FIGURE 256 The pressure of air leads to surprisingly large forces, especially for large objects that enclose a vacuum. This was regularly demonstrated in the years from 1654 onwards by Otto von Guericke with the help of his so-called *Magdeburg hemispheres* and, above all, the various vacuum pumps that he invented (© Deutsche Post, Otto-von-Guericke-Gesellschaft, Deutsche Fotothek).

hemispheres; enjoy performing the experiment yourself.

LAMINAR AND TURBULENT FLOW

Like all motion, fluid motion obeys energy conservation. For fluids in which no energy is transformed into heat, the conservation of energy is particularly simple. Motion that does not generate heat is motion without vortices; such fluid motion is called *laminar*. In the special case that, in addition, the speed of the fluid does not depend on time at all positions, the fluid motion is called *stationary*. Non-laminar flow is called *turbulent*. Figure 251 and Figure 257 show examples.

For motion that is both laminar and stationary, energy conservation can be expressed with the help of speed v and pressure p:

$$\tfrac{1}{2}\rho v^2 + p + \rho g h = \text{const} \tag{111}$$

where h is the height above ground, ρ is the mass density and $g = 9.8\,\mathrm{m/s^2}$ is the gravitational acceleration. This is called *Bernoulli's equation*.* In this equation, the last term is only important if the fluid rises against the ground. The first term is the kinetic energy

---

* Daniel Bernoulli (b. 1700 Bâle, d. 1782 Bâle), important mathematician and physicist. Also his father Jo-

FIGURE 257 Left: *non-stationary* and *stationary* laminar flows; right: an example of *turbulent* flow (© Martin Thum, Steve Butler).

(per volume) of the fluid, and the other two terms are potential energies (per volume). Indeed, the second term is the potential energy (per volume) resulting from the compression of the fluid. This is due to a further way to define pressure:

Challenge 590 e

▷ ***Pressure*** is potential energy per volume.

Energy conservation implies that the lower the pressure is, the larger the speed of a fluid becomes. We can use this relation to measure the speed of a stationary water flow in a tube. We just have to narrow the tube somewhat at one location along the tube, and then measure the pressure difference before and at the tube restriction. The speed v far from the constriction is then given as $v = k\sqrt{p_1 - p_2}$. (What is the constant k?) A device using this method is called a Venturi gauge.

Challenge 591 s

Now think about flowing water. If the geometry is kept fixed and the water speed is increased – or the relative speed of a body in water is increased – at a certain speed value, we observe a transition: the water loses its clarity, the flow is not stationary and not laminar any more. We can observe the transition whenever we open a water tap: at a certain speed, the flow changes from laminar to turbulent. From this point onward, Bernoulli's equation (111) is *not* valid any more.

A precise description of turbulence has not yet been achieved. This might be the toughest of all open problems in physics. When the young Werner Heisenberg was asked to continue research on turbulence, he refused – rightly so – saying it was too difficult; he

---

hann and his uncle Jakob were famous mathematicians, as were his brothers and some of his nephews. Daniel Bernoulli published many mathematical and physical results. In physics, he studied the separation of compound motion into translation and rotation. In 1738 he published the *Hydrodynamique*, in which he deduced all results from a single principle, namely the conservation of energy. The so-called *Bernoulli equation* states how the pressure of a fluid decreases when its speed increases. He studied the tides and many complex mechanical problems, and explained the Boyle–Mariotte gas 'law'. For his publications he won the prestigious prize of the French Academy of Sciences – a forerunner of the Nobel Prize – ten times.

FIGURE 258 The *moth* sailing class: a 30 kg boat that sails above the water using *hydrofoils*, i.e., underwater wings (© Bladerider International).

turned to something easier and he discovered and developed quantum theory instead. Turbulence is such a vast topic, with many of its concepts still not settled, that despite the number and importance of its applications, only now, at the beginning of the twenty-first century, are its secrets beginning to be unravelled. Ref. 273

It is thought that the equations of motion describing fluids in full generality, the so-called *Navier–Stokes equations*, are sufficient to understand turbulence.* But the mathematics behind these equations is mind-boggling. There is even a prize of one million dollars offered by the Clay Mathematics Institute for the completion of certain steps on the way to solving the equations.

Important systems which show laminar flow, vortices and turbulence at the same time are wings and sails. (See Figure 258.) All wings and sails work best in laminar mode. The essence of a wing is that it imparts air a downward velocity with as little turbulence as possible. (The aim to minimize turbulence is the reason that wings are curved. If the engine is very powerful, a flat wing at an angle also keeps an aeroplane in the air. However, the fuel consumption increases dramatically. On the other hand, strong turbulence is of advantage for landing safely.) Around a wing of a flying bird or aeroplane, the downward velocity of the trailing air leads to a centrifugal force acting on the air that passes above the wing. This leads to a lower pressure, and thus to lift. (Wings thus do *not* rely on the Bernoulli equation, where lower pressure *along* the flow leads to higher air speed, as unfortunately, many books used to say. Above a wing, the higher speed is related to lower pressure *across* the flow.) Ref. 274

The different speeds of the air above and below the wing lead to vortices at the end of every wing. These vortices are especially important for the take-off of any insect, bird and aeroplane. More aspects of wings are explored later on. Vol. V, page 278

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* They are named after Claude Navier (b. 1785 Dijon, d. 1836 Paris), important engineer and bridge builder, and Georges Gabriel Stokes (b. 1819 Skreen, d. 1903 Cambridge), important physicist and mathematician.

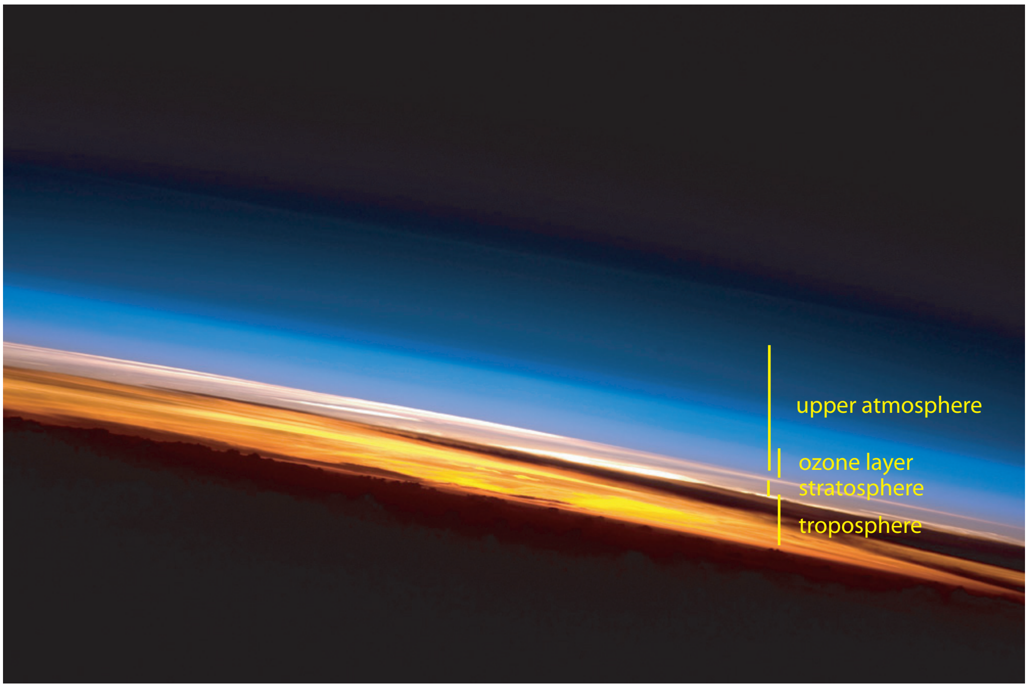

FIGURE 259 Several layers of the atmosphere are visble in this sunset photograph taken from the International Space Station, flying at several hundred km of altitude (courtesy NASA).

THE ATMOSPHERE

The atmosphere, a thin veil around our planet that is shown in Figure 259, keeps us alive. The atmosphere is a fluid that surrounds the Earth and consists of $5 \cdot 10^{18}$ kg of gas. The density decreases with height: 50 % of the mass is below 5.6 km of height, 75 % within 11 km, 90 % within 16 km and 99,999 97 % within 100 km.*

At sea level, the atmospheric density is, on average, $1.29\,\mathrm{kg/m^3}$ – about 1/800th of that of water – and the pressure is 101.3 kPa; both values decrease with altitude. The composition of the atmosphere at sea level is given on page 516. Also the composition varies with altitude; furthermore, the composition depends on the weather and on the pollution level.

The structure of the atmosphere is given in Table 45. The atmosphere ceases to behave as a gas above the thermopause, somewhere between 500 and 1000 km; above that altitude, there are no atomic collisions any more. In fact, we could argue that the atmosphere ceases to behave as an everyday gas above 150 km, when no audible sound is transmitted any more, not even at 20 Hz, due to the low atomic density.

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* The last height is called the *Kármán line*; it is the conventional height at which a flying system cannot use lift to fly any more, so that it is often used as boundary between aeronautics and astronautics.

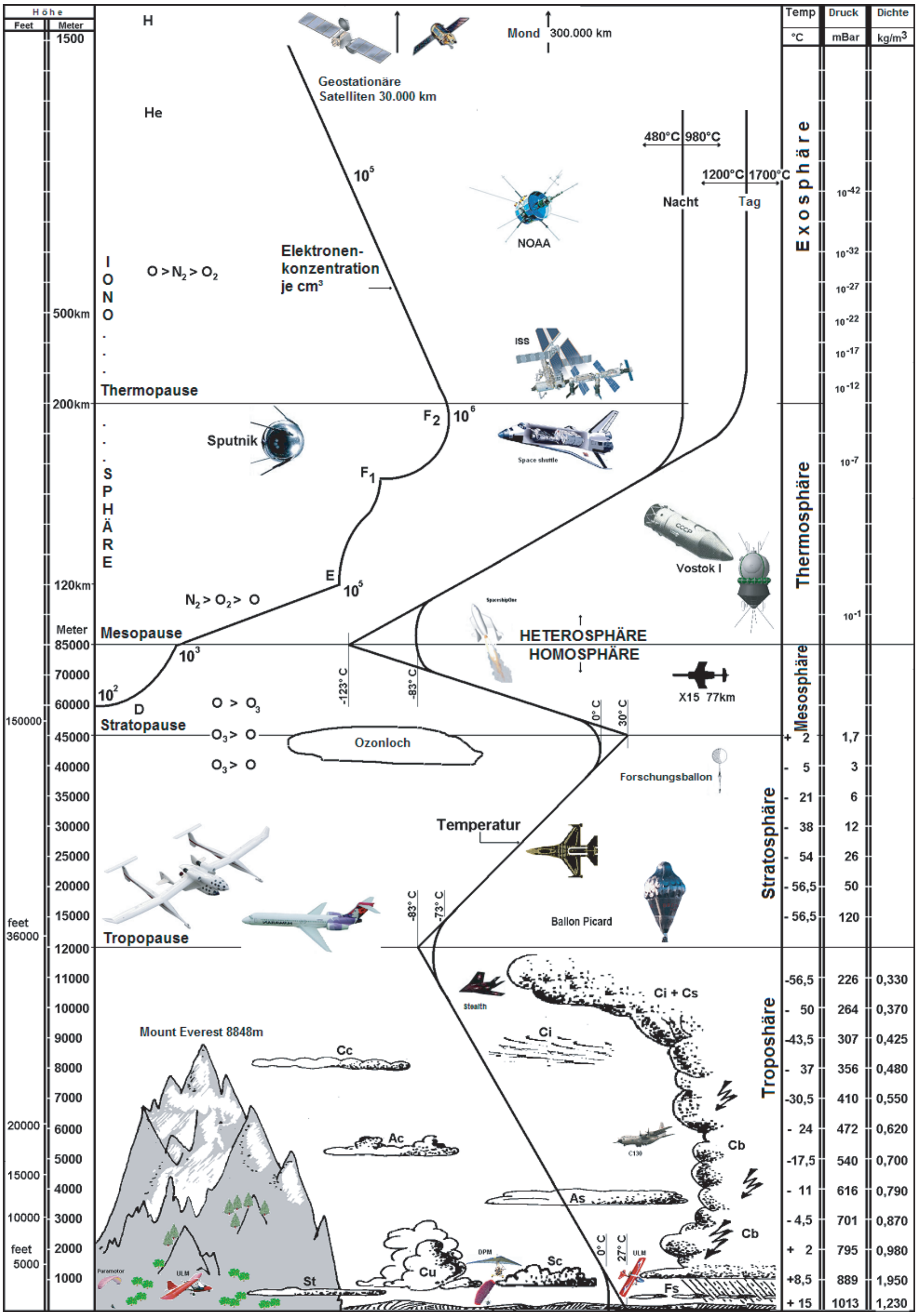

FIGURE 260 The main layers of the atmosphere (© Sebman81).

TABLE 45 The layers of the atmosphere.

Layer	Altitude	Details
Exosphere	> 500 to about 10 000 km	mainly composed of hydrogen and helium, includes the *magnetosphere*, temperature above 1000°C, contains many artificial satellites and sometimes aurora phenomena, includes, at its top, the luminous *geocorona*
Boundary: *thermopause* or *exobase*	between 500 and 1000 km	above: no 'gas' properties, no atomic collisions; below: gas properties, friction for satellites; altitude varies with solar activity
Thermosphere	from 85 km to thermopause	composed of oxygen, helium, hydrogen and ions, temperature of up to 2500°C, pressure 1 to 10 µPa; infrasound speed around 1000 m/s; no transmission of sound above 20 Hz at altitudes above 150 km; contains the International Space Station and many satellites; featured the Sputnik and the Space Shuttle
Heterosphere	all above turbopause	separate concept that includes all layers that show diffusive mixing, i.e., most of the thermosphere and the exosphere
Boundary: *turbopause* or *homopause*	100 km	boundary between diffusive mixing (above) and turbulent mixing (below)
Homosphere	everything below turbopause	separate concept that includes the lowest part of the thermosphere and all layers below it
Boundary: *mesopause*	85 km	temperature between −100°C and −85°C, lowest temperature 'on' Earth; temperature depends on season; contains ions, includes a sodium layer that is used to make guide stars for telescopes
Mesosphere	from stratopause to mesopause	temperature decreases with altitude, mostly hydrogen, contains noctilucent clouds, sprites, elves, ions; burns most meteors, shows atmospheric tides and a circulation from summer to winter pole
Ionosphere or *magnetosphere*	60 km to 1000 km	a separate concept that includes all layers that contain ions, thus the exosphere, the thermosphere and a large part of the mesosphere
Boundary: *stratopause* (or mesopeak)	50 to 55 km	maximum temperature between stratosphere and mesosphere; pressure around 100 Pa, temperature −15°C to −3°C
Stratosphere	up to the stratopause	stratified, no weather phenomena, temperature increases with altitude, dry, shows quasi-biennial oscillations, contains the *ozone layer* in its lowest 20 km, as well as aeroplanes and some balloons

TABLE 45 (Continued) The layers of the atmosphere.

Layer	Altitude	Details
Boundary: *tropopause*	6 to 9 km at the poles, 17 to 20 km at the equator	temperature −50 °C, temperature gradient vanishes, no water any more
Troposphere	up to the tropopause	contains water and shows weather phenomena; contains life, mountains and aeroplanes; makes stars flicker; temperature generally decreases with altitude; speed of sound is around 340 m/s
Boundary: *planetary boundary layer* or *peplosphere*	0.2 to 2 km	part of the troposphere that is influenced by friction with the Earth's surface; thickness depends on landscape and time of day

The physics of blood and breath

Fluid motion is of vital importance. There are at least four fluid circulation systems inside the human body. First, *blood* flows through the blood system by the heart. Second, *air* is circulated inside the lungs by the diaphragm and other chest muscles. Third, *lymph* flows through the lymphatic vessels, moved passively by body muscles. Fourth, the *cerebrospinal fluid* circulates around the brain and the spine, moved by motions of the head. For this reason, medical doctors like the simple statement: every illness is ultimately due to bad circulation.

Challenge 592 e

Why do living beings have circulation systems? Circulation is necessary because diffusion is too slow. Can you detail the argument? We now explore the two main circulation systems in the human body.

Blood keeps us alive: it transports most chemicals required for our metabolism to and from the various parts of our body. The flow of blood is almost always laminar; turbulence only exists in the venae cavae, near the heart. The heart pumps around 80 ml of blood per heartbeat, about 5 l/min. At rest, a heartbeat consumes about 1.2 J. The consumption is sizeable, because the dynamic viscosity of blood ranges between $3.5 \cdot 10^{-3}$ Pa s (3.5 times higher than water) and 10^{-2} Pa s, depending on the diameter of the blood vessel; it is highest in the tiny capillaries. The speed of the blood is highest in the aorta, where it flows with 0.5 m/s, and lowest in the capillaries, where it is as low as 0.3 mm/s. As a result, a substance injected in the arm arrives in the feet between 20 and 60 s after the injection.

Challenge 593 ny

In fact, all animals have similar blood circulation speeds, usually between 0.2 m/s and 0.4 m/s. Why?

To achieve blood circulation, the heart produces a (systolic) pressure of about 16 kPa, corresponding to a height of about 1.6 m of blood. This value is needed by the heart to pump blood through the brain. When the heart relaxes, the elasticity of the arteries keeps the (diastolic) pressure at around 10 kPa. These values are measured at the height of the heart.* The values vary greatly with the position and body orientation at which they are

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* The blood pressure values measured on the two upper arms also differ; for right-handed people, the pressure in the right arm is higher.

measured: the systolic pressure at the feet of a standing adult reaches 30 kPa, whereas it is 16 kPa in the feet of a lying person. For a standing human, the pressure in the veins in the foot is 18 kPa, larger than the systolic pressure in the heart. The high pressure values in the feet and legs are one of the reasons that lead to varicose veins. Nature uses many tricks to avoid problems with blood circulation in the legs. Humans leg veins have *valves* to prevent the blood from flowing downwards; giraffes have extremely thin legs with strong and tight skin in the legs for the same reason. The same happens for other large animals.

At the end of the capillaries, the pressure is only around 2 kPa. The lowest blood pressure is found in veins that lead back from the head to the heart, where the pressure can even be slightly negative. Because of blood pressure, when a patient receives an (intravenous) *infusion*, the bag must have a minimum height above the infusion point where the needle enters the body; values of about 0.8 to 1 m cause no trouble. (Is the height difference also needed for person-to-person transfusions of blood?) Since arteries have higher blood pressure, for the more rare arterial infusions, hospitals usually use arterial pumps, to avoid the need for unpractical heights of 2 m or more.

Ref. 275 Recent research has demonstrated what was suspected for a long time: in the capillaries, the red blood cells change shape and motion. The shape change depends on the capillary diameter and the flow speed. In large vessels, red blood cells usually tumble in the bloodstream. In smaller blood vessels, they roll and in still smaller ones they deform in various ways. These changes explain how blood flows more easily, i.e., with lower viscosity, in thinner vessels, It is even conjectured that disturbing these shape changes might be related to specific symptoms and illnesses.

The physics of breathing is equally interesting. A human cannot breathe at any depth under water, even if he has a tube going to the surface, as shown in Figure 261. At a few metres of depth, trying to do so is inevitably *fatal*! Even at a depth of 50 cm only, the human body can only breathe in this way for a few minutes and can get badly hurt for life. Why? Challenge 594 s

Inside the lungs, the gas exchange with the blood occurs in around 300 millions of little spheres, the *alveoli*, with a diameter between 0.2 and 0.6 mm. They are shown in Figure 245. To avoid that the large ones grow and the small ones collapse – as in the experiment of Figure 276– the alveoli are covered with a phospholipid surfactant that reduces their surface tension. In newborns, the small radius of the alveoli and the low level of surfactant is the reason that the first breaths, and sometimes also the subsequent ones, require a large effort.

Page 348

Page 389

We need around 2 % of our energy for breathing alone. The speed of air in the throat is 3 m/s for normal breathing; when coughing, it can be as high as 50 m/s. The flow of air in the bronchi is turbulent; the noise can be heard in a quiet environment. In normal breathing, the breathing muscles, in the thorax and in the belly, exchange 0.5 l of air; in a deep breath, the volume can reach 4 l.

Breathing is especially tricky in unusual situations. After scuba diving* at larger depths than a few meters for more than a few minutes, it is important to rise slowly, to

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* Originally, 'scuba' is the abbreviation of 'self-contained underwater breathing apparatus'. The central device in it, the 'aqua lung', was invented by Emile Gagnan and Jacques Cousteau; it keeps the air pressure always at the same level as the water pressure.

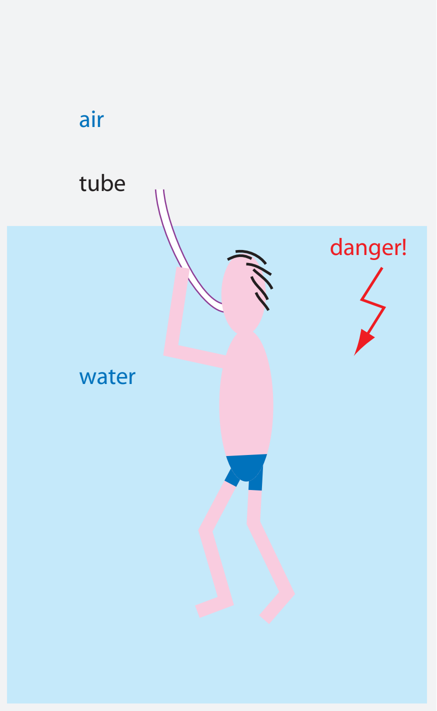

FIGURE 261 Attention, danger! Trying to do this will destroy your lung irreversibly and possibly kill you.

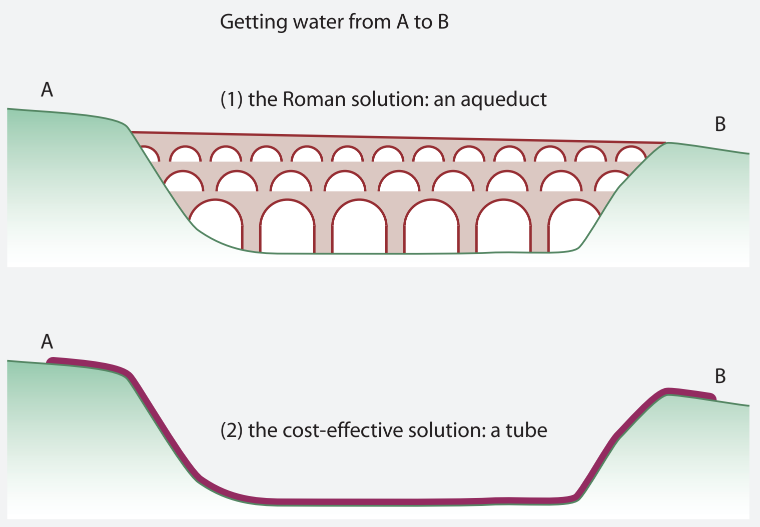

FIGURE 262 Wasting money because of lack of knowledge about fluids.

Challenge 595 e

avoid a potentially fatal embolism. Why? The same can happen to participants in high-altitude flights with balloons or aeroplanes, to high-altitude parachutists and to cosmonauts.

Curiosities and fun challenges about fluids

What happens if people do not know the rules of nature? The answer is the same since 2000 years ago: taxpayers' money is wasted or health is in danger. One of the oldest examples, the aqueducts from Roman time, is shown in Figure 262. Aqueducts only exist because Romans did not know how fluids move. The figure tells why there are no aqueducts any more.

We note that using a 1 or 2 m water hose in the way shown in Figure 262 or in Figure 255 to transport gasoline can be dangerous. Why? Challenge 596 s

* *

Take an empty milk carton, and make a hole on one side, 1 cm above the bottom. Then make two holes above it, each 5 cm above the previous one. If you fill the carton with water and put it on a table, which of the three streams will reach further away? And if you put the carton on the edge of the table, so that the streams fall down on the floor? Challenge 597 e

* *

Your bathtub is full of water. You have an unmarked 3-litre container and an unmarked 5-litre container. How can you get 4 litres of water from the bathtub? Challenge 598 e

* *

What is the easiest way to create a supersonic jet of air? Simply drop a billiard ball into a bucket full of water. It took a long time to discover this simple method. Enjoy researching the topic. Ref. 276

* *

Fluids are important for motion. Spiders have muscles to flex their legs, but no muscles to extend them. How do they extend their legs?

In 1944, Ellis discovered that spiders extend their legs by hydraulic means: they increase the pressure of a fluid inside their legs; this pressure stretches the leg like the water pressure stiffens a garden hose. If you prefer, spider legs thus work a bit like the arm of an excavator. That is why spiders have bent legs when they are dead. The fluid mechanism works well: it is also used by jumping spiders. Ref. 277

* *

Where did the water in the oceans – whose amount is illustrated in Figure 263 – come from? Interestingly enough, this question is not fully settled! In the early age of the Earth, the high temperatures made all water evaporate and escape into space. So where did today's water come from? (For example, could the hydrogen come from the radioactivity of the Earth's core?) The most plausible proposal is that the water comes from comets. Comets are made, to a large degree, of ice. Comets hitting the Earth in the distant past seem to have formed the oceans. In 2011, it was shown for the first time, by the Herschel infrared space telescope of the European Space Agency, that comets from the Kuiper belt – in contrast to comets from the inner Solar System – have ice of the same oxygen isotope composition as the Earth's oceans. The comet origin of oceans seems settled. Ref. 278

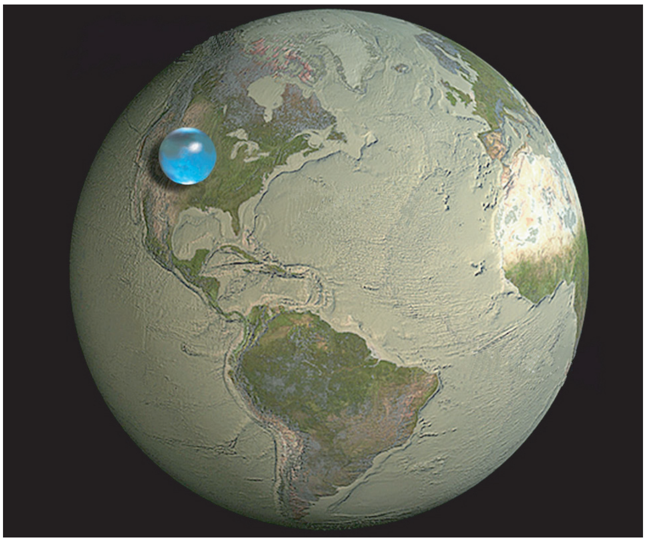

FIGURE 263 All the water on Earth would form a sphere with a radius of about 700 km, as illustrated in this computer-generated graphic. (© Jack Cook, Adam Nieman, Woods Hole Oceanographic Institution, Howard Perlman, USGS)

* *

The physics of underwater diving, in particular the physics of apnoea diving, is full of wonders and of effects that are not yet understood. For example, every apnoea champion knows that it is quite hard to hold your breath for five or six minutes while sitting in a chair. But if the same is done in a swimming pool, the feat becomes readily achievable for modern apnoea champions. It is still not fully clear why this is the case. Ref. 279

There are many apnoea diving disciplines. In 2009, the no-limit apnoea diving record is at the incredible depth of 214 m, achieved by Herbert Nitsch. The record static apnoea time is over eleven minutes, and, with hyperventilation with pure oxygen, over 22 minutes. The dynamic apnoea record, without fins, is 213 m.

When an apnoea diver reaches a depth of 100 m, the water pressure corresponds to a weight of over 11 kg on each square centimetre of his skin. To avoid the problems of ear pressure compensation at great depths, a diver has to flood the mouth and the trachea with water. His lungs have shrunk to one eleventh of their original size, to the size of apples. The water pressure shifts almost all blood from the legs and arms into the thorax and the brain. At 150 m, there is no light, and no sound – only the heartbeat. And the heartbeat is slow: there is only a beat every seven or eight seconds. He becomes relaxed and euphoric at the same time. None of these fascinating observations is fully understood.

Sperm whales, *Physeter macrocephalus*, can stay below water more than half an hour, and dive to a depth of more than 3000 m. Weddell seals, *Leptonychotes weddellii*, can stay below water for an hour and a half. The mechanisms are unclear, but seem to involve haemoglobine and neuroglobine. The research into the involved mechanisms is interesting because it is observed that diving capability strengthens the brain. For example, bowhead whales, *Balaena mysticetus*, do not suffer strokes nor brain degeneration, even though they reach over 200 years in age.

* *

Apnoea records show the beneficial effects of oxygen on human health. An *oxygen bottle* is therefore a common item in professional first aid medical equipment.

* *

What is the speed record for motion underwater? Probably only few people know: it is a military secret. In fact, the answer needs to be split into two. The fastest published speed for a *projectile* underwater, almost fully enclosed in a gas bubble, is 1550 m/s, faster than the speed of sound in water, achieved over a distance of a few metres in a military laboratory in the 1990s. The fastest system *with an engine* seems to be a torpedo, also moving mainly in a gas bubble, that reaches over 120 m/s, thus faster than any formula 1 racing car. The exact speed achieved is higher and secret, as the method of enclosing objects underwater in gas bubbles, called *supercavitation*, is a research topic of military engineers all over the world.

The fastest fish, the sailfish *Istiophorus platypterus*, reaches 22 m/s, but speeds up to 30 m/s are suspected. Underwater, the fastest *manned objects* are military submarines, whose speeds are secret, but believed to be around 21 m/s. (All military naval engineers in this world, with the enormous budgets they have, are not able to make submarines that are faster than fish. The reason that aeroplanes are faster than birds is evident: aeroplanes were not developed by military engineers, but by civilian engineers.) The fastest human-powered submarines reach around 4 m/s. We can estimate that if human-powered submarine developers had the same development budget as military engineers, their machines would probably be faster than nuclear submarines.

There are no record lists for swimming under water. Underwater swimming is known to be faster than above-water breast stroke, backstroke or dolphin stroke: that is the reason that swimming underwater over long distances is forbidden in competitions in these styles. However, it is not known whether crawl-style records are faster or slower than records for the fastest swimming style below water. Which one is faster in your own case?

Challenge 599 e

* *

How much water is necessary to moisten the air in a room in winter? At 0°C, the vapour pressure of water is 6 mbar, 20°C it is 23 mbar. As a result, heating air in the winter gives at most a humidity of 25 %. To increase the humidity by 50 %, about 1 litre of water per 100 m^3 is needed.

Challenge 600 e

* *

Surface tension can be dangerous. A man coming out of a swimming pool is wet. He carries about half a kilogram of water on his skin. In contrast, a wet insect, such as a house fly, carries many times its own weight. It is unable to fly and usually dies. Therefore, most insects stay away from water as much as they can – or at least use a long proboscis.

* *

The human heart pumps blood at a rate of about 0.1 l/s. A typical *capillary* has the diameter of a red blood cell, around 7 μm, and in it, the blood moves at a speed of half a millimetre per second. How many capillaries are there in a human?

Challenge 601 s

* *

You are in a boat on a pond with a stone, a bucket of water and a piece of wood. What happens to the water level of the pond after you throw the stone in it? After you throw the water into the pond? After you throw the piece of wood? Challenge 602 s

* *

Challenge 603 s A ship leaves a river and enters the sea. What happens?

* *

Put a rubber air balloon over the end of a bottle and let it hang inside the bottle. How much can you blow up the balloon inside the bottle? Challenge 604 e

* *

Put a rubber helium balloon in your car. You accelerate and drive around bends. In which direction does the balloon move? Challenge 605 s

* *

Put a small paper ball into the neck of a horizontal bottle and try to blow it into the bottle. The paper will fly *towards* you. Why? Challenge 606 e

* *

It is possible to blow an egg from one egg cup to a second one just behind it. Can you perform this trick? Challenge 607 e

* *

In the seventeenth century, engineers who needed to pump water faced a challenge. To pump water from mine shafts to the surface, no water pump managed more than 10 m of height difference. For twice that height, one always needed two pumps in series, connected by an intermediate reservoir. Why? How then do trees manage to pump water upwards for larger heights? Challenge 608 s

* *

When hydrogen and oxygen are combined to form water, the amount of hydrogen needed is exactly twice the amount of oxygen, if no gas is to be left over after the reaction. How does this observation confirm the existence of atoms? Challenge 609 s

* *

Challenge 610 s How are alcohol-filled chocolate pralines made? Note that the alcohol is not injected into them afterwards, because there would be no way to keep the result tight enough.

* *

How often can a stone jump when it is thrown over the surface of water? The present world record was achieved in 2002: 40 jumps. More information is known about the previous world record, achieved in 1992: a palm-sized, triangular and flat stone was thrown Ref. 280 with a speed of 12 m/s (others say 20 m/s) and a rotation speed of about 14 revolutions per second along a river, covering about 100 m with 38 jumps. (The sequence was filmed

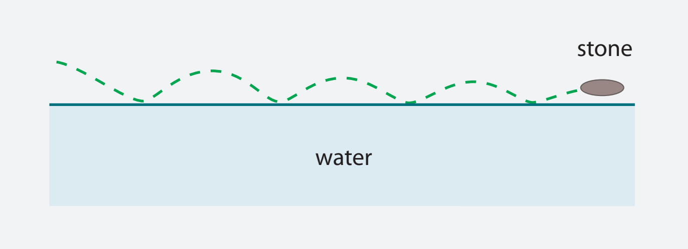

FIGURE 264 What is your personal stone-skipping record?

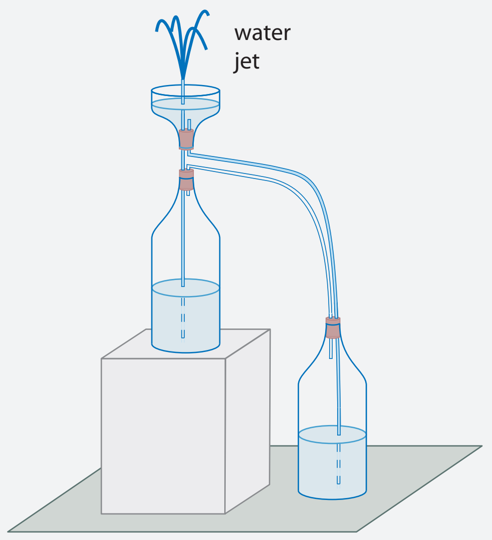

FIGURE 265 Heron’s fountain in operation.

with a video recorder from a bridge.)

What would be necessary to increase the number of jumps? Can you build a machine that is a better thrower than yourself? Challenge 611 r

* *

The most abundant component of air is nitrogen (about 78 %). The second component is oxygen (about 21 %). What is the third one? Challenge 612 s

* *

Which everyday system has a pressure lower than that of the atmosphere and usually kills a person if the pressure is raised to the usual atmospheric value? Challenge 613 s

* *

Water can flow uphill: Heron’s fountain shows this most clearly. Heron of Alexandria (*c.* 10 to *c.* 70) described it 2000 years ago; it is easily built at home, using some plastic bottles and a little tubing. How does it work? How is it started? Challenge 614 s

* *

A light bulb is placed, underwater, in a stable steel cylinder with a diameter of 16 cm. An original Fiat Cinquecento car (500 kg) is placed on a piston pushing onto the water surface. Will the bulb resist? Challenge 615 s

* *

Challenge 616 s What is the most dense gas? The most dense vapour?

* *

Every year, the Institute of Maritime Systems of the University of Rostock organizes a contest. The challenge is to build a paper boat with the highest carrying capacity. The paper boat must weigh at most 10 g and fulfil a few additional conditions; the carrying capacity is measured by pouring small lead shot onto it, until the boat sinks. The 2008 record stands at 5.1 kg. Can you achieve this value? (For more information, see the www.paperboat.de website.) Challenge 617 e

* *

Challenge 618 s Is it possible to use the wind to move against the wind, head-on?

* *

Measuring wind speed is an important task. Two methods allow measuring the wind speed at an altitude of about 100 m above the ground: *sodar*, i.e., sound detection and ranging, and *lidar*, i.e., light detection and ranging. Two typical devices are shown in Figure 266. Sodar works also for clear air, whereas lidar needs aerosols.

* *

A modern version of an old question – originally posed by the physicist Daniel Colladon (b. 1802 Geneva, d. 1893 Geneva) – is the following version by Yakov Perelman. A ship of mass m in a river is pulled by horses walking along the river bank attached by ropes. If the river is of superfluid helium, meaning that there is no friction between ship and river, what energy is necessary to pull the ship upstream along the river until a height h has been gained? Challenge 619 s

* *

An urban legend pretends that at the bottom of large waterfalls there is not enough air to breathe. Why is this wrong? Challenge 620 e

* *

The Swiss physicist and inventor Auguste Piccard (b. 1884 Basel, d. 1962 Lausanne) was a famous explorer. Among others, he explored the stratosphere: he reached the record height of 16 km in his *aerostat*, a hydrogen gas balloon. Inside the airtight cabin hanging under his balloon, he had normal air pressure. However, he needed to introduce several ropes attached at the balloon into the cabin, in order to be able to pull and release them, as they controlled his balloon. How did he get the ropes into the cabin while at the same time preventing air from leaving? Challenge 621 s

* *

FIGURE 266 Two wind measuring systems: a sodar system and a lidar system (© AQSystems, Leosphere).

A human in air falls with a limiting speed of about 50 m/s (the precise value depends on clothing). How long does it take to fall from a plane at 3000 m down to a height of 200 m? Challenge 622 e

* *

To get an idea of the size of Avogadro's and Loschmidt's number, two questions are usually asked. First, on average, how many molecules or atoms that you breathe in with every breath have previously been exhaled by Caesar? Second, on average, how many atoms of Jesus do you eat every day? Even though the Earth is large, the resulting numbers are still telling. Challenge 623 s

* *

A few drops of tea usually flow along the underside of the spout of a teapot (or fall onto the table). This phenomenon has even been simulated using supercomputer simula-

FIGURE 267 A water droplet on a pan: an example of the Leidenfrost effect (© Kenji Lopez-Alt).

Ref. 281 tions of the motion of liquids, by Kistler and Scriven, using the Navier–Stokes equations. Teapots are still shedding drops, though.

* *

The best giant soap bubbles can be made by mixing 1.5 l of water, 200 ml of corn syrup and 450 ml of washing-up liquid. Mix everything together and then let it rest for four hours. You can then make the largest bubbles by dipping a metal ring of up to 100 mm diameter into the mixture. But why do soap bubbles burst? Challenge 624 s

* *

A drop of water that falls into a pan containing moderately hot oil evaporates immediately. However, if the oil is really hot, i.e., above 210 °C, the water droplet dances on the oil surface for a considerable time. Cooks test the temperature of oil in this way. Why does this so-called *Leidenfrost effect* take place? Challenge 625 s The effect is named after the theologian and physician Johann Gottlob Leidenfrost (b. 1715 Rosperwenda, d. 1794 Duisburg). For an instructive and impressive demonstration of the Leidenfrost effect with water droplets, see the video featured at www.thisiscolossal.com/2014/03/water-maze/. The video also shows water droplets running uphill and running through a maze.

The Leidenfrost effect also allows one to plunge the bare hand into molten lead or liquid nitrogen, to keep liquid nitrogen in one's mouth, to check whether a pressing iron is hot, or to walk over hot coal – if one follows several safety rules, as explained by Jearl Walker. Ref. 282 (Do *not* try this yourself! Many things can go wrong.) The main condition is that the hand, the mouth or the feet must be wet. Walker lost two teeth in a demonstration and badly burned his feet in a walk when the condition was not met. You can see some videos of the effect for a hand in liquid nitrogen on www.popsci.com/diy/article/2010-08/cool-hand-theo and for a finger in molten lead on www.popsci.com/science/

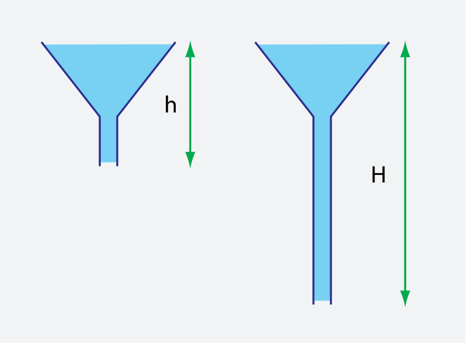

FIGURE 268 Which funnel empties more rapidly?

article/2012-02/our-columnist-tests-his-trust-science-dipping-his-finger-molten-lead.

* *

Challenge 626 s Why don't air molecules fall towards the bottom of the container and stay there?

* *

Challenge 627 s Ref. 283 Which of the two water funnels in Figure 268 is emptied more rapidly? Apply energy conservation to the fluid's motion (the Bernoulli equation) to find the answer.

* *

Challenge 628 s As we have seen, fast flow generates an underpressure. How do fish prevent their eyes from popping when they swim rapidly?

* *

Challenge 629 ny Golf balls have dimples for the same reasons that tennis balls are hairy and that shark and dolphin skin is not flat: deviations from flatness reduce the flow resistance because many small eddies produce less friction than a few large ones. Why?

* *

Challenge 630 s The recognized record height reached by a helicopter is 12 442 m above sea level, though 12 954 m has also been claimed. (The first height was reached in 1972, the second in 2002, both by French pilots in French helicopters.) Why, then, do people still continue to use their legs in order to reach the top of Mount Sagarmatha, the highest mountain in the world?

* *

Challenge 631 e A loosely knotted sewing thread lies on the surface of a bowl filled with water. Putting a bit of washing-up liquid into the area surrounded by the thread makes it immediately become circular. Why?

* *

Challenge 632 s How can you put a handkerchief under water using a glass, while keeping it dry?

* *

Are you able to blow a ping-pong ball out of a funnel? What happens if you blow through a funnel towards a burning candle?

* *

The fall of a leaf, with its complex path, is still a topic of investigation. We are far from being able to predict the time a leaf will take to reach the ground; the motion of the air around a leaf is not easy to describe. One of the simplest phenomena of hydrodynamics remains one of its most difficult problems.

* *

Fluids exhibit many interesting effects. Soap bubbles in air are made of a thin spherical film of liquid with air on both sides. In 1932, anti-bubbles, thin spherical films of air Ref. 284 with liquid on both sides, were first observed. In 2004, the Belgian physicist Stéphane Dorbolo and his team showed that it is possible to produce them in simple experiments, and in particular, in Belgian beer.

* *

Have you ever dropped a Mentos candy into a Diet Coca Cola bottle? You will get an Challenge 633 e interesting effect. (Do it at your own risk...) Is it possible to build a rocket in this way?

* *

Challenge 634 e A needle can swim on water, if you put it there carefully. Just try, using a fork. Why does it float?

* *

The Rhine emits about 2 300 m^3/s of water into the North Sea, the Amazon River about Challenge 635 e 120 000 m^3/s into the Atlantic. How much is this less than $c^3/4G$?

* *

Fluids exhibit many complex motions. To see an overview, have a look at the beautiful Challenge 636 e collection on the website serve.me.nus.edu.sg/limtt. Among fluid motion, vortex rings, as emitted by smokers or volcanoes, have often triggered the imagination. (See Figure 269.) One of the most famous examples of fluid motion is the leapfrogging of vortex rings, shown in Figure 270. Lim Tee Tai explains that more than two leapfrogs are extremely hard to achieve, because the slightest vortex ring misalignment leads to the collapse of Ref. 285 the system.

* *

A surprising effect can be observed when pouring *shampoo* on a plate: sometimes a thin stream is ejected from the region where the shampoo hits the plate. This so-called *Kaye effect* is best enjoyed in the beautiful movie produced by the University of Twente found on the youtube.com/watch?v=GX4_3cV_3Mw website.

* *

Challenge 637 e Most mammals take around 30 seconds to urinate. Can you find out why?

FIGURE 269 A smoke ring, around 100 m in size, ejected from Mt. Etna's Bocca Nova in 2000 (© Daniela Szczepanski at www.vulkanarchiv.de and www.vulkane.net).

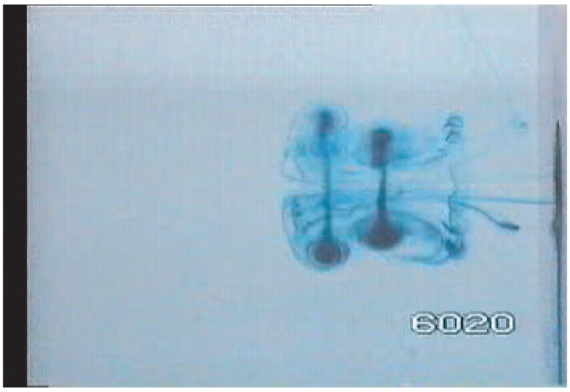

FIGURE 270 Two leapfrogging vortex rings. (QuickTime film © Lim Tee Tai)

* *

Aeroplanes toilets are dangerous places. In the 1990s, a fat person sat on the toilet seat and pushed the 'flush' button while sitting. (Never try this yourself.) The underpressure exerted by the toilet was so strong that it pulled out the intestine and the person had to be brought into hospital. (Everything ended well, by the way.)

* *

If one surrounds water droplets with the correct type of dust, the droplets can roll along inclined planes. They can roll with a speed of up to 1 m/s, whereas on the same surface, water would flow a hundred times more slowly. When the droplets get too fast, they become flat discs; at even higher speeds, they get a doughnut shape. Such droplets can even jump and swim. Ref. 286

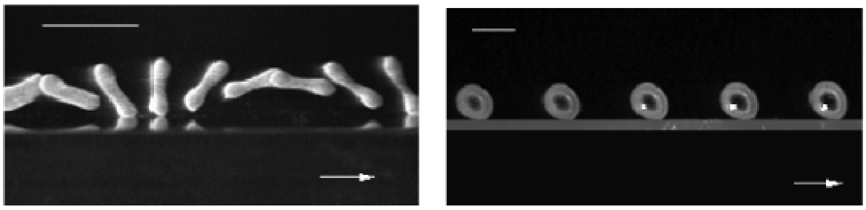

FIGURE 271 Water droplets covered with pollen rolling over inclined planes at 35 degrees (inclination not shown) have unexpected shapes. The grey lines have a length of 1 cm; the spherical droplets had initial radii of 1.3 mm and 2.5 mm, respectively, and the photographs were taken at 9 ms and 23 ms time intervals (© David Quéré).

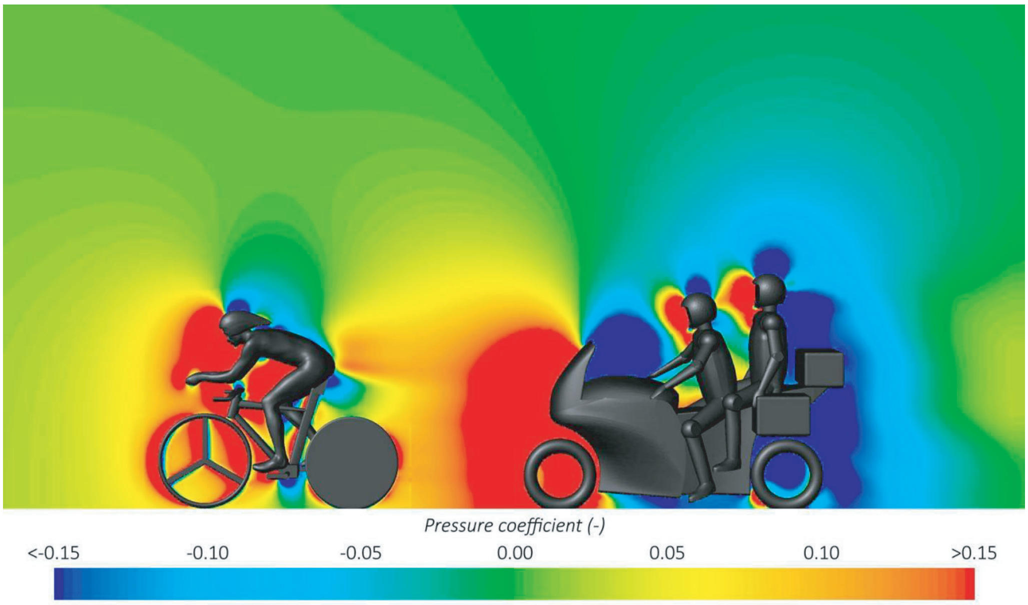

FIGURE 272 A computer visualization of the pressure regions surrounding a racing bicycle and a motorcycle following it (© Technical University Eindhoven).

* *

It is well known that it is easier to ride a bicycle behind a truck, a car or a motorcycle, because wind drag is lower in such situations. Indeed, the cycling speed record, achieved by Fred Rompelberg in 1990, is 74.7 m/s, or 268.8 km/h, and was achieved with a car driving in front of the bicycle. It is more surprising that a motorcycle driving *behind* a bicycle also reduces the drag, up to nine per cent. In 2016, Bert Blocken and his team confirmed this effect in the wind tunnel and with numeric simulations. The effect is thus sufficient to determine the winner of racing prologues or time trials. In fact, three motorcycles riding in a row behind the bicycle can reduce the drag for the bicycle riding in front by up to 14 per cent. The effect occurs because the following motorcycles cause a reduction of the low-pressure region – shown in red in Figure 272 – behind the rider; Ref. 298

this reduces the aerodynamic drag for the bicycle rider.

* *

All fluids can sustain vortices. Forexample, in air there are cyclones, tornados and sea hoses. But vortices also exist in the sea. They have a size between 100 m and 10 km; their short life last from a few hours up to a day. Sea vortices mix the water layers in the sea and they transport heat, nutrients and seaweed. Research on their properties and effects is just at its beginning.

Summary on fluids

The motion of a fluid is the motion of its constituent particles. The motion of fluids allows for swimming, flying, breathing, blood circulation, vortices and turbulence. Fluid motion can be *laminar* or *turbulent*. Laminar flow that lacks any internal friction is described by Bernoulli's equation, i.e., by energy conservation. Laminar flow with internal friction is beyond the scope of this text; so is turbulent flow. The exact description of turbulent fluid motion is the most complicated problem of physics and not yet fully solved.

CHAPTER 13

ON HEAT AND MOTION REVERSAL INVARIANCE

Spilled milk never returns into its container by itself. Any hot object, left alone, starts to cool down with time; it never heats up. These and many other observations show that numerous processes in nature are *irreversible*. On the other hand, for every a stone that flies along a path, it is possible to throw a stone that follows the path in the reverse direction. The motion of stones is *reversible*. Further observations show that irreversibility is only found in systems composed of a *many* particles, and that all irreversible systems involve *heat*.

Our everyday experience, including human warmth, ovens and stoves, shows that heat flows. *Heat moves.* But the lack of reversibility also shows that heat moves in a special way. Since heat appears in many-particle systems, we are led to explore the next global approach for the description of motion: *statistical physics*. Statistical physics, which includes *thermodynamics*, the study of heat and temperature, explains the origin of many material properties, and also the observed irreversibility of heat flow.

Does irreversibility mean that motion, at a fundamental level, is not invariant under reversal, as Nobel Prize winner Ilya Prigogine, one of the fathers of self-organization, thought? In this chapter, we show that despite his other achievements, he was wrong.[**] To deduce this result, we first need to know the basic facts about temperature and heat;
Ref. 287 then we discuss irreversibility and motion reversal.

TEMPERATURE

Macroscopic bodies, i.e., bodies made of many atoms, have *temperature*. Only bodies made of few atoms do not have a temperature. Ovens have high temperature, and refrigerators low temperature. Temperature changes have important effects: matter changes from solid to liquid to gaseous to plasma state. With a change in temperature, matter also changes size, colour, magnetic properties, stiffness and many other properties.

Temperature is an aspect of the *state* of a body. In other words, two otherwise identical bodies can be characterized and distinguished by their temperature. This is well-known to criminal organizations around the world that rig lotteries. When a blind-folded child is asked to draw a numbered ball from a set of such balls, such as in Figure 274, it is often told beforehand to draw only *hot* or *cold* balls. The blindfolding also helps to hide the

---

[**] Many even less serious thinkers often ask the question in the following term: is motion *time-invariant*? The cheap press goes even further, and asks whether motion has an 'arrow' or whether time has a preferred
Page 48 'direction of flow'. We have already shown above that this is nonsense. We steer clear of such phrases in the following.

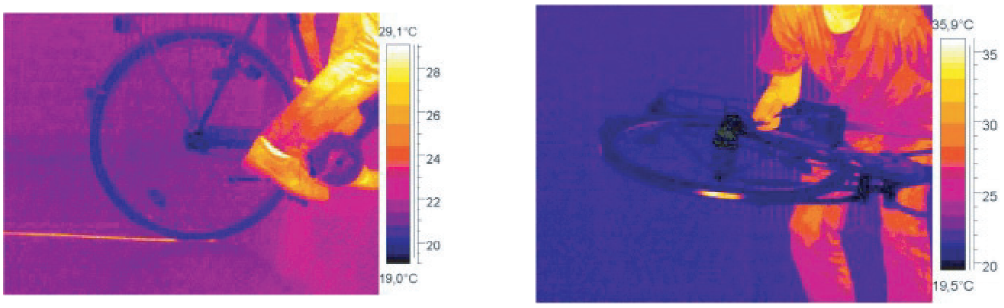

FIGURE 273 Braking generates heat on the floor and in the tire (© Klaus-Peter Möllmann and Michael Vollmer).

FIGURE 274 A rigged lottery shows that temperature is an aspect of the state of a body (© ISTA).

Ref. 289 tears due to the pain.

The temperature of a macroscopic body is an aspect of its state. In particular, the temperature is an *intensive* quantity or variable. In short, temperature describes the intensity of heat flow. An overview of measured temperature values is given in Table 46.

We observe that any two bodies in contact tend towards the *same* temperature: temperature is *contagious*. In other words, temperature describes a situation of *equilibrium*. Temperature thus behaves like pressure and any other intensive variable: it is the same for all parts of a system. We call *heating* the increase of temperature, and *cooling* its decrease.

How is temperature measured? The eighteenth century produced the clearest answer: temperature is best defined and measured by the *expansion of gases*. For the simplest, so-called *ideal* gases, the product of pressure p and volume V is proportional to temperature:

$$pV \sim T \;. \qquad (112)$$

The proportionality constant is fixed by the *amount* of gas used. (More about it shortly.) The ideal gas relation allows us to determine temperature by measuring pressure and volume. This is the way (absolute) temperature has been defined and measured for about a century. To define the *unit* of temperature, we only have to fix the amount of gas used. It is customary to fix the amount of gas at 1 mol; for example, for oxygen this is 32 g. The proportionality constant for 1 mol, called the *ideal gas constant* R, is defined to be

Ref. 288
Page 453

$$R = 8.314\,462\,618\,\mathrm{J/(mol\,K)}\ . \qquad (113)$$

This numerical value has been chosen in order to yield the best approximation to the independently defined Celsius temperature scale. Fixing the ideal gas constant in this way defines 1 K, or one Kelvin, as the unit of temperature.

In general, if we need to determine the temperature of an object, we thus need to take a mole of an ideal gas, put it in contact with the object, wait a while, and then measure the pressure and the volume of the gas. The ideal gas relation (112) then gives the temperature of the object.

For every substance, a characteristic material property is the triple point. The *triple point* is the only temperature at which the solid, liquid, and gaseous phase coexists. For water, it lies at 0.01 °C (and at a partial vapour pressure of 611.657 Pa). Using the triple point of water, we can define the Kelvin in simple terms: a temperature increase of one Kelvin is the temperature increase that makes an ideal gas at the triple point of water increase in volume, keeping the pressure fixed, by a fraction of 1/273.16 or 0.366 09 %.

Challenge 638 e

Most importantly, the ideal gas relation shows that there is a lowest temperature in nature, namely that temperature at which an ideal gas would have a vanishing volume. For an ideal gas, this would happen at $T = 0$ K. In reality, other effects, like the volume of the atoms themselves, prevent the volume of a real gas from ever reaching zero exactly.

▷ The unattainability of zero temperature is called the *third principle of thermodynamics*.

In fact, the temperature values achieved by a civilization can be used as a measure of its technological achievements. We can define in this way the *Bronze Age* (1.1 kK, 3500 BCE), the *Iron Age* (1.8 kK, 1000 BCE), the *Electric Age* (3 kK from *c.* 1880) and the *Atomic Age* (several MK, from 1944) in this way. Taking into account also the quest for lower temperatures, we can define the *Quantum Age* (4 K, starting 1908). All these thoughts lead to a simple question: what *exactly* are heating and cooling? What happens in these processes?

Ref. 290

TABLE 46 Some temperature values.

OBSERVATION	TEMPERATURE
Lowest,but unattainable, temperature	0 K = −273.15°C
In the context of lasers, it can make (almost) sense to talk about negative temperature.	
Temperature a perfect vacuum would have at Earth's surface Vol. V, Page 146	40 zK
Sodium gas in certain laboratory experiments – coldest matter system achieved by man and possibly in the universe	0.45 nK
Temperature of neutrino background in the universe	*c.* 2 K

TABLE 46 (Continued) Some temperature values.

Observation	Temperature
Temperature of (photon) cosmic background radiation in the universe	2.7 K
Liquid helium	4.2 K
Oxygen triple point	54.3584 K
Liquid nitrogen	77 K
Coldest weather ever measured (Antarctic)	185 K = −88°C
Freezing point of water at standard pressure	273.15 K = 0.00°C
Triple point of water	273.16 K = 0.01°C
Average temperature of the Earth's surface	287.2 K
Smallest uncomfortable skin temperature	316 K (10 K above normal)
Interior of human body	310.0 ± 0.5 K = 36.8 ± 0.5°C
Temperature of most land mammals	310 ± 3 K = 36.8 ± 2°C
Hottest weather ever measured	343.8 K = 70.7°C
Boiling point of water at standard pressure	373.13 K or 99.975°C
Temperature of hottest living things: thermophile bacteria	395 K = 122°C
Large wood fire, liquid bronze	*c.* 1100 K
Freezing point of gold	1337.33 K
Liquid, pure iron	1810 K
Bunsen burner flame	up to 1870 K
Light bulb filament	2.9 kK
Melting point of hafnium carbide	4.16 kK
Earth's centre	5(1) kK
Sun's surface	5.8 kK
Air in lightning bolt	30 kK
Hottest star's surface (centre of NGC 2240)	250 kK
Space between Earth and Moon (no typo)	up to 1 MK
Centre of white dwarf	5 to 20 MK
Sun's centre	20 MK
Centre of the accretion disc in X-ray binary stars	10 to 100 MK
Inside the JET fusion tokamak	100 MK
Centre of hottest stars	1 GK
Maximum temperature of systems without electron–positron pair generation	ca. 6 GK
Universe when it was 1 s old	100 GK
Hagedorn temperature	1.9 TK
Heavy ion collisions – highest man-made value	up to 3.6 TK
Planck temperature – nature's upper temperature limit	10^{32} K

FIGURE 275 Thermometers: a Galilean thermometer (left), the row of infrared sensors in the jaw of the emerald tree boa *Corallus caninus*, an infrared thermometer to measure body temperature in the ear, a nautical thermometer using a bimetal, a mercury thermometer, and a thermocouple that is attached to a voltmeter for read-out (© Wikimedia, Ron Marcus, Braun GmbH, Universum, Wikimedia, Thermodevices).

Thermal energy

Around us, friction slows down moving bodies, and, while doing so, heats them up. The ‘creation’ of heat by friction can be observed in many experiments. An example is shown Ref. 291 in Figure 273. Such experiments show that heat can be generated from friction, just by continuous rubbing, *without any limit*. This endless ‘creation’ of heat implies that heat is neither a material fluid nor a substance extracted from the body – which in this case would be consumed after a certain time – but something else. Indeed, today we know that heat, even though it behaves in some ways like a fluid, is due to the disordered motion of particles. The conclusion from all these explorations is:

▷ Friction is the transformation of mechanical (i.e., ordered) energy into (disordered) *thermal energy*, i.e., into the disordered motion of the particles making up a material.

Heating and cooling is thus the flow of disordered energy. In order to increase the temperature of 1 kg of water by 1 K using friction, 4.2 kJ of mechanical energy must be supplied. The first to measure this quantity with precision was, in 1842, the physician Julius Robert Mayer (b. 1814 Heilbronn, d. 1878 Heilbronn). He described his experiments as proofs of the conservation of energy; indeed, he was the first person to state energy conservation! It is something of an embarrassment to modern physics that a medical doctor was the first to show the conservation of energy, and furthermore, that he was ridiculed by most physicists of his time. Worse, conservation of energy was accepted by scientists only when it was publicized many years later by two authorities: Hermann von Helmholtz – himself also a physician turned physicist – and William Thomson, who also cited similar, but later experiments by James Joule.* All of them acknowledged Mayer's priority. Marketing by William Thomson eventually led to the naming of the unit of energy after Joule. In summary, two medical doctors proved to all experts on motion:

▷ In a closed system, the sum of mechanical energy and thermal energy is constant. This is called the *first principle of thermodynamics*.

Equivalently, it is impossible to produce mechanical energy without paying for it with some other form of energy. This is an important statement because among others it means that humanity will stop living one day. Indeed, we live mostly on energy from the Sun; since the Sun is of finite size, its energy content will eventually be consumed. Can you estimate when this will happen? Challenge 639 s

Page 110 The first principle of thermodynamics, the conservation of energy, implies:

▷ There is *no* perpetuum mobile 'of the first kind'.

In particular, no machine can run without energy input. For this very reason, we need food to eat: the energy in the food keeps us alive. If we stop eating, we die. The conservation of energy also makes most so-called 'wonders' impossible: in nature, energy cannot be created, but is conserved.

Thermal energy is a form of energy. Thermal energy can be stored, accumulated, transferred, transformed into mechanical energy, electrical energy or light. In short, thermal energy can be transformed into motion, into work, and thus into money.

The first principle of thermodynamics also allows us to formulate what a car engine achieves. Car engines are devices that transform hot matter – the hot exploding fuel inside the cylinders – into motion of the car wheels. Car engines, like steam engines, are thus examples of *heat engines*.

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* Hermann von Helmholtz (b. 1821 Potsdam, d. 1894 Berlin), important scientist. William Thomson-Kelvin (b. 1824 Belfast, d. 1907 Netherhall), important physicist. James Prescott Joule (b. 1818 Salford, d. 1889 Sale), physicist. Joule is pronounced so that it rhymes with 'cool', as his descendants like to stress. (The pronunciation of the name 'Joule' varies from family to family.)

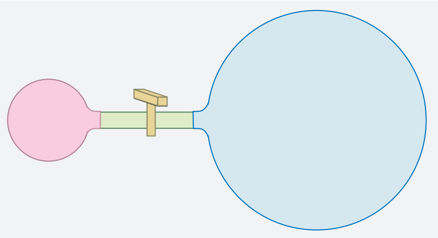

FIGURE 276 Which balloon wins when the tap is opened? Note: this is also how aneurysms grow in arteries.

The study of heat and temperature is called *thermostatics* if the systems concerned are at equilibrium, and *thermodynamics* if they are not. In the latter case, we distinguish situations *near* equilibrium, when equilibrium concepts such as temperature can still be used, from situations *far* from equilibrium, where such concepts often cannot be applied. Page 415

Does it make sense to distinguish between thermal energy and heat? It does. Many older texts use the term 'heat' to mean the *same* as thermal energy. However, this is confusing to students and experts:

▷ In this text, 'heat' is used, in accordance with modern approaches, as the everyday term for *entropy*.

Both thermal energy and heat flow from one body to another, and both accumulate. Both have no measurable mass.* Both the amount of thermal energy and the amount of heat inside a body increase with increasing temperature. The precise relation will be given shortly. But heat has many other interesting properties and stories to tell. Of these, two are particularly important: first, heat is due to particles; and second, heat is at the heart of the difference between past and future. These two stories are intertwined.

Why do balloons take up space? – The end of continuity

Heat properties are material-dependent. Studying thermal properties therefore should enable us to understand something about the constituents of matter. Now, the simplest materials of all are gases.** Gases need space: any amount of gas has pressure and volume. It did not take a long time to show that gases *could not* be continuous. One of the first scientists to think about gases as made up of atoms or molecules was Daniel Bernoulli. Bernoulli reasoned that if gases are made up of small particles, with mass and momentum, he should be able to make quantitative predictions about the behaviour of gases, and check them with experiments. If the particles fly around in a gas, then the *pressure* of a gas in a container is produced by the steady flow of particles hitting the wall. Bernoulli understood that if he reduced the volume to one-half, the particles in the gas would need only to travel half as long to hit a wall: thus the pressure of the gas

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* This might change in future, when mass measurements improve in precision, thus allowing the detection of relativistic effects. In this case, temperature increase may be detected through its related mass increase. However, such changes are noticeable only with twelve or more digits of precision in mass measurements. Vol. II, page 71

** By the way, the word *gas* is a modern construct. It was coined by the alchemist and physician Johan Baptista van Helmont (b. 1579 Brussels, d. 1644 Vilvoorde), to sound similar to 'chaos'. It is one of the few words which have been invented by one person and then adopted all over the world.

FIGURE 277 What happened here? (© Johan de Jong)

would double. He also understood that if the temperature of a gas is increased while its volume is kept constant, the speed of the particles would increase. Combining these results, Bernoulli concluded that if the particles are assumed to behave as tiny, hard and perfectly elastic balls, the pressure p, the volume V and the temperature T must be related by Challenge 640 s

$$pV = nRT = kNT \,. \tag{114}$$

In this so-called *ideal gas relation*, R is the *ideal gas constant*; it has the value $R = 8.314\,462\,618\,\mathrm{J/(mol\,K)}$ and n is the number of moles. In the alternative writing of the gas relation, N is the number of particles contained in the gas and k is the Boltzmann constant, one of the fundamental constants of nature, is given by $k = 1.380\,648\,52(79) \cdot 10^{-23}\,\mathrm{J/K}$. (More about this constant is told below.) Page 392 A gas made of particles with such a textbook behaviour is called an *ideal gas*. Relation (114), often also called the *ideal gas 'law'*, was known before Bernoulli; the relation has been confirmed by experiments at room and higher temperatures, for all known gases.

Bernoulli derived the ideal gas relation, with a specific prediction for the proportionality constant R, from the single assumption that gases are made of small particles with mass. This derivation provides a clear argument for the existence of atoms and for their behaviour as normal, though small objects. And indeed, we have already seen above how N can be determined experimentally. Page 340

The ideal gas model helps us to answer questions such as the one illustrated in Figure 276. Two *identical* rubber balloons, one filled up to a larger size than the other, are connected via a pipe and a valve. The valve is opened. Which one deflates? Challenge 641 s

The ideal gas relation states that hotter gases, at given pressure, need more volume. Challenge 642 e The relation thus explains why winds and storms exist, why hot air balloons rise – even those of Figure 277 – why car engines work, why the ozone layer is destroyed by certain gases, or why during the extremely hot summer of 2001 in the south of Turkey, oxygen masks were necessary to walk outside during the day.

The ideal gas relation also explains why on the 21st of August 1986, over a thousand people and three-thousand livestock were found dead in their homes in Cameroon. They were living below a volcano whose crater contains a lake, *Lake Nyos*. It turns out that the volcano continuously emits carbon dioxide, or CO_2, into the lake. The carbon dioxide is usually dissolved in the water. But in August 1986, an unknown event triggered the release of a bubble of around one million tons of CO_2, about a cubic kilometre, into the atmosphere. Because carbon dioxide ($2.0\,\mathrm{kg/m^3}$) is denser than air ($1.2\,\mathrm{kg/m^3}$), the gas flowed down into the valleys and villages below the volcano. The gas has no colour and smell, and it leads to asphyxiation. It is unclear whether the outgassing system installed in the lake after the event is sufficiently powerful to avoid a recurrence of the event.

Using the ideal gas relation you are now able to explain why balloons increase in size as they rise high up in the atmosphere, even though the air is colder there. The largest balloon built so far had a diameter, at high altitude, of 170 m, but only a fraction of that value at take-off. How much?

Challenge 643 ny

Now you can also take up the following challenge: how can you measure the weight of a car or a bicycle with a ruler only?

Challenge 644 s

The picture of gases as being made of hard constituents without any long-distance interactions breaks down at very low temperatures. However, the ideal gas relation (114) can be improved to overcome these limitations, by taking into account the deviations due to interactions between atoms or molecules. This approach is now standard practice and allows us to measure temperatures even at extremely low values. The effects observed below 80 K, such as the solidification of air, frictionless transport of electrical current, or frictionless flow of liquids, form a fascinating world of their own, the beautiful domain of low-temperature physics. The field will be explored later on.

Ref. 292

Ref. 293

Vol. V, page 108

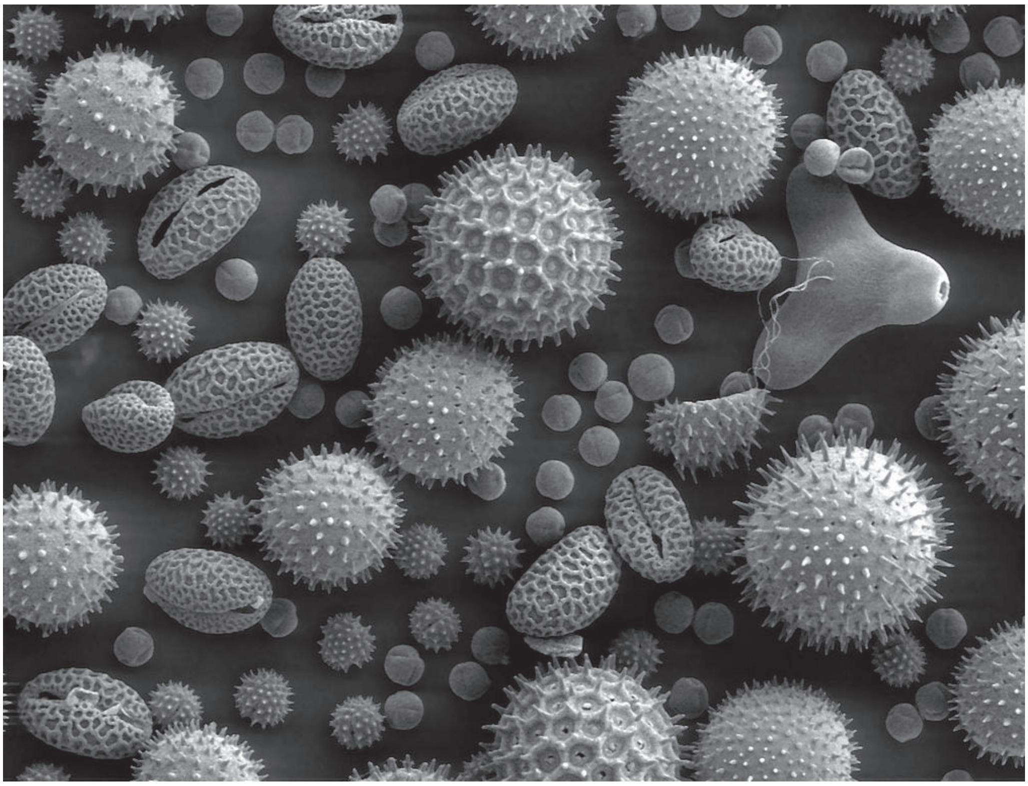
Vol. V, page 103

Not long after Bernoulli, chemists found strong arguments confirming the existence of atoms. They discovered that chemical reactions occur under 'fixed proportions': only specific ratios of amounts of chemicals react. Many researchers, including John Dalton, deduced that this property occurs because in chemistry, all reactions occur atom by atom. For example, two hydrogen atoms and one oxygen atoms form one water molecule in this way – even though these terms did not exist at the time. The relation is expressed by the chemical formula H_2O. These arguments are strong, but did not convince everybody. Finally, the existence of atoms was confirmed by observing the effects of their motion even more directly.

Brownian motion

If fluids are made of particles moving randomly, this random motion should have observable effects. An example of the observed motion is shown in Figure 280. The particles seem to follow a random zig-zag movement. The first description is by Lucretius, in the year 60 BCE, in his poem *De rerum natura*. In it, Lucretius tells about a common observation: in air that is illuminated by the Sun, dust particles seem to dance.

In 1785, Jan Ingenhousz saw that coal dust particles never come to rest. Indeed, under a microscope it is easy to observe that coal dust or other small particles in or on a liquid never come to rest. Ingenhousz discovered what is called *Brownian motion* today. 40 years after him, the botanist Robert Brown was the first Englishman to repeat the observation, this time for small particles floating in vacuoles *inside* pollen. Further ex-



Vol. III, page 183 FIGURE 278 An image of pollen grains – field size about 0.3 mm – made with an electron microscope (Dartmouth College Electron Microscope Facility).

periments by many other researchers showed that the observation of a random motion is independent of the type of particle and of the type of liquid. In other words, Ingenhousz had discovered a fundamental form of *noise* in nature.

Around 1860, the random motion of particles in liquids was attributed by various researchers to the molecules of the liquid that were colliding with the particles. In 1905 and 1906, Marian von Smoluchowski and, independently, Albert Einstein argued that this attribution could be tested experimentally, even though at that time nobody was able to observe molecules directly. The test makes use of the specific properties of thermal noise. Ref. 294

It had already been clear for a long time that if molecules, i.e., indivisible matter particles, really existed, then thermal energy had to be disordered motion of these constituents and temperature had to be the average energy per degree of freedom of the constituents. Bernoulli's model of Figure 279 implies that for monatomic gases the kinetic energy $T_{\rm kin}$ per particle is given by Page 393 Challenge 645 ny

$$T_{\rm kin} = \frac{3}{2}kT \qquad (115)$$

where T is temperature. The so-called *Boltzmann constant* $k = 1.4 \cdot 10^{-23}$ J/K is the standard conversion factor between temperature and energy.* At a room temperature

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* The Boltzmann constant k was discovered and named by Max Planck, in the same work in which he also

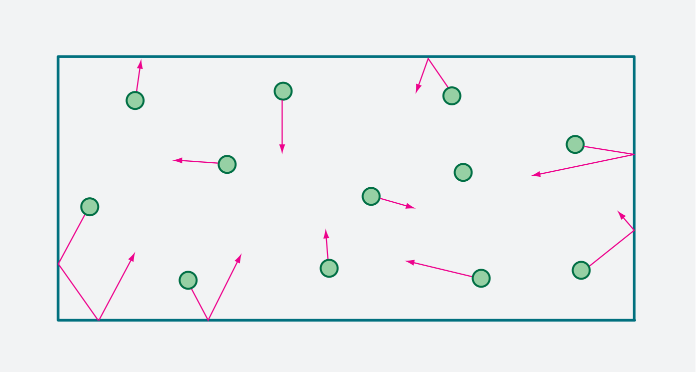

FIGURE 279 The basic idea of statistical mechanics about gases: gases are systems of moving particles, and pressure is due to their collisions with the container.

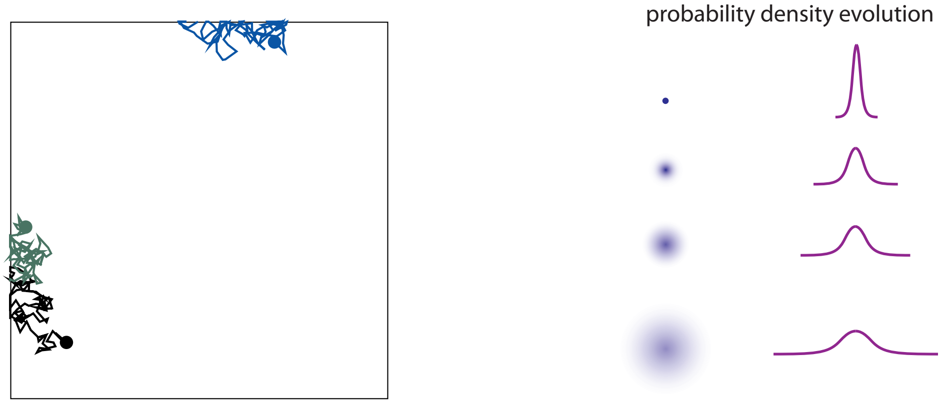

FIGURE 280 Example paths for particles in Brownian motion and their displacement distribution.

of 293 K, the kinetic energy of a particle is thus 6 zJ.

If you use relation (115) to calculate the speed of air molecules at room temperature, you get values of several hundred metres per second, about the speed of sound! Given this large speed, why does smoke from a candle take so long to diffuse through a room that has no air currents? Rudolph Clausius (b. 1822 Köslin, d. 1888 Bonn) answered this question in the mid-nineteenth century: smoke diffusion is slowed by the collisions with air molecules, in the same way as pollen particles collide with molecules in liquids. Since flows are usually more effective than diffusion, the materials that show no flows at all are those where the importance of diffusion is most evident: solids. Metal hardening and semiconductor production are examples. Challenge 646 e

The description of Brownian motion can be tested by following the displacement of pollen particles under the microscope. At first sight, we might guess that the average

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discovered what is now called Planck's constant $\hbar$, the quantum of action. For more details on Max Planck, see later on. Vol. III, page 149

Planck named the Boltzmann constant after the important physicist Ludwig Boltzmann (b. 1844 Vienna, d. 1906 Duino), who is most famous for his work on thermodynamics. Boltzmann explained all thermodynamic phenomena and observables, above all entropy itself, as results of the behaviour of molecules. It seems that Boltzmann committed suicide partly because of the animosities of his fellow physicists towards his ideas and himself. Nowadays, his work is standard textbook material.

distance the pollen particle has moved after n collisions should be zero, because the molecule velocities are random. However, this is wrong, as experiment shows.

An increasing average *square* displacement, written $\langle d^2 \rangle$, is observed for the pollen particle. It cannot be predicted in which direction the particle will move, but it does move. If the distance the particle moves after one collision is l, the average square displacement after n collisions is given, as you should be able to show yourself, by Challenge 647 ny

$$\langle d^2 \rangle = nl^2 . \quad (116)$$

For molecules with an average velocity v over time t this gives

$$\langle d^2 \rangle = nl^2 = vlt . \quad (117)$$

In other words, the average square displacement increases proportionally with time. Of course, this is only valid because the liquid is made of separate molecules. Repeatedly measuring the position of a particle should give the distribution shown in Figure 280 for the probability that the particle is found at a given distance from the starting point. This is called the *(Gaussian) normal distribution*. In 1908, Jean Perrin* performed extensive experiments in order to test this prediction. Ref. 295 He found that equation (117) corresponded completely with observations, thus convincing everybody that Brownian motion is indeed due to collisions with the molecules of the surrounding liquid, as had been expected.** Perrin received the 1926 Nobel Prize in Physics for these experiments.

Einstein also showed that the same experiment could be used to determine the number of molecules in a litre of water (or equivalently, the Boltzmann constant k). Can you work out how he did this? Challenge 648 d

Why stones can be neither smooth nor fractal, nor made of little hard balls

The exploration of temperature yields another interesting result. Researchers first studied gases, and measured how much energy was needed to heat them by 1 K. The result is simple: all gases share only a few values, when the number of molecules N is taken into account. Monatomic gases (in a container with constant volume and at sufficient temperature) require $3Nk/2$, diatomic gases (and those with a linear molecule) $5Nk/2$, and almost all other gases $3Nk$, where $k = 1.4 \cdot 10^{-23}$ J/K is the Boltzmann constant. Page 392

The explanation of this result was soon forthcoming: each thermodynamic degree of freedom*** contributes the energy $kT/2$ to the total energy, where T is the temperature.

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* Jean Perrin (b. 1870 Lille, d. 1942 New York), important physicist, devoted most of his career to the experimental proof of the atomic hypothesis and the determination of Avogadro's number; in pursuit of this aim he perfected the use of emulsions, Brownian motion and oil films. His Nobel Prize speech (nobelprize.org/physics/laureates/1926/perrin-lecture.html) tells the interesting story of his research. He wrote the influential book *Les atomes* and founded the Centre National de la Recherche Scientifique. He was the first to speculate, in 1901, that an atom might be similar to a small solar system.

Ref. 296 ** In a delightful piece of research, Pierre Gaspard and his team showed in 1998 that Brownian motion is also chaotic, in the strict physical sense given later on. Page 424

*** A *thermodynamic degree of freedom* is, for each particle in a system, the number of dimensions in which

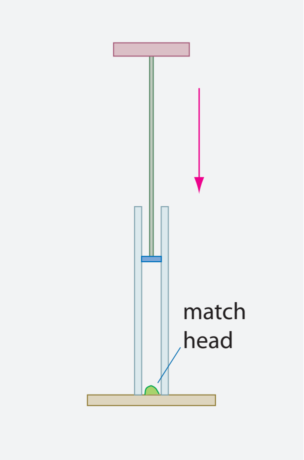

FIGURE 281 The fire pump.

So the number of degrees of freedom in physical bodies is finite. Bodies are not continuous, nor are they fractals: if they were, their specific thermal energy would be infinite. Matter is indeed made of small basic entities.

All degrees of freedom contribute to the specific thermal energy. At least, this is what classical physics predicts. Solids, like stones, have 6 thermodynamic degrees of freedom and should show a specific thermal energy of $3Nk$. At high temperatures, this is indeed observed. But measurements of solids at room temperature yield lower values, and the lower the temperature, the lower the values become. Even gases show values lower than those just mentioned, when the temperature is sufficiently low. In other words, molecules and atoms behave differently at low energies: atoms are not immutable little hard balls. The deviation of these values is one of the first hints of quantum theory.

Entropy

> – It's irreversible.
> – Like my raincoat!
>
> Mel Brooks, *Spaceballs*, 1987

Every domain of physics describes change in terms of three quantities: energy, as well as an intensive and an extensive quantity characteristic of the domain. In the domain of thermal physics, the intensive quantity is temperature. What is the corresponding extensive quantity? Ref. 299

The obvious guess would be 'heat'. Unfortunately, the quantity that physicists usually call 'heat' is not the same as what we call 'heat' in our everyday speech. For this historical reason, we need to introduce a new term. The extensive quantity corresponding to what we call 'heat' in everyday speech is called *entropy* in physics.[*]

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it can move plus the number of dimensions in which it is kept in a potential. Atoms in a solid have six, particles in monatomic gases have only three; particles in diatomic gases or rigid linear molecules have five. The number of degrees of freedom of larger molecules depends on their shape. Ref. 297

[*] The term 'entropy' was invented by the physicist Rudolph Clausius (b. 1822 Köslin, d. 1888 Bonn) in 1865. He formed it from the Greek ἐν 'in' and τρόπος 'direction', to make it sound similar to 'energy'. The term

Entropy describes the *amount* of everyday heat. Entropy is measured in joule per kelvin or J/K; some example values (per amount of matter) are listed in Table 47 and Table 48. Entropy describes everyday heat in the same way as momentum describes everyday motion. Correspondingly, temperature describes the *intensity* of everyday heat or entropy, in the same way that speed describes the intensity of motion.

When two objects of different speeds collide, a flow of momentum takes place between them. Similarly, when two objects differing in temperature are brought into contact, an entropy flow takes place between them. We now define the concept of entropy – 'everyday heat' – more precisely and explore its properties in some more detail.

Entropy measures the degree to which energy is *mixed up* inside a system, that is, the degree to which energy is spread or shared among the components of a system. When all components of a system – usually the molecules or atoms – move in the same way, *in concert*, the entropy of the system is low. When the components of the system move completely independently, *randomly*, the entropy is large. In short, entropy measures the amount of disordered energy content per temperature in a system. That is the reason that it is measured in J/K.

Entropy is an extensive quantity, like charge and momentum. It is measured by transferring it to a measurement apparatus. The simplest measurement apparatus is a mixture of water and ice. When an amount S of entropy is transferred to the mixture, the amount of melted ice is a measure of the transferred entropy.

More precisely, the entropy ΔS flowing into a system is measured by measuring the energy E flowing into the system, and recording the temperature T that occurs during the process:

$$\Delta S = \int_{T_{\text{start}}}^{T_{\text{end}}} \frac{dE}{T} \ . \tag{118}$$

Often, this can be approximated as $\Delta S = P\,\Delta t/\overline{T}$, where P is the power of the heating device, Δt is the heating time, and $\overline{T}$ is the average temperature.

Since entropy measures an amount, an extensive quantity, and not an intensity, entropy *adds up* when identical systems are composed into one. When two one-litre bottles of water at the same temperature are poured together, the entropy of the water adds up. Again, this corresponds to the behaviour of momentum: it also adds up when systems are composed.

Like any other extensive quantity, entropy can be accumulated in a body, and entropy can flow into or out of bodies. When we transform water into steam by heating it, we say that we add entropy to the water. We also add entropy when we transform ice into liquid water. After either transformation, the added entropy is contained in the warmer phase. Indeed, we can *measure* the entropy we add by measuring how much ice melts or how much water evaporates. In short, entropy is the exact term for what we call 'heat' in everyday speech.

Whenever we dissolve a block of salt in water, the entropy of the total system must increase, because the disorder increases. We now explore this process.

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entropy has always had the meaning given here.

In contrast, what physicists traditionally called 'heat' is a form of energy and not an extensive quantity in general.

TABLE 47 Some measured specific entropy values.

Process/System	Entropy value
Carbon, solid, in diamond form	2.43 J/K mol
Carbon, solid, in graphite form	5.69 J/K mol
Melting of ice	1.21 kJ/K kg = 21.99 J/K mol
Iron, solid, under standard conditions	27.2 J/K mol
Magnesium, solid, under standard conditions	32.7 J/K mol
Water, liquid, under standard conditions	70.1(2) J/K mol
Boiling of 1 kg of liquid water at 101.3 kPa	6.03 kJ/K= 110 J/K mol
Helium gas under standard conditions	126.15 J/K mol
Hydrogen gas under standard conditions	130.58 J/K mol
Carbon gas under standard conditions	158 J/K mol
Water vapour under standard conditions	188.83 J/K mol
Oxygen O_2 under standard conditions	205.1 J/K mol
C_2H_6 gas under standard conditions	230 J/K mol
C_3H_8 gas under standard conditions	270 J/K mol
C_4H_{10} gas under standard conditions	310 J/K mol
C_5H_{12} gas under standard conditions	348.9 J/K mol
$TiCl_4$ gas under standard conditions	354.8 J/K mol

TABLE 48 Some typical entropy values per particle at *standard* temperature and pressure as multiples of the Boltzmann constant.

Material	Entropy per particle
Monatomic solids	0.3 k to 10 k
Diamond	0.29 k
Graphite	0.68 k
Lead	7.79 k
Monatomic gases	15-25 k
Helium	15.2 k
Radon	21.2 k
Diatomic gases	15 k to 30 k
Polyatomic solids	10 k to 60 k
Polyatomic liquids	10 k to 80 k
Polyatomic gases	20 k to 60 k
Icosane	112 k

Entropy from particles

Once it had become clear that heat and temperature are due to the motion of microscopic particles, people asked what entropy *was* microscopically. The answer can be formulated in various ways. The two most extreme answers are:

- ▷ Entropy measures the (logarithm of the) number W of possible microscopic states. A given macroscopic state can have many microscopic realizations. The logarithm of this number, multiplied by the Boltzmann constant k, gives the entropy.*
- ▷ Entropy is the expected number of yes-or-no questions, multiplied by $k \ln 2$, the answers of which would tell us everything about the system, i.e., about its microscopic state.

In short, the higher the entropy, the more microstates are possible. Through either of these definitions, entropy measures the quantity of randomness in a system. In other words, entropy measures the transformability of energy: higher entropy means lower transformability. Alternatively, entropy measures the *freedom* in the choice of microstate that a system has. High entropy means high freedom of choice for the microstate. For example, when a molecule of glucose (a type of sugar) is produced by photosynthesis, about 40 bits of entropy are released. This means that after the glucose is formed, 40 additional yes-or-no questions must be answered in order to determine the full microscopic state of the system. Physicists often use a macroscopic unit; most systems of interest are large, and thus an entropy of 10^{23} bits is written as 1 J/K. (This is only approximate. Can you find the precise value?) Challenge 649 ny

To sum up, entropy is thus a specific measure for the characterization of the disorder of thermal systems. Ref. 300 Three points are worth making here. First of all, entropy is not *the* measure of disorder, but *one* measure of disorder. It is therefore *not* correct to use entropy as a *synonym* for the concept of disorder, as is often done in popular literature. Entropy is only defined for systems that have a temperature, in other words, only for systems that are in or near equilibrium. (For systems far from equilibrium, no measure of disorder has been found yet; probably none is possible.) In fact, the use of the term entropy has degenerated so much that sometimes one has to call it *thermodynamic* entropy for clarity.

Secondly, entropy is related to information *only if* information is defined also as $-k \ln W$. To make this point clear, take a book with a mass of one kilogram. At room temperature, its entropy content is about 4 kJ/K. The printed information inside a book, say 500 pages of 40 lines with each containing 80 characters out of 64 possibilities, corresponds to an entropy of $4 \cdot 10^{-17}$ J/K. In short, what is usually called 'information' in everyday life is a negligible fraction of what a physicist calls information. Entropy is defined using the *physical* concept of information.

Finally, entropy is *not* a measure for what in normal life is called the *complexity* of a situation. Ref. 301 In fact, nobody has yet found a quantity describing this everyday notion. The task is surprisingly difficult. Have a try! Challenge 650 ny

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* When Max Planck went to Austria to search for the anonymous tomb of Boltzmann in order to get him buried in a proper grave, he inscribed the formula $S = k \ln W$ on the tombstone. (Which physicist would finance the tomb of another, nowadays?)

In summary, if you hear the term entropy used with a different meaning than the expression $S = k \ln W$, beware. Somebody is trying to get you, probably with some ideology.

THE CHARACTERISTIC ENTROPY OF NATURE – THE QUANTUM OF INFORMATION

Before we complete our discussion of thermal physics we must point out in another way the importance of the Boltzmann constant k. We have seen that this constant appears whenever the granularity of matter plays a role; it expresses the fact that matter is made of small basic entities. The most striking way to put this statement is the following:

▷ There is a characteristic entropy change in nature for single particles: $\Delta S \approx k$.

This result is almost 100 years old; it was stated most clearly (with a different numerical Ref. 302 factor) by Leo Szilard. The same point was made by Léon Brillouin (again with a different Ref. 303 numerical factor). The statement can also be taken as the *definition* of the Boltzmann constant k.

The existence of a characteristic entropy change in nature is a powerful statement. It eliminates the possibility of the continuity of matter and also that of its fractality. A characteristic entropy implies that matter is made of a finite number of small components. The characteristic entropy expresses the fact that matter is made of particles.* The characteristic entropy also shows that Galilean physics cannot be correct: Galilean physics assumes that arbitrarily small quantities do exist. The entropy limit is the first of several limits to motion that we will encounter in our adventure. After we have found all limits, we can start the final leg that leads to the unified description of motion.

The existence of a characteristic quantity also implies a limit on the precision of measurements. Measurements cannot have infinite precision. This limitation is usually stated in the form of an indeterminacy relation. Indeed, the existence of a characteristic entropy can be rephrased as an indeterminacy relation between the temperature T and the inner energy U of a system:

$$\Delta \frac{1}{T} \, \Delta U \geqslant \frac{k}{2} \, . \qquad (119)$$

Ref. 305 This relation** was given by Niels Bohr; it was discussed by Werner Heisenberg, who Vol. VI, page 31 called it one of the basic indeterminacy relations of nature. The Boltzmann constant (di- Ref. 306 vided by 2) thus fixes the smallest possible entropy value in nature. For this reason, Gilles Ref. 303 Cohen-Tannoudji calls it the *quantum of information* and Herbert Zimmermann calls it the *quantum of entropy*.

The relation (119) points towards a more general pattern. For every minimum value for an observable, there is a corresponding indeterminacy relation. We will come across this

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[*] The chracteristic entropy change implies that matter is made of tiny spheres; the minimum *action*, which we will encounter in quantum theory, implies that these spheres are actually small clouds.

Ref. 304 [**] It seems that the historical value for the right-hand side, k, has to be corrected to $k/2$, for the same reason that the quantum of action $\hbar$ appears with a factor 1/2 in Heisenberg's indeterminacy relations.

Vol. IV, page 15 several times in the rest of our adventure, most importantly in the case of the quantum of action and Heisenberg's indeterminacy relation.

The existence of a characteristic entropy has numerous consequences. First of all, it sheds light on the third principle of thermodynamics. A characteristic entropy implies that absolute zero temperature is not achievable. Secondly, a characteristic entropy explains why entropy values are finite instead of infinite. Thirdly, it fixes the absolute value of entropy for every system; in continuum physics, entropy, like energy, is only defined up to an additive constant. The quantum of entropy settles all these issues.

The existence of a characteristic value for an observable implies that an indeterminacy relation appears for any two quantities whose product yields that observable. For example, entropy production rate and time are such a pair. Indeed, an indeterminacy relation connects the entropy production rate $P = \mathrm{d}S/\mathrm{d}t$ and the time t:

$$\Delta P \ \Delta t \geqslant \frac{k}{2} \ . \qquad (120)$$

Ref. 306, Ref. 304 From this and the previous relation (119) it is possible to deduce all of statistical physics, i.e., the precise theory of thermostatics and thermodynamics. We will not explore this further here. (Can you show that the third principle follows from the existence of a characteristic entropy?) Challenge 651 ny We will limit ourselves to one of the cornerstones of thermodynamics: the second principle.

Is everything made of particles?

> “A physicist is the atom's way of knowing about atoms.”
>
> George Wald

Ref. 307

Historically, the study of statistical mechanics has been of fundamental importance for physics. It provided the first demonstration that physical objects are made of interacting particles. The story of this topic is in fact a long chain of arguments showing that all the properties we ascribe to objects – including size, stiffness, colour, mass density, magnetism, thermal or electrical conductivity – result from the interactions of the many particles they consist of.

▷ All objects are made of interacting particles.

This discovery has often been called the main result of modern science.

Page 356 How was this discovery made? Table 43 listed the main extensive quantities used in physics. Extensive quantities are able to flow. It turns out that all flows in nature are *composed* of elementary processes, as shown in Table 49. We have already seen that flows of mass, volume, charge, entropy and substance are composed. Later, quantum theory will show the same for flows of angular momentum and of the nuclear quantum numbers.

▷ All flows are made of particles.

The success of this idea has led many people to generalize it to the statement: ‘Everything

TABLE 49 Some minimum flow values found in nature.

Observation	Minimum flow
Matter flow	one molecule or one atom or one particle
Volume flow	one molecule or one atom or one particle
Angular momentum flow	Planck's quantum of action
Chemical amount of substance	one molecule, one atom or one particle
Entropy flow	the minimum entropy
Charge flow	one elementary charge
Light flow	one single photon, Planck's quantum of action

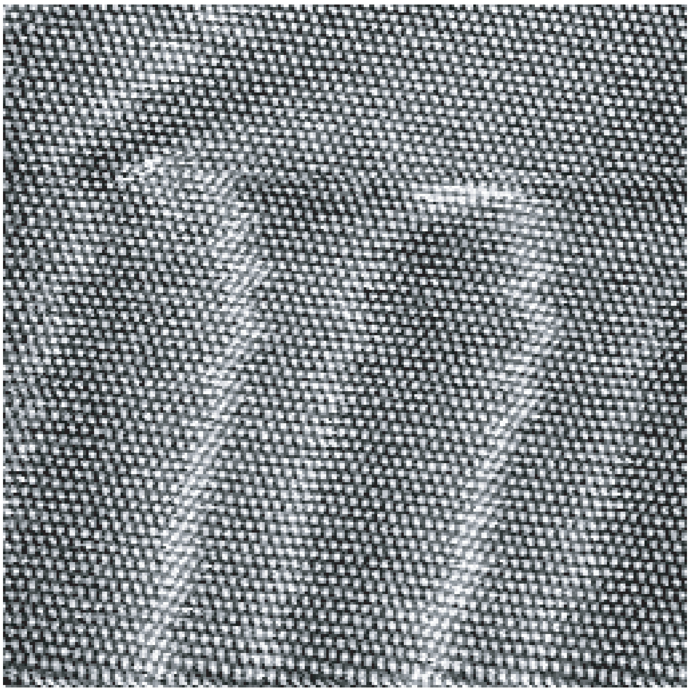

FIGURE 282 A 111 crystal surface of a gold single crystal, every bright dot being an atom, with a surface dislocation (© CNRS).

we observe is made of parts.' This approach has been applied with success to chemistry with molecules, materials science and geology with crystals, electricity with electrons, atoms with elementary particles, space with points, time with instants, light with photons, biology with cells, genetics with genes, neurology with neurons, mathematics with sets and relations, logic with elementary propositions, and even to linguistics with morphemes and phonemes. All these sciences have flourished on the idea that everything is made of *related parts*. The basic idea seems so self-evident that we find it difficult even to formulate an alternative; try!

Ref. 308

Challenge 652 e

However, in the case of the *whole* of nature, the idea that nature is a sum of related parts is incorrect and only approximate. It turns out to be a prejudice, and a prejudice so entrenched that it retarded further developments in physics in the latter decades of the twentieth century. In particular, it does *not* apply to elementary particles or to space:

Vol. VI, page 106

▷ Elementary particles and space are not made of parts.

Finding the correct description for the whole of nature, for elementary particles and for space is the biggest challenge of our adventure. It requires a complete change in thinking habits. There is a lot of fun ahead.

> “Jede Aussage über Komplexe läßt sich in eine Aussage über deren Bestandteile und in diejenigen Sätze zerlegen, welche die Komplexe vollständig beschreiben.[*]”
>
> Ludwig Wittgenstein, *Tractatus*, 2.0201

The second principle of thermodynamics

In contrast to several other important extensive quantities, entropy is *not* conserved. On the one hand, in closed systems, entropy accumulates and never decreases; the sharing or mixing of energy among the components of a system cannot be undone. On the other hand, the sharing or mixing can increase spontaneously over time. Entropy is thus only ‘half conserved’. What we call thermal equilibrium is simply the result of the highest possible mixing. Entropy allows us to define the concept of *equilibrium* more precisely as the state of maximum entropy, or maximum energy sharing among the components of a system. In short, the entropy of a closed system increases until it reaches the maximum possible value, the equilibrium value.

The non-conservation of entropy has far-reaching consequences. When a piece of rock is detached from a mountain, it falls, tumbles into the valley, heating up a bit, and eventually stops. The opposite process, whereby a rock cools and tumbles upwards, is never observed. Why? We could argue that the opposite motion does not contradict any rule or pattern about motion that we have deduced so far. Challenge 653 s

Rocks never fall upwards because mountains, valleys and rocks are made of *many* particles. Motions of many-particle systems, especially in the domain of thermodynamics, are called *processes*. Central to thermodynamics is the distinction between *reversible* processes, such as the flight of a thrown stone, and *irreversible* processes, such as the afore-mentioned tumbling rock. Irreversible processes are all those processes in which friction and its generalizations play a role. Irreversible processes are those processes that increase the sharing or mixing of energy. They are important: if there were no friction, shirt buttons and shoelaces would not stay fastened, we could not walk or run, coffee machines would not make coffee, and maybe most importantly of all, we would have no memory. Ref. 309 Vol. IV, page 159

Irreversible processes, in the sense in which the term is used in thermodynamics, transform macroscopic motion into the disorganized motion of all the small microscopic components involved: they increase the sharing and mixing of energy. Irreversible processes are therefore not *strictly* irreversible – but their reversal is *extremely* improbable. We can say that entropy measures the ‘amount of irreversibility’: it measures the degree of mixing or decay that a collective motion has undergone.

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[*] ‘Every statement about complexes can be resolved into a statement about their constituents and into the propositions that describe the complexes completely.’

Entropy is not conserved. Indeed, entropy – ‘heat’ – can appear out of nowhere, spontaneously, because energy sharing or mixing can happen by itself. For example, when two different liquids of the same temperature are mixed – such as water and sulphuric acid – the final temperature of the mix can differ. Similarly, when electrical current flows through material at room temperature, the system can heat up or cool down, depending on the material.

All experiments on heat agree with the so-called *second principle of thermodynamics*, which states:

▷ The entropy in a closed system tends towards its maximum.

In sloppy terms, ‘entropy ain’t what it used to be.’ In this statement, a *closed system* is a system that does not exchange energy or matter with its environment. Can you think of an example? Challenge 654 s

In a closed system, entropy never decreases. Even everyday life shows us that in a closed system, such as a room, the disorder increases with time, until it reaches some maximum. To reduce disorder, we need effort, i.e., work and energy. In other words, in order to reduce the disorder in a system, we need to connect the system to an energy source in some clever way. For this reason, refrigerators need electrical current or some other energy source.

Ref. 310 In 1866, Ludwig Boltzmann showed that the second principle of thermodynamics results from the principle of least action. Can you imagine and sketch the general ideas? Challenge 655 ny

Because entropy never decreases in closed systems, *white colour does not last*. Whenever disorder increases, the colour white becomes ‘dirty’, usually grey or brown. Perhaps for this reason white objects, such as white clothes, white houses and white underwear, are valued in our society. White objects defy decay.

The second principle implies that heat cannot be transformed to work completely. In other words, every heat engine needs cooling: that is the reason for the holes in the front of cars. The first principle of thermodynamics then states that the mechanical power of a heat engine is the difference between the inflow of thermal energy at high temperature and the outflow of thermal energy at low temperature. If the cooling is insufficient – for example, because the weather is too hot or the car speed too low – the power of the engine is reduced. Every driver knows this from experience.

In summary, the concept of entropy, corresponding to what is called ‘heat’ in everyday life – but *not* to what is traditionally called ‘heat’ in physics! – describes the randomness of the internal motion in matter. Entropy is not conserved: in a closed system, entropy never decreases, but it can increase until it reaches a maximum value. The non-conservation of entropy is due to the many components inside everyday systems. The large number of components lead to the non-conservation of entropy and therefore explain, among many other things, that many processes in nature never occur backwards, even though they could do so in principle.

WHY CAN'T WE REMEMBER THE FUTURE?

> It's a poor sort of memory which only works backwards.
>
> Lewis Carroll, *Alice in Wonderland*

Page 40

When we first discussed time, we ignored the difference between past and future. But obviously, a difference exists, as we do not have the ability to remember the future. This is not a limitation of our brain alone. All the devices we have invented, such as tape recorders, photographic cameras, newspapers and books, only tell us about the past. Is there a way to build a video recorder with a 'future' button? Such a device would have to solve a deep problem: how would it distinguish between the near and the far future? It does not take much thought to see that any way to do this would conflict with the second principle of thermodynamics. That is unfortunate, as we would need precisely the same device to show that there is faster-than-light motion. Can you find the connection?

Challenge 656 e

Challenge 657 ny

In summary, the future cannot be remembered because entropy in closed systems tends towards a maximum. Put even more simply, memory exists because the brain is made of many particles, and so the brain is limited to the past. However, for the most simple types of motion, when only a few particles are involved, the difference between the past and the future disappears. For few-particle systems, there is no difference between times gone by and times approaching. We could say that the future differs from the past only in our brain, or equivalently, only because of friction. Therefore the difference between the past and the future is not mentioned frequently in this walk, even though it is an essential part of our human experience. But the fun of the present adventure is precisely to overcome our limitations.

FLOW OF ENTROPY

We know from daily experience that transport of an extensive quantity always involves friction. Friction implies the generation of entropy. In particular, the flow of entropy itself produces additional entropy. For example, when a house is heated, entropy is produced in the wall. Heating means keeping a temperature difference ΔT between the interior and the exterior of the house. The heat flow J traversing a square metre of a wall is given by

$$J = \kappa \Delta T = \kappa (T_\mathrm{i} - T_\mathrm{e}) \qquad (121)$$

where κ is a constant characterizing the ability of the wall to conduct heat. While conducting heat, the wall also *produces* entropy. The entropy production σ is proportional to the difference between the interior and the exterior entropy flows. In other words, one has

$$\sigma = \frac{J}{T_\mathrm{e}} - \frac{J}{T_\mathrm{i}} = \kappa \frac{(T_\mathrm{i} - T_\mathrm{e})^2}{T_\mathrm{i} T_\mathrm{e}} \; . \qquad (122)$$

Note that we have assumed in this calculation that everything is near equilibrium in each slice parallel to the wall, a reasonable assumption in everyday life. A typical case of a good wall has $\kappa = 1\,\mathrm{W/m^2K}$ in the temperature range between 273 K and 293 K. With this value, one gets an entropy production of

$$\sigma = 5 \cdot 10^{-3}\,\mathrm{W/m^2K}\,. \qquad (123)$$

Can you compare the amount of entropy that is produced in the flow with the amount that is transported? In comparison, a good goose-feather duvet has $\kappa = 1.5\,\mathrm{W/m^2K}$, which in shops is also called 15 tog.* Challenge 658 ny

The insulation power of materials is usually measured by the constant $\lambda = \kappa d$ which is independent of the thickness d of the insulating layer. Values in nature range from about 2000 W/K m for diamond, which is the best conductor of all, down to between 0.1 W/K m and 0.2 W/K m for wood, between 0.015 W/K m and 0.05 W/K m for wools, cork and foams, and the small value of $5 \cdot 10^{-3}$ W/K m for krypton gas.

Entropy can be transported in three ways: through *heat conduction*, as just mentioned, via *convection*, used for heating houses, and through *radiation*, which is possible also through empty space. For example, the Earth radiates about $1.2\,\mathrm{W/m^2K}$ into space, in total thus about 0.51 PW/K. The entropy is (almost) the same that the Earth receives from the Sun. If more entropy had to be radiated away than received, the temperature of the surface of the Earth would have to increase. This is called the *greenhouse effect* or *global warming*. Let's hope that it remains small in the near future.

Do isolated systems exist?

In all our discussions so far, we have assumed that we can distinguish the system under investigation from its environment. But do such *isolated* or *closed* systems, i.e., systems not interacting with their environment, actually exist? Probably our own human condition was the original model for the concept: we do experience having the possibility to act independently of our environment. An isolated system may be simply defined as a system not exchanging any energy or matter with its environment. For many centuries, scientists saw no reason to question this definition.

The concept of an isolated system had to be refined somewhat with the advent of quantum mechanics. Nevertheless, the concept of an isolated system provides useful and precise descriptions of nature also in the quantum domain, if some care is used. Only in the final part of our walk will the situation change drastically. There, the investigation of whether the universe is an isolated system will lead to surprising results. (What do you think? A strange hint: your answer is almost surely wrong.) We'll take the first steps towards the answer shortly. Challenge 659 s

Curiosities and fun challenges about reversibility and heat

Running backwards is an interesting sport. The 2006 world records for running backwards can be found on www.recordholders.org/en/list/backwards-running.html. You will be astonished how much these records are faster than your best personal *forward*-running time. Challenge 660 e

---

* The unit tog is not as bad as the official unit (not a joke) BthU · h/sqft/cm/°F used in some remote provinces of our galaxy.

* *

Ref. 313 In 1912, Emile Borel noted that if a gram of matter on Sirius was displaced by one centimetre, it would change the gravitational field on Earth by a tiny amount only. But this tiny change would be sufficient to make it impossible to calculate the path of molecules in a gas after a fraction of a second.

* *

If heat really is the disordered motion of atoms, a big problem appears. When two atoms collide head-on, in the instant of smallest distance, neither atom has velocity. Where does the kinetic energy go? Obviously, it is transformed into potential energy. But that implies that atoms can be deformed, that they have internal structure, that they have parts, and thus that they can in principle be split. In short, if heat is disordered atomic motion, *atoms are not indivisible*! In the nineteenth century this argument was put forward in order to show that heat cannot be atomic motion, but must be some sort of fluid. But since we know that heat really is kinetic energy, *atoms must be divisible*, even though their name means 'indivisible'. We do not need an expensive experiment to show this! We will discover more about them later on in our exploration. Vol. IV, page 79

* *

Compression of air increases its temperature. This is shown directly by the fire pump, a variation of a bicycle pump, shown in Figure 281. (For a working example, see the web page www.de-monstrare.nl). A match head at the bottom of an air pump made of transparent material is easily ignited by the compression of the air above it. The temperature of the air after compression is so high that the match head ignites spontaneously.

* *

In the summer, temperature of the air can easily be measured with a clock. Indeed, the rate of chirping of most crickets depends on temperature. For example, for a cricket species most common in the United States, by counting the number of chirps during 8 seconds and adding 4 yields the air temperature in degrees Celsius. Ref. 311

* *

How long does it take to cook an egg? This issue has been researched in extensive detail; of course, the time depends on what type of cooked egg you want, how large it is, and whether it comes from the fridge or not. There is even a formula for calculating the cooking time! Egg white starts hardening at 62°C, the yolk starts hardening at 65°C. The best-tasting hard eggs are formed at 69°C, half-hard eggs at 65°C, and soft eggs at 63°C. If you cook eggs at 100°C (for a long time) , the white gets the consistency of rubber and the yolk gets a green surface that smells badly, because the high temperature leads to the formation of the smelly H_2S, which then bonds to iron and forms the green FeS. Note that when temperature is controlled, the time plays no role; 'cooking' an egg at 65°C for 10 minutes or 10 hours gives the *same* result. Ref. 312

* *

It is possible to cook an egg in such a way that the white is hard but the yolk remains liquid. Can you achieve the opposite? Research has even shown how you can cook an Challenge 661 s

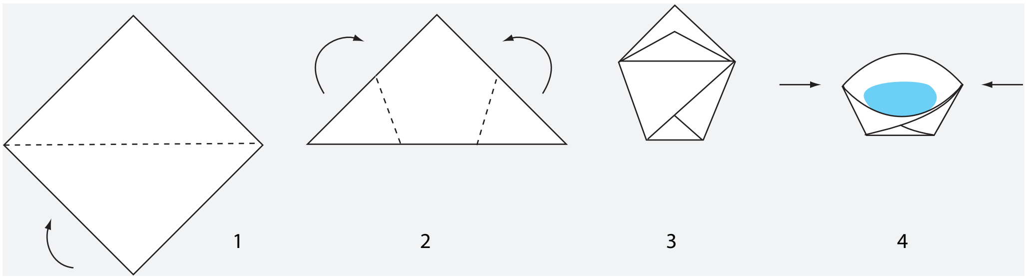


FIGURE 283 Can you boil water in this paper cup?

Challenge 662 e egg so that the yolk remains at the centre. Can you imagine the method?

* *

Not only gases, but also most other materials expand when the temperature rises. As a result, the electrical wires supported by pylons hang much lower in summer than in winter. True? Challenge 663 s

* *

Ref. 314 The following is a famous problem asked by Fermi. Given that a human corpse cools down four hours after death, what is the minimum number of calories needed per day in our food? Challenge 664 ny

* *

The energy contained in thermal motion is not negligible. For example, a 1 g bullet travelling at the speed of sound has a kinetic energy of only 0.04 kJ= 0.01 kcal. What is its thermal energy content? Challenge 665 e

* *

Challenge 666 s How does a typical, 1500 m^3 hot-air balloon work?

* *

If you do not like this text, here is a proposal. You can use the paper to make a cup, as shown in Figure 283, and boil water in it over an open flame. However, to succeed, you have to be a little careful. Can you find out in what way? Challenge 667 s

* *

Mixing 1 kg of water at 0°C and 1 kg of water at 100°C gives 2 kg of water at 50°C. What is the result of mixing 1 kg of *ice* at 0°C and 1 kg of water at 100°C? Challenge 668 s

* *

Temperature has many effects. In the past years, the World Health Organization found that drinking liquids that are hotter than 65°C – including coffee, chocolate or tea – causes cancer in the oesophagus, independently of the type of liquid.

* *

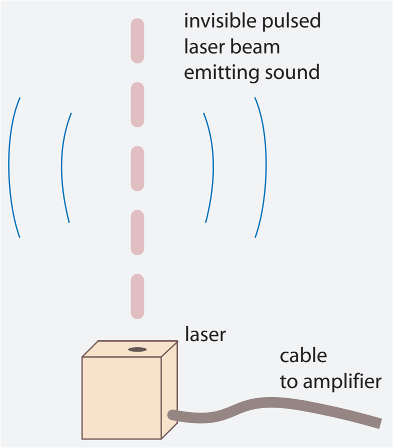


FIGURE 284 The invisible loudspeaker.

Ref. 315 The highest recorded air temperature in which a man has survived is 127°C. This was tested in 1775 in London, by the secretary of the Royal Society, Charles Blagden, together with a few friends, who remained in a room at that temperature for 45 minutes. Interestingly, the raw steak which he had taken in with him was cooked ('well done') when he and his friends left the room. What condition had to be strictly met in order to avoid cooking the people in the same way as the steak? Challenge 669 s

* *

Is the *Boltzmann constant* k really the smallest possible value for entropy in nature? How then can the entropy per particle of Krypton be as low as $0.3k$ per particle? The answer to this paradox is that a single free particle has more entropy, in fact more than k, than a bound one. The limit entropy k is thus valid for a physical system, such as the crystal as a whole, but not separately for the entropy value for each bound particle that is part of a system.

* *

The influential astronomer Anders Celsius (b. 1701 Uppsala, d. 1744 Uppsala) originally set the freezing point of water at 100 degrees and the boiling point at 0 degrees. Shortly afterwards, the scale was reversed to the one in use now. However, this is not the whole Ref. 316 story. With the official definition of the kelvin and the degree Celsius, at the standard pressure of 101 325 Pa, water boils at 99.974°C. Can you explain why it is not 100°C any more? Challenge 670 s

* *

Challenge 671 s Can you fill a bottle precisely with 1 ± 10^{-30} kg of water?

* *

One gram of fat, either butter or human fat, contains 38 kJ of chemical energy (or, in ancient units more familiar to nutritionists, 9 kcal). That is the same value as that of petrol. Why are people and butter less dangerous than petrol? Challenge 672 s

* *

In 1992, the Dutch physicist Martin van der Mark invented a loudspeaker which works by heating air with a laser beam. He demonstrated that with the right wavelength and with a suitable modulation of the intensity, a laser beam in air can generate sound. The effect at the basis of this device, called the *photoacoustic effect*, appears in many materials. The best laser wavelength for air is in the infrared domain, on one of the few absorption lines of water vapour. In other words, a properly modulated infrared laser beam that shines through the air generates sound. Such light can be emitted from a small matchbox-sized semiconductor laser hidden in the ceiling and shining downwards. The sound is emitted in all directions perpendicular to the beam. Since infrared laser light is not (usually) visible, Martin van der Mark thus invented an invisible loudspeaker! Unfortunately, the efficiency of present versions is still low, so that the power of the speaker is not yet sufficient for practical applications. Progress in laser technology should change this, so that in the future we should be able to hear sound that is emitted from the centre of an otherwise empty room.

* *

Challenge 673 s A famous exam question: How can you measure the height of a building with a barometer, a rope and a ruler? Find at least six different ways.

* *

Challenge 674 s What is the approximate probability that out of one million throws of a coin you get exactly 500 000 heads and as many tails? You may want to use Stirling's formula $n! \approx \sqrt{2\pi n}\,(n/e)^n$ to calculate the result.[*]

* *

Challenge 675 s Does it make sense to talk about the entropy of the universe?

* *

Challenge 676 ny Can a helium balloon lift the tank which filled it?

* *

All friction processes, such as osmosis, diffusion, evaporation, or decay, are *slow*. They take a characteristic time. It turns out that any (macroscopic) process with a time-scale is irreversible. This is no real surprise: we know intuitively that undoing things always takes more time than doing them. That is again the second principle of thermodynamics.

* *

Ref. 317 It turns out that *storing* information is possible with negligible entropy generation. However, *erasing* information requires entropy. This is the main reason why computers, as well as brains, require energy sources and cooling systems, even if their mechanisms would otherwise need no energy at all.

---

[*] There are many improvements to Stirling's formula. A simple one is Gosper's formula $n! \approx \sqrt{(2n+1/3)\pi}\,(n/e)^n$. Another is $\sqrt{2\pi n}\,(n/e)^n e^{1/(12n+1)} < n! < \sqrt{2\pi n}\,(n/e)^n e^{1/(12n)}$.

∗ ∗

When mixing hot rum and cold water, how does the increase in entropy due to the mixing compare with the entropy increase due to the temperature difference? Challenge 677 ny

∗ ∗

Why aren't there any small humans, say 10 mm in size, as in many fairy tales? In fact, there are no warm-blooded animals of that size at all. Why not? Challenge 678 s

∗ ∗

Shining a light onto a body and repeatedly switching it on and off produces sound. This is called the *photoacoustic effect*, and is due to the thermal expansion of the material. By changing the frequency of the light, and measuring the intensity of the noise, one reveals a characteristic photoacoustic spectrum for the material. This method allows us to detect gas concentrations in air of one part in 10^9. It is used, among other methods, to study the gases emitted by plants. Plants emit methane, alcohol and acetaldehyde in small quantities; the photoacoustic effect can detect these gases and help us to understand the processes behind their emission.

∗ ∗

What is the rough probability that all oxygen molecules in the air would move away from a given city for a few minutes, killing all inhabitants? Challenge 679 ny

∗ ∗

If you pour a litre of water into the sea, stir thoroughly through all the oceans and then take out a litre of the mixture, how many of the original atoms will you find? Challenge 680 ny

∗ ∗

Challenge 681 s How long would you go on breathing in the room you are in if it were airtight?

∗ ∗

Heat loss is a larger problem for smaller animals, because the surface to volume ratio increases when size decreases. As a result, small animals are found in hot climate, large animals are found in cold climates. This is true for bears, birds, rabbits, insects and many other animal families. For the same reason, small living beings need high amounts of food per day, when calculated in body weight, whereas large animals need far less food.

∗ ∗

What happens if you put some ash onto a piece of sugar and set fire to the whole? (Warning: this is dangerous and not for kids.) Challenge 682 s

∗ ∗

Entropy calculations are often surprising. For a system of N particles with two states each, there are $W_{\text{all}} = 2^N$ states. For its most probable configuration, with exactly half the particles in one state, and the other half in the other state, we have $W_{\text{max}} = N!/((N/2)!)^2$. Now, for a macroscopic system of particles, we might typically have $N = 10^{24}$. That gives $W_{\text{all}} \gg W_{\text{max}}$; indeed, the former is 10^{12} times larger than the latter. On the other hand, Challenge 683 ny we find that $\ln W_{\text{all}}$ and $\ln W_{\text{max}}$ agree for the first 20 digits! Even though the configuration with exactly half the particles in each state is much more rare than the general case, where the ratio is allowed to vary, the entropy turns out to be the same. Why? Challenge 684 ny

* *

If heat is due to motion of atoms, our built-in senses of heat and cold are simply detectors of motion. How could they work? Challenge 685 ny

By the way, the senses of smell and taste can also be seen as motion detectors, as they signal the presence of molecules flying around in air or in liquids. Do you agree? Challenge 686 e

* *

The Moon has an atmosphere, although an extremely thin one, consisting of sodium (Na) and potassium (K). This atmosphere has been detected up to nine Moon radii from its surface. The atmosphere of the Moon is generated at the surface by the ultraviolet radiation from the Sun. Can you estimate the Moon's atmospheric density? Challenge 687 s

* *

Does it make sense to add a line in Table 43 for the quantity of physical action? A column? Why? Challenge 688 ny

* *

Diffusion provides a length scale. For example, insects take in oxygen through their skin. As a result, the interiors of their bodies cannot be much more distant from the surface than about a centimetre. Can you list some other length scales in nature implied by diffusion processes? Challenge 689 s

* *

Rising warm air is the reason why many insects are found in tall clouds in the evening. Many insects, especially that seek out blood in animals, are attracted to warm and humid air.

* *

Thermometers based on mercury can reach 750°C. How is this possible, given that mercury boils at 357°C? Challenge 690 s

* *

What does a burning candle look like in weightless conditions? Challenge 691 s

* *

It is possible to build a power station by building a large chimney, so that air heated by the Sun flows upwards in it, driving a turbine as it does so. It is also possible to make a power station by building a long vertical tube, and letting a gas such as ammonia rise into it which is then liquefied at the top by the low temperatures in the upper atmosphere; as it falls back down a second tube as a liquid – just like rain – it drives a turbine. Why are such schemes, which are almost completely non-polluting, not used yet? Challenge 692 s

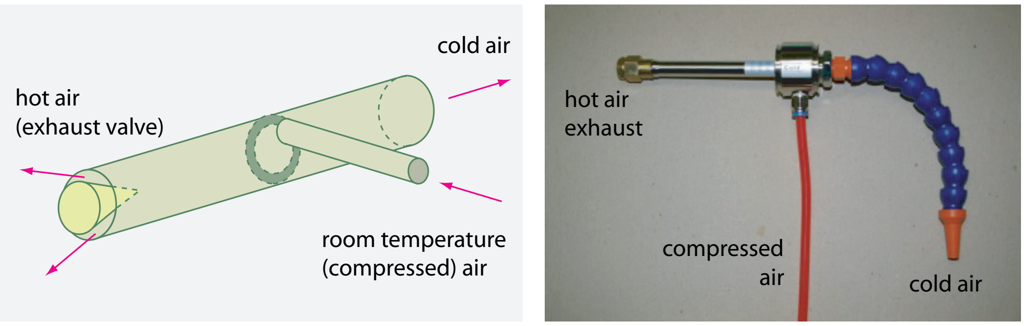


FIGURE 285 The design of the Wirbelrohr or Ranque–Hilsch vortex tube, and a commercial version, about 40 cm in size, used to cool manufacturing processes (© Coolquip).

* *

One of the most surprising devices ever invented is the *Wirbelrohr* or Ranque–Hilsch vortex tube. By blowing compressed air at room temperature into it at its midpoint, two flows of air are formed at its ends. One is extremely cold, easily as low as −50°C, and one extremely hot, up to 200°C. No moving parts and no heating devices are found inside. How does it work? Challenge 693 s

* *

Thermoacoustic engines, pumps and refrigerators provide many strange and fascinating applications of heat. For example, it is possible to use loud sound in closed metal chambers to move heat from a cold place to a hot one. Such devices have few moving parts and are being studied in the hope of finding practical applications in the future. Ref. 318

* *

Does a closed few-particle system contradict the second principle of thermodynamics? Challenge 694 s

* *

What happens to entropy when gravitation is taken into account? We carefully left gravitation out of our discussion. In fact, gravitation leads to many new problems – just try to think about the issue. For example, Jacob Bekenstein has discovered that matter reaches its highest possible entropy when it forms a black hole. Can you confirm this? Challenge 695 s

* *

The numerical values – but not the units! – of the Boltzmann constant $k = 1.38 \cdot 10^{-23}\,\mathrm{J/K}$ and the combination h/ce – where h is Planck's constant, c the speed of light and e the electron charge – agree in their exponent and in their first three digits. How can you dismiss this as mere coincidence? Challenge 696 s

* *

Mixing is not always easy to perform. The experiment of Figure 286 gives completely different results with water and glycerine. Can you guess them? Challenge 697 s

* *

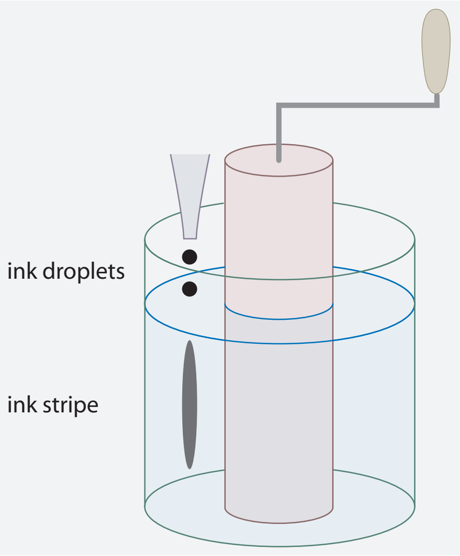


FIGURE 286 What happens to the ink stripe if the inner cylinder is turned a few times in one direction, and then turned back by the same amount?

Challenge 698 s How can you get rid of chewing gum in clothes?

* *

With the knowledge that air consists of molecules, Maxwell calculated that ''each particle makes 8 077 200 000 collisions per second''. How did he do this? Challenge 699 ny

* *

A perpetuum mobile 'of the second kind' is a machine that converts heat into motion without the use of a second, cooler bath. Entropy implies that such a device does not exist. Can you show this? Challenge 700 e

* *

There are less-well known arguments proving the existence of atoms. In fact, two everyday observations prove the existence of atoms: reproduction and memory. Why? Challenge 701 ny

* *

In the context of lasers and of spin systems, it is fun to talk about negative temperature. Why is this not really sensible? Challenge 702 s

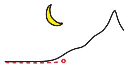
Summary on heat and time-invariance

Microscopic motion due to gravity and electric interactions, thus all microscopic motion in everyday life, is *reversible*: such motion can occur backwards in time. In other words, motion due to gravity and electromagnetism is *symmetric under motion reversal* or, as is often incorrectly stated, under 'time reversal'.

Nevertheless, everyday motion is *irreversible*, because there are no completely closed systems in everyday life. Lack of closure leads to fluctuations; fluctuations lead to friction.

Equivalently, irreversibility results from the extremely low probability of motion reversal in many-particle systems. Macroscopic irreversibility does not contradict microscopic reversibility.

For these reasons, in everyday life, entropy in closed systems never decreases. This leads to a famous issue: how can biological evolution be reconciled with entropy increase? Let us have a look.



Chapter 14

SELF-ORGANIZATION AND CHAOS – THE SIMPLICITY OF COMPLEXITY

> To speak of non-linear physics is like calling zoology the study of non-elephant animals.
>
> Stanislaw Ulam

Ref. 319

Page 242

Ref. 320

Challenge 703 s

In our list of global descriptions of motion, the study of self-organization is the high point. Self-organization is the appearance of order. In physics, *order* is a term that includes *shapes*, such as the complex symmetry of snowflakes; *patterns*, such as the stripes of zebras and the ripples on sand; and *cycles*, such as the creation of sound when singing. When we look around us, we note that every example of what we call *beauty* is a combination of shapes, patterns and cycles. (Do you agree?) Self-organization can thus be called the study of the origin of beauty. Table 50 shows how frequently the appearance of order shapes our environment.

TABLE 50 Some rhythms, patterns and shapes observed in nature.

Observation	Driving ‘force’	Restoring ‘force’	Typ. scale
Fingerprint	chemical reactions	diffusion	0.1 mm
Clock ticking	falling weight	friction	1 s
Chalk squeaking due to stick-slip instability	motion	friction	600 Hz
Musical note generation in violin	bow motion	friction	600 Hz
Musical note generation in flute	air flow	turbulence	400 Hz
Train oscillations transversally to the track	motion	friction	0.3 Hz
Flow structures in waterfalls and fountains	water flow	turbulence	10 cm
Jerky detachment of scotch tape	pulling speed	sticking friction	0.1 Hz
Radius oscillations in spaghetti and polymer fibre production	extrusion speed	friction	10 cm
Patterns on buckled metal plates and foils	deformation	stiffness	depend on thickness

TABLE 50 (Continued) Some rhythms, patterns and shapes observed in nature.

OBSERVATION	DRIVING 'FORCE'	RESTORING 'FORCE'	TYP. SCALE
Flapping of flags in steady wind	air flow	stiffness	20 cm
Dripping of water tap	water flow	surface tension	1 Hz
Bubble stream from a beer glass irregularity	dissolved gas pressure	surface tension	0.1 Hz, 1 mm
Raleigh–Bénard instability	temperature gradient	diffusion	0.1 Hz, 1 mm
Couette–Taylor flow	speed gradient	friction	0.1 Hz, 1 mm
Bénard–Marangoni flow, sea wave generation	surface tension	viscosity	0.1 Hz, 1 mm
Karman wakes, Emmon spots, Osborne Reynolds flow	momentum	viscosity	from mm to km
Regular bangs in a car exhaustion pipe	flow	pressure resonances	0.3 Hz
Regular cloud arrangements	flow	diffusion	0.5 km
El Niño	flow	diffusion	5 to 7 years
Wine arcs on glass walls	surface tension	binary mixture	0.1 Hz, 1 mm
Ferrofluids surfaces in magnetic fields	magnetic energy	gravity	3 mm
Patterns in liquid crystals	electric energy	stress	1 mm, 3 s
Flickering of aging fluorescence lights	electron flow	diffusion	1 Hz
Surface instabilities of welding	electron flow	diffusion	1 cm
Tokamak plasma instabilities	electron flow	diffusion	10 s
Snowflake formation and other dendritic growth processes	concentration gradient	surface diffusion	10 μm
Solidification interface patterns, e.g. in CBr_4	entropy flow	surface tension	1 mm
Periodic layers in metal corrosion	concentration gradients	diffusion	10 μm
Hardening of steel by cold working	strain	dislocation motion	5 μm
Labyrinth structures in proton irradiated metals	particle flow	dislocation motion	5 μm
Patterns in laser irradiated Cd-Se alloys	laser irradiation	diffusion	50 μm

TABLE 50 (Continued) Some rhythms, patterns and shapes observed in nature.

Observation	Driving 'force'	Restoring 'force'	Typ. Scale
Dislocation patterns and density oscillations in fatigued Cu single crystals	strain	dislocation motion	10 μm 100 s
Laser light emission, its cycles and chaotic regimes	pumping energy	light losses	10 ps to 1 ms
Rotating patterns from shining laser light on the surface of certain electrolytes	light energy	diffusion	1 mm
Belousov-Zhabotinski reaction patterns and cycles	concentration gradients	diffusion	1 mm, 10 s
Flickering of a burning candle	heat and concentration gradients	thermal and substance diffusion	0.1 s
Regular sequence of hot and cold flames in carbohydrate combustion	heat and concentration gradients	thermal and substance diffusion	1 cm
Feedback whistle from microphone to loudspeaker	amplifiers	electric losses	1 kHz
Any electronic oscillator in radio sets, television sets, computers, mobile phones, etc.	power supply	resistive losses	1 kHz to 30 GHz
Periodic geyser eruptions	underground heating	evaporation	10 min
Periodic earthquakes at certain faults	tectonic motion	ruptures	1 Ms
Hexagonal patterns in basalt rocks	heating	heat diffusion	1 m
Hexagonal patterns on dry soil	regular temperature changes	water diffusion	0.5 m
Periodic intensity changes of the Cepheids and other stars	nuclear fusion	energy emission	3 Ms
Convection cells on the surface of the Sun	nuclear fusion	energy emission	1000 km
Formation and oscillations of the magnetic field of the Earth and other celestial bodies	charge separation due to convection and friction	resistive losses	100 ka
Wrinkling/crumpling transition	strain	stiffness	1 mm

TABLE 50 (Continued) Some rhythms, patterns and shapes observed in nature.

Observation	Driving 'force'	Restoring 'force'	Typ. Scale
Patterns of animal furs	chemical concentration	diffusion	1 cm
Growth of fingers and limbs	chemical concentration	diffusion	1 cm
Symmetry breaking in embryogenesis, such as the heart on the left	probably molecular chirality plus chemical concentration	diffusion	1 m
Cell differentiation and appearance of organs during growth	chemical concentration	diffusion	10 μm to 30 m
Prey–predator oscillations	reproduction	hunger	3 to 17 a
Thinking	neuron firing	heat dissipation	1 ms, 100 μm

Appearance of order

The appearance of order is a general observation across nature. *Fluids* in particular exhibit many phenomena where order appears and disappears. Examples include the more or less regular flickering of a burning candle, the flapping of a flag in the wind, the regular stream of bubbles emerging from small irregularities in the surface of a beer or champagne glass, and the regular or irregular dripping of a water tap. Figure 251 shows some additional examples, and so do the figures in this chapter. Other examples include the appearance of clouds and of regular cloud arrangements in the sky. It can be fascinating to ponder, during an otherwise boring flight, the mechanisms behind the formation of the cloud shapes and patterns you see from the aeroplane. A typical cloud has a mass density of 0.3 to 5 g/m^3, so that a large cloud can contain several hundred tons of water.

Page 355

Challenge 704 e

Other cases of self-organization are mechanical, such as the formation of mountain ranges when continents move, the creation of earthquakes, or the formation of laughing folds at the corners of human eyes.

All *growth* processes are self-organization phenomena. The appearance of order is found in the cell differentiation in an embryo inside a woman's body; the formation of colour patterns on tigers, tropical fish and butterflies; the symmetrical arrangements of flower petals; the formation of biological rhythms; and so on.

Page 46

Have you ever pondered the incredible way in which teeth grow? A practically inorganic material forms shapes in the upper and the lower rows fitting exactly into each other. How this process is controlled is still a topic of research. Also the formation, before and after birth, of neural networks in the brain is another process of self-organization. Even the physical processes at the basis of thinking, involving changing electrical signals, is to be described in terms of self-organization.

Vol. III, page 242

Biological evolution is a special case of growth. Take the evolution of animal shapes. It turns out that snake tongues are forked because that is the most efficient shape for following chemical trails left by prey and other snakes of the same species. (Snakes smell

Ref. 321

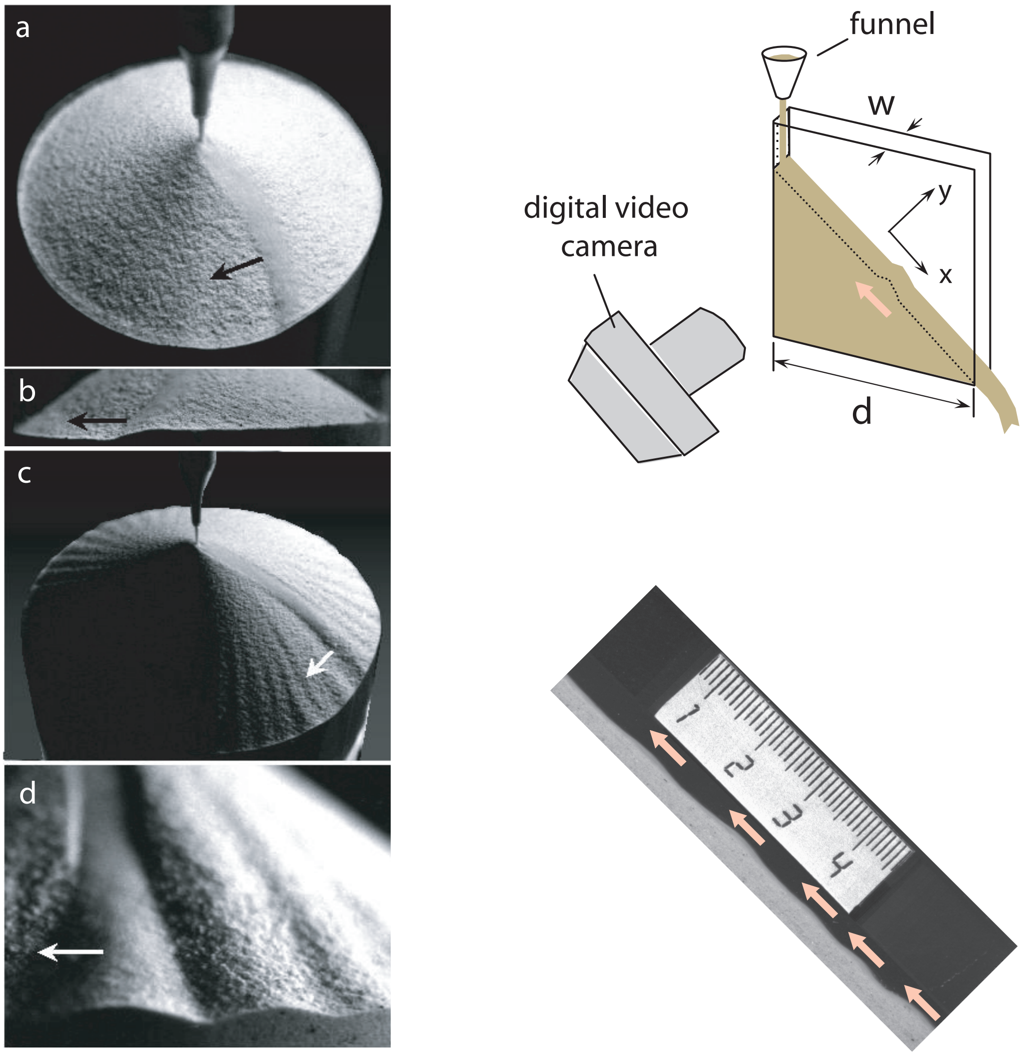


FIGURE 287 Examples of self-organization for sand: spontaneous appearance of a temporal cycle (a and b), spontaneous appearance of a periodic pattern (b and c), spontaneous appearance of a spatiotemporal pattern, namely solitary waves (right) (© Ernesto Altshuler et al.).

with the help of their tongue.) How many tips would the tongues of flying reptiles need, Challenge 705 e such as flying dragons?

Ref. 322 The fixed number of fingers in human hands are also consequence of self-Vol. III, page 298 organization. The number of petals of flowers may or may not be due to self-organization.

Studies into the conditions required for the appearance or disappearance of order have shown that their description requires only a few common concepts, independently of the details of the physical system. This is best seen looking at a few simple examples.

TABLE 51 Patterns and a cycle on horizontal sand and on sand-like surfaces in the sea and on land.

Pattern/cycle	Period	Amplitude	Origin
Under water			
Ripples	5 cm	5 mm	water waves
Megaripples	1 m	0.1 m	tides
Sand waves	100 to 800 m	5 m	tides
Sand banks	2 to 10 km	2 to 20 m	tides
In air			
Ripples	0.1 m	0.05 m	wind
Singing sand	65 to 110 Hz	up to 105 dB	wind on sand dunes, avalanches making the dune vibrate
Road corrugations	0.3 to 0.9 m	0.05 m	wheels
Ski moguls	5 to 6 m	up to 1 m	skiers
Elsewhere			
On Mars	a few km	few tens of m	wind

FIGURE 288 Road corrugations (courtesy David Mays).

Self-organization in sand

All the richness of self-organization reveals itself in the study of plain sand. Why do sand dunes have ripples, as does the sand floor at the bottom of the sea? How do avalanches occur on steep heaps of sand? How does sand behave in hourglasses, in mixers, or in vibrating containers? The results are often surprising.

An overview of self-organization phenomena in sand is given in Table 51. For example, as recently as 2006, the Cuban research group of Ernesto Altshuler and his colleagues discovered solitary waves on sand flows (shown in Figure 287). Ref. 323 They had already discovered the revolving river effect on sand piles, shown in the same figure, in 2002. Even

FIGURE 289 Oscillons formed by shaken bronze balls; horizontal size is about 2 cm (© Paul Umbanhowar)

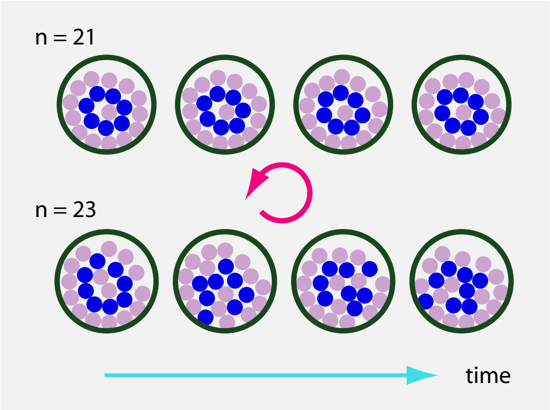


FIGURE 290 Magic numbers: 21 spheres, when swirled in a dish, behave differently from non-magic numbers, like 23, of spheres (redrawn from photographs © Karsten Kötter).

more surprisingly, these effects occur only for Cuban sand, and a few rare other types of sand. The reasons are still unclear.

Ref. 324 Similarly, in 1996 Paul Umbanhowar and his colleagues found that when a flat container holding tiny bronze balls (around 0.165 mm in diameter) is shaken up and down in vacuum at certain frequencies, the surface of this bronze 'sand' forms stable heaps. They are shown in Figure 289. These heaps, so-called *oscillons*, also bob up and down. The oscillons can move and interact with one another.

Oscillons in bronze sand are a simple example of a general effect in nature:

▷ *Discrete* systems with non-linear interactions can exhibit *localized excitations*. Ref. 325

This fascinating topic is just beginning to be researched. It might well be that one day it will yield results relevant to our understanding of the growth of organisms.

Sand shows many other pattern-forming processes.

- A mixture of sand and sugar, when poured onto a heap, forms regular layered structures that in cross-section look like zebra stripes.
- Horizontally rotating cylinders with binary mixtures inside them separate the mixture out over time.
- Take a container with two compartments separated by a 1 cm wall. Fill both halves with sand and rapidly shake the whole container with a machine. Over time, all the sand will spontaneously accumulate in one half of the container.
- In sand, people have studied the various types of sand dunes that 'sing' when the wind blows over them. Ref. 326

FIGURE 291 Self-organization: a growing snowflake. (QuickTime film © Kenneth Libbrecht)

Ref. 327 Challenge 706 s Page 319

— Also the corrugations formed by traffic on roads without tarmac, the washboard roads shown in Figure 288, are an example of self-organization. These corrugation patterns often move, over time, *against* the traffic direction. Can you explain why? The moving ski moguls mentioned above also belong here.

In fact, the behaviour of sand and dust is proving to be such a beautiful and fascinating topic that the prospect of each human returning to dust does not look so grim after all.

Self-organization of spheres

A stunningly simple and beautiful example of self-organization is the effect discovered in 1999 by Karsten Kötter and his group. They found that the behaviour of a set of spheres swirled in a dish depends on the number of spheres used. Usually, all the spheres get continuously mixed up. But for certain 'magic' numbers, such as 21, stable ring patterns emerge, for which the outside spheres remain outside and the inside ones remain inside. The rings, best seen by colouring the spheres, are shown in Figure 290. Ref. 328

Conditions for the appearance of order

The many studies of self-organizing systems have changed our understanding of nature in a number of ways. First of all, they have shown that patterns and shapes are similar to cycles: all are *due to motion*. Without motion, and thus without history, there is no order, neither patterns nor shapes nor rhythms. Every pattern has a history; every pattern is a result of *motion*. As an example, Figure 291 shows how a snowflake grows. Ref. 329

Secondly, patterns, shapes and rhythms are due to the organized motion of *large numbers of small constituents*. Systems which self-organize are always composite: they are *co-operative structures*.

Thirdly, all these systems obey evolution equations which are *non-linear* in the macroscopic configuration variables. Linear systems do not self-organize.

Fourthly, the appearance and disappearance of order depends on the strength of a driving force or driving process, the so-called *order parameter*.

Finally, all order and all structures appear when two general types of motion compete with each other, namely a 'driving', energy-adding process, and a *'dissipating', braking mechanism.* Thermodynamics thus plays a role in all self-organization. Self-organizing systems are always *dissipative systems*, and are always far from equilibrium. When the driving and the dissipation are of the same order of magnitude, and when the key behaviour of the system is not a linear function of the driving action, order may appear.*

The mathematics of order appearance

Every pattern, every shape and every rhythm or cycle can be described by some observable A that describes the amplitude of the pattern, shape or rhythm. For example, the amplitude A can be a length for sand patterns, or a chemical concentration for biological systems, or an air pressure for sound appearance.

Order appears when the amplitude A differs from zero. To understand the appearance of order, we have to understand the evolution of the amplitude A. The study of order has shown that this amplitude always follows similar evolution equations, *independently* of the physical mechanism of the system. This surprising result unifies the whole field of self-organization.

All self-organizing systems at the onset of order appearance can be described by equations for the pattern amplitude A of the general form

$$\frac{\partial A(t,x)}{\partial t} = \lambda A - \mu |A|^2 A + \kappa \, \Delta A + \text{higher orders} \, . \tag{124}$$

Here, the observable A – which can be a real or a complex number, in order to describe phase effects – is the observable that appears when order appears, such as the oscillation amplitude or the pattern amplitude. The first term λA is the *driving term*, in which λ is a parameter describing the strength of the driving. The next term is a typical *non-linearity* in A, with μ a parameter that describes its strength, and the third term $\kappa \, \Delta A = \kappa(\partial^2 A/\partial x^2 + \partial^2 A/\partial y^2 + \partial^2 A/\partial z^2)$ is a typical diffusive and thus *dissipative* term.

We can distinguish two main situations. In cases where the dissipative term plays no role ($\kappa = 0$), we find that when the driving parameter λ increases above zero, a *temporal* oscillation appears, i.e., a stable *limit cycle* with non-vanishing amplitude. Challenge 707 ny In cases where the diffusive term does play a role, equation (124) describes how an amplitude for a *spatial* oscillation appears when the driving parameter λ becomes positive, as the solution $A = 0$ then becomes spatially unstable. Challenge 708 ny

In both cases, the onset of order is called a *bifurcation*, because at this critical value of the driving parameter λ the situation with amplitude zero, i.e., the homogeneous (or unordered) state, becomes unstable, and the ordered state becomes stable. *In non-linear systems, order is stable.* This is the main conceptual result of the field. Equation (124) and its numerous variations allow us to describe many phenomena, ranging from spirals,

---

* To describe the 'mystery' of human life, terms like 'fire', 'river' or 'tree' are often used as analogies. These are all examples of self-organized systems: they have many degrees of freedom, have competing driving and braking forces, depend critically on their initial conditions, show chaos and irregular behaviour, and sometimes show cycles and regular behaviour. Humans and human life resemble them in all these respects; thus there is a solid basis for their use as metaphors. We could even go further and speculate that pure beauty *is* pure self-organization. The lack of beauty indeed often results from a disturbed equilibrium between external braking and external driving.

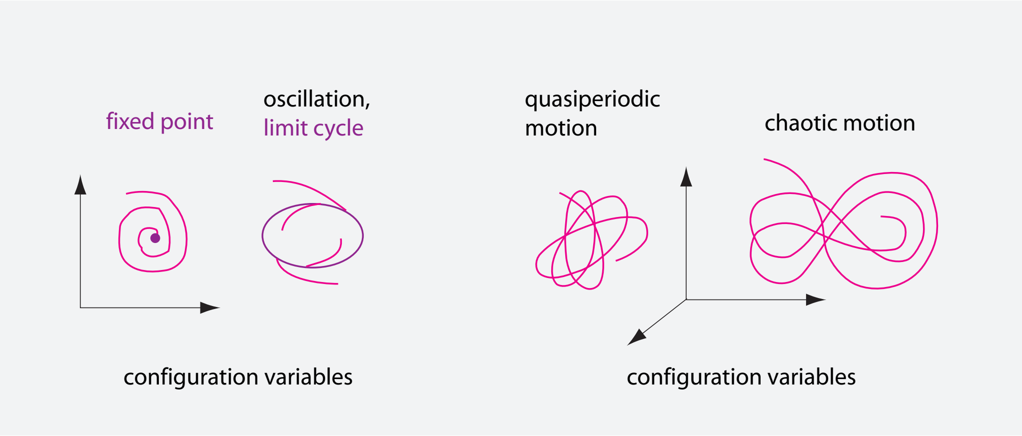


FIGURE 292 Examples of different types of motion in configuration space.

Ref. 330 waves, hexagonal patterns, and topological defects, to some forms of turbulence. For every physical system under study, the main task is to distil the observable A and the parameters λ, μ and κ from the underlying physical processes.

In summary, the appearance of order can be described to generally valid equations. Self-organization is, fundamentally, a simple process. In short: beauty is simple.

Self-organization is a vast field which is yielding new results almost by the week. To discover new topics of study, it is often sufficient to keep your eye open; most effects are comprehensible without advanced mathematics. Enjoy the hunting! Challenge 709 e

CHAOS

Most systems that show self-organization also show another type of motion. When the driving parameter of a self-organizing system is increased to higher and higher values, order becomes more and more irregular, and in the end one usually finds *chaos*.

For physicists, chaotic motion is the most irregular type of motion.* Chaos can be defined independently of self-organization, namely as that motion of systems for which small changes in initial conditions evolve into large changes of the motion (exponentially with time). This is illustrated in Figure 293. More precisely,

> ▷ (Physical) *chaos* is irregular motion characterized by a positive *Lyapounov exponent* in the presence of a strictly valid evolution.

A simple chaotic system is the damped pendulum above three magnets. Figure 294 shows how regions of predictability (around the three magnet positions) gradually change into a chaotic region, i.e., a region of effective unpredictability, for higher initial amplitudes. The weather is also a chaotic system, as are dripping water-taps, the fall of dice, and many

---

* On the topic of chaos, see the beautiful book by Heinz-Otto Peitgen, Hartmut Jürgens & Dietmar Saupe, *Chaos and Fractals*, Springer Verlag, 1992. It includes stunning pictures, the necessary mathematical background, and some computer programs allowing personal exploration of the topic. ‘Chaos’ is an old word: according to Greek mythology, the first goddess, Gaia, i.e., the Earth, emerged from the chaos existing at the beginning. She then gave birth to the other gods, the animals and the first humans.

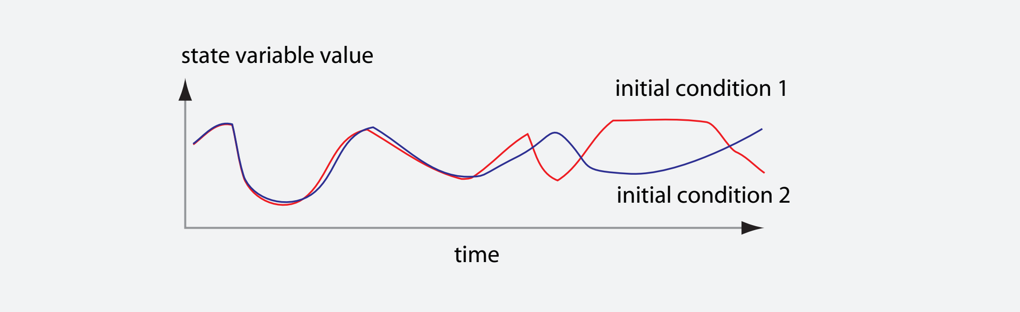


FIGURE 293 Chaos as sensitivity to initial conditions.

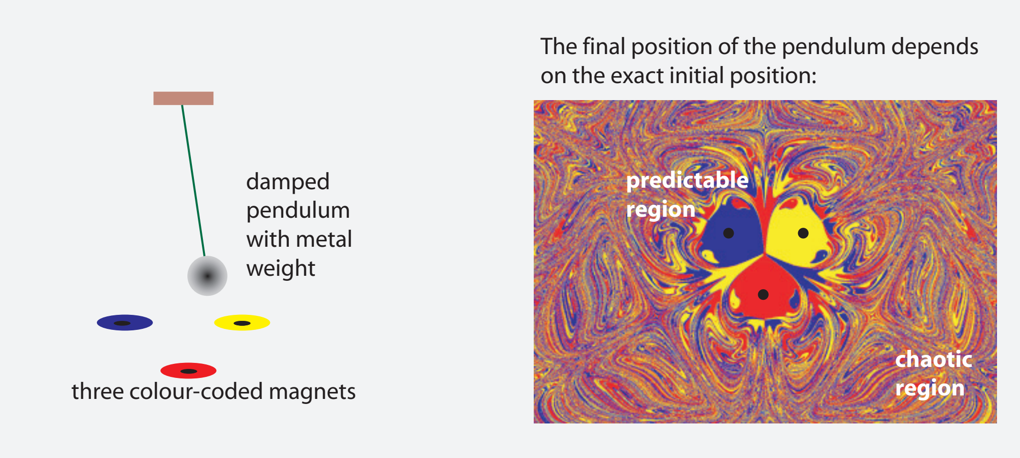


FIGURE 294 A simple chaotic system: a metal pendulum over three magnets (fractal © Paul Nylander).

other everyday systems. For example, research on the mechanisms by which the heart beat is generated has shown that the heart is not an oscillator, but a chaotic system with irregular cycles. This allows the heart to be continuously ready for demands for changes in beat rate which arise once the body needs to increase or decrease its efforts. Ref. 331

There is chaotic motion also in machines: chaos appears in the motion of trains on the rails, in gear mechanisms, and in fire-fighter's hoses. The precise study of the motion in a zippo cigarette lighter will probably also yield an example of chaos. The mathematical description of chaos – simple for some textbook examples, but extremely involved for others – remains an important topic of research. Challenge 710 ny

Incidentally, can you give a simple argument to show that the so-called *butterfly effect* does not exist? This 'effect' is often cited in newspapers. The claim is that non-linearities imply that a small change in initial conditions can lead to large effects; thus a butterfly wing beat is alleged to be able to induce a tornado. Even though non-linearities do indeed lead to growth of disturbances, the butterfly 'effect' has *never* been observed. Thus it does *not* exist. This 'effect' exists only to sell books and to get funding. Challenge 711 s

All the steps from disorder to order, quasiperiodicity and finally to chaos, are examples of self-organization. These types of motion, illustrated in Figure 292, are observed in many fluid systems. Their study should lead, one day, to a deeper understanding of the

Ref. 332 mysteries of turbulence. Despite the fascination of this topic, we will not explore it further, because it does not help solving the mystery of motion.

EMERGENCE

> “From a drop of water a logician could predict an Atlantic or a Niagara.”
>
> Arthur Conan Doyle, *A Study in Scarlet*

Self-organization is of interest also for a more general reason. It is sometimes said that our ability to formulate the patterns or rules of nature from observation does not imply the ability to predict *all* observations from these rules. According to this view, so-called ‘emergent’ properties exist, i.e., properties appearing in complex systems as something *new* that cannot be deduced from the properties of their parts and their interactions. (The ideological backdrop to this view is obvious; it is the latest attempt to fight the idea of determinism.) The study of self-organization has definitely settled this debate. The properties of water molecules do allow us to predict Niagara Falls.* Similarly, the diffusion of signal molecules do determine the development of a single cell into a full human being: in particular, cooperative phenomena determine the places where arms and legs are formed; they ensure the (approximate) right–left symmetry of human bodies, prevent mix-ups of connections when the cells in the retina are wired to the brain, and explain the Ref. 334 fur patterns on zebras and leopards, to cite only a few examples. Similarly, the mechanisms at the origin of the heart beat and many other cycles have been deciphered. Several cooperative phenomena in fluids have been simulated even down to the molecular level.

Self-organization provides general principles which allow us in principle to predict the behaviour of complex systems of any kind. They are presently being applied to the most complex system in the known universe: the human brain. The details of how it learns to coordinate the motion of the body, and how it extracts information from the images in the eye, are being studied intensely. The ongoing work in this domain is fasRef. 335 cinating. (A neglected case of self-organization is *laughter*, but also *humour* itself.) If you Challenge 713 e plan to become a scientist, consider taking this path.

Self-organization research provided the final arguments that confirmed what J. Offrey de la Mettrie stated and explored in his famous book *L’homme machine* in 1748: humans are complex machines. Indeed, the lack of understanding of complex systems in the past was due mainly to the restrictive teaching of the subject of motion, which usually concentrated – as we do in this walk – on examples of motion in *simple* systems. The concepts of self-organization allow us to understand and to describe what happens during the functioning and the growth of organisms.

Even though the subject of self-organization provides fascinating insights, and will do so for many years to come, we now leave it. We continue with our own adventure,

---

* Already small versions of Niagara Falls, namely dripping water taps, show a large range of cooperative Ref. 333 phenomena, including the chaotic, i.e., non-periodic, fall of water drops. This happens when the water flow Challenge 712 e rate has the correct value, as you can verify in your own kitchen.

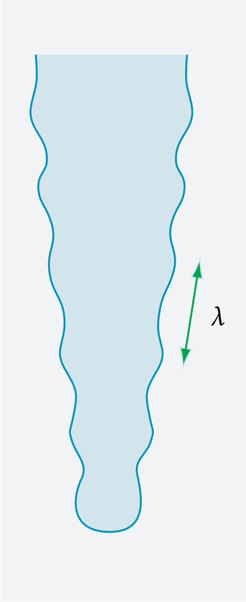


FIGURE 295 The wavy surface of icicles.

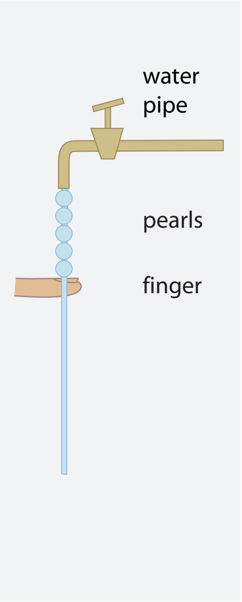


FIGURE 296 Water pearls.

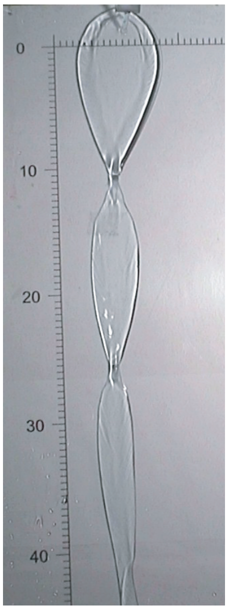


FIGURE 297 A braiding water stream (© Vakhtang Putkaradze).

namely to explore the *basics* of motion.

> Ich sage euch: man muss noch Chaos in sich haben, um einen tanzenden Stern gebären zu können. Ich sage euch: ihr habt noch Chaos in euch.*
>
> Friedrich Nietzsche, *Also sprach Zarathustra*.

CURIOSITIES AND FUN CHALLENGES ABOUT SELF-ORGANIZATION

Ref. 336 Challenge 714 ny All icicles have a wavy surface, with a crest-to-crest distance of about 1 cm, as shown in Figure 295. The distance is determined by the interplay between water flow and surface cooling. How? (Indeed, stalactites do not show the effect.)

* *

Challenge 715 ny When a fine stream of water leaves a water tap, putting a finger in the stream leads to a wavy shape, as shown in Figure 296. Why?

* *

The research on sand has shown that it is often useful to introduce the concept of *granular temperature*, which quantifies how fast a region of sand moves. Research into this field is still in full swing.

---

[*] 'I tell you: one must have chaos inside oneself, in order to give birth to a dancing star. I tell you: you still have chaos inside you.'

FIGURE 298 The Belousov-Zhabotinski reaction: the liquid periodically changes colour, both in space and time (© Yamaguchi University).

* *

When water emerges from a oblong opening, the stream forms a braid pattern, as shown in Figure 297. This effect results from the interplay and competition between inertia and surface tension: inertia tends to widen the stream, while surface tension tends to narrow it. Predicting the distance from one narrow region to the next is still a topic of research. Ref. 337

If the experiment is done in free air, without a plate, one usually observes an additional effect: there is a *chiral* braiding at the narrow regions, induced by the asymmetries of the water flow. You can observe this effect in the toilet! Scientific curiosity knows no limits: are you a right-turner or a left-turner, or both? On every day? Challenge 716 e

* *

When wine is made to swirl in a wine glass, after the motion has calmed down, the wine flowing down the glass walls forms little arcs. Can you explain in a few words what forms them? Challenge 717 ny

* *

How does the average distance between cars parked along a street change over time, assuming a constant rate of cars leaving and arriving? Challenge 718 ny

* *

A famous case of order appearance is the *Belousov-Zhabotinski reaction*. This mixture of chemicals spontaneously produces spatial and temporal patterns. Thin layers produce slowly rotating spiral patterns, as shown in Figure 298; Large, stirred volumes oscillate back and forth between two colours. A beautiful movie of the oscillations can be found on www.uni-r.de/Fakultaeten/nat_Fak_IV/Organische_Chemie/Didaktik/Keusch/D-oscill-d.htm. The exploration of this reaction led to the Nobel Prize in Chemistry for Ilya Prigogine in 1997.

* *

Gerhard Müller has discovered a simple but beautiful way to observe self-organization in solids. His system also provides a model for a famous geological process, the formation of

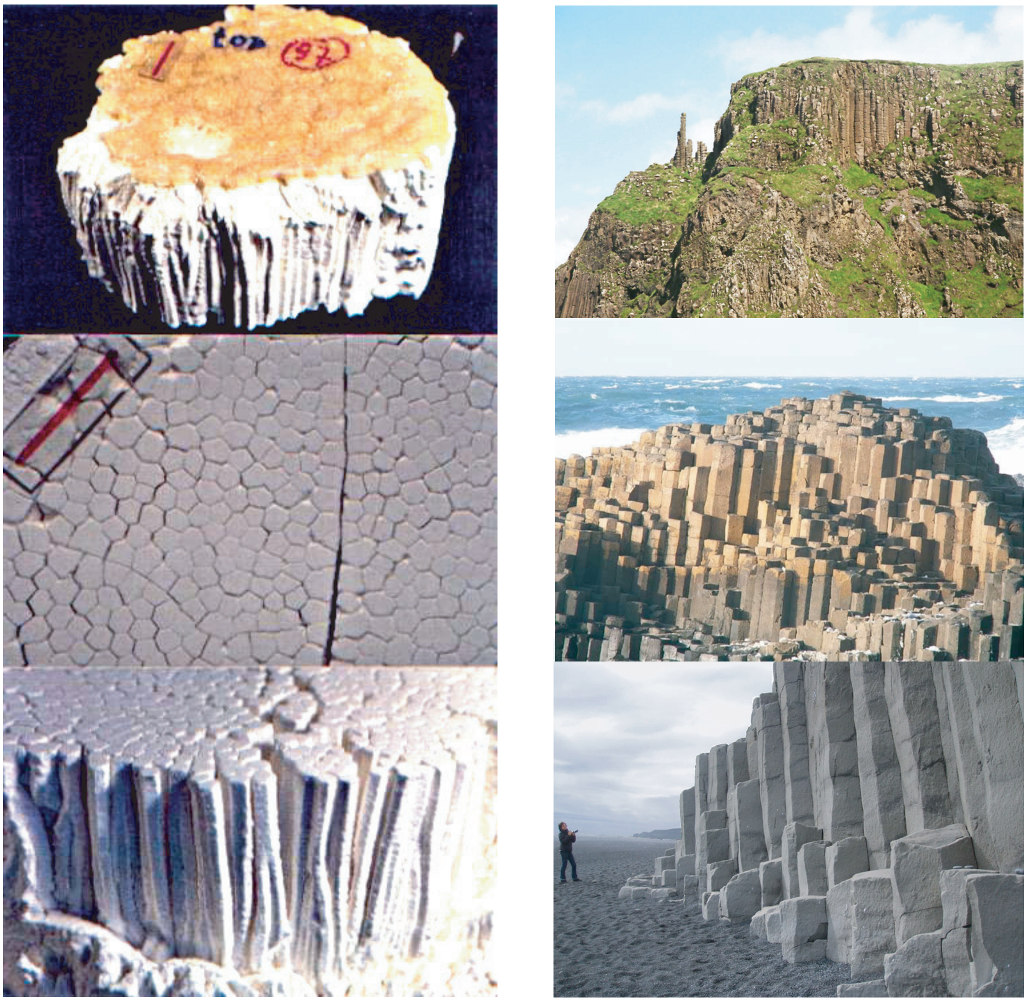


FIGURE 299 A famous correspondence: on the left, hexagonal columns in starch, grown in a kitchen pan (the red lines are 1 cm in length), and on the right, hexagonal columns in basalt, grown from lava in Northern Ireland (top right, view of around 300 m, and middle right, view of around 40 m) and in Iceland (view of about 30 m, bottom right) (© Gerhard Müller, Raphael Kessler - www.raphaelk.co.uk, Bob Pohlad, and Cédric Hüsler).

hexagonal columns in basalt, such as the Giant's Causeway in Northern Ireland. Similar formations are found in many other places of the Earth. Just take some rice flour or Ref. 338 corn starch, mix it with about half the same amount of water, put the mixture into a Challenge 719 e pan and dry it with a lamp: hexagonal columns form. The analogy with basalt structures is possible because the drying of starch and the cooling of lava are diffusive processes governed by the same equations, because the boundary conditions are the same, and because both materials respond to cooling with a small reduction in volume.

* *

Water flow in pipes can be laminar (smooth) or turbulent (irregular and disordered).

The transition depends on the diameter d of the pipe and the speed v of the water. The transition usually happens when the so-called *Reynolds number* – defined as $\mathrm{Re} = vd/\eta$ becomes greater than about 2000. (The Reyonolds number is one of the few physical observables with a conventional abbreviation made of two letters.) Here, η is the *kinematic viscosity* of the water, around $1\,\mathrm{mm}^2/\mathrm{s}$; in contrast, the *dynamic viscosity* is defined as Ref. 339 $\mu = \eta\rho$, where ρ is the density of the fluid. A high Reynolds number means a high ratio between inertial and dissipative effects and specifies a turbulent flow; a low Reynolds number is typical of *viscous* flow.

Modern, careful experiments show that with proper handling, laminar flows can be produced up to $\mathrm{Re} = 100\,000$. A linear analysis of the equations of motion of the fluid, the Navier–Stokes equations, even predicts stability of laminar flow for *all* Reynolds numbers. This riddle was solved only in the years 2003 and 2004. First, a complex mathematical analysis showed that the laminar flow is not always stable, and that the transition to turbulence in a long pipe occurs with travelling waves. Then, in 2004, careful experiments showed that these travelling waves indeed appear when water is flowing through a pipe at large Reynolds numbers. Ref. 340

* *

For more beautiful pictures on self-organization in fluids, see the mentioned serve.me.nus.edu.sg/limtt website.

* *

Chaos can also be observed in simple (and complicated) electronic circuits. If the electronic circuit that you have designed behaves erratically, check this option!

* *

Also *dance* is an example of self-organization. This type of self-organization takes place in the brain. Like for all complex movements, learning them is often a challenge. Nowadays Ref. 341 there are beautiful books that tell how physics can help you improve your dancing skills and the grace of your movements.

* *

Do you want to enjoy working on your PhD? Go into a scientific toy shop, and look for any toy that moves in a complex way. There are high chances that the motion is chaotic; explore the motion and present a thesis about it. For example, go to the extreme: explore the motion of a hanging rope whose upper end is externally driven. This simple system is fascinating in its range of complex motion behaviours.

* *

Self-organization is also observed in liquid corn starch–water mixtures. Enjoy the film at www.youtube.com/watch?v=f2XQ97XHjVw and watch even more bizarre effects, for humans walking over a pool filled with the liquid, on www.youtube.com/watch?v=nq3ZjY0Uf-g.

* *

Snowflakes and snow crystals have already been mentioned as examples of self-

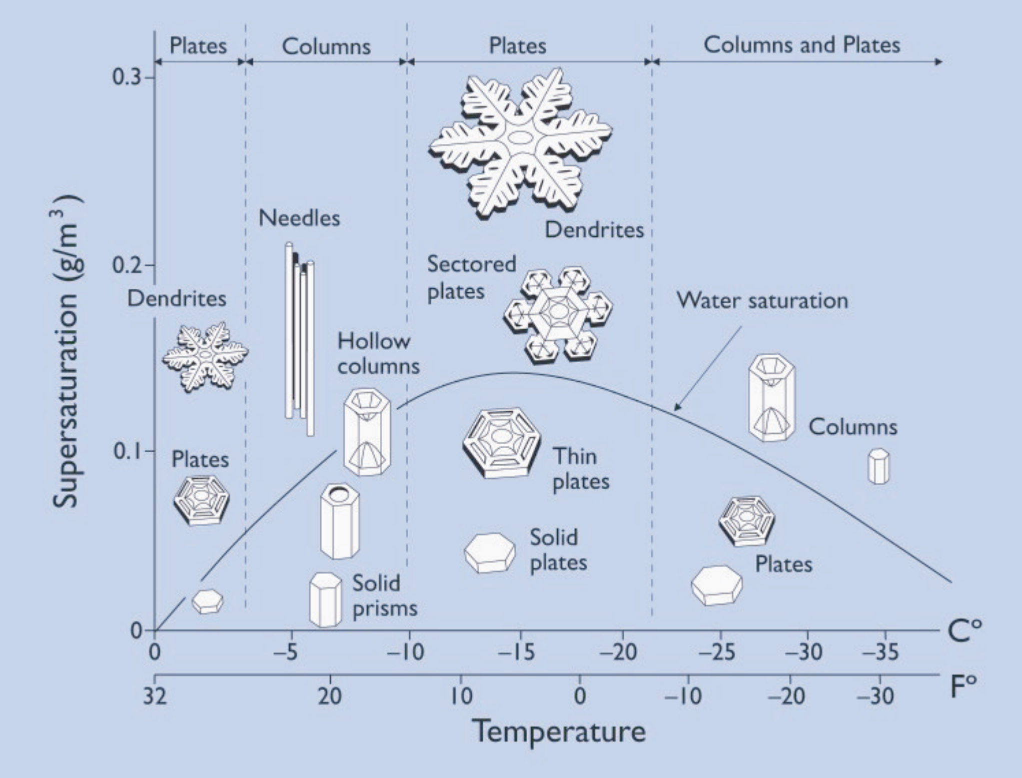


FIGURE 300 How the shape of snow crystals depend on temperature and saturation (© Kenneth Libbrecht).

organization. Figure 300 shows the general connection. To learn more about this fascinating topic, explore the wonderful website snowcrystals.com by Kenneth Libbrecht. A complete classification of snow crystals has also been developed. Ref. 342

* *

A famous example of self-organization whose mechanisms are not well-known so far is the *hiccup*. It is known that the vagus nerve plays a role in it. Like many other examples of self-organization, it takes quite some energy to get rid of a hiccup. Modern experimental research has shown that orgasms, which strongly stimulate the vagus nerve, are excellent ways to overcome hiccups. One of these researchers won the 2006 IgNobel Prize for medicine for his work.

* *

Another important example of self-organization is the *weather*. If you want to know more about the known connections between the weather and the quality of human life on Earth, free of any ideology, read the wonderful book by Reichholf. It explains how the weather between the continents is connected and describes how and why the weather changed in the last one thousand years. Ref. 343

FIGURE 301 A typical swarm of starlings that visitors in Rome can observe every autumn (© Andrea Cavagna, Physics Today).

* *

Does self-organization or biological evolution contradict the second principle of thermodynamics? Of course not.

Self-organizing systems are, by definition, open systems, and often far from equilibrium. The concept of entropy cannot be defined, in general, for such systems, and the second principle of thermodynamics does not apply to them. However, entropy can be defined for systems *near* equilibrium. In such systems the second principle of thermodynamics must be modified; it is then possible to explore and confirm that self-organization does not contradict but in fact *follows* from the modified second principle.

In particular, evolution does not contradict thermodynamics, as the Earth is not a closed thermodynamic system in equilibrium. Statements of the opposite are only made by crooks. Ref. 344

* *

Ref. 345 In 2015, three physicists predicted that there is a largest Lyapunov exponent in nature,

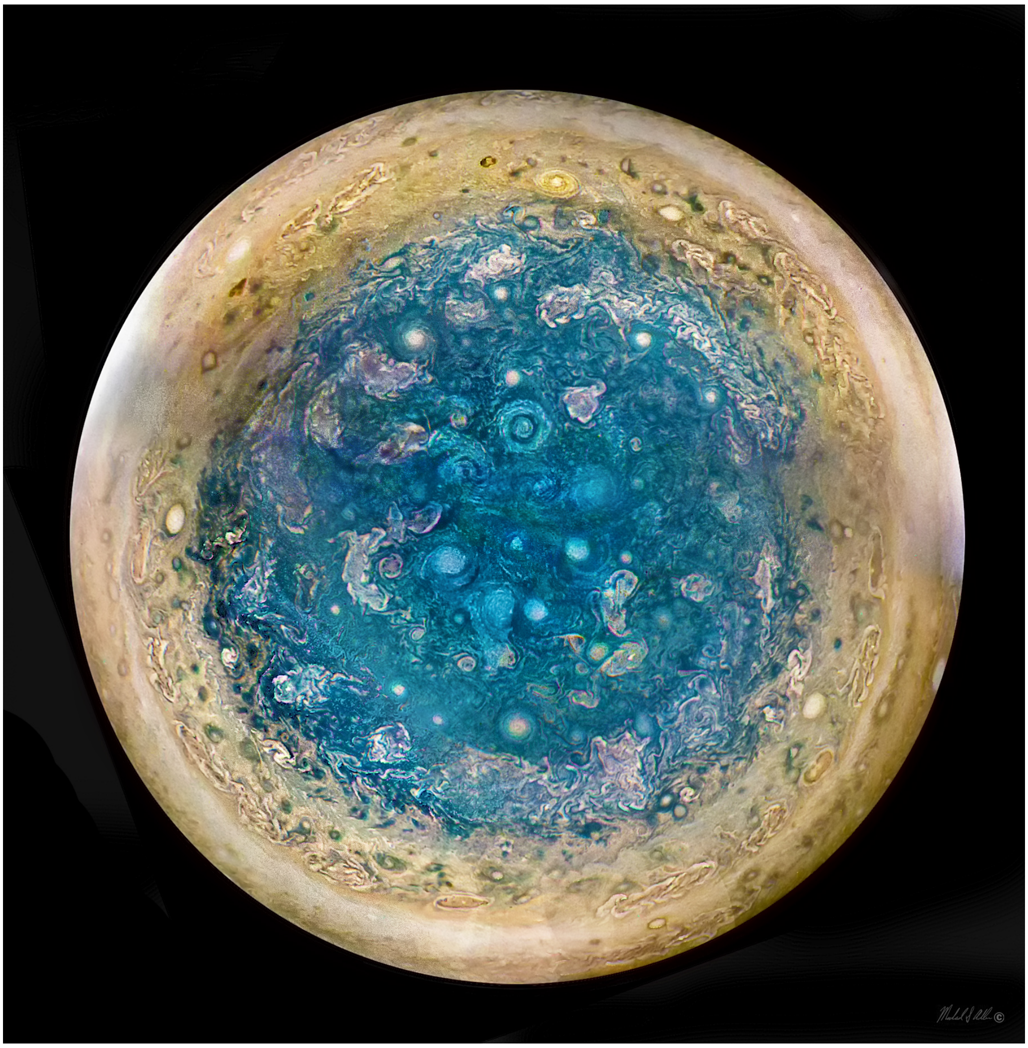


FIGURE 302 An image of the south pole of Jupiter, taken in 2019 by the Juno space probe. The cyclones have a typical diameter of 10 000 km (courtesy Msadler13, Wikimedia).

so that one always has

$$\lambda \leqslant e\pi \frac{kT}{\hbar} \tag{125}$$

where T is temperature, k is Boltzmann's constant, and $\hbar$ is the quantum of action. The growth of disorder is thus limited.

* *

Are systems that show self-organization the most complex ones that can be studied with evolution equations? No. The most complex systems are those that consist of many interacting self-organizing systems. The obvious example are *swarms*. Swarms of birds, as Ref. 346 shown in Figure 301, of fish, of insects and of people – for example in a stadium or in

Ref. 347 cars on a highway – have been studied extensively and are still a subject of research. Their beauty is fascinating.

The other example of many interconnected self-organized systems is the brain; the exploration of how the interconnected neurons work to produce our *thoughts* will occupy researchers for many years. We will explore some aspects in the next volumes.

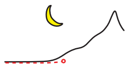
* *

Another example of the interplay between fluids, gravity, self-organisation and beauty is shown in Figure 302.

SUMMARY ON SELF-ORGANIZATION AND CHAOS

Appearance of *order*, in form of patterns, shapes and cycles, is not due to a decrease in entropy, but to a competition between driving causes and dissipative effects in open systems. Such appearance of order is a common process, is often automatic, and is predictable with (quite) simple equations. Also the growth of living systems and biological evolution are examples of appearance of order.

Chaos, the sensitivity of evolution to initial conditions, is common in strongly driven open systems. Chaos is at the basis of everyday chance, and often is described by simple equations as well. In nature, complexity is apparent. Motion is simple.



Chapter 15

FROM THE LIMITATIONS OF PHYSICS TO THE LIMITS OF MOTION

> “I only know that I know nothing.”
> Socrates, as cited by Plato

We have explored, in our environment, the concept of motion. We called this exploration of moving objects and fluids *Galilean physics*. We found that in everyday life, motion is *predictable*: nature shows no surprises and no miracles. In particular, we have found six important aspects of this predictability:

1. Everyday motion is *continuous*. Motion allows us to define space and time.
2. Everyday motion *conserves* mass, momentum, energy and angular momentum. Nothing appears out of nothing.
3. Everyday motion is *relative*: motion depends on the observer.
4. Everyday motion is *reversible*: everyday motion can occur backwards.
5. Everyday motion is *mirror-invariant*: everyday motion can occur in a mirror-reversed way.

The final property is the most important and, in addition, contains the five previous ones:

6. **Everyday motion is** ***lazy***: motion happens in a way that minimizes change, i.e., physical action.

This Galilean description of nature made engineering possible: textile machines, steam engines, combustion motors, kitchen appliances, watches, many children toys, fitness machines, medical devices and all the progress in the quality of life that came with these devices are due to the results of Galilean physics. But despite these successes, Socrates’ saying, cited above, still applies to Galilean physics: we still know almost nothing. Let us see why.

Research topics in classical dynamics

Even though mechanics and thermodynamics are now several hundred years old, research into its details is still ongoing. For example, we have already mentioned above that it is unclear whether the Solar System is stable. The long-term future of the planets is unknown! In general, the behaviour of few-body systems interacting through gravitation is still a research topic of mathematical physics. Answering the simple question of how long a given set of bodies gravitating around each other will stay together is a formidable challenge. The history of this so-called *many-body problem* is long and involved.

Ref. 348

TABLE 52 Examples of errors in state-of-the art measurements (numbers in brackets give one standard deviation in the last digits), partly taken from physics.nist.gov/constants.

Observation	Measurement	Precision / accuracy
Highest precision achieved: ratio between the electron magnetic moment and the Bohr magneton μ_e/μ_B	$-1.001\,159\,652\,180\,76(24)$	$2.6 \cdot 10^{-13}$
High precision: Rydberg constant	$10\,973\,731.568\,539(55)\ \mathrm{m}^{-1}$	$5.0 \cdot 10^{-12}$
High precision: astronomical unit	$149\,597\,870.691(30)$ km	$2.0 \cdot 10^{-10}$
Industrial precision: typical part dimension tolerance in car engine	2 µm of 20 cm	$5 \cdot 10^{-6}$
Low precision: gravitational constant G	$6.674\,28(67) \cdot 10^{-11}\ \mathrm{Nm}^2/\mathrm{kg}^2$	$1.0 \cdot 10^{-4}$
Everyday precision: human circadian clock governing sleep	15 h to 75 h	2

Interesting progress has been achieved, but the final answer still eludes us.

Many challenges remain in the fields of self-organization, of non-linear evolution equations and of chaotic motion. In these fields, *turbulence* is a famous example: a precise description of turbulence has not yet been achieved, despite intense efforts. Large prizes are offered for its solution.

Many other challenges motivate numerous researchers in mathematics, physics, chemistry, biology, medicine and the other natural sciences. But apart from these research topics, classical physics leaves unanswered several basic questions.

What is contact?

> “Democritus declared that there is a unique sort of motion: that ensuing from collision.”
> Simplicius, *Commentary on the Physics of Aristotle*, 42, 10

Ref. 349

Of the questions unanswered by classical physics, the details of *contact* and *collisions* are among the most pressing. Indeed, we defined mass in terms of velocity changes during collisions. But why do objects change their motion in such instances? Why are collisions between two balls made of chewing gum different from those between two stainless-steel balls? What happens during those moments of contact? Page 100

Contact is related to material properties, which in turn influence motion in a complex way. The complexity is such that the sciences of material properties developed independently from the rest of physics for a long time; for example, the techniques of metallurgy (often called the oldest science of all), of chemistry and of cooking were related to the properties of motion only in the twentieth century, after having been independently pursued for thousands of years. Since material properties determine the essence of contact, we *need* knowledge about matter and about materials to understand the notion of mass, of contact and thus of motion. The parts of our adventure that deal with quantum theory will reveal these connections.

WHAT DETERMINES PRECISION AND ACCURACY?

Ref. 350 Precision has its own fascination. How many digits of π, the ratio between circumference and diameter of a circle, do you know by heart? What is the largest number of digits of π you have calculated yourself? Challenge 720 e

Is it possible to draw or cut a rectangle for which the ratio of lengths is a number, e.g. of the form 0.131520091514001315211420010914..., whose digits encode a full book? (A simple method would code a space as 00, the letter 'a' as 01, 'b' as 02, 'c' as 03, etc. Even more interestingly, could the number be printed inside its own book?) Challenge 721 s

Why are so many measurement results, such as those of Table 52, of *limited* precision, even if the available financial budget for the measurement apparatus is almost unlimited? These are all questions about precision.

When we started climbing Motion Mountain, we explained that gaining height means increasing the *precision* of our description of nature. To make even this statement itself more precise, we distinguish between two terms: *precision* is the degree of reproducibility; *accuracy* is the degree of correspondence to the actual situation. Both concepts apply to measurements,* to statement and to physical concepts.

Statements with false accuracy and false precision abound. What should we think of a car company – Ford – who claim that the drag coefficient c_w of a certain model is 0.375? Or of the official claim that the world record in fuel consumption for cars is 2315.473 km/l? Or of the statement that 70.3 % of all citizens share a certain opinion? One lesson we learn from investigations into measurement errors is that we should never provide more digits for a result than we can put our hand into fire for. Challenge 722 s

Page 453 In short, precision and accuracy are *limited*. At present, the record number of reliable digits ever measured for a physical quantity is 13. Why so few? Galilean physics doesn't provide an answer at all. What is the maximum number of digits we can expect in measurements; what determines it; and how can we achieve it? These questions are still open at this point in our adventure. They will be covered in the parts on quantum theory.

In our walk we aim for highest possible precision and accuracy. Therefore, concepts have mainly to be *precise*, and descriptions have to be *accurate*. Any inaccuracy is a proof of lack of understanding. To put it bluntly, in our adventure, 'inaccurate' means *wrong*. Increasing the accuracy and precision of our description of nature implies leaving behind us all the mistakes we have made so far. This quest raises several issues.

CAN ALL OF NATURE BE DESCRIBED IN A BOOK?

> “Darum kann es in der Logik auch *nie*
> Überraschungen geben.** ”
> Ludwig Wittgenstein, *Tractatus*, 6.1251

Could the perfect physics publication, one that describes *all* of nature, exist? If it does, it must also describe itself, its own production – including its readers and its author – and most important of all, its own contents. Is such a book possible? Using the concept of information, we can state that such a book must contain all information contained in

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* For measurements, both precision and accuracy are best described by their *standard deviation*, as explained on page 459.

** 'Hence there can *never* be surprises in logic.'

the universe. Is this possible? Let us check the options.

If nature requires an *infinitely* long book to be fully described, such a publication obviously cannot exist. In this case, only approximate descriptions of nature are possible and a perfect physics book is impossible.

If nature requires a *finite* amount of information for its description, there are two options. One is that the information of the universe is so large that it cannot be summarized in a book; then a perfect physics book is again impossible. The other option is that the universe does contain a finite amount of information and that it can be summarized in a few short statements. This would imply that the rest of the universe would not add to the information already contained in the perfect physics book.

We note that the answer to this puzzle also implies the answer to another puzzle: whether a brain can contain a full description of nature. In other words, the real question is: can we understand nature? Can we reach our aim to understand motion? We usually believe this. But the arguments just given imply that we effectively believe that the universe does not contain more information than what our brain could contain or even contains already. What do you think? We will solve this puzzle later in our adventure. Until then, do make up your own mind.

Vol. VI, page 111

Challenge 723 e

SOMETHING IS WRONG ABOUT OUR DESCRIPTION OF MOTION

> “Je dis seulement qu’il n’y a point d’espace, où il n’y a point de matière; et que l’espace lui-même n’est point une réalité absolue. *”
>
> Leibniz

We described nature in a rather simple way. *Objects* are permanent and massive entities localized in space-time. *States* are changing properties of objects, described by position in space and instant in time, by energy and momentum, and by their rotational equivalents. *Time* is the relation between events measured by a clock. *Clocks* are devices in undisturbed motion whose position can be observed. *Space* and position is the relation between objects measured by a metre stick. *Metre sticks* are devices whose shape is subdivided by some marks, fixed in an invariant and observable manner. *Motion* is change of position with time (times mass); it is determined, does not show surprises, is conserved (even in death), and is due to gravitation and other interactions.

Even though this description works rather well in practice, it contains a circular definition. Can you spot it? Each of the two central concepts of motion is defined with the help of the other. Physicists worked for about 400 years on classical mechanics without noticing or wanting to notice the situation. Even thinkers with an interest in discrediting science did not point it out. Can an exact science be based on a circular definition? Obviously yes, and physics has done quite well so far. Is the situation unavoidable in principle?

Challenge 724 s

Challenge 725 s

Undoing the circular definition of Galilean physics is one of the aims of the rest of our walk. The search for a solution is part of the last leg of our adventure. To achieve

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* ‘I only say that there is no space where there is no matter; and that space itself is not an absolute reality.’ Gottfried Wilhelm Leibniz writes this already in 1716, in section 61 of his famous fifth letter to Clarke, the assistant and spokesman of Newton. Newton, and thus Clarke, held the opposite view; and as usual, Leibniz was right.

the solution, we need to increase substantially the level of precision in our description of motion.

Whenever precision is increased, imagination is restricted. We will discover that many types of motion that seem possible are not. Motion is *limited*. Nature limits speed, size, acceleration, mass, force, power and many other quantities. Continue reading the other parts of this adventure only if you are prepared to exchange fantasy for precision. It will be no loss because exploring the precise working of nature will turn out to be more fascinating than any fantasy.

Why is measurement possible?

In the description of gravity given so far, the one that everybody learns – or should learn – at school, acceleration is connected to mass and distance via $a = GM/r^2$. That's all. But this simplicity is deceiving. In order to check whether this description is correct, we have to measure lengths and times. However, it is *impossible* to measure lengths and time intervals with any clock or any ruler based on the gravitational interaction alone! Challenge 726 s Try to conceive such an apparatus and you will inevitably be disappointed. You always need a non-gravitational method to start and stop the stopwatch. Similarly, when you measure length, e.g. of a table, you have to hold a ruler or some other device near it. The interaction necessary to line up the ruler and the table cannot be gravitational.

A similar limitation applies even to mass measurements. Try to measure mass using Challenge 727 s gravitation alone. Any scale or balance needs other – usually mechanical, electromagnetic or optical – interactions to achieve its function. Can you confirm that the same Challenge 728 s applies to speed and to angle measurements? In summary, whatever method we use,

▷ In order to measure velocity, length, time, and mass, interactions other than gravity are needed.

Our ability to measure shows that gravity is not all there is. And indeed, we still need to understand charge and colours.

In short, Galilean physics does not explain our ability to measure. In fact, it does not even explain the existence of measurement standards. Why do objects have fixed lengths? Why do clocks work with regularity? Galilean physics cannot explain these observations; we will need relativity and quantum physics to find out.

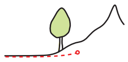
Is motion unlimited?

Galilean physics suggests that linear motion could go on forever. In fact, Galilean physics tacitly assumes that the universe is infinite in space and time. Indeed, finitude of any kind contradicts the Galilean description of motion. On the other hand, we know from observation that the universe is *not* infinite: if it were infinite, the night would not be Challenge 729 e dark.

Galilean physics also suggests that speeds can have any value. But the existence of infinite speeds in nature would not allow us to define time sequences. Clocks would be impossible. In other words, a description of nature that allows unlimited speeds is not precise. Precision and measurements require limits.

Because Galilean physics disregards limits to motion, Galilean physics is inaccurate, and thus wrong.

To achieve the highest possible precision, and thus to find the correct description of motion, we need to discover all of *motion's limits*. So far, we have discovered one: there is a smallest entropy in nature. We now turn to another, more striking limit: the speed limit for energy, objects and signals. To observe and understand the speed limit, the next volume explores the most rapid motion of energy, objects and signals that is known: the motion of light.



APPENDIX A

NOTATION AND CONVENTIONS

Newly introduced concepts are indicated, throughout this text, by *italic typeface*. New definitions are also referred to in the index. In this text, naturally we use the international SI units; they are defined in Appendix B. Page 453 Experimental results are cited with limited precision, usually only two digits, as this is almost always sufficient for our purposes. High-precision reference values for important quantities can also be found in Appendix B. Additional precision values on composite Vol. V, page 342 physical systems are given in volume V.

But the information that is provided in this volume uses some additional conventions that are worth a second look.

THE LATIN ALPHABET

Ref. 351

> What is written without effort is in general read without pleasure.
>
> Samuel Johnson

Books are collections of symbols. *Writing* was probably invented between 3400 and 3300 BCE by the Sumerians in Mesopotamia (though other possibilities are also discussed). It then took over a thousand years before people started using symbols to represent sounds instead of concepts: this is the way in which the first alphabet was created. This happened between 2000 and 1600 BCE (possibly in Egypt) and led to the Semitic alphabet. The use of an alphabet had so many advantages that it was quickly adopted in all neighbouring cultures, though in different forms. As a result, the Semitic alphabet is the forefather of all alphabets used in the world.

This text is written using the Latin alphabet. At first sight, this seems to imply that its pronunciation *cannot* be explained in print, in contrast to the pronunciation of other alphabets or of the International Phonetic Alphabet (IPA). (They can be explained using the alphabet of the main text.) However, it *is* in principle possible to write a text that describes exactly how to move lips, mouth and tongue for each letter, using physical concepts where necessary. The descriptions of pronunciations found in dictionaries make indirect use of this method: they refer to the memory of pronounced words or sounds found in nature.

Historically, the Latin alphabet was derived from the Etruscan, which itself was a derivation of the Greek alphabet. An overview is given in Figure 303. There are two main forms of the Latin alphabet.

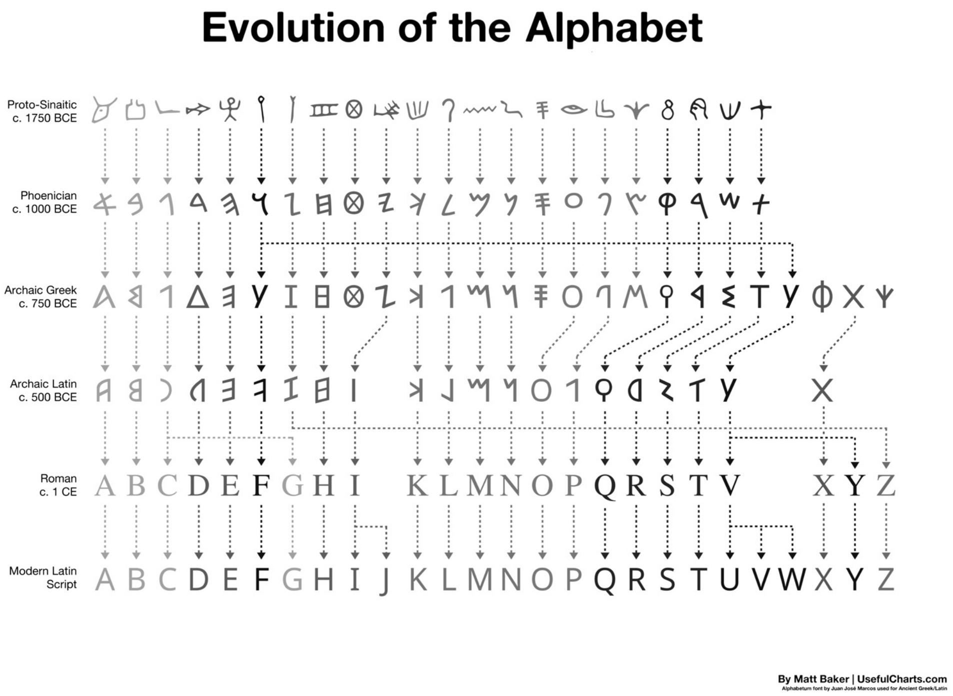


FIGURE 303 A summary of the history of the Latin alphabet (© Matt Baker at usefulcharts.com).

The *ancient* Latin alphabet,
used from the sixth century BCE onwards:

A B C D E F Z H I K L M N O P Q R S T V X

The *classical* Latin alphabet,
used from the second century BCE until the eleventh century:

A B C D E F G H I K L M N O P Q R S T V X Y Z

The letter G was added in the third century BCE by the first Roman to run a fee-paying school, Spurius Carvilius Ruga. He added a horizontal bar to the letter C and substituted the letter Z, which was not used in Latin any more, for this new letter. In the second century BCE, after the conquest of Greece, the Romans included the letters Y and Z from the Greek alphabet at the end of their own (therefore effectively reintroducing the Z) in order to be able to write Greek words. This classical Latin alphabet was stable for the next thousand years.*

The classical Latin alphabet was spread around Europe, Africa and Asia by the Romans during their conquests; due to its simplicity it began to be used for writing in nu-

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* To meet Latin speakers and writers, go to www.alcuinus.net.

merous other languages. Most modern 'Latin' alphabets include a few other letters. The letter W was introduced in the eleventh century in French and was then adopted in most European languages.* The letter U was introduced in the mid-fifteenth century in Italy, Ref. 352 the letter J at the end of that century in Spain, to distinguish certain sounds which had previously been represented by V and I. The distinction proved a success and was already common in most European languages in the sixteenth century. The contractions æ and œ date from the Middle Ages. The German alphabet includes the *sharp s*, written ß, a contraction of 'ss' or 'sz', and the Nordic alphabets added *thorn*, written Þ or þ, and *eth*, Ref. 353 written Ð or ð, both taken from the futhorc,** and other signs.

Lower-case letters were not used in classical Latin; they date only from the Middle Ages, from the time of Charlemagne. Like most accents, such as ê, ç or ä, which were also first used in the Middle Ages, lower-case letters were introduced to save the then expensive paper surface by shortening written words.

> Outside a dog, a book is a man's best friend.
> Inside a dog, it's too dark to read.
>
> Groucho Marx

The Greek alphabet

The Greek alphabet is central to modern culture and civilization. It is at the origin of the Etruscan alphabet, from which the Latin alphabet was derived. The Greek alphabet was itself derived from the Phoenician or a similar northern Semitic alphabet in the Ref. 355 tenth century BCE. The Greek alphabet, for the first time, included letters also for vowels, which the Semitic alphabets lacked (and often still lack).

In the Phoenician alphabet and in many of its derivatives, such as the Greek alphabet, each letter has a proper name. This is in contrast to the Etruscan and Latin alphabets. The first two Greek letter names are, of course, the origin of the term *alphabet* itself.

In the tenth century BCE, the Ionian or *ancient* (eastern) Greek alphabet consisted of the upper-case letters only. In the sixth century BCE several letters were dropped, while a few new ones and the lower-case versions were added, giving the *classical* Greek alphabet. Later, accents, subscripts and breathings were introduced. Table 53 also gives the values signified by the letters when they were used as numbers. For this special use, the obsolete ancient letters were kept during the classical period; thus they also acquired lower-case forms.

The Latin correspondence in the table is the standard classical one, used for writing Greek words. The question of the correct *pronunciation* of Greek has been hotly debated

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* In Turkey, still in 2013, you can be convoked in front of a judge if you use the letters w, q or x in an official letter; these letters only exist in the Kurdish language, not in Turkish. Using them is 'unturkish' behaviour, support of terrorism and punishable by law. It is not generally known how physics and mathematics teachers cope with this situation.

** The Runic script, also called *Futhark* or *Futhorc*, a type of alphabet used in the Middle Ages in Germanic, Anglo–Saxon and Nordic countries, probably also derives from the Etruscan alphabet. The name derives from the first six letters: f, u, th, a (or o), r, k (or c). The third letter is the letter thorn mentioned above; it is often written 'Y' in Old English, as in 'Ye Olde Shoppe.' From the runic alphabet Old English also took the Ref. 354 letter *wyn* to represent the 'w' sound, and the already mentioned eth. (The other letters used in Old English – not from futhorc – were the *yogh*, an ancient variant of g, and the ligatures æ or Æ, called *ash*, and œ or Œ, called *ethel*.)

TABLE 53 The ancient and classical Greek alphabets, and the correspondence with Latin letters and Indian digits.

Anc.	Class.		Name	Corresp.		Anc.	Class.		Name	Corresp.	
Α	Α	α	alpha	a	1	Ν	Ν	ν	nu	n	50
Β	Β	β	beta	b	2	Ξ	Ξ	ξ	xi	x	60
Γ	Γ	γ	gamma	g, n[1]	3	Ο	Ο	ο	omicron	o	70
Δ	Δ	δ	delta	d	4	Π	Π	π	pi	p	80
Ε	Ε	ε	epsilon	e	5	Ϙ ϙ, ϟ			qoppa[3]	q	90
F ϝ, ϛ			digamma,	w	6	Ρ	Ρ	ρ	rho	r, rh	100
			stigma[2]			Σ	Σ	σ, ς	sigma[4]	s	200
Ζ	Ζ	ζ	zeta	z	7	Τ	Τ	τ	tau	t	300
Η	Η	η	eta	e	8		Υ	υ	upsilon	y, u[5]	400
Θ	Θ	θ	theta	th	9		Φ	φ	phi	ph, f	500
Ι	Ι	ι	iota	i, j	10		Χ	χ	chi	ch	600
Κ	Κ	κ	kappa	k	20		Ψ	ψ	psi	ps	700
Λ	Λ	λ	lambda	l	30		Ω	ω	omega	o	800
Μ	Μ	μ	mu	m	40	Ͳ ϡ			sampi[6]	s	900

The regional archaic letters yot, sha and san are not included in the table. The letter san was the ancestor of sampi.

1. Only if before velars, i.e., before kappa, gamma, xi and chi.
2. ‘Digamma’ is the name used for the F-shaped form. It was mainly used as a letter (but also sometimes, in its lower-case form, as a number), whereas the shape and name ‘stigma’ is used only for the number. Both names were derived from their respective shapes; in fact, the stigma is a medieval, uncial version of the digamma. The name ‘stigma’ is derived from the fact that the letter looks like a sigma with a tau attached under it – though unfortunately not in all modern fonts. The original letter name, also giving its pronunciation, was ‘waw’.
3. The version of qoppa that looks like a reversed and rotated z is still in occasional use in modern Greek. Unicode calls this version ‘koppa’.
4. The second variant of sigma is used only at the end of words.
5. Uspilon corresponds to ‘u’ only as the second letter in diphthongs.
6. In older times, the letter sampi was positioned between pi and qoppa.

in specialist circles; the traditional *Erasmian* pronunciation does not correspond either to the results of linguistic research nor to modern Greek. In classical Greek, the sound that sheep make was βη–βη. (Erasmian pronunciation wrongly insists on a narrow η; modern Greek pronunciation is different for β, which is now pronounced ‘v’, and for η, which is now pronounced as ‘i:’ – a long ‘i’.) Obviously, the pronunciation of Greek varied from region to region and over time. For Attic Greek, the main dialect spoken in the classical period, the question is now settled. Linguistic research has shown that chi, phi and theta were less aspirated than usually pronounced in English and sounded more like the initial sounds of ‘cat’, ‘perfect’ and ‘tin’; moreover, the zeta seems to have been pronounced more like ‘zd’ as in ‘buzzed’. As for the vowels, contrary to tradition, epsilon is closed and short whereas eta is open and long; omicron is closed and short whereas omega is wide and long, and upsilon is really a sound like a French ‘u’ or German ‘ü.’

The Greek vowels can have rough or smooth *breathings*, *subscripts*, and acute, grave, circumflex or diaeresis *accents*. Breathings – used also on ρ – determine whether the letter is aspirated. Accents, which were interpreted as stresses in the Erasmian pronunciation, actually represented pitches. Classical Greek could have up to three of these added signs per letter; modern Greek never has more than one.

Another descendant of the Greek alphabet* is the *Cyrillic alphabet*, which is used with slight variations in many Slavic languages, such as Russian and Bulgarian. There is no standard transcription from Cyrillic to Latin, so that often the same Russian name is spelled differently in different countries or even in the same country on different occasions.

TABLE 54 The beginning of the Hebrew abjad.

Letter	Names	Correspondence	
א	aleph	a	1
ב	beth	b	2
ג	gimel	g	3
ד	daleth	d	4
etc.			

THE HEBREW ALPHABET AND OTHER SCRIPTS

The Phoenician alphabet is also the origin of the Hebrew consonant alphabet or *abjad*. Its first letters are given in Table 54. Only the letter aleph is commonly used in mathematics, though others have been proposed. Ref. 356 Vol. III, page 288

Around one hundred writing systems are in use throughout the world. Experts classify them into five groups. *Phonemic alphabets*, such as Latin or Greek, have a sign for each consonant and vowel. *Abjads* or consonant alphabets, such as Hebrew or Arabic, have a sign for each consonant (sometimes including some vowels, such as aleph), and do not write (most) vowels; most abjads are written from right to left. *Abugidas*, also called *syllabic alphabets* or *alphasyllabaries*, such as Balinese, Burmese, Devanagari, Tagalog, Thai, Tibetan or Lao, write consonants and vowels; each consonant has an inherent vowel which can be changed into the others by diacritics. *Syllabaries*, such as Hiragana or Ethiopic, have a sign for each syllable of the language. Finally, *complex scripts*, such as Chinese, Mayan, or Egyptian hieroglyphs, use signs which have both sound and meaning. Writing systems can have text flowing from right to left, from bottom to top, and can count book pages in the opposite sense to this book.

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* The Greek alphabet is also the origin of the *Gothic alphabet*, which was defined in the fourth century by Wulfila for the Gothic language, using also a few signs from the Latin and futhorc scripts.

The Gothic alphabet is not to be confused with the so-called *Gothic letters*, a style of the *Latin* alphabet used all over Europe from the eleventh century onwards. In Latin countries, Gothic letters were replaced in the sixteenth century by the *Antiqua*, the ancestor of the type in which this text is set. In other countries, Gothic letters remained in use for much longer. They were used in type and handwriting in Germany until 1941, when the National Socialist government suddenly abolished them, in order to comply with popular demand. They remain in sporadic use across Europe. In many physics and mathematics books, Gothic letters are used to denote vector quantities.

Even though there are about 6000 languages on Earth, there are only about one hundred writing systems in use today. About fifty other writing systems have fallen out of use.[*]For physical and mathematical formulae, though, the sign system used in this text, based on Latin and Greek letters, written from left to right and from top to bottom, is a standard the world over. It is used independently of the writing system of the text containing it. Ref. 357

NUMBERS AND THE INDIAN DIGITS

Both the digits and the method used in this text to write numbers originated in India. The Indian digits date from around 600 BCEand were brought to the Mediterranean by Arabic mathematicians in the Middle Ages. The number system used in this text is thus much younger than the alphabet. The Indian numbers were made popular in Europe by Leonardo of Pisa, called Fibonacci,[**] in his book *Liber Abaci* or 'Book of Calculation', which he published in 1202. That book revolutionized mathematics. Anybody with paper and a pen (the pencil had not yet been invented) was now able to calculate and write down numbers as large as reason allowed, or even larger, and to perform calculations with them. Fibonacci's book started: Ref. 358

> Novem figure indorum he sunt 9 8 7 6 5 4 3 2 1. Cum his itaque novem figuris, et cum hoc signo 0, quod arabice zephirum appellatur, scribitur quilibet numerus, ut inferius demonstratur.[***]

The Indian method of writing numbers, the *Indian number system*, introduced two innovations: a large one, the *positional system*, and a small one, the digit zero. [****] The positional system, as described by Fibonacci, was so much more efficient that it completely replaced the previous *Roman number system*, which writes 1996 as IVMM or MCMIVC or MCMXCVI, as well as the *Greek number system*, in which the Greek letters were used for numbers in the way shown in Table 53, thus writing 1996 as ͵αϡϙς′. Compared to these systems, the Indian numbers are a much better technology.

The Indian number system proved so practical that calculations done on paper completely eliminated the need for the *abacus*, which therefore fell into disuse. The abacus is still in use in countries, for example in Asia, America or Africa, and by people who do not use a positional system to write numbers. It is also useful for the blind.

The Indian number system also eliminated the need for systems to represent numbers with fingers. Such ancient systems, which could show numbers up to 10 000 and more,

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[*] A well-designed website on the topic is www.omniglot.com. The main present and past writing systems are encoded in the Unicode standard, which at present contains 52 writing systems. See www.unicode.org. An eccentric example of a writing system is the alphabet proposed by George Bernard Shaw, presented at en.wikipedia.org/wiki/Shavian_alphabet.

[**] Leonardo di Pisa, called Fibonacci (b. *c.* 1175 Pisa, d. 1250 Pisa), was the most important mathematician of his time.

[***] 'The nine figures of the Indians are: 9 8 7 6 5 4 3 2 1. With these nine figures, and with this sign 0 which in Arabic is called zephirum, any number can be written, as will be demonstrated below.'

[****] It is thus *not* correct to call the digits 0 to 9 *Arabic*. Both the digits used in Arabic texts and the digits used in Latin texts such as this one derive from the *Indian digits*. You can check this by yourself: only the digits 0, 2, 3 and 7 resemble those used in Arabic writing, and then only if they are turned clockwise by 90°.

have left only one trace: the term ‘digit’ itself, which derives from the Latin word for finger.

The power of the positional number system is often forgotten. For example, only a positional number system allows mental calculations and makes calculating prodigies possible.[*]

THE SYMBOLS USED IN THE TEXT

> “To avoide the tediouse repetition of these
> woordes: is equalle to: I will sette as I doe often
> in woorke use, a paire of paralleles, or Gemowe
> lines of one lengthe, thus: = , bicause noe .2.
> thynges, can be moare equalle”
>
> Robert Recorde[**]

Besides text and numbers, physics books contain other symbols. Most symbols have been developed over hundreds of years, so that only the clearest and simplest are now in use. In this adventure, the symbols used as abbreviations for *physical* quantities are all taken from the Latin or Greek alphabets and are always defined in the context where they are used. The symbols designating units, constants and particles are defined in Appendix B Page 453 and in Appendix B of volume V. The symbols used in this text are those in common use Ref. 356 in the practice and the teaching of physics.

There is even an international standard for the symbols in physical formulae – ISO EN 80000, formerly ISO 31 – but it is shamefully expensive, virtually inaccessible and incredibly useless: the symbols listed in it are those in common use anyway, and their use is not binding anywhere, not even in the standard itself! ISO 80000 is a prime example of bureaucracy gone wrong.

The *mathematical* symbols used in this text, in particular those for mathematical operations and relations, are given in the following list, together with their historical origin. Ref. 360 The details of their history have been extensively studied in by scholars.

TABLE 55 The history of mathematical notation and symbols.

SYMBOL	MEANING	ORIGIN
+, −	plus, minus	Johannes Widmann 1489; the plus sign is derived from Latin ‘et’.
$\sqrt{\quad}$	read as ‘square root’	used by Christoff Rudolff in 1525; the sign evolved from a point.
=	equal to	Robert Recorde 1557

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[*] Currently, the shortest time for finding the thirteenth (integer) root of a hundred-digit (integer) number, a result with 8 digits, is below 4 seconds, and 70.2 seconds for a 200 digit number, for which the result has sixteen digits. Both records are held by Alexis Lemaire. For more about the stories and the methods of Ref. 359 calculating prodigies, see the bibliography.

[**] Robert Recorde (b. *c.* 1510 Tenby, d. 1558 London), mathematician and physician; he died in prison because of debts. The quotation is from his *The Whetstone of Witte*, 1557. An image showing the quote can be found at en.wikipedia.org/wiki/Equals_sign. It is usually suggested that the quote is the first introduction of the equal sign; claims that Italian mathematicians used the equal sign before Recorde are not backed up Ref. 360 by convincing examples.

TABLE 55 (Continued) The history of mathematical notation and symbols.

SYMBOL	MEANING	ORIGIN
{ }, [ ], ( )	grouping symbols	use starts in the sixteenth century
$>, <$	larger than, smaller than	Thomas Harriot 1631
$\times$	multiplied with, times	England *c.* 1600, made popular by William Oughtred 1631
a^n	a to the power n, $a \cdot ... \cdot a$ (n factors)	René Descartes 1637
x, y, z	coordinates, unknowns	René Descartes 1637
$ax+by+c=0$	constants and equations for unknowns	René Descartes 1637
∞	infinity	John Wallis 1655
$\mathrm{d}/\mathrm{d}x$, $\mathrm{d}x$, $\int y\,\mathrm{d}x$	derivative, differential, integral	Gottfried Wilhelm Leibniz 1675
:	divided by	Gottfried Wilhelm Leibniz 1684
$\cdot$	multiplied with, times	Gottfried Wilhelm Leibniz *c.* 1690
a_1, a_n	indices	Gottfried Wilhelm Leibniz *c.* 1690
$\sim$	similar to	Gottfried Wilhelm Leibniz *c.* 1690
π	circle number, 4 arctan 1	William Jones 1706
φx	function of x	Johann Bernoulli 1718
$fx, f(x)$	function of x	Leonhard Euler 1734
e	$\sum_{n=0}^{\infty}\frac{1}{n!} = \lim_{n\to\infty}(1+1/n)^n$	Leonhard Euler 1736
$f'(x)$	derivative of function at x	Giuseppe Lagrangia 1770
$\Delta x, \sum$	difference, sum	Leonhard Euler 1755
$\prod$	product	Carl Friedrich Gauss 1812
i	imaginary unit, $+\sqrt{-1}$	Leonhard Euler 1777
$\neq$	is different from	Leonhard Euler eighteenth century
$\partial/\partial x$	partial derivative, read like ‘d/dx’	it was derived from a cursive form of ‘d’ or of the letter ‘dey’ of the Cyrillic alphabet by Adrien-Marie Legendre in 1786 and made popular by Carl Gustav Jacobi in 1841
$n!$	factorial, $1 \cdot 2 \cdot ... \cdot$	Christian Kramp 1808
Δ	Laplace operator	Robert Murphy 1833
$\lvert x\rvert$	absolute value	Karl Weierstrass 1841
∇	read as ‘nabla’ (or ‘del’)	introduced by William Hamilton in 1853 and Peter Tait in 1867, named after the shape of an old Egyptian musical instrument
$\subset, \supset$	set inclusion	Ernst Schröder in 1890
$\cup, \cap$	set union and intersection	Giuseppe Peano 1888
$\in$	element of	Giuseppe Peano 1888
$\otimes$	dyadic product or tensor product or outer product	unknown
$\langle\psi\rvert, \lvert\psi\rangle$	bra and ket state vectors	Paul Dirac 1930

TABLE 55 (Continued) The history of mathematical notation and symbols.

Symbol	Meaning	Origin
∅	empty set	André Weil as member of the Nicolas Bourbaki group in the early twentieth century
$[x]$	the measurement unit of a quantity x	twentieth century

Ref. 361

Other signs used here have more complicated origins. The & sign is a contraction of Latin *et* meaning ‘and’, as is often more clearly visible in its variations, such as *&*, the common italic form.

Ref. 362

Ref. 363

Each of the punctuation signs used in sentences with modern Latin alphabets, such as , . ; : ! ? ‘ ’ » « – ( ) ... has its own history. Many are from ancient Greece, but the question mark is from the court of Charlemagne, and exclamation marks appear first in the sixteenth century.* The @ or *at-sign* probably stems from a medieval abbreviation of Latin *ad*, meaning ‘at’, similarly to how the & sign evolved from Latin *et*. In recent years, the *smiley* :-) and its variations have become popular. The smiley is in fact a new version of the ‘point of irony’ which had been proposed in 1899, without success, by the poet Alcanter de Brahm (b. 1868 Mulhouse, d. 1942 Paris).

Ref. 364

The section sign § dates from the thirteenth century in northern Italy, as was shown by the palaeographer Paul Lehmann. It was derived from ornamental versions of the capital letter C for *capitulum*, i.e., ‘little head’ or ‘chapter.’ The sign appeared first in legal texts, where it is still used today, and then spread into other domains.

The paragraph sign ¶ was derived from a simpler ancient form looking like the Greek letter Γ, a sign which was used in manuscripts from ancient Greece until well into the Middle Ages to mark the start of a new text paragraph. In the Middle Ages it took the modern form, probably because a letter c for *caput* was added in front of it.

Ref. 365

One of the most important signs of all, the *white space* separating words, was due to Celtic and Germanic influences when these people started using the Latin alphabet. It became commonplace between the ninth and the thirteenth century, depending on the language in question.

CALENDARS

The many ways to keep track of time differ greatly from civilization to civilization. The most common calendar, and the one used in this text, is also one of the most absurd, as it is a compromise between various political forces who tried to shape it.

In ancient times, independent localized entities, such as tribes or cities, preferred *lunar* calendars, because lunar timekeeping is easily organized locally. This led to the use of the month as a calendar unit. Centralized states imposed *solar* calendars, based on the year. Solar calendars require astronomers, and thus a central authority to finance them. For various reasons, farmers, politicians, tax collectors, astronomers, and some, but not all, religious groups wanted the calendar to follow the solar year as precisely as possible. The compromises necessary between days and years are the origin of leap days.

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* On the parenthesis see the beautiful book by J. Lennard, *But I Digress*, Oxford University Press, 1991.

The compromises necessary between months and year led to the varying lengths of the months; they are different in different calendars. The most commonly used year–month structure was organized over 2000 years ago by Caesar, and is thus called the *Julian calendar*.

The system was destroyed only a few years later: August was lengthened to 31 days when it was named after Augustus. Originally, the month was only 30 days long; but in order to show that Augustus was as important as Caesar, after whom July is named, all month lengths in the second half of the year were changed, and February was shortened by one additional day.

Ref. 366 The *week* is an invention of the Babylonians. One day in the Babylonian week was 'evil' or 'unlucky', so it was better to do nothing on that day. The modern week cycle with its resting day descends from that superstition. (The way astrological superstition and astronomy cooperated to determine the order of the weekdays is explained in the section on gravitation.) Page 221 Although about three thousand years old, the week was fully included into the Julian calendar only around the year 300, towards the end of the Western Roman Empire. The final change in the Julian calendar took place between 1582 and 1917 (depending on the country), when more precise measurements of the solar year were used to set a new method to determine leap days, a method still in use today. Together with a reset of the date and the fixation of the week rhythm, this standard is called the *Gregorian calendar* or simply the *modern calendar*. It is used by a majority of the world's population.

Despite its complexity, the modern calendar does allow you to determine the day of the week of a given date in your head. Just execute the following six steps:

1. take the last two digits of the year, and divide by 4, discarding any fraction;
2. add the last two digits of the year;
3. subtract 1 for January or February of a leap year;
4. add 6 for 2000s or 1600s, 4 for 1700s or 2100s,
   2 for 1800s and 2200s, and 0 for 1900s or 1500s;
5. add the day of the month;
6. add the month key value, namely 144 025 036 146 for JFM AMJ JAS OND.

The remainder after division by 7 gives the day of the week, with the correspondence 1-2-3-4-5-6-0 meaning Sunday-Monday-Tuesday-Wednesday-Thursday-Friday-Saturday.*

When to start counting the years is a matter of choice. The oldest method not attached to political power structures was that used in ancient Greece, when years were counted from the first Olympic games. People used to say, for example, that they were born in the first year of the twenty-third Olympiad. Later, political powers always imposed the counting of years from some important event onwards.** Maybe reintroducing the

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* Remembering the intermediate result for the current year can simplify things even more, especially since the dates 4.4, 6.6, 8.8, 10.10, 12.12, 9.5, 5.9, 7.11, 11.7 and the last day of February all fall on the same day of the week, namely on the year's intermediate result plus 4.

** The present counting of years was defined in the Middle Ages by setting the date for the foundation of Rome to the year 753 BCE, or 753 *before the Common Era*, and then counting backwards, so that the BCE years behave almost like negative numbers. However, the year 1 follows directly after the year 1 BCE: there was no year 0.

Some other standards set by the Roman Empire explain several abbreviations used in the text:

- c. is a Latin abbreviation for *circa* and means 'roughly';

Olympic counting is worth considering?

PEOPLE NAMES

In the Far East, such as *Corea**, *Japan* or *China*, family names are put in front of the given name. For example, the first Japanese winner of the Nobel Prize in Physics was Yukawa Hideki. In *India*, often, but not always, there is no family name; in those cases, the father's first name is used. In *Russia*, the family name is rarely used in conversation; instead, the first name of the father is. For example, Lev Landau was addressed as Lev Davidovich ('son of David'). In addition, Russian transliteration is not standardized; it varies from country to country and from tradition to tradition. For example, one finds the spellings Dostojewski, Dostoevskij, Dostoïevski and Dostoyevsky for the same person. In the *Netherlands*, the official given names are never used; every person has a semi-official first name by which he is called. For example, Gerard 't Hooft's official given name is Gerardus. In *Germany*, some family names have special pronunciations. For example, Voigt is pronounced 'Fohgt'. In *Italy*, during the Middle Age and the Renaissance, people were called by their first name only, such as Michelangelo or Galileo, or often by first name plus a personal surname that was not their family name, but was used like one, such as Niccolò Tartaglia or Leonardo Fibonacci. In *ancient Rome*, the name by which people are known is usually their surname. The family name was the middle name. For example, Cicero's family name was Tullius. The law introduced by Cicero was therefore known as 'lex Tullia'. In *ancient Greece*, there were no family names. People had only one name. In the English language, the Latin version of the Greek name is used, such as Democritus.

ABBREVIATIONS AND EPONYMS OR CONCEPTS?

Sentences like the following are the scourge of modern physics:

> The EPR paradox in the Bohm formulation can perhaps be resolved using the GRW approach, with the help of the WKB approximation of the Schrödinger equation.

- i.e. is a Latin abbreviation for *id est* and means 'that is';
- e.g. is a Latin abbreviation for *exempli gratia* and means 'for the sake of example';
- ibid. is a Latin abbreviation for *ibidem* and means 'at that same place';
- inf. is a Latin abbreviation for *infra* and means '(see) below';
- op. cit. is a Latin abbreviation for *opus citatum* and means 'the cited work';
- et al. is a Latin abbreviation for *et alii* and means 'and others'.

By the way, *idem* means 'the same' and *passim* means 'here and there' or 'throughout'. Many terms used in physics, like frequency, acceleration, velocity, mass, force, momentum, inertia, gravitation and temperature, are derived from Latin. In fact, it is arguable that the language of science has been Latin for over two thousand years. In Roman times it was Latin vocabulary with Latin grammar, in modern times it switched to Latin vocabulary with French grammar, then for a short time to Latin vocabulary with German grammar, after which it changed to Latin vocabulary with British/American grammar.

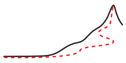
Many units of measurement also date from Roman times, as explained in the next appendix. Even the infatuation with Greek technical terms, as shown in coinages such as 'gyroscope', 'entropy' or 'proton', dates from Roman times. Ref. 367

* Corea was temporarily forced to change its spelling to 'Korea' by the Japanese Army because the generals could not bear the fact that Corea preceded Japan in the alphabet. This is not a joke.

Using such vocabulary is the best way to make language unintelligible to outsiders. (In fact, the sentence is nonsense anyway, because the 'GRW approach' is false.) First of all, the sentence uses abbreviations, which is a shame. On top of this, the sentence uses people's names to characterize concepts, i.e., it uses *eponyms*. Originally, eponyms were intended as tributes to outstanding achievements. Today, when formulating radical new laws or variables has become nearly impossible, the spread of eponyms intelligible to a steadily decreasing number of people simply reflects an increasingly ineffective drive to fame.

Eponyms are a proof of scientist's lack of imagination. We avoid them as much as possible in our walk and give *common* names to mathematical equations or entities wherever possible. People's names are then used as appositions to these names. For example, 'Newton's equation of motion' is never called 'Newton's equation'; 'Einstein's field equations' is used instead of 'Einstein's equations'; and 'Heisenberg's equation of motion' is used instead of 'Heisenberg's equation'.

However, some exceptions are inevitable: certain terms used in modern physics have no real alternatives. The Boltzmann constant, the Planck scale, the Compton wavelength, the Casimir effect and Lie groups are examples. In compensation, the text makes sure that you can look up the definitions of these concepts using the index. In addition, the text tries to provide pleasurable reading. Ref. 368



Appendix B

UNITS, MEASUREMENTS AND CONSTANTS

Measurements are comparisons with standards. Standards are based on *units*. Many different systems of units have been used throughout the world. Most of these standards confer power to the organization in charge of them. Such power can be misused; this is the case today, for example in the computer industry, and was so in the distant past. The solution is the same in both cases: organize an independent and global standard. For measurement units, this happened in the eighteenth century: in order to avoid misuse by authoritarian institutions, to eliminate problems with differing, changing and irreproducible standards, and – this is not a joke – to simplify tax collection and to make it more just, a group of scientists, politicians and economists agreed on a set of units. It is called the *Système International d'Unités*, abbreviated *SI*, and is defined by an international treaty, the 'Convention du Mètre'. The units are maintained by an international organization, the 'Conférence Générale des Poids et Mesures', and its daughter organizations, the 'Commission Internationale des Poids et Mesures' and the 'Bureau International des Poids et Mesures' (BIPM). All originated in the times just before the French revolution. Ref. 369

SI UNITS

All SI units are built from seven *base units*. Their simplest definitions, translated from French into English, are the following ones, together with the dates of their formulation and a few comments:

- 'The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.' (1967) The 2019 definition is equivalent, but much less clear.*
- 'The *metre* is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.' (1983) The 2019 definition is equivalent, but much less clear.*
- 'The *kilogram*, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,15 \cdot 10^{-34}$ when expressed in the unit J · s, which is equal to $\mathrm{kg} \cdot \mathrm{m}^2 \cdot \mathrm{s}^{-1}$.' (2019)*
- 'The *ampere*, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,634 \cdot 10^{-19}$ when expressed in the unit C, which is equal to A · s.' (2019)* This definition is equivalent to: One ampere is $6.241\,509\,074... \cdot 10^{18}$ elementary charges per second.
- 'The *kelvin*, symbol K, is the SI unit of thermodynamic temperature. It is defined by

taking the fixed numerical value of the Boltzmann constant k to be $1.380649 \cdot 10^{-23}$ when expressed in the unit $\mathrm{J \cdot K^{-1}}$.' (2019)*

- 'The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \cdot 10^{23}$ elementary entities.' (2019)*
- 'The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ hertz and has a radiant intensity in that direction of (1/683) watt per steradian.' (1979) The 2019 definition is equivalent, but much less clear.*

We note that both time and length units are defined as certain properties of a standard example of motion, namely light. In other words, also the Conférence Générale des Poids et Mesures makes the point that the observation of motion is a *prerequisite* for the definition and construction of time and space. *Motion is the fundament of every observation and of all measurement.* By the way, the use of light in the definitions had been proposed already in 1827 by Jacques Babinet.**

From these basic units, all other units are defined by multiplication and division. Thus, all SI units have the following properties:

- SI units form a system with *state-of-the-art precision*: all units are defined with a precision that is higher than the precision of commonly used measurements. Moreover, the precision of the definitions is regularly being improved. The present relative uncertainty of the definition of the second is around 10^{-14}, for the metre about 10^{-10}, for the kilogram about 10^{-9}, for the ampere 10^{-7}, for the mole less than 10^{-6}, for the kelvin 10^{-6} and for the candela 10^{-3}.
- SI units form an *absolute* system: all units are defined in such a way that they can be reproduced in every suitably equipped laboratory, independently, and with high precision. This avoids as much as possible any error or misuse by the standard-setting organization. In fact, the SI units are as now as near as possible to Planck's natural units, which are presented below. In practice, the SI is now an international standard defining the numerical values of the seven constants $\Delta\nu_{\mathrm{Cs}}$, c, $\hbar$, e, k, N_{A} and K_{cd}. After over 200 years of discussions, the CGPM has little left to do.
- SI units form a *practical* system: the base units are quantities of everyday magnitude. Frequently used units have standard names and abbreviations. The complete list includes the seven base units just given, the supplementary units, the derived units and the admitted units.

The *supplementary* SI units are two: the unit for (plane) angle, defined as the ratio of arc length to radius, is the *radian* (rad). For solid angle, defined as the ratio of the subtended area to the square of the radius, the unit is the *steradian* (sr).

The *derived* units with special names, in their official English spelling, i.e., without capital letters and accents, are:

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* The symbols of the seven units are s, m, kg, A, K, mol and cd. The full official definitions are found at www.bipm.org. For more details about the levels of the caesium atom, consult a book on atomic physics. The Celsius scale of temperature θ is defined as: $\theta/°\mathrm{C} = T/\mathrm{K} - 273.15$; note the small difference with the number appearing in the definition of the kelvin. In the definition of the candela, the frequency of the light corresponds to 555.5 nm, i.e., green colour, around the wavelength to which the eye is most sensitive. Ref. 370

** Jacques Babinet (1794–1874), French physicist who published important work in optics.

NAME	ABBREVIATION
hertz	$\mathrm{Hz} = 1/\mathrm{s}$
pascal	$\mathrm{Pa} = \mathrm{N/m^2} = \mathrm{kg/m\,s^2}$
watt	$\mathrm{W} = \mathrm{kg\,m^2/s^3}$
volt	$\mathrm{V} = \mathrm{kg\,m^2/As^3}$
ohm	$\Omega = \mathrm{V/A} = \mathrm{kg\,m^2/A^2s^3}$
weber	$\mathrm{Wb} = \mathrm{Vs} = \mathrm{kg\,m^2/As^2}$
henry	$\mathrm{H} = \mathrm{Vs/A} = \mathrm{kg\,m^2/A^2s^2}$
lumen	$\mathrm{lm} = \mathrm{cd\,sr}$
becquerel	$\mathrm{Bq} = 1/\mathrm{s}$
sievert	$\mathrm{Sv} = \mathrm{J/kg} = \mathrm{m^2/s^2}$

NAME	ABBREVIATION
newton	$\mathrm{N} = \mathrm{kg\,m/s^2}$
joule	$\mathrm{J} = \mathrm{Nm} = \mathrm{kg\,m^2/s^2}$
coulomb	$\mathrm{C} = \mathrm{As}$
farad	$\mathrm{F} = \mathrm{As/V} = \mathrm{A^2s^4/kg\,m^2}$
siemens	$\mathrm{S} = 1/\Omega$
tesla	$\mathrm{T} = \mathrm{Wb/m^2} = \mathrm{kg/As^2} = \mathrm{kg/Cs}$
degree Celsius	°C (see definition of kelvin)
lux	$\mathrm{lx} = \mathrm{lm/m^2} = \mathrm{cd\,sr/m^2}$
gray	$\mathrm{Gy} = \mathrm{J/kg} = \mathrm{m^2/s^2}$
katal	$\mathrm{kat} = \mathrm{mol/s}$

We note that in all definitions of units, the kilogram only appears to the powers of 1, 0 and −1. Can you try to formulate the reason? Challenge 730 s

The *admitted* non-SI units are *minute*, *hour*, *day* (for time), *degree* $1° = \pi/180\,\mathrm{rad}$, *minute* $1' = \pi/10\,800\,\mathrm{rad}$, *second* $1'' = \pi/648\,000\,\mathrm{rad}$ (for angles), *litre*, and *tonne*. All other units are to be avoided.

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called *prefixes*:*

POWER	NAME	
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	Mega	M
10^{9}	Giga	G
10^{12}	Tera	T
10^{15}	Peta	P

POWER	NAME	
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

POWER	NAME	
10^{18}	Exa	E
10^{21}	Zetta	Z
10^{24}	Yotta	Y
unofficial:		
10^{27}	Xenta	X
10^{30}	Wekta	W
10^{33}	Vendekta	V
10^{36}	Udekta	U

POWER	NAME	
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y
Ref. 371		
10^{-27}	xenno	x
10^{-30}	weko	w
10^{-33}	vendeko	v
10^{-36}	udeko	u

- SI units form a *complete* system: they cover in a systematic way the full set of observables of physics. Moreover, they fix the units of measurement for all other sciences as well.

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* Some of these names are invented (yocto to sound similar to Latin *octo* 'eight', zepto to sound similar to Latin *septem*, yotta and zetta to resemble them, exa and peta to sound like the Greek words ἑξάκις and πεντάκις for 'six times' and 'five times', the unofficial ones to sound similar to the Greek words for nine, ten, eleven and twelve); some are from Danish/Norwegian (atto from *atten* 'eighteen', femto from *femten* 'fifteen'); some are from Latin (from *mille* 'thousand', from *centum* 'hundred', from *decem* 'ten', from *nanus* 'dwarf'); some are from Italian (from *piccolo* 'small'); some are Greek (micro is from μικρός 'small', deca/deka from δέκα 'ten', hecto from ἑκατόν 'hundred', kilo from χίλιοι 'thousand', mega from μέγας 'large', giga from γίγας 'giant', tera from τέρας 'monster').

Challenge 731 e Translate: I was caught in such a traffic jam that I needed a microcentury for a picoparsec and that my car's average fuel consumption was huge: it was two tenths of a square millimetre.

- SI units form a *universal* system: they can be used in trade, in industry, in commerce, at home, in education and in research. They could even be used by extraterrestrial civilizations, if they existed.
- SI units form a *self-consistent* system: the product or quotient of two SI units is also an SI unit. This means that in principle, the same abbreviation, e.g. 'SI', could be used for every unit.

The SI units are not the only possible set that could fulfil all these requirements, but they are the only existing system that does so.*

The meaning of measurement

Challenge 732 e

Every measurement is a comparison with a standard. Therefore, any measurement requires *matter* to realize the standard (even for a speed standard), and *radiation* to achieve the comparison. The concept of measurement thus assumes that matter and radiation exist and can be clearly separated from each other.

Every measurement is a comparison. Measuring thus implies that space and time exist, and that they differ from each other.

Every measurement produces a measurement result. Therefore, every measurement implies the *storage* of the result. The process of measurement thus implies that the situation before and after the measurement can be distinguished. In other terms, every measurement is an *irreversible* process.

Every measurement is a process. Thus every measurement takes a certain amount of time and a certain amount of space.

All these properties of measurements are simple but important. Beware of anybody who denies them.

Curiosities and fun challenges about units

Not using SI units can be expensive. In 1999, NASA lost a satellite on Mars because some software programmers had used provincial units instead of SI units in part of the code. As a result of using feet instead of meters, the Mars Climate Orbiter crashed into the planet, instead of orbiting it; the loss was around 100 million euro.**

* *

The second does not correspond to 1/86 400th of the day any more, though it did in the year 1900; the Earth now takes about 86 400.002 s for a rotation, so that the *International Earth Rotation Service* must regularly introduce a leap second to ensure that the Sun is at

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[*] Apart from international units, there are also *provincial* units. Most provincial units still in use are of Roman origin. The mile comes from *milia passum*, which used to be one thousand (double) strides of about 1480 mm each; today a nautical mile, once defined as minute of arc on the Earth's surface, is defined as exactly 1852 m. The inch comes from *uncia/onzia* (a twelfth – now of a foot). The pound (from *pondere* 'to weigh') is used as a translation of *libra* – balance – which is the origin of its abbreviation lb. Even the habit of counting in dozens instead of tens is Roman in origin. These and all other similarly funny units – like the system in which all units start with 'f', and which uses furlong/fortnight as its unit of velocity – are now officially defined as multiples of SI units.

[**] This story revived an old but false urban legend claiming that only three countries in the world do not use SI units: Liberia, the USA and Myanmar.

the highest point in the sky at 12 o'clock sharp.* The time so defined is called *Universal Time Coordinate*. The speed of rotation of the Earth also changes irregularly from day to day due to the weather; the average rotation speed even changes from winter to summer because of the changes in the polar ice caps; in addition that average decreases over time, because of the friction produced by the tides. The rate of insertion of leap seconds is therefore higher than once every 500 days, and not constant in time.

* *

The most precise clock ever built, using microwaves, had a stability of 10^{-16} during a Ref. 372 running time of 500 s. For longer time periods, the record in 1997 was about 10^{-15}; but Ref. 373 values around 10^{-17} seem within technological reach. The precision of clocks is limited for short measuring times by *noise*, and for long measuring times by *drifts*, i.e., by systematic effects. The region of highest stability depends on the clock type; it usually lies between 1 ms for optical clocks and 5000 s for masers. Pulsars are the only type of clock for which this region is not known yet; it certainly lies at more than 20 years, the time elapsed at the time of writing since their discovery.

* *

The least precisely measured of the fundamental constants of physics are the gravitational constant G and the strong coupling constant α_s. Even less precisely known are the age of Page 466 the universe and its density (see Table 60).

* *

The precision of mass measurements of solids is limited by such simple effects as the adsorption of water. Can you estimate the mass of a monolayer of water – a layer with Challenge 733 s thickness of one molecule – on a metal weight of 1 kg?

* *

In the previous millennium, thermal energy used to be measured using the unit *calorie*, written as cal. 1 cal is the energy needed to heat 1 g of water by 1 K. To confuse matters, 1 kcal was often written 1 Cal. (One also spoke of a large and a small calorie.) The value of 1 kcal is 4.1868 kJ.

* *

SI units are adapted to humans: the values of heartbeat, human size, human weight, human temperature and human substance are no more than a couple of orders of magnitude near the unit value. SI units thus (roughly) confirm what Protagoras said 25 centuries ago: 'Man is the measure of all things.'

* *

Some units systems are particularly badly adapted to humans. The most infamous is shoe

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* Their website at hpiers.obspm.fr gives more information on the details of these insertions, as does maia.usno.navy.mil, one of the few useful military websites. See also www.bipm.fr, the site of the BIPM.

size S. It is a pure number calculated as

$$\begin{aligned} S_{\text{France}} &= 1.5\,\text{cm}^{-1}(l + (1 \pm 1)\,\text{cm}) \\ S_{\text{central Europe}} &= 1.5748\,\text{cm}^{-1}(l + (1 \pm 1)\,\text{cm}) \\ S_{\text{Anglo-saxon men}} &= 1.181\,\text{cm}^{-1}(l + (1 \pm 1)\,\text{cm}) - 22 \end{aligned} \qquad (126)$$

where l is the length of a foot and the correction length depends on the manufacturing company. In addition, the Anglo-Saxon formula is not valid for women and children, where the first factor depends, for marketing reasons, both on the manufacturer and on size itself. The ISO standard for shoe size requires, unsurprisingly, to use foot length in millimetres.

* *

Challenge 734 s The table of SI prefixes covers 72 orders of magnitude. How many additional prefixes will be needed? Even an extended list will include only a small part of the infinite range of possibilities. Will the Conférence Générale des Poids et Mesures have to go on forever, defining an infinite number of SI prefixes? Why?

* *

In the 21st century, the textile industry uses three measurement systems to express how fine a fibre is. All three use the linear mass density m/l. The international system uses the unit 1 tex = 1 g/km. Another system is particular to silk and used the *Denier* as unit: 1 den = 1/9 g/km. The third system uses the unit *Number English*, defined as the number of hanks of cotton that weigh one pound. Here, a *hank* is 7 leas. A *lea* is 120 yards and a *yard* is three feet. In addition, the defining number of hanks differs for linnen and differs again for wool, and in addition it depends on the treatment method of the wool. Reading about textile units makes every comedy show feel like a boring lullaby.

* *

The French philosopher Voltaire, after meeting Newton, publicized the now famous story that the connection between the fall of objects and the motion of the Moon was discovered by Newton when he saw an apple falling from a tree. More than a century later, just before the French Revolution, a committee of scientists decided to take as the unit of force precisely the force exerted by gravity on a *standard apple*, and to name it after the English scientist. After extensive study, it was found that the mass of the standard apple was 101.9716 g; its weight was called 1 newton. Since then, visitors to the museum in Sèvres near Paris have been able to admire the standard metre, the standard kilogram and the standard apple.*

---

* To be clear, this is a joke; no standard apple exists. It is *not* a joke however, that owners of several apple trees in Britain and in the US claim descent, by rerooting, from the original tree under which Newton had Ref. 374 his insight. DNA tests have even been performed to decide if all these derive from the same tree. The result was, unsurprisingly, that the tree at MIT, in contrast to the British ones, is fake.

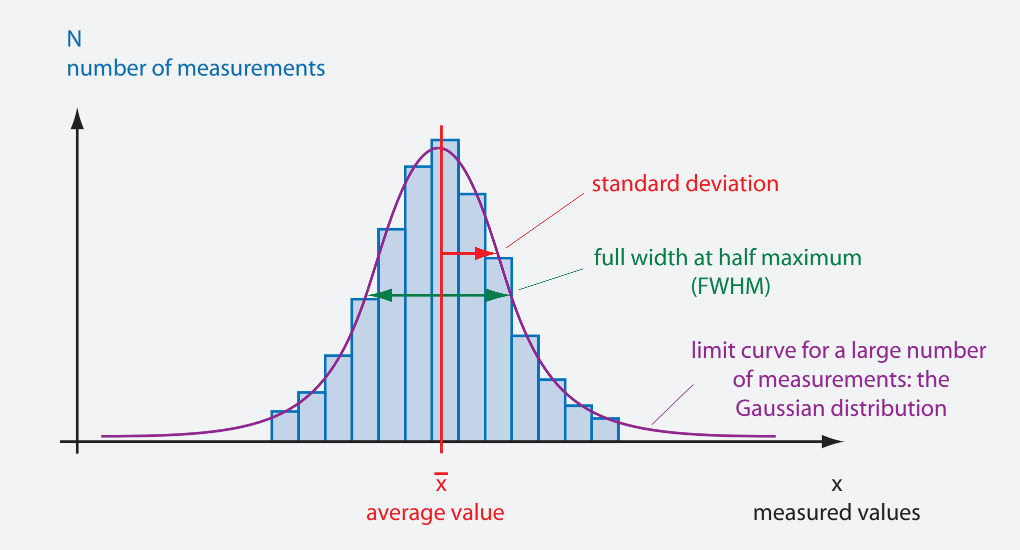

FIGURE 304 A precision experiment and its measurement distribution. The precision is high if the width of the distribution is narrow; the accuracy is high if the centre of the distribution agrees with the actual value.

PRECISION AND ACCURACY OF MEASUREMENTS

Measurements are the basis of physics. Every measurement has an *error*. Errors are due to lack of precision or to lack of accuracy. *Precision* means how well a result is reproduced when the measurement is repeated; *accuracy* is the degree to which a measurement corresponds to the actual value.

Lack of precision is due to accidental or *random errors*; they are best measured by the *standard deviation*, usually abbreviated σ; it is defined through

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 , \qquad (127)$$

where $\bar{x}$ is the average of the measurements x_i. (Can you imagine why $n-1$ is used in Challenge 735 s the formula instead of n?)

For most experiments, the distribution of measurement values tends towards a normal distribution, also called *Gaussian distribution*, whenever the number of measurements is increased. The distribution, shown in Figure 304, is described by the expression

$$N(x) \approx \mathrm{e}^{-\frac{(x-\bar{x})^2}{2\sigma^2}} . \qquad (128)$$

The square σ^2 of the standard deviation is also called the *variance*. For a Gaussian distri- Challenge 736 e bution of measurement values, 2.35σ is the full width at half maximum.

Lack of accuracy is due to *systematic errors*; usually these can only be estimated. This estimate is often added to the random errors to produce a *total experimental error*, some- Ref. 375 times also called *total uncertainty*. The *relative* error or uncertainty is the ratio between the error and the measured value.

For example, a professional measurement will give a result such as 0.312(6) m. The number between the parentheses is the standard deviation σ, in units of the last digits. As above, a Gaussian distribution for the measurement results is assumed. Therefore, a value of 0.312(6) m implies that the actual value is expected to lie

Challenge 737 e

- within 1σ with 68.3 % probability, thus in this example within 0.312 ± 0.006 m;
- within 2σ with 95.4 % probability, thus in this example within 0.312 ± 0.012 m;
- within 3σ with 99.73 % probability, thus in this example within 0.312 ± 0.018 m;
- within 4σ with 99.9937 % probability, thus in this example within 0.312 ± 0.024 m;
- within 5σ with 99.999 943 % probability, thus in this example within 0.312 ± 0.030 m;
- within 6σ with 99.999 999 80 % probability, thus in this example within 0.312 ± 0.036 m;
- within 7σ with 99.999 999 999 74 % probability, thus in this example within 0.312 ± 0.041 m.

Challenge 738 s

(Do the latter numbers make sense?)

Note that standard deviations have one digit; you must be a world expert to use two, and a fool to use more. If no standard deviation is given, a (1) is assumed. As a result, among professionals, 1 km and 1000 m are *not* the same length!

What happens to the errors when two measured values A and B are added or subtracted? If all measurements are independent – or *uncorrelated* – the standard deviation of the sum *and* that of difference is given by $\sigma = \sqrt{\sigma_A^2 + \sigma_B^2}$. For both the product or ratio of two measured and uncorrelated values C and D, the result is $\rho = \sqrt{\rho_C^2 + \rho_D^2}$, where the ρ terms are the *relative* standard deviations.

Challenge 739 s

Assume you measure that an object moves 1 m in 3 s: what is the measured speed value?

LIMITS TO PRECISION

What are the limits to accuracy and precision? There is no way, even in principle, to measure a length x to a *precision* higher than about 61 digits, because in nature, the ratio between the largest and the smallest measurable length is $\Delta x/x > l_{\text{Pl}}/d_{\text{horizon}} = 10^{-61}$. (Is this ratio valid also for force or for volume?) In the final volume of our text, studies of clocks and metre bars strengthen this theoretical limit.

Challenge 740 e
Vol. VI, page 94

But it is not difficult to deduce more stringent practical limits. No imaginable machine can measure quantities with a higher precision than measuring the diameter of the Earth within the smallest length ever measured, about 10^{-19} m; that is about 26 digits of precision. Using a more realistic limit of a 1000 m sized machine implies a limit of 22 digits. If, as predicted above, time measurements really achieve 17 digits of precision, then they are nearing the practical limit, because apart from size, there is an additional practical restriction: cost. Indeed, an additional digit in measurement precision often means an additional digit in equipment cost.

PHYSICAL CONSTANTS

In physics, general observations are deduced from more fundamental ones. As a consequence, many measurements can be deduced from more fundamental ones. The most fundamental measurements are those of the physical constants.

The following tables give the world's best values of the most important physical constants and particle properties – in SI units and in a few other common units – as published in the standard references. The values are the world averages of the best measurements made up to the present. As usual, experimental errors, including both random and estimated systematic errors, are expressed by giving the standard deviation in the last digits. In fact, behind each of the numbers in the following tables there is a long story which is worth telling, but for which there is not enough room here.

Ref. 376

Ref. 377

In principle, *all* quantitative properties of matter can be calculated with quantum theory – more precisely, equations of the standard model of particle – and a set of *basic* physical constants that are given in the next table. For example, the colour, density and elastic properties of any material can be predicted, in principle, in this way.

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TABLE 57 Basic physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.[a]
Constants that define the SI measurement units			
Vacuum speed of light [c]	c	299 792 458 m/s	0
Original Planck constant [c]	h	$6.626\,070\,15 \cdot 10^{-34}$ Js	0
Reduced Planck constant, quantum of action	$\hbar$	$1.054\,571\,817 ... \cdot 10^{-34}$ Js	0
Positron charge [c]	e	0.160 217 6634 aC	0
Boltzmann constant [c]	k	$1.380\,649 \cdot 10^{-23}$ J/K	0
Avogadro's number	N_A	$6.022\,140\,76 \cdot 10^{23}$ 1/mol	0
Constant that *should* define the SI measurement units			
Gravitational constant	G	$6.674\,30(15) \cdot 10^{-11}$ Nm²/kg²	$2.2 \cdot 10^{-5}$
Other fundamental constants			
Number of space-time dimensions		3 + 1	0 [b]
Fine-structure constant [d] or	$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$	$1/137.035\,999\,084(21)$	$1.5 \cdot 10^{-10}$
e.m. coupling constant	$= g_{em}(m_e^2c^2)$	$= 0.007\,297\,352\,5693(11)$	$1.5 \cdot 10^{-10}$
Fermi coupling constant [d] or	$G_F/(\hbar c)^3$	$1.166\,3787(6) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$	$5.1 \cdot 10^{-7}$
weak coupling constant	$\alpha_w(M_Z) = g_w^2/4\pi$	$1/30.1(3)$	$1 \cdot 10^{-2}$
Strong coupling constant [d]	$\alpha_s(M_Z) = g_s^2/4\pi$	0.1179(10)	$8.5 \cdot 10^{-3}$
Weak mixing angle	$\sin^2\theta_W(\overline{MS})$	0.231 22(4)	$1.7 \cdot 10^{-4}$
	$\sin^2\theta_W$ (on shell) $= 1 - (m_W/m_Z)^2$	0.222 90(30)	$1.3 \cdot 10^{-3}$
CKM quark mixing matrix	$\lvert V\rvert$	$\begin{pmatrix} 0.97383(24) & 0.2272(10) & 0.00396(9) \\ 0.2271(10) & 0.97296(24) & 0.04221(80) \\ 0.00814(64) & 0.04161(78) & 0.999100(34) \end{pmatrix}$	
Jarlskog invariant	J	$3.08(18) \cdot 10^{-5}$	
PMNS neutrino mixing m.	$\lvert P\rvert$	$\begin{pmatrix} 0.82(2) & 0.55(4) & 0.150(7) \\ 0.37(13) & 0.57(11) & 0.71(7) \\ 0.41(13) & 0.59(10) & 0.69(7) \end{pmatrix}$	

TABLE 57 (Continued) Basic physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.[a]
Electron mass	m_e	$9.109\,383\,7015(28) \cdot 10^{-31}$ kg	$3.0 \cdot 10^{-10}$
		$5.485\,799\,090\,65(16) \cdot 10^{-4}$ u	$2.9 \cdot 10^{-11}$
		$0.510\,998\,950\,00(15)$ MeV	$3.0 \cdot 10^{-10}$
Muon mass	m_μ	$1.883\,531\,627(42) \cdot 10^{-28}$ kg	$2.2 \cdot 10^{-8}$
		$105.658\,3755(23)$ MeV	$2.2 \cdot 10^{-8}$
Tau mass	m_τ	$1.776\,82(12)\,\mathrm{GeV}/c^2$ $6.8 \cdot 10^{-5}$	
El. neutrino mass	m_{ν_e}	$< 2\,\mathrm{eV}/c^2$	
Muon neutrino mass	m_{ν_μ}	$< 2\,\mathrm{eV}/c^2$	
Tau neutrino mass	m_{ν_τ}	$< 2\,\mathrm{eV}/c^2$	
Up quark mass	u	$21.6(+0.49/-0.26)\,\mathrm{MeV}/c^2$	
Down quark mass	d	$4.67(+0.48/-0.17)\,\mathrm{MeV}/c^2$	
Strange quark mass	s	$93(+11/-5)\,\mathrm{MeV}/c^2$	
Charm quark mass	c	$1.27(2)\,\mathrm{GeV}/c^2$	
Bottom quark mass	b	$4.18(3)\,\mathrm{GeV}/c^2$	
Top quark mass	t	$172.9(0.4)\,\mathrm{GeV}/c^2$	
Photon mass	γ	$< 2 \cdot 10^{-54}$ kg	
W boson mass	$W^\pm$	$80.379(12)\,\mathrm{GeV}/c^2$	
Z boson mass	Z^0	$91.1876(21)\,\mathrm{GeV}/c^2$	
Higgs mass	H	$125.10(14)\,\mathrm{GeV}/c^2$	
Gluon mass	$\mathrm{g}_{1...8}$	$c.\ 0\,\mathrm{MeV}/c^2$	

a. Uncertainty: standard deviation of measurement errors.
b. Measured from to 10^{-19} m to 10^{26} m.
c. Defining constant.
d. All coupling constants depend on the 4-momentum transfer, as explained in the section on renormalization. *Fine-structure constant* is the traditional name for the electromagnetic coupling constant $g_{\rm em}$ in the case of a 4-momentum transfer of $Q^2 = m_e^2 c^2$, which is the smallest one possible. At higher momentum transfers it has larger values, e.g., $g_{\rm em}(Q^2 = M_W^2 c^2) \approx 1/128$. In contrast, the strong coupling constant has lover values at higher momentum transfers; e.g., $\alpha_s(34\,\mathrm{GeV}) = 0.14(2)$.

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Why do all these basic constants have the values they have? For any basic constant *with a dimension*, such as the quantum of action $\hbar$, the numerical value has only historical meaning. It is $1.054 \cdot 10^{-34}$ Js because of the SI definition of the joule and the second. The question why the value of a *dimensional* constant is not larger or smaller therefore always requires one to understand the origin of some *dimensionless* number giving the ratio between the constant and the corresponding *natural unit* that is defined with c, G, k, N_A and $\hbar$. Details and values for the natural units are given in the dedicated section.

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In other words, understanding the sizes of atoms, people, trees and stars, the duration of molecular and atomic processes, or the mass of nuclei and mountains, implies understanding the ratios between these values and the corresponding natural units. The key to understanding nature is thus the understanding of all measurement ratios, and thus of all dimensionless constants. This quest, including the understanding of the fine-structure constant α itself, is completed only in the final volume of our adventure.

The basic constants yield the following useful high-precision observations.

TABLE 58 Derived physical constants.

Quantity	Symbol	Value in SI units	Uncert.
Vacuum permeability	μ_0	1.256 637 062 12(19) µH/m	$1.5 \cdot 10^{-10}$
Vacuum permittivity	$\varepsilon_0 = 1/\mu_0 c^2$	8.854 187 8128(13) pF/m	$1.5 \cdot 10^{-10}$
Vacuum impedance	$Z_0 = \sqrt{\mu_0/\varepsilon_0}$	376.730 313 668(57) Ω	$1.5 \cdot 10^{-10}$
Loschmidt's number at 273.15 K and 101 325 Pa	N_L	2.686 780 111... $\cdot 10^{25}$ 1/m^3	0
Faraday's constant	$F = N_\mathrm{A} e$	96 485.332 12... C/mol	0
Universal gas constant	$R = N_\mathrm{A} k$	8.314 462 618... J/(mol K)	0
Molar volume of an ideal gas at 273.15 K and 101 325 Pa	$V = RT/p$	22.413 969 54... l/mol	0
Rydberg constant [a]	$R_\infty = m_\mathrm{e} c \alpha^2/2h$	10 973 731.568 160(21) m^{-1}	$1.9 \cdot 10^{-12}$
Conductance quantum	$G_0 = 2e^2/h$	77.480 917 29... µS	0
Magnetic flux quantum	$\varphi_0 = h/2e$	2.067 833 848... fWb	0
Josephson frequency ratio	$2e/h$	483.597 8484... THz/V	0
Von Klitzing constant	$h/e^2 = \mu_0 c/2\alpha$	25 812.807 45... Ω	0
Bohr magneton	$\mu_\mathrm{B} = e\hbar/2m_\mathrm{e}$	9.274 010 0783(28) yJ/T	$3.0 \cdot 10^{-10}$
Classical electron radius	$r_\mathrm{e} = e^2/4\pi\varepsilon_0 m_\mathrm{e} c^2$	2.817 940 3262(13) fm	$4.5 \cdot 10^{-10}$
Compton wavelength	$\lambda_\mathrm{C} = h/m_\mathrm{e} c$	2.426 310 238 67(73) pm	$3.0 \cdot 10^{-10}$
of the electron	$\lambda\!\!\!^{-}_\mathrm{c} = \hbar/m_\mathrm{e} c = r_\mathrm{e}/\alpha$	0.386 159 267 96(12) pm	$3.0 \cdot 10^{-10}$
Bohr radius [a]	$a_\infty = r_\mathrm{e}/\alpha^2$	52.917 721 0903(80) pm	$1.5 \cdot 10^{-10}$
Quantum of circulation	$h/2m_\mathrm{e}$	3.636 947 5516(11) cm^2/s	$3.0 \cdot 10^{-10}$
Specific positron charge	e/m_e	175.882 001 076(55) GC/kg	$3.0 \cdot 10^{-10}$
Cyclotron frequency of the electron	$f_\mathrm{c}/B = e/2\pi m_\mathrm{e}$	27.992 489 872(9) GHz/T	$3.0 \cdot 10^{-10}$
Electron magnetic moment	μ_e	−9.284 764 7043(28) yJ/T	$3.0 \cdot 10^{-10}$
	$\mu_\mathrm{e}/\mu_\mathrm{B}$	−1.001 159 652 181 28(18)	$1.7 \cdot 10^{-13}$
	$\mu_\mathrm{e}/\mu_\mathrm{N}$	−1 838.281 971 88(11) $\cdot 10^3$	$6.0 \cdot 10^{-11}$
Electron g-factor	g_e	−2.002 319 304 362 56(35)	$1.7 \cdot 10^{-13}$
Muon–electron mass ratio	m_μ/m_e	206.768 2830(46)	$2.2 \cdot 10^{-8}$
Muon magnetic moment	μ_μ	−4.490 448 30(10) $\cdot 10^{-26}$ J/T	$2.2 \cdot 10^{-8}$
Muon g-factor	g_μ	−2.002 331 8418(13)	$6.3 \cdot 10^{-10}$
Atomic mass unit	$1\,\mathrm{u} = m_{^{12}\mathrm{C}}/12$	1.660 539 066 60(50) $\cdot 10^{-27}$ kg	$3.0 \cdot 10^{-10}$
Proton mass	m_p	1.672 621 923 69(51) $\cdot 10^{-27}$ kg	$3.1 \cdot 10^{-10}$
		1.007 276 466 621(53) u	$5.3 \cdot 10^{-11}$
		938.272 088 16(29) MeV	$3.1 \cdot 10^{-10}$
Proton–electron mass ratio	$m_\mathrm{p}/m_\mathrm{e}$	1 836.152 673 43(11)	$6.0 \cdot 10^{-11}$
Specific proton charge	e/m_p	9.578 833 1560(29) $\cdot 10^7$ C/kg	$3.1 \cdot 10^{-10}$

TABLE 58 (Continued) Derived physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.
Proton Compton wavelength	$\lambda_{C,p} = h/m_p c$	1.321 409 855 39(40) fm	$3.1 \cdot 10^{-10}$
Nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,7461(15) \cdot 10^{-27}$ J/T	$3.1 \cdot 10^{-10}$
Proton magnetic moment	μ_p	$1.410\,606\,797\,36(60) \cdot 10^{-26}$ J/T	$4.2 \cdot 10^{-10}$
	μ_p/μ_B	$1.521\,032\,202\,30(46) \cdot 10^{-3}$	$3.0 \cdot 10^{-10}$
	μ_p/μ_N	2.792 847 344 63(82)	$2.9 \cdot 10^{-10}$
Proton gyromagnetic ratio	$\gamma_p = 2\mu_p/h$	42.577 478 518(18) MHz/T	$4.2 \cdot 10^{-10}$
Proton g factor	g_p	5.585 694 6893(16)	$2.9 \cdot 10^{-10}$
Neutron mass	m_n	$1.674\,927\,498\,04(95) \cdot 10^{-27}$ kg	$5.7 \cdot 10^{-10}$
		1.008 664 915 95(43) u	$4.8 \cdot 10^{-10}$
		939.565 420 52(54) MeV	$5.7 \cdot 10^{-10}$
Neutron–electron mass ratio	m_n/m_e	1 838.683 661 73(89)	$4.8 \cdot 10^{-10}$
Neutron–proton mass ratio	m_n/m_p	1.001 378 419 31(49)	$4.9 \cdot 10^{-10}$
Neutron Compton wavelength	$\lambda_{C,n} = h/m_n c$	1.319 590 905 81(75) fm	$5.7 \cdot 10^{-10}$
Neutron magnetic moment	μ_n	$-0.966\,236\,51(23) \cdot 10^{-26}$ J/T	$2.4 \cdot 10^{-7}$
	μ_n/μ_B	$-1.041\,875\,63(25) \cdot 10^{-3}$	$2.4 \cdot 10^{-7}$
	μ_n/μ_N	−1.913 042 73(45)	$2.4 \cdot 10^{-7}$
Stefan–Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	56.703 744 19... nW/m^{2}K^4	0
Wien's displacement constant	$b = \lambda_{max} T$	2.897 771 955... mmK	0
		58.789 257 57... GHz/K	0
Electron volt	eV	0.160 217 6634... aJ	0
Bits to entropy conversion const.	$k \ln 2$	10^{23} bit = 0.956 994... J/K	0
TNT energy content		3.7 to 4.0 MJ/kg	$4 \cdot 10^{-2}$

a. For infinite mass of the nucleus.

Some useful properties of our local environment are given in the following table.

TABLE 59 Astronomical constants.

QUANTITY	SYMBOL	VALUE
Tropical year 1900 [a]	a	31 556 925.974 7 s
Tropical year 1994	a	31 556 925.2 s
Mean sidereal day	d	$23^h 56' 4.090\,53''$
Average distance Earth–Sun [b]		149 597 870.691(30) km
Astronomical unit [b]	AU	149 597 870 691 m
Light year, based on Julian year [b]	al	9.460 730 472 5808 Pm
Parsec	pc	30.856 775 806 Pm = 3.261 634 al
Earth's mass	$M_{♁}$	$5.973(1) \cdot 10^{24}$ kg
Geocentric gravitational constant	GM	$3.986\,004\,418(8) \cdot 10^{14}$ m^3/s^2
Earth's gravitational length	$l_{♁} = 2GM/c^2$	8.870 056 078(16) mm

TABLE 59 (Continued) Astronomical constants.

QUANTITY	SYMBOL	VALUE
Earth's equatorial radius [c]	$R_{\text{♁}\mathrm{eq}}$	6378.1366(1) km
Earth's polar radius [c]	$R_{\text{♁}\mathrm{p}}$	6356.752(1) km
Equator–pole distance [c]		10 001.966 km (average)
Earth's flattening [c]	$e_{\text{♁}}$	1/298.25642(1)
Earth's av. density	$\rho_{\text{♁}}$	5.5 $\mathrm{Mg/m^3}$
Earth's age	$T_{\text{♁}}$	4.50(4) Ga = 142(2) Ps
Earth's normal gravity	g	9.806 65 $\mathrm{m/s^2}$
Earth's standard atmospher. pressure	p_0	101 325 Pa
Moon's radius	$R_{\text{☾}\mathrm{v}}$	1738 km in direction of Earth
Moon's radius	$R_{\text{☾}\mathrm{h}}$	1737.4 km in other two directions
Moon's mass	$M_{\text{☾}}$	$7.35 \cdot 10^{22}$ kg
Moon's mean distance [d]	$d_{\text{☾}}$	384 401 km
Moon's distance at perigee [d]		typically 363 Mm, historical minimum 359 861 km
Moon's distance at apogee [d]		typically 404 Mm, historical maximum 406 720 km
Moon's angular size [e]		average 0.5181° = 31.08′, minimum 0.49°, maximum 0.55°
Moon's average density	$\rho_{\text{☾}}$	3.3 $\mathrm{Mg/m^3}$
Moon's surface gravity	$g_{\text{☾}}$	1.62 $\mathrm{m/s^2}$
Moon's atmospheric pressure	$p_{\text{☾}}$	from 10^{-10} Pa (night) to 10^{-7} Pa (day)
Jupiter's mass	$M_{\text{♃}}$	$1.90 \cdot 10^{27}$ kg
Jupiter's radius, equatorial	$R_{\text{♃}}$	71.398 Mm
Jupiter's radius, polar	$R_{\text{♃}}$	67.1(1) Mm
Jupiter's average distance from Sun	$D_{\text{♃}}$	778 412 020 km
Jupiter's surface gravity	$g_{\text{♃}}$	24.9 $\mathrm{m/s^2}$
Jupiter's atmospheric pressure	$p_{\text{♃}}$	from 20 kPa to 200 kPa
Sun's mass	$M_{\odot}$	$1.988\,43(3) \cdot 10^{30}$ kg
Sun's gravitational length	$2GM_{\odot}/c^2$	2.953 250 08(5) km
Heliocentric gravitational constant	$GM_{\odot}$	$132.712\,440\,018(8) \cdot 10^{18}\ \mathrm{m^3/s^2}$
Sun's luminosity	$L_{\odot}$	384.6 YW
Solar equatorial radius	$R_{\odot}$	695.98(7) Mm
Sun's angular size		0.53° average; minimum on fourth of July (aphelion) 1888″, maximum on fourth of January (perihelion) 1952″
Sun's average density	$\rho_{\odot}$	1.4 $\mathrm{Mg/m^3}$
Sun's average distance	AU	149 597 870.691(30) km
Sun's age	$T_{\odot}$	4.6 Ga
Solar velocity around centre of galaxy	$v_{\odot\mathrm{g}}$	220(20) km/s

TABLE 59 (Continued) Astronomical constants.

QUANTITY	SYMBOL	VALUE
Solar velocity against cosmic background	$v_{\odot\mathrm{b}}$	370.6(5) km/s
Sun’s surface gravity	$g_\odot$	$274\,\mathrm{m/s^2}$
Sun’s lower photospheric pressure	$p_\odot$	15 kPa
Distance to Milky Way’s centre		8.0(5) kpc = 26.1(1.6) kal
Milky Way’s age		13.6 Ga
Milky Way’s size		*c.* 10^{21} m or 100 kal
Milky Way’s mass		10^{12} solar masses, *c.* $2 \cdot 10^{42}$ kg
Most distant galaxy cluster known	SXDF-XCLJ 0218-0510	$9.6 \cdot 10^9$ al

a. Defining constant, from vernal equinox to vernal equinox; it was once used to define the second. (Remember: π seconds is about a nanocentury.) The value for 1990 is about 0.7 s less, corresponding to a slowdown of roughly 0.2 ms/a. (Watch out: why?) There is even an empirical formula for the change of the length of the year over time.

Challenge 741 s

Ref. 378

b. The truly amazing precision in the average distance Earth–Sun of only 30 m results from time averages of signals sent from Viking orbiters and Mars landers taken over a period of over twenty years. Note that the International Astronomical Union distinguishes the average distance Earth–Sun from the *astronomical unit* itself; the latter is defined as a fixed and exact length. Also the *light year* is a unit defined as an exact number by the IAU. For more details, see www.iau.org/public/measuring.

c. The shape of the Earth is described most precisely with the World Geodetic System. The last edition dates from 1984. For an extensive presentation of its background and its details, see the www.wgs84.com website. The International Geodesic Union refined the data in 2000. The radii and the flattening given here are those for the ‘mean tide system’. They differ from those of the ‘zero tide system’ and other systems by about 0.7 m. The details constitute a science in itself.

d. Measured centre to centre. To find the precise position of the Moon in the sky at a given date, see the www.fourmilab.ch/earthview/moon_ap_per.html page. For the planets, see the page www.fourmilab.ch/solar/solar.html and the other pages on the same site.

e. Angles are defined as follows: 1 degree = $1^\circ = \pi/180$ rad, 1 (first) minute = $1' = 1^\circ/60$, 1 second (minute) = $1'' = 1'/60$. The ancient units ‘third minute’ and ‘fourth minute’, each 1/60th of the preceding, are not in use any more. (‘Minute’ originally means ‘very small’, as it still does in modern English.)

Some properties of nature at large are listed in the following table. (If you want a challenge, can you determine whether any property of the universe itself is listed?)

Challenge 742 s

TABLE 60 Cosmological constants.

QUANTITY	SYMBOL	VALUE
Cosmological constant	Λ	*c.* $1 \cdot 10^{-52}\,\mathrm{m^{-2}}$
Age of the universe [a] (determined from space-time, via expansion, using general relativity)	t_0	$4.333(53) \cdot 10^{17}\,\mathrm{s} = 13.8(0.1) \cdot 10^9\,\mathrm{a}$

TABLE 60 (Continued) Cosmological constants.

QUANTITY	SYMBOL	VALUE
Age of the universe[a]	t_0	over $3.5(4) \cdot 10^{17}$ s $= 11.5(1.5) \cdot 10^9$ a
(determined from matter, via galaxies and stars, using quantum theory)		
Hubble parameter[a]	H_0	$2.3(2) \cdot 10^{-18}\,\mathrm{s}^{-1} = 0.73(4) \cdot 10^{-10}\,\mathrm{a}^{-1}$
	$= h_0 \cdot 100\,\mathrm{km/s\,Mpc} = h_0 \cdot 1.0227 \cdot 10^{-10}\,\mathrm{a}^{-1}$	
Reduced Hubble parameter[a]	h_0	0.71(4)
Deceleration parameter[a]	$q_0 = -(\ddot{a}/a)_0/H_0^2$	−0.66(10)
Universe's horizon distance[a]	$d_0 = 3ct_0$	$40.0(6) \cdot 10^{26}$ m = 13.0(2) Gpc
Universe's topology		trivial up to 10^{26} m
Number of space dimensions		3, for distances up to 10^{26} m
Critical density	$\rho_c = 3H_0^2/8\pi G$	$h_0^2 \cdot 1.878\,82(24) \cdot 10^{-26}\,\mathrm{kg/m^3}$
of the universe		$= 0.95(12) \cdot 10^{-26}\,\mathrm{kg/m^3}$
(Total) density parameter[a]	$\Omega_0 = \rho_0/\rho_c$	1.02(2)
Baryon density parameter[a]	$\Omega_{B0} = \rho_{B0}/\rho_c$	0.044(4)
Cold dark matter density parameter[a]	$\Omega_{CDM0} = \rho_{CDM0}/\rho_c$	0.23(4)
Neutrino density parameter[a]	$\Omega_{\nu 0} = \rho_{\nu 0}/\rho_c$	0.001 to 0.05
Dark energy density parameter[a]	$\Omega_{X0} = \rho_{X0}/\rho_c$	0.73(4)
Dark energy state parameter	$w = p_X/\rho_X$	−1.0(2)
Baryon mass	m_b	$1.67 \cdot 10^{-27}$ kg
Baryon number density		$0.25(1)\,/\mathrm{m^3}$
Luminous matter density		$3.8(2) \cdot 10^{-28}\,\mathrm{kg/m^3}$
Stars in the universe	n_s	$10^{22\pm1}$
Baryons in the universe	n_b	$10^{81\pm1}$
Microwave background temperature[b]	T_0	2.725(1) K
Photons in the universe	n_γ	10^{89}
Photon energy density	$\rho_\gamma = \pi^2 k^4/15T_0^4$	$4.6 \cdot 10^{-31}\,\mathrm{kg/m^3}$
Photon number density		$410.89\,/\mathrm{cm^3}$ or $400\,/\mathrm{cm^3}(T_0/2.7\,\mathrm{K})^3$
Density perturbation amplitude	$\sqrt{S}$	$5.6(1.5) \cdot 10^{-6}$
Gravity wave amplitude	$\sqrt{T}$	$< 0.71\sqrt{S}$
Mass fluctuations on 8 Mpc	σ_8	0.84(4)
Scalar index	n	0.93(3)
Running of scalar index	$\mathrm{d}n/\mathrm{d}\ln k$	−0.03(2)
Planck length	$l_{Pl} = \sqrt{\hbar G/c^3}$	$1.62 \cdot 10^{-35}$ m
Planck time	$t_{Pl} = \sqrt{\hbar G/c^5}$	$5.39 \cdot 10^{-44}$ s
Planck mass	$m_{Pl} = \sqrt{\hbar c/G}$	21.8 µg
Instants in history[a]	t_0/t_{Pl}	$8.7(2.8) \cdot 10^{60}$
Space-time points inside the horizon[a]	$N_0 = (R_0/l_{Pl})^3 \cdot (t_0/t_{Pl})$	$10^{244\pm1}$
Mass inside horizon	M	$10^{54\pm1}$ kg

a. The index 0 indicates present-day values.

b. The radiation originated when the universe was 380 000 years old and had a temperature of about 3000 K; the fluctuations ΔT_0 which led to galaxy formation are today about $16 \pm 4\,\mu\mathrm{K} = 6(2) \cdot 10^{-6}\,T_0$.

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USEFUL NUMBERS

Ref. 350

e	$2.71828\,18284\,59045\,23536\,02874\,71352\,66249\,77572\,47093\,69995_9$
π	$3.14159\,26535\,89793\,23846\,26433\,83279\,50288\,41971\,69399\,37510_5$
π^2	$9.86960\,44010\,89358\,61883\,44909\,99876\,15113\,53136\,99407\,24079_0$
γ	$0.57721\,56649\,01532\,86060\,65120\,90082\,40243\,10421\,59335\,93992_3$
ln 2	$0.69314\,71805\,59945\,30941\,72321\,21458\,17656\,80755\,00134\,36025_5$
ln 10	$2.30258\,50929\,94045\,68401\,79914\,54684\,36420\,76011\,01488\,62877_2$
$\sqrt{10}$	$3.16227\,76601\,68379\,33199\,88935\,44432\,71853\,37195\,55139\,32521_6$

Appendix C

SOURCES OF INFORMATION ON MOTION

> “No place affords a more striking conviction of the vanity of human hopes than a public library.”
> Samuel Johnson

> “In a consumer society there are inevitably two kinds of slaves: the prisoners of addiction and the prisoners of envy.”
> Ivan Illich**

In the text, good books that introduce neighbouring domains are presented in the bibliography. The bibliography also points to journals and websites, in order to satisfy more intense curiosity about what is encountered in this adventure. All citations can also be found by looking up the author in the name index. To find additional information, either libraries or the internet can help.

In a library, review articles of recent research appear in journals such as Reviews of Modern Physics, Reports on Progress in Physics, Contemporary Physics and Advances in Physics. Good pedagogical introductions are found in the American Journal of Physics, the European Journal of Physics and Physik in unserer Zeit.

Overviews on research trends occasionally appear in magazines such as Physics World, Physics Today, Europhysics Journal, Physik Journal and Nederlands tijdschrift voor natuurkunde. For coverage of all the sciences together, the best sources are the magazines Nature, New Scientist, Naturwissenschaften, La Recherche and Science News.

Research papers on the foundations of motion appear mainly in Physics Letters B, Nuclear Physics B, Physical Review D, Physical Review Letters, Classical and Quantum Gravity, General Relativity and Gravitation, International Journal of Modern Physics and Modern Physics Letters. The newest results and speculative ideas are found in conference proceedings, such as the Nuclear Physics B Supplements. Research articles also appear in Fortschritte der Physik, European Physical Journal, La Rivista del Nuovo Cimento, Europhysics Letters, Communications in Mathematical Physics, Journal of Mathematical Physics, Foundations of Physics, International Journal of Theoretical Physics and Journal of Physics G.

There are only a few internet physics journals of quality: one is Living Reviews in Relativity, found at www.livingreviews.org, the other is the New Journal of Physics, which can be found at the www.njp.org website. There are, unfortunately, also many internet physics journals that publish incorrect research. They are easy to spot: they ask for money

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** Ivan Illich (b. 1926 Vienna, d. 2002 Bremen), theologian and social and political thinker.

to publish a paper.

By far the simplest way to keep in touch with ongoing research on motion and modern physics is to use the *internet*, the international computer network. To start using it, ask a friend who knows.*

In the last decade of the twentieth century, the internet expanded into a combination of library, business tool, discussion platform, media collection, garbage collection and, above all, addiction provider. Do not use it too much. Commerce, advertising and – unfortunately – addictive material for children, youth and adults, as well as crime of all kind are also an integral part of the web. With a personal computer, a modem and free browser software, you can look for information in millions of pages of documents or destroy your professional career through addiction. The various parts of the documents are located in various computers around the world, but the user does not need to be aware of this.**

Most theoretical physics papers are available free of charge, as *preprints*, i.e., before official publication and checking by referees, at the arxiv.org website. A service for finding subsequent preprints that cite a given one is also available.

Research papers on the description of motion appear *after* this text is published can also be found via www.webofknowledge.com a site accessible only from libraries. It allows one to search for all publications which *cite* a given paper.

Searching the web for authors, organizations, books, publications, companies or simple keywords using search engines can be a rewarding experience or an episode of addiction, depending entirely on yourself. A selection of interesting servers about motion is given below.

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* It is also possible to use the internet and to download files through FTP with the help of email only. But the tools change too often to give a stable guide here. Ask your friend.

** Several decades ago, the provocative book by Ivan Illich, *Deschooling Society*, Harper & Row, 1971, listed four basic ingredients for any educational system:

1. access to *resources* for learning, e.g. books, equipment, games, etc. at an affordable price, for everybody, at any time in their life;
2. for all who want to learn, access to *peers* in the same learning situation, for discussion, comparison, cooperation and competition;
3. access to *elders*, e.g. teachers, for their care and criticism towards those who are learning;
4. exchanges between students and *performers* in the field of interest, so that the latter can be models for the former. For example, there should be the possibility to listen to professional musicians and reading the works of specialist writers. This also gives performers the possibility to share, advertise and use their skills.

Illich develops the idea that if such a system were informal – he then calls it a ‘learning web’ or ‘opportunity web’ – it would be superior to formal, state-financed institutions, such as conventional schools, for the development of mature human beings. These ideas are deepened in his following works, *Deschooling Our Lives*, Penguin, 1976, and *Tools for Conviviality*, Penguin, 1973.

Today, any networked computer offers *email* (electronic mail), FTP (file transfers to and from another computer), access to discussion groups on specific topics, such as particle physics, and the *world-wide web*. In a rather unexpected way, all these facilities of the internet have transformed it into the backbone of the ‘opportunity web’ discussed by Illich. However, as in any school, it strongly depends on the user’s discipline whether the internet actually does provide a learning web or an entry into addiction.

TABLE 61 Some interesting sites on the world-wide web.

Topic	Website address
General knowledge	
Innovation in science and technology	www.innovations-report.de
Book collections	www.ulib.org books.google.com
Entertaining science education by Theodore Gray	www.popsci.com/category/popsci-authors/theodore-gray
Entertaining and professional science education by Robert Krampf	thehappyscientist.com
Science Frontiers	www.science-frontiers.com
Science Daily News	www.sciencedaily.com
Science News	www.sciencenews.org
Encyclopedia of Science	www.daviddarling.info
Interesting science research	www.max-wissen.de
Quality science videos	www.vega.org.uk
ASAP Science videos	plus.google.com/101786231119207015313/posts
Physics	
Learning physics with toys from rubbish	www.arvindguptatoys.com
Official SI unit website	www.bipm.fr
Unit conversion	www.chemie.fu-berlin.de/chemistry/general/units.html
Particle data	pdg.web.cern.ch
Engineering data and formulae	www.efunda.com
Information on relativity	math.ucr.edu/home/baez/relativity.html
Research preprints	arxiv.org www.slac.stanford.edu/spires
Abstracts of papers in physics journals	www.osti.gov
Many physics research papers	sci-hub.tv, sci-hub.la libgen.pw, libgen.io
Physics news, weekly	www.aip.org/physnews/update
Physics news, daily	phys.org
Physics problems by Yacov KantorKantor, Yacov	www.tau.ac.il/~kantor/QUIZ/
Physics problems by Henry Greenside	www.phy.duke.edu/~hsg/physics-challenges/challenges.html
Physics ‘question of the week’	www.physics.umd.edu/lecdem/outreach/QOTW/active
Physics ‘miniproblem’	www.nyteknik.se/miniproblemet
Physikhexe	physik-verstehen-mit-herz-und-hand.de/html/de-6.html

Topic	Website address
Magic science tricks	www.sciencetrix.com
Physics stack exchange	physics.stackexchange.com
‘Ask the experts’	www.sciam.com/askexpert_directory.cfm
Nobel Prize winners	www.nobel.se/physics/laureates
Videos of Nobel Prize winner talks	www.mediatheque.lindau-nobel.org
Pictures of physicists	www.if.ufrj.br/famous/physlist.html
Physics organizations	www.cern.ch
	www.hep.net
	www.nikhef.nl
	www.het.brown.edu/physics/review/index.html
Physics textbooks on the web	www.physics.irfu.se/CED/Book
	www.biophysics.org/education/resources.htm
	www.lightandmatter.com
	www.physikdidaktik.uni-karlsruhe.de/index_en.html
	www.feynmanlectures.info
	hyperphysics.phy-astr.gsu.edu/hbase/hph.html
	www.motionmountain.net
Three beautiful French sets of notes on classical mechanics and particle theory	feynman.phy.ulaval.ca/marleau/notesdecours.htm
The excellent *Radical Freshman Physics* by David Raymond	www.physics.nmt.edu/~raymond/teaching.html
Physics course scripts from MIT	ocw.mit.edu/courses/physics/
Physics lecture scripts in German and English	www.akleon.de
‘World lecture hall’	wlh.webhost.utexas.edu
Optics picture of the day	www.atoptics.co.uk/opod.htm
Living Reviews in Relativity	www.livingreviews.org
Wissenschaft in die Schulen	www.wissenschaft-schulen.de
Videos of Walter Lewin’sIndexLewin, Walter physics lectures	ocw.mit.edu/courses/physics/8-01-physics-i-classical-mechanics-fall-1999/
Physics videos of Matt Carlson	www.youtube.com/sciencetheater
Physics videos by the University of Nottingham	www.sixtysymbols.com
Physics lecture videos	www.coursera.org/courses?search=physics
	www.edx.org/course-list/allschools/physics/allcourses

Mathematics

Topic	Website address
'Math forum' internet resource collection	mathforum.org/library
Biographies of mathematicians	www-history.mcs.st-andrews.ac.uk/BiogIndex.html
Purdue math problem of the week	www.math.purdue.edu/academics/pow
Macalester College maths problem of the week	mathforum.org/wagon
Mathematical formulae	dlmf.nist.gov
Weisstein's World of Mathematics	mathworld.wolfram.com
Functions	functions.wolfram.com
Symbolic integration	www.integrals.com
Algebraic surfaces	www.mathematik.uni-kl.de/~hunt/drawings.html
Math lecture videos, in German	www.j3l7h.de/videos.html
Gazeta Matematica, in Romanian	www.gazetamatematica.net
Astronomy	
ESA	sci.esa.int
NASA	www.nasa.gov
Hubble space telescope	hubble.nasa.gov
Sloan Digital Sky Survey	skyserver.sdss.org
The 'cosmic mirror'	www.astro.uni-bonn.de/~dfischer/mirror
Solar System simulator	space.jpl.nasa.gov
Observable satellites	liftoff.msfc.nasa.gov/RealTime/JPass/20
Astronomy picture of the day	antwrp.gsfc.nasa.gov/apod/astropix.html
The Earth from space	www.visibleearth.nasa.gov
From Stargazers to Starships	www.phy6.org/stargaze/Sintro.htm
Current solar data	www.n3kl.org/sun
Specific topics	
Sonic wonders to visit in the world	www.sonicwonders.org
Encyclopedia of photonics	www.rp-photonics.com
Chemistry textbook, online	chemed.chem.wisc.edu/chempaths/GenChem-Textbook
Minerals	webmineral.com www.mindat.org
Geological Maps	onegeology.org
Optical illusions	www.sandlotscience.com
Rock geology	sandatlas.org
Petit's science comics	www.jp-petit.org
Physical toys	www.e20.physik.tu-muenchen.de/~cucke/toylinke.htm

Topic	Website address
Physics humour	www.dctech.com/physics/humor/biglist.php
Literature on magic	www.faqs.org/faqs/magic-faq/part2
music library, searchable by tune	imslp.org
Making paper aeroplanes	www.pchelp.net/paper_ac.htm www.ivic.qc.ca/~aleexpert/aluniversite/klinevogelmann.html
Small flying helicopters	pixelito.reference.be
Science curiosities	www.wundersamessammelsurium.info
Ten thousand year clock	www.longnow.org
Gesellschaft Deutscher Naturforscher und Ärzte	www.gdnae.de
Pseudoscience	suhep.phy.syr.edu/courses/modules/PSEUDO/pseudo_main.html
Crackpots	www.crank.net
Periodic table with videos for each element	www.periodicvideos.com
Mathematical quotations	math.furman.edu/mwoodard/~mquot.html
The ‘World Question Center’	www.edge.org/questioncenter.html
Plagiarism	www.plagiarized.com
Hoaxes	www.museumofhoaxes.com
Encyclopedia of Earth	www.eoearth.org
This is colossal	thisiscolossal.com

Do you want to study physics without actually going to university? Nowadays it is possible to do so via email and internet, in German, at the University of Kaiserslautern.[*] In the near future, a nationwide project in Britain should allow the same for English-speaking students. As an introduction, use the latest update of this physics text!

> “Das Internet ist die offenste Form der geschlossenen Anstalt.[**]”
> Matthias Deutschmann

> “Si tacuisses, philosophus mansisses.[***]”
> After Boethius.

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[*] See the www.fernstudium-physik.de website.

[**] ‘The internet is the most open form of a closed institution.’

[***] ‘If you had kept quiet, you would have remained a philosopher.’ After the story Boethius (*c.* 480–*c.* 525) tells in *De consolatione philosophiae*, 2.7, 67 ff.

CHALLENGE HINTS AND SOLUTIONS

> Never make a calculation before you know the answer.
>
> John Wheeler’s motto

John Wheeler wanted people to estimate, to try and to guess; but not saying the guess out loud. A correct guess reinforces the physics instinct, whereas a wrong one leads to the pleasure of surprise. Guessing is thus an important first step in solving every problem.

Teachers have other criteria to keep in mind. Good problems can be solved on different levels of difficulty, can be solved with words or with images or with formulae, activate knowledge, concern real-world applications, and are open.

Challenge 1, page 10: Do not hesitate to be demanding and strict. The next edition of the text will benefit from it.

Challenge 2, page 16: There are many ways to distinguish real motion from an illusion of motion: for example, only real motion can be used to set something else into motion. In addition, the motion illusions of the figures show an important failure; nothing moves if the head and the paper remain fixed with respect to each other. In other words, the illusion only *amplifies* existing motion, it does not *create* motion from nothing.

Challenge 3, page 17: Without detailed and precise experiments, both sides can find examples to prove their point. Creation is supported by the appearance of mould or bacteria in a glass of water; creation is also supported by its opposite, namely traceless disappearance, such as the disappearance of motion. However, conservation is supported and creation falsified by all those investigations that explore assumed cases of appearance or disappearance in full detail.

Challenge 4, page 19: The amount of water depends on the shape of the bucket. The system chooses the option (tilt or straight) for which the centre of gravity is lowest.

Challenge 5, page 20: To simplify things, assume a cylindrical bucket. If you need help, do the experiment at home. For the reel, the image is misleading: the rim on which the reel advances has a *larger* diameter than the section on which the string is wound up. The wound-up string does not touch the floor, like for the reel shown in Figure 305.

Challenge 6, page 19: Political parties, sects, helping organizations and therapists of all kinds are typical for this behaviour.

Challenge 7, page 24: The issue is not yet completely settled for the motion of empty space, such as in the case of gravitational waves. Thus, the motion of empty space might be an exception. In any case, empty space is not made of small particles of finite size, as this would contradict the transversality of gravity waves.

Challenge 8, page 26: Holes are not physical systems, because in general they cannot be tracked.

Challenge 9, page 26: The circular definition is: objects are defined as what moves with respect

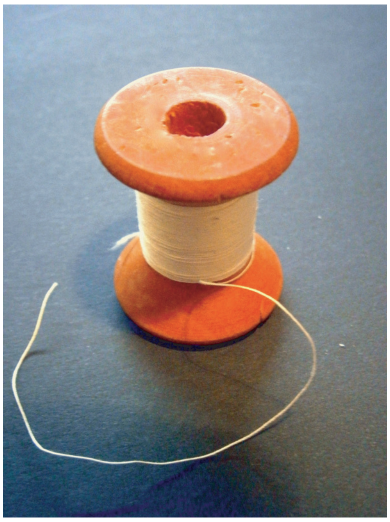

FIGURE 305 The assumed shape for the reel puzzle.

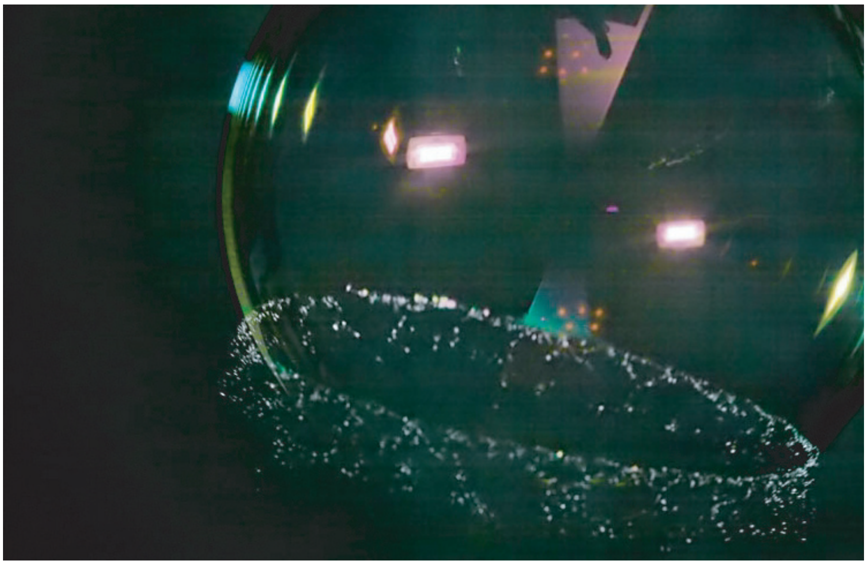

FIGURE 306 A soap bubble while bursting (© Peter Wienerroither).

to the background, and the background is defined as what stays when objects change. We shall return to this important issue several times in our adventure. It will require a certain amount of patience to solve it, though. Page 438

Challenge 10, page 28: No, the universe does not have a state. It is not measurable, not even in principle. See the discussion on the issue in volume IV, on quantum theory. Vol. IV, page 169

Challenge 11, page 28: The final list of intrinsic properties for physical systems found in nature is given in volume V, in the section on particle physics. And of course, the universe has no intrinsic, permanent properties. None of them are measurable for the universe as a whole, not even in principle. Vol. V, page 262

Challenge 12, page 31: Hint: yes, there is such a point.

Challenge 13, page 31: See Figure 306 for an intermediate step. A bubble bursts at a point, and then the rim of the hole increases rapidly until it disappears on the antipodes. During that process the remaining of the bubble keeps its spherical shape, as shown in the figure. For a film of the process, see www.youtube.com/watch?v=dIZwQ24_OU0 (or search for 'bursting soap bubble'). In other words, the final droplets that are ejected stem from the point of the bubble which is opposite to the point of puncture; they are never ejected from the centre of the bubble.

Challenge 14, page 31: A ghost can be a moving image; it cannot be a moving object, as objects cannot interpenetrate. Vol. IV, page 136

Challenge 15, page 31: If something could stop moving, motion could disappear into nothing. For a precise proof, one would have to show that no atom moves any more. So far, this has never been observed: motion is conserved. (Nothing in nature can disappear into nothing.)

Challenge 16, page 31: This would indeed mean that space is infinite; however, it is impossible to observe that something moves 'forever': nobody lives that long. In short, there is no way to prove that space is infinite in this way. In fact, there is no way to prove that space is infinite in any other way either.

Challenge 17, page 31: The necessary rope length is nh, where n is the number of wheels/pulleys. And yes, the farmer is indeed doing something sensible.

Challenge 19, page 31: How would you measure this?

Challenge 20, page 31: The number of reliable digits of a measurement result is a simple quantification of precision. More details can be found by looking up 'standard deviation' in the index.

Challenge 21, page 31: No; memory is needed for observation and measurements. This is the case for humans and measurement apparatus. Quantum theory will make this particularly clear.

Challenge 22, page 31: Note that you never have observed zero speed. There is always some measurement error which prevents one to say that something is zero. No exceptions!

Challenge 23, page 32: $(2^{64} - 1) = 18\,446\,744\,073\,700\,551\,615$ grains of wheat, with a grain weight of 40 mg, are 738 thousand million tons. Given a world harvest in 2006 of 606 million tons, the grains amount to about 1200 years of the world's wheat harvests.

The grain number calculation is simplified by using the formula $1 + m + m^2 + m^3 + ...m^n = (m^{n+1} - 1)/(m - 1)$, that gives the sum of the so-called *geometric sequence*. The name is historical and is used as a contrast to the *arithmetic sequence* $1 + 2 + 3 + 4 + 5 + ...n = n(n + 1)/2$. Can you prove the two expressions?

The chess legend is mentioned first by Ibn Khallikan (b. 1211 Arbil, d. 1282 Damascus). King Shiram and king Balhait, also mentioned in the legend, are historical figures that lived between the second and fourth century CE. The legend appears to have combined two different stories. Indeed, the calculation of grains appears already in the year 947, in the famous text *Meadows of Gold and Mines of Precious Stones* by Al-Masudi (b. *c.* 896 Baghdad, d. 956 Cairo).

Challenge 24, page 32: In clean experiments, the flame leans forward. But such experiments are not easy, and sometimes the flame leans backward. Just try it. Can you explain both observations?

Challenge 25, page 32: Accelerometers are the simplest motion detectors. They exist in form of piezoelectric devices that produce a signal whenever the box is accelerated and can cost as little as one euro. Another accelerometer that might have a future is an interference accelerometer that makes use of the motion of an interference grating; this device might be integrated in silicon. Other, more precise accelerometers use gyroscopes or laser beams running in circles.

Velocimeters and position detectors can also detect motion; they need a wheel or at least an optical way to look out of the box. Tachographs in cars are examples of velocimeters, computer mice are examples of position detectors.

A cheap enough device would be perfect to measure the speed of skiers or skaters. No such device exists yet.

Challenge 26, page 32: The ball rolls (or slides) towards the centre of the table, as the table centre is somewhat nearer to the centre of the Earth than the border; then the ball shoots over, performing an oscillation around the table centre. The period is 84 min, as shown in challenge 405. (This has never been observed, so far. Why?)

Challenge 27, page 32: Only if the acceleration never vanishes. Accelerations can be felt. Accelerometers are devices that measure accelerations and then deduce the position. They are used in aeroplanes when flying over the Atlantic. If the box does not accelerate, it is impossible to say whether it moves or sits still. It is even impossible to say in which direction one moves. (Close your eyes in a train at night to confirm this.)

Challenge 28, page 32: The block moves twice as fast as the cylinders, independently of their radius.

Challenge 29, page 32: This method is known to work with other fears as well.

Challenge 30, page 33: Three couples require 11 passages. Two couples require 5. For four or more couples there is no solution. What is the solution if there are n couples and $n - 1$ places on the boat?

Challenge 31, page 33: Hint: there is an infinite number of such shapes. These curves are called also *Reuleaux curves*. Another hint: The 20 p and 50 p coins in the UK have such shapes. And yes, other shapes than cylinders are also possible: take a twisted square bar, for example.

Challenge 32, page 33: If you do not know, ask your favourite restorer of old furniture.

Challenge 33, page 33: For this beautiful puzzle, see arxiv.org/abs/1203.3602.

Challenge 34, page 33: Conservation, relativity and minimization are valid generally. In some rare processes in nuclear physics, motion invariance (reversibility) is broken, as is mirror invariance. Continuity is known not to be valid at smallest length and time intervals, but no experiment has yet probed those domains, so that continuity is still valid in practice.

Challenge 35, page 34: In everyday life, this is correct; what happens when quantum effects are taken into account?

Challenge 36, page 36: Take the average distance change of two neighbouring atoms in a piece of quartz over the last million years. Do you know something still slower?

Challenge 37, page 37: There is only one way: compare the velocity to be measured with the speed of light – using cleverly placed mirrors. In fact, almost all physics textbooks, both for schools and for university, start with the definition of space and time. Otherwise excellent relativity textbooks have difficulties avoiding this habit, even those that introduce the now standard k-calculus (which is in fact the approach mentioned here). Starting with speed is the most logical and elegant approach. But it is possible to compare speeds without metre sticks and clocks. Can you devise a method?

Challenge 38, page 37: There is no way to sense your own motion if you are in a vacuum. No way in principle. This result is often called the *principle of relativity*. Page 156

In fact, there is a way to measure your motion in space (though not in vacuum): measure your speed with respect to the cosmic background radiation. So we have to be careful about what is implied by the question.

Challenge 39, page 37: The wing load W/A, the ratio between weight W and wing area A, is obviously proportional to the third root of the weight. (Indeed, $W \sim l^3$, $A \sim l^2$, l being the dimension of the flying object.) This relation gives the green trend line.

The wing load W/A, the ratio between weight W and wing area A, is, like all forces in fluids, proportional to the square of the cruise speed v: we have $W/A = v^2 0.38\,\mathrm{kg/m^3}$. The unexplained

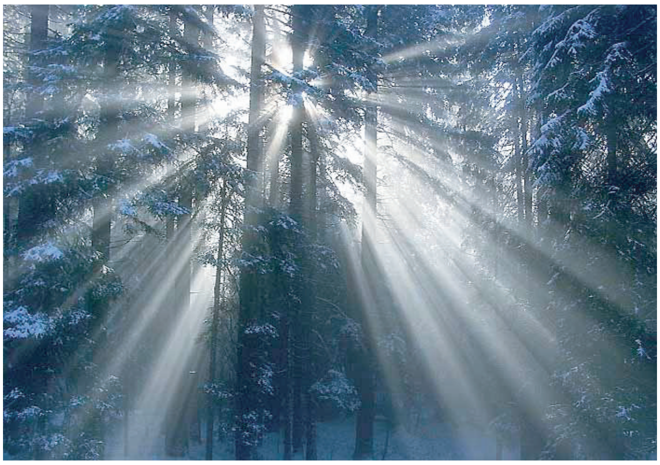

FIGURE 307 Sunbeams in a forest (© Fritz Bieri and Heinz Rieder).

factor contains the density of air and a general numerical coefficient that is difficult to calculate. This relation connects the upper and lower horizontal scales in the graph.

As a result, the cruise speed scales as the *sixth root* of weight: $v \sim W^{1/6}$. In other words, an Airbus A380 is 750 000 million times heavier than a fruit fly, but only a hundred times as fast.

Challenge 41, page 41: Equivalently: do points in space exist? The final part of our adventure explores this issue in detail.

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Challenge 42, page 42: All electricity sources must use the same phase when they feed electric power into the net. Clocks of computers on the internet must be synchronized.

Challenge 43, page 42: Note that the shift increases quadratically with time, not linearly.

Challenge 44, page 43: Galileo measured time with a scale (and with other methods). His stopwatch was a water tube that he kept closed with his thumb, pointing into a bucket. To start the stopwatch, he removed his thumb, to stop it, he put it back on. The volume of water in the bucket then gave him a measure of the time interval. This is told in his famous book GALILEO GALILEI, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attenenti alla mecanica e i movimenti locali*, usually simply called the 'Discorsi', which he published in 1638 with Louis Elsevier in Leiden, in the Netherlands.

Challenge 45, page 44: Natural time is measured with natural motion. Natural motion is the motion of light. Natural time is thus defined with the help of the motion of light.

Challenge 46, page 48: There is no way to define a local time at the poles that is consistent with all neighbouring points. (For curious people, check the website www.arctic.noaa.gov/gallery_np.html.)

Challenge 48, page 50: The forest is full of light and thus of light rays: they are straight, as shown by the sunbeams in Figure 307.

Challenge 49, page 50: One pair of muscles moves the lens along the third axis by deforming the eye from prolate to spherical to oblate.

Challenge 50, page 50: You can solve this problem by trying to think in four dimensions. (Train using the well-known three-dimensional projections of four-dimensional cubes.) Try to imagine how to switch the sequence when two pieces cross. Note: it is usually *not* correct, in this domain, to use time instead of a fourth *spatial* dimension!

Challenge 51, page 52: Measure distances using light.

Challenge 54, page 56: It is easier to work with the unit torus. Take the unit interval $[0, 1]$ and equate the endpoints. Define a set B in which the elements are a given real number b from the interval plus all those numbers that differ from that real by a rational number. The unit circle can be thought as the union of all the sets B. (In fact, every set B is a shifted copy of the rational numbers $\mathbb{Q}$.) Now build a set A by taking one element from each set B. Then build the set family consisting of the set A and its copies A_q shifted by a rational q. The union of all these sets is the unit torus. The set family is countably infinite. Then divide it into *two* countably infinite set families. It is easy to see that each of the two families can be renumbered and its elements shifted in such a way that each of the two families forms a unit torus.

Ref. 44 Mathematicians say that there is no countably infinitely additive measure of $\mathbb{R}^n$ or that sets such as A are non-measurable. As a result of their existence, the 'multiplication' of lengths is possible. Later on, we shall explore whether bread or *gold* can be multiplied in this way.

Challenge 55, page 56: Hint: start with triangles.

Challenge 56, page 56: An example is the region between the x-axis and the function which assigns 1 to every transcendental and 0 to every non-transcendental number.

Challenge 57, page 57: We use the definition of the function of the text. The dihedral angle of a regular tetrahedron is an irrational multiple of π, so the tetrahedron has a non-vanishing Dehn invariant. The cube has a dihedral angle of $\pi/2$, so the Dehn invariant of the cube is 0. Therefore, the cube is not equidecomposable with the regular tetrahedron.

Challenge 58, page 58: If you think you can show that empty space is continuous, you are wrong. Check your arguments. If you think you can prove the opposite, you *might* be right – but only if you already know what is explained in the final part of the text. If that is not the case, check your arguments. In fact, time is neither discrete nor continuous.

Challenge 60, page 59: Obviously, we use light to check that the plumb line is straight, so the two definitions must be the same. This is the case because the field lines of gravity are also possible paths for the motion of light. However, this is not always the case; can you spot the exceptions?

Another way to check straightness is along the surface of calm water.

A third, less precise way is to make use of the straightness sensors on the brain. The human brain has a built-in faculty to determine whether an object seen with the eyes is straight. There are special cells in the brain that fire when this is the case. Any book on vision perception tells more about this topic.

Challenge 61, page 60: The hollow Earth theory is correct if the distance formula is used consistently. In particular, one has to make the assumption that objects get smaller as they approach the centre of the hollow sphere. Good explanations of all events are found on www.geocities.com/inversedearth. Quite some material can be found on the internet, also under the names of celestrocentric system, inner world theory or concave Earth theory. There is no way to prefer one description over the other, except possibly for reasons of simplicity or intellectual laziness.

Challenge 63, page 61: A hint is given in Figure 308. For the measurement of the speed of light with almost the same method, see volume II, on page 20.

Challenge 64, page 61: A fast motorbike is faster: a motorbike driver can catch an arrow, a stunt that was shown on the German television show 'Wetten dass' in the year 2001.

Challenge 65, page 61: The 'only' shape that prevents a cover to fall into the hole beneath is a circular shape. Actually, slight deviations from the circular shape are also allowed.

Challenge 68, page 61: The walking speed of older men depends on their health. If people walk faster than 1.4 m/s, they are healthy. The study concluded that the grim reaper walks with a preferred speed of 0.82 m/s, and with a maximum speed of 1.36 m/s. Ref. 379

Challenge 69, page 62: 72 stairs.