5. Apply the first law of thermodynamics to the closed system, eliminating the terms that are not applicable to the system.
6. Solve for the unknowns by combining the first law of thermodynamics with the ideal gas law, thermodynamic tables, and other physical laws as appropriate.

The following examples demonstrate how to apply the first law of thermodynamics to closed systems.

Example 1

Consider the vapour compression refrigeration cycle consisting of a compressor, condenser, expansion device, and evaporator as shown. The compressor must consume work, W_{in}, from an external energy source such as electricity. The evaporator and condenser absorb and reject heat, Q_H and Q_L, respectively. What is the relation between W_{in}, Q_H, and Q_L?

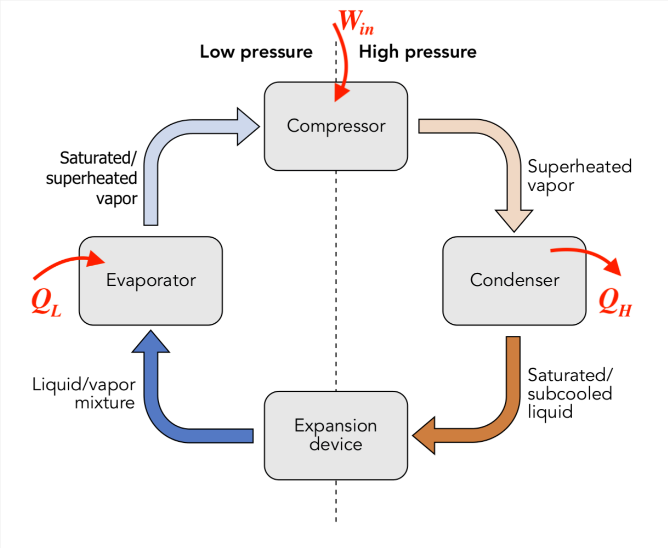

***Figure 4.4.e1** Vapor compression refrigeration cycle consisting of a compressor, condenser, expansion device, and evaporator*

<u>*Solution*</u>:

The vapour compression refrigeration cycle can be regarded as a closed system with the initial and final states being identical; therefore, $\Delta U = 0$.

$$\because \Delta U = 0 = Q - W$$

$$\therefore Q_L - Q_H - (-W_{in}) = 0$$

$$\therefore Q_H = Q_L + W_{in}$$

Note the sign convention for heat: in (+), out (-) and for work: in (-), out (+). This relation can be interpreted as the total energy transferred out of the cycle remains the same as the total energy transferred into the cycle.

Example 2

A rigid tank has two rooms, both containing R134a at the following initial states.

Room A: m= 2 kg, P=200 kPa, v=0.132 m^3/kg

Room B: m= 3 kg, P=500 kPa, T=100°C

A crack is developed in the partition between the two rooms, which allows R134a in the tank to mix. Assume the mixing takes place slowly until R134a in the whole tank reaches a uniform state at 50°C. Find the heat transfer during this process. The process can be treated as a quasi-equilibrium process.

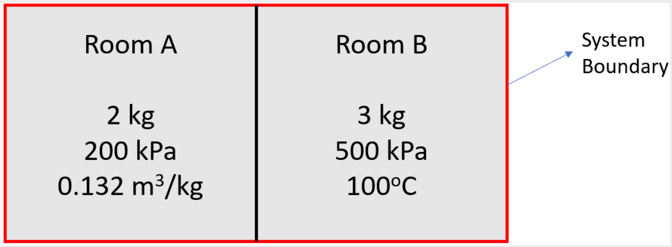

Figure 4.4.e2 *A rigid tank with two rooms*

<u>*Solution:*</u>

First, set the whole rigid tank as the closed system.

Because the volume of the tank remains constant, the boundary work during the mixing process is zero; therefore, from the first law of thermodynamics,

$$\because \Delta U = Q - W \quad \text{and} \quad W = 0$$
$$\therefore \Delta U = Q$$

The heat transfer during the process depends on the internal energies of the initial and final states.

\begin{align*} \Delta U &= U_3 - (U_1 + U_2) \\&= (m_1 + m_2)u_{3} - (m_{1}u_{1} + m_{2}u_{2}) \end{align*}

where the subscripts 1, 2, and 3 represent the initial states of R134a in rooms A and B, and the final state of R134a in the whole tank, respectively.

Second, find the specific internal energies, u_1, u_2 , and u_3.

Room A at the initial state: P =200 kPa, v = 0.132 m^3/kg

From Table C1, at T = -10°C, P_{sat} = 200.6 kPa, v_g = 0.09959 m^3/kg. Since $v > v_g$, R134a at this state is a superheated vapour.

From Table C2, at P = 200 kPa, T = 60°C, v = 0.132057 m^3/kg ≈ 0.132 m^3/kg

$$u_1 = 427.51 \text{ kJ/kg}$$
$$\mathbb{V}_A = m_1 v_1 = 2 \times 0.132 = 0.264 \text{ m}^3$$

Room B at the initial state: P =500 kPa, T = 100°C

From Table C1, at T = 100°C, P_{sat} = 3972.38 kPa. Since $P < P_{sat}$, R134a at this state is a superheated vapour.

From Table C2, at P =500 kPa, T = 100 °C

$$u_2 = 459.65 \text{ kJ/kg}$$

$$v_2 = 0.058054 \text{ m}^3\text{/kg}$$

$$\mathbb{V}_B = m_2 v_2 = 3 \times 0.058054 = 0.1742 \text{ m}^3$$

The final state of R134a in the whole tank: T = 50°C

$$v_3 = \frac{\mathbb{V}_{tot}}{m_{tot}} = \frac{\mathbb{V}_A + \mathbb{V}_B}{m_1 + m_2} = \frac{0.264 + 0.1742}{2 + 3} = 0.0876 \text{ m}^3\text{/kg}$$

From Table C1, at T = 50°C, v_g = 0.015089 m^3/kg. Since $v_3 > v_g$, R134a at the final state is a superheated vapour.

From Table C2,

At P = 200 kPa, T = 50°C, v = 0.127663 m^3/kg, u = 419.29 kJ/kg

At P = 300 kPa, T = 50°C, v = 0.083723 m^3/kg, u = 418.19 kJ/kg

Use linear interpolation,

$$\because \frac{P_3 - 200}{300 - 200} = \frac{0.0876 - 0.127663}{0.083723 - 0.127663} = \frac{u_3 - 419.29}{418.19 - 419.29}$$

$$\therefore P_3 = 291.2 \text{ kPa}$$ and $$u_3 = 418.287 \text{ kJ/kg}$$

Last, substitute u_1, u_2 and u_3 into the simplified first law,

\begin{align*} Q &= \Delta U = (m_1 + m_2)u_{3} - (m_{1}u_{1} + m_{2}u_{2}) \\& = 5 \times 418.287 - (2 \times 427.51 + 3 \times 459.65) \\&= -142.535 \ \rm{kJ}\end{align*}

During the mixing process, the heat is transferred from the tank to the surroundings; therefore, the sign for the heat transfer is negative.

Example 3

Consider 0.5 kg of ammonia in a piston-cylinder device initially at P_1=100 kPa, T_1=0^{o}C. The ammonia is compressed until its pressure reaches P_2=150 kPa in a polytropic process with n=1.25. Calculate the heat transfer in this process.

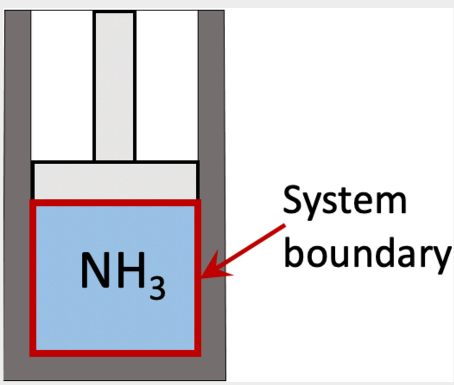

Figure 4.4.e3 *Ammonia in a piston-cylinder device*

<u>*Solution:*</u>

First, set ammonia in the piston-cylinder as the closed system. From the first law of thermodynamics,

$$\because \Delta U = Q - W$$

\therefore Q = W + \Delta U = W + m(u_2 - u_1)

Second, consider the boundary work in a polytropic process. The specific volumes are unknowns

$$W=\frac{P_2\mathbb{V}_2-P_1\mathbb{V}_1}{1-n}=\frac{m\left(P_2v_2-P_1v_1\right)}{1-n}$$

Third, find the specific volumes and specific internal energies at both initial and final states.

At the initial state: P_1 = 100 kPa, T_1 = 0°C. From Table B1, P_{sat} = 429.39 kPa at 0°C. Since $P_1 < P_{sat}$, ammonia is a superheated vapour.

From Table B2,

v_1 = 1.31365 m^3/kg, u_1 = 1504.29 kJ/kg.

At the final state P_2 = 150 kPa. The process is polytropic with n =1.25.

$$\because P_1v_1^n=P_2v_2^n$$

$$\therefore v_2=v_1\left(\frac{P_1}{P_2}\right)^{1/n}=1.31365\times\left(\frac{100}{150}\right)^{1/1.25}=0.94974\ \mathrm{m}^3/\mathrm{kg}$$

From Table B1: at T = -25 °C and P = 151.47 kPa ≈ 150 kPa, v_g = 0.771672 m^3/kg. Since $v_2 > v_g$, ammonia at the final state is a superheated vapour.

From Table B2,

At P = 150 kPa, T = 20°C, v = 0.938100 m^3/kg, u = 1535.05 kJ/kg

At P =150 kPa, T = 30°C, v = 0.972207 m^3/kg, u = 1551.95 kJ/kg

Use linear interpolation,

$$\because \frac{T_2 - 20}{30 - 20} = \frac{0.94974 - 0.938100}{0.972207 - 0.938100} = \frac{u_2 - 1535.05}{1551.95 - 1535.05}$$

$$\therefore T_2 = 23.4\,^\circ\mathrm{C} \quad \text{and}$$

$$u_2 = 1540.82\ \mathrm{kJ/kg}$$

Last, the heat transfer in this process can now be found from

$$\begin{aligned} Q = W + \Delta U &= m\frac{P_2 v_2 - P_1 v_1}{1-n} + m(u_2 - u_1) \\ &= 0.5\left[\frac{150 \times 0.94974 - 100 \times 1.31365}{1 - 1.25} + (1540.82 - 1504.29)\right] \\ &= -3.928\ \mathrm{kJ} \end{aligned}$$

During this process heat is rejected to the surroundings; therefore, the sign for heat transfer is negative.

Example 4

A piston-cylinder device contains steam initially at 200°C and 200 kPa. The steam is first cooled isobarically to saturated liquid, then isochorically until its pressure reaches 25 kPa.

1. Sketch the whole process on the $P - v$ and $T - v$ diagrams
2. Calculate the specific heat transfer in the whole process

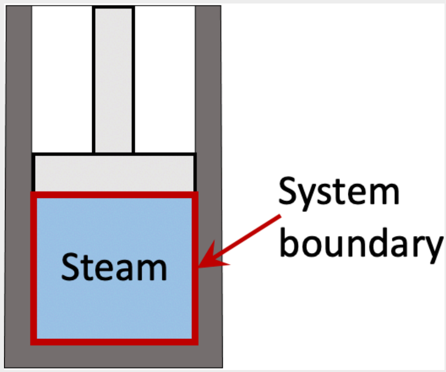

***Figure 4.4.e4** Steam in a piston-cylinder device*

<u>*Solution:*</u>

1. $P - v$ and $T - v$ diagrams

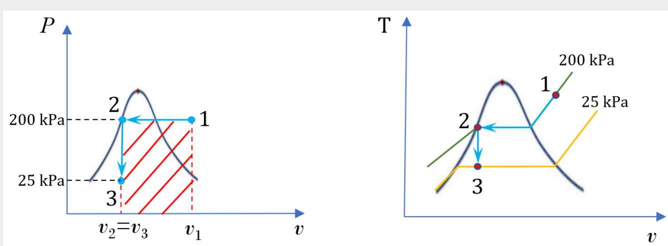

***Figure 4.4.e5** P-v and T-v diagrams of the whole process*

2. Calculate the specific heat transfer

First, set the steam in the piston-cylinder device as a closed system. From the first law of thermodynamics,

$$\because \Delta u = q - w$$

$$\therefore q = \Delta u + w = (u_3 - u_1) + w$$

Second, analyze the processes.

The process is isobaric from state 1 to state 2, then isochoric from state 2 to state 3. The specific work is the shaded area of the rectangle shown in the $P - v$ diagram; therefore,

$$w = P_1(v_3 - v_1) \quad \text{and} \quad v_2 = v_3$$

Third, determine the specific volumes and specific internal energies at states 1 and 3.

At state 1, P_1 = 200 kPa and T_1 = 200°C.

From Table A2,

v_1 = 1.08048 m^3/kg, u_1 = 2654.63 kJ/kg

State 2 is saturated liquid water at P_2 = 200 kPa.

From Table A1,

At T = 120 °C, P = 198.67 kPa, v_f = 0.001060 m^3/kg

At T = 125 °C, P = 232.24 kPa, v_f = 0.001065 m^3/kg

Use linear interpolation,

$$\because \frac{v_2 - 0.001060}{0.001065 - 0.001060} = \frac{T_2 - 120}{125 - 120} = \frac{200 - 198.67}{232.24 - 198.67}$$

$$\therefore v_2 = 0.0010602 \text{ m}^3/\text{kg}$$

and $$T_2 = 120.2\,^\circ\text{C}$$

At state 3, P_3 = 25 kPa. $v_3 = v_2$ = 0.0010602 m^3/kg.

From Table A1, $v_f < v_3 < v_g$; therefore, state 3 is a two phase mixture of saturated liquid and saturated vapour with $T_3 = T_{sat} \approx 65^oC$.

v_f = 0.001020 m^3/kg, v_g = 6.19354 m^3/kg

u_f = 272.09 kJ/kg, u_g = 2462.42 kJ/kg

The quality and specific internal energy of the two phase mixture are

$$x = \frac{v_3 - v_f}{v_g - v_f} = \frac{0.0010602 - 0.001020}{6.19354 - 0.001020} = 6.5 \times 10^{-6}$$

$$\begin{aligned} u_3 &= u_f + x(u_g - u_f) \\ &= 272.09 + 6.5 \times 10^{-6}(0.0010602 - 1.08048) \\ &= 272.10 \text{ kJ/kg} \end{aligned}$$

Note that state 3 is almost a saturated liquid with very small quality; therefore, $u_3 \approx u_g$.

Last, calculate the specific boundary work and specific heat transfer in this whole process

$$\begin{aligned} w &= P_1(v_3 - v_1) \\ &= 200 \times (0.0010602 - 1.08048) \\ &= -215.884 \text{ kJ/kg} \end{aligned}$$

\begin{align*} q &=(u_3 - u_1) + w \\&= (272.10 - 2654.63) + (-215.884) \\&= -2598.4 \ \rm{kJ/kg}\end{align*}

In this cooling process, the volume decreases, resulting in a negative specific boundary work. The temperature and the internal energy decrease too. As a result, the specific heat transfer is negative indicating a heat loss from the system to its surroundings.

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://pressbooks.bccampus.ca/thermo1/?p=1454#h5p-35

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4.5 Chapter review

Energy can be transferred to and from a closed system by two mechanisms: heat transfer and work. Both heat transfer and work have a significant effect on the total energy as well as the internal energy stored in a system, as expressed in the first law of thermodynamics.

When applying the first law of thermodynamics to a closed system, it is important to evaluate the internal energy of the system at various states of a process, and the boundary work and heat transfer during the process.

- The internal energy is a property of a system. It is a state function and is independent of the process path. In general, the specific internal energy can be found by using thermodynamic tables. For ideal gases, solids or liquids, the specific internal energy can be calculated by using the constant-volume specific heat.
- Heat transfer and boundary work are not properties of a system. They are boundary phenomena and path functions. Their magnitudes depend on the initial and final states as well as the path of a process.
- The boundary work and specific boundary work in a process can be expressed graphically as the area under the process curve in the $P-\mathbb{V}$ and $P-v$ diagrams, respectively; therefore, these diagrams are often used when evaluating the boundary work in a process.

4.6 Key equations

Constant-volume specific heat	$C_v = \left(\frac{\partial u}{\partial T}\right)_v$
Change in specific internal energy for **all fluids**	$\Delta u = u_2 - u_1$
Change in specific internal energy for **ideal gases**	$\Delta u = C_v\left(T_2 - T_1\right)$
Specific heat transfer	$q = \frac{Q}{m}$
Boundary work	${}_1W_2 = \int_1^2 Pd\mathbb{V}$
Specific boundary work	${}_1w_2 = \int_1^2 Pdv$
Spring force	$F = Kx$
Spring work	$W_{spring} = \int_1^2 Fdx = \frac{1}{2}K\left(x_2^2 - x_1^2\right)$
The first law of thermodynamics for **closed systems**	$\Delta U = U_2 - U_1 = {}_1Q_2 - {}_1W_2$, assuming $\Delta KE = \Delta PE = 0$

Equations for polytropic Processes

Process function	$Pv^n = \text{constant}$
Boundary work for **real gases**	If $n \neq 1$, $${}_1W_2 = \frac{P_2\mathbb{V}_2 - P_1\mathbb{V}_1}{1-n}$$ If $n = 1$, {}_{1}W_{2}={P}_\mathbf{1}\mathbb{V}_\mathbf{1}{ln}{\displaystyle\frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}={P}_\mathbf{2}\mathbb{V}_\mathbf{2}{ln}{\displaystyle\frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}\\ {}_{1}W_{2}={P}_\mathbf{1}\mathbb{V}_\mathbf{1}{ln}{\displaystyle\frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}={P}_\mathbf{2}\mathbb{V}_\mathbf{2}{ln}{\displaystyle\frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}\\
Specific boundary work for **real gases**	If $n \neq 1$ $${}_1w_2 = \frac{P_2v_2 - P_1v_1}{1-n}$$ If $n = 1$, $${}_1w_2 = P_1v_1 ln\frac{v_2}{v_1} = P_2v_2 ln\frac{v_2}{v_1}$$ $${}_1w_2 = P_1v_1 ln\frac{P_1}{P_2} = P_2v_2 ln\frac{P_1}{P_2}$$

Boundary work for **ideal gases**	If $n \neq 1$ $${}_1W_2 = \frac{P_2\mathbb{V}_2 - P_1\mathbb{V}_1}{1-n}$$ If $n = 1,$ {}_{1}W_{2}={P}_\mathbf{1}\mathbb{V}_\mathbf{1}{ln}{\displaystyle\ frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}={P}_\mathbf{2}\mathbb{V}_\mathbf{2}{ln}{\displaystyle\ frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}\\ {}_{1}W_{2}={P}_\mathbf{1}\mathbb{V}_\mathbf{1}{ln}{\displaystyle\ frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}={P}_\mathbf{2}\mathbb{V}_\mathbf{2}{ln}{\displaystyle\ frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}\\ \\ {}_{1}W_{2}={{mRT}}{ln}{\displaystyle\ frac{\mathbb{V}_\mathbf{2}}{\mathbb{V}_\mathbf{1}}}={{mRT}}{ln}{\displaystyle\ frac{{P}_\mathbf{1}}{{P}_\mathbf{2}}}\\ (T in Kelvin)
Specific boundary work for **ideal gases**	If $n \neq 1$ $${}_1w_2 = \frac{P_2v_2 - P_1v_1}{1-n}$$ If $n = 1,$ $${}_1w_2 = P_1v_1ln\frac{v_2}{v_1} = P_2v_2ln\frac{v_2}{v_1}$$ $${}_1w_2 = P_1v_1ln\frac{P_1}{P_2} = P_2v_2ln\frac{P_1}{P_2}$$ $${}_1w_2 = RTln\frac{v_2}{v_1} = RTln\frac{P_1}{P_2}$$ (T in Kelvin)

5. THE FIRST LAW OF THERMODYNAMICS FOR A CONTROL VOLUME

5.0 Chapter introduction and learning objectives

Many thermal devices, such as compressors, turbines, and heat exchangers can be modelled as open systems. A common feature of these devices is that they all have inlets and outlets, through which a working fluid transfers both mass and energy into and out of the devices. This chapter extends the concept of energy conservation to open systems with a focus on steady-state, steady flows (SSSF). Examples are given to illustrate the applications of the first law of thermodynamics in typical SSSF devices such as turbines, compressors, heat exchangers, expansion valves, and mixing chambers.

Learning Objectives

After completing the chapter, you should be able to

- Determine the enthalpy of real substances by using thermodynamic tables
- Calculate the enthalpy of ideal gases, solids, and liquids by using constant-pressure specific heat
- Calculate mass flow rate and volume flow rate
- Explain the differences between steady and transient flows
- Explain the physical meanings of mass and energy conservation

- Apply the conservation equations of mass and energy to steady-state, steady flow devices

5.1 Enthalpy

Enthalpy is an important thermodynamic property for the analysis of energy conservation in open systems. It combines the internal energy and flow work associated with the flowing fluid (see Section 2.2.5 for details). The following sections explain how to determine the **specific enthalpy** at a given state.

5.1.1 Using thermodynamic tables to determine specific enthalpy h

As described in Chapter 2, thermodynamic tables can be used to determine thermodynamic properties, such as pressure, temperature, specific volume, specific internal energy, specific enthalpy, and specific entropy of a pure substance at a given condition. After the specific enthalpy is found, the enthalpy can then be calculated by using the following equation:

$$H = mh$$

where

H: enthalpy, in kJ or J
h: specific enthalpy, in kJ/kg or J/kg
m: mass of the system, in kg

Example 1

Find the missing properties of R134a and ammonia at the given conditions.

	Substance	**T, °C**	**P, kPa**	***h*, kJ/kg**	***x***	**Phase**
1	R134a	20		380		
2	Ammonia	-20	200			

Solution

1. R134a at T = 20°C has a specific enthalpy of h = 380 kJ/kg

From Appendix C, Table C1, at T = 20°C, h_f = 227.47 kJ/kg, h_g = 409.75 kJ/kg. Since $h_f < h < h_g$, R134a at this state is a two-phase mixture of saturated liquid and saturated vapour with a pressure of $P = P_{sat}$ = 0.57171 MPa = 571.71 kPa and a quality of

$$x = \frac{h - h_f}{h_g - h_f} = \frac{380 - 227.47}{409.75 - 227.47} = 0.83679$$

2. Ammonia at a temperature of T = -20°C and a pressure of P = 200 kPa

From Appendix B, Table B1, at T = -20°C, P_{sat} = 0.19008 MPa = 190.08 kPa. Since $P > P_{sat}$, ammonia at this state is a compressed liquid with $h \approx h_f$ = 251.71 kJ/kg.

In summary,

	Substance	T, °C	P, kPa	h, kJ/kg	x	Phase
1	R134a	20	571.71	380	0.83679	Two-phase mixture
2	Ammonia	-20	200	251.71	n.a.	Compressed liquid

5.1.2 Constant-pressure specific heat

Constant-pressure specific heat is defined as the energy required to raise the temperature of a unit mass (i.e., 1 kg) of a substance by 1 degree (i.e., 1 ^{o}C, or 1 K) in an isobaric process. Mathematically, it is expressed as

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p$$

where

C_p: constant-pressure specific heat, in kJ/kgK
h: specific enthalpy, in kJ/kg
T: temperature, in K or ^{o}C

The constant-pressure specific heat of selected substances can be found in Appendix G. For example, the constant-pressure specific heat of air at 300 K is 1.005 kJ/kgK, see Table G1. Let us consider one kilogram of air originally at 300 K in a piston-cylinder device. It will require 1.005 kJ of heat for the air temperature to increase from 300 K to 301 K if the piston-cylinder device is heated to allow the air to expand in an isobaric process.

It is important to note that both specific heats, C_v and C_p, are properties of a substance. Although they are typically measured in isochoric and isobaric processes, respectively, their applications are NOT limited to isochoric or isobaric processes. For ideal gases, C_v and C_p can be used to determine Δu and Δh, respectively, in **ANY** processes.

5.1.3 Using C_p to calculate Δh for ideal gases

The specific enthalpy of an ideal gas is a function of temperature only, $h = f(T)$; therefore,

$$C_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{dh}{dT}\right)_p = f(T)$$

The change of specific enthalpy of an ideal gas within a small temperature range can be calculated as

$$\Delta h = h_2 - h_1 = C_p(T_2 - T_1)$$

where

C_p: constant-pressure specific heat in a small temperature range, in kJ/kgK

h: specific enthalpy, in kJ/kg

T: temperature, in K or ^{o}C

Subscripts 1 and 2 represent states 1 and 2 in a process, respectively.

The above formula is a simple, approximate method to estimate the change of specific enthalpy of an ideal gas due to temperature variations. It is reasonably accurate and may be used when the thermodynamic tables of an ideal gas are not available. Table G1 lists the constant-pressure specific heat of selected substances at

300 K. Strictly speaking, since $C_p = f(T)$, C_p at a different temperature should be calculated according to that specific temperature. However, Table G1 is often used in approximate calculations as long as the temperature variations of the ideal gas remain in a small range.

The specific heat ratio, k, of an ideal gas is defined as the ratio of C_p to C_v.

$$k = \frac{C_p}{C_v}$$

The following equations relate C_v, C_p, k, and R. Detailed derivations are omitted here.

$$C_p = C_v + R \qquad C_p = \frac{kR}{k-1}$$

$$C_v = \frac{R}{k-1}$$

5.1.4 Using C_p to calculate Δh for solids and liquids

Liquids and solids are generally treated as incompressible substances because their volumes do not change with pressure or temperature significantly. For liquids and solids, the difference between the constant-volume specific heat and the constant-pressure specific heat is typically negligible; therefore,

$$C_p \approx C_v$$

$$\Delta h \approx \Delta u \approx C_p(T_2 - T_1)$$

Example 2

Consider a piston-cylinder device containing 2 kg of oxygen at 300 K, 200 kPa. The piston-cylinder device is made of 0.5 kg of aluminum. How much heat is required for the oxygen to reach a temperature of 600 K in an isobaric process? Assume the oxygen and the piston-cylinder are always in thermal equilibrium during the isobaric process. The oxygen can be treated as an ideal gas in this heating process.

Solution

First, set up a closed system as shown outlined in blue, which consists of the piston-cylinder and O_2.

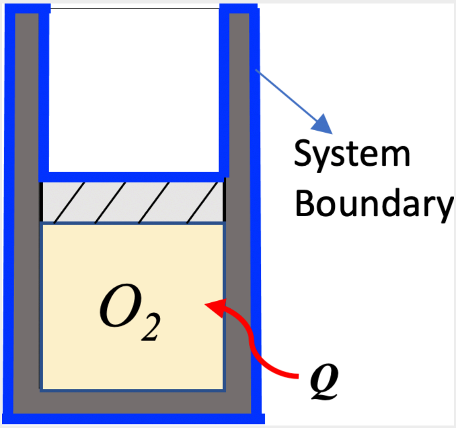

***Figure 5.1.e1** Piston-cylinder device containing oxygen*

Second, apply the first law of thermodynamics to the closed system. Note that, in the heating process, the temperatures and internal energies of both O_2 and the piston-cylinder increase.

$$\because \Delta U_{tot} = Q_{tot} - W_{O_2} \quad \text{and}$$
$$\Delta U_{tot} = \Delta U_{O_2} + \Delta U_{al}$$
$$\therefore Q_{tot} = \Delta U_{tot} + W_{O_2} = \Delta U_{al} + (\Delta U_{O_2} + W_{O_2})$$

Third, calculate the change in internal energy of the piston-cylinder. From Table G3, the constant-pressure specific heat for aluminum is $C_{p,al}$ = 0.897 kJ/kgK.

$$\begin{aligned}\Delta U_{al} &= m_{al}\Delta u_{al} = m_{al}\Delta h_{al} \\ &= m_{al}C_{p,al}(T_2 - T_1) \\ &= 0.5 \times 0.897 \times (600 - 300) = 134.55 \text{ kJ}\end{aligned}$$

Fourth, analyze the boundary work done by O_2 and the change in internal energy of O_2 in this isobaric process, where $P_1 = P_2 = P$.

\begin{align*} \because \; \; W_{O_{2}} &= P(\mathbb{V}_2 - \mathbb{V}_1) \; \; \; \; \Delta U_{O_{2}} = U_2 - U_1 \\ H &=U+P\mathbb{V} \end{align*}

$$\begin{aligned} \therefore \; \Delta U_{O_2} + W_{O_2} &= (U_2 - U_1) + P(\mathbb{V}_2 - \mathbb{V}_1) \\ &= (U_2 + P_2\mathbb{V}_2) - (U_1 + P_1\mathbb{V}_1) \\ &= H_2 - H_1 \end{aligned}$$

$\therefore \Delta U_{O_2} + W_{O_2} = \Delta H_{O_2}$ (for isobaric process)

From Table G1, the constant-pressure specific heat for oxygen is C_{p,O_2} = 0.918 kJ/kgK.

$$\begin{aligned} \therefore \Delta H_{O_2} &= m_{O_2} C_{p,O_2} (T_2 - T_1) \\ &= 2 \times 0.918 \times (600 - 300) = 550.8 \text{ kJ} \end{aligned}$$

Last, calculate the total heat transfer in this process.

$$\begin{aligned} Q_{tot} &= \Delta U_{al} + \Delta H_{O_2} \\ &= 134.55 + 550.8 = 685.35 \text{ kJ} \end{aligned}$$

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://pressbooks.bccampus.ca/thermo1/?p=1658#h5p-36

5.2 Mass and energy conservation equations in a control volume

5.2.1 Steady flow and transient flow

An open system allows both mass and energy to transfer across its boundary. Many thermal devices, such as compressors, turbines, and heat exchangers have inlets and outlets and can be modelled as open systems. Figure 5.2.1 is a schematic drawing of an open system with one inlet and one outlet. A **control volume** (C.V.), shown as the dash-lined rectangle in Figure 5.2.1, is selected for the analysis of the change of properties in the open system. A working fluid flows into and out of the control volume through the inlet and outlet. In addition, energy transfer occurs between the system and its surroundings in the form of heat and work. As a result, both mass and energy within the control volume may change over time.

If neither the mass nor the energy within the control volume change with respect to time, i.e., $\dfrac{dm_{CV}}{dt}=0$ and $\dfrac{dE_{CV}}{dt}=0$, the flow is called a **steady flow**. In a steady flow, the thermodynamic properties within a control volume do not change with respect to time; but they do not need to remain uniform everywhere within the control volume. The properties may vary from point to point, but at any given point, they must remain the same during the entire process. Many devices may be treated as steady flow devices after they have been in operation for a certain period of time under the same operating condition.

In a **transient flow**, the mass and energy within a control volume change with respect to time, i.e., $\frac{dm_{CV}}{dt} \neq 0$ and $\frac{dE_{CV}}{dt} \neq 0$. Consequently, other thermodynamic properties may also change with respect to time. Flow through a device during its start-up and shut-down periods is usually treated as a transient flow.

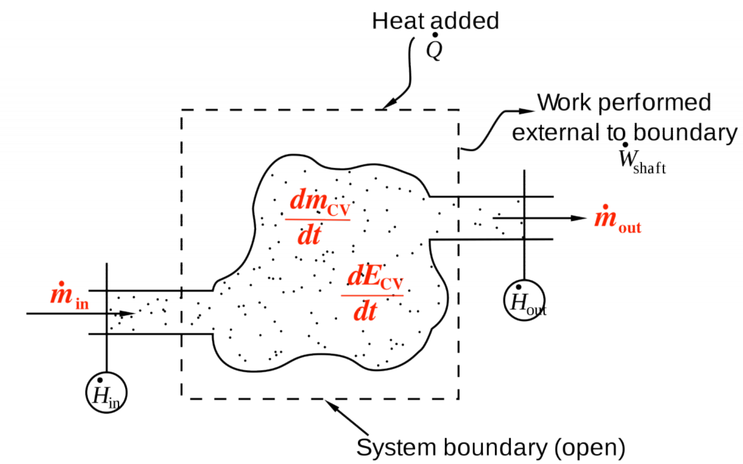

***Figure* 5.2.1** *Flow through a control volume showing mass and energy transfers*

5.2.2 Mass conservation equation

The mass flow rate and volume flow rate are defined as the mass and volume of a fluid flowing through an inlet or outlet per unit time, respectively. They are expressed as

$$\dot{\mathbb{V}} = \frac{d\mathbb{V}}{dt} = \dot{m}v = V_{avg,\ n}A$$

$$\dot{m} = \frac{dm}{dt} = \rho\dot{\mathbb{V}} = \rho\, V_{avg,\ n}A$$

where

A: cross-sectional area of the inlet or outlet, in m^2
m: mass of the flow, in kg
$\dot{m}$: mass flow rate, in kg/s
$\mathbb{V}$: volume of the flow, in m^3
$\dot{\mathbb{V}}$: volume flow rate, in m^3/s
$V_{avg,n}$: average velocity normal to the cross-sectional area A, in m/s
ρ: density of the working fluid, in kg/m^3
v: specific volume of the working fluid, in m^3/kg

The conservation of mass, also called the continuity equation, states that ***mass cannot be created or destroyed. The time rate of change of massin a control volume at a certain time equalsthe netmass flow rate intothe control volume at that time.***

$$\Delta \text{mass} = +\text{in} - \text{out}$$

$$\frac{dm_{CV}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e$$

Since $\dfrac{dm_{CV}}{dt} = 0$ for steady flows, the mass conservation equation for steady flows is, therefore, written as

$$\sum \dot{m}_i = \sum \dot{m}_e$$

where $\dot{m}_i$ and $\dot{m}_e$ represent the mass flow rates through the inlets and outlets of a control volume, respectively.

5.2.3 Energy conservation equation

The exchange of energy between a control volume and its surroundings is achieved via three mechanisms: (1) heat transfer, (2) work, and (3) mass transfer. The conservation of energy in a control volume states that ***the time rate of change of energyin a control volume at a certain time equalsthe netrate of energy transfer intothe control volume at that time via the three mechanisms:heat transfer, work, and mass transfer.***

$$\Delta\text{energy} = +\text{in} - \text{out}$$

$$\frac{dE_{CV}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i(h_i + \frac{1}{2}V_i^2 + gz_i) - \sum \dot{m}_e(h_e + \frac{1}{2}V_e^2 + gz_e)$$

Since $\dfrac{dE_{CV}}{dt} = 0$ for steady flows, the energy conservation equation for steady flows is, therefore, written as

$$\dot{Q}_{cv} + \sum \dot{m}_i \left(h_i + \frac{1}{2}V_i^2 + gz_i\right) = \dot{W}_{cv} + \sum \dot{m}_e \left(h_e + \frac{1}{2}V_e^2 + gz_e\right)$$

where

h: specific enthalpy, in J/kg
$\dot{m}$: mass flow rate, in kg/s
$\dot{Q}_{cv}$: rate of heat transfer, in W, across the boundary of a control volume

V: average velocity of the working fluid through an inlet or outlet, in m/s

$\dot{W}_{cv}$: work, in W, across the boundary of a control volume

z: elevation of an inlet or outlet, in m

Subscripts, i and e, refer to the inlet and outlet of the control volume, respectively.

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
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5.3 Applications of the mass and energy conservation equations in steady flow devices

5.3.1 Turbines and compressors

Turbines and compressors are common energy conversion devices, in which the energy of a working fluid is converted to mechanical energy or vice versa. A typical turbine consists of rotor blades attached to a shaft, which is free to rotate. Figure 5.3.1 shows the main components of a hydraulic turbine. During the operation of a turbine, a working fluid (e.g., steam, gas, wind or water) flows continuously over a row of highly curved blades, creating a torque on the blades. In this process, the energy of the working fluid is converted to the mechanical energy of the rotating shaft. Consequently, the pressure and temperature of the fluid gradually decrease in the turbine. Figure 5.3.2 illustrates the energy conversion in a wind turbine. The wind energy is first converted to the mechanical energy of the shaft and gearbox, then converted to the electrical energy through the generator.

Compressors work in the opposite way to turbines. Compressors consume shaft power in order to increase the pressure of a working fluid, typically a gas. In a dynamic compressor, such as a centrifugal compressor, the increase of the gas pressure is achieved by forcing the gas into narrow passages formed by adjacent blades, which rotate around a shaft. Figure 5.3.3 shows a centrifugal compressor driven by an electric motor. In a reciprocating compressor, the increase of the gas pressure is achieved by the decrease of the gas volume as the piston, driven by a crank-shaft, compresses the gas. Figure 5.3.4 animates the expansion and compression processes in a reciprocating compressor. In both dynamic and reciprocating compressors, the mechanical energy of the rotating shaft or crank-shaft is converted to the energy stored in the fluid. As a result, the pressure of the fluid increases in the compressor.

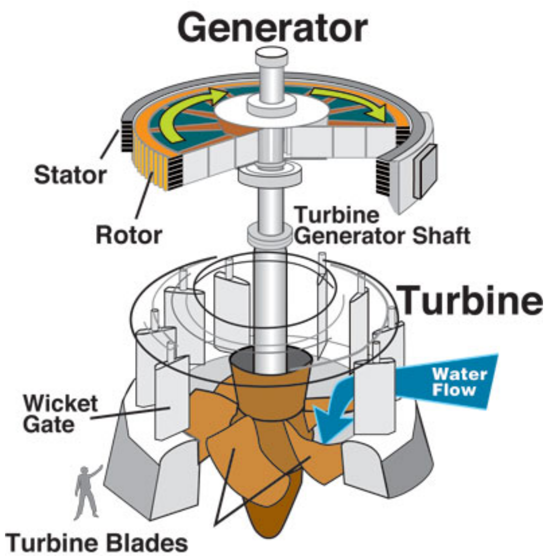

***Figure* 5.3.1** *Hydraulic turbine*

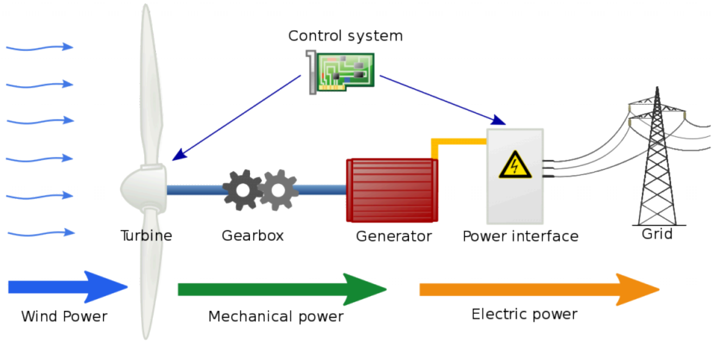

Figure 5.3.2 *Wind turbine schematic showing the energy conversion*

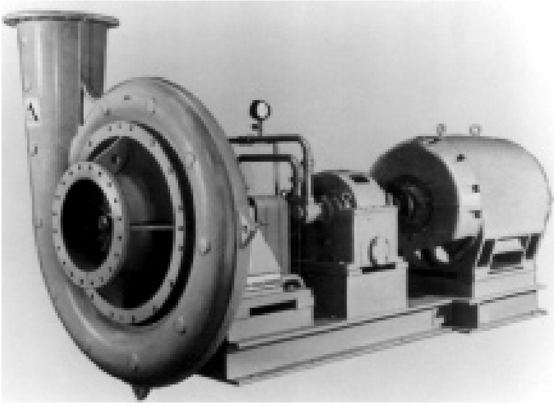

Figure 5.3.3 *Centrifugal compressor*

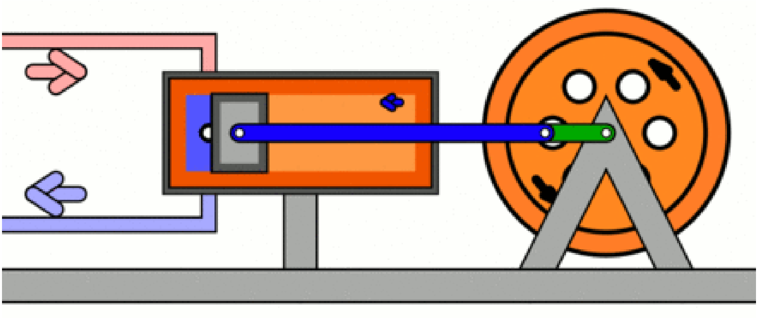

Figure 5.3.4 *Reciprocating compressor: animation of the compression and expansion processes*

Figure 5.3.5 illustrates the flow of energy through a turbine and a compressor. A typical turbine or compressor has one inlet and one outlet. We may apply the following assumptions to typical turbines and compressors:

- The flow through a turbine or a compressor is steady.
- The turbine or compressor is well-insulated: $\dot{Q}_{cv} = 0$
- The changes in potential and kinetic energies are negligible compared to the change in enthalpy: $\Delta PE = 0$, $\Delta KE = 0$

With these assumptions, the mass and energy conservation equations can be simplified as follows,

Turbine:

$$\dot{m}_i = \dot{m}_e = \dot{m}$$
$$\dot{W}_{shaft} = \dot{m}(h_i - h_e)$$

Compressor:

$$\dot{m}_i = \dot{m}_e = \dot{m}$$
$$\dot{W}_{shaft} = \dot{m}(h_e - h_i)$$

where

h: specific enthalpy of the working fluid, in kJ/kg

$\dot{m}$: mass flow rate of the working fluid through a turbine or compressor, in kg/m^3

$\dot{W}_{shaft}$: shaft power of the turbine or compressor, in kW

Subscripts, i and e, represent the inlet and exit of the turbine or compressor, respectively.

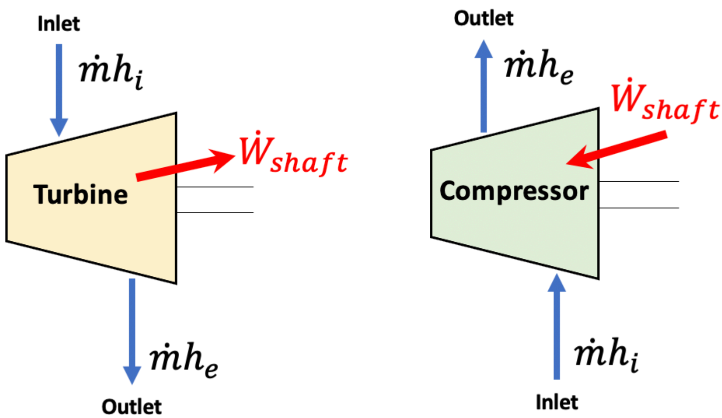

***Figure* 5.3.5** *Flow of energy through a turbine and a compressor*

Example 1

The inlet and outlet conditions of an air compressor are P_1=100 kPa, T_1=20°C, and P_2=300 kPa, respectively. The mass flow rate of air through this compressor is 0.015 kg/s. How much power input from the shaft is required to drive this compressor? Apply the following assumptions in your analysis: (1) the flow is steady with negligible changes in the kinetic and potential energies; (2) the compressor is well-insulated; (3) air is an ideal gas; and (4) the process is polytropic with *n*=1.35.

Solution:

Analysis: the shaft power of the compressor is proportional to the change in enthalpy of air in the compressor. Because air is treated as an ideal gas, the change in enthalpy can be calculated in terms of the change in temperature. Therefore, we need to find the temperature at the outlet of the compressor first.

Air is an ideal gas and the process is polytropic; therefore,

$$Pv = RT$$
$$Pv^n = \text{constant}$$

Combine the above two equations and eliminate v,

$$P^{n-1} = \frac{R^n}{\text{constant}} \times T^n$$

Apply the above equation to the process between the inlet condition, state 1, and the outlet condition, state 2. Note that R is the gas constant of air,

$$\because \left(\frac{P_2}{P_1}\right)^{n-1} = \left(\frac{T_2}{T_1}\right)^n$$

$$\therefore T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\dfrac{n-1}{n}}$$

$$= (273.15 + 20)\left(\frac{300}{100}\right)^{\dfrac{1.35-1}{1.35}}$$

$$= 389.75 \text{ K} = 116.6\ ^{\text{o}}\text{C}$$

From Table G1, C_p = 1.005 kJ/kgK for air. Apply the energy conservation equation to the compressor considering the first two assumptions in the question descriptions.

$$\begin{aligned}\dot{W}_{shaft} &= \dot{m}(h_2 - h_1) \\ &= \dot{m}C_p(T_2 - T_1) \\ &= 0.015 \times 1.005 \times (116.6 - 20) = 1.456 \text{ kW}\end{aligned}$$

The compressor consumes 1.456 kW of power when operating under the given conditions.

5.3.2 Throttling valves

Throttling valves are also called thermal expansion valves. They are used in vapour compression refrigeration systems to regulate the pressure in the evaporator as well as the superheat of the refrigerant flowing out of the evaporator. Throttling valves may be constructed as a porous plug, capillary tubes, or an adjustable valve such as an orifice, ball, or poppet valve. By restricting the refrigerant flow through the valve, a considerable pressure drop can be achieved. As the pressure of the refrigerant decreases, its corresponding saturation temperature decreases accordingly.

Therefore, a phase change often occurs as the refrigerant passes through a throttling valve. Figure 5.3.6 shows a thermal expansion valve in a vapour compression refrigeration system. The following assumptions are commonly used for the analysis of the mass and energy conservation in a typical throttling valve.

- The flow through a throttling valve is steady.
- The flow through a throttling valve is fast enough so that the heat transfer between the refrigerant and its surroundings is negligible. Therefore, the flow is commonly treated as adiabatic: $\dot{Q}_{cv} = 0$
- No work exchange occurs between the refrigerant and its surroundings: $\dot{W}_{cv} = 0$
- The changes in potential and kinetic energies are negligible compared to the change in enthalpy: $\Delta PE = 0$, $\Delta KE = 0$

Based on these assumptions, the mass and energy conservation equations in throttling valves can be written as

$$\dot{m}_i = \dot{m}_e$$
$$h_i = h_e$$

where

h: specific enthalpy of the refrigerant, in kJ/kg

$\dot{m}$: mass flow rate of the refrigerant through a throttling valve, in kg/m^3

Subscripts, i and e, represent the inlet and exit of the throttling valve, respectively.

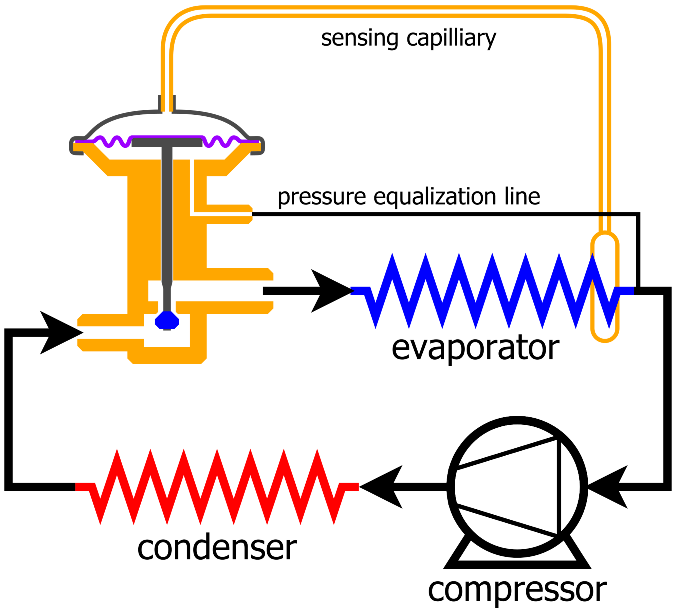

Figure 5.3.6 *Thermal expansion valve in a vapour compression refrigeration system*

Example 2

A simplified transcritical CO_2 refrigeration cycle consists of four processes: compression (1⟶2), gas cooling (2⟶3), expansion (3⟶4), and evaporation (4⟶1), as shown in the $P-h$ diagram, Figure 5.3.e1. The CO_2 gas enters the expansion valve at 10 MPa, 20^{o}C (state 3) and is throttling to a pressure of 3 MPa (state 4). Determine the quality and temperature of CO_2 at state 4.

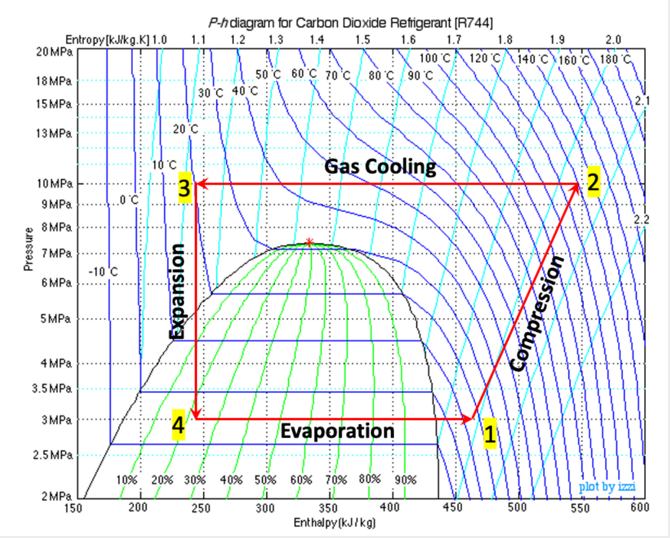

Figure 5.3.e1 *P-h diagram for* CO_2

<u>*Solution:*</u>

The specific enthalpy remains constant in the expansion process; therefore $h_3 = h_4$.

At state 3, $P_3 = 10$ MPa, $T_3 = 20^oC$. From Table D2, $h_3 = 242.70$ kJ/kg

At state 4, $P_4 = 3$ MPa, $h_4 = h_3 = 242.70$ kJ/kg

From Table D1,

At T = -6^{o}C , P = 2.96316 MPa, h_f = 185.71 kJ/kg, h_g = 433.79 kJ/kg

At T = -4^{o}C, P = 3.13027 MPa, h_f = 190.40 kJ/kg, h_g = 432.95 kJ/kg

State 4 must be a two-phase mixture and its saturation temperature must be -6^{o}C < T_4 < -4^{o}C. Use linear interpolation to determine the saturation temperature, T_4, the specific enthalpy of the saturated liquid, $h_{f,4}$, and the specific enthalpy of the saturated vapour, $h_{g,4}$, of CO_2 at P_4 = 3 MPa.

$$\because \frac{T_4-(-6)}{(-4)-(-6)}=\frac{3-2.96316}{3.13027-2.96316}$$

$$\therefore T_4=-5.56\,^{\circ}\mathrm{C}$$

$$\because \frac{h_{f,4}-185.71}{190.40-185.71}=\frac{h_{g,4}-433.79}{432.95-433.79}=\frac{3-2.96316}{3.13027-2.96316}$$

$$\therefore h_{f,4}=186.74\ \mathrm{kJ/kg}$$

$$h_{g,4}=433.60\ \mathrm{kJ/kg}$$

The quality of the two phase mixture at state 4 is

$$x_4=\frac{h_4-h_{f,4}}{h_{g,4}-h_{f,4}}=\frac{242.70-186.74}{433.60-186.74}=0.2267$$

5.3.3 Nozzles and diffusers

Nozzles and diffusers are devices used to accelerate or decelerate flow by gradually changing their cross-sectional areas. They are widely used in many engineering systems, such as piping systems, HVAC (heating, ventilating, and air conditioning) systems, and steam and gas engines. Figure 5.3.7 illustrates a turbojet engine, which consists of a centrifugal compressor, a combustion chamber, a turbine, and a nozzle section. The gas leaving the turbine is accelerated in the nozzle section, creating a high velocity jet, and thus a high thrust.

The flow in nozzles and diffusers can be subsonic or supersonic. In this book, we will only discuss subsonic nozzles and subsonic diffusers; thereafter, the terms "nozzle" and "diffuser" refer to subsonic nozzle and subsonic diffuser, respectively.

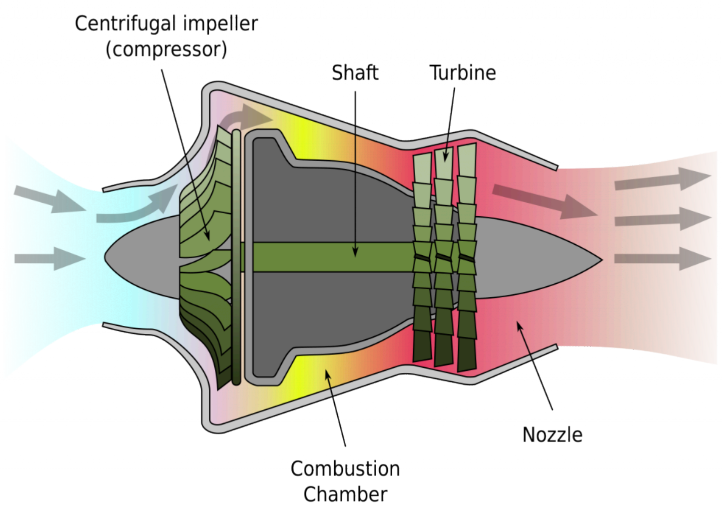

***Figure* 5.3.7** *Schematic of turbojet engine consisting of a centrifugal compressor, a combustion chamber, a turbine, and a nozzle section*

In a nozzle, the flow velocity increases and the pressure decreases as the cross-sectional area of the nozzle decreases. On the contrary, in a diffuser, the flow velocity decreases and the pressure increases as the cross-sectional area of the diffuser increases. Figure 5.3.8 shows a schematic of a nozzle and a diffuser. The following assumptions are commonly made when analyzing the mass and energy conservation in a typical nozzle or diffuser.

- The flow through a nozzle or diffuser is steady.
- The flow through a nozzle or diffuser is fast enough so that the heat transfer between the fluid and its surroundings is negligible. Therefore, the flow is commonly treated as adiabatic: $\dot{Q}_{cv} = 0$
- No work exchange occurs between the fluid and its surroundings: $\dot{W}_{cv} = 0$
- The change in potential energy is negligible compared to the changes in enthalpy and in kinetic energy: $\Delta PE = 0$

With these assumptions, the mass and energy conservation equations in nozzles and diffusers can be written as

$$\dot{m}_i = \dot{m}_e$$

$$h_i + \frac{1}{2}V_i^2 = h_e + \frac{1}{2}V_e^2$$

where

h: specific enthalpy of the fluid, in J/kg. Note: J/kg = m^2/s^2.

$\dot{m}$: mass flow rate of the fluid through a nozzle or diffuser, in kg/m^3

V: velocity of the fluid, in m/s

Subscripts, i and e, represent the inlet and exit of the nozzle or diffuser, respectively.

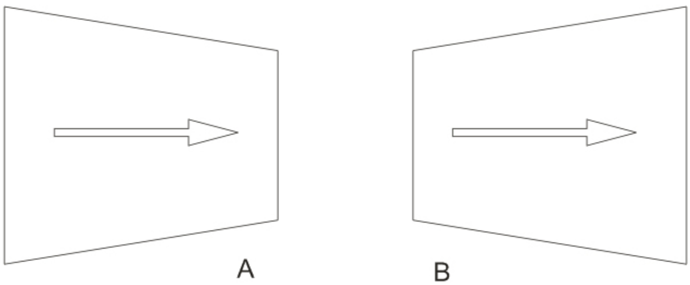

Figure 5.3.8 *Nozzle* (A) *and diffuser* (B)

Example 3

The inlet and outlet diameters of a hydrogen nozzle are 44 mm and 20 mm, respectively. Hydrogen enters the nozzle at -30^{o}C, 5 bar, and 2 g/s, and exits the nozzle at 2 bar. Assuming the process is adiabatic and hydrogen is an ideal gas, what is the temperature and velocity of hydrogen at the nozzle outlet?

Solution:

Analysis: the energy conservation equation provides the relationship between the temperature, specific enthalpy, and velocity of the hydrogen. We will apply the mass and energy conservation equations to find the velocity and temperature at the nozzle outlet.

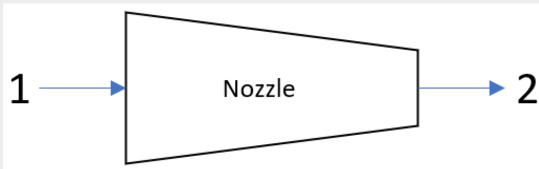

***Figure 5.3.e2** Flow through a nozzle*

From Table G1: R = 4.124 kJ/kgK and C_p = 14.307 kJ/kgK for hydrogen.

Apply the mass flow rate and ideal gas law to the inlet of the nozzle:

$$\because \dot{m} = \rho_1 V_1 A_1 \quad \text{and} \quad P_1 = \rho_1 R T_1$$

$$\therefore V_1 = \frac{\dot{m}_1 R T_1}{P_1 A_1}$$

$$= \frac{0.002 \times 4.124 \times (273.15 - 30)}{500 \times \pi \times 0.022^2} = 2.638 \text{ m/s}$$

Apply the mass conservation equation and the ideal gas law to the inlet and outlet of the nozzle:

$$\because \dot{m}_1 = \dot{m}_2$$

$$\therefore \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

and $P_1 = \rho_1 R T_1 \ , \ P_2 = \rho_2 R T_2$

$$\therefore V_2 = V_1 \frac{A_1 P_1 T_2}{A_2 P_2 T_1} = V_1 \frac{P_1 T_2}{P_2 T_1} \times \left(\frac{D_1}{D_2}\right)^2$$
$$= 2.638 \times \frac{5 \times T_2}{2 \times T_1} \times \left(\frac{44}{20}\right)^2 = 31.92 \frac{T_2}{T_1}$$

Apply the energy conservation equation to the inlet and outlet of the nozzle:

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2$$

and $\Delta h = C_p(T_2 - T_1)$ for ideal gases

therefore,

$$C_p T_1 + \frac{1}{2}V_1^2 = C_p T_2 + \frac{1}{2}V_2^2$$

Substitute $V_2 = 31.92\frac{T_2}{T_1}$ and rearrange

$$1 + \frac{V_1^2}{2C_p T_1} = \frac{T_2}{T_1} + \frac{1}{2C_p T_1}\left(31.92\frac{T_2}{T_1}\right)^2$$

Substitute V_1 = 2.638 m/s, T_1 =273.15 – 30 = 243.15 K, C_p = 14307 J/kgK into the above relation, and rearrange,

$$0.00014643\left(\frac{T_2}{T_1}\right)^2 + \frac{T_2}{T_1} - 1 = 0$$

$$\therefore \frac{T_2}{T_1} = 0.99985$$

$$\therefore T_2 = 0.99985 \times 243.15 = 243.11\ K = -30.04\ ^\circ\mathrm{C}$$

$$\therefore V_2 = 31.92\frac{T_2}{T_1} = 31.91 \text{ m/s}$$

In this nozzle, the velocity increases significantly as the pressure decreases. The temperature remains almost constant.

Important note:

- ***The temperatures in the calculations must be in Kelvin because the ideal gas law is used to derive other equations.***
- ***The constant-pressure specific heat must be in J/kgK to ensure the consistency of the units.***

5.3.4 Ducts and pipes

Ducts and pipes are used to transport liquids and gases. Their configurations vary largely with applications; therefore, it is impractical to apply general assumptions to all ducts and pipes. Ducts and pipes may be analyzed specifically with the mass and energy conservation equations in Section 5.2.

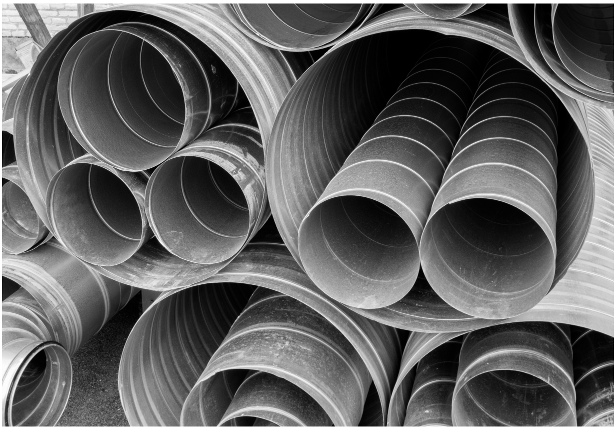

***Figure 5.3.9** HVAC pipes*

Example 4

Consider a vertical pipe section exposed to the ambient. The pipe has a constant diameter of 100 mm. Its inlet is 5 m above its outlet. Water at 10^{o}C and 100 kPa enters the pipe at a velocity of 2 m/s, and exits at 12^{o}C and 100 kPa. Assuming steady flow, what is the rate of heat transfer absorbed by the water in this pipe section?

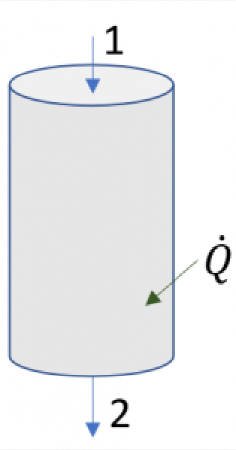

***Figure 5.3.e3** Flow through a vertical pipe section*

<u>*Solution:*</u>

From Table A1, water is a compressed liquid at both inlet and outlet conditions, therefore,

$$v_1 \approx v_2 = v_f = 0.001 \text{ m}^3/\text{kg}$$

$$\rho_1 \approx \rho_2 = \frac{1}{v_f} = 1000 \text{ kg/m}^3$$

Mass flow rate is

$$\dot{m} = \rho_1 V_1 A_1 = 1000 \times 2 \times \pi \times 0.05^2 = 15.7 \text{ kg/s}$$

Apply the energy conservation equation to the pipe section

$$\dot{Q} + \dot{m}\left(h_1 + \frac{1}{2}V_1^2 + gz_1\right) = \dot{m}\left(h_2 + \frac{1}{2}V_2^2 + gz_2\right)$$

Since the diameter of the pipe section and the density of water remain constant in the pipe, the inlet and outlet velocity must be the same based on the mass conservation equation. From Table G2: C_p =4.181 kJ/kgK = 4181 J/kgK for water.

$$\because V_1 = V_2$$

$$\begin{aligned}\therefore \dot{Q} &= \dot{m}(h_2 - h_1 + gz_2 - gz_1)\\ &= \dot{m}Cp(T_2 - T_1) + \dot{m}g(z_2 - z_1)\\ &= 15.7 \times 4181 \times (12 - 10) + 15.7 \times 9.81 \times (-5)\\ &= 130580 \text{ W} = 130.6 \text{ kW}\end{aligned}$$

The water absorbs 130.6 kW of heat from the ambient.

Important note:

- ***The constant-pressure specific heat must be in J/kgK in the calculations to ensure the consistency of the units.***

5.3.5 Mixing chambers

Mixing chambers are devices used to mix multiple streams of a fluid to achieve a desirable temperature or phase in the mixed flow. Figure 5.3.10 is a schematic of a mixing chamber in HVAC systems, where the outdoor fresh air mixes with the indoor return air in order to achieve a desirable air temperature and a targeted fresh air to return air ratio. The common assumptions for analyzing the mass and energy conservation in a mixing chamber are listed below:

- The flow through a mixing chamber is steady.

- No work exchange occurs between the mixing chamber and its surroundings: $\dot{W}_{cv} = 0$
- The changes in potential and kinetic energies are negligible: $\Delta PE = 0$, $\Delta KE = 0$
- The pressure variation in a mixing chamber is usually negligible.

With these assumptions, the mass and energy conservation equations in mixing chambers can be written as

$$\sum \dot{m}_i = \sum \dot{m}_e$$
$$\dot{Q}_{cv} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

where

h: specific enthalpy of the fluid, in kJ/kg
$\dot{m}$: mass flow rate, in kg/m^3
$\dot{Q}_{cv}$: rate of heat transfer entering the mixing chamber, in kW

Subscripts, i and e, represent the inlets and exits of the mixing chamber, respectively.

***Figure* 5.3.10** *Schematic of a mixing chamber in HVAC (heating, ventilating and air conditioning) system*

Example 5

Consider a well-insulated mixing chamber in an HVAC system, see Figure 5.3.10. The cold outdoor air at -10^{o}C and 100 kPa mixes with the return air at 22^{o}C and 100 kPa. The mass flow rates of the outdoor air and return air are 0.5 kg/s and 3 kg/s, respectively. Assume the flow is steady and the air is an ideal gas under the given conditions. What is the air temperature leaving the mixing chamber?

Solution:

Apply both mass and energy conservation equations to the mixing chamber:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

\Delta h = C_p \Delta T (for ideal gases)

From the above three equations,

\begin{align*} T_3 &= \dfrac{\dot{m}_{1}T_2 + \dot{m}_{2}T_2}{\dot{m}_1 + \dot{m}_2} \\&= \dfrac{0.5 \times (-10) + 3 \times (22)}{0.5 + 3} = 17.43 \ {}^ \rm{o} \rm{C} \end{align*}

The mixed air leaves the mixing chamber at a temperature of 17.43^{o}C. In this calculation, the temperatures can be expressed either in ^{o}C or Kelvin as as long as the consistent units are used.

5.3.6 Heat exchangers

Heat exchangers are used to transfer heat between two flowing fluids. They can be found in many applications, such as HVAC systems, automotive systems, power generation plants, and chemical processing. Heat exchangers come in different types. For example, Figure 5.3.11 shows a tube-within-shell heat exchanger, where heat is transferred between the shell-side fluid and the tube-side fluid. The tube-within-shell heat exchanger is a typical indirect heat exchanger, in which the two fluids don't mix during the heat transfer process.

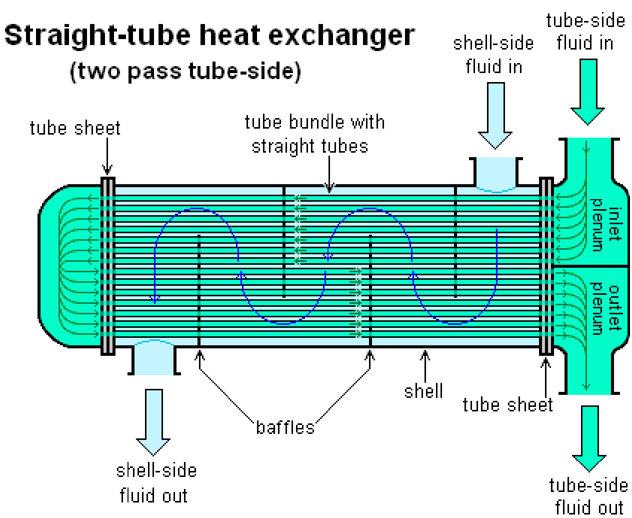

***Figure* 5.3.11** *Schematic of a straight tube heat exchanger*

Below are the common assumptions for analyzing the mass and energy conservation in an indirect heat exchanger:

- The flows through an indirect heat exchanger are steady.
- Each flow passage has only one inlet and one outlet.
- No work exchange occurs between the heat exchanger and its surroundings: $\dot{W}_{cv} = 0$
- The changes in potential and kinetic energies are negligible: $\Delta PE = 0$, $\Delta KE = 0$
- The pressure variation in a heat exchanger is usually negligible.

With these assumptions, the mass and energy conservation equations for an indirect heat exchanger can be written as

$$\dot{m}_{i,hot} = \dot{m}_{e,hot}$$
$$\dot{m}_{i,cold} = \dot{m}_{e,cold}$$
$$\dot{Q}_{cv} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$$

where

h: specific enthalpy of the fluid, in kJ/kg
$\dot{m}$: mass flow rate, in kg/m^3
$\dot{Q}_{cv}$: rate of heat transfer entering the heat exchanger, in kW

Subscripts, *i* and *e*, represent the inlet and exit of the heat exchanger, respectively.

Subscripts, *hot* and *cold*, represent the hot and cold streams, respectively, through the heat exchanger.

It is important to note that, although the mass and energy conservation equations derived for mixing chambers and heat exchangers look very similar, there exists a major difference between these two devices. In a mixing chamber, all flow streams mix and exit as a single stream; therefore, both mass and energy conservation equations are applied to the entire mixing chamber consisting of all flow streams. On the contrary, in an indirect heat exchanger, the cold and hot streams do NOT mix; therefore, the mass conservation equation must be applied to each flow stream individually. The energy conservation equation may be applied to the entire heat exchanger or each flow stream individually.

Example 6

Consider the evaporator of a vapour compression refrigeration system using R134a as the working fluid. R134a enters the evaporator at -10^oC with a quality of 0.1 and exits as a saturated vapour. Air enters the evaporator in a separate stream at 25^oC and exits at 5^oC. Assume (1) the evaporator is well-insulated; (2) the flow is steady; (3) the pressure remains constant in the evaporator; and (4) air is an ideal gas. What is the mass flow rate of air required in this system if the mass flow rate of R134a is 2 kg/s?

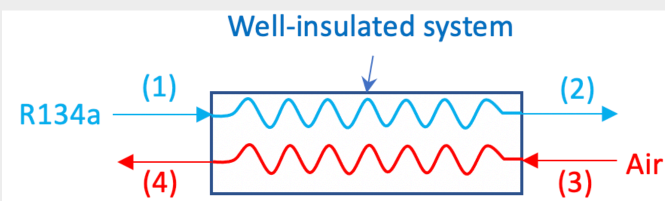

Figure 5.3.e4 *Heat exchanger*

Solution:

Apply the mass conservation equations to the R134a and air streams, separately.

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_{R134a}$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_{air}$$

Apply the energy conservation equation to the evaporator.

$$\because \dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$
$$\therefore \dot{m}_{air}(h_3 - h_4) = \dot{m}_{R134a}(h_2 - h_1)$$
$$\therefore \dot{m}_{air} = \dot{m}_{R134a}\frac{(h_2 - h_1)}{(h_3 - h_4)}$$

Find the specific enthalpies of R134a at states 1 and 2 using thermodynamic tables.

At state 1, T_1 = -10$^{\text{o}}$C, x= 0.1. From Table C1, h_f = 186.70 kJ/kg, h_g = 392.67 kJ/kg

$$\begin{aligned} h_1 &= h_f + x(h_g - h_f) \\ &= 186.70 + 0.1 \times (392.67 - 186.70) = 207.297 \text{ kJ/kg} \end{aligned}$$

At state 2, R134a is a saturated vapour at T_2 = -10$^{\text{o}}$C; therefore,

$$h_2 = h_g = 392.67 \text{ kJ/kg}$$

From Table G1, $C_{p,air}$ = 1.005 kJ/kgK. Therefore,

$$\begin{aligned} \dot{m}_{air} &= \dot{m}_{R134a}\frac{(h_2 - h_1)}{C_{p,air}(T_3 - T_4)} \\ &= 2 \times \frac{392.67 - 207.297}{1.005 \times (25 - 5)} = 18.445 \text{ kg/s} \end{aligned}$$

The system requires an air supply at a rate of 18.445 kg/s.

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
https://pressbooks.bccampus.ca/thermo1/?p=1662#h5p-38

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5.4 Chapter review

In this chapter, we have introduced the mass and energy conservation equations for control volumes. The general conservation equations for control volumes are explained in Section 5.2. When applying these conservation principles to a thermal device, it is important to evaluate the operation of the device, set up an appropriate control volume, and make appropriate assumptions. This is an important skill that students need to develop through practice.

Section 5.3 demonstrates how to simplify the conservation equations for common steady-state steady-flow devices such as turbines and compressors, throttling valves, pipes and ducts, nozzles and diffusers, mixing chambers, and heat exchangers. It is important to understand that, the equations in Section 5.3 are derived for these devices with certain assumptions. When applying these derived equations to a specific thermal device, a careful evaluation of the assumptions must be done to ensure the derived equations are appropriate for the given conditions.

5.5 Key equations

Constant-pressure and constant-volume specific heats

Constant-pressure specific heat	$C_p = \left(\dfrac{\partial h}{\partial T}\right)_p$
Constant-volume specific heat	$C_v = \left(\dfrac{\partial u}{\partial T}\right)_v$
Relations between C_p and C_v for **ideal gases**	$k = \dfrac{C_p}{C_v} \qquad C_p = C_v + R$ C_v=\displaystyle\frac{R}{k-1} \qquad C_p=\displaystyle\frac{kR}{k-1}

Specific enthalpy

Change in specific enthalpy	$\Delta h = h_2 - h_1$
Change in specific enthalpy for **ideal gases**	$\Delta h = h_2 - h_1 = C_p(T_2 - T_1)$ (assuming constant C_p in the temperature range)
Relation between Δh and Δu for **solids and liquids**	$\Delta h \approx \Delta u \approx C_p(T_2 - T_1)$

Mass conservation equations in a control volume

Volume flow rate	$\dot{\mathbb{V}} = \frac{d\mathbb{V}}{dt} = V_{avg,\ n}A = \dot{m}v$
Mass flow rate	$\dot{m} = \frac{dm}{dt} = \rho\, V_{avg,\ n}A = \rho\dot{\mathbb{V}}$
Transient flow	$\frac{dm_{CV}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e \neq 0$
Steady flow	$\frac{dm_{CV}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e = 0$

Energy conservation equations in a control volume

Transient flow	$\frac{dE_{CV}}{dt} \neq 0$ $\frac{dE_{CV}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i(h_i + \frac{1}{2}V_i^2 + gz_i) - \sum \dot{m}_e(h_e + \frac{1}{2}V_e^2 + gz_e)$
Steady flow	$\frac{dE_{CV}}{dt} = 0$ {\dot{Q}}_{cv} +\sum{{\dot{m}}_i\ left(h_i+\displaystyle\frac{1}{2}V_i^2+gz_i\right) \\={\dot{W}}_{cv} +\sum{{\dot{m}}_e\ left(h_e+\displaystyle\frac{1}{2}V_e^2+gz_e\right)}}

Mass and energy conservation equations for steady-state, steady-flow (SSSF) devices

SSSF device	Assumptions	Mass conservation	Energy conservation
Expansion device	Adiabatic flow; Negligible work transfer with the surroundings; Negligible changes in kinetic and potential energies	$\dot{m}_i = \dot{m}_e$	$h_i = h_e$
Nozzle and diffuser	Adiabatic flow; Negligible work transfer with the surroundings; Negligible change in potential energy	$\dot{m}_i = \dot{m}_e$	$h_i + \frac{1}{2}V_i^2 = h_e + \frac{1}{2}V_e^2$
Mixing chamber	Negligible work transfer with the surroundings; Negligible changes in kinetic and potential energies	$\sum \dot{m}_i = \sum \dot{m}_e$	$\dot{Q}_{cv} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$
Heat exchanger	Negligible work transfer with the surroundings; Negligible changes in kinetic and potential energies	$\dot{m}_i = \dot{m}_e$ (for each of the hot and cold streams, separately)	$\dot{Q}_{cv} + \sum \dot{m}_i h_i = \sum \dot{m}_e h_e$
Turbine	Adiabatic flow; Negligible changes in kinetic and potential energies	$\dot{m}_i = \dot{m}_e = \dot{m}$	$\dot{W}_{shaft} = \dot{m}(h_i - h_e)$

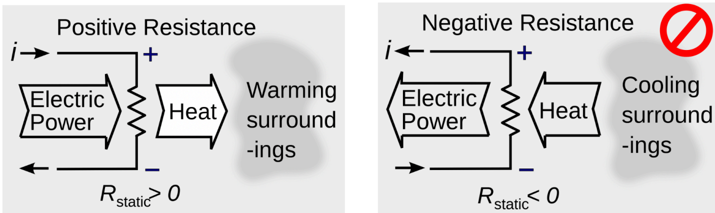
Compressor	Adiabatic flow; Negligible changes in kinetic and potential energies	$\dot{m}_i = \dot{m}_e = \dot{m}$	$\dot{W}_{shaft} = \dot{m}(h_e - h_i)$

6. ENTROPY AND THE SECOND LAW OF THERMODYNAMICS

6.0 Chapter introduction and learning objectives

Heat and work are two forms of energy. From the first law of thermodynamics, we may say “1 kJ of heat = 1 kJ of work”, but how do we interpret the “=” sign here? Does it mean that heat and work are equivalent?

Let us consider a daily example. In winter, you might need an electric resistance heater to warm up your room. From Figure 6.0.1 (a), the electric charges do work on the resistor as they flow in the direction of lower electric potential. In this process, the electric energy from the circuit is converted to heat, which is then dissipated in your room, or the surroundings. As a result, the room temperature will tend to rise. If the heater consumes 1 kW of electric power, it will generate and dissipate approximately 1 kW of heat to the surroundings. In terms of the amount of energy conversion, we can safely say that 1 kW of electric energy is no different from 1 kW of heat. However, is this energy conversion process reversible? If we simply cool the surroundings and provide 1 kW of heat to the resistor, is it possible to generate 1 kW of electric power from this 1 kW of heat? Unfortunately, the answers to both questions are no, see Figure 6.0.1 (b).



(a) Work can be converted to heat **directly and completely**

(b) Heat can NOT be converted to work **directly and completely**

Figure 6.0.1 *Electric resistance heater illustrating heat-and-work conversion and irreversible process*

This example illustrates an important concept of heat-work conversion. Although heat and work can be converted between each other, they are not equivalent. Work can always be converted to heat *completely* and *directly*; but heat cannot be converted to work *completely* and *directly* even under an ideal condition, because heat is a form of energy of "low" quality (a more "random", "disordered" system) and work is a form of energy of "high" quality (a highly "ordered" system). A natural, spontaneous process has a tendency to move toward a greater degree of randomness or disorder in a system. Therefore, an energy conversion process in nature tends to occur in the direction of converting energy from a "high" quality form to a "low" quality form. The first law of thermodynamics addresses the heat and work conversion quantitatively, but has no indication on whether or under what condition such conversion can actually happen. Satisfying the first law of thermodynamics does not ensure a process can take place in reality. Whether a process can happen or not is determined by the second law of thermodynamics.

This chapter introduces the concepts of reversible and irreversible processes, Carnot cycles, entropy generation and the second law of thermodynamics, and the applications of the second law to both closed and open systems.

Learning Objectives

After completing the chapter, you should be able to

- Demonstrate an understanding of reversible and irreversible processes and Carnot cycles
- Calculate the thermal efficiency of heat engines and the coefficient of performance (COP) of refrigerators and heat pumps
- Calculate the change in entropy in real substances, ideal gases, solids, and liquids
- Explain entropy generation, entropy transfer mechanisms, and the second law of thermodynamics
- Apply the second law of thermodynamics to closed systems
- Apply the second law of thermodynamics to various steady flow devices

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6.1 Heat engine

A heat engine is a continuously operating device that produces work by transferring heat from a **heat source** (high-temperature body) to a **heat sink** (low-temperature body) using a working fluid. In a heat engine cycle, a working fluid may remain as a single-phase fluid or experience phase changes.

A steam engine is a type of heat engine commonly used in steam power generating plants. It operates on Rankine cycles and uses water as the working fluid. We will use a steam engine to illustrate how heat is converted to work in heat engines. A typical steam engine consists of four main equipment: boiler, turbine, condenser, and pump, as shown in Figure 6.1.1. The *T*-s diagram in Figure 6.1.2 illustrates the four processes in a Rankine cycle:

1. Water at a low pressure and a low temperature (state 1) is pumped to a boiler. The pump consumes power, $\dot{W}_{pump}$, in order to maintain a continuous supply of water to the boiler while increasing the pressure of the water entering the boiler (state 2). Process 1-2 may be assumed adiabatic.
2. In the boiler, the liquid water absorbs heat, $\dot{Q}_H$, from an external heat source and changes into high-temperature, high-pressure steam (state 3). The pressure drop in the boiler is usually negligible; therefore, process 2-3 may be assumed isobaric.
3. The high-temperature, high-pressure steam then expands in the turbine, making the turbine rotate continuously, and thus generating mechanical power, $\dot{W}_{turbine}$. During the expansion process, the temperature and pressure of the steam decrease. Consequently, the steam leaving the turbine (state 4) becomes a low-temperature, low-pressure, two-phase mixture.

Process 3-4 may be assumed adiabatic.

4. The steam leaving the turbine then enters a condenser and is condensed to a saturated or compressed liquid (state 1). During this process, heat, $\dot{Q}_L$, is removed from the steam. The pressure drop in the condenser is usually negligible; therefore, process 4-1 may be assumed isobaric.

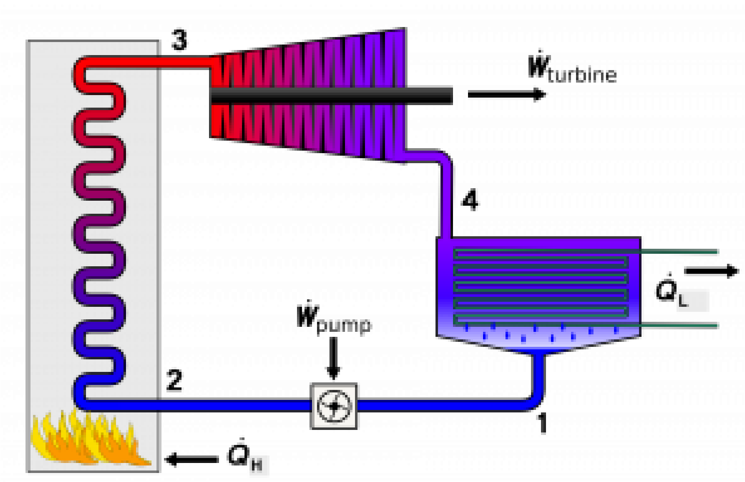

***Figure 6.1.1** Rankine cycle*

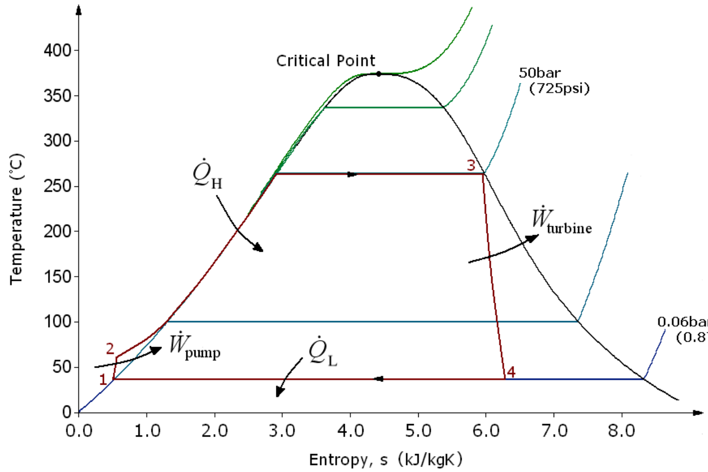

Figure 6.1.2 *T-s diagram of a Rankine cycle*

Figure 6.1.3 is a simplified schematic for analyzing the energy conservation in heat engines. Applying the first law of thermodynamics to the cycle, we can write

$$\dot{Q}_{H} - \dot{Q}_{L} = \dot{W}_{turbine} - \dot{W}_{pump} = \dot{W}_{net, out}$$

Clearly, the heat removed by the condenser, $\dot{Q}_L$, cannot be converted to useful work. It is *wasted* in order to complete the cycle. In other words, a heat engine cannot convert *all* the heat supplied by the heat source (e.g., boiler) to useful work, even under ideal conditions. Thermal efficiency is a dimensionless parameter used to measure the performance of a heat engine.

$$\eta_{th} = \frac{desired\ output}{required\ input} = \frac{\dot{W}_{net,\ out}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}$$

where

$\dot{Q}_H$: heat absorbed from the heat source, in kW

$\dot{Q}_L$: heat rejected to the heat sink, in kW

$\dot{W}_{net,\ out}$: net work output from the heat engine, in kW

η_{th}: thermal efficiency of the heat engine, dimensionless

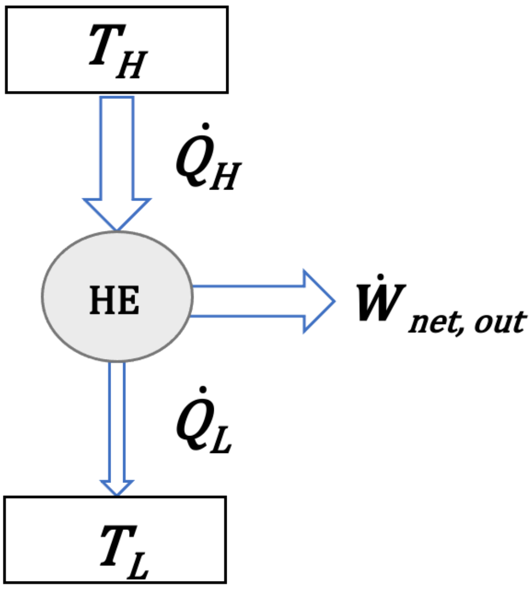

Figure 6.1.3 *Schematic of a heat engine*

Example 1

Figures 6.1.e1 and 6.1.e2 illustrate a Rankine cycle consisting of a two-stage steam engine and a feedwater heater. The steam engine is enclosed in the red outlines in Figure 6.1.e1. The two stages of the turbine are labelled as HE1 and HE2, respectively. In stage 1, the steam absorbs heat, $\dot{Q}_H$, from the boiler and generates a power, $\dot{W}_1$. A portion of the exhaust steam from stage 1 then enters stage 2, further generating a power, $\dot{W}_2$. The remaining exhaust steam from stage 1 is used to preheat the feed water. If the thermal efficiencies of the two turbine stages are $\eta_{th,1}$ and $\eta_{th,2}$, what is the overall thermal efficiency of the cycle as a function of $\eta_{th,1}$ and $\eta_{th,2}$? Assume 90% of the exhaust steam exiting from stage 1 enters stage 2 and generates the power, $\dot{W}_2$.

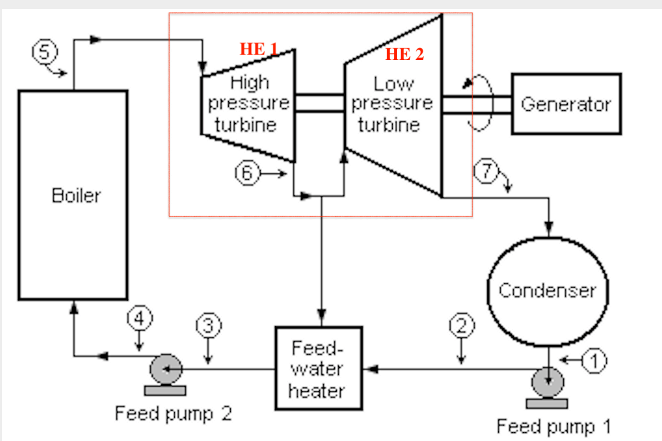

Figure 6.1.e1 *Two-stage steam turbine with a feed-water heater*

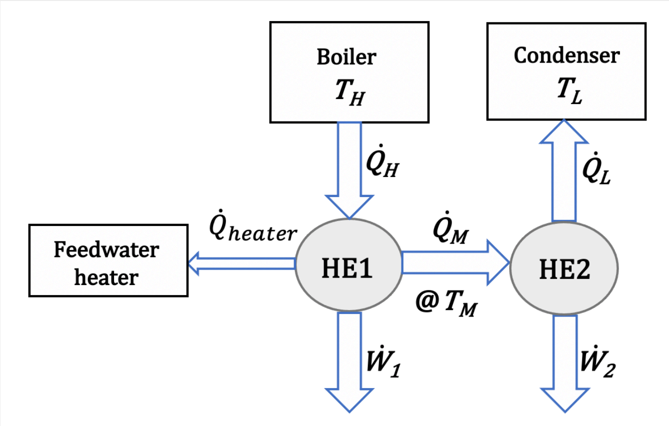

Figure 6.1.e2 *Schematic of the two-stage heat engine*

Solution:

The thermal efficiency of the first and second stages of the steam turbine can be written as

$$\eta_{th,1} = \frac{\dot{W}_1}{\dot{Q}_H}$$

$$\eta_{th,2} = \frac{\dot{W}_2}{\dot{Q}_M}$$

The desired output of the cycle is the total power generated by the turbine and the required energy input comes from the boiler; therefore,

$$\eta_{th} = \frac{\dot{W}_{tot}}{\dot{Q}_H} = \frac{\dot{W}_1 + \dot{W}_2}{\dot{Q}_H}$$

Apply the first law of thermodynamics to the first stage, HE1. Note that 90% of the exhaust steam from stage 1 enters stage 2; therefore,

$$\dot{Q}_M + \dot{Q}_{Heater} = \dot{Q}_H - \dot{W}_1$$

and

$$\dot{Q}_M = 0.9(\dot{Q}_H - \dot{W}_1)$$

Combine the above equations and rearrange,

$$\begin{aligned}
\because \eta_{th} &= \frac{\dot{W}_1}{\dot{Q}_H} + \frac{\dot{W}_2}{\dot{Q}_H} \\
&= \eta_{th,1} + \frac{\eta_{th,2}\dot{Q}_M}{\dot{Q}_H} \\
&= \eta_{th,1} + \frac{\eta_{th,2} \times 0.9(\dot{Q}_H - \dot{W}_1)}{\dot{Q}_H} \\
\therefore \eta_{th} &= \eta_{th,1} + 0.9\eta_{th,2}(1 - \eta_{th,1})
\end{aligned}$$

Practice Problems

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6.2 Refrigerator and heat pump

6.2.1 Refrigerator

A refrigerator is a cyclic device, which absorbs heat from a **heat sink** and reject heat to a **heat source** by consuming work. The working fluid is called refrigerant, which usually undergoes phase changes in the cycle.

A typical vapour-compression refrigeration system consists of mainly four pieces of equipment, as shown in Figure 6.2.1:

1. An evaporator, through which the low-pressure, low-temperature refrigerant absorbs heat from a heat sink (e.g., a freezer compartment or a space to be refrigerated) and changes from a two-phase mixture to a saturated or superheated vapour.
2. A compressor, which is used to increase the pressure and temperature of the refrigerant vapour by consuming work.
3. A condenser, through which heat is rejected to a heat source (e.g. kitchen or outdoor air). At the exit of the condenser, the refrigerant is typically a two-phase mixture or a liquid.
4. An expansion valve, which is used to reduce the pressure and temperature of the refrigerant in order to achieve a liquid-vapour mixture of desirable quality at the exit of the expansion valve.

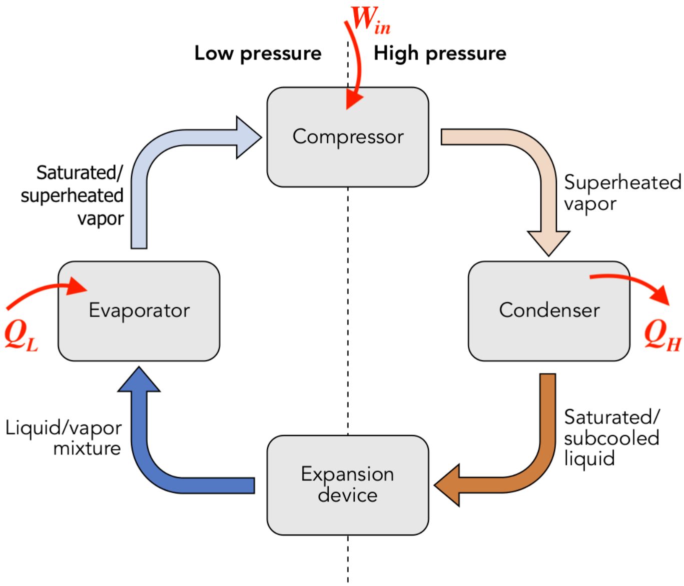

Figure 6.2.1 *Vapor compression refrigeration cycle consisting of a compressor, condenser, expansion device, and evaporator*

As heat cannot be transferred from a low-temperature body to a high-temperature body spontaneously in nature, refrigerators must *consume work* in order to operate between a **heat sink** and a **heat source**, even under ideal conditions.

Figure 6.2.2 is a schematic for analyzing the energy conservation in a refrigerator. The same schematic may be used to represent a heat pump by replacing the notation "Ref" with "HP" in the circular area.

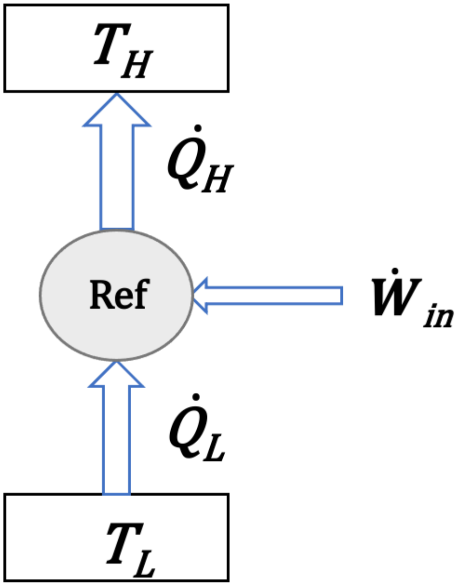

***Figure 6.2.2** Schematic of a refrigerator (or a heat pump)*

Applying the first law of thermodynamics to the cycle, we can write

$$\dot{Q}_H - \dot{Q}_L = \dot{W}_{in}$$

The main purpose of refrigerators is to remove heat, $\dot{Q}_L$, from a **heat sink** (cold space); therefore, we are interested in the amount of heat that can be removed per unit of power consumption. The dimensionless ratio of $\dot{Q}_L/\dot{W}_{in}$ is called the coefficient of performance, COP_R, of the refrigerator. It is an indicator of the performance of a refrigerator.

$$COP_R = \frac{desired\ output}{required\ input} = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} = \frac{1}{\dot{Q}_H/\dot{Q}_L - 1}$$

Refrigerators are typically designed to consume a power $\dot{W}_{in} < \dot{Q}_L$; therefore, $COP_R > 1$ in a well-designed refrigerator. A higher COP_R indicates a better performance.

6.2.2 Heat pump

A heat pump uses the same vapour compression refrigeration cycle, see Figure 6.2.1, as a refrigerator. It absorbs heat from a **heat sink** (e.g., outdoor air in the winter) and delivers (more) heat to a **heat source** (e.g., indoor air) by consuming work. Applying the first law of thermodynamics to the heat pump cycle, we can derive

$$\dot{W}_{in} = \dot{Q}_H - \dot{Q}_L$$

Different from refrigerators, the main purpose of heat pumps is to add heat, $\dot{Q}_H$, to a heat source, such as an indoor space of a building. Therefore, we are interested in the amount of heat, $\dot{Q}_H$, that can be transferred from the condenser to the heat source per unit power consumption. The dimensionless ratio of $\dot{Q}_H/\dot{W}_{in}$ is called the coefficient of performance, COP_{HP}, of heat pump. It is an indicator of the performance of a heat pump. As $\dot{Q}_H > \dot{W}_{in}$, $COP_{HP} > 1$.

$$COP_{HP} = \frac{desired\ output}{required\ input} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{\dot{Q}_H/\dot{Q}_L}{\dot{Q}_H/\dot{Q}_L - 1}$$

Example 1

Consider a vapour-compression refrigeration cycle, Figure 6.2.1. The working fluid is ammonia.

- At the compressor inlet, P_1=0.2 MPa, T_1=-10^{o}C, $\dot{m}_1 = 0.1\ kg/s$
- At the compressor outlet, P_2=1.4 MPa, T_2=150^{o}C
- At the condenser outlet, P_3=1.4 MPa, T_3=35^{o}C
- At the evaporator inlet, P_4=0.2 MPa

Assuming that the heat transfer and pressure drops in the connecting pipes are negligible, and $\Delta PE = 0$, $\Delta KE = 0$,

(1) determine the rate of heat absorbed by the evaporator, $\dot{Q}_L$, the power required by the compressor, $\dot{W}_{in}$, and the rate of heat rejected by the condenser, $\dot{Q}_H$.

(2) determine COP_R of the refrigeration cycle

(3) If the same vapour refrigeration cycle were used as a heat pump, what would the COP_{HP} be?

Solution:

(1) The rates of heat transfer in the evaporator and condenser and the power consumption in the compressor depend on the changes of enthalpies in these devices. First, determine the specific enthalpy of each state from Tables B1 and B2.

- State 1, compressor inlet: P_1 = 0.2 MPa, T_1 = -10^{o}C. From Table B2, h_1 = 1603.63 kJ/kg.
- State 2, compressor outlet: P_2 = 1.4 MPa, T_2 = 150^{o}C. From Table B2, h_2 = 1940.26 kJ/kg

- State 3, condenser outlet: P_3 = 1.4 MPa, T_2 = 35^oC. From Table B1, ammonia at this state is a compressed liquid; therefore, $h_3 \approx h_{f@35^oC}$ = 509.23 kJ/kg
- State 4, evaporator inlet: P_4 = 0.2 MPa. Typically, the expansion valve is assumed well-insulated; therefore, the expansion process is adiabatic, and $h_4 = h_3$ = 509.24 kJ/kg.

The rate of the heat absorbed by the evaporator

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.1 \times (1603.63 - 509.23) = 109.44\ kW$$

The power consumption by the compressor

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = 0.1 \times (1940.26 - 1603.63) = 33.66\ kW$$

The rate of the heat rejected by the condenser

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = 0.1 \times (1940.26 - 509.23) = 143.10\ kW$$

From the above calculations, it is easy to verify that

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{in}$$

(2) COP of the refrigeration cycle

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{109.44}{33.66} = 3.25$$

(3) If the same cycle is used as a heat pump, its COP will be

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{143.10}{33.66} = 4.25$$

Comment:

It is noted that both COP_R and COP_{HP} are greater than 1. By consuming a certain amount of power, the cycle transfers a much larger amount of heat (desirable output) either by the evaporator or by the condenser. In addition, it is always true that

$$COP_{HP} = COP_R + 1$$

because

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{in}} = \frac{\dot{Q}_L + \dot{W}_{in}}{\dot{W}_{in}} = COP_R + 1$$

Practice Problems

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6.3 The second law of thermodynamics: Kelvin-Planck and Clausius statements

The first law of thermodynamics focuses on energy conservation. It does not describe any restrictions or possibilities for a process to take place. A process satisfying the first law of thermodynamics may or may not be achievable in reality. In fact, whether a process is possible is governed by both the first and the second laws of thermodynamics. There are two classical statements of the second law of thermodynamics; one imposes the limits on the operation of heat engines, and the other on the operation of refrigerators and heat pumps.

Kelvin-Planck Statement: ***it is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce only a net amount of work.*** Figure 6.3.1 illustrates the Kelvin-Planck statement. Any device violating the Kelvin-Planck statement would produce a net work output equivalent to the amount of the heat received by the device, or $\dot{W} = \dot{Q}_H$, thus resulting in a thermal efficiency $\eta_{th} = \dot{W}/\dot{Q}_H = 100\%$.

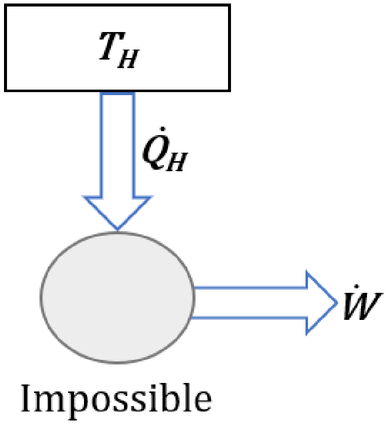

Figure 6.3.1 *Schematic illustrating Kelvin-Planck statement*

Clausius statement: ***it is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body***. Figure 6.3.2 illustrates the Clausius statement. Any device violating the Clausius statement would have zero work input and nonzero heat transfer received and delivered by the device, or $\dot{W} = 0$, $\dot{Q}_H = \dot{Q}_L \neq 0$, and thus a $COP = \dot{Q}_L / \dot{W} = \infty$.

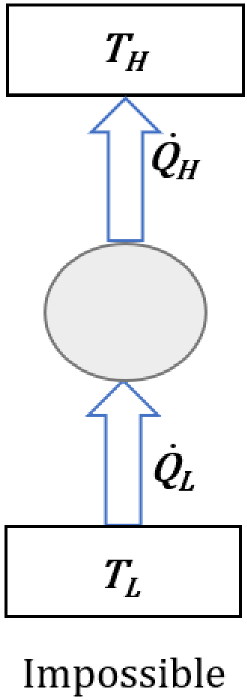

***Figure 6.3.2** Schematic illustrating the Clausius statement*

Although the Kelvin–Planck and Clausius statements refer to different cycles, they are equivalent in their consequences. Any device that violates the Kelvin–Planck statement must violate the Clausius statement, and vice versa.

Let us consider a combined system, which consists of a heat pump and an imagined engine operating between a heat source at a higher temperature T_H and a heat sink at a lower temperature T_L, as shown in Figure 6.3.3.

1. Let us assume the imagined engine violates the Kelvin-Planck statement; therefore, it can convert all of the heat absorbed from the heat source to useful power, or $\dot{W} = \dot{Q}$. The power generated from the imagined engine is then used to drive the heat pump.
2. For the heat pump, $COP = \dot{Q}_H / \dot{W}$. Therefore, the heat released by the heat pump to the heat source is $\dot{Q}_H = COP \cdot \dot{Q}$.
3. Applying the first law to the heat pump, the heat absorbed by the heat pump from the heat sink must be $\dot{Q}_L = \dot{Q}_H - \dot{Q} = (COP - 1)\dot{Q}$.
4. Now let us compare the <u>net</u> heat absorbed by the combined system from the heat sink, $\dot{Q}_L = (COP - 1)\dot{Q}$, and the <u>net</u> heat rejected by the combined system to the heat source, $\dot{Q}_H - \dot{Q} = (COP - 1)\dot{Q}$. They are exactly the same! What the combined system does is to transfer heat from the heat sink (lower-temperature body) to the heat source (higher-temperature body) without consuming any power at all!

It is apparent that the combined system violates the Clausius statement because the imagined engine violates the Kelvin-Planck statement. Since the Kelvin-Planck and Clausius statements are equivalent, both of them may be used to describe the second law of thermodynamics.

A real process or device must satisfy both the first and the second laws of thermodynamics: the first law sets a constraint on the amount of energy that must be conserved in a process or a device; the second law indicates whether a process is possible in reality. Any device that violates either the first or the second law is called a perpetual-motion machine, which attempts to produce work from nothing or convert heat *completely* to useful work. In fact, no perpetual-motion machine can actually work.

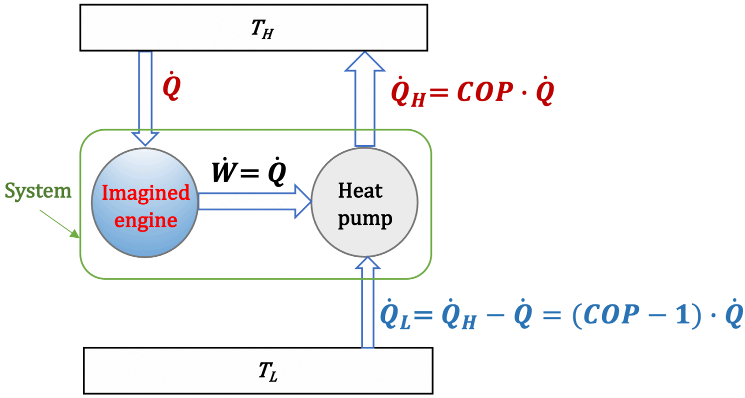

Figure 6.3.3 *Schematic showing the equivalence of the Kelvin-Planck and Clausius statements*

Example 1

Consider the following scenarios proposed for a heat engine or a refrigerator. For each of the scenarios, determine if the proposed heat engine or refrigerator satisfies the first and second laws of thermodynamics.

1. Proposed heat engine:
$$\dot{Q}_H = 10\,\text{kW}, \dot{Q}_L = 3\,\text{kW}, \dot{W}_{net,out} = 5\,\text{kW}$$
2. Proposed refrigerator:
$$\dot{Q}_H = 9\,\text{kW}, \dot{Q}_L = 4\,\text{kW}, \dot{W}_{in} = 0\,\text{kW}$$

Solution:

(1) Proposed heat engine:

\dot{Q}_H = 10 \ {\rm{kW}}, \dot{Q}_L = 3 \ {\rm{kW}}, \dot{W}_{net,out} = 5 \ {\rm{kW}}

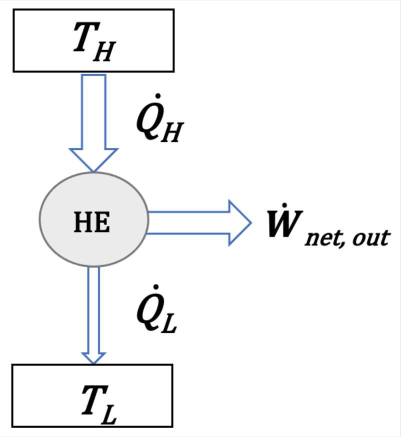

Figure 6.1.3 *Schematic of a heat engine (reproduced)*

The first law requires $\dot{Q}_H = \dot{Q}_L + \dot{W}_{net,out}$, and the second law requires $\dot{Q}_L > 0$. Since $10 \neq 3 + 5$ and $\dot{Q}_L = 3 \text{ kW} > 0$, the proposed heat engine doesn't satisfy the first law of thermodynamics; but it satisfies the second law of thermodynamics or the Kelvin-Planck statement.

(2) Proposed refrigerator:

\dot{Q}_{H} = 9 \ {\rm{kW}}, \dot{Q}_{L} = 4 \ {\rm{kW}}, \dot{W}_{in} = 0 \ {\rm{kW}}

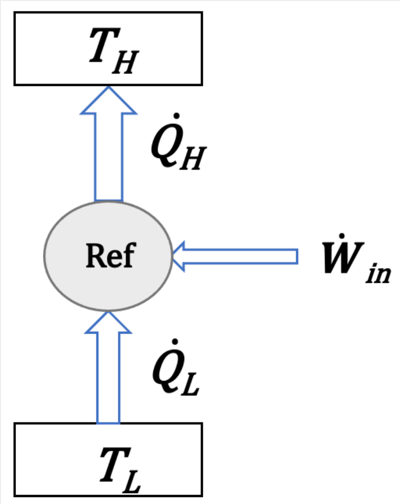

Figure 6.2.2 *Schematic of a refrigerator (reproduced)*

The first law requires $\dot{Q}_H = \dot{Q}_L + \dot{W}_{in}$, and the second law requires $\dot{W}_{in} > 0$. Since $9 \neq 4 + 0$ and $\dot{W}_{in} = 0$, the proposed refrigerator satisfies neither the first nor the second laws of thermodynamics. It violates the Clausius statement.

Practice Problems

An interactive H5P element has been excluded from this version of the text. You can view it online here:
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6.4 Carnot cycles

6.4.1 Irreversibilities

The Kelvin-Planck and Clausius statements of the second law indicate that for heat engines, $\eta_{th} < 1$, and for refrigerators and heat pumps, $COP < \infty$. But can $\eta_{th} \to 1$ and $COP \to \infty$ in an ideal condition? How is an ideal process or cycle defined? What is the theoretical limit of η_{th} or COP in an ideal cycle? To answer these questions, we will first need to understand the concepts of reversible processes and irreversibilities.

A **reversible process** is a process that can be reversed *without leaving any change in either the system or its surroundings*, which means both the system and its surroundings always return to their original states during a reversible process. A fictitious, frictionless pendulum can be treated as a reversible process, see Figure 6.4.1; it will never stop and always returns to its original state. However, in reality, friction always exists. A pendulum will gradually slow down and eventually stop.

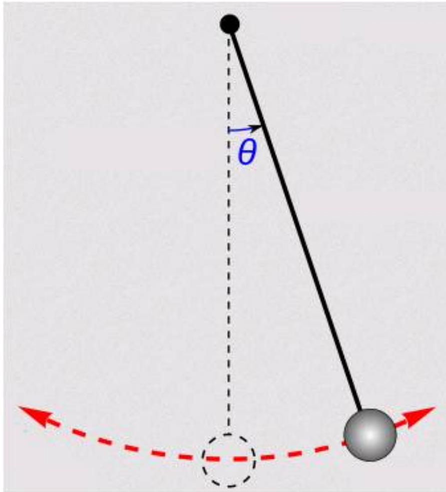

Figure 6.4.1 *Frictionless pendulum as an example of a reversible process*

Factors that render a process irreversible are called **irreversibilities**. Friction, unrestrained expansion, mixing of fluids, heat transfer through a finite temperature difference, electric resistance, inelastic deformation of solids, and chemical reactions are some examples of irreversibilities. All processes occurring in nature are irreversible due to the existence of irreversibilities. Some processes are more irreversible than others. A reversible process is commonly used as an idealized model to which an actual process can be compared. The efficiency of a reversible process is the theoretical limit that an actual, irreversible process can possibly achieve in a nearly ideal condition.

6.4.2 Carnot heat engine

A Carnot heat engine is an idealized cycle, which consists of four reversible processes. Figures 6.4.2–6.4.4 show the schematic of a Carnot heat engine and its pressure-volume, $P - \mathbb{V}$, and temperature-entropy, $T - S$, diagrams. The cycle consists of the following four reversible processes:

1. Process A⟶B: a reversible isothermal heat addition process of Q_H from a heat source at constant temperature T_H.
2. Process B⟶C: a reversible adiabatic expansion process, during which the temperature drops from T_H to T_L.
3. Process C⟶D: a reversible isothermal heat removal process of Q_L from a heat sink at constant temperature T_L.
4. Process D⟶A: a reversible adiabatic compression process, in which the temperature increases from T_L to T_H.

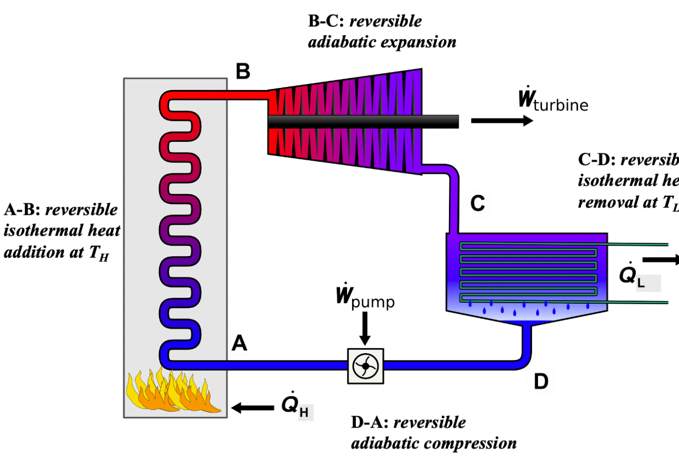

Figure 6.4.2 *Carnot heat engine*

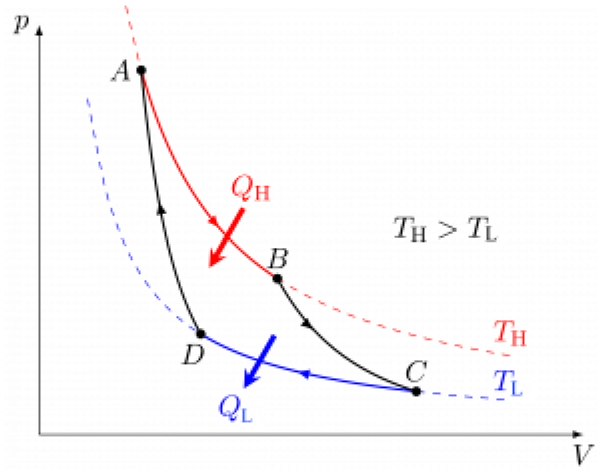

***Figure 6.4.3** Carnot heat engine: [latex]P-\mathbb{V}[/latex] diagram*

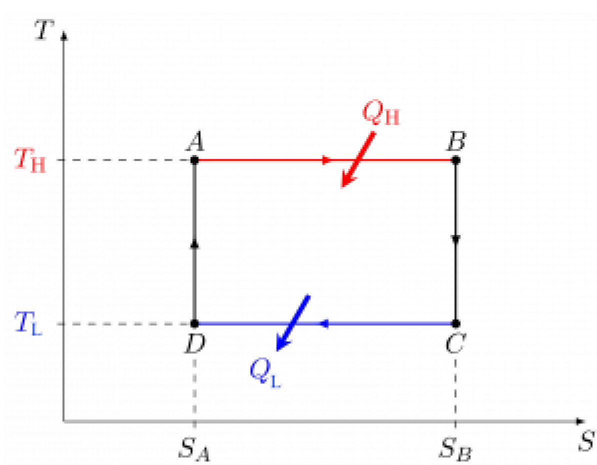

***Figure 6.4.4** Carnot heat engine: T-S diagram*

Because the Carnot heat engine cycle is an ideal cycle consisting of only reversible processes, it produces the maximum power output and has the maximum thermal efficiency among all heat engines operating between the same heat source at T_H and the same heat sink at T_L. The thermal efficiency of a Carnot heat engine can be expressed as

$$\eta_{th,\ rev} = 1 - \left(\frac{Q_L}{Q_H}\right)_{rev} = 1 - \frac{T_L}{T_H}$$

where

T_H: absolute temperature of the heat source, in Kelvin

T_L: absolute temperature of the heat sink, in Kelvin

Q_H: heat supplied by the heat source to the Carnot heat engine, in kJ

Q_L: heat rejected from the Carnot heat engine to the heat sink, in kJ

$\eta_{th,\ rev}$: thermal efficiency of the Carnot heat engine, dimensionless

The thermodynamic temperature scale, or absolute temperature scale is defined so that $\left(\frac{Q_L}{Q_H}\right)_{rev} = \frac{T_L}{T_H}$.

In summary, the thermal efficiency of an actual heat engine is always less than that of the Carnot heat engine. It is impossible for any heat engine to achieve a better performance than the Carnot heat engine operating between the same heat source at T_H and the same heat sink at T_L.

- Irreversible, actual heat engine: $\eta_{th} < \eta_{th,\ rev}$
- Carnot heat engine: $\eta_{th} = \eta_{th,\ rev}$
- Impossible device: $\eta_{th} > \eta_{th,\ rev}$

For a heat engine operating between a heat source at T_H and a heat sink at T_L, either increasing T_H or decreasing T_L can improve its thermal efficiency. In practice, much effort is made to increase the temperature of the heat source T_H within the material limits because the temperature of the heat sink T_L is usually constrained by the ambient temperature. In many large steam power plants, the temperature of the inlet steam (heat source) is in the range of 300~600°C (573~873 K) in order to improve the thermal efficiency of the heat engines.

Example 1

Consider a proposal to build a 500-MW steam power plant operating on a Rankine cycle by a lake. The steam generated by the boiler is at 250°C. The condenser is to be cooled by the lake water at a flow rate of 20 m^3/s. As the cooling water will be discharged to the lake, the design team needs to evaluate its impact on the local aquatic ecosystem. Assume the cooling water supplied from the lake to the condenser is at a constant temperature of 18°C.

1. If the cycle were reversible (Carnot cycle), what is the thermal efficiency of the power plant? Estimate the temperature rise of the discharge water to the lake.
2. If the cycle were reversible (Carnot cycle), and the steam temperature increases to 300°C, how would the thermal efficiency of the power plant change?
3. The above two scenarios are ideal Carnot cycles. In fact, the projected thermal efficiency of the actual

cycle is about 35%. Estimate the temperature rise of the discharge water to the lake in this case.

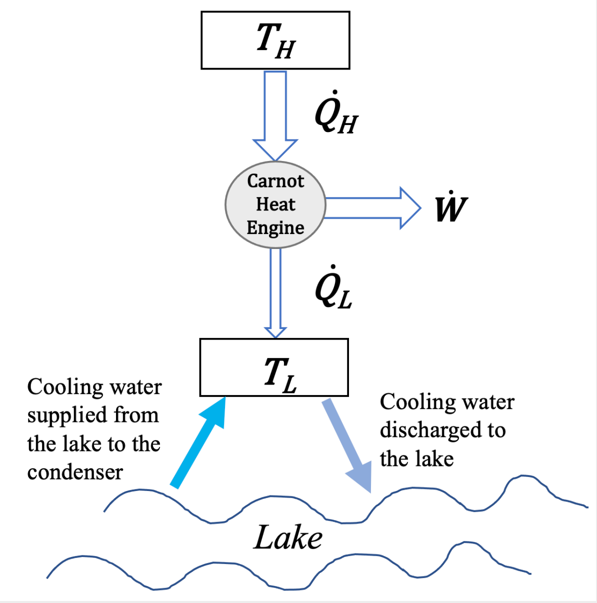

Figure 6.4.e1 *A schematic of a Carnot heat engine. Heat is rejected to a nearby lake.*

<u>*Solution:*</u>

1. The power plant is assumed to operate on a Carnot cycle between a heat source at the steam temperature of T_H = 250°C and a heat sink at the cooling water temperature of T_L = 18°C. The thermal efficiency of the Carnot cycle is

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{273.15 + 18}{273.15 + 250} = 0.443$$

$$\eta_{th,rev} = \frac{\dot{W}}{\dot{Q}_H} \text{ and}$$

$$\dot{Q}_H = \dot{Q}_L + \dot{W}$$

Therefore, the heat rejected to the discharge water to the lake is

$$\dot{Q}_L = \dot{W}\left(\frac{1}{\eta_{th,rev}} - 1\right)$$
$$= 500\left(\frac{1}{0.443} - 1\right) = 627.48 \text{ MW}$$

From Table G2, C_p = 4.181 kJ/kgK for water, and the density of water is $\rho = 1000 \text{ kg/m}^3$; therefore,

$$\because \dot{Q}_L = \dot{m}C_p\Delta T = \rho\dot{\mathbb{V}}C_p\Delta T$$

$$\therefore \Delta T = \frac{\dot{Q}_L}{\rho\dot{\mathbb{V}}C_p} = \frac{627.48 \times 10^3}{1000 \times 20 \times 4.181} = 7.5\ ^\circ\text{C}$$

2. For a Carnot cycle with T_H = 300°C and T_L = 18°C,

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{273.15 + 18}{273.15 + 300} = 0.492$$

Comment:

Compare the two Carnot cycles, increasing the heat source temperature T_H can increase the thermal efficiency of the cycle. Similarly, decreasing the heat sink temperature T_L can also increase the thermal efficiency of the cycle.

3. The thermal efficiency of an actual cycle is always less than that of the Carnot cycle operating between the same

heat source and heat sink. If the projected thermal efficiency of the actual cycle is 35%, then the heat absorbed by the discharge water will be

$$\dot{Q}_L = \dot{W}\left(\frac{1}{\eta_{th}} - 1\right)$$
$$= 500 \times \left(\frac{1}{0.35} - 1\right) = 928.57\ \mathrm{MW}$$

The temperature rise of the discharge water will be

$$\Delta T = \frac{\dot{Q}_L}{\rho\dot{\mathbb{V}}C_p} = \frac{928.57}{1000 \times 20 \times \times 4.181} = 11.1\ ^{\circ}\mathrm{C}$$

Comment:

The actual cycle will cause a much higher temperature rise in the discharge water than the Carnot cycle due to the existence of the irreversibilities. A large amount of heat rejection to the lake could have a negative impact on the aquatic life and should be monitored closely.

6.4.3 Carnot refrigerator and Carnot heat pump

If the four processes, compression, evaporation, expansion and condensation, in a vapour-compression refrigeration system are all reversible, the cycle is reversible and is called the Carnot refrigeration cycle. A refrigerator or a heat pump operating in such a cycle is called a Carnot refrigerator or a Carnot heat pump.

Among all refrigerators (or heat pumps) operating between the same heat source at T_H and the same heat sink at T_L, the Carnot refrigerator (or heat pump) consumes the least amount of power, thus achieving the highest coefficient of performance, $COP_{R,\ rev}$ or $COP_{HP,\ rev}$, as expressed below:

- Carnot refrigerator:
$$COP_{R,\ rev}=\frac{T_L}{T_H-T_L}=\frac{1}{T_H/T_L-1}$$
- Carnot heat pump:
$$COP_{HP,\ rev}=\frac{T_H}{T_H-T_L}=\frac{1}{1-T_L/T_H}$$

where

$COP_{R,\ rev}$: coefficient of performance of a Carnot refrigerator, dimensionless

$COP_{HP,\ rev}$: coefficient of performance of a Carnot heat pump, dimensionless

T_H: absolute temperature of the heat source, in Kelvin

T_L: absolute temperature of the heat sink, in Kelvin

Example 2

A heat pump provides 20 kW of heating to a house in winter. The house must be maintained at 24^{o}C. If the COP of the heat pump is 5.5, and the outdoor temperature is 0^{o}C, answer the following questions.

1. How much power is required to drive the heat

pump? Compare the power consumption of this heat pump and an electric resistance heater providing the same amount of heating to the house.

2. If the cycle were a Carnot cycle, what would the power consumption be?
3. If the outdoor temperature decreases, will the COP of the heat pump increase, remain the same, or decrease?

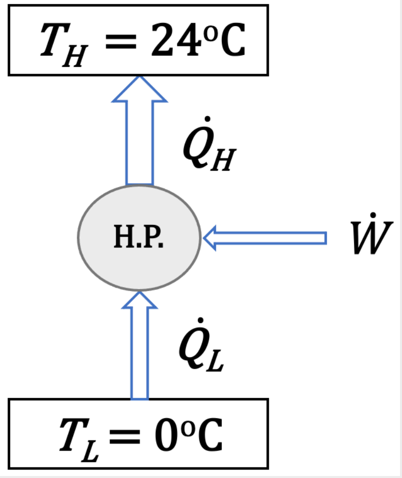

Figure 6.4.e2 *Schematic of a heat pump*

<u>*Solution:*</u>

1. The heat pump provides 20 kW of heating to the house; therefore, $\dot{Q}_H = 20\ \mathrm{kW}$

$$\because\ COP_{HP} = \frac{\dot{Q}_H}{\dot{W}} = 5.5$$

$$\therefore\ \dot{W} = \frac{\dot{Q}_H}{COP_{HP}} = \frac{20}{5.5} = 3.636\ \mathrm{kW}$$

If an electric resistance heater is used to provide 20 kW of heating to the house, the electric resistance heater will consume 20 kW of electric power, assuming the efficiency of the heater is 100%. Therefore,

$$\Delta\dot{W} = 20 - 3.636 = 16.364\ \mathrm{kW}$$

Compare the power consumption of the heat pump and electric heater, using heat pump will save 16.354 kW of power.

2. If the cycle were a Carnot cycle,

$$\begin{aligned} COP_{HP,rev} &= \frac{T_H}{T_H - T_L} \\ &= \frac{273.15+24}{(273.15+24)-(273.15+0)} = 12.38 \end{aligned}$$

$$\because\ COP_{HP,rev} = \frac{\dot{Q}_H}{\dot{W}_{rev}}$$

$$\therefore\ \dot{W}_{rev} = \frac{\dot{Q}_H}{COP_{HP,rev}} = \frac{20}{12.38} = 1.615\ \mathrm{kW}$$

The actual cycle achieves a much smaller COP and consumes a much greater power than the Carnot cycle due to the irreversibilities.

3. Since

$$COP_{HP,rev} = \frac{T_H}{T_H - T_L} = \frac{1}{1 - T_L/T_H}$$

As T_H is a constant room temperature, $COP_{HP,rev}$ will decrease if the outdoor temperature T_L decreases. Because the Carnot cycle always has the maximum COP compared to any real cycle operating between the same heat source and heat sink, the COP of a real heat pump will also decrease if the outdoor temperature decreases; therefore, heat pumps are preferably used in mild winter conditions.

Practice Problems

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6.5 Entropy and entropy generation

6.5.1 The inequality of Clausius

The inequality of Clausius states that for any cycle, reversible or irreversible, there exists the following relation:

$$\oint \frac{\delta Q}{T} \leq 0 \qquad (\text{“=” for reversible cycles; “<” for irreversible cycles})$$

where δQ represents the differential amount of heat transfer into or out of a system through an infinitesimal part of the system boundary. δQ is positive for heat transfer into the system and is negative for heat transfer out of the system. T is the absolute temperature at the infinitesimal part of the system boundary, where the heat transfer occurs. The cyclic integral symbol $\oint$ indicates that the integration must be done for the entire cycle. In other words, all heat transfer into and out of the system, as well as their corresponding boundary temperatures, must be considered in the integral.

The inequality of Clausius applies to all cycles. We will prove it by using the heat engine cycle, Figure 6.1.3, as an example. For a reversible heat engine cycle operating between a heat source at a constant temperature of T_H and a heat sink at a constant temperature of T_L, the cyclic integral can be written as

$$\oint\left(\frac{\delta Q}{T}\right)_{rev}=\left(\frac{Q_H}{T_H}\right)_{rev}+\left(\frac{-Q_L}{T_L}\right)_{rev}$$

Note that for a reversible cycle,

$$\left(\frac{T_H}{T_L}\right)_{rev}=\left(\frac{Q_H}{Q_L}\right)_{rev}$$

Therefore, the following equation exists for a reversible cycle.

$$\oint\left(\frac{\delta Q}{T}\right)_{rev}=0$$

For an irreversible cycle operating between the same two heat reservoirs at constant temperatures of T_H and T_L, we assume that the heat absorbed from the heat source, Q_H, remains the same as that in the reversible cycle,

$$\because Q_{H,rev}=Q_{H,irrev}=Q_H \quad \text{and} \quad W_{rev}>W_{irrev}$$

$$\therefore Q_{L,rev}<Q_{L,irrev}$$

$$\therefore \left(\frac{Q_H}{Q_L}\right)_{irrev}<\left(\frac{Q_H}{Q_L}\right)_{rev} \quad \text{and} \quad \left(\frac{Q_H}{Q_L}\right)_{rev}=\left(\frac{T_H}{T_L}\right)_{rev}$$

$$\therefore\left(\dfrac{Q_H}{Q_L}\right)_{irrev} < \left(\dfrac{T_H}{T_L}\right)_{rev}$$ and $$\left(\dfrac{T_H}{T_L}\right)_{rev} = \left(\dfrac{T_H}{T_L}\right)_{irrev}$$

$$\therefore\left(\dfrac{Q_H}{T_H}\right)_{irrev} < \left(\dfrac{Q_L}{T_L}\right)_{irrev}$$

Therefore,

$$\displaystyle\oint\left(\dfrac{\delta Q}{T}\right)_{irrev} = \left(\dfrac{Q_H}{T_H}+\dfrac{-Q_L}{T_L}\right)_{irrev} = \left(\dfrac{Q_H}{T_H}\right)_{irrev} - \left(\dfrac{Q_L}{T_L}\right)_{irrev} < 0$$

Now, we have proven the inequality of Clausius for heat engine cycles. A similar procedure may be applied to prove the inequality of Clausius for refrigerator and heat pump cycles.

6.5.2 Definition of entropy

Why is the inequality of Clausius important? The cyclic integral is either equal to or less than zero depending on the nature of the cycle: reversible or irreversible. The inequality of Clausius provides a basis for introducing the concepts of entropy and entropy generation. Both concepts are important in the second law of thermodynamics.

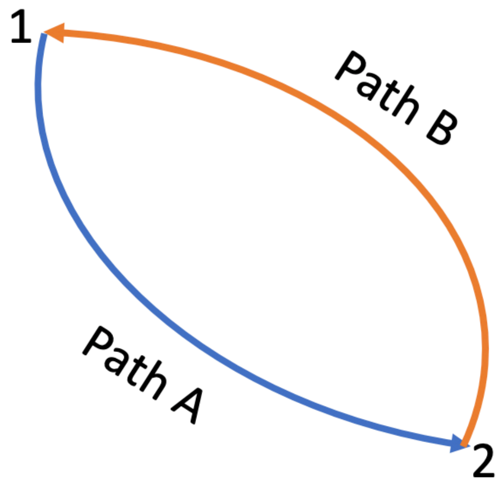

Figure 6.5.1 A *reversible cycle consisting of path* A *and path* B

Let us apply the inequality of Clausius to a reversible cycle consisting of two reversible processes 1⟶2 via path A and 2⟶1 via path B, see Figure 6.5.1.

$$\because \oint \left(\frac{\delta Q}{T}\right)_{rev} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{pathA} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{pathB}$$
$$= \int_1^2 \left(\frac{\delta Q}{T}\right)_{pathA} - \int_1^2 \left(\frac{\delta Q}{T}\right)_{pathB} = 0$$

$$\therefore \int_1^2 \left(\frac{\delta Q}{T}\right)_{pathA} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{pathB}$$

The above equation indicates that the integral between the two states 1 and 2 of any reversible processes depends only on the two states, not on the paths; therefore, the integral $\int_1^2 \left(\frac{\delta Q}{T}\right)_{rev}$ is a state function and must be related to a thermodynamic property. We define such thermodynamic property as **entropy,** S, and the change in entropy between two states can be expressed as

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev}$$

The infinitesimal change of entropy in a reversible process can thus be written as

$$dS = \left(\frac{\delta Q}{T}\right)_{rev}$$

where S is the entropy and T is the absolute temperature. The common SI units for entropy are kJ/K or J/K. It is important to note that entropy, S, is a state function; ΔS in a process depends on the initial and final states, not on the path of the process.

Entropy is an extensive property; its corresponding intensive property is called **specific entropy**, $s = \frac{S}{m}$, and its common SI units are kJ/kgK or J/kgK.

Example 1

Consider a reversible process and an irreversible process from states 1 to 2, as shown in the T-S diagram, Figure 6.5.e1. Answer the following questions

(1) Is the change in entropy, ΔS, the same or different in these two processes?

(2) Is it possible to show the heat transfer of the reversible process in the T-S diagram?

(3) Is it possible to show the heat transfer of the irreversible process in the T-S diagram?

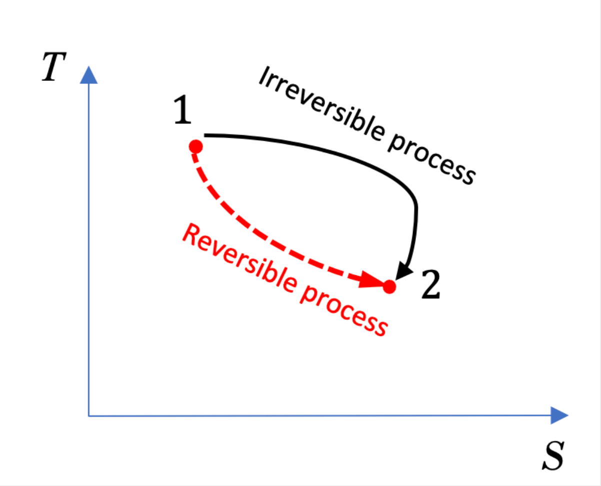

Figure 6.5.e1 *T-S diagram for a reversible process and an irreversible process with the same initial and final states*

Solution:

(1) Entropy is a state function. The two processes have the same initial and final states, therefore, the same ΔS.

(2) From the definition of entropy, the heat transfer in the reversible process can be found from

$$Q_{rev} = \int_1^2 (\delta Q)_{rev} = \int_1^2 (TdS)_{rev}$$

This integral can be shown graphically as the shaded area under the T-S curve of the reversible process, see Figure 6.5.e2.

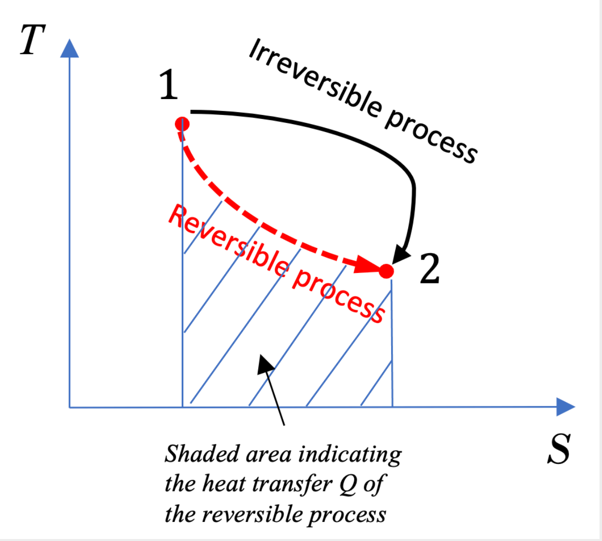

Figure 6.5.e2 *T-S diagram: the shaded area represents the heat transfer of a reversible process.*

(3) The heat transfer of the irreversible process cannot be simply calculated without additional information, and it cannot be shown in the T-S diagram.

Example 2

A **reversible** process from states 1⟶2 in a piston-cylinder is shown in Figure 6.5.e3. Determine whether the change in specific internal energy $\Delta u = u_2 - u_1$, specific work w, and specific heat transfer q are positive, zero, or negative.

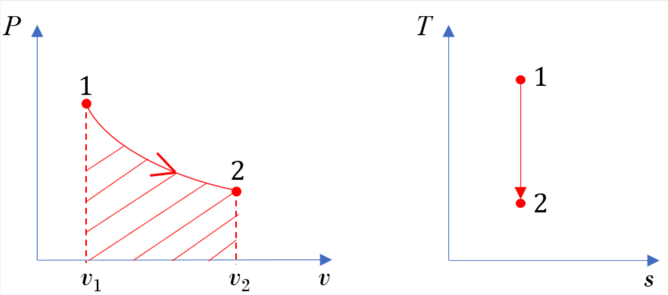

***Figure 6.5.e3** P-v and T-s diagrams of a reversible process*

Solution:

The $P - v$ diagram shows an expansion process in the piston-cylinder. Its specific boundary work can be shown as the shaded area in the $P - v$ diagram, see Figure 6.5.e3. It can also be expressed as

$$_1w_2 = \int_1^2 Pdv > 0$$

From the definition of entropy,

$$dS = \left(\frac{\delta Q}{T}\right)_{rev}$$

The process is reversible; therefore,

$$(\delta Q)_{rev} = TdS$$

\therefore \; {}_{1}Q_{2} = Q_{rev} = \displaystyle\int_{1}^{2} (\delta Q)_{rev} = \displaystyle\int_{1}^{2} TdS

The specific heat transfer is

$${}_{1}q_{2} = \frac{{}_{1}Q_{2}}{m} = \int_{1}^{2} Tds$$

From the $T-s$ diagram in Figure 6.5.e3, $s_1 = s_2$; therefore,

$${}_{1}q_{2} = 0$$

The specific heat transfer in a reversible process can be shown graphically as the area under the process line in the $T-s$ diagram. Note: *this statement is only true for reversible processes*; *it is not valid for irreversible processes*!

Apply the first law of thermodynamics to the piston-cylinder (closed system),

$$\Delta u = {}_{1}q_{2} - {}_{1}w_{2} = 0 - {}_{1}w_{2} < 0$$

In conclusion, the reversible expansion process illustrated in the $P-v$ and $T-s$ diagrams in Figure 6.5.e3 has a positive boundary work and zero heat transfer (adiabatic). The specific internal energy decreases in the process.

6.5.3 Entropy generation, S_{gen}

Entropy generation is another important concept in the second law of thermodynamics. Let us consider a cycle consisting of two processes; process 2⟶1 is a reversible process and process 1⟶2 can be any process, either reversible or irreversible, see Figure 6.5.2. We will apply the definition of entropy and the inequality of Clausius in the following derivations.

$$\because \Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev}$$

$$\begin{aligned}\therefore \oint \frac{\delta Q}{T} &= \int_1^2 \left(\frac{\delta Q}{T}\right) + \int_2^1 \left(\frac{\delta Q}{T}\right)_{rev} \\ &= \int_1^2 \left(\frac{\delta Q}{T}\right) - \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev} \\ &= \int_1^2 \left(\frac{\delta Q}{T}\right) - (S_2 - S_1) \le 0\end{aligned}$$

$$\therefore (S_2 - S_1) \ge \int_1^2 \left(\frac{\delta Q}{T}\right)$$

We may change the above inequality to an equation by introducing entropy generation, $S_{gen} \ge 0$, to the right side; therefore,

\Delta S = (S_2-S_1) = \displaystyle\int_{1}^{2}\left(\displaystyle\frac{\delta Q}{T}\right) + S_{gen} \hspace{3em} \left(S_{gen} \ge 0\right)

This relation is valid for all processes with the "$=$" sign for reversible processes, and the "$>$" sign for irreversible processes. The differential form of the relation can be expressed as

$$dS=\displaystyle\frac{\delta Q}{T}+\delta S_{gen} \hspace{3em} \left(\delta S_{gen} \ge 0\right)$$

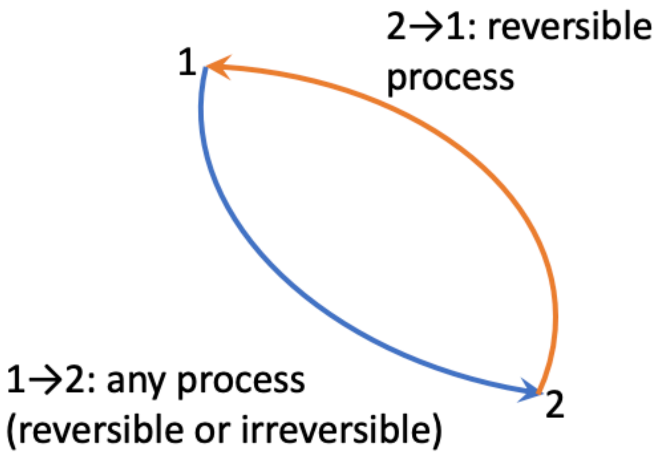

***Figure 6.5.2** A cycle consisting of reversible and/or irreversible processes for introducing entropy generation*

It is important to note that S_{gen} and ΔS are different concepts.

- Entropy generation, S_{gen}, is a measure of the irreversibilities in a process. S_{gen} is NOT a property of the system. It depends on the path of a process; the more irreversible a process is, the larger S_{gen} is.
- Entropy S is a thermodynamic property of the system. It is a

state function. ΔS depends on the initial and final states only, not on the path of a process.
- In general, $\Delta S \neq S_{gen}$. ΔS may be positive or negative, but S_{gen} must be positive for irreversible processes or zero for reversible processes.

Why is entropy generation important? how does it play a role in the universe? The universe is everything, including all the matter and energy that could possibly exist in all space and time. We may treat the universe as an isolated system because nothing exists outside the universe. From the entropy generation, $S_{gen} \geq 0$, we can prove that the entropy in the universe always increases due to the existence of irreversibilities in nature and in all human activities.

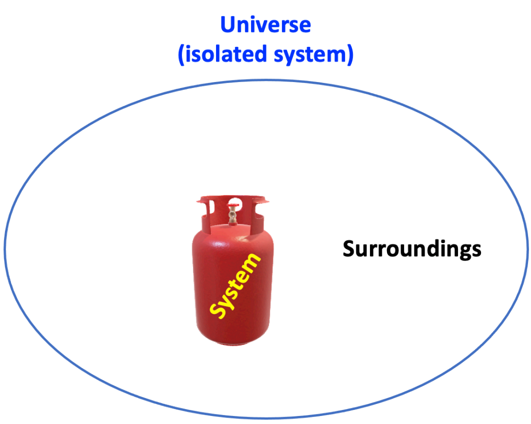

Figure 6.5.3 *The universe as an isolated system*

Let us define a system in the universe and everything outside the system boundary as the surroundings, see Figure 6.5.3. The change of entropy in the universe can be written as

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur}$$

where ΔS_{univ} , ΔS_{sys}, and ΔS_{sur} represent the changes of entropy in the universe, the system, and its surroundings, respectively.

The change of entropy in the universe can be written in terms of entropy generation as

$$\Delta S_{univ} = \int_1^2 \left(\frac{\delta Q}{T}\right) + (S_{gen})_{univ} \quad \text{and}$$
$$(S_{gen})_{univ} \geq 0$$

Since the universe is an isolated system, heat transfer across the universe boundary $Q = 0$. In addition, since all real processes happening in the universe are irreversible, we can drop the "$=$" sign in the inequality; therefore,

$$(S_{gen})_{univ} = \Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur} > 0$$

The above relation indicates that the entropy generation in the universe is always a positive number due to the irreversibilities in all real processes. As a result, the entropy in the universe always increases. This concept can be expressed in a general format as follows if we divide the universe into a number of subsystems.

$$(S_{gen})_{univ} = \Delta S_{univ} = \Delta S_{sys,1} + \Delta S_{sys,2} + \ldots + \Delta S_{sys,n} = \sum_{i=1}^{n} \Delta S_{sys,i} > 0$$

where $\Delta S_{sys,i}$ is the change of entropy in the subsystem, i, in the universe.

6.5.4 The second law of thermodynamics

expressed in terms of entropy generation, S_{gen}

The second law of thermodynamics was previously described with the Kelvin-Planck and Clausius statements. In fact, any device that violates the Kelvin-Planck or Clausius statements must have $S_{gen} < 0$.

Take a heat pump as an example. We may prove that any heat pump that violates the Clausius statement would have $S_{gen} < 0$. A heat pump that violates the Clausius statement would have $Q_H = Q_L$ from the first law of thermodynamics, see Figure 6.5.4. Note that entropy is a state function. For the heat pump cycle, the initial and final states are the same; therefore, $\Delta S = 0$. The change of entropy in the heat pump cycle can be expressed as follows,

$$\because \Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right) + S_{gen} = \frac{-Q_H}{T_H} + \frac{Q_L}{T_L} + S_{gen} = 0$$

and $T_H > T_L$

$$\therefore S_{gen} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = Q_H\left(\frac{1}{T_H} - \frac{1}{T_L}\right) < 0$$

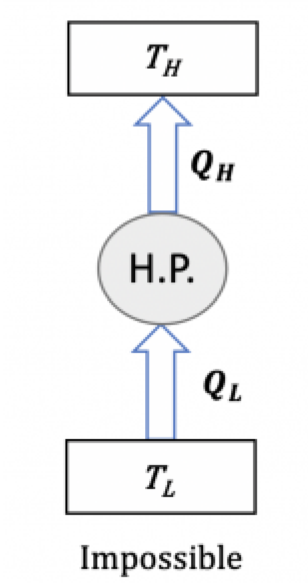

Figure 6.5.4 A *heat pump violating the Clausius statement*

Now, we have proven that any heat pump that violates the Clausius statement would have $S_{gen} < 0$. A similar procedure may be applied to prove that any heat engine that violates the Kelvin-Planck statement would have $S_{gen} < 0$.

In summary, the second law of thermodynamics requires that any process or cycle proceeds in the direction that obeys $S_{gen} \geq 0$, in which the "$=$" sign applies to the ideal Carnot cycles and the "$>$" sign applies to any real, irreversible cycles or processes.

- Actual, irreversible process or cycle: $S_{gen} > 0$
- Carnot, reversible process or cycle: $S_{gen} = 0$

- Impossible process or cycle: $S_{gen} < 0$

Practice Problems

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https://pressbooks.bccampus.ca/thermo1/?p=1835#h5p-44

6.6 The second law of thermodynamics for closed systems

The change of entropy in a system is caused by entropy transfer and entropy generation.

$$\Delta\text{entropy} = +\text{in} - \text{out} + \text{gen}$$

Entropy can be transferred to a system via two mechanisms: (1) heat transfer and (2) mass transfer. It is noted that work is a form of energy transfer; it does NOT contribute to entropy transfer!

For a closed system, entropy is transferred only by heat transfer, see Figure 6.6.1; therefore, the second law of thermodynamics for a closed system undergoing a process from states 1 to 2 can be written as

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{gen} \cong \sum \frac{Q_k}{T_k} + S_{gen} \qquad (S_{gen} \geq 0)$$

This equation is also referred to as the entropy balance equation for closed systems. The integral $\displaystyle\int_1^2 \frac{\delta Q}{T}$ represents the entropy transfer caused by the heat transfer between the system and its surroundings (i.e., heat source and heat sink). Because this integral is difficult to calculate, a common practice is to approximate it with $\displaystyle\sum \frac{Q_k}{T_k}$, where Q_k is the heat transfer that takes place at location k of the system boundary, which has a constant temperature of T_k. If heat transfer occurs in multiple locations, all locations must be considered. Attention should be paid to T_k, which represents the temperature of the system boundary or the surroundings if the boundary and the surroundings are in thermal equilibrium. T_k is NOT the temperature of the system itself!

$$T_k = T_{reserviour} \quad \text{or}$$
$$T_k = T_{boundary}$$
$$T_k \neq T_{system}$$

The entropy balance equation is often used together with the first law of thermodynamics, thermodynamic tables (for real substances), or ideal gas equations (for ideal gases). We will demonstrate the applications of the entropy balance equation in Section 6.8.

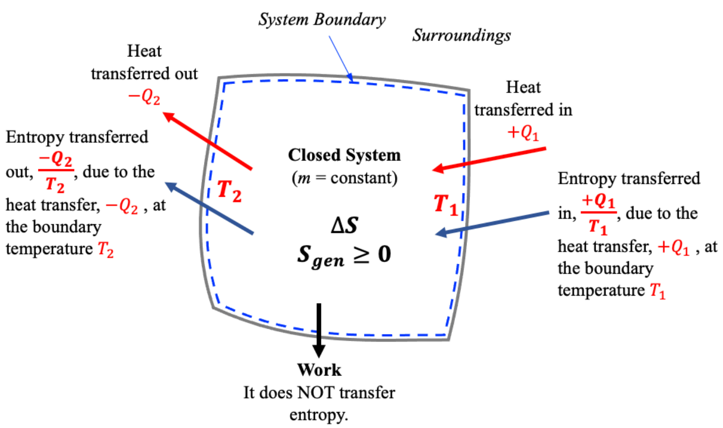

***Figure 6.6.1** Entropy transferred into and out of a closed system due to heat transfer*

Practice Problems

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6.7 Specific entropy of a state

6.7.1 Determining the specific entropy of pure substances by using thermodynamic tables

The specific entropy of a pure substance can be found from thermodynamic tables if the tables are available. The procedures are explained in Section 2.4. In addition to the $P-v$ and $T-v$ diagrams, the $T-s$ diagram is commonly used to illustrate the relation between temperature and specific entropy of a pure substance. Figure 6.7.1 shows the $T-s$ diagram for water.

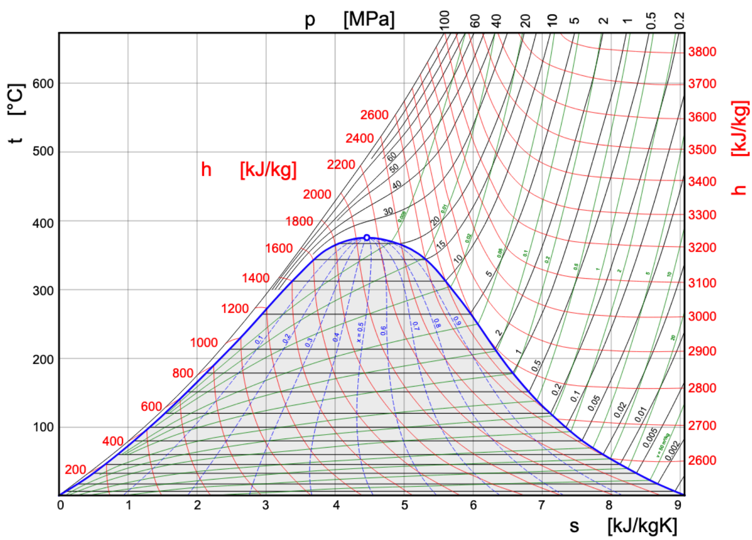

Figure 6.7.1 *T-s diagram for water*

Example 1

Fill in the table.

Substa nce	T, ^oC	P, kPa	v, m^3/kg	Quality *x*	s
Water	250		0.02		
R134a	-2	100			

<u>*Solution:*</u>

Water: T = 250 ^oC, v = 0.2 m^3/kg

From Table A1: T = 250 ^oC, v_f = 0.001252 m^3/kg, v_g = 0.050083 m^3/kg

Since v_f < v < v_g, water at the given state is a two phase mixture; the saturation pressure is P_{sat} = 3976.17 kPa, and s_f = 2.7935 kJ/kgK, s_g = 6.0721 kJ/kgK

The quality is

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.02 - 0.001252}{0.050083 - 0.001252} = 0.383936$$

The specific entropy is

$$\begin{aligned} s &= s_f + x(s_g - s_f) \\ &= 2.7935 + 0.383936 \times (6.0721 - 2.7935) = 4.0523 \text{ kJ/kgK} \end{aligned}$$

R134a: T = -2 ^oC, P = 100 kPa

From Table C1: by examining the saturation pressures at 0 ^oC and – 5 ^oC, we can estimate that the saturation pressure for T = -2 ^oC is about 270 kPa; therefore, R134a at the given state is a superheated vapour.

From Table C2,

P = 100 kPa, T = -10 ^oC, v= 0.207433 m^3/ kg, s = 1.7986 kJ/kgK

P = 100 kPa, T = 0 ^oC, v= 0.216303 m^3/ kg, s = 1.8288 kJ/kgK

Use linear interpolation to find v and s at T = -2 ^oC.

$$\because \frac{v-0.207433}{0.2160303-0.207433}=\frac{s-1.7986}{1.8288-1.7986}=\frac{-2-(-10)}{0-(-10)}$$

\therefore v = 0.214529 \ \rm{m^3/kg}

and $$s = 1.8228\ \mathrm{kJ/kgK}$$

In summary,

Substance	T ^oC	P kPa	v m^3/kg	Quality x	s kJ/kg-K	Phase
Water	250	3976.17	0.02	0.383936	4.0523	two-phase
R134a	-2	100	0.214529	n.a.	1.8228	superheated vapour

Example 2

A rigid tank contains 3 kg of R134a initially at 0^{o}C, 200 kPa. R134a is now cooled until its temperature drops to -20^{o}C. Determine the change in entropy, ΔS, of R134a during this process. Is $\Delta S = S_{gen}$?

Solution:

The initial state is at T_1 = 0^{o}C and P_1 =200 kPa. From Table C2 in Appendix C,

s_1 = 1.7654 kJ/kgK, v_1 = 0.104811 m^3/kg

The tank is rigid; therefore, v_2 = v_1 = 0.104811 m^3/kg.

From Table C1, at T_2 = -20^{o}C:

v_f = 0.000736 m^3/kg, v_g = 0.147395 m^3/kg

s_f = 0.9002 kJ/kgK, s_g = 1.7413 kJ/kgK

Because v_f < v_2 < v_g, the final state is a two-phase mixture.

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.104811 - 0.000736}{0.147395 - 0.000736} = 0.70964$$

$$\begin{aligned} s_2 &= s_f + x_2(s_g - s_f) \\ &= 0.9002 + 0.70964 \times (1.7413 - 0.9002) = 1.4971 \text{ kJ/kgK} \end{aligned}$$

The total entropy change is

$$\Delta S = m(s_2 - s_1) = 3 \times (1.4971 - 1.7654) = -0.8049 \text{ kJ/K}$$

It is important to note that $\Delta S \neq S_{gen}$ in general. The total entropy of R134a decreases in this cooling process, but the entropy generation is always greater than zero in a real process.

6.7.2 Determining the specific entropy of solids and liquids

The specific entropy of a solid or a liquid depends mainly on the temperature. The change of specific entropy in a process from states 1 to 2 can be calculated as,

$$s_2 - s_1 = C_p ln \frac{T_2}{T_1}$$

where

s: specific entropy, in kJ/kgK

C_p: specific heat, in kJ/kgK. Note that for solids and liquids, $C_p = C_v$. Table G2 and Table G3 list the specific heats of selected solids and liquids, respectively.

T: absolute temperature, in Kelvin

6.7.3 Determining the specific entropy of ideal gases

The specific entropy of an ideal gas is a function of both temperature and pressure. Here we will introduce a simplified method for calculating the change of the specific entropy of an ideal gas in a process by assuming constant specific heats. This method is reasonably accurate for a process undergoing a small temperature change.

$$s_2 - s_1 = C_p ln \frac{T_2}{T_1} - R ln \frac{P_2}{P_1}$$
$$s_2 - s_1 = C_v ln \frac{T_2}{T_1} + R ln \frac{v_2}{v_1}$$

where

C_p, C_v and R are the constant-pressure specific heat, constant-volume specific heat, and gas constant, respectively, in kJ/kgK. Table G1 lists these properties of selected ideal gases.

T: absolute temperature, in Kelvin
P: pressure, in kPa
s: specific entropy, in kJ/kgK
v: specific volume, in m^3/kg

Example 3

Air is compressed from an initial state of 100 kPa, 27^{o}C to a final state of 600 kPa, 67^{o}C. Treat air as an ideal gas. Calculate the change of specific entropy, Δs, in this process. Is $\Delta s = s_{gen}$?

<u>*Solution:*</u>

From Table G1: C_p = 1.005 kJ/kgK, R = 0.287 kJ/kgK

$$\Delta s = s_2 - s_1 = C_p ln\frac{T_2}{T_1} - Rln\frac{P_2}{P_1}$$
$$= 1.005ln\frac{273.15+67}{273.15+27} - 0.287ln\frac{600}{100} = -0.3885 \text{ kJ/kgK}$$

It is important to note that $\Delta s \neq s_{gen}$ in general. The specific entropy decreases in this process, but the rate of entropy generation is always greater than zero in a real process.

6.7.4 Isentropic relations for an ideal gas

If a process is reversible and adiabatic, it is called an **isentropic process** and its entropy remains constant. An isentropic process is an idealized process. It is commonly used as a basis for evaluating real processes. The concept of isentropic applies to all substances including ideal gases. The following isentropic relations, however, are ONLY valid for ideal gases.

$$Pv^k = \text{constant} \quad \text{and}$$
$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^k = \left(\frac{T_2}{T_1}\right)^{k/(k-1)}$$

where

$k = \dfrac{C_p}{C_v}$: specific heat ratio. The k values of selected ideal gases can be found in Table G1.

T: absolute temperature, in Kelvin

P: pressure, in kPa

v: specific volume, in m^3/kg

It is noted that the isentropic relation $Pv^k = \text{constant}$ for ideal gases is actually a special case of the polytropic relation $Pv^n = \text{constant}$ with $n = k = \dfrac{C_p}{C_v}$.

Example 4

Derive the isentropic relation $Pv^k = \text{constant}$

<u>*Solution:*</u>

For an ideal gas undergoing an isentropic process,

$$\Delta s = s_2 - s_1 = C_p ln\frac{T_2}{T_1} - Rln\frac{P_2}{P_1} = 0$$

Substitute $C_p = \dfrac{kR}{k-1}$ in the above equation and rearrange,

$$\because \frac{k}{k-1} ln\frac{T_2}{T_1} = ln\frac{P_2}{P_1}$$

$$\therefore ln\left(\frac{T_2}{T_1}\right)^{\dfrac{k}{k-1}} = ln\frac{P_2}{P_1}$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\dfrac{k}{k-1}}$$

Combine with the ideal gas law, $Pv = RT$,

$$\therefore \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{P_2 v_2}{P_1 v_1}\right)^{\frac{k}{k-1}}$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$$

$$\therefore Pv^k = \text{constant}$$ and

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

Practice Problems

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6.8 Applications of the second law of thermodynamics in closed systems

The first and second laws of thermodynamics are often used together with the thermodynamic tables or ideal gas equations in thermal analysis. The following strategy may be adapted when solving problems requiring the second law of thermodynamics:

1. Identify the process(es), e.g., isobaric, isothermal, isochoric, or isentropic process.
2. Determine the initial, final, and any intermediate states. Find the properties, such as, P, T, v, u, s, by using the thermodynamic tables or equations, e.g., for an ideal gas, solid, or liquid.
3. Determine the heat transfer, Q, or other unknowns by applying the first law of thermodynamics for closed systems.
4. Determine the entropy generation, S_{gen}, or other unknowns by applying the second law of thermodynamics for closed systems.

Example 1

A piston-cylinder contains ammonia at 2000 kPa, 80°C. The piston is loaded with a linear spring, see Figure 6.8.e1. The outside ambient is at 15°C. The ammonia is now cooled down to saturated liquid at 15°C. Assuming the cylinder is always at the ambient temperature during the cooling process, determine the specific boundary work, the specific heat transfer, and the specific entropy generation in the process.

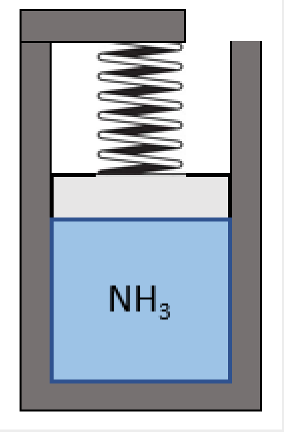

Figure 6.8.e1 A *piston-cylinder device containing pressurized ammonia*

<u>*Solution:*</u>

Analysis:

- Ammonia in the piston-cylinder device can be treated as a closed system.
- As the piston is loaded with a linear spring, the

pressure of ammonia changes linearly with its specific volume, see the $P - v$ diagram, Figure 6.8.e2. Please refer to example 5 in Section 4.3 for a detailed analysis. The specific boundary work can be found from

$$_1w_2 = \int_1^2 P dv = \frac{1}{2}(P_1 + P_2)(v_2 - v_1)$$

- Apply both the first and second laws of thermodynamics to ammonia, we can then find the specific heat transfer and specific entropy generation.

The first law: $\Delta u = {}_1q_2 - {}_1w_2$

The second law: $\Delta s = \sum \frac{_1q_2}{T_{surr}} + s_{gen}$

- We will need to determine the following properties to complete the calculations.

State 1: P_1, v_1, u_1, s_1

State 2: P_2, v_2, u_2, s_2

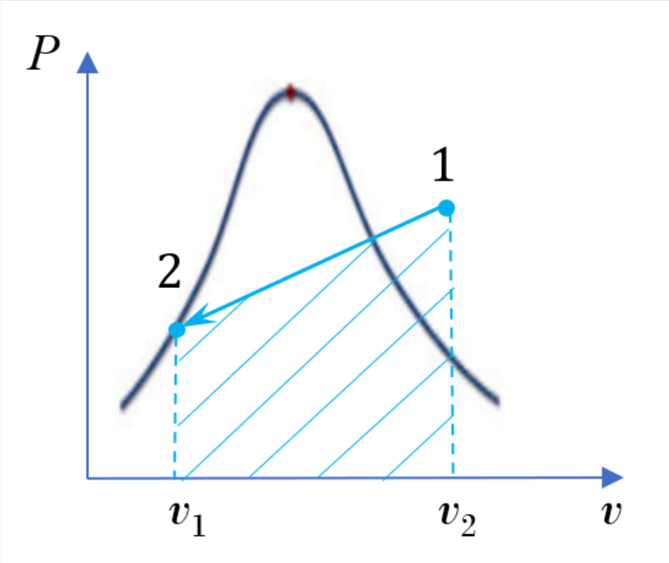

***Figure 6.8.e2** P-v diagram of ammonia in a piston-cylinder device*

Now let us solve the problem in detail.

From Table B1: T_1 = 80°C, P_{sat} = 4.14197 MPa. Ammonia in state 1 is a superheated vapour because P_1 = 2000 kPa = 2 MPa < P_{sat}. From Table B2: for state 1 at T_1 = 80°C, P_1 = 2000 kPa, v_1 = 0.075952 m^3/kg, u_1 = 1583.81 kJ/kg, s_1 = 5.8292 kJ/kgK

State 2 is a saturated liquid at T_2 = 15°C. From Table B1, P_2 = 728.53 kPa, $v_2 = v_f$ = 0.001619 m^3/kg, u_2 = 412.06 kJ/kg, $s_2 = s_f$ = 1.7197 kJ/kgK

The specific boundary work is

$$\begin{aligned} {}_1w_2 &= \frac{1}{2}(P_1 + P_2)(v_2 - v_1) \\ &= \frac{1}{2}(2000 + 728.53)(0.001619 - 0.075952) = -101.41 \text{ kJ/kg} \end{aligned}$$

The specific heat transfer is

$$\begin{aligned} {}_1q_2 &= \Delta u + {}_1w_2 \\ &= (412.06 - 1583.81) + (-101.41) = -1273.16 \text{ kJ/kg} \end{aligned}$$

The specific entropy generation is

$$\begin{aligned} s_{gen} &= \Delta s - \sum \frac{{}_1q_2}{T_{surr}} \\ &= (s_2 - s_1) - \sum \frac{{}_1q_2}{T_{surr}} \\ &= (1.7197 - 5.8292) - \frac{-1273.16}{(273.15 + 15)} = 0.3089 \text{ kJ/kgK} > 0 \end{aligned}$$

Comment:

- The cooling process is irreversible; therefore, the specific entropy generation, $s_{gen} > 0$.
- When applying the second law of thermodynamics, it is important to note that T_{surr} is the absolute temperature (in Kelvin) of the system boundary or the surroundings if the boundary is in thermal equilibrium with the surroundings.

Example 2

Three kilograms of CO_2 at 150 kPa, 300 K is mixed with two kilograms of CO_2 at 220 kPa, 500 K, in a rigid, well-insulated tank, see Figure 6.8.e3. Find the final state (P, *T*) and the entropy generation in this process. Assume CO_2 is an ideal gas in this mixing process.

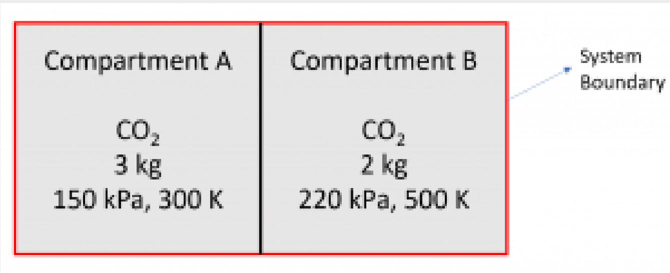

Figure 6.8.e3 *A rigid tank with two compartments containing* CO_2

<u>*Solution*</u>*:*

Analysis:

- CO_2 in the whole tank can be treated as a closed system.
- The mixing occurs after the partition is removed. As the tank is well-insulated, the heat transfer between the system and the surroundings is zero in this mixing process: ${}_{1}Q_{2}=0$
- Since the tank is rigid, the total volume of CO_2 remains constant; therefore, the boundary work is zero in the mixing process: ${}_{1}W_{2}=0$
- Apply the first law to the system (whole tank)

$$\Delta U = {}_1Q_2 - {}_1W_2 = 0$$

- The entropy generation can be found by applying the second law to the system.

$$\because \Delta S = \sum \frac{{}_1Q_2}{T_{surr}} + S_{gen} \quad \text{and}$$

$${}_1Q_2 = 0$$

$$\therefore S_{gen} = \Delta S$$

- To complete the calculation, we will apply the ideal gas relations to determine the final pressure, temperature, Δu, and Δs.

Now let us solve the problem in detail.

From Table G1 for CO_2: R = 0.1889 kJ/kgK, C_p = 0.846 kJ/kgK, C_v = 0.657 kJ/kgK.

First, the volumes of compartments A and B at the initial state, state 1, can be found from the ideal gas law.

$$\because P\mathbb{V} = mRT$$

$$\therefore \mathbb{V}_A = \frac{m_{1A}RT_{1A}}{P_{1A}} = \frac{3 \times 0.1889 \times 300}{150} = 1.1334 \text{ m}^3$$

$$\therefore \mathbb{V}_B = \frac{m_{1B}RT_{1B}}{P_{1B}} = \frac{2 \times 0.1889 \times 500}{220} = 0.8586 \text{ m}^3$$

The total volume of the tank is

$$\mathbb{V}_{tot} = \mathbb{V}_A + \mathbb{V}_B = 1.1334 + 0.8586 = 1.992 \text{ m}^3$$

Next, the final temperature can be calculated by applying the first law to the whole tank. Note that the tank is rigid and well insulated; therefore, $\Delta U = 0$.

$$\because \Delta U = (m_{1A} + m_{1B})u_2 - (m_{1A}u_{1A} + m_{1B}u_{1B}) = 0$$

$$\therefore m_{1A}(u_2 - u_{1A}) + m_{1B}(u_2 - u_{1B}) = 0$$

$$\text{and} \quad \Delta u = C_v \Delta T$$

$$\therefore m_{1A}C_v(T_2 - T_{1A}) + m_{1B}C_v(T_2 - T_{1B}) = 0$$

$$\therefore T_2 = \frac{m_{1A}T_{1A} + m_{1B}T_{1B}}{m_{1A} + m_{1B}}$$

$$= \frac{3 \times 300 + 2 \times 500}{3 + 2} = 380 \text{ K}$$

Then, the final pressure can be determined from the ideal gas law.

$$P_2 = \frac{(m_{1A} + m_{1B})RT_2}{\mathbb{V}_{tot}}$$

$$= \frac{(3 + 2) \times 0.1889 \times 380}{1.992} = 180.17 \text{ kPa}$$

Last, the entropy generation can be calculated from the second law.

$$S_{gen} = \Delta S$$

$$= (m_{1A} + m_{1B})s_2 - (m_{1A}s_{1A} + m_{1B}s_{1B})$$

$$= m_{1A}(s_2 - s_{1A}) + m_{1B}(s_2 - s_{1B})$$

\begin{align*} s_2 - s_{1A} &= C_pln\dfrac{T_2}{T_{1A}} - Rln\dfrac{P_2}{P_{1A}} \\&= 0.846ln\dfrac{380}{300} - 0.1889ln\dfrac{180.17}{150} = 0.16536 \ \rm{kJ/kgK} \ >\ 0\end{align*}

$$
\begin{aligned}
s_2 - s_{1B} &= C_p ln\frac{T_2}{T_{1B}} - Rln\frac{P_2}{P_{1B}} \\
&= 0.846 ln\frac{380}{500} - 0.1889 ln\frac{180.17}{220} = -0.19445 \text{ kJ/kgK} \ < \ 0
\end{aligned}
$$

$$
\begin{aligned}
\therefore S_{gen} &= m_{1A}(s_2 - s_{1A}) + m_{1B}(s_2 - s_{1B}) \\
&= 3 \times 0.16536 + 2 \times (-0.19445) = 0.1072 \text{ kJ/K}
\end{aligned}
$$

Comment:

The entropy in a process can increase or decrease. In this example, the entropy of CO_2 originally in compartment A increases and the entropy of CO_2 originally in compartment B decreases in this mixing process, but the entropy generation $S_{gen} \geq 0$ because the mixing process is irreversible.

Practice Problems

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https://pressbooks.bccampus.ca/thermo1/?p=1838#h5p-47

6.9 The second law of thermodynamics for open systems

Entropy can be transferred to a system via two mechanisms: (1) heat transfer and (2) mass transfer. For open systems, the second law of thermodynamics is often written in the rate form; therefore, we are interested in the time rate of entropy transfer due to heat transfer and mass transfer.

$$\dot{S}_{heat}=\frac{dS_{heat}}{dt}\cong\sum\frac{\dot{Q}_k}{T_k}$$

$$\dot{S}_{mass}=\sum\frac{dS_{mass}}{dt}=\sum\dot{m}_k s_k$$

where

$\dot{m}$: rate of mass transfer

$\dot{Q}_k$: rate of heat transfer via the location k of the system boundary, which is at a temperature of T_k in Kelvin

$\dot{S}_{heat}$: time rate of entropy transfer due to heat transfer

$\dot{S}_{mass}$: time rate of entropy transfer that accompanies the mass transfer into or out of a control volume

s_k: specific entropy of the fluid

Applying the entropy balance equation, $\Delta\text{entropy}=+\text{in}-\text{out}+\text{gen}$, to a control volume, see Figure 6.9.1, we can write the following equations:

- General equation for both steady and transient flow devices

$$\frac{dS_{c.v.}}{dt} = \left(\sum \dot{m}_i s_i + \sum \frac{\dot{Q}_{c.v.}}{T}\right) - \left(\sum \dot{m}_e s_e\right) + \dot{S}_{gen} \qquad (\dot{S}_{gen} \ge 0)$$

- For steady-state, steady-flow devices, $\dfrac{dS_{c.v.}}{dt} = 0$; therefore,

\displaystyle \sum{{\dot{{m}}}_{e}{s}_{e}}-\sum{{\dot{{m}}}_{i}{s}_{i}}=\displaystyle \sum\dfrac{{\dot{{Q}}}_{{c}.{v}.}}{{T}}+{\dot{{S}}}_{{gen}}\\ \ \ \ \ ({\dot{{S}}}_{{gen}} \ge 0)

- For steady and **isentropic** flow devices, $\dot{Q}_{c.v.} = 0$ and $\dot{S}_{gen} = 0$; therefore,

$$\sum \dot{m}_e s_e = \sum \dot{m}_i s_i$$

where

$\dot{m}$: rate of mass transfer of the fluid entering or leaving the control volume via the inlet i or exit e, in kg/s

$\dot{Q}_{c.v.}$: rate of heat transfer into the control volume via the system boundary (at a constant T), in kW

$S_{c.v.}$: entropy in the control volume, in kJ/K

$\dfrac{dS_{c.v.}}{dt}$: time rate of change of entropy in the control volume, in kW/K

$\dot{S}_{gen}$: time rate of entropy generation in the process, in kW/K

s: specific entropy of the fluid entering or leaving the control volume via the inlet i or exit e, in kJ/kgK

T: absolute temperature of the system boundary, in Kelvin

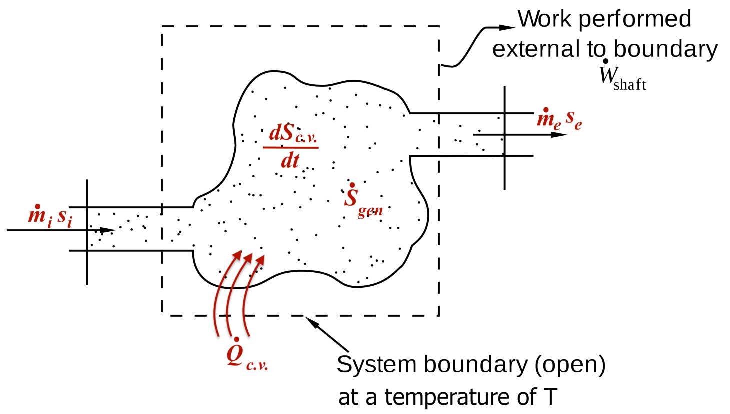

Figure 6.9.1 *Flow through a control volume, showing the entropy transfers and entropy generation*

Example 1

The diagrams in Figure 6.9.e1 show a reversible process in a steady-state, single flow of air. The letters i and e represent the initial and final states, respectively. Treat air as an ideal gas and assume $\Delta KE = \Delta PE = 0$. Are the change in specific enthalpy $\Delta h = h_e - h_i$, specific work w, and specific heat transfer q positive, zero, or negative values? What is the relation between w and q?

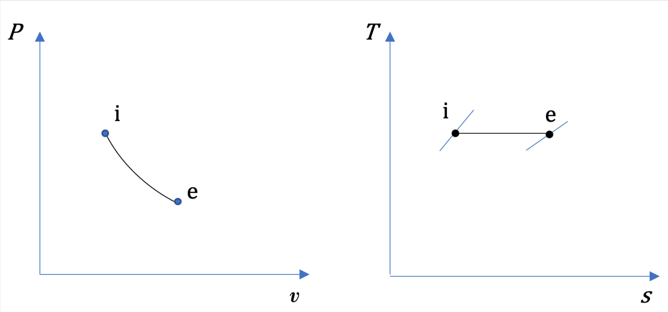

Figure 6.9.e1 *T-s and P-v diagrams of a reversible process for an ideal gas*

<u>*Solution:*</u>

The specific work can be evaluated mathematically and graphically.

(1) Mathematically,

$$\because v_e > v_i$$

$$\therefore w = \int_i^e Pdv \ > \ 0$$

(2) Graphically, the specific work is the area under the process curve in the $P - v$ diagram; therefore w is positive, see Figure 6.9.e2.

In a similar fashion, the specific heat transfer can also be evaluated graphically and mathematically.

(1) Graphically,

$$\because ds = \left(\frac{\delta q}{T}\right)_{rev}$$

\therefore q_{rev} = \displaystyle \int_{i}^{e}{Tds} = T(s_e-s_i)\ >\ 0

For a reversible process, the area under the process curve in the $T-s$ diagram represents the specific heat transfer of the reversible process; therefore $q=q_{rev}$ is positive, see Figure 6.9.e2.

(2) The same conclusion, $q_{rev}>0$, can also be derived from the second law of thermodynamics mathematically, as follows.

$$\dot{m}(s_e-s_i)=\sum\frac{\dot{Q}}{T_{surr}}+\dot{S}_{gen}$$

For a reversible process, $\dot{S}_{gen}$ = 0, and the fluid is assumed to be always in thermal equilibrium with the system boundary, or $T=T_{surr}$; therefore,

$$q_{rev}=\frac{\dot{Q}}{\dot{m}}=T(s_e-s_i)>0$$

The change in specific enthalpy can then be evaluated. For an ideal gas,

$$\Delta h=h_e-h_i=C_p(T_e-T_i)$$

$$\because T_e=T_i$$

$$\therefore h_e=h_i \text{ and } \Delta h=0$$

Now, we can determine the relation between w and q_{rev} from the first law of thermodynamics for control volumes.

$$\because \dot{m}(h_e - h_i) = \dot{Q}_{rev} - \dot{W} = 0$$

$$\therefore \dot{Q}_{rev} = \dot{W}$$

$$\therefore q_{rev} = w$$

In this reversible process, the specific heat transfer and specific work must be the same. Graphically, the two areas under the $P - v$ and $T - s$ diagrams must be the same.

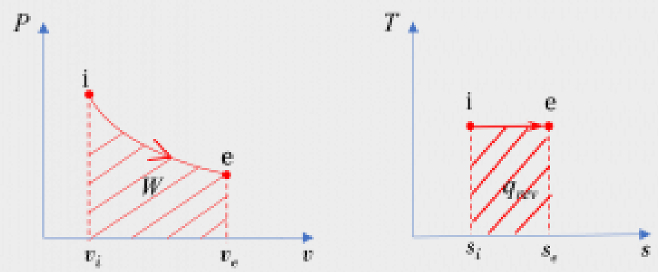

Figure 6.9.e2 *T-s and P-v diagrams, showing the solutions for a reversible process of an ideal gas*

Practice Problems

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6.10 Applications of the second law of thermodynamics in open systems

When solving problems in a control volume, the first and second laws of thermodynamics are often used together with the continuity equation and the thermodynamic tables or the ideal gas equation. The following strategy may be adapted:

1. Set up a proper control volume to enclose the device of interest and identify the flow condition, i.e., steady or transient flow, through the control volume.
2. Determine the relations among the mass flow rates at the inlet(s) and outlet(s) by using the continuity equation.
3. Find the fluid properties at the inlet(s) and outlet(s), such as, P, T, v, h, s, by using the thermodynamic tables or equations, e.g., for an ideal gas, solid or liquid.
4. Determine the rate of heat transfer, $\dot{Q}$, or other unknowns by applying the first law of thermodynamics for open systems;
5. Determine the rate of entropy generation, $\dot{S}_{gen}$, or other unknowns by applying the second law of thermodynamics for open systems.

Example 1

Steam is used to provide heating to air in a building through a well-insulated heat exchanger, see Figure 6.10.e1.

- Saturated steam at 100^{o}C enters the heat exchanger at a mass flow rate of 0.5 kg/s and leaves the heat exchanger as a saturated liquid at 100^{o}C
- Air enters the heat exchanger at 5^{o}C, 101 kPa, and leaves at 25^{o}C, 101 kPa.

Assume air is an ideal gas. Determine the mass flow rate of air and $\dot{S}_{gen}$ in this process.

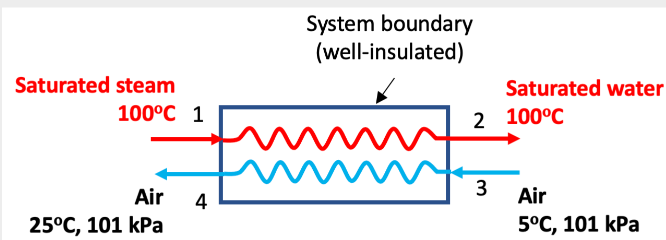

Figure 6.10.e1 *Well-insulated heat exchanger*

<u>*Solution:*</u>

Analysis

- The mass flow rate of air can be found by using the first law of thermodynamics

$$\because \dot{m}_w(h_1 - h_2) = \dot{m}_a(h_4 - h_3) = \dot{m}_a C_p(T_4 - T_3)$$

$$\therefore \dot{m}_{a} = \dot{m}_{w} \dfrac{h_1 - h_2}{C_p(T_4 - T_3)}$$

- The rate of entropy generation can be found by using the second law of thermodynamics. Note that the heat exchanger is well-insulated.

$$\because \sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

and $\dot{Q}_k = 0$

\begin{align*}\therefore \dot{S}_{gen} &= \sum\dot{m}_{e}s_{e} - \sum\dot{m}_{i}s_{i} \\&= (\dot{m}_{w}s_{2} + \dot{m}_{a}s_{4}) - (\dot{m}_{w}s_{1} + \dot{m}_{a}s_{3}) \\&= \dot{m}_{w}(s_{2}-s_{1}) + \dot{m}_{a}(s_{4} - s_{3}) \end{align*}

- The following properties are needed in order to complete the calculations:
  - For the steam-water stream: h_1 , h_2 , s_1 , s_2
  - For the air stream: s_3 , s_4

Now, let us solve the problem in detail.

From Table A1, we can find the specific enthalpies and specific entropies of the saturated steam and saturated liquid water at 100^{o}C.

$h_1 = h_g$ = 2675.57 kJ/kg , $h_2 = h_f$ = 419.17 kJ/kg

$s_1 = s_g$ = 7.3541 kJ/kgK, $s_2 = s_f$ = 1.3072 kJ/kgK

From Table G1 for air: C_p = 1.005 kJ/kgK, R = 0.287 kJ/kgK

$$\because s_4 - s_3 = C_p ln \frac{T_4}{T_3} - R ln \frac{P_4}{P_3}$$

and $P_4 = P_3$

$$\therefore s_4 - s_3 = 1.005 ln \frac{273.15+25}{273.15+5} = 0.06978 \text{ kJ/kgK}$$

Now, we can complete the calculations. The mass flow rate of air is

$$\begin{aligned} \dot{m}_a &= \dot{m}_w \frac{h_1 - h_2}{C_p(T_4 - T_3)} \\ &= 0.5 \times \frac{2675.57 - 419.17}{1.005(25-5)} = 56.13 \text{ kg/s} \end{aligned}$$

The rate of entropy generation is

$$\begin{aligned} \dot{S}_{gen} &= \dot{m}_w(s_2 - s_1) + \dot{m}_a(s_4 - s_3) \\ &= 0.5 \times (1.3072 - 7.3541) + 56.13 \times 0.06978 \\ &= 0.8934 \text{ kW/K } > 0 \end{aligned}$$

Comment:

The heat transfer process is irreversible; therefore, the rate of entropy generation is greater than zero.

Example 2

R134a at 300 kPa, 20°C is heated to 50°C isobarically in a heat exchanger. The mass flow rate of R134a is 1.5 kg/s. The heat at a rate of $\dot{Q}_H$ is supplied by a heat pump, which absorbs heat at a rate of $\dot{Q}_L$ from the ambient at 280 K, see Figure 6.10.e2. If $COP_{HP} = 5$, and there is no heat loss in the heat exchanger, find the power input, $\dot{W}_{HP}$, and the rate of heat transfer, $\dot{Q}_L$. Is this setup consisting of the heat exchanger and heat pump possible?

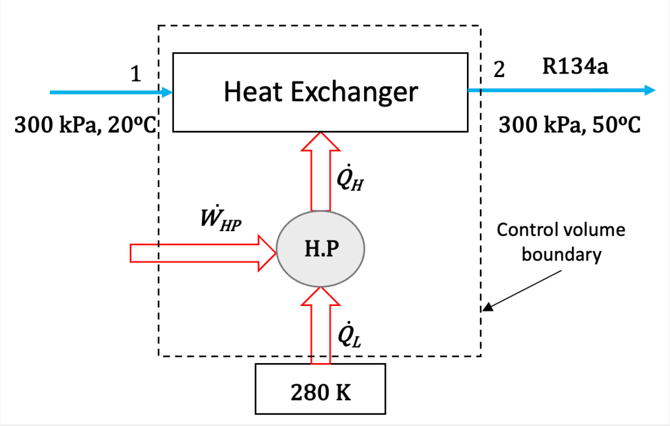

Figure 6.10.e2 *A device consisting of a heat exchanger and a heat pump*

<u>*Solution:*</u>

The control volume is set up to enclose both the heat pump and heat exchanger, as shown in Figure 6.10.e2.

From Table C1, we can tell that, at the given pressures and temperatures, R134a remains a superheated vapour at both the inlet and the outlet of the heat exchanger. The specific enthalpies and specific entropies of R134a can be found in Table C2.

- At the inlet, P_1 = 300 kPa, T_1= 20^{o}C; therefore, h_1 = 416.24 kJ/kg, s_1 = 1.7876 kJ/kgK
- At the outlet, P_2 = 300 kPa, T_2 = 50^{o}C; therefore, h_2 = 443.31 kJ/kg, s_1 = 1.8755 kJ/kgK

Apply the first law of thermodynamics to the control volume.

$$\dot{m}h_1 + \dot{Q}_L + \dot{W}_{HP} = \dot{m}h_2$$

For the heat pump

$$COP_{HP} = \frac{\dot{Q}_H}{\dot{W}_{HP}} \quad \text{and}$$

$$\dot{Q}_L + \dot{W}_{HP} = \dot{Q}_H$$

Combine the above three relations, we can derive

$$\dot{W}_{HP} = \frac{\dot{m}(h_2 - h_1)}{COP_{HP}}$$
$$= \frac{1.5 \times (443.31 - 416.24)}{5} = 8.121 \text{ kW}$$

$$\dot{Q}_L = (COP_{HP} - 1)\dot{W}_{HP}$$
$$= (5 - 1) \times 8.121 = 32.484 \text{ kW}$$

Apply the second law of thermodynamics to the control volume.

$$\because \dot{m}(s_2 - s_1) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

$$\therefore \dot{S}_{gen} = \dot{m}(s_2 - s_1) - \frac{\dot{Q}_L}{T_{surr}}$$
$$= 1.5 \times (1.8755 - 1.7876) - \frac{32.484}{280} = 0.0158 \text{ kW/K} \ > 0$$

Comment:

The rate of entropy generation in this control volume (consisting of both heat exchanger and heat pump) is positive; therefore, it is very likely that this setup can work in reality, and the processes in the heat exchanger and heat pump are irreversible.

To further verify this, we can calculate the rate of entropy generation for each individual device, i.e., heat exchanger and heat pump, separately. However, with the given information, there exists a challenge to evaluate the amount of entropy transfer due to the heat transfer provided by the heat pump to the heat exchanger. Strictly speaking, $\int \frac{\dot{Q}}{T_b}$ should be evaluated because the temperature of the heat exchanger varies, leading to a possible variation of temperature, T_b, at the boundary between the heat pump and the heat exchanger. The relation between the instantaneous rate of heat transfer from the heat pump to the heat exchanger, $\dot{Q}$, and the boundary temperature, T_b, must be obtained to enable a detailed analysis.

Here, let us perform a simplified calculation to demonstrate the concept. Assume that T_b remains constant as the average temperature between the inlet and the outlet of the heat exchanger, thus

$$T_b = 35^{\circ}\text{C} = 308.15\ \text{K}.$$

Apply the second law of thermodynamics to the heat pump. Note that $\dot{Q}_H = \dot{Q}_L + \dot{W}_{HP}$, and $\Delta s = 0$ for the heat pump cycle.

$$\begin{aligned}\dot{S}_{gen,HP} &= \dot{m}\Delta s - \frac{\dot{Q}_L}{T_{surr}} + \frac{\dot{Q}_H}{T_b} \\ &= 0 - \frac{32.484}{280} + \frac{32.484 + 8.121}{308.15} = 0.015755 \text{ kW/K } > 0\end{aligned}$$

Furthermore, the COP of the Carnot cycle operating between the heat sink of 280 K and the heat source of 308.15 K is

COP_{HP, rev} = \dfrac{308.15}{308.15-280}=10.9 > 5=COP_{HP}

Because $\dot{S}_{gen,HP} > 0$ and $COP_{HP} < COP_{HP,rev}$, this heat pump cycle is an irreversible, realistic cycle.

Similarly, let us apply the second law of thermodynamics to the heat exchanger.

$$\begin{aligned}\dot{S}_{gen,HE} &= \dot{m}(s_2 - s_1) - \frac{\dot{Q}_H}{T_b} \\ &= 1.5 \times (1.8755 - 1.7876) - \frac{32.484 + 8.121}{308.15} \\ &= 8 \times 10^{-5} \text{ kW/K} \approx 0\end{aligned}$$

Because $\dot{S}_{gen,HE} \approx 0$, the process in this heat exchanger is almost ideal and reversible. It is noted that heat losses are typically unavoidable in real devices, but they are not considered in this example. Due to heat losses from the heat exchanger, the real process will have $\dot{S}_{gen,HE} > 0$. Consequently, the outlet temperature may not be able to reach 50^{o}C.

Example 3

Consider an adiabatic two-stage compressor having an intercooler, as shown in Figure 6.10.e3.

- Air at 10°C, 101 kPa enters the first stage of the compressor and is compressed to 300 kPa, 150°C.
- After being cooled to 30°C in a well-insulated intercooler, the air then enters the second stage cylinder and is compressed to 900 kPa.

Assume that (1) the pressure drop in the intercooler is negligible, (2) air is an ideal gas, and (3) the two compression processes are polytropic with the same polytropic exponent. Determine

1. The total specific work of the two-stage compressor.
2. The specific entropy generation in each of the two stages.
3. The total specific entropy generation in the system consisting of the two-stage compressor and the intercooler, assuming a constant temperature of 30°C at the system boundary.
4. Assume there is no intercooler. The air from the first stage enters directly to the second stage and is compressed to 900 kPa, what is the air temperature at the exit of the second stage? What is the total specific work of the compressor? Assume the polytropic exponent remains the same in both stages.

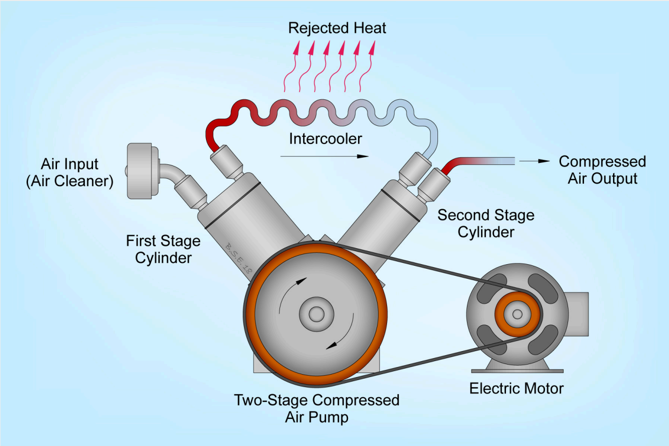

Figure 6.10.e3 *Two stage compressor with an intercooler*

<u>*Solution:*</u>

(1) Assume air is an ideal gas. From Table G1 for air: R = 0.287 kJ/kgK, C_p = 1.005 kJ/kgK.

The compressor is adiabatic with the following inlet and outlet conditions for each stage:

- Stage 1
  - Inlet P_1 = 101 kPa, T_1 = 10°C
  - Outlet P_2 = 300 kPa, T_2 =150°C

  Therefore, the specific work of stage 1 is

$$\begin{aligned} w_1 &= h_2 - h_1 = C_p(T_2 - T_1) \\ &= 1.005 \times (150 - 10) = 140.7 \text{ kJ/kg} \end{aligned}$$

  The polytropic exponent can be found by

combining the polytropic relation and the ideal gas law.

$$\because P_1 v_1^n = P_2 v_2^n \quad \text{and}$$
$$Pv = RT$$

$$\therefore \dfrac{P_2}{P_1} = \left(\dfrac{T_2}{T_1}\right)^{\dfrac{n}{n-1}}$$

$$\therefore n = \dfrac{ln\dfrac{P_2}{P_1}}{ln\left(\dfrac{P_2 T_1}{P_1 T_2}\right)}$$

$$= \dfrac{ln\dfrac{300}{101}}{ln\left[\dfrac{300 \times (273.15+10)}{101 \times (273.15+150)}\right]} = 1.58486$$

- Stage 2
  - Inlet P_3 = 300 kPa, T_3 = 30°C
  - Outlet P_4 = 900 kPa

The air temperature at the outlet of stage 2 can be found from the polytropic relation.

$$\because \dfrac{P_4}{P_3} = \left(\dfrac{T_4}{T_3}\right)^{\dfrac{n}{n-1}}$$

\begin{align*} \therefore T_4 &= T_3 \left(\dfrac{P_4}{P_3}\right)^{\dfrac{n-1}{n}} \\&= (273.15 + 30) \times \dfrac{900}{300}^{\dfrac{1.58486 -1}{1.58486}} \\&= 454.7 \ \rm{K} = 181.56^ \rm{o} \rm{C}\end{align*}

Therefore, the specific work of stage 2 is

$$w_2 = h_4 - h_3 = C_p(T_4 - T_3)$$
$$= 1.005(181.56 - 30) = 152.31 \text{ kJ/kg}$$

The total specific work of the two stage compressor is

$$w_{tot} = w_1 + w_2 = 140.7 + 152.31 = 293 \text{ kJ/kg}$$

(2) Both stages are adiabatic; therefore, for each stage

$$\because \dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$

and $\dot{Q}_k = 0$

$$\therefore s_{gen} = \frac{\dot{S}_{gen}}{\dot{m}} = s_e - s_i = C_p ln\frac{T_e}{T_i} - Rln\frac{P_e}{P_i}$$

For stage 1,

$$s_{gen,1} = C_p ln\frac{T_2}{T_1} - Rln\frac{P_2}{P_1}$$
$$= 1.005ln\frac{(273.15 + 150)}{(273.15 + 10)} - 0.287ln\frac{300}{101}$$
$$= 0.0913 \text{ kJ/kgK}$$

For stage 2,

$$s_{gen,2} = C_p ln\frac{T_4}{T_3} - Rln\frac{P_4}{P_3}$$
$$= 1.005ln\frac{454.7}{(273.15 + 30)} - 0.287ln\frac{900}{300}$$
$$= 0.0921 \text{ kJ/kgK}$$

(3) To calculate the total specific entropy generation in

the system consisting of the compressor and the heat exchanger, we can set up the control volume as shown in Figure 6.10.e4.

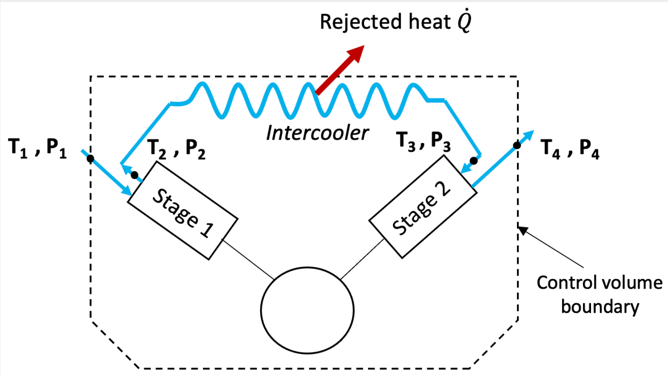

Figure 6.10.e4 *Schematic of a two-stage compressor with an intercooler*

The specific heat transfer rejected from the cooler to the surroundings is

$$q = \frac{\dot{Q}}{\dot{m}} = h_2 - h_3 = C_p(T_2 - T_3)$$
$$= 1.005 \times (150 - 30) = 120.6 \text{ kJ/kg}$$

Apply the second law of thermodynamics to the control volume as outlined in Figure 6.10.e4, we can find the total specific entropy generation.

$$\because \dot{m}(s_e - s_i) = \sum \frac{\dot{Q}_k}{T_k} + \dot{S}_{gen}$$