

FIGURE 7.23 Energy consumed by humans is converted to work, thermal energy, and stored fat. By far the largest fraction goes to thermal energy, although the fraction varies depending on the type of physical activity.

Power Consumed at Rest

The *rate* at which the body uses food energy to sustain life and to do different activities is called the **metabolic rate**. The total energy conversion rate of a person *at rest* is called the **basal metabolic rate (BMR)** and is divided among various systems in the body, as shown in [Table 7.4](#). The largest fraction goes to the liver and spleen, with the brain coming next. Of course, during vigorous exercise, the energy consumption of the skeletal muscles and heart increase markedly. About 75% of the calories burned in a day go into these basic functions. The BMR is a function of age, gender, total body weight, and amount of muscle mass (which burns more calories than body fat). Athletes have a greater BMR due to this last factor.

Organ	Power consumed at rest (W)	Oxygen consumption (mL/min)	Percent of BMR
Liver & spleen	23	67	27
Brain	16	47	19
Skeletal muscle	15	45	18
Kidney	9	26	10
Heart	6	17	7
Other	16	48	19
Totals	85 W	250 mL/min	100%

TABLE 7.4 Basal Metabolic Rates (BMR)

Energy consumption is directly proportional to oxygen consumption because the digestive process is basically one of oxidizing food. We can measure the energy people use during various activities by measuring their oxygen use. (See [Figure 7.24](#).) Approximately 20 kJ of energy are produced for each liter of oxygen consumed, independent of the type of food. [Table 7.5](#) shows energy and oxygen consumption rates (power expended) for a variety of activities.

Power of Doing Useful Work

Work done by a person is sometimes called **useful work**, which is *work done on the outside world*, such as lifting weights. Useful work requires a force exerted through a distance on the outside world, and so it excludes internal work, such as that done by the heart when pumping blood. Useful work does include that done in climbing stairs or accelerating to a full run, because these are accomplished by exerting forces on the outside world. Forces exerted by the body are nonconservative, so that they can change the mechanical energy ($KE + PE$) of the system worked upon, and this is often the goal. A baseball player throwing a ball, for example, increases both the ball's kinetic and potential energy.

If a person needs more energy than they consume, such as when doing vigorous work, the body must draw upon the chemical energy stored in fat. So exercise can be helpful in losing fat. However, the amount of exercise needed to produce a loss in fat, or to burn off extra calories consumed that day, can be large, as [Example 7.13](#) illustrates.

EXAMPLE 7.13

Calculating Weight Loss from Exercising

If a person who normally requires an average of 12,000 kJ (3000 kcal) of food energy per day consumes 13,000 kJ per day, they will steadily add body fat. How much bicycling per day is required to work off this extra 1000 kJ?

Solution

[Table 7.5](#) states that 400 W are used when cycling at a moderate speed. The time required to work off 1000 kJ at this rate is then

$$\text{Time} = \frac{\text{energy}}{\left(\frac{\text{energy}}{\text{time}}\right)} = \frac{1000 \text{ kJ}}{400 \text{ W}} = 2500 \text{ s} = 42 \text{ min.} \quad 7.75$$

Discussion

If this person uses more energy than they consume, the person's body will obtain the needed energy by metabolizing body fat. If the person uses 13,000 kJ but consumes only 12,000 kJ, then the amount of fat loss will be

$$\text{Fat loss} = (1000 \text{ kJ}) \left(\frac{1.0 \text{ g fat}}{39 \text{ kJ}} \right) = 26 \text{ g,} \quad 7.76$$

assuming the energy content of fat to be 39 kJ/g.

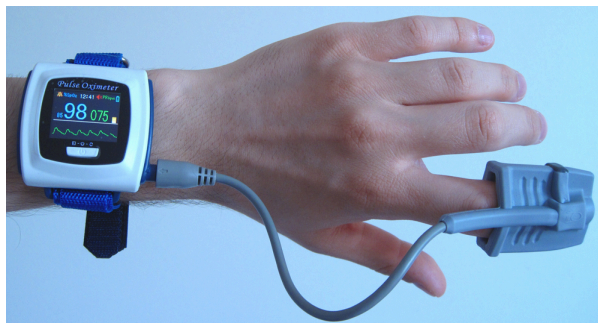


FIGURE 7.24 A pulse oximeter is an apparatus that measures the amount of oxygen in blood. A knowledge of oxygen and carbon dioxide levels indicates a person's metabolic rate, which is the rate at which food energy is converted to another form. Such measurements can indicate the level of athletic conditioning as well as certain medical problems. (credit: UusiAjaja, Wikimedia Commons)

Activity	Energy consumption in watts	Oxygen consumption in liters O ₂ /min
Sleeping	83	0.24
Sitting at rest	120	0.34
Standing relaxed	125	0.36
Sitting in class	210	0.60

TABLE 7.5 Energy and Oxygen Consumption Rates² (Power)

² for a 76-kg male with typical metabolic functions

Activity	Energy consumption in watts	Oxygen consumption in liters O ₂ /min
Walking (5 km/h)	280	0.80
Cycling (13–18 km/h)	400	1.14
Shivering	425	1.21
Playing tennis	440	1.26
Swimming breaststroke	475	1.36
Ice skating (14.5 km/h)	545	1.56
Climbing stairs (116/min)	685	1.96
Cycling (21 km/h)	700	2.00
Running cross-country	740	2.12
Playing basketball	800	2.28
Cycling, professional racer	1855	5.30
Sprinting	2415	6.90

TABLE 7.5 Energy and Oxygen Consumption Rates² (Power)

All bodily functions, from thinking to lifting weights, require energy. (See [Figure 7.25](#).) The many small muscle actions accompanying all quiet activity, from sleeping to head scratching, ultimately become thermal energy, as do less visible muscle actions by the heart, lungs, and digestive tract. Shivering, in fact, is an involuntary response to low body temperature that pits muscles against one another to produce thermal energy in the body (and do no work). The kidneys and liver consume a surprising amount of energy, but the biggest surprise of all is that a full 25% of all energy consumed by the body is used to maintain electrical potentials in all living cells. (Nerve cells use this electrical potential in nerve impulses.) This bioelectrical energy ultimately becomes mostly thermal energy, but some is utilized to power chemical processes such as in the kidneys and liver, and in fat production.

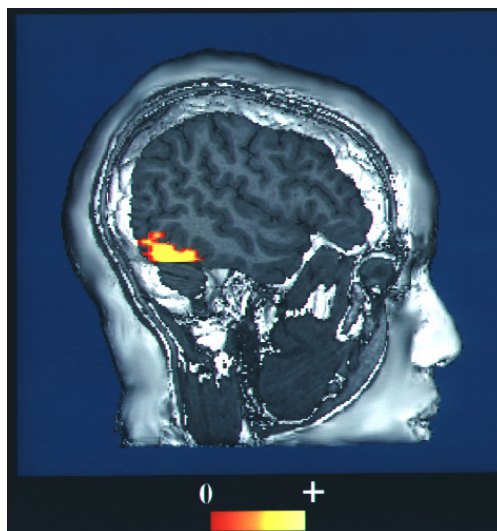


FIGURE 7.25 This fMRI scan shows an increased level of energy consumption in the vision center of the brain. Here, the patient was being

asked to recognize faces. (credit: NIH via Wikimedia Commons)

7.9 World Energy Use

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the distinction between renewable and nonrenewable energy sources.
- Explain why the inevitable conversion of energy to less useful forms makes it necessary to conserve energy resources.

Energy is an important ingredient in all phases of society. We live in a very interdependent world, and access to adequate and reliable energy resources is crucial for economic growth and for maintaining the quality of our lives. But current levels of energy consumption and production are not sustainable. Depending on the data source, estimates indicate that about 31–35% of the world’s energy comes from oil, and much of that goes to transportation uses. This is a reduction by a few percentage points from ten years ago. Oil prices are dependent as much upon new (or foreseen) discoveries as they are upon political events and situations around the world. The U.S., with 4.25% of the world’s population, consumes 21% of the world’s oil production per year.

Renewable and Nonrenewable Energy Sources

The principal energy resources used in the world are shown in [Figure 7.26](#). The fuel mix has changed over the years but now is dominated by oil, although natural gas and solar contributions are increasing. **Renewable forms of energy** are those sources that cannot be used up, such as water, wind, solar, and biomass. About 85% of our energy comes from nonrenewable **fossil fuels**—oil, natural gas, coal. The likelihood of a link between global warming and fossil fuel use, with its production of carbon dioxide through combustion, has made, in the eyes of many scientists, a shift to non-fossil fuels of utmost importance—but it will not be easy.

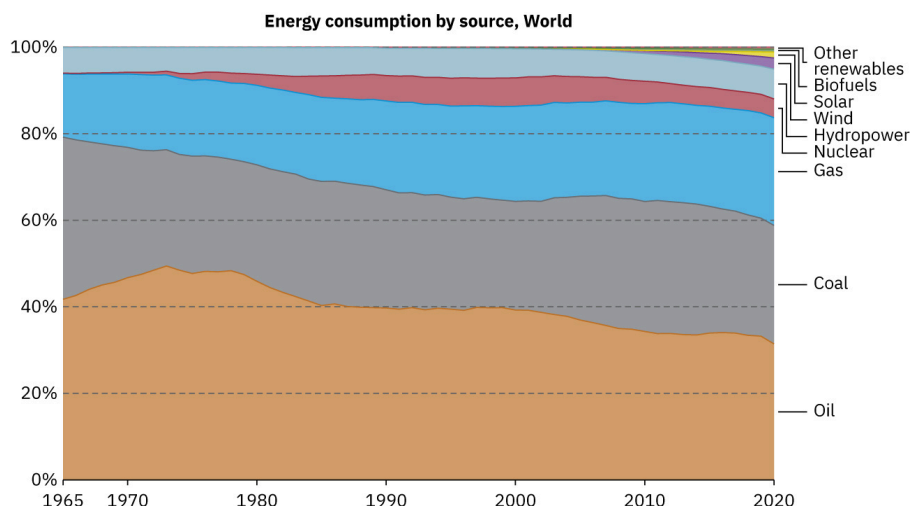


FIGURE 7.26 World energy consumption by source, in billions of kilowatt-hours: 2006. (credit: KVDP)

The World’s Growing Energy Needs

World energy consumption continues to rise, especially in the developing countries. (See [Figure 7.27](#).) Global demand for energy has tripled in the past 50 years and might triple again in the next 30 years. While much of this growth will come from the rapidly booming economies of China and India, many of the developed countries, especially those in Europe, are hoping to meet their energy needs by expanding the use of renewable sources. Although presently only a small percentage, renewable energy is growing very fast, especially wind energy. For example, Germany plans to meet 65% of its power and 30% of its overall energy needs with renewable resources by the year 2030. (See [Figure 7.28](#).) Energy is a key constraint in the rapid economic growth of China and India. In 2003, China surpassed Japan as the world’s second largest consumer of oil. However, over 1/3 of this is imported. Unlike most Western countries, coal dominates the commercial energy resources of China, accounting for 2/3 of its energy consumption. In 2009 China surpassed the United States as the largest generator of CO₂. In India, the main energy resources are biomass (wood and dung) and coal. Half of India’s oil is imported. About 70% of India’s

electricity is generated by highly polluting coal. Yet there are sizeable strides being made in renewable energy. India has a rapidly growing wind energy base, and it has the largest solar cooking program in the world. China has invested substantially in building solar collection farms as well as hydroelectric plants.

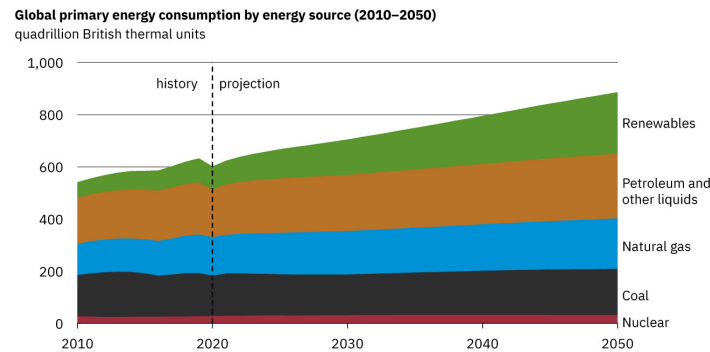


FIGURE 7.27 Past and projected world energy use (source: Based on data from U.S. Energy Information Administration, 2021)



FIGURE 7.28 Solar cell arrays at a power plant in California (credit: Bureau of Land Management, Flickr)

[Table 7.6](#) displays the 2020 commercial energy mix by country for some of the prime energy users in the world. While non-renewable sources dominate, some countries get a sizeable percentage of their electricity from renewable resources. For example, about two-thirds of New Zealand's electricity demand is met by hydroelectric. Only 10% of the U.S. electricity is generated by renewable resources, primarily hydroelectric. It is difficult to determine total sources and consumers of energy in many countries, and estimates vary somewhat by data source and type of measurement.

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Australia	5.6	1.8%	1.5%	1.7%	0%	0.1%	0.5%
Brazil	12	4.6%	1.2%	0.6%	0.1%	3.5%	2%
China	145.5	28.5%	11.9%	82.3%	3.3%	11.7%	7.8%
Egypt	3.7	1.3%	2.1%	0.03%	0%	0.1%	0.1%
Germany	12.1	4.2%	3.1%	1.9%	0.6%	0.2%	2.2%
India	31.99	9%	2.2%	17.5%	0.4%	1.5%	1.4%
Indonesia	8.1	2.8%	1.5%	3.3%	0%	0.2%	0.4%

TABLE 7.6 Energy Consumption—Selected Countries (2020)

Country	Consumption, in EJ (10^{18} J)	Oil	Natural Gas	Coal	Nuclear	Hydro	Other Renewables
Japan	17	6.5%	3.8%	4.6%	0.4%	0.7%	1.1%
United Kingdom	6.9	2.4%	2.6%	0.2%	0.5%	0.1%	1.2%
Russia	28.3	6.4%	14.8%	3.2%	1.9%	1.9%	0.5%
U.S.	87.8	32.5%	30%	9.2%	7.4%	2.6%	6.2%
World	557.1	174.2%	137.6%	151.4%	24%	38.2%	31.7%

TABLE 7.6 Energy Consumption—Selected Countries (2020)

Energy and Economic Well-being

Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (gross domestic product) per capita, are matched by higher levels of energy consumption per capita. This is borne out in [Figure 7.29](#). Increased efficiency of energy use will change this dependency. A global problem is balancing energy resource development against the harmful effects upon the environment in its extraction and use.

New and diversified energy sources do, however, greatly increase economic opportunity and stability. First, the extensive employment opportunities in renewable energy make it one of the most sustainable and secure fields to enter. Second, renewable energy provides countries and localities with increased levels of resiliency in the face of natural disasters, conflict, or other disruptions. The 21st century has already seen major economic impacts from energy disruptions: Hurricane Katrina, Superstorm Sandy, various wildfires, Hurricane Maria, and the 2021 Texas Winter Storm demonstrate the vulnerability of United States power systems. Diversifying energy sources through renewables and other fossil-fuel alternatives brings power grids and transportation systems back online much more quickly, saving lives and enabling a more swift return to economic operations. And as critical emerging information infrastructure, such as data centers, requires more of the world's energy, supplying those growing systems during normal operations and crises will be increasingly important.

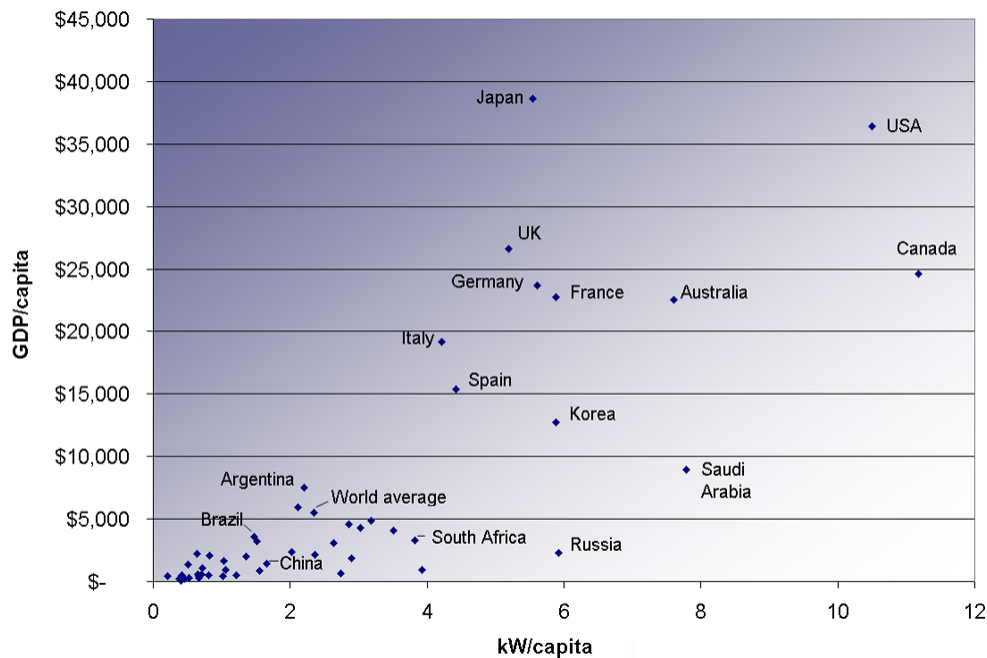


FIGURE 7.29 Power consumption per capita versus GDP per capita for various countries. Note the increase in energy usage with increasing GDP. (2007, credit: Frank van Mierlo, Wikimedia Commons)

Conserving Energy

As we finish this chapter on energy and work, it is relevant to draw some distinctions between two sometimes misunderstood terms in the area of energy use. As has been mentioned elsewhere, the “law of the conservation of energy” is a very useful principle in analyzing physical processes. It is a statement that cannot be proven from basic principles, but is a very good bookkeeping device, and no exceptions have ever been found. It states that the total amount of energy in an isolated system will always remain constant. Related to this principle, but remarkably different from it, is the important philosophy of energy conservation. This concept has to do with seeking to decrease the amount of energy used by an individual or group through (1) reduced activities (e.g., turning down thermostats, driving fewer kilometers) and/or (2) increasing conversion efficiencies in the performance of a particular task—such as developing and using more efficient room heaters, cars that have greater miles-per-gallon ratings, energy-efficient compact fluorescent lights, etc.

Since energy in an isolated system is not destroyed or created or generated, one might wonder why we need to be concerned about our energy resources, since energy is a conserved quantity. The problem is that the final result of most energy transformations is waste heat transfer to the environment and conversion to energy forms no longer useful for doing work. To state it in another way, the potential for energy to produce useful work has been “degraded” in the energy transformation. (This will be discussed in more detail in [Thermodynamics](#).)

Glossary

basal metabolic rate the total energy conversion rate of a person at rest

chemical energy the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

conservation of mechanical energy the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

conservative force a force that does the same work for any given initial and final configuration, regardless of the path followed

efficiency a measure of the effectiveness of the input of energy to do work; useful energy or work divided by the total input of energy

electrical energy the energy carried by a flow of charge

energy the ability to do work

fossil fuels oil, natural gas, and coal

friction the force between surfaces that opposes one sliding on the other; friction changes mechanical energy into thermal energy

gravitational potential energy the energy an object has due to its position in a gravitational field

horsepower an older non-SI unit of power, with $1 \text{ hp} = 746 \text{ W}$

joule SI unit of work and energy, equal to one newton-meter

kilowatt-hour ($\text{kW} \cdot \text{h}$) unit used primarily for electrical energy provided by electric utility companies

kinetic energy the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

law of conservation of energy the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

mechanical energy the sum of kinetic energy and potential energy

metabolic rate the rate at which the body uses food energy to sustain life and to do different activities

net work work done by the net force, or vector sum of all the forces, acting on an object

nonconservative force a force whose work depends on the path followed between the given initial and final configurations

nuclear energy energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

potential energy energy due to position, shape, or configuration

potential energy of a spring the stored energy of a spring as a function of its displacement; when Hooke's law applies, it is given by the expression $\frac{1}{2}kx^2$ where x is the distance the spring is compressed or extended and k is the spring constant

power the rate at which work is done

radiant energy the energy carried by electromagnetic waves

renewable forms of energy those sources that cannot be used up, such as water, wind, solar, and biomass

thermal energy the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

useful work work done on an external system

watt (W) SI unit of power, with $1 \text{ W} = 1 \text{ J/s}$

work the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

work-energy theorem the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

Section Summary

7.1 Work: The Scientific Definition

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force \mathbf{F} does on an object is the product of the magnitude F of the force, times the magnitude d of the displacement, times the cosine of the angle θ between them. In symbols, $W = Fd \cos \theta$.
- The SI unit for work and energy is the joule (J), where $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.

- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

7.2 Kinetic Energy and the Work-Energy Theorem

- The net work W_{net} is the work done by the net

force acting on an object.

- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

7.3 Gravitational Potential Energy

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g , is $\Delta\text{PE}_g = mgh$, with h being the increase in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g , have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta\text{KE} = -\Delta\text{PE}_g$.

7.4 Conservative Forces and Potential Energy

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- The potential energy of a spring is $\text{PE}_s = \frac{1}{2}kx^2$, where k is the spring's force constant and x is the displacement from its undeformed position.
- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is constant. In equation form,

$$\left. \begin{array}{l} \text{KE} + \text{PE} = \text{constant} \\ \text{or} \\ \text{KE}_i + \text{PE}_i = \text{KE}_f + \text{PE}_f \end{array} \right\} \text{(conservative forces only)}$$

where i and f denote initial and final values. This is known as the conservation of mechanical energy.

7.5 Nonconservative Forces

- A nonconservative force is one for which work depends on the path.

- Friction is an example of a nonconservative force that changes mechanical energy into thermal energy.
- Work W_{nc} done by a nonconservative force changes the mechanical energy of a system. In equation form, $W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE}$ or, equivalently, $\text{KE}_i + \text{PE}_i + W_{\text{nc}} = \text{KE}_f + \text{PE}_f$.
- When both conservative and nonconservative forces act, energy conservation can be applied and used to calculate motion in terms of the known potential energies of the conservative forces and the work done by nonconservative forces, instead of finding the net work from the net force, or having to directly apply Newton's laws.

7.6 Conservation of Energy

- The law of conservation of energy states that the total energy is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- When all forms of energy are considered, conservation of energy is written in equation form as $\text{KE}_i + \text{PE}_i + W_{\text{nc}} + \text{OE}_i = \text{KE}_f + \text{PE}_f + \text{OE}_f$, where **OE** is all **other forms of energy** besides mechanical energy.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.
- Energy is often utilized to do work, but it is not possible to convert all the energy of a system to work.
- The efficiency Eff of a machine or human is defined to be $\text{Eff} = \frac{W_{\text{out}}}{E_{\text{in}}}$, where W_{out} is useful work output and E_{in} is the energy consumed.

7.7 Power

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t , $P = W/t$.
- The SI unit for power is the watt (W), where $1 \text{ W} = 1 \text{ J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1 \text{ hp} = 746 \text{ W}$.

7.8 Work, Energy, and Power in Humans

- The human body converts energy stored in food into work, thermal energy, and/or chemical energy that is stored in fatty tissue.
- The *rate* at which the body uses food energy to sustain life and to do different activities is called the metabolic rate, and the corresponding rate

when at rest is called the basal metabolic rate (BMR)

- The energy included in the basal metabolic rate is divided among various systems in the body, with the largest fraction going to the liver and spleen, and the brain coming next.
- About 75% of food calories are used to sustain basic body functions included in the basal metabolic rate.
- The energy consumption of people during various activities can be determined by measuring their oxygen use, because the digestive process is basically one of oxidizing food.

7.9 World Energy Use

- The relative use of different fuels to provide energy has changed over the years, but fuel use is currently dominated by oil, although natural gas

and solar contributions are increasing.

- Although non-renewable sources dominate, some countries meet a sizeable percentage of their electricity needs from renewable resources.
- The United States obtains only about 10% of its energy from renewable sources, mostly hydroelectric power.
- Economic well-being is dependent upon energy use, and in most countries higher standards of living, as measured by GDP (Gross Domestic Product) per capita, are matched by higher levels of energy consumption per capita.
- Even though, in accordance with the law of conservation of energy, energy can never be created or destroyed, energy that can be used to do work is always partly converted to less useful forms, such as waste heat to the environment, in all of our uses of energy for practical purposes.

Conceptual Questions

7.1 Work: The Scientific Definition

1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.
2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.
3. Describe a situation in which a force is exerted for a long time but does no work. Explain.

7.2 Kinetic Energy and the Work-Energy Theorem

4. The person in [Figure 7.30](#) does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

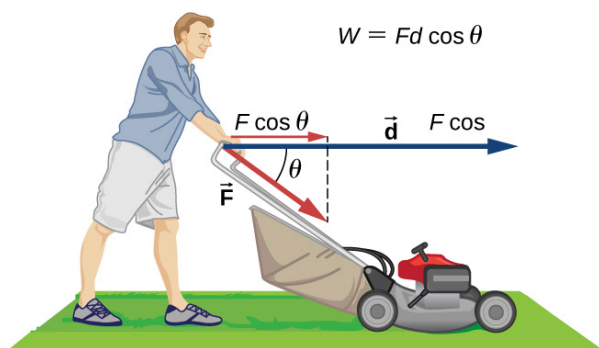


FIGURE 7.30

5. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

6. When solving for speed in [Example 7.4](#), we kept only the positive root. Why?

7.3 Gravitational Potential Energy

7. In [Example 7.7](#), we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s uphill instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that its final speed is the same as its initial speed. Explain in terms of conservation of energy.
8. Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

7.4 Conservative Forces and Potential Energy

9. What is a conservative force?
10. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.
11. Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

12. What is the relationship of potential energy to conservative force?

7.6 Conservation of Energy

13. Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See [Figure 7.31](#).)

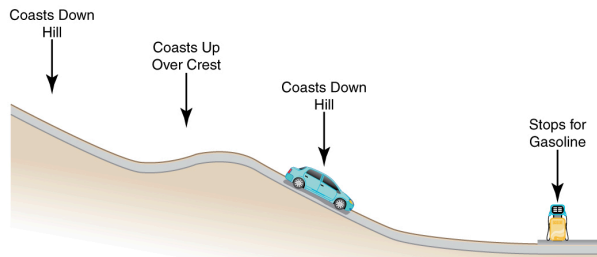


FIGURE 7.31 A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

14. Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.
15. Do devices with efficiencies of less than one violate the law of conservation of energy? Explain.
16. List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.
17. List the energy conversions that occur when riding a bicycle.

Problems & Exercises

7.1 Work: The Scientific Definition

1. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.
2. A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task. (Neglect friction in your calculations.)

7.7 Power

18. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.
19. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?
20. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

7.8 Work, Energy, and Power in Humans

21. Explain why it is easier to climb a mountain on a zigzag path rather than one straight up the side. Is your increase in gravitational potential energy the same in both cases? Is your energy consumption the same in both?
22. Do you do work on the outside world when you rub your hands together to warm them? What is the efficiency of this activity?
23. Shivering is an involuntary response to lowered body temperature. What is the efficiency of the body when shivering, and is this a desirable value?
24. Discuss the relative impact of dieting and exercise, noting that most athletic activities consume food energy at a rate of 400 to 500 W, while a single cup of yogurt can contain 1360 kJ (325 kcal).

7.9 World Energy Use

25. What is the difference between energy conservation and the law of conservation of energy? Give some examples of each.
26. If the efficiency of a coal-fired electrical generating plant is 35%, then what do we mean when we say that energy is a conserved quantity?
3. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the elevator car by the gravitational force in this process? (c) What is the total work done on the elevator car?

- Suppose a car travels 108 km at a speed of 30.0 m/s, and uses 2.0 gal of gasoline. Only 30% of the gasoline goes into useful work by the force that keeps the car moving at constant speed despite friction. (See [Table 7.1](#) for the energy content of gasoline.) (a) What is the magnitude of the force exerted to keep the car moving at constant speed? (b) If the required force is directly proportional to speed, how many gallons will be used to drive 108 km at a speed of 28.0 m/s?
- Calculate the work done by an 85.0-kg man who pushes a crate 4.00 m up along a ramp that makes an angle of 20.0° with the horizontal. (See [Figure 7.32](#).) He exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.

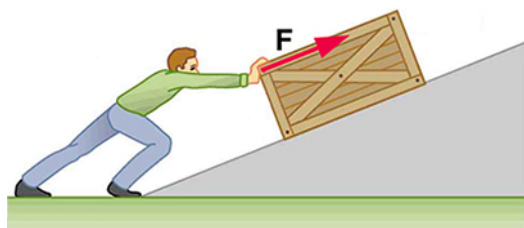


FIGURE 7.32 A man pushes a crate up a ramp.

- How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in [Figure 7.33](#)? Assume no friction acts on the wagon.

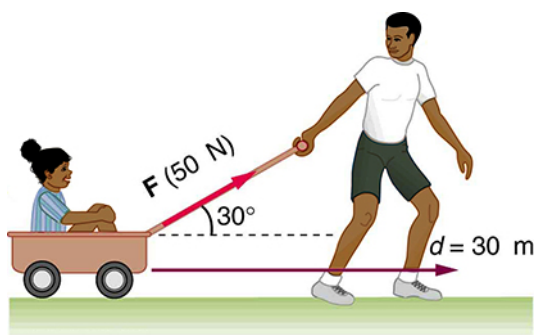


FIGURE 7.33 The boy does work on the system of the wagon and the child when he pulls them as shown.

- A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction 25.0° below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?

- Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed, as shown in [Figure 7.34](#). The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

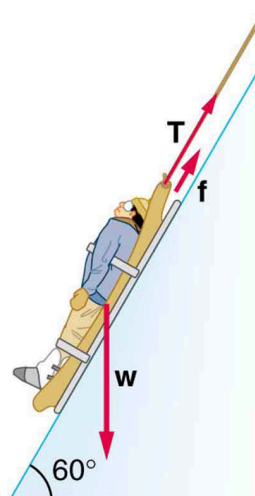


FIGURE 7.34 A rescue sled and victim are lowered down a steep slope.

7.2 Kinetic Energy and the Work-Energy Theorem

- Compare the kinetic energy of a 20,000-kg truck moving at 110 km/h with that of an 80.0-kg astronaut in orbit moving at 27,500 km/h.
- (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.
- Confirm the value given for the kinetic energy of an aircraft carrier in [Table 7.1](#). You will need to look up the definition of a nautical mile (1 knot = 1 nautical mile/h).
- (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 90.0 km/h in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

- 13.** A car's bumper is designed to withstand a 4.0-km/h (1.12-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.12 m/s.
- 14.** Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?
- 15.** Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.
- 17.** (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about 7×10^9 kg and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person?
- 18.** Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?
- 19.** In [Example 7.7](#), we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta PE \gg KE_i$. Confirm this statement by taking the ratio of ΔPE to KE_i . (Note that mass cancels.)
- 20.** A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in [Figure 7.36](#). Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

7.3 Gravitational Potential Energy

- 16.** A hydroelectric power facility (see [Figure 7.35](#)) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume 50.0 km^3 (mass = 5.00×10^{13} kg), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.



FIGURE 7.35 Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)



FIGURE 7.36 A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

- 21.** In a downhill ski race, surprisingly, little advantage is gained by getting a running start. (This is because the initial kinetic energy is small compared with the gain in gravitational potential energy on even small hills.) To demonstrate this, find the final speed and the time taken for a skier who skies 70.0 m along a 30° slope neglecting friction: (a) Starting from rest. (b) Starting with an initial speed of 2.50 m/s. (c) Does the answer surprise you? Discuss why it is still advantageous to get a running start in very competitive events.

7.4 Conservative Forces and Potential Energy

22. A 5.00×10^5 -kg subway train is brought to a stop from a speed of 0.500 m/s in 0.400 m by a large spring bumper at the end of its track. What is the force constant k of the spring?
23. A pogo stick has a spring with a force constant of 2.50×10^4 N/m, which can be compressed 12.0 cm. To what maximum height can a child jump on the stick using only the energy in the spring, if the child and stick have a total mass of 40.0 kg? Explicitly show how you follow the steps in the [Problem-Solving Strategies for Energy](#).

7.5 Nonconservative Forces

24. A 60.0-kg skier with an initial speed of 12.0 m/s coasts up a 2.50-m-high rise as shown in [Figure 7.37](#). Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.0800. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)

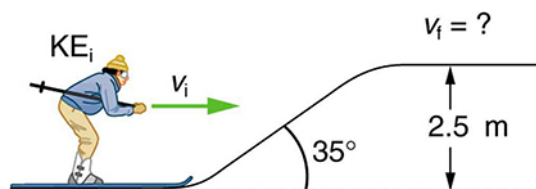


FIGURE 7.37 The skier's initial kinetic energy is partially used in coasting to the top of a rise.

25. (a) How high a hill can a car coast up (engine disengaged) if work done by friction is negligible and its initial speed is 110 km/h? (b) If, in actuality, a 750-kg car with an initial speed of 110 km/h is observed to coast up a hill to a height 22.0 m above its starting point, how much thermal energy was generated by friction? (c) What is the average force of friction if the hill has a slope 2.5° above the horizontal?

7.6 Conservation of Energy

26. Using values from [Table 7.1](#), how many DNA molecules could be broken by the energy carried by a single electron in the beam of an old-fashioned TV tube? (These electrons were not dangerous in themselves, but they did create dangerous x rays. Later model tube TVs had shielding that absorbed x rays before they escaped and exposed viewers.)

27. Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.
28. If the energy in fusion bombs were used to supply the energy needs of the world, how many of the 9-megaton variety would be needed for a year's supply of energy (using data from [Table 7.1](#))? This is not as far-fetched as it may sound—there are thousands of nuclear bombs, and their energy can be trapped in underground explosions and converted to electricity, as natural geothermal energy is.
29. (a) Use of hydrogen fusion to supply energy is a dream that may be realized in the next century. Fusion would be a relatively clean and almost limitless supply of energy, as can be seen from [Table 7.1](#). To illustrate this, calculate how many years the present energy needs of the world could be supplied by one millionth of the oceans' hydrogen fusion energy. (b) How does this time compare with historically significant events, such as the duration of stable economic systems?

7.7 Power

30. The Crab Nebula (see [Figure 7.38](#)) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from [Table 7.3](#), calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.



FIGURE 7.38 Crab Nebula (credit: ESO, via Wikimedia Commons)

- 31.** Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from [Table 7.3](#): (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.
- 32.** A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?
- 33.** What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is \$0.0900 per kW · h?
- 34.** A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is \$0.110 per kW · h?
- 35.** (a) What is the average power consumption in watts of an appliance that uses 5.00 kW · h of energy per day? (b) How many joules of energy does this appliance consume in a year?
- 36.** (a) What is the average useful power output of a person who does 6.00×10^6 J of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)
- 37.** A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?
- 38.** (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?
- 39.** (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is \$0.0900 per kW · h?
- 40.** (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply 8.00×10^4 J run a pocket calculator that consumes energy at the rate of 1.00×10^{-3} W?
- 41.** (a) How long would it take a 1.50×10^5 -kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)
- 42.** Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the [Problem-Solving Strategies for Energy](#).
- 43.** (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be 4.00×10^{26} W.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of 1.30 kW/m^2 reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs (1.05×10^{20} J)? Australia's energy needs (5.4×10^{18} J)? China's energy needs (6.3×10^{19} J)? (These energy consumption values are from 2006.)

7.8 Work, Energy, and Power in Humans

44. (a) How long can you rapidly climb stairs (116/min) on the 93.0 kcal of energy in a 10.0-g pat of butter? (b) How many flights is this if each flight has 16 stairs?
45. (a) What is the power output in watts and horsepower of a 70.0-kg sprinter who accelerates from rest to 10.0 m/s in 3.00 s? (b) Considering the amount of power generated, do you think a well-trained athlete could do this repetitively for long periods of time?
46. Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)



FIGURE 7.39 Shot putter at the Dornoch Highland Gathering in 2007. (credit: John Haslam, Flickr)

47. (a) What is the efficiency of an out-of-condition person who does 2.10×10^5 J of useful work while metabolizing 500 kcal of food energy? (b) How many food calories would a well-conditioned athlete metabolize in doing the same work with an efficiency of 20%?
48. Energy that is not utilized for work or heat transfer is converted to the chemical energy of body fat containing about 39 kJ/g. How many grams of fat will you gain if you eat 10,000 kJ (about 2500 kcal) one day and do nothing but sit relaxed for 16.0 h and sleep for the other 8.00 h? Use data from [Table 7.5](#) for the energy consumption rates of these activities.
49. Using data from [Table 7.5](#), calculate the daily energy needs of a person who sleeps for 7.00 h, walks for 2.00 h, attends classes for 4.00 h, cycles for 2.00 h, sits relaxed for 3.00 h, and studies for 6.00 h. (Studying consumes energy at the same rate as sitting in class.)
50. What is the efficiency of a subject on a treadmill who puts out work at the rate of 100 W while consuming oxygen at the rate of 2.00 L/min? (Hint: See [Table 7.5](#).)
51. Shoveling snow can be extremely taxing because the arms have such a low efficiency in this activity. Suppose a person shoveling a footpath metabolizes food at the rate of 800 W. (a) What is her useful power output? (b) How long will it take her to lift 3000 kg of snow 1.20 m? (This could be the amount of heavy snow on 20 m of footpath.) (c) How much waste heat transfer in kilojoules will she generate in the process?
52. Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the magnitude of the force produced if an 80.0-kg person jumps from a 0.600-m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the magnitude of the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.
53. Jogging on hard surfaces with insufficiently padded shoes produces large forces in the feet and legs. (a) Calculate the magnitude of the force needed to stop the downward motion of a jogger's leg, if his leg has a mass of 13.0 kg, a speed of 6.00 m/s, and stops in a distance of 1.50 cm. (Be certain to include the weight of the 75.0-kg jogger's body.) (b) Compare this force with the weight of the jogger.
54. (a) Calculate the energy in kJ used by a 55.0-kg person who does 50 deep knee bends in which their center of mass is lowered and raised 0.400 m. (They do work in both directions.) You may assume their efficiency is 20%. (b) What is the average power consumption rate in watts if they do this in 3.00 min?

55. Kanellos Kanellopoulos flew 119 km from Crete to Santorini, Greece, on April 23, 1988, in the *Daedalus 88*, an aircraft powered by a bicycle-type drive mechanism (see [Figure 7.40](#)). His useful power output for the 234-min trip was about 350 W. Using the efficiency for cycling from [Table 7.2](#), calculate the food energy in kilojoules he metabolized during the flight.



FIGURE 7.40 The *Daedalus 88* in flight. (credit: NASA photo by Beasley)

56. The swimmer shown in [Figure 7.41](#) exerts an average horizontal backward force of 80.0 N with his arm during each 1.80 m long stroke. (a) What is his work output in each stroke? (b) Calculate the power output of his arms if he does 120 strokes per minute.

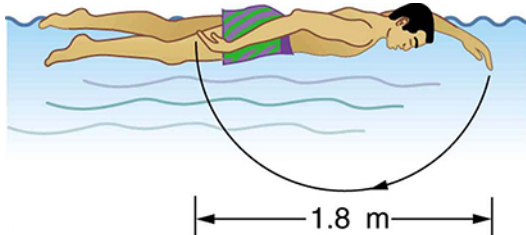


FIGURE 7.41

57. Mountain climbers carry bottled oxygen when at very high altitudes. (a) Assuming that a mountain climber uses oxygen at twice the rate for climbing 116 stairs per minute (because of low air temperature and winds), calculate how many liters of oxygen a climber would need for 10.0 h of climbing. (These are liters at sea level.) Note that only 40% of the inhaled oxygen is utilized; the rest is exhaled. (b) How much useful work does the climber do if he and his equipment have a mass of 90.0 kg and he gains 1000 m of altitude? (c) What is his efficiency for the 10.0-h climb?

58. The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about 7×10^9 kg. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see [Figure 7.42](#)), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

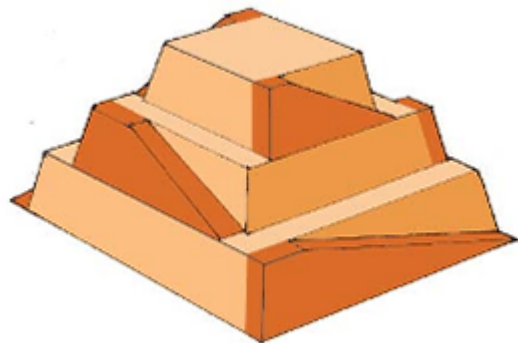


FIGURE 7.42 Ancient pyramids were probably constructed using ramps as simple machines. (credit: Franck Monnier, Wikimedia Commons)

59. (a) How long can you play tennis on the 800 kJ (about 200 kcal) of energy in a candy bar? (b) Does this seem like a long time? Discuss why exercise is necessary but may not be sufficient to cause a person to lose weight.

7.9 World Energy Use

60. Integrated Concepts

(a) Calculate the force the woman in [Figure 7.43](#) exerts to do a push-up at constant speed, taking all data to be known to three digits. (b) How much work does she do if her center of mass rises 0.240 m? (c) What is her useful power output if she does 25 push-ups in 1 min? (Should work done lowering her body be included? See the discussion of useful work in [Work, Energy, and Power in Humans](#).)

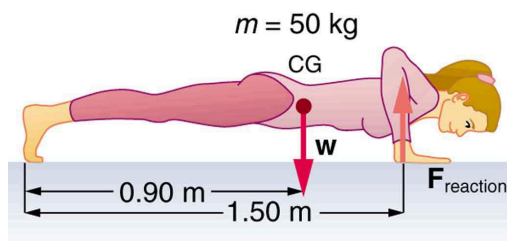


FIGURE 7.43 Forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (CG).

61. Integrated Concepts

A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

62. Integrated Concepts

The 70.0-kg swimmer in [Figure 7.41](#) starts a race with an initial velocity of 1.25 m/s and exerts an average force of 80.0 N backward with his arms during each 1.80 m long stroke. (a) What is his initial acceleration if water resistance is 45.0 N? (b) What is the subsequent average resistance force from the water during the 5.00 s it takes him to reach his top velocity of 2.50 m/s? (c) Discuss whether water resistance seems to increase linearly with velocity.

63. Integrated Concepts

A toy gun uses a spring with a force constant of 300 N/m to propel a 10.0-g steel ball. If the spring is compressed 7.00 cm and friction is negligible: (a) How much force is needed to compress the spring? (b) To what maximum height can the ball be shot? (c) At what angles above the horizontal may a child aim to hit a target 3.00 m away at the same height as the gun? (d) What is the gun's maximum range on level ground?

64. Integrated Concepts

(a) What force must be supplied by an elevator cable to produce an acceleration of 0.800 m/s^2 against a 200-N frictional force, if the mass of the loaded elevator is 1500 kg? (b) How much work is done by the cable in lifting the elevator 20.0 m? (c) What is the final speed of the elevator if it starts from rest? (d) How much work went into thermal energy?

65. Unreasonable Results

A car advertisement claims that its 900-kg car accelerated from rest to 30.0 m/s and drove 100 km, gaining 3.00 km in altitude, on 1.0 gal of gasoline. The average force of friction including air resistance was 700 N. Assume all values are known to three significant figures. (a) Calculate the car's efficiency. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

66. Unreasonable Results

Body fat is metabolized, supplying 9.30 kcal/g, when dietary intake is less than needed to fuel metabolism. The manufacturers of an exercise bicycle claim that you can metabolize 0.500 kg of fat per day by vigorously exercising for 2.00 h per day on their machine. (a) How many kcal are supplied by the metabolization of 0.500 kg of fat? (b) Calculate the kcal/min that you would have to utilize to metabolize fat at the rate of 0.500 kg in 2.00 h. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?

67. Construct Your Own Problem

Consider a person climbing and descending stairs. Construct a problem in which you calculate the long-term rate at which stairs can be climbed considering the mass of the person, his ability to generate power with his legs, and the height of a single stair step. Also consider why the same person can descend stairs at a faster rate for a nearly unlimited time in spite of the fact that very similar forces are exerted going down as going up. (This points to a fundamentally different process for descending versus climbing stairs.)

68. Construct Your Own Problem

Consider humans generating electricity by pedaling a device similar to a stationary bicycle. Construct a problem in which you determine the number of people it would take to replace a large electrical generation facility. Among the things to consider are the power output that is reasonable using the legs, rest time, and the need for electricity 24 hours per day. Discuss the practical implications of your results.

69. Integrated Concepts

A 105-kg basketball player crouches down 0.400 m while waiting to jump. After exerting a force on the floor through this 0.400 m, their feet leave the floor and their center of gravity rises 0.950 m above its normal standing erect position. (a) Using energy considerations, calculate the player's velocity when they leave the floor. (b) What average force did the player exert on the floor? (Do not neglect the force to support their weight as well as that to accelerate them.) (c) What was the player's power output during the acceleration phase?

70. Critical Thinking Two boxes of the same mass are pressed against pistons that compress and release the boxes on frictionless surfaces. Box A's piston has a constant force F over a distance x before the box is released while box B starts at zero force and increases linearly to a force of $2F$ before releasing the box at a distance of $1/2d$. (a) Which of these boxes has the greatest speed after releasing from the piston? (b) Based on experimental data for trials similar to box A, an equation that fits the data is suggested for the change in kinetic energy, ΔKE , which may not be correct: $\Delta KE = KF^{1/2}x^{1/2}$, where K is a constant with appropriate units, F is the applied force, and x is the distance the piston moves in contact with the box. Is this equation consistent with your answer from part a? Explain why or why not. Does this equation make sense? Explain why or why not. (c) If the applied force of the piston is proportional to the distance in contact with the box, graph the work done on the box vs. the distance in contact with the piston and box.

CHAPTER 8

Linear Momentum and Collisions



FIGURE 8.1 Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzie, Flickr)

CHAPTER OUTLINE

8.1 Linear Momentum and Force

8.2 Impulse

8.3 Conservation of Momentum

8.4 Elastic Collisions in One Dimension

8.5 Inelastic Collisions in One Dimension

8.6 Collisions of Point Masses in Two Dimensions

8.7 Introduction to Rocket Propulsion

INTRODUCTION TO LINEAR MOMENTUM AND COLLISIONS We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature,

and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/8-introduction-to-linear-momentum-and-collisions\)](https://openstax.org/books/college-physics-2e/pages/8-introduction-to-linear-momentum-and-collisions)

8.1 Linear Momentum and Force

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton’s second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

Linear Momentum

The scientific definition of linear momentum is consistent with most people’s intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system’s mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\mathbf{p} = m\mathbf{v}. \quad 8.1$$

Momentum is directly proportional to the object’s mass and also its velocity. Thus the greater an object’s mass or the greater its velocity, the greater its momentum. Momentum \mathbf{p} is a vector having the same direction as the velocity \mathbf{v} . The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.

Linear Momentum

Linear momentum is defined as the product of a system’s mass multiplied by its velocity:

$$\mathbf{p} = m\mathbf{v}. \quad 8.2$$



EXAMPLE 8.1

Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player’s momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p . (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv \quad 8.3$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player’s mass and speed into the equation.

$$p_{\text{player}} = (110 \text{ kg})(8.00 \text{ m/s}) = 880 \text{ kg} \cdot \text{m/s} \quad 8.4$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball’s mass and speed into the equation.

$$p_{\text{ball}} = (0.410 \text{ kg})(25.0 \text{ m/s}) = 10.3 \text{ kg} \cdot \text{m/s} \quad 8.5$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9. \quad 8.6$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}, \quad 8.7$$

where \mathbf{F}_{net} is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} \quad 8.8$$

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}). \quad 8.9$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta \mathbf{v}. \quad 8.10$$

So that for constant mass, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}. \quad 8.11$$

Because $\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \quad 8.12$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail; however, the relationship between momentum and force remains useful

when mass is constant, such as in the following example.

EXAMPLE 8.2

Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}. \quad 8.13$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_f - v_i). \quad 8.14$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_f - v_i) \\ &= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.306 \text{ kg} \cdot \text{m/s} \approx 3.3 \text{ kg} \cdot \text{m/s} \end{aligned} \quad 8.15$$

Now the magnitude of the net external force can be determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306 \text{ kg} \cdot \text{m/s}}{5.0 \times 10^{-3} \text{ s}} \\ &= 661 \text{ N} \approx 660 \text{ N}, \end{aligned} \quad 8.16$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = ma$, but one additional step would be required compared with the strategy used in this example.

8.2 Impulse

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In [Example 8.1](#), a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could

cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta\mathbf{p}$.

By rearranging the equation $\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t}$ to be

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}} \Delta t, \quad 8.17$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\mathbf{F}_{\text{net}} \Delta t$ is given the name **impulse**. Impulse is the same as the change in momentum.

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}} \Delta t \quad 8.18$$

The quantity $\mathbf{F}_{\text{net}} \Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

EXAMPLE 8.3

Calculating Magnitudes of Impulses: Two Billiard Balls Striking a Rigid Wall

Two identical billiard balls strike a rigid wall with the same speed, and are reflected without any change of speed. The first ball strikes perpendicular to the wall. The second ball strikes the wall at an angle of 30° from the perpendicular, and bounces off at an angle of 30° from perpendicular to the wall.

- Determine the direction of the force on the wall due to each ball.
- Calculate the ratio of the magnitudes of impulses on the two balls by the wall.

Strategy for (a)

In order to determine the force on the wall, consider the force on the ball due to the wall using Newton's second law and then apply Newton's third law to determine the direction. Assume the x -axis to be normal to the wall and to be positive in the initial direction of motion. Choose the y -axis to be along the wall in the plane of the second ball's motion. The momentum direction and the velocity direction are the same.

Solution for (a)

The first ball bounces directly into the wall and exerts a force on it in the $+x$ direction. Therefore the wall exerts a force on the ball in the $-x$ direction. The second ball continues with the same momentum component in the y direction, but reverses its x -component of momentum, as seen by sketching a diagram of the angles involved and keeping in mind the proportionality between velocity and momentum.

These changes mean the change in momentum for both balls is in the $-x$ direction, so the force of the wall on each ball is along the $-x$ direction.

Strategy for (b)

Calculate the change in momentum for each ball, which is equal to the impulse imparted to the ball.

Solution for (b)

Let u be the speed of each ball before and after collision with the wall, and m the mass of each ball. Choose the x -axis and y -axis as previously described, and consider the change in momentum of the first ball which strikes perpendicular to the wall.

$$p_{xi} = mu; p_{yi} = 0 \quad 8.19$$

$$p_{xf} = -mu; p_{yf} = 0 \quad 8.20$$

Impulse is the change in momentum vector. Therefore the x -component of impulse is equal to $-2mu$ and the y -component of impulse is equal to zero.

Now consider the change in momentum of the second ball.

$$p_{xi} = mu \cos 30^\circ; p_{yi} = -mu \sin 30^\circ \quad 8.21$$

$$p_{xf} = -mu \cos 30^\circ; p_{yf} = -mu \sin 30^\circ \quad 8.22$$

It should be noted here that while p_x changes sign after the collision, p_y does not. Therefore the x -component of impulse is equal to $-2mu \cos 30^\circ$ and the y -component of impulse is equal to zero.

The ratio of the magnitudes of the impulse imparted to the balls is

$$\frac{2mu}{2mu \cos 30^\circ} = \frac{2}{\sqrt{3}} = 1.155. \quad 8.23$$

Discussion

The direction of impulse and force is the same as in the case of (a); it is normal to the wall and along the negative x -direction. Making use of Newton's third law, the force on the wall due to each ball is normal to the wall along the positive x -direction.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt . *Forces are usually not constant.* Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. [Figure 8.2](#) shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2 . That area is equal to the area inside the rectangle bounded by F_{eff} , t_1 , and t_2 . Thus the impulses and their effects are the same for both the actual and effective forces.

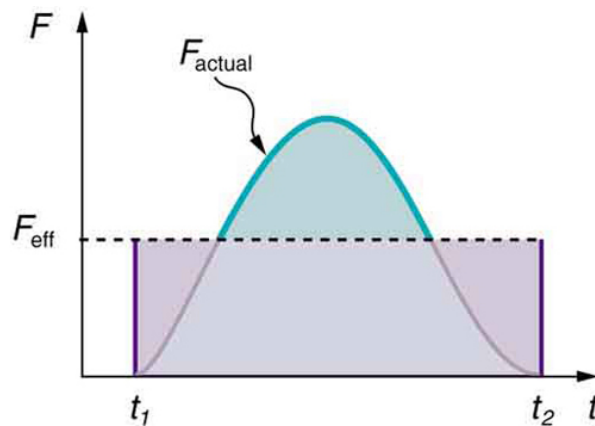


FIGURE 8.2 A graph of force versus time with time along the x -axis and force along the y -axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

Making Connections: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

Making Connections: Constant Force and Constant Acceleration

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

8.3 Conservation of Momentum

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the principle of conservation of momentum.
- Derive an expression for the conservation of momentum.
- Explain conservation of momentum with examples.
- Explain the principle of conservation of momentum as it relates to atomic and subatomic particles.

Momentum is an important quantity because it is conserved. Yet it was not conserved in the examples in [Impulse](#) and [Linear Momentum and Force](#), where large changes in momentum were produced by forces acting on the system of interest. Under what circumstances is momentum conserved?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—for example, one car bumping into another, as shown in [Figure 8.3](#). Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

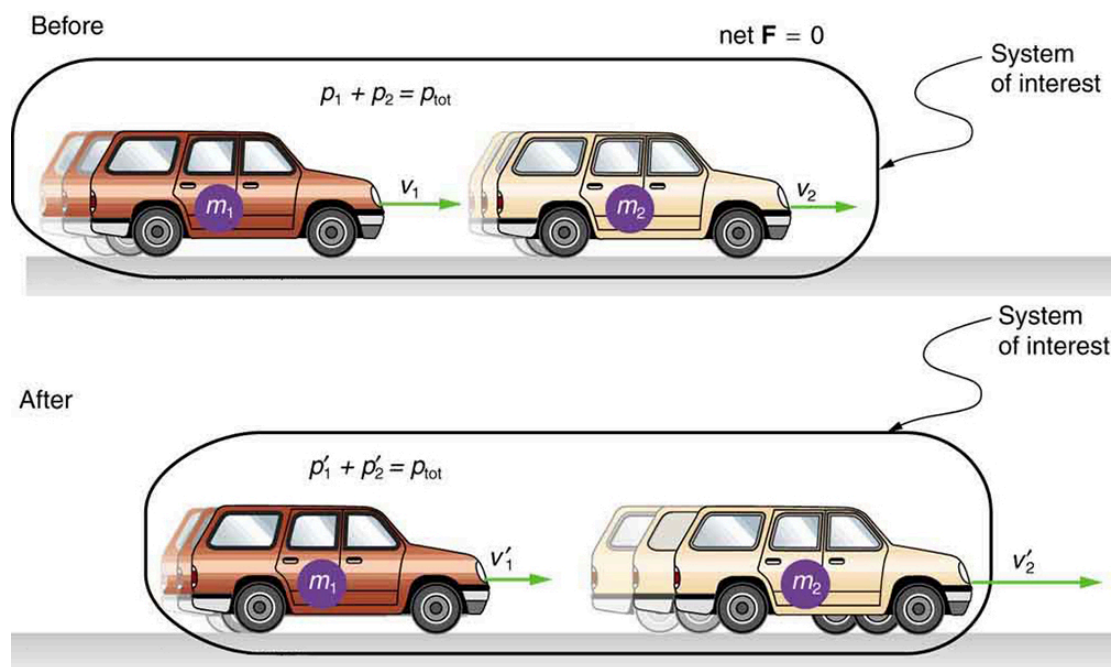


FIGURE 8.3 A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v'_1 and the second speeds up to a velocity of v'_2 . The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t, \quad 8.24$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Intuitively, it seems obvious that the collision time is the same for both cars, but it is only true for objects traveling at ordinary speeds. This assumption must be modified for objects travelling near the speed of light, without affecting the result that momentum is conserved.

Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t, \quad 8.25$$

where F_2 is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1. \quad 8.26$$

Thus, the changes in momentum are equal and opposite, and

$$\Delta p_1 + \Delta p_2 = 0. \quad 8.27$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = \text{constant}, \quad 8.28$$

$$p_1 + p_2 = p'_1 + p'_2, \quad 8.29$$

where p'_1 and p'_2 are the momenta of cars 1 and 2 after the collision. (We often use primes to denote the final state.)

This result—that momentum is conserved—has validity far beyond the preceding one-dimensional case. It can be similarly shown that total momentum is conserved for any isolated system, with any number of objects in it. In equation form, the **conservation of momentum principle** for an isolated system is written

$$\mathbf{p}_{\text{tot}} = \text{constant}, \quad 8.30$$

or

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}, \quad 8.31$$

where \mathbf{p}_{tot} is the total momentum (the sum of the momenta of the individual objects in the system) and \mathbf{p}'_{tot} is the total momentum some time later. (The total momentum can be shown to be the momentum of the center of mass of the system.) An **isolated system** is defined to be one for which the net external force is zero ($\mathbf{F}_{\text{net}} = 0$).

Conservation of Momentum Principle

$$\mathbf{p}_{\text{tot}} = \text{constant}$$

$$\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}} \text{ (isolated system)}$$

8.32

Isolated System

An isolated system is defined to be one for which the net external force is zero ($\mathbf{F}_{\text{net}} = 0$).

Perhaps an easier way to see that momentum is conserved for an isolated system is to consider Newton's second law in terms of momentum, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}_{\text{tot}}}{\Delta t}$. For an isolated system, ($\mathbf{F}_{\text{net}} = 0$); thus, $\Delta \mathbf{p}_{\text{tot}} = 0$, and \mathbf{p}_{tot} is constant.

We have noted that the three length dimensions in nature— x , y , and z —are independent, and it is interesting to note that momentum can be conserved in different ways along each dimension. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See [Figure 8.4](#).) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

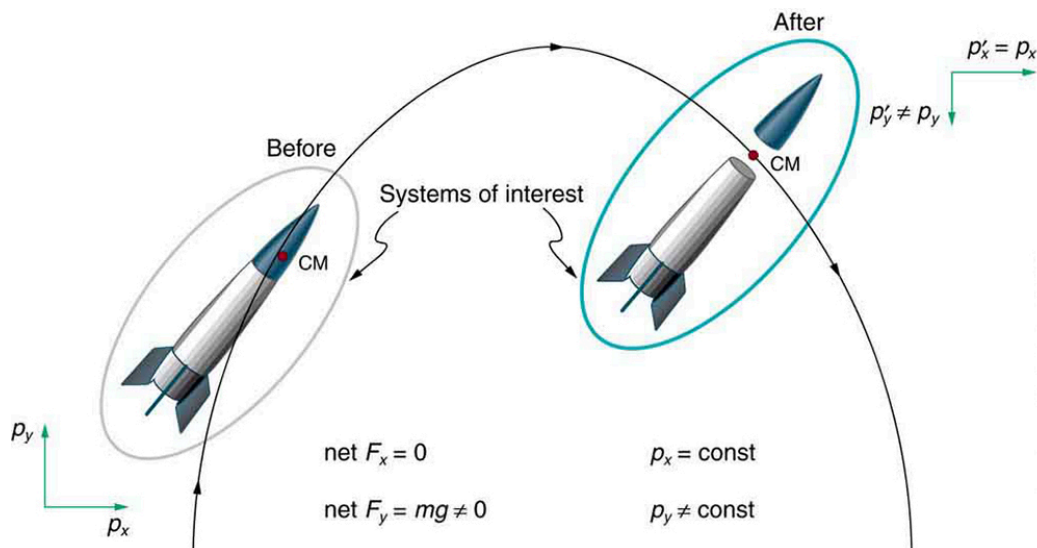


FIGURE 8.4 The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $F_{x-\text{net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $F_{y-\text{net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Making Connections: Conservation of Momentum and Collision

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Subatomic Collisions and Momentum

The conservation of momentum principle not only applies to the macroscopic objects, it is also essential to our explorations of atomic and subatomic particles. Giant machines hurl subatomic particles at one another, and researchers evaluate the results by assuming conservation of momentum (among other things).

On the small scale, we find that particles and their properties are invisible to the naked eye but can be measured with our instruments, and models of these subatomic particles can be constructed to describe the results. Momentum is found to be a property of all subatomic particles including massless particles such as photons that compose light. Momentum being a property of particles hints that momentum may have an identity beyond the description of an object's mass multiplied by the object's velocity. Indeed, momentum relates to wave properties and plays a fundamental role in what measurements are taken and how we take these measurements. Furthermore, we find that the conservation of momentum principle is valid when considering systems of particles. We use this principle to analyze the masses and other properties of previously undetected particles, such as the nucleus of an atom and the existence of quarks that make up particles of nuclei. [Figure 8.5](#) below illustrates how a particle scattering backward from another implies that its target is massive and dense. Experiments seeking evidence that

quarks make up protons (one type of particle that makes up nuclei) scattered high-energy electrons off of protons (nuclei of hydrogen atoms). Electrons occasionally scattered straight backward in a manner that implied a very small and very dense particle makes up the proton—this observation is considered nearly direct evidence of quarks. The analysis was based partly on the same conservation of momentum principle that works so well on the large scale.

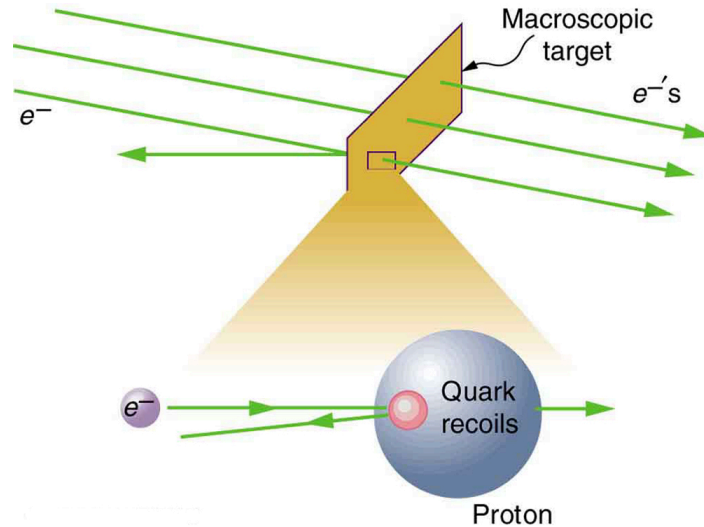


FIGURE 8.5 A subatomic particle scatters straight backward from a target particle. In experiments seeking evidence for quarks, electrons were observed to occasionally scatter straight backward from a proton.

8.4 Elastic Collisions in One Dimension

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe an elastic collision of two objects in one dimension.
- Define internal kinetic energy.
- Derive an expression for conservation of internal kinetic energy in a one dimensional collision.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves internal kinetic energy. **Internal kinetic energy** is the sum of the kinetic energies of the objects in the system. [Figure 8.6](#) illustrates an elastic collision in which internal kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An **elastic collision** is one that conserves internal kinetic energy.

Internal Kinetic Energy

Internal kinetic energy is the sum of the kinetic energies of the objects in the system.

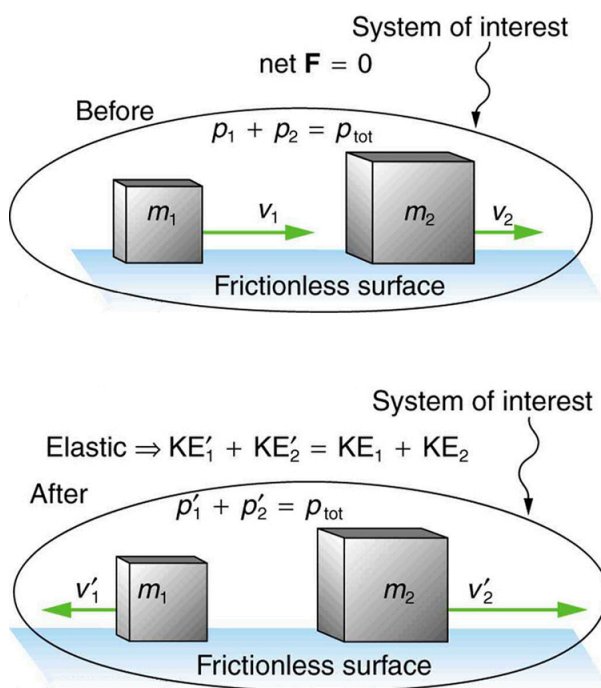


FIGURE 8.6 An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0) \quad 8.33$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0), \quad 8.34$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves internal kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v'_1{}^2 + \frac{1}{2} m_2 v'_2{}^2 \quad (\text{two-object elastic collision}) \quad 8.35$$

expresses the equation for conservation of internal kinetic energy in a one-dimensional collision.

EXAMPLE 8.4

Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500 \text{ kg}, \quad m_2 = 3.50 \text{ kg}, \quad v_1 = 4.00 \text{ m/s}, \quad \text{and} \quad v_2 = 0. \quad 8.36$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in [Figure 8.6](#) where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \quad 8.37$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \quad 8.38$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2. \quad 8.39$$

Solving the first equation (momentum equation) for v'_2 , we obtain

$$v'_2 = \frac{m_1}{m_2} (v_1 - v'_1). \quad 8.40$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2 , leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00 \text{ m/s} \quad 8.41$$

and

$$v'_1 = -3.00 \text{ m/s}. \quad 8.42$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution ($v'_1 = -3.00$ m/s) is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2} (v_1 - v'_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}} [4.00 - (-3.00)] \text{ m/s} \quad 8.43$$

or

$$v'_2 = 1.00 \text{ m/s}. \quad 8.44$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

PHET EXPLORATIONS

Collision Lab

Investigate collisions on an air hockey table. Set up your own experiments: vary the number of discs, masses and initial conditions. Is momentum conserved? Is kinetic energy conserved? Vary the elasticity and see what happens.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/8-4-elastic-collisions-in-one-dimension\)](https://openstax.org/books/college-physics-2e/pages/8-4-elastic-collisions-in-one-dimension)



8.5 Inelastic Collisions in One Dimension

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to sports.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, internal kinetic energy is conserved. An **inelastic collision** is one in which the internal kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add internal kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some internal kinetic energy into heat transfer. Or it may convert stored energy into internal kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Inelastic Collision

An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).

[Figure 8.7](#) shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total internal kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving momentum. But the internal kinetic energy is zero after the collision. A collision in which the objects stick together is sometimes called a **perfectly inelastic collision** because it reduces internal kinetic energy more than does any other type of inelastic collision. In fact, such a collision reduces internal kinetic energy to the minimum it can have while still conserving momentum.

Perfectly Inelastic Collision

A collision in which the objects stick together is sometimes called “perfectly inelastic.”

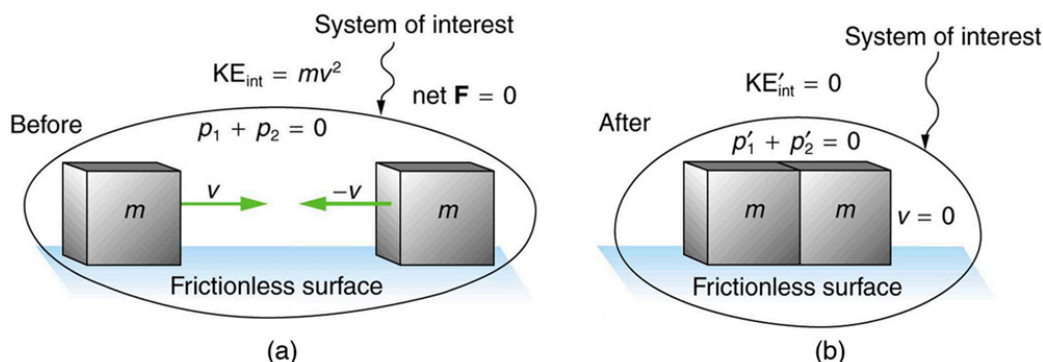


FIGURE 8.7 An inelastic one-dimensional two-object collision. Momentum is conserved, but internal kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic

collision), and so their final velocity is zero. The internal kinetic energy of the system changes in any inelastic collision and is reduced to zero in this example.

EXAMPLE 8.5

Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See [Figure 8.8](#))

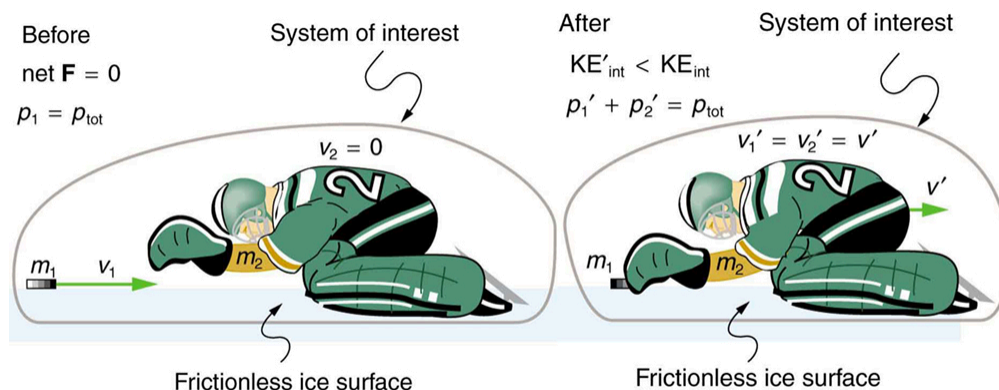


FIGURE 8.8 An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2 \quad 8.45$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \quad 8.46$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2) v'. \quad 8.47$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1. \quad 8.48$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{0.150 \text{ kg} + 70.0 \text{ kg}} \right) (35.0 \text{ m/s}) = 7.48 \times 10^{-2} \text{ m/s}. \quad 8.49$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$\begin{aligned} KE_{\text{int}} &= \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J.} \end{aligned} \quad 8.50$$

After the collision, the internal kinetic energy is

$$\begin{aligned} KE'_{\text{int}} &= \frac{1}{2}(m + M)v^2 = \frac{1}{2}(70.15 \text{ kg})(7.48 \times 10^{-2} \text{ m/s})^2 \\ &= 0.196 \text{ J.} \end{aligned} \quad 8.51$$

The change in internal kinetic energy is thus

$$\begin{aligned} KE'_{\text{int}} - KE_{\text{int}} &= 0.196 \text{ J} - 91.9 \text{ J} \\ &= -91.7 \text{ J} \end{aligned} \quad 8.52$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial internal kinetic energy is lost in this perfectly inelastic collision. KE_{int} is mostly converted to thermal energy and sound.

During some collisions, the objects do not stick together and less of the internal kinetic energy is removed—such as happens in most automobile accidents. Alternatively, stored energy may be converted into internal kinetic energy during a collision. [Figure 8.9](#) shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring. [Example 8.6](#) deals with data from such a collision.

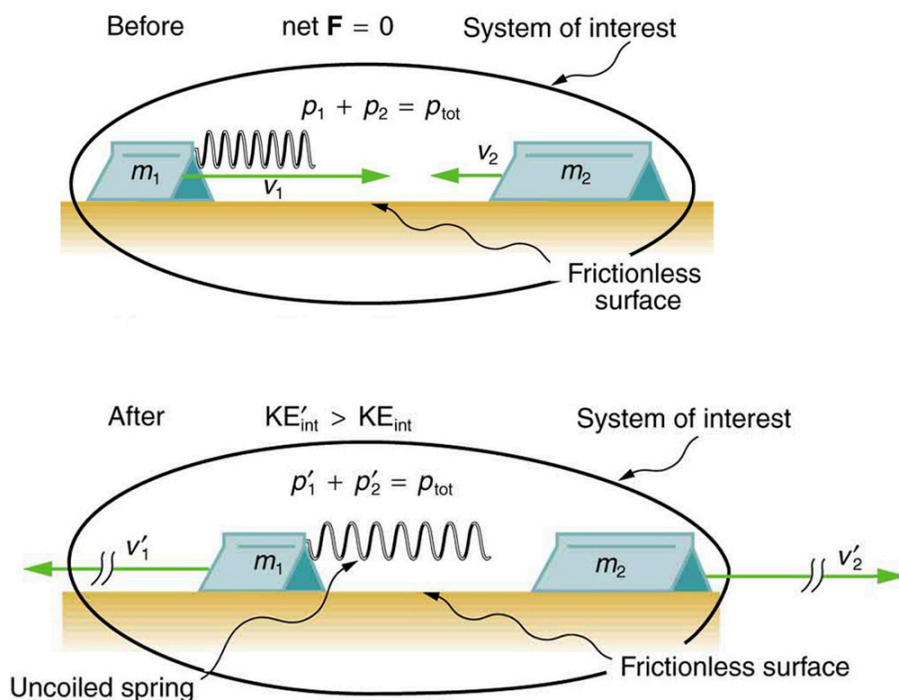


FIGURE 8.9 An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in [Example 8.6](#), the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

Collisions are particularly important in sports and the sporting and leisure industry utilizes elastic and inelastic collisions. Let us look briefly at tennis. Recall that in a collision, it is momentum and not force that is important. So, a heavier tennis racquet will have the advantage over a lighter one. This conclusion also holds true for other sports—a lightweight bat (such as a softball bat) cannot hit a hardball very far.

The location of the impact of the tennis ball on the racquet is also important, as is the part of the stroke during which the impact occurs. A smooth motion results in the maximizing of the velocity of the ball after impact and reduces sports injuries such as tennis elbow. A tennis player tries to hit the ball on the “sweet spot” on the racquet, where the vibration and impact are minimized and the ball is able to be given more velocity. Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

Take-Home Experiment—Bouncing of Tennis Ball

1. Find a racquet (a tennis, badminton, or other racquet will do). Place the racquet on the floor and stand on the handle. Drop a tennis ball on the strings from a measured height. Measure how high the ball bounces. Now ask a friend to hold the racquet firmly by the handle and drop a tennis ball from the same measured height above the racquet. Measure how high the ball bounces and observe what happens to your friend’s hand during the collision. Explain your observations and measurements.
2. The coefficient of restitution (c) is a measure of the elasticity of a collision between a ball and an object, and is defined as the ratio of the speeds after and before the collision. A perfectly elastic collision has a c of 1. For a ball bouncing off the floor (or a racquet on the floor), c can be shown to be $c = (h/H)^{1/2}$ where h is the height to which the ball bounces and H is the height from which the ball is dropped. Determine c for the cases in Part 1 and for the case of a tennis ball bouncing off a concrete or wooden floor ($c = 0.85$ for new tennis balls used on a tennis court).



EXAMPLE 8.6

Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in [Figure 8.9](#), two carts collide inelastically. Cart 1 (denoted m_1) carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to internal kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in [Figure 8.9](#)) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the internal kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \quad 8.53$$

The only unknown in this equation is v'_2 . Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350 \text{ kg})(2.00 \text{ m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s}) - (0.350 \text{ kg})(-4.00 \text{ m/s})}{0.500 \text{ kg}} \\ &= 3.70 \text{ m/s}. \end{aligned} \quad 8.54$$

Solution for (b)

The internal kinetic energy before the collision is

$$\begin{aligned}
 \text{KE}_{\text{int}} &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\
 &= \frac{1}{2}(0.350 \text{ kg})(2.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(-0.500 \text{ m/s})^2 \\
 &= 0.763 \text{ J}.
 \end{aligned}
 \tag{8.55}$$

After the collision, the internal kinetic energy is

$$\begin{aligned}
 \text{KE}'_{\text{int}} &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \\
 &= \frac{1}{2}(0.350 \text{ kg})(-4.00 \text{ m/s})^2 + \frac{1}{2}(0.500 \text{ kg})(3.70 \text{ m/s})^2 \\
 &= 6.22 \text{ J}.
 \end{aligned}
 \tag{8.56}$$

The change in internal kinetic energy is thus

$$\begin{aligned}
 \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22 \text{ J} - 0.763 \text{ J} \\
 &= 5.46 \text{ J}.
 \end{aligned}
 \tag{8.57}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The internal kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

8.6 Collisions of Point Masses in Two Dimensions

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss two dimensional collisions as an extension of one dimensional analysis.
- Define point masses.
- Derive an expression for conservation of momentum along x -axis and y -axis.
- Describe elastic collisions of two objects with equal mass.
- Determine the magnitude and direction of the final velocity given initial velocity, and scattering angle.

In the previous two sections, we considered only one-dimensional collisions; during such collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and we shall see that their study is an extension of the one-dimensional analysis already presented. The approach taken (similar to the approach in discussing two-dimensional kinematics and dynamics) is to choose a convenient coordinate system and resolve the motion into components along perpendicular axes. Resolving the motion yields a pair of one-dimensional problems to be solved simultaneously.

One complication arising in two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass by one another, they will spin in circles. We will not consider such rotation until later, and so for now we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum \mathbf{p} is conserved. The simplest collision is one in which one of the particles is initially at rest. (See [Figure 8.10](#).) The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in [Figure 8.10](#). Because momentum is conserved, the components of momentum along the x - and y -axes (p_x and p_y) will also be conserved, but with the chosen coordinate system, p_y is initially zero and p_x is the momentum of the incoming particle. Both facts simplify the analysis. (Even with the simplifying assumptions of point masses, one particle initially at rest, and a convenient coordinate system, we still gain new insights into nature from the analysis of two-dimensional collisions.)

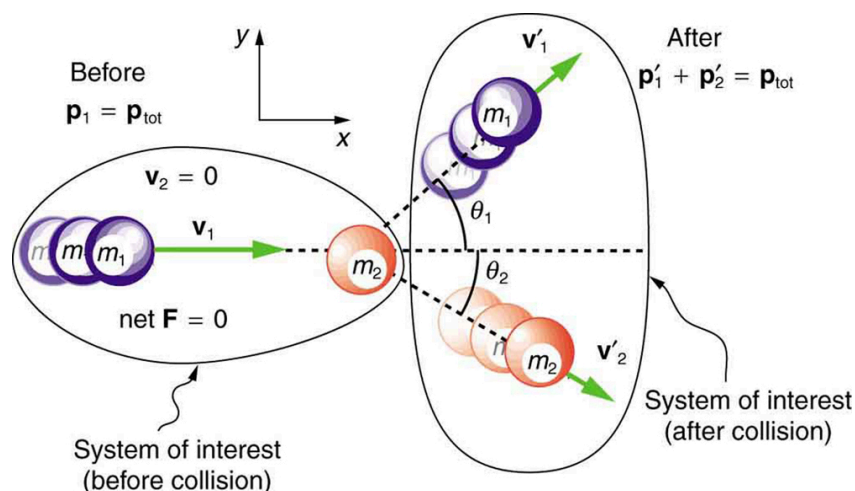


FIGURE 8.10 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and v_1 is parallel to the x -axis. This coordinate system is sometimes called the laboratory coordinate system, because many scattering experiments have a target that is stationary in the laboratory, while particles are scattered from it to determine the particles that make-up the target and how they are bound together. The particles may not be observed directly, but their initial and final velocities are.

Along the x -axis, the equation for conservation of momentum is

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}. \quad 8.58$$

Where the subscripts denote the particles and axes and the primes denote the situation after the collision. In terms of masses and velocities, this equation is

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}. \quad 8.59$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}. \quad 8.60$$

The components of the velocities along the x -axis have the form $v \cos \theta$. Because particle 1 initially moves along the x -axis, we find $v_{1x} = v_1$.

Conservation of momentum along the x -axis gives the following equation:

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2, \quad 8.61$$

where θ_1 and θ_2 are as shown in [Figure 8.10](#).

Conservation of Momentum along the x -axis

$$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2 \quad 8.62$$

Along the y -axis, the equation for conservation of momentum is

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y} \quad 8.63$$

or

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}. \quad 8.64$$

But v_{1y} is zero, because particle 1 initially moves along the x -axis. Because particle 2 is initially at rest, v_{2y} is also zero. The equation for conservation of momentum along the y -axis becomes

$$0 = m_1 v'_{1y} + m_2 v'_{2y}. \quad 8.65$$

The components of the velocities along the y -axis have the form $v \sin \theta$.

Thus, conservation of momentum along the y -axis gives the following equation:

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2. \quad 8.66$$

Conservation of Momentum along the y -axis

$$0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2 \quad 8.67$$

The equations of conservation of momentum along the x -axis and y -axis are very useful in analyzing two-dimensional collisions of particles, where one is originally stationary (a common laboratory situation). But two equations can only be used to find two unknowns, and so other data may be necessary when collision experiments are used to explore nature at the subatomic level.

EXAMPLE 8.7

Determining the Final Velocity of an Unseen Object from the Scattering of Another Object

Suppose the following experiment is performed. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45.0° with its incoming direction.

The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2 and θ_2) of the 0.400-kg object after the collision.

Strategy

Momentum is conserved because the surface is frictionless. The coordinate system shown in [Figure 8.11](#) is one in which m_2 is originally at rest and the initial velocity is parallel to the x -axis, so that conservation of momentum along the x - and y -axes is applicable.

Everything is known in these equations except v'_2 and θ_2 , which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the x - and y -directions.

Solution

Solving $m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$ for $v'_2 \cos \theta_2$ and $0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2$ for $v'_2 \sin \theta_2$ and taking the ratio yields an equation (in which θ_2 is the only unknown quantity. Applying the identity ($\tan \theta = \frac{\sin \theta}{\cos \theta}$), we obtain:

$$\tan \theta_2 = \frac{v'_1 \sin \theta_1}{v'_1 \cos \theta_1 - v_1}. \quad 8.68$$

Entering known values into the previous equation gives

$$\tan \theta_2 = \frac{(1.50 \text{ m/s})(0.7071)}{(1.50 \text{ m/s})(0.7071) - 2.00 \text{ m/s}} = -1.129. \quad 8.69$$

Thus,

$$\theta_2 = \tan^{-1}(-1.129) = 311.5^\circ \approx 312^\circ. \quad 8.70$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in [Figure 8.11](#), as expected (this angle is in the fourth quadrant). Either equation for the x - or y -axis can now be used to solve for v'_2 , but the latter equation is easiest because it has fewer terms.

$$v'_2 = -\frac{m_1}{m_2} v'_1 \frac{\sin \theta_1}{\sin \theta_2} \quad 8.71$$

Entering known values into this equation gives

$$v'_2 = -\left(\frac{0.250 \text{ kg}}{0.400 \text{ kg}}\right)(1.50 \text{ m/s})\left(\frac{0.7071}{-0.7485}\right). \quad 8.72$$

Thus,

$$v'_2 = 0.886 \text{ m/s}. \quad 8.73$$

Discussion

It is instructive to calculate the internal kinetic energy of this two-object system before and after the collision. (This calculation is left as an end-of-chapter problem.) If you do this calculation, you will find that the internal kinetic energy is less after the collision, and so the collision is inelastic. This type of result makes a physicist want to explore the system further.

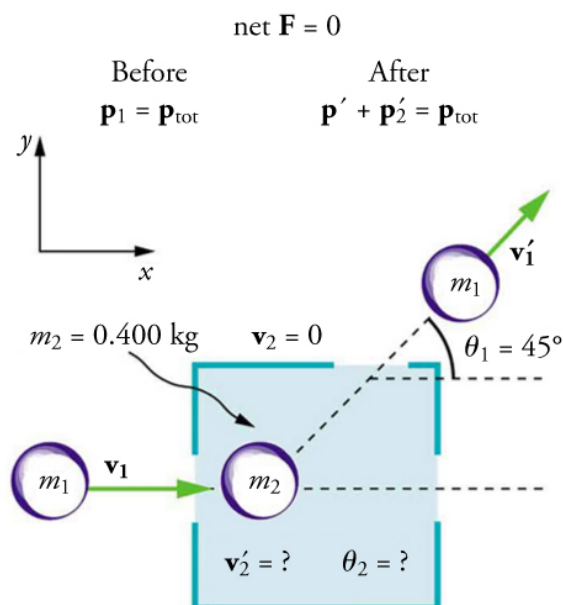


FIGURE 8.11 A collision taking place in a dark room is explored in [Example 8.7](#). The incoming object m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Elastic Collisions of Two Objects with Equal Mass

Some interesting situations arise when the two colliding objects have equal mass and the collision is elastic. This situation is nearly the case with colliding billiard balls, and precisely the case with some subatomic particle collisions. We can thus get a mental image of a collision of subatomic particles by thinking about billiards (or pool). (Refer to [Figure 8.10](#) for masses and angles.) First, an elastic collision conserves internal kinetic energy. Again, let us assume object 2 (m_2) is initially at rest. Then, the internal kinetic energy before and after the collision of two objects that have equal masses is

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2. \quad 8.74$$

Because the masses are equal, $m_1 = m_2 = m$. Algebraic manipulation (left to the reader) of conservation of momentum in the x - and y -directions can show that

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 + mv_1'v_2' \cos(\theta_1 - \theta_2). \quad 8.75$$

(Remember that θ_2 is negative here.) The two preceding equations can both be true only if

$$mv_1'v_2' \cos(\theta_1 - \theta_2) = 0. \quad 8.76$$

There are three ways that this term can be zero. They are

- $v'_1 = 0$: head-on collision; incoming ball stops
- $v'_2 = 0$: no collision; incoming ball continues unaffected
- $\cos(\theta_1 - \theta_2) = 0$: angle of separation ($\theta_1 - \theta_2$) is 90° after the collision

All three of these ways are familiar occurrences in billiards and pool, although most of us try to avoid the second. If you play enough pool, you will notice that the angle between the balls is very close to 90° after the collision, although it will vary from this value if a great deal of spin is placed on the ball. (Large spin carries in extra energy and a quantity called *angular momentum*, which must also be conserved.) The assumption that the scattering of billiard balls is elastic is reasonable based on the correctness of the three results it produces. This assumption also implies that, to a good approximation, momentum is conserved for the two-ball system in billiards and pool. The problems below explore these and other characteristics of two-dimensional collisions.

Connections to Nuclear and Particle Physics

Two-dimensional collision experiments have revealed much of what we know about subatomic particles, as we shall see in [Medical Applications of Nuclear Physics](#) and [Particle Physics](#). Ernest Rutherford, for example, discovered the nature of the atomic nucleus from such experiments.

8.7 Introduction to Rocket Propulsion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State Newton's third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket and discuss the factors that affect the acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton's third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun's recoil or kick.

Making Connections: Take-Home Experiment—Propulsion of a Balloon

Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon's direction change? Explain your answer.

[Figure 8.12](#) shows a rocket accelerating straight up. In part (a), the rocket has a mass m and a velocity v relative to Earth, and hence a momentum mv . In part (b), a time Δt has elapsed in which the rocket has ejected a mass Δm of hot gas at a velocity v_e relative to the rocket. The remainder of the mass ($m - \Delta m$) now has a greater velocity ($v + \Delta v$). The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time Δt , producing a negative impulse $\Delta p = -mg\Delta t$. (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket's thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over Δt , and equating this change to the impulse, the

following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g \quad 8.77$$

“The rocket” is that part of the system remaining after the gas is ejected, and g is the acceleration due to gravity.

Acceleration of a Rocket

Acceleration of a rocket is

$$a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g, \quad 8.78$$

where a is the acceleration of the rocket, v_e is the exhaust velocity, m is the mass of the rocket, Δm is the mass of the ejected gas, and Δt is the time in which the gas is ejected.

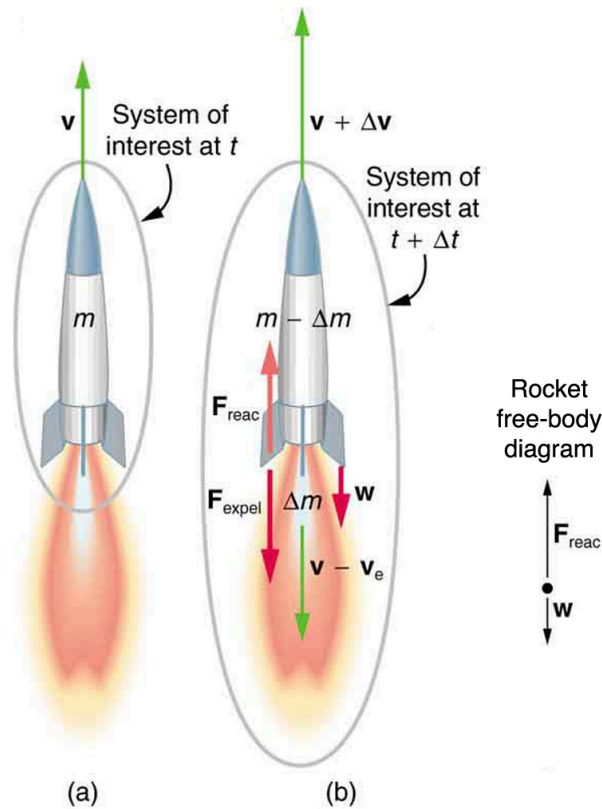


FIGURE 8.12 (a) This rocket has a mass m and an upward velocity v . The net external force on the system is $-mg$, if air resistance is neglected. (b) A time Δt later the system has two main parts, the ejected gas and the remainder of the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket. First, the greater the exhaust velocity of the gases relative to the rocket, v_e , the greater the acceleration is. The practical limit for v_e is about 2.5×10^3 m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor $\Delta m/\Delta t$ in the equation. The quantity $(\Delta m/\Delta t)v_e$, with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass m of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass m decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity v_e of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

EXAMPLE 8.8

Calculating Acceleration: Initial Acceleration of a Moon Launch

A Saturn V's mass at liftoff was 2.80×10^6 kg, its fuel-burn rate was 1.40×10^4 kg/s, and the exhaust velocity was 2.40×10^3 m/s. Calculate its initial acceleration.

Strategy

This problem is a straightforward application of the expression for acceleration because a is the unknown and all of the terms on the right side of the equation are given.

Solution

Substituting the given values into the equation for acceleration yields

$$\begin{aligned} a &= \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g \\ &= \frac{2.40 \times 10^3 \text{ m/s}}{2.80 \times 10^6 \text{ kg}} (1.40 \times 10^4 \text{ kg/s}) - 9.80 \text{ m/s}^2 \\ &= 2.20 \text{ m/s}^2. \end{aligned} \quad 8.79$$

Discussion

This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because m decreases while v_e and $\frac{\Delta m}{\Delta t}$ remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was 3.36×10^7 N.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln \frac{m_0}{m_r}, \quad 8.80$$

where $\ln(m_0/m_r)$ is the natural logarithm of the ratio of the initial mass of the rocket (m_0) to what is left (m_r) after all of the fuel is exhausted. (Note that v is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about 11.2×10^3 m/s, and assuming an exhaust velocity $v_e = 2.5 \times 10^3$ m/s.

$$\ln \frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2 \times 10^3 \text{ m/s}}{2.5 \times 10^3 \text{ m/s}} = 4.48 \quad 8.81$$

Solving for m_0/m_r gives

$$\frac{m_0}{m_r} = e^{4.48} = 88. \quad 8.82$$

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}. \quad 8.83$$

This result means that only $1/88$ of the mass is left when the fuel is burnt, and $87/88$ of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass m_f remaining can only be about $m_0/180$. It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See [Figure 8.13](#)) The shuttle's need to be operated by humans, however, made it at least as costly for launching satellites as expendable, unpiloted rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.

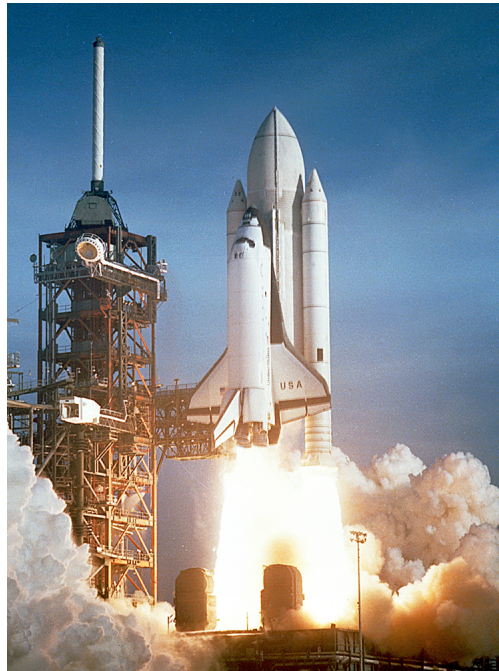


FIGURE 8.13 The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)

Glossary

change in momentum the difference between the final and initial momentum; the mass times the change in velocity

conservation of momentum principle when the net external force is zero, the total momentum of the system is conserved or constant

elastic collision a collision that also conserves internal kinetic energy

impulse the average net external force times the time it acts; equal to the change in momentum

inelastic collision a collision in which internal kinetic energy is not conserved

internal kinetic energy the sum of the kinetic energies of the objects in a system

isolated system a system in which the net external force is zero

linear momentum the product of mass and velocity

perfectly inelastic collision a collision in which the colliding objects stick together

point masses structureless particles with no rotation or spin

quark fundamental constituent of matter and an elementary particle

second law of motion physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Section Summary

8.1 Linear Momentum and Force

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum \mathbf{p} is defined to be $\mathbf{p} = m\mathbf{v}$, where m is the mass of the system and \mathbf{v} is its velocity.
- The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$, \mathbf{F}_{net} is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change time.

8.2 Impulse

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts: $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.
- Forces are usually not constant over a period of time.

8.3 Conservation of Momentum

- The conservation of momentum principle is written $\mathbf{p}_{\text{tot}} = \text{constant}$ or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$ (isolated system), \mathbf{p}_{tot} is the initial total momentum and \mathbf{p}'_{tot} is the

total momentum some time later.

- An isolated system is defined to be one for which the net external force is zero ($\mathbf{F}_{\text{net}} = 0$).
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.
- Conservation of momentum applies only when the net external force is zero.
- The conservation of momentum principle is valid when considering systems of particles.

8.4 Elastic Collisions in One Dimension

- An elastic collision is one that conserves internal kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

8.5 Inelastic Collisions in One Dimension

- An inelastic collision is one in which the internal kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces internal kinetic energy more than does any other type of inelastic collision.
- Sports science and technologies also use physics concepts such as momentum and rotational motion and vibrations.

8.6 Collisions of Point Masses in Two Dimensions

- The approach to two-dimensional collisions is to choose a convenient coordinate system and break

the motion into components along perpendicular axes. Choose a coordinate system with the x -axis parallel to the velocity of the incoming particle.

- Two-dimensional collisions of point masses where mass 2 is initially at rest conserve momentum along the initial direction of mass 1 (the x -axis), stated by $m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$ and along the direction perpendicular to the initial direction (the y -axis) stated by $0 = m_1 v'_{1y} + m_2 v'_{2y}$.
- The internal kinetic before and after the collision of two objects that have equal masses is $\frac{1}{2} m v_1^2 = \frac{1}{2} m v'^2_1 + \frac{1}{2} m v'^2_2 + m v'_1 v'_2 \cos(\theta_1 - \theta_2)$.
- Point masses are structureless particles that

cannot spin.

8.7 Introduction to Rocket Propulsion

- Newton's third law of motion states that to every action, there is an equal and opposite reaction.
- Acceleration of a rocket is $a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g$.
- A rocket's acceleration depends on three main factors. They are
 1. The greater the exhaust velocity of the gases, the greater the acceleration.
 2. The faster the rocket burns its fuel, the greater its acceleration.
 3. The smaller the rocket's mass, the greater the acceleration.

Conceptual Questions

8.1 Linear Momentum and Force

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?
2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?
3. Professional Application
Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.
4. How can a small force impart the same momentum to an object as a large force?

8.2 Impulse

5. Professional Application
Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.
6. While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?
7. Professional Application
Tennis racquets have "sweet spots." If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

8.3 Conservation of Momentum

8. Professional Application
If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.
9. Under what circumstances is momentum conserved?
10. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?
11. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.
12. Professional Application
Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.
13. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.
14. Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

8.4 Elastic Collisions in One Dimension

15. What is an elastic collision?

8.5 Inelastic Collisions in One Dimension

16. What is an inelastic collision? What is a perfectly inelastic collision?

17. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?
18. A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

8.6 Collisions of Point Masses in Two Dimensions

19. [Figure 8.14](#) shows a cube at rest and a small object heading toward it. (a) Describe the directions (angle θ_1) at which the small object can emerge after colliding elastically with the cube. How does θ_1 depend on b , the so-called impact parameter? Ignore any effects that might be due to rotation after the collision, and assume that the cube is much more massive than the small object. (b) Answer the same questions if the small object instead collides with a massive sphere.

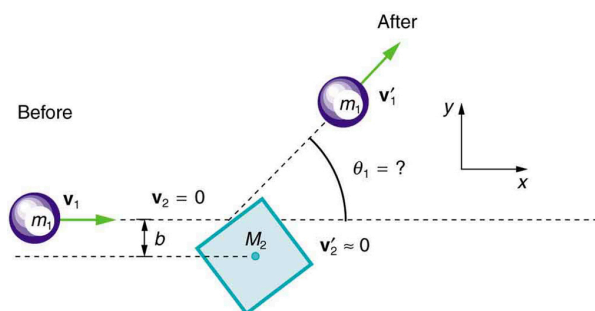


FIGURE 8.14 A small object approaches a collision with a much more massive cube, after which its velocity has the direction θ_1 . The angles at which the small object can be scattered are determined by the shape of the object it strikes and the impact parameter b .

Problems & Exercises

8.1 Linear Momentum and Force

1. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s. (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

8.7 Introduction to Rocket Propulsion

20. Professional Application
Suppose a fireworks shell explodes, breaking into three large pieces for which air resistance is negligible. How is the motion of the center of mass affected by the explosion? How would it be affected if the pieces experienced significantly more air resistance than the intact shell?
21. Professional Application
During a visit to the International Space Station, an astronaut was positioned motionless in the center of the station, out of reach of any solid object on which he could exert a force. Suggest a method by which he could move himself away from this position, and explain the physics involved.
22. Professional Application
It is possible for the velocity of a rocket to be greater than the exhaust velocity of the gases it ejects. When that is the case, the gas velocity and gas momentum are in the same direction as that of the rocket. How is the rocket still able to obtain thrust by ejecting the gases?

2. (a) What is the mass of a large ship that has a momentum of $1.60 \times 10^9 \text{ kg} \cdot \text{m/s}$, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s.

3. (a) At what speed would a 2.00×10^4 -kg airplane have to fly to have a momentum of 1.60×10^9 kg · m/s (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.
4. (a) What is the momentum of a garbage truck that is 1.20×10^4 kg and is moving at 30.0 m/s? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?
5. A runaway train car that has a mass of 15,000 kg travels at a speed of 5.4 m/s down a track. Compute the time required for a force of 1500 N to bring the car to rest.
6. The mass of Earth is 5.972×10^{24} kg and its orbital radius is an average of 1.496×10^{11} m. Calculate its linear momentum.

8.2 Impulse

7. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600.0 m/s in a time of 2.00 ms (milliseconds)?
8. Professional Application
A car moving at 10.0 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70.0 kg.
9. A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.
10. Professional Application
A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).
11. Professional Application
Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.
12. Professional Application
One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of 4.00×10^3 m/s, given the collision lasts 6.00×10^{-8} s.
13. Professional Application
A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.
14. Professional Application
Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

- 15.** A cruise ship with a mass of 1.00×10^7 kg strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)
- 16.** Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of 1.76×10^4 N for 5.50×10^{-2} s.
- 17.** Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.
- 18.** A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?
- 19.** Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.
- 20.** A ball with an initial velocity of 10 m/s moves at an angle 60° above the $+x$ -direction. The ball hits a vertical wall and bounces off so that it is moving 60° above the $-x$ -direction with the same speed. In terms of m , the mass of the ball, what is the impulse delivered by the wall?
- 21.** When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.
- 22.** A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55° above the horizontal. In terms of m , the mass of the ball, what is the impulse delivered by the foot (magnitude and direction)?

8.3 Conservation of Momentum

- 23. Professional Application**
Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s. (The minus indicates direction of motion.) What is their final velocity?
- 24.** Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?
- 25. Professional Application**
Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.
- 26.** What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.
- 27.** A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

8.4 Elastic Collisions in One Dimension

- 28.** Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.
- 29. Professional Application**
Two piloted satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of 4.00×10^3 kg, and the second a mass of 7.50×10^3 kg. If the two satellites collide elastically rather than dock, what is their final relative velocity?

- 30.** A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

8.5 Inelastic Collisions in One Dimension

- 31.** A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?
- 32.** During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?
- 33.** Professional Application
Using mass and speed data from [Example 8.1](#) and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?
- 34.** A battleship that is 6.00×10^7 kg and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.
- 35.** Professional Application
Two piloted satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of 4.00×10^3 kg, and the second a mass of 7.50×10^3 kg. (a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.
- 36.** Professional Application
A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?
- 37.** Professional Application
Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?
- 38.** A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

39. Professional Application

One of the waste products of a nuclear reactor is plutonium-239 (^{239}Pu). This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus ($^4\text{He} + ^{235}\text{U}$), the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is 8.40×10^{-13} J and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is 6.68×10^{-27} kg, while that of the uranium is 3.92×10^{-25} kg (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

40. Professional Application

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of 5.00×10^{12} kg (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is 7.36×10^{22} kg)? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

41. Professional Application

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of -3.50 m/s. What is their velocity just after impact if they cling together?

42. What is the speed of a garbage truck that is 1.20×10^4 kg and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

43. During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

44. (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

8.6 Collisions of Point Masses in Two Dimensions

45. Two identical pucks collide on an air hockey table. One puck was originally at rest. (a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0° , what is the velocity (magnitude and direction) of the second puck? (You may use the result that $\theta_1 - \theta_2 = 90^\circ$ for elastic collisions of objects that have identical masses.) (b) Confirm that the collision is elastic.

46. Confirm that the results of the example [Example 8.7](#) do conserve momentum in both the x - and y -directions.

47. A 3000-kg cannon is mounted so that it can recoil only in the horizontal direction. (a) Calculate its recoil velocity when it fires a 15.0-kg shell at 480 m/s at an angle of 20.0° above the horizontal. (b) What is the kinetic energy of the cannon? This energy is dissipated as heat transfer in shock absorbers that stop its recoil. (c) What happens to the vertical component of momentum that is imparted to the cannon when it is fired?

48. Professional Application

A 5.50-kg bowling ball moving at 9.00 m/s collides with a 0.850-kg bowling pin, which is scattered at an angle of 85.0° to the initial direction of the bowling ball and with a speed of 15.0 m/s. (a) Calculate the final velocity (magnitude and direction) of the bowling ball. (b) Is the collision elastic? (c) Linear kinetic energy is greater after the collision. Discuss how spin on the ball might be converted to linear kinetic energy in the collision.

49. Professional Application

Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei (${}^4\text{He}$) from gold-197 nuclei (${}^{197}\text{Au}$). The energy of the incoming helium nucleus was 8.00×10^{-13} J, and the masses of the helium and gold nuclei were 6.68×10^{-27} kg and 3.29×10^{-25} kg, respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of 120° during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?

50. Professional Application

Two cars collide at an icy intersection and stick together afterward. The first car has a mass of 1200 kg and is approaching at 8.00 m/s due south. The second car has a mass of 850 kg and is approaching at 17.0 m/s due west. (a) Calculate the final velocity (magnitude and direction) of the cars. (b) How much kinetic energy is lost in the collision? (This energy goes into deformation of the cars.) Note that because both cars have an initial velocity, you cannot use the equations for conservation of momentum along the x -axis and y -axis; instead, you must look for other simplifying aspects.

51. Starting with equations

$m_1 v_1 = m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2$ and $0 = m_1 v'_1 \sin \theta_1 + m_2 v'_2 \sin \theta_2$ for conservation of momentum in the x - and y -directions and assuming that one object is originally stationary, prove that for an elastic collision of two objects of equal masses,

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v'_1{}^2 + \frac{1}{2} m v'_2{}^2 + m v'_1 v'_2 \cos(\theta_1 - \theta_2)$$

as discussed in the text.

52. Integrated Concepts

A 90.0-kg ice hockey player hits a 0.150-kg puck, giving the puck a velocity of 45.0 m/s. If both are initially at rest and if the ice is frictionless, how far does the player recoil in the time it takes the puck to reach the goal 15.0 m away?

8.7 Introduction to Rocket Propulsion**53. Professional Application**

Antiballistic missiles (ABMs) are designed to have very large accelerations so that they may intercept fast-moving incoming missiles in the short time available. What is the takeoff acceleration of a 10,000-kg ABM that expels 196 kg of gas per second at an exhaust velocity of 2.50×10^3 m/s?

54. Professional Application

What is the acceleration of a 5000-kg rocket taking off from the Moon, where the acceleration due to gravity is only 1.6 m/s^2 , if the rocket expels 8.00 kg of gas per second at an exhaust velocity of 2.20×10^3 m/s?

55. Professional Application

Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of 2.00×10^3 m/s. You may assume the gravitational force is negligible at the probe's location.

56. Professional Application

Ion-propulsion rockets have been proposed for use in space. They employ atomic ionization techniques and nuclear energy sources to produce extremely high exhaust velocities, perhaps as great as 8.00×10^6 m/s. These techniques allow a much more favorable payload-to-fuel ratio. To illustrate this fact: (a) Calculate the increase in velocity of a 20,000-kg space probe that expels only 40.0-kg of its mass at the given exhaust velocity. (b) These engines are usually designed to produce a very small thrust for a very long time—the type of engine that might be useful on a trip to the outer planets, for example. Calculate the acceleration of such an engine if it expels 4.50×10^{-6} kg/s at the given velocity, assuming the acceleration due to gravity is negligible.

57. Derive the equation for the vertical acceleration of a rocket.**58. Professional Application**

(a) Calculate the maximum rate at which a rocket can expel gases if its acceleration cannot exceed seven times that of gravity. The mass of the rocket just as it runs out of fuel is 75,000-kg, and its exhaust velocity is 2.40×10^3 m/s. Assume that the acceleration of gravity is the same as on Earth's surface (9.80 m/s^2). (b) Why might it be necessary to limit the acceleration of a rocket?

- 59.** Given the following data for a fire extinguisher-toy wagon rocket experiment, calculate the average exhaust velocity of the gases expelled from the extinguisher. Starting from rest, the final velocity is 10.0 m/s. The total mass is initially 75.0 kg and is 70.0 kg after the extinguisher is fired.
- 60.** How much of a single-stage rocket that is 100,000 kg can be anything but fuel if the rocket is to have a final speed of 8.00 km/s, given that it expels gases at an exhaust velocity of 2.20×10^3 m/s?
- 61.** Professional Application
(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?
- 62.** Unreasonable Results
Squids have been reported to jump from the ocean and travel 30.0 m (measured horizontally) before re-entering the water. (a) Calculate the initial speed of the squid if it leaves the water at an angle of 20.0° , assuming negligible lift from the air and negligible air resistance. (b) The squid propels itself by squirting water. What fraction of its mass would it have to eject in order to achieve the speed found in the previous part? The water is ejected at 12.0 m/s; gravitational force and friction are neglected. (c) What is unreasonable about the results? (d) Which premise is unreasonable, or which premises are inconsistent?
- 63.** Construct Your Own Problem
Consider an astronaut in deep space cut free from her space ship and needing to get back to it. The astronaut has a few packages that she can throw away to move herself toward the ship. Construct a problem in which you calculate the time it takes her to get back by throwing all the packages at one time compared to throwing them one at a time. Among the things to be considered are the masses involved, the force she can exert on the packages through some distance, and the distance to the ship.
- 64.** Construct Your Own Problem
Consider an artillery projectile striking armor plating. Construct a problem in which you find the force exerted by the projectile on the plate. Among the things to be considered are the mass and speed of the projectile and the distance over which its speed is reduced. Your instructor may also wish for you to consider the relative merits of depleted uranium versus lead projectiles based on the greater density of uranium.

CHAPTER 9

Statics and Torque



FIGURE 9.1 On a short time scale, rocks like these in Australia's Kalbarri National Park are static, or motionless relative to the Earth. (credit: Richard Liblanc/Flickr)

CHAPTER OUTLINE

- 9.1 The First Condition for Equilibrium**
- 9.2 The Second Condition for Equilibrium**
- 9.3 Stability**
- 9.4 Applications of Statics, Including Problem-Solving Strategies**
- 9.5 Simple Machines**
- 9.6 Forces and Torques in Muscles and Joints**

INTRODUCTION TO STATICS AND TORQUE What might desks, bridges, buildings, trees, and mountains have in common—at least in the eyes of a physicist? The answer is that they are ordinarily motionless relative to the Earth. Furthermore, their acceleration is zero because they remain motionless. That means they also have something in common with a car moving at a constant velocity, because anything with a constant velocity also has an acceleration of zero. Now, the important part—Newton's second law states that net $\mathbf{F} = m\mathbf{a}$, and so the net external force is zero for all stationary objects and for all objects moving at constant velocity. There are forces acting, but they are balanced. That is, they are in *equilibrium*.

Statics

Statics is the study of forces in equilibrium, a large group of situations that makes up a special case of Newton's second law. We have already considered a few such situations; in this chapter, we cover the topic more thoroughly, including consideration of such possible effects as the rotation and deformation of an object by the

forces acting on it.

How can we guarantee that a body is in equilibrium and what can we learn from systems that are in equilibrium? There are actually two conditions that must be satisfied to achieve equilibrium. These conditions are the topics of the first two sections of this chapter.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/9-introduction-to-statics-and-torque\)](https://openstax.org/books/college-physics-2e/pages/9-introduction-to-statics-and-torque)

9.1 The First Condition for Equilibrium

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State the first condition of equilibrium.
- Explain static equilibrium.
- Explain dynamic equilibrium.

The first condition necessary to achieve equilibrium is the one already mentioned: the net external force on the system must be zero. Expressed as an equation, this is simply

$$\text{net } \mathbf{F} = 0 \quad 9.1$$

Note that if net \mathbf{F} is zero, then the net external force in *any* direction is zero. For example, the net external forces along the typical x - and y -axes are zero. This is written as

$$\text{net } F_x = 0 \text{ and } \text{net } F_y = 0 \quad 9.2$$

[Figure 9.2](#) and [Figure 9.3](#) illustrate situations where net $\mathbf{F} = 0$ for both **static equilibrium** (motionless), and **dynamic equilibrium** (constant velocity).

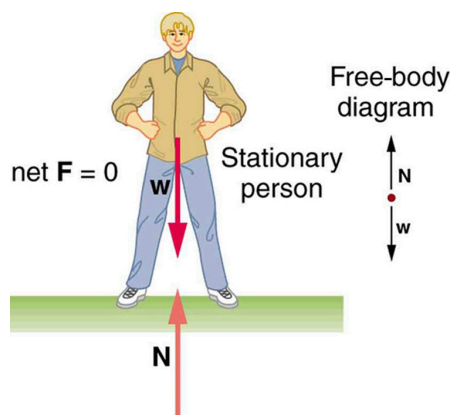


FIGURE 9.2 This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.

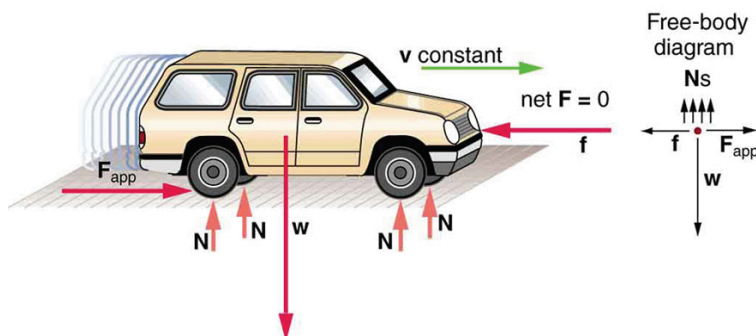


FIGURE 9.3 This car is in dynamic equilibrium because it is moving at constant velocity. There are horizontal and vertical forces, but the net external force in any direction is zero. The applied force \mathbf{F}_{app} between the tires and the road is balanced by air friction, and the weight of the car is supported by the normal forces, here shown to be equal for all four tires.

However, it is not sufficient for the net external force of a system to be zero for a system to be in equilibrium. Consider the two situations illustrated in [Figure 9.4](#) and [Figure 9.5](#) where forces are applied to an ice hockey stick

lying flat on ice. The net external force is zero in both situations shown in the figure; but in one case, equilibrium is achieved, whereas in the other, it is not. In [Figure 9.4](#), the ice hockey stick remains motionless. But in [Figure 9.5](#), with the same forces applied in different places, the stick experiences accelerated rotation. Therefore, we know that the point at which a force is applied is another factor in determining whether or not equilibrium is achieved. This will be explored further in the next section.

Equilibrium: remains stationary

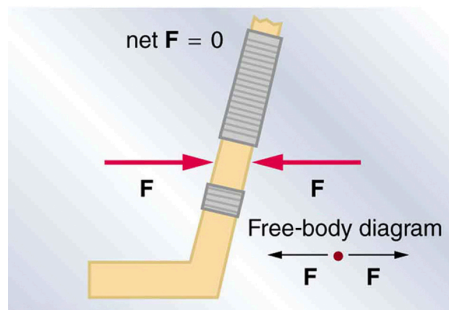


FIGURE 9.4 An ice hockey stick lying flat on ice with two equal and opposite horizontal forces applied to it. Friction is negligible, and the gravitational force is balanced by the support of the ice (a normal force). Thus, $\text{net } \mathbf{F} = 0$. Equilibrium is achieved, which is static equilibrium in this case.

Nonequilibrium: rotation accelerates

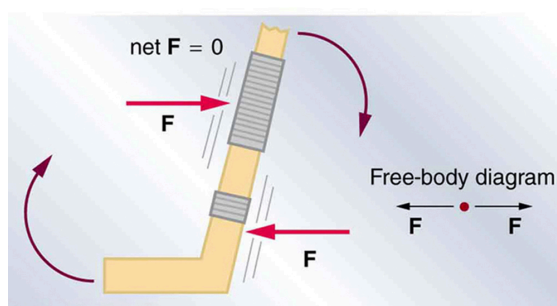


FIGURE 9.5 The same forces are applied at other points and the stick rotates—in fact, it experiences an accelerated rotation. Here $\text{net } \mathbf{F} = 0$ but the system is *not* at equilibrium. Hence, the $\text{net } \mathbf{F} = 0$ is a necessary—but not sufficient—condition for achieving equilibrium.



PHET EXPLORATIONS

Torque

Investigate how torque causes an object to rotate. Discover the relationships between angular acceleration, moment of inertia, angular momentum and torque.

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9.2 The Second Condition for Equilibrium

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State the second condition that is necessary to achieve equilibrium.
- Explain torque and the factors on which it depends.
- Describe the role of torque in rotational mechanics.

Torque

The second condition necessary to achieve equilibrium involves avoiding accelerated rotation (maintaining a

constant angular velocity). A rotating body or system can be in equilibrium if its rate of rotation is constant and remains unchanged by the forces acting on it. To understand what factors affect rotation, let us think about what happens when you open an ordinary door by rotating it on its hinges.

Several familiar factors determine how effective you are in opening the door. See [Figure 9.6](#). First of all, the larger the force, the more effective it is in opening the door—obviously, the harder you push, the more rapidly the door opens. Also, the point at which you push is crucial. If you apply your force too close to the hinges, the door will open slowly, if at all. Most people have been embarrassed by making this mistake and bumping up against a door when it did not open as quickly as expected. Finally, the direction in which you push is also important. The most effective direction is perpendicular to the door—we push in this direction almost instinctively.

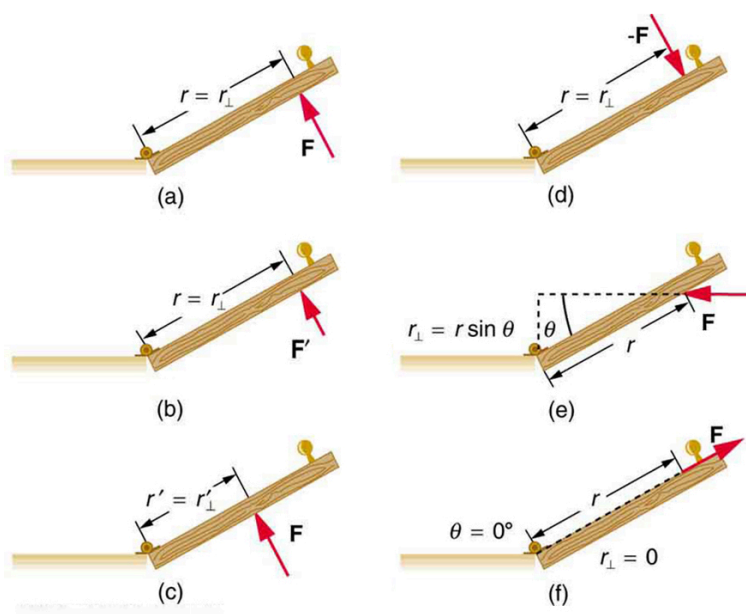


FIGURE 9.6 Torque is the turning or twisting effectiveness of a force, illustrated here for door rotation on its hinges (as viewed from overhead). Torque has both magnitude and direction. (a) Counterclockwise torque is produced by this force, which means that the door will rotate in a counterclockwise due to \mathbf{F} . Note that r_{\perp} is the perpendicular distance of the pivot from the line of action of the force. (b) A smaller counterclockwise torque is produced by a smaller force \mathbf{F}' acting at the same distance from the hinges (the pivot point). (c) The same force as in (a) produces a smaller counterclockwise torque when applied at a smaller distance from the hinges. (d) The same force as in (a), but acting in the opposite direction, produces a clockwise torque. (e) A smaller counterclockwise torque is produced by the same magnitude force acting at the same point but in a different direction. Here, θ is less than 90° . (f) Torque is zero here since the force just pulls on the hinges, producing no rotation. In this case, $\theta = 0^{\circ}$.

The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque** is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time). In equation form, the magnitude of torque is defined to be

$$\tau = rF \sin \theta \quad 9.3$$

where τ (the Greek letter tau) is the symbol for torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between the force and the vector directed from the point of application to the pivot point, as seen in [Figure 9.6](#) and [Figure 9.7](#). An alternative expression for torque is given in terms of the **perpendicular lever arm** r_{\perp} as shown in [Figure 9.6](#) and [Figure 9.7](#), which is defined as

$$r_{\perp} = r \sin \theta \quad 9.4$$

so that

$$\tau = r_{\perp} F. \quad 9.5$$

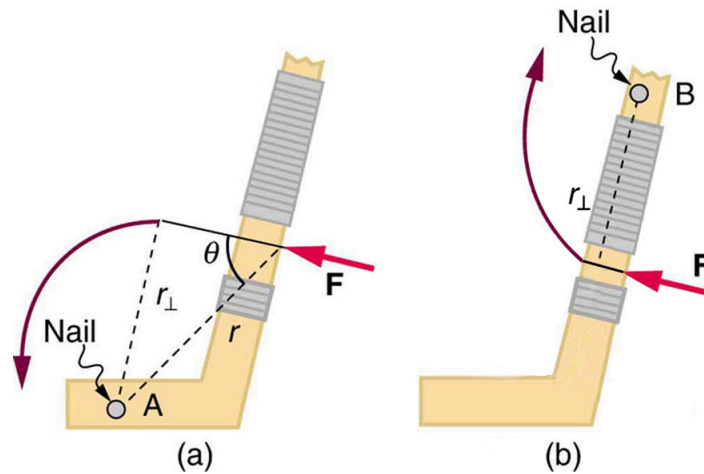


FIGURE 9.7 A force applied to an object can produce a torque, which depends on the location of the pivot point. (a) The three factors \mathbf{r} , \mathbf{F} , and θ for pivot point A on a body are shown here— \mathbf{r} is the distance from the chosen pivot point to the point where the force \mathbf{F} is applied, and θ is the angle between \mathbf{F} and the vector directed from the point of application to the pivot point. If the object can rotate around point A, it will rotate counterclockwise. This means that torque is counterclockwise relative to pivot A. (b) In this case, point B is the pivot point. The torque from the applied force will cause a clockwise rotation around point B, and so it is a clockwise torque relative to B.

The perpendicular lever arm r_{\perp} is the shortest distance from the pivot point to the line along which \mathbf{F} acts; it is shown as a dashed line in [Figure 9.6](#) and [Figure 9.7](#). Note that the line segment that defines the distance r_{\perp} is perpendicular to \mathbf{F} , as its name implies. It is sometimes easier to find or visualize r_{\perp} than to find both r and θ . In such cases, it may be more convenient to use $\tau = r_{\perp}F$ rather than $\tau = rF \sin \theta$ for torque, but both are equally valid.

The **SI unit of torque** is newtons times meters, usually written as $\text{N} \cdot \text{m}$. For example, if you push perpendicular to the door with a force of 40 N at a distance of 0.800 m from the hinges, you exert a torque of $32 \text{ N} \cdot \text{m}$ ($0.800 \text{ m} \times 40 \text{ N} \times \sin 90^\circ$) relative to the hinges. If you reduce the force to 20 N, the torque is reduced to $16 \text{ N} \cdot \text{m}$, and so on.

The torque is always calculated with reference to some chosen pivot point. For the same applied force, a different choice for the location of the pivot will give you a different value for the torque, since both r and θ depend on the location of the pivot. Any point in any object can be chosen to calculate the torque about that point. The object may not actually pivot about the chosen “pivot point.”

Note that for rotation in a plane, torque has two possible directions. Torque is either clockwise or counterclockwise relative to the chosen pivot point, as illustrated for points B and A, respectively, in [Figure 9.7](#). If the object can rotate about point A, it will rotate counterclockwise, which means that the torque for the force is shown as counterclockwise relative to A. But if the object can rotate about point B, it will rotate clockwise, which means the torque for the force shown is clockwise relative to B. Also, the magnitude of the torque is greater when the lever arm is longer.

Now, *the second condition necessary to achieve equilibrium* is that *the net external torque on a system must be zero*. An external torque is one that is created by an external force. You can choose the point around which the torque is calculated. The point can be the physical pivot point of a system or any other point in space—but it must be the same point for all torques. If the second condition (net external torque on a system is zero) is satisfied for one choice of pivot point, it will also hold true for any other choice of pivot point in or out of the system of interest. (This is true only in an inertial frame of reference.) The second condition necessary to achieve equilibrium is stated in equation form as

$$\text{net } \tau = 0 \quad 9.6$$

where net means total. Torques, which are in opposite directions are assigned opposite signs. A common convention is to call counterclockwise (ccw) torques positive and clockwise (cw) torques negative.

When two children balance a seesaw as shown in [Figure 9.8](#), they satisfy the two conditions for equilibrium. Most people have perfect intuition about seesaws, knowing that the lighter child must sit farther from the pivot and that a heavier child can keep a lighter one off the ground indefinitely.

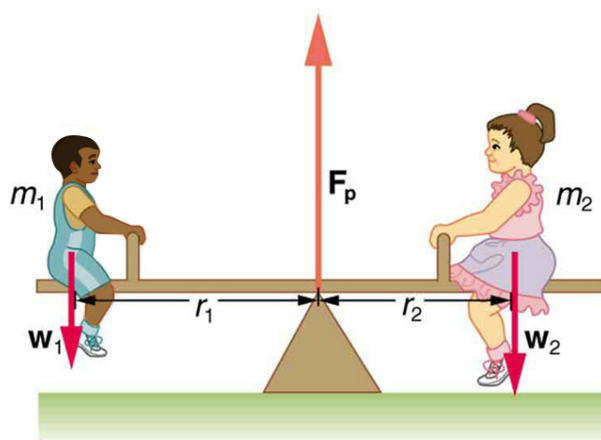


FIGURE 9.8 Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

EXAMPLE 9.1

She Saw Torques On A Seesaw

The two children shown in [Figure 9.8](#) are balanced on a seesaw of negligible mass. (This assumption is made to keep the example simple—more involved examples will follow.) The first child has a mass of 26.0 kg and sits 1.60 m from the pivot. (a) If the second child has a mass of 32.0 kg, how far is she from the pivot? (b) What is F_p , the supporting force exerted by the pivot?

Strategy

Both conditions for equilibrium must be satisfied. In part (a), we are asked for a distance; thus, the second condition (regarding torques) must be used, since the first (regarding only forces) has no distances in it. To apply the second condition for equilibrium, we first identify the system of interest to be the seesaw plus the two children. We take the supporting pivot to be the point about which the torques are calculated. We then identify all external forces acting on the system.

Solution (a)

The three external forces acting on the system are the weights of the two children and the supporting force of the pivot. Let us examine the torque produced by each. Torque is defined to be

$$\tau = rF \sin \theta. \quad 9.7$$

Here $\theta = 90^\circ$, so that $\sin \theta = 1$ for all three forces. That means $r_\perp = r$ for all three. The torques exerted by the three forces are first,

$$\tau_1 = r_1 w_1 \quad 9.8$$

second,

$$\tau_2 = -r_2 w_2 \quad 9.9$$

and third,

$$\begin{aligned} \tau_p &= r_p F_p \\ &= 0 \cdot F_p \\ &= 0. \end{aligned} \quad 9.10$$

Note that a minus sign has been inserted into the second equation because this torque is clockwise and is therefore negative by convention. Since F_p acts directly on the pivot point, the distance r_p is zero. A force acting on the pivot cannot cause a rotation, just as pushing directly on the hinges of a door will not cause it to rotate. Now, the second condition for equilibrium is that the sum of the torques on both children is zero. Therefore

$$\tau_2 = -\tau_1, \quad 9.11$$

or

$$r_2 w_2 = r_1 w_1. \quad 9.12$$

Weight is mass times the acceleration due to gravity. Entering mg for w , we get

$$r_2 m_2 g = r_1 m_1 g. \quad 9.13$$

Solve this for the unknown r_2 :

$$r_2 = r_1 \frac{m_1}{m_2}. \quad 9.14$$

The quantities on the right side of the equation are known; thus, r_2 is

$$r_2 = (1.60 \text{ m}) \frac{26.0 \text{ kg}}{32.0 \text{ kg}} = 1.30 \text{ m}. \quad 9.15$$

As expected, the heavier child must sit closer to the pivot (1.30 m versus 1.60 m) to balance the seesaw.

Solution (b)

This part asks for a force F_p . The easiest way to find it is to use the first condition for equilibrium, which is

$$\text{net } \mathbf{F} = 0. \quad 9.16$$

The forces are all vertical, so that we are dealing with a one-dimensional problem along the vertical axis; hence, the condition can be written as

$$\text{net } F_y = 0 \quad 9.17$$

where we again call the vertical axis the y -axis. Choosing upward to be the positive direction, and using plus and minus signs to indicate the directions of the forces, we see that

$$F_p - w_1 - w_2 = 0. \quad 9.18$$

This equation yields what might have been guessed at the beginning:

$$F_p = w_1 + w_2. \quad 9.19$$

So, the pivot supplies a supporting force equal to the total weight of the system:

$$F_p = m_1 g + m_2 g. \quad 9.20$$

Entering known values gives

$$\begin{aligned} F_p &= (26.0 \text{ kg})(9.80 \text{ m/s}^2) + (32.0 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 568 \text{ N}. \end{aligned} \quad 9.21$$

Discussion

The two results make intuitive sense. The heavier child sits closer to the pivot. The pivot supports the weight of the two children. Part (b) can also be solved using the second condition for equilibrium, since both distances are known, but only if the pivot point is chosen to be somewhere other than the location of the seesaw's actual pivot!

Several aspects of the preceding example have broad implications. First, the choice of the pivot as the point around which torques are calculated simplified the problem. Since F_p is exerted on the pivot point, its lever arm is zero. Hence, the torque exerted by the supporting force F_p is zero relative to that pivot point. The second condition for equilibrium holds for any choice of pivot point, and so we choose the pivot point to simplify the solution of the problem.

Second, the acceleration due to gravity canceled in this problem, and we were left with a ratio of masses. *This will not always be the case.* Always enter the correct forces—do not jump ahead to enter some ratio of masses.

Third, the weight of each child is distributed over an area of the seesaw, yet we treated the weights as if each force

were exerted at a single point. This is not an approximation—the distances r_1 and r_2 are the distances to points directly below the **center of gravity** of each child. As we shall see in the next section, the mass and weight of a system can act as if they are located at a single point.

Finally, note that the concept of torque has an importance beyond static equilibrium. *Torque plays the same role in rotational motion that force plays in linear motion.* We will examine this in the next chapter.

Take-Home Experiment

Take a piece of modeling clay and put it on a table, then mash a cylinder down into it so that a ruler can balance on the round side of the cylinder while everything remains still. Put a penny 8 cm away from the pivot. Where would you need to put two pennies to balance? Three pennies?

9.3 Stability

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State the types of equilibrium.
- Describe stable and unstable equilibriums.
- Describe neutral equilibrium.

It is one thing to have a system in equilibrium; it is quite another for it to be stable. The toy doll perched on the man's hand in [Figure 9.9](#), for example, is not in stable equilibrium. There are *three types of equilibrium: stable, unstable, and neutral.* Figures throughout this module illustrate various examples.

[Figure 9.9](#) presents a balanced system, such as the toy doll on the man's hand, which has its center of gravity (cg) directly over the pivot, so that the torque of the total weight is zero. This is equivalent to having the torques of the individual parts balanced about the pivot point, in this case the hand. The cgs of the arms, legs, head, and torso are labeled with smaller type.

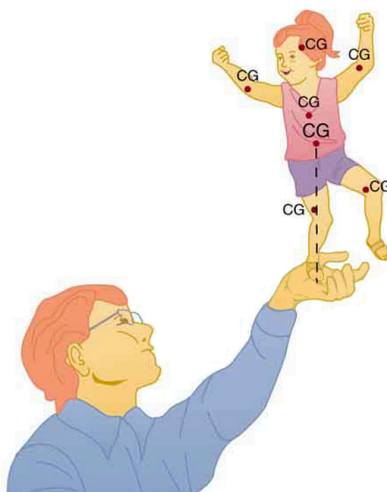


FIGURE 9.9 A man balances a toy doll on one hand.

A system is said to be in **stable equilibrium** if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a marble at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements. For another example of stable equilibrium, see the pencil in [Figure 9.10](#).

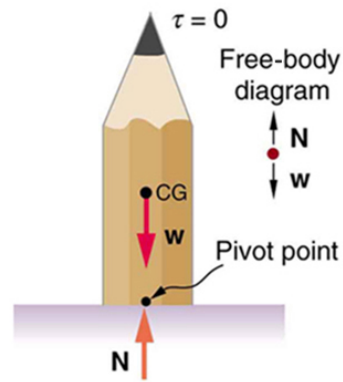


FIGURE 9.10 This pencil is in the condition of equilibrium. The net force on the pencil is zero and the total torque about any pivot is zero.

A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ball resting on top of a hill. Once displaced, it accelerates away from the crest. See the next several figures for examples of unstable equilibrium.

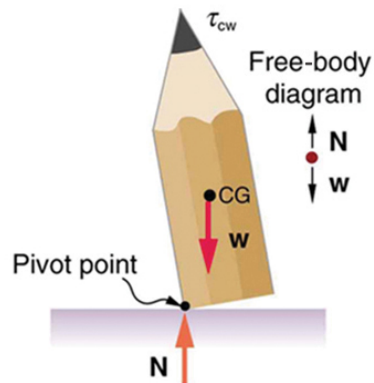


FIGURE 9.11 If the pencil is displaced slightly to the side (counterclockwise), it is no longer in equilibrium. Its weight produces a clockwise torque that returns the pencil to its equilibrium position.

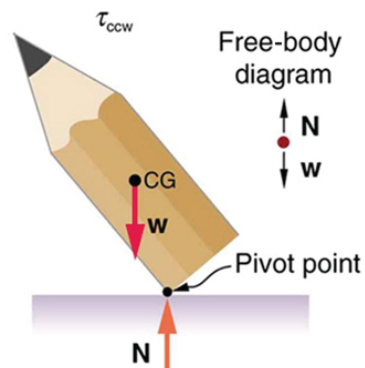


FIGURE 9.12 If the pencil is displaced too far, the torque caused by its weight changes direction to counterclockwise and causes the displacement to increase.

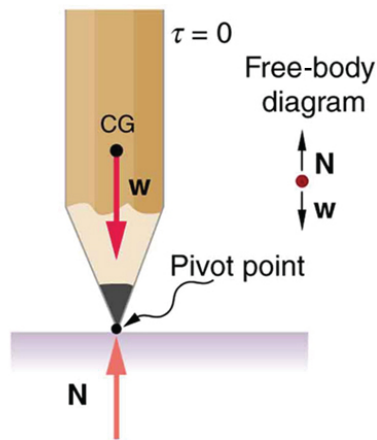


FIGURE 9.13 This figure shows unstable equilibrium, although both conditions for equilibrium are satisfied.

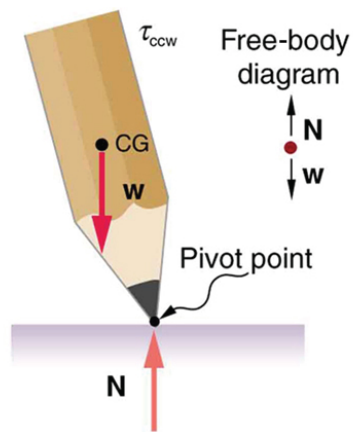


FIGURE 9.14 If the pencil is displaced even slightly, a torque is created by its weight that is in the same direction as the displacement, causing the displacement to increase.

A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A marble on a flat horizontal surface is an example. Combinations of these situations are possible. For example, a marble on a saddle is stable for displacements toward the front or back of the saddle and unstable for displacements to the side. [Figure 9.15](#) shows another example of neutral equilibrium.

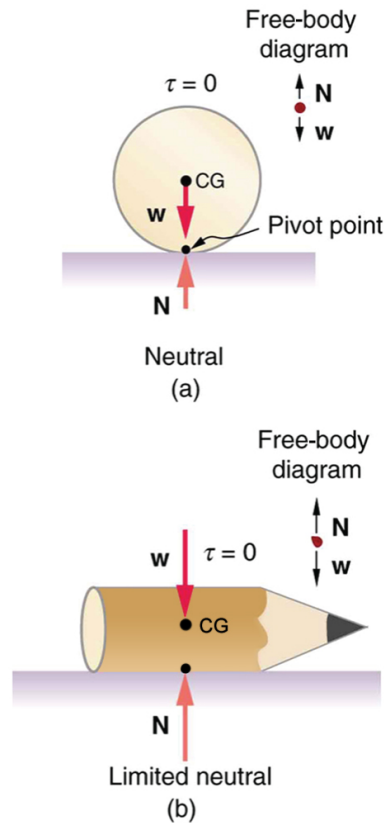


FIGURE 9.15 (a) Here we see neutral equilibrium. The cg of a sphere on a flat surface lies directly above the point of support, independent of the position on the surface. The sphere is therefore in equilibrium in any location, and if displaced, it will remain put. (b) Because it has a circular cross section, the pencil is in neutral equilibrium for displacements perpendicular to its length.

When we consider how far a system in stable equilibrium can be displaced before it becomes unstable, we find that some systems in stable equilibrium are more stable than others. The pencil in [Figure 9.10](#) and the person in [Figure 9.16\(a\)](#) are in stable equilibrium, but become unstable for relatively small displacements to the side. The critical point is reached when the cg is no longer *above* the base of support. Additionally, since the cg of a person's body is above the pivots in the hips, displacements must be quickly controlled. This control is a central nervous system function that is developed when we learn to hold our bodies erect as infants. For increased stability while standing, the feet should be spread apart, giving a larger base of support. Stability is also increased by lowering one's center of gravity by bending the knees, as when a football player prepares to receive a ball or braces themselves for a tackle. A cane, a crutch, or a walker increases the stability of the user, even more as the base of support widens. Usually, the cg of a female is lower (closer to the ground) than a male. Young children have their center of gravity between their shoulders, which increases the challenge of learning to walk.

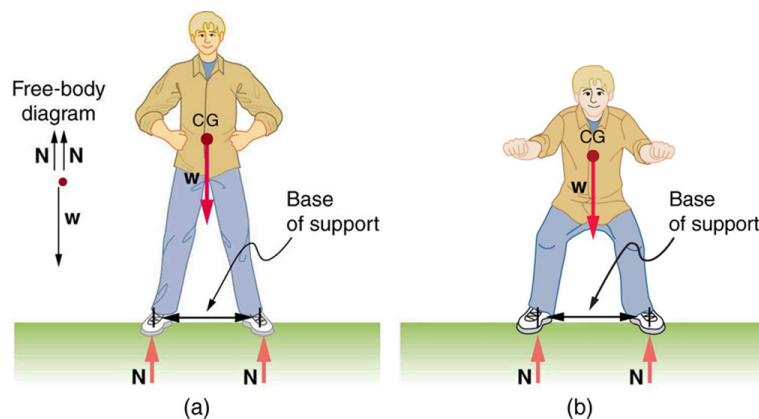


FIGURE 9.16 (a) The center of gravity of an adult is above the hip joints (one of the main pivots in the body) and lies between two narrowly-separated feet. Like a pencil standing on its eraser, this person is in stable equilibrium in relation to sideways displacements, but relatively small displacements take his cg outside the base of support and make him unstable. Humans are less stable relative to forward and

backward displacements because the feet are not very long. Muscles are used extensively to balance the body in the front-to-back direction. (b) While bending in the manner shown, stability is increased by lowering the center of gravity. Stability is also increased if the base is expanded by placing the feet farther apart.

Animals such as chickens have easier systems to control. [Figure 9.17](#) shows that the cg of a chicken lies below its hip joints and between its widely separated and broad feet. Even relatively large displacements of the chicken's cg are stable and result in restoring forces and torques that return the cg to its equilibrium position with little effort on the chicken's part. Not all birds are like chickens, of course. Some birds, such as the flamingo, have balance systems that are almost as sophisticated as that of humans.

[Figure 9.17](#) shows that the cg of a chicken is below the hip joints and lies above a broad base of support formed by widely-separated and large feet. Hence, the chicken is in very stable equilibrium, since a relatively large displacement is needed to render it unstable. The body of the chicken is supported from above by the hips and acts as a pendulum between the hips. Therefore, the chicken is stable for front-to-back displacements as well as for side-to-side displacements.

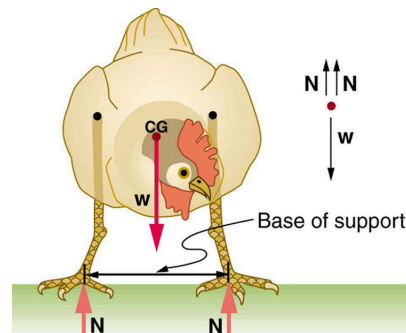


FIGURE 9.17 The center of gravity of a chicken is below the hip joints. The chicken is in stable equilibrium. The body of the chicken is supported from above by the hips and acts as a pendulum between them.

Engineers and architects strive to achieve extremely stable equilibriums for buildings and other systems that must withstand wind, earthquakes, and other forces that displace them from equilibrium. Although the examples in this section emphasize gravitational forces, the basic conditions for equilibrium are the same for all types of forces. The net external force must be zero, and the net torque must also be zero.

Take-Home Experiment

Stand straight with your heels, back, and head against a wall. Bend forward from your waist, keeping your heels and bottom against the wall, to touch your toes. Can you do this without toppling over? Explain why and what you need to do to be able to touch your toes without losing your balance. Is it easier for a woman to do this?

9.4 Applications of Statics, Including Problem-Solving Strategies

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Discuss the applications of Statics in real life.
- State and discuss various problem-solving strategies in Statics.

Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We begin with a discussion of problem-solving strategies specifically used for statics. Since statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in [Problem-Solving Strategies](#), still apply.

Problem-Solving Strategy: Static Equilibrium Situations

1. The first step is to determine whether or not the system is in **static equilibrium**. This condition is always the case when the *acceleration of the system is zero and accelerated rotation does not occur*.

2. It is particularly important to *draw a free body diagram for the system of interest*. Carefully label all forces, and note their relative magnitudes, directions, and points of application whenever these are known.
3. Solve the problem by applying either or both of the conditions for equilibrium (represented by the equations $\text{net } \mathbf{F} = 0$ and $\text{net } \boldsymbol{\tau} = 0$, depending on the list of known and unknown factors. If the second condition is involved, *choose the pivot point to simplify the solution*. Any pivot point can be chosen, but the most useful ones cause torques by unknown forces to be zero. (Torque is zero if the force is applied at the pivot (then $r = 0$), or along a line through the pivot point (then $\theta = 0$)). Always choose a convenient coordinate system for projecting forces.
4. *Check the solution to see if it is reasonable* by examining the magnitude, direction, and units of the answer. The importance of this last step never diminishes, although in unfamiliar applications, it is usually more difficult to judge reasonableness. These judgments become progressively easier with experience.

Now let us apply this problem-solving strategy for the pole vaulter shown in the three figures below. The pole is uniform and has a mass of 5.00 kg. In [Figure 9.18](#), the pole's cg lies halfway between the vaulter's hands. It seems reasonable that the force exerted by each hand is equal to half the weight of the pole, or 24.5 N. This obviously satisfies the first condition for equilibrium ($\text{net } \mathbf{F} = 0$). The second condition ($\text{net } \boldsymbol{\tau} = 0$) is also satisfied, as we can see by choosing the cg to be the pivot point. The weight exerts no torque about a pivot point located at the cg, since it is applied at that point and its lever arm is zero. The equal forces exerted by the hands are equidistant from the chosen pivot, and so they exert equal and opposite torques. Similar arguments hold for other systems where supporting forces are exerted symmetrically about the cg. For example, the four legs of a uniform table each support one-fourth of its weight.

In [Figure 9.18](#), a pole vaulter holding a pole with its cg halfway between his hands is shown. Each hand exerts a force equal to half the weight of the pole, $F_R = F_L = w/2$. (b) The pole vaulter moves the pole to his left, and the forces that the hands exert are no longer equal. See [Figure 9.18](#). If the pole is held with its cg to the left of the person, then he must push down with his right hand and up with his left. The forces he exerts are larger here because they are in opposite directions and the cg is at a long distance from either hand.

Similar observations can be made using a meter stick held at different locations along its length.

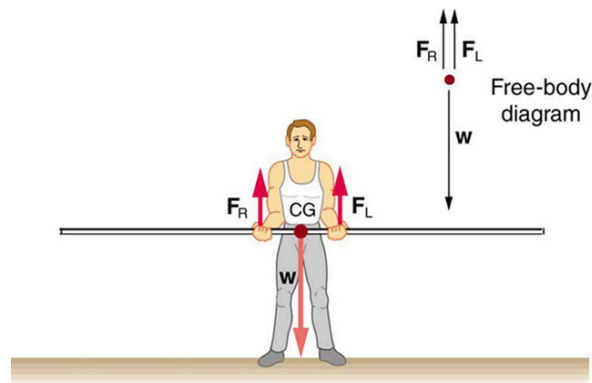


FIGURE 9.18 A pole vaulter holds a pole horizontally with both hands.

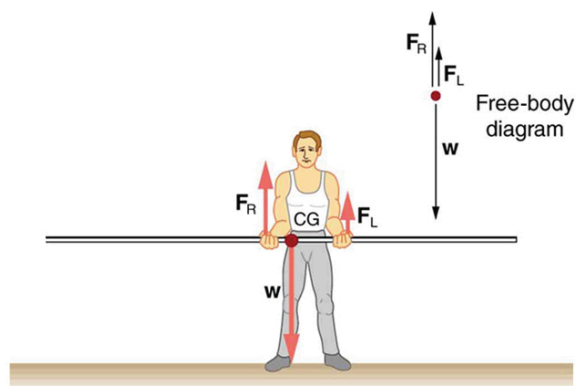


FIGURE 9.19 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is near his right hand.

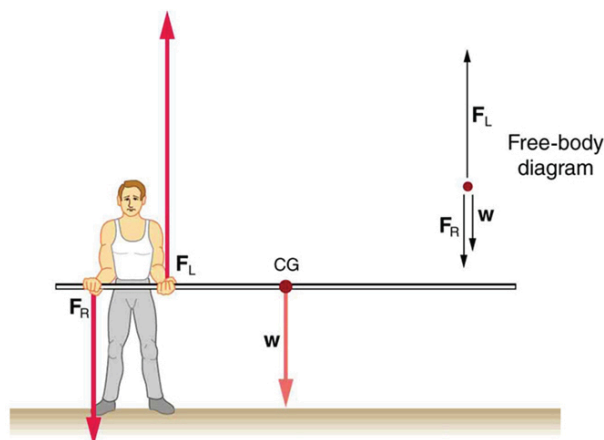


FIGURE 9.20 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

If the pole vaulter holds the pole as shown in [Figure 9.19](#), the situation is not as simple. The total force he exerts is still equal to the weight of the pole, but it is not evenly divided between his hands. (If $F_L = F_R$, then the torques about the cg would not be equal since the lever arms are different.) Logically, the right hand should support more weight, since it is closer to the cg. In fact, if the right hand is moved directly under the cg, it will support all the weight. This situation is exactly analogous to two people carrying a load; the one closer to the cg carries more of its weight. Finding the forces F_L and F_R is straightforward, as the next example shows.

If the pole vaulter holds the pole from near the end of the pole ([Figure 9.20](#)), the direction of the force applied by the right hand of the vaulter reverses its direction.

EXAMPLE 9.2

What Force Is Needed to Support a Weight Held Near Its CG?

For the situation shown in [Figure 9.19](#), calculate: (a) F_R , the force exerted by the right hand, and (b) F_L , the force exerted by the left hand. The hands are 0.900 m apart, and the cg of the pole is 0.600 m from the left hand.

Strategy

[Figure 9.19](#) includes a free body diagram for the pole, the system of interest. There is not enough information to use the first condition for equilibrium ($\text{net } \mathbf{F} = 0$), since two of the three forces are unknown and the hand forces cannot be assumed to be equal in this case. There is enough information to use the second condition for equilibrium ($\text{net } \boldsymbol{\tau} = 0$) if the pivot point is chosen to be at either hand, thereby making the torque from that hand zero. We choose to locate the pivot at the left hand in this part of the problem, to eliminate the torque from the left hand.

Solution for (a)

There are now only two nonzero torques, those from the gravitational force (τ_w) and from the push or pull of the

right hand (τ_R). Stating the second condition in terms of clockwise and counterclockwise torques,

$$\text{net } \tau_{\text{cw}} = -\text{net } \tau_{\text{ccw}}. \quad 9.22$$

or the algebraic sum of the torques is zero.

Here this is

$$\tau_R = -\tau_w \quad 9.23$$

since the weight of the pole creates a counterclockwise torque and the right hand counters with a clockwise torque. Using the definition of torque, $\tau = rF \sin \theta$, noting that $\theta = 90^\circ$, and substituting known values, we obtain

$$(0.900 \text{ m})(F_R) = (0.600 \text{ m})(mg). \quad 9.24$$

Thus,

$$\begin{aligned} F_R &= (0.667)(5.00 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 32.7 \text{ N}. \end{aligned} \quad 9.25$$

Solution for (b)

The first condition for equilibrium is based on the free body diagram in the figure. This implies that by Newton's second law:

$$F_L + F_R - mg = 0 \quad 9.26$$

From this we can conclude:

$$F_L + F_R = w = mg \quad 9.27$$

Solving for F_L , we obtain

$$\begin{aligned} F_L &= mg - F_R \\ &= mg - 32.7 \text{ N} \\ &= (5.00 \text{ kg})(9.80 \text{ m/s}^2) - 32.7 \text{ N} \\ &= 16.3 \text{ N} \end{aligned} \quad 9.28$$

Discussion

F_L is seen to be exactly half of F_R , as we might have guessed, since F_L is applied twice as far from the cg as F_R .

If the pole vaulter holds the pole as he might at the start of a run, shown in [Figure 9.20](#), the forces change again. Both are considerably greater, and one force reverses direction.

Take-Home Experiment

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!



PHET EXPLORATIONS

Balancing Act

Play with objects on a teeter totter to learn about balance. Test what you've learned by trying the Balance Challenge game.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/9-4-applications-of-statics-including-](https://openstax.org/books/college-physics-2e/pages/9-4-applications-of-statics-including-)

9.5 Simple Machines

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe different simple machines.
- Calculate the mechanical advantage.

Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we apply the force. The word for “machine” comes from the Greek word meaning “to help make things easier.” Levers, gears, pulleys, wedges, and screws are some examples of machines. Energy is still conserved for these devices because a machine cannot do more work than the energy put into it. However, machines can reduce the input force that is needed to perform the job. The ratio of output to input force magnitudes for any simple machine is called its **mechanical advantage** (MA).

$$MA = \frac{F_o}{F_i} \quad 9.29$$

One of the simplest machines is the lever, which is a rigid bar pivoted at a fixed place called the fulcrum. Torques are involved in levers, since there is rotation about a pivot point. Distances from the physical pivot of the lever are crucial, and we can obtain a useful expression for the MA in terms of these distances.

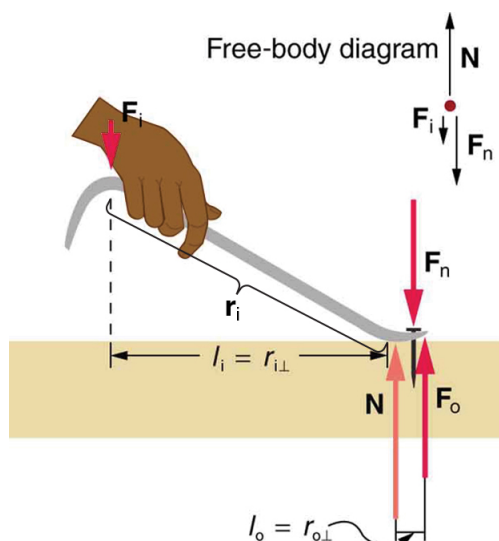


FIGURE 9.21 A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail (\mathbf{F}_o) is not a force on the nail puller. The reaction force the nail exerts back on the puller (\mathbf{F}_n) is an external force and is equal and opposite to \mathbf{F}_o . The perpendicular lever arms of the input and output forces are l_i and l_o .

Figure 9.21 shows a lever type that is used as a nail puller. Crowbars, seesaws, and other such levers are all analogous to this one. \mathbf{F}_i is the input force and \mathbf{F}_o is the output force. There are three vertical forces acting on the nail puller (the system of interest) – these are \mathbf{F}_i , \mathbf{F}_n , and \mathbf{N} . \mathbf{F}_n is the reaction force back on the system, equal and opposite to \mathbf{F}_o . (Note that \mathbf{F}_o is not a force on the system.) \mathbf{N} is the normal force upon the lever, and its torque is zero since it is exerted at the pivot. The torques due to \mathbf{F}_i and \mathbf{F}_n must be equal to each other if the nail is not moving, to satisfy the second condition for equilibrium (net $\tau = 0$). (In order for the nail to actually move, the torque due to \mathbf{F}_i must be ever-so-slightly greater than torque due to \mathbf{F}_n .) Hence,

$$l_i F_i = l_o F_o \quad 9.30$$

Notice that \mathbf{r}_i is the distance from the pivot point to the point where the input force \mathbf{F}_i is applied, and \mathbf{r}_o (not labeled on the diagram) is the distance from the pivot point to the point where the output force \mathbf{F}_o is applied. The distances l_i and l_o are the perpendicular components of the distances from where the input and output forces are applied to the pivot, as shown in the figure. Rearranging the last equation gives

$$\frac{F_o}{F_i} = \frac{l_i}{l_o}. \quad 9.31$$

What interests us most here is that the magnitude of the force exerted by the nail puller, F_o , is much greater than the magnitude of the input force applied to the puller at the other end, F_i . For the nail puller,

$$\text{MA} = \frac{F_o}{F_i} = \frac{l_i}{l_o}. \quad 9.32$$

This equation is true for levers in general. For the nail puller, the MA is certainly greater than one. The longer the handle on the nail puller, the greater the force you can exert with it.

Two other types of levers that differ slightly from the nail puller are a wheelbarrow and a shovel, shown in [Figure 9.22](#). All these lever types are similar in that only three forces are involved – the input force, the output force, and the force on the pivot – and thus their MAs are given by $\text{MA} = \frac{F_o}{F_i}$ and $\text{MA} = \frac{d_1}{d_2}$, with distances being measured relative to the physical pivot. The wheelbarrow and shovel differ from the nail puller because both the input and output forces are on the same side of the pivot.

In the case of the wheelbarrow, the output force or load is between the pivot (the wheel's axle) and the input or applied force. In the case of the shovel, the input force is between the pivot (at the end of the handle) and the load, but the input lever arm is shorter than the output lever arm. In this case, the MA is less than one.

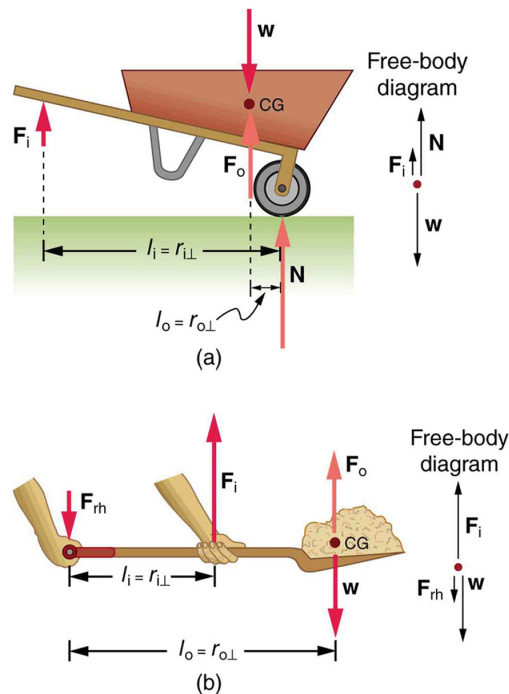


FIGURE 9.22 (a) In the case of the wheelbarrow, the output force or load is between the pivot and the input force. The pivot is the wheel's axle. Here, the output force is greater than the input force. Thus, a wheelbarrow enables you to lift much heavier loads than you could with your body alone. (b) In the case of the shovel, the input force is between the pivot and the load, but the input lever arm is shorter than the output lever arm. The pivot is at the handle held by the right hand. Here, the output force (supporting the shovel's load) is less than the input force (from the hand nearest the load), because the input is exerted closer to the pivot than is the output.

EXAMPLE 9.3

What is the Advantage for the Wheelbarrow?

In the wheelbarrow of [Figure 9.22](#), the load has a perpendicular lever arm of 7.50 cm, while the hands have a perpendicular lever arm of 1.02 m. (a) What upward force must you exert to support the wheelbarrow and its load if their combined mass is 45.0 kg? (b) What force does the wheelbarrow exert on the ground?

Strategy

Here, we use the concept of mechanical advantage.

Solution

(a) In this case, $\frac{F_o}{F_i} = \frac{l_i}{l_o}$ becomes

$$F_i = F_o \frac{l_o}{l_i}. \quad 9.33$$

Adding values into this equation yields

$$F_i = (45.0 \text{ kg})(9.80 \text{ m/s}^2) \frac{0.075 \text{ m}}{1.02 \text{ m}} = 32.4 \text{ N}. \quad 9.34$$

The free-body diagram (see [Figure 9.22](#)) gives the following normal force: $F_i + N = W$. Therefore, $N = (45.0 \text{ kg})(9.80 \text{ m/s}^2) - 32.4 \text{ N} = 409 \text{ N}$. N is the normal force acting on the wheel; by Newton's third law, the force the wheel exerts on the ground is 409 N.

Discussion

An even longer handle would reduce the force needed to lift the load. The MA here is $MA = 1.02/0.0750 = 13.6$.

Another very simple machine is the inclined plane. Pushing a cart up a plane is easier than lifting the same cart straight up to the top using a ladder, because the applied force is less. However, the work done in both cases (assuming the work done by friction is negligible) is the same. Inclined lanes or ramps were probably used during the construction of the Egyptian pyramids to move large blocks of stone to the top.

A crank is a lever that can be rotated 360° about its pivot, as shown in [Figure 9.23](#). Such a machine may not look like a lever, but the physics of its actions remain the same. The MA for a crank is simply the ratio of the radii r_i/r_o . Wheels and gears have this simple expression for their MAs too. The MA can be greater than 1, as it is for the crank, or less than 1, as it is for the simplified car axle driving the wheels, as shown. If the axle's radius is 2.0 cm and the wheel's radius is 24.0 cm, then $MA = 2.0/24.0 = 0.083$ and the axle would have to exert a force of 12,000 N on the wheel to enable it to exert a force of 1000 N on the ground.

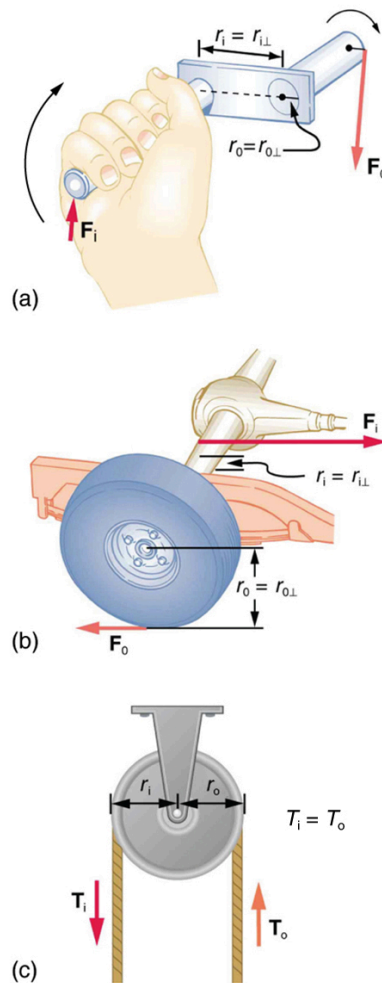


FIGURE 9.23 (a) A crank is a type of lever that can be rotated 360° about its pivot. Cranks are usually designed to have a large MA. (b) A simplified automobile axle drives a wheel, which has a much larger diameter than the axle. The MA is less than 1. (c) An ordinary pulley is used to lift a heavy load. The pulley changes the direction of the force T exerted by the cord without changing its magnitude. Hence, this machine has an MA of 1.

An ordinary pulley has an MA of 1; it only changes the direction of the force and not its magnitude. Combinations of pulleys, such as those illustrated in [Figure 9.24](#), are used to multiply force. If the pulleys are friction-free, then the force output is approximately an integral multiple of the tension in the cable. The number of cables pulling directly upward on the system of interest, as illustrated in the figures given below, is approximately the MA of the pulley system. Since each attachment applies an external force in approximately the same direction as the others, they add, producing a total force that is nearly an integral multiple of the input force T .

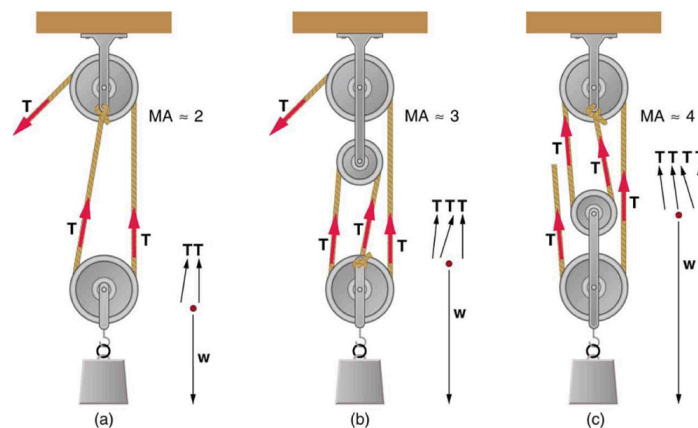


FIGURE 9.24 (a) The combination of pulleys is used to multiply force. The force is an integral multiple of tension if the pulleys are

frictionless. This pulley system has two cables attached to its load, thus applying a force of approximately $2T$. This machine has $MA \approx 2$. (b) Three pulleys are used to lift a load in such a way that the mechanical advantage is about 3. Effectively, there are three cables attached to the load. (c) This pulley system applies a force of $4T$, so that it has $MA \approx 4$. Effectively, four cables are pulling on the system of interest.

9.6 Forces and Torques in Muscles and Joints

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the forces exerted by muscles.
- State how a bad posture causes back strain.
- Discuss the benefits of skeletal muscles attached close to joints.
- Discuss various complexities in the real system of muscles, bones, and joints.

Muscles, bones, and joints are some of the most interesting applications of statics. There are some surprises. Muscles, for example, exert far greater forces than we might think. [Figure 9.25](#) shows a forearm holding a book and a schematic diagram of an analogous lever system. The schematic is a good approximation for the forearm, which looks more complicated than it is, and we can get some insight into the way typical muscle systems function by analyzing it.

Muscles can only contract, so they occur in pairs. In the arm, the biceps muscle is a flexor—that is, it closes the limb. The triceps muscle is an extensor that opens the limb. This configuration is typical of skeletal muscles, bones, and joints in humans and other vertebrates. Most skeletal muscles exert much larger forces within the body than the limbs apply to the outside world. The reason is clear once we realize that most muscles are attached to bones via tendons close to joints, causing these systems to have mechanical advantages much less than one. Viewing them as simple machines, the input force is much greater than the output force, as seen in [Figure 9.25](#).

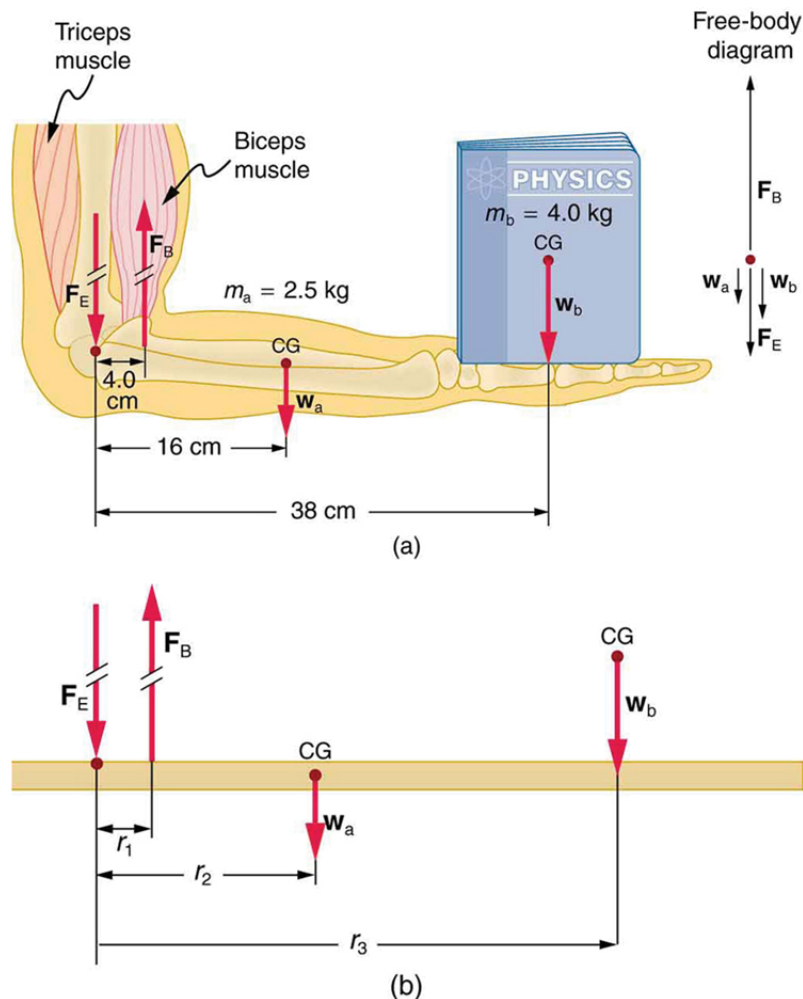


FIGURE 9.25 (a) The figure shows the forearm of a person holding a book. The biceps exert a force F_B to support the weight of the forearm

and the book. The triceps are assumed to be relaxed. (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in [Example 9.4](#).

EXAMPLE 9.4

Muscles Exert Bigger Forces Than You Might Think

Calculate the force the biceps muscle must exert to hold the forearm and its load as shown in [Figure 9.25](#), and compare this force with the weight of the forearm plus its load. You may take the data in the figure to be accurate to three significant figures.

Strategy

There are four forces acting on the forearm and its load (the system of interest). The magnitude of the force of the biceps is F_B ; that of the elbow joint is F_E ; that of the weights of the forearm is w_a , and its load is w_b . Two of these are unknown (F_B and F_E), so that the first condition for equilibrium cannot by itself yield F_B . But if we use the second condition and choose the pivot to be at the elbow, then the torque due to F_E is zero, and the only unknown becomes F_B .

Solution

The torques created by the weights are clockwise relative to the pivot, while the torque created by the biceps is counterclockwise; thus, the second condition for equilibrium (net $\tau = 0$) becomes

$$r_2 w_a + r_3 w_b = r_1 F_B. \quad 9.35$$

Note that $\sin \theta = 1$ for all forces, since $\theta = 90^\circ$ for all forces. This equation can easily be solved for F_B in terms of known quantities, yielding

$$F_B = \frac{r_2 w_a + r_3 w_b}{r_1}. \quad 9.36$$

Entering the known values gives

$$F_B = \frac{(0.160 \text{ m})(2.50 \text{ kg})(9.80 \text{ m/s}^2) + (0.380 \text{ m})(4.00 \text{ kg})(9.80 \text{ m/s}^2)}{0.0400 \text{ m}} \quad 9.37$$

which yields

$$F_B = 470 \text{ N}. \quad 9.38$$

Now, the combined weight of the arm and its load is $(6.50 \text{ kg})(9.80 \text{ m/s}^2) = 63.7 \text{ N}$, so that the ratio of the force exerted by the biceps to the total weight is

$$\frac{F_B}{w_a + w_b} = \frac{470}{63.7} = 7.38. \quad 9.39$$

Discussion

This means that the biceps muscle is exerting a force 7.38 times the weight supported.

In the above example of the biceps muscle, the angle between the forearm and upper arm is 90° . If this angle changes, the force exerted by the biceps muscle also changes. In addition, the length of the biceps muscle changes. The force the biceps muscle can exert depends upon its length; it is smaller when it is shorter than when it is stretched.

Very large forces are also created in the joints. In the previous example, the downward force F_E exerted by the humerus at the elbow joint equals 407 N, or 6.38 times the total weight supported. (The calculation of F_E is straightforward and is left as an end-of-chapter problem.) Because muscles can contract, but not expand beyond their resting length, joints and muscles often exert forces that act in opposite directions and thus subtract. (In the above example, the upward force of the muscle minus the downward force of the joint equals the weight supported—that is, $470 \text{ N} - 407 \text{ N} = 63 \text{ N}$, approximately equal to the weight supported.) Forces in muscles and

joints are largest when their load is a long distance from the joint, as the book is in the previous example.

In racquet sports such as tennis the constant extension of the arm during game play creates large forces in this way. The mass times the lever arm of a tennis racquet is an important factor, and many players use the heaviest racquet they can handle. It is no wonder that joint deterioration and damage to the tendons in the elbow, such as “tennis elbow,” can result from repetitive motion, undue torques, and possibly poor racquet selection in such sports. Various tried techniques for holding and using a racquet or bat or stick not only increases sporting prowess but can minimize fatigue and long-term damage to the body. For example, tennis balls correctly hit at the “sweet spot” on the racquet will result in little vibration or impact force being felt in the racquet and the body—less torque as explained in [Collisions of Extended Bodies in Two Dimensions](#). Twisting the hand to provide top spin on the ball or using an extended rigid elbow in a backhand stroke can also aggravate the tendons in the elbow.

Training coaches and physical therapists use the knowledge of relationships between forces and torques in the treatment of muscles and joints. In physical therapy, an exercise routine can apply a particular force and torque which can, over a period of time, revive muscles and joints. Some exercises are designed to be carried out under water, because this requires greater forces to be exerted, further strengthening muscles. However, connecting tissues in the limbs, such as tendons and cartilage as well as joints are sometimes damaged by the large forces they carry. Often, this is due to accidents, but heavily muscled athletes, such as weightlifters, can tear muscles and connecting tissue through effort alone.

The back is considerably more complicated than the arm or leg, with various muscles and joints between vertebrae, all having mechanical advantages less than 1. Back muscles must, therefore, exert very large forces, which are borne by the spinal column. Discs crushed by mere exertion are very common. The jaw is somewhat exceptional—the masseter muscles that close the jaw have a mechanical advantage greater than 1 for the back teeth, allowing us to exert very large forces with them. A cause of stress headaches is persistent clenching of teeth where the sustained large force translates into fatigue in muscles around the skull.

[Figure 9.26](#) shows how bad posture causes back strain. In part (a), we see a person with good posture. Note that her upper body’s cg is directly above the pivot point in the hips, which in turn is directly above the base of support at her feet. Because of this, her upper body’s weight exerts no torque about the hips. The only force needed is a vertical force at the hips equal to the weight supported. No muscle action is required, since the bones are rigid and transmit this force from the floor. This is a position of unstable equilibrium, but only small forces are needed to bring the upper body back to vertical if it is slightly displaced. Bad posture is shown in part (b); we see that the upper body’s cg is in front of the pivot in the hips. This creates a clockwise torque around the hips that is counteracted by muscles in the lower back. These muscles must exert large forces, since they have typically small mechanical advantages. (In other words, the perpendicular lever arm for the muscles is much smaller than for the cg.) Poor posture can also cause muscle strain for people sitting at their desks using computers. Special chairs are available that allow the body’s CG to be more easily situated above the seat, to reduce back pain. Prolonged muscle action produces muscle strain. Note that the cg of the entire body is still directly above the base of support in part (b) of [Figure 9.26](#). This is compulsory; otherwise the person would not be in equilibrium. We lean forward for the same reason when carrying a load on our backs, to the side when carrying a load in one arm, and backward when carrying a load in front of us, as seen in [Figure 9.27](#).

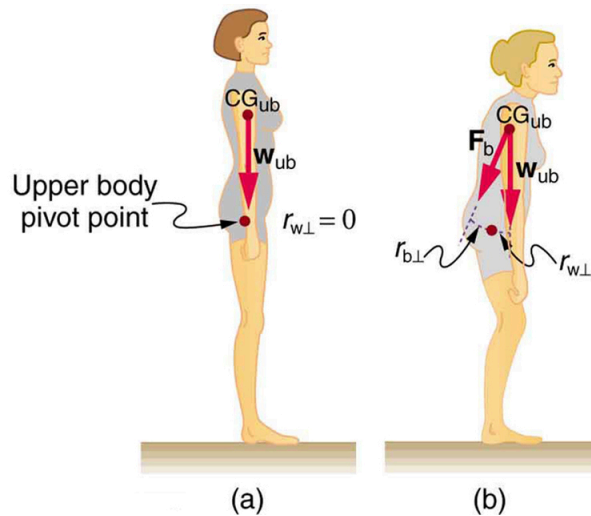


FIGURE 9.26 (a) Good posture places the upper body's cg over the pivots in the hips, eliminating the need for muscle action to balance the body. (b) Poor posture requires exertion by the back muscles to counteract the clockwise torque produced around the pivot by the upper body's weight. The back muscles have a small effective perpendicular lever arm, $r_{b\perp}$, and must therefore exert a large force F_b . Note that the legs lean backward to keep the cg of the entire body above the base of support in the feet.

You have probably been warned against lifting objects with your back. This action, even more than bad posture, can cause muscle strain and damage discs and vertebrae, since abnormally large forces are created in the back muscles and spine.

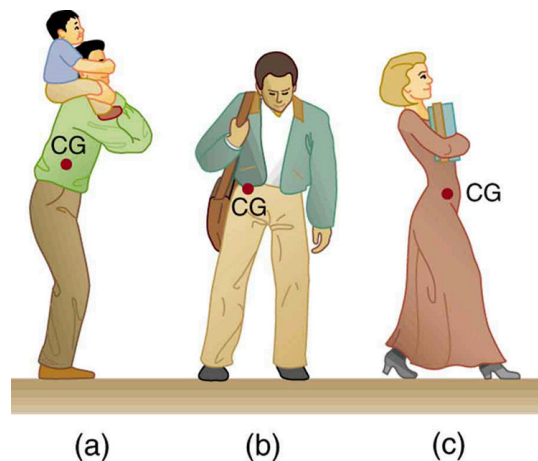


FIGURE 9.27 People adjust their stance to maintain balance. (a) A father carrying his son piggyback leans forward to position their overall cg above the base of support at his feet. (b) A student carrying a shoulder bag leans to the side to keep the overall cg over their feet. (c) Another student carrying a load of books in her arms leans backward for the same reason.

EXAMPLE 9.5

Do Not Lift with Your Back

Consider the person lifting a heavy box with his back, shown in [Figure 9.28](#). (a) Calculate the magnitude of the force F_B —in the back muscles that is needed to support the upper body plus the box and compare this with his weight. The mass of the upper body is 55.0 kg and the mass of the box is 30.0 kg. (b) Calculate the magnitude and direction of the force F_V —exerted by the vertebrae on the spine at the indicated pivot point. Again, data in the figure may be taken to be accurate to three significant figures.

Strategy

By now, we sense that the second condition for equilibrium is a good place to start, and inspection of the known values confirms that it can be used to solve for F_B —if the pivot is chosen to be at the hips. The torques created by w_{ub} and w_{box} —are clockwise, while that created by F_B —is counterclockwise.

Solution for (a)

Using the perpendicular lever arms given in the figure, the second condition for equilibrium (net $\tau = 0$) becomes

$$(0.350 \text{ m})(55.0 \text{ kg})(9.80 \text{ m/s}^2) + (0.500 \text{ m})(30.0 \text{ kg})(9.80 \text{ m/s}^2) = (0.0800 \text{ m})F_B. \quad 9.40$$

Solving for F_B yields

$$F_B = 4.20 \times 10^3 \text{ N}. \quad 9.41$$

The ratio of the force the back muscles exert to the weight of the upper body plus its load is

$$\frac{F_B}{w_{\text{ub}} + w_{\text{box}}} = \frac{4200 \text{ N}}{833 \text{ N}} = 5.04. \quad 9.42$$

This force is considerably larger than it would be if the load were not present.

Solution for (b)

More important in terms of its damage potential is the force on the vertebrae \mathbf{F}_V . The first condition for equilibrium (net $\mathbf{F} = 0$) can be used to find its magnitude and direction. Using y for vertical and x for horizontal, the condition for the net external forces along those axes to be zero

$$\text{net } F_y = 0 \text{ and net } F_x = 0. \quad 9.43$$

Starting with the vertical (y) components, this yields

$$F_{Vy} - w_{\text{ub}} - w_{\text{box}} - F_B \sin 29.0^\circ = 0. \quad 9.44$$

Thus,

$$\begin{aligned} F_{Vy} &= w_{\text{ub}} + w_{\text{box}} + F_B \sin 29.0^\circ \\ &= 833 \text{ N} + (4200 \text{ N}) \sin 29.0^\circ \end{aligned} \quad 9.45$$

yielding

$$F_{Vy} = 2.87 \times 10^3 \text{ N}. \quad 9.46$$

Similarly, for the horizontal (x) components,

$$F_{Vx} - F_B \cos 29.0^\circ = 0 \quad 9.47$$

yielding

$$F_{Vx} = 3.67 \times 10^3 \text{ N}. \quad 9.48$$

The magnitude of \mathbf{F}_V is given by the Pythagorean theorem:

$$F_V = \sqrt{F_{Vx}^2 + F_{Vy}^2} = 4.66 \times 10^3 \text{ N}. \quad 9.49$$

The direction of \mathbf{F}_V is

$$\theta = \tan^{-1} \left(\frac{F_{Vy}}{F_{Vx}} \right) = 38.0^\circ. \quad 9.50$$

Note that the ratio of F_V to the weight supported is

$$\frac{F_V}{w_{\text{ub}} + w_{\text{box}}} = \frac{4660 \text{ N}}{833 \text{ N}} = 5.59. \quad 9.51$$

Discussion

This force is about 5.6 times greater than it would be if the person were standing erect. The trouble with the back is not so much that the forces are large—because similar forces are created in our hips, knees, and ankles—but that our spines are relatively weak. Proper lifting, performed with the back erect and using the legs to raise the body and

load, creates much smaller forces in the back—in this case, about 5.6 times smaller.

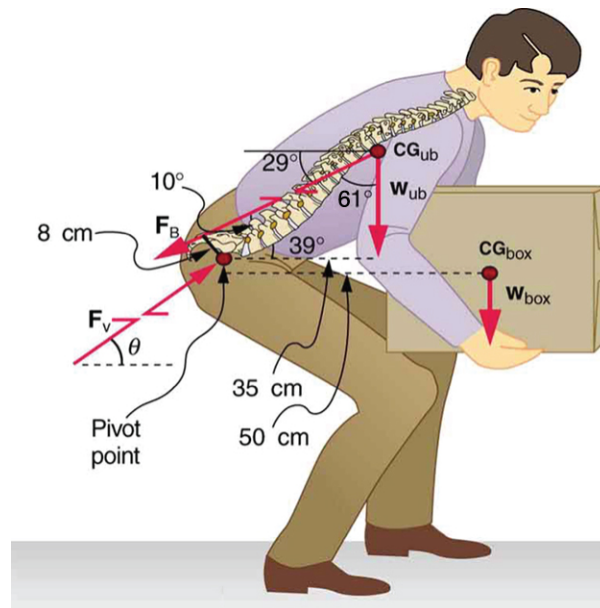


FIGURE 9.28 This figure shows that large forces are exerted by the back muscles and experienced in the vertebrae when a person lifts with their back, since these muscles have small effective perpendicular lever arms. The data shown here are analyzed in the preceding example, [Example 9.5](#).

What are the benefits of having most skeletal muscles attached so close to joints? One advantage is speed because small muscle contractions can produce large movements of limbs in a short period of time. Other advantages are flexibility and agility, made possible by the large numbers of joints and the ranges over which they function. For example, it is difficult to imagine a system with biceps muscles attached at the wrist that would be capable of the broad range of movement we vertebrates possess.

There are some interesting complexities in real systems of muscles, bones, and joints. For instance, the pivot point in many joints changes location as the joint is flexed, so that the perpendicular lever arms and the mechanical advantage of the system change, too. Thus the force the biceps muscle must exert to hold up a book varies as the forearm is flexed. Similar mechanisms operate in the legs, which explain, for example, why there is less leg strain when a bicycle seat is set at the proper height. The methods employed in this section give a reasonable description of real systems provided enough is known about the dimensions of the system. There are many other interesting examples of force and torque in the body—a few of these are the subject of end-of-chapter problems.

Glossary

center of gravity the point where the total weight of the body is assumed to be concentrated

dynamic equilibrium a state of equilibrium in which the net external force and torque on a system moving with constant velocity are zero

mechanical advantage the ratio of output to input forces for any simple machine

neutral equilibrium a state of equilibrium that is independent of a system's displacements from its original position

perpendicular lever arm the shortest distance from the pivot point to the line along which \mathbf{F} lies

SI units of torque newton times meters, usually written as N·m

stable equilibrium a system, when displaced, experiences a net force or torque in a direction opposite to the direction of the displacement

static equilibrium a state of equilibrium in which the net external force and torque acting on a system is zero

static equilibrium equilibrium in which the acceleration of the system is zero and accelerated rotation does not occur

torque turning or twisting effectiveness of a force

unstable equilibrium a system, when displaced, experiences a net force or torque in the same direction as the displacement from equilibrium

Section Summary

9.1 The First Condition for Equilibrium

- Statics is the study of forces in equilibrium.
- Two conditions must be met to achieve equilibrium, which is defined to be motion without linear or rotational acceleration.
- The first condition necessary to achieve equilibrium is that the net external force on the system must be zero, so that net $\mathbf{F} = 0$.

9.2 The Second Condition for Equilibrium

- The second condition assures those torques are also balanced. Torque is the rotational equivalent of a force in producing a rotation and is defined to be

$$\tau = rF \sin \theta$$

where τ is torque, r is the distance from the pivot point to the point where the force is applied, F is the magnitude of the force, and θ is the angle between \mathbf{F} and the vector directed from the point where the force acts to the pivot point. The perpendicular lever arm r_{\perp} is defined to be

$$r_{\perp} = r \sin \theta$$

so that

$$\tau = r_{\perp} F.$$

- The perpendicular lever arm r_{\perp} is the shortest distance from the pivot point to the line along which F acts. The SI unit for torque is newton-meter (N·m). The second condition necessary to achieve equilibrium is that the net external torque on a system must be zero:

$$\text{net } \tau = 0$$

By convention, counterclockwise torques are positive, and clockwise torques are negative.

9.3 Stability

- A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite the direction of the displacement.
- A system is in unstable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in the same direction as the displacement from equilibrium.
- A system is in neutral equilibrium if its equilibrium is independent of displacements from its original position.

9.4 Applications of Statics, Including Problem-Solving Strategies

- Statics can be applied to a variety of situations, ranging from raising a drawbridge to bad posture and back strain. We have discussed the problem-solving strategies specifically useful for statics. Statics is a special case of Newton's laws, both the general problem-solving strategies and the special strategies for Newton's laws, discussed in [Problem-Solving Strategies](#), still apply.

9.5 Simple Machines

- Simple machines are devices that can be used to multiply or augment a force that we apply – often at the expense of a distance through which we have to apply the force.
- The ratio of output to input forces for any simple machine is called its mechanical advantage
- A few simple machines are the lever, nail puller, wheelbarrow, crank, etc.

9.6 Forces and Torques in Muscles and Joints

- Statics plays an important part in understanding everyday strains in our muscles and bones.
- Many lever systems in the body have a mechanical advantage of significantly less than one, as many

of our muscles are attached close to joints.

- Someone with good posture stands or sits in such a way that the person's center of gravity lies directly above the pivot point in the hips, thereby avoiding back strain and damage to disks.

Conceptual Questions

9.1 The First Condition for Equilibrium

1. What can you say about the velocity of a moving body that is in dynamic equilibrium? Draw a sketch of such a body using clearly labeled arrows to represent all external forces on the body.
2. Under what conditions can a rotating body be in equilibrium? Give an example.

9.2 The Second Condition for Equilibrium

3. What three factors affect the torque created by a force relative to a specific pivot point?
4. A wrecking ball is being used to knock down a building. One tall unsupported concrete wall remains standing. If the wrecking ball hits the wall near the top, is the wall more likely to fall over by rotating at its base or by falling straight down? Explain your answer. How is it most likely to fall if it is struck with the same force at its base? Note that this depends on how firmly the wall is attached at its base.
5. Mechanics sometimes put a length of pipe over the handle of a wrench when trying to remove a very tight bolt. How does this help? (It is also hazardous since it can break the bolt.)

9.3 Stability

6. A round pencil lying on its side as in [Figure 9.12](#) is in neutral equilibrium relative to displacements perpendicular to its length. What is its stability relative to displacements parallel to its length?
7. Explain the need for tall towers on a suspension bridge to ensure stable equilibrium.

9.4 Applications of Statics, Including Problem-Solving Strategies

8. When visiting some countries, you may see a person balancing a load on the head. Explain why the center of mass of the load needs to be directly above the person's neck vertebrae.

9.5 Simple Machines

9. Scissors are like a double-lever system. Which of the simple machines in [Figure 9.21](#) and [Figure 9.22](#) is most analogous to scissors?
10. Suppose you pull a nail at a constant rate using a nail puller as shown in [Figure 9.21](#). Is the nail puller in equilibrium? What if you pull the nail with some acceleration – is the nail puller in equilibrium then? In which case is the force applied to the nail puller larger and why?
11. Explain the mechanical advantage of a wheelbarrow and how it explains why moving a heavy load is easier with a wheelbarrow.
12. How does a pulley enable a person to lift a load as heavy as a piano with little effort?

9.6 Forces and Torques in Muscles and Joints

13. Why are the forces exerted on the outside world by the limbs of our bodies usually much smaller than the forces exerted by muscles inside the body?
14. Explain why the forces in our joints are several times larger than the forces we exert on the outside world with our limbs. Can these forces be even greater than muscle forces?
15. Certain types of dinosaurs were bipedal (walked on two legs). What is a good reason that these creatures invariably had long tails if they had long necks?
16. Swimmers and athletes during competition need to go through certain postures at the beginning of the race. Consider the balance of the person and why start-offs are so important for races.
17. If the maximum force the biceps muscle can exert is 1000 N, can we pick up an object that weighs 1000 N? Explain your answer.
18. Suppose the biceps muscle was attached through tendons to the upper arm close to the elbow and the forearm near the wrist. What would be the advantages and disadvantages of this type of construction for the motion of the arm?

19. Explain one of the reasons why pregnant women often suffer from back strain late in their pregnancy.

Problems & Exercises

9.2 The Second Condition for Equilibrium

- (a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850 m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?
- When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt. (a) How much torque are you exerting in newton \times meters (relative to the center of the bolt)? (b) Convert this torque to footpounds.
- Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 17.5 N at a distance of 0.600 m from the hinges, and the second child pushes at a distance of 0.450 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.
- Use the second condition for equilibrium (net $\tau = 0$) to calculate F_p in [Example 9.1](#), employing any data given or solved for in part (a) of the example.
- Repeat the seesaw problem in [Example 9.1](#) with the center of mass of the seesaw 0.160 m to the left of the pivot (on the side of the lighter child) and assuming a mass of 12.0 kg for the seesaw. The other data given in the example remain unchanged. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium.

9.3 Stability

6. Suppose a horse leans against a wall as in [Figure 9.29](#), emulating a scene in the comedy movie *Cat Ballou*. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal in magnitude and opposite in direction to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.

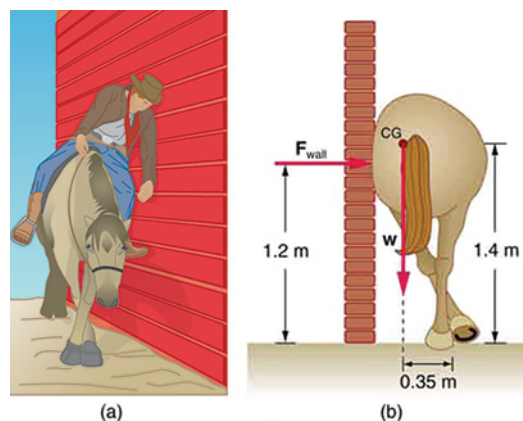


FIGURE 9.29

- Two children of mass 20.0 kg and 30.0 kg sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3.00 m, at what distance from the pivot point is the small child sitting in order to maintain the balance?
- (a) Calculate the magnitude and direction of the force on each foot of the horse in [Figure 9.29](#) (two are on the ground), assuming the center of mass of the horse is midway between the feet. The total mass of the horse and rider is 500 kg. (b) What is the minimum coefficient of friction between the hooves and ground? Note that the force exerted by the wall is horizontal.
- A person carries a plank of wood 2.00 m long with one hand pushing down on it at one end with a force F_1 and the other hand holding it up at .500 m from the end of the plank with force F_2 . If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces F_1 and F_2 ?

10. A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in [Figure 9.30](#). The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

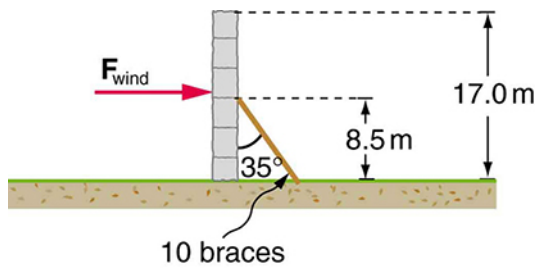


FIGURE 9.30

11. (a) What force must be exerted by the wind to support a 2.50-kg chicken in the position shown in [Figure 9.31](#)? (b) What is the ratio of this force to the chicken's weight? (c) Does this support the contention that the chicken has a relatively stable construction?

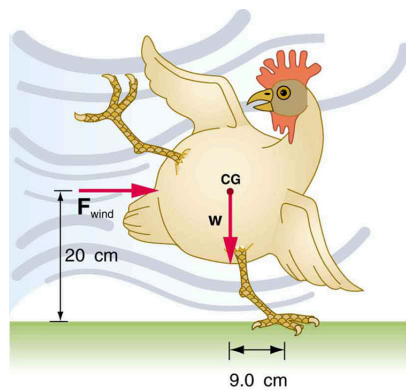
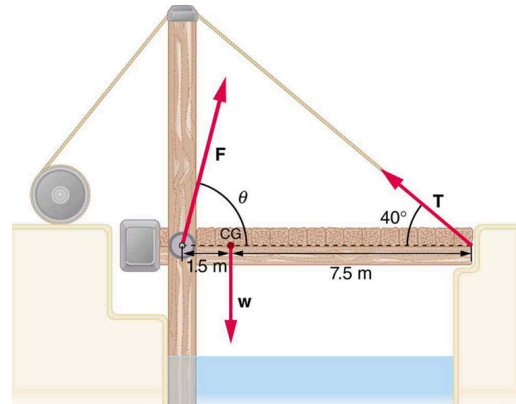


FIGURE 9.31

12. Suppose the weight of the drawbridge in [Figure 9.32](#) is supported entirely by its hinges and the opposite shore, so that its cables are slack. The mass of the bridge is 2500 kg. (a) What fraction of the weight is supported by the opposite shore if the point of support is directly beneath the cable attachments? (b) What is the direction and magnitude of the force the hinges exert on the bridge under these circumstances?

FIGURE 9.32 A small drawbridge, showing the forces on the hinges (F), its weight (w), and the tension in its wires (T).

13. Suppose a 900-kg car is on the bridge in [Figure 9.32](#) with its center of mass halfway between the hinges and the cable attachments. (The bridge is supported by the cables and hinges only.) (a) Find the force in the cables. (b) Find the direction and magnitude of the force exerted by the hinges on the bridge.

14. A sandwich board advertising sign is constructed as shown in [Figure 9.33](#). The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?

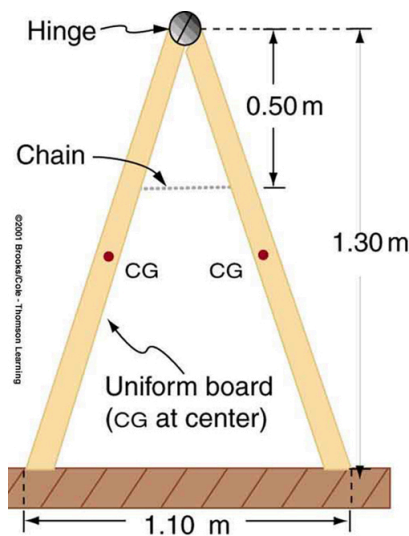


FIGURE 9.33 A sandwich board advertising sign demonstrates tension.

15. (a) What minimum coefficient of friction is needed between the legs and the ground to keep the sign in [Figure 9.33](#) in the position shown if the chain breaks? (b) What force is exerted by each side on the hinge?
16. An athlete is attempting to perform splits. From the information given in [Figure 9.34](#), calculate the magnitude and direction of the force exerted on each foot by the floor.

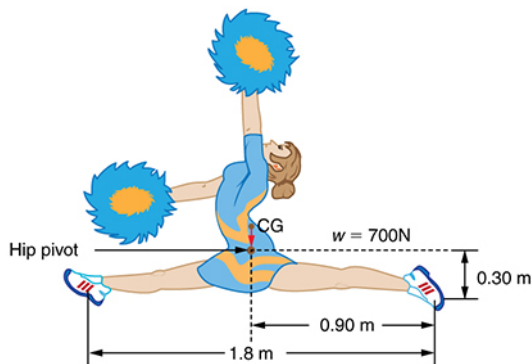


FIGURE 9.34 An athlete performs full split. The center of gravity and the various distances from it are shown.

9.4 Applications of Statics, Including Problem-Solving Strategies

17. To get up on the roof, a person (mass 70.0 kg) places a 6.00-m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?
18. In [Figure 9.20](#), the cg of the pole held by the pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by (a) his right hand and (b) his left hand. (c) If each hand supports half the weight of the pole in [Figure 9.18](#), show that the second condition for equilibrium (net $\tau = 0$) is satisfied for a pivot other than the one located at the center of gravity of the pole. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium described above.

9.5 Simple Machines

19. What is the mechanical advantage of a nail puller—similar to the one shown in [Figure 9.21](#)—where you exert a force 45 cm from the pivot and the nail is 1.8 cm on the other side? What minimum force must you exert to apply a force of 1250 N to the nail?
20. Suppose you needed to raise a 250-kg mower a distance of 6.0 cm above the ground to change a tire. If you had a 2.0-m long lever, where would you place the fulcrum if your force was limited to 300 N?
21. a) What is the mechanical advantage of a wheelbarrow, such as the one in [Figure 9.22](#), if the center of gravity of the wheelbarrow and its load has a perpendicular lever arm of 5.50 cm, while the hands have a perpendicular lever arm of 1.02 m? (b) What upward force should you exert to support the wheelbarrow and its load if their combined mass is 55.0 kg? (c) What force does the wheel exert on the ground?
22. A typical car has an axle with 1.10 cm radius driving a tire with a radius of 27.5 cm. What is its mechanical advantage assuming the very simplified model in [Figure 9.23\(b\)](#)?
23. What force does the nail puller in [Exercise 9.19](#) exert on the supporting surface? The nail puller has a mass of 2.10 kg.

24. If you used an ideal pulley of the type shown in [Figure 9.24](#)(a) to support a car engine of mass 115 kg, (a) What would be the tension in the rope? (b) What force must the ceiling supply, assuming you pull straight down on the rope? Neglect the pulley system's mass.
25. Repeat [Exercise 9.24](#) for the pulley shown in [Figure 9.24](#)(c), assuming you pull straight up on the rope. The pulley system's mass is 7.00 kg.

9.6 Forces and Torques in Muscles and Joints

26. Verify that the force in the elbow joint in [Example 9.4](#) is 407 N, as stated in the text.
27. Two muscles in the back of the leg pull on the Achilles tendon as shown in [Figure 9.35](#). What total force do they exert?

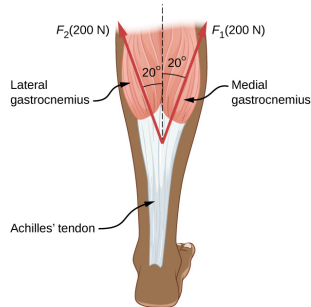


FIGURE 9.35 The Achilles tendon of the posterior leg serves to attach plantaris, gastrocnemius, and soleus muscles to calcaneus bone.

28. The upper leg muscle (quadriceps) exerts a force of 1250 N, which is carried by a tendon over the kneecap (the patella) at the angles shown in [Figure 9.36](#). Find the direction and magnitude of the force exerted by the kneecap on the upper leg bone (the femur).

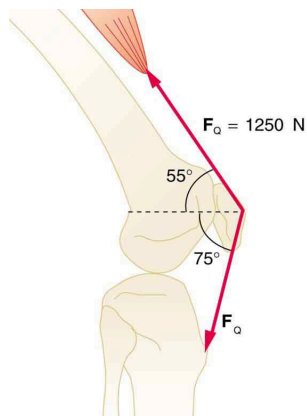


FIGURE 9.36 The knee joint works like a hinge to bend and straighten the lower leg. It permits a person to sit, stand, and pivot.

29. A device for exercising the upper leg muscle is shown in [Figure 9.37](#), together with a schematic representation of an equivalent lever system. Calculate the force exerted by the upper leg muscle to lift the mass at a constant speed. Explicitly show how you follow the steps in the Problem-Solving Strategy for static equilibrium in [Applications of Statistics, Including Problem-Solving Strategies](#).

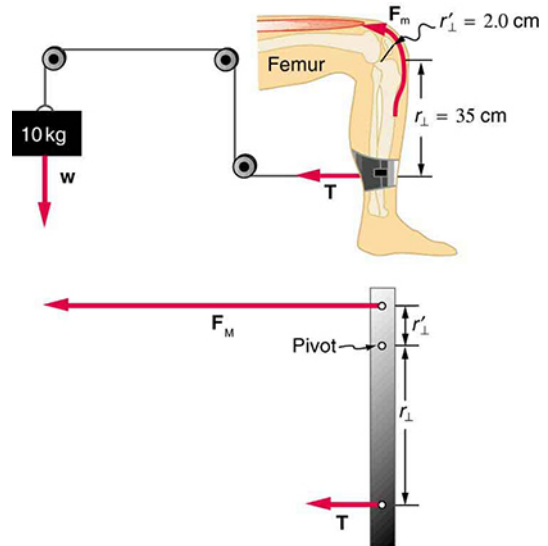


FIGURE 9.37 A mass is connected by pulleys and wires to the ankle in this exercise device.

30. A person working at a drafting board may hold her head as shown in [Figure 9.38](#), requiring muscle action to support the head. The three major acting forces are shown. Calculate the direction and magnitude of the force supplied by the upper vertebrae F_V to hold the head stationary, assuming that this force acts along a line through the center of mass as do the weight and muscle force.

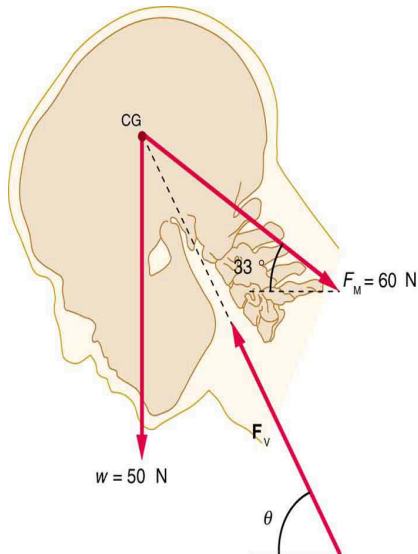


FIGURE 9.38

31. We analyzed the biceps muscle example with the angle between forearm and upper arm set at 90° . Using the same numbers as in [Example 9.4](#), find the force exerted by the biceps muscle when the angle is 120° and the forearm is in a downward position.

32. Even when the head is held erect, as in [Figure 9.39](#), its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?

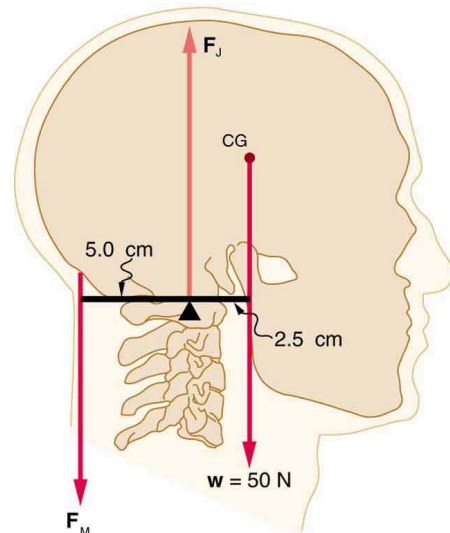


FIGURE 9.39 The center of mass of the head lies in front of its major point of support, requiring muscle action to hold the head erect. A simplified lever system is shown.

33. A 75-kg man stands on his toes by exerting an upward force through the Achilles tendon, as in [Figure 9.40](#). (a) What is the force in the Achilles tendon if he stands on one foot? (b) Calculate the force at the pivot of the simplified lever system shown—that force is representative of forces in the ankle joint.

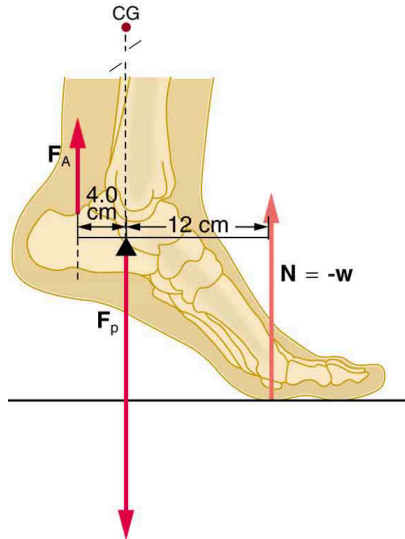


FIGURE 9.40 The muscles in the back of the leg pull the Achilles tendon when one stands on one's toes. A simplified lever system is shown.

34. A father lifts his child as shown in [Figure 9.41](#). What force should the upper leg muscle exert to lift the child at a constant speed?

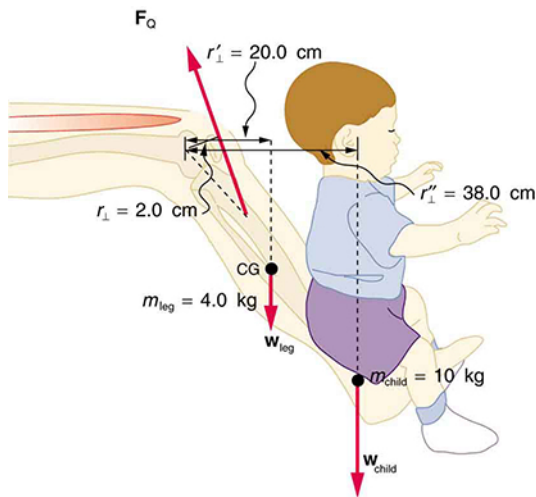


FIGURE 9.41 A child being lifted by a father's lower leg.

35. Unlike most of the other muscles in our bodies, the masseter muscle in the jaw, as illustrated in [Figure 9.42](#), is attached relatively far from the joint, enabling large forces to be exerted by the back teeth. (a) Using the information in the figure, calculate the force exerted by the lower teeth on the bullet. (b) Calculate the force on the joint.

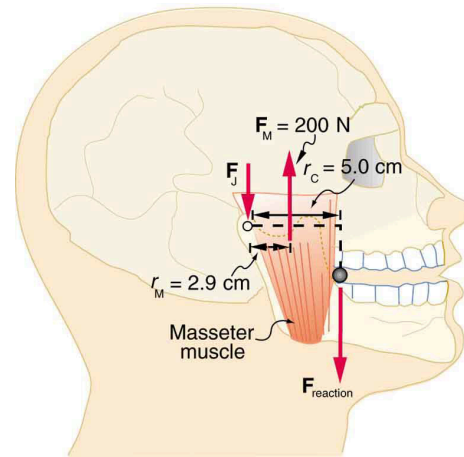


FIGURE 9.42 A person clenching a bullet between his teeth.

36. **Integrated Concepts**
Suppose we replace the 4.0-kg book in [Exercise 9.31](#) of the biceps muscle with an elastic exercise rope that obeys Hooke's Law. Assume its force constant $k = 600 \text{ N/m}$. (a) How much is the rope stretched (past equilibrium) to provide the same force F_B as in this example? Assume the rope is held in the hand at the same location as the book. (b) What force is on the biceps muscle if the exercise rope is pulled straight up so that the forearm makes an angle of 25° with the horizontal? Assume the biceps muscle is still perpendicular to the forearm.

- 37.** (a) What force should the woman in [Figure 9.43](#) exert on the floor with each hand to do a push-up? Assume that she moves up at a constant speed. (b) The triceps muscle at the back of her upper arm has an effective lever arm of 1.75 cm, and she exerts force on the floor at a horizontal distance of 20.0 cm from the elbow joint. Calculate the magnitude of the force in each triceps muscle, and compare it to her weight. (c) How much work does she do if her center of mass rises 0.240 m? (d) What is her useful power output if she does 25 pushups in one minute?

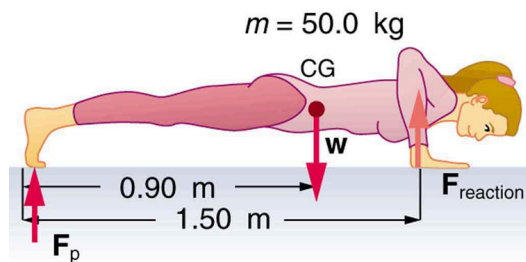


FIGURE 9.43 A woman doing pushups.

- 38.** You have just planted a sturdy 2-m-tall palm tree in your front lawn for your mother's birthday. Your brother kicks a 500 g ball, which hits the top of the tree at a speed of 5 m/s and stays in contact with it for 10 ms. The ball falls to the ground near the base of the tree and the recoil of the tree is minimal. (a) What is the force on the tree? (b) The length of the sturdy section of the root is only 20 cm. Furthermore, the soil around the roots is loose and we can assume that an effective force is applied at the tip of the 20 cm length. What is the effective force exerted by the end of the tip of the root to keep the tree from toppling? Assume the tree will be uprooted rather than bend. (c) What could you have done to ensure that the tree does not uproot easily?
- 39. Unreasonable Results**
Suppose two children are using a uniform seesaw that is 3.00 m long and has its center of mass over the pivot. The first child has a mass of 30.0 kg and sits 1.40 m from the pivot. (a) Calculate where the second 18.0 kg child must sit to balance the seesaw. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?
- 40. Construct Your Own Problem**
Consider a method for measuring the mass of a person's arm in anatomical studies. The subject lies on her back, extends her relaxed arm to the side and two scales are placed below the arm. One is placed under the elbow and the other under the back of her hand. Construct a problem in which you calculate the mass of the arm and find its center of mass based on the scale readings and the distances of the scales from the shoulder joint. You must include a free body diagram of the arm to direct the analysis. Consider changing the position of the scale under the hand to provide more information, if needed. You may wish to consult references to obtain reasonable mass values.

CHAPTER 10

Rotational Motion and Angular Momentum



FIGURE 10.1 The mention of a tornado conjures up images of raw destructive power. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. They descend from clouds in funnel-like shapes that spin violently, particularly at the bottom where they are most narrow, producing winds as high as 500 km/h. (credit: Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

CHAPTER OUTLINE

10.1 Angular Acceleration

10.2 Kinematics of Rotational Motion

10.3 Dynamics of Rotational Motion: Rotational Inertia

10.4 Rotational Kinetic Energy: Work and Energy Revisited

10.5 Angular Momentum and Its Conservation

10.6 Collisions of Extended Bodies in Two Dimensions

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

INTRODUCTION TO ROTATIONAL MOTION AND ANGULAR MOMENTUM Why do tornadoes spin at all? And why do tornadoes spin so rapidly? The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner as seen in [Figure 10.2](#). The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion. These and other aspects of rotational motion are covered in this chapter. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogs in linear motion. First, we look at angular acceleration—the rotational analog of linear acceleration.



FIGURE 10.2 This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. (credit: Luu, Wikimedia Commons)

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/10-introduction-to-rotational-motion-and-angular-momentum\)](https://openstax.org/books/college-physics-2e/pages/10-introduction-to-rotational-motion-and-angular-momentum)

10.1 Angular Acceleration

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe uniform circular motion.
- Explain non-uniform circular motion.
- Calculate angular acceleration of an object.
- Observe the link between linear and angular acceleration.

[Uniform Circular Motion and Gravitation](#) discussed only uniform circular motion, which is motion in a circle at constant speed and, hence, constant angular velocity. Recall that angular velocity ω was defined as the time rate of change of angle θ :

$$\omega = \frac{\Delta\theta}{\Delta t}, \quad 10.1$$

where θ is the angle of rotation as seen in [Figure 10.3](#). The relationship between angular velocity ω and linear velocity v was also defined in [Rotation Angle and Angular Velocity](#) as

$$v = r\omega \quad 10.2$$

or

$$\omega = \frac{v}{r}, \quad 10.3$$

where r is the radius of curvature, also seen in [Figure 10.3](#). According to the sign convention, the counter clockwise direction is considered as positive direction and clockwise direction as negative

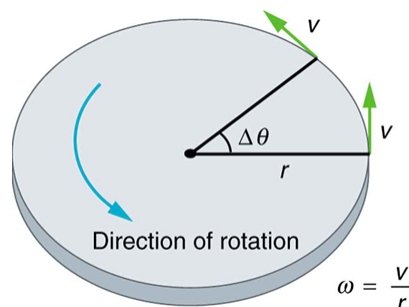


FIGURE 10.3 This figure shows uniform circular motion and some of its defined quantities.

Angular velocity is not constant when a skater pulls in her arms, when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an **angular acceleration**, in which ω changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration α is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

$$\alpha = \frac{\Delta\omega}{\Delta t}, \quad 10.4$$

where $\Delta\omega$ is the **change in angular velocity** and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2 . If ω increases, then α is positive. If ω decreases, then α is negative.

EXAMPLE 10.1

Calculating the Angular Acceleration and Deceleration of a Bike Wheel

Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in rad/s^2 . (b) If she now slams on the brakes, causing an angular acceleration of -87.3 rad/s^2 , how long does it take the wheel to stop?

Strategy for (a)

The angular acceleration can be found directly from its definition in $\alpha = \frac{\Delta\omega}{\Delta t}$ because the final angular velocity and time are given. We see that $\Delta\omega$ is 250 rpm and Δt is 5.00 s.

Solution for (a)

Entering known information into the definition of angular acceleration, we get

$$\begin{aligned} \alpha &= \frac{\Delta\omega}{\Delta t} \\ &= \frac{250 \text{ rpm}}{5.00 \text{ s}}. \end{aligned} \quad 10.5$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert $\Delta\omega$ from rpm to rad/s:

$$\begin{aligned} \Delta\omega &= 250 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} \\ &= 26.2 \frac{\text{rad}}{\text{s}}. \end{aligned} \quad 10.6$$

Entering this quantity into the expression for α , we get

$$\begin{aligned} \alpha &= \frac{\Delta\omega}{\Delta t} \\ &= \frac{26.2 \text{ rad/s}}{5.00 \text{ s}} \\ &= 5.24 \text{ rad/s}^2. \end{aligned} \quad 10.7$$

Strategy for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for Δt , yielding

$$\Delta t = \frac{\Delta\omega}{\alpha}. \quad 10.8$$

Solution for (b)

Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is -26.2 rad/s , and α is given to be -87.3 rad/s^2 . Thus,

$$\begin{aligned}\Delta t &= \frac{-26.2 \text{ rad/s}}{-87.3 \text{ rad/s}^2} \\ &= 0.300 \text{ s.}\end{aligned}\quad 10.9$$

Discussion

Note that the angular acceleration as the girl spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero. In both cases, the relationships are analogous to what happens with linear motion. For example, there is a large deceleration when you crash into a brick wall—the velocity change is large in a short time interval.

If the bicycle in the preceding example had been on its wheels instead of upside-down, it would first have accelerated along the ground and then come to a stop. This connection between circular motion and linear motion needs to be explored. For example, it would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in [Figure 10.4](#). Thus, linear acceleration is called **tangential acceleration** a_t .

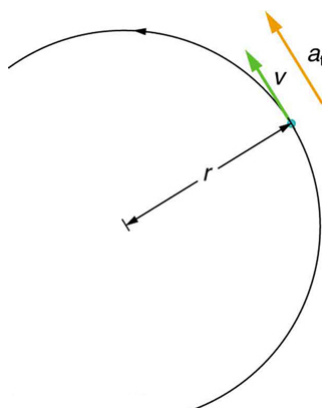


FIGURE 10.4 In circular motion, linear acceleration a , occurs as the magnitude of the velocity changes: a is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration a_t .

Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction. We know from [Uniform Circular Motion and Gravitation](#) that in circular motion centripetal acceleration, a_c , refers to changes in the direction of the velocity but not its magnitude. An object undergoing circular motion experiences centripetal acceleration, as seen in [Figure 10.5](#). Thus, a_t and a_c are perpendicular and independent of one another. Tangential acceleration a_t is directly related to the angular acceleration α and is linked to an increase or decrease in the velocity, but not its direction.

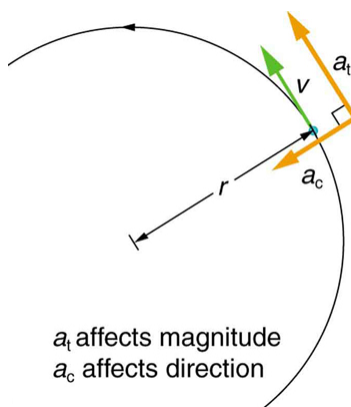


FIGURE 10.5 Centripetal acceleration a_c occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus perpendicular to each other.

Now we can find the exact relationship between linear acceleration a_t and angular acceleration α . Because linear acceleration is proportional to a change in the magnitude of the velocity, it is defined (as it was in [One-Dimensional](#)

[Kinematics](#)) to be

$$a_t = \frac{\Delta v}{\Delta t}. \quad 10.10$$

For circular motion, note that $v = r\omega$, so that

$$a_t = \frac{\Delta(r\omega)}{\Delta t}. \quad 10.11$$

The radius r is constant for circular motion, and so $\Delta(r\omega) = r(\Delta\omega)$. Thus,

$$a_t = r \frac{\Delta\omega}{\Delta t}. \quad 10.12$$

By definition, $\alpha = \frac{\Delta\omega}{\Delta t}$. Thus,

$$a_t = r\alpha, \quad 10.13$$

or

$$\alpha = \frac{a_t}{r}. \quad 10.14$$

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a car's drive wheels, the greater the acceleration of the car. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration α .



EXAMPLE 10.2

Calculating the Angular Acceleration of a Motorcycle Wheel

A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels? (See [Figure 10.6](#).)

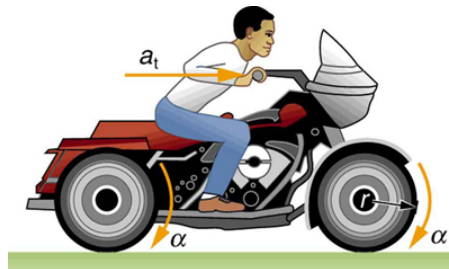


FIGURE 10.6 The linear acceleration of a motorcycle is accompanied by an angular acceleration of its wheels.

Strategy

We are given information about the linear velocities of the motorcycle. Thus, we can find its linear acceleration a_t . Then, the expression $\alpha = \frac{a_t}{r}$ can be used to find the angular acceleration.

Solution

The linear acceleration is

$$\begin{aligned} a_t &= \frac{\Delta v}{\Delta t} \\ &= \frac{30.0 \text{ m/s}}{4.20 \text{ s}} \\ &= 7.14 \text{ m/s}^2. \end{aligned} \quad 10.15$$

We also know the radius of the wheels. Entering the values for a_t and r into $\alpha = \frac{a_t}{r}$, we get

$$\begin{aligned}
 \alpha &= \frac{a_t}{r} \\
 &= \frac{7.14 \text{ m/s}^2}{0.320 \text{ m}} \\
 &= 22.3 \text{ rad/s}^2.
 \end{aligned}
 \tag{10.16}$$

Discussion

Units of radians are dimensionless and appear in any relationship between angular and linear quantities.

So far, we have defined three rotational quantities— θ , ω , and α . These quantities are analogous to the translational quantities x , v , and a . [Table 10.1](#) displays rotational quantities, the analogous translational quantities, and the relationships between them.

Rotational	Translational	Relationship
θ	x	$\theta = \frac{x}{r}$
ω	v	$\omega = \frac{v}{r}$
α	a	$\alpha = \frac{a_t}{r}$

TABLE 10.1 Rotational and Translational Quantities

Making Connections: Take-Home Experiment

Sit down with your feet on the ground on a chair that rotates. Lift one of your legs such that it is unbent (straightened out). Using the other leg, begin to rotate yourself by pushing on the ground. Stop using your leg to push the ground but allow the chair to rotate. From the origin where you began, sketch the angle, angular velocity, and angular acceleration of your leg as a function of time in the form of three separate graphs. Estimate the magnitudes of these quantities.

✓ CHECK YOUR UNDERSTANDING

Angular acceleration is a vector, having both magnitude and direction. How do we denote its magnitude and direction? Illustrate with an example.

Solution

The magnitude of angular acceleration is α and its most common units are rad/s^2 . The direction of angular acceleration along a fixed axis is denoted by a + or a – sign, just as the direction of linear acceleration in one dimension is denoted by a + or a – sign. For example, consider a gymnast doing a forward flip. Her angular momentum would be parallel to the mat and to her left. The magnitude of her angular acceleration would be proportional to her angular velocity (spin rate) and her moment of inertia about her spin axis.

Ladybug Revolution

Join the ladybug in an exploration of rotational motion. Rotate the merry-go-round to change its angle, or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's x,y position, velocity, and acceleration using vectors or graphs.

[Click to view content \(https://openstax.org/l/28ladybugrevolutionrotation\)](https://openstax.org/l/28ladybugrevolutionrotation).

10.2 Kinematics of Rotational Motion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Observe the kinematics of rotational motion.
- Derive rotational kinematic equations.
- Evaluate problem solving strategies for rotational kinematics.

Just by using our intuition, we can begin to see how rotational quantities like θ , ω , and α are related to one another. For example, if a motorcycle wheel has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. In more technical terms, if the wheel's angular acceleration α is large for a long period of time t , then the final angular velocity ω and angle of rotation θ are large. The wheel's rotational motion is exactly analogous to the fact that the motorcycle's large translational acceleration produces a large final velocity, and the distance traveled will also be large.

Kinematics is the description of motion. The **kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time. Let us start by finding an equation relating ω , α , and t . To determine this equation, we recall a familiar kinematic equation for translational, or straight-line, motion:

$$v = v_0 + at \quad (\text{constant } a) \quad 10.17$$

Note that in rotational motion $a = a_t$, and we shall use the symbol a for tangential or linear acceleration from now on. As in linear kinematics, we assume a is constant, which means that angular acceleration α is also a constant, because $a = r\alpha$. Now, let us substitute $v = r\omega$ and $a = r\alpha$ into the linear equation above:

$$r\omega = r\omega_0 + r\alpha t. \quad 10.18$$

The radius r cancels in the equation, yielding

$$\omega = \omega_0 + \alpha t \quad (\text{constant } \alpha), \quad 10.19$$

where ω_0 is the initial angular velocity. This last equation is a *kinematic relationship* among ω , α , and t —that is, it describes their relationship without reference to forces or masses that may affect rotation. It is also precisely analogous in form to its translational counterpart.

Making Connections

Kinematics for rotational motion is completely analogous to translational kinematics, first presented in [One-Dimensional Kinematics](#). Kinematics is concerned with the description of motion without regard to force or mass. We will find that translational kinematic quantities, such as displacement, velocity, and acceleration have direct analogs in rotational motion.

Starting with the four kinematic equations we developed in [One-Dimensional Kinematics](#), we can derive the following four rotational kinematic equations (presented together with their translational counterparts):

Rotational	Translational	
$\theta = \bar{\omega}t$	$x = \bar{v}t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α, a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	(constant α, a)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	(constant α, a)

TABLE 10.2 Rotational Kinematic Equations

In these equations, the subscript 0 denotes initial values (θ_0 , x_0 , and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity \bar{v} are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}. \quad 10.20$$

The equations given above in [Table 10.2](#) can be used to solve any rotational or translational kinematics problem in which a and α are constant.

Problem-Solving Strategy for Rotational Kinematics

1. Examine the situation to determine that rotational kinematics (rotational motion) is involved. Rotation must be involved, but without the need to consider forces or masses that affect the motion.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A sketch of the situation is useful.
3. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).
4. Solve the appropriate equation or equations for the quantity to be determined (the unknown). It can be useful to think in terms of a translational analog because by now you are familiar with such motion.
5. Substitute the known values along with their units into the appropriate equation, and obtain numerical solutions complete with units. Be sure to use units of radians for angles.
6. Check your answer to see if it is reasonable: Does your answer make sense?



EXAMPLE 10.3

Calculating the Acceleration of a Fishing Reel

A deep-sea fisherman hooks a big fish that swims away from the boat pulling the fishing line from his fishing reel. The whole system is initially at rest and the fishing line unwinds from the reel at a radius of 4.50 cm from its axis of rotation. The reel is given an angular acceleration of 110 rad/s^2 for 2.00 s as seen in [Figure 10.7](#).

- (a) What is the final angular velocity of the reel?
- (b) At what speed is fishing line leaving the reel after 2.00 s elapses?
- (c) How many revolutions does the reel make?
- (d) How many meters of fishing line come off the reel in this time?

Strategy

In each part of this example, the strategy is the same as it was for solving problems in linear kinematics. In particular, known values are identified and a relationship is then sought that can be used to solve for the unknown.

Solution for (a)

Here α and t are given and ω needs to be determined. The most straightforward equation to use is $\omega = \omega_0 + \alpha t$ because the unknown is already on one side and all other terms are known. That equation states that

$$\omega = \omega_0 + \alpha t. \quad 10.21$$

We are also given that $\omega_0 = 0$ (it starts from rest), so that

$$\omega = 0 + (110 \text{ rad/s}^2)(2.00\text{s}) = 220 \text{ rad/s}. \quad 10.22$$

Solution for (b)

Now that ω is known, the speed v can most easily be found using the relationship

$$v = r\omega, \quad 10.23$$

where the radius r of the reel is given to be 4.50 cm; thus,

$$v = (0.0450 \text{ m})(220 \text{ rad/s}) = 9.90 \text{ m/s.} \quad 10.24$$

Note again that radians must always be used in any calculation relating linear and angular quantities. Also, because radians are dimensionless, we have $\text{m} \times \text{rad} = \text{m}$.

Solution for (c)

Here, we are asked to find the number of revolutions. Because $1 \text{ rev} = 2\pi \text{ rad}$, we can find the number of revolutions by finding θ in radians. We are given α and t , and we know ω_0 is zero, so that θ can be obtained using $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$.

$$\begin{aligned} \theta &= \omega_0 t + \frac{1}{2} \alpha t^2 \\ &= 0 + (0.500)(110 \text{ rad/s}^2)(2.00 \text{ s})^2 = 220 \text{ rad.} \end{aligned} \quad 10.25$$

Converting radians to revolutions gives

$$\theta = (220 \text{ rad}) \frac{1 \text{ rev}}{2\pi \text{ rad}} = 35.0 \text{ rev.} \quad 10.26$$

Solution for (d)

The number of meters of fishing line is x , which can be obtained through its relationship with θ :

$$x = r\theta = (0.0450 \text{ m})(220 \text{ rad}) = 9.90 \text{ m.} \quad 10.27$$

Discussion

This example illustrates that relationships among rotational quantities are highly analogous to those among linear quantities. We also see in this example how linear and rotational quantities are connected. The answers to the questions are realistic. After unwinding for two seconds, the reel is found to spin at 220 rad/s, which is 2100 rpm. (No wonder reels sometimes make high-pitched sounds.) The amount of fishing line played out is 9.90 m, about right for when the big fish bites.

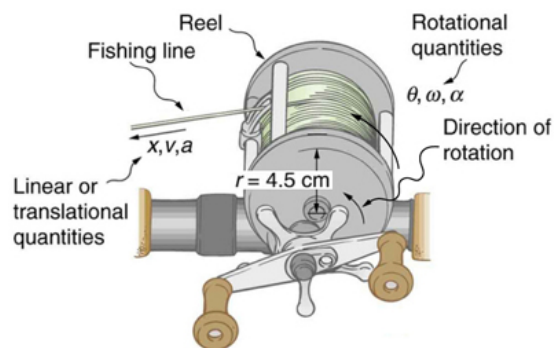


FIGURE 10.7 Fishing line coming off a rotating reel moves linearly. [Example 10.3](#) and [Example 10.4](#) consider relationships between rotational and linear quantities associated with a fishing reel.

EXAMPLE 10.4

Calculating the Duration When the Fishing Reel Slows Down and Stops

Now let us consider what happens if the fisherman applies a brake to the spinning reel, achieving an angular acceleration of -300 rad/s^2 . How long does it take the reel to come to a stop?

Strategy

We are asked to find the time t for the reel to come to a stop. The initial and final conditions are different from those in the previous problem, which involved the same fishing reel. Now we see that the initial angular velocity is $\omega_0 = 220 \text{ rad/s}$ and the final angular velocity ω is zero. The angular acceleration is given to be $\alpha = -300 \text{ rad/s}^2$.

Examining the available equations, we see all quantities but t are known in $\omega = \omega_0 + \alpha t$, making it easiest to use this equation.

Solution

The equation states

$$\omega = \omega_0 + \alpha t. \quad 10.28$$

We solve the equation algebraically for t , and then substitute the known values as usual, yielding

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 220 \text{ rad/s}}{-300 \text{ rad/s}^2} = 0.733 \text{ s}. \quad 10.29$$

Discussion

Note that care must be taken with the signs that indicate the directions of various quantities. Also, note that the time to stop the reel is fairly small because the acceleration is rather large. Fishing lines sometimes snap because of the accelerations involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration.



EXAMPLE 10.5

Calculating the Slow Acceleration of Trains and Their Wheels

Large freight trains accelerate very slowly. Suppose one such train accelerates from rest, giving its 0.350-m-radius wheels an angular acceleration of 0.250 rad/s^2 . After the wheels have made 200 revolutions (assume no slippage): (a) How far has the train moved down the track? (b) What are the final angular velocity of the wheels and the linear velocity of the train?

Strategy

In part (a), we are asked to find x , and in (b) we are asked to find ω and v . We are given the number of revolutions θ , the radius of the wheels r , and the angular acceleration α .

Solution for (a)

The distance x is very easily found from the relationship between distance and rotation angle:

$$\theta = \frac{x}{r}. \quad 10.30$$

Solving this equation for x yields

$$x = r\theta. \quad 10.31$$

Before using this equation, we must convert the number of revolutions into radians, because we are dealing with a relationship between linear and rotational quantities:

$$\theta = (200 \text{ rev}) \frac{2\pi \text{ rad}}{1 \text{ rev}} = 1257 \text{ rad}. \quad 10.32$$

Now we can substitute the known values into $x = r\theta$ to find the distance the train moved down the track:

$$x = r\theta = (0.350 \text{ m})(1257 \text{ rad}) = 440 \text{ m}. \quad 10.33$$

Solution for (b)

We cannot use any equation that incorporates t to find ω , because the equation would have at least two unknown values. The equation $\omega^2 = \omega_0^2 + 2\alpha\theta$ will work, because we know the values for all variables except ω :

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad 10.34$$

Taking the square root of this equation and entering the known values gives

$$\begin{aligned}\omega &= [0 + 2(0.250 \text{ rad/s}^2)(1257 \text{ rad})]^{1/2} \\ &= 25.1 \text{ rad/s.}\end{aligned}\quad 10.35$$

We can find the linear velocity of the train, v , through its relationship to ω :

$$v = r\omega = (0.350 \text{ m})(25.1 \text{ rad/s}) = 8.77 \text{ m/s.}\quad 10.36$$

Discussion

The distance traveled is fairly large and the final velocity is fairly slow (just under 32 km/h).

There is translational motion even for something spinning in place, as the following example illustrates. [Figure 10.8](#) shows a fly on the edge of a rotating microwave oven plate. The example below calculates the total distance it travels.

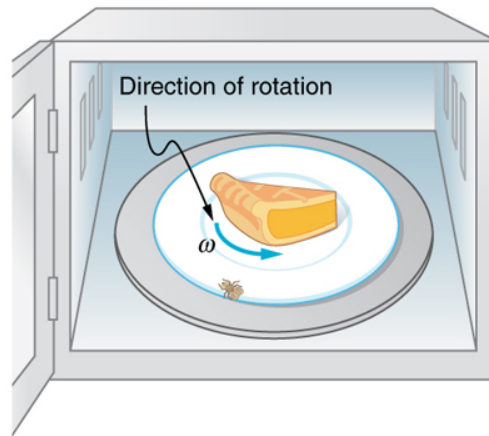


FIGURE 10.8 The image shows a microwave plate. The fly makes revolutions while the food is heated (along with the fly).



EXAMPLE 10.6

Calculating the Distance Traveled by a Fly on the Edge of a Microwave Oven Plate

A person decides to use a microwave oven to reheat some lunch. In the process, a fly accidentally flies into the microwave and lands on the outer edge of the rotating plate and remains there. If the plate has a radius of 0.15 m and rotates at 6.0 rpm, calculate the total distance traveled by the fly during a 2.0-min cooking period. (Ignore the start-up and slow-down times.)

Strategy

First, find the total number of revolutions θ , and then the linear distance x traveled. $\theta = \bar{\omega}t$ can be used to find θ because $\bar{\omega}$ is given to be 6.0 rpm.

Solution

Entering known values into $\theta = \bar{\omega}t$ gives

$$\theta = \bar{\omega}t = (6.0 \text{ rpm})(2.0 \text{ min}) = 12 \text{ rev.}\quad 10.37$$

As always, it is necessary to convert revolutions to radians before calculating a linear quantity like x from an angular quantity like θ :

$$\theta = (12 \text{ rev}) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 75.4 \text{ rad.}\quad 10.38$$

Now, using the relationship between x and θ , we can determine the distance traveled:

$$x = r\theta = (0.15 \text{ m})(75.4 \text{ rad}) = 11 \text{ m.}\quad 10.39$$

Discussion

Quite a trip (if it survives)! Note that this distance is the total distance traveled by the fly. Displacement is actually zero for complete revolutions because they bring the fly back to its original position. The distinction between total distance traveled and displacement was first noted in [One-Dimensional Kinematics](#).

✓ CHECK YOUR UNDERSTANDING

Rotational kinematics has many useful relationships, often expressed in equation form. Are these relationships laws of physics or are they simply descriptive? (Hint: the same question applies to linear kinematics.)

Solution

Rotational kinematics (just like linear kinematics) is descriptive and does not represent laws of nature. With kinematics, we can describe many things to great precision but kinematics does not consider causes. For example, a large angular acceleration describes a very rapid change in angular velocity without any consideration of its cause.

10.3 Dynamics of Rotational Motion: Rotational Inertia

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in [Figure 10.9](#). In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

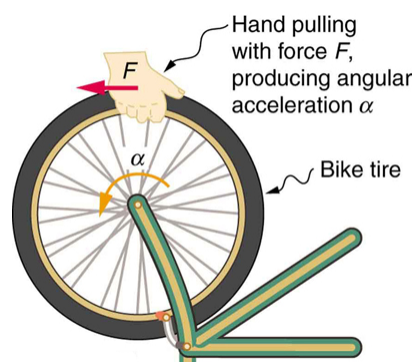


FIGURE 10.9 Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in [Figure 10.10](#). Because the force is perpendicular to r , an acceleration $a = \frac{F}{m}$ is obtained in the direction of F . We can rearrange this equation such that $F = ma$ and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

$$F = mr\alpha. \quad 10.40$$

Recall that **torque** is the turning effectiveness of a force. In this case, because \mathbf{F} is perpendicular to r , torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r , we get torque on the left-hand side. That is,

$$rF = mr^2\alpha \quad 10.41$$

or

$$\tau = mr^2\alpha. \quad 10.42$$

This last equation is the rotational analog of Newton's second law ($F = ma$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the **rotational inertia** or **moment of inertia** of a point mass m a distance r from the center of rotation.

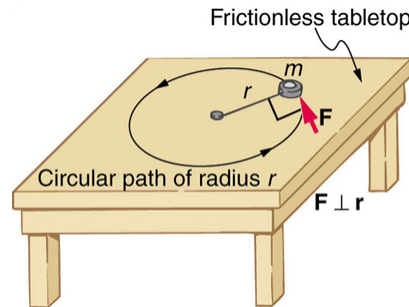


FIGURE 10.10 An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force F is applied to the object perpendicular to the radius r , causing it to accelerate about the pivot point. The force is kept perpendicular to r .

Making Connections: Rotational Motion Dynamics

Dynamics for rotational motion is completely analogous to linear or translational dynamics. Dynamics is concerned with force and mass and their effects on motion. For rotational motion, we will find direct analogs to force and mass that behave just as we would expect from our earlier experiences.

Rotational Inertia and Moment of Inertia

Before we can consider the rotation of anything other than a point mass like the one in [Figure 10.10](#), we must extend the idea of rotational inertia to all types of objects. To expand our concept of rotational inertia, we define the **moment of inertia** I of an object to be the sum of mr^2 for all the point masses of which it is composed. That is, $I = \sum mr^2$. Here I is analogous to m in translational motion. Because of the distance r , the moment of inertia for any object depends on the chosen axis. Actually, calculating I is beyond the scope of this text except for one simple case—that of a hoop, which has all its mass at the same distance from its axis. A hoop's moment of inertia around its axis is therefore MR^2 , where M is its total mass and R its radius. (We use M and R for an entire object to distinguish them from m and r for point masses.) In all other cases, we must consult [Figure 10.11](#) (note that the table is piece of artwork that has shapes as well as formulae) for formulas for I that have been derived from integration over the continuous body. Note that I has units of mass multiplied by distance squared ($\text{kg} \cdot \text{m}^2$), as we might expect from its definition.

The general relationship among torque, moment of inertia, and angular acceleration is

$$\text{net } \tau = I\alpha \quad 10.43$$

or

$$\alpha = \frac{\text{net } \tau}{I}, \quad 10.44$$

where net τ is the total torque from all forces relative to a chosen axis. For simplicity, we will only consider torques exerted by forces in the plane of the rotation. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = I\alpha$, $\alpha = \frac{\text{net } \tau}{I}$ is the rotational analog to Newton's second law and is very generally applicable. This equation is actually valid for *any* torque, applied to *any* object, relative to *any* axis.

As we might expect, the larger the torque is, the larger the angular acceleration is. For example, the harder a child

pushes on a merry-go-round, the faster it accelerates. Furthermore, the more massive a merry-go-round, the slower it accelerates for the same torque. The basic relationship between moment of inertia and angular acceleration is that the larger the moment of inertia, the smaller is the angular acceleration. But there is an additional twist. The moment of inertia depends not only on the mass of an object, but also on its *distribution* of mass relative to the axis around which it rotates. For example, it will be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge. The mass is the same in both cases, but the moment of inertia is much larger when the children are at the edge.

Take-Home Experiment

Cut out a circle that has about a 10 cm radius from stiff cardboard. Near the edge of the circle, write numbers 1 to 12 like hours on a clock face. Position the circle so that it can rotate freely about a horizontal axis through its center, like a wheel. (You could loosely nail the circle to a wall.) Hold the circle stationary and with the number 12 positioned at the top, attach a lump of blue putty (sticky material used for fixing posters to walls) at the number 3. How large does the lump need to be to just rotate the circle? Describe how you can change the moment of inertia of the circle. How does this change affect the amount of blue putty needed at the number 3 to just rotate the circle? Change the circle's moment of inertia and then try rotating the circle by using different amounts of blue putty. Repeat this process several times.

Problem-Solving Strategy for Rotational Dynamics

1. *Examine the situation to determine that torque and mass are involved in the rotation.* Draw a careful sketch of the situation.
2. *Determine the system of interest.*
3. *Draw a free body diagram.* That is, draw and label all external forces acting on the system of interest.
4. *Apply $\text{net } \tau = I\alpha$, $\alpha = \frac{\text{net } \tau}{I}$, the rotational equivalent of Newton's second law, to solve the problem.* Care must be taken to use the correct moment of inertia and to consider the torque about the point of rotation.
5. *As always, check the solution to see if it is reasonable.*

Making Connections

In statics, the net torque is zero, and there is no angular acceleration. In rotational motion, net torque is the cause of angular acceleration, exactly as in Newton's second law of motion for rotation.

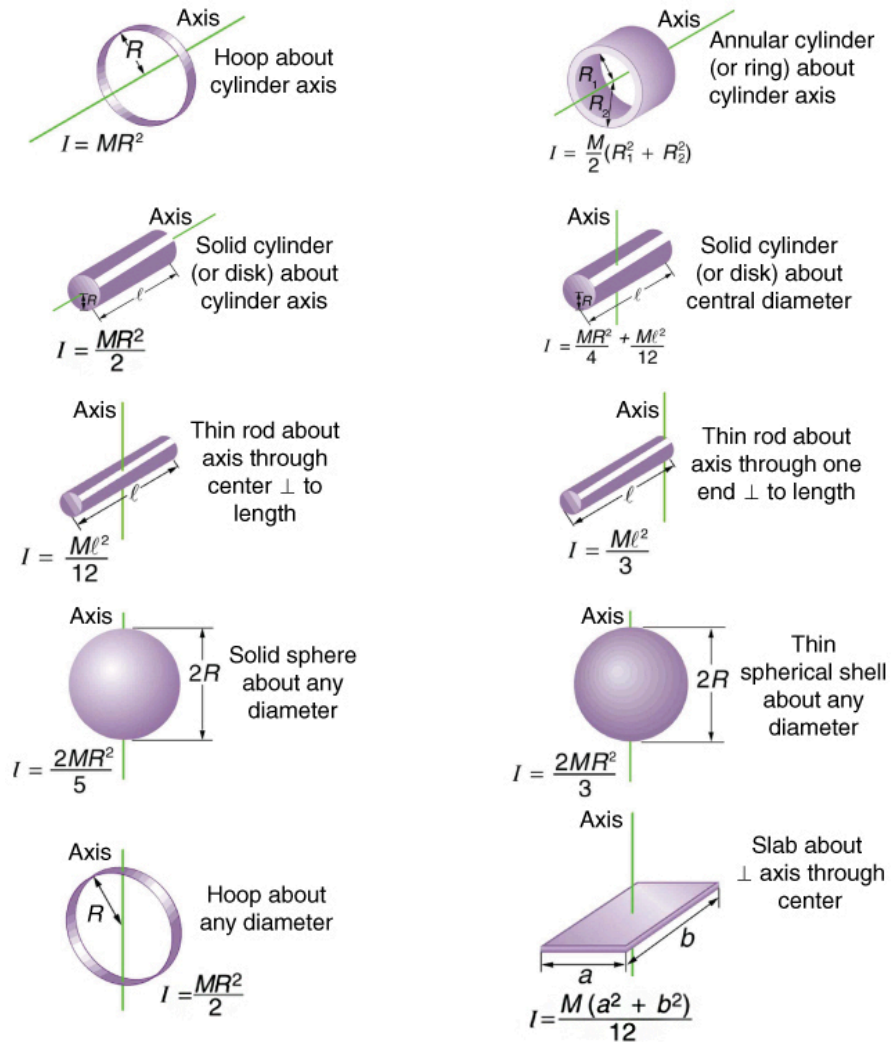


FIGURE 10.11 Some rotational inertias.

EXAMPLE 10.7

Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in [Figure 10.12](#). He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible retarding friction.

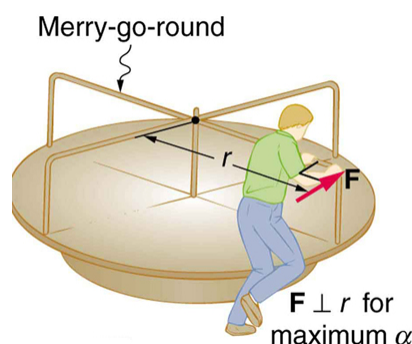


FIGURE 10.12 A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Strategy

Angular acceleration is given directly by the expression $\alpha = \frac{\text{net } \tau}{I}$:

$$\alpha = \frac{\tau}{I}. \quad 10.45$$

To solve for α , we must first calculate the torque τ (which is the same in both cases) and moment of inertia I (which is greater in the second case). To find the torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF \sin \theta = (1.50 \text{ m})(250 \text{ N}) = 375 \text{ N} \cdot \text{m}. \quad 10.46$$

Solution for (a)

The moment of inertia of a solid disk about this axis is given in [Figure 10.11](#) to be

$$\frac{1}{2}MR^2, \quad 10.47$$

where $M = 50.0 \text{ kg}$ and $R = 1.50 \text{ m}$, so that

$$I = (0.500)(50.0 \text{ kg})(1.50 \text{ m})^2 = 56.25 \text{ kg} \cdot \text{m}^2. \quad 10.48$$

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375 \text{ N} \cdot \text{m}}{56.25 \text{ kg} \cdot \text{m}^2} = 6.67 \frac{\text{rad}}{\text{s}^2}. \quad 10.49$$

Solution for (b)

We expect the angular acceleration for the system to be less in this part, because the moment of inertia is greater when the child is on the merry-go-round. To find the total moment of inertia I , we first find the child's moment of inertia I_c by considering the child to be equivalent to a point mass at a distance of 1.25 m from the axis. Then,

$$I_c = MR^2 = (18.0 \text{ kg})(1.25 \text{ m})^2 = 28.13 \text{ kg} \cdot \text{m}^2. \quad 10.50$$

The total moment of inertia is the sum of moments of inertia of the merry-go-round and the child (about the same axis). To justify this sum to yourself, examine the definition of I :

$$I = 28.13 \text{ kg} \cdot \text{m}^2 + 56.25 \text{ kg} \cdot \text{m}^2 = 84.38 \text{ kg} \cdot \text{m}^2. \quad 10.51$$

Substituting known values into the equation for α gives

$$\alpha = \frac{\tau}{I} = \frac{375 \text{ N} \cdot \text{m}}{84.38 \text{ kg} \cdot \text{m}^2} = 4.44 \frac{\text{rad}}{\text{s}^2}. \quad 10.52$$

Discussion

The angular acceleration is less when the child is on the merry-go-round than when the merry-go-round is empty, as expected. The angular accelerations found are quite large, partly due to the fact that friction was considered to be

negligible. If, for example, the father kept pushing perpendicularly for 2.00 s, he would give the merry-go-round an angular velocity of 13.3 rad/s when it is empty but only 8.89 rad/s when the child is on it. In terms of revolutions per second, these angular velocities are 2.12 rev/s and 1.41 rev/s, respectively. The father would end up running at about 50 km/h in the first case. Summer Olympics, here he comes! Confirmation of these numbers is left as an exercise for the reader.

✓ CHECK YOUR UNDERSTANDING

Torque is the analog of force and moment of inertia is the analog of mass. Force and mass are physical quantities that depend on only one factor. For example, mass is related solely to the numbers of atoms of various types in an object. Are torque and moment of inertia similarly simple?

Solution

No. Torque depends on three factors: force magnitude, force direction, and point of application. Moment of inertia depends on both mass and its distribution relative to the axis of rotation. So, while the analogies are precise, these rotational quantities depend on more factors.

10.4 Rotational Kinetic Energy: Work and Energy Revisited

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Derive the equation for rotational work.
- Calculate rotational kinetic energy.
- Demonstrate the Law of Conservation of Energy.

In this module, we will learn about work and energy associated with rotational motion. [Figure 10.13](#) shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable **rotational kinetic energy**.

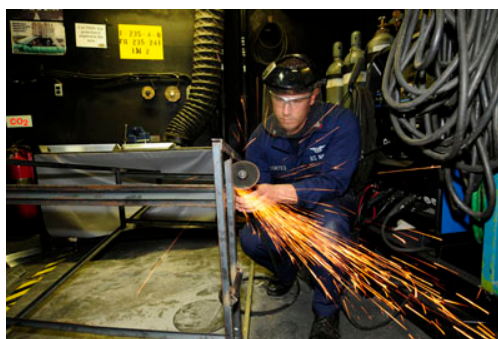


FIGURE 10.13 The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell)

Work must be done to rotate objects such as grindstones or merry-go-rounds. Work was defined in [Uniform Circular Motion and Gravitation](#) for translational motion, and we can build on that knowledge when considering work done in rotational motion. The simplest rotational situation is one in which the net force is exerted perpendicular to the radius of a disk (as shown in [Figure 10.14](#)) and remains perpendicular as the disk starts to rotate. The force is parallel to the displacement, and so the net work done is the product of the force times the arc length traveled:

$$\text{net } W = (\text{net } F)\Delta s. \quad 10.53$$

To get torque and other rotational quantities into the equation, we multiply and divide the right-hand side of the equation by r , and gather terms:

$$\text{net } W = (r \text{ net } F) \frac{\Delta s}{r}. \quad 10.54$$

We recognize that $r \text{ net } F = \text{net } \tau$ and $\Delta s/r = \theta$, so that

$$\text{net } W = (\text{net } \tau)\theta. \quad 10.55$$

This equation is the expression for rotational work. It is very similar to the familiar definition of translational work as force multiplied by distance. Here, torque is analogous to force, and angle is analogous to distance. The equation $\text{net } W = (\text{net } \tau)\theta$ is valid in general, even though it was derived for a special case.

To get an expression for rotational kinetic energy, we must again perform some algebraic manipulations. The first step is to note that $\text{net } \tau = I\alpha$, so that

$$\text{net } W = I\alpha\theta. \quad 10.56$$

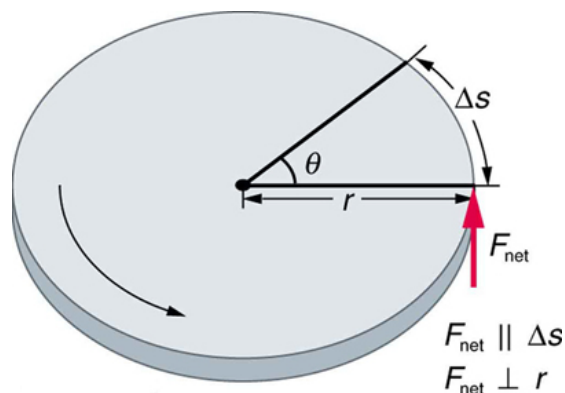


FIGURE 10.14 The net force on this disk is kept perpendicular to its radius as the force causes the disk to rotate. The net work done is thus $(\text{net } F)\Delta s$. The net work goes into rotational kinetic energy.

Making Connections

Work and energy in rotational motion are completely analogous to work and energy in translational motion, first presented in [Uniform Circular Motion and Gravitation](#).

Now, we solve one of the rotational kinematics equations for $\alpha\theta$. We start with the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta. \quad 10.57$$

Next, we solve for $\alpha\theta$:

$$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}. \quad 10.58$$

Substituting this into the equation for net W and gathering terms yields

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2. \quad 10.59$$

This equation is the **work-energy theorem** for rotational motion only. As you may recall, net work changes the kinetic energy of a system. Through an analogy with translational motion, we define the term $(\frac{1}{2})I\omega^2$ to be **rotational kinetic energy** KE_{rot} for an object with a moment of inertia I and an angular velocity ω :

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \quad 10.60$$

The expression for rotational kinetic energy is exactly analogous to translational kinetic energy, with I being analogous to m and ω to v . Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in [Figure 10.15](#).

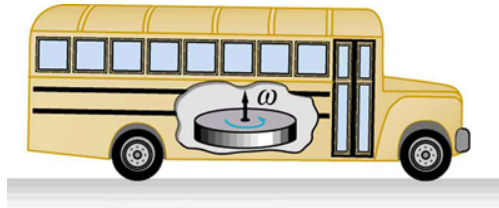


FIGURE 10.15 Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into KE_{rot} . It can also convert translational kinetic energy, when the bus stops, into KE_{rot} . The flywheel's energy can then be used to accelerate, to go up another hill, or to keep the bus from slowing down due to friction.

EXAMPLE 10.8

Calculating the Work and Energy for Spinning a Grindstone

Consider a person who spins a large grindstone by placing her hand on its edge and exerting a force through part of a revolution as shown in [Figure 10.16](#). In this example, we verify that the work done by the torque she exerts equals the change in rotational energy. (a) How much work is done if she exerts a force of 200 N through a rotation of 1.00 rad (57.3°)? The force is kept perpendicular to the grindstone's 0.320-m radius at the point of application, and the effects of friction are negligible. (b) What is the final angular velocity if the grindstone has a mass of 85.0 kg? (c) What is the final rotational kinetic energy? (It should equal the work.)

Strategy

To find the work, we can use the equation $\text{net } W = (\text{net } \tau)\theta$. We have enough information to calculate the torque and are given the rotation angle. In the second part, we can find the final angular velocity using one of the kinematic relationships. In the last part, we can calculate the rotational kinetic energy from its expression in $KE_{\text{rot}} = \frac{1}{2}I\omega^2$.

Solution for (a)

The net work is expressed in the equation

$$\text{net } W = (\text{net } \tau)\theta, \quad 10.61$$

where $\text{net } \tau$ is the applied force multiplied by the radius (rF) because there is no retarding friction, and the force is perpendicular to r . The angle θ is given. Substituting the given values in the equation above yields

$$\begin{aligned} \text{net } W &= rF\theta = (0.320 \text{ m})(200 \text{ N})(1.00 \text{ rad}) \\ &= 64.0 \text{ N} \cdot \text{m}. \end{aligned} \quad 10.62$$

Noting that $1 \text{ N} \cdot \text{m} = 1 \text{ J}$,

$$\text{net } W = 64.0 \text{ J}. \quad 10.63$$

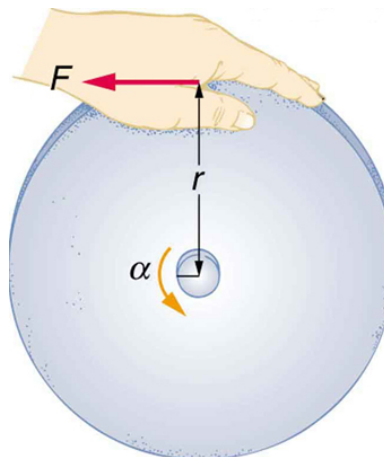


FIGURE 10.16 A large grindstone is given a spin by a person grasping its outer edge.

Solution for (b)

To find ω from the given information requires more than one step. We start with the kinematic relationship in the equation

$$\omega^2 = \omega_0^2 + 2\alpha\theta. \quad 10.64$$

Note that $\omega_0 = 0$ because we start from rest. Taking the square root of the resulting equation gives

$$\omega = (2\alpha\theta)^{1/2}. \quad 10.65$$

Now we need to find α . One possibility is

$$\alpha = \frac{\text{net } \tau}{I}, \quad 10.66$$

where the torque is

$$\text{net } \tau = rF = (0.320 \text{ m})(200 \text{ N}) = 64.0 \text{ N} \cdot \text{m}. \quad 10.67$$

The formula for the moment of inertia for a disk is found in [Figure 10.11](#):

$$I = \frac{1}{2}MR^2 = 0.5(85.0 \text{ kg})(0.320 \text{ m})^2 = 4.352 \text{ kg} \cdot \text{m}^2. \quad 10.68$$

Substituting the values of torque and moment of inertia into the expression for α , we obtain

$$\alpha = \frac{64.0 \text{ N} \cdot \text{m}}{4.352 \text{ kg} \cdot \text{m}^2} = 14.7 \frac{\text{rad}}{\text{s}^2}. \quad 10.69$$

Now, substitute this value and the given value for θ into the above expression for ω :

$$\omega = (2\alpha\theta)^{1/2} = \left[2 \left(14.7 \frac{\text{rad}}{\text{s}^2} \right) (1.00 \text{ rad}) \right]^{1/2} = 5.42 \frac{\text{rad}}{\text{s}}. \quad 10.70$$

Solution for (c)

The final rotational kinetic energy is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \quad 10.71$$

Both I and ω were found above. Thus,

$$\text{KE}_{\text{rot}} = (0.5)(4.352 \text{ kg} \cdot \text{m}^2)(5.42 \text{ rad/s})^2 = 64.0 \text{ J}. \quad 10.72$$

Discussion

The final rotational kinetic energy equals the work done by the torque, which confirms that the work done went into rotational kinetic energy. We could, in fact, have used an expression for energy instead of a kinematic relation to solve part (b). We will do this in later examples.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

Problem-Solving Strategy for Rotational Energy

1. Determine that energy or work is involved in the rotation.
2. Determine the system of interest. A sketch usually helps.
3. Analyze the situation to determine the types of work and energy involved.
4. For closed systems, mechanical energy is conserved. That is, $KE_i + PE_i = KE_f + PE_f$. Note that KE_i and KE_f may each include translational and rotational contributions.
5. For open systems, mechanical energy may not be conserved, and other forms of energy (referred to previously as OE), such as heat transfer, may enter or leave the system. Determine what they are, and calculate them as necessary.
6. Eliminate terms wherever possible to simplify the algebra.
7. Check the answer to see if it is reasonable.



EXAMPLE 10.9

Calculating Helicopter Energies

A typical small rescue helicopter, similar to the one in [Figure 10.17](#), has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2. \quad 10.73$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find KE_{rot} . The angular velocity ω is

$$\omega = \frac{300 \text{ rev}}{1.00 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1.00 \text{ min}}{60.0 \text{ s}} = 31.4 \frac{\text{rad}}{\text{s}}. \quad 10.74$$

The moment of inertia of one blade will be that of a thin rod rotated about its end, found in [Figure 10.11](#). The total I is four times this moment of inertia, because there are four blades. Thus,

$$I = 4 \frac{M\ell^2}{3} = 4 \times \frac{(50.0 \text{ kg})(4.00 \text{ m})^2}{3} = 1067 \text{ kg} \cdot \text{m}^2. \quad 10.75$$

Entering ω and I into the expression for rotational kinetic energy gives

$$\begin{aligned} KE_{\text{rot}} &= 0.5(1067 \text{ kg} \cdot \text{m}^2)(31.4 \text{ rad/s})^2 \\ &= 5.26 \times 10^5 \text{ J} \end{aligned} \quad 10.76$$

Solution for (b)

Translational kinetic energy was defined in [Uniform Circular Motion and Gravitation](#). Entering the given values of mass and velocity, we obtain

$$KE_{\text{trans}} = \frac{1}{2}mv^2 = (0.5)(1000 \text{ kg})(20.0 \text{ m/s})^2 = 2.00 \times 10^5 \text{ J}. \quad 10.77$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00 \times 10^5 \text{ J}}{5.26 \times 10^5 \text{ J}} = 0.380. \quad 10.78$$

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

$$KE_{\text{rot}} = PE_{\text{grav}} \quad 10.79$$

or

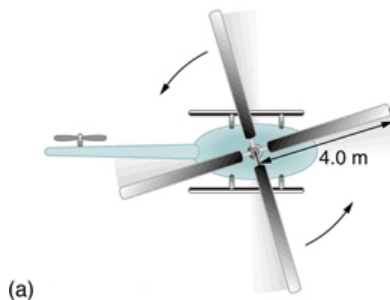
$$\frac{1}{2}I\omega^2 = mgh. \quad 10.80$$

We now solve for h and substitute known values into the resulting equation

$$h = \frac{\frac{1}{2}I\omega^2}{mg} = \frac{5.26 \times 10^5 \text{ J}}{(1000 \text{ kg})(9.80 \text{ m/s}^2)} = 53.7 \text{ m}. \quad 10.81$$

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.



(a)



(b)

FIGURE 10.17 The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. (credit: 111 Emergency, Flickr)

Making Connections

Conservation of energy includes rotational motion, because rotational kinetic energy is another form of KE. [Uniform Circular Motion and Gravitation](#) has a detailed treatment of conservation of energy.

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy PE_{grav} , which is converted entirely to KE, provided each rolls without slipping. KE, however, can take the form of KE_{trans} or KE_{rot} , and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in [Figure 10.18](#).

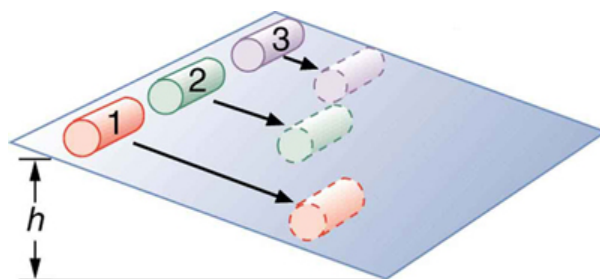


FIGURE 10.18 Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

$$PE_i = KE_f. \quad 10.82$$

More specifically,

$$PE_{\text{grav}} = KE_{\text{trans}} + KE_{\text{rot}} \quad 10.83$$

or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \quad 10.84$$

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

Take-Home Experiment

Locate several cans each containing different types of food. First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. You could also do this experiment by collecting several empty cylindrical containers of the same size and filling them with different materials such as wet or dry sand.

EXAMPLE 10.10

Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with v as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \quad 10.85$$

Before we can solve for v , we must get an expression for I from [Figure 10.11](#). Because v and ω are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v/R$ into the expression. These substitutions yield

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right). \quad 10.86$$

Interestingly, the cylinder's radius R and mass m cancel, yielding

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2. \quad 10.87$$

Solving algebraically, the equation for the final velocity v gives

$$v = \left(\frac{4gh}{3}\right)^{1/2}. \quad 10.88$$

Substituting known values into the resulting expression yields

$$v = \left[\frac{4(9.80 \text{ m/s}^2)(2.00 \text{ m})}{3}\right]^{1/2} = 5.11 \text{ m/s}. \quad 10.89$$

Discussion

Because m and R cancel, the result $v = \left(\frac{4}{3}gh\right)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}mv^2 = mgh$ and $v = (2gh)^{1/2}$, which is 22% greater than $(4gh/3)^{1/2}$. That is, the cylinder would go faster at the bottom.

CHECK YOUR UNDERSTANDING

Analogy of Rotational and Translational Kinetic Energy Is rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

Solution

Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An

example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

10.5 Angular Momentum and Its Conservation

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

Why does Earth keep on spinning? What started it spinning to begin with? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster? Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

By now the pattern is clear—every rotational phenomenon has a direct translational analog. It seems quite reasonable, then, to define **angular momentum** L as

$$L = I\omega. \quad 10.90$$

This equation is an analog to the definition of linear momentum as $p = mv$. Units for linear momentum are $\text{kg} \cdot \text{m/s}$ while units for angular momentum are $\text{kg} \cdot \text{m}^2/\text{s}$. As we would expect, an object that has a large moment of inertia I , such as Earth, has a very large angular momentum. An object that has a large angular velocity ω , such as a centrifuge, also has a rather large angular momentum.

Making Connections

Angular momentum is completely analogous to linear momentum, first presented in [Uniform Circular Motion and Gravitation](#). It has the same implications in terms of carrying rotation forward, and it is conserved when the net external torque is zero. Angular momentum, like linear momentum, is also a property of the atoms and subatomic particles.



EXAMPLE 10.11

Calculating Angular Momentum of the Earth

Strategy

No information is given in the statement of the problem; so we must look up pertinent data before we can calculate $L = I\omega$. First, according to [Figure 10.11](#), the formula for the moment of inertia of a sphere is

$$I = \frac{2MR^2}{5} \quad 10.91$$

so that

$$L = I\omega = \frac{2MR^2\omega}{5}. \quad 10.92$$

Earth's mass M is 5.979×10^{24} kg and its radius R is 6.376×10^6 m. The Earth's angular velocity ω is, of course, exactly one revolution per day, but we must convert ω to radians per second to do the calculation in SI units.

Solution

Substituting known information into the expression for L and converting ω to radians per second gives

$$\begin{aligned}
 L &= 0.4(5.979 \times 10^{24} \text{ kg})(6.376 \times 10^6 \text{ m})^2 \left(\frac{1 \text{ rev}}{\text{d}}\right) \\
 &= 9.72 \times 10^{37} \text{ kg} \cdot \text{m}^2 \cdot \text{rev/d}.
 \end{aligned}
 \tag{10.93}$$

Substituting 2π rad for 1 rev and 8.64×10^4 s for 1 day gives

$$\begin{aligned}
 L &= (9.72 \times 10^{37} \text{ kg} \cdot \text{m}^2) \left(\frac{2\pi \text{ rad/rev}}{8.64 \times 10^4 \text{ s/d}}\right) (1 \text{ rev/d}) \\
 &= 7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}.
 \end{aligned}
 \tag{10.94}$$

Discussion

This number is large, demonstrating that Earth, as expected, has a tremendous angular momentum. The answer is approximate, because we have assumed a constant density for Earth in order to estimate its moment of inertia.

When you push a merry-go-round, spin a bike wheel, or open a door, you exert a torque. If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases. The greater the net torque, the more rapid the increase in L . The relationship between torque and angular momentum is

$$\text{net } \tau = \frac{\Delta L}{\Delta t}.
 \tag{10.95}$$

This expression is exactly analogous to the relationship between force and linear momentum, $F = \Delta p/\Delta t$. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ is very fundamental and broadly applicable. It is, in fact, the rotational form of Newton's second law.



EXAMPLE 10.12

Calculating the Torque Putting Angular Momentum Into a Rotating Food Tray

Figure 10.19 shows a rotating food tray, often called a lazy Susan, being turned by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?

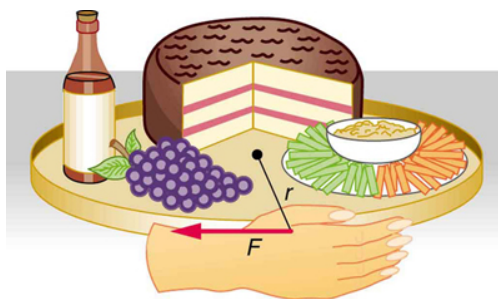


FIGURE 10.19 A partygoer exerts a torque on a lazy Susan to make it rotate. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ gives the relationship between torque and the angular momentum produced.

Strategy

We can find the angular momentum by solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL , and using the given information to calculate the torque. The final angular momentum equals the change in angular momentum, because the lazy Susan starts from rest. That is, $\Delta L = L$. To find the final velocity, we must calculate ω from the definition of L in $L = I\omega$.

Solution for (a)

Solving $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for ΔL gives

$$\Delta L = (\text{net } \tau)\Delta t.
 \tag{10.96}$$

Because the force is perpendicular to r , we see that net $\tau = rF$, so that

$$\begin{aligned} L &= rF\Delta t = (0.260 \text{ m})(2.50 \text{ N})(0.150 \text{ s}) \\ &= 9.75 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s}. \end{aligned} \quad 10.97$$

Solution for (b)

The final angular velocity can be calculated from the definition of angular momentum,

$$L = I\omega. \quad 10.98$$

Solving for ω and substituting the formula for the moment of inertia of a disk into the resulting equation gives

$$\omega = \frac{L}{I} = \frac{L}{\frac{1}{2}MR^2}. \quad 10.99$$

And substituting known values into the preceding equation yields

$$\omega = \frac{9.75 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s}}{(0.500)(4.00 \text{ kg})(0.260 \text{ m})^2} = 0.721 \text{ rad/s}. \quad 10.100$$

Discussion

Note that the imparted angular momentum does not depend on any property of the object but only on torque and time. The final angular velocity is equivalent to one revolution in 8.71 s (determination of the time period is left as an exercise for the reader), which is about right for a lazy Susan.

EXAMPLE 10.13

Calculating the Torque in a Kick

The person whose leg is shown in [Figure 10.20](#) kicks his leg by exerting a 2000-N force with his upper leg muscle. The effective perpendicular lever arm is 2.20 cm. Given the moment of inertia of the lower leg is $1.25 \text{ kg} \cdot \text{m}^2$, (a) find the angular acceleration of the leg. (b) Neglecting the gravitational force, what is the rotational kinetic energy of the leg after it has rotated through 57.3° (1.00 rad)?

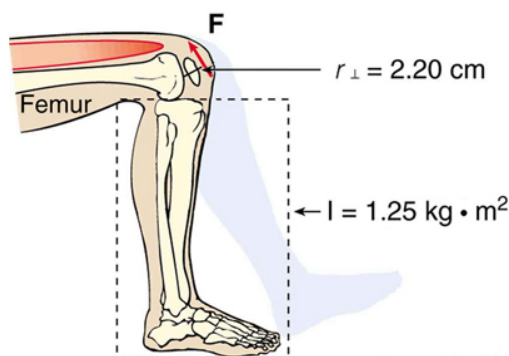


FIGURE 10.20 The muscle in the upper leg gives the lower leg an angular acceleration and imparts rotational kinetic energy to it by exerting a torque about the knee. \mathbf{F} is a vector that is perpendicular to r . This example examines the situation.

Strategy

The angular acceleration can be found using the rotational analog to Newton's second law, or $\alpha = \text{net } \tau/I$. The moment of inertia I is given and the torque can be found easily from the given force and perpendicular lever arm. Once the angular acceleration α is known, the final angular velocity and rotational kinetic energy can be calculated.

Solution to (a)

From the rotational analog to Newton's second law, the angular acceleration α is

$$\alpha = \frac{\text{net } \tau}{I}. \quad 10.101$$

Because the force and the perpendicular lever arm are given and the leg is vertical so that its weight does not create a torque, the net torque is thus

$$\begin{aligned} \text{net } \tau &= r_{\perp} F \\ &= (0.0220 \text{ m})(2000 \text{ N}) \\ &= 44.0 \text{ N} \cdot \text{m}. \end{aligned} \quad 10.102$$

Substituting this value for the torque and the given value for the moment of inertia into the expression for α gives

$$\alpha = \frac{44.0 \text{ N} \cdot \text{m}}{1.25 \text{ kg} \cdot \text{m}^2} = 35.2 \text{ rad/s}^2. \quad 10.103$$

Solution to (b)

The final angular velocity can be calculated from the kinematic expression

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad 10.104$$

or

$$\omega^2 = 2\alpha\theta \quad 10.105$$

because the initial angular velocity is zero. The kinetic energy of rotation is

$$\text{KE}_{\text{rot}} = \frac{1}{2} I \omega^2 \quad 10.106$$

so it is most convenient to use the value of ω^2 just found and the given value for the moment of inertia. The kinetic energy is then

$$\begin{aligned} \text{KE}_{\text{rot}} &= 0.5(1.25 \text{ kg} \cdot \text{m}^2)(70.4 \text{ rad}^2/\text{s}^2) \\ &= 44.0 \text{ J} \end{aligned} \quad 10.107$$

Discussion

These values are reasonable for a person kicking his leg starting from the position shown. The weight of the leg can be neglected in part (a) because it exerts no torque when the center of gravity of the lower leg is directly beneath the pivot in the knee. In part (b), the force exerted by the upper leg is so large that its torque is much greater than that created by the weight of the lower leg as it rotates. The rotational kinetic energy given to the lower leg is enough that it could give a ball a significant velocity by transferring some of this energy in a kick.

Making Connections: Conservation Laws

Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conservation of Angular Momentum

We can now understand why Earth keeps on spinning. As we saw in the previous example, $\Delta L = (\text{net } \tau)\Delta t$. This equation means that, to change angular momentum, a torque must act over some period of time. Because Earth has a large angular momentum, a large torque acting over a long time is needed to change its rate of spin. So what external torques are there? Tidal friction exerts torque that is slowing Earth's rotation, but tens of millions of years must pass before the change is very significant. Recent research indicates the length of the day was 18 h some 900 million years ago. Only the tides exert significant retarding torques on Earth, and so it will continue to spin, although ever more slowly, for many billions of years.

What we have here is, in fact, another conservation law. If the net torque is *zero*, then angular momentum is constant or *conserved*. We can see this rigorously by considering net $\tau = \frac{\Delta L}{\Delta t}$ for the situation in which the net torque is zero. In that case,

$$\text{net } \tau = 0 \quad 10.108$$

implying that

$$\frac{\Delta L}{\Delta t} = 0. \quad 10.109$$

If the change in angular momentum ΔL is zero, then the angular momentum is constant; thus,

$$L = \text{constant (net } \tau = 0) \quad 10.110$$

or

$$L = L' (\text{net } \tau = 0). \quad 10.111$$

These expressions are the **law of conservation of angular momentum**. Conservation laws are as scarce as they are important.

An example of conservation of angular momentum is seen in [Figure 10.21](#), in which an ice skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice and because the friction is exerted very close to the pivot point. (Both F and r are small, and so τ is negligibly small.) Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that

$$L = L'. \quad 10.112$$

Expressing this equation in terms of the moment of inertia,

$$I\omega = I'\omega', \quad 10.113$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity ω' must increase to keep the angular momentum constant. The change can be dramatic, as the following example shows.

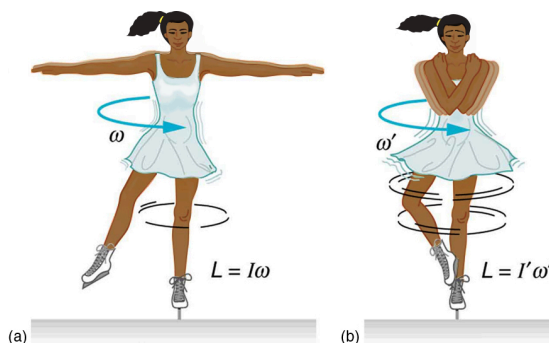


FIGURE 10.21 (a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.



EXAMPLE 10.14

Calculating the Angular Momentum of a Spinning Skater

Suppose an ice skater, such as the one in [Figure 10.21](#), is spinning at 0.800 rev/s with her arms extended. She has a moment of inertia of $2.34 \text{ kg} \cdot \text{m}^2$ with her arms extended and of $0.363 \text{ kg} \cdot \text{m}^2$ with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

Strategy

In the first part of the problem, we are looking for the skater's angular velocity ω' after she has pulled in her arms. To find this quantity, we use the conservation of angular momentum and note that the moments of inertia and initial angular velocity are given. To find the initial and final kinetic energies, we use the definition of rotational kinetic energy given by

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2. \quad 10.114$$

Solution for (a)

Because torque is negligible (as discussed above), the conservation of angular momentum given in $I\omega = I'\omega'$ is applicable. Thus,

$$L = L' \quad 10.115$$

or

$$I\omega = I'\omega' \quad 10.116$$

Solving for ω' and substituting known values into the resulting equation gives

$$\begin{aligned} \omega' &= \frac{I}{I'}\omega = \left(\frac{2.34 \text{ kg}\cdot\text{m}^2}{0.363 \text{ kg}\cdot\text{m}^2}\right)(0.800 \text{ rev/s}) \\ &= 5.16 \text{ rev/s.} \end{aligned} \quad 10.117$$

Solution for (b)

Rotational kinetic energy is given by

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2. \quad 10.118$$

The initial value is found by substituting known values into the equation and converting the angular velocity to rad/s:

$$\begin{aligned} KE_{\text{rot}} &= (0.5)(2.34 \text{ kg}\cdot\text{m}^2)((0.800 \text{ rev/s})(2\pi \text{ rad/rev}))^2 \\ &= 29.6 \text{ J.} \end{aligned} \quad 10.119$$

The final rotational kinetic energy is

$$KE_{\text{rot}'} = \frac{1}{2} I' \omega'^2. \quad 10.120$$

Substituting known values into this equation gives

$$\begin{aligned} KE_{\text{rot}'} &= (0.5)(0.363 \text{ kg}\cdot\text{m}^2)[(5.16 \text{ rev/s})(2\pi \text{ rad/rev})]^2 \\ &= 191 \text{ J.} \end{aligned} \quad 10.121$$

Discussion

In both parts, there is an impressive increase. First, the final angular velocity is large, although most world-class skaters can achieve spin rates about this great. Second, the final kinetic energy is much greater than the initial kinetic energy. The increase in rotational kinetic energy comes from work done by the skater in pulling in her arms. This work is internal work that depletes some of the skater's food energy.

There are several other examples of objects that increase their rate of spin because something reduced their moment of inertia. Tornadoes are one example. Storm systems that create tornadoes are slowly rotating. When the radius of rotation narrows, even in a local region, angular velocity increases, sometimes to the furious level of a tornado. Earth is another example. Our planet was born from a huge cloud of gas and dust, the rotation of which came from turbulence in an even larger cloud. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result. (See [Figure 10.22](#).)

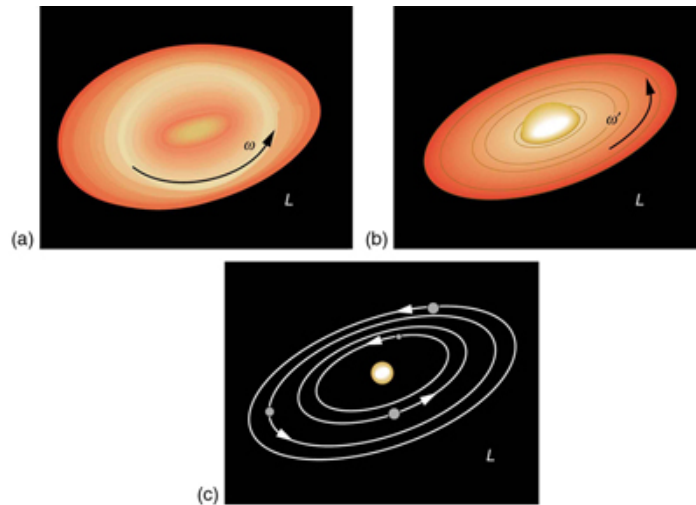


FIGURE 10.22 The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

✓ CHECK YOUR UNDERSTANDING

Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

Solution

Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

10.6 Collisions of Extended Bodies in Two Dimensions

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Observe collisions of extended bodies in two dimensions.
- Examine collision at the point of percussion.

Bowling pins are sent flying and spinning when hit by a bowling ball—angular momentum as well as linear momentum and energy have been imparted to the pins. (See [Figure 10.23](#)). Many collisions involve angular momentum. Cars, for example, may spin and collide on ice or a wet surface. Baseball pitchers throw curves by putting spin on the baseball. A tennis player can put a lot of top spin on the tennis ball which causes it to dive down onto the court once it crosses the net. We now take a brief look at what happens when objects that can rotate collide.

Consider the relatively simple collision shown in [Figure 10.24](#), in which a disk strikes and adheres to an initially motionless stick nailed at one end to a frictionless surface. After the collision, the two rotate about the nail. There is an unbalanced external force on the system at the nail. This force exerts no torque because its lever arm r is zero. Angular momentum is therefore conserved in the collision. Kinetic energy is not conserved, because the collision is inelastic. It is possible that momentum is not conserved either because the force at the nail may have a component in the direction of the disk's initial velocity. Let us examine a case of rotation in a collision in [Example 10.15](#).



FIGURE 10.23 The bowling ball causes the pins to fly, some of them spinning violently. (credit: Tinou Bao, Flickr)

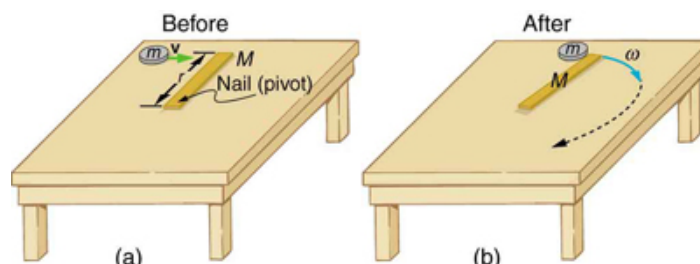


FIGURE 10.24 (a) A disk slides toward a motionless stick on a frictionless surface. (b) The disk hits the stick at one end and adheres to it, and they rotate together, pivoting around the nail. Angular momentum is conserved for this inelastic collision because the surface is frictionless and the unbalanced external force at the nail exerts no torque.

EXAMPLE 10.15

Rotation in a Collision

Suppose the disk in [Figure 10.24](#) has a mass of 50.0 g and an initial velocity of 30.0 m/s when it strikes the stick that is 1.20 m long and 2.00 kg.

- What is the angular velocity of the two after the collision?
- What is the kinetic energy before and after the collision?
- What is the total linear momentum before and after the collision?

Strategy for (a)

We can answer the first question using conservation of angular momentum as noted. Because angular momentum is $I\omega$, we can solve for angular velocity.

Solution for (a)

Conservation of angular momentum states

$$L = L', \quad 10.122$$

where primed quantities stand for conditions after the collision and both momenta are calculated relative to the pivot point. The initial angular momentum of the system of stick-disk is that of the disk just before it strikes the stick. That is,

$$L = I\omega, \quad 10.123$$

where I is the moment of inertia of the disk and ω is its angular velocity around the pivot point. Now, $I = mr^2$ (taking the disk to be approximately a point mass) and $\omega = v/r$, so that

$$L = mr^2 \frac{v}{r} = mvr. \quad 10.124$$

After the collision,

$$L' = I' \omega' . \quad 10.125$$

It is ω' that we wish to find. Conservation of angular momentum gives

$$I' \omega' = mvr . \quad 10.126$$

Rearranging the equation yields

$$\omega' = \frac{mvr}{I'} , \quad 10.127$$

where I' is the moment of inertia of the stick and disk stuck together, which is the sum of their individual moments of inertia about the nail. [Figure 10.11](#) gives the formula for a rod rotating around one end to be $I = Mr^2/3$. Thus,

$$I' = mr^2 + \frac{Mr^2}{3} = \left(m + \frac{M}{3}\right)r^2 . \quad 10.128$$

Entering known values in this equation yields,

$$I' = (0.0500 \text{ kg} + 0.667 \text{ kg})(1.20 \text{ m})^2 = 1.032 \text{ kg} \cdot \text{m}^2 . \quad 10.129$$

The value of I' is now entered into the expression for ω' , which yields

$$\begin{aligned} \omega' &= \frac{mvr}{I'} = \frac{(0.0500 \text{ kg})(30.0 \text{ m/s})(1.20 \text{ m})}{1.032 \text{ kg} \cdot \text{m}^2} \\ &= 1.744 \text{ rad/s} \approx 1.74 \text{ rad/s} . \end{aligned} \quad 10.130$$

Strategy for (b)

The kinetic energy before the collision is the incoming disk's translational kinetic energy, and after the collision, it is the rotational kinetic energy of the two stuck together.

Solution for (b)

First, we calculate the translational kinetic energy by entering given values for the mass and speed of the incoming disk.

$$\text{KE} = \frac{1}{2}mv^2 = (0.500)(0.0500 \text{ kg})(30.0 \text{ m/s})^2 = 22.5 \text{ J} \quad 10.131$$

After the collision, the rotational kinetic energy can be found because we now know the final angular velocity and the final moment of inertia. Thus, entering the values into the rotational kinetic energy equation gives

$$\begin{aligned} \text{KE}' &= \frac{1}{2}I'\omega'^2 = (0.5)(1.032 \text{ kg} \cdot \text{m}^2)\left(1.744 \frac{\text{rad}}{\text{s}}\right)^2 \\ &= 1.57 \text{ J} . \end{aligned} \quad 10.132$$

Strategy for (c)

The linear momentum before the collision is that of the disk. After the collision, it is the sum of the disk's momentum and that of the center of mass of the stick.

Solution of (c)

Before the collision, then, linear momentum is

$$p = mv = (0.0500 \text{ kg})(30.0 \text{ m/s}) = 1.50 \text{ kg} \cdot \text{m/s} . \quad 10.133$$

After the collision, the disk and the stick's center of mass move in the same direction. The total linear momentum is that of the disk moving at a new velocity $v' = r\omega'$ plus that of the stick's center of mass,

which moves at half this speed because $v_{\text{CM}} = \left(\frac{r}{2}\right)\omega' = \frac{v'}{2}$. Thus,

$$p' = mv' + Mv_{\text{CM}} = mv' + \frac{Mv'}{2} . \quad 10.134$$

Gathering similar terms in the equation yields,

$$p' = \left(m + \frac{M}{2}\right)v' \quad 10.135$$

so that

$$p' = \left(m + \frac{M}{2}\right)r\omega'. \quad 10.136$$

Substituting known values into the equation,

$$p' = (1.050 \text{ kg})(1.20 \text{ m})(1.744 \text{ rad/s}) = 2.20 \text{ kg} \cdot \text{m/s}. \quad 10.137$$

Discussion

First note that the kinetic energy is less after the collision, as predicted, because the collision is inelastic. More surprising is that the momentum after the collision is actually greater than before the collision. This result can be understood if you consider how the nail affects the stick and vice versa. Apparently, the stick pushes backward on the nail when first struck by the disk. The nail's reaction (consistent with Newton's third law) is to push forward on the stick, imparting momentum to it in the same direction in which the disk was initially moving, thereby increasing the momentum of the system.

The above example has other implications. For example, what would happen if the disk hit very close to the nail? Obviously, a force would be exerted on the nail in the forward direction. So, when the stick is struck at the end farthest from the nail, a backward force is exerted on the nail, and when it is hit at the end nearest the nail, a forward force is exerted on the nail. Thus, striking it at a certain point in between produces no force on the nail. This intermediate point is known as the *percussion point*.

An analogous situation occurs in tennis as seen in [Figure 10.25](#). If you hit a ball with the end of your racquet, the handle is pulled away from your hand. If you hit a ball much farther down, for example, on the shaft of the racquet, the handle is pushed into your palm. And if you hit the ball at the racquet's percussion point (what some people call the "sweet spot"), then little or *no* force is exerted on your hand, and there is less vibration, reducing chances of a tennis elbow. The same effect occurs for a baseball bat.

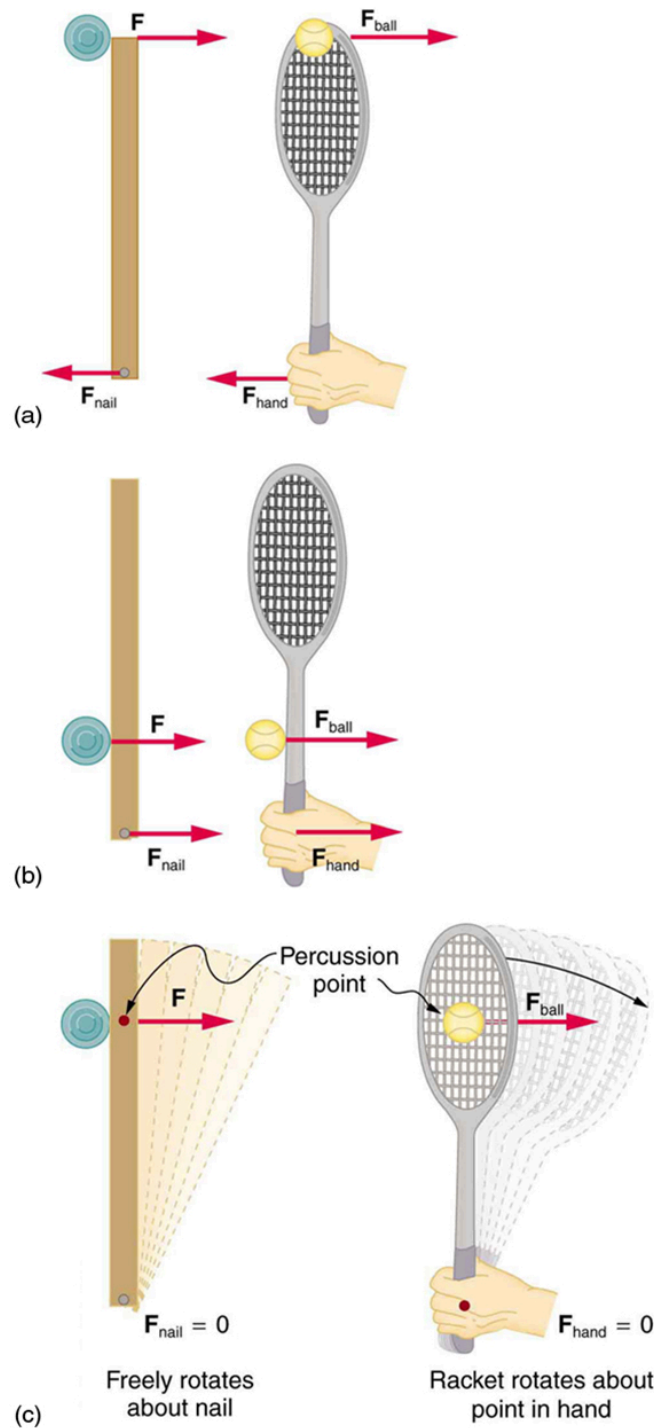


FIGURE 10.25 A disk hitting a stick is compared to a tennis ball being hit by a racquet. (a) When the ball strikes the racquet near the end, a backward force is exerted on the hand. (b) When the racquet is struck much farther down, a forward force is exerted on the hand. (c) When the racquet is struck at the percussion point, no force is delivered to the hand.

✓ CHECK YOUR UNDERSTANDING

Is rotational kinetic energy a vector? Justify your answer.

Solution

No, energy is always scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Describe the right-hand rule to find the direction of angular velocity, momentum, and torque.
- Explain the gyroscopic effect.
- Study how Earth acts like a gigantic gyroscope.

Angular momentum is a vector and, therefore, *has direction as well as magnitude*. Torque affects both the direction and the magnitude of angular momentum. What is the direction of the angular momentum of a rotating object like the disk in [Figure 10.26](#)? The figure shows the **right-hand rule** used to find the direction of both angular momentum and angular velocity. Both \mathbf{L} and $\boldsymbol{\omega}$ are vectors—each has direction and magnitude. Both can be represented by arrows. The right-hand rule defines both to be perpendicular to the plane of rotation in the direction shown. Because angular momentum is related to angular velocity by $\mathbf{L} = I\boldsymbol{\omega}$, the direction of \mathbf{L} is the same as the direction of $\boldsymbol{\omega}$. Notice in the figure that both point along the axis of rotation.

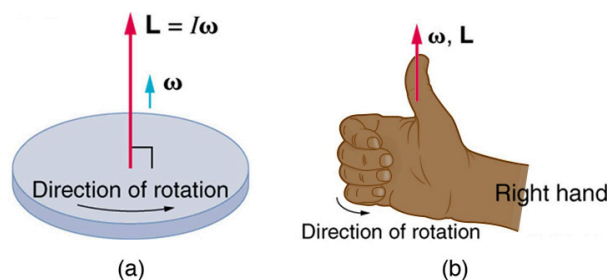


FIGURE 10.26 Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity $\boldsymbol{\omega}$ and angular momentum \mathbf{L} are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation as shown.

Now, recall that torque changes angular momentum as expressed by

$$\text{net } \boldsymbol{\tau} = \frac{\Delta \mathbf{L}}{\Delta t}. \quad 10.138$$

This equation means that the direction of $\Delta \mathbf{L}$ is the same as the direction of the torque $\boldsymbol{\tau}$ that creates it. This result is illustrated in [Figure 10.27](#), which shows the direction of torque and the angular momentum it creates.

Let us now consider a bicycle wheel with a couple of handles attached to it, as shown in [Figure 10.28](#). (This device is popular in demonstrations among physicists, because it does unexpected things.) With the wheel rotating as shown, its angular momentum is to the woman's left. Suppose the person holding the wheel tries to rotate it as in the figure. Her natural expectation is that the wheel will rotate in the direction she pushes it—but what happens is quite different. The forces exerted create a torque that is horizontal toward the person, as shown in [Figure 10.28](#)(a). This torque creates a change in angular momentum $\Delta \mathbf{L}$ in the same direction, perpendicular to the original angular momentum \mathbf{L} , thus changing the direction of \mathbf{L} but not the magnitude of \mathbf{L} . [Figure 10.28](#) shows how $\Delta \mathbf{L}$ and \mathbf{L} add, giving a new angular momentum with direction that is inclined more toward the person than before. The axis of the wheel has thus moved *perpendicular to the forces exerted on it*, instead of in the expected direction.

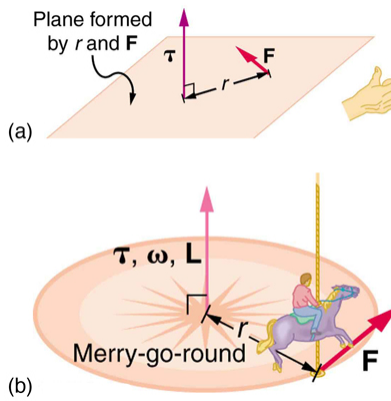


FIGURE 10.27 In figure (a), the torque is perpendicular to the plane formed by r and \mathbf{F} and is the direction your right thumb would point to if you curled your fingers in the direction of \mathbf{F} . Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.

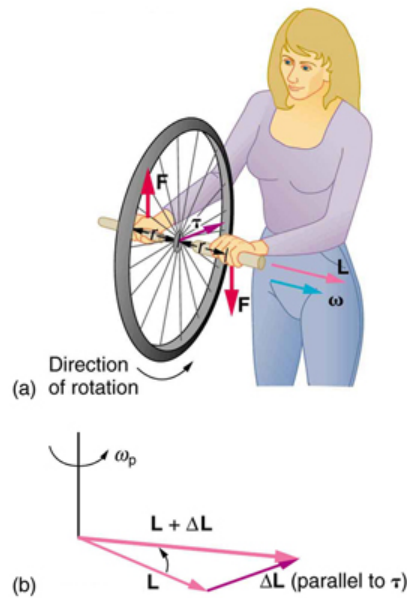


FIGURE 10.28 In figure (a), a person holding the spinning bike wheel lifts it with her right hand and pushes down with her left hand in an attempt to rotate the wheel. This action creates a torque directly toward her. This torque causes a change in angular momentum $\Delta\mathbf{L}$ in exactly the same direction. Figure (b) shows a vector diagram depicting how $\Delta\mathbf{L}$ and \mathbf{L} add, producing a new angular momentum pointing more toward the person. The wheel moves toward the person, perpendicular to the forces she exerts on it.

This same logic explains the behavior of gyroscopes. [Figure 10.29](#) shows the two forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the torque is changed, but not its magnitude. The gyroscope *precesses* around a vertical axis, since the torque is always horizontal and perpendicular to \mathbf{L} . If the gyroscope is *not* spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$), and it rotates around a horizontal axis, falling over just as we would expect.

Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star. But Earth is slowly precessing (once in about 26,000 years) due to the torque of the Sun and the Moon on its nonspherical shape.

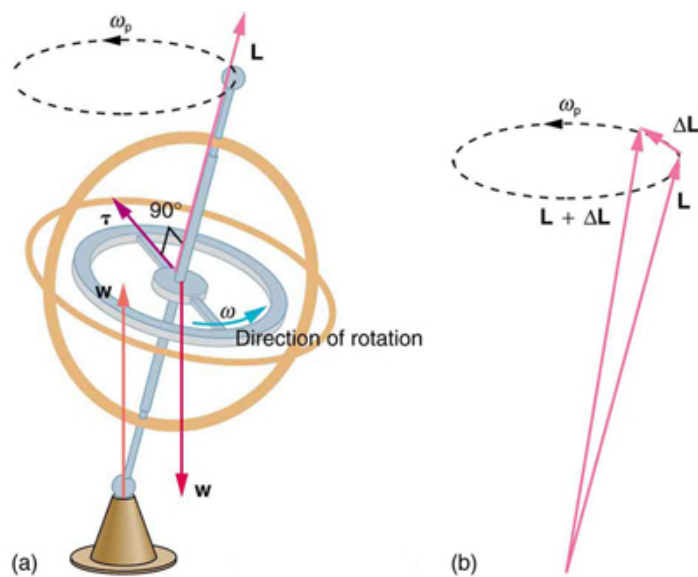


FIGURE 10.29 As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum $\Delta\mathbf{L}$ that is also horizontal. In figure (b), $\Delta\mathbf{L}$ and \mathbf{L} add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

✓ CHECK YOUR UNDERSTANDING

Rotational kinetic energy is associated with angular momentum? Does that mean that rotational kinetic energy is a vector?

Solution

No, energy is always a scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

Glossary

- angular acceleration** the rate of change of angular velocity with time
- angular momentum** the product of moment of inertia and angular velocity
- change in angular velocity** the difference between final and initial values of angular velocity
- kinematics of rotational motion** describes the relationships among rotation angle, angular velocity, angular acceleration, and time
- law of conservation of angular momentum** angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system
- moment of inertia** mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = mr^2$ and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia

- right-hand rule** direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation
- rotational inertia** resistance to change of rotation. The more rotational inertia an object has, the harder it is to rotate
- rotational kinetic energy** the kinetic energy due to the rotation of an object. This is part of its total kinetic energy
- tangential acceleration** the acceleration in a direction tangent to the circle at the point of interest in circular motion
- torque** the turning effectiveness of a force
- work-energy theorem** if one or more external forces act upon a rigid object, causing its kinetic energy to change from KE_1 to KE_2 , then the work W done by the net force is equal to the change in kinetic energy

Section Summary

10.1 Angular Acceleration

- Uniform circular motion is the motion with a constant angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\alpha = \frac{\Delta\omega}{\Delta t}$.
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $a_t = \frac{\Delta v}{\Delta t}$.
- For circular motion, note that $v = r\omega$, so that $a_t = \frac{\Delta(r\omega)}{\Delta t}$.
- The radius r is constant for circular motion, and so $\Delta(r\omega) = r\Delta\omega$. Thus, $a_t = r\frac{\Delta\omega}{\Delta t}$.
- By definition, $\Delta\omega/\Delta t = \alpha$. Thus, $a_t = r\alpha$
or
 $\alpha = \frac{a_t}{r}$.

10.2 Kinematics of Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time.
- Starting with the four kinematic equations we developed in the [One-Dimensional Kinematics](#), we

can derive the four rotational kinematic equations (presented together with their translational counterparts) seen in [Table 10.2](#).

- In these equations, the subscript 0 denotes initial values (x_0 and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity \bar{v} are defined as follows:

$$\bar{\omega} = \frac{\omega_0 + \omega}{2} \text{ and } \bar{v} = \frac{v_0 + v}{2}.$$

10.3 Dynamics of Rotational Motion: Rotational Inertia

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r , an acceleration $a = F/m$ is obtained in the direction of F . We can rearrange this equation such that $F = ma$, and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

$$F = mra$$

- Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r , torque is simply $\tau = rF$. If we multiply both sides of the equation above by r , we get torque on the left-

hand side. That is,

$$rF = mr^2\alpha$$

or

$$\tau = mr^2\alpha.$$

- The moment of inertia I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,

$$I = \sum mr^2.$$

- The general relationship among torque, moment of inertia, and angular acceleration is

$$\tau = I\alpha$$

or

$$\alpha = \frac{\text{net } \tau}{I}.$$

10.4 Rotational Kinetic Energy: Work and Energy Revisited

- The rotational kinetic energy KE_{rot} for an object with a moment of inertia I and an angular velocity ω is given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.
- The equation for the **work-energy theorem** for rotational motion is,

$$\text{net } W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2.$$

10.5 Angular Momentum and Its Conservation

- Every rotational phenomenon has a direct translational analog, likewise angular momentum L can be defined as $L = I\omega$.
- This equation is an analog to the definition of

Conceptual Questions

10.1 Angular Acceleration

1. Analogies exist between rotational and translational physical quantities. Identify the rotational term analogous to each of the following: acceleration, force, mass, work, translational kinetic energy, linear momentum, impulse.

linear momentum as $p = mv$. The relationship between torque and angular momentum is $\text{net } \tau = \frac{\Delta L}{\Delta t}$.

- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

10.6 Collisions of Extended Bodies in Two Dimensions

- Angular momentum L is analogous to linear momentum and is given by $L = I\omega$.
- Angular momentum is changed by torque, following the relationship $\text{net } \tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum is conserved if the net torque is zero $L = \text{constant}$ ($\text{net } \tau = 0$) or $L = L'$ ($\text{net } \tau = 0$). This equation is known as the law of conservation of angular momentum, which may be conserved in collisions.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

- Torque is perpendicular to the plane formed by r and \mathbf{F} and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of \mathbf{F} . The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to \mathbf{L} . If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$), and it rotates about a horizontal axis, falling over just as we would expect.
- Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star.

2. Explain why centripetal acceleration changes the direction of velocity in circular motion but not its magnitude.
3. In circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction. Explain your answer.

4. Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience nonzero tangential acceleration, centripetal acceleration, or both when: (a) The plate starts to spin? (b) The plate rotates at constant angular velocity? (c) The plate slows to a halt?

10.3 Dynamics of Rotational Motion: Rotational Inertia

5. The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $ML^2/3$. Why is this moment of inertia greater than it would be if you spun a point mass M at the location of the center of mass of the rod (at $L/2$)? (That would be $ML^2/4$.)
6. Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius? Why is the moment of inertia of a spherical shell that has a mass M and a radius R greater than that of a solid sphere that has the same mass and radius?
7. Give an example in which a small force exerts a large torque. Give another example in which a large force exerts a small torque.
8. While reducing the mass of a racing bike, the greatest benefit is realized from reducing the mass of the tires and wheel rims. Why does this allow a racer to achieve greater accelerations than would an identical reduction in the mass of the bicycle's frame?



FIGURE 10.30 The image shows a side view of a racing bicycle. Can you see evidence in the design of the wheels on this racing bicycle that their moment of inertia has been purposely reduced? (credit: Jesús Rodríguez)

9. A ball slides up a frictionless ramp. It is then rolled without slipping and with the same initial velocity up another frictionless ramp (with the same slope angle). In which case does it reach a greater height, and why?

10.4 Rotational Kinetic Energy: Work and Energy Revisited

10. Describe the energy transformations involved when a yo-yo is thrown downward and then climbs back up its string to be caught in the user's hand.
11. What energy transformations are involved when a dragster engine is revved, its clutch let out rapidly, its tires spun, and it starts to accelerate forward? Describe the source and transformation of energy at each step.
12. The Earth has more rotational kinetic energy now than did the cloud of gas and dust from which it formed. Where did this energy come from?

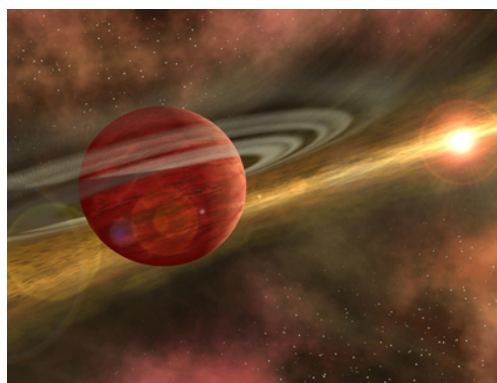


FIGURE 10.31 An immense cloud of rotating gas and dust contracted under the influence of gravity to form the Earth and in the process rotational kinetic energy increased. (credit: NASA)

10.5 Angular Momentum and Its Conservation

13. When you start the engine of your car with the transmission in neutral, you notice that the car rocks in the opposite sense of the engine's rotation. Explain in terms of conservation of angular momentum. Is the angular momentum of the car conserved for long (for more than a few seconds)?

14. Suppose a child walks from the outer edge of a rotating merry-go-round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.

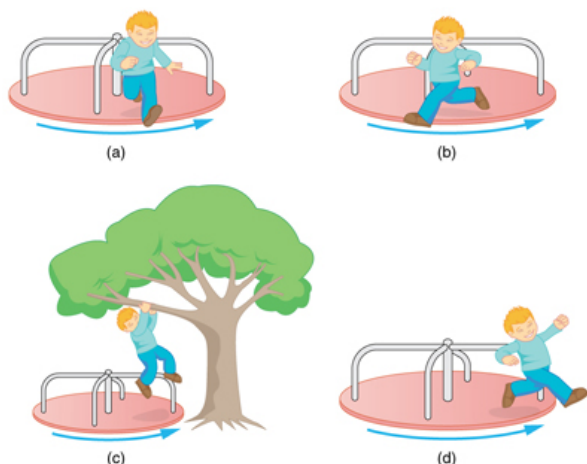


FIGURE 10.32 A child may jump off a merry-go-round in a variety of directions.

15. Suppose a child gets off a rotating merry-go-round. Does the angular velocity of the merry-go-round increase, decrease, or remain the same if: (a) He jumps off radially? (b) He jumps backward to land motionless? (c) He jumps straight up and hangs onto an overhead tree branch? (d) He jumps off forward, tangential to the edge? Explain your answers. (Refer to [Figure 10.32](#)).
16. Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton's third law why the helicopter body rotates in the opposite direction to the blades.
17. Whenever a helicopter has two sets of lifting blades, they rotate in opposite directions (and there will be no tail propeller). Explain why it is best to have the blades rotate in opposite directions.
18. Describe how work is done by a skater pulling in her arms during a spin. In particular, identify the force she exerts on each arm to pull it in and the distance each moves, noting that a component of the force is in the direction moved. Why is angular momentum not increased by this action?
19. When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.
20. Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?
21. Jet turbines spin rapidly. They are designed to fly apart if something makes them seize suddenly, rather than transfer angular momentum to the plane's wing, possibly tearing it off. Explain how flying apart conserves angular momentum without transferring it to the wing.
22. An astronaut tightens a bolt on a satellite in orbit. He rotates in a direction opposite to that of the bolt, and the satellite rotates in the same direction as the bolt. Explain why. If a handhold is available on the satellite, can this counter-rotation be prevented? Explain your answer.
23. Competitive divers pull their limbs in and curl up their bodies when they do flips. Just before entering the water, they fully extend their limbs to enter straight down. Explain the effect of both actions on their angular velocities. Also explain the effect on their angular momenta.

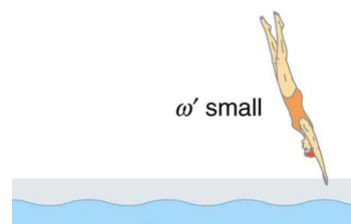


FIGURE 10.33 The diver spins rapidly when curled up and slows when she extends her limbs before entering the water.

24. Draw a free body diagram to show how a diver gains angular momentum when leaving the diving board.

25. In terms of angular momentum, what is the advantage of giving a football or a rifle bullet a spin when throwing or releasing it?

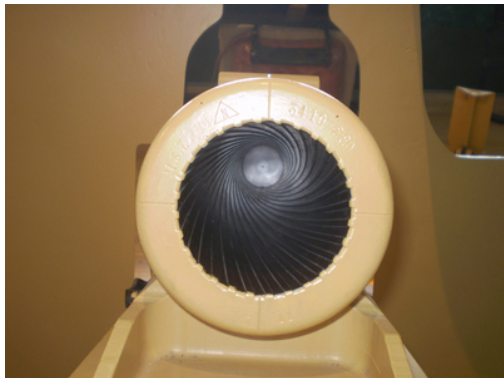


FIGURE 10.34 The image shows a view down the barrel of a cannon, emphasizing its rifling. Rifling in the barrel of a canon causes the projectile to spin just as is the case for rifles (hence the name for the grooves in the barrel). (credit: Elsie esq., Flickr)

10.6 Collisions of Extended Bodies in Two Dimensions

26. Describe two different collisions—one in which angular momentum is conserved, and the other in which it is not. Which condition determines whether or not angular momentum is conserved in a collision?

Problems & Exercises

10.1 Angular Acceleration

- At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?
- Integrated Concepts**
An ultracentrifuge accelerates from rest to 100,000 rpm in 2.00 min. (a) What is its angular acceleration in rad/s^2 ? (b) What is the tangential acceleration of a point 9.50 cm from the axis of rotation? (c) What is the radial acceleration in m/s^2 and multiples of g of this point at full rpm?
- Integrated Concepts**
You have a grindstone (a disk) that is 90.0 kg, has a 0.340-m radius, and is turning at 90.0 rpm, and you press a steel axe against it with a radial force of 20.0 N. (a) Assuming the kinetic coefficient of friction between steel and stone is 0.20, calculate the angular acceleration of the grindstone. (b) How many turns will the stone make before coming to rest?

- Suppose an ice hockey puck strikes a hockey stick that lies flat on the ice and is free to move in any direction. Which quantities are likely to be conserved: angular momentum, linear momentum, or kinetic energy (assuming the puck and stick are very resilient)?
- While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

- While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.
- Gyroscopes used in guidance systems to indicate directions in space must have an angular momentum that does not change in direction. Yet they are often subjected to large forces and accelerations. How can the direction of their angular momentum be constant when they are accelerated?

4. Unreasonable Results

You are told that a basketball player spins the ball with an angular acceleration of 100 rad/s^2 . (a) What is the ball's final angular velocity if the ball starts from rest and the acceleration lasts 2.00 s? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

10.2 Kinematics of Rotational Motion

- With the aid of a string, a gyroscope is accelerated from rest to 32 rad/s in 0.40 s. (a) What is its angular acceleration in rad/s^2 ? (b) How many revolutions does it go through in the process?
- Suppose a piece of dust finds itself on a CD. If the spin rate of the CD is 500 rpm, and the piece of dust is 4.3 cm from the center, what is the total distance traveled by the dust in 3 minutes? (Ignore accelerations due to getting the CD rotating.)

7. A gyroscope slows from an initial rate of 32.0 rad/s at a rate of 0.700 rad/s^2 .
- How long does it take to come to rest?
 - How many revolutions does it make before stopping?
8. During a very quick stop, a car decelerates at 7.00 m/s^2 .
- What is the angular acceleration of its 0.280-m -radius tires, assuming they do not slip on the pavement?
 - How many revolutions do the tires make before coming to rest, given their initial angular velocity is 95.0 rad/s ?
 - How long does the car take to stop completely?
 - What distance does the car travel in this time?
 - What was the car's initial velocity?
 - Do the values obtained seem reasonable, considering that this stop happens very quickly?



FIGURE 10.35 Yo-yos are amusing toys that display significant physics and are engineered to enhance performance based on physical laws. (credit: Beyond Neon, Flickr)

9. Everyday application: Suppose a yo-yo has a center shaft that has a 0.250 cm radius and that its string is being pulled.
- If the string is stationary and the yo-yo accelerates away from it at a rate of 1.50 m/s^2 , what is the angular acceleration of the yo-yo?
 - What is the angular velocity after 0.750 s if it starts from rest?
 - The outside radius of the yo-yo is 3.50 cm . What is the tangential acceleration of a point on its edge?

10.3 Dynamics of Rotational Motion: Rotational Inertia

10. This problem considers additional aspects of example [Calculating the Effect of Mass Distribution on a Merry-Go-Round](#). (a) How long does it take the father to give the merry-go-round an angular velocity of 1.50 rad/s ? (b) How many revolutions must he go through to generate this velocity? (c) If he exerts a slowing force of 300 N at a radius of 1.35 m , how long would it take him to stop them?
11. Calculate the moment of inertia of a skater given the following information. (a) The 60.0-kg skater is approximated as a cylinder that has a 0.110-m radius. (b) The skater with arms extended is approximately a cylinder that is 52.5 kg , has a 0.110-m radius, and has two 0.900-m -long arms which are 3.75 kg each and extend straight out from the cylinder like rods rotated about their ends.
12. The triceps muscle in the back of the upper arm extends the forearm. This muscle in a professional boxer exerts a force of $2.00 \times 10^3 \text{ N}$ with an effective perpendicular lever arm of 3.00 cm , producing an angular acceleration of the forearm of 120 rad/s^2 . What is the moment of inertia of the boxer's forearm?
13. A soccer player extends her lower leg in a kicking motion by exerting a force with the muscle above the knee in the front of her leg. She produces an angular acceleration of 30.00 rad/s^2 and her lower leg has a moment of inertia of $0.750 \text{ kg} \cdot \text{m}^2$. What is the force exerted by the muscle if its effective perpendicular lever arm is 1.90 cm ?
14. Suppose you exert a force of 180 N tangential to a 0.280-m -radius 75.0-kg grindstone (a solid disk). (a) What torque is exerted? (b) What is the angular acceleration assuming negligible opposing friction? (c) What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

15. Consider the 12.0 kg motorcycle wheel shown in [Figure 10.36](#). Assume it to be approximately an annular ring with an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. (a) If the drive chain exerts a force of 2200 N at a radius of 5.00 cm, what is the angular acceleration of the wheel? (b) What is the tangential acceleration of a point on the outer edge of the tire? (c) How long, starting from rest, does it take to reach an angular velocity of 80.0 rad/s?

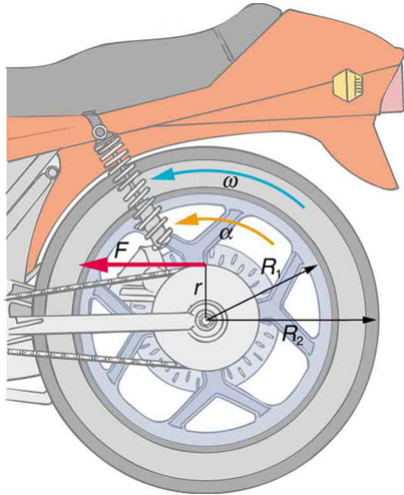


FIGURE 10.36 A motorcycle wheel has a moment of inertia approximately that of an annular ring.

16. Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of 4.00×10^7 N (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Explicitly show how you follow the steps found in [Problem-Solving Strategy for Rotational Dynamics](#).
17. An automobile engine can produce $200 \text{ N} \cdot \text{m}$ of torque. Calculate the angular acceleration produced if 95.0% of this torque is applied to the drive shaft, axle, and rear wheels of a car, given the following information. The car is suspended so that the wheels can turn freely. Each wheel acts like a 15.0 kg disk that has a 0.180 m radius. The walls of each tire act like a 2.00-kg annular ring that has inside radius of 0.180 m and outside radius of 0.320 m. The tread of each tire acts like a 10.0-kg hoop of radius 0.330 m. The 14.0-kg axle acts like a rod that has a 2.00-cm radius. The 30.0-kg drive shaft acts like a rod that has a 3.20-cm radius.
18. Starting with the formula for the moment of inertia of a rod rotated around an axis through one end perpendicular to its length ($I = M\ell^2/3$), prove that the moment of inertia of a rod rotated about an axis through its center perpendicular to its length is $I = M\ell^2/12$. You will find the graphics in [Figure 10.11](#) useful in visualizing these rotations.
19. **Unreasonable Results**
A gymnast doing a forward flip lands on the mat and exerts a $500\text{-N} \cdot \text{m}$ torque to slow their angular velocity to zero. Their initial angular velocity is 10.0 rad/s, and their moment of inertia is $0.050 \text{ kg} \cdot \text{m}^2$. (a) What time is required for the gymnast to exactly stop their spin? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?
20. **Unreasonable Results**
An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel, which can accelerate the car from rest to a speed of 30.0 m/s. The flywheel is a disk with a 0.150-m radius. (a) Calculate the angular velocity the flywheel must have if 95.0% of its rotational energy is used to get the car up to speed. (b) What is unreasonable about the result? (c) Which premise is unreasonable or which premises are inconsistent?

10.4 Rotational Kinetic Energy: Work and Energy Revisited

21. This problem considers energy and work aspects of [Example 10.7](#)—use data from that example as needed. (a) Calculate the rotational kinetic energy in the merry-go-round plus child when they have an angular velocity of 20.0 rpm. (b) Using energy considerations, find the number of revolutions the father will have to push to achieve this angular velocity starting from rest. (c) Again, using energy considerations, calculate the force the father must exert to stop the merry-go-round in two revolutions
22. What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?
23. (a) Calculate the rotational kinetic energy of Earth on its axis. (b) What is the rotational kinetic energy of Earth in its orbit around the Sun?
24. Calculate the rotational kinetic energy in the motorcycle wheel ([Figure 10.36](#)) if its angular velocity is 120 rad/s. Assume $M = 12.0 \text{ kg}$, $R_1 = 0.280 \text{ m}$, and $R_2 = 0.330 \text{ m}$.

- 25.** A baseball pitcher throws the ball in a motion where there is rotation of the forearm about the elbow joint as well as other movements. If the linear velocity of the ball relative to the elbow joint is 20.0 m/s at a distance of 0.480 m from the joint and the moment of inertia of the forearm is $0.500 \text{ kg} \cdot \text{m}^2$, what is the rotational kinetic energy of the forearm?
- 26.** While punting a football, a kicker rotates her leg about the hip joint. The moment of inertia of the leg is $3.75 \text{ kg} \cdot \text{m}^2$ and its rotational kinetic energy is 175 J. (a) What is the angular velocity of the leg? (b) What is the velocity of tip of the punter's shoe if it is 1.05 m from the hip joint? (c) Explain how the football can be given a velocity greater than the tip of the shoe (necessary for a decent kick distance).
- 27.** A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg. (a) Calculate the angular velocity the flywheel must have to contain enough energy to take the bus from rest to a speed of 20.0 m/s, assuming 90.0% of the rotational kinetic energy can be transformed into translational energy. (b) How high a hill can the bus climb with this stored energy and still have a speed of 3.00 m/s at the top of the hill? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Rotational Energy](#).
- 28.** A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches. (b) Repeat the calculation for the same ball if it slides up the hill without rolling.
- 29.** While exercising in a fitness center, a man lies face down on a bench and lifts a weight with one lower leg by contacting the muscles in the back of the upper leg. (a) Find the angular acceleration produced given the mass lifted is 10.0 kg at a distance of 28.0 cm from the knee joint, the moment of inertia of the lower leg is $0.900 \text{ kg} \cdot \text{m}^2$, the muscle force is 1500 N, and its effective perpendicular lever arm is 3.00 cm. (b) How much work is done if the leg rotates through an angle of 20.0° with a constant force exerted by the muscle?
- 30.** To develop muscle tone, a woman lifts a 2.00-kg weight held in her hand. She uses her biceps muscle to flex the lower arm through an angle of 60.0° . (a) What is the angular acceleration if the weight is 24.0 cm from the elbow joint, her forearm has a moment of inertia of $0.250 \text{ kg} \cdot \text{m}^2$, and the net force she exerts is 750 N at an effective perpendicular lever arm of 2.00 cm? (b) How much work does she do?
- 31.** Consider two cylinders that start down identical inclines from rest except that one is frictionless. Thus one cylinder rolls without slipping, while the other slides frictionlessly without rolling. They both travel a short distance at the bottom and then start up another incline. (a) Show that they both reach the same height on the other incline, and that this height is equal to their original height. (b) Find the ratio of the time the rolling cylinder takes to reach the height on the second incline to the time the sliding cylinder takes to reach the height on the second incline. (c) Explain why the time for the rolling motion is greater than that for the sliding motion.
- 32.** What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest, and has a final velocity of 6.00 m/s? Express the moment of inertia as a multiple of MR^2 , where M is the mass of the object and R is its radius.
- 33.** Suppose a 200-kg motorcycle has two wheels like, [the one described in Problem 10.15](#) and is heading toward a hill at a speed of 30.0 m/s. (a) How high can it coast up the hill, if you neglect friction? (b) How much energy is lost to friction if the motorcycle only gains an altitude of 35.0 m before coming to rest?
- 34.** In softball, the pitcher throws with the arm fully extended (straight at the elbow). In a fast pitch the ball leaves the hand with a speed of 139 km/h. (a) Find the rotational kinetic energy of the pitcher's arm and ball together given that the arm's moment of inertia is $0.720 \text{ kg} \cdot \text{m}^2$ and the ball leaves the hand at a distance of 0.600 m from the pivot at the shoulder. (b) What force did the muscles exert to cause the arm to rotate if their effective perpendicular lever arm is 4.00 cm and the ball is 0.156 kg?

- 35. Construct Your Own Problem**
Consider the work done by a spinning skater pulling his arms in to increase his rate of spin. Construct a problem in which you calculate the work done with a “force multiplied by distance” calculation and compare it to the skater’s increase in kinetic energy.

10.5 Angular Momentum and Its Conservation

- 36.** (a) Calculate the angular momentum of the Earth in its orbit around the Sun.
(b) Compare this angular momentum with the angular momentum of Earth on its axis.
- 37.** (a) What is the angular momentum of the Moon in its orbit around Earth?
(b) How does this angular momentum compare with the angular momentum of the Moon on its axis? Remember that the Moon keeps one side toward Earth at all times.
(c) Discuss whether the values found in parts (a) and (b) seem consistent with the fact that tidal effects with Earth have caused the Moon to rotate with one side always facing Earth.
- 38.** Suppose you start an antique car by exerting a force of 300 N on its crank for 0.250 s. What angular momentum is given to the engine if the handle of the crank is 0.300 m from the pivot and the force is exerted to create maximum torque the entire time?
- 39.** A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.
- 40.** Three children are riding on the edge of a merry-go-round that is 100 kg, has a 1.60-m radius, and is spinning at 20.0 rpm. The children have masses of 22.0, 28.0, and 33.0 kg. If the child who has a mass of 28.0 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm?
- 41.** (a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is $0.400 \text{ kg} \cdot \text{m}^2$. (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?
- 42. Construct Your Own Problem**
Consider the Earth-Moon system. Construct a problem in which you calculate the total angular momentum of the system including the spins of the Earth and the Moon on their axes and the orbital angular momentum of the Earth-Moon system in its nearly monthly rotation. Calculate what happens to the Moon’s orbital radius if the Earth’s rotation decreases due to tidal drag. Among the things to be considered are the amount by which the Earth’s rotation slows and the fact that the Moon will continue to have one side always facing the Earth.

10.6 Collisions of Extended Bodies in Two Dimensions

- 43.** Repeat [Example 10.15](#) in which the disk strikes and adheres to the stick 0.100 m from the nail.
- 44.** Repeat [Example 10.15](#) in which the disk originally spins clockwise at 1000 rpm and has a radius of 1.50 cm.
- 45.** Twin skaters approach one another as shown in [Figure 10.37](#) and lock hands. (a) Calculate their final angular velocity, given each had an initial speed of 2.50 m/s relative to the ice. Each has a mass of 70.0 kg, and each has a center of mass located 0.800 m from their locked hands. You may approximate their moments of inertia to be that of point masses at this radius. (b) Compare the initial kinetic energy and final kinetic energy.

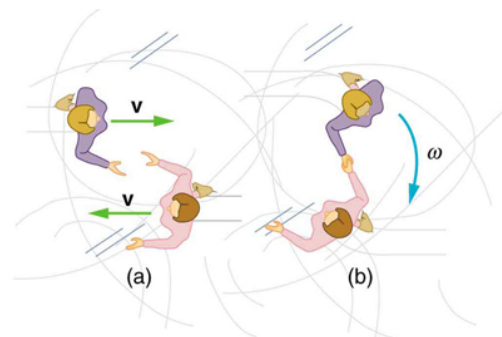


FIGURE 10.37 Twin skaters approach each other with identical speeds. Then, the skaters lock hands and spin.

46. Suppose a 0.250-kg ball is thrown at 15.0 m/s to a motionless person standing on ice who catches it with an outstretched arm as shown in [Figure 10.38](#).

- (a) Calculate the final linear velocity of the person, given his mass is 70.0 kg.
 (b) What is his angular velocity if each arm is 5.00 kg? You may treat the ball as a point mass and treat the person's arms as uniform rods (each has a length of 0.900 m) and the rest of his body as a uniform cylinder of radius 0.180 m. Neglect the effect of the ball on his center of mass so that his center of mass remains in his geometrical center.
 (c) Compare the initial and final total kinetic energies.

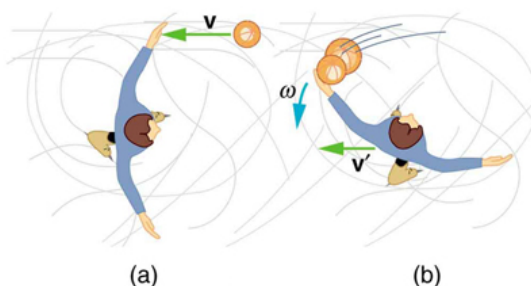


FIGURE 10.38 The figure shows the overhead view of a person standing motionless on ice about to catch a ball. Both arms are outstretched. After catching the ball, the skater recoils and rotates.

47. Repeat [Example 10.15](#) in which the stick is free to have translational motion as well as rotational motion.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

48. Integrated Concepts

The axis of Earth makes a 23.5° angle with a direction perpendicular to the plane of Earth's orbit. As shown in [Figure 10.39](#), this axis precesses, making one complete rotation in 25,780 y.

- (a) Calculate the change in angular momentum in half this time.
 (b) What is the average torque producing this change in angular momentum?
 (c) If this torque were created by a single force (it is not) acting at the most effective point on the equator, what would its magnitude be?

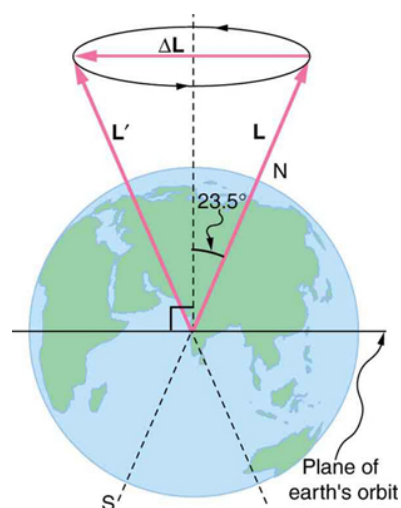


FIGURE 10.39 The Earth's axis slowly precesses, always making an angle of 23.5° with the direction perpendicular to the plane of Earth's orbit. The change in angular momentum for the two shown positions is quite large, although the magnitude L is unchanged.

CHAPTER 11

Fluid Statics



FIGURE 11.1 The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)

CHAPTER OUTLINE

11.1 What Is a Fluid?

11.2 Density

11.3 Pressure

11.4 Variation of Pressure with Depth in a Fluid

11.5 Pascal's Principle

11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

11.7 Archimedes' Principle

11.8 Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

11.9 Pressures in the Body

INTRODUCTION TO FLUID STATICS Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some of the laws that govern their behavior are the topics of this chapter. [Fluid Dynamics and Its Biological and Medical Applications](#) explores aspects of fluid flow.

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11.1 What Is a Fluid?

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

Matter most commonly exists as a solid, liquid, gas, or plasma; these states are known as the common *phases of*

matter. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held, and plasmas also have neither definite shape nor volume. (See [Figure 11.2](#).) Liquids, gases, and plasmas are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity which is discussed in detail in [Viscosity and Laminar Flow; Poiseuille's Law](#). We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.

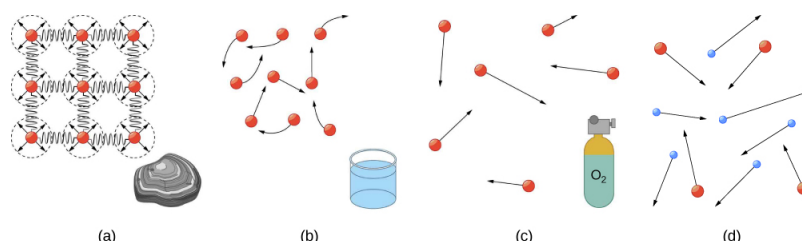


FIGURE 11.2 (a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely. (d) A plasma is composed of electrons, protons, and ions that, like gases, are spaced far apart and move about freely.

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

Connections: Submicroscopic Explanation of Solids and Liquids

Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This submicroscopic explanation is one theme of this text and is highlighted in the Things Great and Small features in [Conservation of Momentum](#). See, for example, microscopic description of collisions and momentum or microscopic description of pressure in a gas. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* and charged particles in *plasmas* are separated by distances that are large compared with the size of the particles. The forces between the particles are therefore very weak, except when they collide with one another. Gases and plasmas thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between the particles. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. Plasmas are difficult to contain because they have so much energy. When discussing how substances flow, we shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.


PHET EXPLORATIONS
States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/11-1-what-is-a-fluid\)](https://openstax.org/books/college-physics-2e/pages/11-1-what-is-a-fluid)



11.2 Density

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See [Figure 11.3](#).)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V}, \quad 11.1$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Density

Density is mass per unit volume.

$$\rho = \frac{m}{V}, \quad 11.2$$

where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg/m^3 , representative values are given in [Table 11.1](#). The metric system was originally devised so that water would have a density of 1 g/cm^3 , equivalent to 10^3 kg/m^3 . Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3 .

Substance	$\rho(\times 10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$
Solids		Liquids		Gases	
Aluminum	2.7	Water (4°C)	1.000	Air	1.29×10^{-3}

TABLE 11.1 Densities of Various Substances

Substance	$\rho(\times 10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$
Brass	8.44	Blood	1.05	Carbon dioxide	1.98×10^{-3}
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	1.25×10^{-3}
Gold	19.32	Mercury	13.6	Hydrogen	0.090×10^{-3}
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	0.18×10^{-3}
Lead	11.3	Gasoline	0.68	Methane	0.72×10^{-3}
Polystyrene	0.10	Glycerin	1.26	Nitrogen	1.25×10^{-3}
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	1.98×10^{-3}
Uranium	18.70			Oxygen	1.43×10^{-3}
Concrete	2.30–3.0			Steam (100° C)	0.60×10^{-3}
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth's crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				
Silver	10.49				

TABLE 11.1 Densities of Various Substances

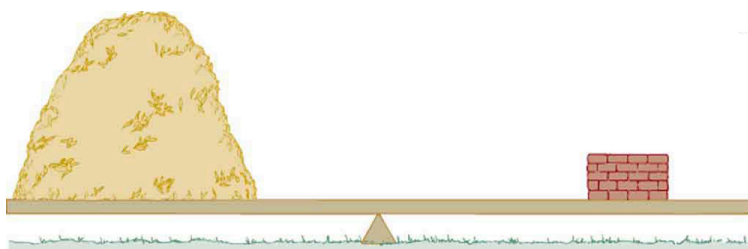


FIGURE 11.3 A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining [Table 11.1](#), the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Take-Home Experiment Sugar and Salt

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

EXAMPLE 11.1

Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of 50.0 km^2 and an average depth of 40.0 m . What mass of water is held behind the dam? (See [Figure 11.4](#) for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)

Strategy

We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in [Table 11.1](#). Then the mass m can be found from the definition of density

$$\rho = \frac{m}{V}. \quad 11.3$$

Solution

Solving equation $\rho = m/V$ for m gives $m = \rho V$.

The volume V of the reservoir is its surface area A times its average depth h :

$$\begin{aligned} V &= Ah = (50.0 \text{ km}^2)(40.0 \text{ m}) \\ &= \left[(50.0 \text{ km}^2) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right)^2 \right] (40.0 \text{ m}) = 2.00 \times 10^9 \text{ m}^3 \end{aligned} \quad 11.4$$

The density of water ρ from [Table 11.1](#) is $1.000 \times 10^3 \text{ kg/m}^3$. Substituting V and ρ into the expression for mass gives

$$\begin{aligned} m &= (1.00 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^9 \text{ m}^3) \\ &= 2.00 \times 10^{12} \text{ kg}. \end{aligned} \quad 11.5$$

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $mg = 1.96 \times 10^{13}$ N, where g is the acceleration due to the Earth's gravity (about 9.80 m/s^2). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.



FIGURE 11.4 Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

11.3 Pressure

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

$$P = \frac{F}{A} \quad 11.6$$

where F is a force applied to an area A that is perpendicular to the force.

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

$$P = \frac{F}{A}. \quad 11.7$$

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in [Figure 11.5](#). The SI unit for pressure is the *pascal*, where

$$1 \text{ Pa} = 1 \text{ N/m}^2. \quad 11.8$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

$$100 \text{ mb} = 1 \times 10^4 \text{ Pa}. \quad 11.9$$

Pounds per square inch (lb/in^2 or psi) is still sometimes used as a measure of tire pressure, and millimeters of

mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.

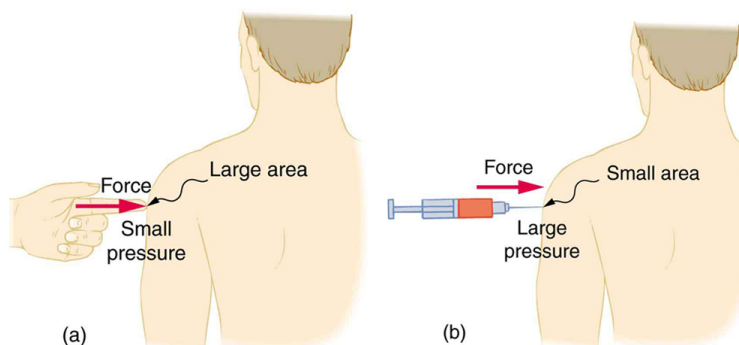


FIGURE 11.5 (a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

EXAMPLE 11.2

Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads 6.90×10^6 Pa. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P = \frac{F}{A}$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

$$F = PA. \quad 11.10$$

Here, the pressure P is given, as is the area of the end of the cylinder A , given by $A = \pi r^2$. Thus,

$$\begin{aligned} F &= (6.90 \times 10^6 \text{ N/m}^2)(3.14)(0.0750 \text{ m})^2 \\ &= 1.22 \times 10^5 \text{ N}. \end{aligned} \quad 11.11$$

Discussion

Wow! No wonder the tank must be strong. Since we found $F = PA$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in [Figure 11.6](#), for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See [Figure 11.7](#).)

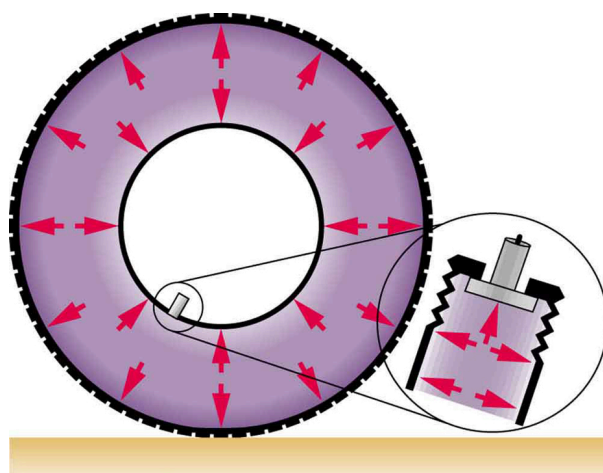


FIGURE 11.6 Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.

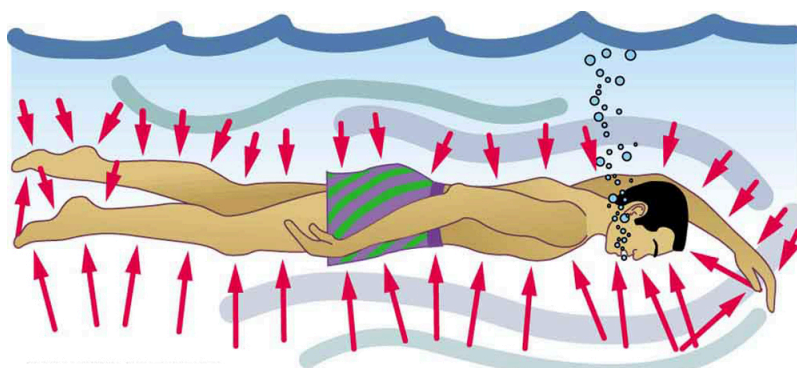


FIGURE 11.7 Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.



PHET EXPLORATIONS

Gas Properties

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.

[Click to view content \(https://openstax.org/books/college-physics-2e/pages/11-3-pressure\)](https://openstax.org/books/college-physics-2e/pages/11-3-pressure)



11.4 Variation of Pressure with Depth in a Fluid

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a

meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in [Figure 11.8](#). Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

$$P = \frac{mg}{A}. \quad 11.12$$

We can find the mass of the fluid from its volume and density:

$$m = \rho V. \quad 11.13$$

The volume of the fluid V is related to the dimensions of the container. It is

$$V = Ah, \quad 11.14$$

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

$$m = \rho Ah. \quad 11.15$$

If we enter this into the expression for pressure, we obtain

$$P = \frac{(\rho Ah)g}{A}. \quad 11.16$$

The area cancels, and rearranging the variables yields

$$P = h\rho g. \quad 11.17$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P = h\rho g$ represents the pressure due to the weight of any fluid of *average density* ρ at any depth h below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. [Example 11.4](#) illustrates this situation.

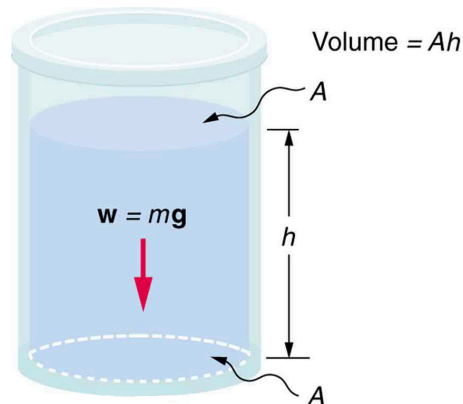


FIGURE 11.8 The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.



EXAMPLE 11.3

Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In [Example 11.1](#), we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See [Figure 11.9](#).) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be 1.96×10^{13} N).

Strategy for (a)

The average pressure \bar{P} due to the weight of the water is the pressure at the average depth \bar{h} of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

$$\bar{P} = \bar{h}\rho g. \quad 11.18$$

Entering the density of water from [Table 11.1](#) and taking \bar{h} to be the average depth of 40.0 m, we obtain

$$\begin{aligned} \bar{P} &= (40.0 \text{ m})\left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right) \\ &= 3.92 \times 10^5 \frac{\text{N}}{\text{m}^2} = 392 \text{ kPa}. \end{aligned} \quad 11.19$$

Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

$$F = \bar{P}A. \quad 11.20$$

Solution for (b)

We have already found the value for \bar{P} . The area of the dam is $A = 80.0 \text{ m} \times 500 \text{ m} = 4.00 \times 10^4 \text{ m}^2$, so that

$$\begin{aligned} F &= (3.92 \times 10^5 \text{ N/m}^2)(4.00 \times 10^4 \text{ m}^2) \\ &= 1.57 \times 10^{10} \text{ N}. \end{aligned} \quad 11.21$$

Discussion

Although this force seems large, it is small compared with the $1.96 \times 10^{13} \text{ N}$ weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.

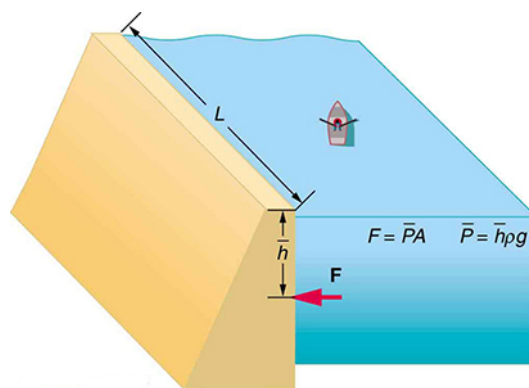


FIGURE 11.9 The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

Atmospheric pressure is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure* P_{atm} , measured to be

$$1 \text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5 \text{ N/m}^2 = 101 \text{ kPa}. \quad 11.22$$

This relationship means that, on average, at sea level, a column of air above 1.00 m^2 of the Earth's surface has a weight of $1.01 \times 10^5 \text{ N}$, equivalent to 1 atm. (See [Figure 11.10](#).)

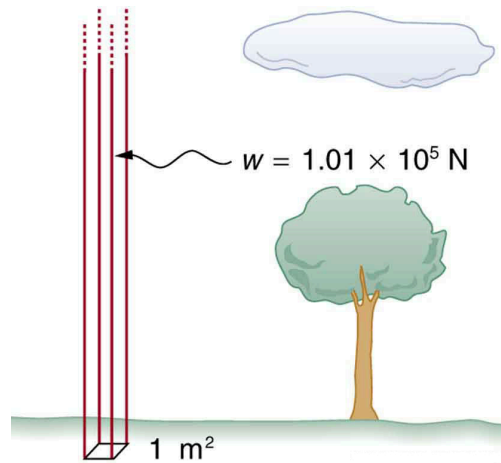


FIGURE 11.10 Atmospheric pressure at sea level averages $1.01 \times 10^5 \text{ Pa}$ (equivalent to 1 atm), since the column of air over this 1 m^2 , extending to the top of the atmosphere, weighs $1.01 \times 10^5 \text{ N}$.

EXAMPLE 11.4

Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in [Table 11.1](#).

Strategy

If we solve $P = h\rho g$ for density, we see that

$$\bar{\rho} = \frac{P}{hg}. \quad 11.23$$

We then take P to be atmospheric pressure, h is given, and g is known, and so we can use this to calculate $\bar{\rho}$.

Solution

Entering known values into the expression for $\bar{\rho}$ yields

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{ N/m}^2}{(120 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} = 8.59 \times 10^{-2} \text{ kg/m}^3. \quad 11.24$$

Discussion

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in [Table 11.1](#) as 1.29 kg/m^3 —about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

EXAMPLE 11.5

Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

Strategy

We begin by solving the equation $P = h\rho g$ for depth h :

$$h = \frac{P}{\rho g}. \quad 11.25$$

Then we take P to be 1.00 atm and ρ to be the density of the water that creates the pressure.

Solution

Entering the known values into the expression for h gives

$$h = \frac{1.01 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m}. \quad 11.26$$

Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in [Pascal's Principle](#) that fluid pressures always add in this way.

11.5 Pascal's Principle

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define pressure.
- State Pascal's principle.
- Understand applications of Pascal's principle.
- Derive relationships between forces in a hydraulic system.

Pressure is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal's principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's Principle

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that *the total pressure in a fluid is the sum of the pressures from different sources*. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in [Figure 11.11](#).

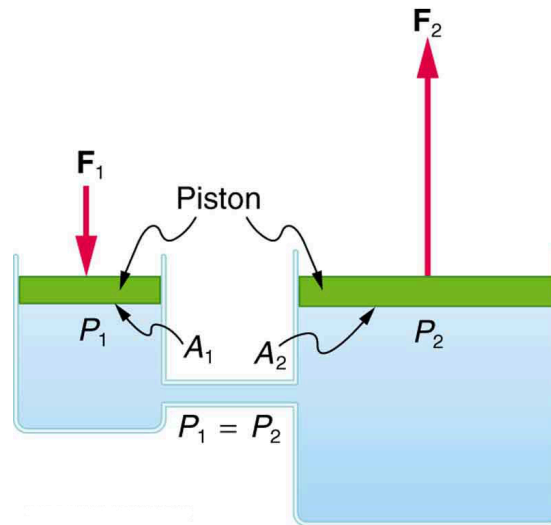


FIGURE 11.11 A typical hydraulic system with two fluid-filled cylinders, capped with pistons and connected by a tube called a hydraulic line. A downward force F_1 on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force F_2 on the right piston that is larger than F_1 because the right piston has a larger area.

Relationship Between Forces in a Hydraulic System

We can derive a relationship between the forces in the simple hydraulic system shown in [Figure 11.11](#) by applying Pascal's principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to F_1 acting on area A_1 is simply $P_1 = \frac{F_1}{A_1}$, as defined by $P = \frac{F}{A}$. According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure P_2 is felt at the other piston that is equal to P_1 . That is $P_1 = P_2$.

But since $P_2 = \frac{F_2}{A_2}$, we see that $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in [Figure 11.11](#) and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.



EXAMPLE 11.6

Calculating Force of Wheel Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in [Figure 11.12](#).

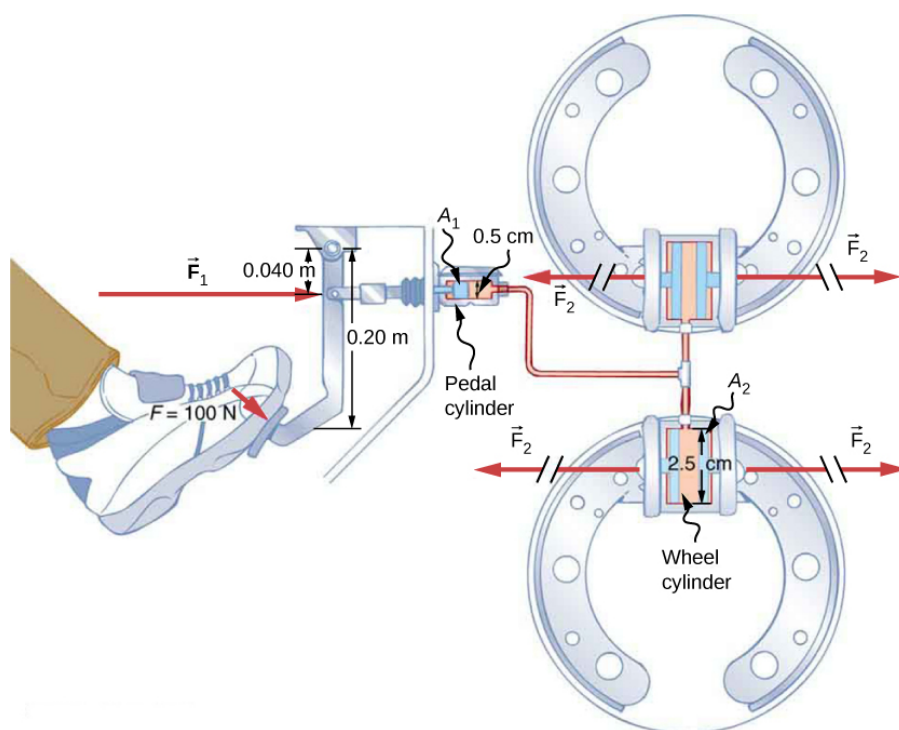


FIGURE 11.12 Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical wheel cylinders receives the same pressure and, therefore, creates the same force output F_2 . The circular cross-sectional areas of the pedal and wheel cylinders are represented by A_1 and A_2 , respectively

A force of 100 N is applied to the brake pedal, which acts on the pedal cylinder through a lever. A force of 500 N is exerted on the pedal cylinder. (The reader can verify that the force is 500 N using techniques of statics from [Applications of Statics, Including Problem-Solving Strategies](#).) Pressure created in the pedal cylinder is transmitted to four wheel cylinders. The pedal cylinder has a diameter of 0.500 cm, and each wheel cylinder has a diameter of 2.50 cm. Calculate the force F_2 created at each of the wheel cylinders.

Strategy

We are given the force F_1 that is applied to the pedal cylinder. The cross-sectional areas A_1 and A_2 can be calculated from their given diameters. Then $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ can be used to find the force F_2 . Manipulate this algebraically to get F_2 on one side and substitute known values:

Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} F_1 = \frac{(1.25 \text{ cm})^2}{(0.250 \text{ cm})^2} \times 500 \text{ N} = 1.25 \times 10^4 \text{ N}. \quad 11.27$$

Discussion

This value is the force exerted by each of the four wheel cylinders. Note that we can add as many wheel cylinders as we wish. If each has a 2.50-cm diameter, each will exert 1.25×10^4 N.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the wheel cylinder moves through a smaller distance than the pedal cylinder. Furthermore, the more wheels added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

Making Connections: Conservation of Energy

Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and other similar hydraulic systems use pumps to supply extra energy when needed.

11.6 Gauge Pressure, Absolute Pressure, and Pressure Measurement

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define gauge pressure and absolute pressure.
- Understand the working of aneroid and open-tube barometers.

If you limp into a gas station with a nearly flat tire, you will notice the tire gauge on the airline reads nearly zero when you begin to fill it. In fact, if there were a gaping hole in your tire, the gauge would read zero, even though atmospheric pressure exists in the tire. Why does the gauge read zero? There is no mystery here. Tire gauges are simply designed to read zero at atmospheric pressure and positive when pressure is greater than atmospheric.

Similarly, atmospheric pressure adds to blood pressure in every part of the circulatory system. (As noted in [Pascal's Principle](#), the total pressure in a fluid is the sum of the pressures from different sources—here, the heart and the atmosphere.) But atmospheric pressure has no net effect on blood flow since it adds to the pressure coming out of the heart and going back into it, too. What is important is how much *greater* blood pressure is than atmospheric pressure. Blood pressure measurements, like tire pressures, are thus made relative to atmospheric pressure.

In brief, it is very common for pressure gauges to ignore atmospheric pressure—that is, to read zero at atmospheric pressure. We therefore define **gauge pressure** to be the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

Gauge Pressure

Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal's principle. The total pressure, or **absolute pressure**, is thus the sum of gauge pressure and atmospheric pressure: $P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$ where P_{abs} is absolute pressure, P_{g} is gauge pressure, and P_{atm} is atmospheric pressure. For example, if your tire gauge reads 34 psi (pounds per square inch), then the absolute pressure is 34 psi plus 14.7 psi (P_{atm} in psi), or 48.7 psi (equivalent to 336 kPa).

Absolute Pressure

Absolute pressure is the sum of gauge pressure and atmospheric pressure.

For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is $P_{\text{g}} = -P_{\text{atm}}$ (this makes P_{abs} zero). There is no theoretical limit to how large a gauge pressure can be.

There are a host of devices for measuring pressure, ranging from tire gauges to blood pressure cuffs. Pascal's principle is of major importance in these devices. The undiminished transmission of pressure through a fluid allows precise remote sensing of pressures. Remote sensing is often more convenient than putting a measuring device into a system, such as a person's artery.

[Figure 11.13](#) shows one of the many types of mechanical pressure gauges in use today. In all mechanical pressure

gauges, pressure results in a force that is converted (or transduced) into some type of readout.

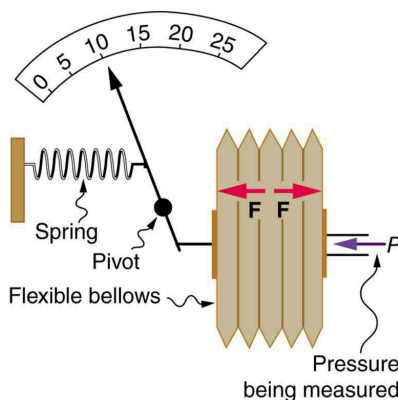


FIGURE 11.13 This aneroid gauge utilizes flexible bellows connected to a mechanical indicator to measure pressure.

An entire class of gauges uses the property that pressure due to the weight of a fluid is given by $P = h\rho g$. Consider the U-shaped tube shown in [Figure 11.14](#), for example. This simple tube is called a *manometer*. In [Figure 11.14\(a\)](#), both sides of the tube are open to the atmosphere. Atmospheric pressure therefore pushes down on each side equally so its effect cancels. If the fluid is deeper on one side, there is a greater pressure on the deeper side, and the fluid flows away from that side until the depths are equal.

Let us examine how a manometer is used to measure pressure. Suppose one side of the U-tube is connected to some source of pressure P_{abs} such as the toy balloon in [Figure 11.14\(b\)](#) or the vacuum-packed peanut jar shown in [Figure 11.14\(c\)](#). Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In [Figure 11.14\(b\)](#), P_{abs} is greater than atmospheric pressure, whereas in [Figure 11.14\(c\)](#), P_{abs} is less than atmospheric pressure. In both cases, P_{abs} differs from atmospheric pressure by an amount $h\rho g$, where ρ is the density of the fluid in the manometer. In [Figure 11.14\(b\)](#), P_{abs} can support a column of fluid of height h , and so it must exert a pressure $h\rho g$ greater than atmospheric pressure (the gauge pressure P_g is positive). In [Figure 11.14\(c\)](#), atmospheric pressure can support a column of fluid of height h , and so P_{abs} is less than atmospheric pressure by an amount $h\rho g$ (the gauge pressure P_g is negative). A manometer with one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is $P_g = h\rho g$ and is found by measuring h .

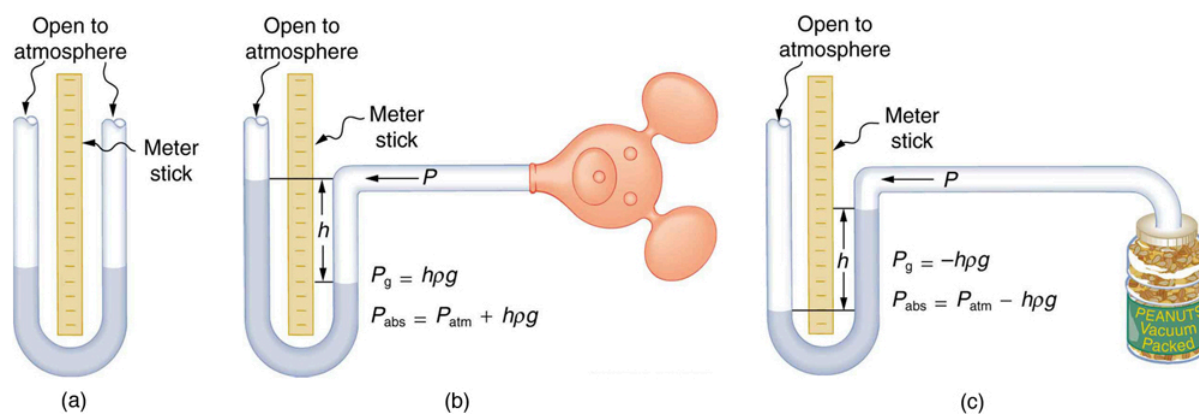


FIGURE 11.14 An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and there will be flow from the deeper side. (b) A positive gauge pressure $P_g = h\rho g$ transmitted to one side of the manometer can support a column of fluid of height h . (c) Similarly, atmospheric pressure is greater than a negative gauge pressure P_g by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

Mercury manometers are often used to measure arterial blood pressure. An inflatable cuff is placed on the upper arm as shown in [Figure 11.15](#). By squeezing the bulb, the person making the measurement exerts pressure, which is transmitted undiminished to both the main artery in the arm and the manometer. When this applied pressure exceeds blood pressure, blood flow below the cuff is cut off. The person making the measurement then slowly lowers the applied pressure and listens for blood flow to resume. Blood pressure pulsates because of the pumping action of the heart, reaching a maximum, called **systolic pressure**, and a minimum, called **diastolic pressure**, with

each heartbeat. Systolic pressure is measured by noting the value of h when blood flow first begins as cuff pressure is lowered. Diastolic pressure is measured by noting h when blood flows without interruption. The typical blood pressure of a young adult raises the mercury to a height of 120 mm at systolic and 80 mm at diastolic. This is commonly quoted as 120 over 80, or 120/80. The first pressure is representative of the maximum output of the heart; the second is due to the elasticity of the arteries in maintaining the pressure between beats. The density of the mercury fluid in the manometer is 13.6 times greater than water, so the height of the fluid will be $1/13.6$ of that in a water manometer. This reduced height can make measurements difficult, so mercury manometers are used to measure larger pressures, such as blood pressure. The density of mercury is such that $1.0 \text{ mm Hg} = 133 \text{ Pa}$.

Systolic Pressure

Systolic pressure is the maximum blood pressure.

Diastolic Pressure

Diastolic pressure is the minimum blood pressure.



FIGURE 11.15 In routine blood pressure measurements, an inflatable cuff is placed on the upper arm at the same level as the heart. Blood flow is detected just below the cuff, and corresponding pressures are transmitted to a mercury-filled manometer. (credit: U.S. Army photo by Spc. Micah E. Clare\4TH BCT)



EXAMPLE 11.7

Calculating Height of IV Bag: Blood Pressure and Intravenous Infusions

Intravenous infusions are usually made with the help of the gravitational force. Assuming that the density of the

fluid being administered is 1.00 g/ml, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if the blood pressure in the vein is 18 mm Hg above atmospheric pressure? Assume that the IV bag is collapsible.

Strategy for (a)

For the fluid to just enter the vein, its pressure at entry must exceed the blood pressure in the vein (18 mm Hg above atmospheric pressure). We therefore need to find the height of fluid that corresponds to this gauge pressure.

Solution

We first need to convert the pressure into SI units. Since 1.0 mm Hg = 133 Pa,

$$P = 18 \text{ mm Hg} \times \frac{133 \text{ Pa}}{1.0 \text{ mm Hg}} = 2400 \text{ Pa.} \quad 11.28$$

Rearranging $P_g = h\rho g$ for h gives $h = \frac{P_g}{\rho g}$. Substituting known values into this equation gives

$$\begin{aligned} h &= \frac{2400 \text{ N/m}^2}{(1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} \\ &= 0.24 \text{ m.} \end{aligned} \quad 11.29$$

Discussion

The IV bag must be placed at 0.24 m above the entry point into the arm for the fluid to just enter the arm. Generally, IV bags are placed higher than this. You may have noticed that the bags used for blood collection are placed below the donor to allow blood to flow easily from the arm to the bag, which is the opposite direction of flow than required in the example presented here.

A *barometer* is a device that measures atmospheric pressure. A mercury barometer is shown in [Figure 11.16](#). This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that $h\rho g = P_{\text{atm}}$. When atmospheric pressure varies, the mercury rises or falls, giving important clues to weather forecasters. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers are so common that units of mm Hg are often quoted for atmospheric pressure and blood pressures. [Table 11.2](#) gives conversion factors for some of the more commonly used units of pressure.

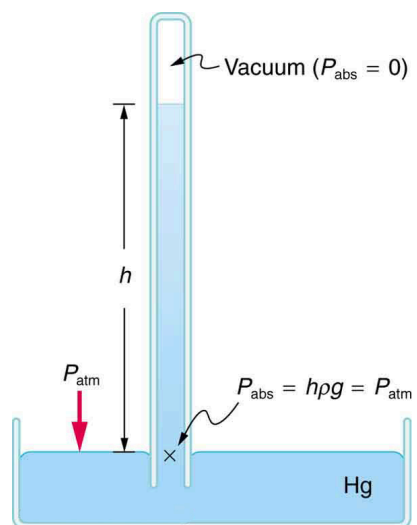


FIGURE 11.16 A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height h because the pressure above the mercury is zero.

Conversion to N/m ² (Pa)	Conversion from atm
1.0 atm = 1.013×10^5 N/m ²	1.0 atm = 1.013×10^5 N/m ²
1.0 dyne/cm ² = 0.10 N/m ²	1.0 atm = 1.013×10^6 dyne/cm ²
1.0 kg/cm ² = 9.8×10^4 N/m ²	1.0 atm = 1.013 kg/cm ²
1.0 lb/in. ² = 6.90×10^3 N/m ²	1.0 atm = 14.7 lb/in. ²
1.0 mm Hg = 133 N/m ²	1.0 atm = 760 mm Hg
1.0 cm Hg = 1.33×10^3 N/m ²	1.0 atm = 76.0 cm Hg
1.0 cm water = 98.1 N/m ²	1.0 atm = 1.03×10^3 cm water
1.0 bar = 1.000×10^5 N/m ²	1.0 atm = 1.013 bar
1.0 millibar = 1.000×10^2 N/m ²	1.0 atm = 1013 millibar

TABLE 11.2 Conversion Factors for Various Pressure Units

11.7 Archimedes' Principle

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Define buoyant force.
- State Archimedes' principle.
- Understand why objects float or sink.
- Understand the relationship between density and Archimedes' principle.

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See [Figure 11.17](#).)

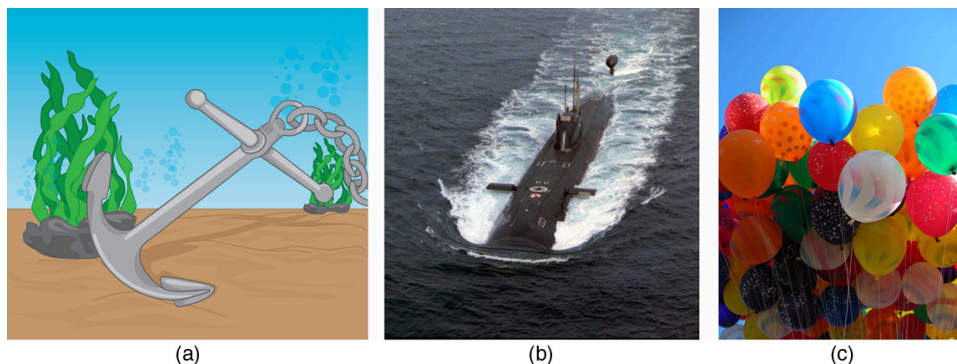


FIGURE 11.17 (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit: Crystl)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See [Figure 11.18](#).) If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than

the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

Buoyant Force

The buoyant force is the net upward force on any object in any fluid.

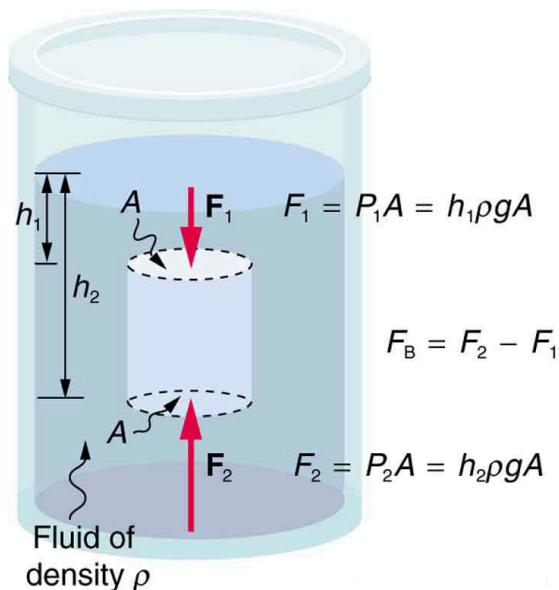


FIGURE 11.18 Pressure due to the weight of a fluid increases with depth since $P = h\rho g$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force F_B . (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in [Figure 11.19](#).

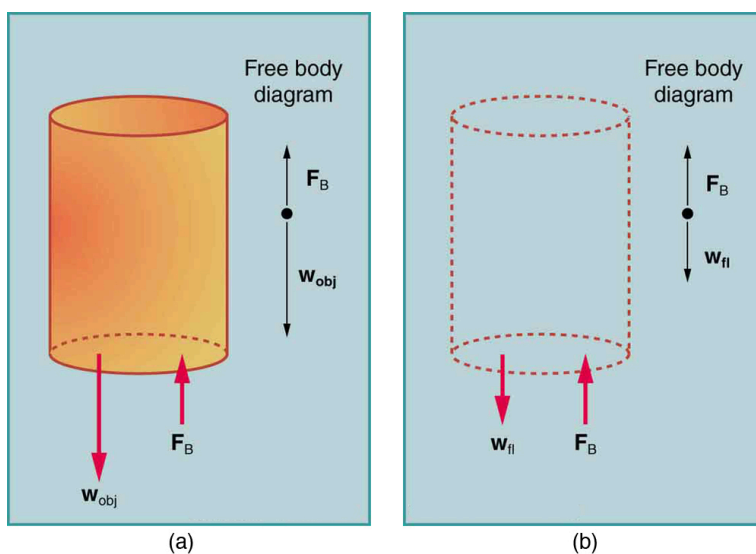


FIGURE 11.19 (a) An object submerged in a fluid experiences a buoyant force F_B . If F_B is greater than the weight of the object, the object will rise. If F_B is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight w_{fl} . Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is, $F_B = w_{fl}$, a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight w_{fl} . This weight is supported by the surrounding fluid, and so the buoyant force must equal w_{fl} , the weight of the fluid displaced by the object. It is a tribute to the genius of the

Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl}, \quad 11.30$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl}, \quad 11.31$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm ... High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.



EXAMPLE 11.8

Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons (1.00×10^7 kg) of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace 1.00×10^5 m³ of water?

Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in [Table 11.1](#). We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

Solution for (a)

First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{st} = \frac{m_{st}}{\rho_{st}} = \frac{1.00 \times 10^7 \text{ kg}}{7.8 \times 10^3 \text{ kg/m}^3} = 1.28 \times 10^3 \text{ m}^3. \quad 11.32$$

Because the steel is completely submerged, this is also the volume of water displaced, V_w . We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$\begin{aligned} m_w &= \rho_w V_w = (1.000 \times 10^3 \text{ kg/m}^3)(1.28 \times 10^3 \text{ m}^3) \\ &= 1.28 \times 10^6 \text{ kg.} \end{aligned} \quad 11.33$$

By Archimedes' principle, the weight of water displaced is $m_w g$, so the buoyant force is

$$\begin{aligned} F_B &= w_w = m_w g = (1.28 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 1.3 \times 10^7 \text{ N.} \end{aligned} \quad 11.34$$

The steel's weight is $m_w g = 9.80 \times 10^7 \text{ N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$\begin{aligned} m_w &= \rho_w V_w = (1.000 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^5 \text{ m}^3) \\ &= 1.00 \times 10^8 \text{ kg.} \end{aligned} \quad 11.35$$

The maximum buoyant force is the weight of this much water, or

$$\begin{aligned} F_B &= w_w = m_w g = (1.00 \times 10^8 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 9.80 \times 10^8 \text{ N.} \end{aligned} \quad 11.36$$

Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Making Connections: Take-Home Investigation

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this "boat," what shape of the boat would allow it to hold the most "cargo" when placed in water? Test your prediction.

Density and Archimedes' Principle

Density plays a crucial role in Archimedes' principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object's density is related to that of the fluid. In [Figure 11.20](#), for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}. \quad 11.37$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl} . Now we can obtain the relationship

between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\bar{\rho}_{\text{obj}}}, \quad 11.38$$

where $\bar{\rho}_{\text{obj}}$ is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}. \quad 11.39$$

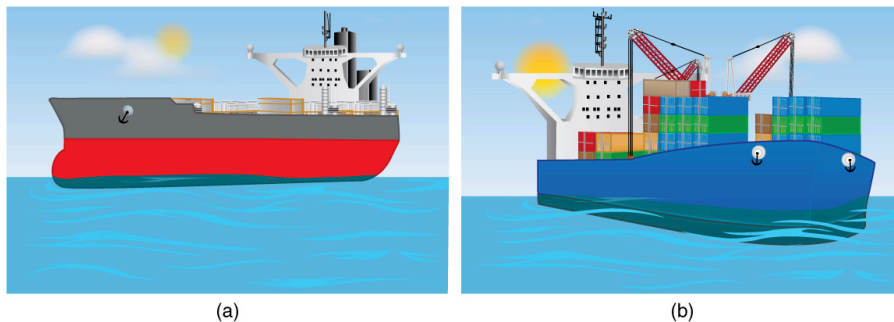


FIGURE 11.20 An unloaded ship (a) floats higher in the water than a loaded ship (b).

We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

$$\text{specific gravity} = \frac{\bar{\rho}}{\rho_{\text{w}}}, \quad 11.40$$

where $\bar{\rho}$ is the average density of the object or substance and ρ_{w} is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for ρ . If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in [Figure 11.21](#).

Specific Gravity

Specific gravity is the ratio of the density of an object to a fluid (usually water).

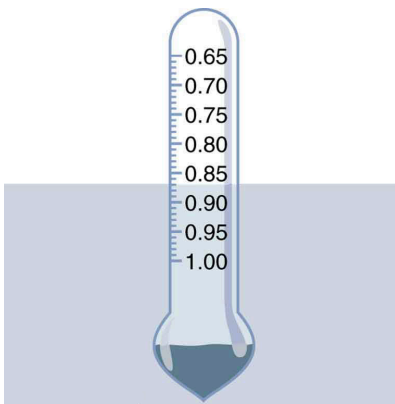


FIGURE 11.21 This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

EXAMPLE 11.9

Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}} \quad 11.41$$

for the density of the object. This yields

$$\bar{\rho}_{\text{obj}} = \bar{\rho}_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}. \quad 11.42$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

Solution

Entering the known values into the expression for her density, we obtain

$$\bar{\rho}_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3} \right) = 970 \frac{\text{kg}}{\text{m}^3}. \quad 11.43$$

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See [Figure 11.22](#).)

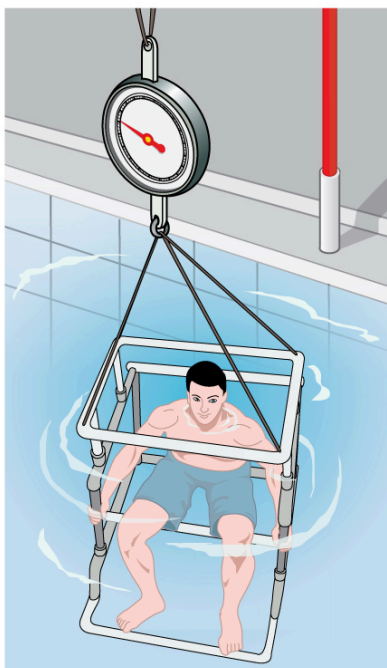


FIGURE 11.22 Subject in a hydrostatic weighing device, where they are weighed while completely submerged as part of a body density determination. The subject must completely empty their lungs and hold a metal weight in order to sink. Corrections are made for the residual air in the lungs (measured separately) and the metal weight. Their corrected submerged weight, their weight in air, and pinch tests of strategic fatty areas are used to calculate the percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a “lava lamp,” to name a few. Less obvious