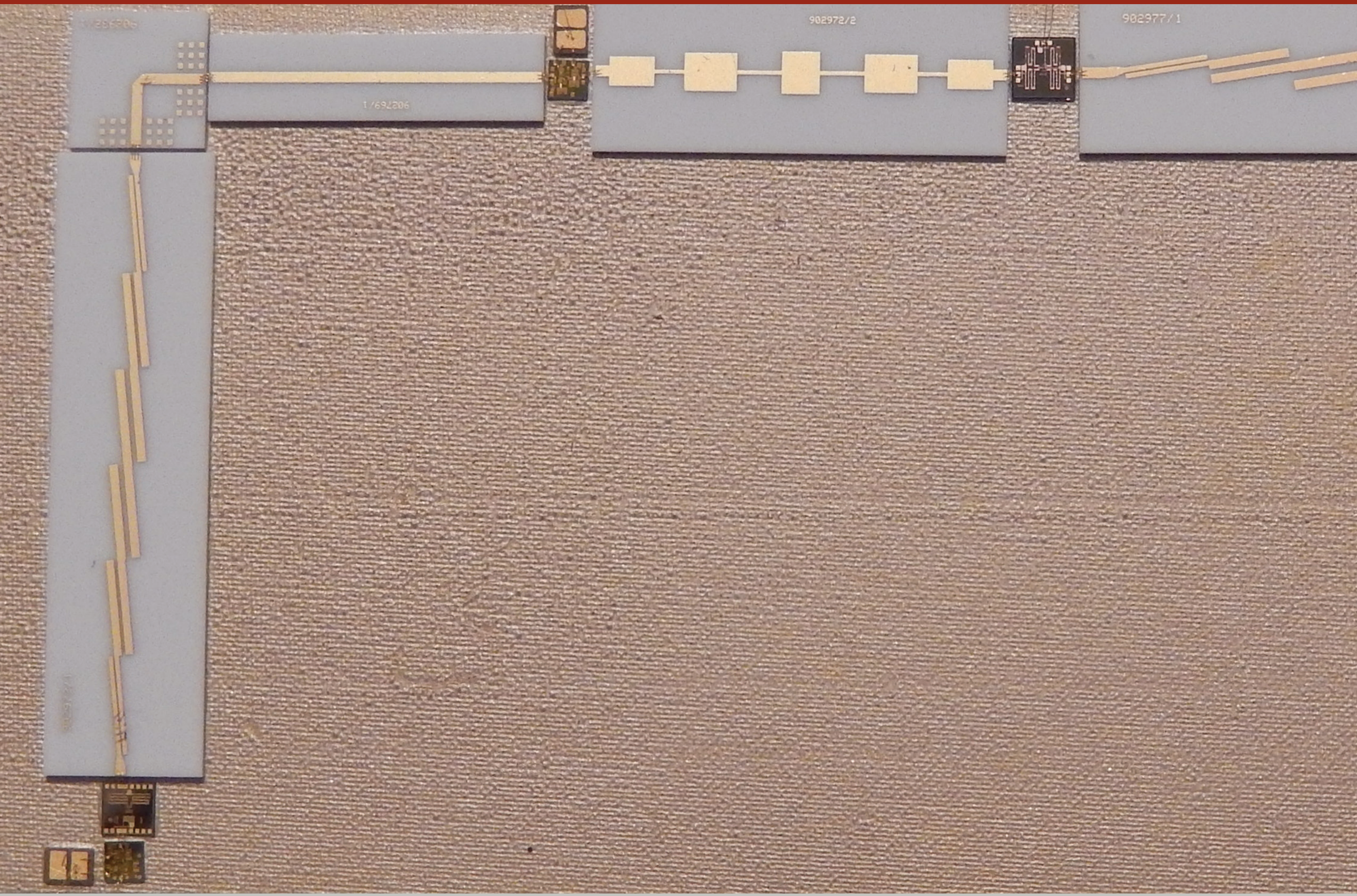
FUNDAMENTALS OF MICROWAVE AND RF DESIGN



Michael Steer

Third Edition

Fundamentals of Microwave and RF Design

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To my brother and sister from the start,
and the brothers and sisters I have gathered along the way

Paul, Donna, Ruth, Eric, Juliet, Ian, Paul, Tamara, Joseph, Louise, Patricia and Robert

Preface

The objective in writing this textbook is that the student will acquire the skills to be proficient at RF and microwave module design. The distinguishing feature of RF and microwave design is that distributed effects due to the finite delay of electrical signals must be accounted for. Sometimes this requires a design approach that avoids problems due to distributed effects. But very often these effects provide novel circuit functions that have no equivalent at lower frequencies. Distributed effects must be understood so that microwave circuits can be designed to avoid problems such as multimoding. Also understanding distributed effects leads to an understanding and appreciation of the vast trove of unique microwave elements that can be employed in system design. An understanding of microwave network theory will be gained and this leads to development of the skills required to design matched circuits that maximize microwave power transfer. Coupled with filtering, matching maximizes the all important signal-to-interference ratio system metric. Microwave engineering has several 'barriers-to-entry' and one of these is the use of graphical techniques in distributed circuit design. The microwave system designer must develop expertise in Smith chart-based design. A second barrier to entry is the need to embrace forward- and backward-traveling waves. This is the way the world works, and the finite speed of information transfer due to signals traveling as electromagnetic waves is a central concept in microwave engineering. This book leaves the design of modules themselves for more specialized microwave design and that is the province of a companion book series.

This book is derived from a multi volume book series with an emphasis in this *Fundamentals* book being on presenting material, the fundamentals, required to cross the threshold to RF and microwave design. The series itself comprises textbooks for several postgraduate classes and is also a comprehensive reference library on microwave engineering. However the series is too detailed for a first course on microwave engineering. But since *The Fundamentals of RF and Microwave Design* closely parallels the book series, referencing the book series will be familiar and welcoming.

The books in the Microwave and RF Design series are authored by Michael Steer, published by North Carolina State University, and distributed by the University of North Carolina Press are:

- Microwave and RF Design: Radio Systems, Volume 1
- Microwave and RF Design: Transmission Lines, Volume 2
- Microwave and RF Design: Networks, Volume 3
- Microwave and RF Design: Modules, Volume 4
- Microwave and RF Design: Amplifiers and Oscillators, Volume 5

They are available in low cost paperback format and as open access ebooks. Go to `https://www.lib.ncsu.edu/do/open-education` for more details.

As much as possible this *Fundamentals* book parallels the book series using common section names for example. This will help when consulting the series. Even after my many decades in the field, I still fall back on my undergraduate texts as first resources before searching out more detailed and up-to-date information. Perhaps this is because I am well calibrated to these books, and also because one never forgets the first way material is introduced. My hope is that this will also be true for the reader.

Supplementary Materials

Supplementary materials available to qualified instructors adopting the book include PowerPoint slides and solutions to the end-of-chapter problems. Requests should be directed to the author. Access to downloads of the books, additional material and YouTube videos are available at `https://www.lib.ncsu.edu/do/open-education`

Acknowledgments

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The book was produced using LaTeX and open access fonts, line art was drawn using xfig and inkscape, and images were edited in gimp. So thanks to the many volunteers who developed these packages.

My family, Mary, Cormac, Fiona, and Killian, gracefully put up with my absence for innumerable nights and weekends, many more than I could have ever imagined. I truly thank them. I also thank my academic sponsor, Dr. Ross Lampe, Jr., whose support of the university and its mission enabled me to pursue high risk and high reward endeavors including this book.

Michael Steer
North Carolina State University
Raleigh, North Carolina
mbs@ncsu.edu

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Contents

CHAPTER 1

Introduction to Microwave Engineering

1.1 RF and Microwave Engineering

Radio communications is the main driver of RF system development, leading to RF technology evolution at an unprecedented pace. A radio signal is a signal that is coherently generated, radiated by a transmit antenna, propagated through the air, collected by a receive antenna, and then amplified and information extracted. The radio spectrum is part of the electromagnetic (EM) spectrum exploited by humans for communications. A broad categorization of the EM spectrum is shown in Table 1-1. Today radios operate from 3 Hz (for submarine communications) to 300 GHz (proposed for 6G cellular communications).

Name or band	Frequency	Wavelength
Radio frequency	3 Hz – 300 GHz	100 000 km – 1 mm
Microwave	300 MHz – 300 GHz	1 m – 1 mm
Millimeter (mm) band	110 – 300 GHz	2.7 mm – 1.0 mm
Infrared	300 GHz – 400 THz	1 mm – 750 nm
Far infrared	300 GHz – 20 THz	1 mm – 15 μm
Long-wavelength infrared	20 THz – 37.5 THz	15–8μm
Mid-wavelength infrared	37.5 – 100 THz	8–3 μm
Short-wavelength infrared	100 THz – 214 THz	3–1.4 μm
Near infrared	214 THz – 400 THz	1.4 μm – 750 nm
Visible	400 THz – 750 THz	750 – 400 mm
Ultraviolet	750 THz – 30 PHz	400 – 10 nm
X-Ray	30 PHz – 30 EHz	10 – 0.01 nm
Gamma Ray	$>$ 15 EHz	$<$ 0.02 nm

Gigahertz, GHz = 10^9 Hz; terahertz, THz = 10^{12} Hz; pentahertz, PHz = 10^{15} Hz; exahertz, EHz = 10^{18} Hz.

Table 1-1: Broad electromagnetic spectrum divisions.

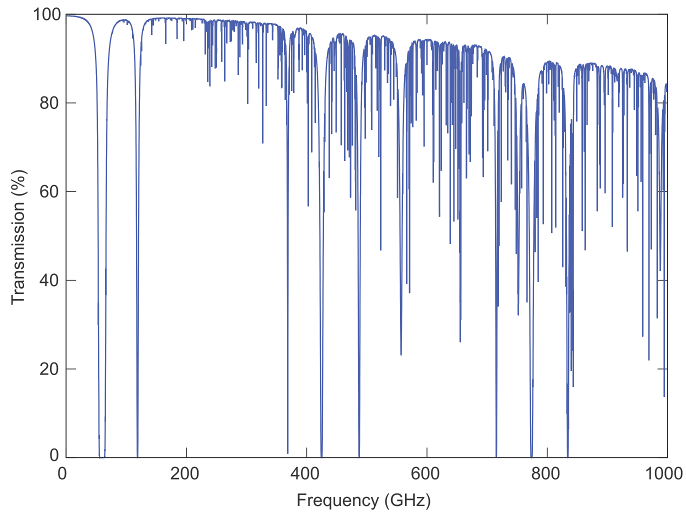


Figure 1-1: Atmospheric transmission at Mauna Kea, with a height of 4.2 km, on the Island of Hawaii where the atmospheric pressure is 60% of that at sea level and the air is dry with a precipitable water vapor level of 0.001 mm. After [1].

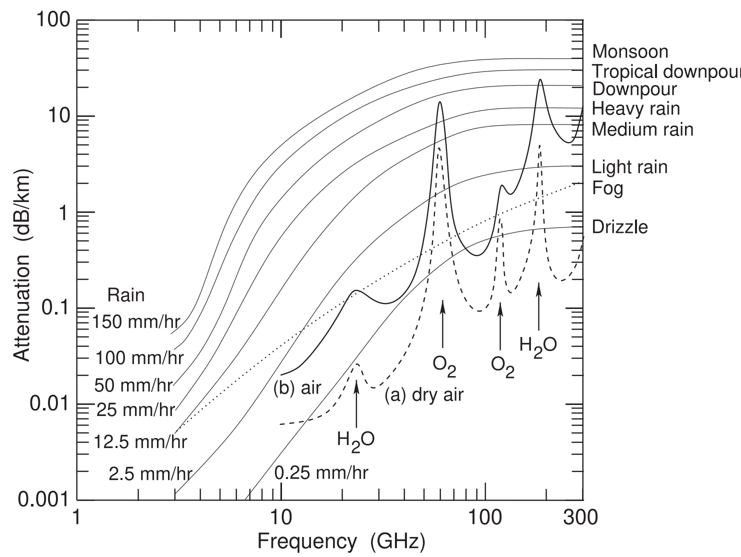


Figure 1-2: Excess attenuation due to atmospheric conditions showing the effect of rain on RF transmission at sea level. Curve (a) is atmospheric attenuation, due to excitation of molecular resonances, of very dry air at 0 °C, curve (b) is for typical air (i.e. less dry) at 20 °C. The attenuation shown for fog and rain is additional (in dB) to the atmospheric absorbtion shown as curve (b).

Propagating RF signals in air are absorbed by molecules in the atmosphere primarily by molecular resonances such as the bending and stretching of bonds which converts EM energy into heat. The transmittance of radio signals versus frequency in dry air at an altitude of 4.2 km is shown in Figure 1-1 and there are many transmission holes due to molecular resonances. The lowest frequency molecular resonance in dry air is the oxygen resonance centered at 60 GHz, but below that the absorption in dry air is very small. Attenuation increases with higher water vapor pressure peakin at 22 GHz and broadening due to the close packing of molecules in air. The effect of water is seen in Figure 1-2 and it is seen that rain and water vapor have little effect on cellular communications which are below 5 GHz except for millimeter-wave 5G.

RF signals diffract and so can bend around structures and penetrate into valleys. The ability to diffract reduces with increasing frequency. However, as frequency increases the size of antennas decreases and the capacity to carry information increases. A very good compromise for mobile communications is at UHF, 300 MHz to 4 GHz, where antennas are of convenient size and there is a good ability to diffract around objects and even penetrate walls.

1.1.1 Electromagnetic Fields

Communicating using EM signals built from an understanding of magnetic induction based on the experiments of Faraday in 1831 [2] in which he investigated the relationship of magnetic fields and currents, and now known as Faraday's law. This and the other static field laws are not enough to describe radio waves. The required description is embodied in Maxwell's equations and after these were developed it took little time before radio was invented.

1.1.2 Static Field Laws

There are two components of the EM field, the **electric field**, E, with units of volts per meter (V/m), and the **magnetic field**, H, with units of amperes per meter (A/m). There are also two flux quantities with the first being D, the **electric flux** density, with units of coulombs per square meter (C/m^2), and the other is B, the **magnetic flux** density, with units of teslas (T). B and H, and D and E, are related to each other by the properties of the medium, which are embodied in the quantities μ and ε (with the caligraphic letter, e.g. $\mathcal{B}$, denoting a time domain quantity):

$$\bar{\mathcal{B}} = \mu\bar{\mathcal{H}} \quad (1.1) \qquad \bar{\mathcal{D}} = \varepsilon\bar{\mathcal{E}}, \quad (1.2)$$

where the over bar denotes a vector quantity, and μ is the **permeability** of the medium and describes the ability to store **magnetic energy** in a region. The permeability in free space (or vacuum) is $\mu_0 = 4\pi \times 10^{-7}$ H/m and then

$$\bar{\mathcal{B}} = \mu_0\bar{\mathcal{H}}. \quad (1.3)$$

The other material quantity is the **permittivity**, ε, and in a vacuum

$$\bar{\mathcal{D}} = \varepsilon_0\bar{\mathcal{E}}, \quad (1.4)$$

where ε_0 = 8.854×10^{-12} F/m is the permittivity of a vacuum. The **relative permittivity**, ε_r, the **relative permeability**, μ_r, are defined as

$$\varepsilon_r = \varepsilon/\varepsilon_0 \qquad \text{and} \qquad \mu_r = \mu/\mu_0. \quad (1.5)$$

Biot-Savart Law

The Biot–Savart law relates current to magnetic field as, see Figure 1-3,

$$d\bar{H} = \frac{I d\boldsymbol{\ell} \times \hat{a}_R}{4\pi R^2}, \quad (1.6)$$

with units of amperes per meter in the SI system. In Equation (1.6) $d\bar{H}$ is the incremental static H field, I is current, $d\boldsymbol{\ell}$ is the vector of the length of a filament of current I, $\hat{a}_R$ is the unit vector in the direction from the current filament to the magnetic field, and R is the distance between the filament and the magnetic field. The $d\bar{H}$ field is directed at right angles to $\hat{a}_R$ and the current filament. So Equation (1.6) says that a filament of current produces a

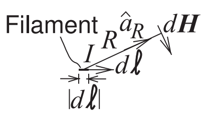


Figure 1-3: Diagram illustrating the Biot-Savart law. The law relates a static filament of current to the incremental H field at a distance.

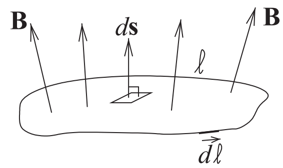


Figure 1-4: Diagram illustrating Faraday's law. The contour ℓ encloses the surface.

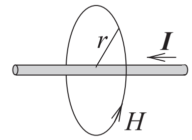


Figure 1-5: Diagram illustrating Ampere's law. Ampere's law relates the current, I, on a wire to the magnetic field around it, H.

magnetic field at a point. The total magnetic field from a current on a wire or surface can be found by modeling the wire or surface as a number of current filaments, and the total magnetic field at a point is obtained by integrating the contributions from each filament.

Faraday's Law of Induction

Faraday's law relates a time-varying magnetic field to an induced voltage drop, V, around a closed path, which is now understood to be $\oint_\ell \bar{\mathcal{E}} \cdot d\boldsymbol{\ell}$, that is, the closed contour integral of the electric field,

$$V = \oint_\ell \bar{\mathcal{E}} \cdot d\boldsymbol{\ell} = -\oint_s \frac{\partial \bar{\mathcal{B}}}{\partial t} \cdot d\mathbf{s}, \tag{1.7}$$

and this has the units of volts in the SI unit system. The operation described in Equation (1.7) is illustrated in Figure 1-4.

Ampere's Circuital Law

Ampere's circuital law, often called just Ampere's law, relates direct current and the static magnetic field $\bar{\mathcal{H}}$, see Figure 1-5:

$$\oint_\ell \bar{H} \cdot d\boldsymbol{\ell} = I_{\text{enclosed}}. \tag{1.8}$$

That is, the integral of the magnetic field around a loop is equal to the current enclosed by the loop. Using symmetry, the magnitude of the magnetic field at a distance r from the center of the wire shown in Figure 1-5 is

$$H = |I|/(2\pi r). \tag{1.9}$$

Gauss's Law

The final static EM law is Gauss's law, which relates the static electric flux density vector, $\bar{D}$, to charge. With reference to Figure 1-6, Gauss's law in integral form is

$$\oint_s \bar{D} \cdot d\mathrm{s} = \int_v \rho_v \cdot dv = Q_{\text{enclosed}}. \tag{1.10}$$

This states that the integral of the constant electric flux vector, $\bar{D}$, over a closed surface is equal to the total charge enclosed by the surface, Q_{enclosed}.

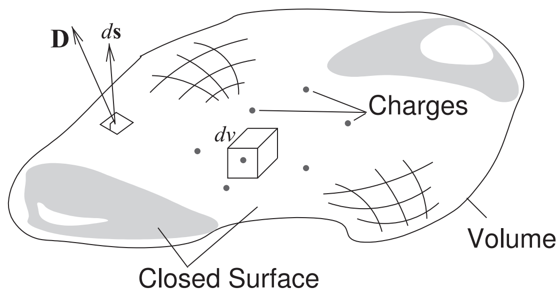


Figure 1-6: Diagram illustrating Gauss's law. Charges are distributed in the volume enclosed by the closed surface. An incremental area is described by the vector $d\mathbf{S}$, which is normal to the surface and whose magnitude is the area of the incremental area.

Gauss's Law of Magnetism

Gauss's law of magnetism parallels Gauss's law which applies to electric fields and charges. In integral form the law is

$$\oint_s \bar{B} \cdot d\mathrm{s} = 0. \tag{1.11}$$

This states that the integral of the constant magnetic flux vector, $\bar{D}$, over a closed surface is zero reflecting the fact that magnetic charges do not exist.

1.1.3 Maxwell's Equations

The essential step in the invention of radio was the development of Maxwell's equations in 1861. Before Maxwell's equations were postulated, several static EM laws were known. Taken together they cannot describe the propagation of EM signals, but they can be derived from Maxwell's equations. Maxwell's equations cannot be derived from the static electric and magnetic field laws. Maxwell's equations embody additional insight relating spatial derivatives to time derivatives, which leads to a description of propagating fields. Maxwell's equations are

$$\nabla \times \bar{\mathcal{E}} = -\frac{\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}} \tag{1.12}$$

$$\nabla \cdot \bar{\mathcal{D}} = \rho_V \tag{1.13}$$

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}} \tag{1.14}$$

$$\nabla \cdot \bar{\mathcal{B}} = \rho_{mV}. \tag{1.15}$$

The additional quantities in Equations (1.12)–(1.15) are

- $\bar{\mathcal{J}}$, the **electric current** density, with units of amperes per square meter ($\mathrm{A/m^2}$);
- ρ_V, the **electric charge** density, with units of coulombs per cubic meter ($\mathrm{C/m^3}$);
- ρ_{mV}, the magnetic charge density, with units of webers per cubic meter ($\mathrm{Wb/m^3}$); and
- $\bar{\mathcal{M}}$, the magnetic current density, with units of volts per square meter ($\mathrm{V/m^2}$).

Magnetic charges do not exist, but their introduction through the **magnetic charge** density, ρ_{mV}, and the **magnetic current** density, $\bar{\mathcal{M}}$, introduce an aesthetically appealing symmetry to Maxwell's equations. Maxwell's equations are differential equations, and as with most differential equations, their solution is obtained with particular boundary conditions, which in radio engineering are imposed by conductors.

Maxwell's equations have three types of derivatives. First, there is the time derivative, $\partial/\partial t$. Then there are two spatial derivatives, $\nabla\times$, called **curl**, capturing the way a field circulates spatially (or the amount that it curls up on itself), and $\nabla\cdot$, called the **div** operator, describing the spreading-out of a

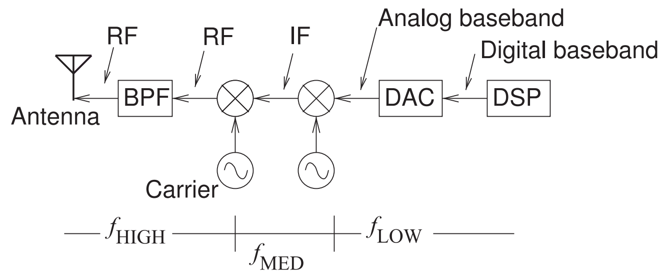


Figure 1-7: A simple transmitter with low, $f_{\rm LOW}$, medium, $f_{\rm MED}$, and high frequency, $f_{\rm HIGH}$, sections. The mixers can be idealized as multipliers that boost the frequency of the input baseband or IF signal by the frequency of the carrier.

field. In rectangular coordinates, curl, $\nabla\times$, describes how much a field circles around the x, y, and z axes. That is, the curl describes how a field circulates on itself. So Equation (1.12) relates the amount an electric field circulates on itself to changes of the B field in time. So a spatial derivative of electric fields is related to a time derivative of the magnetic field. Also in Equation (1.14) the spatial derivative of the magnetic field is related to the time derivative of the electric field. These are the key elements that result in self-sustaining propagation.

Div, $\nabla\cdot$, describes how a field spreads out from a point. How fast a field varies with time, $\partial\bar{\mathcal{B}}/\partial t$ and $\partial\bar{\mathcal{D}}/\partial t$, depends on frequency. The more interesting derivatives are $\nabla\times\bar{\mathcal{E}}$ and $\nabla\times\bar{\mathcal{H}}$ which describe how fast a field can change spatially—this depends on wavelength relative to geometry. If the cross-sectional dimensions of a transmission line are less than a wavelength ($\lambda/2$ or $\lambda/4$ in different circumstances when there are conductors), then it will be impossible for the fields to curl up on themselves and so there will be only one solution (with no or minimal variation of the E and H fields) or, in some cases, no solution to Maxwell's equations.

1.2 Radio Architecture

A radio device is comprised of reasonably well-defined units, see Figure 1-7. The analog baseband signal can have frequency components that range from DC to many megahertz.

Up until the mid-1970s most wireless communications were based on centralized high-power transmitters and reception was expected until the signal level fell below a noise threshold. These systems were particularly sensitive to interference, therefore systems transmitting at the same frequency were geographically separated so that signals fell below the background noise threshold before there was a chance of interfering with a neighboring system operating at the same frequency. This situation is illustrated in Figure 1-8.

Cellular communications is based on the concept of cells in which a terminal unit communicates with a basestation at the center of a cell. For communication in closely spaced cells to work, interference from other radios must be managed. This is facilitated using the ability to recover from errors available with error correction schemes.

In 1981 the U.S. FCC defined cellular radio as "a high capacity land mobile system in which assigned spectrum is divided into discrete channels which are assigned in groups to geographic cells covering a cellular geographic area. The discrete channels are capable of being reused within the service area." The key attributes here are (a) the concept of cells arranged in clusters and the total number of channels available is divided among the cells in

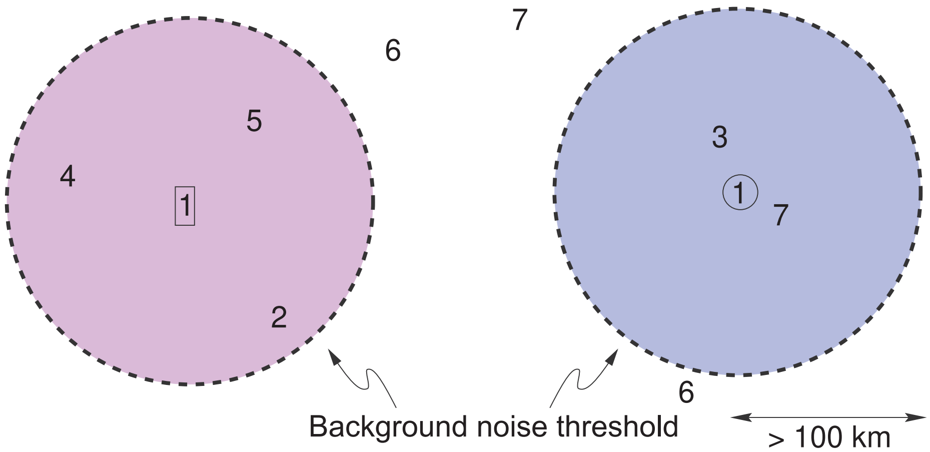


Figure 1-8: Interference in a conventional radio system. The two transmitters, 1, are at the centers of the coverage circles defined by the background noise threshold.

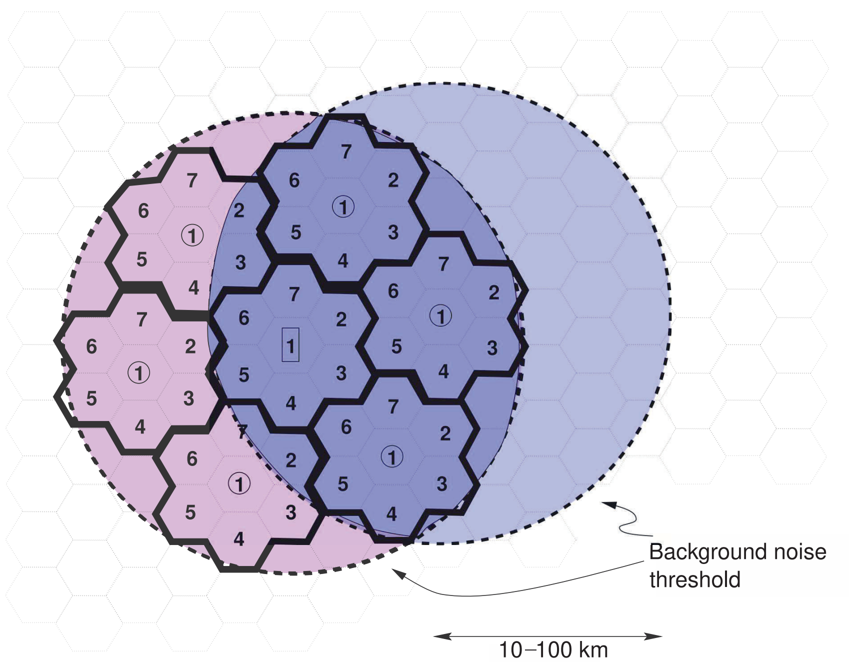


Figure 1-9: Interference in a cellular radio system.

a cluster and the full set is repeated in each cluster, e.g. in Figure 1-9 a 7-cell cluster is shown; and (b) frequency reuse. with frequencies used in one cell are reused in the corresponding cell in another cluster. As the cells are relatively close, it is important to dynamically control the power radiated by each radio, as radios in one cell will produce interference in other clusters.

Achieving maximum frequency reuse is essential. In a cellular system, there is a radical departure in concept from this. Consider the interference in a cellular system as shown in Figure 1-9. The signals in corresponding cells in different clusters interfere with each other and the interference is much larger than that of the background noise. Error correction coding to correct errors resulting from interference.

1.3 RF Power Calculations

1.3.1 RF Propagation

As an RF signal propagates away from a transmitter the power density reduces conserving the power in the EM wave. In the absence of obstacles and without atmospheric attenuation the total power passing through the surface of a sphere centered on a transmitter is equal to the power transmitted. Since the area of the sphere of radius r is $4\pi r^2$, the power density, e.g. in W/m^2, at a distance r drops off as $1/r^2$. With obstacles the EM wave can be further attenuated.

EXAMPLE 1.1 Signal Propagation

A signal is received at a distance r from a transmitter and the received power drops off as $1/r^2$. When $r = 1$ km, 100 nW is received. What is r when the received power is 100 fW?

Solution:

The signal collected by the receiver is proportional to the power density of the EM signal. The received signal power $P_r = k/r^2$ where k is a constant. This leads to

$$\frac{P_r(1\ \text{km})}{P_r(r)} = \frac{100\ \text{nW}}{100\ \text{fW}} = 10^6 = \frac{kr^2}{k(1\ \text{km})^2} = \frac{r^2}{(10^3\ \text{m})^2}; \quad r = \sqrt{10^{12}\ \text{m}^2} = 1000\ \text{km} \quad (1.16)$$

1.3.2 Logarithm

A cellular phone can reliably receive a signal as small as 100 fW and the signal to be transmitted could be 1 W. So the same circuitry can encounter signals differing in power by a factor of 10^{13}. To handle such a large range of signals a logarithmic scale is used.

Logarithms are used in RF engineering to express the ratio of powers using reasonable numbers. Logarithms are taken with respect to a base b such that if $x = b^y$, then $y = \log_b(x)$. In engineering, $\log(x)$ is the same as $\log_{10}(x)$, and $\ln(x)$ is the same as $\log_e(x)$ and is called the natural logarithm (e = 2.71828 ...). In physics and mathematics $\log x$ (and programs such as MATLAB) means $\ln x$, so be careful. Common formulas involving logarithms are given in Table 1-2.

Table 1-2: Common logarithm formulas. In engineering $\log x \equiv \log_{10} x$ and $\ln x \equiv \log_2 x$.

Description	Formula	Example
Equivalence	$y = \log_b(x) \longleftrightarrow x = b^y$	$\log(1000) = 3$ and $10^3 = 1000$
Product	$\log_b(xy) = \log_b(x) + \log_b(y)$	$\log(0.13 \cdot 978) = \log(0.13) + \log(978) = -0.8861 + 2.990 = 2.104$
Ratio	$\log_b(x/y) = \log_b(x) - \log_b(y)$	$\ln(8/2) = \ln(8) - \ln(2) = 3 - 1 = 2$
Power	$\log_b(x^p) = p\log_b(x)$	$\ln(3^2) = 2\ln(3) = 2 \cdot 1.0986 = 2.197$
Root	$\log_b(\sqrt[p]{x}) = \frac{1}{p}\log_b(x)$	$\log(\sqrt[3]{20}) = \frac{1}{3}\log(20) = 0.4337$
Change of base	$\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$	$\ln(100) = \frac{\log(100)}{\log(2)} = \frac{2}{0.30103} = 6.644$

1.3.3 Decibels

RF signal levels are expressed in terms of the power of a signal. While power can be expressed in absolute terms, e.g. watts (W), it is more useful to use a logarithmic scale. The ratio of two power levels P and P_{REF} in bels (B) is

$$P(B) = \log\left(\frac{P}{P_{\mathrm{REF}}}\right), \tag{1.17}$$

where P_{REF} is a reference power. Here $\log x$ is the same as $\log_{10} x$. Human senses have a logarithmic response and the minimum resolution tends to be about 0.1 B, so it is most common to use decibels (dB); 1 B = 10 dB. Common designations are shown in Table 1-3. Also, 1 mW = 0 dBm is a very common power level in RF and microwave power circuits where the m in dBm refers to the 1 mW reference. As well, dBW is used, and this is the power ratio with respect to 1 W with 1 W = 0 dBW = 30 dBm.

Table 1-3: Common power designations: (a) reference powers, P_{REF}; (b) power ratios in decibels(dB); and (c) powers in dBm and watts.

(a)

P_{REF}	Bell units	Decibel units
1 W	BW	dBW
1 mW = 10^{-3} W	Bm	dBm
1 fW = 10^{-15} W	Bf	dBf

(b)

Power ratio	in dB
10^{-6}	−60
0.001	−30
0.1	−20
1	0
10	10
1000	30
10^{6}	60

(c)

Power	Absolute power
−120 dBm	10^{-12} mW = 10^{-15} W = 1 fW
0 dBm	1 mW
10 dBm	10 mW
20 dBm	100 mW = 0.1 W
30 dBm	1000 mW = 1 W
40 dBm	10^{4} mW = 10 W
50 dBm	10^{5} mW = 100 W
−90 dBm	10^{-9} mW = 10^{-12} W = 1 pW
−60 dBm	10^{-6} mW = 10^{-9} W= 1 nW
−30 dBm	0.001 mW = 1 µW
−20 dBm	0.01 mW = 10 µW
−10 dBm	0.1 mW = 100 µW

EXAMPLE 1.2 Power Gain

An amplifier has a power gain of 1200. What is the power gain in decibels? If the input power is 5 dBm, what is the output power in dBm?

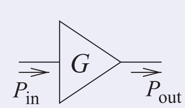
Solution:

Power gain in decibels, $G_{\mathrm{dB}} = 10 \log 1200 = 30.79$ dB.

The output power is $P_{\mathrm{out}}|_{\mathrm{dBm}} = P_{\mathrm{dB}} + P_{\mathrm{in}}|_{\mathrm{dBm}} = 30.79 + 5 = 35.79$ dBm.

EXAMPLE 1.3 Gain Calculations

A signal with a power of 2 mW is applied to the input of an amplifier that increases the power of the signal by a factor of 20.



(a) What is the input power in dBm?

$$P_{\mathrm{in}} = 2\ \mathrm{mW} = 10 \cdot \log\left(\frac{2\ \mathrm{mW}}{1\ \mathrm{mW}}\right) = 10 \cdot \log(2) = 3.010\ \mathrm{dBm} \approx 3.0\ \mathrm{dBm}. \tag{1.18}$$

(b) What is the gain, G, of the amplifier in dB?
The amplifier gain (by default this is power gain) is

$$G = 20 = 10 \cdot \log(20) \text{ dB} = 10 \cdot 1.301 \text{ dB} = 13.0 \text{ dB}. \quad (1.19)$$

(c) What is the output power of the amplifier?

$$G = \frac{P_{out}}{P_{in}}, \quad \text{and in decibels } G|_{dB} = P_{out}|_{dBm} - P_{in}|_{dBm} \quad (1.20)$$

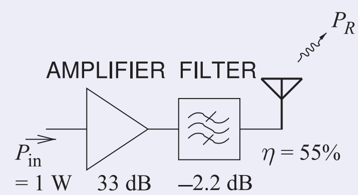
Thus the output power in dBm is

$$P_{out}|_{dBm} = G|_{dB} + P_{in}|_{dBm} = 13.0 \text{ dB} + 3.0 \text{ dBm} = 16.0 \text{ dBm}. \quad (1.21)$$

Note that dB and dBm are dimensionless but they do have meaning; dB indicates a power ratio but dBm refers to a power. Quantities in dB and one quantity in dBm can be added or subtracted to yield dBm, and the difference of two quantities in dBm yields a power ratio in dB.

In Examples 1.2 and 1.3 two digits following the decimal point were used for the output power expressed in dBm. This corresponds to an implied accuracy of about 0.01% or 4 significant digits of the absolute number. This level of precision is typical for the result of an engineering calculation.

EXAMPLE 1.4 Power Calculations



The output stage of an RF front end consists of an amplifier followed by a filter and then an antenna. The amplifier has a gain of 33 dB, the filter has a loss of 2.2 dB, and of the power input to the antenna, 45% is lost as heat due to resistive losses. If the power input to the amplifier is 1 W, then:

(a) What is the power input to the amplifier expressed in dBm?
$P_{in} = 1 \text{ W} = 1000 \text{ mW}, \quad P_{dBm} = 10 \log(1000/1) = 30 \text{ dBm}.$

(b) Express the loss of the antenna in dB.
45% of the power input to the antenna is dissipated as heat.
The antenna has an efficiency, η, of 55% and so $P_2 = 0.55 P_1$.
Loss $= P_1/P_2 = 1/0.55 = 1.818 = 2.60$ dB.

(c) What is the total gain of the RF front end (amplifier + filter + antenna)?

$$\begin{aligned} \text{Total gain} &= (\text{amplifier gain})_{dB} + (\text{filter gain})_{dB} - (\text{loss of antenna})_{dB} \\ &= (33 - 2.2 - 2.6) \text{ dB} = 28.2 \text{ dB} \end{aligned} \quad (1.22)$$

(d) What is the total power radiated by the antenna in dBm?

$$\begin{aligned} P_R &= P_{in}|_{dBm} + (\text{amplifier gain})_{dB} + (\text{filter gain})_{dB} - (\text{loss of antenna})_{dB} \\ &= 30 \text{ dBm} + (33 - 2.2 - 2.6) \text{ dB} = 58.2 \text{ dBm}. \end{aligned} \quad (1.23)$$

(e) What is the total power radiated by the antenna?

$$P_R = 10^{58.2/10} = (661 \times 10^3) \text{ mW} = 661 \text{ W}. \quad (1.24)$$

1.4 SI Units

The main SI units used in microwave engineering are given in Table 1-4.

- Symbols for units are written in upright roman font and are lowercase unless the symbol is derived from the name of a person. An exception is the use of L for liter to avoid possible confusion with l.

Table 1-4: Main SI units used in RF and microwave engineering.

SI unit	Name	Usage	In terms of fundamental units
A	ampere	current (abbreviated as amp)	Fundamental unit
cd	candela	luminous intensity	Fundamental unit
C	coulomb	charge	$\mathrm{A{\cdot}s}$
F	farad	capacitance	$\mathrm{kg^{-1}{\cdot}m^{-2}{\cdot}A^{-2}{\cdot}s^{4}}$
g	gram	weight	=kg/1000
H	henry	inductance	$\mathrm{kg{\cdot}m^{2}{\cdot}A^{-2}{\cdot}s^{-2}}$
J	joule	unit of energy	$\mathrm{kg{\cdot}m^{2}{\cdot}s^{-2}}$
K	kelvin	thermodynamic temperature	Fundamental unit
kg	kilogram	SI fundamental unit	Fundamental unit
m	meter	length	Fundamental unit
mol	mole	amount of substance	Fundamental unit
N	newton	unit of force	$\mathrm{kg{\cdot}m{\cdot}s^{-2}}$
Ω	ohm	resistance	$\mathrm{kg{\cdot}m^{2}{\cdot}A^{-2}{\cdot}s^{-3}}$
Pa	pascal	pressure	$\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-2}}$
s	second	time	Fundamental unit
S	siemen	admittance	$\mathrm{kg^{-1}{\cdot}m^{-2}{\cdot}A^{2}{\cdot}s^{3}}$
V	volt	voltage	$\mathrm{kg{\cdot}m^{2}{\cdot}A^{-1}{\cdot}s^{-3}}$
W	watt	power	$J{\cdot}s^{-1}$

- A space separates a value from the symbol for the unit (e.g., 5.6 kg). There is an exception for degrees, with the symbol °, e.g. $45°$.

When SI units are multiplied a center dot is used. For example, newton meters is written N·m. When a unit is derived from the ratio of symbols then either a solidus (/) or a negative exponent is used; the symbol for velocity (meters per second) is either m/s or $\mathrm{m{\cdot}s^{-1}}$. The use of multiple solidi for a combination symbol is confusing and must be avoided. So the symbol for acceleration is $\mathrm{m{\cdot}s^{-2}}$ or m/s^2 and not m/s/s.

Consider calculation of the thermal resistance of a rod of cross-sectional area A and length ℓ:

$$R_{\mathrm{TH}} = \frac{\ell}{kA}. \tag{1.25}$$

If $A = 0.3\ \mathrm{cm}^2$ and $\ell = 2$ mm, the thermal resistance is

$$\begin{aligned} R_{\mathrm{TH}} &= \frac{(2\ \mathrm{mm})}{(237\ \mathrm{kW{\cdot}m^{-1}{\cdot}K^{-1}})\cdot(0.3\ \mathrm{cm}^2)} \\ &= \frac{(2\cdot 10^{-3}\ \mathrm{m})}{237\cdot(10^3\cdot \mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}})\cdot 0.3\cdot(10^{-2}\cdot\mathrm{m})^2} \\ &= \frac{2\cdot 10^{-3}}{237\cdot 10^3\cdot 0.3\cdot 10^{-4}}\cdot\frac{\mathrm{m}}{\mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}{\cdot}m^2}} \\ &= 2.813\cdot 10^{-4}\ \mathrm{K{\cdot}W^{-1}} = 281.3\ \mathrm{\mu K/W}. \end{aligned} \tag{1.26}$$

This would be an error-prone calculation if the thermal conductivity was taken as 237 kW/m/K.

SI prefixes are given in Table 1-5 and indicate the multiple of a unit (e.g., 1 pA is 10^{-12} amps). (Source: 2015 ISO/IEC 8000 [3].) In 2009 new definitions of the prefixes for bits and bytes were adopted [3] removing the confusion over the earlier use of quantities such as kilobit to represent either 1,000 bits or 1,024 bits. Now kilobit (kbit) always means 1,000 bits and a new term kibibit (Kibit) means 1,024 bit. Also the now obsolete usage of kbps is replaced by kbit/s (kilobit per second). The prefix k stands for kilo (i.e. 1,000) and Ki is the symbol for the binary prefix kibi- (i.e. 1,024). The symbol for byte (= 8 bits) is "B".

Table 1-5: SI prefixes.

SI Prefixes			SI Prefixes			Prefixes for bits and bytes		
Symbol	Factor	Name	Symbol	Factor	Name	Name		
10^{-24}	y	yocto	10^{1}	da	deca	kilobit	kbit	1000 bit
10^{-21}	z	zepto	10^{2}	h	hecto	megabit	Mbit	1000 kbit
10^{-18}	a	atto	10^{3}	k	kilo	gigabit	Gbit	1000 Mbit
10^{-15}	f	femto	10^{6}	M	mega	terabit	Tbit	1000 Gbit
10^{-12}	p	pico	10^{9}	G	giga	kibibit	Kibit	1024 bit
10^{-9}	n	nano	10^{12}	T	tera	mebibit	Mibit	1024 Kibit
10^{-6}	μ	micro	10^{15}	P	peta	gibibit	Gibit	1024 Mibit
10^{-3}	m	milli	10^{18}	E	exa	tebibit	Tibit	1024 Gibit
10^{-2}	c	centi	10^{21}	Z	zetta	kilobyte	kB	1000 B
10^{-1}	d	deci	10^{24}	Y	yotta	kibibyte	KiB	1024 B

1.5 Summary

The RF spectrum is used to support a tremendous range of applications, including voice and data communications, satellite-based navigation, radar, weather radar, mapping, environmental monitoring, air traffic control, police radar, perimeter surveillance, automobile collision avoidance, and many military applications.

In RF and microwave engineering there are always considerable approximations made in design, partly because of necessary simplifications that must be made in modeling, but also because many of the material properties required in a detailed design can only be approximately known. Most RF and microwave design deals with frequency-selective circuits often relying on line lengths that have a length that is a particular fraction of a wavelength. Many designs can require frequency tolerances of as little as 0.1%, and filters can require even tighter tolerances. It is therefore impossible to design exactly. Measurements are required to validate and iterate designs. Conceptual understanding is essential; the designer must be able to relate measurements, which themselves have errors, with computer simulations. The ability to design circuits with good tolerance to manufacturing variations and perhaps circuits that can be tuned by automatic equipment are skills developed by experienced designers.

1.6 References

[1] "Atmospheric microwave transmittance at mauna kea, wikipedia creative commons."

[2] "IEEE Virtual Museum," at http://www.ieee-virtual-museum.org Search term: 'Faraday'.

[3] "ISO/IEC 8000," intenational Standard Organization (ISO), International Electrotechnical Commission standard including standards on on letter symbols to be used in electrical technology. http://www.iso.org.

1.7 Exercises

1. What is the wavelength in free space of a signal at 4.5 GHz?
2. Consider a monopole antenna that is a quarter of a wavelength long. How long is the antenna if it operates at 3 kHz?
3. Consider a monopole antenna that is a quarter of a wavelength long. How long is the antenna if it operates at 500 MHz?
4. Consider a monopole antenna that is a quarter of a wavelength long. How long is the antenna if it operates at 2 GHz?
5. A dipole antenna is half of a wavelength long. How long is the antenna at 2 GHz?
6. A dipole antenna is half of a wavelength long. How long is the antenna at 1 THz?
7. A transmitter transmits an FM signal with a bandwidth of 100 kHz and the signal is received by a receiver at a distance r from the transmitter. When $r = 1$ km the signal power received by the receiver is 100 nW. When the receiver moves further away from the transmitter the power received drops off as $1/r^2$. What is r in kilometers when the received power is 100 pW. [Parallels Example 1.1]
8. A transmitter transmits an AM signal with a bandwidth of 20 kHz and the signal is received by a receiver at a distance r from the transmitter. When $r = 10$ km the signal power received is 10 nW. When the receiver moves further away from the transmitter the power received drops off as $1/r^2$. What is r in kilometers when the received power is equal to the received noise power of 1 pW? [Parallels Example 1.1]
9. The logarithm to base 2 of a number x is 0.38 (i.e., $\log_2(x) = 0.38$). What is x?
10. The natural logarithm of a number x is 2.5 (i.e., $\ln(x) = 2.5$). What is x?
11. The logarithm to base 2 of a number x is 3 (i.e., $\log_2(x) = 3$). What is $\log_2(\sqrt[2]{x})$?
12. What is $\log_3(10)$?
13. What is $\log_{4.5}(2)$?
14. Without using a calculator evaluate $\log\{[\log_3(3x) - \log_3(x)]\}$.
15. A 50 Ω resistor has a sinusoidal voltage across it with a peak voltage of 0.1 V. The RF voltage is $0.1\cos(\omega t)$, where ω is the radian frequency of the signal and t is time.
    (a) What is the power dissipated in the resistor in watts?
    (b) What is the power dissipated in the resistor in dBm?
16. The power of an RF signal is 10 mW. What is the power of the signal in dBm?
17. The power of an RF signal is 40 dBm. What is the power of the signal in watts?
18. An amplifier has a power gain of 2100.
    (a) What is the power gain in decibels?
    (b) If the input power is -5 dBm, what is the output power in dBm? [Parallels Example 1.2]
19. An amplifier has a power gain of 6. What is the power gain in decibels? [Parallels Example 1.2]
20. A filter has a loss factor of 100. [Parallels Example 1.2]
    (a) What is the loss in decibels?
    (b) What is the gain in decibels?
21. An amplifier has a power gain of 1000. What is the power gain in dB? [Parallels Example 1.2]
22. An amplifier has a gain of 14 dB. The input to the amplifier is a 1 mW signal, what is the output power in dBm?
23. An RF transmitter consists of an amplifier with a gain of 20 dB, a filter with a loss of 3 dB and then that is then followed by a lossless transmit antenna. If the power input to the amplifier is 1 mW, what is the total power radiated by the antenna in dBm? [Parallels Example 1.4]
24. The final stage of an RF transmitter consists of an amplifier with a gain of 30 dB and a filter with a loss of 2 dB that is then followed by a transmit antenna that looses half of the RF power as heat. [Parallels Example 1.4]
    (a) If the power input to the amplifier is 10 mW, what is the total power radiated by the antenna in dBm?
    (b) What is the radiated power in watts?
25. A 5 mW-RF signal is applied to an amplifier that increases the power of the RF signal by a factor of 200. The amplifier is followed by a filter that losses half of the power as heat.
    (a) What is the output power of the filter in watts?
    (b) What is the output power of the filter in dBW?
26. The power of an RF signal at the output of a receive amplifier is 1 μW and the noise power at the output is 1 nW. What is the output signal-to-noise ratio in dB?

27. The power of a received signal is 1 pW and the received noise power is 200 fW. In addition the level of the interfering signal is 100 fW. What is the signal-to-noise ratio in dB? Treat interference as if it is an additional noise signal.age gain of 1 has an input impedance of 100 Ω, a zero output impedance, and drives a 5 Ω load. What is the power gain of the amplifier?
28. A transmitter transmits an FM signal with a bandwidth of 100 kHz and the signal power received by a receiver is 100 nW. In the same bandwidth as that of the signal the receiver receives 100 pW of noise power. In decibels, what is the ratio of the signal power to the noise power, i.e. the signal-to-noise ratio (SNR) received by the receiver?
29. An amplifier with a voltage gain of 20 has an input resistance of 100 Ω and an output resistance of 50 Ω. What is the power gain of the amplifier in decibels? [Parallels Example 1.0]
30. An amplifier with a voltage gain of 1 has an input resistance of 100 Ω and an output resistance of 5 Ω. What is the power gain of the amplifier in decibels? Explain why there is a power gain of more than 1 even though the voltage gain is 1. [Parallels Example 1.0]
31. An amplifier has a power gain of 1900.
    (a) What is the power gain in decibels?
    (b) If the input power is -8 dBm, what is the output power in dBm? [Parallels Example 1.2]
32. An amplifier has a power gain of 20.
    (a) What is the power gain in decibels?
    (b) If the input power is -23 dBm, what is the output power in dBm? [Parallels Example 1.2]
33. An amplifier has a voltage gain of 10 and a current gain of 100.
    (a) What is the power gain as an absolute number?
    (b) What is the power gain in decibels?
    (c) If the input power is -30 dBm, what is the output power in dBm?
    (c) What is the output power in mW?
34. An amplifier with 50 Ω input impedance and 50 Ω load impedance has a voltage gain of 100. What is the (power) gain in decibels?
35. An attenuator reduces the power level of a signal by 75%. What is the (power) gain of the attenuator in decibels?

1.7.1 Exercises By Section

†challenging

§1.2	1, 2, 3, 4, 5, 6, 7, 8	18, 19, 20, 21, 22, 23†, 24†, 25†
§1.3	9, 10, 11, 12, 13, 14, 15, 16, 17	26, 27, 28, 29, 30, 31, 32, 33, 34, 35

1.7.2 Answers to Selected Exercises

4	3.25 cm	17	10 W	23	50.12 mW
12	2.096	19	7.782 dB	24(b)	3.162 W
16	10 dBm	22	1.301		

CHAPTER 2

Antennas and the RF Link

2.1 Introduction

An antenna interfaces circuits with free-space with a transmit antenna converting a guided wave signal on a transmission line to an electromagnetic (EM) wave propagating in free space, while a receive antenna is a transducer that converts a free-space EM wave to a guided wave on a transmission line and eventually to a receiver circuit. Together the transmit and receive antennas are part of the RF link. The RF link is the path between the output of the transmitter circuit and the input of the receiver circuit (see Figure 2-1). Usually this path includes the cable from the transmitter to the transmit antenna, the transmit antenna itself, the propagation path, the receive antenna, and the transmission line connecting the receive antenna to the receiver circuit. The received signal is much smaller than the transmitted signal. The overwhelming majority of the loss is from the propagation path as the EM signal spreads out, and usually diffracts, reflects, and is partially blocked by objects such as hills and buildings.

The first half of this chapter is concerned with the properties of antennas. One of the characteristics of antennas is that the energy can be focused in a particular direction, a phenomenon captured by the concept of antenna gain, which can partially compensate for path loss. The second half of this chapter considers modeling the RF link and the geographical arrangement of antennas that manage interference from other radios while providing support for as many users as possible.

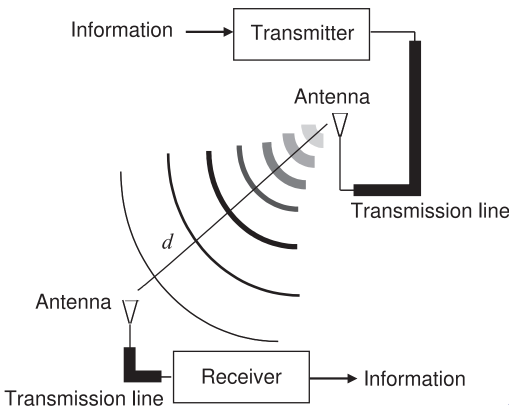
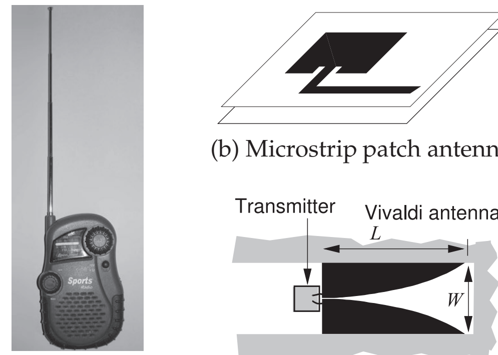


Figure 2-1: RF link.



(a) Monopole antenna

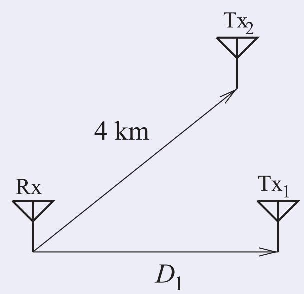
(b) Microstrip patch antenna

(c) Vivaldi antenna

Figure 2-2: Representative resonant, (a) and (b), and traveling-wave, (c), antennas.

EXAMPLE 2.1 Interference

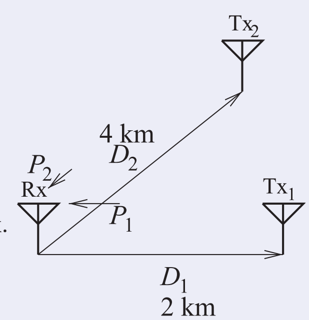
In the figure there are two transmitters, Tx_1 and Tx_2, operating at the same power level, and one receiver, Rx. Tx_1 is an intentional transmitter and its signal is intended to be received at Rx. Tx_1 is separated from Rx by $D_1 = 2$ km. Tx_2 uses the same frequency channel as Tx_1, and as far as RX is concerned it transmits an interfering signal. Assume that the antennas are omnidirectional (i.e., they transmit and receive signals equally in all directions) and that the transmitted power density drops off as $1/d^2$, where d is the distance from the transmitter. Calculate the signal-to-interference ratio (SIR) at Rx.



Solution:

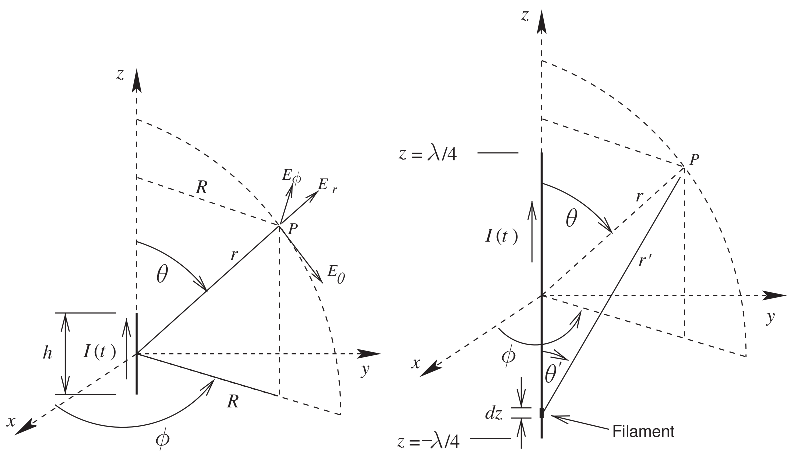
$D_1 = 2$ km and $D_2 = 4$ km.
P_1 is the signal power transmitted by Tx1 and received at Rx.
P_2 is the interference power transmitted by Tx2 and received at Rx.

So SIR $= \dfrac{P_1}{P_2} = \left(\dfrac{D_2}{D_1}\right)^2 = 4 = 6.02$ dB.



2.2 RF Antennas

Antennas are of two fundamental types: **resonant** and **traveling-wave antennas**, see Figure 2-2. Resonant antennas establish a standing wave of current with required resonance usually established when the antenna section is either a quarter- or half-wavelength long. These antennas are also known as standing-wave antennas. Resonant antennas are inherently narrowband because of the resonance required to establish a large standing wave of current. Figures 2-2(a and b) show two representative resonant antennas. The physics of the operation of a resonant antenna is understood by considering the time domain. First consider the physical operation of a transmit antenna. When a sinusoidal voltage is applied to the conductor



(a) Current filament ($h \ll \lambda$) (b) Half-wavelength long wire

Figure 2-3: Wire antennas. The distance from the center of an antenna to the field point P is r. $R = r \sin\theta$.

of an antenna, charges, i.e. free electrons, accelerate or decelerate under the influence of an applied voltage source which typically arrives at the antenna from a traveling wave voltage on a transmission line. When the charges accelerate (or decelerate) they produce an EM field which radiates away from the antenna. At a point on the antenna there is a current sinusoidally varying in time, and the net acceleration of the charges (in charge per second per second) is also sinusoidal with an amplitude that is directly proportional to the amplitude of the current sinewave. Hence having a large standing wave of current, when the antenna resonates sinusoidally, results in a large charge acceleration and hence large radiated fields.

When an EM field impinges upon a conductor the field causes charges to accelerate and hence induce a voltage that propagates on a transmission line connected to the receive antenna. Traveling-wave antennas, an example of one is shown in Figures 2-2(c), operate as extended lines that gradually flare out so that a traveling wave on the original transmission line transitions into free space. Traveling-wave antennas tend to be two or more wavelengths long at the lowest frequency of operation. While relatively long, they are broadband, many 3 or more octaves wide (e.g. extending from 500 MHz to 4 GHz or more). These antennas are sometimes referred to as **aperture antennas**.

2.3 Resonant Antennas

With a resonant antenna the current on the antenna is directly related to the amplitude of the radiated EM field. Resonance ensures that the standing wave current on the antenna is high.

2.3.1 Radiation from a Current Filament

The fields radiated by a resonant antenna are most conveniently calculated by considering the distribution of current on the antenna. The analysis begins by considering a short filament of current, see Figure 2-3(a). Considering the sinusoidal steady state at radian frequency ω, the current on the filament with phase χ is $I(t) = |I_0|\cos(\omega t + \chi)$, so that $I_0 = |I_0|\mathrm{e}^{-\jmath\chi}$ is the phasor of the current on the filament. The length of the filament is h, but it has no other dimensions, that is, it is considered to be infinitely thin.

Resonant antennas are conveniently modeled as being made up of an array of current filaments with spacings and lengths being a tiny fraction of a wavelength. Wire antennas are even simpler and can be considered to be a line of current filaments. Ramo, Whinnery, and Van Duzer [1] calculated the spherical EM fields at the point P with the spherical coordinates (ϕ, θ, r) generated by the z-directed current filament centered at the origin in Figure 2-3. The total EM field components in phasor form are

$$H_\phi = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{\jmath k}{r} + \frac{1}{r^2} \right) \sin\theta, \quad \bar{H}_\phi = H_\phi \hat{\phi} \tag{2.1}$$

$$E_r = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{2\eta}{r^2} + \frac{2}{\jmath\omega\varepsilon_0 r^3} \right) \cos\theta, \quad \bar{E}_r = E_r \hat{\mathbf{r}} \tag{2.2}$$

$$E_\theta = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{\jmath\omega\mu_0}{r} + \frac{1}{\jmath\omega\varepsilon_0 r^3} + \frac{\eta}{r^2} \right) \sin\theta, \quad \bar{E}_\theta = E_\theta \hat{\theta}, \tag{2.3}$$

where η is the free-space characteristic impedance. The variable k is called the **wavenumber** and $k = 2\pi/\lambda = \omega\sqrt{\mu_0\varepsilon_0}$. The $\mathrm{e}^{-\jmath kr}$ terms describe the variation of the phase of the field as the field propagates away from the filament. Equations (2.1)–(2.3) are the complete fields with the $1/r^2$ and $1/r^3$ dependence describing the near-field components. In the far field, i.e. $r \gg \lambda$, the components with $1/r^2$ and $1/r^3$ dependence become negligible and the field components left are the propagating components H_ϕ and E_θ:

$$H_\phi = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{\jmath k}{r} \right) \sin\theta, \quad E_r = 0, \quad \text{and} \quad E_\theta = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{\jmath\omega\mu_0}{r} \right) \sin\theta. \tag{2.4}$$

Now consider the fields in the plane normal to the filament, that is, with $\theta = \pi/2$ radians so that $\sin\theta = 1$. The fields are now

$$H_\phi = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{\jmath k}{r} \right) \quad \text{and} \quad E_\theta = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \left(\frac{\jmath\omega\mu_0}{r} \right) \tag{2.5}$$

and the wave impedance is

$$\eta = \frac{E_\theta}{H_\phi} = \frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \frac{\jmath\omega\mu_0}{r} \left(\frac{I_0 h}{4\pi} \mathrm{e}^{-\jmath kr} \frac{\jmath k}{r} \right)^{-1} = \frac{\omega\mu_0}{k}. \tag{2.6}$$

Note that the strength of the fields is directly proportional to the magnitude of the current. This proves to be very useful in understanding spurious radiation from microwave structures. Now $k = \omega\sqrt{\mu_0\epsilon_0}$, so

$$\eta = \frac{\omega\mu_0}{\omega\sqrt{\mu_0\epsilon_0}} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\ \Omega, \tag{2.7}$$

as expected. Thus an antenna can be viewed as having the inherent function of an impedance transformer converting from the lower characteristic impedance of a transmission line (often $50\ \Omega$) to the $377\ \Omega$ characteristic impedance of free space.

Further comments can be made about the propagating fields (Equation (2.4)). The EM field propagates in all directions except not directly in line with the filament. For fixed r, the amplitude of the propagating field increases sinusoidally with respect to θ until it is maximum in the direction normal to the filament.

The power radiated is obtained using the **Poynting vector**, which is the cross-product of the propagating electric and magnetic fields. From this the time-average propagating power density is (with the SI units of $\mathrm{W/m^2}$)

$$P_R = \frac{1}{2}\Re\left(E_\theta H_\phi^*\right) = \frac{\eta k^2 |I_0|^2 h^2}{32\pi^2 r^2}\sin^2\theta, \tag{2.8}$$

and the power density is proportional to $1/r^2$. In Equation (2.8) $\Re(\cdots)$ indicates that the real part is taken.

2.3.2 Finite-Length Wire Antennas

The EM wave propagated from a wire of finite length is obtained by considering the wire as being made up of many filaments and the field is then the superposition of the fields from each filament. As an example, consider the antenna in Figure 2-3(b) where the wire is half a wavelength long. As a good approximation the current on the wire is a standing wave and the current on the wire is in phase so that the current phasor is

$$I(z) = I_0 \cos(kz). \tag{2.9}$$

From Equation (2.4) and referring to Figure 2-3 the fields in the far field are

$$H_\phi = \int_{-\lambda/4}^{\lambda/4} \frac{I_0\cos(kz)}{4\pi} \mathrm{e}^{-\jmath k r'}\left(\frac{\jmath k}{r'}\right)\sin\theta'\,dz \tag{2.10}$$

$$E_\theta = \int_{-\lambda/4}^{\lambda/4} \frac{I_0\cos(kz)}{4\pi} \mathrm{e}^{-\jmath k r'}\left(\frac{\jmath\omega\mu_0}{r'}\right)\sin\theta'\,dz, \tag{2.11}$$

where θ' is the angle from the filament to the point P. Now $k = 2\pi/\lambda$ and at the ends of the wire $z = \pm\lambda/4$ where $\cos(kz) = \cos(\pm\pi/2) = 0$. Evaluating the equations is analytically involved and will not be done here. The net result is that the fields are further concentrated in the plane normal to the wire. At large r, of at least several wavelengths distant from the antenna, only the field components decreasing as $1/r$ are significant. At large r the phase differences of the contributions from the filaments is significant and results in shaping of the fields. The geometry to be used in calculating the far field is shown in Figure 2-4(a). The phase contribution of each filament, relative to that at $z = 0$, is $(kz\sin\theta)/\lambda$ and Equations (2.10) and (2.11) become

$$H_\phi = I_0\left(\frac{\jmath k}{4\pi r}\right)\sin(\theta)\,\mathrm{e}^{-\jmath k r}\int_{-\lambda/4}^{\lambda/4}\frac{\sin(kz)}{4\pi}\sin(z\sin(\theta))dz \tag{2.12}$$

$$E_\theta = I_0\left(\frac{\jmath\omega\mu_0}{4\pi r}\right)\sin(\theta)\,\mathrm{e}^{-\jmath k r}\int_{-\lambda/4}^{\lambda/4}\sin(kz)\sin(z\sin(\theta))dz. \tag{2.13}$$

Figure 2-4(b) is a plot of the near-field electric field in the y-z plane calculating E_r and E_θ (recall that $E_\phi = 0$) every $90°$. Further from the antenna the E_r component rapidly reduces in size, and E_θ dominates.

A summary of the implications of the above equations are, first, that the strength of the radiated electric and magnetic fields are proportional to the

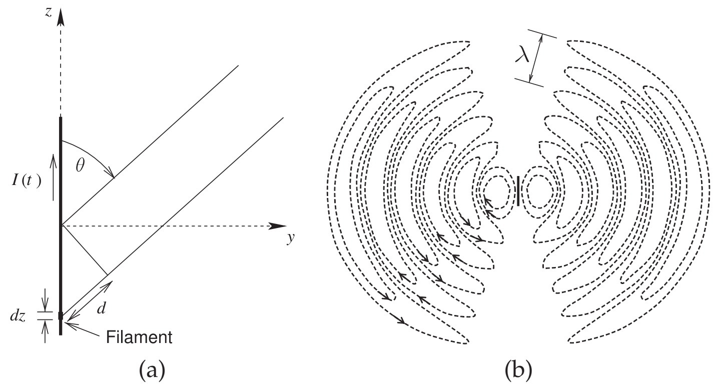


Figure 2-4: Wire antenna: (a) geometry for calculating contributions from current filaments of length dz with cordinate z ($d = -z\sin\theta$); and (b) instantaneous electric field in the y-z plane due to a $\lambda/2$ long current element. There is also a magnetic field.

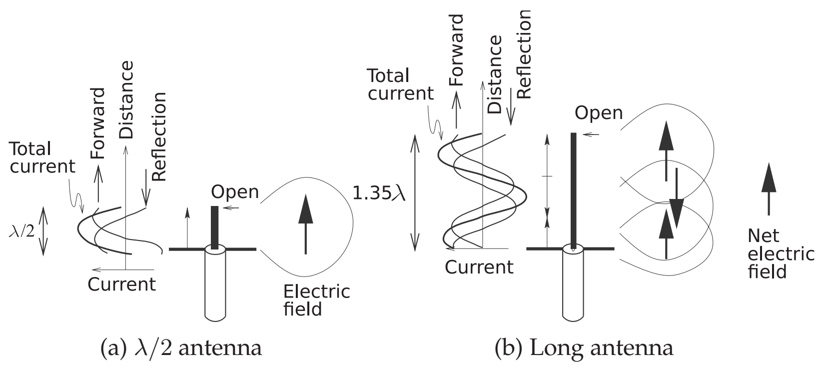


Figure 2-5: Monopole antenna showing total current and forward- and backward-traveling currents: (a) a $\frac{1}{2}\lambda$-long antenna; and (b) a relatively long antenna.

current on the wire antenna. So establishing a standing current wave and hence magnifying the current is important to the efficiency of a wire antenna. A second result is that the power density of freely propagating EM fields in the far field is proportional to $1/r^2$, where r is the distance from the antenna. A third interpretation is that the longer the antenna, the flatter the radiated transmission profile; that is, the radiated energy is more tightly confined to the x-y (i.e. $\Theta = 0$) plane. For the wire antenna the peak radiated field is in the plane normal to the antenna, and thus the wire antenna is generally oriented vertically so that transmission is in the plane of the earth and power is not radiated unnecessarily into the ground or into the sky.

To obtain an efficient resonant antenna, all of the current should be pointed in the same direction at a particular time. One way of achieving this is to establish a standing wave, as shown in Figure 2-5(a). At the open-circuited end, the current reflects so that the total current at the end of the wire is zero. The initial and reflected current waves combine to create a standing wave. Provided that the antenna is sufficiently short, all of the total current—the standing wave—is pointed in the same direction. The optimum length is about a half wavelength. If the wire is longer, the contributions to the field from the oppositely directed current segments cancel (see Figure 2-5(b)).

In Figure 2-5(a) a coaxial cable is attached to the monopole antenna below the ground plane and often a series capacitor between the cable and the antenna provides a low level of coupling leading to a larger standing wave. The capacitor also approximately matches the characteristic impedance of the cable to the input impedance Z_{in} of the antenna. If the length of the monopole is reduced to one-quarter wavelength long it is again resonant, and the input impedance, Z_{in}, is found to be $36\ \Omega$. Then a $50\ \Omega$ cable can

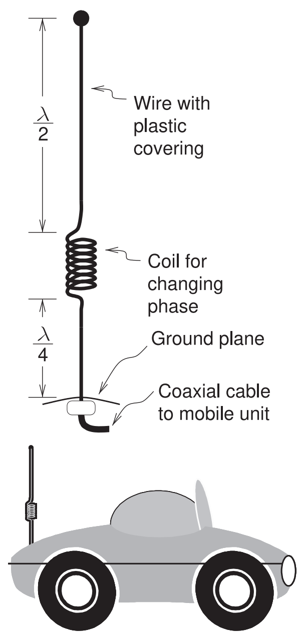


Figure 2-6: Mobile antenna with phasing coil extending the effective length of the antenna.

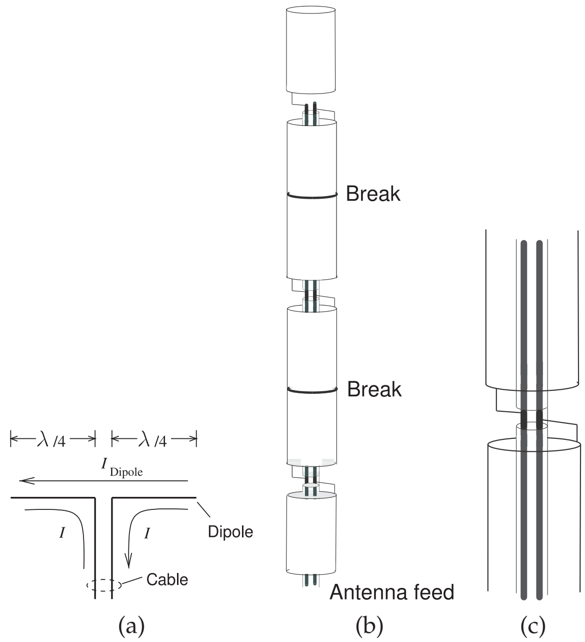


Figure 2-7: Dipole antenna: (a) current distribution; (b) stacked dipole antenna; and (c) detail of the connection in a stacked dipole antenna.

be directly connected to the antenna and there is only a small mismatch and nearly all the power is transferred to the antenna and then radiated.

Another variation on the monopole is shown in Figure 2-6, where the key component is the phasing coil. The phasing coil (with a wire length of $\lambda/2$) rotates the electrical angle of the current phasor on the line so that the current on the $\lambda/4$ segment is in the same direction as on the $\lambda/2$ segment. The result is that the two straight segments of the loaded monopole radiate a more tightly confined EM field. The phasing coil itself does not radiate (much).

Another ingenious solution to obtaining a longer effective wire antenna with same-directed current (and hence a more tightly confined RF beam) is the stacked dipole antenna (Figure 2-7). The basis of the antenna is a dipole as shown in Figure 2-7(a). The cable has two conductors that have equal amplitude currents, I, but flowing as shown. The wire section is coupled to the cable so that the currents on the two conductors realize a single effective current I_{dipole} on the dipole antenna. The stacked dipole shown in Figure 2-7(b) takes this geometric arrangement further. Now the radiating element is hollow and a coaxial cable is passed through the antenna elements and the half-wavelength sections are fed separately to effectively create a wire antenna that is several wavelengths long with the current pointing in one direction. Most cellular antennas using wire antennas are stacked dipole antennas.

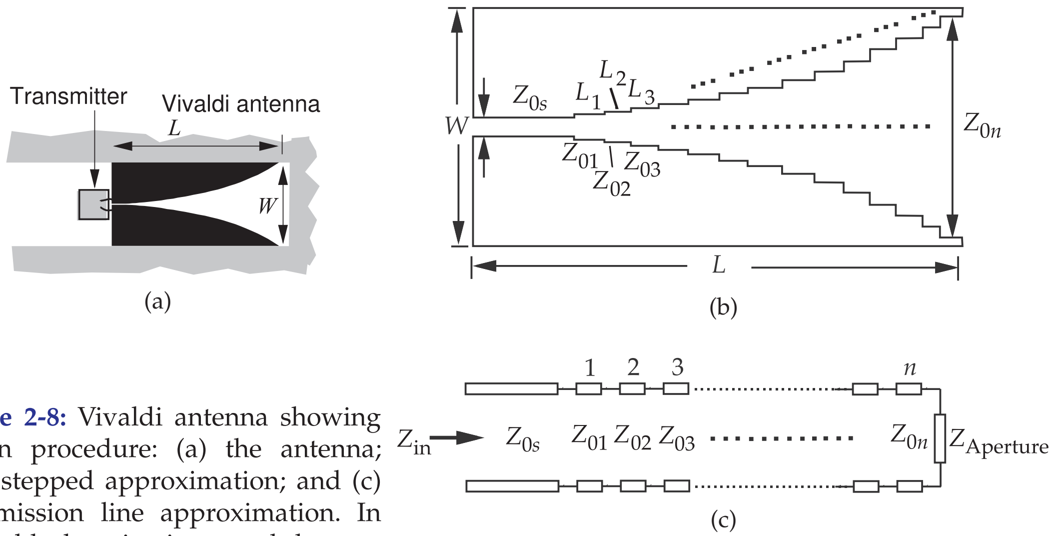


Figure 2-8: Vivaldi antenna showing design procedure: (a) the antenna; (b) a stepped approximation; and (c) transmission line approximation. In (a) the black region is a metal sheet.

Summary

Standing waves of current can be realized by resonant structures other than wires. A microstrip patch antenna, see Figure 2-2(b), is an example, but the underlying principle is that an array of current filaments generates EM components that combine to create a propagating field. Resonant antennas are inherently narrowband because of the reliance on the establishment of a standing wave. A relative bandwidth of 5%–10% is typical.

2.4 Traveling-Wave Antennas

Traveling-wave antennas have the characteristics of broad bandwidth and large size. These antennas begin as a transmission line structure that flares out slowly, providing a low reflection transition from a transmission line to free space. The bandwidth can be very large and is primarily dependent on how gradual the transition is.

One of the more interesting traveling-wave antennas is the **Vivaldi antenna** of Figure 2-8(a). The Vivaldi antenna is an extension of a slotline in which the fields are confined in the space between two metal sheets in the same plane. The slotline spacing increases gradually in an exponential manner, much like that of a Vivaldi violin (from which it gets its name), over a distance of a wavelength or more. A circuit model is shown in Figures 2-8(b and c) where the antenna is modeled as a cascade of many transmission lines of slowly increasing characteristic impedance, Z_0. Since the Z_0 progression is gradual there are low-level reflections at the transmission line interfaces. The forward-traveling wave on the antenna continues to propagate with a negligible reflected field. Eventually the slot opens sufficiently that the effective impedance of the slot is that of free space and the traveling wave continues to propagate in air.

The other traveling-wave antennas work similarly and all are at least a wavelength long, with the central concept being a gradual taper from the characteristic impedance of the originating transmission line to free space. The final aperture is at least one-half wavelength across so that the fields can curl on themselves (i.e. loop back on themselves) and are self-supporting as they leave the antenna.

2.5 Antenna Parameters

This section introduces a number of antenna metrics that are used to characterize antenna performance.

2.5.1 Radiation Density and Radiation Intensity

Antennas do not radiate equally in all directions concentrating radiated power in one direction called the **main** (or **major**) **lobe** of the antenna. This focusing effect is called **directivity**. The power in a particular direction is characterized by the radiation density and the radiation intensity metrics. The radiation density, S_r, is the power per unit area with the SI units of W/m^2, and will be maximum in the main lobe. Referring to Figure 2-9 with an antenna located at the center of the sphere of radius r and radiating a total power P_r, S_r is the incremental radiated power dP_r passing through the incremental shaded region of area, dA:

$$S_r = \frac{dP_r}{dA}. \tag{2.14}$$

S_r reduces with distance falling off as $1/r^2$ in free space. For a practical antenna S_r will vary across the surface of the sphere. The total power radiated is the closed integral over the surface S of the sphere:

$$P_r = \oint_S dP_r = \oint_S S_r dA. \tag{2.15}$$

An alternative measure of power concentration is the radiation intensity U which is in terms of the incremental solid angle $d\Omega$ subtended by dA so that $d\Omega = dA/r^2$ and (with the SI units of W/steradian or W/sr)

$$U = \frac{dP_r}{d\Omega} = \frac{dP_r}{dA} r^2 = r^2 S_r. \tag{2.16}$$

Isotropic Antenna

It is useful to reference the directivity of an antenna with respect to a fictitious isotropic antenna that has no loss and radiates equally in all directions so that S_r is only a function of r. Then integrating over the surface of the sphere

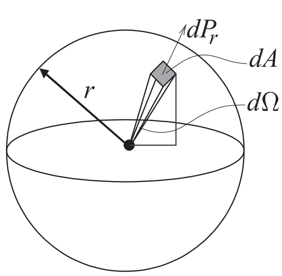


Figure 2-9: Free-space spreading loss. The incremental power, dP_r, intercepted by the shaded region of incremental area dA is proportional to $1/r^2$. The solid angle subtended by the shaded area is the incremental solid angle $d\Omega$. The integral of dA over the surface of the sphere, i.e. the area of the sphere is $4\pi r^2$. The total solid angle subtended by the sphere is the integral of $d\Omega$ over the sphere and is 4π steradians (or 4π sr).

yields the total radiated power

$$P_r|_{\text{Isotropic}} = \oint_S dP_r = \oint_S S_r dA = S_r \oint_S dA = S_r 4\pi r^2 = 4\pi U. \quad (2.17)$$

Since the isotropic antenna has no loss the power input to the antenna P_{IN} is equal to the power radiated $P_r = P_{\text{IN}}$. Thus for an isotropic antenna

$$S_r = \frac{P_r}{4\pi r^2} = \frac{P_{\text{IN}}}{4\pi r^2} \quad (2.18) \quad \text{and} \quad U|_{\text{Isotropic}} = r^2 S_r = \frac{P_{\text{IN}}}{4\pi r^2}. \quad (2.19)$$

Antenna Efficiency

Antenna efficiency, sometimes called the **radiation efficiency**, describes losses in an antenna principally due to resistive (I^2R) losses. Resonant antennas work by creating a large current that is maximized through the generation of a standing wave at resonance. There is a lot of current, and even just a little resistance results in substantial resistive loss. The power that is reflected from the input of the antenna is usually small. The total radiated power (in all directions), P_r, is the power input to the antenna less losses. The antenna efficiency, η_A is therefore defined as

$$\eta_A = P_r/P_{\text{IN}}, \quad (2.20)$$

where P_{IN} is the power input to the antenna and $\eta_A < 1$ and usually expressed as a percentage. Antenna efficiency is very close to one for many antennas, but can be 50% for microstrip patch antennas.

Antenna loss refers to the same mechanism that gives rise to antenna efficiency. Thus an antenna with an antenna efficiency of 50% has an antenna loss of 3 dB. Generally losses are resistive due to I^2R loss and mismatch loss of the antenna that occurs when the input impedance is not matched to the impedance of the cable connected to the antenna. Because of confusion with antenna gain (they are not the opposite of each other) the use of the term 'antenna loss' is discouraged and instead 'antenna efficiency' preferred.

2.5.2 Directivity and Antenna Gain

The directivity of an antenna, D, is the ratio of the radiated power density to that of an isotropic antenna with the same total radiated power P_r:

$$D = \frac{S_r}{S_r|_{\text{Isotropic}}} = \frac{U}{U|_{\text{Isotropic}}} \quad (2.21)$$

where S_r and U refer to the actual antenna and the power densities and intensities are measured at the same distance from the antennas. For an actual antenna D is dependent on the direction from the antenna, see Figure 2-10. The maximum value of D will be in the direction of the main lobe of the antenna and this is called **directivity gain**.

The focusing property of an antenna is characterized by comparing the radiated power density to that of an isotropic antenna with the same input power. The antenna gain, G_A, is the maximum value of D when the power input $P_{\text{IN}} = P_r/\eta_A$ to the antenna and the isotropic antenna are the same:

$$G_A = \eta_A \max(D). \quad (2.22)$$

Antenna	Type	Figure	Gain (dBi)	Notes
Lossless isotropic antenna			0	
$\lambda/2$ dipole	Resonant	2-7(a)	2	$R_{\text{in}} = 73\ \Omega$
3λ diameter parabolic dish	Traveling	—	38	$R_{\text{in}} =$ match
Patch	Resonant	2-2(b)	9	$R_{\text{in}} =$ match
Vivaldi	Traveling	2-2(c)	10	$R_{\text{in}} =$ match
$\lambda/4$ monopole on ground	Resonant	2-5(a)	2	$R_{\text{in}} = 36\ \Omega$
$5/8\lambda$ monopole on ground	Resonant	2-5(a)	3	Matching required

Table 2-1: Several antenna systems. R_{in} = match for resonant antennas indicates that the antenna can be designed to have an input impedance matching that of a feed cable. Traveling-wave antennas are intrinsically matched.

Losses in the antenna are accounted for by the efficiency term η_A.

In Equation (2.22) G_A is a gain factor and is often expressed in terms of decibels (taking 10 times the log of G_A) but dBi (with 'i' standing for 'with-respect-to isotropic') is used to indicate that it is not a power gain in the same sense as amplifier gain. G_A instead is the ratio of power densities for two different antennas. For example, an antenna that focuses power in one direction increasing the peak radiated power density by a factor of 20 relative to that of an isotropic antenna thus has an antenna gain of 13 dBi. With care G_A can be often used in calculations of power as as with amplifier gain.

Since it is almost impossible to calculate internal antenna losses, antenna gain is invariably only measured. The input power to an antenna can be measured and the peak radiated power density, $P_D|_{\text{Maximum}}$, measured in the far field at several wavelengths distant (at $r \gg \lambda$). This is compared to the power density from an ideal isotropic antenna at the same distance with the same input power. Antenna gain is determined from

$$G_A = \frac{\text{Maximum radiated power per unit area}}{\text{Maximum radiated power per unit area for an isotropic antenna}} \tag{2.23}$$

$$\begin{aligned} &= \frac{S_r|_{\text{Maximum}}}{S_r|_{\text{Isotropic}}} = 4\pi r^2 \frac{P_D|_{\text{Maximum}}}{P_{\text{IN}}} \\ &= 4\pi \frac{\text{Maximum radiated power per unit solid angle}}{\text{Total input power to the antenna}} \\ &= 4\pi \frac{(dP_r/d\Omega)|_{\text{Maximum}}}{P_{\text{IN}}} = 4\pi r^2 \frac{(dP_r/dA)|_{\text{Maximum}}}{P_{\text{IN}}}. \end{aligned} \tag{2.24}$$

The antenna gains of common resonant and traveling-wave antennas are given in Table 2-1. In free space the antenna gain determined using Equation (2.22) is independent of distance. Antenna gain is measured on an antenna range using a calibrated receive antenna and care taken to avoid reflections from objects, especially from the ground.

The losses of an antenna are incorporated in the antenna gain which is defined in terms of the power input to the antenna, see Equation (2.24). Thus in calculations of radiated power using antenna gain, there is no need to separately account for resistive losses in the antenna.

In summary, antennas concentrate the radiated power in one direction so that the density of the power radiated in the direction of the peak field is higher than the power density from an isotropic antenna. Power radiated from a base station antenna, such as that shown in Figure 2-11, is concentrated in a region that looks like a toroid or, more closely, a balloon squashed

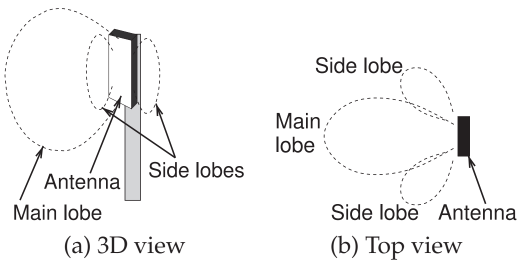


Figure 2-10: Field pattern produced by a microstrip antenna.

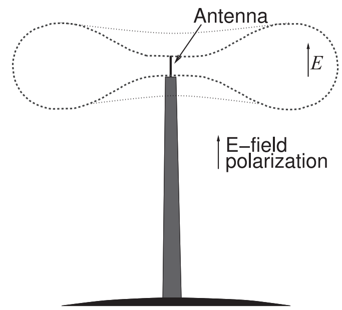


Figure 2-11: A base station transmitter pattern.

at its north and south poles. Then the antenna does not radiate much power into space and will concentrate power in a region skimming the surface of the earth. Antenna gain is a measure of the effectiveness of an antenna to concentrate power in one direction. Thus, in free space where power spreads out by $1/r^2$, the maximum power density (in SI units of W/m^2) at a distance r is

$$P_D = \frac{G_A P_{\text{IN}}}{4\pi d^2}, \tag{2.25}$$

where $4\pi d^2$ is the area of a sphere of radius d and P_{IN} is the input power.

Measurements of antenna gain are used to derive antenna efficiency. It is impossible to measure or simulate the resistive and dielectric losses of an antenna directly. Antenna efficiency is obtained using theoretical calculations of antenna gain assuming no losses in the antenna itself. This is compared to the measured antenna gain yielding the antenna efficiency.

EXAMPLE 2.2 Antenna Gain

A base station antenna has an antenna gain, G_A, of 11 dBi and a 40 W input. The transmitted power density falls off with distance d as $1/d^2$. What is the peak power density at 5 km?

Solution:

A sphere of radius 5 km has an area $A = 4\pi r^2 = 3.142 \cdot 10^8$ m^2; $G_A = 11$ dBi = 12.6.
In the direction of peak radiated power, the power density at 5 km is

$$P_D = \frac{P_{\text{in}} G_A}{A} = \frac{40 \cdot 12.6 \text{ W}}{3.142 \cdot 10^8 \text{m}^2} = 1.603\ \mu\text{W/m}^2.$$

EXAMPLE 2.3 Antenna Efficiency

A antenna has an antenna gain of 13 dBi and an antenna efficiency of 50% and all of the loss is due to resistive losses and resistance of metals is proportional to temperature. The RF signal input to the antenna has a power of 40 W.

(a) What is the input power in dBm?

P_{in} = 40 W = 46.02 dBm.

(b) What is the total power transmitted in dBm?

$$P_{\text{Radiated}} = 50\% \text{ of } P_{\text{IN}} = 20 \text{ W or } 43.01 \text{ dBm.}$$
$$\text{Alternatively, } P_{\text{Radiated}} = 46.02 \text{ dBm} - 3 \text{ dB} = 43.02 \text{ dBm.}$$

(c) If the antenna is cooled to near absolute zero so that it is lossless, what would the antenna gain be?

The antenna gain would increase by 3 dB and antenna gain incorporates both directivity and antenna losses. So the gain of the cooled antenna is 16 dBi.

2.5.3 Effective Isotropic Radiated Power

A transmit antenna does not radiate power equally in all directions and for a receiver in the main lobe of the transmit antenna it is as though there is an isotropic transmit antenna with a much higher input power. This concept is incorporated in the effective isotropic radiated power (**EIRP**):

$$\text{EIRP} = P_{\text{IN}} G_A. \tag{2.26}$$

This is the total power that would be radiated by an isotropic antenna producing the same (peak) power density as the actual antenna.

2.5.4 Effective Aperture Size

Effective aperture size is defined so that the power density at a receive antenna when multiplied by its effective aperture size, A_R, yields the power output from the antenna at its connector. An antenna has an effective size that is more than its actual physical size because of its influence on the EM fields around it. The effective aperture size of an antenna is the area of the surface that captures all of the power passing through it and delivers this power to the output terminals of the antenna.

The effective aperture area of a receive antenna, A_R, is related to the receive antenna gain, G_R, as follows [2, 3] (note that A_e is often used if it is not necessary to distinguish antennas):

$$A_R = \frac{G_R \lambda^2}{4\pi}, \tag{2.27}$$

where λ is the wavelength of the radio signal. The effective aperture area of an antenna can have little to do with its physical size; e.g., a wire antenna has almost no physical size but has a significant effective aperture size.

If S_r is the transmitted power density at the receive antenna, the power received is

$$P_R = P_D A_R = P_D \frac{G_R \lambda^2}{4\pi}. \tag{2.28}$$

The power density at a distance d (ignoring multipath effects),is

$$S_r = \frac{P_T G_T}{4\pi d^2}, \tag{2.29}$$

where P_T is the power input to the transmit antenna with antenna gain G_T. The power delivered by the receive antenna is

$$P_R = S_r A_R = \frac{P_T G_T}{4\pi d^2}\frac{G_R \lambda^2}{4\pi} = P_T G_T G_R \left(\frac{\lambda}{4\pi d}\right)^2. \tag{2.30}$$

2.5.5 Summary

This section introduced several metrics for characterizing antennas:

Metric	Equation	Description
S_r	(2.14)	Radiated power density, W/m^2
U	(2.16)	Radiation intensity W/sr
η_A	(2.27)	Antenna efficiency
D	(2.21)	Antenna directivity
G_A	(2.23)	Antenna gain,used with a transmit antenna
A_e	(2.27)	Effective aperture area, used with a receive antenna
EIRP	(2.26)	Equivalent isotropic radiated power

EXAMPLE 2.4 Point-to-Point Communication

In a point-to-point communication system, a parabolic receive antenna has an antenna gain of 60 dBi. If the signal is 60 GHz and the power density at the receive antenna is 1 pW/cm^2, what is the power at the output of the receive antenna connected to the RF electronics?

Solution:

The first step is determining the effective aperture area, A_R, of the antenna. At 60 GHz $\lambda = 5$ mm. Note that G_R = 60 dBi = 10^6. From Equation (2.27),

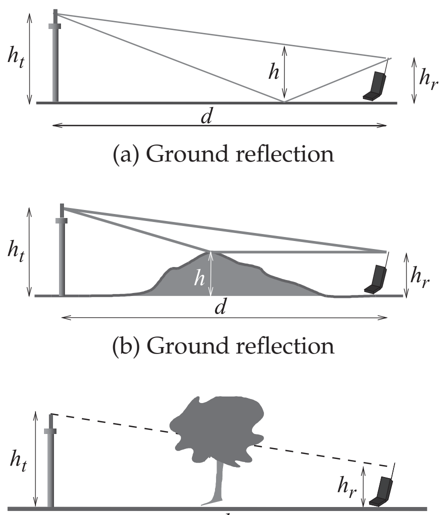
$$A_R = \frac{G_R \lambda^2}{4\pi} = \frac{10^6 \cdot 0.005^2}{4\pi} = 1.989\ \text{m}^2. \tag{2.31}$$

Using Equation (2.28) P_D = 1 pW/cm^2 = 10 nW/m^2, the total power delivered to the RF receiver electronics (at the output of the receive antenna) is

$$P_R = P_D A_R = 10\ \text{nW} \cdot \text{m}^{-2} \cdot 1.989\ \text{m}^2 = 19.89\ \text{nW}. \tag{2.32}$$

2.6 The RF Link

The RF link is between a transmit antenna and a receive antenna. Sometimes the RF link includes the antenna, this will be clear from the context, but usually it includes the antennas. The principle source of link loss is the spreading out of the EM field as it propagates. In the absence obstructions the power density reduces as $1/d^2$, where d is distance, and this is the **line-of-sight** (**LOS**) situation. In this section the propagation path is first described along with its impairments including propagation on multiple paths between a transmit antenna and a receive antenna.

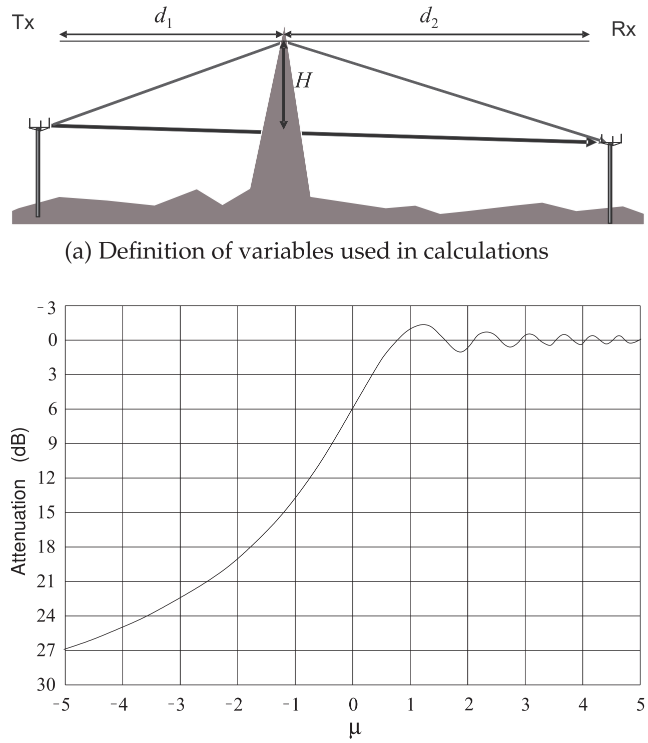


(a) Ground reflection

(b) Ground reflection

(c) Scattering through tree

Figure 2-12: Common paths contributing to multipath propagation.



(a) Definition of variables used in calculations

(b) Chart for determining attenuation using Equation (2.33)

Figure 2-13: Knife-edge diffraction.

2.6.1 Propagation Path

When the radiated signal reflects and diffracts there are multiple propagation paths that result in fading as the paths constructively and destructively combine at the receiver. Of these destructive combining is much worse as it can reduce a signal level below what it would be if propagation was in free space. In urban areas, 10 or 20 paths can have significant powers [4].

Common paths encountered in cellular radio are shown in Figure 2-12. As a rough guide, in the single-digit gigahertz range each diffraction and scattering event reduces the signal received by 20 dB. The knife-edge diffraction scenario is shown in more detail in Figure 2-13. This case is fairly easy to analyze and can be used to estimate the effects of individual obstructions. The diffraction model is derived from the theory of half-infinite screen diffraction [5]. First, calculate the parameter ν from the geometry of the path using

$$\nu = -H\sqrt{\frac{2}{\lambda}\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}. \tag{2.33}$$

Next, consult the plot in Figure 2-13(b) to obtain the diffraction loss (or attenuation). This loss should be added (using decibels) to the otherwise determined path loss to obtain the total path loss.

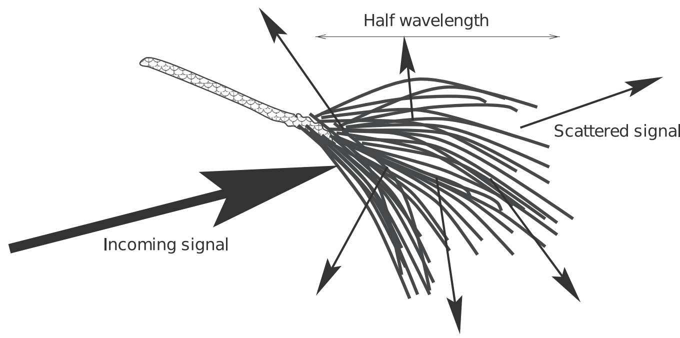


Figure 2-14: Pine needles scattering an incoming EM signal.

2.6.2 Resonant Scattering

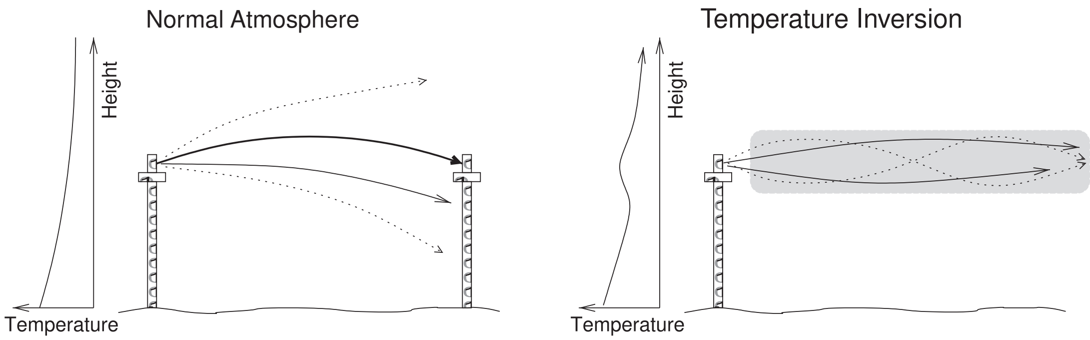
Propagation is rarely from point to point, i.e. non-LOS (**NLOS**), as the path is often obstructed. One type of event that reduces transmission is scattering. The level of the effect depends on the size of the objects causing scattering. Here the effect of the pine needles of Figure 2-14 will be considered. The pine needles (as most objects in the environment) conduct electricity, especially when wet. When an EM field is incident an individual needle acts as a wire antenna, with the current maximum when the pine needle is one-half wavelength long. At this length, the "needle" antenna supports a standing wave and will re-radiate the signal in all directions. This is scattering, and there is a considerable loss in the direction of propagation of the original fields. The effect of scattering is frequency and size dependent. A typical pine needle is 15 cm long, which is exactly $\lambda/2$ at 1 GHz, and so a stand of pine trees have an extraordinary impact on cellular communications at 1 GHz. As a rough guide, 20 dB of a signal is lost when passing through a small stand of pine trees.

2.6.3 Fading

Fading refers to the variation of the received signal with time or when the position of transmit or receive antennas is changed. The most important fading types are flat fading, multipath fading, and rain fading.

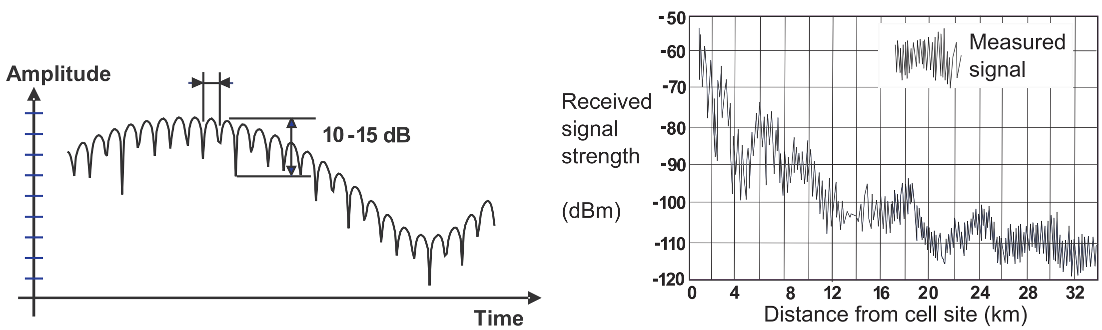
Flat Fading

Temperature variations of the atmosphere between the transmit and receive antennas give rise to what is called flat fading and sometimes called **thermal fading**. This fading is called flat because it is independent of frequency. One form of flat fading is due to refraction, which occurs when different layers of the atmosphere have different densities and thus dielectric permittivities increasing or decreasing away from the surface of the earth. The temperature profile can increase away from the earth surface or reduce depending on whether the temperature of the earth is higher than that of the air and is commonly associated with the beginning and end of the day. This causes refraction of the propagating wave, , see Figure 2-15(a). **Temperature inversions** can also occur where the temperature profile producing a layer with a relatively higher permittivity. RF energy gets trapped in this layer, reflecting from the top and bottom of the inversion layer, see Figure 2-15(b). This is called **ducting**. In point-to-point communication systems the transmit and receive antennas are mounted high on towers and then



(a) Atmospheric refraction (b) Atmospheric ducting

Figure 2-15: Fading resulting from ducting: (a) normal atmospheric refraction (normally the temperature of air drops with increasing height and the lower refractive index at high heights results in a concave refraction); and (b) atmospheric ducting (resulting from temperature inversion inducing an air layer with higher dielectric constant than the surrounding air).



(a) (b)

Figure 2-16: Fast and slow fading: (a) in time as a radio and obstructions move; and (b) in distance.

reflection from ground objects is often small. In such cases flat fading is the most commonly observed phenomena and minor fluctuations of several decibels in receive signal level are common throughout the day. However, when temperature variations are extreme, ducting can severely impact communications reducing signal levels by up to 20 dB.

Shadow Fading

Shadow fading occurs when the LOS path is blocked by an obstruction such as a building or hill. This results in relatively slow fades with the amplitude response varying in time and distance as shown in Figure 2-16. This figure shows both fast fades that are 10 to 15 dB deep and slow or shadow fades that are 20 to 30 dB deep.

Multipath Fading

Multipath describes the situation situations where there are many reflections that combine destructively and constructively. Multipath fading is also called **fast fading**, as the characteristics of the channel can change significantly in a few milliseconds. Multipath fading of 20 dB can occur for a small percentage of the time on time scales of many seconds when there are few propagation paths (e.g. in a rural area) to a large percentage of the time many times per second in a dense urban environment where there are many paths. Constructive combining does increase the signal level momentarily, but there is no advantage to this. Destructive combining can result in deep fades of 20 dB impacting communications and forcing the communication system to accommodate either by using higher average powers or using strategies such as multiple antennas or spreading the communication signal over a wide bandwidth since fades tend to be 500 kHz to 1 MHz wide at all frequencies.

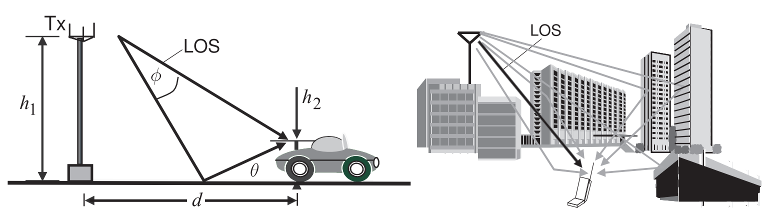
When the signal on one of the paths dominates, this is usually the LOS path, fading is called **Rician fading**. With LOS and a single ground reflection, the situation is the classic Rician fading shown in Figure 2-17(a). Here the ground changes the phase of the signal upon reflection by 180°. When the receiver is a long way from the base station, the lengths of the two paths are almost identical and the level of the signals in the two paths are almost the same. The net result is that these two signals almost cancel, and so instead of the power falling off by $1/d^2$, it falls off by $1/d^3$. When there are many paths and all have similar amplitude signals, fading is called **Rayleigh fading**. In an urban area such as that shown in Figure 2-17(b), there are many significant multipaths and the power falls off by $1/d^4$ and sometimes faster.

Rain Fading

Rain fading is due to both the amount of rain and the size of individual rain drops and fading occurs over periods of minutes to hours. Propagation through the atmosphere is affected by absorption by molecules in the air, fog, and rain, and by scattering by rain drops. Figure 1-2 shows the attenuation in decibels per kilometer from 3 GHz to 300 GHz. The attenuation due to rain increases with frequency and this derives largely from scattering.

Summary of Fading

The fades of most concern in a mobile wireless system are the deep fades resulting from destructive interference of multiple reflections. These fades vary rapidly (over a few milliseconds) if a handset is moving at vehicular speeds but occur slowly when the transmitter and receiver are fixed. Fades can be viewed as deep amplitude modulation, and so so modulation is restricted to phase shift keying schemes when a transmitter and receive



(a) Rician fading (b) Rayleigh fading

Figure 2-17: Multipath propagation: (a) line of sight (LOS) and ground reflection paths only; and (b) in an urban environment.

moving at vehicular speeds relative to each other.

2.6.4 Link Loss and Path Loss

With transmit and receive antennas included in the RF link, the usual case, link loss is defined as the ratio of the power input to the transmit antenna, P_T, to the power delivered by the receive antenna, P_R. Rearranging Equation (2.30) the total line-of-sight (LOS) link loss, $L_{\text{LINK,LOS}}$, between the input of the transmit antenna and the output of the receive antenna separated by distance d is (in decibels):

$$L_{\text{LINK,LOS}}|_{\text{dB}} = 10\log\left(\frac{P_T}{P_R}\right) = 10\log\left(\frac{P_T}{P_D A_R}\right) \tag{2.34}$$

$$= 10\log\left[P_T\left(\frac{4\pi d^2}{P_T G_T}\right)\left(\frac{4\pi}{\lambda^2 G_R}\right)\right] \tag{2.35}$$

$$= 10\log\left[\left(\frac{1}{G_T G_R}\right)\left(\frac{4\pi d}{\lambda}\right)^2\right] \tag{2.36}$$

$$= -10\log G_T - 10\log G_R + 20\log\left(\frac{4\pi d}{\lambda}\right). \tag{2.37}$$

The last term includes d and is called the LOS path loss (in decibels):

$$L_{\text{PATH,LOS}}|_{\text{dB}} = 20\log\left(\frac{4\pi d}{\lambda}\right). \tag{2.38}$$

This is the preferred form of the expression for path loss, as it can be used directly in calculating link loss using the antenna gains of the transmit and receive antennas without the exercise of calculating the **effective aperture size** of the receive antenna.

Multipath effects result in losses that are proportional to d^n [6, 7] so that the general path loss, including multipath effects, is (in decibels)

$$\begin{aligned} L_{\text{PATH}}|_{\text{dB}} &= L_{\text{PATH,LOS}}|_{\text{dB}} + \text{excess loss}|_{\text{dB}} \\ &= 20\log\left(\frac{4\pi d}{\lambda}\right) + 10(n-2)\log\left(\frac{d}{1\text{ m}}\right) \\ &= 20\log\left[\frac{4\pi(1\text{ m})}{\lambda}\right] + 10(2)\log\left(\frac{d}{1\text{ m}}\right) + 10(n-2)\log\left(\frac{d}{1\text{ m}}\right) \\ &= 10n\log[d/(1\text{ m})] + C, \end{aligned} \tag{2.39}$$

where the distance d and wavelength λ are in meters, and C is a constant that captures the effect of wavelength. Here,

$$C = 20\log\left[4\pi(1\text{ m})/\lambda\right]. \tag{2.40}$$

Combining this with Equation (2.37) yields the link loss:

$$L_{\text{LINK}}|_{\text{dB}} = -G_T|_{\text{dB}} - G_R|_{\text{dB}} + 10n\log[d/(1\text{ m})] + C. \tag{2.41}$$

EXAMPLE 2.5 Link Loss

A 5.6 GHz communication system uses a transmit antenna with an antenna gain G_T of 35 dB and a receive antenna with an antenna gain G_R of 6 dB. If the distance between the antennas is 200 m, what is the link loss if the power density reduces as $1/d^3$? The link loss here is between the input to the transmit antenna and the output from the receive antenna.

Solution:

The link loss is provided by Equation (2.41),

$$L_{\mathrm{LINK}}|_{\mathrm{dB}} = -G_T - G_R + 10n\log[d/(1\text{ m})] + C,$$

and C comes from Equation (2.40), where λ = 5.36 cm. So

$$C = 20\log\left(\frac{4\pi}{\lambda}\right) = 20\log\left(\frac{4\pi}{0.0536}\right) = 47.4\text{ dB}.$$

With n = 3 and d = 200 m,

$$L_{\mathrm{LINK}}|_{\mathrm{dB}} = -35 - 6 + 10\cdot 3\cdot\log(200) + 47.4\text{ dB} = 75.4\text{ dB}.$$

EXAMPLE 2.6 Radiated Power Density

In free space, radiated power density drops off with distance d as $1/d^2$. However, in a terrestrial environment there are multiple paths between a transmitter and a receiver, with the dominant paths being the direct LOS path and the path involving reflection off the ground. Reflection from the ground partially cancels the signal in the direct path, and in a semi-urban environment results in an attenuation loss of 40 dB per decade of distance (instead of the 20 dB per decade of distance roll-off in free space). Consider a transmitter that has a power density of 1 W/m^2 at a distance of 1 m from the transmitter.

(a) The power density falls off as $1/d^n$, where d is distance and n is an index. What is n?
(b) At what distance from the transmit antenna will the power density reach 1 μW·m^{-2}?

Solution:

(a) Power drops off by 40 dB per decade of distance. 40 dB corresponds to a factor of 10,000 (= 10^4). So, at distance d, the power density $P_D(d) = k/d^n$ (k is a constant).
At a decade of distance, $10d$, $P_D(10d) = k/(10d)^n = P_D(d)/10000$, thus

$$\frac{k}{10^n d^n} = \frac{1}{10,000}\frac{k}{d^n}; \qquad 10^n = 10,000 \Rightarrow n = 4.$$

(b) At d = 1 m, $P_D(1\text{ m}) = 1\text{ W/m}^2$. At a distance x,

$$P_D(x) = 1\ \mu\text{W/m}^2 = \frac{k}{x^4}\text{m}^2 = \frac{k}{x^4} \rightarrow x^4 = \frac{1}{10^{-6}} \quad \text{and so} \quad x = 31.6\text{ m}.$$

2.6.5 Propagation Model in the Mobile Environment

RF propagation in the mobile environment cannot be accurately derived. Instead, a fit to measurements is often used. One of the models is the Okumura–Hata model [8], which calculates the path loss as

$$L_{\mathrm{PATH}}|_{(\mathrm{dB})} = 69.55 + 26.16\log f - 13.82\log H + (44.9 - 6.55\log H)\cdot\log d + c, \tag{2.42}$$

where f is the frequency (in MHz), d is the distance between the base station and terminal (in km), H is the effective height of the base station antenna (in m), and c is an environment correction factor (c = 0 dB in a dense urban area,

$c = -5$ dB in an urban area, $c = -10$ dB in a suburban area, and $c = -17$ dB in a rural area, for f = 1 GHz and H=1.5 m).

There are many propagation models for different frequency ranges and different environments. The considerable effort put into developing reliable models is because being able to predict signal coverage is essential to efficient design of basestation layout.

2.7 Antenna Array

An antenna array comprises multiple radiating elements, i.e. individual antennas, and focuses a transmit beam in a desired direction. In Figure 2-20 the field pattern in the plane of the earth produced by an array of 30 antenna elements arranged horizontally is shown. The fields from each antenna element combined to narrow and strengthen the main beam. Side (or grating) lobes are produced and these will result in some interference but their level here is 40 dB below that of the main beam. This is another way of managing interference in a cellular system but the direction to the mobile unit must be known. Antenna arrays are used in 4G and 5G. The affect of the array is to increase the power density of the main beam and the density relative to that of an isotropic antenna is called the directional gain of the array, D_{Array}, and is the product of the antenna gain, G_A, of an individual antenna element and the array gain, G_{Array}:

$$D_{\text{Array}} = G_{\text{Array}} G_A \tag{2.43}$$

The maximum value of G_{Array} is N for an N element array. So the maximum directional gain of the array in dBi is

$$D_{\text{Array}}|_{\text{dBi}} = G_A|_{\text{dBi}} + 10 \log N. \tag{2.44}$$

2.7.1 Multiple Input, Multiple Output

Multiple input, multiple output (**MIMO**, pronounced my-moe) technology uses multiple antennas to transmit and receive signals. The MIMO concept was developed in the 1990s [9, 10] and implemented in 4G and 5G, and a variety of WLAN systems. MIMO relies on signals traveling on multiple paths between an array of transmit antennas and an array of receive antennas In MIMO these paths are used to carry more information with each path propagates an image of one transmitted signal (from one antenna) that differs in both amplitude and phase from the images following other paths. Effectively there are multiple connections between each transmit antenna and each receive antenna, see Figure 2-18. Here a high-speed data-stream is split into several slower data streams, shown in Figure 2-18 as the a, b, and c bitstreams. The distinct bitstreams are separately modulated and sent from their own transmit antenna, with the constellation diagrams of the transmitted modulated signals labeled A, B, and C. The signals from each of the transmit antenna reaches all of the receive antennas by following different uncorrelated paths.

The output of each receive antenna is a linear combination of the multiple transmitted data streams, with the sampled RF phasor diagrams labeled M, N, and O. (It is not really appropriate to call these constellation diagrams.) That is, each receive antenna has a different linear combination of the multiple images. In effect, the output from each receive antenna can be

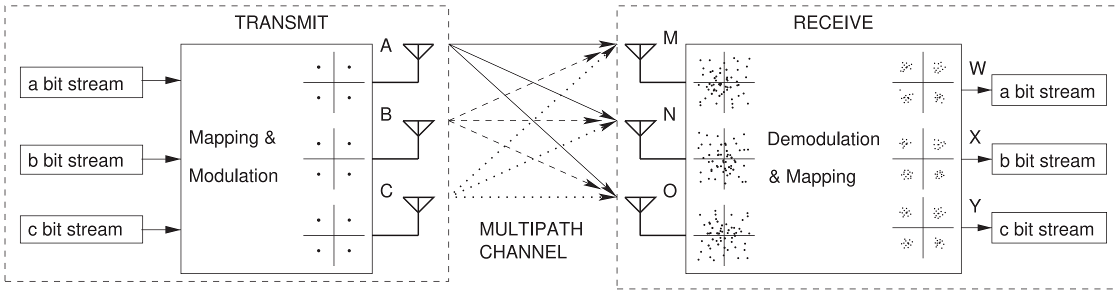


Figure 2-18: A MIMO system showing multiple paths between each transmit antenna and each receive antenna.

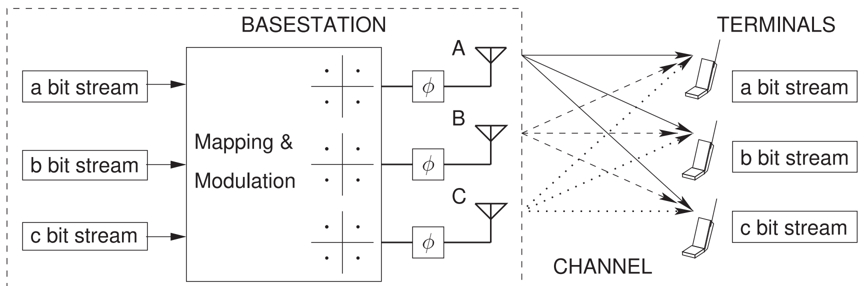


Figure 2-19: A massive MIMO system with beams from the basestation antennas to each terminal unit.

thought of as the solution of linear equations, with each transmit antenna-receive antenna link corresponding to an equation. Continuing the analogy, the signal from each transmit antenna represents a variable. So a set of simultaneous equations can be solved to obtain the original bitstreams. This is accomplished by demodulation and mapping using knowledge of the channel characteristics to yield the original transmitted signals modified by interference. The result is that the constellation diagrams W, X, and Y are obtained. The composite channel can be characterized using known test signals.

The capacity of a MIMO system with high SIR scales approximately linearly with the minimum of M and N, $\min(M, N)$, where M is the number of transmit antennas and N is the number of receive antennas (provided that there is a rich set of paths) [11, 12]. So a system with $M = N = 4$ will have four times the capacity of a system with just one transmit antenna or one receive antenna.

2.7.2 Massive MIMO

MIMO in 4G uses multiple antennas at the basestation and at the mobile terminals to increase overall data rates provided that there are multiple paths between the transmitter and receiver. In 5G there can be a very large number of transmit antennas even though there are few receive antennas per terminal unit as multiple terminal units mean that there can effectively be a very large number of receive antennas, see Figure 2-19.

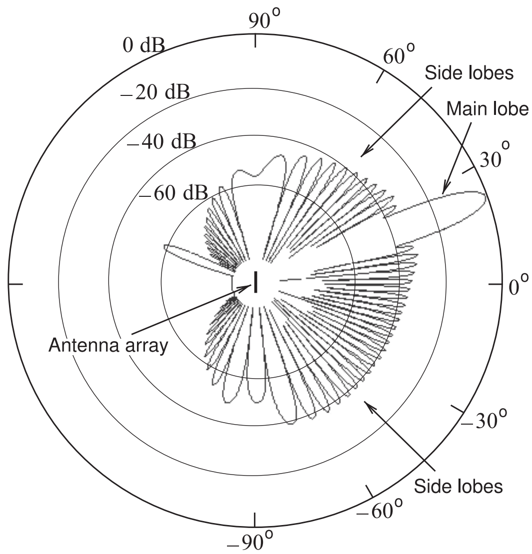


Figure 2-20: Electric field pattern from a 30 element array of antennas spaced 0.65λ appart. The sidelobe levels are about 40 dB below the power level of the main lobe. The same signal is presented to the antenna elements except that phases of the signal at each antennas is adjusted to produce a main beam directed at 20 degrees. The signals to each antenna are thus correlated. After [13].

2.8 Summary

This chapter discussed the impact of imperfections in the RF link from the output of the transmitter to the input of the receiver. Prior to cellular communications becoming so important, only the LOS communication path was considered and the impact of fading, multiple reflections, and delay spread was regarded as detrimental. While many aspects of RF propagation are random, concepts and statistical models have been developed that enable design choices to be made that permit digital communication systems to operate in what would have been regarded as a hostile environment.

2.9 References

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[13] Phased array radiation pattern, Phased_array_radiation_pattern.gif, By Maxter315 [CC BY-SA 4.0 (https://creativecommons.org/licenses/by-sa/4.0)], from Wikimedia Commons.

2.10 Exercises

1. An antenna only radiates 45% of the power input to it. The rest is lost as heat. What input power (in dBm) is required to radiate 30 dBm?
2. The output stage of an RF front end consists of an amplifier followed by a filter and then an antenna. The amplifier has a gain of 27 dB, the filter has a loss of 1.9 dB, and of the power input to the antenna, 35% is lost as heat due to resistive losses. If the power input to the amplifier is 30 dBm, calculate the following:
   (a) What is the power input to the amplifier in watts?
   (b) Express the loss of the antenna in dB.
   (c) What is the total gain of the RF front end (amplifier + filter)?
   (d) What is the total power radiated by the antenna in dBm?
   (e) What is the total power radiated by the antenna in mW?
3. The output stage of an RF front end consists of an amplifier followed by a filter and then an antenna. The amplifier has a gain of 27 dB, the filter has a loss of 1.9 dB, and of the power input to the antenna, 45% is lost as heat due to resistive losses. If the power input to the amplifier is 30 dBm, calculate the following:
   (a) What is the power input to the amplifier in watts?
   (b) Express the loss of the antenna in decibels.
   (c) What is the total gain of the RF front end (amplifier + filter)?
   (d) What is the total power radiated by the antenna in dBm?
   (e) What is the total power radiated by the antenna in milliwatts?
4. Thirty five percent of the power input to an antenna is lost as heat, what is the loss of the antenna in dB.
5. Only 65% of the power input to an antenna is radiated with the rest lost to dissipation in the antenna, what is the gain of the antenna in dB? (This is not the antenna gain.)
6. The efficiency of an antenna is 66%. If the power input to the antenna is 10 W what is the power radiated by the antenna in dBm?
7. An antenna with an input of 1 W operates in free space and has an antenna gain of 12 dBi. What is the maximum power density at 100 m from the antenna?
8. A transmitter has an antenna with an antenna gain of 10 dBi, the resistive losses of the antenna are 50%, and the power input to the antenna is 1 W. What is the EIRP in watts?

   90.] A transmitter has an antenna with an antenna gain of 20 dBi, the resistive losses of the antenna are 50%, and the power input to the antenna is 100 mW. What is the EIRP in watts?
10. An antenna with an antenna gain of 8 dBi radiates 6.67 W. What is the EIRP in watts? Assume that the antenna is 100% efficient.
11. An antenna has an antenna gain of 10 dBi and a 40 W input signal. What is the EIRP in watts?
12. An antenna with 5 W of input power has an antenna gain of 20 dBi and an antenna efficiency of 25% and all of the loss is due to resistive losses in the antenna. [Parallels Example 2.3]
    (a) How much power in dBm is lost as heat in the antenna?
    (b) How much power in dBm is radiated by the antenna?
    (c) What is the EIRP in dBW?
13. An antenna with an efficiency of 50% has an antenna gain of 12 dBi and radiates 100 W. What is the EIRP in watts?
14. An antenna with an efficiency of 75% and an antenna gain of 10 dBi. If the power input to the antenna is 100 W,
    (a) what is the total power in dBm radiated by the antenna?
    (b) what is the EIRP in dBm?
15. The output stage of an RF front end consists of an amplifier followed by a filter and then an antenna. The amplifier has a gain of 27 dB, the filter has a loss of 1.9 dB, and of the power input to the antenna, 45% is lost as heat due to resistive losses. If the power input to the amplifier is 30 dBm, calculate the following:
    (a) What is the power input to the amplifier in watts?
    (b) Express the loss of the antenna in decibels.
    (c) What is the total gain of the RF front end (amplifier + filter)?
    (d) What is the total power radiated by the antenna in dBm?

(e) What is the total power radiated by the antenna in milliwatts?

16. A communication system operating at 2.5 GHz includes a transmit antenna with an antenna gain of 12 dBi and a receive antenna with an effective aperture area of 20 cm^2. The distance between the two antennas is 100 m.
    (a) What is the antenna gain of the receive antenna?
    (b) If the input to the transmit antenna is 1 W, what is the power density at the receive antenna if the power falls off as $1/d^2$, where d is the distance from the transmit antenna?
    (c) Thus what is the power delivered at the output of the receive antenna?

17. Consider a point-to-point communication system. Parabolic antennas are mounted high on a mast so that ground effects do not exist, thus power falls off as $1/d^2$. The gain of the transmit antenna is 20 dBi and the gain of the receive antenna is 15 dBi. The distance between the antennas is 10 km. The effective area of the receive antenna is 3 cm^2. If the power input to the transmit antenna is 600 mW, what is the power delivered at the output of the receive antenna?

18. Consider a 28 GHz point-to-point communication system. Parabolic antennas are mounted high on a mast so that ground effects do not exist, thus power falls off as $1/d^2$. The gain of the transmit antenna is 20 dBi and the gain of the receive antenna is 15 dBi. The distance between the antennas is 10 km. If the power output from the receive antenna is 10 pW, what is the power input to the transmit antenna?

19. An antenna has an effective aperture area of 20 cm^2. What is the antenna gain of the antenna at 2.5 GHz?

20. An antenna operating at 28 GHz has an antenna gain of 50 dBi. What is the effective aperture area of the antenna?

21. A 15 GHz receive antenna has an antenna gain of 20 dBi. If the power density at the receive antenna is 1 nW/cm^2, what is the power at the output of the antenna? [Parallels Example 2.6]

22. Two identical antennas are used in a point-to-point communication system, each having a gain of 50 dBi. The system has an operating frequency of 28 GHz and the antennas are at the top of masts 100 m tall. The RF link between the antennas consists only of the direct line-of-sight path.
    (a) What is the effective aperture area of each antenna?
    (b) How does the power density of the propagating signal rolloff with distance.
    (c) If the separation of the transmit and receive antennas is 10 km, what is the path loss in decibels?
    (d) If the separation of the transmit and receive antennas is 10 km, what is the link loss in decibels?

23. A transmitter and receiver operating at 2 GHz are at the same level, but the direct path between them is blocked by a building and the signal must diffract over the building for a communication link to be established. This is a classic knife-edge diffraction situation. The transmit and receive antennas are each separated from the building by 4 km and the building is 20 m higher than the antennas (which are at the same height). Consider that the building is very thin. It has been found that the path loss can be determined by considering loss due to free-space propagation and loss due to diffraction over the knife edge.
    (a) What is the additional attenuation (in decibels) due to diffraction?
    (b) If the operating frequency is 100 MHz, what is the attenuation (in decibels) due to diffraction?
    (c) If the operating frequency is 10 GHz, what is the attenuation (in decibels) due to diffraction?

24. A hill is 1 km from a transmit antenna and 2 km from a receive antenna. The receive and transmit antennas are at the same height and the hill is 20 m above the height of the antennas. What is the additional loss caused by diffraction over the top of the hill? Treat the hill as causing knife-edge diffraction and the operating frequency is 1 GHz.

25. Two identical antennas are used in a point-to-point communication system, each having a gain of 30 dBi. The system has an operating frequency of 14 GHz and the antennas are at the top of masts 100 m tall. The RF link between the antennas consists only of the direct LOS path.
    (a) What is the effective aperture area of each antenna?
    (b) How does the power density of the propagating signal rolloff with distance?
    (c) If the separation of the transmit and receive antennas is 10 km, what is the path loss? Ignore atmospheric loss.

26. The three main cellular communication bands are centered around 450 MHz, 900 MHz, and 2 GHz. Compare these three bands in terms of

multipath effects, diffraction around buildings, object (such as a wall) penetration, scattering from trees and parts of trees, and ability to follow the curvature of hills. Complete the table below with the relative attributes: high, medium, and low.

Characteristic	450 MHz	900 MHz	2 GHz
Multipath			
Scattering			
Penetration			
Following curvature			
Range			
Antenna size			
Atmospheric loss			

27. Describe the difference in multipath effects in a central city area compared to multipath effects in a desert. Your description should be approximately 4 lines long and not use a diagram

28. Wireless LAN systems can operate at 2.4 GHz, 5.6 GHz, 40 GHz and 60 GHz. Contrast with explanation the performance of these schemes inside a building in terms of range.

29. At 60 GHz the atmosphere strongly attenuates a signal. Discuss the origin of this and indicate an advantage and a disadvantage.

30. Short answer questions. Each part requires a short paragraph of about five lines and a figure, where appropriate, to illustrate your understanding.
    (a) Cellular communications systems use two frequency bands to communicate between the basestation and the mobile unit. The bands are generally separated by 50 MHz or so. Which band (higher or lower) is used for the downlink from the basestation to the mobile unit and what are the reasons behind this choice?
    (b) Describe at least two types of interference in a cellular system from the perspective of a mobile handset.

31. The three main cellular communication bands are centered around 450 MHz, 900 MHz, and 2 GHz. Compare these three bands in terms of multipath effects, diffraction around buildings, object (such as a wall) penetration, scattering from trees and parts of trees, and the ability to follow the curvature of hills. Use a table and indicate the relative attributes: high, medium, and low.

32. Describe Rayleigh fading in approximately 4 lines and without using a diagram.

33. In several sentences and using a diagram describe Rayleigh fading and the impact it has on radio communications.

34. A transmitter and receiver operate at 100 MHz, are at the same level, and are separated by 4 km. The signal must diffract over a building half way between the antennas that is 20 m higher than the direct path between the antennas. What is the attenuation (in decibels) due to diffraction?

35. A transmitter and receiver operate at 10 GHz, are at the same level, and are 4 km apart. The signal must diffract over a building that is half way between the antennas and is 20 m higher than the line between the antennas. What is the attenuation (in dB) due to diffraction?

2.10.1 Exercises By Section

†challenging

§2.3 $1^{\dagger}$

§2.5 $2^{\dagger}, 3^{\dagger}, 4, 5, 6^{\dagger}, 7^{\dagger}, 8^{\dagger}, 19, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21$

§2.6 $22^{\dagger}, 23^{\dagger}, 24^{\dagger}, 25^{\dagger}, 27^{\dagger}, 28, 29, 30^{\dagger}, 31, 32, 33, 34, 35$

2.10.2 Answers to Selected Exercises

2(e) 53.23 dBm

16 0.251 μW

17 14.3 pW

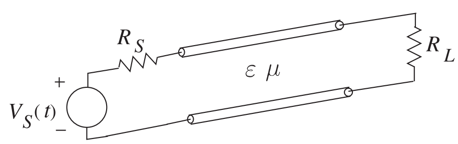
CHAPTER 3

Transmission Lines

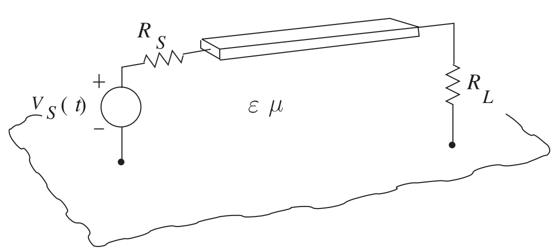
3.1 Introduction

A transmission line stores electric and magnetic energy. As such a line has a circuit form that combines inductors, Ls (for the magnetic energy), capacitors, Cs (for the electric energy), and resistors, Rs (modeling losses), whose values depend on the line geometry and material properties.

The transmission lines considered in this chapter are restricted to just two parallel conductors, as shown in Figure 3-1, with the distance between the two wires (i.e., in the transverse direction) being substantially smaller than the wavelengths of the signals on the line. The correct physical interpretation is that the conductors of a transmission line confine and guide an EM field. The EM field contains the energy of the signal and not the current on the line. However, with electrically small transverse dimensions, a two conductor line



(a) Two-wire line



(b) Strip-above-ground line

Figure 3-1: Two conductor transmission lines.

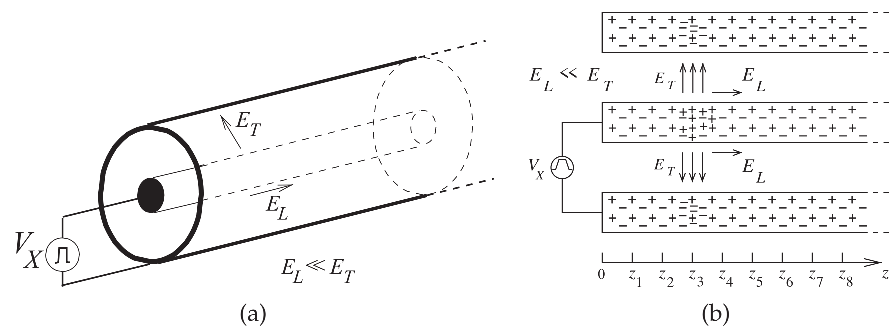


Figure 3-2: A coaxial transmission line: (a) three-dimensional view; (b) the line with pulsed voltage source showing the electric fields at an instant in time as a voltage pulse travels down the line.

may be satisfactorily analyzed on the basis of voltages and currents.

Frequency-domain analysis is the best way to understand transmission lines. This transmission line theory with modern developments is presented in Section 3.2 and useful formulas and concepts are developed in Section 3.3 for lossless transmission lines. Section 3.4 presents several configurations of lossless lines that are particularly useful in microwave circuit design and used in many places in this book series.

3.1.1 When Must a Line be Considered a Transmission Line

The key determinant of whether a transmission line can be considered as an invisible connection between two points is whether the signal anywhere along the interconnect has the same value at a particular instant. If the value of the signal (say, voltage) varies along the line (at an instant), then it may be necessary to consider transmission line effects. A typical criterion used is that if the length of the interconnect is less than 1/20th of the wavelength of the highest-frequency component of a signal, then transmission line effects can be safely ignored and the circuit can be modeled as a single RLC circuit [1].

3.1.2 Movement of a Signal on a Transmission Line

When a positive voltage pulse is applied to the center conductor of a coaxial line, as shown in Figure 3-2(a), an electric field results that is directed from the center conductor to the outer conductor. Referring to Figure 3-2(b), the component of the field that is directed along the shortest path from the center conductor to the outer conductor is denoted E_T, and the subscript T denotes the transverse component of the field as shown. Figure 3-2(b) shows the fields in the structure after the pulse has started moving along the line. This is shown in another view in Figure 3-3 at four different times. The transverse voltage, V_T, is given by E_T integrated along a path between the inner and outer conductors: $V_T \approx E_T(a - b)$. This is a good measure, provided that the transverse dimensions are small compared to a wavelength. The voltage pulse exciting the line has a trapezoidal shape and the voltage between the inner and outer conductor has the same form as E_T. As indicated by Maxwell's equations, a change in time of the electric field results in a spatial

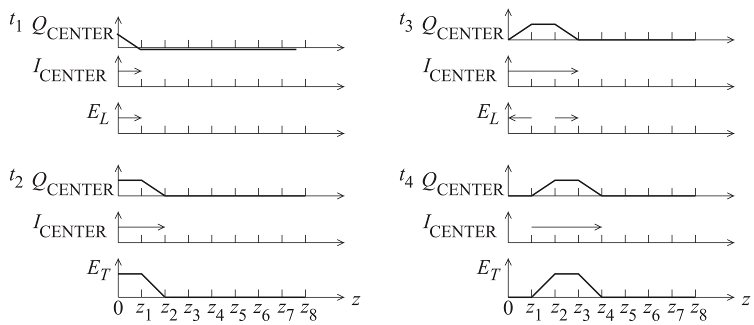


Figure 3-3: Fields, currents, and charges on the coaxial transmission line of Figure 3-2 at times $t_4 > t_3 > t_2 > t_1$. Q_{CENTER} is the net free charge on the center conductor. I_{CENTER} is the current on the center conductor.

change in the magnetic field and hence current. As a result there is a variation of the current in time and this results in a spatial change of the electric field. The chasing from a time variation to a spatial variation and then back to a time variation causes the pulse to move down the line.

The pulse moves down the line at the **group velocity**, which for a lossless coaxial line is the same as the **phase velocity**, v_p.[1] This is determined by the physical properties of the region between the conductors. The permittivity, ε, describes energy storage associated with the electric field, E, and the energy storage associated with the magnetic field, H, is described by the permeability, μ. It has been determined that

$$v_p = 1/\sqrt{\mu\varepsilon}. \tag{3.1}$$

In free space $v_p = c = 1/\sqrt{\mu_0\varepsilon_0}$ = 3×10^8 m/s. The free-space wavelengths, $\lambda_0 = c/f$, at several frequencies, f, are

f	100 MHz	1 GHz	10 GHz
λ_0	3 m	30 cm	3.0 cm

Here λ_0 is used to indicate the wavelength in free space and λ_g, the guide wavelength, is used to denote the wavelength on a transmission line.

EXAMPLE 3.1 Transmission Line Wavelength

A length of coaxial line is filled with a dielectric having a relative permittivity, ε_r, of 20 and is designed to be 1/4 wavelength long at a frequency, f, of 1.850 GHz. ($\mu_r = 1$)

(a) What is the free-space wavelength?
(b) What is the wavelength of the signal in the dielectric-filled coaxial line?
(c) How long is the line?

Solution:

(a) $\lambda_0 = c/f = 3 \times 10^8/1.85 \times 10^9 = 0.162$ m $= 16.2$ cm.
(b) Note that for a dielectric-filled line with $\mu_r = 1$, $\lambda = v_p/f = c/(\sqrt{\varepsilon_r} f) = \lambda_0/\sqrt{\varepsilon_r}$, so $\lambda = \lambda_0/\sqrt{\varepsilon_r} = 16.2$ cm$/\sqrt{20} = 3.62$ cm.
(c) $\lambda_g/4 = 3.62$ cm$/4 = 9.05$ mm.

[1] The phase velocity is the apparent velocity of a point of constant phase on a sinewave and is almost frequency independent for a low-loss coaxial line of small transverse dimensions (less than $\lambda/10$).

3.2 Transmission Line Theory

This section develops the theory of signal propagation on transmission lines. The first section, Section 3.2.1, makes the argument that a circuit with resistors, inductors, and capacitors is a good model for a transmission line. The development of transmission line theory is presented in Section 3.2.2. The dimensions of some of the quantities that appear in transmission line theory are discussed in Section 3.2.3. Section 3.2.4 summarizes the important parameters of a lossless line and then a particularly important line, the microstrip line, is considered in Section 3.2.5.

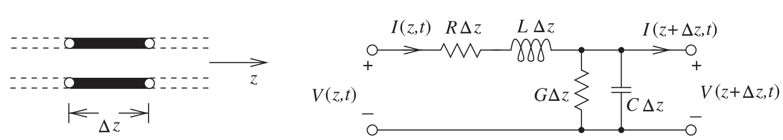
3.2.1 Transmission Line RLGC Model

Regardless of the actual structure, a segment of uniform transmission line (i.e., a line with constant cross section along its length) as shown in Figure 3-4(a) can be modeled by the circuit shown in Figure 3-4(b) with

Resistance along the line	$= R$	all specified per unit length.
Inductance along the line	$= L$	
Conductance shunting the line	$= G$	
Capacitance shunting the line	$= C$	

R, L, G, and C are referred to as resistance, inductance, conductance, and capacitance per unit length. (Sometimes p.u.l. is used as shorthand for **per unit length**.) In the metric system, ohms per meter (Ω/m), henries per meter (H/m), siemens per meter (S/m) and farads per meter (F/m), respectively, are used. The values of R, L, G, and C are affected by the geometry of the transmission line and by the electrical properties of the dielectrics and conductors. C describes the ability to store electrical energy and is mostly due to the properties of the dielectric. G describes loss in the dielectric which derives from conduction in the dielectric and from dielectric relaxation. Most microwave substrates have negligible conductivity so dielectric loss dominates. Dielectric relaxation loss results from the movement of charge centers which result in distortion of the dielectric lattice (if a crystal) or molecular structure. The periodic variation of the E field transfers energy from the EM field to mechanical vibrations. R is due to ohmic loss in the metal more than anything else. L describes the ability to store magnetic energy and is mostly a function of geometry, as most materials used with transmission lines have $\mu_r = 1$ (so no more magnetic energy is stored than in a vacuum).

For most lines the effects due to L and C dominate because of the relatively low series resistance and shunt conductance. The propagation characteristics of the line are described by its loss-free, or lossless, equivalent line, although in practice some information about R or G is necessary to determine power losses. The lossless concept is just a useful and good approximation.



(a) (b)

Figure 3-4: Transmission line segment: (a) of length Δz; and (b) lumped-element model.

3.2.2 Derivation of Transmission Line Properties

In this section the differential equations governing the propagation of signals on a transmission line are derived. These are coupled first-order differential equations and are akin to Maxwell's Equations in one dimension. Solution of the differential equations describes how signals propagate, and leads to the extraction of a few parameters that describe transmission line properties.

Applying **Kirchoff's laws** applied to the model in Figure 3-4(b) and taking the limit as $\Delta z \to 0$ the **transmission line equations**

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} \tag{3.2}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}. \tag{3.3}$$

In sinusoidal steady-state using cosine-based phasors these become

$V(z)$ is a phasor and $v(z,t) = \Re\{V(z)e^{\jmath\omega t}\}$. $\Re\{w\}$ denotes the real part of w, a complex number.

$$\frac{\mathrm{d}V(z)}{\mathrm{d}z} = -(R + \jmath\omega L)I(z) \quad (3.4) \quad \text{and} \quad \frac{\mathrm{d}I(z)}{\mathrm{d}z} = -(G + \jmath\omega C)V(z). \quad (3.5)$$

Eliminating $I(z)$ in the above yields the wave equation for $V(z)$:

$$\frac{\mathrm{d}^2V(z)}{\mathrm{d}z^2} - \gamma^2 V(z) = 0. \quad (3.6) \quad \text{Similarly} \quad \frac{\mathrm{d}^2I(z)}{\mathrm{d}z^2} - \gamma^2 I(z) = 0, \quad (3.7)$$

where the propagation constant is

$$\gamma = \alpha + \jmath\beta = \sqrt{(R + \jmath\omega L)(G + \jmath\omega C)}, \tag{3.8}$$

with SI units of m^{-1} and where α is the attenuation coefficient and has units of nepers per meter (Np/m), and β is the phase-change coefficient, or phase constant, and has units of radians per meter (rad/m or radians/m). Nepers and radians are dimensionless units, but serve as prompts for what is being referred to.

Equations (3.6) and (3.7) are second-order differential equations that have solutions of the form

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (3.9) \quad \text{and} \quad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}. \quad (3.10)$$

The physical interpretation of these solutions is that $V^+(z) = V_0^+ e^{-\gamma z}$ and $I^+(z) = I_0^+ e^{-\gamma z}$ are forward-traveling waves (moving in the $+z$ direction) and $V^-(z) = V_0^- e^{\gamma z}$ and $I^-(z) = I_0^- e^{\gamma z}$ are backward-traveling waves (moving in the $-z$ direction). $V(z)$, V_0^+, V_0^-, $I(z)$, I_0^+ and I_0^- are all phasors. Substituting Equation (3.9) in Equation (3.4) results in

$$I(z) = \frac{\gamma}{R + \jmath\omega L}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]. \tag{3.11}$$

Then from Equations (3.11) and (3.10)

$$I_0^+ = \frac{\gamma}{R + \jmath\omega L}V_0^+ \quad \text{and} \quad I_0^- = \frac{\gamma}{R + \jmath\omega L}(-V_o^-). \tag{3.12}$$

The characteristic impedance is defined as

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{R + \jmath\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \tag{3.13}$$

with the SI unit of ohms (Ω). Equations (3.9) and (3.10) can be rewritten as

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (3.14) \quad \text{and} \quad I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}. \quad (3.15)$$

Converting back to the time domain:

$$v(z,t) = \left|V_0^+\right| \cos(\omega t - \beta z + \varphi^+) e^{-\alpha z} + \left|V_0^-\right| \cos(\omega t + \beta z + \varphi^-) e^{\alpha z}, \quad (3.16)$$

where φ^+ and φ^- are phases of the forward- and backward-traveling waves, respectively. The phasors of the traveling voltage waves are

$$V_0^+(z) = \left|V_0^+\right| e^{\jmath\varphi^+} e^{-\jmath\beta z} \quad \text{and} \quad V_0^-(z) = \left|V_0^-\right| e^{\jmath\varphi^-} e^{\jmath\beta z}. \quad (3.17)$$

The following quantities are defined:

Characteristic impedance: $Z_0 = \sqrt{(R + \jmath\omega L)/(G + \jmath\omega C)}$ (3.18)

Propagation constant: $\gamma = \sqrt{(R + \jmath\omega L)(G + \jmath\omega C)}$ (3.19)

Attenuation constant: $\alpha = \Re\{\gamma\}$ (3.20)

Phase constant: $\beta = \Im\{\gamma\}$ (3.21)

Wavenumber: $k = -\jmath\gamma$ (3.22)

Phase velocity: $v_p = \omega/\beta$ (3.23)

Wavelength: $\lambda = \frac{2\pi}{|\gamma|} = \frac{2\pi}{|k|}$, (3.24)

where $\omega = 2\pi f$ is the radian frequency and f is the frequency with the SI units of hertz (Hz). The wavenumber k as defined here is used in electromagnetics and where wave propagation is concerned. Considering one of the traveling waves, the phase velocity refers to the apparent velocity of which a point of constant phase on the sinewave appears to move.

The important result here is that a voltage wave (and a current wave) can be defined on a transmission line. One more parameter needs to be introduced: the group velocity,

$$v_g = \frac{\partial \omega}{\partial \beta}. \quad (3.25)$$

The group velocity is the velocity of a modulated waveform's envelope and describes how fast information propagates. It is the velocity at which the energy (i.e. information) in the waveform moves. Thus group velocity can never be more than the speed of light in a vacuum, c. Phase velocity, however, can be more than c. If the speed at which information moves varies with frequency, then a signal such as a pulse will spread out. Such a line is said to have dispersion. For a lossless, dispersionless line, the group and phase velocity are the same. If the phase velocity is frequency independent, then β is linearly proportional to ω.

Electrical length is used in designs prior to establishing the physical length of a line. The electrical length is expressed either as a fraction of a wavelength or in degrees (or radians), where a wavelength corresponds to 360° (or 2π radians). If ℓ is its physical length, the electrical length of the line in radians is $\beta\ell$.

EXAMPLE 3.2 Physical and Electrical Length

A transmission line is 10 cm long and at the operating frequency the phase constant β is 30 rad/m. What is the electrical length of the line?

Solution:

The physical length of the line is ℓ = 10 cm = 0.1 m. Then the electrical length of the line is $\ell_e = \beta\ell = (30 \text{ rad/m}) \times 0.1$ m = 3 radians. The electrical length can also be expressed in terms of wavelength noting that 360° corresponds to 2π radians, which also corresponds to λ. Thus $\ell_e = (3 \text{ radians}) = 3 \times 360/(2\pi)$ = 171.9° or $\ell_e = 3/(2\pi)\ \lambda = 0.477\ \lambda$.

EXAMPLE 3.3 *RLGC* Parameters

A transmission line has the $RLGC$ parameters $R = 100\ \Omega$/m, $L = 80$ nH/m, $G = 1.6$ S/m, and $C = 200$ pF/m. Consider a traveling wave at 2 GHz on the line.

(a) What is the attenuation constant?
(b) What is the phase constant?
(c) What is the phase velocity?
(d) What is the characteristic impedance of the line?
(e) What is the group velocity?

Solution:

(a) α: $\gamma = \alpha + \jmath\beta = \sqrt{(R + \jmath\omega L)(G + \jmath\omega C)}$; $\omega = 12.57 \cdot 10^9$ rad/s

$$\gamma = \sqrt{(100 + \jmath\omega \cdot 80 \cdot 10^{-9})(1.6 + \jmath\omega 200 \times 10^{-12})} = (17.94 + \jmath 51.85)\ \text{m}^{-1}$$
$$\alpha = \Re\{\gamma\} = 17.94\ \text{Np/m}$$

(b) Phase constant: $\beta = \Im\{\gamma\} = 51.85$ rad/m

(c) Phase velocity:
$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{12.57 \times 10^9 \text{rad} \cdot \text{s}^{-1}}{51.85\ \text{rad} \cdot \text{m}^{-1}} = 2.42 \times 10^8\ \text{m/s}$$

(d) $Z_0 = (R + \jmath\omega L)/\gamma = (100 + \jmath\omega \cdot 80 \cdot 10^{-9})/(17.94 + \jmath 51.85) = (17.9 + \jmath 4.3)\ \Omega$
Note also that $Z_0 = \sqrt{(R + \jmath\omega L)/(G + j\omega C)}$, which yields the same answer.

(e) Group velocity:
$$v_g = \left.\frac{\partial\omega}{\partial\beta}\right|_{f=2\ \text{GHz}}$$

Numerical derivatives will be used, thus $v_g = \Delta\omega/\Delta\beta$. Now β is already known at 2 GHz. At 1.9 GHz, $\gamma = 17.884 + \jmath 49.397\ \text{m}^{-1}$, and so $\beta = 49.397$ rad/m.

$$v_g = \frac{2\pi(2\ \text{GHz} - 1.9\ \text{GHz})}{\beta(2\ \text{GHz}) - \beta(1.9\ \text{GHz})} = \frac{2\pi(2 - 1.9)10^9\ \text{Hz}}{(51.85 - 49.397)\ \text{m}^{-1}} = 2.563 \times 10^8\ \text{m/s}.$$

(Note that Hz = s^{-1}. Note that $v_g \neq v_p$, and so the transmission line has dispersion.)

3.2.3 Dimensions of γ, α, and β

The SI unit of γ are inverse meters (m^{-1}) and the attenuation constant, α, and the phase constant, β, have, strictly speaking, the same units. However, the convention is to introduce the dimensionless quantities Neper and radian to convey additional information. Thus the attenuation constant α has the units of Nepers per meter (Np/m) and the phase constant β has the units radians per meter (rad/m). The unit Neper comes from the name of the e (= 2.7182818284590452354. . .) symbol (written in upright font and

The name for e derives from John **Napier**, who developed the theory of logarithms [2]. e is sometimes called Euler's constant.

not italics since it is a constant), which is called the Neper. The Neper is used in calculating transmission line signal levels, as in Equations (3.9) and (3.10). The attenuation and phase constants are often separated and then the attenuation constant describes the decrease in signal amplitude as the signal travels down a transmission line. So when $\alpha\ell$ = 1 Np, where ℓ is the length of the line, the signal has decreased to $1/\mathrm{e}$ of its original value, and the power drops to $1/\mathrm{e}^2$ of its original value. The decrease in signal level represents loss and the units of decibels per meter (dB/m) are used with 1 Np = $20\log\mathrm{e}$ = 8.6858896381 dB. So expressing α as 1 Np/m is the same as saying that the attenuation loss is 8.6859 dB/m. To convert from dB to Np multiply by 0.1151. Thus α = x dB/m = $x \times 0.1151$ Np/m.

In engineering $\log x \equiv \log_{10} x$ and $\ln x \equiv \log_{\mathrm{e}} x$.

EXAMPLE 3.4 Transmission Line Characteristics

A line has an attenuation of 10 dB/m and a phase constant of 50 radians/m at 2 GHz.

(a) What is the complex propagation constant of the transmission line?
(b) If the capacitance of the line is 100 pF/m and the conductive loss is zero (i.e., $G = 0$), what is the characteristic impedance of the line?

Solution:

(a) $\alpha|_{\mathrm{Np}} = 0.1151 \times \alpha|_{\mathrm{dB}} =$ 0.1151$\times$ (10 dB/m) = 1.151 Np/m, $\beta = 50$ rad/m
Propagation constant, $\gamma = \alpha + \jmath\beta = (1.151 + \jmath 50)\ \mathrm{m}^{-1}$

(b) $\gamma = \sqrt{(R + \jmath\omega L)\,(G + \jmath\omega C)}$, and $Z_0 = \sqrt{(R + \jmath\omega L)/(G + \jmath\omega C)}$,
therefore $Z_0 = \gamma/(G + \jmath\omega C)$; $\omega = 2\pi \cdot 2 \times 10^9\ s^{-1}$; $G = 0$; $C = 100 \times 10^{-12}$ F,
so $Z_0 = 39.8 - \jmath 0.916\ \Omega$.

3.2.4 Lossless Transmission Line

If the conductor and dielectric are ideal (i.e., lossless), then $R = 0 = G$ and the equations for the transmission line characteristics simplify. The transmission line parameters from Equations (3.13) and (3.19)–(3.24) are then

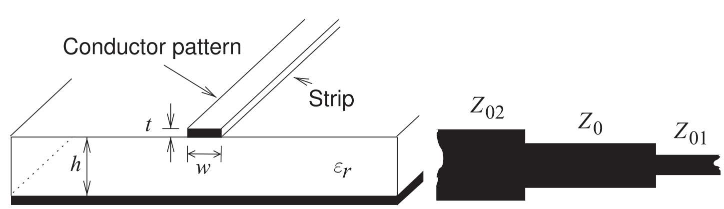
$$Z_0 = \sqrt{\frac{L}{C}} \quad (3.26) \qquad \alpha = 0 \quad (3.27)$$

$$\beta = \omega\sqrt{LC} \quad (3.28) \qquad v_p = 1/\sqrt{LC} \quad (3.29)$$

$$\lambda_g = \frac{2\pi}{\omega\sqrt{LC}} = \frac{v_p}{f}. \quad (3.30)$$

3.2.5 Microstrip Line

A microstrip line is shown in Figure 3-5(a). This is a commonly used transmission line, as it can be cheaply fabricated using printed circuit board techniques. This line consists of a metal-backed substrate of relative permittivity ε_r on top of which is a metal strip. Above that is air. The



(a) Microstrip (b) Layout of microstrip lines

Figure 3-5: Microstrip transmission line. The layout (or top) view is commonly used with circuit designs using microstrip. This is the pattern of the strip where (b) shows three lines of different width.

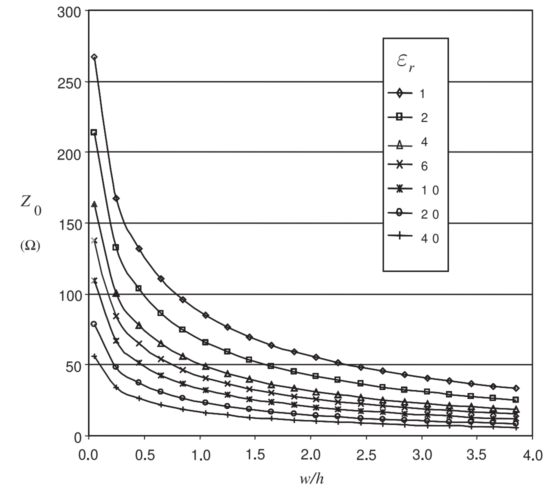


Figure 3-6: Dependence of Z_0 of a microstrip line at 1 GHz for various ε_r and aspect (w/h) ratios. Calculated using EM simulation.

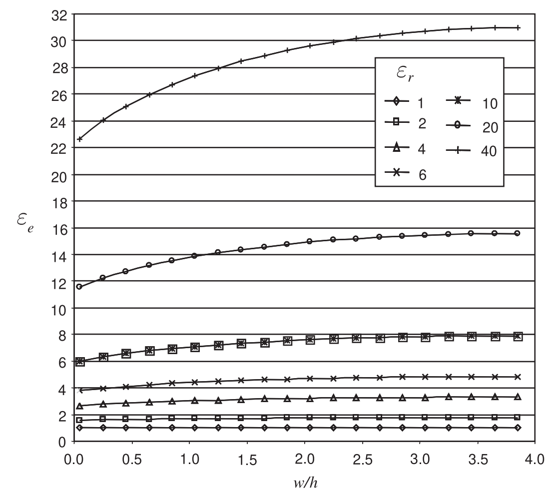


Figure 3-7: Dependence of effective relative permittivity ε_e of a microstrip line at 1 GHz for various permittivities and aspect ratios (w/h).

width of the strip determines the characteristic impedance of the line. The characteristic impedance of microstrip lines having various strip widths is shown in Figure 3-6 for several substrate permittivities. So the wider the strip and the higher the substrate permittivity, the lower the characteristic impedance of the line. The EM fields are partly in air and partly in the dielectric and an effective permittivity must be used when calculating the electrical length of the line. The results of field simulations of the effective permittivity of lines of various widths and with various substrate permittivities are shown in Figure 3-7, where it can be seen that the effective relative permittivity, ε_e, increases for wide strips. This is because more of the EM field is in the substrate. Microstrip transmission line structures are often drawn showing just the layout of the strip, as shown in Figure 3-5(b), where the three lines have different characteristic impedances. The next chapter presents detailed analyses of microstrip and other planar transmission lines.

3.2.6 Summary

The important takeaway from this section is that a signal moves on a transmission line as forward- and backward-traveling waves. The energy transferred is in the traveling waves. The total voltage and current at a point on the line is the sum of the traveling voltage and current waves, respectively, but the total voltage/current view is not sufficient to describe how a transmission line works. Transmission line theory is developed in terms of traveling voltages and current waves and these are akin to a one-dimensional form of Maxwell's equations.

3.3 The Lossless Terminated Line

Microwave engineers want to work with total voltage and current when possible and the art of design synthesis usually requires relating the total voltage and current world of a lumped element circuit to the traveling voltage world of transmission lines. This section develops the important abstractions that enable the total voltage and current view of the world to be used with transmission lines. The first step is in Section 3.3.1 where total voltages and currents are related to forward- and backward-traveling voltages and currents. Insight into traveling waves and reflections is presented in Section 3.3.2. Important abstractions are presented first for the input reflection coefficient of a terminated lossless line in Section 3.3.3 and then for the input impedance of the line in Section 3.3.4. The last section, Section 3.3.5, presents a view of the total voltage on the transmission line and describes the voltage standing wave concept.

3.3.1 Total Voltage and Current on the Line

Consider the terminated line shown in Figure 3-8(a). Assume an incident or forward-traveling wave, with traveling voltage $V_0^+\,\mathrm{e}^{-\jmath\beta z}$ and current $I_0^+\,\mathrm{e}^{-\jmath\beta z}$ propagating toward the load Z_L at $z = 0$. The characteristic impedance of the transmission line is the ratio of the voltage and current traveling waves so that

$$\frac{V_0^+(z)}{I_0^+(z)} = \frac{V_0^+\,\mathrm{e}^{-\jmath\beta z}}{I_0^+\,\mathrm{e}^{-\jmath\beta z}} = \frac{V_0^+(0)}{I_0^+(0)} = \frac{V_0^+}{I_0^+} = Z_0. \tag{3.31}$$

The reflected wave has a similar relationship (but note the sign change):

$$\frac{V_0^-\,\mathrm{e}^{\jmath\beta z}}{-I_0^-\,\mathrm{e}^{\jmath\beta z}} = \frac{V_0^-}{-I_0^-} = Z_0. \tag{3.32}$$

The load Z_L imposes an additional constraint on the relationship of the total voltage and current at $z = 0$:

$$\frac{V_L}{I_L} = \frac{V(z=0)}{I(z=0)} = Z_L. \tag{3.33}$$

When $Z_L \neq Z_0$ there must be a reflected wave with appropriate amplitude to satisfy the above equations. Now the total voltage

$$V(z) = V_0^+\,\mathrm{e}^{-\jmath\beta z} + V_0^-\,\mathrm{e}^{\jmath\beta z}, \tag{3.34}$$

and the total current, $I(z)$, is related to the traveling current waves by

$$I(z) = \frac{V_0^+}{Z_0}\,\mathrm{e}^{-\jmath\beta z} - \frac{V_0^-}{Z_0}\,\mathrm{e}^{\jmath\beta z} = I_0^+\mathrm{e}^{-\jmath\beta z} + I_0^-\,\mathrm{e}^{\jmath\beta z}. \tag{3.35}$$

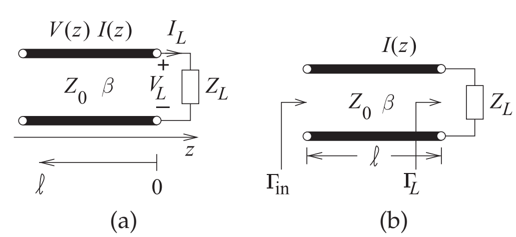


Figure 3-8: A terminated transmission line.

Thus at the termination of the line $(z = 0)$,

$$\frac{V(0)}{I(0)} = Z_L = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}.$$

This can be rearranged as the ratio of the reflected voltage to the incident voltage:

$$\frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

This ratio is defined as the voltage **reflection coefficient** at the load,

$$\Gamma_L = \Gamma_L^V = \frac{V_0^-(0)}{V_0^+(0)} = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}. \tag{3.36}$$

That is, at the load

$$V_0^- = \Gamma_L V_0^+. \tag{3.37}$$

The relationship in Equation (3.36) can be rewritten so that the input load impedance can be obtained from the reflection coefficient:

$$Z_L = Z_0 \frac{1 + \Gamma^V}{1 - \Gamma^V}. \tag{3.38}$$

Similarly, the current reflection coefficient can be written as

$$\Gamma^I = \frac{I_0^-}{I_0^+} = \frac{-Z_L + Z_0}{Z_L + Z_0} = -\Gamma^V. \tag{3.39}$$

The voltage reflection coefficient is used most of the time, so the reflection coefficient, Γ, on its own refers to the voltage reflection coefficient, $\Gamma^V = \Gamma$.

EXAMPLE 3.5 Forward- and Backward-Traveling Waves at an Open Circuit

A lossless transmission line is terminated in an open circuit. What is the relationship between the forward- and backward-traveling voltage waves at the end of the line?

Solution:

At the end of the line the total current is zero, so that $I^+ + I^- = 0$ and so

$$I^- = -I^+. \tag{3.40}$$

The forward- and backward traveling voltages and currents are related to the characteristic impedance by

$$Z_0 = V^+/I^+ = -V^-/I^-, \tag{3.41}$$

Note the change in sign, as a result of the direction of propagation changing but the positive reference for current is in the same direction. Substituting for I^- at the termination,

$$V^+ = -V^- I^+/I^- = -V^- I^+/(-I^+) = V^-. \tag{3.42}$$

Thus the total voltage at the end of the line, V_{TOTAL}, is $V^+ + V^- = 2V^+$. Note that the total voltage at the end of the line is twice the incident (forward-traveling) voltage.

EXAMPLE 3.6 Current Reflection Coefficient

A load consists of a shunt connection of a capacitor of 10 pF and a resistor of 60 Ω. The load terminates a lossless 50 Ω transmission line. The operating frequency is 5 GHz.

(a) What is the impedance of the load?
(b) What is the normalized impedance of the load (normalized to Z_0 of the line)?
(c) What is the reflection coefficient of the load?
(d) What is the current reflection coefficient of the load?

Solution:

(a) $C = 10 \cdot 10^{-12}$ F; $R = 60\ \Omega$; $f = 5 \cdot 10^9$ Hz; $\omega = 2\pi f$; $Z_0 = 50\ \Omega$

$$Z_L = R||C = (1/R + \jmath\omega C)^{-1} = 0.168 - \jmath 3.174\ \Omega.$$

(b) $z_L = Z_L/Z_0 = 3.368 \cdot 10^{-3} - \jmath 0.063$.
(c) This is the voltage reflection coefficient. $\Gamma_L = (z_L - 1)/(z_L + 1) = -0.985 - \jmath 0.126 = 0.993\angle 187.3^\circ$.
(d) $\Gamma_L^I = -\Gamma_L = 0.985 + \jmath 0.126 = 0.993\angle(187.3 - 180)^\circ = 0.993\angle 7.3^\circ$.

3.3.2 Forward- and Backward-Traveling Pulses

Reflections at the end of a line produce a backward-traveling signal. Forward- and backward-traveling pulses are shown in Figure 3-9(a) for the situation where the resistance at the end of the line is lower than the

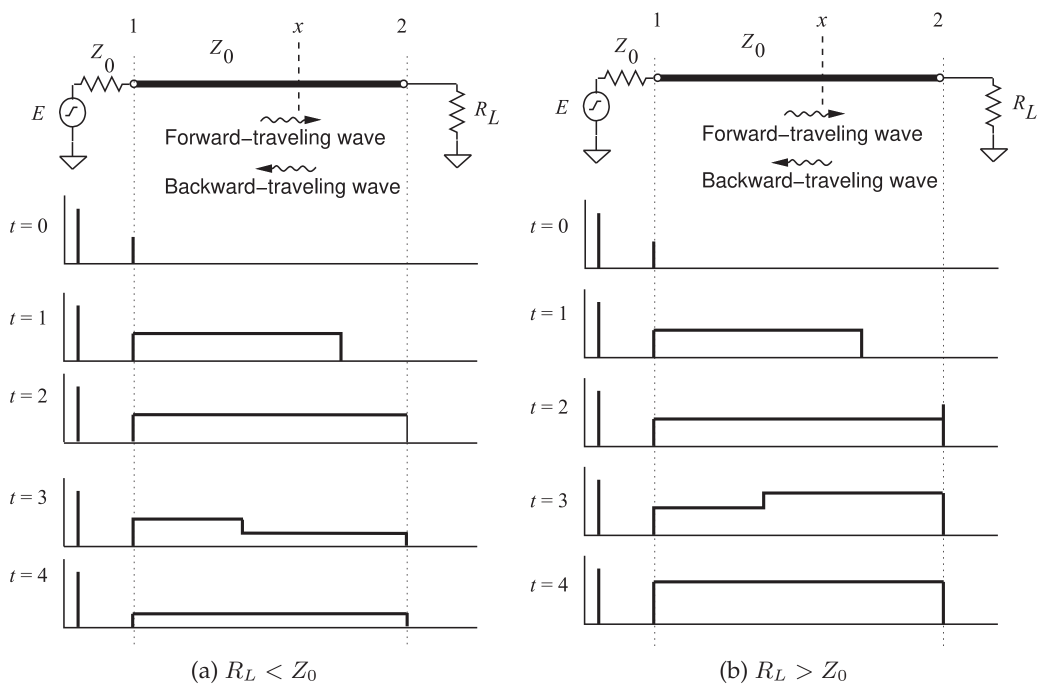


Figure 3-9: Reflection of a voltage pulse from a load: (a) when the resistance of the load, R_L is lower than the characteristic impedance of the line, Z_0; and (b) when R_L is greater than Z_0.

characteristic impedance of the line ($Z_L < Z_0$). The voltage source is a step voltage that is zero for time $t < 0$. At time $t = 0$, the step is applied to the line and it begins traveling down the line, as shown at time $t = 1$. This voltage step moving from left to right is called the forward-traveling voltage wave.

At time $t = 2$, the leading edge of the step reaches the load, and as the load has lower resistance than the characteristic impedance of the line, the total voltage across the load drops below the level of the forward-traveling voltage step. The reflected wave is called the backward-traveling wave and it must be negative, as it adds to the forward-traveling wave to yield the total voltage. Thus the voltage reflection coefficient, Γ, is negative and the total voltage on the line, which is all that can be directly observed, drops. A reflected, smaller, and opposite step signal travels in the backward direction and adds to the forward-traveling step to produce the waveform shown at $t = 3$. The impedance of the source matches the transmission line impedance so that the reflection at the source is zero. The signal on the line at time $t = 4$, the time for round-trip propagation on the line, therefore remains at the lower value. The easiest way to remember the polarity of the reflected pulse is to consider the situation with a short-circuit at the load. Then the total voltage on the line at the load must be zero. The only way this can occur when a signal is incident is if the reflected signal is equal in magnitude but opposite in sign, in this case $\Gamma = -1$. So whenever $|Z_L| < |Z_0|$, the reflected pulse will tend to subtract from the incident pulse.

The opposite situation occurs when the resistance at the load is higher than the characteristic impedance of the line (Figure 3-9(b)). In this case the reflected pulse has the same polarity as the incident signal. Again, to remember this, think of the open-circuited case. The voltage across the load doubles, as the reflected pulse has the same sign as well as magnitude as that of the incident signal, in this case $\Gamma = +1$. This is required so that the total current is zero.

A more illustrative situation is shown in Figure 3-10, where a more complicated signal is incident on a load that has a resistance higher than that of the characteristic impedance of the line. The peaking of the voltage that results at the load is typically the design objective in many long digital interconnects, as less overall signal energy needs to be transmitted down the line, or equivalently a lower current drive capability of the source is required to achieve first incidence switching. This is at the price of having reflected signals on the interconnects, but these are dissipated through a combination of line loss and absorption of the reflected signal at the driver.

3.3.3 Input Reflection Coefficient of a Lossless Line

The reflection coefficient looking into a line varies with position along the line as the forward- and backward-traveling waves change in relative phase. Referring to Figure 3-11, at a distance ℓ from the load (i.e., $z = -\ell$), the input reflection looking into a terminated lossless line is

$$\Gamma_{\text{in}}|_{z=-\ell} = \frac{V^-(z=-\ell)}{V^+(z=-\ell)} = \frac{V^-(z=0)\text{e}^{-\jmath\beta\ell}}{V^+(z=0)\text{e}^{+\jmath\beta\ell}} = \frac{V^-(z=0)}{V^+(z=0)}\frac{\text{e}^{-\jmath\beta\ell}}{\text{e}^{+\jmath\beta\ell}} = \Gamma_L\text{e}^{-\jmath 2\beta\ell} \tag{3.43}$$

Note that Γ_{in} has the same magnitude as Γ_L but rotates in the clockwise direction (becomes increasingly negative) at twice the rate of increase of the

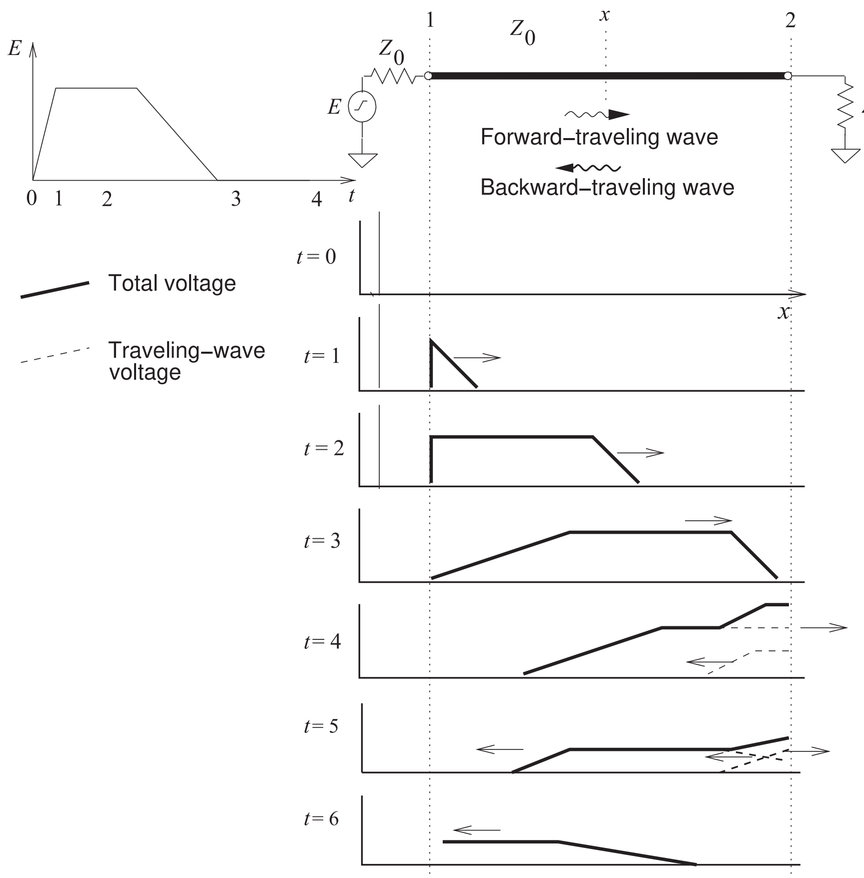


Figure 3-10: Reflection of a pulse from a termination with $Z_L > Z_0$. Z_L and Z_0 are real.

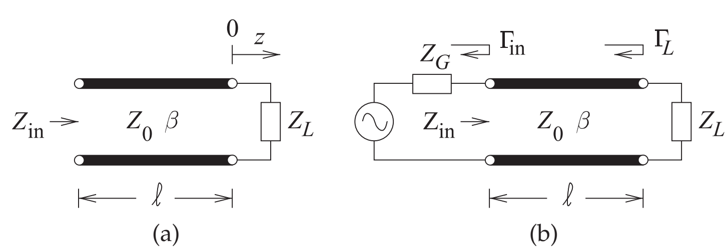


Figure 3-11: Terminated transmission line: (a) a transmission line terminated in a load impedance, Z_L, with an input impedance of Z_{in}; and (b) a transmission line with source impedance Z_G and load Z_L.

electrical length $\beta\ell$.

It is important to graphical concepts introduced later that there be a full appreciation for the angle of Γ_{in} becoming increasingly negative at twice the rate at which the electrical length of the line increases. Figure 3-12 is a way of visualizing this. The transmission line here is $\lambda/4$ long with an electrical length of 90° and is terminated in a load with reflection coefficient $\Gamma_L = +1$. At position $z = 0$ the forward-traveling voltage wave is $v^+(t, 0) = |V^+|\cos(\omega t)$, and this then propagates down the line in the $+z$ direction. The forward-traveling voltage at point $z = \lambda/8$ at $t = 0$ will be the same as the voltage at $z = 0$ at a time one-eighth of a period in the past. The voltage at $z =$

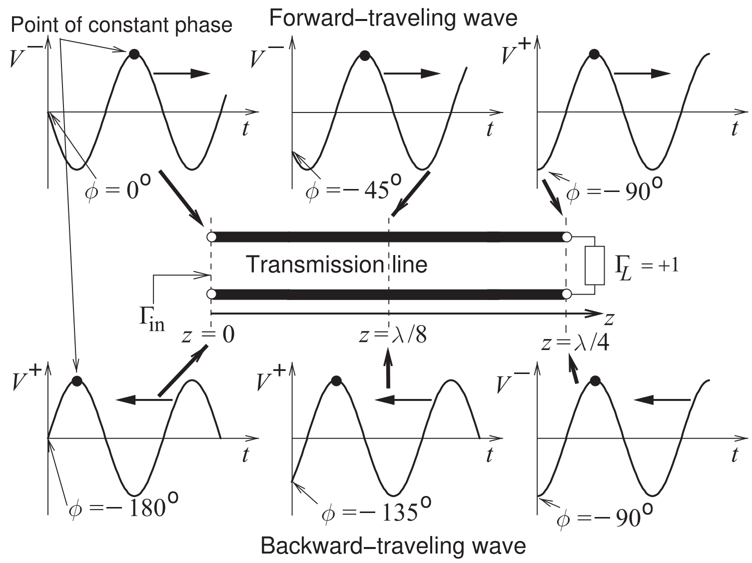


Figure 3-12: The forward-traveling wave $v^+(t,z) = |V^+|\cos(\omega t - \beta z) = |V^+|\cos(\omega t + \phi(z))$ and the backward-traveling wave $v^-(t,z) = |V^+|\cos(\omega t + \beta z) = |V^+|\cos[\omega t + \phi(z)]$. The phase, ϕ, of the forward-traveling wave becomes increasingly negative along the line as z increases, and when reflected the phase ϕ of the backward-traveling wave becomes increasingly negative as the wave moves away from the load (i.e. as z decreases).

$\lambda/8$ is $v^+(t,\lambda/8) = |V^+|\cos(\omega t - 2\pi/8)$, i.e. there is a phase rotation of $-45°$. Then at $z = \lambda/4$, $v^+(t,\lambda/4) = |V^+|\cos(\omega t - 2\pi/4)$, i.e. at time $t = 0$ there is a phase rotation of $-90°$ relative to $v^+(0,0)$, and this is the negative of the electrical length of the line. The voltage wave reflects at the load and becomes a backward-traveling wave. Here $\Gamma_L = +1$ and so, at the load, the phase of the backward- and forward-traveling waves are the same. The backward-traveling wave continues to travel in the $-z$ direction and its phase at $t = 0$ becomes increasingly negative as z gets closer to the input of the line. The phase of the backward-traveling wave at $z = 0$ is rotated $-90°$ with respect to the backward-traveling wave at the load, and has rotated $-180°$ relative to the forward-traveling wave at $z = 0$. For a lossless line, in general, the angle of $\Gamma_{\rm in}$ = [phase of $V^-(z=0)$ relative to the phase of $V^+(z=0)$] + (the phase of Γ_L) = −2(electrical length of the line) + (the phase of Γ_L).

3.3.4 Input Impedance of a Lossless Line

The impedance looking into a lossless line varies with position, as the forward- and backward-traveling waves combine to yield position-dependent total voltage and current. At a distance ℓ from the load (i.e., $z = -\ell$), the input impedance seen looking toward the load is

$$Z_{\rm in}|_{z=-\ell} = \frac{V(z=-\ell)}{I(z=-\ell)} = Z_0 \frac{1 + |\Gamma|\,e^{\jmath(\Theta - 2\beta\ell)}}{1 - |\Gamma|\,e^{\jmath(\Theta - 2\beta\ell)}} = Z_0 \frac{1 + \Gamma_L e^{\jmath(-2\beta\ell)}}{1 - \Gamma_L e^{\jmath(-2\beta\ell)}}. \quad (3.44)$$

Another form is obtained by substituting Equation (3.36) in Equation (3.44):

$$Z_{\rm in} = Z_0 \frac{(Z_L + Z_0)e^{\jmath\beta\ell} + (Z_L - Z_0)e^{-\jmath\beta\ell}}{(Z_L + Z_0)e^{\jmath\beta\ell} - (Z_L - Z_0)e^{-\jmath\beta\ell}} = Z_0 \frac{Z_L\cos(\beta\ell) + \jmath Z_0\cos(\beta\ell)}{Z_0\cos(\beta\ell) + \jmath Z_L\cos(\beta\ell)}$$
$$= Z_0 \frac{Z_L + \jmath Z_0 \tan\beta\ell}{Z_0 + \jmath Z_L \tan\beta\ell}. \quad (3.45)$$

This is the **lossless telegrapher's equation**. The electrical length, $\beta\ell$, is in radians when used in calculations.

3.3.5 Standing Waves and Voltage Standing Wave Ratio

The total voltage on a terminated line is the sum of forward- and backward-traveling waves. This sum produces what is called a standing wave. Figure 3-13 shows the total and traveling waveforms on a line terminated in a reactance and evaluated at times equal to multiples of an eighth of a period. Here the traveling waves have the same amplitude indicating that the termination of the line is reactive, $|\Gamma| = 1$. The interesting property here is that the total voltage appears as a standing wave with fixed points called nodes where the total voltage is always zero. This is more easily seen in Figure 3-14(a), where the total voltage is overlaid for many times. If the termination has resistance, then the magnitude of the backward-traveling wave will be less than that of the forward-traveling wave and the overlaid

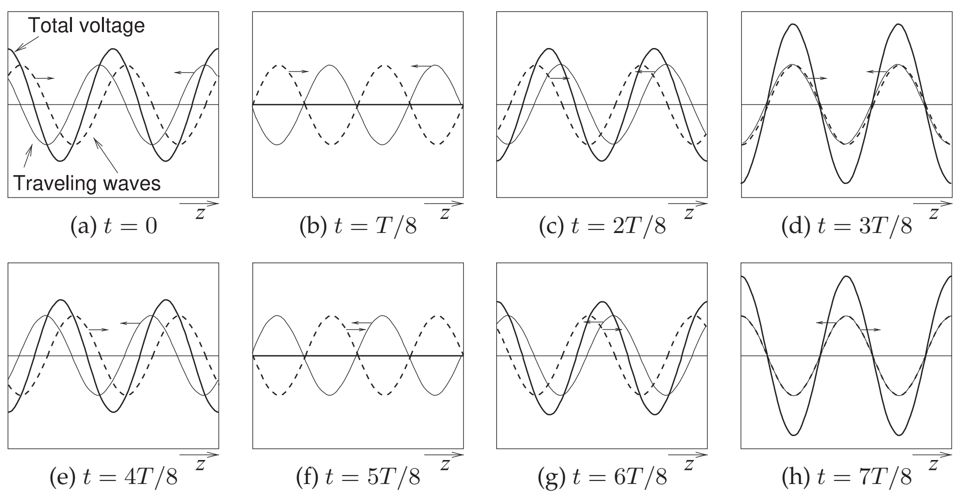


Figure 3-13: Evolution of a standing wave as the sum of forward- and backward-traveling waves (to the right and left, respectively) of equal amplitude evaluated at times t equal to eighths of the period T. At $t = T/8$ and $t = 5T/8$ the total voltage everywhere on the line is zero.

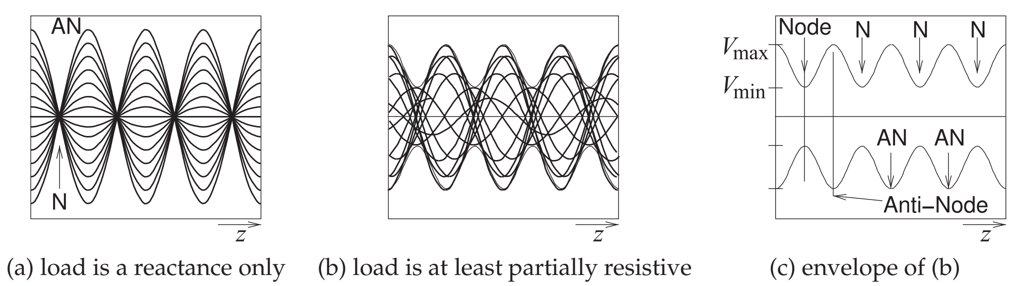


Figure 3-14: Standing waves as an overlay of waveforms at many times: (a) when the forward- and backward-traveling waves have the same amplitude; (b) when the waves have different amplitudes; and (c) the envelope of the standing wave. N is a node (a minimum) and AN is an antinode (a maximum). Nodes, N, are separated by $\lambda/2$. Antinodes, AN, are separated by $\lambda/2$.

total voltage is as shown in Figure 3-14(b). This is still a standing wave, but the minima are now not zero. The envelope of the standing wave is shown in Figure 3-14(c), where there is a maximum amplitude $V_{\max}$ and a minimum amplitude $V_{\min}$.

Now this situation will be examined mathematically to relate the standing wave to the reflection coefficient. If $\Gamma = 0$, then the magnitude of the total voltage on the line, $|V(z)|$, is equal to $\left|V_0^+\right|$ anywhere on the line. For this reason, such a line is said to be "flat." If there is reflection the magnitude of the total voltage on the line is not constant (see Figure 3-14(b)). Thus from Equation (3.43) :

$$|V(z)| = \left|V_0^+\right|\left|1 + \Gamma e^{2\jmath\beta z}\right| = \left|V_0^+\right|\left|1 + \Gamma e^{-2\jmath\beta\ell}\right|, \tag{3.46}$$

where $z = -\ell$ is the positive distance measured from the load at $z = 0$ toward the generator. Or, setting $\Gamma = |\Gamma|e^{\jmath\Theta}$,

$$|V(z)| = \left|V_0^+\right|\left|1 + |\Gamma|e^{\jmath(\Theta - 2\beta\ell)}\right|, \tag{3.47}$$

where Θ is the phase of the reflection coefficient ($\Gamma = |\Gamma|e^{\jmath\Theta}$) at the load. This result shows that the voltage magnitude oscillates with position z along the line. The maximum value occurs when $e^{\jmath(\Theta - 2\beta\ell)} = 1$ and is given by

$$V_{\max} = \left|V_0^+\right|(1 + |\Gamma|). \tag{3.48}$$

Similarly the minimum value of the total voltage magnitude occurs when the phase term is $e^{\jmath(\Theta - 2\beta l)} = -1$, and is given by

$$V_{\min} = \left|V_0^+\right|(1 - |\Gamma|). \tag{3.49}$$

A mismatch can be defined by the **voltage standing wave ratio** (**VSWR**):

$$\text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{(1 + |\Gamma|)}{(1 - |\Gamma|)}. \tag{3.50}$$

$$\text{Also} \quad |\Gamma| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1}. \tag{3.51}$$

Notice that in general Γ is complex, but VSWR is necessarily always real and $1 \leq \text{VSWR} \leq \infty$. For the matched condition, $\Gamma = 0$ and $\text{VSWR} = 1$, and the closer VSWR is to 1, the closer the load is to being matched to the line and the more power is delivered to the load. The magnitude of the reflection coefficient on a line with a short-circuit or open-circuit load is 1, and in both cases the VSWR is infinite.

To determine the position of the standing wave maximum, $\ell_{\max}$, consider Equation (3.47) and note that at the maximum

$$\Theta - 2\beta\ell_{\max} = 2n\pi, \quad n = 0, 1, 2, \ldots . \tag{3.52}$$

Here Θ is the angle of the reflection coefficient at the load:

$$\Theta - 2n\pi = 2\frac{2\pi}{\lambda_g}\ell_{\max} . \tag{3.53}$$

Thus the position of the voltage maxima, $l_{\max}$, normalized to wavelength is

$$\frac{\ell_{\max}}{\lambda_g} = \frac{1}{2}\left(\frac{\Theta}{2\pi} - n\right), \quad n = 0, -1, -2, \ldots . \tag{3.54}$$

Similarly the position of the voltage minima is (using Equation (3.47)):

$$\Theta - 2\beta\ell_{\text{min}} = (2n+1)\pi. \tag{3.55}$$

After rearranging the terms,

$$\frac{\ell_{\text{min}}}{\lambda_g} = \frac{1}{2}\left(\frac{\Theta}{2\pi} - n + \frac{1}{2}\right), \quad n = 0, -1, -2, \ldots. \tag{3.56}$$

Summarizing from Equations (3.54) and (3.56):

1. The distance between two successive maxima is $\lambda_g/2$.
2. The distance between two successive minima is $\lambda_g/2$.
3. The distance between a maximum and an adjacent minimum is $\lambda_g/4$.
4. From the measured VSWR the magnitude of the reflection coefficient $|\Gamma|$ can be found. From the measured ℓ_{max} the angle Θ of Γ can be found. Then from Γ the load impedance can be found.

In a similar manner to that above, the magnitude of the total current on the line is

$$|I(\ell)| = \frac{|V_0^+|}{Z_0}\left|1 - |\Gamma| e^{\jmath(\Theta - 2\beta\ell)}\right|. \tag{3.57}$$

Hence the standing wave current is maximum where the standing-wave voltage amplitude is minimum, and minimum where the standing-wave voltage amplitude is maximum.

Now Z_{in} in Equation (3.45) is a periodic function of ℓ with period $\lambda/2$ and it varies between Z_{max} and Z_{min}, where

$$Z_{\text{max}} = \frac{V_{\text{max}}}{I_{\text{min}}} = Z_0 \times \text{VSWR} \quad \text{and} \quad Z_{\text{min}} = \frac{V_{\text{min}}}{I_{\text{max}}} = \frac{Z_0}{\text{VSWR}}. \tag{3.58}$$

EXAMPLE 3.7 Standing Wave Ratio

In Example 3.6 a load consisted of a capacitor of 10 pF in shunt with a resistor of 60 Ω. The load terminated a lossless 50 Ω transmission line. The operating frequency is 5 GHz.

(a) What is the SWR?

(b) What is the current standing wave ratio (ISWR)? (When SWR is used on its own it is assumed to refer to VSWR.)

Solution:

(a) From Example 3.6 on page 52, $\Gamma_L = 0.993\angle 187.3^\circ$ and so

$$\text{VSWR} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.993}{1-0.993} = 285.$$

(b) ISWR = VSWR = 285.

EXAMPLE 3.8 Standing Waves

A load has an impedance $Z_L = 45 + \jmath 75\ \Omega$ and the system reference impedance, Z_0, is 100 Ω.

(a) What is the reflection coefficient?
(b) What is the current reflection coefficient?
(c) What is the SWR?
(d) What is the ISWR?
(e) The power available from a source with a 100 Ω Thevenin equivalent impedance is 1 mW. The source is connected directly to the load, Z_L. Use the reflection coefficient to calculate the power delivered to Z_L.
(f) What is the total power absorbed by the Thevenin equivalent source impedance?
(g) Discuss the effect on power flow of inserting a lossless 100 Ω transmission line between the source and the load.

Solution:

(a) The voltage reflection coefficient is

$$\begin{aligned}\Gamma_L &= (Z_L - Z_0)/(Z_L + Z_0) = (45 + \jmath 75 - 100)/(45 + \jmath 75 + 100) \\ &= (93.0\angle(2.204\ \text{rads}))/(163.2\angle(0.4773\ \text{rads})) \\ &= 0.570\angle(1.726\ \text{rads}) = 0.570\angle 98.9^\circ = -0.0881 + \jmath 0.563 = \Gamma^V. \end{aligned} \tag{3.59}$$

(b) The current reflection coefficient is

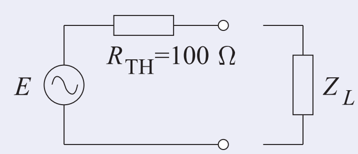
$$\Gamma^I = -\Gamma^V = 0.0881 - \jmath 0.563 = 0.570\angle(98.9^\circ - 180^\circ) = 0.570\angle 81.1^\circ. \tag{3.60}$$

(c) The SWR is the VSWR, so

$$\text{SWR} = \text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma^V|}{1 - |\Gamma^V|} = \frac{1 + 0.570}{1 - 0.570} = 3.65. \tag{3.61}$$

(d) The current SWR is ISWR = VSWR.

(e) To determine the reflection coefficient of the load, begin by developing the Thevenin equivalent circuit of the load. The power available from the source is $P_A = 1\ \text{mW},$ so the Thevenin equivalent circuit is



The power reflected by the load is

$$P_R = P_A \left|\Gamma_L^2\right| = 1\ \text{mW} \cdot (0.570)^2 = 0.325\ \text{mW}$$

and the power delivered to the load is

$$P_D = P_A \left(1 - \left|\Gamma_L^2\right|\right) = 0.675\ \text{mW}.$$

(f) It is tempting to think that the power dissipated in R_{TH} is just P_R. However, this is not correct. Instead, the current in R_{TH} must be determined and then the power dissipated in R_{TH} found. Let the current through $R_{\text{TH}cdot}$ be I, and this is composed of forward- and backward-traveling components:

$$I = I^+ + I^- = (1 + \Gamma_I)I^+,$$

where I^+ is the forward-traveling current wave. Thus

$$P_A = \tfrac{1}{2}|I^+|^2 R_{\text{TH}} = \tfrac{1}{2}|I^+|^2 \times 100 = 1\ \text{mW} = 10^{-3}\ \text{W},$$

so $I^+ = 4.47$ mA, and

$$I = (1 + \Gamma_I)I^+ = (1 + 0.0881 - \jmath 0.563) \times 4.47 \times 10^{-3}\ \text{A}, \qquad |I| = 5.48\ \text{mA}.$$

The power dissipated in R_{TH} is

$$P_{\text{TH}} = \tfrac{1}{2}|I|^2 R_{\text{TH}} = \tfrac{1}{2}(5.48 \times 10^{-3})^2 R_{\text{TH}} = 1.50\ \text{mW}. \tag{3.62}$$

The circuit is that shown in part (e) and so the current in $R_{\rm TH}$ is the same as the current in Z_L. Thus the power delivered to the load Z_L is due to the real part of Z_L:

$$P_D = \tfrac{1}{2}\left|I^2\right|\Re(Z_L) = \tfrac{1}{2}(5.48\times10^{-3})^2\times 45 = 0.676 \text{ mW} \tag{3.63}$$

(g) Inserting a transmission line with the same characteristic impedance as the Thevenin equivalent impedance will have no effect on power flow.

3.3.6 Summary

This section related the physics of traveling voltage and current waves on lossless transmission lines to the total voltage and current view. First the input reflection coefficient of a terminated lossless line was developed and from this the input impedance, which is the ratio of total voltage and total current, derived. At any point along a line the amplitude of total voltage varies sinusoidally, tracing out a standing wave pattern along the line and yielding the VSWR metric which is the ratio of the maximum amplitude of the total voltage to the minimum amplitude of that voltage. This is an important metric that is often used to provide an indication of how good a match, i.e. how small the reflection is, with a VSWR $=$ 1 indicating no reflection and a VSWR $=\infty$ indicating total reflection, i.e. a reflection coefficient magnitude of 1.

3.4 Special Lossless Line Configurations

The lossless transmission line configurations considered in this section are used as circuit elements in RF designs and are used elsewhere in this book series. The first element considered in Section 3.4.1 is a short length of short-circuited line which looks like an inductor. The element considered in Section 3.4.2 is a short length of open-circuited line which looks like a capacitor. Then lengths of short-circuited and open-circuited lines, called stubs, used nearly always as shunt elements to introduce an admittance in a circuit, are described in Sections 3.4.3 and 3.4.4. Another type of element, described in Section 3.4.5, is a short length of line with either high or low characteristic impedance realizing a small series inductor or capacitor respectively. The final element described in Section 3.4.6 is a quarter-wave transformer, a quarter-wavelength long line with a particular characteristic impedance which is used in two ways. It can be used to provide maximum power transfer from a source to a load resistance, and it can invert an impedance, e.g. making a capacitor terminating the line look like an inductor.

3.4.1 Short Length of Short-Circuited Line

A transmission line terminated in a short circuit ($Z_L = 0$) has the input impedance (using Equation (3.45))

$$Z_{\rm in} = \jmath Z_0 \tan(\beta\ell). \tag{3.64}$$

So a short length of short-circuited line, $\ell < \lambda_g/4$, looks like an inductor with inductance L_s,

$$Z_0\tan(\beta\ell) = \omega L_s, \quad \text{and so} \quad L_s = \frac{Z_0}{\omega}\tan\left(\frac{2\pi\ell}{\lambda_g}\right). \tag{3.65}$$

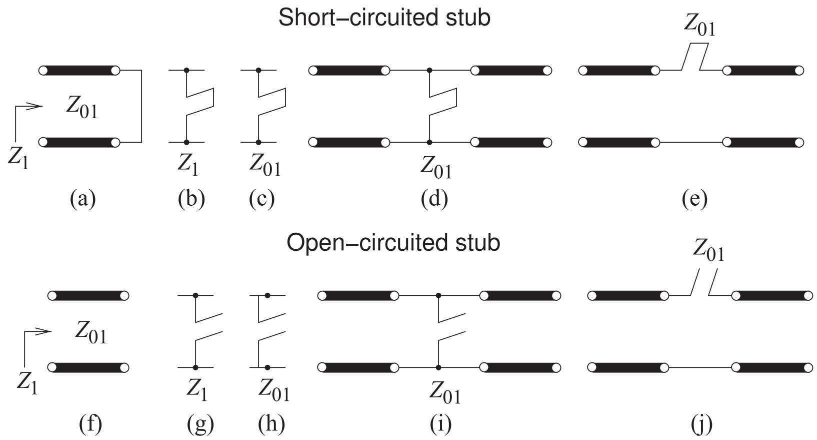


Figure 3-15: Transmission line stubs: (a)–(e) short-circuited stubs; and (f)–(j) open-circuited stubs.

From Equation (3.65) it can be seen that for a given ℓ, L_s is proportional to Z_0. Hence, for a larger values of L_s, sections of transmission line of high characteristic impedance is needed. So microstrip lines with narrow strips can be used to realize inductors in planar microstrip circuits.

3.4.2 Short Length of Open-Circuited Line

An open-circuited line has $Z_L = \infty$ and so (using Equation (3.45))

$$Z_{\text{in}} = -\jmath \frac{Z_0}{\tan \beta\ell} \,. \tag{3.66}$$

For lengths ℓ such that $\ell < \lambda/4$, an open-circuited segment of line realizes a capacitor C_0 for which

$$\frac{1}{\omega C_0} = \frac{Z_0}{\tan \beta\ell} \quad \text{and so} \quad C_0 = \frac{1}{Z_0}\frac{\tan(\beta\ell)}{\omega}. \tag{3.67}$$

From the above relationship, it can be seen that C_0 is inversely proportional to Z_0. Hence, for a larger value of C_0, a section of transmission line with low characteristic impedance is needed.

3.4.3 Short-Circuited Stub

A stub is a section of open-circuited or short-circuited transmission line and is used as a series or shunt element in a microwave circuit. There are several representations. A shorted stub is shown in Figure 3-15(a) as a transmission line with characteristic impedance Z_{01} that is short circuited. The input impedance of the line is Z_1. If the line is lossless, the usual assumption, then Z_{01} will be real and Z_1 will be imaginary. Stubs are commonly used in microwave circuits and generally all stubs in a network have the same length, such as $\lambda/4$ long or $\lambda/8$ long. Which it is is specified in the design. Realistically they do not need to have the same length but there are some special properties for certain lengths, as will become clearer. A cleaner way to indicate a shorted stub is shown in Figure 3-15(b), where the impedance of the stub is as indicated. The absence of a 0 subscript (which would indicate characteristic impedance) means that this is the reactive input impedance of the stub. If a 0 subscript is used, as in Figure 3-15(c), the characteristic impedance of the stub is indicated. If a numerical value is given then an imaginary impedance indicates that the input impedance is being specified, whereas a real impedance indicates the characteristic impedance of the stub. The shorted stub is shown as a shunt element in Figure 3-15(d) and as a series element in Figure 3-15(e). However in nearly all transmission line technologies, including microstrip, only shunt stubs can be realized. The open-circuited stubs with annotations are shown in Figures 3-15(f–j) with similar assignments of meaning. The length of a stub is often indicated by its resonant frequency, f_r. This is the frequency at which the stub is $\lambda/4$ long.

The shorted stub in Figure 3-15(a) has the input impedance (from Equation (3.45))

$$Z_1 = \jmath Z_{01} \tan \beta\ell, \tag{3.68}$$

where ℓ is the physical length of the line. Since the stub is $\lambda/4$ long at f_r, then at frequency f, the input impedance of the stub is

$$Z_1 = \jmath Z_{01} \tan\left(\frac{\pi}{2}\frac{f}{f_r}\right). \tag{3.69}$$

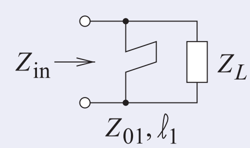
A special situation, and the most commonly used in design, is when the operating frequency is around one-half of the resonant frequency (i.e., $f \approx \frac{1}{2} f_r$). Then the stub is one-eighth of a wavelength long and the argument of the tangent function in Equation (3.69) is approximately $\pi/4$ and Z_1 becomes

$$Z_1 \approx \jmath Z_{01} \tan\left(\frac{\pi}{4}\right) = \jmath Z_{01}, \tag{3.70}$$

thus realizing an inductance at $f = f_r/2$ with a reactance equal to the characteristic impedance of the line.

EXAMPLE 3.9 Short-Circuited Stub

Develop the electrical design of the shunt stub shown with a load of impedance $Z_L = 75 + \jmath 15\ \Omega$ so that the total impedance of the load and stub is real.



Solution:

The short-circuited stub has characteristic impedance Z_{01} and length ℓ_1. Choose $Z_{01} = 75\ \Omega$ (generally this must be between 15 Ω and 100 Ω for most transmission line technologies). The stub needs to be designed so that the susceptances of the stub and load sum to zero. The admittance of the load $Y_L = 1/Z_L = 0.01282 - \jmath 0.002564$ S. The required admittance of the stub is $Y_{\text{STUB}} = \jmath 0.002564$ S so, using Equation (3.68),

$$Z_{\text{STUB}} = 1/Y_{\text{STUB}} = \jmath Z_{01} \tan \beta\ell_1 = -\jmath 390\ \Omega.$$

Therefore, the electrical length of the stub is

$$\beta\ell_1 = \arctan(-\jmath 390/\jmath 75) = -1.381 + n\pi \text{ radians}, n = 0, 1, 2, \ldots. \tag{3.71}$$

The first positive angle is taken so the stub has the shortest length. So

$$\beta\ell_1 = 1.761 \text{ radians} = 100.9^\circ. \tag{3.72}$$

The complete electrical design of the stub is that it is a shunt short-circuited stub with a characteristic impedance of 75 Ω and with an electrical length of 100.9°. The combined impedance of the stub and load is $Z_X = 1/(\Re\{Y_L\}) = 1/0.01282 = 78.00\ \Omega$.

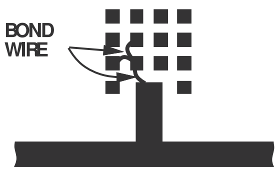


Figure 3-16: Open-circuited stub with variable length realized using wire bonding from the fixed stub to one of the bond pads. The bond pads are on the same layer as the strip metal layer and bonding to them extends the length of the open-circuited stub.

3.4.4 Open-Circuited Stub

An open-circuited transmission line is commonly used as a circuit element called an open stub shown in Figure 3-15(f–j). From Equation (3.45) and noting that $Z_L = \infty$, the open stub input impedance is

$$Z_1 = -\jmath Z_{01} \frac{1}{\tan \beta \ell}. \tag{3.73}$$

With the stub one-quarter wavelength long at the frequency f_r, the input impedance at f_r is a short circuit and the stub is said to be resonant at f_r. Then at a frequency f, the input impedance of the stub is

$$Z_1 = -\jmath Z_{01} \tan^{-1}\left(\frac{\pi}{2}\frac{f}{f_r}\right). \tag{3.74}$$

When $f = \frac{1}{2} f_r$ the stub is one-eighth wavelength long and

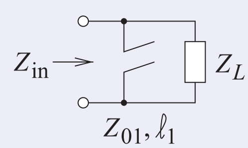
$$Z_1 = -\jmath Z_{01} \frac{1}{\tan\left(\frac{\pi}{4}\right)} = -\jmath Z_{01}. \tag{3.75}$$

So a $\lambda/4$ long open-circuited stub realizes a capacitance with a reactance equal to the characteristic impedance of the line.

If the length of a stub can be changed then the stub can be used as a tuning element. A common microstrip tuning technique is shown in Figure 3-16, where bonding to different pads enables a variable length stub to be realized.

EXAMPLE 3.10 Open-Circuited Stub

Develop the electrical design of the open-circuit stub shown with a load of impedance $Z_L = 75 + \jmath 15\ \Omega$ so that the total impedance of the load and stub is real.



Solution:

The open-circuited stub has characteristic impedance Z_{01} and length ℓ_1. A good choice is to choose Z_{01} around the impedance level of the load as long as it can be realized; so choose $Z_{01} = 75\ \Omega$. The stub needs to be designed so that the susceptances of the stub and load sum to zero. The admittance of the load $Y_L = 1/Z_L = 0.01282 - \jmath 0.002564$ S. The required admittance of the stub is $Y_{\text{STUB}} = \jmath 0.002564$ S, so, using Equation (3.73),

$$Z_{\text{STUB}} = 1/Y_{\text{STUB}} = -\jmath Z_{01}/\tan \beta \ell_1 = -\jmath 390\ \Omega.$$

Therefore, the electrical length of the stub is

$$\beta \ell_1 = \arctan(\jmath 75/\jmath 390) = 0.1900 + n\pi \text{ radians}, n = 0, 1, 2, \ldots. \tag{3.76}$$

The first positive angle is taken so the stub has the shortest length. Then

$$\beta \ell_1 = 0.1900 \text{ radians} = 10.89^\circ. \tag{3.77}$$

The complete electrical design of the stub is that it is a shunt open-circuited stub with a characteristic impedance of 75 Ω and an electrical length of 10.89°. The combined impedance of the stub and load is $Z_X = 1/(\Re\{Y_L\}) = 1/0.01282 = 78.00\ \Omega$.

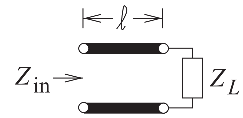


Figure 3-17: An electrically short line.

3.4.5 Electrically Short Lossless Line

Consider the input impedance, Z_{in}, of an electrically short line (i.e., $\beta\ell$ is small) (see Figure 3-17). Using Equation (3.45),

$$Z_{\text{in}} \approx \frac{Z_L + \jmath Z_0(\beta\ell)}{1 + \jmath(Z_L/Z_0)(\beta\ell)} \approx [Z_L + \jmath Z_0(\beta\ell)]\left[1 - \jmath\frac{Z_L}{Z_0}(\beta\ell)\right]. \tag{3.78}$$

Since $Z_0\beta = \sqrt{L/C}(\omega\sqrt{LC}) = \omega L$ and $\beta/Z_0 = (\omega\sqrt{LC})/\sqrt{L/C}) = \omega C$ (where L and C are the inductance and capacitance per unit length of the line), Equation (3.78) can be written as

$$Z_{\text{in}} \approx Z_L\left[1 + (\beta\ell)^2\right] + \jmath\left[\omega(L\ell) - Z_L^2\omega(C\ell)\right]. \tag{3.79}$$

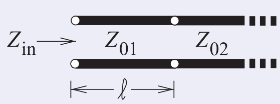
Since $\beta\ell$ is small, $(\beta\ell)^2$ is very small, and so the $(\beta\ell)^2$ term can be ignored. Then the input impedance of an electrically short line terminated in impedance Z_L is

$$Z_{\text{in}} \approx Z_L + \jmath\left[\omega(L\ell) - Z_L^2\omega(C\ell)\right]. \tag{3.80}$$

Some special cases of this result will be considered in the following examples.

EXAMPLE 3.11 Capacitive Transmission Line Segment

This example demonstrates that a predominantly capacitive behavior can be obtained from a short segment of transmission line, the Z_{01} line here, of low characteristic impedance. Consider the transmission line system shown below with lines having the characteristic impedances, Z_{01} and Z_{02}, $Z_{02} \gg Z_{01}$.



The value of Z_{in} is (treating Z_{02} as the load)

$$Z_{\text{in}} = Z_{01}\frac{Z_{02} + \jmath Z_{01}\tan\beta\ell}{Z_{01} + \jmath Z_{02}\tan\beta\ell}. \tag{3.81}$$

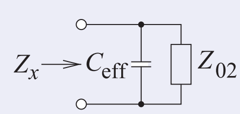
Now $(1 + \jmath x)^{-1} \approx 1 - \jmath x - x^2$. Thus for a short line (and so dropping the $\tan^2(\beta\ell)$ term)

$$Z_{\text{in}} \approx Z_{02} - \jmath\frac{Z_{02}^2}{Z_{01}}\tan(\beta\ell) + \jmath Z_{01}\tan(\beta\ell) = Z_{02} + \jmath Z_{01}\tan(\beta\ell)\left[1 - \frac{Z_{02}^2}{Z_{01}^2}\right]. \tag{3.82}$$

For $Z_{02} \gg Z_{01}$ and for a short line, $\tan(\beta\ell) \approx \beta\ell$, and this becomes

$$Z_{\text{in}} \approx Z_{02} - \jmath\frac{Z_{02}^2}{Z_{01}}\tan(\beta\ell) \approx Z_{02} - \jmath\frac{Z_{02}^2}{Z_{01}}\beta\ell, \tag{3.83}$$

which is capacitive. Now consider the circuit to the right where an effective capacitance C_{eff} is in shunt with a load Z_{02}. This has the input impedance



$$Z_x = \left(\jmath\omega C_{\text{eff}} + \frac{1}{Z_{02}}\right)^{-1} = \frac{Z_{02}}{1 + \jmath\omega C_{\text{eff}}Z_{02}} = Z_{02}[1 - \jmath\omega C_{\text{eff}}Z_{02} - (\jmath\omega C_{\text{eff}}Z_{02})^2 + \dots \,. \tag{3.84}$$

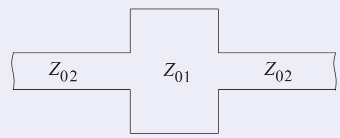
For $\omega C_{\text{eff}}Z_{02} \ll 1$ (i.e. an electrically short line)

$$Z_x \approx Z_{02} - \jmath\omega C_{\text{eff}} Z_{02}^2 \tag{3.85}$$

Equating Equations (3.83) and (3.85), the effective value of the shunt capacitor realized by the short length of low-impedance line, the Z_{01} line, is

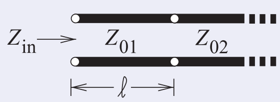
$$C_{\text{eff}} = \frac{1}{\omega Z_{02}^2} \frac{Z_{02}^2 \beta \ell}{Z_{01}} = \frac{\beta}{\omega} \frac{\ell}{Z_{01}}. \tag{3.86}$$

Thus a shunt capacitor can be realized approximately by a low-impedance line embedded between two high-impedance lines. The microstrip layout of this is shown in the figure on the right. Recall that a wide microstrip line has a low characteristic impedance.



EXAMPLE 3.12 Inductive Transmission Line Segment

This example demonstrates that a (predominantly) inductive behavior can be obtained from a segment of transmission line. Consider the transmission line system shown below with lines having two different characteristic impedances, Z_{01} and Z_{02}, $Z_{02} \ll Z_{01}$.



The value of Z_{in} is (using Equation (3.45))

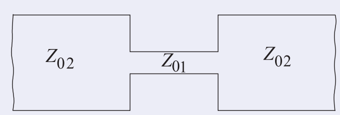
$$Z_{\text{in}} = Z_{01} \frac{Z_{02} + \jmath Z_{01} \tan \beta\ell}{Z_{01} + \jmath Z_{02} \tan \beta\ell}, \tag{3.87}$$

which for a short line can be expressed as

$$Z_{\text{in}} \approx Z_{02}\left[1 + \tan(\beta l)\right] + \jmath Z_{01} \tan(\beta\ell)\ . \tag{3.88}$$

Note that $\jmath Z_{01} \tan(\beta\ell)$ is the dominant part for $\ell < \lambda/8$ and $Z_{02} \ll Z_{01}$.

Thus a microstrip realization of a series inductor is a high-impedance line embedded between two low-impedance lines. A top view of such a configuration in microstrip is shown in the figure. A narrow microstrip line has high characteristic impedance.



The previous two examples showed how a shunt capacitance and series inductance can be realized using short sections of line, the Z_{01} line here, with a high characteristic impedance. This enables realization of some lumped element circuits in microstrip form. A lumped element lowpass filter is shown in Figure 3-18(a) and this can be realized using wide and narrow microstrip lines, as shown in Figure 3-18(b).

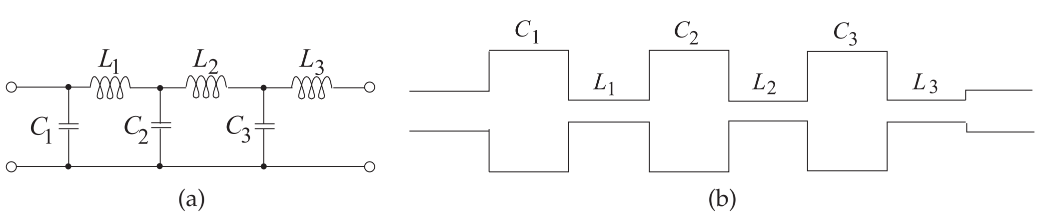


Figure 3-18: A lowpass filter: (a) in the form of an LC ladder network; and (b) realized using microstrip lines.

3.4.6 Quarter-Wave Transformer

Figure 3-19(a) shows a resistive load R_L and a section of transmission line with length $\ell = \lambda_g/4$ (hence the name quarter-wave transformer). The input impedance of the line is

$$Z_{\text{in}} = Z_1 \frac{R_L + \jmath Z_1 \tan(\beta\ell)}{Z_1 + \jmath R_L \tan(\beta\ell)} = Z_1 \frac{R_L + \jmath Z_1 \infty}{Z_1 + \jmath R_L \infty} = \frac{Z_1^2}{R_L} \,. \quad (3.89)$$

The input impedance is matched to the transmission line Z_0 if

$$Z_{\text{in}} = Z_0^* = Z_0, \quad (3.90)$$

since here the characteristic impedance is real. Thus

$$Z_1 = \sqrt{Z_0 R_L} \quad (3.91)$$

and so the one-quarter wavelength long line acts as an ideal impedance transformer.

Another example of the quarter-wave transformer is shown in Figure 3-19(b). The input impedance looking into the quarter-wave transformer (from the left) is given by

$$Z_{\text{in}} = Z_0 \frac{Z_{01} + \jmath Z_0 \tan(\beta\ell)}{Z_0 + \jmath Z_{01} \tan(\beta\ell)} = Z_0 \frac{Z_{01} + \jmath Z_0 \infty}{Z_0 + \jmath Z_{01} \infty} = \frac{Z_0^2}{Z_{01}}. \quad (3.92)$$

Hence a section of transmission line of length $\ell = \lambda_g/4 + n\lambda_g/2$, where $n = 0, 1, 2, \ldots$, can be used to match lines having different impedances, Z_{01} and Z_{02}, by constructing the line so that its characteristic impedance is

$$Z_0 = \sqrt{Z_{01} Z_{02}}. \quad (3.93)$$

Note that for a design center frequency f_0, the matching section provides a perfect match only at the center frequency and at frequencies where $\ell = \lambda_g/4 + n\lambda_g/2$.

A quarter-wave transformer has an interesting property that is widely used. Examine the final result in Equation (3.92), which is repeated here:

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{01}} \,. \quad (3.94)$$

Equation (3.94) indicates that a one-quarter wavelength long line is an impedance inverter presenting, at Port 1, the inverse of the impedance presented at port 2, Z_{01}. This result also applies to complex impedances replacing Z_{01}. This impedance inversion is scaled by the square of the characteristic impedance of the line. This inversion holds in the reverse direction as well.

The layout of a microstrip quarter-wave transformer is shown in Figure 3-20, where $\ell = \lambda_g/4$ and the characteristic impedance of the transformer, Z_0, is the geometric mean of the impedances on either side, that is, $Z_0 = \sqrt{Z_{01} Z_{02}}$.

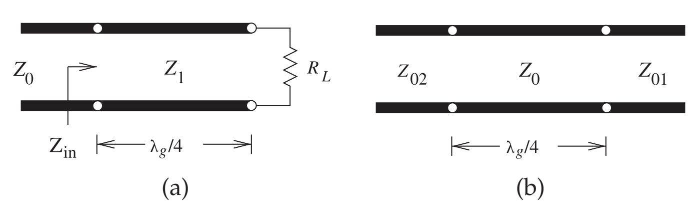


Figure 3-19: The quarter-wave transformer line: (a) transforming a load; and (b) interfacing two lines.

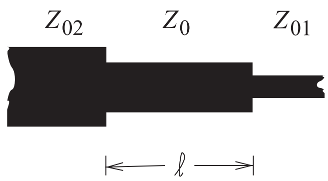


Figure 3-20: Layout of a microstrip quarter-wave transformer.

3.4.7 Summary

The lossless transmission line configurations considered in this section are those most commonly used in microwave circuit design. It is important to note that the stub line is almost always used in shunt configuration to provide an admittance in a circuit. Most transmission line technologies, including coaxial lines and microstrip, only permit shunt stubs. The quarter-wave transformer is a particularly interesting element enabling maximum power transfer from a source to a load that may be different. An interesting feature that is widely exploited is that the quarter-wave transformer inverts an impedance. For example, turning a small resistance into a large resistance, or even turning a small capacitor into a large inductance. These transformations are valid over a moderate bandwidth.

3.4.8 Summary

An earlier section developed the input reflection coefficient and input impedance of a lossless line. Several circuit elements based on a a transmission line were introduced: stubs, short-sections of line have either high or low characteristic impedance, and the quarter-wave transformer.

3.5 Circuit Models of Transmission Lines

Circuit models of transmission lines are required if they are to be used in a circuit simulator. RF and microwave engineering uses two types of simulators. Spice-like simulators use lumped-element transmission line models in which an $RLGC$ model of a short segment of line is replicated for the length of the line. If the ground plane is treated as a universal ground, then the model of a segment of length Δz is as shown in Figure 3-21(a). In this segment $r = R\Delta z$, $l = L\Delta z$, $g = G\Delta z$, and $c = C\Delta z$, where R, L, G, and C are the per unit length parameters of the line. Cascading the segments to get the length of the line yields the complete lumped-element model of the line, as shown in Figure 3-21(b).

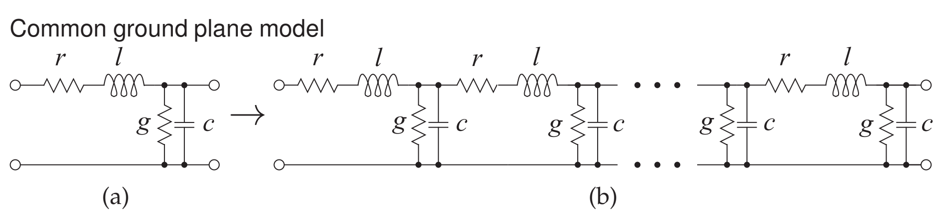


Figure 3-21: Lumped-element transmission line models: (a) model of a short segment (e.g. $\lambda/20$ long); and (b) complete model of a transmission line.

3.6 Summary

In this chapter a classical treatment of transmission lines was presented. Transmission lines are distributed elements and form the basis of microwave circuits. A distinguishing feature is that they support forward- and backward-traveling waves and they can be used to implement circuit functions.

The most important of the formulas presented in this chapter are listed here. Reflection coefficients are referenced to an impedance Z_0, the load impedance is Z_L, and a line has a characteristic impedance Z_0, physical length ℓ, and propagation constant γ (or electrical length in radians of $\beta\ell$ where ℓ is the physical length of the line.

Reflection coefficient of a load impedance Z_L:

$$\Gamma = \Gamma^V = \frac{Z_L - Z_{\rm REF}}{Z_L + Z_{\rm REF}} \qquad (3.36)$$

Load impedance in terms of reflection coefficient Γ:

$$Z_L = Z_{\rm REF}\frac{1+\Gamma}{1-\Gamma} \qquad (3.38)$$

Input reflection coefficient of a lossless line of length ℓ

$$\Gamma_{\rm in} = \Gamma_L e^{-\jmath 2\beta\ell} \qquad (3.43)$$

Reflection coefficient in terms of VSWR

$$|\Gamma| = \frac{\text{VSWR}-1}{\text{VSWR}+1} \qquad (3.51)$$

Input impedance of a lossless line

$$Z_{\rm in} = Z_0\frac{Z_L + \jmath Z_0\tan\beta\ell}{Z_0 + \jmath Z_L \tan\beta\ell} \qquad (3.45)$$

VSWR in terms of reflection coefficient

$$\text{VSWR} = \frac{(1+|\Gamma|)}{(1-|\Gamma|)} \qquad (3.50)$$

3.7 References

[1] T. Edwards and M. Steer, *Foundations for Microstrip Circuit Design*. John Wiley & Sons, 2016.

[2] C. Boyer and U. Merzbach, "Invention of logarithms," in *A History of Mathematics*, 2nd ed. John Wiley & Sons, 1991.

3.8 Exercises

1. A coaxial line is short-circuited at one end and is filled with a dielectric with a relative permittivity of 64. [Parallels Example 3.1]
   (a) What is the free-space wavelength at 18 GHz?
   (b) What is the wavelength in the dielectric-filled coaxial line at 18 GHz?
   (c) The first resonance of the coaxial resonator is at 18 GHz. What is the physical length of the resonator?

2. A transmission line has the following $RLGC$ parameters: $R = 100$ Ω/m, $L = 85$ nH/m, $G = 1$ S/m, and $C = 150$ pF/m. Consider a traveling wave on the transmission line with a frequency of 1 GHz. [Parallels Example 3.3]
   (a) What is the attenuation constant?
   (b) What is the phase constant?
   (c) What is the phase velocity?
   (d) What is the characteristic impedance of the line?
   (e) What is the group velocity?

3. A transmission line has the per-unit length parameters $L = 85$ nH/m, $G = 1$ S/m, and $C = 150$ pF/m. Use a frequency of 1 GHz. [Parallels Example 3.3]
   (a) What is the phase velocity if $R = 0$ Ω/m?
   (b) What is the group velocity if $R = 0$ Ω/m?
   (c) If $R = 10$ kΩ/m what is the phase velocity?
   (d) If $R = 10$ kΩ/m what is the group velocity?

4. A line is 10 cm long and at the operating frequency the phase constant β is 40 rad/m. What is the electrical length of the line? [Parallels Example 3.2]

5. A coaxial transmission line is filled with lossy dielectric material with a relative permittivity of $5 - \jmath 0.2$. If the line is air-filled it would have a characteristic impedance of 100 Ω. What is the input impedance of the line if it is 1 km long? Use reasonable approximations. [Hint: Does the termination matter?]

6. A transmission line has the per unit length parameters $R = 2$ Ω/cm, L=100 nH/m, $G =$

1 mS/m, $C = 200$ pF/m.

(a) What is the propagation constant of the line at 5 GHz?
(b) What is the characteristic impedance of the line at 5 GHz?
(c) Plot the magnitude of the characteristic impedance versus frequency from 100 MHz to 10 GHz.

7. A line is 20 cm long and at 1 GHz the phase constant β is 20 rad/m. What is the electrical length of the line in degrees?

8. What is the electrical length of a line that is a quarter of a wavelength long,

(a) in degrees?
(b) in radians?

9. A lossless transmission line has an inductance of 8 nH/cm and a capacitance of 40 pF/cm.

(a) What is the characteristic impedance of the line?
(b) What is the phase velocity on the line at 1 GHz?

10. A transmission line has an attenuation of 2 dB/m and a phase constant of 25 radians/m at 2 GHz. [Parallels Example 3.4]

(a) What is the complex propagation constant of the transmission line?
(b) If the capacitance of the line is 50 pF·m^{-1} and $G = 0$, what is the characteristic impedance of the line?

11. A very low-loss microstrip transmission line has the following per unit length parameters: $R = 2\ \Omega$/m, $L = 80$ nH/m, $C = 200$ pF/m, and $G = 1\ \mu$S/m.

(a) What is the characteristic impedance of the line if loss is ignored?
(b) What is the attenuation constant due to conductor loss?
(c) What is the attenuation constant due to dielectric loss?

12. A lossless transmission line carrying a 1 GHz signal has the following per unit length parameters: $L = 80$ nH/m, $C = 200$ pF/m.

(a) What is the attenuation constant?
(b) What is the phase constant?
(c) What is the phase velocity?
(d) What is the characteristic impedance of the line?

13. A transmission line has a characteristic impedance Z_0 and is terminated in a load with a reflection coefficient of $0.8\angle 45°$. A forward-traveling voltage wave on the line has a power of 1 dBm.

1. How much power is reflected by the load?
2. What is the power delivered to the load?

14. A transmission line has an attenuation of 0.2 dB/cm and a phase constant of 50 radians/m at 1 GHz.

(a) What is the complex propagation constant of the transmission line?
(b) If the capacitance of the line is 100 pF/m and $G = 0$, what is the complex characteristic impedance of the line?
(c) If the line is driven by a source modeled as an ideal voltage and a series impedance, what is the impedance of the source for maximum transfer of power to the transmission line?
(d) If 1 W is delivered (i.e. in the forward-traveling wave) to the transmission line by the generator, what is the power in the forward-traveling wave on the line at 2 m from the generator?

15. A lossless transmission line is driven by a 1 GHz generator having a Thevenin equivalent impedance of 50 Ω. The transmission line is lossless, has a characteristic impedance of 75 Ω, and is infinitely long. The maximum power that can be delivered to a load attached to the generator is 2 W.

(a) What is the total (phasor) voltage at the input to the transmission line?
(b) What is the magnitude of the forward-traveling voltage wave at the generator side of the line?
(c) What is the magnitude of the forward-traveling current wave at the generator side of the line?

16. A transmission line is terminated in a short circuit. What is the ratio of the forward- and backward-traveling voltage waves at the termination? [Parallels Example 3.5]

17. A 50 Ω transmission line is terminated in a 40 Ω load. What is the ratio of the forward- to the backward-traveling voltage waves at the termination? [Parallels Example 3.5]

18. A 50 Ω transmission line is terminated in an open circuit. What is the ratio of the forward- to the backward-traveling voltage waves at the termination? [Parallels Example 3.5]

19. A line has a characteristic impedance Z_0 and is terminated in a load with a reflection coefficient of 0.8. A forward-traveling voltage wave on the line has a power of 1 W.

(a) How much power is reflected by the load?
(b) What is the power delivered to the load?

20. A load consists of a shunt connection of a capacitor of 10 pF and a resistor of 25 Ω. The load terminates a lossless 50 Ω transmission line. The operating frequency is 1 GHz. [Parallels Example 3.6]
    (a) What is the impedance of the load?
    (b) What is the normalized impedance of the load (normalized to the characteristic impedance of the line)?
    (c) What is the reflection coefficient of the load?
    (d) What is the current reflection coefficient of the load?
    (e) What is the standing wave ratio (SWR)?
    (f) What is the current standing wave ratio (ISWR)?
21. An amplifier is connected to a load by a transmission line matched to the amplifier. If the SWR on the line is 1.5, what percentage of the available amplifier power is absorbed by the load?
22. A load has a reflection coefficient of 0.5 when referred to $50\ \Omega$. The load is at the end of a line with a $50\ \Omega$ characteristic impedance.
    (a) If the line has an electrical length of 45°, what is the reflection coefficient calculated at the input of the line?
    (b) What is the VSWR on the $50\ \Omega$ line?
23. A 100 Ω resistor in parallel with a 5 pF capacitor terminates a 100 Ω transmission line. Calculate the SWR on the line at 2 GHz.
24. A lossless 50 Ω transmission line has a 50 Ω generator at one end and is terminated in $100\ \Omega$. What is the VSWR on the line?
25. A lossless 75 Ω line is driven by a 75 Ω generator. The line is terminated in a load that with a reflection coefficient (referred to 50 Ω) of $0.5 + \jmath 0.5$. What is the VSWR on the line?
26. A load with a 20 pF capacitor in parallel with a 50 Ω resistor terminates a 25 Ω line. The operating frequency is 5 GHz. [Parallels Example 3.7]
    (a) What is the VSWR?
    (b) What is ISWR?
27. A load $Z_L = 55 - \jmath 55\ \Omega$ and the system reference impedance, Z_0, is 50 Ω. [Parallels Example 3.8]
    (a) What is the load reflection coefficient Γ_L?
    (b) What is the current reflection coefficient?
    (c) What is the VSWR on the line?
    (d) What is the ISWR on the line?
    (e) Now consider a source connected directly to the load. The source has a Thevenin equivalent impedance $Z_G = 60\ \Omega$ and an available power of 1 W. Use Γ_L to find the power delivered to Z_L.
    (f) What is the total power absorbed by Z_G?
28. A load of 100 Ω is to be matched to a transmission line with a characteristic impedance of 50 Ω. Use a quarter-wave transformer. What is the characteristic impedance of the quarter-wave transformer?
29. Determine the characteristic impedance of a quarter-wave transformer used to match a load of 50 Ω to a generator with a Thevenin equivalent impedance of 75 Ω.
30. A transmission line is to be inserted between a 5 Ω line and a 50 Ω load so that there is maximum power transfer to the 50 Ω load at 20 GHz.
    (a) How long is the inserted line in terms of wavelengths at 20 GHz?
    (b) What is the characteristic impedance of the line at 20 GHz?

3.8.1 Exercises By Section

†challenging, ‡very challenging

§3.1 1
§3.2 $2^\dagger, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12^\dagger, 13$
§3.3 $14^\dagger, 15, 16, 17, 18, 19^\dagger, 20^\dagger, 21^\dagger, 22, 23, 24, 25, 26, 27$
§3.4 $28, 29, 30$

3.8.2 Answers to Selected Exercises

10 $0.23 + \jmath 25\ \text{m}^{-1}$

29 $61.2\ \Omega$

CHAPTER 4

Planar Transmission Lines

4.1 Introduction

The majority of transmission lines used in high-speed digital, RF, and microwave circuits are planar, as these can be defined using masks, photoresist, and etching of metal sheets. Such lines are called planar transmission lines. A common planar line is the microstrip line shown in Figure 4-1 and in cross section in Figure 4-2. This cross section is typical of what would be found on a semiconductor or **printed circuit board** (**PCB**). Current flows in both the top and bottom conductor, but in opposite directions. The physics is such that if there is a signal current on the top

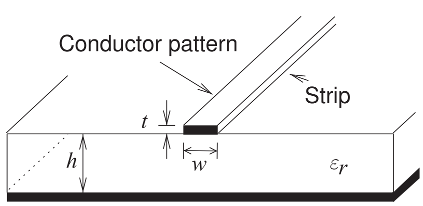


Figure 4-1: Microstrip transmission line.

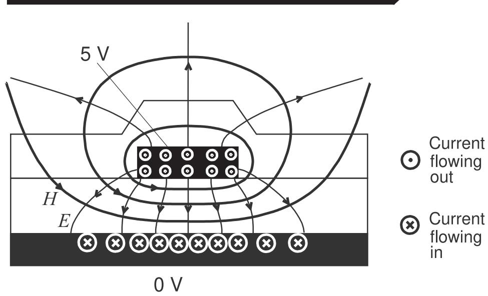


Figure 4-2: Cross-sectional view of a microstrip line showing electric and magnetic field lines and current flow. The electric and magnetic fields are in two mediums—the dielectric and air. If the line is homogeneous (the same dielectric everywhere) the electric and magnetic fields are only in the transverse plane, a field configuration known as the transverse electromagnetic mode (TEM).

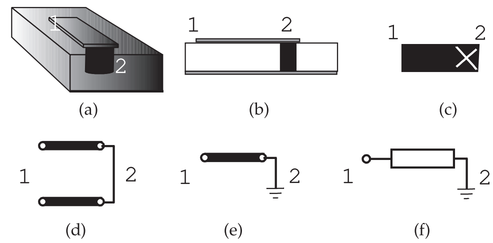


Figure 4-3: Representations of a shorted microstrip line with a short (or via) at port 2: (a) three-dimensional (3D) view indicating via; (b) side view; (c) top view with via indicated by X; (d) schematic representation of transmission line; (e) alternative schematic representation; and (f) representation as a circuit element.

conductor, there must be a return signal current, which will be as close to the signal current as possible to minimize stored energy. The provision of a signal return path is important in maintaining the integrity of (i.e., a predictable signal waveform on) an interconnect.

In the microstrip line, electric field lines start on one of the conductors and finish on the other and are located almost entirely in the plane transverse to the long length of the line. The magnetic field is also mostly confined to the transverse plane, and so this line is referred to as a **transverse electromagnetic** (**TEM**) line. More accurately it is called a **quasi-TEM** line, as the longitudinal fields, particularly in the air region, are not negligible.

Various schematic representations of a microstrip line are used. Consider the representations in Figure 4-3 of a length of microstrip line shorted by a via at the end denoted by "2" (specifically the "2" refers to Port 2).

4.2 Substrates

Planar line design involves choosing both the transmission line structure to use and the substrate. In this section the electrical and magnetic properties of substrate materials will be discussed.

4.2.1 Dielectric Effect

When the fields are in more than one medium (a **nonhomogeneous transmission line**), as for the microstrip line, the effective relative permittivity, $\varepsilon_{r,e}$ (or usually just $\varepsilon_e = \varepsilon_{r,e}$), is used. The characteristics of the line are then more or less the same as for the same structure with a uniform dielectric of permittivity, $\varepsilon_{\text{eff}} = \varepsilon_e\varepsilon_0$. The ε_{eff} changes with frequency as the proportion of energy stored in the different regions changes. This effect is called **dispersion** and causes a pulse to spread out as the different frequency components of a signal travel at different speeds.

4.2.2 Dielectric Loss Tangent, $\tan\delta$

Loss in a dielectric comes from (a) **dielectric damping** (also called **dielectric relaxation**), and (b) conduction losses in the dielectric. Dielectric damping originates from the movement of charge centers resulting in vibration of the lattice and thus energy is lost from the electric field. It is easy to see that this loss increases linearly with frequency and is zero at DC. In the frequency domain loss is incorporating in an imaginary term in the **permittivity**:

$$\varepsilon = \varepsilon_r\varepsilon_0 = \varepsilon' - \jmath\varepsilon'' = \varepsilon_0\left(\varepsilon_r' - \jmath\varepsilon_r''\right). \tag{4.1}$$

If there is no dielectric damping loss, $\varepsilon'' = 0$. The other type of loss is due to the movement of charge carriers in the dielectric. The ability to move charges

Material	$10^4 \tan\delta$ (at 10 GHz)	ε_r
Air (dry)	≈ 0	1
Alumina, 99.5%	1–2	10.1
Sapphire	0.4–0.7	9.4, 11.6
Glass, typical	20	5
Polyimide	50	3.2
Quartz (fused)	1	3.8
FR4 circuit board	100	4.3–4.5
RT-duroid 5880	5–15	2.16–2.24
RT-duroid 6010	10–60	10.2–10.7
AT-1000	20	10.0–13.0
Si (high resistivity)	10–100	11.9
GaAs	6	12.85
InP	10	12.4
SiO_2 (on-chip)	—	4.0–4.2
LTCC (typical, green tape(TM) 951)	15	7.8

Table 4-1: Properties of common substrate materials. The dielectric loss tangent is scaled. For example, for glass, $\tan\delta$ is typically 0.002.

is described by the conductivity, σ, and this loss is independent of frequency. So the energy lost in the dielectric is proportional to $\omega\varepsilon'' + \sigma$ and the energy stored in the electric field is proportional to $\omega\varepsilon'$. Thus a loss tangent, $\tan\delta$, is introduced:

$$\tan\delta = \frac{\omega\varepsilon'' + \sigma}{\omega\varepsilon'}. \tag{4.2}$$

Also the relative permittivity can be redefined as

$$\varepsilon_r = \varepsilon_r' - \jmath\left(\varepsilon_r'' + \frac{\sigma}{\omega\varepsilon_0}\right). \tag{4.3}$$

With the exception of silicon, the loss tangent is very small for dielectrics that are useful at RF and microwave frequencies and so most of the time

$$|\varepsilon_r| \approx \varepsilon_r'. \tag{4.4}$$

Thus

$$\varepsilon_r = \varepsilon_r' - \jmath\left(\varepsilon_r'' + \sigma/(\omega\varepsilon_0)\right) \approx \varepsilon_r'\left(1 - \jmath\tan\delta\right). \tag{4.5}$$

4.2.3 Magnetic Material Effect

Except for very special circumstances substrates used for microstrip lines are non magnetic so $\mu = \mu_0$ and the relative permeability, μ_r, is defined so that

$$\mu = \mu_r\mu_0. \tag{4.6}$$

4.2.4 Substrates for Planar Transmission Lines

The properties of common substrate materials are given in Table 4-1. Crystal substrates have very good dimensional tolerances and uniformity of electrical properties. Many other substrates have high surface roughness and electrical properties that can vary. For example, FR4 is the most common type of PCB substrate and is a weave of fiberglass embedded in resin. So the material is not uniform and there is an unpredictable localized variation in the proportion of resin and glass. High-performance microwave circuit boards have ceramic particles embedded in the resin.

4.3 Planar Transmission Line Structures

Two planar transmission line structures are shown in Figure 4-4. The reason these are so popular is that they can be mass produced. For the microstrip line in Figure 4-4(a) the fabrication process begins with a dielectric sheet with solid metal layers on the top and bottom. One of these is covered with a photosensitive material, called a photoresist, exposed to a prepared pattern that defines the interconnect line network, then the photoresist is developed and the unexposed (or exposed, depending on whether the photoresist is positive or negative) metal on one side is etched away. The stripline in Figure 4-4(b) is fabricated similarly to microstrip but followed by one more step in which a dielectric sheet with a ground plane only is bonded on top.

The most important planar transmission line structures are shown in Figure 4-5. With the homogeneous lines virtually all of the fields are in the plane transverse to the direction of propagation (i.e., the longitudinal direction). Transmission lines where the longitudinal fields are almost insignificant are referred to as supporting a TEM mode, and they are called TEM lines.

The most important inhomogeneous lines are shown in Figure 4-5(a–c). The main difference between the two sets of configurations (homogeneous and inhomogeneous) is the frequency-dependent variation of the EM field distributions with inhomogeneous lines. With inhomogeneous lines, the EM fields are not confined entirely to the transverse plane even if the conductors are perfect. However, they are largely confined to the transverse plane and so these lines are called **quasi-TEM lines**. The actual choice of structure depends on several factors, including operating frequency and the type of substrate and metallization system available. This book focuses on microstrip as it is by far the most commonly used planar transmission line.

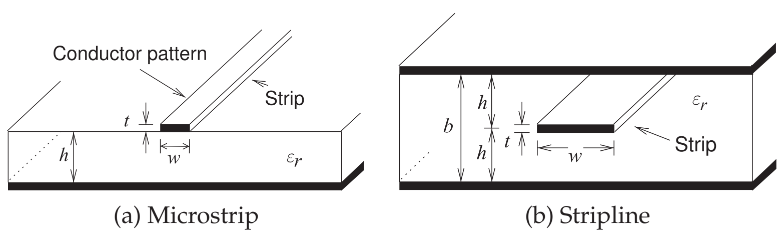


Figure 4-4: Planar transmission lines.

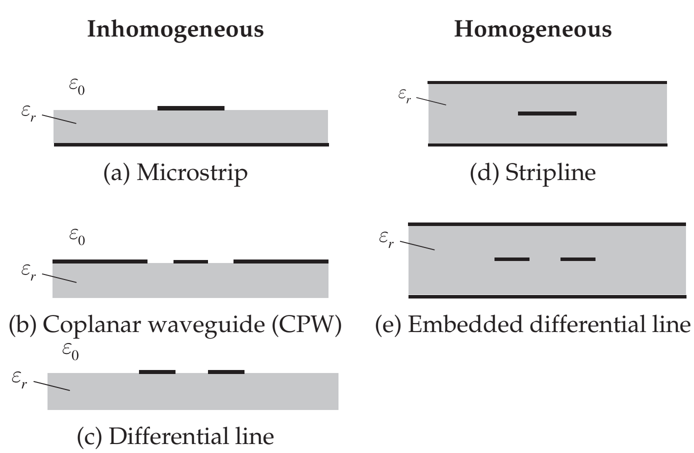


Figure 4-5: Cross sections of several homogeneous and inhomogeneous planar transmission line structures.

4.4 Microstrip Transmission Lines

Microstrip has conductors embedded in two dielectric mediums and cannot support a pure TEM mode. In most practical cases, the dielectric substrate is electrically thin, that is, $h \ll \lambda$. Then the transverse field is dominant and the fields are called quasi-TEM.

4.4.1 Microstrip Line in the Quasi-TEM Approximation

In this section relations are developed based on the principle that the phase velocity of an EM wave in an air-only homogeneous transmission with a TEM field line is just c. As a first step, the potential of the conductor strip is set to V_0 and Laplace's equation is solved using an EM simulator for the electrostatic potential everywhere in the dielectric. Then the per unit length (p.u.l.) electric charge, Q, on the conductor is determined. Using this in the following relation gives the line capacitance:

$$C = \frac{Q}{V_0}.$$

In the next step, the process is repeated with $\varepsilon_r = 1$ to determine C_{air} (the capacitance of the line without a dielectric).

If the microstrip line is now an air-filled lossless TEM structure,

$$v_{p,\text{air}} = c = \frac{1}{LC_{\text{air}}} \qquad (4.7) \qquad \text{and so} \quad L = \frac{1}{c^2 C_{\text{air}}}. \qquad (4.8)$$

L is not affected by the dielectric properties of the medium. L calculated above is the desired p.u.l. inductance of the line with the dielectric as well as in free space. Once L and C have been found, the characteristic impedance can be found using

$$Z_0 = \sqrt{\frac{L}{C}}, \qquad (4.9) \qquad \text{rewritten as} \quad Z_0 = \frac{1}{c}\frac{1}{\sqrt{C\,C_{\text{air}}}}, \qquad (4.10)$$

and the phase velocity is

$$v_p = \frac{1}{\sqrt{LC}} = c\sqrt{\frac{C_{\text{air}}}{C}}. \qquad (4.11)$$

Now the field is distributed in the inhomogeneous medium and in free space, as shown in Figure 4-6(a). So the effective relative permittivity, ε_e, of the equivalent homogeneous microstrip line (see Figure 4-6(b)) is defined by

$$\sqrt{\varepsilon_e} = \frac{c}{v_p}. \qquad (4.12)$$

Combining Equations (4.11) and (4.12), the effective relative permittivity (usually just the term effective permittivity is used) is obtained:

$$\varepsilon_e = \frac{C}{C_{\text{air}}}. \qquad (4.13)$$

The effective permittivity can be interpreted as the permittivity of a homogeneous medium that replaces the air and the dielectric regions of the

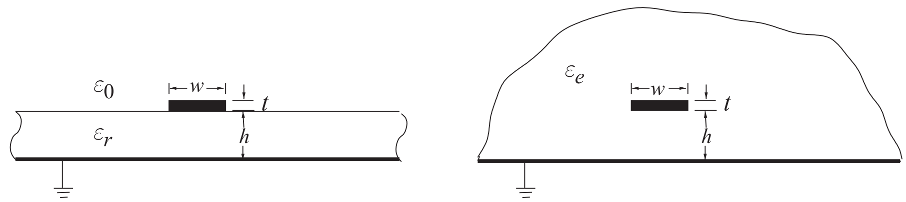


Figure 4-6: Microstrip line: (a) cross section; and (b) eps .. equivalent structure where the strip is embedded in a dielectric of semi-infinite extent with effective relative permittivity ε_e.

microstrip, as shown in Figure 4-6. Since some of the field is in the dielectric and some is in air, the effective relative permittivity must satisfy

$$1 < \varepsilon_e < \varepsilon_r. \tag{4.14}$$

However, the minimum ε_e will be greater than 1 as electrical energy will be distributed in air and dielectric. The wavelength on a transmission line, the guide wavelength λ_g, is related to the free space wavelength by $\lambda_g = \lambda_0/\sqrt{\varepsilon_e}$.

EXAMPLE 4.1 Microstrip Calculations

A microstrip line has a characteristic impedance Z_0 of 50 Ω derived from reflection coefficient measurements and an effective permittivity, ε_e, of 7 derived from measurement of phase velocity. What is the line's per-unit-length inductance, L, and capacitance, C?

Solution: The key equations are $Z_0 = \sqrt{L/C}$, $\varepsilon_e = C/C_{\rm air}$, and in air $v_p = 1/\sqrt{LC_{\rm air}} = c$. Also assume that $\mu_r = 1$ which is the default if not specified otherwise and also that L does not change if only the dielectric is changed. Thus

$$C_{\rm air} = \frac{C}{\varepsilon_e} \quad \text{and then} \quad L = \frac{\varepsilon_e}{c^2 C} \quad \text{so that} \quad Z_0 = \sqrt{\frac{L}{C}} = \frac{\sqrt{\varepsilon_e}}{cC}, \quad \text{that is} \quad C = \frac{\sqrt{\varepsilon_e}}{cZ_0}.$$

So $C = \sqrt{7}/(2.998 \cdot 10^8 \times 50) = 1.765 \cdot 10^{-10} = 176.5\text{pF/m}$ and $L = Z_0^2 C = 44.13\ \mu\text{H/m}$.

4.4.2 Effective Permittivity and Characteristic Impedance

This section presents formulas for the effective permittivity and characteristic impedance of a microstrip line. These formulas are fits to the results of detailed EM simulations. Also, the form of the equations is based on good physical understanding. First, assume that the thickness, t, is zero. This is not a bad approximation, as $t \ll w, h$ for most microwave circuits.

Hammerstad and others provide well-accepted formulas for calculating the effective permittivity and characteristic impedance of microstrip lines [1–3]. Given ε_r, w, and h, the effective relative permittivity is

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{10h}{w}\right)^{-a \cdot b}, \tag{4.15}$$

where

$$a(u)|_{u=w/h} = 1 + \frac{1}{49}\ln\left[\frac{u^4 + \{u/52\}^2}{u^4 + 0.432}\right] + \frac{1}{18.7}\ln\left[1 + \left(\frac{u}{18.1}\right)^3\right] \tag{4.16}$$

$$\text{and} \qquad (\varepsilon_r) = 0.564 \left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right]^{0.053}. \tag{4.17}$$

Take some time to interpret Equation (4.15), the formula for **effective relative permittivity**. If $\varepsilon_r = 1$, then $\varepsilon_e = (1+1)/2 + 0 = 1$, as expected. If ε_r is not that of air, then ε_e will be between 1 and ε_r, dependent on the geometry of the line, or more specifically, the ratio w/h. For a very wide line, $w/h \gg 1$, $\varepsilon_e = (\varepsilon_r+1)/2 + (\varepsilon_r-1)/2 = \varepsilon_r$, corresponding to the EM energy being confined to the dielectric. For a thin line $w/h \ll 1$, $\varepsilon_e = (\varepsilon_r+1)/2$, the average of the dielectric and air permittivities.

Mostly the term "**effective permittivity**" is used to mean effective relative permittivity (check the magnitude).

The characteristic impedance is given by

$$Z_0 = \frac{Z_{01}}{\sqrt{\varepsilon_e}}, \tag{4.18}$$

where the characteristic impedance of the microstrip line in free space is

$$Z_{01} = Z_0|_{(\varepsilon_r=1)} = 60 \ln \left[\frac{F_1 h}{w} + \sqrt{1 + \left(\frac{2h}{w}\right)^2} \right] \tag{4.19}$$

$$\text{and} \qquad F_1 = 6 + (2\pi - 6) \exp\left\{ -(30.666h/w)^{0.7528} \right\}. \tag{4.20}$$

The accuracy of Equation (4.15) is better than 0.2% for $0.01 \le w/h \le 100$ and $1 \le \varepsilon_r \le 128$. Also, the accuracy of Equation (4.19) is better than 0.1% for $w/h < 1000$. Note that Z_0 has a maximum value when w is small and a minimum value when w is large.

Now consider the special case where w is vanishingly small. Then ε_e has its minimum value:

$$\varepsilon_e = \tfrac{1}{2}(\varepsilon_r + 1). \tag{4.21}$$

This leads to an approximate (and convenient) form of Equation (4.15):

$$\varepsilon_e = \frac{(\varepsilon_r+1)}{2} + \frac{(\varepsilon_r-1)}{2} \frac{1}{\sqrt{1+12h/w}}. \tag{4.22}$$

This approximation has its greatest error for low and high ε_r and narrow lines, $w/h \ll 1$, where the maximum error is 1%. Again, Equation (4.18) is used to calculate the characteristic impedance. The more exact analysis, represented by Equation (4.15), was used to develop Table 4-2, which can be used in the design of microstrip.

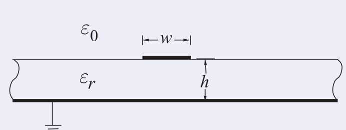
EXAMPLE 4.2 Microstrip Characteristic Impedance Calculation

The strip of a microstrip line has a width of 600 μm and is fabricated on a lossless substrate that is 635 μm thick and has a relative permittivity of 4.1.

(a) What is the effective relative permittivity?
(b) What is the characteristic impedance?
(c) What is the propagation constant at 5 GHz ignoring any losses?

Solution:

Use the formulas for effective permittivity, characteristic impedance, and attenuation constant from Section 4.4.2 with $w = 600$ μm ; $h = 635$ μm ; $\varepsilon_r = 4.1$; $w/h = 600/635 = 0.945$.



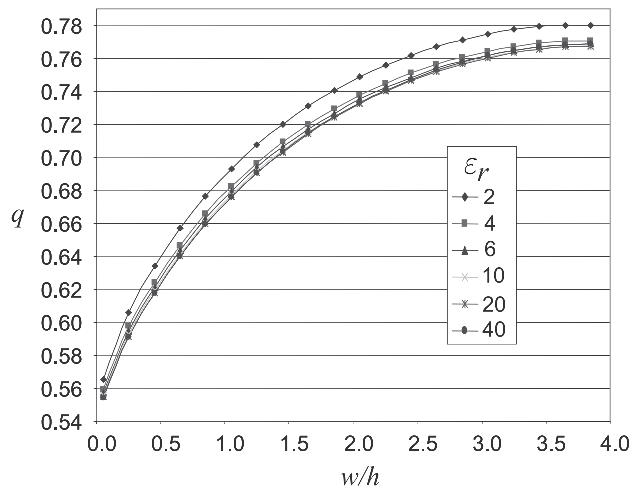


Figure 4-7: Dependence of the q factor of a microstrip line at 1 GHz for various permittivities and aspect (w/h) ratios. (Data obtained from EM field simulations using Sonnet.)

(a)
$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{10h}{w}\right)^{-a \cdot b}$$

From Equations (4.16) and (4.17),

$$a = 1 + \frac{1}{49}\ln\left[\frac{(w/h)^4 + \{w/(52h)\}^2}{(w/h)^4 + 0.432}\right] + \frac{1}{18.7}\ln\left[1 + \left(\frac{w}{18.1h}\right)^3\right] = 0.991,$$

$$b = 0.564\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.053} = 0.541.$$

From Equation (4.15), $\varepsilon_e = 2.967$.

(b) In free space,
$$Z_0|_{\text{air}} = 60\ln\left[\frac{F_1 \cdot h}{w} + \sqrt{1 + \left(\frac{2h}{w}\right)^2}\right],$$

where $F_1 = 6 + (2\pi - 6)\exp\left\{-(30.666h/\omega)^{0.7528}\right\}, \quad Z_0 = Z_0|_{\text{air}}/\sqrt{\varepsilon_e}$

$$Z_0|_{\text{air}} = 129.7\ \Omega \quad \text{and} \quad Z_0 = Z_0|_{\text{air}}/\sqrt{\varepsilon_e} = 75.3\ \Omega.$$

(c) $f = 5$ GHz, $\omega = 2\pi f$, $\gamma = \jmath\omega\sqrt{\mu_0\varepsilon_0\varepsilon_e} = \jmath 180.5/$ m.

4.4.3 Filling Factor

Defining a filling factor, q, provides useful insight into the distribution of energy in an inhomogeneous transmission line. The effective microstrip permittivity is

$$\varepsilon_e = 1 + q(\varepsilon_r - 1), \tag{4.23}$$

where for a microstrip line q has the bounds $\frac{1}{2} \le q \le 1$ and is almost independent of ε_r. A q of 1 indicates that all of the fields are in the dielectric region. The dependence of the q of a microstrip line at 1 GHz for various permittivities and aspect (w/h) ratios is shown in Figure 4-7. Fitting yields:

$$q = \frac{1}{2}\left(1 + \frac{1}{\sqrt{1 + 12h/w}}\right). \tag{4.24}$$

Table 4-2: Microstrip line normalized width $u\ (= w/h)$ and effective permittivity, ε_e, for specified characteristic impedance Z_0. Data derived from the analysis in Section 4.4.2.

Z_0 (Ω)	$\varepsilon_r = 4$ (SiO_2, FR4)		$\varepsilon_r = 10$ (Alumina)		$\varepsilon_r = 11.9$ (Si)	
	u	ε_e	u	ε_e	u	ε_e
140	0.171	2.718	0.028	5.914	0.017	6.907
139	0.176	2.720	0.029	5.917	0.018	6.910
138	0.181	2.722	0.030	5.919	0.019	6.914
137	0.185	2.723	0.031	5.922	0.020	6.919
136	0.190	2.725	0.032	5.924	0.021	6.923
135	0.195	2.727	0.033	5.927	0.022	6.925
134	0.201	2.729	0.035	5.931	0.022	6.927
133	0.206	2.731	0.036	5.933	0.023	6.930
132	0.212	2.733	0.037	5.936	0.024	6.934
131	0.217	2.734	0.038	5.939	0.025	6.937
130	0.223	2.736	0.040	5.942	0.026	6.941
129	0.229	2.738	0.043	5.949	0.028	6.948
128	0.235	2.740	0.044	5.951	0.029	6.951
127	0.241	2.742	0.046	5.955	0.030	6.954
126	0.248	2.744	0.048	5.958	0.031	6.957
125	0.254	2.746	0.050	5.962	0.033	6.963
124	0.261	2.748	0.052	5.966	0.034	6.966
123	0.268	2.750	0.054	5.970	0.035	6.969
122	0.275	2.752	0.056	5.973	0.038	6.977
121	0.283	2.755	0.058	5.977	0.039	6.980
120	0.290	2.757	0.061	5.982	0.041	6.985
119	0.298	2.759	0.063	5.985	0.043	6.990
118	0.306	2.761	0.066	5.990	0.045	6.995
117	0.314	2.763	0.068	5.993	0.047	6.999
116	0.323	2.766	0.071	5.998	0.049	7.004
115	0.331	2.768	0.074	6.003	0.051	7.008
114	0.340	2.771	0.077	6.007	0.053	7.013
113	0.349	2.773	0.080	6.012	0.055	7.017
112	0.359	2.776	0.083	6.016	0.057	7.022
111	0.368	2.778	0.086	6.021	0.060	7.028
110	0.378	2.781	0.089	6.025	0.062	7.032
109	0.389	2.783	0.093	6.031	0.065	7.038
108	0.399	2.786	0.097	6.036	0.068	7.044
107	0.410	2.789	0.100	6.040	0.071	7.050
106	0.421	2.791	0.104	6.046	0.074	7.055
105	0.432	2.794	0.109	6.052	0.077	7.061
104	0.444	2.797	0.113	6.057	0.080	7.066
103	0.456	2.800	0.117	6.062	0.084	7.073
102	0.468	2.803	0.122	6.069	0.087	7.079
101	0.481	2.806	0.127	6.075	0.091	7.085
100	0.494	2.809	0.132	6.081	0.095	7.092
99	0.507	2.812	0.137	6.087	0.099	7.099
98	0.521	2.815	0.143	6.094	0.103	7.105
97	0.535	2.819	0.148	6.100	0.108	7.113
96	0.550	2.822	0.154	6.106	0.112	7.120
95	0.565	2.825	0.160	6.113	0.117	7.127
94	0.580	2.829	0.167	6.121	0.122	7.135
93	0.596	2.832	0.173	6.127	0.128	7.144
92	0.612	2.836	0.180	6.134	0.133	7.151
91	0.629	2.839	0.187	6.142	0.139	7.159
90	0.646	2.843	0.195	6.150	0.145	7.168
89	0.664	2.847	0.202	6.157	0.151	7.176
87	0.701	2.855	0.219	6.173	0.164	7.193
86	0.721	2.859	0.228	6.182	0.171	7.203
85	0.740	2.863	0.237	6.190	0.179	7.213
84	0.761	2.867	0.246	6.198	0.187	7.223
83	0.782	2.872	0.256	6.208	0.195	7.233
82	0.804	2.876	0.266	6.216	0.203	7.242
81	0.826	2.881	0.277	6.226	0.212	7.253
80	0.849	2.885	0.288	6.235	0.221	7.263

Table 4-2 continued.

Z_0 (Ω)	$\varepsilon_r = 4$ (SiO_2, FR4)		$\varepsilon_r = 10$ (Alumina)		$\varepsilon_r = 11.9$ (Si)	
	u	ε_e	u	ε_e	u	ε_e
79	0.873	2.890	0.299	6.245	0.230	7.274
78	0.898	2.895	0.311	6.255	0.240	7.285
77	0.923	2.900	0.324	6.265	0.251	7.297
76	0.949	2.905	0.337	6.276	0.262	7.309
75	0.976	2.910	0.350	6.286	0.273	7.321
74	1.003	2.915	0.364	6.297	0.285	7.333
73	1.032	2.921	0.379	6.309	0.297	7.345
72	1.062	2.926	0.394	6.320	0.310	7.359
71	1.092	2.932	0.410	6.332	0.323	7.371
70	1.123	2.937	0.426	6.344	0.338	7.386
69	1.156	2.943	0.444	6.357	0.352	7.399
68	1.190	2.949	0.462	6.369	0.368	7.414
67	1.224	2.955	0.480	6.382	0.384	7.429
66	1.260	2.961	0.500	6.396	0.400	7.444
65	1.298	2.968	0.520	6.410	0.418	7.460
64	1.336	2.974	0.541	6.424	0.436	7.476
63	1.376	2.980	0.563	6.439	0.455	7.492
62	1.417	2.987	0.586	6.454	0.475	7.509
61	1.460	2.994	0.610	6.470	0.496	7.527
60	1.504	3.001	0.635	6.486	0.518	7.545
59	1.551	3.008	0.661	6.502	0.541	7.564
58	1.598	3.015	0.688	6.519	0.564	7.583
57	1.648	3.022	0.717	6.538	0.589	7.603
56	1.700	3.030	0.746	6.556	0.616	7.624
55	1.753	3.037	0.777	6.575	0.643	7.645
54	1.809	3.045	0.809	6.594	0.672	7.667
53	1.867	3.053	0.843	6.614	0.702	7.690
52	1.927	3.061	0.878	6.635	0.733	7.713
51	1.991	3.069	0.915	6.657	0.766	7.738
50	2.056	3.077	0.954	6.679	0.800	7.763
49	2.125	3.086	0.995	6.702	0.837	7.790
48	2.197	3.094	1.037	6.726	0.875	7.817
47	2.272	3.103	1.081	6.750	0.914	7.845
46	2.350	3.112	1.128	6.775	0.956	7.874
45	2.432	3.121	1.177	6.801	1.000	7.904
44	2.518	3.131	1.229	6.828	1.047	7.936
43	2.609	3.140	1.283	6.856	1.096	7.968
42	2.703	3.150	1.340	6.884	1.147	8.002
41	2.803	3.160	1.400	6.913	1.201	8.036
40	2.908	3.171	1.464	6.944	1.259	8.072
39	3.019	3.181	1.531	6.974	1.319	8.108
38	3.136	3.192	1.602	7.006	1.384	8.147
37	3.259	3.203	1.677	7.039	1.452	8.186
36	3.390	3.214	1.757	7.073	1.524	8.226
35	3.528	3.226	1.841	7.108	1.600	8.268
34	3.675	3.237	1.931	7.143	1.682	8.311
33	3.831	3.250	2.027	7.180	1.769	8.355
32	3.997	3.262	2.129	7.218	1.862	8.402
31	4.174	3.275	2.238	7.258	1.961	8.449
30	4.364	3.288	2.355	7.298	2.067	8.498
29	4.567	3.301	2.480	7.340	2.181	8.549
28	4.785	3.315	2.615	7.384	2.304	8.601
27	5.020	3.329	2.760	7.428	2.436	8.655
26	5.273	3.344	2.917	7.475	2.579	8.712
25	5.547	3.359	3.087	7.523	2.734	8.770
24	5.845	3.374	3.272	7.573	2.902	8.831
23	6.169	3.390	3.474	7.625	3.086	8.894
22	6.523	3.407	3.694	7.679	3.287	8.960
21	6.912	3.424	3.936	7.734	3.508	9.028
20	7.341	3.441	4.203	7.793	3.752	9.100
19	7.815	3.459	4.499	7.854	4.022	9.174
18	8.344	3.478	4.829	7.917	4.323	9.252
17	8.936	3.497	5.199	7.983	4.661	9.334
16	9.603	3.517	5.616	8.053	5.043	9.419
15	10.361	3.538	6.090	8.126	5.476	9.509
14	11.229	3.559	6.633	8.202	5.972	9.604
13	12.233	3.581	7.262	8.282	6.547	9.704
12	13.407	3.604	7.997	8.367	7.219	9.809
11	14.798	3.628	8.868	8.456	8.016	9.920
10	16.471	3.652	9.916	8.550	8.975	10.038

4.5 Microstrip Design Formulas

The formulas developed in Section 4.4.2 enable the electrical characteristics to be determined given the material properties and the physical dimensions of a microstrip line. In design, the physical dimensions must be determined given the desired electrical properties. Several people have developed procedures that can be used to synthesize microstrip lines. This subject is considered in much more depth in [4], and here just one approach is reported. The formulas are useful outside the range indicated, but with reduced accuracy. Again, these formulas are the result of curve fits, but starting with physically based equation forms.

4.5.1 High Impedance

For narrow strips, that is, when $Z_0 > (44 - 2\varepsilon_r)\ \Omega$,

$$\frac{w}{h} = \left(\frac{\exp H'}{8} - \frac{1}{4\exp H'}\right)^{-1}, \tag{4.25}$$

where $$H' = \frac{Z_0\sqrt{2(\varepsilon_r+1)}}{119.9} + \frac{1}{2}\left(\frac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right). \tag{4.26}$$

For $Z_0 > (63 - 2\varepsilon_r)\ \Omega$,

$$\varepsilon_e = \frac{\varepsilon_r+1}{2}\left[1 + \frac{29.98}{Z_0}\left(\frac{2}{\varepsilon_r+1}\right)^{1/2}\left(\frac{\varepsilon_r-1}{\varepsilon_r+1}\right)\left(\ln\frac{\pi}{2} + \frac{1}{\varepsilon_r}\ln\frac{4}{\pi}\right)\right]^2. \tag{4.27}$$

The formula for ε_e is accurate to better than 1% for $Z_0 > (44 - 2\varepsilon_r)\ \Omega$ (i.e $w/h < 1.3$) for $8 < \varepsilon_r < 12$. Overall the synthesis of w/h has an accuracy of better than 1%.

4.5.2 Low Impedance

Strips with low Z_0 are relatively wide and the formulas below can be used when $Z_0 < (44 - 2\varepsilon_r)\ \Omega$. The cross-sectional geometry is given by

$$\frac{w}{h} = \frac{2}{\pi}\left[(d_{\varepsilon_r} - 1) - \ln(2d_{\varepsilon_r} - 1)\right] + \frac{(\varepsilon_r - 1)}{\pi\varepsilon_r}\left[\ln(d_{\varepsilon_r} - 1) + 0.293 - \frac{0.517}{\varepsilon_r}\right], \tag{4.28}$$

where $$d_{\varepsilon_r} = \frac{59.95\pi^2}{Z_0\sqrt{\varepsilon_r}}. \tag{4.29}$$

For $Z_0 < (63 - 2\varepsilon_r)\ \Omega$

$$\varepsilon_e = \frac{\varepsilon_r}{0.96 + \varepsilon_r(0.109 - 0.004\varepsilon_r)[\log(10 + Z_0) - 1]}. \tag{4.30}$$

The expression for ε_e is accurate to better than 1% for $8 < \varepsilon_r < 12$ and $8 \le Z_0 \le (63 - 2\varepsilon_r)\ \Omega$.

EXAMPLE 4.3 Microstrip Design

Design a microstrip line to have a characteristic impedance of 75 Ω at 10 GHz. The microstrip a substrate that is 500 μm thick with a relative permittivity of 5.6. (a) What is the width of the line? (b) What is the effective permittivity of the line?

Solution:

(a) The high-impedance (or narrow-strip) formula (Equation (4.25)) is to be used for $Z_0 > (44 - \varepsilon_r)$ $[= (44 - 5.6) = 38.4]$ Ω.
With $\varepsilon_r = 5.6$ and $Z_0 = 75$ Ω, Equation (4.26) yields $H' = 2.445$. From Equation (4.25), $w/h = 0.704$, thus $w = w/h \times h = 0.704 \times 500$ μm $= 352$ μm.
(b) The effective permittivity formula is Equation (4.27), and so $\varepsilon_e = 3.82$.

4.6 Summary

This chapter considered micostrip, the most important planar transmission line. At frequencies below 1 GHz, economics require that standard FR4 circuit boards be used. The weave of a conventional FR4 circuit board can be a significant fraction of critical transmission line dimensions and so affect electrical performance. Nonwoven substrates and sometimes hard substrates such as alumina ceramic, sapphire, or silicon crystal wafers are often required. These provide higher-dimensional tolerance than can be achieved using conventional woven FR4 substrates. In general, once a design has been optimized in fabrication so that the desired electrical characteristics are obtained, microwave circuits using planar transmission lines can be cheaply and repeatably manufactured in volume.

4.7 References

[1] E. Hammerstad and O. Jensen, "Accurate models for microstrip computer-aided design," in *1980 IEEE MTT-S Int. Microwave Symp. Digest*, May 1980, pp. 407–409.

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[3] E. O. Hammerstad, "Equations for microstrip circuit design," in *5th European Microwave Conf.*, Sep. 1975, pp. 268–272.

[4] T. Edwards and M. Steer, *Foundations for Microstrip Circuit Design*. John Wiley & Sons, 2016.

4.8 Exercises

1. A microstrip line on 250 μm thick GaAs has a minimum and maximum strip widths of 50 μm and 250 μm respectively. What is the range of characteristic impedances that can be used in design?

2. A microstrip line with a substrate having a relative permittivity of 10 has an effective permittivity of 8. What is the wavelength of a 10 GHz signal propagating on the microstrip?

3. A microstrip line has a width of 500 μm and a substrate that is 635 μm thick with a relative permittivity of 20. What is the effective permittivity of the line?

4. The strip of a microstrip has a width of 250 μm and is fabricated on a lossless substrate that is 500 μm thick and has a relative permittivity of 2.3. [Parallels Example 4.2]
   (a) What is the effective relative permittivity of the line?
   (b) What is the characteristic impedance of the line?
   (c) What is the propagation constant at 3 GHz ignoring any losses?
   (d) If the strip has a resistance of 0.5 Ω/cm and the ground plane resistance can be ignored, what is the attenuation constant of the line at 3 GHz?

5. A microstrip line on a 250 μm-thick silicon substrate has a width of 200 μm. Use Table 4-2.
   (a) What is line's effective permittivity.
   (b) What is its characteristic impedance?
6. A 600 μm-wide microstrip line on a 500 μm-thick alumina substrate. Use Table 4-2.
   (a) What is line's effective permittivity.
   (b) What is its characteristic impedance?
7. A microstrip line on a 1 mm-thick FR4 substrate has a width of 0.497 mm. Use Table 4-2.
   (a) What is line's effective permittivity.
   (b) What is its characteristic impedance?
8. Consider a microstrip line on a substrate with a relative permittivity of 12 and thickness of 1 mm.
   (a) What is the minimum effective permittivity of the microstrip line if there is no limit on the minimum or maximum width of the strip?
   (b) What is the maximum effective permittivity of the microstrip line if there is no limit on the minimum or maximum width of the strip?
9. A microstrip line has a width of 1 mm and a substrate that is 1 mm thick with a relative permittivity of 20. What is the geometric filling factor of the line?
10. The substrate of a microstrip line has a relative permittivity of 16 but the calculated effective permittivity is 12. What is the filling factor?
11. A microstrip line has a strip width of 250 μm and a substrate with a relative permittivity of 10 and a thickness of 125 μm. What is the filling factor?
12. A microstrip line has a strip width of 250 μm and a substrate with a relative permittivity of 4 and thickness of 250 μm. Determine the line's filling factor and thus its effective relative permittivity.
13. A microstrip line has a strip with a width of 100 μm and the substrate which is 250 μm thick and a relative permittivity of 8.
   (a) What is the filling factor, q, of the line?
   (b) What is the line's effective relative permittivity?
   (c) What is the characteristic impedance of the line?
14. An inhomogeneous transmission line is fabricated using a medium with a relative permittivity of 10 and has an effective permittivity of 7. What is the fill factor q?
15. A microstrip technology uses a substrate with a relative permittivity of 10 and thickness of 400 μm. The minimum strip width is 20 μm. What is the highest characteristic impedance that can be achieved?
16. A microstrip transmission line has a characteristic impedance of 75 Ω, a strip resistance of 5 Ω/m, and a ground plane resistance of 5 Ω/m. The dielectric of the line is lossless.
   (a) What is the total resistance of the line in Ω/m?
   (b) What is the attenuation constant in Np/m?
   (c) What is the attenuation constant in dB/cm?
17. A microstrip line has a characteristic impedance of 50 Ω, a strip resistance of 10 Ω/m, and a ground plane resistance of 3 Ω/m.
   (a) What is the total resistance of the line in Ω/m?
   (b) What is the attenuation constant in Np/m?
   (c) What is the attenuation constant in dB/cm?
18. A microstrip line has 10 μm-thick gold metallization for both the strip and ground plane. The strip has a width of 125 μm and the substrate is 125 μm thick.
   (a) What is the low frequency resistance (in Ω/m) of the strip?
   (b) What is the low frequency resistance of the ground plane?
   (c) What is the total low frequency resistance of the microstrip line?
19. A 50 Ω microstrip line has 10 μm-thick gold metallization for both the strip and ground plane. The strip has a width of 250 μm and the lossless substrate is 250 μm thick.
   (a) What is the low frequency resistance (in Ω/m) of the strip?
   (b) What is the low frequency resistance of the ground plane?
   (c) What is the total low frequency resistance of the microstrip line?
   (d) What is the attenuation in dB/m of the line at low frequencies?
20. A 50 Ω microstrip line with a lossless substrate has a 0.5 mm-wide strip with a sheet resistance of 1.5 mΩ and the ground plane resistance can be ignored. What is the attenuation constant at 1 GHz? [Parallels Example 4.0]
21. A microstrip line operating at 10 GHz has a substrate with a relative permittivity of 10 and a loss tangent of 0.005. It has a characteristic impedance of 50 Ω and an effective permittivity of 7.
   (a) What is the conductance of the line in S/m?
   (b) What is the attenuation constant in Np/m?
   (c) What is the attenuation constant in dB/cm?

22. A microstrip line has the per unit length parameters L = 2 nH/m and C = 1 pF/m, also at 10 GHz the substrate has a conductance G of 0.001 S/m. The substrate loss is solely due to dielectric relaxation loss and there is no substrate conductive loss. The resistances of the ground and strip are zero.
    (a) What is G at 1 GHz?
    (b) What is the magnitude of the characteristic impedance at 1 GHz?
    (c) What is the dielectric attenuation constant of the line at 1 GHz in dB/m?
23. A microstrip line has the per unit length parameters L = 1 nH/m and C = 1 pF/m, also at 1 GHz the substrate has a conductance G of 0.001 S/m. The substrate loss is solely due to dielectric relaxation loss and there is no substrate conductive loss. The resistance of the strip is 0.5 Ω/m and the resistance of the ground plane is 0.1 Ω/m.
    (a) What is the per unit length resistance of the microstrip line at 1 GHz?
    (b) What is the magnitude of the characteristic impedance at 1 GHz?
    (c) What is the conductive attenuation constant in Np/m?
    (d) What is the dielectric attenuation constant of the line at 1 GHz in dB/m?
24. A microstrip line operating at 2 GHz has perfect metallization for both the strip and ground plane. The strip has a width of 250 μm and the substrate is 250 μm thick with a relative permittivity of 10 and a loss tangent of 0.001.
    (a) What is the filling factor, q, of the line?
    (b) What is the line's effective relative permittivity?
    (c) What is the line's attenuation in Np/m?
    (d) What is the line's attenuation in dB/m?
25. A 50 Ω microstrip line operating at 1 GHz has perfect metallization for both the strip and ground plane. The substrate has a relative permittivity of 10 and a loss tangent of 0.001. Without the dielectric the line has a capacitance of 100 pF/m.
    (a) What is the line conductance in S/m?
    (b) What is the line's attenuation in Np/m?
    (c) What is the line's attenuation in dB/m?
26. Design a microstrip line having a 50 Ω characteristic impedance. The substrate has a permittivity of 2.3 and is 250 μm thick. The operating frequency is 18 GHz. You need to determine the width of the microstrip line.
27. Design a microstrip line to have a characteristic impedance of 65 Ω at 5 GHz. The substrate is 635 μm thick with a relative permittivity of 9.8. Ignore the thickness of the strip. [Parallels Example 4.3]
    (a) What is the width of the line?
    (b) What is the effective permittivity of the line?
28. Design a microstrip line to have a characteristic impedance of 20 Ω. The microstrip is to be constructed on a substrate that is 1 mm thick with a relative permittivity of 12. [Parallels Example 4.3]
    (a) What is the width of the line? Ignore the thickness of the strip and frequency-dependent effects.
    (b) What is the effective permittivity of the line?

4.8.1 Exercises by Section

†challenging

§4.4 $1, 2, 3^{\dagger}, 4^{\dagger}, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22^{\dagger}, 23^{\dagger}, 24, 25$

§4.5 $26^{\dagger}, 27^{\dagger}, 28^{\dagger}$

4.8.2 Answers to Selected Exercises

3	12.75
4(c)	$\jmath 84.1\ \text{m}^{-1}$
16(c)	0.579 dB/m
17(a)	13 Ω/m
22(b)	44.72 Ω
28(b)	9.17

CHAPTER 5

Extraordinary Transmission Line Effects

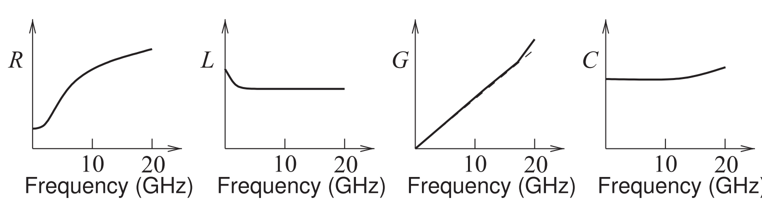
5.1 Introduction

Previous chapters discussed the low-frequency operation of transmission lines. This chapter describes the origins of frequency-dependent behavior. Figure 5-1 shows the typical frequency dependence of a line's $RLGC$ parameters, and, except with semiconductor substrates, G is usually negligible. This is called dispersion and a typical example is the spreading out of a pulse, see Figure 5-2.

The major limitation on the dimensions and maximum operating frequency of a transmission line is determined by the origination of higher-order modes (i.e., orientations of the fields). Different modes on a transmission line travel at different velocities. Thus the problem is that if a signal is split between two modes, then information sent from one end of the line will reach the other end in two packets arriving at different times. Multimoding must always be avoided.

5.2 Frequency-Dependent Characteristics

In this section the origins of the frequency-dependent behavior of a microstrip line are examined. The most important frequency-dependent



(a) R (b) L (c) G (d) C

Figure 5-1: Frequency dependence of transmission line parameters.

effects are

- changes of material properties (permittivity, permeability, and conductivity) with frequency (Section 5.2.1),
- current bunching (discussed in Section 5.2.3),
- skin effect (Sections 5.2.4 and 5.2.5),
- internal conductor inductance variation (Section 5.2.4),
- dielectric dispersion (Section 5.2.6), and
- multimoding (Section 5.3).

5.2.1 Material Dependency

Changes of permittivity, permeability, and conductivity with frequency are properties of the materials used. Fortunately the characteristics microwave materials are almost independent of frequency, at least up to 300 GHz.

5.2.2 Frequency-Dependent Charge Distribution

Skin effect, current bunching, and internal conductor inductance are all due to the necessary delay in transferring EM information from one location to another. This information cannot travel faster than the speed of light in the medium. In a dielectric material the speed of an EM wave will be slower than that in free space by a factor of $\sqrt{\varepsilon_r}$, e.g. $c/3.2$ for $\varepsilon_r = 10$. The velocity in a conductor is extremely low, around $c/1000$, because of high conductivity. In brief, current bunching is due to changes related to the finite velocity of information transfer through the dielectric, and skin effect is due to the very slow speed of information transfer inside a conductor. As frequency increases, only limited information to rearrange charges can be sent before the polarity of the signal reverses and information is 'sent' to reverse the changes. The skin and charge-bunching effects on a microstrip line are illustrated in Figure 5-3.

5.2.3 Current Bunching

Consider the microstrip charge distribution shown in Figure 5-3. The thickness of the microstrip is often a significant fraction of its width, although this is exaggerated here.

The charge distribution shown in Figure 5-3(a) applies when there is a positive DC voltage on the strip and the positive charges on the top conductor arranged with a fairly uniform distribution. The individual positive charges tend to repel each other, but this has little effect on the

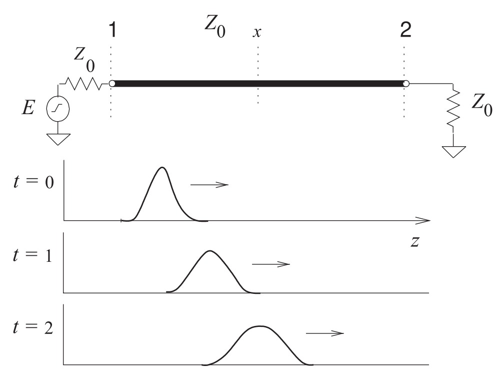


Figure 5-2: Dispersion of a pulse along a line.

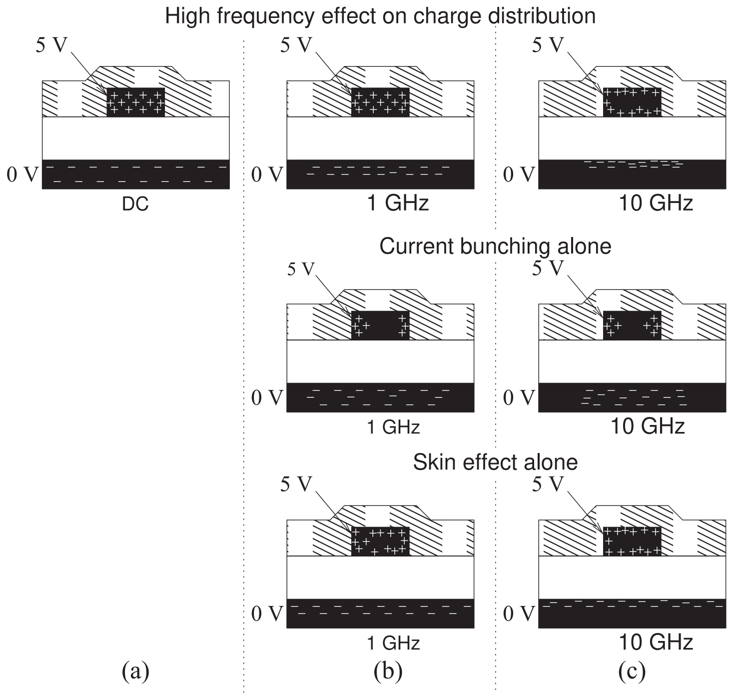


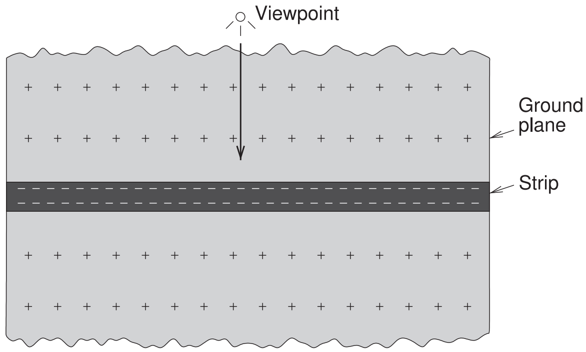
Figure 5-3: Cross-sectional view of the charge distribution on an interconnect at different frequencies. The + and − indicate charge concentrations of different polarity and corresponding current densities. There is no current bunching or skin effect at DC.

charge distribution at low frequencies for practical conductors with finite conductivity. On the ground plane there are balancing negative charges which are uniformly distributed across the whole of the ground plane. The charge distribution at DC, indicates that current would flow uniformly throughout the strip and the return current in the ground plane would be distributed over the whole of the ground plane.

The charge distribution becomes less uniform as frequency increases and eventually the signal changes so quickly that information to rearrange charges on the ground plane is soon (half a period latter) countered by reverse instructions. Thus the charge distribution depends on how fast the signaling changes. One way of looking at this effect is to view the charges on the strip of the microstrip line at one time. This is shown in Figure 5-4 for a DC signal on the line and for a high-frequency signal.

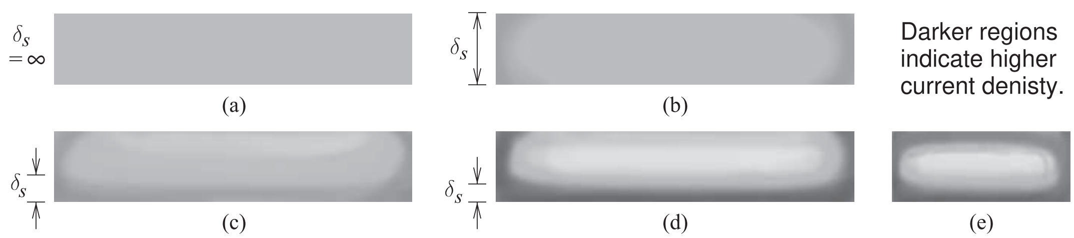
The electric field lines, which must originate and terminate on charges, will concentrate in the substrate more closely under the strip as frequency increases. The two major effects are that the effective permittivity of the microstrip line increases with frequency, and resistive loss increases as the current density in the ground, which corresponds to the net charge density, increases. Thus the line resistance and capacitance increase with frequency, see Figures 5-1(a and d). For the majority of substrates G is due to dielectric relaxation and so increases linearly with frequency with a superlinear increase at very high frequencies when the electric field is more concentrated in the dielectric, see Figure 5-1(c).

In the frequency domain the current bunching effects are seen in the higher-frequency views shown in Figures 5-3(b and c). (The concentration of charges near the metal surface is a separate effect known as the skin effect.)



(a) DC (b) High frequency

Figure 5-4: Current bunching effect in time. Positive and negative charges are shown on the strip and on the ground plane. The viewpoint is on the ground plane.



(a) (b) (c) (d) (e)

Figure 5-5: Cross-sections of the strip of a microstrip line showing the impact of skin effect and current bunching on current density: (a) at dc (uniform current density); (b) the strip thickness, t, is equal to the skin depth, δ_s (i.e. at a low microwave frequency); (c) $t = 3\delta_s$; (d) $t = 5\delta_s$ (i.e. at a high microwave frequency); and (e) $t = 5\delta_s$ for a narrow strip. The plots are the result of 3D simulations of a microstrip line using internal conductor gridding.

5.2.4 Skin Effect and Internal Conductor Inductance

At low frequencies, currents are distributed uniformly throughout a conductor. Thus there are magnetic fields inside the conductor and hence magnetic energy storage. As a result, there is additional inductance. Transferring charge to the interior of conductors is particularly slow, and as the frequency of the signal increases, charges are confined closer to the surface of the metal, see Figure 5-5. This phenomenon is known as the skin effect. With lower internal currents, the internal conductor inductance reduces and the total inductance of the line drops, see Figure 5-1(b). Only above a few gigahertz or so can the line inductance be approximated as being constant for a typical microstrip line.

The skin depth, δ_s, is the distance at which the electric field, or the charge density, reduces to $1/\mathrm{e}$ of its value at the surface. The skin depth is

$$\delta_s = 1/\sqrt{\pi f \mu_0 \sigma_2}. \tag{5.1}$$

Here f is frequency and σ_2 is the conductivity of the conductor. The (real part of the) permittivity and permeability of metals typically used for interconnects (e.g., gold, silver, copper, and aluminum) are that of free space as there is no mechanism to store additional electric or magnetic energy.

5.2.5 Skin Effect and Line Resistance

The skin effect is illustrated in Figure 5-3(b) at 1 GHz. The situation is more extreme as the frequency continues to increase. There are several important consequences of this. On the top conductor, as frequency increases, current flow is mostly concentrated near the surface of the conductors and the effective cross-sectional area of the conductor, as far as the current is concerned, is less. Thus the resistance of the top conductor increases. A more dramatic situation exists for the charge in the ground plane which becomes more concentrated under the strip. In addition to this, charges and current are confined to the skin of the ground conductor so that the frequency-dependent relative change of ground plane resistance with increasing frequency is greater than that of the strip.

The skin effect and current bunching result in frequency dependence of the line resistance, R, with

$$R(f) = \begin{cases} R(0) & f \text{ such that } t \leq 3\delta_s \\ R(0) + R_{\text{skin}}(f) & f \text{ such that } t > 3\delta_s, \end{cases} \tag{5.2}$$

where $R(0) = R_{\text{strip}}(0) + R_{\text{ground}}(0)$ is the resistance of the line at low frequencies. $R(f)$ describes the frequency-dependent line resistance that is due to both the skin effect and current bunching. Approximately,

$$R_{\text{skin}}(f) = R(0)k\sqrt{f}. \tag{5.3}$$

where k is a geometry-dependent constant. Note that it is pointless to make a strip or of ground thicker than three times the skin depth.

EXAMPLE 5.1 Skin Depth

Determine the skin depth for copper (Cu), silver (Ag), aluminum (Al), gold (Au), and titanium (Ti) at 100 MHz, 1 GHz, 10 GHz, and 100 GHz.

Solution:

The skin depth is calculated using Equation (5.1).

Metal	Resistivity ($\text{n}\Omega \cdot \text{m}$)	Conductivity (MS/m)	Skin depth, δ_s (μm)			
			100 MHz	1 GHz	10 GHz	100 GHz
Copper (Cu)	16.78	59.60	6.52	2.06	0.652	0.206
Silver (Ag)	15.87	63.01	6.34	2.01	0.634	0.201
Aluminum (Al)	26.50	37.74	8.19	2.59	0.819	0.259
Gold (Ag)	22.14	45.17	7.489	2.37	0.749	0.237
Titanium (Ti)	4200	0.2381	103.1	32.6	10.3	3.26

5.2.6 Dielectric Dispersion

Dispersion is principally the result of the propagation velocity of a sinusoidal signal being dependent on frequency since the effective permittivity is frequency-dependent. The electric field lines shift as a result with more of the electric energy being in the dielectric as frequency increases. At high frequencies the rearrangement results in the capacitance of the line increases—typically less than 10% from DC to 100 GHz.

The limits of $\varepsilon_{\rm e}(f)$ are readily established; at the low-frequency extreme it reduces to the static TEM value $\varepsilon_{\rm e}$ (or $\varepsilon_{\rm e}(0)$), while as frequency is increased indefinitely, $\varepsilon_e(f)$ approaches the substrate permittivity itself, ε_r. That is,

$$\varepsilon_e\,(f) \rightarrow \begin{cases} \varepsilon_e(0) & \text{as } f \rightarrow 0 \\ \varepsilon_r & \text{as } f \rightarrow \infty, \end{cases} \tag{5.4}$$

where $\varepsilon_e(0)$ is given by Equation 4.18. Detailed analysis yields

$$\varepsilon_e(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_e(0)}{1 + (f/f_a)^m}, \tag{5.5}$$

where m and f_a depend on w, h, and ε_r, see Section 24.3 of [1]. Very approximately $m \approx 2$ and $f_a \approx 50\ GHz$ for a 50 Ω microstrip line with/ a 500 μm substrate. The **frequency-dependent characteristic impedance** is

$$Z_0(f) = Z_0(0)\frac{\sqrt{\varepsilon_e(0)}}{\sqrt{\varepsilon_e(f)}}. \tag{5.6}$$

where $Z_0(0)$ is the low-frequency value given by Equation (4.15).

5.2.7 Summary

For microstrip, with increasing frequency the proportion of signal energy in the air region reduces and the proportion in the dielectric increases. The overall trend is for the fields to be more concentrated in the dielectric as frequency increases and thus effective relative permittivity increases.

5.3 Multimoding on Transmission Lines

Multimoding on a transmission line occurs when there are two or more EM field configurations that can support a propagating wave. Different field configurations travel at different speeds so that the information traveling in two modes will combine randomly and it will be impossible to discern the intended signal. It is critical that the dimensions of a transmission line be small enough to avoid multimoding. Larger dimensions however are more easily manufactured. With microstrip the quasi-TEM mode is supported from DC. Other modes can propagate above a cut-off frequency when transverse dimensions are larger than a quarter or half of a wavelength.

The important concept from EM throry is that electric and magnetic walls in the transverse direction (perpendicular to the direction of propagation) impose boundary conditions on the fields.

5.3.1 Multimoding and Electric and Magnetic Walls

The introduction of a magnetic wall greatly aids in the intuitive understanding of multimoding and in identifying problem situations. A magnetic wall can only be approximated since magnetic charges do not exist. An electric wall requires that the E field be perpendicular and the H field be parallel to the wall, see Figure 5-6. Similarly a magnetic wall requires that the H field be perpendicular and the E field be parallel to the wall, see Figure 5-7. A magnetic wall is approximated at the interface of the dielectric and air, see Figure 5-8(a), and near the edges of the strip, see Figure 5-8(b). The electric and magnetic walls establish boundary conditions for Maxwell's equations with the lowest frequency solutions shown in Table 5-9. With two electric or two magnetic walls, a TEM mode (having no field variations in the transverse plane) can be supported. Modes (other than TEM) with geometric variations in the transverse plane have a critical wavelength, λ_c, and hence a critical frequency, f_c, below which the mode cannot propagate. In Figure 5-9 the distance between the walls is d. For the case of two like walls (Figures 5-9(a and c)), $\lambda_c = 2h$, as one-half sinusoidal variation is required. With unlike walls (see Figure 5-9(b)), the varying modes are supported with just one-quarter sinusoidal variation, and so $\lambda_c = 4h$.

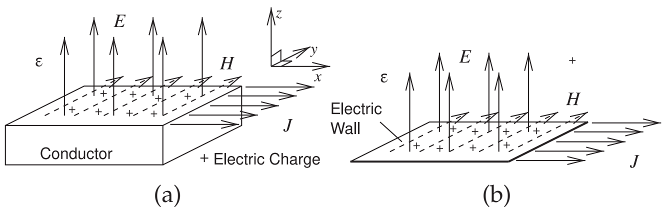


Figure 5-6: Properties of an electric wall: (a) the electrical field is perpendicular to a conductor and the magnetic field is parallel to it; and (b) a conductor can be approximated by an electric wall.

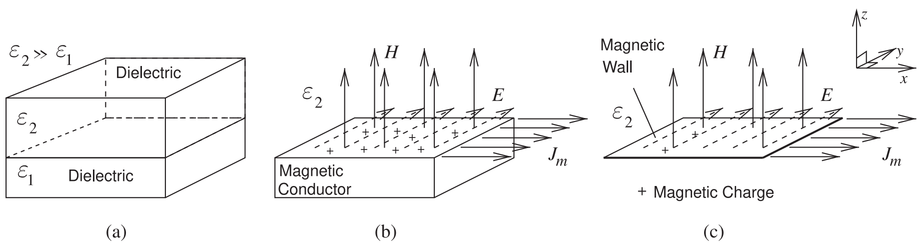


Figure 5-7: Properties of the EM field at a magnetic wall: (a) the interface of two dielectrics of contrasting permittivities approximates a magnetic wall; (b) the dielectric with lower permittivity can be approximated as a magnetic conductor; and (c) a magnetic wall.

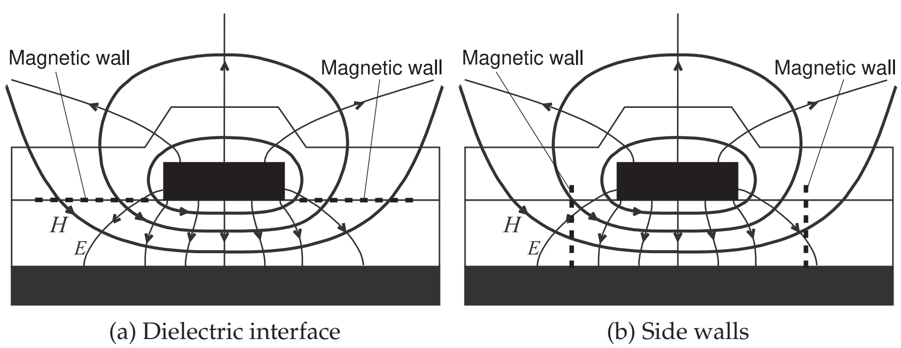


Figure 5-8: Approximate magnetic walls in microstrip where the H field is almost normal and the E field is almost parallel to the wall.

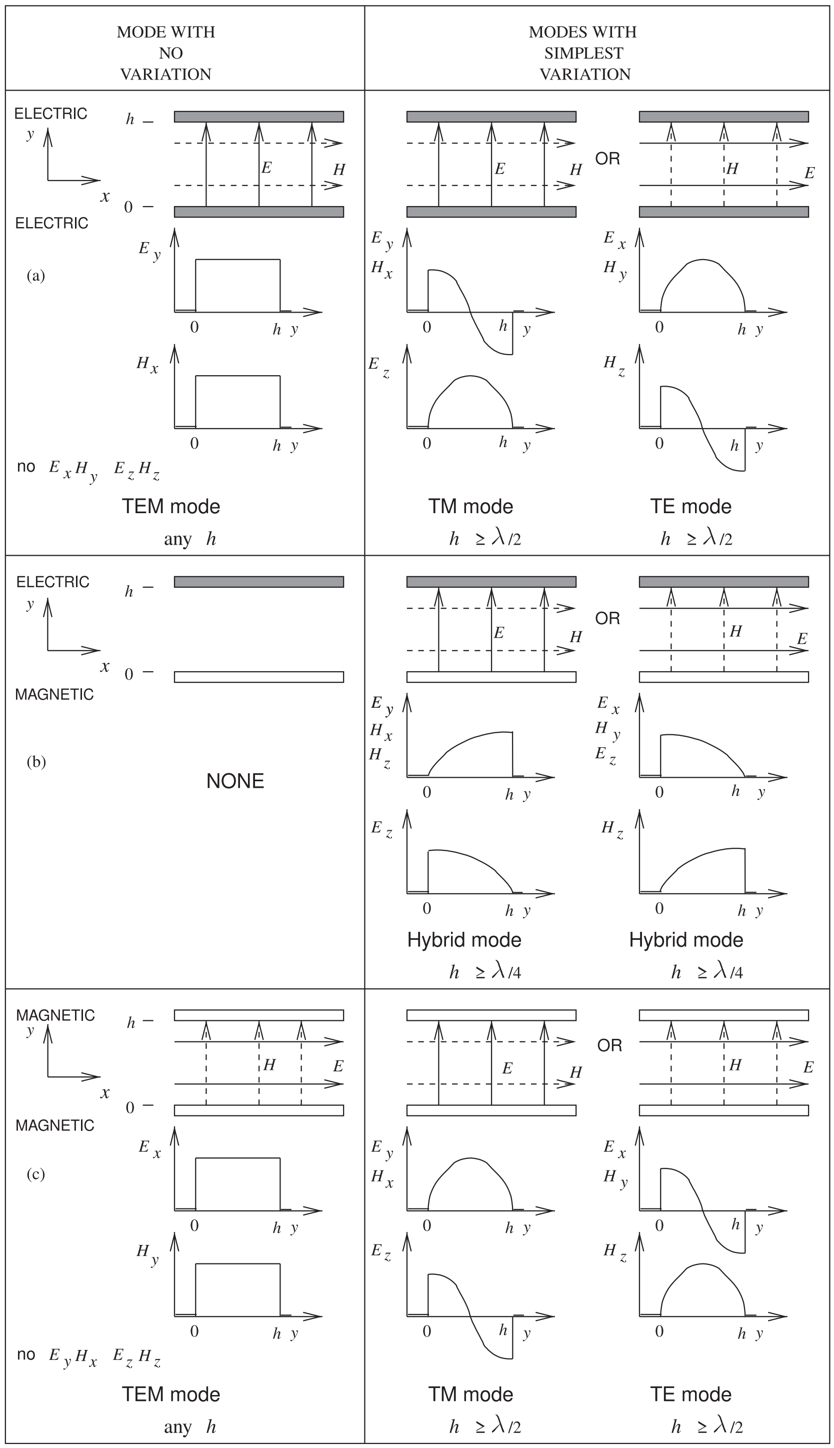


Figure 5-9: Lowest-order modes supported by combinations of electric and magnetic walls.

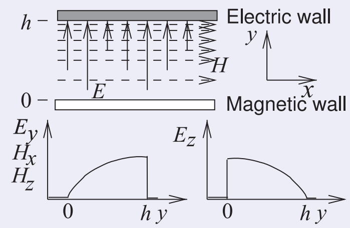
EXAMPLE 5.2 Modes and Electric and Magnetic Walls

A magnetic wall and an electric wall are 1 cm apart and are separated by a lossless material having $\varepsilon_r = 9$. What is the cut-off frequency of the lowest-order mode in this system?

Solution:

The EM field established by the electric and magnetic walls is described in Figure 5-9(b). There is no solution to the Maxwell's equations that has no variation of the EM fields since it is not possible to have a spatially uniform electric field which is perpendicular to an electric wall while also being perpendicular to a parallel magnetic wall. The other solutions of Maxwell's equations require that the fields vary spatially, i.e. curl. Without electric and magnetic walls the minimum distance over which the EM fields will fold back on to themselves is a wavelength. With parallel electric and magnetic wall separated by h the minimum distance for a solution of Maxwell's equations is a quarter wavelength, λ, of the walls as shown on the right in Figure 5-9(b). That is

$$h = \lambda/4 \quad \text{or} \quad \lambda = 4h = 4 \text{ cm} = \lambda_0/(\sqrt{\varepsilon_t \mu_r}). \tag{5.7}$$



Since the relative permeability has not been specified we assume that $\mu_r = 1$ so

$$\lambda_0 = \lambda\sqrt{9} = 12 \text{ cm} = c/f$$

The cut-off frequency is

$$\begin{aligned} f &= (2.998 \times 10^8 \text{ m/s})/(0.12 \text{ m}) \\ &= 2.498 \text{ GHz} \end{aligned} \tag{5.8}$$

5.4 Microstrip Operating Frequency Limitations

The higher-order modes, i.e. other than the quasi-TEM mode, with the lowest cut-off or critical frequency, f_c are usually (a) the lowest-order TM mode and (b) the lowest-order transverse microstrip resonance mode.

5.4.1 Microstrip Dielectric Mode (Slab Mode)

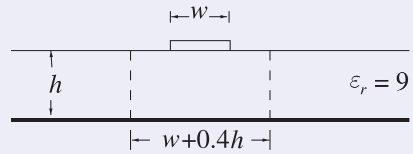
A dielectric on a ground plane with an air region (of a wavelength or more above it) can support a TM mode, variously called a microstrip dielectric mode, **substrate mode**, **microstrip TM mode**, or slab mode. Referring to Figure 5-9(b), the cut-off or critical frequency, $f_{c,\text{SLAB}}$ above which the slab mode exists is when the distance between the electric and magnetic walls $h > \lambda/4$, where $\lambda = \lambda_0/\sqrt{\varepsilon_r}$ is the wavelength in the dielectric, so

$$f_{c,\text{SLAB}} = \frac{c}{4h\sqrt{\varepsilon_r}}. \tag{5.9}$$

Experience is that the the critical frequency is less than this, e.g. 20% less, which is consistent with the distance between the electric and magnetic walls being slightly larger so that the magnetic wall is slightly into the air region. The critical frequency is also affected by discontinuities. This stresses the importance of measurements of a design in development and the design engineer aware of effects such as multimoding that are not fully accounted for in modeling. No software can properly account for all effects in part because EM simulation tools rely on symmetry, smooth surface, and dielectrics and conductors with uniform properties.

EXAMPLE 5.3 Dielectric Mode

The strip of a microstrip has a width of 1 mm and the substrate is 2.5 mm thick with a relative permittivity of 9. What is the lowest frequency that slab mode exists?



Solution:

The critical frequency comes from the thickness and permittivity of the substrate. The slab mode exists when a variation of the magnetic or electric field can be supported between the ground plane and the approximate magnetic wall at the substrate/air interface. This is when $h = \frac{1}{4}\lambda = \lambda_0/(4\sqrt{9}) = 2.5$ mm $\Rightarrow \lambda_0 = 3$ cm. Thus the critical slab mode critical frequency is

$$f_{c,\text{SLAB}} = 10 \text{ GHz}.$$

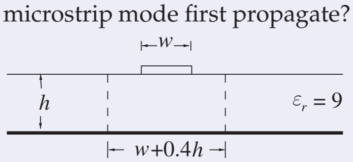
5.4.2 Higher-Order Microstrip Mode

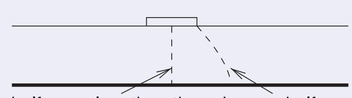
The next highest microstrip mode (or parallel-plate TE mode) above the quasi-TEM mode occurs when there is a half-sinusoidal variation of the electric field between the strip and the ground plane. This corresponds to Figure 5-9(a). However the first higher-order microstrip mode has been found be experiemntally found to exist at a lower frequency than derived using the parallel electrical wall model. This is because the microstrip fields do not follow the shortest distance between the strip and the ground plane. Thus the fields along the longer paths to the sides of the strip can vary at a lower frequency than on the direct path. With experimental support it has been established that the first higher-order microstrip mode can exist at frequencies above [2]

$$f_{\text{Higher-Microstrip}} = \frac{c}{4h\sqrt{\varepsilon_r - 1}}. \tag{5.10}$$

EXAMPLE 5.4 Higher-Order Microstrip Mode

The strip of a microstrip has a width of 1 mm and is fabricated on a lossless substrate that is 2.5 mm thick and has a relative permittivity of 9. At what frequency does the first higher microstrip mode first propagate?





Solution:

The higher-order microstrip mode occurs when a half-wavelength variation of the electric field between the strip and the ground plane can be supported. When $h = \lambda/2 = \lambda_0/(3 \cdot 2) = 2.5$ mm; that is, the mode will occur when $\lambda_0 = 15$ mm. So

$$f_{\text{Higher-Microstrip}} = 20 \text{ GHz}.$$

A better estimate of the frequency where the higher-order microstrip mode becomes a problem is given by Equation (5.10):

$$f_{\text{Higher-Microstrip}} = c/(4h\sqrt{\varepsilon_r - 1}) = 10.6 \text{ GHz}.$$

So two estimates have been calculated for the frequency at which the first higher-order microstrip mode can first exist. The first estimate is approximate and is based on a half-wavelength variation of the electric field confined to the direct path between the strip and the ground plane. The second estimate is more accurate as it considers that on the edge of the strip the fields follow a longer path to the ground plane. It is the half-wavelength variation on this longer path that determines if the higher-order microstrip mode will exist.

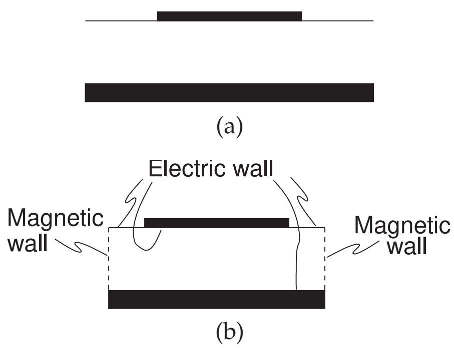


Figure 5-10: Approximation of a microstrip line as a waveguide: (a) cross section of microstrip; and (b) **microstrip waveguide model** having effective width $w + 0.4h$ with magnetic and electric walls.

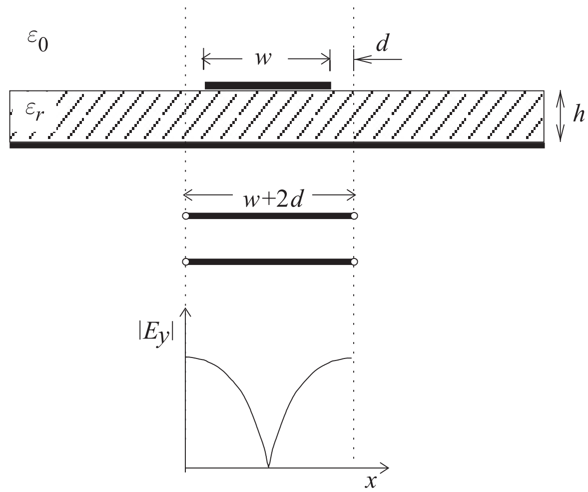


Figure 5-11: Transverse resonance: standing wave ($|E_y|$) and equivalent transmission line of length $w + 2d$, where $d = 0.2h$.

5.4.3 Transverse Microstrip Resonance

For a wide microstrip line, a transverse resonance mode can exist. This is the mode that occurs when EM energy bounces between the edges of the strip, see Figure 5-10. Here the microstrip is modelled by magnetic walls on the sides and an extended electrical wall on the top surface of the dielectric. The transverse resonance mode corresponds to the lowest-order H field variation between the magnetic walls. The equivalent circuit for the transverse-resonant mode is a resonant transmission line of length $w + 2d$, as shown in Figure 5-11, where $d = 0.2h$ accounts for the microstrip side fringing. A half-wavelength must be supported by the length $w + 2d$. Therefore the cutoff half-wavelength is

$$\frac{\lambda_c}{2} = w + 2d = w + 0.4h, \quad \text{that is,} \quad \frac{c}{2f_c\sqrt{\varepsilon_r}} = w + 0.4h. \tag{5.11}$$

Hence the critical frequency for transverse resonance is

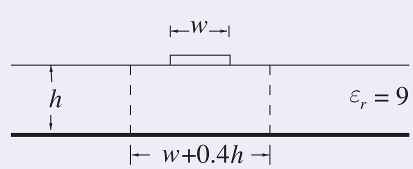
$$f_{c,\mathrm{TRAN}} = \frac{c}{\sqrt{\varepsilon_r}\,(2w + 0.8h)}. \tag{5.12}$$

EXAMPLE 5.5 Transverse Resonance Mode

The strip of a microstrip has a width of 1 mm and is fabricated on a lossless substrate that is 2.5 mm thick and has a relative permittivity of 9.

(a) At what frequency does the transverse resonance first occur?

(b) What is the operating frequency range of the microstrip line?



Solution:

h = 2.5 mm, $w = 1$ mm , $\lambda = \lambda_0/\sqrt{\varepsilon_r} = \lambda_0/3$

(a) The magnetic waveguide model of Figure 5-10 can be used in estimating the frequency at which this occurs. The frequency at which the first transverse resonance mode occurs is when there is a full half-wavelength variation of the magnetic field between the magnetic walls, that is, when $w + 0.4h = \lambda/2 = 2$ mm:

$$\frac{\lambda_0}{3 \cdot 2} = 2\ \text{mm} \Rightarrow \lambda_0 = 12\ \text{mm}, \qquad \text{and so} \qquad f_{c,\text{TRAN}} = 25\ \text{GHz}. \tag{5.13}$$

(b) All of the critical multimoding frequencies must be considered here and the minimum taken: for the slab mode, $f_{c,\text{SLAB}}$ (Equation (5.9)); for the higher-order microstrip mode, $f_{\text{High}-\text{Microstrip}}$ (Equation (5.10)); and for the transverse resonance mode (Equation (5.13)). So the operating frequency range is DC to 10 GHz.

5.4.4 Summary

There are three principal higher-order modes that need to be considered with microstrip transmission lines:

Mode	Critical frequency
Dielectric (or substrate) mode with discontinuity	Equation (5.9)
Higher order microstrip mode	Equation (5.10)
Transverse resonance mode	Equation (5.12)

The lowest frequency determines the upper frequency of transmission line operation with a single mode.

5.5 Summary

Microstrip can support multiple modes, but most are cut off by keeping transverse dimensions small with respect to a wavelength. When it is possible for a second (or higher-order) mode to exist, whether that mode is generated depends on the coupling mechanism between modes. This coupling mechanism is a discontinuity, which of course is common if circuit structures are to be incorporated. A microwave designer must always be aware of frequency dependence and multimoding and choose dimensions to avoid their occurrence except when the use is intentional and controlled.

5.6 References

[1] M. Steer, *Microwave and RF Design, Transmission Lines*, 3rd ed. North Carolina State University, 2019.

[2] T. Edwards and M. Steer, *Foundations for Microstrip Circuit Design*. John Wiley & Sons, 2016.

5.7 Exercises

1. What is the skin depth on a copper microstrip line at 10 GHz? Assume that the conductivity of the deposited copper forming the strip is half that of bulk single-crystal copper. Use the data in Example 5.1.
2. What is the skin depth on a silver microstrip line at 1 GHz? Assume that the conductivity of the fabricated silver conductor is 75% that of bulk single-crystal silver. Use the data in Example 5.1.
3. A magnetic wall and an electric wall are 2 cm apart and are separated by a lossless material having a relative permittivity of 10 and a relative permeability of 23. What is the cut-off frequency of the lowest-order mode in this system?
4. The strip of a microstrip has a width of 600 μm and is fabricated on a lossless substrate that is 1 mm thick and has a relative permittivity of 10.
   (a) Draw the microstrip waveguide model of the microstrip line. Put dimensions on your drawing.
   (b) Sketch the electric field distribution of the first transverse resonance mode and calculate the frequency at which the transverse resonance mode occurs.
   (c) Sketch the electric field distribution of the first higher-order microstrip mode and calculate the frequency at which it occurs.
   (d) Sketch the electric field distribution of the slab mode and calculate the frequency at which it occurs.
5. A microstrip line has a width of 352 μm and is constructed on a substrate that is 500 μm thick with a relative permittivity of 5.6.
   (a) Determine the frequency at which transverse resonance would first occur.
   (b) When the dielectric is slightly less than one-quarter wavelength in thickness the dielectric slab mode can be supported. Some of the fields will appear in the air region as well as in the dielectric, extending the effective thickness of the dielectric. Ignoring the fields in the air (use a one-quarter wavelength criterion), at what frequency will the dielectric slab mode first occur?
6. The strip of a microstrip has a width of 600 μm and uses a lossless substrate that is 635 μm thick and has a relative permittivity of 4.1.
   (a) At what frequency will the first transverse resonance occur?
   (b) At what frequency will the first higher-order microstrip mode occur?
   (c) At what frequency will the slab mode occur?
   (d) Identify the useful operating frequency range of the microstrip.
7. The strip of a microstrip has a width of 500 μm and is fabricated on a lossless substrate that is 635 μm thick and has a relative permittivity of 12. [Parallels Examples 5.3, 5.4, and 5.5]
   (a) At what frequency does the transverse resonance first occur?
   (b) At what frequency does the first higher-order microstrip mode first propagate?
   (c) At what frequency does the substrate (or slab) mode first occur?
8. The strip of a microstrip has a width of 250 μm and uses a lossless substrate that is 300 μm thick and has a relative permittivity of 15.
   (a) At what frequency does the transverse resonance first occur?
   (b) At what frequency does the first higher-order microstrip mode propagate?
   (c) At what frequency does the substrate (or slab) mode first occur?
   (d) What is the highest operating frequency of the microstrip?
9. A microstrip line has a strip width of 100 μm and is fabricated on a substrate that is 150 μm thick and has a relative permittivity of 9.
   (a) Draw the microstrip waveguide model and indicate and calculate the dimensions of the model.
   (b) Based only on the microstrip waveguide model, determine the frequency at which the first transverse resonance occurs?
   (c) Based on the microstrip waveguide model, determine the frequency at which the first higher-order microstrip mode occurs?
   (d) At what frequency will the slab mode occur? For this you cannot use the microstrip waveguide model.
10. A microstrip line has a strip width of 100 μm and is fabricated on a substrate that is 150 μm thick and has a relative permittivity of 9.
    (a) Define the properties of a magnetic wall.
    (b) Identify two situations where a magnetic wall can be used in the analysis of a microstrip line; that is, give two situations where a magnetic wall approximation can be used.
    (c) Draw the microstrip waveguide model and indicate and calculate the dimensions of the model.

11. The strip of a microstrip line has a width of 0.5 mm, and the microstrip substrate is 1 mm thick and has a relative permittivity of 9 and relative permeability of 1.
    (a) Draw the microstrip waveguide model and calculate the dimensions of the model. Clearly show the electric and magnetic walls in the model.
    (b) Use the microstrip waveguide model to calculate the cut off frequency of the transverse resonance mode?
    (c) A substrate mode can also be excited but the cut off frequency of this mode cannot be calculated using the microstrip waveguide model. Provide a brief description of the substrate mode and calculate the lowest frequency at which it can exist.
12. A microstrip technology uses a substrate with a relative permittivity of 10 and thickness of 400 μm. If the operating frequency is 10 GHz, what is the maximum width of the strip from higher-order mode considerations.
13. A microstrip line operating at 18 GHz has a 200 μm thick substrate with a relative permittivity of 20.
    (a) Determine the maximum width of the strip from higher-order mode considerations. Consider the transverse resonance mode, the higher-order microstrip mode, and the slab mode.
    (b) Thus determine the minimum achievable characteristic impedance.
14. A microstrip line has a strip width of 100 μm and is fabricated on a 150 μm-thick lossless substrate with a relative permittivity of 9.
    (a) Define the properties of a magnetic wall.
    (b) Identify two situations where a magnetic wall can be used in determining multimoding on a microstrip line. That is, give two locations where a magnetic wall approximation can be used.
15. Two magnetic walls are separated by 1 mm in a lossless material having a relative permittivity of 9 and a relative permeability of 1.
    (a) What is the wavelength of a 10 GHz signal in this material?
    (b) Now consider a field variation, i.e. a mode and not constant, established by the magnetic walls. Describe this lowest order field variation. That is how does the H field vary or how does the E field vary (one is sufficient)?
    (c) What is the lowest frequency at which a field variation can be supported by those walls in the specified medium?
16. A microstrip line has a strip width of 250 μm and a 300 μm thick substrate with a relative permittivity of 15. At what frequency can the substrate mode first occur?
17. A microstrip line has a strip width of 250 μm and a 300 μm thick substrate with a relative permittivity of 15. At what frequency can a higher-order microstrip mode first propagate?
18. A microstrip line has a strip width of 250 μm and a 300 μm thick substrate with a relative permittivity of 15. At what frequency can transverse resonance first occur?
19. The strip of a microstrip line has a width of 200 μm and the substrate is 400 μm thick and has a relative permittivity of 4.
    (a) Draw the effective waveguide model of a microstrip line with magnetic walls and an effective strip width, w_{eff}.
    (b) What is the effective relative permittivity of the microstrip waveguide model?
    (c) What is w_{eff}?
    (d) Can the lowest frequency at which the transverse resonance mode first occurs be determined from the microstrip waveguide model?

5.7.1 Exercises by Section

†challenging

§5.2 $1, 2$

§5.3 3

§5.4 $4^\dagger, 5^\dagger, 6^\dagger, 7^\dagger, 8^\dagger, 9^\dagger, 10^\dagger, 11^\dagger, 12, 13, 14, 15, 16, 17, 18, 19$

5.7.2 Answers to Selected Exercises

2 $2.315\ \mu\text{m}$

3 247 MHz

4(c) 25 GHz

5(c) DC to 63.4 GHz

6(d) DC $\leq f \leq 48.6$ GHz

8 104.6 GHz

11(b) 55.6 GHz

CHAPTER 6

Coupled Lines and Applications

6.1 Introduction

This chapter describes what happens when two transmission lines are so close together that the fields produced by one line interfere with the other line. Then a portion of the signal energy on one line is transferred to the other resulting in coupling. For microstrip lines the coupling reduces as the lines separate and usually the coupling is small enough to be ignored if the separation is at least three times the height of the strips. Transmission line coupling may be undesirable in many situations, but the phenomenon is exploited to realize many novel RF and microwave elements such as filters. A coupled pair of transmissions lines also enables the forward- and backward-traveling waves on a line to be separately measured.

The chapter begins with a discussion of the physics of coupling which leads to the preferred description of propagation on a pair of **parallel coupled line**s (**PCL**s) as supporting even and odd modes.

6.2 Physics of Coupling

If the fields of one transmission line intersect the region around another transmission line, then some of the energy propagating on the first line appears on the second. This coupling discriminates in terms of forward- and backward-traveling waves. In particular, consider the coupled microstrips shown in Figure 6-1. The direction of propagation comes from $\bar{\mathcal{E}} \times \bar{\mathcal{H}}$. Using the right-hand rule, the direction of propagation of the signal on the left-hand strip is out of the page. This is called the forward-traveling direction. Now consider the fields around the right-hand strip, the inactive line, and note the direction of the E field. The H field immediately to the left of the

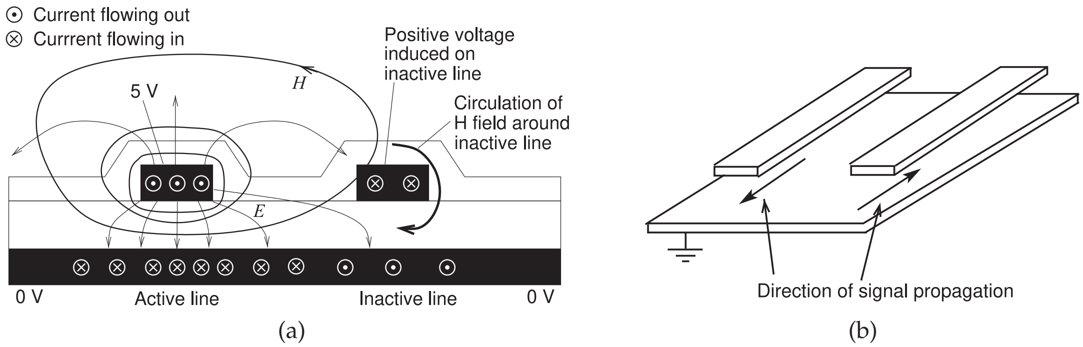


Figure 6-1: Edge-coupled microstrip lines with the left line driven with the forward-traveling field coming out of the page: (a) cross section of microstrip lines as found on a printed circuit board showing a conformal top passivation or solder resist layer; and (b) in perspective showing the direction of propagation of the driven line (left) and the direction of propagation of the main coupled signal on the victim line (right).

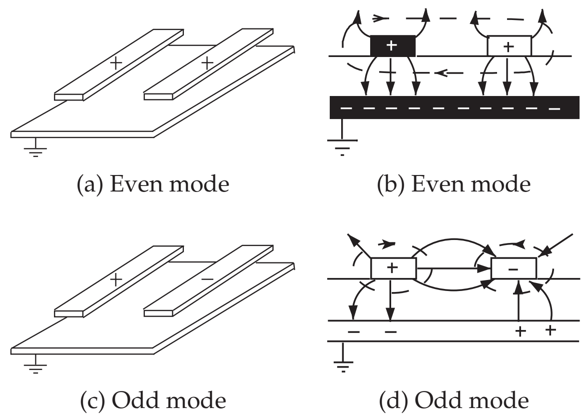


Figure 6-2: Modes on parallel-coupled microstrip lines. In (b) and (d) the electric fields are indicated as arrowed lines and the magnetic field lines are dashed.

right-hand strip is stronger than on the right of the strip. So effectively there is a clockwise circulation of the H field around the right hand strip. By applying $\bar{\mathcal{E}} \times \bar{\mathcal{H}}$ to the signal induced on the right-hand strip, it is seen that the induced signal travels into the page, or in the backward-traveling direction. The coupling with parallel-coupled microstrip lines is called backward-wave coupling. Both the even mode and the odd mode, and both have forward- and backward-traveling components (see Figure 6-2).

In the even mode (Figures 6-2(a and b)), the amplitude and polarity of the voltages on the two signal conductors are the same. In the odd mode (Figures 6-2(c and d)), the voltages on the two signal conductors are equal but have opposite polarity. Any EM field orientation on the coupled lines can be represented as a weighted addition of the even-mode and odd-mode field configurations. The actual fields will be a superposition of the even and odd modes. Also the voltages on the lines are composed of even and odd components.

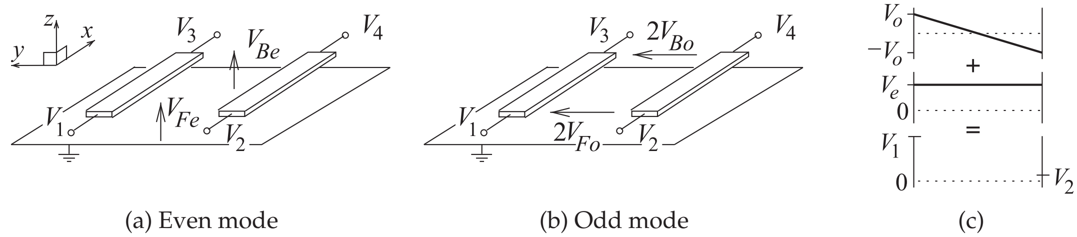


Figure 6-3: Definitions of total voltages and even- and odd-mode voltage phasors on a pair of coupled microstrip lines: (a) even-mode voltage definition; (b) odd-mode voltage definition; (c) depiction of how even- and odd-mode voltages combine to yield the total voltages on individual lines. F indicates front and B indicates back.

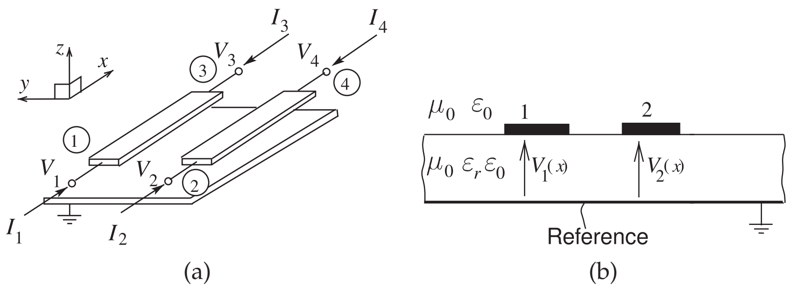


Figure 6-4: Coupled microstrip lines: (a) with total voltage and current phasors at the four terminals; and (b) cross section.

The characteristic impedances of the even and odd modes will differ because of the different field orientations. These are termed the even-mode and odd-mode characteristic impedances, denoted by Z_{0e} and Z_{0o}, respectively. Here the e subscript identifies the even mode and the o subscript identifies the odd mode. With coupled microstrip lines, the phase velocities of the two modes will differ since the two modes have different amounts of energy in the air and in the dielectric, resulting in different effective permittivities of the two modes.

Coupled lines are modeled by determining the propagation characteristics of the even and odd modes. Using the definitions shown in Figures 6-3 and 6-4, the total voltage and current phasors on the original structure are a superposition of the even- and odd-mode solutions:

$$\left.\begin{array}{ll} V_1 = V_{Fe} + V_{Fo} & I_1 = I_{Fe} + I_{Fo} \\ V_2 = V_{Fe} - V_{Fo} & I_2 = I_{Fe} - I_{Fo} \\ V_3 = V_{Be} + V_{Bo} & I_3 = I_{Be} + I_{Bo} \\ V_4 = V_{Be} - V_{Bo} & I_4 = I_{Be} - I_{Bo} \end{array}\right\}, \tag{6.1}$$

with the subscripts F and B indicating the front and back, respectively, of the even- and odd-mode components. Also

$$\left.\begin{array}{ll} V_{Fe} = (V_1 + V_2)/2 & I_{Fe} = (I_1 + I_2)/2 \\ V_{Fo} = (V_1 - V_2)/2 & I_{Fo} = (I_1 - I_2)/2 \\ V_{Be} = (V_3 + V_4)/2 & I_{Be} = (I_3 + I_4)/2 \\ V_{Bo} = (V_3 - V_4)/2 & I_{Bo} = (I_3 - I_4)/2 \end{array}\right\}. \tag{6.2}$$

The forward-traveling components can be written as

$$\left.\begin{array}{ll} V_1^+ = V_{Fe}^+ + V_{Fo}^+ & I_1^+ = I_{Fe}^+ + I_{Fo}^- \\ V_2^+ = V_{Fe}^+ - V_{Fo}^+ & I_2^+ = I_{Fe}^+ - I_{Fo}^- \\ V_3^+ = V_{Be}^+ + V_{Bo}^+ & I_3^+ = I_{Be}^+ + I_{Bo}^- \\ V_4^+ = V_{Be}^+ - V_{Bo}^+ & I_4^+ = I_{Be}^+ - I_{Bo}^- \end{array}\right\}. \tag{6.3}$$

The backward-traveling components are written similarly so that the total front and back even- and odd-mode voltages are

$$\left.\begin{array}{ll} V_{Fe} = V_{Fe}^+ + V_{Fe}^- & V_{Be} = V_{Be}^+ + V_{Be}^- \\ V_{Fo} = V_{Fo}^+ + V_{Fo}^- & V_{Bo} = V_{Bo}^+ + V_{Bo}^- \end{array}\right\}. \tag{6.4}$$

With an ideal voltmeter the V_1, V_2, V_3, and V_4 voltages would be measured from a point on the strip to a point on the ground plane immediately below. The voltages $2V_{Fo}$ and $2V_{Bo}$ are the voltages that would be measured between the strips at the front and back of the lines, respectively (see Figure 6-2). It would not be possible to directly measure V_{Fe} and V_{Be}.

One set of network parameters describing a pair of coupled lines is the port-based admittance matrix equation relating the port voltages, V_1–V_4, to the port currents, I_1–I_4:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}. \tag{6.5}$$

These are port-based y parameters as the currents and voltages are defined for ports. Since microstrip lines are fabricated (most of the time) on non-magnetic dielectrics, then they are reciprocal so that $y_{ij} = y_{ji}$. Coupling from one line to another is described by the terms y_{12} (=y_{21}) and y_{34} (=y_{43}).

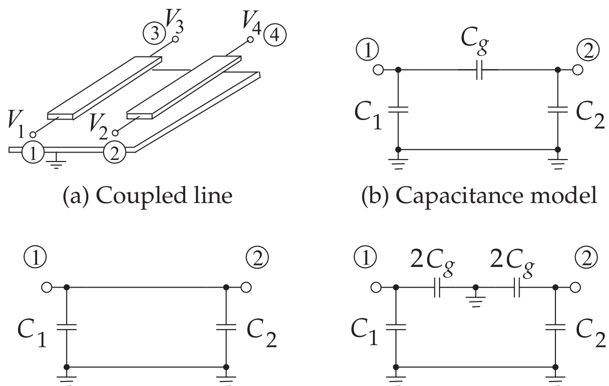
6.2.1 Summary

The important concept introduced in this section is that fields and the propagating waves on a pair of parallel coupled lines can be described as a combination of odd and even modes each of which has forward- and backward-wave versions.

6.3 Low-Frequency Capacitance Model of Coupled Lines

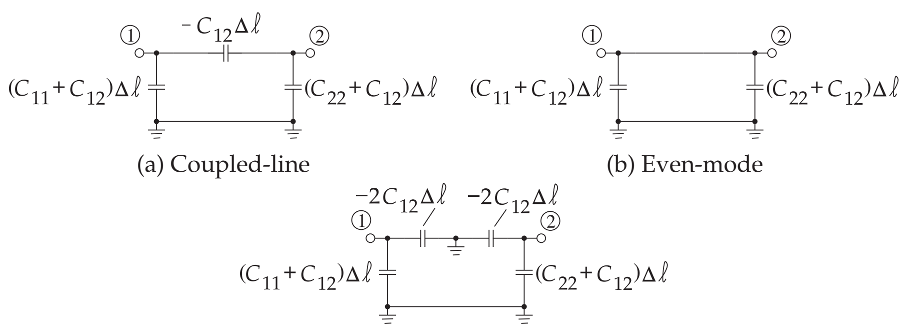
The low-frequency model of a pair of lossless coupled lines comprises only capacitances. A pair of coupled lines, as shown in Figure 6-5(a), has four terminals. At very low frequencies V_1 and V_3 are identical as are voltages V_2 and V_4. So the low-frequency model of the pair of coupled lines has just two terminals in addition to ground, as shown in Figure 6-5(b).

The capacitances in Figure 6-5(b) are the shunt capacitance C_1 and C_2 and the mutual capacitance C_g. In the even mode, the voltages at terminals 1 and 2 are the same so that C_g vanishes, see Figure 6-5(c). In the odd mode, the voltage at terminal 2 is the negative of the voltage at terminal 1. The result is that there is a virtual ground between the terminals. Now a better circuit model is that shown in Figure 6-5(d). This is where the restriction that the lines are of equal width is used. This assumption places the virtual ground



(a) Coupled line (b) Capacitance model (c) Even mode (d) Odd mode

Figure 6-5: Very low frequency models of a pair of coupled lines.



(a) Coupled-line (b) Even-mode (c) Odd-mode

Figure 6-6: Low frequency capacitance models of a pair of coupled lines of length $\Delta\ell$. C_{12} is negative.

between equal-value capacitances. The symmetrical case is the one of most interest.

To proceed, the capacitance model must be put in the form of per unit length capacitances and put in terms of the elements of a capacitance matrix. The indefinite nodal admittance matrix of the low-frequency coupled-line model of Figure 6-5(b) is

$$\mathbf{Y} = \jmath\omega \begin{bmatrix} C_1 + C_g & -C_g \\ -C_g & C_2 - C_g \end{bmatrix} = \jmath\omega \mathbf{C}\Delta\ell, \tag{6.6}$$

where $\Delta\ell$ is the length of the coupled lines, and $\mathbf{C}$ is the per unit length capacitance matrix. Thus the low-frequency capacitance model of a pair of coupled lines of length $\Delta\ell$ and equal width is as shown in Figure 6-6(a). It is found in analysis that C_{12} is negative.

For symmetrical coupled lines (the strips having the same width) the per unit length even- and odd-mode capacitances, as defined in the definition of odd and even modes in Section 6.2, are

$$C_e = C_{11} + C_{12} \quad \text{and} \quad C_o = C_{11} - C_{12}. \tag{6.7}$$

That is, $$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(C_e + C_o) & \frac{1}{2}(C_e - C_o) \\ \frac{1}{2}(C_e - C_o) & \frac{1}{2}(C_e + C_o) \end{bmatrix}. \tag{6.8}$$

6.4 Symmetric Coupled Transmission Lines

In this section, even and odd modes are considered as defining independent transmission lines. The development is restricted to a symmetrical pair of coupled lines. Thus the strips have the same self-inductance, $L_s = L_{11} = L_{22}$, and self-capacitance, $C_s = C_{11} = C_{22}$, where the subscript s stands for "self." $L_m = L_{12} = L_{21}$ and $C_m = C_{12} = C_{21}$ are the mutual inductance and capacitance of the lines, and the subscript m stands for "mutual." The coupled transmission line equations are

$$\frac{dV_1(x)}{dx} = -\jmath\omega L_s I_1(x) - \jmath\omega L_m I_2(x) \tag{6.9}$$
$$\frac{dV_2(x)}{dx} = -\jmath\omega L_m I_1(x) - \jmath\omega L_s I_2(x) \tag{6.10}$$
$$\frac{dI_1(x)}{dx} = -\jmath\omega C_s V_1(x) - \jmath\omega C_m V_2(x) \tag{6.11}$$
$$\frac{dI_2(x)}{dx} = -\jmath\omega C_m V_1(x) - \jmath\omega C_s V_2(x). \tag{6.12}$$

The even mode is defined as the mode corresponding to both conductors being at the same potential and carrying the same currents:[1]

$$V_1 = V_2 = V_e \quad \text{and} \quad I_1 = I_2 = I_e. \tag{6.13}$$

The odd mode is defined as the mode corresponding to the conductors being at opposite potentials relative to the reference conductor and carrying currents of equal amplitude but of opposite sign:[2]

$$V_1 = -V_2 = V_o \quad \text{and} \quad I_1 = -I_2 = I_o. \tag{6.14}$$

The characteristics of the two possible modes of the coupled transmission lines are now described. For the even mode, from Equations (6.9) and (6.10),

$$\frac{d}{dx}\left[V_1(x) + V_2(x)\right] = -\jmath\omega\left[L_m + L_s\right]\left[I_1(x) + I_2(x)\right], \tag{6.15}$$

which becomes

$$\frac{dV_e(x)}{dx} = -\jmath\omega\left(L_s + L_m\right) I_e(x). \tag{6.16}$$

Similarly, using Equations (6.11) and (6.12),

$$\frac{d}{dx}\left[I_1(x) + I_2(x)\right] = -\jmath\omega\left(C_s + C_m\right)\left[V_1(x) + V_2(x)\right], \tag{6.17}$$

which in turn becomes

$$\frac{dI_e(x)}{dx} = -\jmath\omega\left(C_s + C_m\right) V_e(x). \tag{6.18}$$

Defining the even-mode inductance and capacitance, L_e and C_e, respectively, as

$$L_e = L_s + L_m = L_{11} + L_{12} \quad \text{and} \quad C_e = C_s + C_m = C_{11} + C_{12} \tag{6.19}$$

---

[1] Here $I_e = (I_1 + I_2)/2$ and $V_e = (V_1 + V_2)/2$.
[2] Here $I_o = (I_1 - I_2)/2$ and $V_o = (V_1 - V_2)/2$.

leads to the **even-mode telegrapher's equations**:

$$\frac{dV_e(x)}{dx} = -\jmath\omega\, L_e I_e(x) \quad (6.20) \qquad \text{and} \qquad \frac{dI_e(x)}{dx} = -\jmath\omega\, C_e V_e(x). \quad (6.21)$$

From these, the even-mode characteristic impedance can be found,

$$Z_{0e} = \sqrt{\frac{L_e}{C_e}} = \sqrt{\frac{L_s + L_m}{C_s + C_m}}, \tag{6.22}$$

and also the even-mode phase velocity,

$$v_{pe} = \frac{1}{\sqrt{L_e C_e}}. \tag{6.23}$$

The characteristics of the odd-mode operation of the coupled transmission line can be determined in a similar procedure to that used for the even mode. Using Equations (6.9)–(6.12), the **odd-mode telegrapher's equations** become

$$\frac{dV_o(x)}{dx} = -\jmath\omega\,(L_s - L_m)\, I_o(x) \quad (6.24) \qquad \text{and} \qquad \frac{dI_o(x)}{dx} = -\jmath\omega\,(C_s - C_m)\, V_o(x). \quad (6.25)$$

Defining L_o and C_o for the odd mode such that

$$L_o = L_s - L_m = L_{11} - L_{12} \quad \text{and} \quad C_o = C_s - C_m = C_{11} - C_{12}, \tag{6.26}$$

then the odd-mode characteristic impedance is

$$Z_{0o} = \sqrt{\frac{L_o}{C_o}} = \sqrt{\frac{L_s - L_m}{C_s - C_m}} \tag{6.27}$$

and the odd-mode phase velocity is

$$v_{po} = \frac{1}{\sqrt{L_o C_o}}. \tag{6.28}$$

Now for a sanity check. If the individual strips are widely separated, L_m and C_m will become very small and Z_{0e} and Z_{0o} will be almost equal. As the strips become closer, L_m and C_m will become larger and Z_{0e} and Z_{0o} will diverge. This is as expected.

6.5 Directional Coupler

Coupling can be exploited to realize a new type of element called a directional coupler. The schematic of a directional coupler is shown in Figure 6-7(a) and a microstrip realization is shown in Figure 6-7(b). A usable directional coupler has a coupled line length of at least one-quarter

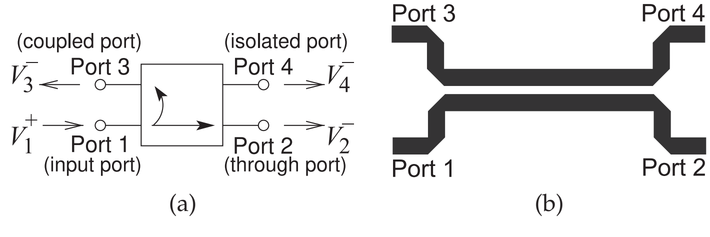


Figure 6-7: Directional couplers: (a) schematic; and (b) backward-coupled microstrip coupler. (Note that not all couplers are backward-wave couplers as shown in (a).)