontal displacement (Δx) of the ball (you should measure the horizontal distance from the end of the barrel of the launcher to the mark on the white paper (see Figure 15.2). Perform 5 trials and record your values in Table 15.1.

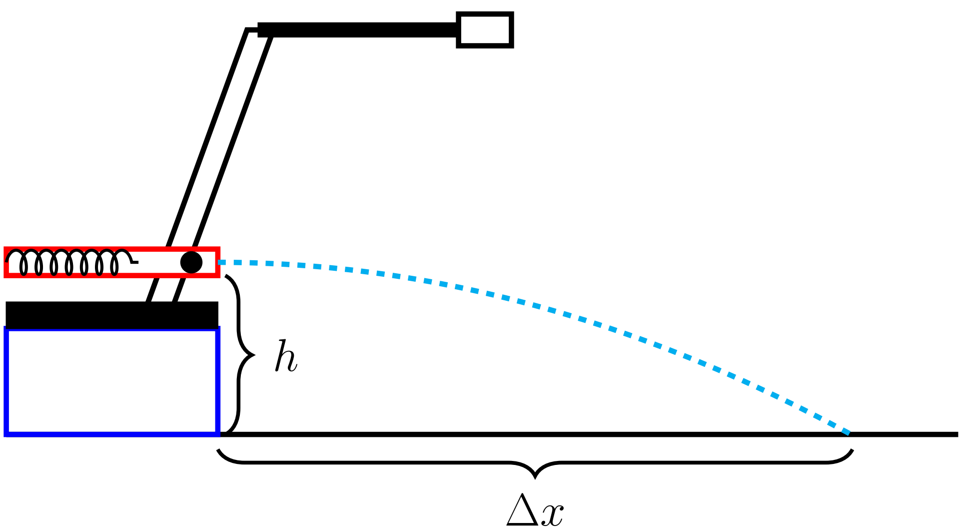


Figure 15.2: Measurements of Projectile Launched Horizontally

Question 1: Determine the average horizontal displacement (Δx) and the average height of the launcher (h). Compute the PRAAD for the horizontal displacement and record your values in Table 15.1.

Question 2: Using your kinematic equations for constant acceleration, show that the initial

Trial	Horizontal Displacement (Δx) (m)	Absolute Deviations (m)	Height of Launcher (h) (m)
1			
2			
3			
4			
5			
Average			
PRAAD (%)			

Table 15.1: Data for Ballistic Pendulum Projectile Launched Horizontally

speed of the ball as it leaves the launcher is given by

$$v_0 = \Delta x \sqrt{\frac{g}{2h}} \tag{15.6}$$

and then use this equation to determine the experimental initial speed of the ball ($v_{0,\text{exp}}$) from your data. Record your value in Table 15.2.

Experimental Initial Speed of Ball ($v_{0,\text{exp}}$) (m/s)	

Table 15.2: Experimental Initial Speed of Ball

15.2.1.2 Determination of the Initial Speed of the Ball by Applying the Laws of Conservation of Momentum and Energy

Prepare the launcher for shooting (see Figure 15.3). Initially, the ball should be placed in the launch tube and the spring should be fully compressed. Make sure you use the supplied plastic rod to compress the spring. The pendulum should initially be freely hanging and the angle indicator set to zero. When the pendulum is still, pull the trigger on the launcher. The ball will embed itself within the pendulum and the pendulum will swing upward. The angle indicator will remain at the location representing the highest position of the pendulum (see Figure 15.4). Record this value to the nearest 0.5° in Table 15.3. Repeat this procedure a total of five times.

Question 3: Compute the average angle position where the pendulum reaches its maximum angular displacement from the vertical (make sure you round to the resolution of the instrument which is 0.5°). Also compute the PRAAD for angular position. Record your values in Table 15.3.

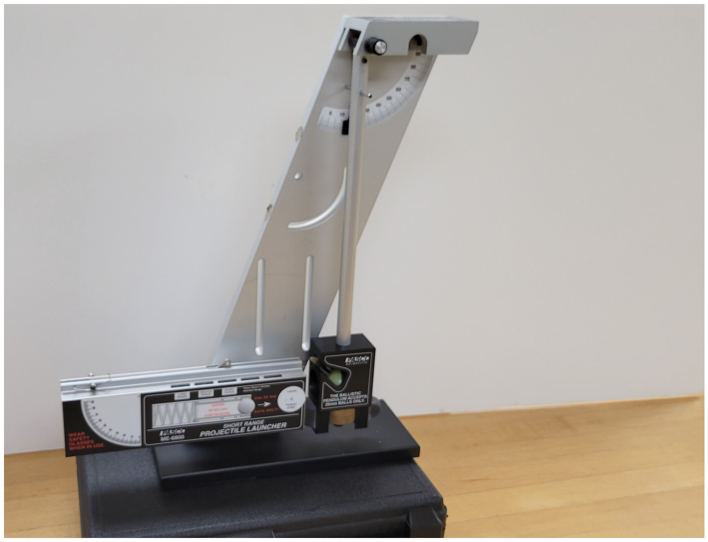


Figure 15.3: Ballistic Pendulum Initial Configuration

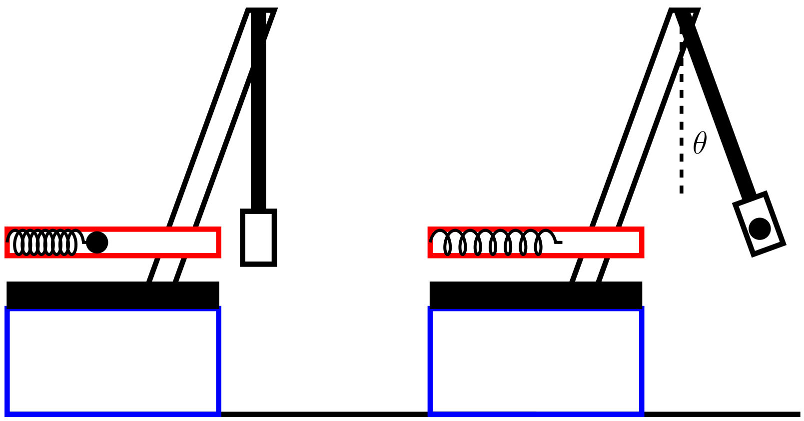


Figure 15.4: Launching Ball into the Pendulum

	θ_1	θ_2	θ_3	θ_4	θ_5	θ_{ave}	PRAAD (%)
Angle (°)							
Abs. Dev.							

Table 15.3: Maximum Angle of Pendulum

Question 4: Remove the pendulum and measure its length (L) which is defined to be from the pivot point to the bottom of the "V" in the base of the pendulum. Calculate the change in height (Δh) using

$$\Delta h = L - L\cos\theta_{\text{ave}} = L(1 - \cos\theta_{\text{ave}})$$

See Figure 15.5 for details. Record your values in Table 15.4.

Question 5: Measure the mass of the ball and the mass of the pendulum using the electronic scale. Estimate the PRAAD for mass in each case by using the resolution of the scale (0.0001 kg)

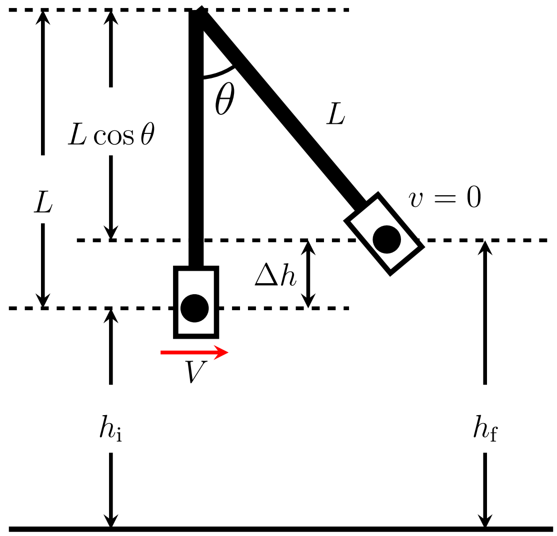


Figure 15.5: Diagram to Compute Change in Height of Pendulum

Length of Pendulum (L) (m)			
Change in Height (Δh) (m)			
Mass of Ball (m_1) (kg)		PRAAD (%)	
Mass of Pendulum (m_2) (kg)		PRAAD (%)	

Table 15.4: Properties of Ball and Pendulum

as the average absolute deviation. Record your values in Table 15.4.

The system of the ball and the pendulum is shown in Figure 15.6a. Refer to the ball as object 1 and the pendulum as object 2. Let $\vec{P}_\text{i}$ be the total initial momentum of the system of ball and pendulum before the collision. $\vec{P}_\text{i}$ has no y-component. That is

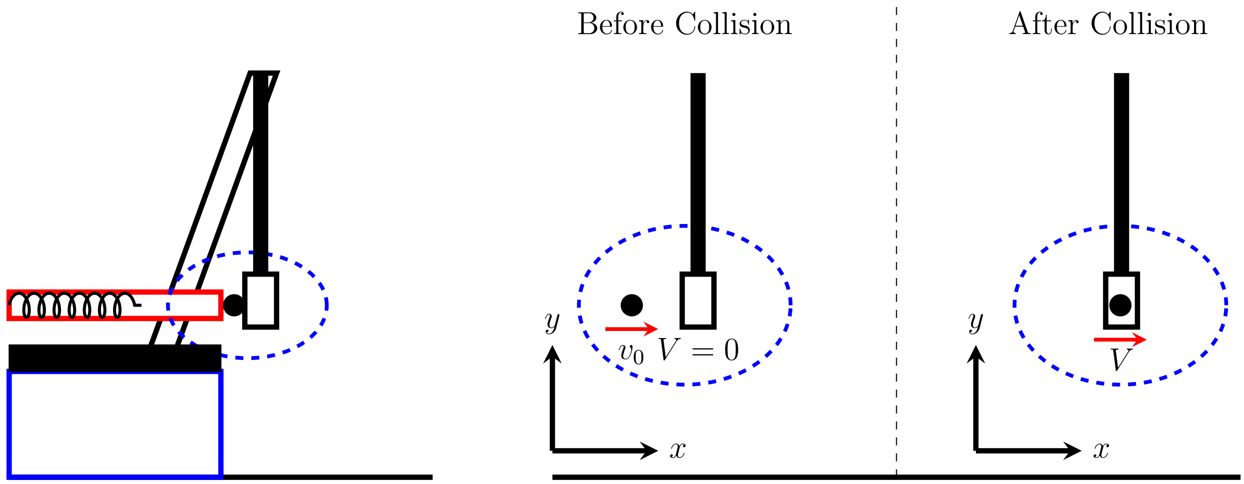
$$P_{\text{i}y} = 0 \tag{15.7}$$

Question 6: Why is $P_{\text{i}y} = 0$? Explain carefully.

Choosing a coordinate system where the x-axis is positive towards the right and the y-axis is positive vertically upwards, the x-component of $\vec{P}_\text{i}$ is then given by

$$P_{\text{i}x} = P_{1x} + P_{2x} = m_1 v_0 = m_2(0) = m_1 v_0 \tag{15.8}$$

See Figure 15.6b.



(a) System of Ball and Pendulum (b) Before Collision and After Collision

Figure 15.6: System of Ball and Pendulum

Question 7: What would the value of P_{1x} have been if the x-axis had been chosen to be positive directed towards the left?

Now consider the system of the ball and the pendulum in some other state f, the state just after the ball has collided with and is embedded in the pendulum (see Figure 15.6b). Let $\vec{P}_f$ be the total momentum of the system of the ball and pendulum in the state f. Here, the ball and pendulum move as a single object of mass $(m_1 + m_2)$ with a velocity $\vec{V}$. $\vec{P}_f$ has no y component. The x component of $\vec{P}_f$ is given by

$$P_{\mathrm{f}x} = (m_1 + m_2)V \tag{15.9}$$

Question 8: Using the law of conservation of momentum, show that

$$v_0 = \left(\frac{m_1 + m_2}{m_1}\right) V \tag{15.10}$$

Next consider the system of pendulum and ball in a later state f', where the pendulum and ball have swung through the maximum vertical distance Δh (see Figure 15.5). The energy of the system in state f' is all in the form of gravitational potential energy; there is no kinetic energy since the system has momentarily come to rest. Therefore, we have that

$$E_{f'} = (m_1 + m_2)g\Delta h \tag{15.11}$$

On the other hand, when the system was in the state f, the total energy of the system was in the form of kinetic energy (we are assuming the potential energy when the pendulum is hanging vertically was chosen to be zero). Therefore, we have

$$E_f = \frac{1}{2}(m_1 + m_2)V^2 \tag{15.12}$$

Question 9: Assuming there was no friction at the pivot of the pendulum and that air resistance is negligible, then mechanical energy is conserved ($E_{f'} = E_f$). Use this fact to show that

$$V = \sqrt{2g\Delta h} \tag{15.13}$$

Question 10: Using Equation 15.13, compute V, the speed of the ball and pendulum just after the collision. Record your value in Table 15.5.

Speed of Ball/Pendulum Just After Collision (V) (m/s)	

Table 15.5: Speed of Ball/Pendulum System After Collision

Question 11: Using Equation 15.10, compute the predicted initial speed of the ball $v_{0,\text{pred}}$ upon being released from the launcher. Record your value in Table 15.6.

Predicted Initial Speed of the Ball ($v_{0,\text{pred}}$) (m/s)	

Table 15.6: Speed of Ball Before Collision

Question 12: Compare the two values of v_0 that you have obtained in this experiment ($v_{0,\text{exp}}$ from question 2 and $v_{0,\text{pred}}$ from question 11) by computing the percent experimental error. Record you values in Table 15.7.

$v_{0,\text{exp}}$ (m/s)	$v_{0,\text{pred}}$ (m/s)	Percent Experimental Error (%)

Table 15.7: Comparison of Initial Speed of Launched Ball

The kinetic energy of the system when it was in state i is given by

$$K_\text{i} = \frac{1}{2} m_1 v_{0,\text{pred}}^2 \tag{15.14}$$

Question 13: Using Equation 15.14, compute the initial kinetic energy of the system K_i. Record your value in Table 15.8.

The kinetic energy of the system when it was in state f is given by

$$K_\text{f} = \frac{1}{2}(m_1 + m_2)V^2 \tag{15.15}$$

Question 14: Using Equation 15.15, compute the final kinetic energy of the system K_f. Record your value in Table 15.8.

Question 15: Compare K_f with K_i. Is K_f less than or greater than K_i.

The collision that you have witnessed in this experiment is known as a completely inelastic collision. In this type of collision, *kinetic energy* is not conserved but *total energy* is conserved. The energy that appears to be missing has taken other forms (mostly thermal energy).

Question 16: Compute the percentage of kinetic energy "lost" using

$$\text{Percent Kinetic Energy Lost} = \frac{K_\mathrm{i} - K_\mathrm{f}}{K_\mathrm{i}} \times 100\%$$

Record your value in Table 15.8.

Initial Kinetic Energy of the System (K_i) (J)	
Final Kinetic Energy of the System (K_f) (J)	
Percent Kinetic Energy "Lost" (%)	

Table 15.8: Kinetic Energy of the System Before and After the Collision

Lab 16

An Experimental Demonstration of the Laws of Conservation of Momentum and Energy - Cart Collisions on an Air Track

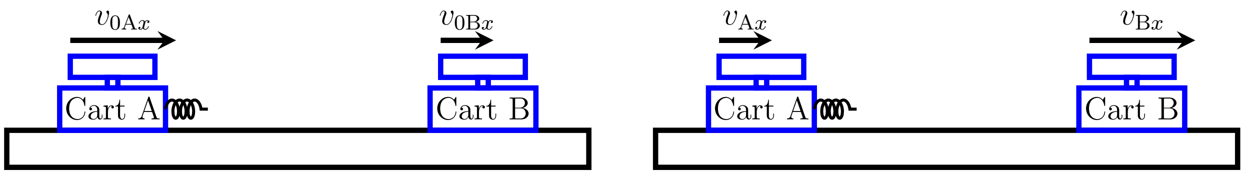
In this experiment, we will continue our study of conservation of momentum and energy by examining one-dimensional collisions between gliders on an air track.

16.1 Theory

16.1.1 Perfectly Elastic Collision

A perfectly elastic collision is one in which the bodies collide and separate without any permanent deformation having occurred during the collision time. As a consequence, the total kinetic energy of the system of colliding bodies is conserved. To approximate a perfectly elastic collision between two carts or gliders, springs, rubber band launchers, magnets, etc. can be placed on the carts. In this lab, we will use a rubber band launcher placed on one cart and a glider bumper placed on the other cart to simulate a perfectly elastic collision (see Figure 16.1).

Figure 16.1: Perfectly Elastic Collision Between Carts On An Air Track



(a) Carts on Air Track Just Before Collision (b) Carts on Air Track Just After Collision

Figure 16.2: Perfectly Elastic Collision of Carts on an Air Track

Conservation of total momentum is expressed by the following equation:

$$m_{\mathrm{A}} v_{\mathrm{A}x} + m_{\mathrm{B}} v_{\mathrm{B}x} = m_{\mathrm{A}} v_{0\mathrm{A}x} + m_{\mathrm{B}} v_{0\mathrm{B}x} \tag{16.1}$$

where $v_{0\mathrm{A}x}$ and $v_{0\mathrm{B}x}$ are the velocities of bodies A and B, respectively, before the collision and $v_{\mathrm{A}x}$ and $v_{\mathrm{B}x}$ are the velocities of bodies A and B, respectively, after the collision (see Figure 16.2).

Because total kinetic energy is also conserved in a perfectly elastic collision we have

$$\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}}^2 + \frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}}^2 = \frac{1}{2} m_{\mathrm{A}} v_{0\mathrm{A}}^2 + \frac{1}{2} m_{\mathrm{B}} v_{0\mathrm{B}}^2 \tag{16.2}$$

where $v_{0\mathrm{A}}$ and $v_{0\mathrm{B}}$ are the speeds of bodies A and B, respectively, before the collision, and v_{A} and v_{B} are the speeds of bodies A and B, respectively, after the collision. Since this is a one dimensional collision where movement is occurring along the x-axis, then $v_{0\mathrm{A}}$ and $v_{0\mathrm{B}}$ equal $v_{0\mathrm{A}x}$ and $v_{0\mathrm{B}x}$, respectively. Also, v_{A} and v_{B} equal $v_{\mathrm{A}x}$ and $v_{\mathrm{B}x}$, respectively. Equation 16.2 can then be rewritten as

$$\frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}x}^2 + \frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}x}^2 = \frac{1}{2} m_{\mathrm{A}} v_{0\mathrm{A}x}^2 + \frac{1}{2} m_{\mathrm{B}} v_{0\mathrm{B}x}^2 \tag{16.3}$$

Because total kinetic energy is conserved, it can be shown that the relative velocity of departure between the two bodies is equal but opposite to the relative velocity of approach between the two bodies. That is,

$$v_{\mathrm{B}x} - v_{\mathrm{A}x} = -\left(v_{0\mathrm{B}x} - v_{0\mathrm{A}x}\right) \tag{16.4}$$

16.1.2 Completely Inelastic Collision

A completely inelastic collision is one in which the bodies are deformed to the extent that they stay stuck together after the collision. Since energy is required to deform the bodies, the total kinetic energy is not conserved during a completely inelastic collision. To create a completely inelastic collision between two carts we will place a needle on one cart and a wax filled receptacle on the other cart (see Figure 16.3).

Conservation of total momentum for a completely inelastic collision is expressed by

$$\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right) V_x = m_{\mathrm{A}} v_{0\mathrm{A}x} + m_{\mathrm{B}} v_{0\mathrm{B}x} \tag{16.5}$$

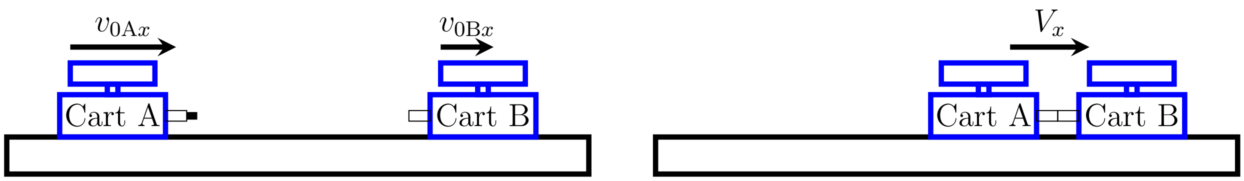
Notice that after the completely inelastic collision, the two carts move together with a common velocity V_x (see Figure 16.4b).

The kinetic energy before and after the collision are given by

$$K_{\text{after}} = \frac{1}{2}\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right) V_x^2 \tag{16.6}$$

$$K_{\text{before}} = \frac{1}{2} m_{\mathrm{A}} v_{0\mathrm{A}x}^2 + \frac{1}{2} m_{\mathrm{B}} v_{0\mathrm{B}x}^2 \tag{16.7}$$

Figure 16.3: Completely Inelastic Collision Between Carts On An Air Track



(a) Carts on Air Track Just Before Collision (b) Carts on Air Track Just After Collision

Figure 16.4: Completely Inelastic Collision of Carts on an Air Track

16.1.3 Semi-Elastic Collision

A semi-elastic collision is one in which the two bodies are deformed during the collision, but they separate after the collision. Because deformation occurs, total kinetic energy is not conserved during a semi-elastic collision. The kinetic energy after and before the collision are given by

$$K_{\text{after}} = \frac{1}{2} m_{\mathrm{A}} v_{\mathrm{A}x}^2 + \frac{1}{2} m_{\mathrm{B}} v_{\mathrm{B}x}^2 \tag{16.8}$$

$$K_{\text{before}} = \frac{1}{2} m_{\mathrm{A}} v_{0\mathrm{A}x}^2 + \frac{1}{2} m_{\mathrm{B}} v_{0\mathrm{B}x}^2 \tag{16.9}$$

However, total momentum of the system is conserved and is given by Equation 16.1.

16.2 Experiment

The purpose of this experiment is to test the Law of Conservation of Total Momentum by studying one-dimensional collisions between two carts on an air track. It is assumed that the air track is level and provides a friction-free surface.

16.2.1 Procedure 1: Equal Masses, One Cart at Rest

Place a 10 cm flag on each cart. Attach a spring or rubber band launcher to one cart and a glider bumper to the other cart. Add slotted weights or paper clips to the carts sufficient such that the carts have approximately equal masses (check this with the electronic balance). Record your masses in Table 16.1. If necessary, level the air track such that the carts stay in place when positioned on the track with the air supply turned on. Position two photogates at positions representing about $\frac{1}{4}$ and $\frac{3}{4}$ of the length of the air track (see Figure 16.5).

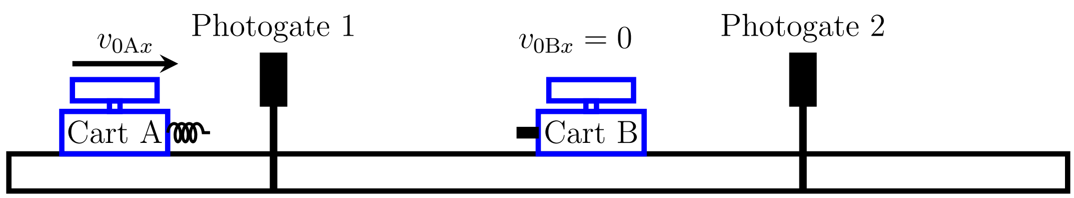


Figure 16.5: Set-up for Procedure 1

Place cart B at rest between the two photogates. Using the rubber band launcher attached to the air track, impart a constant velocity to cart A. Measure the elapsed time for cart A to pass through photogate 1 ($\Delta t_{0\mathrm{A}}$) and the time for cart B to pass through photogate 2 (Δt_{B}). You should find that cart A comes to rest between the photogates after colliding with cart B. Perform three trials and record your data in Table 16.1.

Trial	Elapsed Time at Photogate 1 ($\Delta t_{0\mathrm{A}}$) (s)	Absolute Deviations (s)	Elapsed Time at Photogate 2 (Δt_{B}) (s)	Absolute Deviations (s)
1				
2				
3				
Average				
PRAAD for Elapsed Time (%)				
Mass A (kg)		Mass B (kg)		

Table 16.1: Data for Collisions - Procedure 1

Question 1: Describe the collision.

Question 2: Compute the velocity of each cart before and after the collision using $v_{0\mathrm{A}x} = \frac{\Delta x_{\mathrm{A}}}{\Delta t_{0\mathrm{A}}}$ and $v_{\mathrm{B}x} = \frac{\Delta x_{\mathrm{B}}}{\Delta t_{\mathrm{B}}}$ where $\Delta x_{\mathrm{A}} = 0.1$ m and $\Delta x_{\mathrm{B}} = 0.1$ m and $\Delta t_{0\mathrm{A}}$ and Δt_{B} are the average elapsed times for cart A to pass through the photogate before the collision and for cart B to pass through the photogate after the collision, respectively. Also compute the PRADD for velocity using

$$\text{PRAAD for Velocity} = \text{PRAAD for Displacement} + \text{PRAAD for Elasped Time}$$

PRADD for Displacement can be estimated by using the resolution of the instrument (meter stick) and dividing it by the length of the flag (PRAAD for Displacement $= \frac{0.001}{0.1} \times 100 = 1.0\%$) and PRADD for the elapsed time can be computed directly using your data in Table 16.1. Record your results in Table 16.2.

	Initial Velocity Cart A ($v_{0\mathrm{A}x}$)	Final Velocity Cart A ($v_{\mathrm{A}x}$)	Initial Velocity Cart B ($v_{0\mathrm{B}x}$)	Final Velocity Cart B ($v_{\mathrm{B}x}$)
Velocity (m/s)		0	0	
PRAAD (%)				

Table 16.2: Cart Velocities - Procedure 1

Question 3: Compute the total momentum of the system of two carts before the collision and the total momentum of the two carts after the collision. Is momentum conserved? Record your results in Table 16.3.

Total Momentum Before Collision ($\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$)	Total Momentum After Collision ($\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$)

Table 16.3: Total Momentum Before and After Collision - Procedure 1

Question 4: Compute the total kinetic energy of the system of the two carts before and after the collision. Is total kinetic energy conserved? What type of collision occurred? Record your results in Table 16.4.

Total Kinetic Energy Before Collision (J)	Total Kinetic Energy After Collision (J)

Table 16.4: Total Kinetic Energy Before and After Collision - Procedure 1

16.2.2 Procedure 2: Equal Masses, Both Carts Moving

Again, place a 10 cm flag on each cart and adjust their masses so that they are equal. Record your masses in Table 16.5. Place the carts at each end of the air track and impart a small constant velocity to each cart using the rubber band launchers, such that the carts approach each other and collide in the middle of the track (see Figure 16.6).

Measure the necessary elapsed times to determine the velocities of the carts before and after the collision. To do this, make sure the timer memory switch is on. You will then record the initial time for the carts to pass through the photogates ($\Delta t_{0\mathrm{A}}$ and $\Delta t_{0\mathrm{B}}$) and the total time for the carts

Figure 16.6: Set-up for Procedure 2

to pass through the photogates in both directions ($\Delta t_{\mathrm{A,tot}}$ and $\Delta t_{\mathrm{B,tot}}$). You can then calculate the time it takes the carts to pass through the photogates after the collision (Δt_{A} and Δt_{B}) by subtracting ($\Delta t_{\mathrm{A}} = \Delta t_{\mathrm{A,tot}} - \Delta t_{0\mathrm{A}}$ and $\Delta t_{\mathrm{B}} = \Delta t_{\mathrm{B,tot}} - \Delta t_{0\mathrm{B}}$). Perform three trials and record your data in Table 16.5.

Trial	$\Delta t_{0\mathrm{A}}$ (s)	Dev. (s)	$\Delta t_{\mathrm{A,tot}}$ (s)	Δt_{A} (s)	Dev. (s)	$\Delta t_{0\mathrm{B}}$ (s)	Dev. (s)	$\Delta t_{\mathrm{B,tot}}$ (s)	Δt_{B} (s)	Dev. (s)
1										
2										
3										
Ave.										
PRAAD (%)										
Mass A (kg)				Mass B (kg)						

Table 16.5: Data for Collisions - Procedure 2

Question 5: Describe the collision.

Question 6: Compute the velocity of each cart before and after the collision using $v_{0\mathrm{A}x} = \frac{\Delta x_{\mathrm{A}}}{\Delta t_{0\mathrm{A}}}$ and $v_{\mathrm{A}x} = \frac{\Delta x_{\mathrm{B}}}{\Delta t_{\mathrm{B}}}$, etc. (Hint: Remember that $v_{\mathrm{A}x}$ and $v_{0\mathrm{B}x}$ are negative). Also, compute the PRAAD for each elapsed time and for each velocity. Record your results in Table 16.6.

	Initial Velocity Cart A ($v_{0\mathrm{A}x}$)	Final Velocity Cart A ($v_{\mathrm{A}x}$)	Initial Velocity Cart B ($v_{0\mathrm{B}x}$)	Final Velocity Cart B ($v_{\mathrm{B}x}$)
Velocity (m/s)				
PRAAD (%)				

Table 16.6: Cart Velocities - Procedure 2

Question 7: Compute the total momentum of the system of two carts before the collision and the total momentum of the two carts after the collision. Is momentum conserved? Record your results in Table 16.7.

Total Momentum Before Collision ($\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$)	Total Momentum After Collision ($\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$)

Table 16.7: Total Momentum Before and After Collision - Procedure 2

Question 8: Compute the total kinetic energy of the system of the two carts before and after the collision. Is total kinetic energy conserved? What type of collision occurred? Record your results in Table 16.8.

Total Kinetic Energy Before Collision (J)	Total Kinetic Energy After Collision (J)

Table 16.8: Total Kinetic Energy Before and After Collision - Procedure 2

16.2.3 Procedure 3: Different Masses, One Cart At Rest

Increase the mass of cart B by adding two cylindrical weights to it. Record your masses in Table 16.9. Place cart B in the center of the air track between the two photogates. Impart a small constant velocity to cart A with the use of the rubber band launcher (see Figure 16.7).

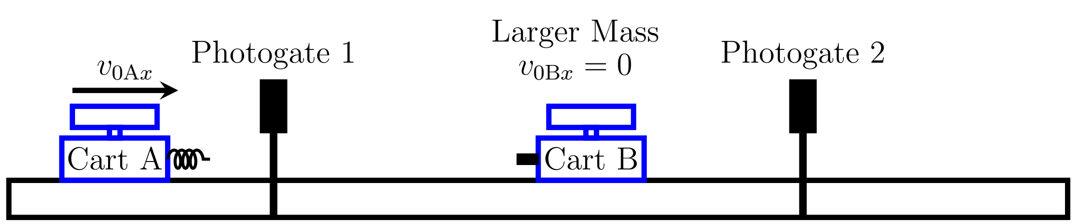


Figure 16.7: Set-up for Procedure 3

Measure the necessary elapsed times to determine the velocities of the carts before and after the collision. To do this, make sure the timer memory switch is on. You will then record the initial time for cart A to pass through the photogates ($\Delta t_{0\mathrm{A}}$) and the total time for the cart A to pass through the photogate in both directions ($\Delta t_{\mathrm{A,tot}}$). You can then calculate the time it takes the cart A to pass through the photogate after the collision (Δt_{A}) by subtracting ($\Delta t_{\mathrm{A}} = \Delta t_{\mathrm{A,tot}} - \Delta t_{0\mathrm{A}}$). Since cart B starts at rest, ($\Delta t_{0\mathrm{B}} = 0$) Also, since cart B only passes through the photogate once, Δt_{B} with be the single elapsed time recorded by the photogate. Perform three trials and record your data in Table 16.9.

Question 9: Describe the collision.

Trial	$\Delta t_{0\text{A}}$ (s)	Dev. (s)	$\Delta t_{\text{A,tot}}$ (s)	Δt_{A} (s)	Dev. (s)	Δt_{B} (s)	Dev. (s)
1							
2							
3							
Ave.							
PRAAD (%)							
Mass A (kg)				Mass B (kg)			

Table 16.9: Data for Collisions - Procedure 3

Question 10: Compute the velocity of each cart before and after the collision using $v_{0\text{A}x} = \frac{\Delta x_{\text{A}}}{\Delta t_{0\text{A}}}$ and $v_{\text{A}x} = \frac{\Delta x_{\text{A}}}{\Delta t_{\text{A}}}$, and $v_{\text{B}x} = \frac{\Delta x_{\text{B}}}{\Delta t_{\text{B}}}$ (Hint: Remember that $v_{\text{A}x}$ is negative. Also, compute the PRAAD for each elapsed time and each velocity. Record your results in Table 16.10.

	Initial Velocity Cart A ($v_{0\text{A}x}$)	Final Velocity Cart A ($v_{\text{A}x}$)	Initial Velocity Cart B ($v_{0\text{B}x}$)	Final Velocity Cart B ($v_{\text{B}x}$)
Velocity (m/s)			0	
PRAAD (%)				

Table 16.10: Cart Velocities - Procedure 3

Question 11: Compute the total momentum of the system of two carts before the collision and the total momentum of the two carts after the collision. Is momentum conserved? Record your results in Table 16.11.

Total Momentum Before Collision ($\text{kg} \cdot \frac{\text{m}}{\text{s}}$)	Total Momentum After Collision ($\text{kg} \cdot \frac{\text{m}}{\text{s}}$)

Table 16.11: Total Momentum Before and After Collision - Procedure 3

Question 12: Compute the total kinetic energy of the system of the two carts before and after the collision. Is total kinetic energy conserved? What type of collision occurred? Record your results in Table 16.12.

Total Kinetic Energy Before Collision (J)	Total Kinetic Energy After Collision (J)

Table 16.12: Total Kinetic Energy Before and After Collision - Procedure 3

16.2.4 Procedure 4: Completely Inelastic Collision

Attach the pin accessory to cart A and the wax receptacle to cart B. Place the 10 cm flag *just* on cart A. Add slotted weights or paper clips so that the masses of both carts are equal. Record your masses in Table 16.13. Place cart B at rest in the center of the air track between the two photogates. Impart a small constant velocity to cart A with the use of the rubber band launcher (see Figure 16.8).

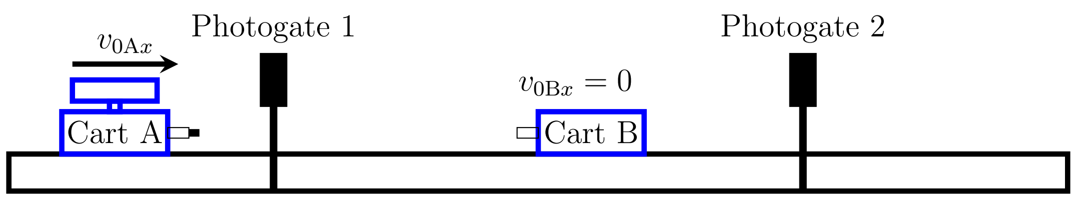


Figure 16.8: Set-up for Procedure 4

Measure the elapsed time for cart A to pass through photogate 1 (Δt_{0A}) and the time for the combination of cart A and cart B (Δt_{AB}) to pass through photogate 2. Perform three trials and record your data in Table 16.13.

Trial	Elapsed Time at Photogate 1 (Δt_{0A}) (s)	Absolute Deviations (s)	Elapsed Time at Photogate 2 (Δt_{AB}) (s)	Absolute Deviations (s)
1				
2				
3				
Average				
PRAAD for Elapsed Time (%)				
Mass A (kg)		Mass B (kg)		

Table 16.13: Data for Collisions - Procedure 4

Question 13: Describe the collision.

Question 14: Compute the velocity of each cart A before and the velocity of the combination of carts A and B after the collision using $v_{0\mathrm{A}x} = \frac{\Delta x_{\mathrm{A}}}{\Delta t_{0\mathrm{A}}}$ and $V_x = \frac{\Delta x_{\mathrm{A}}}{\Delta t_{\mathrm{AB}}}$. Also, compute the PRADD for each each elasped time and each velocity. Record your results in Table 16.14.

	Initial Velocity Cart A ($v_{0\mathrm{A}x}$)	Initial Velocity Cart B ($v_{0\mathrm{B}x}$)	Final Velocity Carts A & B (V_x)
Velocity (m/s)		0	
PRAAD (%)			

Table 16.14: Cart Velocities - Procedure 4

Question 15: Use conservation of total momentum as described by Equation 16.5 to predict the velocity of the two-cart system ($V_{x,\mathrm{pred}}$) after the collision in terms of the velocities of the two carts before the collision. Compare your measured value of the velocity of the two-cart system V_x from the table with the predicted value and compute the percent experimental error. Record your results in Table 16.15.

Predicted Final Velocity ($V_{x,\mathrm{pred}}$) (m/s)	Experimental Final Velocity (V_x) (m/s)	Percent Experimental Error (%)

Table 16.15: Two-Cart Final Velocity Comparisons

Question 16: Compute the total kinetic energy of the system of the two carts before and after the collision (you will need to use Equations 16.6 and 16.7). Is total kinetic energy conserved? What type of collision occurred? Record your results in Table 16.16.

Total Kinetic Energy Before Collision (J)	Total Kinetic Energy After Collision (J)

Table 16.16: Total Kinetic Energy Before and After Collision - Procedure 4

Question 17: Because this was a completely inelastic collision, you should have found that the total kinetic energy of the system after the collision was less than the total kinetic energy before the collision. We sometimes say that kinetic energy was "lost" but actually what is occurring is that some of the initial kinetic energy was converted into other forms of energy (such as thermal energy, sound, etc.) during the collision. To determine how much kinetic energy was "lost", compute

$$\text{Percent Kinetic Energy Lost} = \frac{K_{\text{before}} - K_{\text{after}}}{K_{\text{before}}} \times 100\%$$

Record your value is Table 16.17.

Percent Kinetic Energy Lost (%)	

Table 16.17: Kinetic Energy Lost During The Collision - Procedure 4

Lab 17

An Application of the Impulse-Momentum Theorem - Maximal Vertical Jump

In this experiment, we will estimate the jump height of an individual performing a maximal vertical jump. We will investigate two different jumping techniques, a counter-movement jump and a squat jump. To determine the maximal jump height we will employ the impulse/momentum theorem as well as the time of flight measure.

17.1 Theory

The linear momentum of a particle is given by

$$\vec{p} = m\vec{v} \tag{17.1}$$

where $\vec{p}$ is the linear momentum in units of kg m/s, m is the mass in kg, and $\vec{v}$ is the velocity in m/s. Newton's second law for a particle is given by

$$\vec{F}_{\text{net}} = m\vec{a} \tag{17.2}$$

where $\vec{F}_{\text{net}}$ is the net force acting on the particle in Newtons (N), m is the mass in kg, and $\vec{a}$ is the acceleration in m/s^2. Let's assume that the net force $\vec{F}_{\text{net}}$ is constant and the time interval Δt is small and let's approximate the acceleration by $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. We can then write Newton's second law as

$$\vec{F}_{\text{net}} = m\frac{\Delta\vec{v}}{\Delta t} \tag{17.3}$$

Multiplying both sides by Δt then gives

$$\vec{F}_{\text{net}}\Delta t = m\Delta\vec{v} \tag{17.4}$$

Since we are dealing with classical systems and assume the mass is not changing, we can distribute the mass on the right hand side (ie. bring it into the Δ term) which gives

$$\vec{F}_{\text{net}}\Delta t = \Delta\left(m\vec{v}\right) \tag{17.5}$$

However, notice that the right hand side is just the change in momentum. We now have

$$\vec{F}_{\text{net}}\Delta t = \Delta\vec{p} \tag{17.6}$$

The term on the left hand side $\vec{F}_{\text{net}}\Delta t$ is called the impulse. Equation 17.6 states that the impulse given to a particle is equal to it's change in momentum and is referred to as the impulse-momentum theorem.

Since the impulse-momentum theorem is a vector equation, it also holds true for each of its components. In this lab, we are interested in just the forces in the vertical y direction. The equation would then become

$$\begin{aligned} F_{\text{net},y}\Delta t &= \Delta p_y \\ &= p_{\text{f}y} - p_{\text{i}y} \\ &= mv_{\text{f}y} - mv_{\text{i}y} \\ &= m\left(v_{\text{f}y} - v_{\text{i}y}\right) \end{aligned} \tag{17.7}$$

In most cases, the net force is not a constant but instead varies as a function of time. If we are interested in the total impulse supplied by the force over the entire time period, then we would have to calculate the impulse over small time intervals (where the net force is nearly constant) and then add all of these impulses up to compute the total impulse. The total impulse is then equal to the change in momentum of the particle

$$\sum F_{\text{net},y}\Delta t = m\left(v_{\text{f}y} - v_{\text{i}y}\right) \tag{17.8}$$

where $v_{\text{i}y}$ and $v_{\text{f}y}$ are now the velocities of the particle at the beginning of the total time period and at the end of the total time period, respectively. Computing the total impulse equivalent to approximating the area under the force versus time graph (see Figure 17.1).

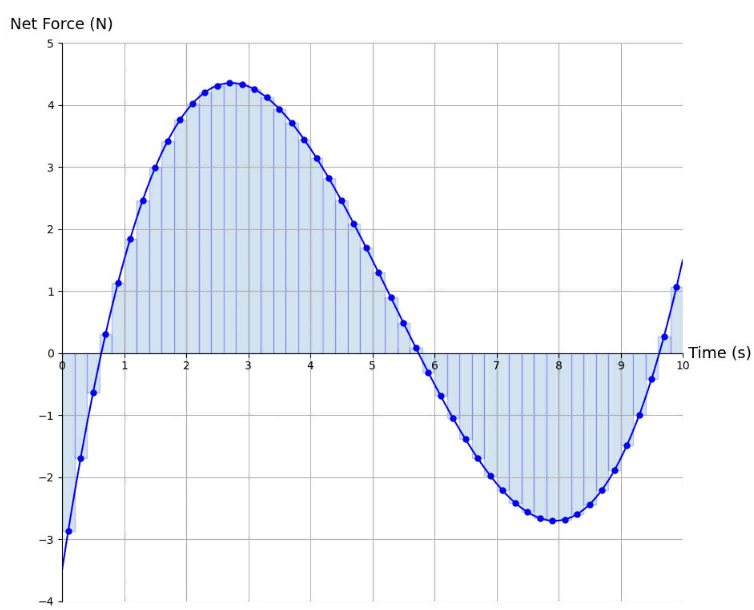


Figure 17.1: Total Impulse Computed from Force versus Time Graph

In this particular example, the area of rectangles of width ($\Delta t = 0.2$ s) and a height equal to the value of the function evaluated at the center of each time interval were used to estimate each impulse. The area of the rectangles was then summed to compute the total impulse over the entire 10 s time interval. The total impulse is then equal to the mass of the particle times the difference of the velocity at 10 s and the velocity at 0 s. Note that when the net force is negative, the impulse is negative. Therefore, you need to subtract the area of the rectangles that are below the time axis. Also note that if you take the limit of the time interval as it approaches zero, then the sum becomes an integral giving

$$\int_{t_\mathrm{i}}^{t_\mathrm{f}} F_{\mathrm{net},y}\, dt = m\left(v_{\mathrm{f}y} - v_{\mathrm{i}y}\right) \tag{17.9}$$

In this lab, we will use Pasco Capstone to collect our data and display our graphs. There is a built in function that will numerically calculate the area under our net force versus time graph (using the trapezoidal rule) which will be a good approximation to the total impulse.

17.1.1 Vertical Jump

The above discussion was true for a single particle of mass m but a human body is much more complicated than a single particle. However, we can model a human body as a series of connected rigid links. We can still apply the impulse-momentum theorem, however, the mass now refers to the total mass of the human body and the velocity refers to the velocity of the center of mass of the body. The forces acting on a human during a vertical jump are the gravitational force $\vec{w} = M\vec{g}$ which acts at the center of mass and the ground reaction force (normal force) $\vec{F}_{\mathrm{GRF}}$ which acts at the feet (see Figure 17.2).

Applying the impulse momentum theorem to the jumper gives

$$\sum F_{\mathrm{net},y}\Delta t = \sum\left(F_{\mathrm{GRF},y} - Mg\right)\Delta t = M\left(v_{\mathrm{cm,f},y} - v_{\mathrm{cm,i},y}\right) \tag{17.10}$$

where $F_{\mathrm{net},y} = F_{\mathrm{GRF},y} - Mg$ is the net force acting on the body, M is the total mass of the body, and $v_{\mathrm{cm,i},y}$ and $v_{\mathrm{cm,f},y}$ are the initial and final velocities of the center of mass, respectively.

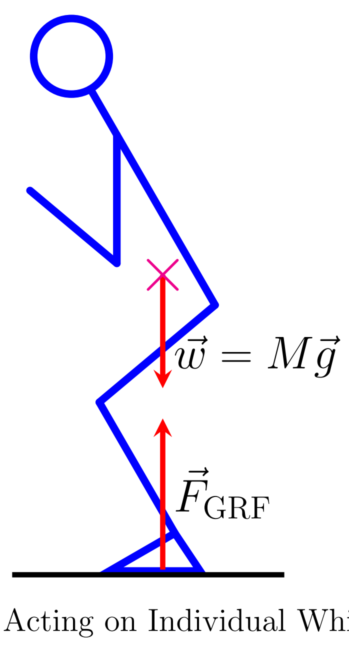


Figure 17.2: Forces Acting on Individual While Performing Jump

The jumper starts at rest so $v_{\text{cm,i},y} = 0$ m/s. Solving for $v_{\text{cm,f},y}$ then gives

$$v_{\text{t}} = v_{\text{cm,f,y}} = \frac{\sum F_{\text{net},y} \Delta t}{M} = \frac{\text{Total Impulse}}{M} \tag{17.11}$$

where $v_{\text{cm,f},y}$ was renamed as v_{t} to represent the take-off velocity of the center of mass as the feet leave the surface of the force platform. The total impulse can be found by computing the area under the net force versus time graph from the beginning of the jump (when the individual is at rest) to when the individual's feet leave the surface.

17.1.2 Calculating Jump Height

We will calculate jump height using two different methods. The first method involves measuring the total time of flight (the total time that the individual is in the air) and then using kinematics to compute the jump height. The second method involves using the impulse-momentum theorem to compute the take-off velocity and then using kinematics to compute the jump height. In both cases, we are computing the change in height Δh of the center of mass measured from when the feet just lose contact with the force platform to the heighest position of the center of mass where it's velocity is instantaneously zero (see Figure 17.3).

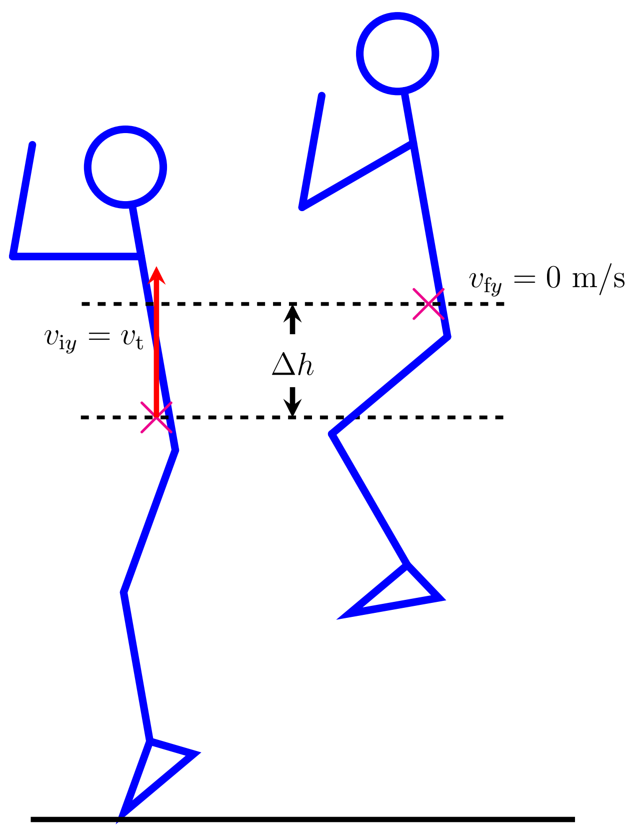


Figure 17.3: Maximal Jump Height

17.1.2.1 Calculating Jump Height Using Time of Flight

This method involves using the time of flight information (this is the time that the individual is in the air and can be determined from the graph of net force versus time). The individual is in the air when the ground reaction force is zero ($F_{\text{GRF}} = 0N$) or equivalently when the net force equals negative body weight ($F_{\text{net},y} = -mg$).

If we consider the total time of flight to be t_{f} then by symmetry, the time to peak t_{p} is half that time ($t_{\text{p}} = \frac{t_{\text{f}}}{2}$). From kinematics, we have

$$\begin{aligned} v_{\text{f}y} &= v_{\text{i}y} - g\Delta t \\ 0 &= v_{\text{i}y} - gt_{\text{p}} \\ v_{\text{i}y} &= gt_{\text{p}} \end{aligned} \tag{17.12}$$

where $v_{\text{f}y}$, the velocity of the center of mass at the highest point is zero and Δt is the time to peak t_{p}. To compute the jump height Δh we then use

$$\begin{aligned} \Delta y &= v_{\text{i}y}\Delta t - \frac{1}{2}g(\Delta t)^2 \\ \Delta y &= gt_{\text{p}}t_{\text{p}} - \frac{1}{2}gt_{\text{p}}^2 \\ \Delta y &= gt_{\text{p}}^2 - \frac{1}{2}gt_{\text{p}}^2 = \frac{1}{2}gt_{\text{p}}^2 \\ \Delta y &= \frac{1}{2}g\left(\frac{t_{\text{f}}}{2}\right)^2 \\ \Delta y &= \frac{gt_{\text{f}}^2}{8} \\ \Delta h &= \frac{gt_{\text{f}}^2}{8} \end{aligned} \tag{17.13}$$

where $v_{\text{i}y} = gt_{\text{p}}$, Δt is the time to peak $t_{\text{p}} = \frac{t_{\text{f}}}{2}$, and Δy is the jump height Δh.

17.1.2.2 Calculating Jump Height Using the Impulse-Momentum Theorem

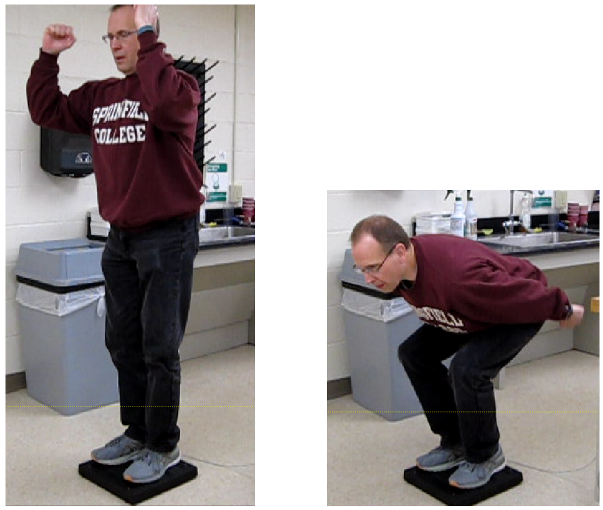
To calculate jump height using the impulse-momentum theorem, we first have to compute the take-off velocity v_{t}. This is done by measuring the net vertical force acting on the individual (using a force platform). We then compute the total impulse over the time interval from when the individual is standing still to when their feet just lose contact with the force platform. v_{t} is then computed using Equation 17.11. Using kinematics we have

$$\begin{aligned} v_{\text{f}y}^2 &= v_{\text{i}y}^2 - 2g\Delta y \\ 0 &= v_{\text{i}y}^2 - 2g\Delta y \\ \Delta y &= \frac{v_{\text{i}y}^2}{2g} \\ \Delta h &= \frac{v_{\text{t}}^2}{2g} \end{aligned} \tag{17.14}$$

where $v_{\text{f}y}$ is 0 m/s, $v_{\text{i}y}$ is the take-off velocity v_{t}, and Δy is the jump height Δh.

17.2 Experiment

In this experiment we will investigate maximal jump heights using two different jumping techniques, a counter-movement jump and a squat jump. In a counter-movement jump, the individual starts at rest in an upright position. In a fluid motion the individual will then move downward to the lowest position where the thighs are nearly parallel to the floor (this is the counter-movement) then explosively push off until they leave the force platform (see Figures 17.4a and 17.4b). In the squat jump, the individual will start the jump at rest in the squat position (with the thighs nearly parallel to the floor) and explosively push off until they leave the force platform (see Figure 17.5).



(a) Start Position (b) Lowest Position

Figure 17.4: Counter-Movement Vertical Jump

Figure 17.5: Squat Jump

17.2.1 Measuring Body Mass

We will first measure the body mass M using the force platform. Make sure the force platform is connected to the GLX datalogger which is then connected to a USB port on your computer.

When you start Pasco Capstone, it should recognize the force platform sensor. Make sure the data collection frequency is set to 2000 Hz (2 KHz). Choose the "graph" icon and set the vertical axis so that it is measuring the vertical force in Newtons. The horizontal axis should default to Time in seconds. Before stepping on the force platform, make sure you tare (zero) it by pressing the tare button on the side of the platform. Then have the subject stand quietly on the force platform. Press the "play" button to record the vertical force as the subject is standing still. Record the vertical force for about 5 - 10 s then press "stop". Change the vertical force scale so it ranges from 0 N to about 100 N above the recorded vertical force. Use the "highlight range of points in active data" tool to highlight a 2 - 3 s region where the subject was standing still. Then use the "display selected statistics for active data tool" to find the individual's mean weight in Newtons (see Figure 17.6).

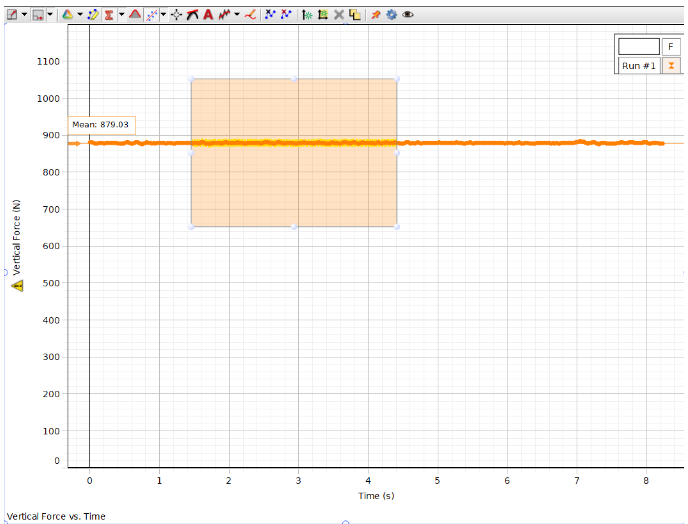


Figure 17.6: Measuring Weight of Subject

Question 1: Compute the mass of the subject using $M = \frac{w}{g}$ where w is the weight of the subject. Record your values in Table 17.1.

Weight of Subject (w) (N)	
Mass of Subject (M) (kg)	

Table 17.1: Weight and Mass of Subject

17.2.2 Counter-Movement Jump

In general, the force platform measures the normal force that the ground exerts on the individual (ground reaction force or GRF). If the ground reaction force is plotted versus time for a counter-movement jump, initially the GRF will be equal to the individuals body weight. However, when computing the total impulse, we need to find the area under the net vertical force versus time graph (the shaded area in pink). To do this, we would have to mathematically subtract the body weight from each GRF value (see Figure 17.7).

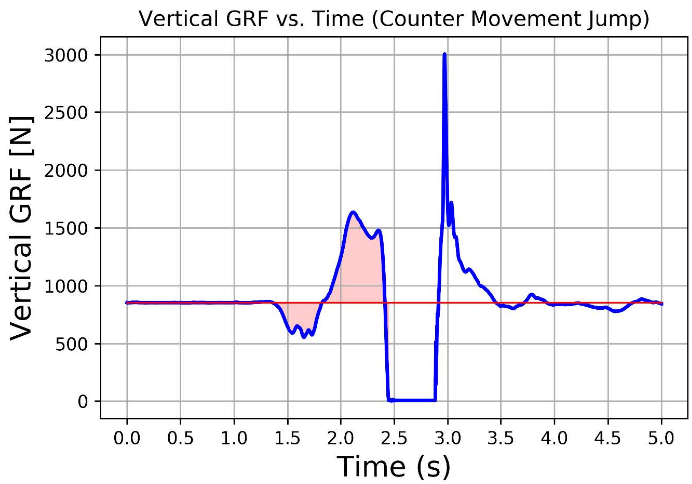


Figure 17.7: Ground Reaction Force for the Counter-Movement Jump

However, when using the force platform and Pasco Capstone, there is an easier way of effectively subtracting out the body weight. Have the subject stand still on the force platform and while the subject is standing still, press the tare button on the platform. The force platform will now record the net vertical force acting on the subject because you have effectively subtracted out the body weight. Have the subject stand still and upright, press the "play" button. Record the force for about 4 - 5 s (it should now read 0 N). Now have the subject perform a maximal vertical jump using the counter-movement technique. Press "stop" after the subject has landed. Your net vertical force versus time graph should look similar to Figure 17.9.

17.2.2.1 Counter-Movement Jump Height Determined by Time of Flight

To measure the time of flight, we need to calculate the time when the subject is in the air. Normally, this is when the GRF is equal to 0 N (see Figure 17.7). However, since we tared the force platform and are now recording the net vertical force, it will be when the net vertical force equals negative the body weight. Zoom in on that section of the graph and then select the "add coordinate tool" icon and select "add multi-coordinates tool". Locate the time when the subject leaves the force platform and the time when the subject again makes contact with the force platform. The difference between these two times is the time of flight (see Figure 17.8). Record your values in Table 17.2.

Initial Time (s)		Final Time (s)		Time of Flight (s)	
Maximal Jump Height Using Time of Flight (Δh) (m)					

Table 17.2: Maximal Counter-Movement Jump Height using Time of Flight

Question 2: Compute the maximal jump height using Equation 17.13 and record your value in Table 17.2.

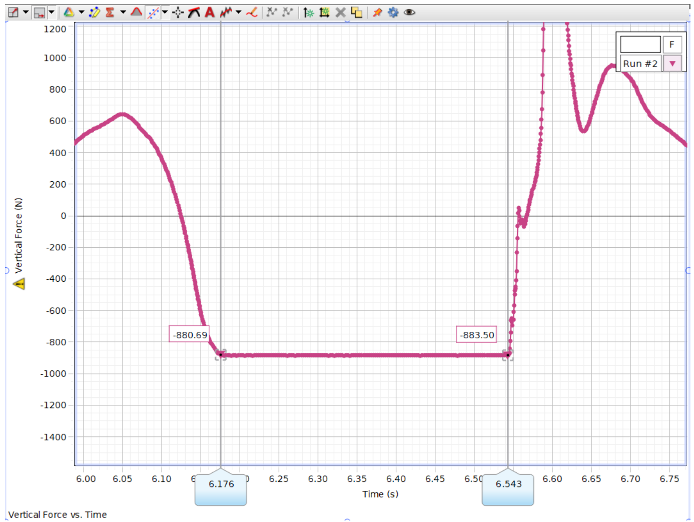


Figure 17.8: Measuring Time of Flight for the Counter-Movement Jump

17.2.2.2 Counter-Movement Jump Height Determined by the Impulse-Momentum Theorem

To determine the jump height using the impulse-momentum theorem we first need to compute the total impulse as the individual is performing the counter-movement and push-off phases of the jump. Zoom back out on the net force versus time graph and using the "highlight range of points in active data tool", highlight the region from when the subject is standing at rest to right when their feet leave the force platform (from 0 N to -w N). Then click on the "display area under highlighted data" icon. This will calculate the area under the graph which is the total impulse in units of Ns (see Figure 17.9).

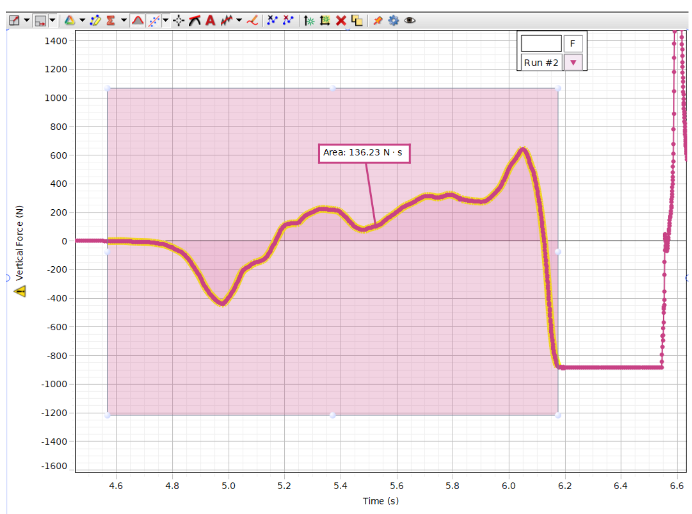


Figure 17.9: Measuring Impulse for the Counter-Movement Jump

Question 3: Compute the take-off velocity v_t using Equation 17.11. Record your value in Table 17.3.

Question 4: Compute the maximal jump height using Equation 17.14. Record your value in

Table 17.3.

Total Impulse During Counter-Movement and Push-off Phase (Ns)	
Take-off Velocity (v_t) (m/s)	
Maximal Jump Height Using Impulse-Momentum (Δh) (m)	

Table 17.3: Maximal Counter-Movement Jump Height using Impulse-Momentum

17.2.3 Squat Jump

We will now measure the maximum jump height using the time of flight and impulse-momentum methods after performing a squat jump. Notice that the vertical ground reaction force versus time graph for the squat jump looks different from that of the counter-movement jump. There is no force pattern representing the counter-movement. Since the subject is pushing off from a static squat position, the GRF immediately exceeds the body weight (see Figure 17.10).

While the subject is standing still on the force platform, press the tare button. This will again ensure that the force platform is measuring the net vertical force acting on the subject and not just the ground reaction force. Have the subject assume a squat position and hold that position. Press the "play" button and record the force for about 4 - 5 s (again, it should read 0 N). Now have the subject explosively push off from the squat position (make sure that the subject does not perform any counter-movement). Press "stop" after the subject has landed. Your net vertical force versus time graph should look similar to Figure 17.11.

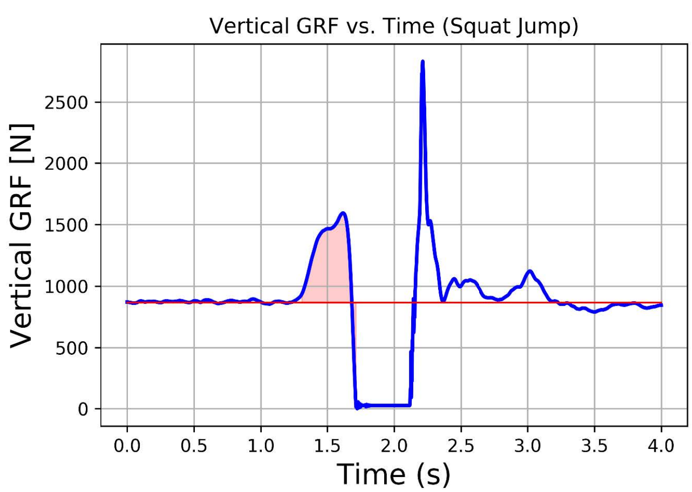


Figure 17.10: Ground Reaction Force for the Squat Jump

17.2.3.1 Squat Jump Height Determined by Time of Flight

Using the same procedures as in Section 17.2.2.1, determine the time when the subject leaves the force platform and the time when the subject again makes contact with the force platform. Compute the time of flight by taking the difference between these two values. Record your data in Table 17.4.

Question 5: Compute the maximal jump height using Equation 17.13 and record your value in Table 17.4.

Initial Time (s)		Final Time (s)		Time of Flight (s)	
Maximal Jump Height Using Time of Flight (Δh) (m)					

Table 17.4: Maximal Squat Jump Height using Time of Flight

17.2.3.2 Squat Jump Height Determined by the Impulse-Momentum Theorem

Using the same procedure as in Section 17.2.2.2, measure the total impulse as the subject is explosively pushing off from the squat position. The graph will now look different as there will be no pattern corresponding to the counter-movement. The net force should initially be 0 N as the subject holds the static squat position then immediately become positive as the subject pushes off from the squat position. The net force will become negative as the subject starts to leave the force platform and their body weight exceeds the ground reaction force (see Figure 17.11).

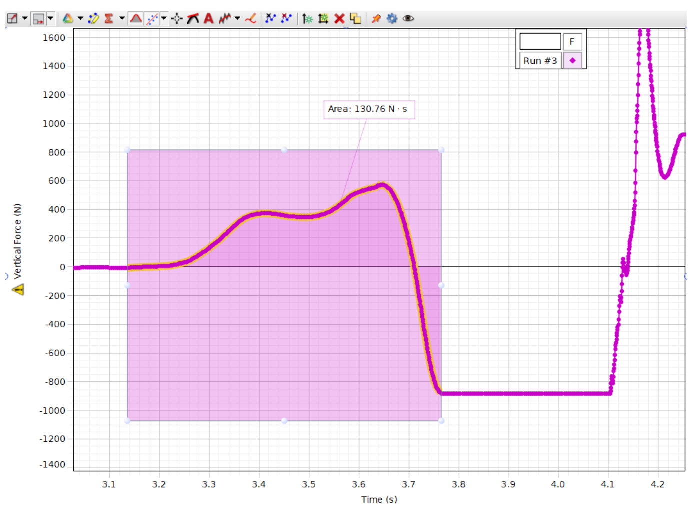


Figure 17.11: Measuring Impulse for the Squat Jump

Question 6: Compute the take-off velocity v_t using Equation 17.11. Record your value in Table 17.5.

Question 7: Compute the maximal jump height using Equation 17.14. Record your value in Table 17.5.

Total Impulse During Push-off Phase From Squat Position (Ns)	
Take-off Velocity (v_t) (m/s)	
Maximal Jump Height Using Impulse-Momentum (Δh) (m)	

Table 17.5: Maximal Squat Jump Height using Impulse-Momentum

Question 8: Was the jump height greater using the time of flight method or the impulse-momentum method? Which one do you think is more accurate (actually represents the displacement of the center of mass from take-off to the highest point)?

Question 9: Was the maximal jump height greater after performing the counter-movement jump or after performing the squat jump? What might be some biomechanical reasons for your finding?

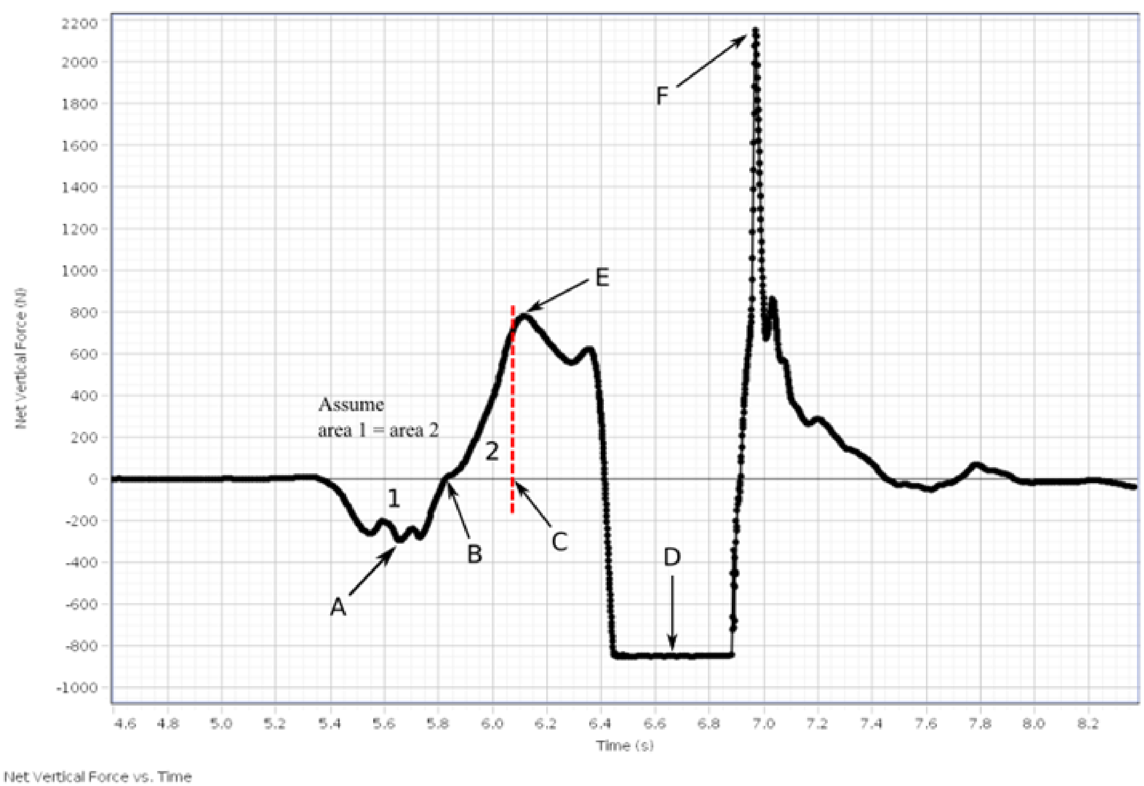


Figure 17.12: Interpreting the Net Vertical Force versus Time Graph for a Counter-Movement Jump

Question 10: Use the net vertical force versus time graph for the counter-movement jump (see Figure 17.12) to match the letter with the correct physical discription of what is occuring to the jumper at that time.

Physical Description	Correct Letter
Subject is in the Air	____________
Subject has the Greatest Negative Acceleration While Performing the Counter-Movement Phase	____________
Subject has an Instantaneous Velocity of Zero While Performing the Counter-Movement Phase	____________
Subject has the Largest Acceleration After Impacting the Force Platform Upon Landing	____________
Subject has the Greatest Positive Acceleration During the Explosive Phase of the Jump	____________
Subject has the Greatest Negative Velocity While Performing the Counter-Movement Phase	____________

Part VI

Circular and Rotational Motion

Lab 18

Uniform Circular Motion

A body accelerates when it experiences a change in its velocity and vice versa. Velocity, recall, is a physical quantity that has both magnitude and direction, i.e., velocity is a vector quantity. Hence, a body accelerates when the magnitude and/or direction of its velocity changes with time. In the simple case of free fall, for example, the velocity changes in magnitude only, but not in direction. When a particle is moving in a circular path, however, its direction of motion is constantly changing. Hence, the velocity of the particle is also changing (see Figure 18.1).

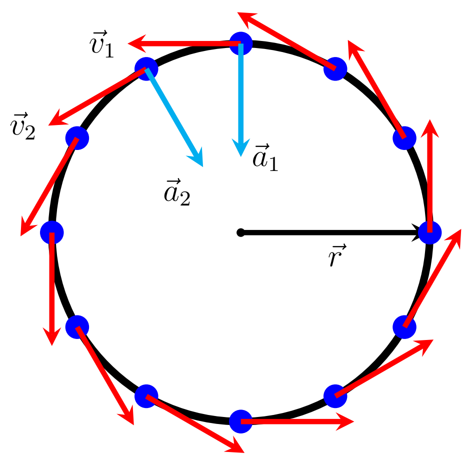


Figure 18.1: Particle Moving in a Circular Path

Suppose now that the particle of Figure 18.1 is moving in a circular path with constant speed, i.e., an equal number of circumferences are travelled in equal amounts of time. Even though the magnitude of the particle's velocity is constant at every position along its circular path (the length of each velocity vector is constant), its direction is constantly changing. Since the direction of the particle's velocity is constantly changing, this means that the velocity of the particle is changing with time; this means that the particle must accelerate and must, therefore, be acted upon by a resultant force. The resultant force that acts on the particle is called the centripetal force. When a particle moves in a circle with a velocity that is constant in magnitude but which changes continuously in direction, we say that the particle is executing uniform circular motion.

The purpose of this experiment is to study uniform circular motion and to compare the observed value of the centripetal force with the calculated value. As a special case of non-uniform circular motion, you will also investigate the motion of a ball as it rounds the top of a vertical curved track. Specifically, you will investigate the "critical" speed that must not be exceeded if the ball is to remain on the track.

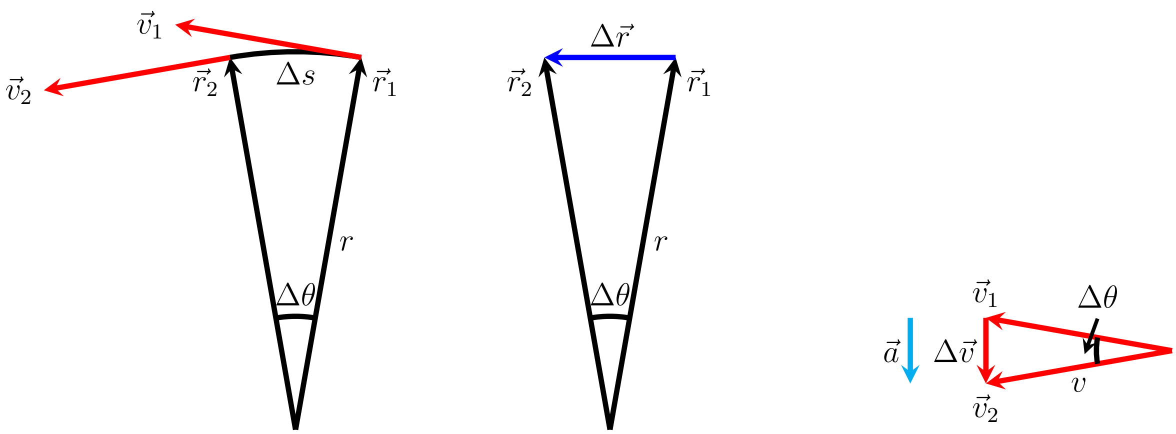


Figure 18.2: Magnitude of Centripetal Acceleration Derivation

18.1 Theory

18.1.1 Centripetal Acceleration and Centripetal Force

It can be shown that a particle that is executing uniform circular motion moves with an acceleration of magnitude

$$a_{\mathrm{c}} = \frac{v^2}{r} \tag{18.1}$$

where a_{c} is the centripetal acceleration, v is the tangential speed of the particle, and r is the radius of the circle. We can derive this expression with the help of Figures 18.2b and 18.2c. The magnitude of the change in position Δr can be written as $\Delta r = v\Delta t$. Notice also that the triangles in Figures 18.2b and 18.2c are similar triangles which means that the ratios of their corresponding sides are proportional. Using the magnitudes of the sides gives

$$\begin{aligned} \frac{\Delta v}{\Delta r} &= \frac{v}{r} \\ \frac{\Delta v}{v\Delta t} &= \frac{v}{r} \\ \frac{\Delta v}{\Delta t} &= \frac{v^2}{r} \\ a &= \frac{v^2}{r} \end{aligned} \tag{18.2}$$

which becomes exact as Δt approaches 0.

The direction of the centripetal acceleration is towards the center of the circle. This can be seen in Figure 18.2c where the change in velocity was computed over a short time interval. Since $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$, the direction of $\vec{a}$ is determined by the direction of $\Delta\vec{v}$ which is toward the center of the circle. Since we know the particle is experiencing an acceleration towards the center of the circle, this means that there is a net force acting on the particle that is also directed towards the center of the circle. The magnitude of this force is given by Newton's second law

$$\vec{F}_{\mathrm{net}} = m\vec{a} \tag{18.3}$$

Since $\vec{a}$ represents the centripetal acceleration and has a magnitude of $a = \frac{v^2}{r}$ the magnitude of the net force is then

$$F_{\text{net}} = F_{\text{c}} = m\frac{v^2}{r} \tag{18.4}$$

where F_{net} is the magnitude of the force which is now labeled as F_{c} since it represents the centripetal force acting on the particle.

It is customary to write the speed of the particle v in terms of the angular speed of the particle ω. The angular speed ω is the number of radians that the particle's motion subtends per second. Thus, if n is the number of revolutions travelled per second by the particle, then since one complete revolution is equal to 2π radians, the angular speed is given as

$$\omega = 2\pi n \tag{18.5}$$

The linear speed v of the particle is expressed in terms of the angular speed w by

$$v = \omega r \tag{18.6}$$

This can be shown by examining Figures 18.2a and 18.2b. The arc length is equal to the radius times the subtended angle

$$\Delta s = r\Delta\theta \tag{18.7}$$

However, for small $\Delta\theta$, the arc length Δs is approximately equal to the magnitude of the change in position $\Delta r = v\Delta t$. Setting these two expressions equal gives

$$\begin{aligned} r\Delta\theta &= v\Delta t \\ \frac{\Delta\theta}{\Delta t} &= \frac{v}{r} \end{aligned} \tag{18.8}$$

If we take the limit as $\Delta t \to 0$, then the left hand side is the angular speed $\lim_{\Delta t\to 0} \frac{\Delta\theta}{\Delta t} = \omega$. This then gives

$$\begin{aligned} \omega &= \frac{v}{r} \\ \text{or} \qquad v &= \omega r \end{aligned} \tag{18.9}$$

If we substitute the expression for v into Equation 18.4, we can write the magnitude of the centripetal force as a function of the angular velocity as

$$F_{\text{c}} = m\omega^2 r \tag{18.10}$$

We will use both forms of the magnitude of the centripetal force (Equations 18.4 and 18.10) where the centripetal force is a function of the linear speed and angular speed, respectively, in this experiment.

18.1.2 Critical Speed

18.1.2.1 Roller Coaster Track

As an interesting application of circular motion in a vertical orbit, consider the motion of a ball as it rolls along a track that has a circular curvature at various locations (see Figure 18.3).

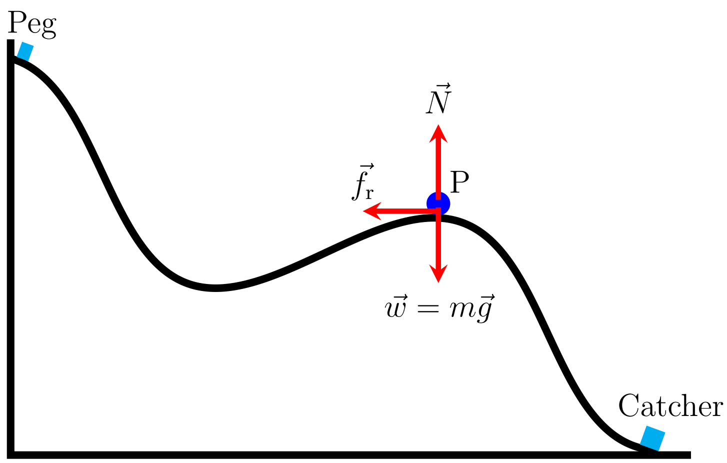


Figure 18.3: Ball Rolling on a Roller Coaster Track

When the metal ball arrives at position P on the track, the external forces that act on the ball are the force of gravity $\vec{w}$, the normal force $\vec{N}$, and the force of rolling friction $\vec{f}_{\rm r}$. Therefore, the resultant force $\vec{F}$ is

$$\vec{F} = \vec{f}_{\rm r} + \vec{N} + \vec{w} \tag{18.11}$$

The component of the resultant force in the direction toward the center of the circular path of the ball is the centripetal force

$$\begin{aligned} \sum F_{\rm c} &= m\frac{v^2}{r} \\ f_{\rm rc} + N_{\rm c} + w_{\rm c} &= m\frac{v^2}{r} \end{aligned} \tag{18.12}$$

where the subscript c indicates the component of the force in the direction toward the center of the circular path, m is the mass of the ball, v is the linear speed of the ball, and r is the radius of the circular path of the ball's center of mass. Therefore, we have

$$\begin{aligned} 0 - N + w &= m\frac{v^2}{r} \\ \text{or} \qquad N &= w - m\frac{v^2}{r} \\ N &= m\left(g - \frac{v^2}{r}\right) \end{aligned} \tag{18.13}$$

Equation 18.12 describes the normal force exerted on the ball by the track when the ball is located at position P. Note that if the linear speed of the ball is large enough, it is possible for N to be equal to zero. This speed is known as the "critical speed". We can compute the critical speed by setting $N = 0$ which gives

$$v_{\rm crit} = \sqrt{g r_{\rm path}} \tag{18.14}$$

where $r_{\rm path}$ is the radius of the path of the center of the mass of the ball at that instant.

18.1.2.2 Loop Track

Notice that instead of having the ball roll over the top of a circular section of track as in a roller coaster, the ball could also roll under a curved section of track as in a loop (see Figure 18.4).

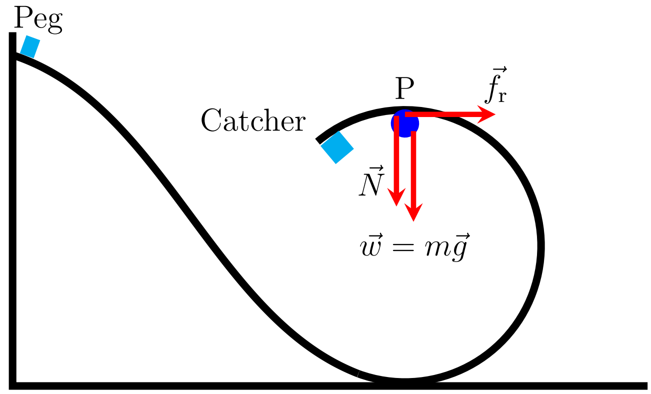


Figure 18.4: Ball Rolling on a Loop Track

The formula for the centripetal force when the ball is at position P is similar to that of the roller coaster expect that the normal force N now points towards the center of the circular path

$$0 + N + w = m\frac{v^2}{r}$$
$$\text{or} \qquad N = m\frac{v^2}{r} - w$$
$$N = m\left(\frac{v^2}{r} - g\right) \tag{18.15}$$

We can then compute the critical speed as before by setting $N = 0$ which gives the same formula

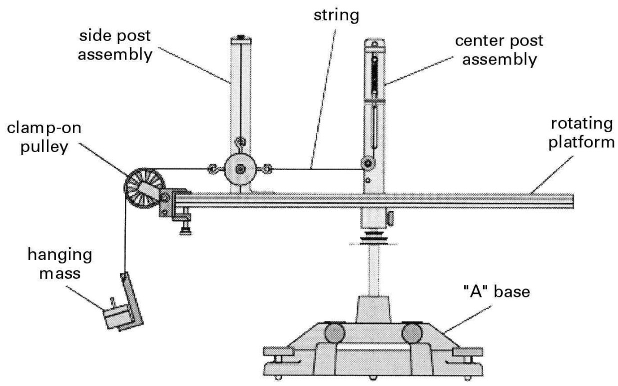
$$v_{\text{crit}} = \sqrt{g r_{\text{path}}} \tag{18.16}$$

18.2 Experiment

18.2.1 Computing Centripetal Force Using the Rotating Apparatus

We will use the rotating platform apparatus (see Figure 18.5) to investigate the centripetal force applied to the hanging object. When the hanging object is rotating, the centripetal force is provided by the spring (hanging from the center of the post) and transmitted by the string to the hanging object.

Set the side post assembly at a radius of 15 cm = 0.15 m from the center post. Remove the hanging object and measure its mass. Record the mass in Table 18.1. Reattach the hanging object to the string so it is hanging from the side post assembly as shown in Figure 18.5. Attach one side of a string to the hanging object, pass it over the pulley, and attach the other side to the hanging mass. Add mass to the hanging mass until the hanging object is hanging vertically from the side post. To determine whether the hanging object is hanging vertically, you should make sure the support string is visually lined up with the vertical line on the side post. Record the mass of the hanging mass in Table 18.1 (note that the blue hanger already has a mass of 5 g = 0.005 kg that needs to be taken into account). This value will be used later to determine the experimental value of the centripetal force.



(a) Rotating Platform Apparatus Diagram (Courtesy of PASCO Scientific)

(b) Rotating Platform Apparatus Set-up

Figure 18.5: Rotating Platform Apparatus

Radius (m)	Mass of Hanging Object (kg)	Mass of Hanging Mass (kg)
0.15		

Table 18.1: Rotating Apparatus Parameters

With the hanging object vertical, adjust the indicator bracket on the center post so that it lines up with the orange indicator. Finally, remove the hanging mass (blue hanger) and pulley. The apparatus is now set up to collect data.

Slowly adjust the DC voltage power supply until the apparatus is rotating at a speed such that the hanging object is hanging vertically (to determine this speed, make sure the orange indicator is lined up with the indicator bracket). Set up the Smart timer so that it can measure the angular speed ω in rad/s. Take 5 measurements of the angular speed and record your values in Table 18.2.

Trial	1	2	3	4	5	Average
Angular Speed (ω) (rad/s)						
Absolute Deviations (rad/s)						
PRAAD for Angular Speed (%)						

Table 18.2: Angular Speed Measurements

Question 1: Compute the average angular speed. Compute the PRADD for angular speed. Record your values in Table 18.2.

Question 2: Calculate the theoretical centripetal force ($F_{\text{c,theory}}$) using Equation 18.10. Estimate the PRAAD for mass m and for radius r by using the resolution of the instrument as the average absolute deviation. PRAAD for Mass $(m) = \frac{0.0001}{m} \cdot 100\%$, PRADD for Radius $(r) = \frac{0.001}{0.15} \cdot 100\%$. Finally, calculate the PRAAD for the centripetal force as

$$\text{PRADD for Centripetal Force } (F_{\text{c}}) = \text{PRAAD for Mass } (m) + \text{PRAAD for Radius } (r) + 2 \cdot \text{PRAAD for Angular Speed } (\omega)$$

Show all calculations and record your values in Table 18.3.

Centripetal Force Theory ($F_{\text{c,theory}}$) (N)	PRAAD for Centripetal Force (%)

Table 18.3: Centripetal Force Theory

Question 3: Compute the experimental value of the centripetal force. This can be found by computing the weight of the hanging mass $F_{\text{c,exp}} = w = mg$, where m is the mass of the hanging mass from Table 18.1. Show your calculation and record your value in Table 18.4.

Centripetal Force Theory ($F_{\text{c,theory}}$) (N)	Centripetal Force Experimental ($F_{\text{c,exp}}$) (N)	Percent Experimental Error (%)

Table 18.4: Comparing Centripetal Force

Question 4: Compare the theoretical centripetal force to the experimental centripetal force by computing the percent experimental error. Show the calculation and record the value in Table 18.4.

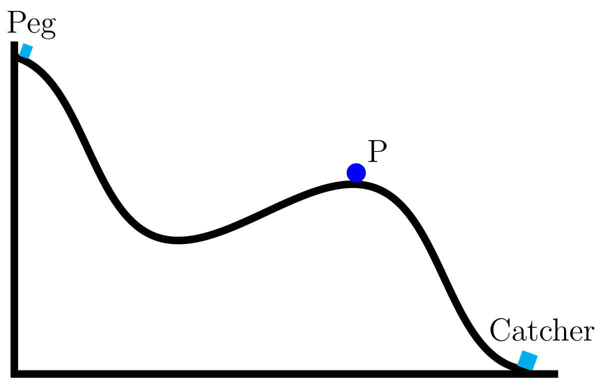
Question 5: How does the centripetal force vary with the angular speed (ω) for a constant radius (r) of the circular path (i.e. is the relation linear, quadratic, cubic, inverse square, etc.?)

Question 6: How does the centripetal force vary with the radius (r) for a constant angular speed (ω)? Hint: Questions 5 and 6 can both be answered by examining Equation 18.10.

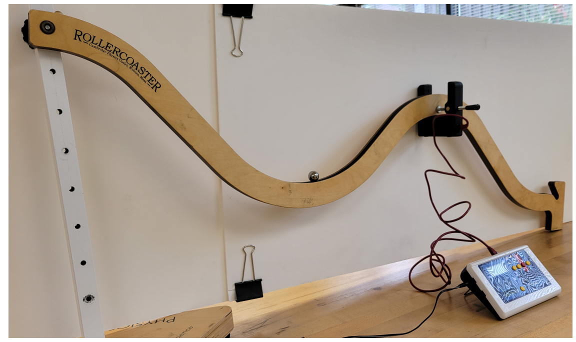
18.2.2 Measurement of Critical Speed on the Roller Coaster Track

Elevate the end of the roller coaster track until you observe the ball to just begin to leave the track at position P (see Figure 18.6).

Secure the photogate clamp at position P so that it fits snugly against the track (see Figure 18.6b). The photogate is then properly positioned to measure the time it takes the diameter of the ball to pass through. Place the CPO timer in interval mode. Release the ball from the position of the peg on the track and measure the time for the ball to pass through the photogate for five trials. Record your data in Table 18.5.



(a) Roller Coaster Track Diagram



(b) Roller Coaster Track Set-Up

Figure 18.6: Roller Coaster Track

Trial	1	2	3	4	5	Average
Elapsed Time (s)						
Absolute Deviations (s)						
PRAAD for Time (%)						

Table 18.5: Roller Coaster Elapsed Time Measurements

Question 7: Compute the average elapsed time and the PRAAD for elapsed time. Show your calculations and record your values in Table 18.5.

Question 8: Measure the diameter of the ball with the Vernier caliper or use the value given in Table 18.6. Determine the experimental critical speed of the ball using $v_{\text{crit,exp}} = \frac{d}{\Delta t_{\text{ave}}}$ where d is the diameter of the ball and t_{ave} is the average time elasped for the ball to pass through the photogate. Record your value in Table 18.7.

Question 9: Determine the radius of the path of the center of mass of the ball while at position P. You can do this by tracing the roller coaster track on a piece of paper over approximately a 10 cm arc length that includes position P at the center of the arc. Then draw two tangent lines to the trace to either side of position P. Construct the perpendicular to each tangent line where it touches the trace. These two perpendicular lines will intersect at the center of the radius of curvature of your trace. Measure the distance from the center of curvature to the trace. This distance will be the radius of curvature of the trace r_{trace} (see Figure 18.7). The radius of the path of the center of mass of the ball at position P is then $r_{\text{path}} = r_{\text{trace}} + r_{\text{ball}}$, where r_{ball} is the radius of the ball (see Figure 18.8). Compute r_{path} and record your values for r_{trace} and r_{path} in Table 18.6.

Question 10: Use Equation 18.16 to determine the predicted value of the critical speed $v_{\text{crit,pred}}$. Compare this predicted value with the experimental value of the critical speed you computed in question 8 by computing the percent experimental error. Record your values in Table 18.7.

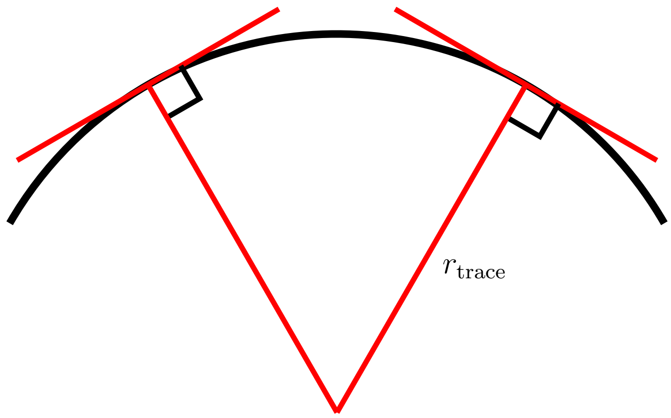


Figure 18.7: Determining r_{trace} for the Curved Tracks

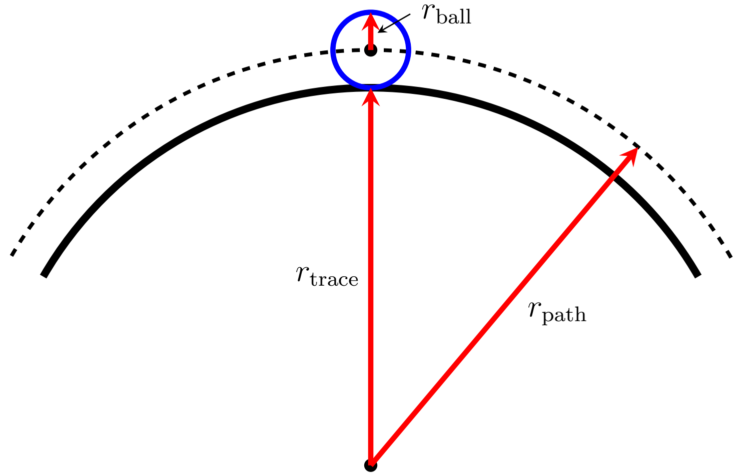


Figure 18.8: Determining r_{path} for the Roller Coaster Track

Radius of Trace of Roller Coaster Track (r_{trace}) (m)	
Radius of Trace of Loop Track (r_{trace}) (m)	
Radius of Path on Roller Coaster Track (r_{path}) (m)	
Radius of Path on Loop Track (r_{path}) (m)	
Diameter of Ball (d_{ball}) (m)	0.0190
Radius of Ball (r_{ball}) (m)	0.0095

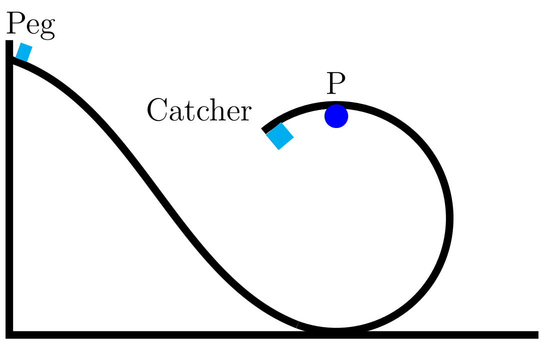
Table 18.6: Track and Ball Parameters

18.2.3 Measurement of Critical Speed on the Loop Track

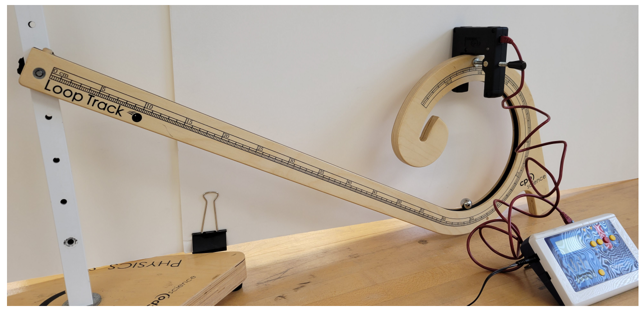
Elevate the end of the loop track until you observe the ball to just begin to leave (fall off) the track at position P (see Figure 18.9).

Critical Speed Predicted ($v_{\text{crit,pred}}$) (m/s)	Critical Speed Experimental ($v_{\text{crit,exp}}$) (m/s)	Percent Experimental Error (%)

Table 18.7: Critical Speed Roller Coaster Track



(a) Loop Track Diagram



(b) Loop Track Set-Up

Figure 18.9: Loop Track

Secure the photogate clamp at position P so that it fits snugly against the track (see Figure 18.9b). The photogate is then properly positioned to measure the time it takes the diameter of the ball to pass through. Place the CPO timer in interval mode. Release the ball from the position of the peg on the track and measure the time for the ball to pass through the photogate for five trials. Record your data in Table 18.8.

Trial	1	2	3	4	5	Average
Elapsed Time (s)						
Absolute Deviations (s)						
PRAAD for Time (%)						

Table 18.8: Loop Elapsed Time Measurements

Question 11: Compute the average elapsed time and the PRAAD for elapsed time. Show your calculations and record your values in Table 18.8.

Question 12: Measure the diameter of the ball with the Vernier caliper or use the given value in Table 18.6. Determine the experimental critical speed of the ball using $v_{\text{crit,exp}} = \frac{d}{\Delta t_{\text{ave}}}$ where d is the diameter of the ball and t_{ave} is the average time elasped for the ball to pass through the photogate. Record your value in Table 18.9.

Question 13: Determine the radius of the path of the center of mass of the ball while at position P as you did in question 9. You can do this by tracing the loop track on a piece of paper over approximately a 10 cm arc length that includes position P at the center of the arc. Then draw two tangent lines to the trace to either side of position P. Construct the perpendicular to each tangent line where it touches the trace. These two perpendicular lines will intersect at the center of the radius of curvature of your trace. Measure the distance from the center of curvature to the trace. This distance will be the radius of curvature of the trace r_{trace} (see Figure 18.7). The radius of the path of the center of mass of the ball at position P is then $r_{\text{path}} = r_{\text{trace}} - r_{\text{ball}}$, where r_{ball} is the radius of the ball (see Figure 18.10). Compute r_{path} and record your values for r_{trace} and r_{path} in Table 18.6. Note that the r_{trace} value for the loop track will be different than that for the roller coaster track. Also note that to compute r_{path} for the loop track, you need to subtract r_{ball} from r_{trace}.

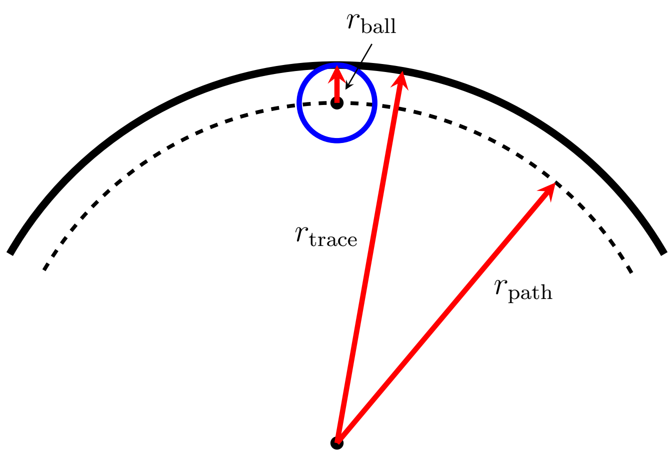


Figure 18.10: Determining r_{path} for the Loop Track

Question 14: Use Equation 18.16 to determine the predicted value of the critical speed $v_{\text{crit,pred}}$. Compare this predicted value with the experimental value of the critical speed you computed in question 12 by computing the percent experimental error. Record your values in Table 18.9.

Critical Speed Predicted ($v_{\text{crit,pred}}$) (m/s)	Critical Speed Experimental ($v_{\text{crit,exp}}$) (m/s)	Percent Experimental Error (%)

Table 18.9: Critical Speed Loop Track

Lab 19

Experimental Studies in Rotational Motion: Part 1

During this laboratory session, you will experiment with bodies that roll or rotate. Bodies that can roll or rotate in addition to translate (i.e., move in a straight line) are said to possess one additional "degree of freedom."

You should already be familiar with the kinematics of the circular motion of a particle, i.e., you are aware of the description of quantities of circular motion, namely angular displacement $\vec{\theta}$, angular velocity $\vec{\omega}$, and angular acceleration $\vec{\alpha}$. The next step is to study the dynamics of the motion. That is, you will attempt to discover the relationship between the rotary motion of a body and the resultant force applied to the body. At your disposal you have your ingenuity and Newton's laws of motion.

Finally, you will apply work-energy concepts to the rolling motion of a ball along a curved track.

19.1 Theory

The bodies whose rotational motion you will observe in this experiment are very good approximations of an ideal object that we call a *rigid body.* A rigid body is an object in which the forces between the atoms making up the object are so strong and of such a nature that any little forces that are needed to move the object do not distort it. That is, the atoms that make up a rigid body maintain fixed positions relative to one another, and thus the shape of a rigid body stays the same as it moves about. It turns out to be quite convenient in the description of the rotational dynamics of a rigid body that is rotating about an axis in an inertial frame of reference to derive a relationship between the torque due to the external resultant force acting on the body $\vec{\tau}_{\text{ext}}$ and the angular acceleration of the body, $\vec{\alpha}$.

Note: An inertial frame of reference is simply one in which Newton's first law holds. For such a body, the torque $\vec{\tau}_{\text{ext}}$ due to the external resultant force applied to the body turns out to be directly proportional to the angular acceleration of the body.

$$\vec{\tau}_{\text{ext}} \propto \vec{\alpha}$$

The constant of proportionality turns out to be a function of both the mass and geometry of the body. This constant of proportionality is given the name moment of inertia, denoted by the symbol I. The reason for the word moment in the name moment of inertia is technical and evolves from the structure of the quantity I. You are already familiar with the term inertia. The mass m of a body and the moment of inertia I of a body are measures of the body's inertia for the case of translatory motion and rotational motion of the body, respectively. The moment of inertia I has units of mass times length squared. The precise relationship between the torque due to the external resultant force acting on a rigid body and the angular acceleration of the body when the body is rotating about an axis in an inertial frame of reference (e.g., a fixed axis of rotation) is

$$\vec{\tau}_{\text{ext}} = I\vec{\alpha} \tag{19.1}$$

This equation is analogous to Newton's Second Law but for rotations. This equation is sometimes referred to as Euler's Second Law.

19.2 Experiment

19.2.1 Basic Concepts of Moment of Inertia

On your bench you should find a wooden disc and a metal ring of the same diameter and of approximately the same mass. Also find two smooth long black boards. Prop the boards up on the 5 kg masses to create a slight incline. Release the wooden disc and metal ring simultaneously from the same position on the inclined boards (see Figure 19.1).

Figure 19.1: Ring and Disk Roll Down an Incline

Question 1: Explain your observation in terms of the moment of inertia of the disc and the moment of inertia of the ring.

Hint: When either the ring or the disc is released on the inclined board, its potential energy at the top of the board is changed into kinetic energy of translation and kinetic energy of rotation at the bottom of the board.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

In this equation, h is the height from which each object is released, v is the linear speed of the center of mass of the object at the bottom of the board, ω is the angular speed of the object about its center of mass at the bottom of the board, m is the mass of the object, and I is the moment of inertia of the object about an axis passing through the center of mass.

Next, place the two black boards on the laboratory bench so that they are horizontal. Place both the disc and the ring on the boards and strike each simultaneously with a rod at the same locations and with some small impulse.

Question 2: Using Euler's Second Law and the concept of inertia, explain your observation.

19.2.2 Advanced Concepts of Moment of Inertia and Its Measurement

19.2.2.1 Moment of Inertia of the Inertia Wheel

Clamped to the edge of your lab bench you will find an apparatus consisting of a large, heavy disc mounted on ball bearings to a bracket. The apparatus is called an inertia wheel. You will find fastened to one side of the large disc a smaller cylindrical hub. The large disc is set into rotation by a falling mass attached to a cord that is wrapped around the hub. A tripping platform holds the mass until it is released by the operator. You will use this apparatus to determine the moment of inertia of the wheel (see Figure 19.2). Below, we outline the theory that you must understand in order to perform this measurement.

Figure 19.2: Inertia Wheel and Hub Apparatus

The torque due to the external resultant force applied to a rigid body rotating about a fixed axis of rotation is related to the angular acceleration of the body according to Equation 19.1. For convenience, the equation is given again below

$$\vec{\tau}_{\text{ext}} = I\vec{\alpha} \tag{19.2}$$

We will use this equation to derive the experimental moment of inertia I_{exp} of the inertia wheel.

It is also possible to calculate the theoretical moment of inertia I_{theory} of the wheel directly from a knowledge of its geometry, mass, and mass distribution. This requires the use of calculus and you will not be required to perform this calculation here. The result of the calculation will be given later.

From Equation 19.2 it should be clear that if the magnitude of $\vec{\tau}_{\mathrm{ext}}$ and $\vec{\alpha}$ are both known, then I can be determined. The idea is to express τ_{ext} and α in terms of measurable quantities, thus giving an experimental determination of I.

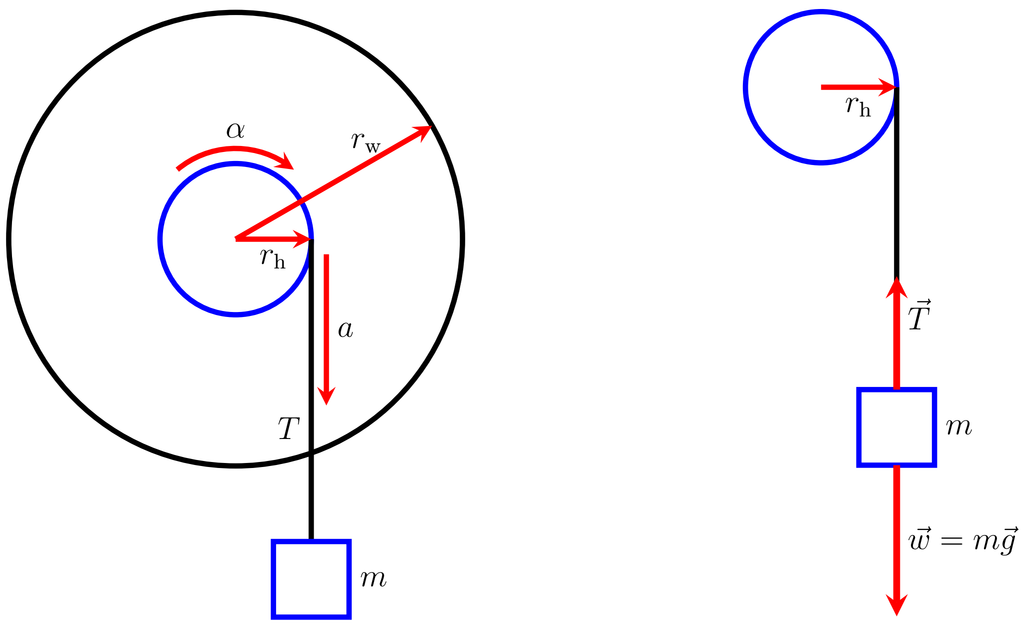
First, the magnitude of the angular acceleration α of the hub is related to the linear acceleration of the rim of the hub a by

$$\alpha = \frac{a}{r_{\mathrm{h}}} \tag{19.3}$$

where r_{h} is the radius of the hub. The linear acceleration of the rim of the hub is the same as the linear acceleration of the descending mass (see Figure 19.3a). When a mass m is attached to the rim of the hub, the tension T in the cord exerts a torque of magnitude $\tau = Tr_{\mathrm{h}}$ on the wheel, and this gives the entire wheel an angular acceleration α. Therefore, we find that

$$Tr_{\mathrm{h}} = I\alpha \tag{19.4}$$

where I is the moment of inertia of the wheel about the fixed axis of rotation passing through its center.



(a) Inertia Wheel with Hub

(b) Forces Acting on Descending Mass

Figure 19.3: Inertia Wheel with Hub and Descending Mass

Consider the descending mass m in Figure 19.3b. Using a coordinate system in which the positive y-axis points downward and applying Newton's second law to the mass m, we find that

$$mg - T = ma \tag{19.5}$$

Solving Equation 19.5 for T, we find that

$$T = m\,(g - a) \tag{19.6}$$

Inserting Equations 19.3 and 19.6 into Equation 19.4, we obtain

$$m\,(g - a)\,r_{\mathrm{h}} = \frac{Ia}{r_{\mathrm{h}}}$$

and solving for I, we get

$$I = \frac{m\,(g-a)\,r_{\mathrm{h}}^2}{a} \tag{19.7}$$

You may use Equation 19.7 to determine the moment of inertia I_{exp} of the wheel experimentally.

Using the mass sets, attach the blue mass hanger to a length of string, making the string just long enough so that the mass hanger will strike the floor before the string completely unwinds from the hub. Tie a small loop in the free end of the string, place the loop over the small pin in the hub, and wind the string around the hub in a neat single layer. Add a mass of 45 g to the 5 g blue hanger so the total mass is 50 g = 0.050 kg. Place the mass on the trip platform. Make sure the large wheel is positioned so that there is no slack in the string.

Trip the platform by pulling the lever. Using a CPO timer (or a smart phone timer), determine the time the mass takes to descend to the floor. It's important to start the timer as soon as the platform is tripped and stop it as the mass hits the floor. Repeat for a total of three time trials. Repeat the entire procedure using masses of 70 g, 90 g, and 110 g. Record your data in Table 19.1.

Measure the displacement Δy from the floor to the bottom of the platform. Measure the diameter of the large metal disk part of the inertia wheel d_{d} and the diameter of the hub d_{h}. Using these values, compute the radius of the metal disk r_{d} and the radius of the hub r_{h}. Record your values in Table 19.2.

Condition	Descending Mass (m) (kg)	t_1 (s)	t_2 (s)	t_3 (s)	t_{ave} (s)	Acceleration (a) ($\mathrm{m/s^2}$)
1	0.050					
2	0.070					
3	0.090					
4	0.110					

Table 19.1: Time Measurements and Linear Acceleration of Descending Mass - Inertia Wheel

Height of Platform (Δy) (m)	Diameter of Metal Disk (d_{d}) (m)	Diameter of Hub (d_{h}) (m)	Radius of Metal Disk (r_{d}) (m)	Radius of Hub (r_{h}) (m)

Table 19.2: Parameters of the Inertia Wheel Apparatus

Question 3: Using kinematics, derive an equation for the constant linear acceleration of the descending mass. Hint: The acceleration will depend on the vertical displacement (height of the platform relative to the floor) and the time. You should find that $a = \frac{2\Delta y}{t_{\mathrm{ave}}^2}$.

Question 4: Using the average of the three time values and the equation derived in question 3, compute the acceleration a for each condition. Record your values in Table 19.1.

Question 5: Using Equation 19.7, your acceleration values from Table 19.1, and the radius of the hub from Table 19.2, compute the experimental moment of inertia I_{exp} of the wheel for each condition. Compute the average value for I_{exp} and the PRAAD for I_{exp}. Record your data in Table 19.3.

Condition	Experimental Moment of Inertia (I_{exp}) (kg m^2)	Absolute Deviations (kg m^2)
1		
2		
3		
4		
Average		
PRAAD (%)		

Table 19.3: Experimental Moment of Inertia of the Wheel

The mass M of the entire inertia wheel is stamped on the apparatus. This is the combined mass of the large metal disk and the hub. A value of 0.070 kg may be used for the hub. The theoretical moment of inertia of the inertia wheel is then found by calculus to be

$$I_{\text{theory}} = \frac{(M - 0.070)\, r_{\text{d}}^2}{2} + \frac{0.070 r_{\text{h}}^2}{2} \tag{19.8}$$

where M is the mass of the entire inertia wheel, r_{d} is the radius of the large metal disk, and r_{h} is the radius of the hub.

Question 6: Use Equation 19.8 to compute the theoretical moment of inertia for the inertia wheel. Record your value in Table 19.4.

Question 7: Compare the theoretical moment of inertia to the average experimental moment of inertia from Table 19.3 by computing the percent experimental error. Record your value in Table 19.4.

Experimental Moment of Inertia (I_{exp}) (kg m^2)	Theoretical Moment of Inertia (I_{theory}) (kg m^2)	Percent Experimental Error (%)

Table 19.4: Comparison of Moment of Inertia of the Inertia Wheel

19.2.2.2 Moment of Inertia of the Meter Stick

(a) Moment of Inertia of Meter Stick - Initial Position

(b) Moment of Inertia of Meter Stick - Mass Descending

Figure 19.4: Moment of Inertia of Rotating Meter Stick

We just found the moment of inertia of a spinning inertia wheel by examining the dynamics of a descending mass connected by a string to the hub of the wheel. We will now perform a similar experiment on a horizontally rotating meter stick. A Pasco Pivot is connected to the top of a vertical rod which is securely clamped into a base. One side of the pivot contains a clamp to hold the meter stick and the other side has a three step pulley. A string that is wound around the largest diameter pulley passes over a super pulley which is also clamped to the rod. A mass hanger is then attached to the free end of the string (see Figure 19.4). This situation is basically identical to the inertia wheel. The mass hanger starts at a certain height above the table and is released from rest. The tension in the string then creates a torque on the pivot pulley which causes an angular acceleration of the meter stick.

Make sure the meter stick is clamped at 50 cm (its center of mass). Wind the string around the pivot pulley so that the mass hanger starts at rest at a height of 0.70 m above the table. For masses of 30 g, 50 g, 70 g, and 90 g, measure the time it takes the mass hanger to reach the table. For each mass, perform three time trials. Note: Remember that the blue hanger has a mass of 5 g, make sure you take that into account (i.e. for the 30 g total mass just add 25 g, etc.). Record your time values in Table 19.5.

Given the diameter of the pivot pulley d_{p}, compute the radius r_{p}. Measure the mass of the meter stick M using an electronic scale. Record your values in Table 19.6.

Question 8: Using the average of the three time values and the equation derived in question

Condition	Descending Mass (m) (kg)	t_1 (s)	t_2 (s)	t_3 (s)	t_{ave} (s)	Acceleration (a) (m/s^2)
1	0.030					
2	0.050					
3	0.070					
4	0.090					

Table 19.5: Time Measurements and Linear Acceleration of Descending Mass - Meter Stick

Starting Height of Bottom of Hanger (Δy) (m)	Diameter of Pivot Pulley (d_p) (m)	Radius of Pivot Pulley (r_p) (m)	Mass of Meter Stick (M) (kg)
0.700	0.048		

Table 19.6: Parameters of the Rotating Meter Stick Apparatus

3, compute the acceleration a for each condition. Record your values in Table 19.5.

Question 9: Using Equation 19.7, your acceleration values from Table 19.5, and the radius of the pivot pulley from Table 19.6, compute the experimental moment of inertia I_{exp} of the meter stick for each condition. Compute the average value for I_{exp} and the PRAAD for I_{exp}. Record your data in Table 19.7. Note that in Equation 19.7, the radius of the hub r_h is now the radius of the pivot pulley r_p.

Condition	Experimental Moment of Inertia (I_{exp}) (kg m^2)	Absolute Deviations (kg m^2)
1		
2		
3		
4		
Average		
PRAAD (%)		

Table 19.7: Experimental Moment of Inertia of the Meter Stick

The theoretical moment of inertia of the meter stick about its center of mass can be found by calculus to be

$$I_{theory} = \frac{1}{12} M L^2 \tag{19.9}$$

where M is the mass of the meter stick and $L = 1$ m is the length of the meter stick.

Question 10: Use Equation 19.9 to compute the theoretical moment of inertia of the meter stick about its center of mass. Record your value in Table 19.8.

Question 11: Compare the theoretical moment of inertia to the average experimental moment of inertia from Table 19.7 by computing the percent experimental error. Record your value in Table 19.8.

Experimental Moment of Inertia (I_{exp}) (kg m^2)	Theoretical Moment of Inertia (I_{theory}) (kg m^2)	Percent Experimental Error (%)

Table 19.8: Comparison of Moment of Inertia of the Inertia Wheel

19.2.3 Work Energy Concepts Applied to Rotational Motion

19.2.3.1 Moment of Inertia of the Ball about its Center of Mass

Position the roller coaster track on the physics stand so that the opposite end of the track rests level on the lab bench (see Figure 19.5).

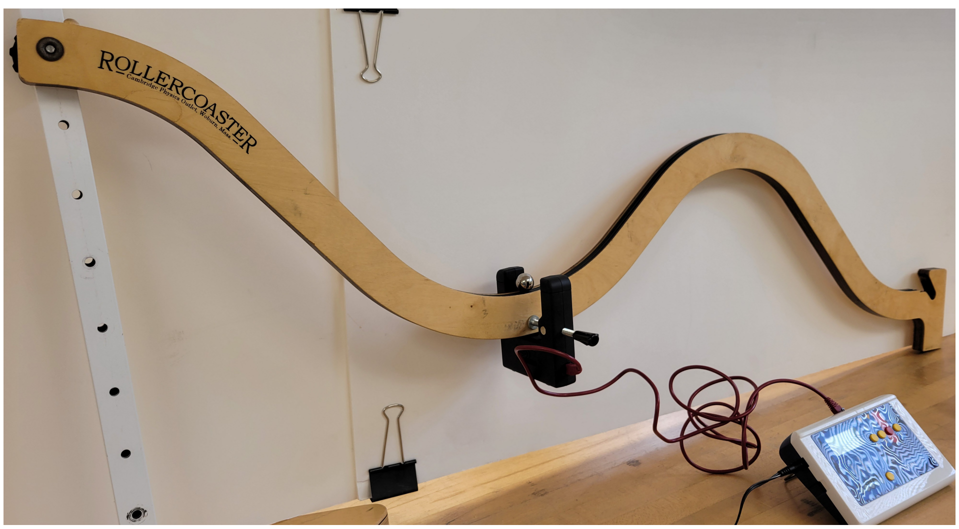


Figure 19.5: Rolling Motion of Ball Along Roller Coaster Track

At any given position, the center of mass of the ball is elevated above the table by a height y (see Figure 19.6). Therefore, the gravitational potential energy of the ball is

$$\text{PE}_{\text{g}} = mgy \tag{19.10}$$

There are two forms of kinetic energy that must be considered here: translational kinetic energy KE_{t} of the center of mass and rotational kinetic energy KE_{r} about the center of mass of the ball. The translational kinetic energy of the center of mass is given by

$$\text{KE}_{\text{t}} = \frac{1}{2}mv^2 \tag{19.11}$$

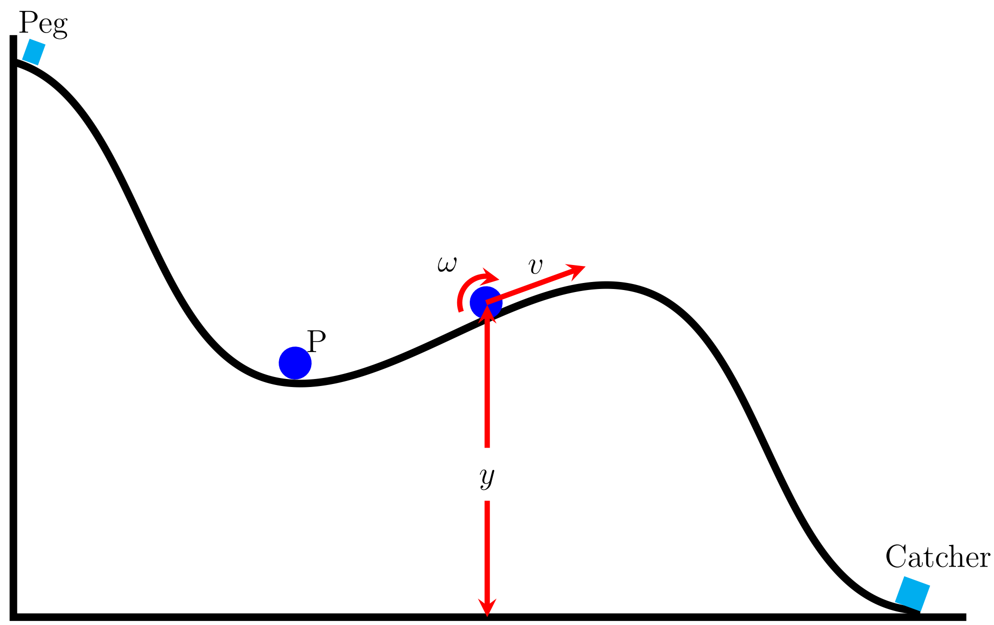


Figure 19.6: Rolling Motion of a Ball Along a Curved Roller Coaster Track

where v is the linear speed of the center of mass of the ball and m is the mass of the ball. The rotational kinetic energy of the ball about its center of mass is given by

$$\mathrm{KE_r} = \frac{1}{2}I\omega^2 \tag{19.12}$$

where ω is the angular speed about the center of mass and I is the moment of inertia of the ball about the center of mass.

Therefore, the total kinetic energy of motion of the ball $\mathrm{KE_m}$ is given by

$$\mathrm{KE_m} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{19.13}$$

As long as the ball does not slip as it rolls along the track, mechanical energy $\mathrm{ME} = \mathrm{KE_m} + \mathrm{PE_g}$ will be conserved. Applying the law of conservation of mechanical energy to the motion of the ball from the peg to position P shown in Figure 19.6, we have

$$\frac{1}{2}mv_{\mathrm{peg}}^2 + \frac{1}{2}I\omega_{\mathrm{peg}}^2 + mgy_{\mathrm{peg}} = \frac{1}{2}mv_{\mathrm{p}}^2 + \frac{1}{2}I\omega_{\mathrm{p}}^2 + mgy_{\mathrm{p}} \tag{19.14}$$

The goal of this experiment is to determine the moment of inertia I of the ball. Secure the photogate clamp at position P so that it fits snugly against the track (see Figure 19.5). The photogate is then properly positioned to measure the time it takes the diameter of the ball to pass through. Place the CPO timer in interval mode. Release the ball from the position of the peg on the track and measure the time for the ball to pass through the photogate for five trials. Record your data in Table 19.9.

Question 12: Compute the average elapsed time and the PRAAD for elapsed time. Show your calculations and record your values in Table 19.9.

Question 13: Measure the diameter of the ball with the Vernier caliper or use the value given by your instructor, then compute the radius of the ball. Determine the average linear speed of the

Trial	1	2	3	4	5	Average
Elapsed Time (Δt) (s)						
Absolute Deviations (s)						
PRAAD for Time (%)						

Table 19.9: Roller Coaster Elapsed Time Measurements

ball at position P using $v = \frac{d}{\Delta t_{\text{ave}}}$ where d is the diameter of the ball and Δt_{ave} is the average time elasped for the ball to pass through the photogate. Record your values in Table 19.10.

Question 14: Compute the angular speed of the ball about its center of mass at position P from the linear speed using $v = \omega r_{\text{b}}$ or $\omega = \frac{v}{r_{\text{b}}}$. Record your value in Table 19.10.

Question 15: Measure the height of the center of mass of the ball above the lab bench at the position of the peg y_{peg} and at position P y_{p}. Measure the mass of the ball m using an electronic scale. Record your values in Table 19.10.

	At Peg	At Position P
Linear Speed v (m/s)	0	
Angular Speed ω (rad/s)	0	
Vertical Position of Ball y (m)		
Diameter of Ball (d_{b}) (m)		
Radius of Ball (r_{b}) (m)		
Mass of Ball (m) (kg)		

Table 19.10: Parameters for Finding the Moment of Inertia of Ball

Question 16: Using Equation 19.14, compute the experimental moment of inertia of the ball about its center of mass I. We will refer to this value as I_{exp}. Record your value in Table 19.11.

Question 17: Compute the theoretical moment of inertia of the ball. Since the ball has the shape of a solid sphere, the moment of inertia about its center of mass is given as

$$I_{\text{theory}} = \frac{2}{5} m r_{\text{b}}^2 \tag{19.15}$$

Compare the theoretical value of the moment of inertia of the ball to the experimental value of the moment of inertia of the ball found in question 16 by computing the percent experimental error. Record your values in Table 19.11.

Experimental Moment of Inertia (I_{exp}) (kg m^2)	Theoretical Moment of Inertia (I_{theory}) (kg m^2)	Percent Experimental Error (%)

Table 19.11: Comparison of Moment of Inertia of the Ball

19.2.3.2 Mechanical Energy of the Ball as it Rolls Along the Track

We will now analyze the mechanical energy of the ball as it rolls along the roller coaster track. Because the ball is a solid sphere of radius r_{b}, I is given by Equation 19.15

$$I = \frac{2}{5} m r_{\text{b}}^2$$

Therefore

$$\text{KE}_{\text{r}} = \frac{1}{2} I \omega^2 = \frac{1}{2} \left(\frac{2}{5} m r_{\text{b}}^2 \right) \omega^2 = \frac{1}{5} m r_{\text{b}}^2 \omega^2$$

However, recall that $v = \omega r_{\text{b}}$ for a ball rolling without slipping. Therefore,

$$\text{KE}_{\text{r}} = \frac{1}{5} \left(m r_{\text{b}}^2 \right) \left(\frac{v}{r_{\text{b}}} \right)^2$$

$$\text{KE}_{\text{r}} = \frac{1}{5} \left(m v^2 \right) \tag{19.16}$$

The total kinetic energy of motion of the ball is then given as

$$\text{KE}_{\text{m}} = \text{KE}_{\text{t}} + \text{KE}_{\text{r}} = \frac{1}{2} m v^2 + \frac{1}{5} m v^2$$

$$\text{KE}_{\text{m}} = \frac{7}{10} m v^2 \tag{19.17}$$

The mechanical energy ME of the ball at each position is the sum of its kinetic energy of motion KE_{m} and its gravitational potential energy PE_{g}. Therefore,

$$\text{ME} = \text{KE}_{\text{m}} + \text{PE}_{\text{g}} = \frac{7}{10} m v^2 + m g y \tag{19.18}$$

Cut a length of string 20 cm in length. Beginning at the position of the peg, you will use this segment of string to define seven positions along the track at intervals of 20 cm (labeled P_1 through P_7) (see Figure 19.7). At each position, you will make the following measurements: The elevation of the center of mass of the ball above the lab bench y and the average of three time trials for the ball to pass through the photogate at that position Δt_{ave}.

Question 18: Make the vertical position and time measurements at each position and record your data in Table 19.12.

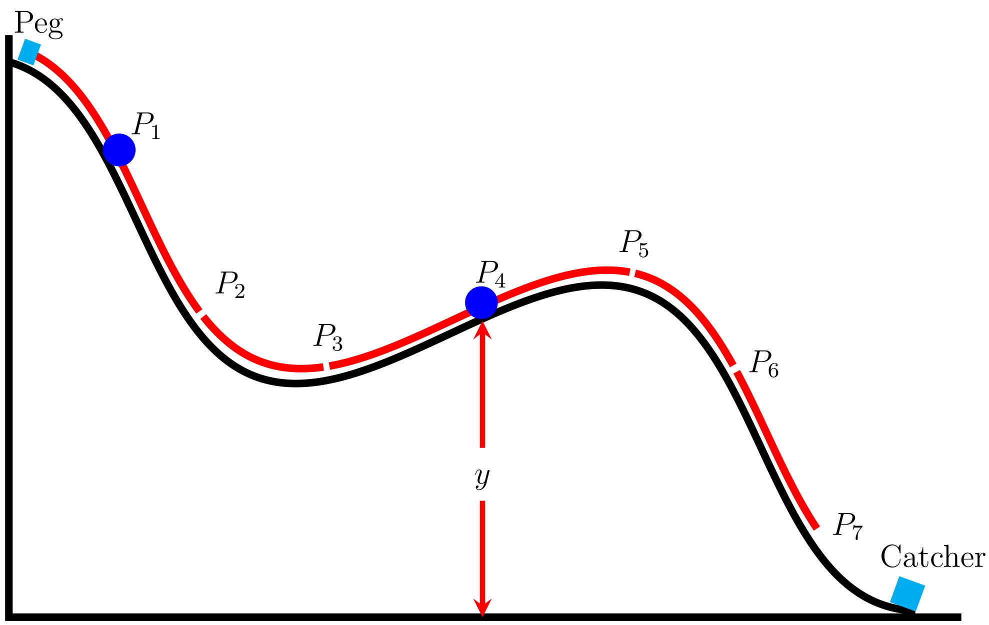


Figure 19.7: Rolling Motion of a Ball Along a Curved Roller Coaster Track

Position	Vertical Position (y) (m)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	Δt_{ave} (s)	Velocity (m/s)
P_1						
P_2						
P_3						
P_4						
P_5						
P_6						
P_7						

Table 19.12: Vertical Position and Average Time Through Photogate at Each Position

Question 19: Determine the average linear speed of the ball at each position P_1 - P_7 using $v = \frac{d}{\Delta t_{\text{ave}}}$ where d is the diameter of the ball and Δt_{ave} is the average time elasped for the ball to pass through the photogate at that position. Record your values in Table 19.12.

Question 20: Measure the mass of the ball on an electronic scale. Using Equations 19.10, 19.11, 19.16, 19.17, and 19.18 compute the gravitational potential energy PE_g, the translational kinetic energy KE_t, the rotational kinetic energy KE_r, the kinetic energy of motion KE_m, and the total mechanical energy ME at each position. Record all of your data in Table 19.13.

Question 21: From the values computed in Table 19.13, graph ME, KE_t, KE_r, KE_m, and PE_g versus the distance traveled along the track s. Since you used a 20 cm (0.20 m) length of string

Position	Translational Kinetic Energy (KE_t) (J)	Rotational Kinetic Energy (KE_r) (J)	Kinetic Energy Motion (KE_m) (J)	Gravitational Potential Energy (PE_g) (J)	Total Mechanical Energy (ME) (J)
P_1					
P_2					
P_3					
P_4					
P_5					
P_6					
P_7					

Table 19.13: Various Energies at Each Position

to mark each position, s for each position should be trivial to obtain. Label your graph with the title "Energy Profile for Rotational Dynamics of a Rolling Sphere". Label the vertical axis "Energy (J)", and label the horizontal axis "Distance Traveled (m)". Make sure to also label each curve appropriately KE_t, KE_r, etc.

Position	Translational Kinetic Energy As a Percentage of Total Kinetic Energy of Motion (%)	Rotational Kinetic Energy As a Percentage of Total Kinetic Energy of Motion (%)
P_1		
P_2		
P_3		
P_4		
P_5		
P_6		
P_7		
Average		

Table 19.14: Translational Kinetic and Rotational Kinetic Energy as a Percentage of the Total Kinetic Energy of Motion

Question 22: For each position noted in Table 19.13, compute the percent contribution of the translational kinetic energy $\mathrm{KE_t}$ and rotational kinetic energy $\mathrm{KE_r}$ to the total kinetic energy of motion $\mathrm{KE_m}$. Record your data in Table 19.14 then compute average values of these percentages.

Question 23: Compare these average percentages with the percentages that you would expect from theoretical considerations. Compare these values by computing the percent experimental error. Record your data in Table 19.15.

	Translational Kinetic Energy As a Percentage of Total Kinetic Energy of Motion (%)	Rotational Kinetic Energy As a Percentage of Total Kinetic Energy of Motion (%)
Experimental Value (%)		
Theoretical Value (%)		
Percent Experimental Error (%)		

Table 19.15: Translational Kinetic and Rotational Kinetic Energy as a Percentage of the Total Kinetic Energy of Motion - Percent Experimental Error

Question 24: Theoretically, if there is no slipping of the ball as it rolls along the track, mechanical energy should be conserved. Even though there is a force of friction when the ball is on a non-flat part of the track (actually, this is a force of static friction at the contact point of the ball and the track), it produces a torque about the center of mass that is made manifest as rotational kinetic energy. Is mechanical energy conserved in your experiment? If not, compute the change in mechanical energy

$$\Delta\mathrm{ME} = \mathrm{ME}_7 - \mathrm{ME}_1$$

that occurred as the ball moved from position 1 to position 7. There are two possible sources for this difference: the work done against the motion of the ball by air resistance and the work done by a force of kinetic friction in the event that the ball slips during its motion. Compute the percent of mechanical energy lost in your experiment using $\frac{\Delta\mathrm{ME}}{\mathrm{ME}_1} \times 100\%$. Using work-energy principles, we can estimate the average combined force of air resistance and kinetic friction f.

$$\begin{aligned} W_f &= \Delta\mathrm{ME} \\ f s \cos 180^\circ &= \Delta\mathrm{ME} \\ f &= -\frac{\Delta\mathrm{ME}}{s} \end{aligned} \tag{19.19}$$

where s is the distance along the track in moving from position 1 to position 7 (120 cm or 1.20 m). Using Equation 19.19, compute the average combined force of air resistance and kinetic friction f.

Lab 20

Experimental Studies in Rotational Motion: Part 2

During the last laboratory session, you were introduced to the dynamics of rotating bodies. In studying the rotational motion of a solid disc and a ring along a board, you discovered that the angular acceleration that the body experiences is directly proportional to the external torque $\vec{\tau}_{\text{ext}}$ applied to the body about an axis fixed in an inertial frame of reference. The constant of proportionality is a measure of the rotational inertia of the body and is called the moment of inertia I of the body. You measured the moment of inertia of a large heavy wheel spinning about a fixed axis passing through its center and the moment of inertia of a meter stick rotating about its center.

In this laboratory session you will make further studies into the rotational motion of bodies. You will first determine the moment of inertia for an aluminum stick, an aluminum hoop, and a masonite disc about fixed axes of rotation by utilizing a device called the physical pendulum. You will find that the moment of inertia of a body depends on the axis of rotation for the body in addition to the geometrical shape, mass, and mass distribution of the body. Next you will demonstrate to yourself a very important conservation theorem, namely, the theorem of conservation of angular momentum. Finally, we will apply the equation

$$\vec{\tau}_{\text{ext}} = I\vec{\alpha} \tag{20.1}$$

to a very interesting and perhaps astonishing phenomenon.

20.1 Theory

20.1.1 Determination of Moments of Inertia of a Body About Various Fixed Axes of Rotation by Utilizing the Device of the Physical Pendulum

A *physical pendulum* is any rigid body mounted so that it can swing about some axis passing through it (see Figure 20.1). You will use the device of the physical pendulum to determine the moment of inertia of an aluminum stick, an aluminum hoop, and a masonite disc about fixed axes of rotation.

The physical pendulum is a generalization of the simple pendulum in which a weightless cord holds a single point particle of mass m. Of course, as we indicated in a previous experiment, the simple pendulum is a mathematical idealization that, however, can be approximated to a very good degree by a small, heavy bob on a thin, light string. Actually, all real pendula are physical pendula.

The period T of a physical pendulum oscillating about a frictionless axis with small amplitude is found to be given by

$$T = 2\pi\sqrt{\frac{I}{Mgd}} \tag{20.2}$$

where I is the moment of inertia of the rigid body, M is the mass of the rigid body, and d is the distance from the pivot point P of the body to the center of mass C (see Figure 20.1).

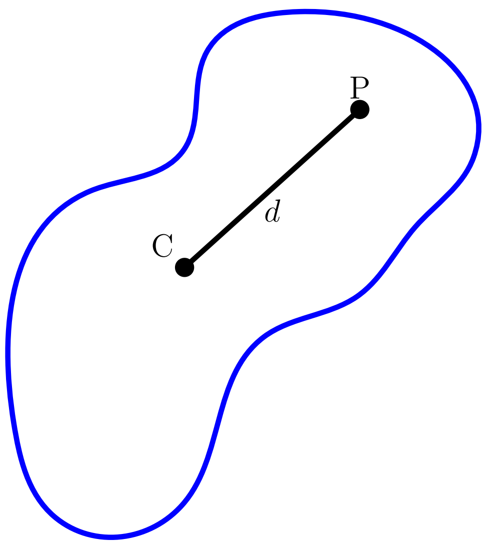


Figure 20.1: A Physical Pendulum. The axis of rotation passes through point P and is perpendicular to the plane of the paper (ie. to the plane of the swing).

Recall that the period of a simple pendulum oscillating about a frictionless axis with a small amplitude is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{20.3}$$

where l is the length of the simple pendulum (the distance from the pivot point to the center of the bob).

One can always find a simple pendulum whose period is equal to that of a particular physical pendulum. It is precisely this fact that you will use to determine the moment of inertia of a rigid body about a fixed axis of rotation. Suppose, for example, that we wanted to determine the moment of inertia of the rigid body shown in Figure 20.1 about the axis through point P. We would take a simple pendulum and adjust its length until its period of oscillation were the same as the period of oscillation of the physical pendulum, which we make using the rigid body. Hence, from Equations 20.2 and 20.3, we write

$$2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{l}{g}}$$

Solving for the moment of inertia I then gives

$$I = Mdl \tag{20.4}$$

The point located a distance $l = \frac{I}{Md}$ from the pivot point P is called the *center of oscillation* of the physical pendulum. It has the special property that if the physical pendulum is allowed to rotate about this point, then the period of oscillation will be the same as it is when the pendulum is rotating about point P. The center of oscillation is not to be confused with the *radius of gyration* for a physical pendulum. The *radius of gyration* is the radial distance from a given axis to a certain point at which the mass of the body could be concentrated without changing the moment of inertia of the body about that axis. That is

$$I = Mk^2 \tag{20.5}$$

where k is the radius of gyration. It follows then that

$$k = \sqrt{\frac{I}{M}} \tag{20.6}$$

The length l in Equation 20.4 is sometimes referred to as the length of the equivalent simple pendulum. You will use Equation 20.4 to find the moment of inertia of the aluminum stick, aluminum hoop, and masonite disc about fixed axes of rotation; you will use Equation 20.6 to determine the corresponding radii of gyration about these axes for each of the aforementioned rigid bodies. A useful theorem in determining the moment of inertia of a body about an arbitrary axis when the moment of inertia of the body about an axis passing through its center of mass is known is called the parallel axis theorem. The parallel axis theorem states that the moment of inertia of a body about any axis is given by

$$I = I_{\text{cm}} + Mh^2 \tag{20.7}$$

where I_{cm} is the moment of inertia of the body about an axis passing through its center of mass and h is the distance between the axis about which the body is rotating and the axis through the center of mass of the body.

20.1.2 The Theorem of Conservation of Angular Momentum

It can be proven that the rate of change of a quantity that we call total *angular momentum* of a body about an axis fixed in an inertial frame of reference or about an axis passing through the center-of-mass is equal to the external torque about that axis. We shall let $\vec{L}$ denote the total angular momentum of a body. Then we have that

$$\vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt} \tag{20.8}$$

Equation 20.8 says that the external torque acting on the body about a given axis is equal to the infinitesimal change of the total angular momentum of the body about that axis with an infinitesimal change in the time t at that time. That is, the external torque at time t is precisely equal to the time rate of change of the total angular momentum at the time t. When we speak of finite changes in the total angular momentum $\vec{L}$ in a finite time interval, we use the Δ notation. That is,

$$\vec{\bar{\tau}}_{\text{ext}} = \frac{\Delta\vec{L}}{\Delta t} \tag{20.9}$$

In Equation 20.9, $\vec{\bar{\tau}}_{\text{ext}}$ is the *average* external torque acting on the body during the time interval Δt. When the *external* torque acting on the body is zero, however, both Equations 20.8 and 20.9 imply that $\vec{L} =$ constant.

That is, the total angular momentum of a body about a given axis remains constant in the absence of an external torque about that axis. We, therefore, say that total angular momentum is *conserved* in the absence of an external torque. Note that since angular momentum is a vector quantity, it must be conserved both in magnitude and direction.

What is angular momentum? For a massive body that is rotating about a given axis in an inertial frame of reference with angular speed ω (see Figure 20.2,) the component of the angular momentum along that axis (say the z axis) is given by

$$L_z = I\omega \tag{20.10}$$

For further details on the definition and use of angular momentum, the student should refer to their textbook.

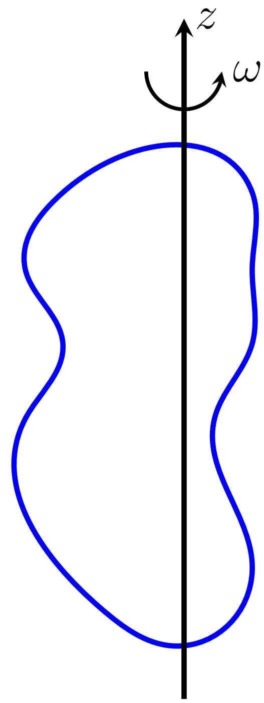


Figure 20.2: Body Rotating Around the z-axis with Angular Speed ω.

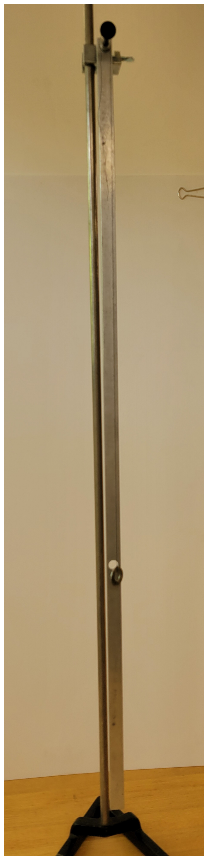
20.2 Experiment

20.2.1 Determination of Moments of Inertia for an Aluminum Stick, Aluminum Hoop, and a Masonite Disc About Fixed Axes of Rotation

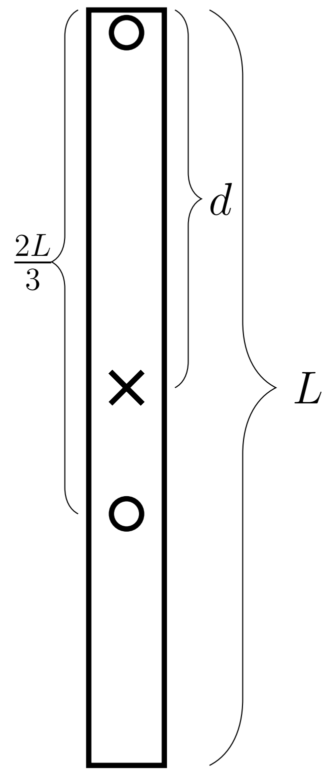
20.2.1.1 Moments of Inertia of an Aluminum Stick about Various Axes of Rotation

Insert a bolt into the aluminum sleeve then place the aluminum stick over the aluminum sleeve (make sure the aluminum stick is hanging from the hole at the end of the stick). Then clamp the bolt to a heavy ring stand. Hang a simple pendulum consisting of a washer tied to a string from the bolt. The length of the simple pendulum can be adjusted by making a few turns of the string around the bolt (see Figure 20.3a).

Set the aluminum stick oscillating at some small amplitude. Having viewed the oscillation of the stick, adjust the length of the simple pendulum so that its period of oscillation is the same as that of the stick. This may require several trials. In each trial set the two pendula into simultaneous motion by pushing them to the side with the flat portion of your hand. Record the length l of the equivalent simple pendulum and the length of the aluminum stick in Table 20.1.



(a) Aluminum Stick with Simple Pendulum



(b) Diagram of the Aluminum Stick

Figure 20.3: Aluminum Stick

Length of Equivalent Simple Pendulum (l) (cm)	
Length of Aluminum Stick (L) (cm)	

Table 20.1: Length of Equivalent Simple Pendulum

Question 1: Having determined the length to which you must adjust the simple pendulum in order too have it oscillate with the same period as the aluminum stick, determine the ratio of the length of the equivalent simple pendulum l to the length of the stick L. Hint: For example, if you find $l = 58$ cm and $L = 100$ cm, then $\frac{l}{L} = \frac{58}{100}$, or $l = 0.58L$.

Question 2: Use Equation 20.4 to determine the mathematical expression for the experimental moment of inertia of the stick about an axis through one of its ends. Express your answer in terms of the mass M of the stick and the length L of the stick (your answer will be in the form $I = aML^2$ where a is a decimal number). Draw and label any figures that may be required in the calculation.