

## CHAPTER 39.

### SLIPPING

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#### SLIPPING

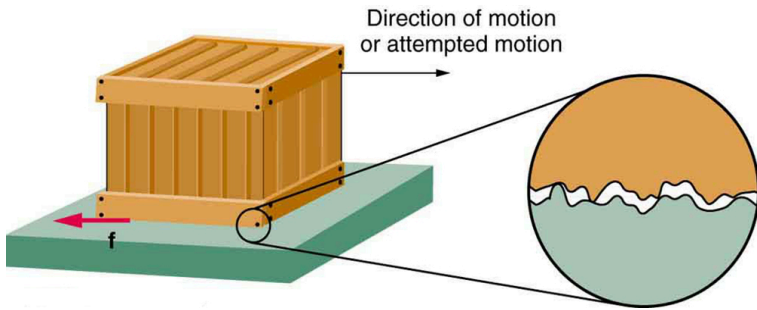
Slipping happens when friction between feet and walking surface is not large enough to prevent your back foot from sliding as it pushes off, or the front foot from sliding when it tries to slow the forward motion of your center of gravity). Together, normal force and friction ( $F_f$ ) provide the forces necessary to support the body and maintain balance. For example, friction prevents crutches from sliding outward when they aren't held perfectly vertical. Friction is also necessary for locomotion, such as walking and running, as we will learn in the Locomotion unit.



Friction between the crutches and the floor prevents the young boy's crutches from sliding outward even when they aren't held straight vertical. This 1942 photo by Fritz Henle was captioned "Nurse training. Using the picture book as bait, the physical therapist encourages a young victim of infantile paralysis [Polio] to learn to use his catches (crutches)." Polio was effectively eradicated from the United States by the polio vaccine, originally developed by Jonas Salk "who never patented the vaccine or earned any money from his discovery, preferring it be distributed as widely as possible." There are two types of vaccine that can prevent polio: inactivated poliovirus vaccine (IPV) and oral poliovirus vaccine (OPV). Only IPV has been used in the United States since 2000 and 99% of children who get all the recommended doses of vaccine will be protected from polio.

## FRICTION

Friction ( $F_f$ ) is the force that resists surfaces sliding against one another. Rub your palms together, the resistance you feel is friction. Complimentary to normal force, which only points perpendicular to surfaces, friction only points parallel to surfaces.



*Frictional forces always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.*

Friction can only exist when two objects are attempting

1. "Photo" by Fritz Henle, Library of Congress is in the Public Domain
2. "History of Salk" by The Salk Institute
3. "What is Polio" by Global Health, Center for Disease Control

to slide past one another, so it is also reactive like normal force. Two surfaces must touch to have friction, so you also can't get friction without normal force. In fact, frictional force is proportional to normal force.

### Reinforcement Activity

Rub your palms together. Now push your palms together hard and try to slide them at the same time.

Now the normal force is larger causing the frictional force to grow in proportion.

### STATIC FRICTION

There are two categories of friction. Static friction ( $F_{f,s}$ ) acts between two surfaces when they are attempting to slide past one another, but have not yet started sliding. Static friction is a reactionary force because it only exists when some other force is pushing an object to attempt to cause it to slide across a surface. Static friction adjusts to maintain equilibrium with whatever other force is doing the pushing or pulling, but static friction has a maximum value. If the applied force gets larger than the maximum static frictional value, then static friction can't maintain equilibrium and the object will slide.

### KINETIC FRICTION

Kinetic friction ( $F_{f,k}$ ) acts whenever two surfaces are sliding past one another, whether or not some other force is pushing the object to keep it sliding. If there is not another force pushing the object to keep it sliding, then kinetic friction will eventually stop the sliding object, but

we will learn more about that later. Static friction is larger than kinetic friction. Choose the friction simulation from the simulation set to see how static and kinetic friction behave.

### Reinforcement Activity

#### FRICITION COEFFICIENT

We now know that friction force is proportional to normal force and that there are two types of friction, static and kinetic. The final concept that affects friction is the roughness, or alternatively the smoothness, of the two surfaces. The coefficient of friction ( $\mu$ ) is a unitless

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number that rates the roughness and is typically determined experimentally. The static frictional force is larger than the kinetic frictional forces because  $\mu_s$  is larger than  $\mu_k$ . Take a look at the table of static and kinetic friction coefficients found below. You can find more values in this massive table of static friction coefficients.

Table of static and kinetic friction coefficients for various surface pairs<sup>4</sup>

System	Static friction, $\mu_s$	Kinetic friction, $\mu_k$
rubber on dry concrete	1.0	0.7
rubber on wet concrete	0.7	0.5
wood on wood	0.5	0.3
waxed wood on wet snow	0.14	0.1
metal on wood	0.5	0.3
steel on steel (dry)	0.6	0.3
steel on steel (oiled)	0.05	0.03
teflon on steel	0.04	0.04
bone lubricated by synovial fluid	0.016	0.015
shoes on wood	0.9	0.7
shoes on ice	0.1	0.05
ice on ice	0.1	0.03
steel on ice	0.4	0.02

4. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Aug 2, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@11.1>.

Notice that two surfaces are always listed in the table; you must have two surfaces to define a  $\mu$ . When someone asks a question like, “what is the  $\mu$  of ice?” they usually mean between ice and ice, but its best to avoid asking such questions and just always reference two surfaces.

## CALCULATING FRICTION FORCES

We can sum up everything we have learned about friction in two equations that relate the friction forces to the friction coefficient for two surfaces and the normal force acting on the surfaces:

Max static friction before release:

$$(1) F_{f,s}^{max} = \mu_s F_N$$

Kinetic friction once moving:

$$(2) F_{f,k} = \mu_k F_N$$

### Everyday Example: Firefighter Physical Ability Test

Firefighter candidates must complete a physical ability test (PAT) that includes dragging a dummy across the floor. The PAT for the city of Lincoln Nebraska specifies that candidates must drag a human form dummy weighing **170 lbs** for 25 feet, around a barrel, and then back across the starting point for a total distance of 50 feet in six minutes or less. The candidates may only drag the dummy using the pull harness attached to the dummy and cannot carry the dummy<sup>5</sup>.

The test is held on a polished concrete floor. The static friction coefficient between cotton clothing and polished concrete is 0.5. If a candidate pulls vertically up on the harness with a force of **70 lbs**

5. "Firefighter Physical Ability Test Candidate Orientation Guide" by Industrial/Organizational Solutions, Inc.

what horizontal pull force must the candidate apply in order to get the dummy moving?

The dummy starts out in static equilibrium so we know the net force must be zero in both the vertical and horizontal directions. First, let's analyze the vertical direction: if the candidate pulls vertically up on the harness with a force of **70 lbs** then the floor must provide a normal force of **100 lbs** to support the dummy.

Now let's analyze the horizontal direction: static friction will match whatever horizontal pull the candidate provides, but in the opposite direction, so that the dummy stays in static equilibrium until the pull exceeds the max static friction force. That's the force the candidate needs to apply to get the dummy moving, so let's find that. We have the friction coefficient and we already found the normal force so we are ready:

$$F_{f,s}^{max} = \mu_s F_N = 0.5 \cdot 100 \text{ lbs} = 50 \text{ lbs}$$

After the dummy starts moving, kinetic friction kicks in so we can use  $\mu_k = 0.4$  to calculate the kinetic frictional force. This force is less than the max static frictional force, so it will require less force to keep the dummy moving than it did to get it started.

$$F_{f,k} = \mu_k F_N = 0.4 \cdot 100 \text{ lbs} = 40 \text{ lbs}$$

## Reinforcement Exercises



*A person clings to a playground fire pole. "Firepole" by Donkeysforever, via Wikimedia Commons is in the Public domain*

The equations given for static and kinetic friction are empirical models that describe the behavior of the forces of friction. While these formulas are very useful for practical purposes, they do not have the status of laws or principles. In fact, there are cases for which these equations are not even good approximations. For instance, neither formula is accurate for surfaces that are well lubricated or sliding at high speeds. Unless specified, we will not be concerned with these exceptions.<sup>6</sup>

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<https://openoregon.pressbooks.pub/bodyphysics/?p=4298>

6. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Aug 2, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@11.1>.

## CHAPTER 40.

### FRICTION IN JOINTS

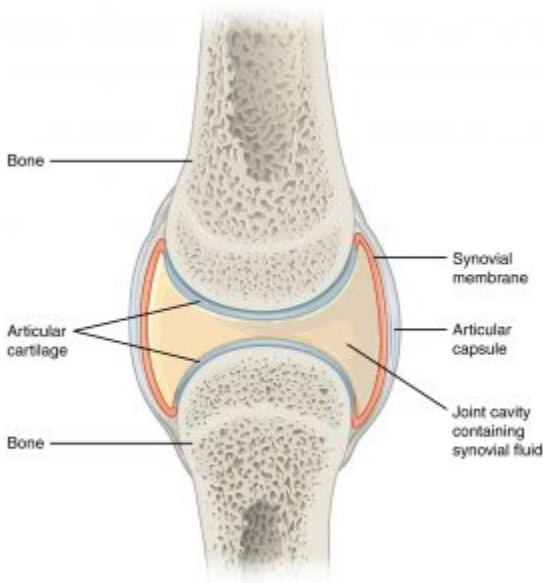
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#### SYNOVIAL JOINT FRICTION

Static and kinetic friction are both present in joints. Static friction must be overcome, by either muscle tension or gravity, in order to move. Once moving, kinetic friction acts to oppose motion, cause wear on joint surfaces, generate thermal energy, and make the body less efficient. (We will examine the efficiency of the body later in this textbook.) The body uses various methods to decrease friction in joints, including synovial fluid, which serves as a lubricant to decrease the friction coefficient between bone surfaces in synovial joints (the majority of joints in the body). Bone surfaces in synovial joints are also covered with a layer of articular cartilage which acts with the synovial fluid to reduce friction and provides something other than the bone surface to wear away over time<sup>1</sup>. We ignored friction when analyzing our forearm as a lever because the frictional forces are relatively small

1. OpenStax, Anatomy & Physiology. OpenStax CNX. Jun 25, 2018 <http://cnx.org/contents/14fb4ad7-39a1-4eee-ab6e-3ef2482e3e22@10.1>.

and because they acted inside the joint, very close to the pivot point so they caused negligible torque.



*Synovial joints allow for smooth movements between the adjacent bones. The joint is surrounded by an articular capsule that defines a joint cavity filled with synovial fluid. The articulating surfaces of the bones are covered by a thin layer of articular cartilage. Ligaments support the joint by holding the bones together and resisting excess or abnormal joint motions. Image Credit: OpenStax Anatomy & Physiology*

### Reinforcement Exercises

Find a value for the kinetic coefficient of friction between ends of a

bone in a synovial joint lubricated by synovial fluid. State your value and your source.

If the normal force between bones in the knee is **160 lbs**, what is the kinetic frictional force between the surfaces of the knee bones?

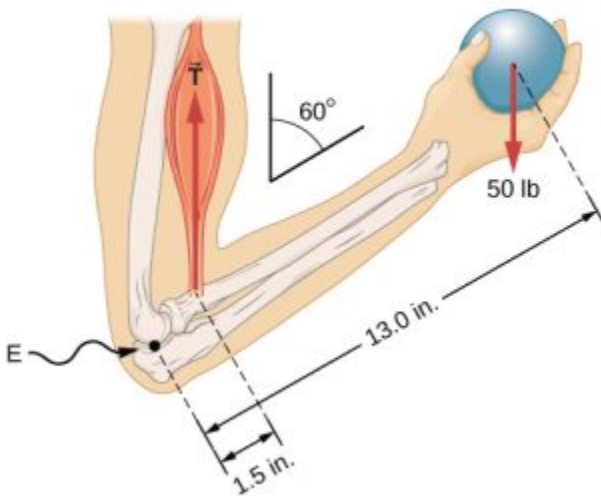
## CHAPTER 41.

### TIPPING

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#### TORQUE

When you hold an object in your hand, the weight of the object tends to cause a rotation of the forearm with the elbow joint acting as the pivot. The tension force applied by your biceps tries to counteract this rotation.



*The elbow joint flexed to form a  $60^\circ$  angle between the upper arm and forearm while the hand holds a 50 lb ball. The weight of the ball exerts a torque on the forearm about the elbow joint.  
Image Credit: Openstax University Physics*

1

When forces applied to an object tend to cause rotation of the object, we say the force is causing a torque. The size of a torque depends on the size of the force, the direction of the force, and the distance from the pivot point to where the force acts.

1. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Jul 11, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@10.18>.

## Reinforcement Activity

### STATIC EQUILIBRIUM

In order for an object to remain still then any torques cancel each other out so that there is no net torque. If the net torque is not zero the the object will begin to rotate rather than remain still. For example, in our example of the forearm holding the ball, the torque due to biceps tension and torque due to ball weight must be equal, but in opposite directions.

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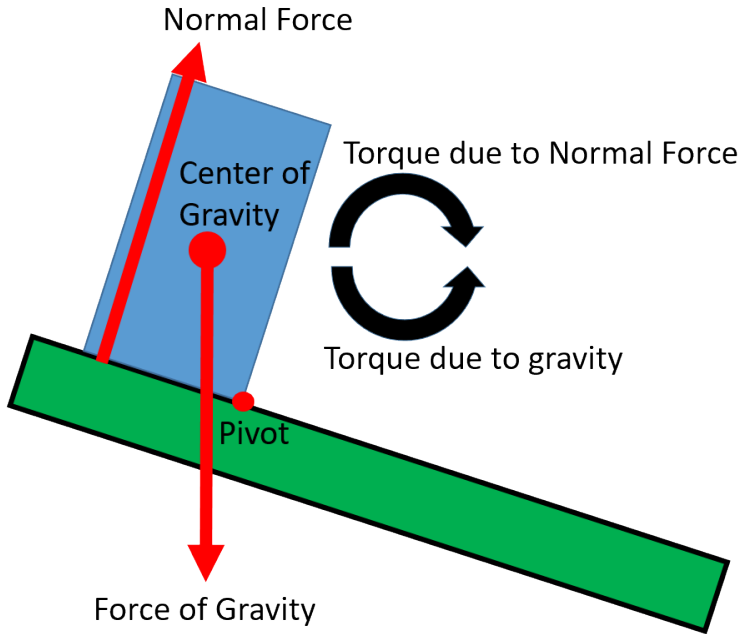
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## Reinforcement Exercises

### TIPPING POINT

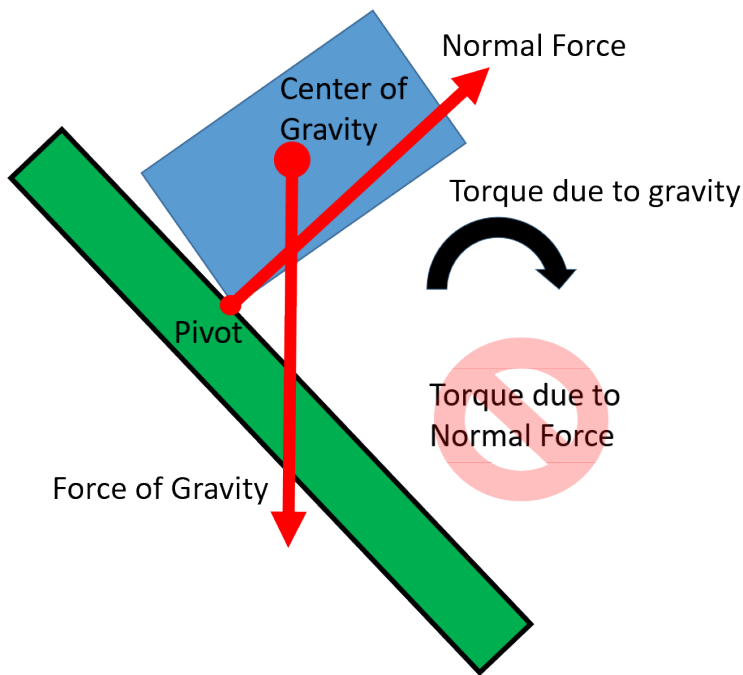
When a body's center of gravity is above the area formed by the support base the normal force can provide the torque necessary to remain in rotational equilibrium.

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*An object in rotational equilibrium. The torque from normal force cancels the torque from gravity. In this case friction (not shown) acts on the bottom surface of the object to keep it from sliding downhill.*

The critical tipping point is reached when the center of gravity passes outside of the support base. Beyond the tipping point, gravity causes rotation away from the support base, so there is no normal force available to cause the torque needed to cancel out the torque caused by gravity. The normal force acting on the pivot point can help support the object's weight, but it can't create a torque because it's not applied at any distance away from the pivot.



*An object out of rotational equilibrium. The normal force acting at the pivot cannot produce a torque to cancel the torque caused by gravity. In this case friction (not shown) acts at the pivot point to keep the object from sliding downhill.*

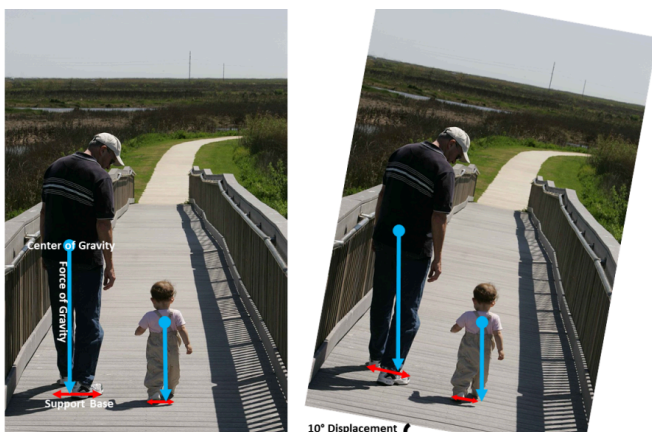
Now with a net torque the object can not be in rotational equilibrium. The object will rotate around the edge of the support base and tip over. We often refer to structures (and bodies) that are relatively resistant to tipping over as having greater stability.

## CHAPTER 42.

### HUMAN STABILITY

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When asking what makes a structure more or less stable, we find that a high center of gravity or a small support base makes a structure less stable. In these cases a small displacement is need in order to move the center of gravity outside the area of support. Structures with a low center of gravity compared to the size of the support area are more stable. One way to visualize stability is to imagine displacement of the center of gravity caused by placing the object on a slope. For example, a  $10^\circ$  displacement angle might displace the center of gravity of a toddler beyond the support base formed by its feet, while an adult would still be in equilibrium.



*Compared to an adult, a smaller displacement will move a toddlers center of gravity outside the base of support. Image adapted from A man and a toddler take a leisurely walk on a boardwalk by Steve Hillibrand via Wikimedia Commons.*

1

The center of gravity of a person's body is above the pivots in the hips, which is relatively high compared to the size of the support base formed by the feet, so displacements must be quickly controlled. This control is a nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by bending the knees, which lowers the center of gravity toward the base of support. A cane, a crutch, or a walker increases the stability of the user by widening the base of support. Due to their disproportionately large heads,

1. "A man and toddler take a leisurely walk on a boardwalk" by Steve Hillibrand, U.S. Fish and Wildlife Service, Wikimedia Commons, is in the Public Domain

young children have their center of gravity between the shoulders, rather than down near the hips, which decreases their stability and increases the likelihood of reaching a tipping point.<sup>2</sup>



*Warning label on a bucket indicating the danger of children falling into a bucket and drowning. This danger is caused by the inherent instability of the toddler body. Image Credit: GodsMoon via Wikimedia Commons.*

3

2. OpenStax, College Physics. OpenStax CNX. Aug 3, 2018 <http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@12.1>.
3. "Drowning Child Warning" by GodsMoon, Wikimedia Commons is licensed under CC BY-SA 2.0

## Reinforcement Exercises

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## CHAPTER 43.

### TRIPPING

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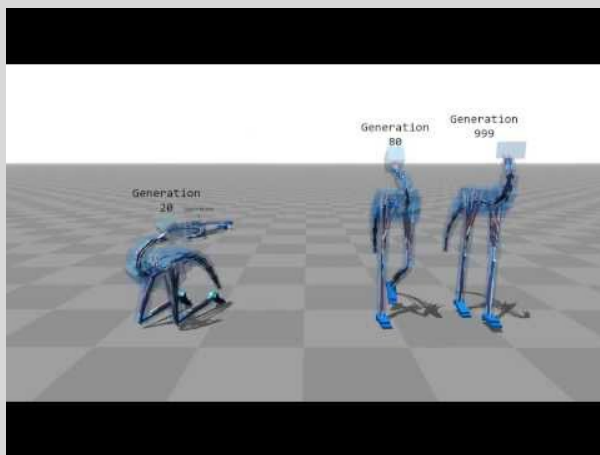
Walking is an act of moving in and out of equilibrium and we will learn more about walking in the unit on locomotion. In order to walk we:

1. Push against the ground with one foot using normal force and friction while leaning forward, lifting the other foot, and moving it forward. This moves the center of gravity passes outside the support base formed by the (now) back foot. Having passed the tipping point, the body would fall except:
2. The front foot lands, slowing the forward motion of your center of gravity and creating a new support base so that you are no longer past the tipping point. The front foot become the back foot and begins to pushes off.
3. Repeat.

Slipping happens when the friction coefficient between feet and walking surface is too small and the frictional force is not large enough to prevent your feet from sliding as the back foot pushes off and/or the front foot tries to slow the forward motion of your center of gravity.

Tripping happens when your foot does not move forward quickly enough to shift your support base below your center of gravity and you either fall over or have to rapidly move your feet into position just in time (stumble). Check out these AI simulations of creatures that employ bipedal motion learning how to walk, and tripping along the way.

To see what AI algorithms can do when given a real physical body to experiment with, check out these robots.



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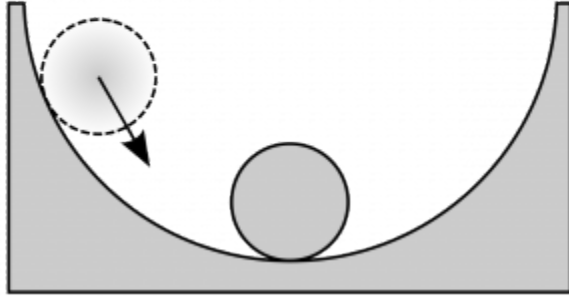
## CHAPTER 44.

### TYPES OF STABILITY

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#### STABLE EQUILIBRIUM

If a structure is pushed out of equilibrium we say it has been displaced from equilibrium. If the object tends to move back toward its equilibrium position then it must be in a region of stable equilibrium and the force that pushed it back is a restoring force.



*A marble in the bottom of a bowl is an example of stable equilibrium. Image credit: "Stable Equilibrium" by Urutseg, via Wikimedia Commons*

1

As your arm hangs from your shoulder, it is in stable equilibrium. If your arm is lifted to the side and then let go it will fall back down to the hanging position. The hanging arm is a stable position because the center of gravity of the arm is located below the base of support, in this case the shoulder. When displaced (lifted a bit) the force of gravity acting on your arm will cause a torque that rotates your arm back down to the hanging position. In such cases, when an object is displaced from the equilibrium position and the resulting net forces (or torques they cause) move the object back toward the equilibrium position then these forces are called restoring forces. The sloth takes advantage of stable equilibrium to

1. "Stable Equilibrium" by Urutseg, Wikimedia Commons is in the Public Domain, CC0

save energy that humans spend on staying upright. If the sloth is displaced in any direction, the force of gravity automatically acts as a restoring force and returns the sloth to its equilibrium position.



*A two-toed sloth hangs from its feet in a stable equilibrium position. Image Credit: Two Toed Sloth by Cliff via Wikimedia Commons*

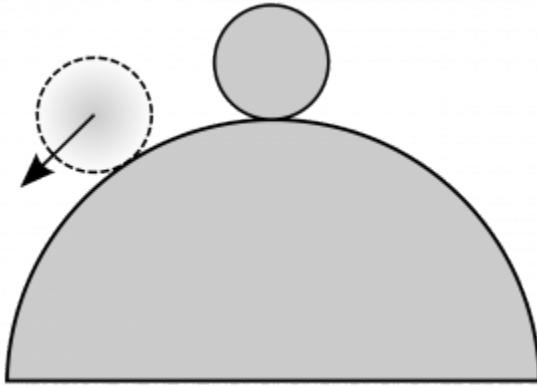
2

## UNSTABLE EQUILIBRIUM

When a system in equilibrium is displaced and the resulting net force pushes the object even further away

2. Two Toed Sloth (*Choloepus didactylus*) By Cliff [CC BY 2.0 (<https://creativecommons.org/licenses/by/2.0>)], via Wikimedia Commons

from the equilibrium position then it must have been in an unstable equilibrium. Technically, real systems cannot spend time at unstable equilibrium point because the tiniest vibration will cause them to move out of equilibrium not to mention that you could never place them perfectly into position in the first place. Trying to balance a marble on a hill is a good example:



*An example of unstable equilibrium is a marble placed on a hill.  
Image Credit: "Unstable Equilibrium" by Urutseg, via  
Wikimedia Commons.*

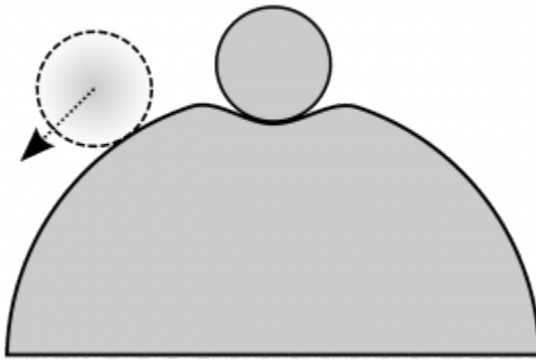
3

## **METASTABLE EQUILIBRIUM**

Some structures that are in stable equilibrium and can be displaced relatively far before they are no longer in equilibrium. Other structures structures that only require

3. "Unstable Equilibrium" by Urutseg, Wikimedia Commons is in the Public Domain, CC0

a small displacement to move out of equilibrium (like toddlers). We often call these systems stable and unstable, but this can be misleading because any standing structure is somewhat stable and a truly unstable structure would not stand still for any time. These structures that are in a stable region, but could be pushed passed a tipping point are known to be in a metastable equilibrium.



*The marble is in meta-stable equilibrium as long as it doesn't move outside the dip in the center. The peak at edge of the dip is analogous to the tipping point for a structure; beyond this point the marble will not move back toward the equilibrium position. Image credit: "Meta-stable Equilibrium" by Urutseg via Wikimedia Commons*

4

Keeping your balance requires that you stay with the the stable region of a metastable equilibrium. For

4. "Meta-stable Equilibrium" by Urutseg, Wikimedia Commons is in the Public Domain, CC0

example, we expect that most people would say the person balancing on their head in the following image is unstable, but that wouldn't be quite accurate. Actually, the person is actively adjusting the shape of their body to shift their center of gravity to remain within the stable region of a metastable equilibrium, though it is a narrow one.



*A person in a barely-stable equilibrium. Image Credit: Usien via Wikimedia Commons.*

## Exercises

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CHAPTER 45.

**THE ANTI-GRAVITY LEAN**

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*GIF animation of the "anti-gravity lean" maneuver in which a person wearing shoes that attach to the floor can lean forward with straight legs and then return to a standing position. Image Credit: Asanagi via Wikimedia Commons*

1

The structures discussed in the previous chapters were resting on the support base, which was not *attached* to the support surface (such as your feet and the ground). Therefore only normal force and friction were available

1. "Anti-gravity Lean" by Asanagi, Wikimedia Commons

to cancel torques caused by gravity and maintain equilibrium. When the support base is attached then tension can help cancel out gravitational torques and the structure can remain in equilibrium even when the center of gravity moves outside the area of support. Such structures are known as cantilevered structures. The animation above shows someone performing the “anti-gravity lean” during which the body is momentarily a cantilevered structure. The maneuver requires that the heels of the shoes be attached to the ground in order to provide a tension force. Cantilevered structures can generate especially large stress and strain on the materials in the structure. For example the Achilles' tendon is severely stressed during the anti-gravity lean. When stress becomes too great then rupture may occur. The following unit will apply what we have learned about static equilibrium to determine the size of forces acting on and within the body when it isn't moving. The unit after that will calculate forces on the body when it is moving.

## CHAPTER 46.

### UNIT 5 REVIEW

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#### Key Terms and Concepts

Center of Gravity

Normal Force

Friction

Coefficient of friction

Reactive forces

Torque

Rotational Equilibrium

Stable Equilibrium

Unstable Equilibrium

Metastable Equilibrium

Stability

## Learner Objectives

1. Define center of gravity, support base, normal force, static friction and kinetic friction.[2]
2. Compare the relative torque applied to objects by various forces.[2]
3. Identify the type of equilibrium exhibited by various structures and rank their relative stability.[2]
4. Apply static equilibrium concepts to determine forces in physical situations, including normal force and friction.[3]

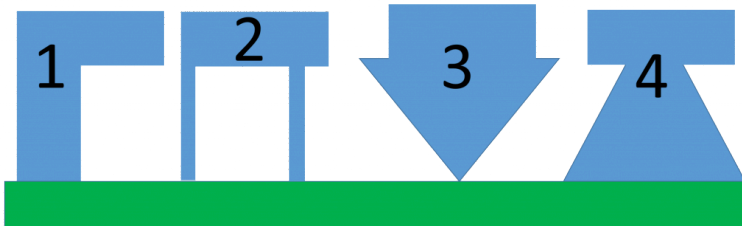
## CHAPTER 47.

### UNIT 5 PRACTICE AND ASSESSMENT

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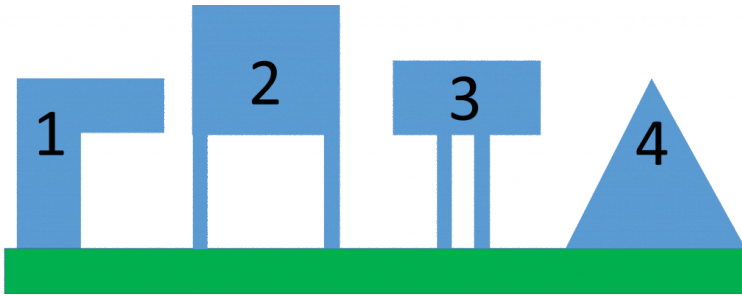
#### Outcome 1

1) Rank the structures below in order of increasing support base width.



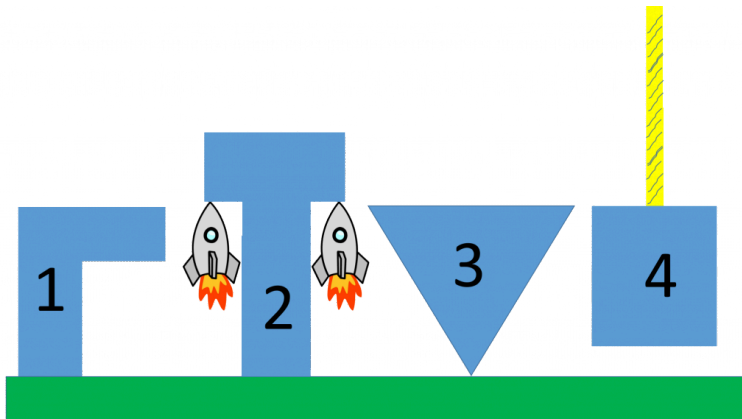
*Four structures of equal height, but varying shape and base width.*

2) Rank the structures below in order of increasing center of gravity height. All four structures are solid and are made of the same material.



*Four structures of equal mass, but varying height and base width.*

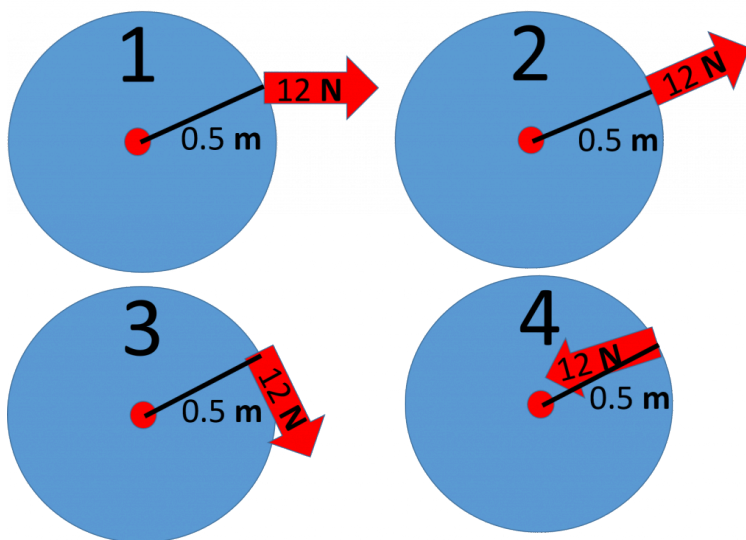
3) Rank the structures below in order of increasing normal force from the ground. All four structures have the same weight and are at rest.



*Four structures of equal weight. The second structure has rockets pushing up on it and the fourth structure is hanging from a rope. Rocket images from <http://wpclipart.com> are in the Public Domain.*

## Outcome 2

5) A child at a playground pushes on a large disk that rotates on an axle through its center. The child tries pushing on the edge of the disk in several different directions, as indicated by the top-down diagrams below. Rank the child's attempts by the amount of torque applied to the disk, from least to greatest.



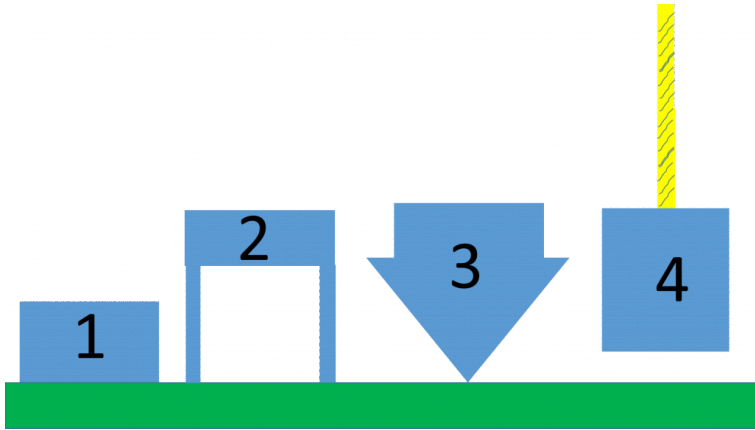
Four disks 0.5 m radius, each with a 12 N force applied at the edge. Disk 1 has the force applied outward at a slight angle to the radius. Disk two has the force applied outward directly along the radius. Disk 3 has the force applied perpendicular to the radius. Disk four has the force applied at a slight angle to the radius, but inward. The angle with the radius is smaller than the angle in disk 1.

6) If the child in the previous problem was able to apply

a 12 N force and the disk had a 0.5 m radius, what would be the value of the torque applied in trial 3?

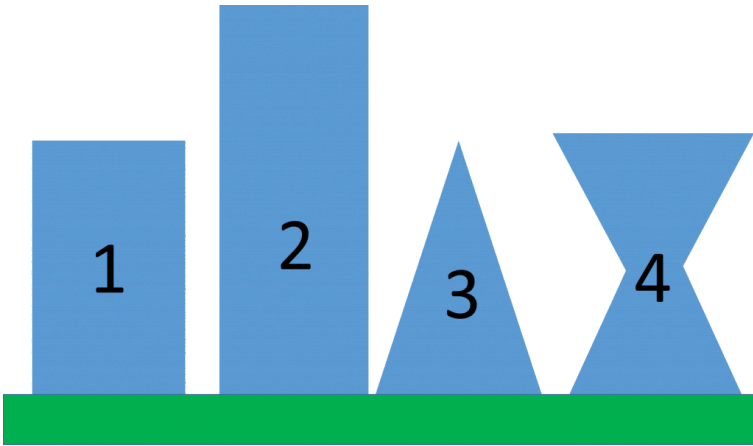
**Outcome 3**

7) State which type of equilibrium is exhibited by each structure below: stable, unstable, or metastable.



*Four structures in static equilibrium*

8) Rank the structures below in order of increasing stability. All structures are solid and made of a single material type.



*Four structures in metastable equilibrium.*

#### **Outcome 4**

9) A car with a weight of 10,000 N is sitting on concrete with the parking brake on.

- a) What is the net force on the car?
- b) What is the net torque on the car?

10) What is the normal force from the concrete on the car from exercise 9?

b) What is the maximum horizontal force that can be applied before the car begins to skid? List your sources for the friction coefficient.

c) After the car begins to skid, how much force is required to keep it moving at constant speed, despite kinetic friction?

d) If you apply only 120 N of horizontal force to the stationary car, what is the static frictional force at that time?

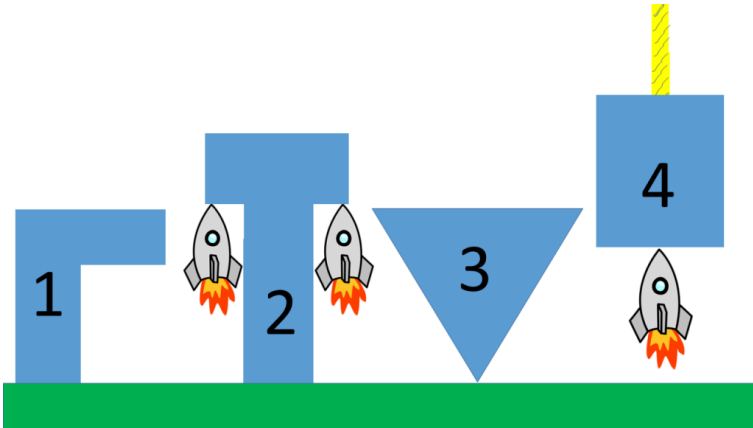
11) Each structure in the following image is at rest.

a) What do you know about the net force on each block?

b) Structure #1 weighs 5000 N. What is the normal force on the structure?

c) Structure # 2 weighs 5000 N. Each rocket is capable of pushing with 1000 N of force. What is the normal force on the structure from the ground?

d) Structure # 4 5000 N. The rocket is capable of pushing with 1000 N of force. What is the tension force provided by the rope?



*Four structures of equal weight. The second structure has rockets pushing up on it and the fourth structure is hanging from a rope. Rocket images from <http://wpclipart.com> are in the Public Domain.*

## PART VI.

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# UNIT 6: STRENGTH AND ELASTICITY OF THE BODY

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### Learner Objectives

1. Identify classes of levers and explain advantages and disadvantages of each classes in terms of mechanical advantage and range of motion.[2]
2. Apply lever and static equilibrium concepts to solve for forces and calculate mechanical advantage in scenarios involving levers. [3]
3. Identify and define the features of a stress-strain curve, including stress, strain, elastic region, elastic modulus, elastic limit, plastic region, ultimate strength, and fracture/rupture.[2]
4. Apply the Hooke's Law along with the definitions of

stress, strain, and elastic modulus to calculate the deformations of structures. [3]

## CHAPTER 48.

### BODY LEVERS

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#### LEVERAGE

Moving patients is a routine part of Jolene's work as a MED floor RN, but in reality there is nothing routine about the biomechanics of lifting and transferring patients. In fact, "disabling back injury and back pain affect 38% of nursing staff" and healthcare makes up the majority of positions in the top ten ranking for risk of back injury, primarily due to moving patients. Spinal load measurements indicated that all of the routine and familiar patient handling tasks tested placed the nurse in a high risk category, even when working with a patient that "[had a mass of] only 49.5 kg and was alert, oriented, and cooperative—not an average patient."<sup>1</sup> People are inherently awkward shapes to move, especially when the patient's bed and other medical equipment cause the nurse to adopt awkward biomechanic positions. The forces required to move people are large to begin with, and the biomechanics of the body can amplify those

1. "Nurses and Preventable Back Injuries" by Deborah X Brown, RN, BSN, American Journal of Critical Care

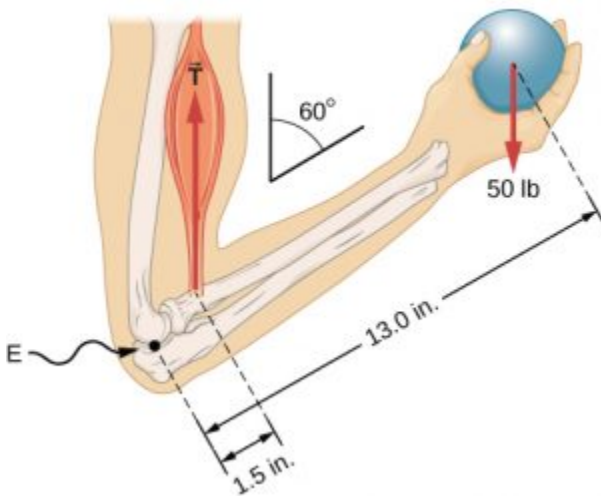
forces by the effects of leverage, or lack thereof. To analyze forces in the body, including the effects of leverage, we must study the properties of levers.

## **LEVER CLASSES**

The ability of the body to both apply and withstand forces is known as strength. One component of strength is the ability apply enough force to move, lift or hold an object with weight, also known as a load. A lever is a rigid object used to make it easier to move a large load a short distance or a small load a large distance. There are three classes of levers, and all three classes are present in the body<sup>23</sup>. For example, the forearm is a 3rd class lever because the biceps pulls on the forearm between the joint (fulcrum) and the ball (load). To see these body levers in action check out this short video animation identifying levers in the body.

2. "Lever of a Human Body" by Alexandra, The Physics Corner

3. "Kinetic Anatomy With Web Resource-3rd Edition " by Robert Behnke , Human Kinetics

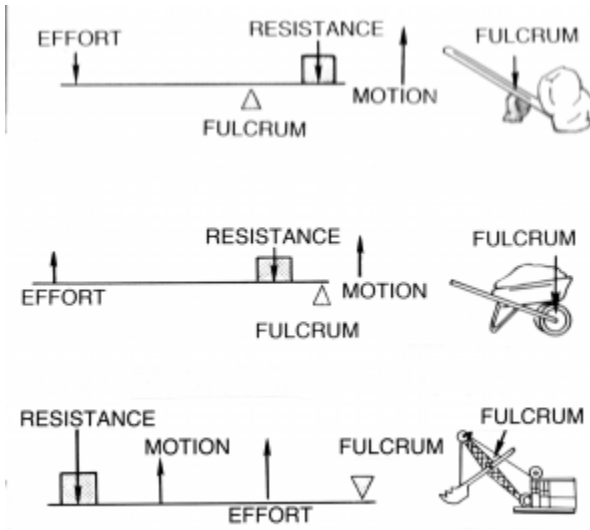


*The elbow joint flexed to form a  $60^\circ$  angle between the upper arm and forearm while the hand holds a 50 lb ball . Image Credit: Openstax University Physics*

4

Using the standard terminology of levers, the forearm is the lever, the biceps tension is the effort, the elbow joint is the fulcrum, and the ball weight is the resistance. When the resistance is caused by the weight of an object we call it the load. The lever classes are identified by the relative location of the resistance, fulcrum and effort. First class levers have the fulcrum in the middle, between the load and resistance. Second class levers have resistance in the middle. Third class levers have the effort in the middle.

4. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Jul 11, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@10.18>.



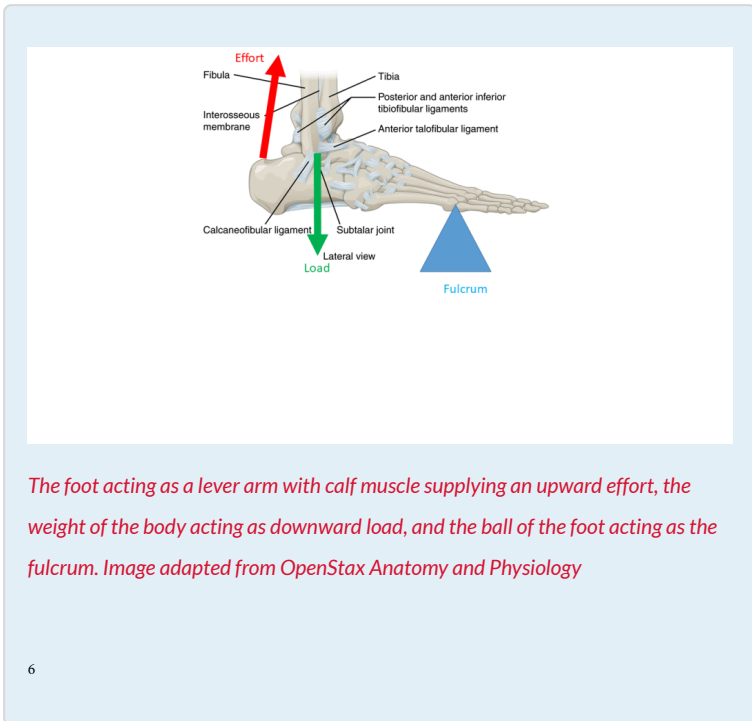
*First (top), second(middle), and third(bottom) class levers and real-world examples of each. Image Credit: Pearson Scott Foresman*

5

### Reinforcement Activity

Identify the class of lever created by the foot and the calf muscle when raising the heel off the ground.

5. "Lever" by Pearson Scott Foresman, Wikimedia Commons is in the Public Domain



## STATIC EQUILIBRIUM IN LEVERS

For all levers the effort and resistance (load) are actually just forces that are creating torques because they are trying to rotate the lever. In order to move or hold a load the torque created by the effort must be large enough to balance the torque caused by the load. Remembering that torque depends on the distance that the force is applied from the pivot, the effort needed to balance the resistance must depend on the distances of the effort and resistance from the pivot. These distances are known as the effort arm and resistance arm (load arm). Increasing the effort arm reduces the size of the effort needed to balance the

load torque. In fact, the ratio of the effort to the load is equal to the ratio of the effort arm to the load arm:

$$(1) \frac{\text{load}}{\text{effort}} = \frac{\text{effort arm}}{\text{load arm}}$$

### Every Day Examples: Biceps Tension

Let's calculate the biceps tension need in our initial body lever example of a holding a 50 **lb** ball in the hand. We are now ready to determine the bicep tension in our forearm problem. The effort arm was 1.5 **in** and the load arm was 13.0 **in**, so the load arm is 8.667 times longer than the effort arm.

$$\frac{13 \text{ in}}{1.5 \text{ in}} = 8.667$$

That means that the effort needs to be 8.667 times larger than the load, so for the 50 **lb** load the bicep tension would need to be 433 **lbs**! That may seem large, but we will find out that such forces are common in the tissues of the body!

#### *\*Adjusting Significant Figures*

Finally, we should make sure our answer has the correct significant figures. The weight of the ball in the example is not written in scientific notation, so it's not really clear if the zeros are placeholders or if they are significant. Let's assume the values were not measured, but were chosen hypothetically, in which case they are exact numbers like in a definition and don't affect the significant figures. The forearm length measurement includes zeros behind the decimal that would be unnecessary for a definition, so they suggest a level of precision in a measurement. We used those values in multiplication and division so we should round the answer to only two significant

figures, because 1.5 **in** only has two (13.0 **in** has three). In that case we round our bicep tension to 430 **lbs**, which we can also write in scientific notation:  $4.3 \times 10^2$  **lbs**.

### *\*Neglecting the Forearm Weight*

Note: We ignored the weight of the forearm in our analysis. If we wanted to include the effect of the weight of the forearm in our example problem we could look up a typical forearm weight and also look up where the center of gravity of the forearm is located and include that load and resistance arm. Instead let's take this opportunity to practice making *justified* assumptions. We know that forearms typically weigh only a few pounds, but the ball weight is 50 **lbs**, so the forearm weight is about an order of magnitude (10x) smaller than the ball weight<sup>7</sup>. Also, the center of gravity of the forearm is located closer to the pivot than the weight, so it would cause significantly less torque. Therefore, it was reasonable to assume the forearm weight was negligible for our purposes.

## MECHANICAL ADVANTAGE

The ratio of load to effort is known as the mechanical advantage (*MA*). For example if you used a second class lever (like a wheelbarrow) to move 200 **lbs** of dirt by lifting with only 50 **lbs** of effort, the mechanical advantage would be four. The mechanical advantage is equal to the ratio of the effort arm to resistance arm.

$$(2) \quad MA = \frac{\text{load}}{\text{effort}} = \frac{\text{effort arm}}{\text{load arm}}$$

7. "Weight, Volume, and Center of Mass of Segments of the Human Body" by Charles E. Clauser, et al, National Technical Information Service, U.S. Department of Commerce

## Reinforcement Activity

Calculate the mechanical advantage of the lever system in our forearm example. [Hint: your answer should be less than one.]

### RANGE OF MOTION

We normally think of levers as helping us to use less effort to hold or move large loads, so our results for the forearm example might seem odd because we had to use a larger effort than the load. The bicep attaches close to the elbow so the effort arm is much shorter than the load arm and the mechanical advantage is less than one. That means the force provided by the bicep has to be much larger than the weight of the ball. That seems like a mechanical disadvantage, so how is that helpful? If we look at how far the weight moved compared to how far the bicep contracted when lifting the weight from a horizontal position we see that the purpose of the forearm lever is to increase range of motion rather than decrease effort required.

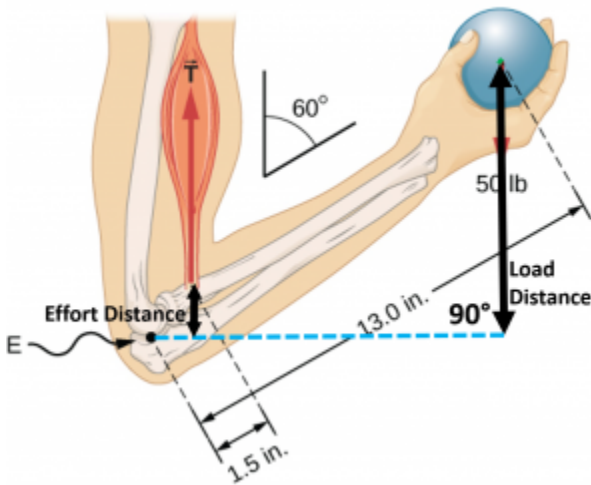
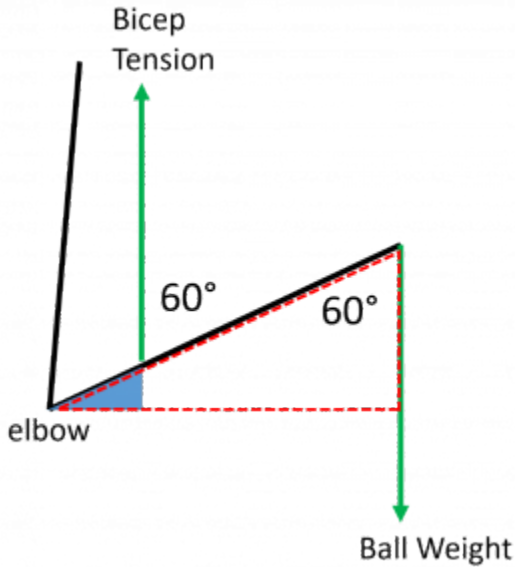


Diagram showing the difference in distance covered by the contracting bicep and the weight in the hand when moving the forearm from horizontal. Image Adapted from Openstax University Physics

Looking at the similar triangles in a stick diagram of the forearm we can see that the ratio of the distances moved by the effort and load must be the same as the ratio of effort arm to resistance arm. That means increasing the effort arm in order to decrease the size of the effort required will also decrease the range of motion of the load by *the same factor*. It's interesting to note that while moving the attachment point of the bicep 20% closer to the hand would make you 20% stronger, you would then be able to move your hand over a 20% smaller range.



*Diagram of the forearm as a lever, showing the similar triangles formed by parts of the forearm as it moves from 90 degrees to 60 degrees from horizontal. The hypotenuse (long side) of the smaller blue triangle is the effort arm and the hypotenuse of the larger dashed red triangle is the load arm. The vertical sides of the triangles are the distances moved by the effort (blue) and the load (dashed red).*

### Reinforcement Exercises

For the case of our example forearm, if the biceps contracts by 2.0 cm, how far will the hand move?

For third class levers the load is always farther from the

fulcrum than the effort, so they will always increase range of motion, but that means they will always increase the amount of effort required by the same factor. Even when the effort is larger than the load as for third class levers, we can still calculate a mechanical advantage, but it will come out to be less than one.

Second class levers always have the load farther from the pivot than the effort, so they will always allow a smaller effort to move a larger load, giving a mechanical advantage greater than one.

First class levers can either provide mechanical advantage or increase range of motion, depending on if the effort arm or load arm is longer, so they can have mechanical advantages of greater, or less, than one.

*A lever cannot provide mechanical advantage and increase range of motion at the same time, so each type of lever has advantages and disadvantages:*

**Comparison of Advantages and Disadvantages of Lever Classes**

Lever Class	Advantage	Disadvantage
3rd	<b>Range of Motion</b> The load moves farther than the effort. <i>(Short bicep contraction moves the hand far)</i>	<b>Effort Required</b> Requires larger effort to hold smaller load. <i>(Bicep tension greater than weight in hand)</i>
2nd	<b>Effort Required</b> Smaller effort will move larger load. <i>(One calf muscle can lift entire body weight)</i>	<b>Range of Motion</b> The load moves a shorter distance than the effort. <i>(Calf muscle contracts farther than the distance that the heel comes off the floor)</i>
1st (effort closer to pivot)	<b>Range of Motion</b> The load moves farther than the effort. <i>(Head moves farther up/down than neck muscles contract)</i>	<b>Effort Required</b> Requires larger effort to hold smaller load.

1st (load closer to pivot)	<b>Effort Required</b> Smaller effort will move larger load.	<b>Range of Motion</b> The load moves shorter distance than the effort.
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### Reinforcement Activity

If you used a wheelbarrow to move 200 lbs of dirt by lifting with 50 lbs of effort, what is the mechanical advantage?

If the handles of the wheelbarrow are 2.0 m from the wheel axle (fulcrum) then how far from the fulcrum is the center of gravity of the the dirt?

To lift the dirt load 3 in, what distance do you have to lift the handles?

Check out the following lever simulation explore how force and distance from fulcrum each affect the equilibrium of the lever. This simulation includes the effects of friction, so you can see how kinetic friction in the joint (pivot) works to stop motion and static friction contributes to maintaining static equilibrium by resisting a start of motion.

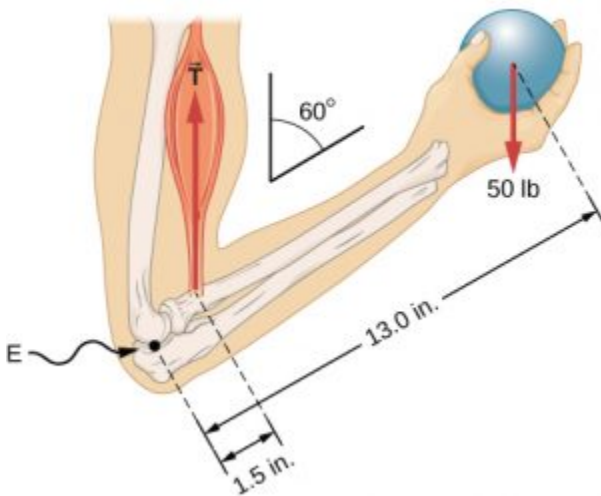
An interactive or media element has been excluded from this version of the text. You can view it online here:  
<https://openoregon.pressbooks.pub/bodyphysics/?p=578>

## CHAPTER 49.

### FORCES IN THE ELBOW JOINT

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In the previous chapter we found the biceps tension force in our example problem to be 430 **lbs**! You may have noticed that when we found the biceps tension we completely ignored the forces acting at the elbow joint. We were allowed to do this because those forces cause no torque. *Forces acting on the fulcrum of a lever don't cause the lever to rotate.* Just because the forces on the elbow don't cause rotation, that doesn't mean they aren't important. Those forces can certainly damage the joint if they get too large. Let's try to find out how big those forces are for our example problem.



*The elbow joint flexed to form a  $60^\circ$  angle between the upper arm and forearm while the hand holds a 50 lb ball . Image Credit: Openstax University Physics*

1

The forearm is holding still and not moving so it must be in static equilibrium and all the vertical forces must be canceling out. If the vertical forces didn't cancel out the forearm would begin to move up or down. We already know that the weight of the ball is **50 lbs** downward and the bicep tension is **433 lbs** upward. The weight cancels 50 **lbs** worth of the muscle tension, leaving behind a remaining **483 lbs** of upward force. The forearm is in static equilibrium, so the vertical force on the end of the forearm at the elbow must cancel out this **483 lbs** upward

1. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Jul 11, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@10.18>.

force, meaning that the vertical force on the elbow end of the forearm is 483 **lbs** downward. This force comes from the upper arm bone (humerus) pushing down on the end of the forearm bones (radius and ulna). Adjusting our significant figures, we should report this force as 480 **lbs**.

### Reinforcement Exercises

Draw a free body diagram of the elbow showing the forces from the ball weight, the bicep tension, and the upper arm pushing on the forearm. The values for all of these forces are given in the previous paragraph.

### HORIZONTAL ELBOW FORCES

The horizontal forces must all cancel out because the forearm is in static equilibrium, but there are no horizontal forces in our example to begin with, so that's it. We're finished analyzing the forces on the forearm while holding a 50 **lb** ball!

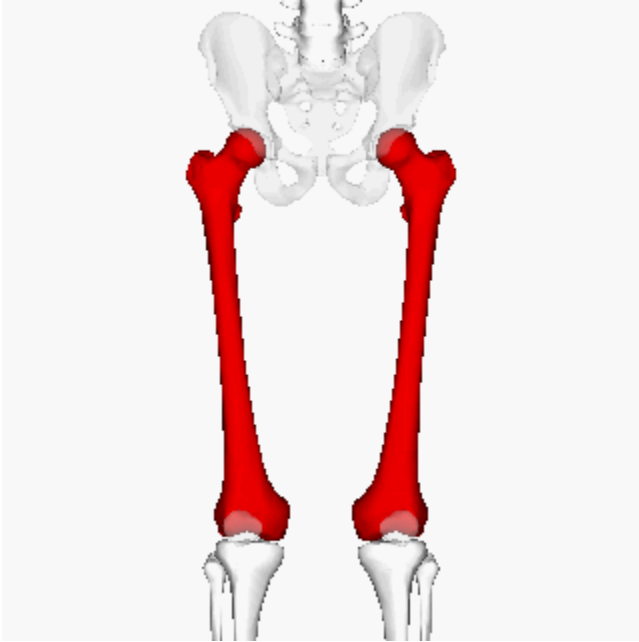
## CHAPTER 50.

# ULTIMATE STRENGTH OF THE HUMAN FEMUR

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### COMPRESSING THE FEMUR

Opposite to tension forces, compression forces are provided by a material in response to being compressed rather than stretched. The resistance of materials to deformation is what causes the normal force (support force) that we introduced in the unit on balance. For example, the femur is compressed while supporting the upper body weight of a person.



*The Human Femur. Image Credit: Anatomography via Wikimedia Commons*

1

“In human anatomy, the femur (thigh bone) is the longest and largest bone. Along with the temporal bone of the skull, it is one of the two strongest bones in the body. The average adult male femur is 48 **cm** (18.9 **in**) in length and 2.34 **cm** (0.92 **in**) in diameter and can support up to 30 times the weight of an adult.”<sup>2</sup>The average weight among adult males in the United States is 196 **lbs** (872 **N**)<sup>3</sup>. According to the statement that the femur can support 30x body weight, the adult male femur can

1. By Anatomography [CC BY-SA 2.1 jp (<https://creativecommons.org/licenses/by-sa/2.1/jp/deed.en>)], via Wikimedia Commons
2. "Femur" by Orthopaedics One

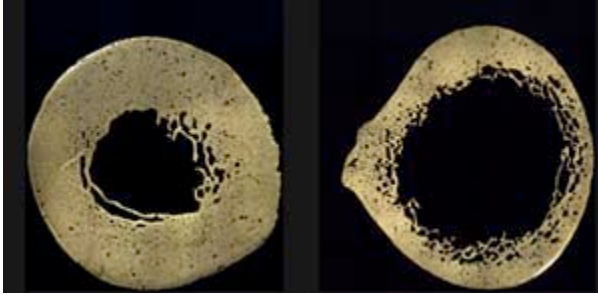
support roughly 6,000 **lbs** of compressive force! Such high forces are rarely generated by the body under its own power, thus motor vehicle collisions are the number one cause of femur fractures<sup>4</sup>.

## **STRESS**

The size of object affects how they deform in response to applied compression and tension forces. For example, the maximum compression or tension forces that a bone can support depends on the size of the bone. More specifically, the more area available for the force to be spread out over, the more force the bone can support. That means the maximum forces bones, (and other objects) can handle are proportional to the cross-sectional area of the bone that is perpendicular (90°) to the direction of the force. For example, the force that the femur can support vertically along its length depends on the area of its horizontal cross-sectional area which is roughly circular and somewhat hollow (bone marrow fills the center space).

3. "Body Measurement" by FastStats, U.S. Centers for Disease Control is in the Public Domain

4. "Femur Shaft Fractures" by OrthoInfo, American Academy of Orthopaedic Surgeons



*These cross sections show the midshaft of the femur of an 84-year-old female with advanced osteoporosis (right), compared to a healthy femur of a 17-year-old female (left).  
Image Credit: Smithsonian National Museum of Natural History*

5

Larger bones and tendons can support more force, so in order to analyze the behavior of the bone material itself we would need to divide the force applied to by the cross-sectional area ( $A_x$ ). The resulting quantity is known as the stress ( $\sigma$ ) on the material. Stress has units of force per area so the SI units are ( $\text{N}/\text{m}^2$ ) which are also known as Pascals. Units of pounds per square inch (**PSI, lbs/in<sup>2</sup>**) are common in the U.S.

$$(1) \quad \text{stress} = \sigma = \frac{F}{A_x}$$

## Reinforcement Exercises

Estimate the compressive stress within a 1.0 cm x 2.0 cm Lego block when you step on it with full body weight in units of `pb_glossary id="3977"`Pascals[/pb\_glossary]. [Hint: We want the result in SI units, so convert the length and width to meters before calculating the cross-sectional area and use SI units for your weight.]

### ULTIMATE STRENGTH OF THE FEMUR

The maximum stress that bone, or any other material, can experience before the material begins fracture or rupture is called the ultimate strength. Notice that material strength is defined in terms of stress, not force, so that we are analyzing the material itself, without including the effect of *how much* material is present. For some materials the ultimate strength is different when the stress is acting to crush the material (compression) versus when the forces are acting to stretch the material under tension, so we often refer to ultimate tensile strength or ultimate compressive strength. For example, the ultimate compressive strength for human femur bone is measured to be 205 **MPa** (205 Million Pascals) under compression along its length. The ultimate tensile strength of femur bone under tension along its length is 135 **MPa**.<sup>6</sup> Along with bone, concrete and chalk are other examples of materials with different compressive and tensile ultimate strengths.

6. "Elastic anisotropy of bone" by Rod Lakes, College of Engineering, University of Wisconsin

## Reinforcement activity

Try to crush a piece of chalk by using your fingers to push on the ends and compress it along the long axis, no bending allowed. Any luck?

Now use your fingers to break the chalk by pulling it apart, straight along the long axis, again no bending allowed. Any luck?

Record your results and explain what they tell you about the compressive ultimate strength and tensile ultimate strength of chalk.

Compare and contrast the behavior you observed for chalk with the known behavior of bone and concrete. Cite your sources.

## Everyday Example: Femur Ultimate Strength

Let's check to see if the measured values for compressive ultimate strength agree with the claim that the human femur can support 30x the adult body weight, or roughly 6,000 lbs

First let's to convert the claimed 6,000 lbs force to Newtons and work in SI units.

$$6,000 \text{ lbs} = 6,000 \text{ lbs} \left( \frac{4.45 \text{ N}}{1 \text{ lb}} \right) = 26,166 \text{ N}.$$

An approximate minimum cross-sectional area of the femur is  $3.2 \times 10^{-4} \text{ m}^2$ . (\*See the bottom of this example if you are interested in learning how we approximated this value). We divide the compressive force by the cross-sectional area to find the compressive stress on the bone.

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{26,166 \text{ N}}{3.2 \times 10^{-4} \text{ m}^2} = 80,000,000 \text{ Pa} = 80 \text{ MPa}$$

Our approximate value for the ultimate strength of bone that would be required to support 30x body weight was 80 MPa, which is actually less than the measured value of 205 MPa, so the claim that the femur can support 30x body weight seems reasonable.

---

*\*This is how we approximated the femur cross-sectional area, skip this if you aren't interested:*

First we divide the 2.34 cm femur diameter quoted earlier by two to find the femur radius, then we convert to standard units of meters.

$$r = \frac{2.34 \text{ cm}}{2} \left( \frac{1 \text{ m}}{100 \text{ cm}} \right) = 0.0117 \text{ m} = 1.17 \times 10^{-2}$$

Using the equation for the area of a circle we calculate the total area of the femur to be:

$$A_{out} = \pi r^2 = \pi (0.0117 \text{ m})^2 = 0.00043 \text{ m}^2 = 4.3 \times 10^{-4} \text{ m}^2$$

Finally we have to subtract off the area of the hollow middle part to get the net bone area. We used a ruler on the above picture of the femur cross-sections to see that the inner radius is roughly half of the outer radius, or  $5.85 \times 10^{-3} \text{ m}$  so we calculate the missing inner area:

$$A_{in} = \pi r^2 = \pi (5.85 \times 10^{-3} \text{ m})^2 = 1.1 \times 10^{-4} \text{ m}^2$$

And subtract off the inner area from the total:

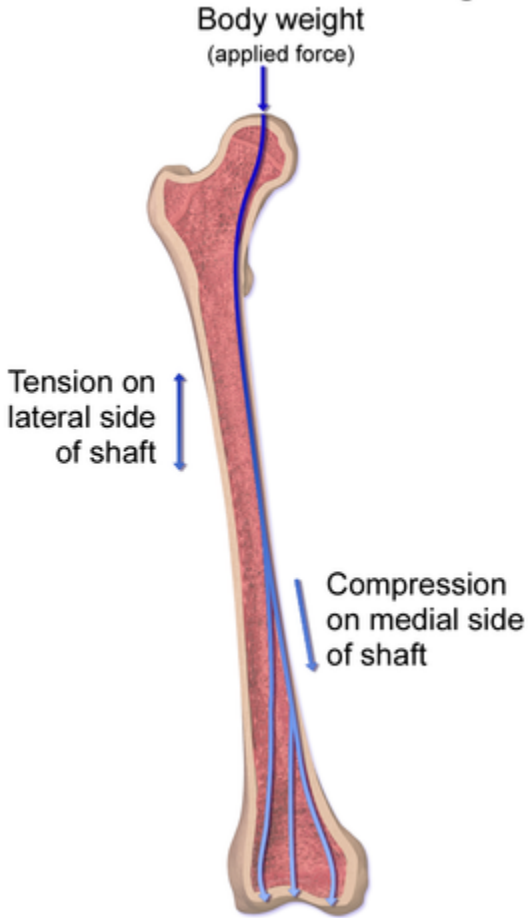
$$A_x = 4.3 \times 10^{-4} \text{ m}^2 - 1.1 \times 10^{-4} \text{ m}^2 = 3.2 \times 10^{-4} \text{ m}^2$$

## TRANSVERSE ULTIMATE STRENGTH

So far we have discussed ultimate strengths along the long axis of the femur, known as the longitudinal direction. Some materials, such as bone and wood, have different ultimate strengths along different axes. The ultimate compressive strength for bone along the short axis (transverse direction) is 131 **MPa**, or about 36% less than the 205 **MPa** longitudinal value. Materials that have different properties along different axes are known as anisotropic. Materials that behave the same in all directions are called isotropic.

An interesting fact to finish up this chapter: when a person stands the femur actually experiences compressive and tensile stresses on different sides of the bone. This occurs because the structure of the hip socket applies the load of the body weight off to the side rather than directly along the long axis of the bone.

## Distribution of Forces on a Long Bone



*Both tension and compressive stresses are applied to the Femur while standing. Image Credit: Blausen Medical via Wikimedia Commons*

7

7. Blausen.com staff (2014). "Medical gallery of Blausen Medical 2014". WikiJournal of Medicine 1 (2). DOI:10.15347/wjm/2014.010. ISSN 2002-4436. [CC BY 3.0 (<https://creativecommons.org/licenses/by/3.0>)], from Wikimedia Commons

## CHAPTER 51.

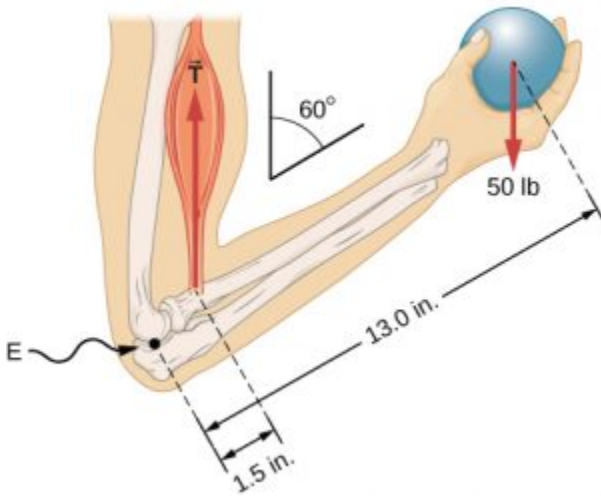
### ELASTICITY OF THE BODY

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#### BICEPS TENSION

Earlier in this unit we found that 430 **lbs** of biceps tension are required to hold a 50 **lb** weight in the hand. Tension forces are restoring forces produced in response to materials being stretched. “The biceps muscle has two tendons that attach the muscle to the shoulder and one tendon that attaches at the elbow. The tendon at the elbow is called the distal biceps tendon. It attaches to a part of the radius bone called the radial tuberosity, a small bump on the bone near your elbow joint.”<sup>1</sup> In addition to the muscle, these tendons are under the same tension as the muscle, and are therefore being stretched. As long as the tendon is not stretched too far, it will behave elastically, meaning that it will return to its original length with no permanent deformation (damage).

1. "Biceps Tendon Tear at the Elbow" by OrthoInfo, American Academy of Orthopedic Surgeons



*The elbow joint flexed to form a  $60^\circ$  angle between the upper arm and forearm while the hand holds a 50 lb ball . Image Credit: Openstax University Physics*

2

## HOOKE'S LAW

When objects, like the distal biceps tendon are only slightly stretched they will behave like springs. In that case the relationship between the tension force and stretch distance follows Hooke's Law and we often call the stretch distance the displacement, ( $\Delta x$ ):

$$(1) F = k\Delta x$$

If we wanted to use Hooke's Law to calculate the stretch of the distal biceps tendon caused by the 430 **lbs** biceps tension for a particular person, we would need to

2. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Jul 11, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@10.18>.

know the spring constant of that person's tendon. The spring constant depends on the stiffness of a particular material, known as the Elastic Modulus, but it also depends on the size and length of the object. For example, a wider tendon will not stretch as much as a narrow one, but a longer tendon would stretch farther than a shorter one. Modeling the tendon as a spring, we can think of stretching a tendon that has twice the cross-sectional area as equivalent to stretching a two of the original springs at the same time, which would require twice the applied force to create the same displacement. We can also think of a tendon with twice the length as equivalent to stretching two of the original springs placed end-to-end. To get the same stretch as a single spring, each spring will only have to stretch half of the total distance, so that would require only half the force to create the same total displacement. Therefore, the spring constant of the tendon (or any object) is proportional to the cross-sectional area ( $A_x$ ) and inversely proportional to its length ( $L$ ).

$$(2) \quad k = E \frac{A_x}{L}$$

Now we can see that the size of an object affects the spring constant. Therefore the force required to achieve a particular stretch is different for objects of different size, even when they are made of the same material. Replacing the spring constant in Hooke's Law with the previous equation shows how force depends on cross-sectional area and length:

$$(3) \quad F = E \frac{A_x}{L} \Delta x$$

## Everyday Examples: Biceps Tendon Stretch

We can use the previous equation to calculate the stretch in the biceps distal tendon for the 430 lb tension force required to hold a 50 lb ball in the hand. First we need to rearrange the equation for the stretch distance by dividing both sides by  $E$ ,  $L$  and  $A_x$ :

$$(4) \quad \frac{F}{E} \frac{L}{A_x} = \Delta x$$

A elastic modulus for tendon is  $1.5 \times 10^9 \text{ N/m}^2$ .<sup>3,4</sup> A typical length of the biceps distal tendon is 6.3 cm and a typical cross-sectional area is  $1.5 \times 10^{-5} \text{ m}^2$ . If we convert the length to meters (0.063 m) and the 430 lb force to Newtons (1913 N) we are ready to find the stretch distance using the previous equation:

$$(5) \quad \Delta x = \frac{F}{E} \frac{L}{A_x} = \frac{1913 \text{ N}}{1.5 \times 10^9 \text{ N/m}^2} \frac{0.063 \text{ m}}{3.8 \times 10^{-5} \text{ m}^2} = 0.002 \text{ m} = 2 \text{ mm}$$

The biceps distal tendon would stretch by an additional 2 mm when placed under the 430 lb tension.

## STRAIN

Notice that the stretch in the biceps tendon that we calculated was dependent on the original length of the tendon. This makes sense because we know it's easier to stretch a long object than a short one. For example, if you tie a body-length section of 1 cm thick nylon rope

3. "Influence of biceps brachii tendon mechanical properties on elbow flexor force steadiness in young and old males" by R. R. Smart, S. Baudry, A. Fedorov, S. L. Kuzyk and J. M. Jakobi, Wiley Online Library
4. "The distal biceps tendon: footprint and relevant clinical anatomy" by Athwal GS, Steinmann SP, Rispoli DM., U.S. National Library of Medicine, National Institutes of Health

to a pole and then pull as hard as you can the stretch will be barely noticeable. If you instead used a rope with 10x body length, you would easily notice the stretch even though both ropes were made of the same material. In order to study the properties of specific materials like tendons, independent of size, we can divide the stretch by the original length. This quantity is known as the strain ( $\epsilon$ ):

$$(6) \text{ strain} \epsilon = \Delta x / L$$

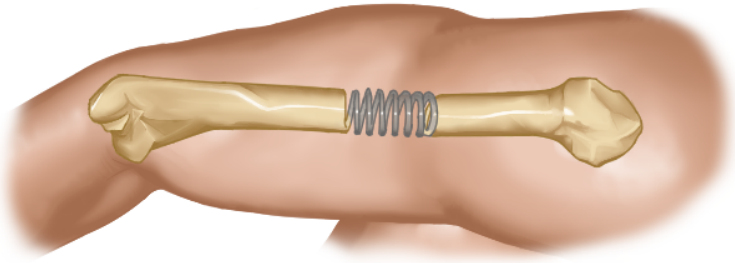
### Reinforcement Exercises

What was the strain experienced by the tendon in the previous example?

We need to be careful with the term strain because it has a different meaning in medical terminology where a strain is an over-stretching or tearing of muscle or tendon, which connect muscle to bones. A sprain is an over-stretching or tearing of ligaments, which connect bones together in joints.<sup>5</sup>

## ELASTIC MODULUS

5. "Sprains and Strains" by Patient care and health information, Mayo Clinic



Artist's conception of the elastic behavior body tissues. "Armcoil" by Sasha Lynch.

Within the linear region we can model materials as springs, just like we did with the biceps distal tendon in the previous chapter. We can start by writing Hooke's Law in terms of the material elastic modulus just as we did for the bicep:

$$(7) \quad F = E \frac{A_x}{L} \Delta x$$

Notice that the right hand side contains our definition of strain, so we can write

$$F = E \cdot A_x \cdot strain$$

If we divide both sides by cross sectional area, we will suddenly have the definition of stress on the left:

$$\frac{F}{A_x} = E \cdot strain$$

So we can write:

$$(8) \quad stress = E \cdot strain$$

We now see that when a material is behaving like a spring, the stress will be proportional to the strain and the elastic modulus of the material will be the proportionality constant that relates the stress and strain. When an object is behaving this way, we say the stress and strain fall within the linear region of the material. To actually find

the elastic modulus of a material experimentally we rearrange the equation:

$$(9) \quad E = \frac{\text{stress}}{\text{strain}}$$

Then we just need to measure how much additional strain is caused by an applied stress (or *vice versa*) then divide the stress by strain to get the elastic modulus. Of course we need to be sure that the material is operating within its linear region, so that it still acts like a spring.

### Reinforcement Exercises

An particular material is withing its linear region and experiences a strain of 0.05 under a new stress of 55,000 Pa.

What is the elastic modulus of the material?

Just as for the ultimate strength, some materials have a different elastic modulus when the stress is applied along different axes, or even between tension and compression along the same axis. For example, the tensile elastic modulus of bone is **16 GPa** ( $16 \times 10^9 \text{ Pa}$ ) compared to **9 GPa** under compression.<sup>6</sup> Check out the engineering toolbox for a massive tensile elastic modulus table. For more information on stress and strain in human tissues, including excellent diagrams, check out posted lecture notes from Professor Tony Leyland at Simon Fraser University.

6. OpenStax, College Physics. OpenStax CNX. Aug 6, 2018 <http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@13.1>.

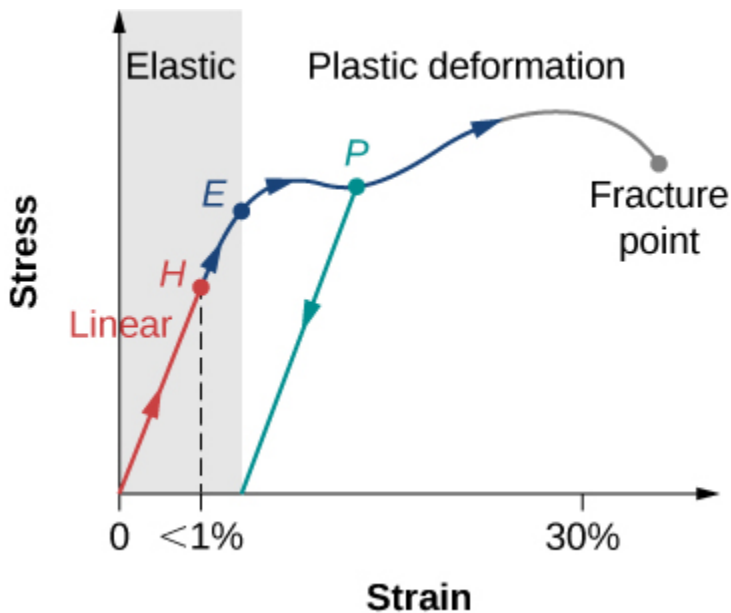
## CHAPTER 52.

# DEFORMATION OF TISSUES

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### STRESS VS. STRAIN CURVES

If you apply some stress to a material and measure the resulting strain, or *vice versa*, you can create a stress vs. strain curve like the one shown below for a typical metal.



Typical stress-strain plot for a metal: The graph ends at the fracture point. The arrows show the direction of changes under an ever-increasing stress. Points H and E are the linearity and elasticity limits, respectively. The green line originating at P illustrates the metal's return to a greater than original length when the stress is removed after entering the plastic region. Image Credit: OpenStax University Physics

1

We see that the metal starts off with stress being proportional to strain, which means that the material is operating in its linear region. We have graphed stress on the vertical axis and strain on the horizontal axis, so the value of *stress/strain* is equal to the *rise/run* of the graph. We saw in the previous chapter that within the linear

1. OpenStax University Physics, University Physics Volume 1. OpenStax CNX. Aug 2, 2018 <http://cnx.org/contents/d50f6e32-0fda-46ef-a362-9bd36ca7c97d@11.1>.

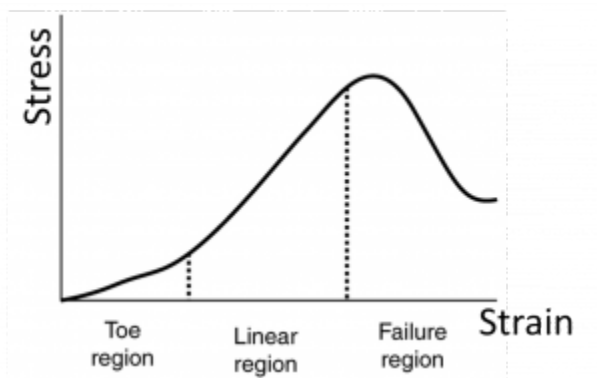
region *stress/strain* is equal to the the elastic modulus and we know the *rise/run* of a graph is the slope, therefore the *elastic modulus of a material is equal to the slope of the linear portion its stress vs. curve*. Let's discuss the important features of the stress vs. strain curve:

1. The absolute highest point on the graph is the ultimate strength, indicating the onset of failure toward fracture or rupture.
2. Notice that after reaching the ultimate strength, but before full failure, the stress can actually decrease as strain increases, this is because the material is changing shape by breaking rather than stretching or compressing the distance between molecules in the material.
3. In the first part of the elastic region, the strain is proportional to the stress, this is known as the linear region. The slope of this region is the elastic modulus.
4. After the stress reaches the linearity limit ( $H$ ) the slope is no longer constant, but the material still behaves elastically.
5. The elastic region ends and the plastic region begins at the yield point ( $E$ ). In the plastic region, a little more stress causes a lot more strain because the material is changing shape at the molecular level. In some cases the stress can actually decrease as strain increases, because the material is changing shape by re-configuring molecules rather than just stretching or compressing the distance between molecules.
6. The green line originating at  $P$  illustrates the metal's return to non-zero strain value when the stress is

removed after being stressed into the plastic region (permanent deformation).

## STRESS AND STRAIN IN TENDONS

Tendons (attaching muscle to bone) and ligaments (attaching bone to bone) have somewhat unique behavior under stress. Functionally, tendons and ligaments must stretch easily at first to allow for flexibility, corresponding to the toe region of the stress-strain curve shown below, but then resist significant stretching under large stress to prevent hyper-extension and dislocation injuries.



*Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region. Image adapted from OpenStax College Physics.*

2

The structure of the tendon creates this specialized behavior. To create the toe region, a small stress causes

2. OpenStax, College Physics. OpenStax CNX. Aug 6, 2018 <http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@13.1>.

the fibers in the tendon begin to align in the direction of the stress, or uncrimp, and the re-alignment provides additional length. Then in the linear region, the fibrils themselves will be stretched.

## STRESS AND STRAIN INJURIES

Stress beyond the yield point will cause permanent deformation and stress beyond the ultimate strength will cause fracture or rupture. These occurrences in body tissues are known as injuries. For example, *sprains* occur when a ligament (connects bone to bone) is torn by a stress greater than its ultimate strength, or even just stretched beyond its elastic region. The same event



A YouTube element has been excluded from this version of the text. You can view it online here: <https://openoregon.pressbooks.pub/bodyphysics/?p=777>

occurring in a tendon (connects muscle to bone) is called known as *strain*.<sup>3</sup> We already know that strain has a different, but related meaning to physicists and engineers, so that discrepancy in terminology is something to watch out for.

### Reinforcement Activity

Hang a rubber band from a cabinet knob, doorknob or other feature. Use a paperclip or tape to hang a plastic cup or baggie or other container to the rubber band. Measure the length of the rubber band. Start adding pennies, five at a time, to the container. Measure the distance the rubber band stretches with each addition and calculate the strain for each case. Do this until you have added 25 pennies and record your results. Now look at the strain values you have and find how much the strain *changed* between each addition of pennies.

The change in stress is the same between each test because you add the same number of pennies each test, but is the change in strain you measured the same each time?

Are you in the linear region throughout this experiment? Explain.

Look up the weight of a penny, measure the cross-sectional area of your rubber band, and calculate the stress you applied with 10 pennies.

Use your stress and strain values for 10 pennies to calculate the elastic modulus of the rubber band.

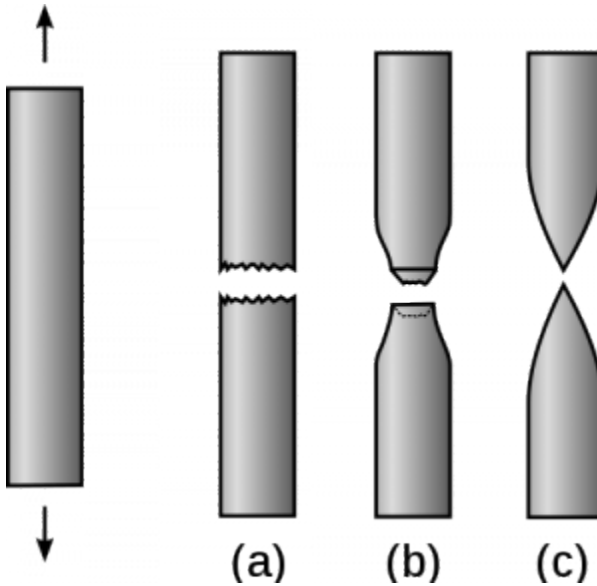
3. "Sprains and Strains" by Patient Care and Health Information, Mayo Clinic

## CHAPTER 53.

### BRITTLE BONES

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Brittle materials have a small plastic region and they begin to fail toward fracture or rupture almost immediately after being stressed beyond their elastic limit. Bone, cast iron, ceramic, and concrete are examples of brittle materials. Materials that have relatively large plastic regions under tensile stress are known as ductile. Examples of ductile materials include aluminum and copper. The following figure shows how brittle and ductile materials change shape under stress. Even the cartilage that makes up tendons and ligaments is relatively brittle because it behaves less like example (c) and more like examples (a) and (b). Luckily, those tissues have adapted to allow the deformation required for typical movement without the brittle nature of the material coming into play. We will learn about that adaptation in the next chapter.



*Profile (a) is an example of the material that fractures with no plastic deformation, i.e., it is a brittle material. Profile (b) is an example of a material that fractures after very little plastic deformation. These two profiles would be classified as having low ductility. Profile (c) in contrast is a material that plastically deforms before fracture. This material has high ductility. Image Credit: Sigmund (Own work) [CC BY-SA 3.0], via Wikimedia Commons*

Materials that are very malleable can undergo significant plastic deformation under compressive stress, as opposed to tensile stress. Very malleable materials can be pounded into thin sheets. Gold is the most malleable metal.<sup>1</sup>

1. "Malleability and Ductility" by John A. Dutton e-Education Institute, Penn State University

## CHAPTER 54.

# EQUILIBRIUM TORQUE AND TENSION IN THE BICEP\*

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### TORQUE ABOUT THE ELBOW

So far we have used lever concepts and static equilibrium to solve for the forces in our forearm example. To gain a deeper understanding of why and how the effort and load forces depend on the effort arm and load arm distances, we can make a closer study of the concepts of torque and static equilibrium. We have already decided that the weight of the ball was pulling the forearm down and trying to rotate it around the elbow joint. When a force tends to start or stop rotating an object then we say the force is causing a torque ( $\tau$ ). In our example, the weight of the ball is causing a torque on the forearm with the elbow joint as the pivot. The size of a torque depends on several things, including the distance from the pivot point to the force that is causing the torque.

## Reinforcement Activity

The torque caused by a force depends on the distance that force acts from the pivot point. To feel this effect for yourself, try this:

Open a door by pushing perpendicular to the door near the handle, which is far from the pivot point at the hinges.

Now apply the same force perpendicular to the door, but right next to the hinges. Does the door open as easily as before, or did you have to push with greater force to make the door rotate?

One method to account for the effect of the distance to pivot when calculating the size of a torque you can first draw the line of action of the force, which just means to extend a line from both ends of the force arrow (vector) in both directions. Next you draw the shortest line that you can from the pivot point to the line of action of the force. This shortest line and the line of action of the force will always be at  $90^\circ$  to each other, so the shortest line is called the *perpendicular distance* ( $d_\perp$ ). The perpendicular distance is also sometimes called the lever arm or *moment arm* or torque arm. We can draw these lines for our example problem:

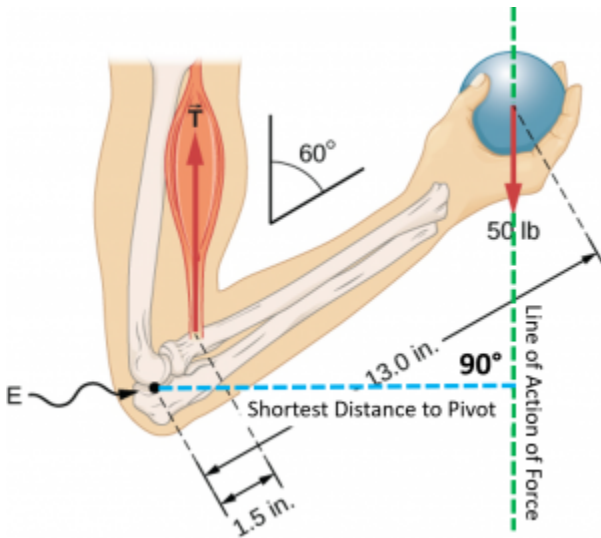


Diagram of the flexed arm showing the line of action of the gravitational force and the perpendicular distance from the pivot to the line of action. Image adapted from Openstax University Physics.

Finally, we can calculate the torque by multiplying the size of the force by the length of the lever arm ( $Fd_{\perp}$ ) and that's it, you get the torque. In symbol form it looks like this:

$$(1) \text{ torque} = \tau = Fd_{\perp}$$

### Reinforcement Activity

If you hold your 0.65 m arm out horizontally with a 12 lb weight in your hand, what is the torque about your shoulder joint caused by the weight? Hint: If your arm is horizontal, and the weight points straight

down, then what is the perpendicular distance from the joint to the weight?

After you have an answer, convert it from **N·m** to **ft·lbs** by using conversion factors between Newtons and pounds and feet and meters.

## STATIC EQUILIBRIUM

For an object to be in static equilibrium both the equilibrium conditions must be met. Writing these conditions on the torque and force in symbol form we have:

$$(2) \tau_{\text{net}} = 0$$

AND

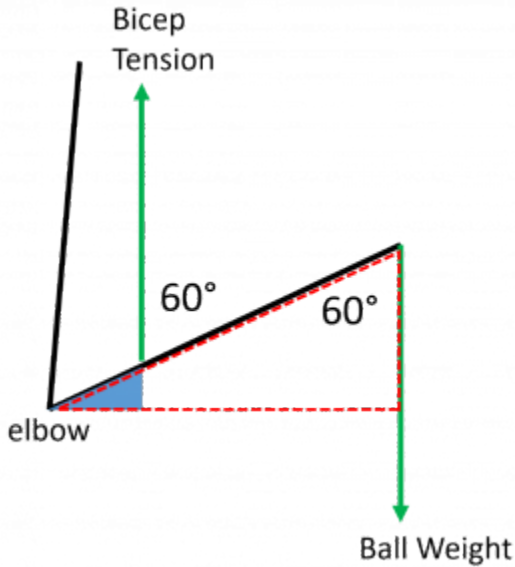
$$(3) \mathbf{F}_{\text{net}} = 0$$

## BICEP TENSION

The torques due to the bicep tension and the ball weight are trying to rotate the elbow in opposite directions, so if the forearm is in static equilibrium the two torques are equal in size they will cancel out and the net torque will be zero.

Looking at our equation for torque, we see that it only depends on the size of the force and the lever arm. That means that if the perpendicular distance to the bicep tension were 10x smaller than the distance to the center of the ball, the bicep tension force will have to be 10x times bigger than the weight of the ball in order to cause the same size torque and maintain rotational equilibrium. To find the bicep tension all we need to do now is determine how many times bigger the is the lever arm for the weight compared to the lever arm for the tension.

You might be thinking, *but we can't use this method, we don't know the perpendicular lengths, they aren't given, we only have the full distances from pivot to ball and pivot to bicep attachment.* Don't worry, if we draw a stick figure diagram we can see two triangles formed by the force action lines, the forearm and the perpendicular distances. The dashed (red) and solid (blue) triangles are similar triangles, which means that their respective sides have the same ratios of lengths.



*Diagram of the forearm as a lever, showing the similar triangles formed by parts of the forearm as it moves from 90 degrees to 60 degrees from horizontal. The hypotenuse (long side) of the smaller blue triangle is the effort arm and the hypotenuse of the larger dashed red triangle is the load arm. The vertical sides of the triangles are the distances moved by the effort (blue) and the load (dashed red)*

The lengths of the long sides of the triangles are 13.0 **in** and 1.5 **in**. Taking the ratio (dividing 13.0 by 1.5) we find that 13.0 **in** is 8.667x longer than 1.5 **in**. The bottom side of the small (solid) triangle must also be 8.667x smaller than the bottom side of the big one (dashed). That means that the lever arm for the bicep is 8.667x smaller than for the weight and so we know the bicep tension must

be 8.667x bigger than the weight of the ball to maintain rotational equilibrium.

The ball weight is **50 lbs**, so the bicep tension must be:

$$8.667 \times 50 \text{ lbs} = 433 \text{ lbs}$$

We've done it! Our result of **433 lbs** seems surprisingly large, but we will see that forces even larger than this are common in the muscles, joints, and tendons of the body.

### SYMBOL FORM

Do you want to see everything we just did to calculate the tension in symbol form? Well, here you go:

The size of the torque due to the ball weight should be the tension multiplied by perpendicular distance to the ball:

$$\tau_g = F_g \cdot d_{\perp,B}$$

The size of the torque due to the bicep tension should be the tension multiplied by perpendicular distance to the bicep attachment:

$$\tau_T = T \cdot d_{\perp,T}$$

In order for net torque to be zero, these torques must be equal in size:

$$T \cdot d_{\perp,T} = F_g \cdot d_{\perp,B}$$

We want the tension, so we divide both sides by  $d_{\perp,T}$ :

$$T = \frac{F_g \cdot d_{\perp,B}}{d_{\perp,T}}$$

From the similar triangles we know that the ratio of perpendicular distances is the same as the ratio of the triangles' long sides:

$$d_{\perp,T} = \frac{13.0 \text{ in}}{1.5 \text{ in}} = 8.667$$

Finally we find the tension:

$$8.667 \times 50 \text{ lbs} = 433 \text{ lbs}$$

## CHAPTER 55.

### ALTERNATIVE METHOD FOR CALCULATING TORQUE AND TENSION\*

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If you would rather not think about finding lever arms, you can instead calculate the size of the torque as the size of the force multiplied by the full distance to the pivot, and by the sine of the angle between the force and that full distance. Written in equation form it looks like this:

$$(1) \text{ torque} = \tau = F \cdot d \cdot \sin\theta$$

#### Reinforcement Activity

The torque caused by a force depends on the angle between the line of action of the force acts and the line from where the force is applied to the pivot point. To feel this effect for yourself, try this:

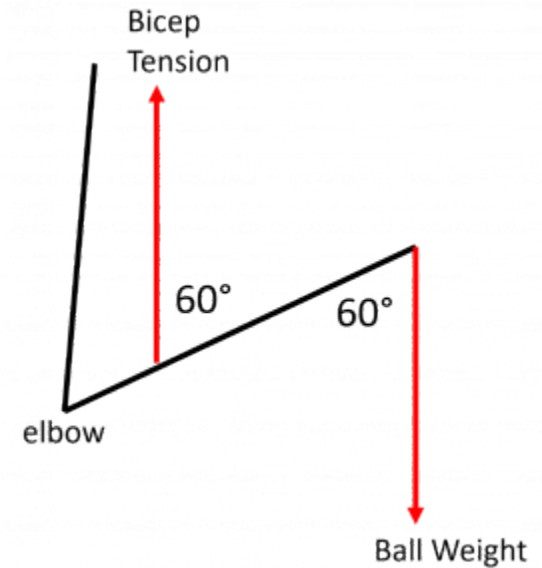
Rotate a door by pushing at  $90^\circ$  to the door right at the outer edge.

Now apply the same force on the door, still on the very edge, but instead of pushing in a direction  $90^\circ$  to the door, push along the door, straight in toward the hinges. Does the door swing as it did before?

In the second case, the angle between the force direction and the distance to the pivot was  $0^\circ$  (they were parallel). Use the previous

equation to show that the torque must be zero any time the line of action of the force goes straight through the rotation point (pivot).

Now, we know the force is 50 **lbs**, the distance from the pivot to the weight is 13.0 **in** length of the forearm and from the diagram we see the angle between the weight of the ball and the forearm distance is  $60^\circ$  (the same as the bicep-forearm angle because they are alternate interior angles). This is easier to see if we draw a stick figure diagram:



*Stick diagram of a flexed arm holding a ball showing the bicep tension and weight and the angles between the forces and the forearm.*

Now we can calculate the torque due to the ball weight  $\tau_b$  as:

$$\begin{aligned}\tau_b &= F \cdot d \cdot \sin\theta \\ &= 50 \text{ lbs} \cdot 13 \text{ in} \cdot \sin(60^\circ) \\ &= 563 \text{ in} \cdot \text{lbs}\end{aligned}$$

We have calculated the torque on the forearm due to the weight of the ball. You may be used to hearing about torque in  $\text{ft} \cdot \text{lbs}$  rather than  $\text{in} \cdot \text{lbs}$ , but we can always convert units later if we desire. For now, let's keep working on finding the muscle tension.

We already know the torque due to the weight of the ball is  $563 \text{ in} \cdot \text{lbs}$  so we just need to make sure that the tension in the biceps is large enough to cause the same torque even though it acts closer to the pivot. The biceps muscle torque,  $\tau_m$  is:

$$\tau_m = T \cdot d \cdot \sin\theta$$

We just need to make this equal to the ball-weight-torque:

$$T \cdot d \cdot \sin\theta = 563 \text{ in} \cdot \text{lbs}$$

Then we divide both sides by  $d$  and  $\sin\theta$  to isolate the bicep tension:

$$T = \frac{563 \text{ in} \cdot \text{lbs}}{d \cdot \sin\theta}$$

Finally we put in our values for  $d$  and  $\theta$ . Our original diagram gave us the distance as from bicep attachment to the pivot as  $1.5 \text{ in}$  and from our stick diagram we can see that the angle between the biceps tension and the distance is  $180^\circ - 60^\circ = 120^\circ$ . We are ready to find the biceps tension value.

$$T = \frac{563 \cancel{\text{in}} \cdot \text{lbs}}{1.5 \cancel{\text{in}} \cdot \sin(120^\circ)}$$

$$T = 433 \text{ lbs}$$

Our result of  $433 \text{ lbs}$  seems surprisingly large, but we

will see that forces even larger than this are common in the muscles, joints, and tendons of the body.

## CHAPTER 56.

### UNIT 6 REVIEW

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#### Key Takeaways

Effort

Resistance (Load)

Fulcrum

Pivot

Lever Arm

Effort Arm

Resistance (Load) Arm

Lever Classes

Mechanical Advantage

Range of Motion

Tension

Compression

Stress

Strain

Elastic Modulus

Ultimate Strength

Linear Region

Elastic Region

Elastic Limit

Plastic Region

Yield Point

Brittle

Ductile

### Learner Objectives

1. Identify classes of levers and explain advantages and disadvantages of each classes in terms of mechanical advantage and range of motion.[2]
2. Apply lever and static equilibrium concepts to solve for forces and calculate mechanical advantage in scenarios involving levers.[3]
3. Identify and define the features of a stress-strain curve, including stress, strain, elastic region, elastic modulus, elastic limit, plastic region, ultimate strength, and fracture/rupture.[2]
4. Apply the Hooke's Law along with the definitions of stress, strain, and elastic modulus to calculate the deformations of structures.[3]

## CHAPTER 57.

### UNIT 6 PRACTICE AND ASSESSMENT

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#### **Outcome 1**

1) Consider the following items:

- Pliers
- Tweezers
- Shovel

(a) For each case, draw a stick figure of the tool and label the fulcrum, effort, load, effort arm, and load arm.

(b) State the class of lever for each item above.

2) For each item in the list in Exercise 4), state whether the tool is providing mechanical advantage or increasing range of motion. Explain how you know.

## Outcome 2

3) When a person raises their heels off the ground, the foot acts like a lever.

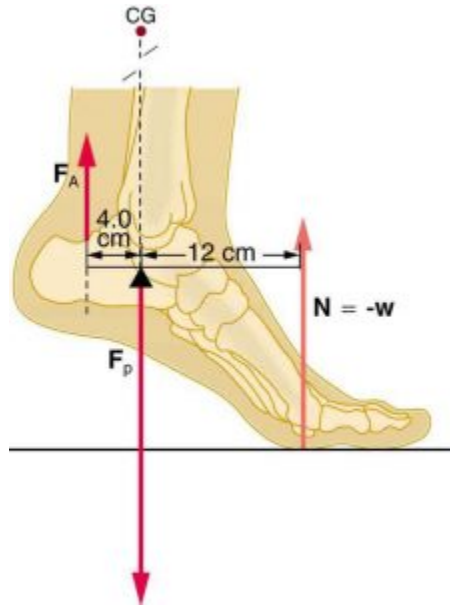
(a) Typically we consider the foot as a second class lever, but if we treat the ankle bone as the fulcrum, the tension in the calf muscle as the effort, and the normal force from the floor as the resistance, what class of lever is this system?

(b) Calculate the mechanical advantage of this system.

(c) Calculate the tension applied by the calf muscles ( $F_A$ ) to lift a person with weight of 637 N.

(d) Calculate the force in the ankle joint between the foot and the lower leg bones ( $F_P$ ). [*Hint: Both the normal force from the floor and the calf tension point upward. In order for the foot to be in static equilibrium, the force of the lower leg pushing down on the foot must cancel out both of those upward forces.*]

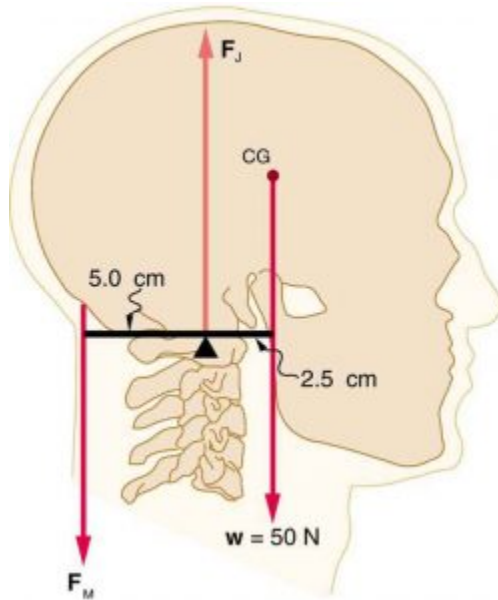
(e) Convert your previous two answers (calf tension and force on ankle) to pounds.



*The foot acting as a lever arm. Image Credit:  
OpenStax College Physics*

1

- 4) The head and neck are also a lever system.
- State the class of this lever system.
  - Calculate the mechanical advantage of this system.
  - Calculate the force of tension in the neck muscles ( $F_M$ ) to hold the head in the position shown in the diagram.
  - Calculate the force on the head-neck joint ( $F_J$ ).
  - Convert your previous two answers to pounds.

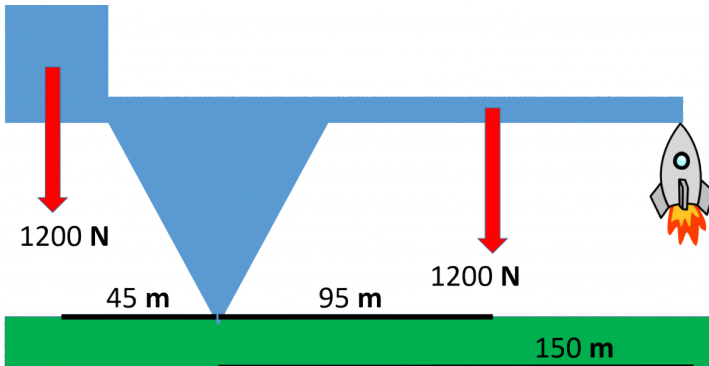


*The head and neck acting as a lever system. Image  
Credit: OpenStax College Physics*

2

5) The structure in the following image remains at rest. What do you know about the force and net torque on the structure?

2. OpenStax, College Physics. OpenStax CNX. Aug 3, 2018 <http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a@11.42>



An inverted triangular structure at rest with a block weighting one side, an arm weighting the other, and a rocket pushing up on the arm. Rocket images from <http://wpclipart.com> are in the Public Domain.

6) An engineer performing an inspection on the structure from the previous exercise and measures **45 m** from beneath the center of gravity of the block to the point where the structure contacts the ground. The block weighs **1200 N**. She then measures the distance to the beneath the center of gravity of the arm to be **95 m**. The arm weighs **1200 N** as well. Finally she measures the distance to beneath the rocket to be **150 m** from the contact point. She then calculates the force being provided by the rocket.

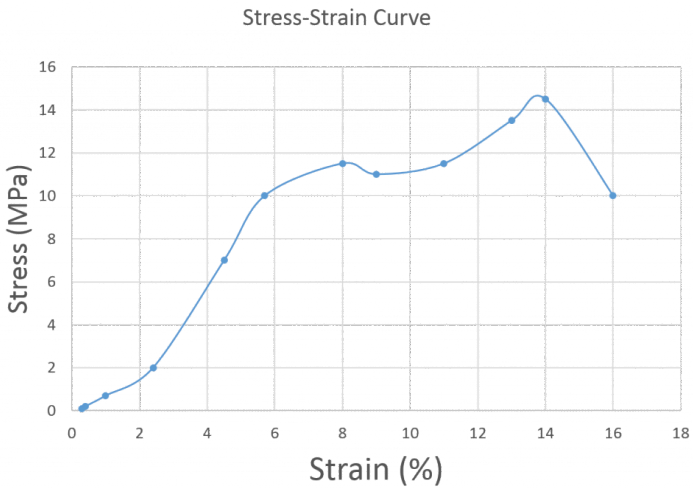
(a) what value does the engineer get when calculating the force provided by the rocket?

(b) The engineer then calculates the normal force on the structure from the ground, what value does she get?

### Outcome 3

7) Label the following features in the stress-strain curve of a hypothetical material seen below:

- Toe region
- Elastic region
- Yield point
- Plastic Region
- Ultimate Strength
- Rupture Point
- Failure Region



*Data adapted with permission from rubber band stress-strain data originally acquired by Umpqua Community College Students: Brittany Watts, Ashlie DeHart, Hanna Wicks and Juan Martinez.*

8) Use the data in the previous graph to determine the elastic modulus of the hypothetical material. Be sure to convert the strain from % stretch back to fractional stretch before doing your calculations.

#### **Outcome 4**

9) A person with a weight of 715 N hangs from a climbing rope 9.2 mm in diameter.

- a) What is the cross-sectional area of the rope in  $\text{m}^2$ ?
- b) What is the stress applied to the rope?

10) A particular 60 m climbing rope stretches by 0.15 m when a 715 N person hangs from it.

- a) What is the strain in the rope?
- b) What is the strain in the rope as a percentage?

11) Answer the following questions regarding the material used to create the created the stress-strain graph above.

a) How much force could be applied to a 2 m x 2 m x 10 m long block of this material before reaching the ultimate strength?

b) When operating in the elastic region, how much additional stress would be required to cause an additional strain of 0.01?

c) What force would cause that amount of stress you found in part b on the 2 m x 2 m block?

d) What actual length would the 10 m long material stretch when put under the strain of 0.01?

e) What is the effective spring constant of this  $2 \text{ m} \times 2 \text{ m} \times 10 \text{ m}$  long block of this material?

## PART VII.

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# UNIT 7: THE BODY IN MOTION

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### Learner Objectives

1. Define position, velocity, and acceleration and explain how they are related.
2. Calculate the drag force on objects moving through fluids.
3. Translate motion graphs into descriptions of motion in terms of position, velocity and acceleration. Translate descriptions of motion into motion graphs.
4. Apply kinematics and Newton's First and Second Laws of Motion to analyze and predict 1-D motion.



## CHAPTER 58.

### FALLING

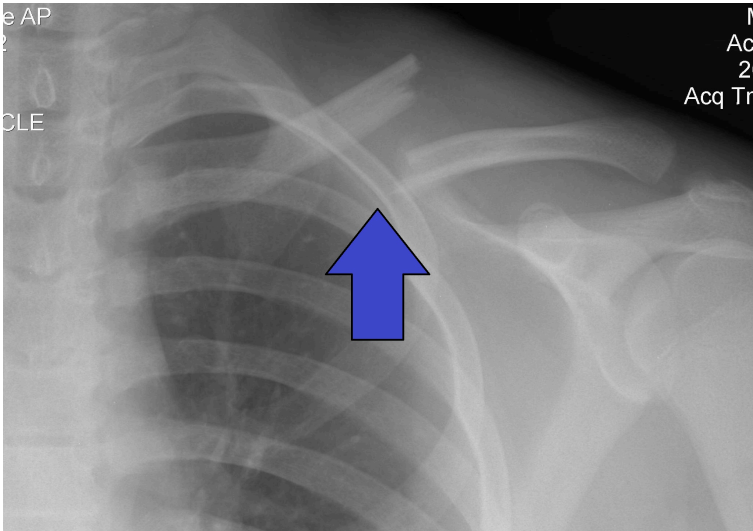
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#### PATIENT FALLS

In the previous unit on the Strength and Elasticity we learned that lifting and holding heavy objects places quite large force (and resulting stress) on the body, so moving patients puts Jolene at risk for injury. Jolene must assume that risk because even those forces are small compared to forces experienced when impacting a hard surface during a fall. Therefore, patient falls must be avoided.

- “Falls with serious injury are consistently among the Top 10 sentinel events reported to The Joint Commission’s Sentinel Event database [...] with the majority of these falls occurring in hospitals.
- Every year in the United States, hundreds of thousands of patients fall in hospitals, with 30-50 percent resulting in injury.
- Injured patients require additional treatment and sometimes prolonged hospital stays that increase medical costs by an average of \$14,000.”

1



*X-Ray image showing a fractured clavicle (collar bone). Clavicle fractures are a one of the most common injuries resulting from falls. This particular fracture occurred during a car accident. of Image Credit: Clavicle Fracture Left uploaded by Majorkev via Wikimedia Commons*

2

Impacts due to falls are not the only source of large forces. In fact, any situation involving a rapid change in motion will produce relatively large forces. These include car accidents, collisions between people, jumping, landing, and explosive body movements. As a result, medical professionals and first-responders often treat patients who experience *mechanisms of injury* (MOI) that

1. "Preventing falls and fall-related injuries in health care facilities" by Sentinel Event Alert, The Joint Commission
2. Majorkev at English Wikipedia [CC BY 3.0 (<https://creativecommons.org/licenses/by/3.0/>)], via Wikimedia Commons

involve rapid changes in motion as having spinal and/or internal injuries until confirmed otherwise by medical imaging or complete examination. Before we can analyze the forces associated with rapid changes in motion, we must also learn how to quantify motion itself. Falling provides an excellent place to begin the study of motion, so let's start there.

## SKYDIVING FREE FALL



*Skydivers adjust body orientation to tune fall speed and adjust their relative vertical positions. Image credit: Skydive Miami by Norcal21jg, via Wikimedia Commons*

3

The time a skydiver spends between leaving the aircraft and opening a parachute is often called the “free fall” time. During a recreational skydive the “free fall” time is about one minute. The current record “free fall” time of about 5 minutes was set by Alan Eustace in 2014 when he

3. "Skydive Miami" by By Norcal21jg, from Wikimedia Commons is in the Public Domain

fell from an altitude of more than 135,000 feet. According to the Paragon Space Development Corporation, “Eustace reached top speeds of over 800 miles per hour. He was going so fast that his body broke the sound barrier, creating a sonic boom that could be heard on the ground.” The jump broke the previous record of 127,852 feet set by Felix Baumgartner in 2012. The 2012 jump was sponsored by GoPro cameras and the video has a much higher production value than the more recent 2014 jump:

### **PHYSICS FREE FALL**

Now that we have introduced the skydiver’s use of the



A YouTube element has been excluded from this version of the text. You can view it online here: <https://openoregon.pressbooks.pub/bodyphysics/?p=1010>

term free fall, we need to recognize that physics uses the term free fall in a completely different way, so we will need to be careful to avoid confusion. In physics, and in this book, we use the term free fall to describe the motion of an object when gravity is the only force acting on the object, or any other forces are small enough compared to gravity that we can ignore them without introducing too much error. Skydivers experience gravity and significant air resistance, so they are not actually in free fall.

## CHAPTER 59.

### DRAG FORCES ON THE BODY

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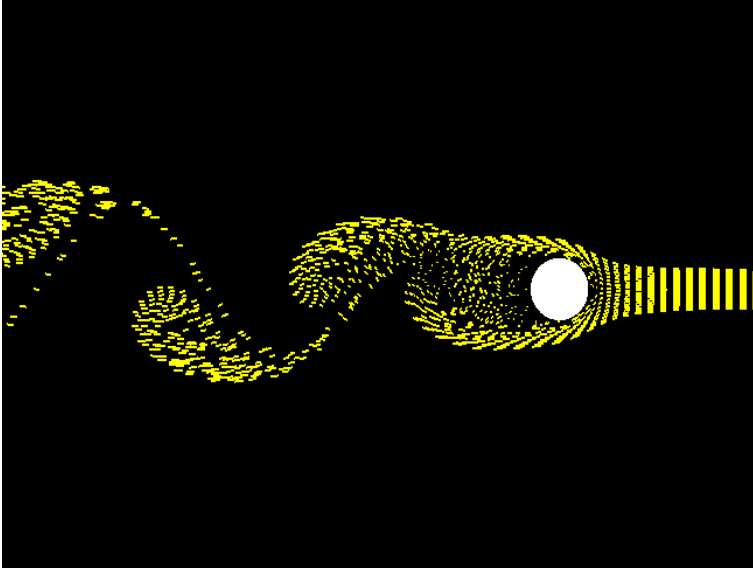
*A skydiver maintains a horizontal (flat) body position with arms and legs spread, which reduces the terminal velocity and increases the fall time. Image Credit: "Gabriel Skydiving" By Gabriel Christian Brown, via Wikimedia Commons*

1

Correct and thoughtful body orientation is an

important part of skydiving because the orientation of the body affects the amount of air resistance experienced by the body. In turn, the air resistance affects the terminal speed, as we will see in the next chapter.

## DRAG



*Simulation of fluid flowing around a sphere. "Drag of a Sphere" by Glenn Research Center Learning Technologies Project, NASA, via GIPHY is in the Public Domain, CCO*

2

Air resistance limits the terminal speed that a falling body can reach. Air resistance is an example of the drag force, which is force that objects feel when they move

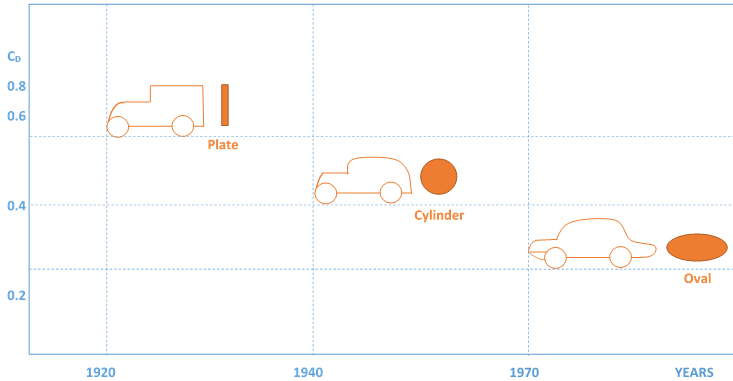
1. "Gabriel Skydiving" By Gabriel Christian Brown [CC BY-SA 4.0 (<https://creativecommons.org/licenses/by-sa/4.0>)], from Wikimedia Commons
2. "Drag of a Sphere" by Glenn Research Center Learning Technologies Project, NASA, via GIPHY is in the Public Domain, CCO

through a fluid (liquid or gas). Similar to kinetic friction, drag force is reactive because it only exists when the object is moving and it points in the opposite direction to the object's motion through the fluid. Drag force can be broken into two types: form drag and skin drag. Form drag is caused by the resistance of fluids (liquids or gases) to being pushed out of the way by an object in motion through the fluid. Form drag is similar to the normal force provided by the resistance of solids to being deformed, only the fluid actually moves instead of just deforming. Skin drag is essentially a kinetic frictional force caused by the sliding of the fluid along the surface of the object.

The drag force depends the density of the fluid ( $\rho$ ), the maximum cross-sectional area of the object ( $A_x$ ), and the drag coefficient ( $C_d$ ), which accounts for the shape of the object. Objects with a low drag coefficient are often referred to as having an aerodynamic or streamlined shape. Finally, the drag force depends on the on the speed ( $v$ ) of the object through the fluid. If the fluid is not not very viscous then drag depends on  $v^2$ , but for viscous fluids the force depends just on  $v$ . In typical situations air is not very viscous so the complete formula for air resistance force is:

$$(1) \quad F_d = \frac{1}{2} C_d \rho A_x v^2$$

The image below illustrates how the shape of an object, in this case a car, affects the drag coefficient. The table that follows provides drag coefficient values for a variety of objects.



*Drag coefficients of cars (vertical axis on left) have changed over time (horizontal axis). Image Credit: Drag of Car by Eshaan 1992 via Wikimedia Commons*

3

Object	Drag Coefficient (C)
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

3. Drag of Car By Eshaan 1992 [CC BY-SA 3.0 (<https://creativecommons.org/licenses/by-sa/3.0/>)], from Wikimedia Commons

### Reinforcement Exercises

Which body orientation would put the largest drag force on a human body moving vertically through a fluid?

- body horizontal and sideways (side first)
- body vertical with arms in (feet first)
- body flat with arms out (front first)

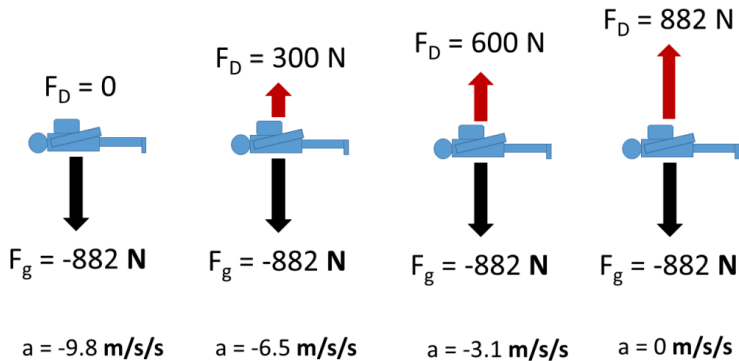
## CHAPTER 60.

### PHYSICAL MODEL FOR TERMINAL VELOCITY

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After jumping, a skydiver begins gaining speed which increases the air resistance they experience. Eventually they will move fast enough that the air resistance is equal in size to their weight, but in opposite direction so they have no net force. This processes is illustrated by free body diagrams for a skydiver with 90 **kg** mass in the following image:

Mass = 90 kg Weight = 882 N



*Free body diagrams of a person with 90 kg mass during a skydive. The initial speed is zero, so drag force is zero. As speed increases, the drag force grows, eventually cancelling out the person's weight. At that point acceleration is zero and terminal velocity is reached.*

## DYNAMIC EQUILIBRIUM

With a net force of zero the skydiver must be in equilibrium, but they are not in static equilibrium because they are not static (motionless). Instead they are in dynamic equilibrium, which means that they are moving, but the motion isn't changing because all the forces are still balanced (net force is zero). This concept is summarized by Newton's First Law, which tells us that an object's motion will not *change* unless it experiences a net force. Newton's first law is sometimes called the *Law of Inertia* because inertia is the name given to an object's tendency to resist changes in motion. Newton's First Law applies to objects that are not moving and to objects that are already moving. Regarding the skydiver, we are applying Newton's First Law to translational

motion (back and forth, up and down), but it also holds for the effect of net torques on changes in rotational motion. Changes in motion are known as accelerations and we will learn more about how net forces cause translational accelerations in upcoming chapters.

### Everyday Example: Head Injuries

The diagram illustrates the mechanism of a concussion. On the left, a sagittal view of a human head shows a red starburst representing an impact on the side of the skull. A red arrow points from the impact site towards the brain. A text box above this part reads: "Concussion: A traumatic brain injury that changes the way your brain functions." Below this part, another text box states: "The brain is made up of soft tissue and is protected by blood and spinal fluid. When the skull is jolted too fast or is impacted by something, the brain shifts and hits against the skull." A red arrow points from the brain in the first diagram to a second diagram on the right. In the second diagram, a red area on the brain's surface indicates injury. A text box above this part reads: "This can lead to bruising and swelling of the brain, tearing of blood vessels and injury to nerves, causing the concussion." Below this part, another text box states: "Most concussions are mild and can be treated with appropriate care. But left untreated, it can be deadly."

Diagram of a concussion. "Concussion Anatomy" by Max Andrews via wikimedia commons.

1

When the head is travelling in a certain direction with constant speed the brain and skull are moving together. If an impact causes the the motion of the skull to change suddenly, the brain tends to continue its original motion according to Newton's First Law of Motion. The resulting impact between the fragile brain and the hard skull may

1. Concussion Anatomy by Max Andrews - Own work. This file was derived from: Concussion mechanics.svg, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=19490504>

result in a concussion. Recent research has shown that even without the occurrence of concussions, the damage caused by sub-concussive events like this can accumulate to cause Chronic Traumatic Encephalopathy (CTE)<sup>2</sup>.

### Reinforcement Exercises

Using above the statement of Newton's first law as it applies to net forces and translational motion as a template, write out Newton's 1st Law as it applies to torques and changes in rotational motion

## DEPENDENCE OF TERMINAL VELOCITY ON MASS

We already know from our experimental work during the Unit 3 lab that increasing mass leads to increasing terminal speed. We can now understand that this behavior occurs because greater mass leads to a greater weight and thus a greater speed required before the drag force (air resistance) is large enough to balance out the weight and dynamic equilibrium is achieved.

### Everyday Example: Tandem Skydive

First-time skydivers are typically attached to an instructor (tandem skydiving). During a tandem skydive the bodies are stacked, so the shape and cross-sectional area of the object don't change much, but the mass does. As a consequence, the terminal speed for tandem

2. "Concussion, microvascular injury, and early tauopathy in young athletes after impact head injury and an impact concussion mouse model" by Chad A Tagge, et. al, Brain, Oxford Academic

diving would be high enough to noticeably reduce the fall time and possibly be dangerous. Increasing the air resistance to account for the extra mass is accomplished by deploying a small drag chute that trails behind the skydivers, as seen in the photo below.



*Tandem skydivers with a small speed-limiting drag chute trailing behind. Image Credit: Fallschirm Tandemsprung bei Jochen Schweizer By Jochen Schweizer via Wikimedia Commons*

3

## A PHYSICAL MODEL FOR TERMINAL VELOCITY

When the skydiver has reached terminal speed and remains in a state of dynamic equilibrium, we know the size of the drag force must be equal to the skydiver's weight, but in the opposite direction. This concept will allow us to determine how the skydiver's mass should affect terminal speed. We start by equating the air resistance with the weight:

3. By Jochen Schweizer GmbH [CC BY-SA 4.0 (<https://creativecommons.org/licenses/by-sa/4.0/>)], from Wikimedia Commons

$$F_d = F_g$$

Then we insert the formulas for air resistance and for weight of an object near Earth's surface. We designate the speed in the resulting equation  $v_T$  because these two forces are only equal at terminal speed.

$$\frac{1}{2}C_d\rho A_x v_T^2 = mg$$

We then need to solve the above equation for the terminal speed.

$$(1) \quad v_T = \sqrt{\frac{2mg}{C_d\rho A_x}}$$

### Everyday Examples: Terminal Speed of the Human Body

Let's estimate the terminal speed of the human body. We start with the previous equation:

$$v_T = \sqrt{\frac{2mg}{C_d\rho A_x}}$$

We need to know the mass, drag coefficient, density of air, and cross-sectional area of the human body. Let's use the authors 80 kg mass and the density of air near the Earth's surface at standard pressure and temperature,  $\rho = 1.2 \text{ kg/m}^3$ . Drag coefficient and cross sectional area depend on body orientation, so let's assume a standard skydiving posture: flat, horizontal, with arms and legs spread. In this case the drag coefficient will likely be 0.4-1.3. A reasonable value would be  $C_d = 1^4$ . To approximate the cross-sectional area we can use the authors average width of 0.3 m and height of 1.5 m for an area of  $A_x = 0.3\text{m} \times 1.5\text{m} = 0.45 \text{ m}^2$

4. "Drag Coefficient" by Engineering Toolbox

Inserting these values into our terminal speed equation we have:

$$v_T = \sqrt{\frac{2(80 \text{ kg})(9.8 \text{ m/s}^2)}{1(1.2 \text{ kg/m}^3)(0.45 \text{ m}^2)}} = 54 \text{ m/s} = 120 \text{ MPH}$$

### Reinforcement Exercises

You already have data on how the terminal speed depends on mass. We acquired this data using coffee filters in the Unit 3 Lab. Looking back at that data, does that data support our physical model for terminal speed? [Hint: If our empirical model (fit equation) suggests that terminal speed depends on mass in the same way as the physical model then yes, our data supports our physical model. Our physical model says that the terminal speed depends on the square root of the mass. Does your empirical fit equation support that result?]

## ACCELERATION DURING A SKYDIVE

We have now analyzed the skydive after terminal speed was reached. Prior to this point the forces of drag and weight are not equal, therefore the skydiver is not in dynamic equilibrium and speed will change over time. In order to analyze the early part of the skydive we need to quantify changes motion and learn how those changes are related to the net force. The next chapters will help us with those two goals.

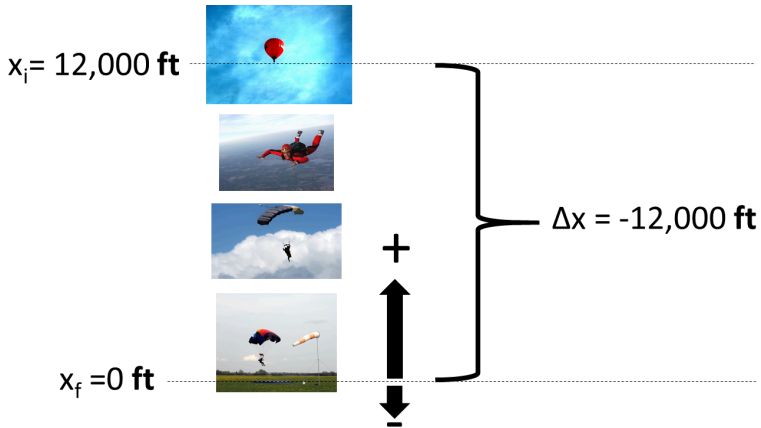
## CHAPTER 61.

### ANALYZING MOTION

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#### POSITION

Position describes the location of an object according to a choice of zero point and positive direction. The zero point is called the origin and upwards is commonly used as the positive direction when analyzing vertical motion. For example, with upward positive, a skydiver in a stationary balloon at an altitude of 12,000 ft would have a position of 12,000 ft, if we called the ground the origin. If we chose 12000 ft as the origin then the position of the skydiver would be zero. If we chose 24,000 ft as the origin, the skydiver would have a position of -12,000 ft. It doesn't matter where you put the origin or which direction is positive as long as you keep them both consistent throughout your analysis.



*Initial position, final position, and displacement of a skydiver from jump to landing. Ground level was chosen as the origin and upwards as the positive direction. Image adapted from Balloon over Straubing, Germany by Runologe, "Gabriel Skydiving" By Gabriel Christian Brown, "EOD parachute jump" Petty Officer 3rd Class Daniel Rolston via and Parachute precise landing by Masur, all via Wikimedia Commons*

1234

Let's say we placed the origin at the ground and chose upwards as positive, as in the diagram above. If we are analyzing the motion of the skydiver starting just as they jump to just as they land, then their initial position ( $x_i$ ) would be 12,000 ft and their final position ( $x_f$ ) would be 0 ft. The change in position would be -12,000 ft because they moved 12,000 ft downward, which is the negative

1. Balloon over Straubing, Germany by Runologe, via wikimedia commons
2. "Gabriel Skydiving" By Gabriel Christian Brown [CC BY-SA 4.0] (<https://creativecommons.org/licenses/by-sa/4.0/>), from Wikimedia Commons
3. "EOD parachute jump" Petty Officer 3rd Class Daniel Rolston (<https://www.dvidshub.net/image/1465626>) [Public domain], via Wikimedia Commons
4. Parachute precise landing by Masur [Public domain], via Wikimedia Commons

direction. We call the change in position the displacement ( $\Delta\mathbf{x}$ ) and we calculate the displacement as:

$$(1) \quad \Delta\mathbf{x} = \mathbf{x}_f - \mathbf{x}_i$$

For our skydiver example we have:

$$(2) \quad \Delta\mathbf{x} = 0\text{ft} - 12,000\text{ft} = -12,000\text{ft}$$

## VECTORS

As we analyze motion we are beginning to see that it's important to keep track of directions for different quantities of motion like position and displacement, just like we do for forces. Just as with forces, we will make the symbols for these vectors **bold** when writing equations to remind ourselves that these quantities include directions.

## DISTANCE AND DISPLACEMENT

It may seem odd that we have introduced displacement as a new word for distance that something travels, but there is actually an important distinction between the two terms. The distance and displacement are sometimes equal, but not always. For example, the distance our skydiver traveled from balloon to ground was 12,000 **ft**, but their displacement was -12,000 **ft**. If we analyze the motion of the skydiver starting from when they got into the balloon on the ground to when they landed after the jump then the distance traveled by the skydiver would be 24,000 **ft**. However, the displacement would be 0 **ft** because their initial and final positions were the same. *The distance traveled can be greater than, or equal to the displacement, but it can never be less.* This distinction arises because direction matters in calculating displacement, but not in measuring distance.

## Reinforcement Exercise

You throw a ball 3 m into the air and it returns to your hand. What are the displacement of the ball and distance traveled by the ball for:

- The first half of the ball's trip (from your hand to the ball's peak height).
- The second half of the ball's trip (from peak height back to your hand).
- The entire round trip of the ball.

## VELOCITY

### INSTANTANEOUS SPEED AND VELOCITY

The maximum speed reached by a body (or any object) falling under the influence of both gravitational force and air resistance is often called terminal velocity or terminal speed. In everyday life we often use speed and velocity to mean the same thing, but they actually have different meanings in physics. Velocity is the rate at which the position is changing and speed is the rate at which distance is covered. Objects cannot travel negative distances so the speed will always be positive. However, position can become more negative, as was the case for our example skydiver, so velocity can be negative. The speed at any instant in time is known as the instantaneous speed. The instantaneous velocity is just the instantaneous speed with a direction included. For example, if at some point our skydiver reached a terminal speed of 89 **MPH**, then their terminal velocity would be

89 **MPH** *downward* or **-89 MPH** for our choice of downward as the negative direction.

#### INITIAL AND FINAL VELOCITY

Just as we defined initial position and final position for the section of an object's motion that we are analyzing, we can also define initial velocity and final velocity. For example, if we analyze the skydiver's motion from jump until they reach an example terminal speed of **180 MPH**, then the initial velocity of our skydiver was zero and the final velocity was **-180 MPH**.

#### AVERAGE VELOCITY

Sometimes we are interested in the average velocity over some amount of time rather than the instantaneous velocity at a single time. To calculate the average velocity for a section of an objects motion we need to divide the change in position (displacement) by the time interval ( $\Delta t$ ) over which the it occurred.

$$(3) \quad v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

Velocities will be negative when the displacement is negative, as was the case for our skydiver's trip from balloon to ground. The negative displacement of our skydiver would result in a negative average velocity during their trip from balloon to ground. This makes sense, as we should be expecting a negative velocity for our skydiver because downward was chosen as our negative direction and the skydiver was moving downward.

## AVERAGE SPEED AND VELOCITY

Sometimes average speed and average velocity are the same, but sometimes they are not. Speed is the rate at which distance is traveled so to calculate average speed we divide the distance traveled by the time required for the travel. Remembering that we use displacement rather than distance in calculating average velocity, we can see that speed and velocity are different. For example the velocity of the skydiver in our example is negative on the way down because displacement is negative, however we cannot say the diver actually traveled a negative distance, so the average speed is positive.

### Everyday Examples

Let's imagine the skydiver in our example rode a hot air balloon upward for 21 minutes, then jumped and fell for 2.0 minutes, then opened their parachute and drifted downward for 5.0 minutes before landing. Let's calculate the average speed and average velocity for the entire trip in feet per minute.

The average speed is the total distance covered divided by the total time, which would be 24,000 ft divided by 27 minutes for an average speed of: **860 ft/min**.

The average velocity would be the total displacement divided by the total time. The skydiver started and ended the trip on the ground, so the total displacement for the round trip was zero, therefore the average velocity for the trip was *zero*! Comparing this average velocity to the average speed of **860 ft/min** we can really see why its important to distinguish between instantaneous vs. average and speed vs. velocity.

## Reinforcement Activities

If the parachute of the skydiver in the previous everyday example opened at 2900 ft, what would be their average speed during the remaining 5.0 minutes it took to hit the ground? What would be their average velocity?

## CHAPTER 62.

# ACCELERATED MOTION

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### ACCELERATION

After the air resistance becomes large enough to balance out a skydiver's weight, they will have no net force. From Newton's First Law we already know that an object's inertia prevents a change in velocity unless it experience a net force, so from that point when the forces are balanced and onward, the skydiver continues at a constant velocity until they open their parachute.

During the initial part of skydive, before the drag force is large enough to balance out the weight, there is a net force so their velocity changes. The rate at which the velocity changes is known as the acceleration. Note that students often confuse velocity and acceleration because they are both rates of change, so to be specific: *velocity defines the rate at which the position is changing and acceleration defines the rate at which the velocity is changing.* We can calculate the average acceleration ( $\mathbf{a}$ ) during a certain time interval ( $\Delta t$ ) by subtracting the initial velocity ( $\mathbf{v}_i$ ) from the final velocity ( $\mathbf{v}_f$ ) to get the change in velocity ( $\Delta \mathbf{v}$ ) and then dividing by time interval ( $\Delta t$ ):

$$(1) \mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}$$

### Everyday Example

Let's calculate the average acceleration during the roughly 2 seconds it takes a parachute to fully open and slow a skydiver from 120 MPH to 6.0 MPH. First let's remember that the skydiver is moving in our negative direction so the initial and final velocities should be negative. Also, lets convert to meters per second:  
 $\mathbf{v}_f = -6.0 \text{ mph} = -2.7 \text{ m/s}$  and  
 $\mathbf{v}_i = \mathbf{v}_f = -120 \text{ mph} = -54 \text{ m/s}$ .

Starting with our definition of acceleration:

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}$$

Inserting our values:

$$\mathbf{a} = \frac{-2.7 \text{ m/s} - (-54 \text{ m/s})}{2 \text{ s}}$$

The two negatives in front of the 54 m/s make a positive, and then we calculate a value.

$$\mathbf{a} = \frac{-2.7 \text{ m/s} + (54 \text{ m/s})}{2 \text{ s}} = 26 \text{ m/s/s}$$

We now get a chance to see that the units of acceleration are m/s/s or equivalently  $\text{m/s}^2$

## ACCELERATION DIRECTION

The direction of acceleration depends on the direction of the change in velocity. If the velocity becomes more

negative, then acceleration must be negative. This is the case for our skydiver during the first part of the jump; their speed is increasing in the negative direction, so their velocity is becoming more negative and therefore acceleration is negative. Conversely, if an object moves in the negative direction, but slows down, the acceleration is positive, *even though the velocity is still negative!* This was the case for our skydiver just after opening their parachute, when they still moved downward, but were slowing down. Slowing down in the negative direction means the velocity is becoming less negative, so the acceleration must be positive. All of the possible combinations of velocity direction and speed change and the resulting acceleration are summarized in the following chart:

**Table Showing Possible Acceleration Directions**

Initial direction of motion (initial velocity direction)	Speed change	Direction of Acceleration
positive	speeding up	positive
positive	slowing down	negative
negative	speeding up	negative
negative	slowing down	positive

### Reinforcement Exercises

A car is moving in the positive direction and slams on the brakes.  
What direction is the acceleration?

A car is moving in the negative direction and slams on the brakes.  
What direction is the acceleration?

A car is moving in the negative direction and speeding up. What direction is the acceleration?

## CHAPTER 63.

# ACCELERATING THE BODY

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### NEWTON'S SECOND LAW OF MOTION

Newton's First Law tells us that we need a net force in order to create an acceleration. As you might expect, a larger net force will cause a larger acceleration, but the more matter you are trying to accelerate the larger force will be required. Newton's Second Law summarizes all of that into a single equation relating the net force, mass, and acceleration:

$$(1) \mathbf{F}_{\text{net}} = m\mathbf{a}$$

### FINDING NET FORCE FROM ACCELERATION

#### Everyday Example: Parachute Opening

In the previous chapter we found that if opening a parachute slows a skydiver from  $54 \text{ m/s}$  to  $2.7 \text{ m/s}$  in just  $2 \text{ s}$  of time then they experienced an average upward acceleration of  $26 \text{ m/s/s}$ . If the mass of our example skydiver is  $85 \text{ kg}$ , what is the average net force on the person? What is the average force on them from the harness?

We start with Newton's Second Law of Motion

$$(2) \mathbf{F_{net} = ma}$$

Enter in our values:

$$(3) \mathbf{F_{net} = (85 \text{ kg})(26 \text{ m/s/s}) = 2200 \text{ N}}$$

The person experiences an average net force of 2200 N upward during chute opening. When the chute begins to open they are still moving near terminal velocity so air resistance is nearly balancing their weight and the harness provides most of the extra 2200 N upward force on the person. That force is 2.7 times their body weight ( $F_g = 85 \text{ kg} \times 9.8 \text{ m/s/s} = 833 \text{ N}$ ).

### Reinforcement Exercises: Failed Chute Opening

If the skydiver in the previous example experienced a failed chute opening and hit the ground so that they came to an abrupt stop in a time of only 0.2 s (instead of slowing to 2.7 m/s in 2 s) what is their acceleration?

What is the average net force on them during the stop?

The normal force from the ground must be large enough to cancel the skydiver's weight and still provide the upward net force you found above. How big is the normal force from the ground?

How many times larger is this normal force than their weight?

## Reinforcement Exercises: Baby Toss



You'd like to accelerate a  $7.6 \text{ kg}$  baby from rest to  $1.5 \text{ m/s}$  over  $1.0 \text{ s}$ .  
What is the baby's acceleration?

What net force is required?

Draw a free body diagram of the situation, that shows the baby's weight and the force you are providing.

Considering the baby's weight, what actual force do you need to provide for the baby to experience the net force you calculated above?

Once the baby is in the air, what acceleration will they have? (Assume they are moving slowly enough that air resistance is negligible).

Using the definition of average acceleration, find the time that the baby was in the air before their velocity reached zero at the top of the toss.

What was the total "hang time" for the baby? (Finish this unit to learn how we calculate the height reached by the baby in this case).

When you catch the baby do you keep your arms held rigid or do you move your hands downward with the baby as you make the catch? Explain why in terms of Newton's Second Law.

## FINDING ACCELERATION FROM NET FORCE

If we know the net force and want to find the acceleration, we can solve Newton's Second Law in terms of the acceleration instead:

$$(4) \quad \mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$$

Now we see that larger net forces create larger accelerations and larger masses reduce the size of the acceleration. In fact, an object's mass is a direct measure of an object's resistance to changing its motion, or its inertia.

### Reinforcement Exercises

You slide a box across the floor by applying a 220 N force to the right. Kinetic friction applies a reactive 170 N force on the box to the left.

What is the size and direction of the net force on the box?

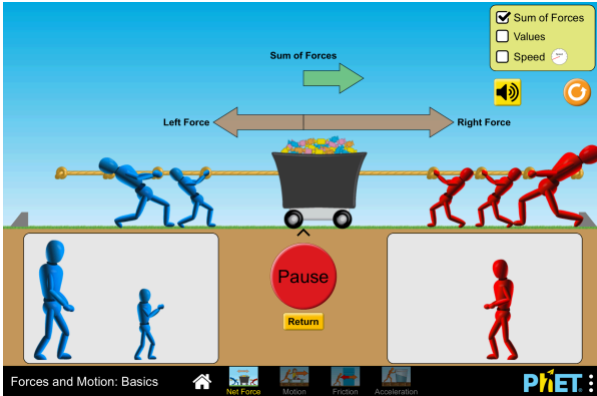
The box has a mass of 25 kg. What is the size and direction of the acceleration on the box?

The box is not accelerating in the vertical direction, so what is the net vertical force? [Hint: Forces in different dimensions (vertical and horizontal) don't affect one another and Newton's Second Law applies separately to each dimension.]

How big is the normal force on the box?

What is the value of the kinetic friction coefficient?

Check out this simulation to see how forces combine to create net forces and accelerations:



## FREE-FALL ACCELERATION

In the absence of air resistance, heavy objects do not fall faster than lighter ones and all objects will fall with the same acceleration. Need experimental evidence? Check out this video:

It's an interesting quirk of our universe that the same property of an object, specifically its mass, determines both the force of gravity on it and its resistance to accelerations, or inertia. Said another way, the inertial mass and the gravitational mass are equivalent. That is why the free-fall acceleration for all objects has a magnitude of  $9.8 \text{ m/s}^2$ , as we will show in the following example.

### Everyday Example: Free-Falling

Let's calculate the initial acceleration of our example skydiver the moment they jump. At this moment they have the force of gravity pulling them down, but they have not yet gained any speed, so the air resistance (drag force) is zero. The net force is then just gravity,

because it is the only force, so they are in free-fall for this moment.  
Starting with Newton's Second Law:

$$(5) \quad \mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$$

And inserting our known formula for calculating force of gravity near the surface of Earth and including a negative sign because down is our negative direction, ( $\mathbf{F}_g = -mg$ ):

$$(6) \quad \mathbf{a} = \frac{-mg}{m}$$

We see that the mass cancels out,



A YouTube element has been excluded from this version of the text. You can view it online here: <https://openoregon.pressbooks.pub/bodyphysics/?p=4790>

$$(7) \quad \mathbf{a} = \frac{-mg}{m} = -g = -9.8 \frac{\mathbf{m}}{\mathbf{s}^2}$$

We see that our acceleration is negative, which makes sense because the acceleration is downward. We also see that the size, or magnitude, of the acceleration is  $g = 9.8 \text{ m/s}^2$ . We have just shown that *in the absence of air resistance, all objects falling near the surface of Earth will experience an acceleration equal in size to  $9.8 \text{ m/s}^2$ , regardless of their mass and weight.* Whether the free-fall acceleration is  $-9.8 \text{ m/s/s}$  or  $+9.8 \text{ m/s/s}$  depends on if you chose downward to be the negative or positive direction.

## CHAPTER 64.

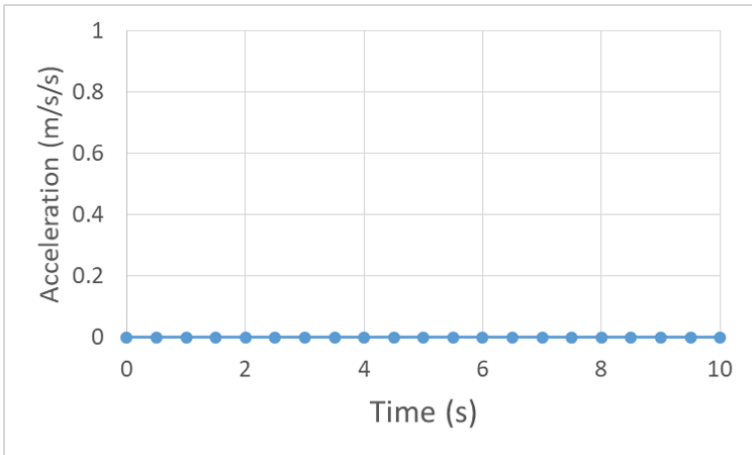
# GRAPHING MOTION

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### BASIC MOTION GRAPHS

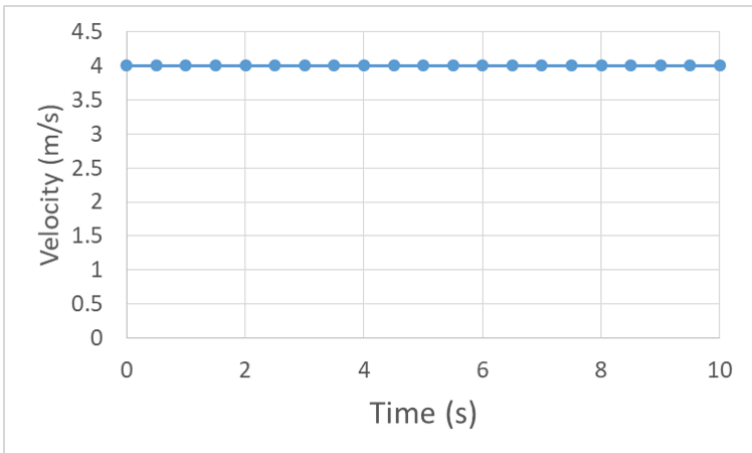
Motion graphs are a useful tool for visualizing and communicating information about an object's motion. Our goal is to create motion graphs for our example skydiver, but first let's make sure we get the basic idea.

We will start by looking at the motion graphs of an object with an initial position of  $2 \text{ m}$  and constant velocity of  $4 \text{ m/s}$ . An object moving at constant velocity has zero acceleration, so the graph of acceleration vs. time just remains at zero:



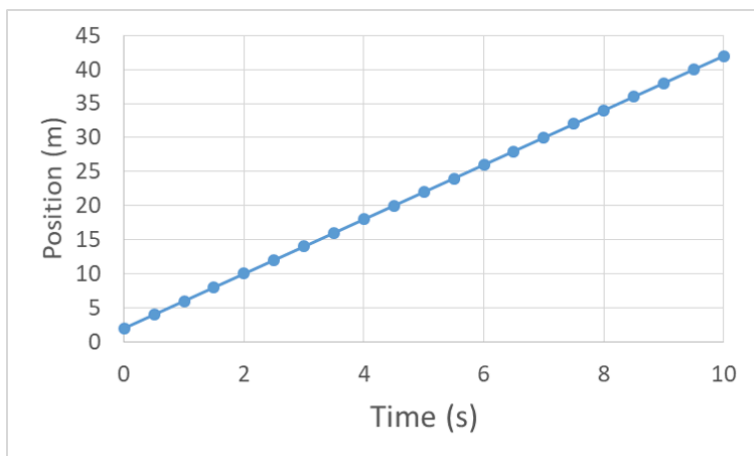
*The acceleration vs. time graph for an object with constant velocity is flat at zero.*

The velocity is constant, so the graph of velocity vs. time will remain at the **4 m/s** value:



*The velocity vs. time graph is flat (constant) at 4 m/s.*

Velocity is the rate at which position changes, so the position v. time graph should change at a constant rate, starting from the initial position (in our example, 2 **m**). The slope of a motion graph tells us the rate of change of the variable on the vertical axis, so we can understand velocity as the slope of the position vs. time graph.



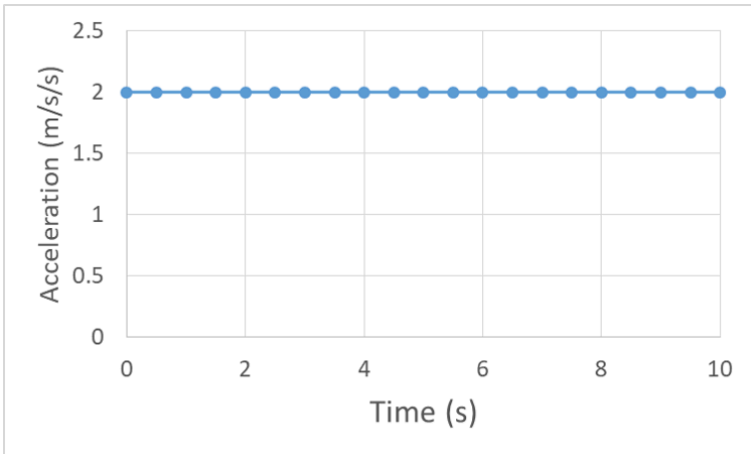
*The position vs. time graph is linear with a slope that is equal to the 4 m/s velocity and intercept that is equal to the 2 m initial position.*

### Reinforcement Exercises

What should be the value for the slope of the position vs. time graph of our example object? Calculate the slope of the position vs. time graph above and compare to your previous answer. [Hint: The initial position is 2 m and the position at a time of 2 s is 10 m. Slope is calculated as rise over run.]

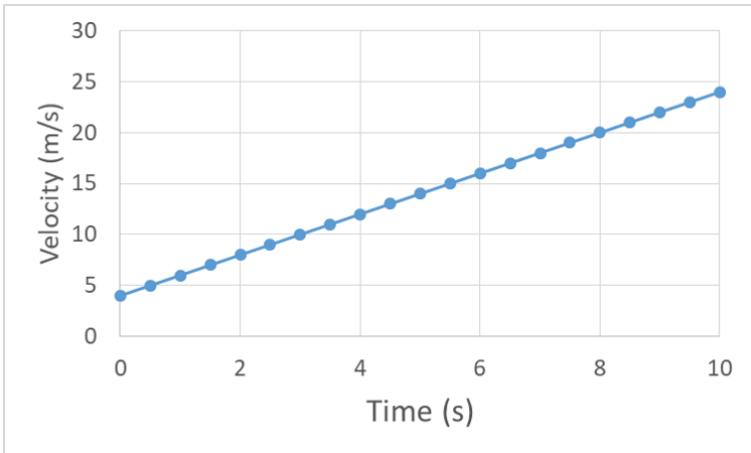
Now let's look at motion graphs for an object with

constant acceleration. Let's give our object the same initial position of  $2 \text{ m}$ , and initial velocity of  $4 \text{ m/s}$ , and now a constant acceleration of  $2 \text{ m/s/s}$ . The acceleration vs. time remains constant at  $2 \text{ m/s/s}$ :



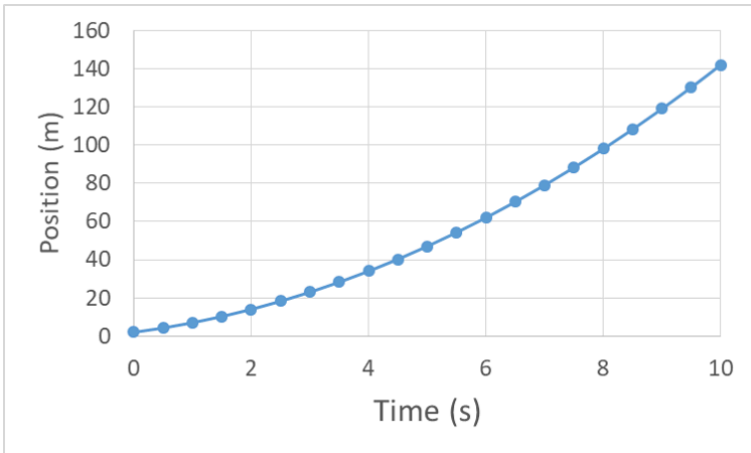
*The acceleration vs. time graph is flat at the acceleration value, in this example  $2 \text{ m/s/s}$*

Acceleration is the rate at which velocity changes, so acceleration is the slope of the velocity vs. time graph. For our constant  $2 \text{ m/s/s}$  acceleration the velocity graph should have a constant slope of  $2 \text{ m/s/s}$ :



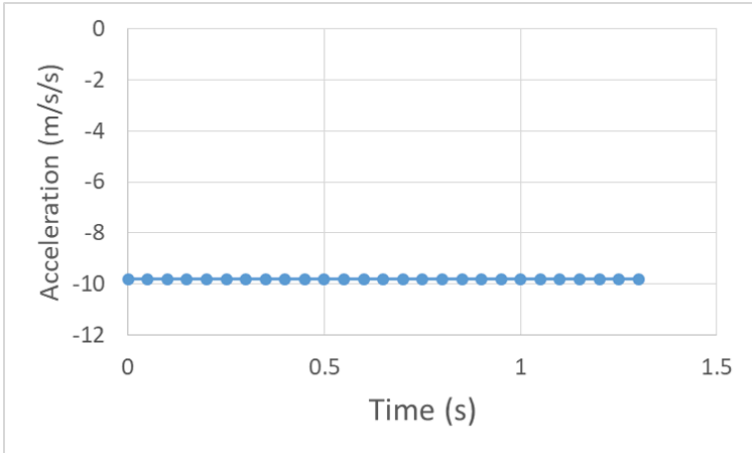
*The velocity vs. time graph is linear with a slope equal to the 2 m/s/s acceleration value and intercept equal to the initial velocity value of 4 m/s.*

Finally, if the velocity is changing at a constant rate, then the slope of the position graph, which represents the velocity, must also be changing at a constant rate. The result of a changing slope is a curved graph, a curve with a constantly changing slope is a *parabolic* curve.



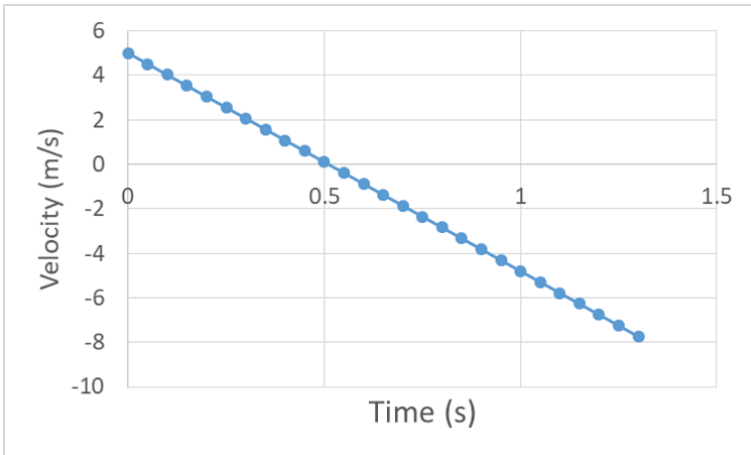
*The position vs. time graph of an object with constant acceleration is a parabolic curve. The curvature is upward for positive acceleration and downward for negative accelerations. The intercept is the initial position, in this example 2 m.*

We haven't made motion graphs for the situation of constant position because they are relatively unexciting. The position graph is constant at the initial value of position, the velocity graph is constant at zero and the acceleration graph is also constant at zero. Let's end this section with some interesting graphs – those of an object that changes direction. For example, an object thrown into the air with an initial velocity of  $5 \text{ m/s}$ , from an initial position of  $2 \text{ m}$  that then falls to the ground at  $0 \text{ m}$ . Neglecting air resistance, the acceleration will be constant at negative  $g$ , or  $-9.8 \text{ m/s}^2$ .



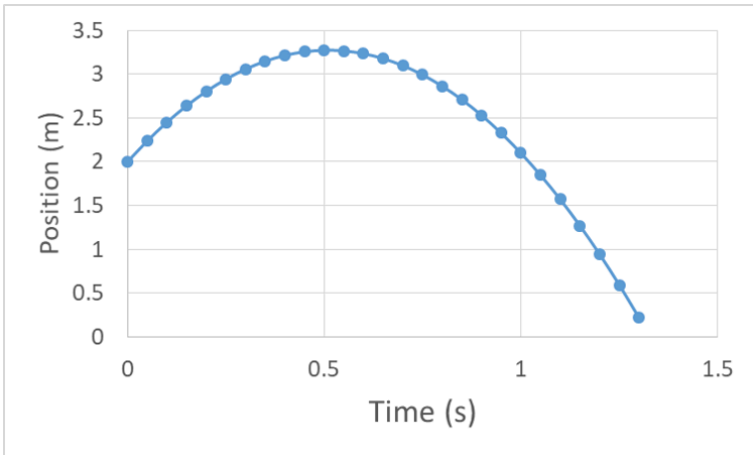
*The acceleration vs. time graph for an object is flat at  $-9.8 \text{ m/s/s}$  (for a choice of downward as the negative direction).*

The velocity will be positive, but slowing down toward zero, cross through zero as the object turns around, and then begin increasing in the negative direction.



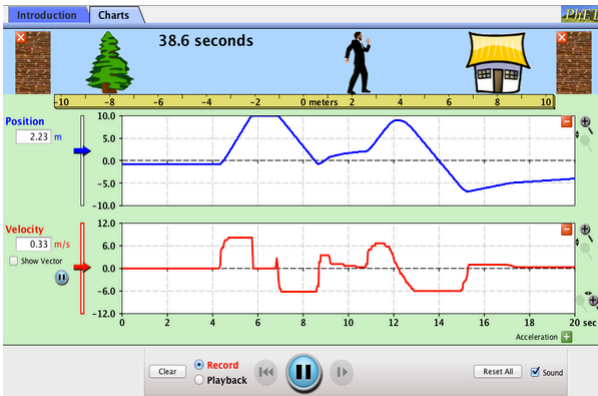
*The velocity vs. time graph starts at 5 m/s and decreases linearly crossing through zero at roughly 0.5 s and then becoming more negative with time in linear fashion and reaching - 5 m/s just after 1 s. The slope is -9.8 m/s/s.*

The position will increase as the object moves upward, then decrease as it falls back down, in a parabolic fashion because the slope is changing at a constant rate (acceleration is constant so velocity changes at a constant rate, so the slope of the position graph changes constantly).



The position vs. time graph is a parabola with downward curvature starting at 2 m, peaking near 3.3 m at roughly 0.5 s, passing back through 2 m just after 1 s, and hitting the ground just after 1.3 s.

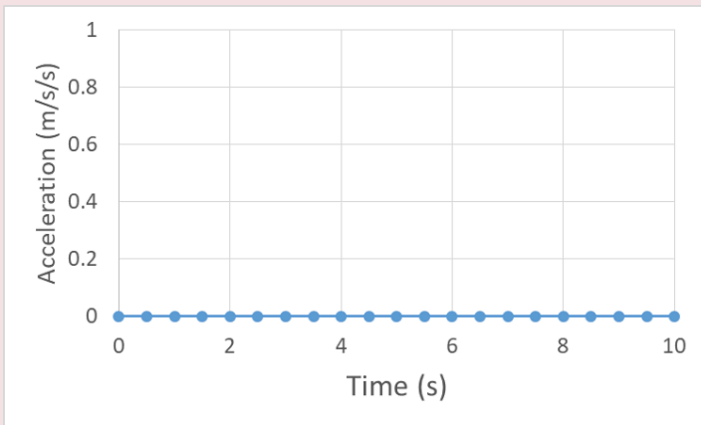
Check out this interactive simulation of a moving person and the associated motion graphs:



## Everyday Example: Terminal Velocity

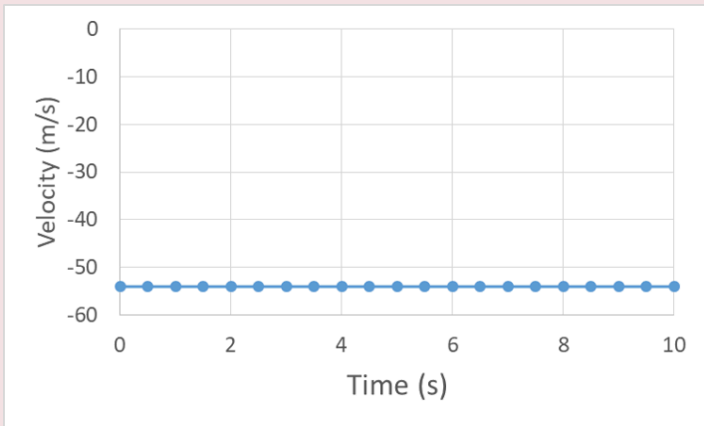
Let's look at the motion graphs for our skydiver while they are at a terminal velocity of **-120 MPH**, which is about **54 m/s**. Let's set our initial position for this analysis to be the position where they hit terminal velocity.

Acceleration is zero because they are at terminal velocity:



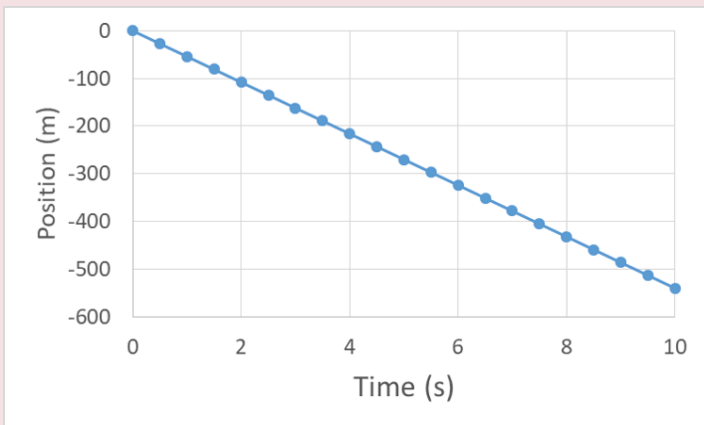
*Acceleration vs. time graph is constant (flat) at zero.*

Velocity is constant, but negative:



*Velocity vs. time graph is constant near -52 m/s.*

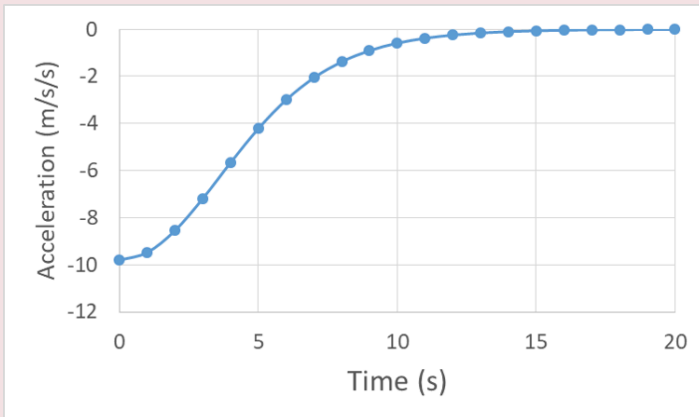
And position changes at a constant rate, becoming more negative with time.



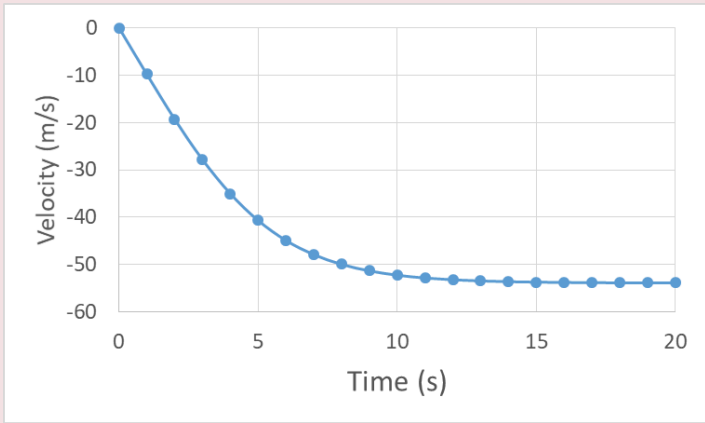
*Position vs time graph decreases linearly from zero to -520 m after 10 s.*

## Everyday Example: Full Skydive

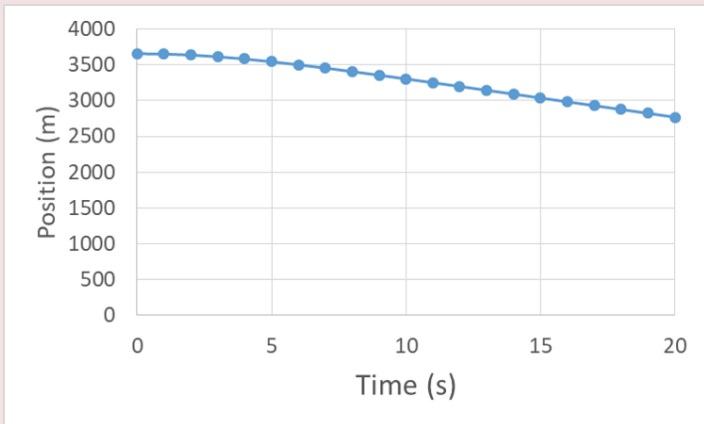
Now let's look at the motion graphs for our skydiver prior to reaching terminal velocity, starting from the initial jump.



The acceleration vs. time curve starts at  $-9.8 \text{ m/s/s}$  because in the first instant there is no drag force so the diver is momentarily in free-fall. As speed is gained, the drag force increases, cancelling out more of the weight, so the acceleration trends toward zero and becomes indistinguishable from zero near 15 s.



*The velocity vs. time curve starts at zero and because the initial speed was zero. Velocity remains negative because the motion is downward, but the slope is not constant like it would in free-fall because the acceleration is not constant like in free-fall. That is because the drag force is growing as the velocity increases, eventually become as large as the weight, so the velocity eventually begins to level off and approach a constant 52 m/s.*



The position vs time curve starts at 3660 m and decreases toward zero with a negative and gradually steepening slope (moving down and speeding up). After after 20 s the skydiver nears position 2750 m and the slope becomes constant at of 52 m/s, indicating terminal velocity. Note that we have converted our [pb\_glossary id="4047"]initial position[/pb\_glossary] of 12,000 ft to the equivalent 3660 m.

## CHAPTER 65.

# QUANTITATIVE MOTION ANALYSIS

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### KINEMATICS

We now know to find average acceleration of an object by finding the net force and applying Newton's Second Law. Once the acceleration is known, we can figure out how the velocity and position change over time. That process is known as kinematics and the equations we use to relate acceleration, velocity, position, and time are known as the kinematic equations. Let's take a look at a few of them one-by-one.

Based on our definition of acceleration as the rate of change of the velocity we can calculate the change in velocity during a time interval as the acceleration multiplied by the length of the time interval:

$$(1) \quad \Delta \mathbf{v} = \mathbf{a}\Delta t$$

#### Reinforcement Exercises

If a person has an acceleration of  $5.0 \text{ m/s/s}$ , how much does their velocity change in  $3.0 \text{ s}$ ?

We can find the current velocity by adding the expression for change in velocity to the initial velocity:

$$(2) \quad v_f = v_i + a\Delta t$$

### Reinforcement Exercises

If the person in the previous exercise has an initial velocity of 2.0 m/s, what is their new velocity after the 3.0 s?

We can calculate the average velocity during the interval as the average of the initial and final velocities:

$$(3) \quad v_{\text{ave}} = \frac{v_i + v_f}{2}$$

### Reinforcement Exercises

What is the average velocity of the person in the previous exercise?

Using the definition of velocity as the rate of change of position we can calculate the change in position during a time interval as the average velocity during the interval multiplied by the length of the time interval.

$$(4) \quad \Delta x = v_{\text{ave}}\Delta t$$

### Reinforcement Exercises

What is the change in position of the person in the previous exercises?

Adding the above expression for change in position to the initial position allows us to calculate the final position after any time:

$$(5) \quad \mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_{\text{ave}}\Delta t$$

### Reinforcement Exercises

If the person in the previous exercises started at a position of 4 m/s, what is their final position?

We can combine everything the from previous steps into a single equation that can save some time on some problems. It looks like this:

$$(6) \quad \mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_i\Delta t + \frac{1}{2}\mathbf{a}(\Delta t)^2$$

To get the above equation we used equation (3) to replace the average velocity with the expression for average velocity:

$$(7) \quad \mathbf{x}_f = \mathbf{x}_i + \frac{\mathbf{v}_i + \mathbf{v}_f}{2}\Delta t$$

Using equation (2) we can then replace the final velocity:

$$(8) \quad \mathbf{x}_f = \mathbf{x}_i + \frac{(\mathbf{v}_i + \mathbf{v}_i + \mathbf{a}\Delta t)}{2}\Delta t$$

After some simplification we are there:

$$(9) \quad \mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_i\Delta t + \frac{1}{2}\mathbf{a}(\Delta t)^2$$

## Reinforcement Exercises

A car accelerates from rest at  $3 \text{ m/s/s}$ . What is the car's velocity after  $5 \text{ s}$  and how far does the car move in the first  $5 \text{ s}$ ? [Hint: The car starts from rest so the initial velocity is zero.]

## Everyday Example

After leaving a friend's 3rd story apartment you get to your car and realize that you have left your keys in the apartment. You call your friend and ask them to drop the keys out the window to you. We want to figure out how long it will take the keys to reach you and how fast they will be falling when they get there. The third story window is about  $35 \text{ ft}$  off the ground. We can convert to meters and use our previously stated acceleration for falling objects,  $g = 9.8 \text{ m/s/s}$ , or we can stick with feet and use  $g = 32 \text{ ft/s/s}$ , so let's do that.

Starting from our last equation from the work we did above:

$$\mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_i \Delta t + \frac{1}{2} \mathbf{a} (\Delta t)^2$$

We choose upward as our positive direction and the ground as our origin, therefore our initial position is  $35 \text{ ft}$  and our final position is  $0 \text{ ft}$ . The keys are dropped from rest, so our initial velocity is zero. Putting the zeros into the equation above we have:

$$0 = \mathbf{x}_i + 0 + \frac{1}{2} \mathbf{a} (\Delta t)^2$$

Now we can isolate the time variable:

$$(10) \quad t^2 = \frac{-2\mathbf{x}_i}{\mathbf{a}}$$

Take the square root to find the time

$$(11) \quad t = \sqrt{\frac{-2x_i}{a}}$$

Entering our known values we can find the fall time. We will use  $-32 \text{ ft/s/s}$  for our acceleration because the acceleration due to gravity is downward and we have chosen upward as the positive direction.

$$(12) \quad t = \sqrt{\frac{-2(35 \text{ ft})}{-32 \text{ ft/s/s}}} = 1.479 \text{ s} = 1.5 \text{ s}$$

Lastly, we can find the velocity of the keys using equation (2) above

$$(13) \quad \mathbf{v}_f = \mathbf{v}_i + \mathbf{a}\Delta t = 0 + (32 \text{ ft/s/s})(1.479 \text{ s}) = 47 \text{ ft/s}$$

The final velocity of  $47 \text{ ft/s}$  is about  $32 \text{ MPH}$ . If the keys smack your hand at that speed, it will hurt. There are techniques you could use to prevent injury in such a situation, and those techniques will be the topic of the next Unit.

In solving the previous example we found an equation to calculate the time required for an object with a certain acceleration to reach a final position of zero when starting from a known initial position. Among other things, this allows us to calculate the time required to fall to the ground from a certain starting height. That equation will come up often, so let's write it out here:

$$(14) \quad t = \sqrt{\frac{-2x_i}{a}}$$

If acceleration is set to  $-9.8 \text{ m/s/s}$  (or  $-g$ ), then this equation calculates the free-fall time for a choice of negative as the downward direction.

## Reinforcement Exercises

Calculate the time required for a person to fall from a height of 0.75 m. From this height, the person will not move fast enough for drag force to become important, so you may assume they are in free-fall.

### THE JERK

We have learned in the last few chapters that our example skydiver has an initial acceleration of  $9.8 \text{ m/s/s}$  and an acceleration of zero after reaching terminal velocity, so between those points the acceleration must be changing. The rate of change of the acceleration is known as the *jerk*, but we won't deal with jerk in this textbook and will instead focus on motion with constant acceleration. However, if we really want to analyze our skydiver's full motion, we will need to somehow deal with a changing acceleration. That's what the next chapter is all about.

## CHAPTER 66.

### FALLING INJURIES

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We started out this chapter by claiming that we would eventually be able to analyze the forces on the body during an impact with a hard surface after a fall. We have reached that point. Let's do it.

#### Everyday Examples: Forces during a Fall

An 80kg person falls 0.80 m from a hospital bed onto a concrete floor. First-off, how much time do they have to reach out and grab something?

In the previous chapter we found an equation for calculating the fall time when starting from rest:

$$(1) \quad t = \sqrt{\frac{-2x_i}{a}}$$

Entering our values:

$$(2) \quad t = \sqrt{\frac{-2(0.80 \text{ m})}{-9.8 \text{ m/s/s}}} = 0.404 \text{ s}$$

If you do the lab at the end of this Unit you will find that 0.4 s is near the limit of human reaction time. If reaction time was impaired for any reason, which is common in hospital patients, it's likely that the person would hit the ground without grabbing something to slow down.

How fast will the person be moving when they hit the floor (what is the impact speed)? Using another kinematic equation:

$$(3) \quad \mathbf{v}_f = \mathbf{v}_i + \mathbf{a}\Delta t$$

And entering our values:

$$(4) \quad \mathbf{v}_f = 0 + (-9.8 \text{ m/s/s})(0.404 \text{ s}) = -3.96 \text{ m/s}$$

The velocity comes out negative as expected because they are moving downward. The hard floor will bring them to a stop in just a fraction of a second, a reasonable estimate would be about 0.2 s (more on this in the next unit). What is the person's average acceleration during impact?

Using the same equation as before:

$$(5) \quad \mathbf{v}_f = \mathbf{v}_i + \mathbf{a}\Delta t$$

But now solving for acceleration:

$$(6) \quad \mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}$$

And entering our values:

$$(7) \quad \mathbf{a} = \frac{0 - (-3.96 \text{ m/s})}{0.2 \text{ s}} = 19.8 \text{ m/s/s}$$

Now we are ready to calculate the average net force on the person.

We'll start from Newton's Second Law:

$$(8) \quad \mathbf{F}_{\text{net}} = m\mathbf{a}$$

Entering our values:

$$(9) \quad \mathbf{F}_{\text{net}} = (80 \text{ kg})(19.8 \text{ m/s/s}) = 1584 \text{ N}$$

Finally, what force does the floor apply (as a normal force) to the person's back to achieve that net force, despite their weight?

We recognize that the the net force is the result of the upward normal force plus the downward weight.

$$(10) \mathbf{F}_{\text{net}} = \mathbf{F}_{\text{N}} + \mathbf{F}_{\text{g}}$$

We solve for the normal force:

$$(11) \mathbf{F}_{\text{N}} = \mathbf{F}_{\text{net}} - \mathbf{F}_{\text{g}}$$

Now we need to calculate the weight, keeping in mind that is negative because it is downward:

$$(12) \mathbf{F}_{\text{g}} = -mg = -(80 \text{ kg})(9.8 \text{ m/s/s}) = -784 \text{ N}$$

Finally entering values for net force and weight to get the normal force:

$$(13) \mathbf{F}_{\text{N}} = 1584 \text{ N} - (-784 \text{ N}) = 2368 \text{ N}$$

That is more than three times the body weight. We will see in the next chapter that the peak force is actually much greater than the average force during impacts like this, so in fact this situation is actually worse than our calculations indicate. Now we see why patient falls must be avoided.

## CHAPTER 67.

# NUMERICAL SIMULATION OF SKYDIVING MOTION\*

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Our goal for this chapter is to understand how we created the previously shown graphs of acceleration, velocity, and position of our example skydiver, even though the net force and acceleration changes throughout. We will use a numerical simulation that ties together just about everything we have learned so far in this unit to achieve this goal. We already know that the initial velocity is zero and therefore the initial drag force is zero. With no drag force in the first moment of the jump, the diver is in free fall and the acceleration is just  $g$  in the downward direction, or  $-9.8 \text{ m/s/s}$ . We can then calculate the velocity after a short time interval  $\Delta t$  as:

$$v_1 = 0 + g\Delta t$$

We have made the assumption that the acceleration during this interval was constant, even though it wasn't, but if we choose a time interval that is very small compared to the time over which the acceleration changes significantly, then our result is a good approximation. A time interval of one second will satisfy

this condition in our case, so we now calculate the velocity at the end of the first one second interval:

$$\mathbf{v}_1 = 0 + g(1\text{ s})$$

Now that we have a velocity we can calculate the air resistance at the start of the second interval using our previously stated values for human drag coefficient, cross-sectional area, and the standard value for air density:

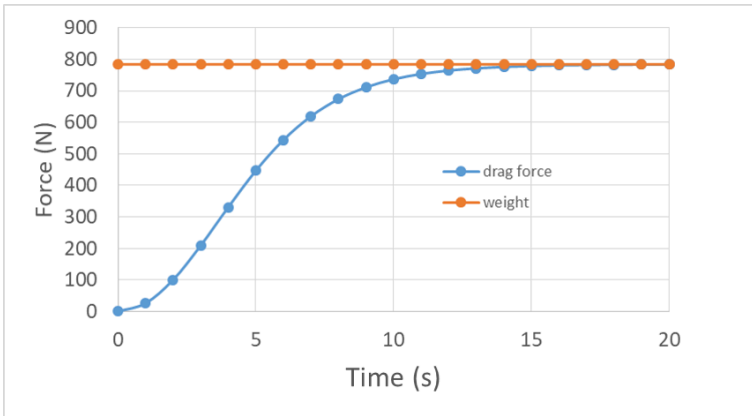
$$\mathbf{F}_{d,2} = \frac{1}{2}C_d\rho A_x v_1^2 = \frac{1}{2}(1)(1.2\text{ kg/m}^3)(0.45\text{m}^2)(.98\text{ m/s})^2 = 0.259\text{ N}$$

Now that we have a drag force due to air resistance we can use Newton's Second Law to calculate the acceleration at the start of the second interval. We have only two forces, drag and gravity and we will use our previously stated skydiver mass of 80 kg:

$$a_2 = \frac{\mathbf{F}_{\text{net}}}{m} = \frac{\mathbf{F}_{d,2} + \mathbf{F}_g}{m} = \frac{\mathbf{F}_{d,2} - mg}{m} = \frac{25.9\text{ N} - (80\text{ kg})(9.8\text{ m/s/s})}{80\text{ kg}} = 9.48\text{ m/s/s}$$

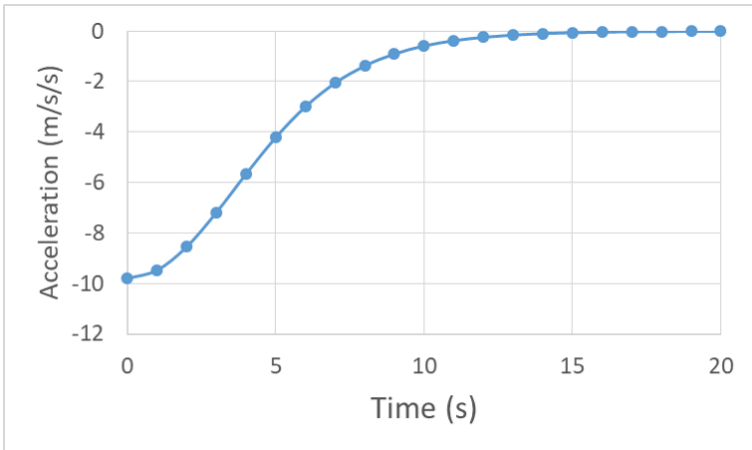
Now we just continue this iterative process of using acceleration and velocity values from the previous interval to calculate new velocity, drag force, and acceleration for next interval.

Using the data produced by the simulation we can graph the drag force. Showing the weight on the same graph we can see how the drag force approaches the weight.



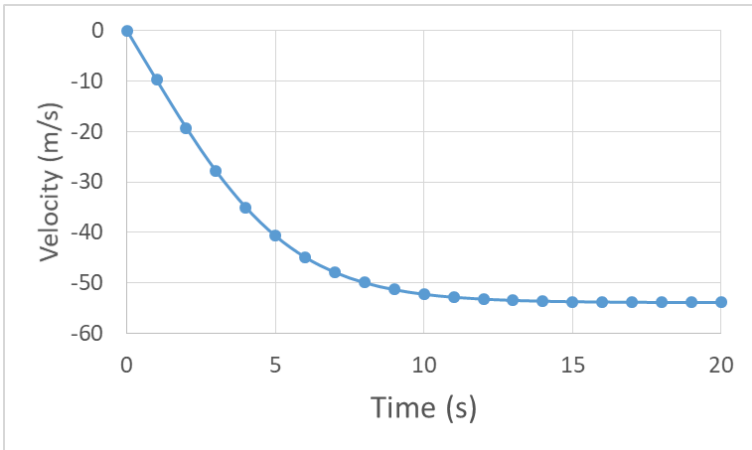
*An example force vs. time curve for both drag and weight during a skydive. Weight is constant at 800 N. Drag starts at zero, increases with increasing slope (upward curvature), reaches an inflection point near 4 s, and continues to increase, but now with decreasing slope, and becomes indistinguishable from the weight value near 15 s.*

We can also use the data to create motion graphs for the skydive and see that the acceleration gradually transitions from  $-9.8 \text{ m/s/s}$  to zero as drag force increases.



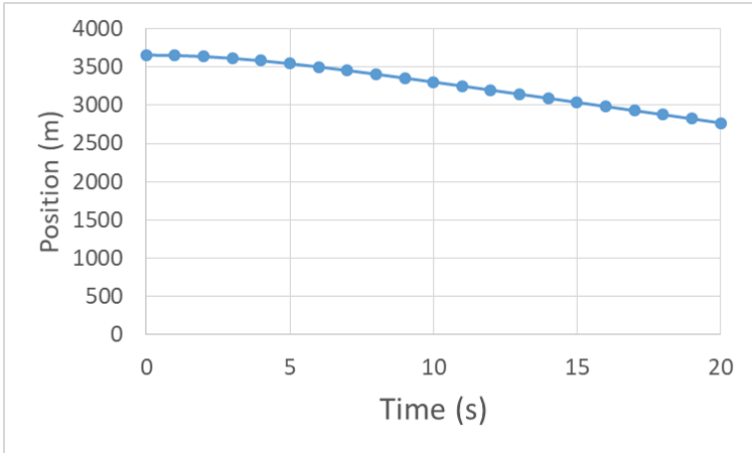
*The acceleration vs. time curve replicates the shape of the force curve, but starts at  $-9.8 \text{ m/s/s}$ , increases toward zero with increasing slope (upward curvature), reaches an inflection point near 4 s, and continues to increase, but now with decreasing slope, and becomes indistinguishable from zero near 15 s.*

We see that velocity is always negative and the speed is always increasing, but the slope becomes less steep because the acceleration is decreasing with time:



*The velocity vs. time curve starts at zero and increases roughly linearly in the negative direction until near 4 s when it begins to level off and approach a constant 52 m/s.*

Finally we can see that the position graph eventually becomes linear as terminal velocity is reached. (Note that we have converted our initial position of 12,000 **ft** to the equivalent 3660 **m**)



*The position vs time curve starts at 3660 m and decreases toward zero with a negative and gradually steepening slope, nearing position 2750 m and slope of 52 m/s after 20 s.*

## CHAPTER 68.

### UNIT 7 REVIEW

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#### Key Terms and Concepts

Drag force

Drag coefficient

Air resistance

Dynamic equilibrium

Newton's First Law (Law of Inertia)

Inertia

Terminal Speed

Position

Displacement

Speed

Velocity

Acceleration

Newton's Second Law

Numerical Simulation

## Learner Outcomes

1. Define position, velocity, and acceleration and explain how they are related.
2. Calculate the drag force on objects moving through fluids.
3. Translate motion graphs into descriptions of motion in terms of position, velocity and acceleration. Translate descriptions of motion into motion graphs.
4. Apply kinematics and Newton's First and Second Laws of Motion to analyze and predict 1-D motion.

## CHAPTER 69.

### UNIT 7 PRACTICE AND ASSESSMENT

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#### **Outcome 1**

- 1) Explain the difference between distance and displacement.
- 2) Explain how velocity relates to position.
- 3) Explain how acceleration relates to velocity.

#### **Outcome 2**

4) Calculate the drag force on a swimmer moving through water at  $0.75 \text{ m/s}$ . The drag coefficient for a human in the prone position is roughly 0.25. Look up the density of water in standard units and cite your source. Estimate the cross-sectional area of a human for this situation by using your own body or average human body measurements (cite your source).

5) The swimmer above is moving at a constant speed. What is the size and direction of the average force applied to the swimmer by the water due to their swimming motion?

### Outcomes 3, 4

6) A toddler runs away from a parent at  $0.3 \text{ m/s}$  for  $3 \text{ s}$ , stops for  $2 \text{ s}$  to see if they are being chased.

a) Draw a velocity vs. time graph for the toddler's motion

b) Draw an acceleration vs. time graph for the toddler's motion

c) Draw a position vs. time graph for the toddler's motion (you will need to calculate the displacements that occur during each interval in order to draw this graph).

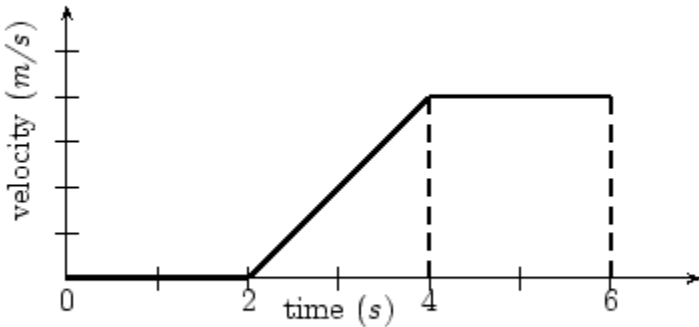
7) Upon realizing they might be chased after the  $2 \text{ s}$  stop, the toddler from the previous exercise begins slowly walking away and increasing speed into a run, reaching a speed of  $0.4 \text{ m/s}$  only  $3 \text{ s}$  later.

a) Complete the acceleration vs. time graph for the toddler's motion, now including this new motion. You may draw a new graph or add to your previous graph in a different color. (You will need to calculate the acceleration during this last part of the toddler's motion in order to complete this graph).

b) Complete the velocity vs. time graph for the toddler's motion. You may draw a new graph or add to your previous graph in a different color. (You will need to use the acceleration you found above to calculate a change in velocity to complete this graph).

c) Complete the position vs. time graph for the toddler's motion. You may draw a new graph or add to your previous graph in a different color. (You will need to use the acceleration you found above to calculate displacements to complete this graph).

8) Describe the motion depicted by the following velocity vs. time graph. The vertical axis tick marks indicate 1 **m/s** intervals, starting from zero **m/s** at the horizontal axis.



*Velocity vs. time graph. The vertical axis marks indicate 1 m/s intervals. Image Credit: Uploaded by Riaan at English Wikibooks.*

1

9) Draw the acceleration vs. time graph associated with the velocity vs time graph above.

10) Draw the position vs. time graph associated with the previous velocity and acceleration vs. time graphs.

#### **Outcome 4**

11) A person with mass of 65 **kg** is out walking two dogs

1. Velocity Graph Uploaded by Riaan at English Wikibooks and transferred from en.wikibooks to Commons., GFDL, is licensed under CC BY-NC-SA 4.0

and stops to talk with a friend. Suddenly the dogs pull in opposite directions. Dog 1 pulls with a force of 500 N to the right. Dog 2 pulls with 300 N to the left.

- a) Draw a free body diagram of the dog walker.
- b) What is the net force on the dog walker?
- c) What is the acceleration of the dog walker, including direction.
- d) What distance will the dog walker have moved in 3 s?
- e) What will the velocity of the dog walker be after 3 s?

## PART VIII.

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# UNIT 8: LOCOMOTION

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### Learning Objectives

1. Describe how locomotion occurs in terms of Newton's Laws of Motion.[2]
2. Apply Newton's Second and Third Laws of motion to analyze the motion of objects.[3]
3. Apply the Law of Conservation of Momentum to analyze the motion of objects.[3]
4. Evaluate and explain strategies for reducing the forces experienced by objects undergoing collisions.[2]



## CHAPTER 70.

### OVERCOMING INERTIA

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Typically an RN like Jolene will walk several miles over the course of a 12 hour shift on the MED floor. Her average speed ( $v_{ave}$ ) can be calculated as the distance covered divided by the time she worked. If she walks three miles, then her average speed would be:

$$(1) \quad v_{ave} = \frac{\text{distance}}{\text{timeinterval}} = \frac{3 \text{ miles}}{12 \text{ hours}} = 0.25 \text{ mph}$$

Jolene's average speed is very different from her instantaneous speed at any one moment in time, which could be anything from zero to about 4.5 **mph** (she tries to avoid running in the hospital). Jolene's instantaneous speed and direction of motion change often as she starts, stops and turns corners. The process of generating, maintaining, and changing motion is known as locomotion.

### NEWTON'S THIRD LAW OF MOTION

Newton's First Law tells us that Jolene must experience a net force in order to initiate a change in motion, also known as a change in velocity. We know that Newton's Second Law tells us how to calculate the net force Jolene

needs in order to achieve a particular amount of velocity change each second (id="4053"]acceleration[/pb\_glossary]). However, Jolene can't apply a net force to herself, so how exactly does Jolene control how much net force she experiences? Newton's Third Law provides the answer. The forces that Jolene experiences must be supplied by the objects around her. The size of the force that Jolene receives from another object, such as the floor or wall, is determined by how hard she pushes against that object. In fact, anytime one object puts a force on a second object, the first object will receive an equal force back, but in the opposite direction. This result is known as Newton's Third Law of Motion. The capacity for using the laws of motion to generate, maintain, and change motion is known as locomotion.

### Examples

The astronaut in the video above starts out in static equilibrium relative to the space station. Then she pushed against the wall. The resistance of the wall to being deformed caused it to apply a reactionary normal force back on her. That unbalanced normal force destroyed her state of static equilibrium, overcame her inertia, and caused her velocity to change relative to the station. This example is a unique form of locomotion, but all locomotion depends on this same process of pushing on an object in order receive a push back form the object.

## Reinforcement Activity

If the astronaut in the previous video pushes against the wall with 3 N of force, what is the force applied back to her by the wall?

If the astronaut has a mass of 60 kg, what is her acceleration?

### THIRD LAW PAIR FORCES

The equal and opposite forces referenced in Newton's Third Law are known as third law pair forces (or third law pairs).

Other Third Law pair forces include:



A YouTube element has been excluded from this version of the text. You can view it online here:  
<https://openoregon.pressbooks.pub/bodyphysics/?p=4615>

- The Earth pulls down on you due to gravity and you pull back up on the Earth due to gravity.
- A falling body pushing air out of its way and air resistance pushing back on the body.
- You pull on a rope and the rope pulls back against your hand via tension.
- You push on the wall, and the wall pushes back with a normal force.
- A rocket engine pushes hot gasses out the back, and the gasses push back on the rocket in the forward direction.
- You push your hand along the wall surface, and the wall pushes back on your hand due to kinetic friction.
- You push your foot against the ground as you walk, and the floor pushes back against your foot due to friction (static if your foot doesn't slip, kinetic if it does).

You may have noticed that in each of the cases above there were two objects listed. This is because *Newton's Third Law pairs must act on different objects*. Therefore, Third Law pair forces cannot be drawn on the same free body diagram and *can never cancel each other out*. (Imagine if they did act on the same object, then they would *always* balance each other out and no object could *ever* have a net force, so no object could ever accelerate!)

### Reinforcement Exercises

An insect collides with a jet moving at 500 **mph**. Which feels a greater force, the bug or the jet, or neither? Explain how you know.

Explain why the jet does not accelerate (and is fine), but the bug accelerates a lot (and is not fine).

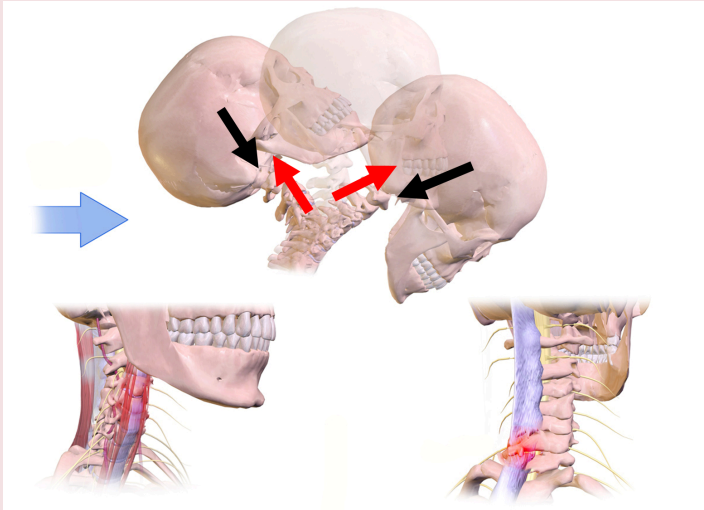
### Reinforcement Exercises

Draw the free body diagrams necessary to show each force in the Third Law pairs listed above. How many free body diagrams will you need to draw for each Third Law pair? [Hint: keep in mind the rule about free body diagrams and Third Law pairs.]

### Everyday Example: Headrest

The headrest in your car is not actually designed as a place to rest your head. Its real purpose is to prevent injury. If someone rear-ends your car it will accelerate forward. As a result your body is accelerated forward by normal force and friction from the seat. If the head rest were not there, your head would momentarily remain in place due to inertia as your body moved forward. The lag in head position gives the impression that the head snapped back, but really the body moved forward and left the head behind. Your head does remain attached to your accelerating body though, so the tissues in your neck must provide the large force required to accelerate the head along with the body. According to Newton's Third Law, the

tissues of the neck will feel an equal and opposite force to that large force they apply to the head. That large force may damage the tissue (cause a stress larger than the yield stress of the tissue).



*Top: Forces on the head from the neck (black) and on the neck from the head (red) during rapid forward-back motion of the head relative to the body. Bottom: Sites of whiplash injury. Image Credit: 3rd Law Whiplash is a derivative of Whiplash Injury by BruceBlaus, via Wikimedia Commons*

1

The headrest provides a normal force on your head so that it accelerates along with the body, keeping your head above your shoulders and your neck in a safe position. You can see the importance of the headrest in these crash-test videos:

The headrest doesn't necessarily reduce the acceleration felt by the head as much as provide the force needed to accelerate the head

1. 3rd Law Whiplash is a derivative of Whiplash Injury by BruceBlaus [CC BY-SA 4.0 (<https://creativecommons.org/licenses/by-sa/4.0/>)], via Wikimedia Commons

along with the body, so that the neck doesn't have to, thus reducing the third law pair forces between the head and neck.

## FALLING AS LOCOMOTION

Notice that the list of third-law pair forces includes the force of gravity on the Earth from you and the force of gravity on you from the Earth (weight), so in fact falling is a form of locomotion. That means that throughout the previous unit on falling we were already studying locomotion, although falling is sort of an uncontrolled, or passive form of locomotion. The next few chapters will



A YouTube element has been excluded from this version of the text. You can view it online here: <https://openoregon.pressbooks.pub/bodyphysics/?p=4615>

help us examine active forms of locomotion like walking, jumping and driving.

## CHAPTER 71.

# LOCOMOTION

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### ACTIVE LOCOMOTION

Locomotion is based on a combination of Newton's Laws. A body will move with a constant velocity until it experiences a net force (Newton's First Law). A change in body motion is achieved by initiating muscle contraction in order to apply a force against another object, such as the floor or the wall. The other object then applies an equal and opposite reaction force against the body (Newton's Third Law). That reaction force acting on the body adds together with all of the other forces on the body to determine the net force. The net force causes an acceleration that depends on the the body mass according to Newton's Second Law.

#### Everyday Examples: Responding to a Code Without Slipping

Jolene is walking with a speed of a 2.5 **mph** down the hospital corridor when a code is called over the intercom. She stops then turns around and starts walking the other direction, toward the room

where the code was called, at a speed of **4.0 mph**. If Jolene's mass is **61 kg** and she tried to make that move very quickly, in only **0.75 s**, for example, what net force would be applied to her?

We have only been given speeds in the problem, but in order to analyze velocity we need to define direction of motion. Let's assign Jolene's initial direction as the negative direction so her initial velocity was **-2.5 mph**. In that case her final velocity was **+4.2 mph**. We can calculate her change in velocity as:

$$(1) \quad \Delta v = v_f - v_i = 4.0 \text{ mph} - (-2.5 \text{ mph}) = 6.5 \text{ mph}$$

If we convert our answer to units of **m/s** we get: **6.5 mph = 2.9 m/s**. (You can check this yourself using unit analysis or an online unit converter).

Next we calculate Jolene's acceleration using the definition of average acceleration:

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Entering our values:

$$\mathbf{a} = \frac{2.9 \text{ m/s}}{0.75 \text{ s}} = 3.9 \text{ m/s/s}$$

We find that Jolene's acceleration is **3.9 m/s/s**, which means that her velocity becomes **3.9 m/s** more positive each second. Notice that she was originally moving in the negative direction, so having a velocity that is becoming more positive means that she must have slowed down.

Now we can use Newton's Second Law to calculate the average net force required to provide this acceleration:

$$\mathbf{F}_{\text{ave}} = m\mathbf{a}$$

We are now ready to enter our values:

$$F_{\text{ave}} = (61 \text{ kg})(3.9 \text{ m/s}) = 240 \text{ N}$$

Friction is the only horizontal force contributing to the horizontal acceleration Jolene experiences, so the net force would just be equal to the friction force. We can figure out if Jolene would slip by comparing the required net force to the maximum possible static friction force.

First we start with the equation for max static friction force:

$$F_{f,s} = \mu_s F_N$$

Then we divide by the normal force to solve for the friction coefficient:

$$\mu_s = \frac{F_{f,s}}{F_N}$$

Assuming the floor is level and only Jolene's horizontal motion is changing, then the normal force must be balancing Jolene's weight (if these were not balanced then she would have a vertical acceleration as well). For this special case we substitute her weight for the normal force:

$$\mu_s = \frac{F_{f,s}}{mg}$$

Finally we enter our values:

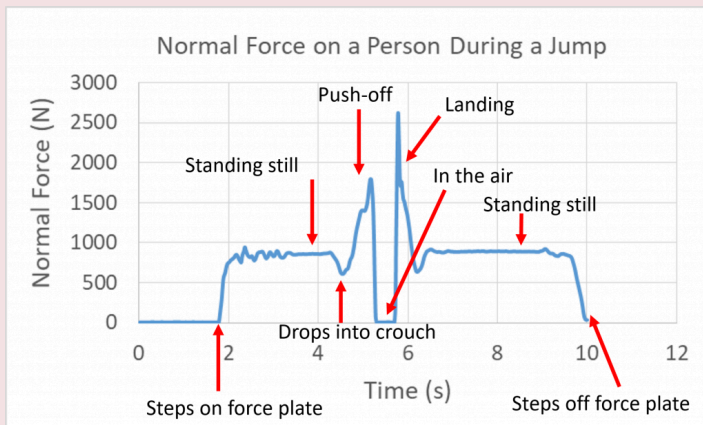
$$\mu_s = \frac{240 \text{ N}}{(61 \text{ kg})(9.8 \text{ m/s/s})} = 0.4$$

Looking up friction coefficients we find that the rubber-concrete friction coefficient is typically 0.6 or greater, so Jolene could likely make this move without slipping.

## JUMPING

### Everyday Example: Jumping

When a person stands still on flat ground, they are in static equilibrium and the normal force pushing up from the ground is equal to their weight. In order to jump, the muscles of the leg contract to push down against the floor. This downward push results in an equal reactive normal force from the floor, so that now the normal force is greater than body weight a there is an upward net force. Now with an upward net force, the body will experience an upward acceleration. The following graph of the normal force on person was created by jumping and landing on a force plate, which is essentially a extra-tough digital scale that records the normal force that it provides over time.



*Normal force from the ground on a person during a jump as recorded by a digital force plate during the experiment described in the following text.*

First the jumper steps on the force plate and it reads their weight of

about 800 N. Then the plate reading dips a couple hundred Newtons because the upward normal force is reduced as they drop down into a crouch, which makes sense because they must accelerate downward to drop down. Next the force peaks to near 1700 N they push off hard to stop their downward motion and initiate upward motion. Throughout this stage the normal force is greater than the weight in order to create this upward acceleration. The normal force drops to zero as the body leaves contact with the ground. While in the air gravity provides the net downward force so the acceleration is downward, and the body's upward velocity slows. (After leaving the ground the acceleration is  $-g = -9.8 \text{ m/s}^2$ ). The body eventually turns around as the upward velocity reaches zero, and then begins to move back toward the ground. Landing requires an upward acceleration to stop the downward velocity, so a large upward normal force of over 2500 N is produced. The jumper is attempting a "soft landing" and so continues into a crouching position during landing, which causes another couple hundred Newton dip in the normal force at the end of landing. Finally, the normal force equals weight again after landing is complete. Upcoming chapters will talk more about "soft landings" and other methods of injury prevention.

## DRIVING

### Everyday Example: Driving

In order for your car to accelerate along a flat road, it must have a net horizontal force. What force acts on your car to provide this net force? Remember the force must be on the car from another object, the car can't put a net force on itself (so the answer can't be the engine or any part of the car). Gravity and normal force are external forces on a car, but those forces act vertically and cannot contribute

to a horizontal acceleration. The force that acts on your car in the horizontal direction is friction. When the tires attempt to rotate, they push against the ground via friction, and the ground pushes back with an equal reactionary friction force, according to Newton's Third Law. Then, according to Newton's Second Law, that friction force acting on the car causes it to accelerate. The purpose of the throttle, engine, and drive train is to cause the tires to push back on the road in order to receive the forward push from the road in return.



*Locomotion in cars is created by the reactionary friction force on the car tires from the road in response to the friction force from the tires on the road. Forward acceleration occurs when the force on the tires from the road is larger than the drag force, providing a net force in the forward direction. Image adapted from Fifth generation Chevrolet Malibu on Interstate 85 in Durham, North Carolina by Ildar Sagdejev (Specious) via wikimedia commons*

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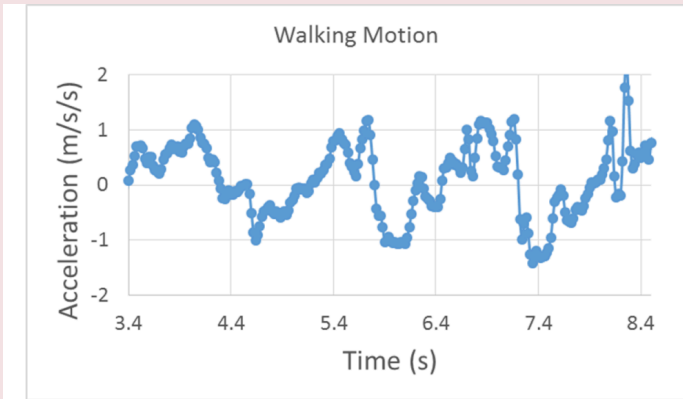
1. Adapted from Fifth generation Chevrolet Malibu on Interstate 85 in Durham, North Carolina by Ildar Sagdejev (Specious) - Own work, CC BY-SA 3.0, via wikimedia commons

Also, your car pushes on the air to move it out the way, so according to Newton's 3rd Law, the air puts a drag force back on your car. When the drag force and the friction force on the car are equal the car reaches a constant velocity, sort of like terminal velocity. If you want the car to go faster, you need to increase the friction force on the car from the road, which you achieve by increasing how hard the tires push against the road via the throttle, engine, and drive train successively.

## WALKING AND RUNNING

### Every Day Examples: Walking and Running

When walking or running you push horizontally against the floor and the floor pushes back, providing you with the net force necessary to create acceleration. Walking at constant average speed is achieved by alternating of forward acceleration caused when the floor pushes forward on your back foot, and backward acceleration caused by the floor pushing back on your front foot. These accelerations average out to zero so velocity appears constant, but if you use a sensor capable of taking several measurements per second, then you can see the oscillatory nature of walking motion:



*Acceleration vs. time curve for a person walking.*

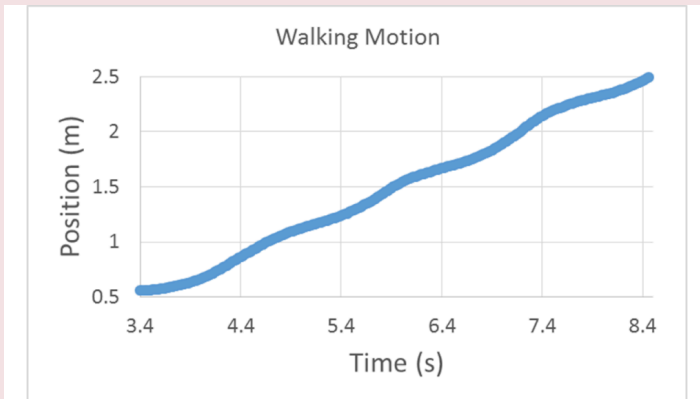
An initial positive acceleration corresponds with the change in velocity away from zero during the first step. Afterward, constant walking motion results in acceleration that oscillates from positive to negative and averages out to zero.



*Velocity vs. time curve for a person walking.*

The oscillating accelerations result in a velocity that alternates

between slowing down and speeding up, however we can see that the velocity stays positive so you are always making progress in the direction you intend to walk. We also see that over a full gait cycle average velocity is constant, near  $0.4 \text{ m/s}$  for this example, which agrees with the zero average acceleration in the previous graph.



*Position vs. time curve of a person walking.*

As you make progress, the position increases roughly linearly with an average slope equal to your constant average velocity, in this case  **$0.4 \text{ m/s}$** . However the oscillations in the instantaneous velocity are noticeable as slight variations in the slope of the position graph.

## CHAPTER 72.

### LOCOMOTION INJURIES

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#### PREVENTING INJURIES

Injuries can be caused by large forces in the absence of acceleration, such as crushing injuries, and those types of injuries can be analyzed using concepts of static equilibrium. However, more often injuries are caused by large forces associated with large accelerations experienced during locomotion. Therefore, we can reduce the likelihood of injuries by:

1. Increasing the ultimate strength of body tissues so that they can handle larger stress and thus larger force.
2. Increasing the cross-sectional area of body parts so that the stress remains below the tissue's ultimate strength, even for larger forces.
3. Decrease the size of forces and resulting stress experienced by the body.

The first two options are controlled by genetics and regular exposure to large, but not injury inducing forces, also known as exercise.<sup>1</sup> The next few chapters will focus

on the third option, which is achieved through thoughtful movement.

### Everyday Examples: Landing after a Jump

You naturally tend to bend your knees when landing after a jump, rather than keep your knees locked and your legs rigid. The reason is that rigid legs bring you to an abrupt stop, but bending your knees allows you to spread the landing out over a longer time, which we have just learned, reduces the average force.

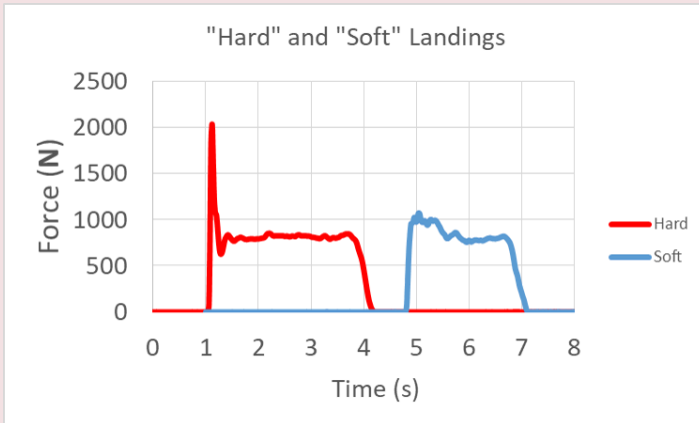


*Stiff and bent-leg landings that produced the force vs. time data shown below.*

The force vs. time graphs show the normal force applied to a person

1. Carlson, K. J., & Marchi, D. (2014). *Reconstructing Mobility*. New York: Springer. doi:10.1007/978-1-4899-7460-0

when landing on one foot after stepping off from a 0.1 m height as seen in the previous GIF. The graph on the left was the more rigid leg landing (it didn't feel good) and the graph on the right was a bent-knee landing.



*Force vs. time data for a stiff-legged landing (red) and crouching landing (blue).*

Notice that the stiff-legged "hard" landing nearly doubled the peak force applied to the body.

When thinking about how to reduce acceleration and the associated forces we should reexamine the definition of average acceleration:

$$(1) \quad \mathbf{a}_{\text{ave}} = \frac{\Delta \mathbf{v}}{\Delta t}$$

From the above definitions we see that there are really two options for reducing accelerations. We can reduce the amount that velocity changes, or we can increase the time over which the velocity changes (or both). Of course we could simply stay away from any situation that could lead to large changes in velocity, but then we would have

to avoid riding in cars, running, heights, or any situation in which we might trip and fall down. Avoiding large velocity changes all together would not be practical or fun, so we need to make sure those changes happen in a controlled way (slowly). Driving without looking at your phone is a good start.

## SOFTENING IMPACTS

Assuming we do experience large changes in velocities, we need to focus on increasing the time over which the velocity changes in order to reduce the average acceleration and average net force. Said another way, *use small net force experienced over a long time to produce the same velocity change as a large net force experienced over a short time.*

### Reinforcement Activity

Throw an egg up into the air and catch it. Did you move your hands with the egg as you caught it on the return, or did you hold your hands still as the egg arrived?

Most of us have learned from experience how to reduce the force on objects as we control their motion. In my experience, toddlers do not apply this technique.

The people in this video are well aware of techniques for reducing forces by extending impact time.

We see from the graph in the previous everyday example that the force varies during an impact and it's often the peak force that we are really worried about with regard