

- How can you make letters on the screen as in Fig. 2.5: A letter *m*? A broad curvy *m*? A narrow sharp peaked *m*? A letter *w*? Any other letters? Use your hand to warm the probe and a cup of tap water to cool the probe quickly.

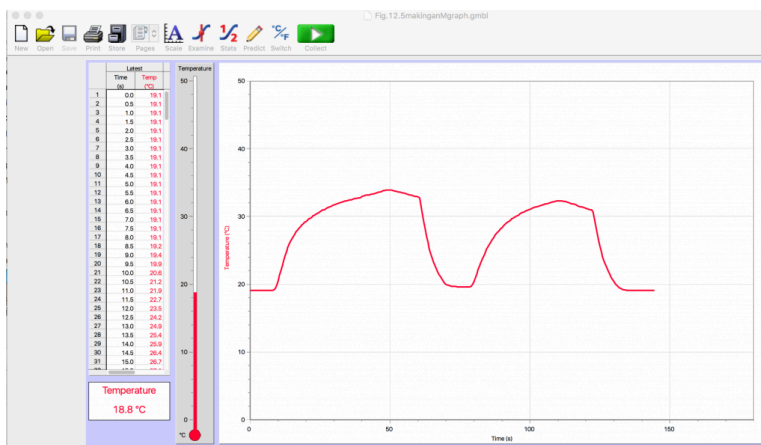


Fig. 2.5 Using a temperature probe to make a letter *m* on the computer screen. ©Vernier Software & Technology-used with permission.

After a while, add a second probe and a cup of hot water as well as cold.

- What questions can you ask and answer by varying the temperatures of the two temperature probes?

Artists may enjoy creating colorful designs with one probe drawing a red line and the other probe drawing a blue one to represent temperatures changing in different ways.

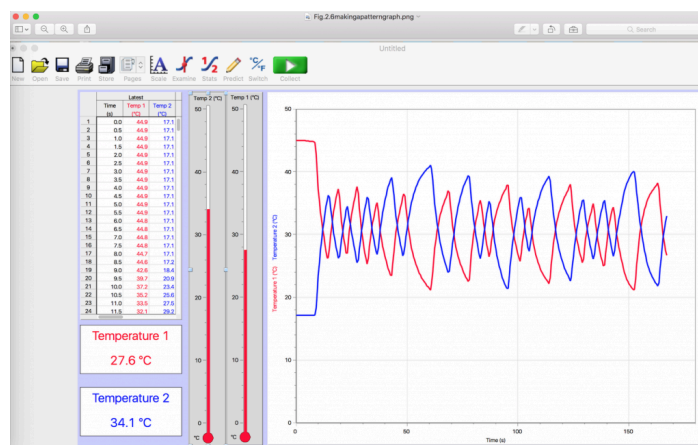


FIG. 2.6 Making a design with two temperature probes and hot and cold water. ©Vernier Software & Technology—used with permission.

- Discuss with your group members how to interpret the graph that you are seeing. How, for example, might students have made the graph shown in Fig. 2.6?

B. Exploring thermal phenomena with everyday materials

One activity most children enjoy at home, at school, and at the beach is playing with water. Explorations that involve mixing hot and cold water can be a logistic challenge but provide an engaging context for additional explorations of thermal phenomena.

For a series of explorations in mixing hot and cold water, use for each group of students:

- two temperature probes (or regular thermometers)
- clear plastic cup with lines indicating 1, 2, 3, or 4 parts for measuring (or a set of measuring cups),
- large Styrofoam cup or ceramic coffee mug for hot water,
- clear plastic cup for cold water,
- quart Styrofoam or plastic containers for storing hot and cold water,
- large container for storing water when finished with a trial

- tray large enough for holding the cups involved in mixing hot and cold water,
- towels in case things spill.

Place the tray and containers of water on a separate area or table from the computer to protect the computer in case of spills.

Question 2.7 What happens when you mix various amounts of hot and cold water?

- Explore thermal phenomena more systematically by mixing hot and cold water. In the *Before* section of a new physics notebook page sketch predictions for graphs that will appear on the computer screen when you mix hot and cold water in the following ways:
 - a: Equal amounts at same temperature
 - b: Equal amounts at unequal temperatures
 - c: Unequal amounts at same temperature
 - d: Unequal amounts at unequal temperatures: more hot than cold
 - e: more cold than hot
- Discuss and agree on the details of how you will be doing the mixing:
 - What type of cup will you use for the hot water. Why?
 - Will you pour the cold water into the hot water or pour the hot water into the cold? Why?
 - How will you measure how much hot and cold water you are using?
- Place one temperature probe in the hot water; the other in the cold water. Click on the green box at the top of the computer screen to start the program.

As you pour the cold water into the hot water, also transfer its temperature probe so

that both temperature probes are now in the mixture of hot and cold water.

If you are using bulb and tube thermometers, place one in the hot water and one in the cold water. Record both temperatures. Also record the temperature of the mixture of hot and cold water.

- In the *During* section of your physics notebook page, record the graphs obtained by mixing the various amounts of hot and cold water specified above.
- Also record and define any new vocabulary relevant to this exploration.

- Discuss with your group members how to describe these graphs in words. What central ideas about mixing hot and cold water can you generate from this exploration?

- In the *After* section of your physics notebook page, state central ideas that have emerged from this exploration and the evidence on which they are based.
- State a rationale that explains how the evidence supports these ideas and why these are important.
- Add a reflection about what you want to remember about this exploration, how you might use this in your own classroom, what you learned about science learning and teaching...
- What are you still wondering?

A useful way to organize outcomes is to record the set up, evidence, and relevant vocabulary for central ideas that emerge from explorations and discussions, as in the continuation of Table II.1.

TABLE II.1 Developing central ideas about thermal phenomena (continued)

TABLE II.1 Explorations of thermal phenomena (continued)			
Sketch of set up/ graph	Evidence	Central Ideas	Vocabulary
		When liquids are mixed, they reach an equilibrium temperature.	
	Report findings for: <ul style="list-style-type: none"> • equal amounts, same T • equal amounts at different temperatures • unequal amounts, same T; • unequal amounts at different temperatures: • more hot than cold • more cold than hot 	The equilibrium temperature depends on the initial temperatures and amounts of the liquids.	

- Complete the continuation of Table II.1 and write a summary of what you learned from this exploration.

After completing Table II.1 and your summary, look at an example of student work, nuances about exploring thermal phenomena by mixing hot and cold water, and some thoughts about the nature of science in this context.

1. Example of student work about mixing hot and cold water

A student summarized the exploration involving mixing hot and cold water as follows: *(The table) shows the experiments that we conducted in order to answer our questions and it also displays the ideas that were developed based upon these experiments.*

Table 5b Additional Powerful Ideas about Thermal Phenomena			
Sketch of set up	Evidence	Powerful Idea	Vocabulary
	I mixed 2 parts hot water with 2 parts hot water and the temperature didn't change	When mixing two amounts of water at the same temperature, the equilibrium temperature of the mixture is the same as the initial temperature whether the amounts are equal or unequal.	
	I mixed 2 parts hot water with 1 part hot water and the temperature didn't change		
	I mixed 2 parts hot water with 2 parts cold water and the temperature was in the middle of their initial temperatures	When mixing equal amounts of hot and cold water, the equilibrium temperature of the mixture is the average of the initial temperatures.	
	I mixed 2 parts cold water with 1 part hot water and the temperature was closer to the initial temperature of the cold water	When mixing unequal amounts of hot and cold water, the equilibrium temperature of the mixture is closer to the initial temperature of the larger amount of water.	

FIG. 2.7 Student's entries describing the mixing of hot and cold water exploration.

For the sketch of the set up in the first row, this student drew two containers representing cups of water and labeled them "2 hot" and "2 hot ." The student wrote, "I mixed 2 parts hot water with 2 parts hot water and the temperature didn't change."

In the second row, the student drew two containers and labeled them "2 hot" and "1 hot." The student wrote, "I mixed 2 parts hot water with 1 part hot water and the temperature didn't change."

In the third row, the student drew two containers and labeled them "2 hot" and "2 cold." The student wrote, "I mixed 2 parts hot water with 2 parts cold water and the temperature was in the middle of their initial temperatures."

In the fourth row, the student drew two containers and labeled them "2 cold" and "1 hot." The student wrote, "I mixed 2 parts cold water with 1 part hot water and the temperature was closer to the initial temperature of the cold water."

The student wrote the following rationales for the ideas claimed in the third column of the table:

When mixing two amounts of water at the same temperature, the equilibrium temperature of the mixture is the same as the initial temperature, whether the amounts are equal or unequal. (Figure 2.8) shows the graphs for mixing equal and unequal amounts of water at the same temperature. The first graph shows mixing

equal amounts of water at the same temperature. I mixed 2 parts hot water with 2 parts hot water and the temperature of the water did not change from the initial temperature of the hot water. The second graph shows mixing unequal amounts of water at the same temperature. I mixed 2 parts hot water with 1 part hot water and the temperature of the water did not change from the initial temperature of the hot water. The graphs in (Figure 2.8) are evidence for the idea that when mixing two amounts of water at the same temperature, the equilibrium temperature of the mixture is the same as the initial temperature, whether the amounts are equal or unequal.

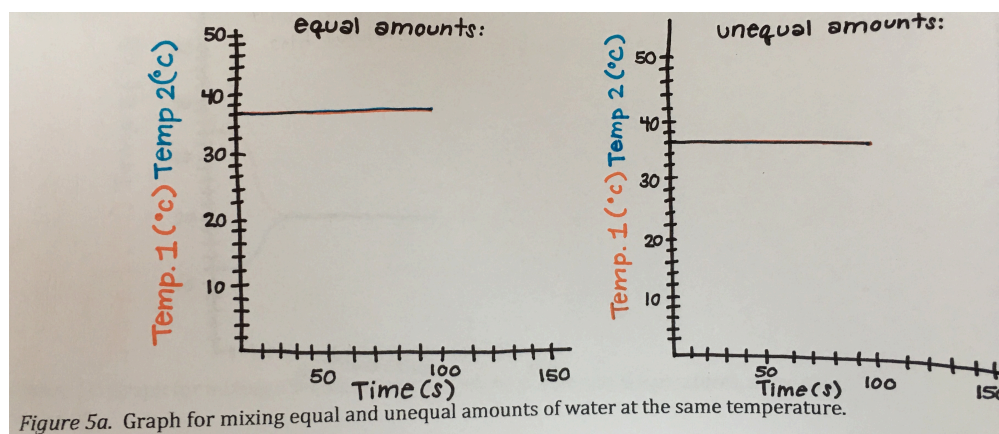


Figure 5a. Graph for mixing equal and unequal amounts of water at the same temperature.
 FIG. 2.8 Mixing equal and unequal amounts at the same temperature.

When mixing equal amounts of hot and cold water, the equilibrium temperature of the mixture is the average of the initial temperatures. (Figure 2.9) shows the graph for mixing equal amounts of hot and cold water. I mixed 2 parts hot water with 2 parts cold water and the temperature of the water was right in the middle of their initial temperatures. The graph in (Figure 2.9) is evidence for the idea that when mixing equal amounts of hot and cold water, the equilibrium temperature of the mixture is the average of the initial temperatures.

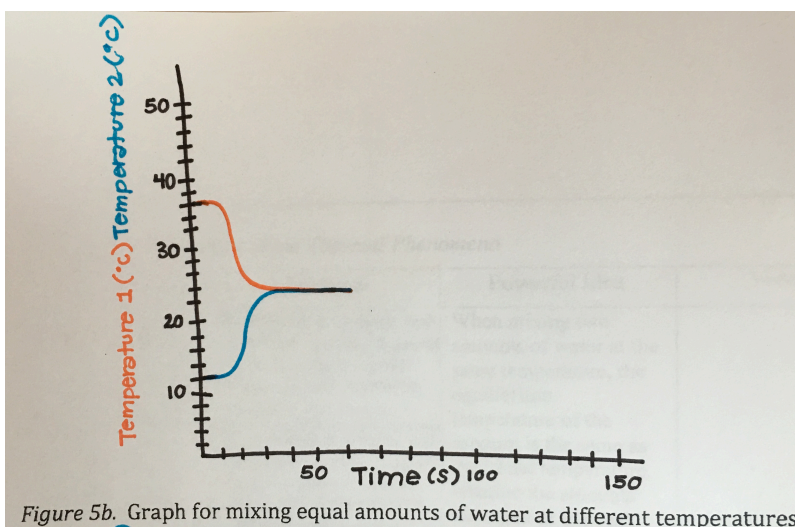


FIG. 2.9 Mixing equal amounts of hot and cold water.

When mixing unequal amounts of hot and cold water, the equilibrium temperature of the mixture is closer to the initial temperature of the larger amount of water. (Figure 2.10) shows the graph for mixing more hot water than cold water. I mixed 2 parts hot water with 1 part cold water and the temperature of the water was closer to the initial temperature of the hot water. (Figure 2.11) shows the graph for mixing more cold water than hot water. I mixed 2 parts cold water with 1 part hot water and the temperature of the water was closer to the initial temperature of the cold water. The graphs in (Figs. 2.10 and 2.11) are evidence for the idea that when mixing unequal amounts of hot and cold water, the equilibrium temperature of the mixture is closer to the initial temperature of larger amount of water.

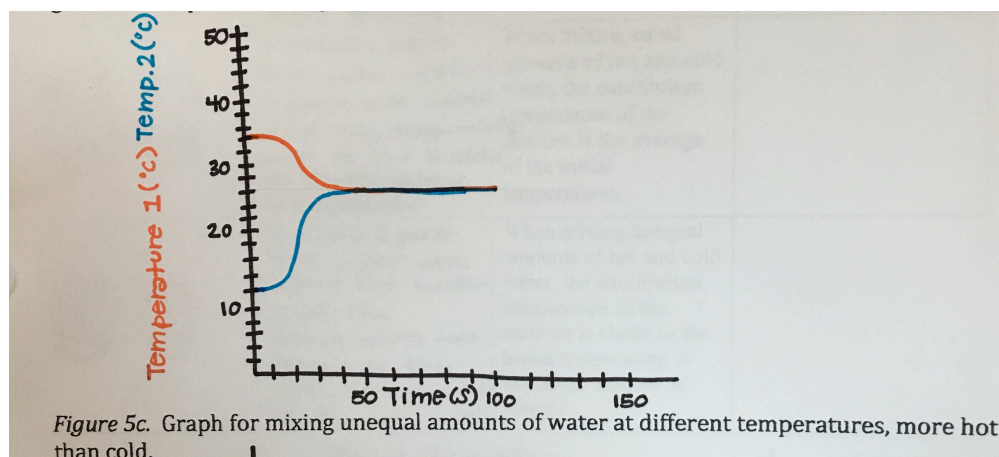


FIG. 2.10 Mixing more hot than cold.

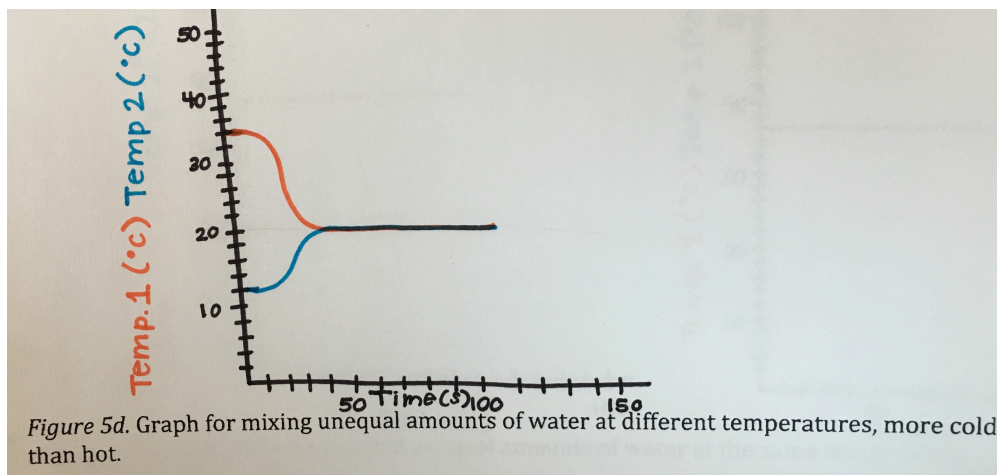


FIG. 2.11 Mixing more cold than hot.

Physics Student, Spring 2016

This was the first experience for these students in drawing and interpreting temperature graphs in this course. This student was careful to label the vertical axis with equal-distant marks representing a temperature scale and to color code the lines representing the changing temperatures for the two temperature probes. The student also indicated on the horizontal axis the time in seconds that the mixing took place.

2. Nuances about exploring thermal phenomena by mixing hot and cold water

Adults are unlikely to have difficulty understanding that mixing equal or unequal amounts of water at the same temperature will result in a mixture at that same temperature. These scenarios are included here because this does not necessarily seem obvious to children. They may be so accustomed to adding numbers that they may predict that they need to add the two temperatures if they are adding the two amounts of water together. The flat line on the resulting graph also is helpful even for adult students in that it makes clear that such a flat line represents something that is not changing, in this case the temperature of the water.

Mixing equal amounts of hot and cold water can be problematic. One needs to take care in measuring the equal amounts. With small amounts, such as a half of cup each, small errors in measuring can bias the result. The expectation is that the final temperature,

called the **equilibrium temperature**, will be half way between the two initial temperatures when mixing equal amounts at unequal temperatures.

In Fig. 2.12, does it look like the hot water changed temperature a little more than the cold? Why might that have happened?

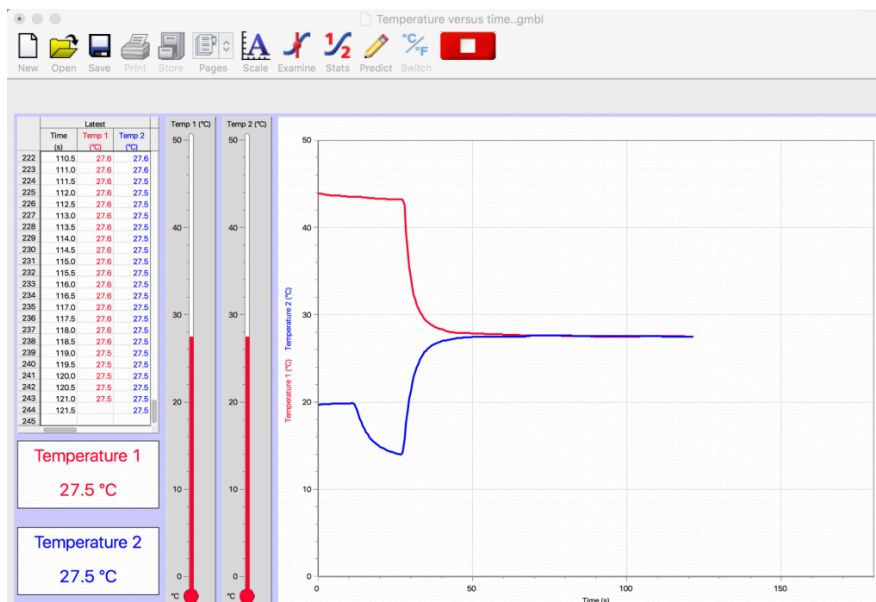


FIG. 2.12 Mixing intended equal amounts of water at unequal temperatures. ©Vernier Software & Technology—used with permission.

It may be that there were not exactly equal amounts of hot and cold water. Another possibility is that the amounts were equal but the hot water was poured into the cold water so some of the energy in the hot water flowed into the air and container as well as into the cold water. This would reduce the energy transferred to the cold water and look like a slightly smaller amount of hot water was used.

Figure 2.13 and Fig. 2.14 show the result of mixing different amounts of hot and cold water at different temperatures.

- In each of these experiments, which had the bigger temperature change? Which the smaller temperature change?
- In each of these experiments, which do you think was the bigger amount of water? The hot or the cold water?

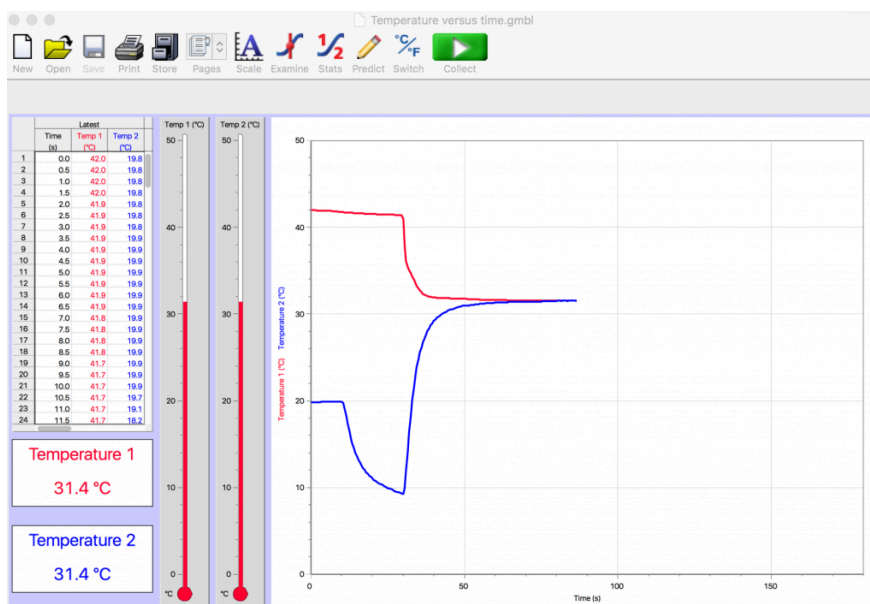


FIG. 2.13 Mixing unequal amounts of water at unequal temperatures. ©Vernier Software & Technology-used with permission.

In Fig. 2.13, it looks like the change in temperature of the hot water was smaller than the change in temperature of the cold water. Consider a bathtub with very hot water, much too hot for a bath. What to do? Add some cold water to cool things down, but not too much! If you mix a little cold water with a lot of hot water, which will change temperature the most?

The temperature of this small amount of cold water will change a lot as it mixes in with a large mass of hot water. The change in temperature is smaller for the larger mass of hot water. The change in temperature is larger for the smaller mass of cold water. Therefore, there must have been more hot water than cold in this scenario.

Figure 2.14 also shows the result of mixing different amounts of hot and cold water. Which had the bigger temperature change here? Which had the smaller temperature change? Which do you think was the bigger amount of water? The hot or the cold water?

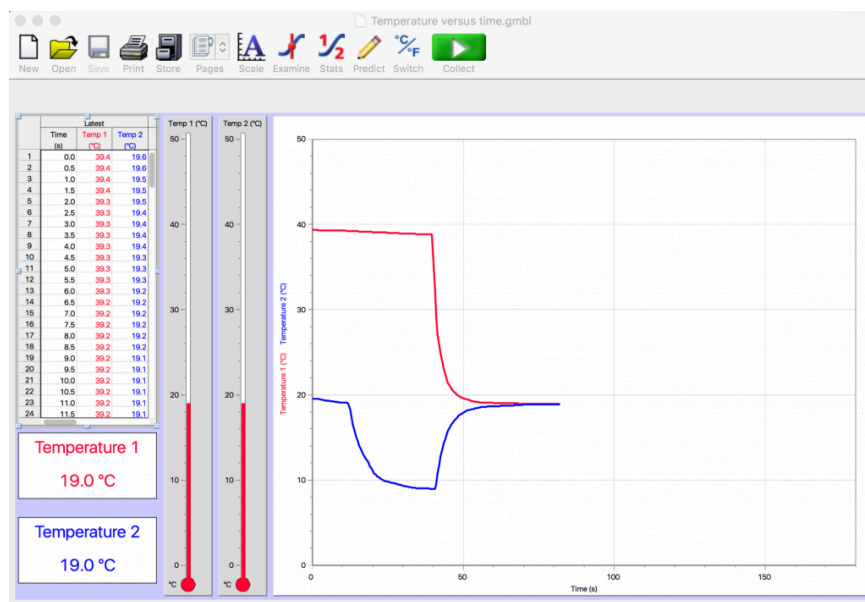


FIG. 2.14 Mixing unequal amounts of water at unequal temperatures. ©Vernier Software & Technology-used with permission.

In Fig. 2.14 the hot water had the bigger temperature change and the cold water had the smaller change in temperature. In this scenario, there must have been more cold than hot because the equilibrium temperature was nearer to the initial cold temperature.

A goal of this course is to build skills in interpreting line graphs in this way, to be able to “tell the story” of what happened by looking at the shapes of the lines and interpreting what must have happened in order for the lines to have formed in that way.

In Figs. 2.12 – 2.14, for example, the horizontal axis is representing the increase in time from left to right. The vertical axis is representing the temperature of the temperature probes.

- What does the red line at the beginning of the experiment indicate was happening to the temperature probe for the hot water?
- What does the blue line at the beginning of the experiment indicate was happening to the temperature probe for the cold water?
- What does the overlapping of the red and blue lines near the end of the mixing process indicate about what happened when these cups of hot and cold water were mixed together?

The almost flat red line at the beginning of the experiment suggests that the

temperature of the hot water was not changing much as the temperature probe was sitting in the cup of hot water. There is a slight slant downward of the red line indicating that the hot water was cooling slightly in its cup. Then there is a very sharp change in the hot temperature recorded by this probe; the red line plunges almost instantly to a much lower temperature, suggesting that the sharp bend occurred the moment that the cold water was poured into the cup of hot water.

The straight blue line near the beginning of the experiment suggests that the temperature probe for the cold water was initially not in the cup of cold water but was outside of the cup, perhaps at room temperature when the computer program was started. The curved blue line represents the drop in temperature when the probe was then placed in the cup of cold water.

In Fig. 2.12 and Fig. 2.13, the temperature probe for the cold water was in the cold water long enough that the blue line has almost flattened out, indicating that the probe was near to reading the temperature of the cold water before mixing began. In Fig. 2.14, the blue line becomes almost flat. It is important to stir both cups of water and to wait until both lines are flat before pouring the cold water into the hot water. Record both temperatures just before mixing, in order to be sure that the initial readings of the temperatures of the hot and cold water are accurate.

The overlapping lines for both temperature probes at the end of the mixing process indicate that the two temperature probes were reading the same temperature, the *equilibrium temperature* reached after the mixing was complete. This assumes that the probes had been *calibrated*, that they had indicated the same temperature when placed in the same cup of water at the beginning of the experiment. If the two probes did not indicate the same temperature when placed in the same cup of water at the beginning of the experiment, the computer program needed to be instructed to change the calibration so that the readings of the two probes would agree when reading the same temperature.

Note how quickly the temperatures change as the mixing begins. Also note the shape of the curves as the temperatures change more gradually as the mixture comes to equilibrium. When drawing graphs, also take care to match the time when the lines start changing drastically, as temperature readings by both the hot and cold probes will start changing at the same instant, at the moment that the cold water is poured into the hot.

Figuring out how best to measure and mix the hot and cold water in order to maximize the accuracy of results is an example of an *engineering design* problem. The possible effects of pouring hot water into cold versus pouring cold water into hot, of stirring or not stirring before and after mixing, of calibrating the thermometers, and of timing the recording of initial conditions are all issues that need to be identified with solutions

developed and optimized. (See Appendix I, *Engineering Design in the Next Generation Science Standards*, <https://www.nextgenscience.org/resources/ngss-appendices>.)

3. *Some thoughts about the nature of science in this context*

An important aspect of doing science is being able to represent and interpret changes that are occurring. Visual displays such as graphs can make aspects visible that might otherwise go unnoticed. Figures 2.13 and 2.14, for example, show dramatic differences in the stability of the initial temperatures of the hot and cold temperature probes. The initial almost flat line for the hot probe indicates a stable initial condition but the initial curved line for the cold probe indicates a dramatically changing initial condition. Such differences could affect the results. It is important in doing careful experiments that both the hot and cold temperature probes show a flat line, indicating a stable initial temperature, before mixing occurs.

These graphs show how the hot and cold water temperatures changed with time; such line graphs show how one quantity, represented by the vertical axis, changes with another quantity, represented by the horizontal axis. Learning to “tell the story” of such line graphs is an important skill in many fields. Being able to interpret such visual displays of data can enhance one’s ability to influence and/or evaluate claims being made, whether in science, business, or personal contexts.

The use of digital temperature probes in this unit illustrate the affordances provided by technologies in collecting and interpreting data. As indicated in the *US Next Generation Science Standards*, *scientific knowledge is based on empirical evidence*. Third to fifth grade students, for example, should understand that *scientists use tools and technologies to make accurate measurements and observations* (NGSS, *Lead States*, 2013, *Appendix H*). (See: <https://www.nextgenscience.org/resources/ngss-appendices>.)

VI. Developing an Additional Central Idea about Thermal Phenomena and Its Mathematical Representations

Being able to tell a story about what is happening by looking at a graph is a useful skill whenever one is monitoring a numerical quantity. Graphs can show at a glance how something is changing, whether that involves finances, populations, sales, or some other quantity of interest. Being able to relate quantities algebraically also is helpful if one wants to make numerical predictions based on evidence rather than intuition. This section builds on skills in interpreting qualitative line graphs, such as those shown in Figs. 2.12–2.14. You also will develop quantitative skills for generating and justifying relevant equations, solving for an unknown, and calculating a quantity of interest.

A. Interpreting features of line graphs

“Telling the story” represented by a line graph involves looking at the shape of the graph and interpreting its features.

Question 2.8 How can you tell what is happening by interpreting the shape of a line graph?

Look at the shapes of the lines in the graphs shown in Figs. 2.15 and 2.16. What was happening if these graphs are representing the mixing of hot and cold water? Which of these graphs is intended to represent mixing more hot water than cold? Which is intended to represent mixing more cold water than hot?

For these graphs, an important feature is the length of the line representing the changing temperature of the hot water as compared to the length of the line representing the changing temperature of the cold water.

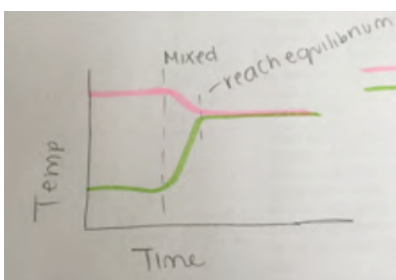


FIG. 2.15 Graph representing the mixing of hot and cold water. More hot water or more cold water?



FIG. 2.16 Graph representing the mixing of hot and cold water. More hot water or more cold water?

Note that in Fig. 2.15, the student drew a long vertical dashed line to represent the time when the mixing of hot and cold water started, shown by the pink line bending down and the green line bending up. The initial bends are not as steep as shown in Fig. 2.13 but the lines start bending roughly at the same time, represented by the vertical line. The long vertical dashed line matches where both of the lines representing the hot- and cold-water temperatures started changing direction, representing that the temperatures of both the hot and cold water started changing at the same moment. The short vertical dashed line indicates roughly the time when the temperatures of the hot and cold water stopped changing as well, shown by the pink and green lines leveling off at about the same height above the horizontal axis, with that height representing the equilibrium temperature. The student wrote “*reach equilibrium*” to label what was happening there. The equilibrium is

closer to the initial hot temperature, indicating that this was a case of more hot water mixing with cold.

In Fig. 2.16, however, the student did not match the lines representing times when the hot and cold water started changing direction. This graph suggests a story in which the cold water started warming up first and almost reached the equilibrium temperature before the hot water started cooling off. This is different from the story told by the lines in Fig. 2.14. It is important when drawing line graphs to check the story that your lines are telling. Are the details of your lines accurately telling the story of what is happening? The equilibrium temperature is closer to the initial cold temperature, indicating that this was a case of more cold water mixing with hot.

B. Identifying patterns in data

General thinking about a situation can help you decide how to set up a series of experiments to collect enough data to be able to perceive a pattern in the data. Once you have identified a pattern, try to represent that pattern mathematically. If the pattern can be expressed by an algebraic equation, you can use that the relationship to make predictions or to estimate a quantity of interest. That process is the focus of this section.

1. *Designing a series of experiments to identify patterns in data*

Question 2.9 *When mixing hot and cold water, how are the amounts of hot and cold water related to how much their temperatures change?*

- To explore what happens quantitatively when you mix various amounts of hot and cold water, first decide what data to record and how you want to keep track of these data.
 - If you want to double, triple, and quadruple an amount of water for a series of experiments, how might you do that?

One way would be to use mass. If you have a balance, you could measure 100 grams, 200 grams, 300 grams, or 400 grams of water.

Another way would be to use volume. If you have a 500-milliliter measuring cup, you could measure 100ml, 200ml, 300ml, or 400 ml of water

Equivalently, use any container that you can mark with 1, 2, 3, 4 levels to measure multiples of the initial amount.

- How will you measure the initial and final temperatures?

Two regular bulb and tube thermometers will work fine. Using two digital temperature probes connected to a computer, however, will allow you to build skill in interpreting the graphs that appear as you mix the various amounts.

- What combinations of hot and cold water will you try and how will you record your findings?
- Another issue is how best to carry out your experiments: What might affect how much the temperatures of the hot and cold water change?
 - How can you be sure that you are measuring the amounts of water consistently with your measuring device?
 - How can you guard against some of the energy from the hot water flowing out into the cup and air instead of into the cold water?
 - How can you be sure that the temperatures you record for the hot and cold water are their actual temperatures at the moment just before you mix them?
 - How can you be sure the hot and cold water are well mixed before you record the equilibrium temperature?
- In the *Before* section of your physics notebook page, describe your experimental design – how you plan to conduct these experiments. This also is a place to record your initial thoughts about what to do with your data once obtained and to make predictions for the results.
 - What data do you plan to record and how do you plan to keep track of these data?
 - How might you use these data to predict the changes in temperature for each

experiment?

- How might you use the amounts of hot and cold water and their initial temperatures to predict whether the equilibrium temperature is likely to be closer to the initial hot temperature, closer to the initial cold temperature, or half way in-between their temperatures?
 - How do you think the equilibrium temperature relates to the initial temperatures of the hot and cold water?
 - How might you use the amounts of hot and cold water to be mixed, their initial temperatures, and the likely equilibrium temperature in order to predict likely changes in temperatures of the hot and of the cold water?
- Explain your reasoning for the process you will use to make these predictions.

This process of designing your exploration by deciding on what to do, what data to record, how to use these data, and predicting what you expect to occur is an important aspect of learning to do science. This open-ended process contrasts with one in which a laboratory manual or teacher already has made most of those decisions for you. The intent here is to model the experimental process as well as the conceptual process of developing central ideas and the mathematical process of representing what is happening through graphs and algebraic equations.

- In the *During* section of your notebook page, create a table to record the findings for each experiment in a clear way.
- Also note any new words or familiar vocabulary that you have noticed as having special meanings in this context.
- Discuss your findings and formulate a relevant central idea.
- Sometimes one feels overwhelmed in the midst of an exploration, particularly if one is trying to figure out relationships among variables rather than simply following a lab manual's verification recipe. If that happens, it can be helpful to ask:
 - What are we doing?
 - Why are we doing this?
 - How is that helping us?

- In the **After** section of the physics notebook page, report this central idea and the evidence on which it is based.
- Write a rationale that explains how the evidence supports the central idea and why this is important.
- Also reflect upon this exploration such as what connections can you make to other experiences? How might you use what you learned in your own classroom?
- What are you still wondering?

After class, write a summary based on your physics notebook pages and handouts before reviewing the following discussion of comparing ratios of hot and cold water and their changes in temperature. To maximize learning, it is important to work through issues in conversation with your group members and with guidance as needed from your instructor in class before reading this text. The purpose of the sections below is to help clarify any details that may be puzzling.

2. Recording and analyzing data

There are many ways to record and analyze data. As an example, consider Fig. 2.14, repeated from above, for an experiment in which two parts of cold water were mixed with one part of hot water.

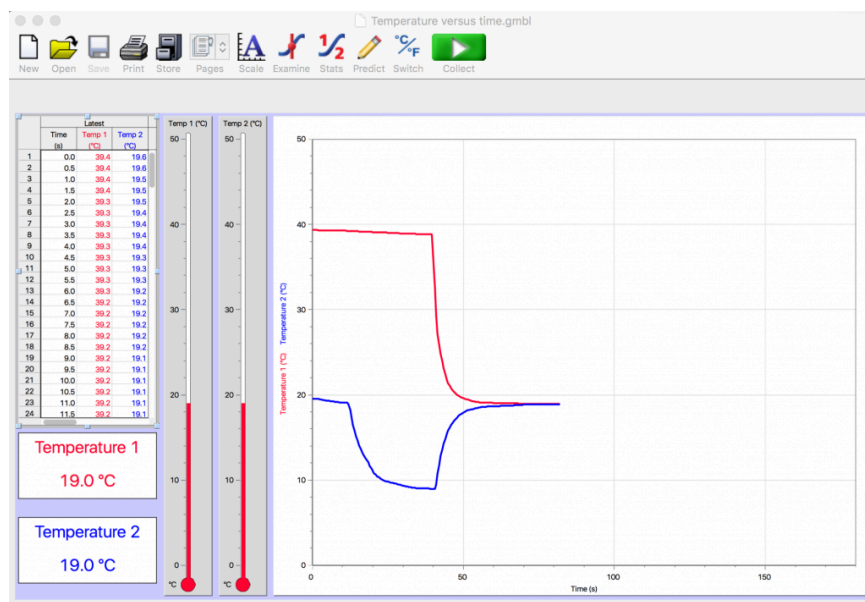


FIG. 2.14 (repeated). Mixing unequal amounts of water at unequal temperatures. ©Vernier Software & Technology-used with permission.

The vertical axis represents temperature in degrees Celsius; the horizontal axis represents time in seconds. The computer display indicates that the mixture of the hot and cold water reached an equilibrium temperature of 19.0 °C. The precision of the digital thermometers is nice but not necessary. If we had been using regular thermometers rather than digital thermometers, probably both regular thermometers would have indicated an equilibrium temperature of about 19°C. That level of precision would be enough for the purposes of this experiment. The graph on the computer is useful, however, in providing an informative visual display of what is happening.

Looking at the initial red line, which is just below the 40°C grid line, what is an estimate of the initial temperature of the hot water?

Looking at the lowest point of the initial blue line, which is just below the 10°C grid line, what is an estimate of the initial temperature of the cold water?

The ratio of parts of cold water to hot water is 2/1. The final temperature of the hot water was about 19°C; the initial temperature of the hot water was about 39°C. The final temperature of the cold water was about 19°C; the initial temperature of the cold water was about 9°C. Although the digital temperature probes provide more precise readings, we are choosing to record readings at $\pm 1^\circ\text{C}$ here to ease visual inspection of the calculations in Table II.2.

A table would be helpful in recording and analyzing such data.

Table II.2 Reporting data and analyzing experiments mixing hot and cold water

TABLE II.2 Reporting data and analyzing experiments mixing hot and cold water									
Exp.	Part hot	Part cold	Ratio parts hot to cold	Final T hot in °C: T_{hf}	Initial T hot in °C: T_{hi}	Change in T hot water: ΔT_h	Final T cold in °C: T_{cf}	Initial T cold in °C: T_{ci}	Change in T cold water: ΔT_c
Fig. 2.14	1	2	1/2	19°C	39°C		19°C	9°C	

A similar table can be made for other combinations, such as mixing 1 part hot water with 4 parts cold, 2 parts hot water with 1 part cold, or 4 parts hot water with 1 part cold. Assembling such data for multiple experiments makes possible looking for patterns in the data across multiple contexts.

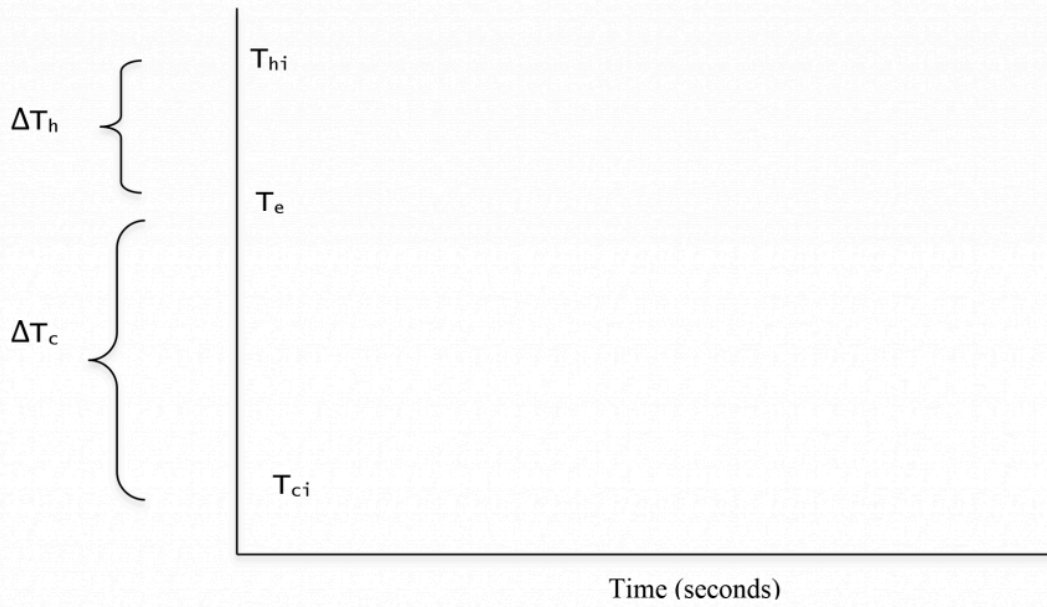
For the experiment represented in Fig. 2.14, what are the changes in temperature for the hot and cold water? When describing changes in temperature in everyday life, we usually indicate whether the temperature increases or decreases: “it will be 5 degrees *warmer* today than yesterday,” “the cake *cooled* to room temperature,” or “her temperature is *back down* to normal.” Mathematically, we describe whether a temperature change is hotter or colder with a positive or negative sign. Therefore, we mathematically define a change in temperature as the final temperature T_f minus the initial temperature T_i and use ΔT , delta T, to represent a change in temperature:

$$\text{change in temperature} = \Delta T = T_f - T_i$$

When the temperature increases, the change in temperature is positive. When the temperature decreases, the change in temperature is negative. When mixing hot and cold water, the final temperature for both the hot and the cold water is the equilibrium temperature T_e so for both changes in temperature, subtract the initial temperature from the equilibrium temperature:

$$\text{change in temperature} = \Delta T = T_e - T_i$$

Figure 2.17 provides a visual way to think about the relationships among the initial, final, and equilibrium temperatures when mixing hot and cold water.



where: $\Delta T_h = T_e - T_{hi}$ $\Delta T_c = T_e - T_{ci}$

Fig. 2.17a. Graph of **temperature versus time** for mixing more hot- than cold- water

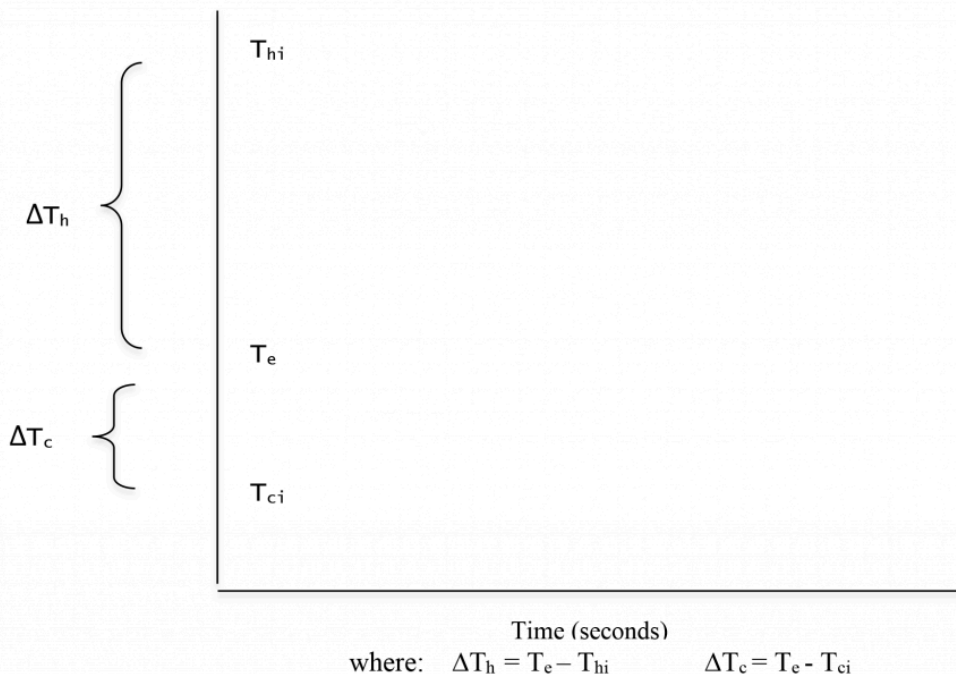


Fig. 2.17b. Graph of **temperature versus time** for mixing more cold- than hot- water

For the hot water:

$$\Delta T_h = T_{hf} - T_{hi} = T_e - T_{hi} = 19^\circ\text{C} - 39^\circ\text{C} = -20^\circ\text{C}$$

The minus sign indicates that ΔT_h , the change in the hot water temperature, was a decrease in temperature. All ΔT_h in these mixing hot and cold water experiments will have a negative value as the hot water will always be at a higher initial temperature than the equilibrium temperature. Subtracting the higher initial temperature from this lower equilibrium temperature will always yield a negative value.

For the cold water:

$$\Delta T_c = T_{cf} - T_{ci} = T_e - T_{ci} = 19^\circ\text{C} - 9^\circ\text{C} = +10^\circ\text{C}$$

The plus sign indicates that ΔT_c , the change in the cold water temperature, was an increase in temperature. All ΔT_c in these mixing hot and cold water experiments will have a positive value as the cold water will always be at a lower initial temperature than the equilibrium temperature. Subtracting the lower initial temperature from this higher equilibrium temperature will always yield a positive value

It can be helpful, when working with problems involving mixing hot and cold water, to sketch a graph that illustrates what is happening as in Fig. 2.18.

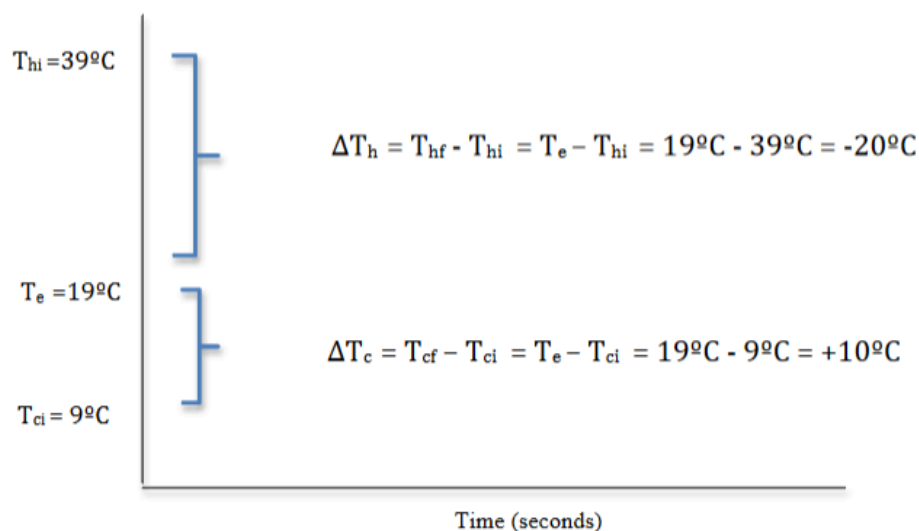


FIG. 2.18 Form of graph of temperature versus time for mixing 1 part hot and 2 parts cold water.

Entering these findings in the table and highlighting these changes in temperature and the parts of hot and cold water creates a visual display that can help identify patterns in these data. The magnitude of the change in temperature of the hot water would be twice as much as magnitude of the change in temperature of the cold water.

Calculating the change in temperatures of the hot water and the cold water completes Table II.2:

Table II.2 (continued) Reporting data and analyzing experiments mixing hot and cold water

Table II.2 (continued) Reporting data and analyzing experiments mixing hot and cold water									
Exp.	Part hot	Part cold	Ratio parts hot to cold	Final T hot in °C: T _e	Initial T hot in °C: T _{hi}	Change in T hot ΔT _h = T _e - T _{hi}	Final T cold in °C: T _e	Initial T cold in °C: T _{ci}	Change in T cold ΔT _c = T _e - T _{ci}
Fig. 2.14	1	2	1/2	19°C	39 °C	- 20 °C	19 °C	9 °C	+10 °C

3. Interpreting findings

How are the highlighted quantities in Table II.2 related mathematically?

Given that we have been comparing what happens when mixing various ratios of the amounts of hot and cold water, it seems reasonable to compare the changes in temperatures as ratios as well:

$$\frac{\text{Part hot}}{\text{Part cold}} \quad \text{equals:} \quad \frac{\text{Change in temperature hot}}{\text{Change in temperature cold}} \quad \text{or}$$

$$\frac{\text{Change in temperature cold}}{\text{Change in temperature hot}} \quad ?$$

For the experiment represented in Fig. 2.14, what does the ratio $\frac{\text{Part hot}}{\text{Part cold}}$ equal ?

$$\frac{\text{Part hot}}{\text{Part cold}} = \frac{1}{2} = \frac{-20^{\circ}C}{+10^{\circ}C} \quad ?$$

or

$$\frac{\text{Part hot}}{\text{Part cold}} = \frac{1}{2} = \frac{+10^{\circ}C}{-20^{\circ}C} \quad ?$$

The results of this experiment indicate that the ratio of the amount of hot water to the amount of cold water (1 to 2) seems to be related to the inverse ratio of the change in cold temperature to the change in hot temperature.

There is a problem, however, in that the ratio of the temperatures is negative but the magnitudes match; they are both of magnitude 1/2. The magnitude of a quantity is its numerical value without reference to whether the value is positive or negative.

Sometimes students also are puzzled by what their results seem to be saying. Most ratios that students have worked with in other contexts seem to have been *direct proportions*, as in the pinhole camera, where the entities are in the same order: $H/D = h/d$ or expressed another way: $h/H = d/D$, the ratio of corresponding heights in similar triangles is equal to the ratio of corresponding distances.

This new relationship between ratios is representing an *inverse proportion*:

$$\frac{\text{Property 1 of Top Entity}}{\text{Property 1 of Bottom Entity}} \quad \text{is related to} \quad \frac{\text{Property 2 of Bottom Entity}}{\text{Property 2 of Top Entity}}$$

If a result seems wrong, consider whether something has gone wrong or whether this experimental result is what is really happening. Based on the data presented above, the ratio of a small amount of hot water to a large amount of cold water is related to the magnitude of the ratio of a small change in temperature of the large amount of cold water to the magnitude of a large change in temperature of the small amount of hot water. This is a description of what the graph showed in Fig. 2.14. In this case, the temperature of

a large amount of cold water changed a little; the temperature of a small amount of hot water changed a lot.

In addition to considering whether a puzzling result seems logical and in agreement with other ways to represent the data obtained in an experiment, it is helpful to test whether the same result occurs in other contexts. This result was based on an experiment mixing one part hot water and two parts cold water. Would the same pattern occur with other combinations such as two parts hot water and one part cold? What about 4 parts hot water and 1 part cold? Or 1 part hot and 4 parts cold water. Such multiple trials make possible looking for patterns in data and testing whether the relationships one infers are valid. Looking for patterns in data is an example of a *crosscutting concept* that occurs across many different science domains as discussed in Appendix G of the Next Generation Science Standards (NGSS Lead States, 2013) <https://www.nextgenscience.org/resources/ngss-appendices>.

4. *Developing an algebraic representation of the findings*

There are several steps needed to express this relationship appropriately in symbols. We have chosen to express the amounts of water in terms of their masses. For the experiment shown in Fig. 2.14, the ratio of the mass of hot water to the mass of cold water was 1 to 2:

$$\frac{\text{mass of hot water}}{\text{mass of cold water}} = \frac{m_h}{m_c} = \frac{1}{2}$$

which is a positive number.

The ratio of the changes in temperatures can be written as:

$$\frac{\text{Change in temperature of the cold water}}{\text{Change in temperature of temperature of the hotwater}} = \frac{\Delta T_c}{\Delta T_h} = \frac{+10^\circ C}{-20^\circ C} = \frac{-1}{2}$$

which is a negative number.

Because ΔT_h is always negative and ΔT_c is always positive, the ratio of changes in temperature will always be a negative number. In order to set it equal to a positive number, a minus sign needs to be added to the expression. Therefore, the results of this experiment can be expressed algebraically as:

$$\frac{m_h}{m_c} = \frac{-\Delta T_c}{\Delta T_h}$$

The results also can be expressed as

$$\frac{m_c}{m_h} = \frac{-\Delta T_h}{\Delta T_c}$$

The magnitudes of the ratios in the experiment represented by the graph in Fig. 2.14 turned out to be exactly equal at the level of precision we used. That is unusual in these experiments. Typically these experimental results are only approximately equal. Therefore we choose to represent these experimental results with the symbol \approx that represents “approximately equal”:

$$\frac{m_h}{m_c} \approx \frac{-\Delta T_c}{\Delta T_h}$$

and

$$\frac{m_c}{m_h} \approx \frac{-\Delta T_h}{\Delta T_c}$$

The ratio of the mass of the hot water to the mass of the cold water is approximately equal to minus the ratio of the change in temperature of the cold water to the change in temperature of the hot water where change in temperature equals the final temperature minus the initial temperature ($\Delta T = T_f - T_i$).

One also can refer to the *magnitude* of the changes in temperature, the numerical values of the changes, without reference to whether they are an increase or decrease in temperature. The magnitude of a quantity is represented by vertical bars on both sides of its symbol: $|\Delta T_h|$ represents the magnitude of the change in temperature of the hot water, 20°C in this case. The ratio of the mass of hot water to the mass of cold water is approximately equal to the ratio of the magnitude of the change in temperature of the cold water to the magnitude of the change in temperature of the hot water. If stated in terms of magnitudes, the minus sign is not needed but vertical bars should be placed of both sides of the expressions with ΔT symbols:

$$\frac{m_h}{m_c} \approx \frac{|\Delta T_c|}{|\Delta T_h|}$$

and

$$\frac{m_c}{m_h} \approx \frac{|\Delta T_h|}{|\Delta T_c|}$$

If you choose to use “magnitudes” be sure to indicate that with both the verbal and symbolic statements of this relationship.

c. Recognizing the importance of systems thinking

Comparing the results for several experiments mixing various amounts of hot and cold water can increase confidence in the interpretation developed above, that the amounts of hot and cold water are inversely proportional to the magnitudes of their changes in temperature. Any differences in findings from multiple experiments would need consideration: what are the components of the system, how might they be interacting, how might the processes involved be affecting what is happening?

Question 2.10 What is the role of systems thinking in interpreting experimental results?

A student reported four experiments in which one set of ratios were equal but three of the four had slightly lower changes in temperature by the cold water than expected. The student reflected upon the role of systems thinking in explaining these findings.

1. *Example of student work reflecting upon the role of systems thinking*

In class, we mixed varying unequal amounts of hot and cold water. Our goal was to find a pattern in order to develop a mathematical representation of thermal phenomena...First, I mixed 2 parts hot water with 1 part cold water...Next, I mixed 1 part hot water with 2 parts cold water...Next, I mixed 4 parts hot water with 1 part cold water...Finally, I mixed 1 part hot water with 4 parts cold water...The experimental evidence that I found is evident in all four of the different unequal amounts of hot water and cold water that I experimented with. These four different trials all produced approximately equal ratios, which is experimental evidence that when mixing unequal amounts of hot and cold water, the ratio of the amount of hot water to the amount of cold water is approximately equal to the ratio of the (magnitude of the) change in temperature of the cold water to the (magnitude of the) change in temperature of the hot water..

Various aspects of the situation may affect how close the mathematical model matches what actually happened. The cups of hot and cold water form a system with energy flowing from the hot water to the cold water. If it is a closed system, all of the energy lost by the hot water will be gained by the cold water. If it is an open system,

some of the energy may flow out and into the environment, such as into the air or the material of the containers. Also, if there is an energy source nearby, some energy may flow into this system. Because we are trying to understand what is happening within our system of the cups of hot and cold water, it is important for us to keep the system as closed as possible.

Because we were trying to find a pattern revolving around how the masses of the hot and cold water related to their changes in temperature when they were mixed, the most accurate results were found when as little energy as possible flowed out of the hot water and into the environment. So, the importance of system thinking in this exploration is that in order to move from the general claim that we developed to the precise mathematical statements, we needed to isolate the system of hot and cold water as much as possible.

Physics student, Spring 2016

In three of the four experiments discussed, the cold water did not change temperature quite as much as expected. This was a consistent effect, suggesting that the system was not a closed system consisting only of the hot and cold water. Some of the energy from the hot water likely flowed into the surrounding air and material forming the containers rather than into warming the cold water. This flow of energy into the surroundings, rather than into the cold water, would reduce the change in temperature for the cold water.

This is an example of the development of systems thinking, that one needs to consider all elements of a system and to observe carefully what is occurring. According to the *Next Generation Science Standards (NGSS Lead States, 2013)*, thinking about *systems and systems models* is common across many science disciplines as described in Appendix G – Crosscutting Concepts (<https://www.nextgenscience.org/resources/ngss-appendices>).

What seems remarkable is that the ratios found in these experiments can be so close, given how simple the equipment and procedures are. We have not asked students to take the next step of doing multiple trials of each experiment in order to estimate uncertainties. The emphasis in this course has been on fostering conceptual understanding of the shapes of qualitative graphs while briefly modeling here a quantitative approach to looking for patterns in the data. With limited time available, our choice has been to balance this experimentally obtained insight about the inverse relationship between the amounts of hot and cold water and their changes in temperature with one based on theoretical considerations.

VII. Developing a Mathematical Representation of Thermal Phenomena Based on Theoretical Considerations

The experimental approach discussed above focused only upon the relative masses of the hot and cold water and their relative changes in temperature. No mention was made of the energy flowing from the hot to the cold water. An earlier section in this unit (II.B, Table II.1, page [195](#)) had developed the following central idea:

A temperature difference implies a flow of energy from hotter objects to colder objects.

In this section, we consider how to express theoretical ways of thinking about that transfer of energy from the hot water into the cold water. This involves use of the Law of Conservation of Energy, that the total energy in a closed system does not change, that energy lost by the hot water is gained by the cold water and its surroundings.

A. Considering what happens when energy flows from hot water into cold

Question 2.11 What theoretical considerations can provide insights into what is happening when energy flows from the hot water into the cold water?

Energy is measured in different units in different contexts. (see <https://www.aps.org/policy/reports/popa-reports/energy/units.cfm> . In this course, we use the energy unit of a *calorie*.

A calorie is the amount of energy needed to change the temperature of one gram of water by one degree Celsius at standard atmosphere pressure and 20°C. This amount of energy is very small; the calories typically discussed in the context of food are kilocalories, 1000 of the calories discussed here. A kilocalorie is the energy needed to change the temperature of one kilogram of water by one degree Celsius at standard atmospheric pressure and 20°C.

There are many different materials one could use in exploring thermal phenomena. **The amount of energy needed to raise the temperature of one gram of a material by one degree Celsius is called the material's specific heat.**

Water is the standard. **One calorie of energy is needed to raise a mass of one gram of water by one degree Celsius** at standard atmospheric pressure and 20°C. The symbol for specific heat is c . In this course, water's specific heat is written $c_w = 1.0 \text{ cal}/(\text{g}^\circ\text{C})$.

Note that the units for mass and temperature both are in the denominator of this expression for specific heat. This is a mathematical way of stating that the change in energy for water is one calorie to change each gram by one Celsius degree. The dimensions of specific heat, c , are:

Dimensions of specific heat: $\frac{(\text{energy})}{(\text{mass})(\text{temperature})}$

The units are: $\frac{(\text{calories})}{(\text{grams})(\text{degrees Celsius})}$

To express mathematically how much energy is gained or lost when something is warmed or cooled:

- What might be the effect of how much “stuff” is involved?
- What might be the effect of the kind of “stuff” one has?
- What might be the effect of how much the temperature changes?
- How can you combine measures of these effects mathematically to estimate the energy lost by the hot water?

Complete Table II.3 and write a summary about expressing these theoretical considerations mathematically.

Table II.3 Developing a mathematical expression for change in energy

Table II.3 Developing a mathematical expression for change in energy			
Sketch	Mathematical representation	Theoretical consideration	Vocabulary
	m	The bigger the mass of material, the more energy is lost or gained	Mass
	c	The energy lost or gained also depends upon a <u>property of the material</u> , how much energy is needed to change the temperature of a mass of one gram by one degree Celsius, its <i>specific heat</i> .	Specific heat
	ΔT	The bigger the temperature change, the more energy is lost or gained.	ΔT , delta T, represents "change in temperature" $\Delta T = T_f - T_i$
	m c ΔT	The change in energy equals the product of mass, specific heat, and change in temperature.	Change in energy

After completing Table II.3, look at an example of student work and consider nuances about mathematical representations of thermal phenomena based upon theoretical considerations.

1. Example of student work about developing a mathematical expression for a change in energy

This section is an example of using mathematics to express some theoretical insights. The ratios above do not directly address what is happening conceptually in terms of energy transfers when mixing hot and cold water. The considerations that one must make, including how much material one has, what kind of material this

is, and how much the temperature changes, are indicated in (the table shown in Fig. 2.19 below.)

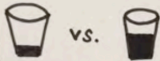

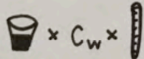
Development of Law of Conservation of Energy and Its Mathematical Representation			
Sketch	Mathematical representation	Theoretical consideration	Vocabulary
	m	The bigger the mass of material, the more energy is lost or gained	Mass
It takes 1 calorie to change 1 gram by 1°C.	c	The energy lost or gained also depends upon a property of the material, its <i>specific heat</i> .	Specific heat
$\Delta T = 35^\circ\text{C}$ vs. $\Delta T = 5^\circ\text{C}$ 	ΔT	The bigger the temperature change, the more energy is lost or gained.	Δ delta (represents "change")
 $m_h \times c_w \times \Delta T_h$	$m_h c_w \Delta T_h$	Energy lost by the hot object equals the product of its mass, specific heat, and change in temperature.	Energy

Fig. 2.19 Student's entries describing exploration of changes in energy.

The bigger the mass of material, the more energy is lost or gained. The first row in (the table) presents this idea. For this experiment, the more water that there was, the more energy was lost or gained. This is because the more mass that a material has, the more energy it will take to change things about the material, such as its temperature. Since it takes 1 calorie to change 1 gram of water 1°C, the greater the mass of the material, the more calories were needed, which means that more energy was lost or gained.

The energy lost or gained also depends upon a property of the material, its specific heat. The second row in (the table) shows this idea. Specific heat is defined as, c = the amount of energy (lost or gained) to change the temperature of one gram of material by one degree Celsius. Water is the standard, c_w = one calorie of energy that is lost or gained when the temperature of one gram of water changes by one degree Celsius (at standard atmospheric pressure and 20°C).

The bigger the change in temperature, the more energy is lost or gained. This idea is displayed in the third row in (the table). Since it takes 1 calorie to change 1 gram of water 1°C, the greater the temperature change, the more calories were needed, meaning that more energy was lost or gained.

Energy lost by the hot object equals the product of its mass, specific heat, and change in temperature. This idea is presented in the fourth row of (the table). These variables are multiplied to express mathematically the energy lost by the hot water $m_h c_w \Delta T_h$, because in order to know how much energy is lost, you have to know how much material is losing energy, the specific heat of that material, and the change in temperature of that material. If you know all of that information, you are able to multiply the three values together in order to determine the amount of energy that is lost.

Physics student, spring 2016

2. An analogy to specific heat and the mathematical expression for change in energy

This progression of mathematical thinking can seem challenging. However, exploring both experimental and theoretical approaches to relating quantities is an important part of modeling phenomena in science.

The process of developing such mathematical expressions can increase comfort with using subscripts, in interpreting expressions involving multiple symbols, and in deepening understanding of what multiplication represents in viewing $m_h c_w \Delta T_h$ as a meaningful expression for how much a mass m_h of hot water, with specific heat c_w , changes in energy when it cools by ΔT_h degrees in temperature when mixed with cold water.

Sometimes it helps to consider an analogous situation:

- How would you estimate the budget for a project if the basic cost, c , is the number of dollars paid for each hour worked by each person?

What would be the estimated budget for the project, for example, if you hired 5 people at a cost of \$15/hour for each person who worked for 2 hours?

You would need: $(5 \text{ people}) \left(\frac{\$ 15}{\text{each person, each hour}} \right) (2 \text{ hours}) = \150

Note that multiplication is indicated here by parentheses () rather than by x's in order to avoid confusion with situations in which an x represents an unknown, which often occurs in algebraic equations.

In general, if n = the number of people, c = the cost for each person for each hour, and Δt

= the estimated duration (final clock reading – initial clock reading), i.e., the time in hours needed for this number of people to complete the project, the budget would be:

$$\text{Number of dollars needed} = \text{(number of people)} \left(\frac{\text{dollars}}{\text{person hour}} \right) \text{(estimated number of hours)}$$

Budget needed for the project = $nc\Delta t$

The mathematical reasoning for energy transfer is exactly the same:

- How would you calculate the change in energy if the mass of the material was m , the specific heat of a material is “ c ,” which is the number of calories needed for an energy change of one gram for each degree C change in temperature, and the material changed in temperature ΔT ?

If m = mass in the number of grams,

c = energy in calories needed for each gram for each degree C change in temperature, and

ΔT = change in temperature in number of degrees C,

the change in energy would be:

$$\text{change in energy} = mc\Delta T$$

Although it may be hard to view $mc\Delta T$ as a meaningful expression for change in energy, one can focus on dimensions and see this as:

$$\text{change in energy} = \frac{(\text{mass}) (\text{energy}) (\text{temperature})}{(\text{mass})(\text{temperature})} = \text{energy}$$

or focus on units and see this as:

$$\text{change in energy} = \frac{\text{(number of grams)} (\text{number of calories}) (\text{number of degrees C change in } T)}{\text{(gram)(degree C)}} = \text{number of calories}$$

It can be helpful in working thermal problems, to step back and evaluate the dimensions on both sides of an equation to check that one has not made an algebraic mistake in manipulating an equation to solve for an unknown.

B. Considering the Law of Conservation of Energy

Question 2.12 How does the energy gained by the cold water compare to the energy lost by the hot water, assuming no energy is gained by the surrounding environment?

The total change in energy should equal the change in energy of the hot water plus the change in energy of the cold water plus any change in energy of the surroundings. This is a statement based on the *Law of Conservation of Energy*. The word “conservation” here means that the total amount of energy does not change when energy flows from one part of a system to another or from one form into another.

This means that the total change in energy when hot and cold water mix should equal 0. Total change in energy = change in energy of hot water + change in energy of cold water + any energy flowing into the surroundings.

If no energy flows into the surroundings:

$$m_h c_w \Delta T_h + m_c c_w \Delta T_c = 0 \quad \text{Law of Conservation of Energy}$$

Note that because $\Delta T_h = T_{hf} - T_{hi}$ and the final temperature is always less than the initial temperature for the hot water, ΔT_h is always negative, and therefore $m_h c_w \Delta T_h$ is always negative, which can be interpreted as the energy lost by the hot water.

The energy lost by the hot water + energy gained by the cold water = 0, makes sense as long as none of the energy lost by the hot water flows into the air or the containers. This also means that the magnitude of the energy lost by the hot water equals the magnitude of the energy gained by the cold water as long as none of the energy lost by the hot water flows into the air or the containers.

Question 2.13 How are these experimental and theoretical approaches related?

According to the Law of Conservation of Energy, the total energy does not change: Total change in energy = change of energy for hot water + change in energy for cold water = 0 if no energy flows into the surroundings.

$$m_h c_w \Delta T_h + m_c c_w \Delta T_c = 0 \quad \text{Law of Conservation of Energy}$$

This algebraic relationship also can be written as: $m_h c_w \Delta T_h = - m_c c_w \Delta T_c$

This equation tells us that the change in energy of the hot water is *equal and opposite* to the change in energy of the cold water

This equation can be simplified by dividing both sides by c_w , m_c and ΔT_h in order to make ratios of m 's and ΔT 's:

$$\frac{m_h}{m_c} = -\frac{\Delta T_c}{\Delta T_h}$$

This is the equation developed experimentally with approximate results! Thus the theoretical approach confirms the relationship evident in the patterns of data obtained by measuring temperature changes when mixing various configurations of hot and cold water.

One can work forwards from the experimental result by multiplying both sides by m_c and ΔT_h

and including the specific heat of water on both sides to obtain the theoretical result:

$$m_h c_w \Delta T_h = -m_c c_w \Delta T_c$$

One also can express this relationship in terms of magnitudes:

$$|m_h c_w \Delta T_h| = |m_c c_w \Delta T_c|$$

Any of these approaches can be used in estimating a thermal quantity of interest.

VIII. Using Mathematical Representations to Estimate a Quantity of Interest

Qualitative approaches, such as interpreting the shapes of graphs, were helpful in gaining a conceptual understanding about what is happening when mixing hot and cold water. Quantitative approaches make possible the making of predictions and estimation of quantities that may be of interest.

A. Solving a thermal math problem

In this section, we demonstrate how to solve thermal math problems in the context of mixing hot and cold water. We assume that the specific heat of water is the same over the range of temperatures between freezing at 0°C and boiling at 100°C . We also assume that the specific heat of water is the same as the specific heat of fluids such as tea, milk, and cocoa when working thermal energy problems involving mixing these substances. The same mathematical process also applies in more complex situations in which the specific heat of the hot material differs, however, from the specific heat of the cold material, when, for example, a piece of hot metal is submerged into a cooling bath.

Question 2.14 How can one use mathematical representations of thermal phenomena to estimate a quantity of interest?

After developing both experimental and theoretical ways to describe what happens when mixing hot and cold water, one can use that knowledge to generate and solve thermal math problems.

To make up a thermal math problem, decide on a scenario and specify three of the four variables involved if only considering changes in temperature: mass of hot water, mass of cold water, magnitude in change in temperature of hot water, or magnitude of

change in temperature of cold water. Also include information about one of the initial temperatures or equilibrium temperatures. Use mass units (grams, kilograms) or parts rather than volume units. If the scenario involves materials with different specific heats, be sure to include that information in stating the problem and in calculating the answer.

As with solving pinhole math problems, the goal in solving a thermal math problem is not the “answer.” The goal is to build your ability to help someone else understand what to do and why. Start by helping the learner to understand what is happening by describing the scenario verbally with words and visually with a sketch. Next review the physics involved by stating what the relevant central ideas are. Also draw a qualitative graph and use it and the central ideas to explain what is happening. Then describe the graph mathematically, being clear about what each symbol represents. Justify the equation that relates the quantities represented; write the equation in both words and symbols. Finally, solve for the unknown in symbols before substituting values. After calculating an answer, be sure to also discuss why that answer seems reasonable.

In facilitating a conversation with someone about thermal phenomena, ask questions rather than tell answers throughout this process. In solving a thermal math problem for homework, follow the format provided here:

Format for Solving a Thermal Math Problem

- a. **State the problem** in words
- b. **Make a sketch** of the amounts to be mixed
- c. **Review what you know** about the physics of this phenomenon: summarize the conceptual model by stating the relevant central ideas
- d. **Draw a graph representing the problem:** use a ruler to make straight perpendicular axes, draw flat horizontal lines to represent temperatures that are not changing before the two substances are mixed, add the shared equilibrium temperature after the mixing, check that you have drawn the appropriately shaped graph for mixing more hot than cold or more cold than hot or equal amounts. Draw the graph to indicate whether the changes in temperature happen very quickly or are gradual processes.
- e. **Tell the ‘story’ of the graph** with the relevant central ideas to **explain** why the equilibrium temperature is likely to be where you have drawn it (nearer the temperature of the initial hot water, initial cold water, or in the middle.)
- f. **Represent this scenario mathematically: State the equation in words** that relates the masses of the substances mixed and their changes in temperature. Use the

experimental form of the equation derived from your exploration, including specific heats if the materials differ, or the theoretical form based on the Conservation of Energy. **Justify the use of the equal sign** accordingly.

- g. **Define symbols, state equation in symbols**, and express how you are envisioning this equation.
 - h. **Solve for the unknown** in symbols
 - i. **Record given values and estimate any needed**
 - j. **Substitute values and calculate answer**
 - k. **Check answer**: why does the number you get from the calculation seem reasonable?
- Complete the process of generating and solving a thermal math problem before looking at a slightly modified example of student work.

1. *Example of student work generating and solving a thermal math problem*

The problem stated in words is: If you have 180 grams of hot tea, and you want to cool it down by 20 degrees Celsius by adding 60 grams of cold water, how much will the temperature of the cold-water change? Assume all of the energy lost by the tea is gained by the cold water and that the specific heats of tea and water are the same. The initial temperature of the cold water is 15 degrees Celsius. After finding the change in temperature of the cold water, find the equilibrium temperature and the initial temperature of the hot tea.

(As shown in Fig. 2.20), below is a sketch of the situation.

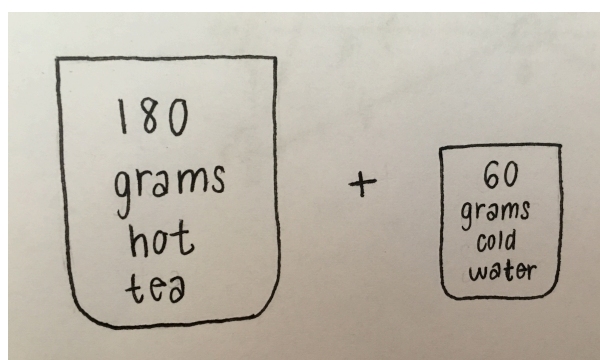


Fig. 2.20 Student sketch of the situation for this problem.

(As shown in Fig 2.21), below is a graph that represents the problem. The first relevant idea is, when mixing unequal amounts of hot and cold water, the ratio of the amount of hot water to the amount of cold water is approximately equal to minus the ratio of the change in temperature of the cold water to the change in temperature of the hot water. Next, the energy lost by the hot object equals the product of its mass, specific heat, and change in temperature. Finally, energy is conserved: energy lost = energy gained, so when mixing hot and cold water, the (magnitude of the) energy lost by the hot water equals (the magnitude of) the energy gained by the cold water.

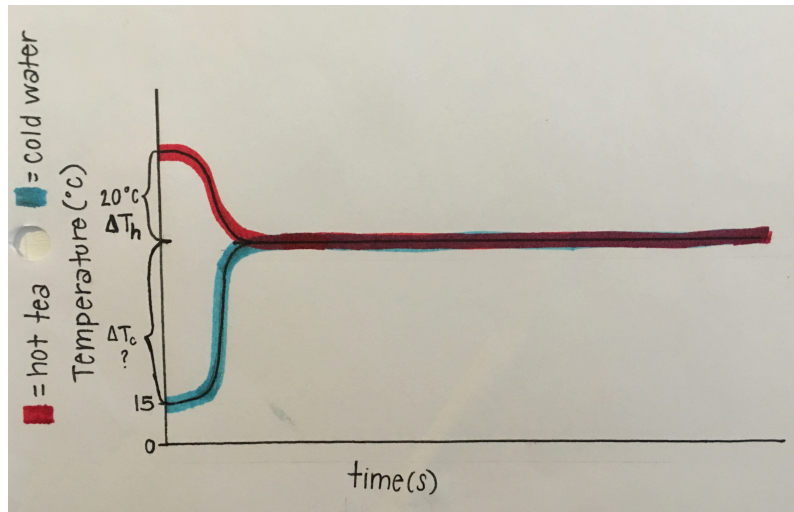


Fig. 2.21 Student representation of the problem graphically.

Symbols that are defined for the relevant quantities are m_h = mass of hot tea, m_c = mass of cold water, ΔT_c = change in Temperature of cold water, and ΔT_h = change in Temperature of hot tea. An algebraic equation that relates these quantities in symbols is

$$\frac{\text{mass of hot tea}}{\text{mass of cold water}} \approx \frac{-\Delta T_c}{\Delta T_h}$$

The same equation in words is

$$\frac{\text{Mass of hot tea}}{\text{Mass of cold water}} \approx \frac{-\text{change in Temperature of cold water}}{\text{change in Temperature of hot tea}}$$

This equation is justified because it is equivalent to the theoretical statement of the Law of Conservation of Energy - $(m_h c_w \Delta T_h) = (m_c c_w \Delta T_c)$, assuming that no energy is gained by the environment such as the cup and air.

The unknown and the quantity to be calculated is the change in temperature of the cold water, ΔT_c . The symbols for the quantities I have provided in the problem

statement and their numerical values are $m_h = 180$ grams, $m_c = 60$ grams, and $\Delta T_h = -20^\circ\text{C}$.

The equation solved algebraically for the unknown is $\Delta T_c = -\Delta T_h \frac{(m_h)}{(m_c)}$.

When I substitute the values above for the known quantities and then calculate the answer I get

$$\Delta T_c = -(-20^\circ\text{C}) \frac{(180 \text{ grams})}{(60 \text{ grams})} = 60^\circ\text{C}$$

This answer is reasonable because there was a lot more hot water than cold water which means that the temperature of the mixture will be closer to the initial temperature of the hot water than the initial temperature of the cold water. This means that the cold water should have a greater temperature change than the hot water. The hot water had a temperature change of -20°C , so this answer of the cold water having a temperature change of 60°C is reasonable.

The initial temperature of the cold water was 15°C . Since we know the initial temperature of the cold water, we are able to use equations to solve for the initial temperature of the hot water and for the equilibrium temperature. I can now write an algebraic equation for the unknown of the equilibrium temperature, T_e . That equation is $T_e = T_c + \Delta T_c$. When I substitute the values in I get $T_e = 15^\circ\text{C} + 60^\circ\text{C} = 75^\circ\text{C}$.

Then, I can write an algebraic equation for the unknown of the initial temperature of the hot tea. That equation is $\Delta T_h = T_e - T_h$ so $T_h = T_e - \Delta T_h$. When I substitute the values in I get $T_h = 75^\circ\text{C} - (-) 20^\circ\text{C} = 95^\circ\text{C}$. This initial hot temperature is 95°C which is below boiling point and the initial cold temperature is 15°C which is above freezing, which both seem reasonable. Cooling tea down to 75°C also seems reasonable because this is below boiling point but above freezing. If a student spills the tea, it is not hot enough to burn them.

Physics Student, Spring 2016

Actually 75°C is hot enough to burn one severely. Recommendations are for 'hot' liquids to be at 60°C or less. <https://www.ncbi.nlm.nih.gov/pubmed/18226454>

IX. Engaging Friends or Family Members in Exploring Thermal Phenomena

Question 2.15 What can you learn about science learning and teaching by engaging a friend or family member in learning about thermal phenomena?

- Invite a friend or family member to explore thermal phenomena with you.
- What does this person already know about thermal phenomena in the context of mixing hot and cold water?
- Help this learner do some systems thinking about how energy is conserved when energy flows from hot to cold water.
- Make up a conservation of energy problem and invite your learner to think aloud with you about how to solve it.
- Include the wording of your conservation of energy problem and its mathematical solution.
- Create an environment in which your learner feels comfortable enough to ask questions. Describe what this learner asked and said.
- Also describe what this learner did and found.
- In addition, discuss a NGSS science or engineering practice in which you engaged your learner while learning about thermal phenomena.
- Reflect on what you learn about teaching science through this experience in facilitating science learning.
- Post on the class electronic discussion board. Read your classmates' postings to learn from their experiences.

Complete this experience in learning and teaching physics before reading the examples of student work about designing and solving thermal math problems.

1. Examples of student work about designing and solving thermal math problems with friends and/or family members

A student engaged her roommate in some qualitative thinking to develop the relationship between masses of hot and cold water and their changes in temperature:

I explored thermal phenomena with my roommate M. Before experimenting, I asked her what she thought would happen when we mixed the hot and cold water and she said that it would become warm water. When I asked why her explanation was just that the cold cools down the hot water while the hot warms up the cold water.

I asked her to try mixing different amounts of water and predict what would happen. We kept all of the results labeled in different cups to compare their temperatures. We did not have a thermometer so we just had to base everything on touch. I asked her to line up the cups from hottest to coldest based on feeling.

M noticed that the hottest had the least amount of cold water in the mixture and the coldest had the least amount of hot water. Since we were not able to measure our temperatures at home, I showed Morgan the data we had collected in class. I did not give her the equation we came up with because I wanted to see where she would go with the information.

She asked me if the temperature changes always need to add up to a specific number. I asked her to try it with different numbers than what we got to see if that was the case. She realized that was wrong because if there is a smaller gap between the temperatures, their temperature differences would be smaller.

M was stumped so I tried having her look at the whole numbers without the decimals. She then realized the big numbers were double the small numbers. After looking at all of the data a little longer she realized that if the amount of hot water was double the cold water, that the temperature change of the cold water would be double that of the hot water.

I then challenged her to imagine what the temperature change would be like if it was 4 parts hot water and 1 part cold water. Immediately M said that the temperature change of the cold water would be 4 times greater than that of the hot.

I asked her if she could come up with an equation and she said it would be hot water divided by cold water would be equal to hot water temperature change divided by cold water temperature change. I asked her to test it with the data. She then

realized the second equation would need to be cold temperature change over hot temperature change.

Once she had figured this out I asked her to figure out the temperature change of cold water when 2g of cold water was mixed with 8g of hot water. I also give her the temperature change of the hot water being 5C. She then figured out that the temperature change of the cold water would be 20C.

I used the same examples that had been used in class because I thought it was a good progression of testing understanding. I learned the importance of teaching in steps and building off previous knowledge.

Physics student, Fall 2016

Another student engaged her boyfriend in some interesting thinking about aspects that might affect what happens:

When I asked my boyfriend what happens when you add 4 parts cold water to 2 parts hot water, he said “the temperature increases and the mixture gets warmer, because it’s closer to the cold water. The heat goes into the cold water”. I said, “Right! Because energy transfers from hot water to cold water.”

When I asked him what other factors might affect the transfer of hot water to cold, specifically when you pour the water in, he responded that the container, the air, and the surface the cups are sitting on all could affect the heat transfer.

Then we started talking about how these things may be affected. The amount both containers would take in heat, would matter on what the material it was made out of. M said, “If it was a thermos, it wouldn’t take in much heat, because the heat would be insulated in the cup.”

We also talked about the temperature of the air where you were pouring. Like if it was in Alaska, or Mexico the air surroundings would take in either more or less depending on which.

Next I gave him a word problem on solving for the change in heat of hot water. “What is the change of the hot water, when you have 30 g of hot tea and add 20g of cold water to make it drinkable and the change in temperature of the cold water is 15 degrees.

He was very confused by this complicated problem and thought that we should match the A temperature with A change in temperature and B temp with B change, to get a new temperature of C. But he did not know where to take it from there.

I asked him if he remembered anything about specific heat and showed him the equation to solve for the change in temp of hot water. He solved the algebra easily,

but said he probably couldn't come up with the equation on his own and I agreed that I wasn't able to come up with it on my own either.

He was on the same page as me about this problem, that the algebra made sense and was easy, but it was the comprehending the bigger picture that was hard to grasp, and thinking up the equation on your own.

I learned that sometimes teachers and learners have the same questions, and that makes it easier to discuss and engage in thorough investigations. Also I learned both having questions lessens the "authoritarian" all knower- teacher type that teachers sometimes possess. We engaged in discussion, using math to solve problems, and analyzed and constructing explanations and designing solutions for the science and engineering practice standards.

Physics student, Fall 2015

X. Making Connections to Educational Policies

What does *doing* science and engineering involve? Unit 1 introduced common practices such as collecting, analyzing and interpreting data as well as engaging in argument from such evidence. *Doing* science and engineering also involves ways of thinking that bridge across different science domains. What concepts, for example, do biologists, physicists, and chemical engineers all use in their studies?

A. Learning about the US Next Generation Science Standards: *Crosscutting Concepts*

Many US states have adopted the *Next Generation Science Standards* (NGSS Lead States, 2013) for guiding science instruction in their schools. In addition to the science and engineering practices introduced in Unit 1, these standards articulate a group of concepts that are common across many science disciplines. Both the science and engineering practices and these crosscutting concepts are intended to help students learn about and participate in the nature of science.

Question 2.16 What relevant crosscutting concepts have you used in exploring light and thermal phenomena?

- Go to <https://www.nextgenscience.org/get-to-know>
- Click on Appendix G, scroll down the first page to the list of seven crosscutting concepts that scientists and engineers use across many different contexts:
 - **Patterns.** Observed patterns of forms and events guide organization and classification, and they prompt questions about relationships and the factors that influence them.
 - **Cause and effect:** Mechanism and explanation. Events have causes, sometimes simple, sometimes multifaceted. A major activity of science is investigating and

explaining causal relationships and the mechanisms by which they are mediated. Such mechanisms can then be tested across given contexts and used to predict and explain events in new contexts.

- **Scale, proportion, and quantity.** In considering phenomena, it is critical to recognize what is relevant at different measures of size, time, and energy and to recognize how changes in scale, proportion, or quantity affect a system's structure or performance.
 - **Systems and system models.** Defining the system under study—specifying its boundaries and making explicit a model of that system—provides tools for understanding and testing ideas that are applicable throughout science and engineering.
 - **Energy and matter:** Flows, cycles, and conservation. Tracking fluxes of energy and matter into, out of, and within systems helps one understand the systems' possibilities and limitations.
 - **Structure and function.** The way in which an object or living thing is shaped and its substructure determine many of its properties and functions.
 - **Stability and change.** For natural and built systems alike, conditions of stability and determinants of rates of change or evolution of a system are critical elements of study
- Scan the discussion of these crosscutting concepts to see what they are and read about any that you find particularly interesting.
 - Put a check in Table II.4 to indicate if you have used a crosscutting concept while exploring light and thermal phenomena in this course.
 - For each context, choose one or more crosscutting concepts and describe an example of what you did and learned.

TABLE II.4 Crosscutting concepts (NGSS Lead States, 2013)

TABLE II.4 Crosscutting concepts (NGSS Lead States, 2013)

	Unit 1 Explaining pinhole phenomena and estimating the size of an object	Unit 1 Exploring reflection, refraction, dispersion and explaining rainbows	Unit 2 Distinguishing between heat and temperature	Unit 2 Exploring transfer of energy when mixing hot and cold water
1. Patterns				
2. Cause and Effect				
3. Scale, proportion, and quantity				
4. Systems and system models				
5. Energy and matter: Flows, cycles, and conservation				
6. Structure and function				
7. Stability and Change				

B. Reflecting upon this exploration of thermal phenomena

This unit began by considering what students already knew about thermal phenomena. Children and adults have many experiences in which high temperatures describe hot days and low temperatures describe cold days. They also have many experiences touching objects of many kinds in which some feel colder than others. These prior experiences often lead students to rank paper or Styrofoam, wood, aluminum and steel in order from higher to lower temperatures because these materials feel so different to touch. Most students are surprised to find that items made of these materials all are at the same temperature when measured by a thermometer, rather than by their hands, if the materials have been in the same room for a long time.

Understanding often emerges when someone mentions the idea of *room temperature*, that objects that have been in the same room for a long time are at the same temperature. A helpful nudge usually occurs when someone comments that maybe the materials differ in what happens when one touches them, that metal differs from paper or Styrofoam in some way.

Eventually someone articulates the idea that energy is flowing from one's hand into the metals, that metal is a conductor, and one's hand is losing energy quickly so one's hand feels cold. Paper and Styrofoam, however, are insulators; very little energy is flowing from one's hand into the paper or Styrofoam, so one's hand stays warm. The materials feel different because of their differences in the property of *thermal conductivity*, in how well energy flows into and throughout the material, but their temperatures are the same if left for a long time in the same room.

This process is an example of what scientists and engineers do when they observe something that seems puzzling. They try to figure out what might be happening in order to explain that puzzle; they ponder how to reconcile different ideas that seem applicable but do not agree. This process of refining one's ideas often involves separating and clarifying the differences between some closely related concepts that seem initially to be the same such as *heat* and *temperature*.

After developing relevant central ideas, this unit also modeled the next step, of figuring out how to represent mathematically the relationships one has been exploring with both experimental and theoretical approaches. A series of experiments led to the inference of

an *inverse relationship* between the amounts of hot and cold water and their changes in temperature when mixed. Many students express surprise at this inverse relationship as their prior experiences with mathematical ratios typically have been with direct relationships, such as the equal ratios of heights and distances involved in pinhole phenomena. A theoretical approach based on the *Law of Conservation of Energy*, however, confirms this inverse relationship between the masses of hot and cold water and their respective changes in temperature when mixed together. A refinement involves recognizing that the property of *specific heat* also affects how much energy is absorbed or released when the temperature of a material changes. Development of an algebraic equation representing these relationships makes possible numerical predictions and estimates of quantities of interest.

Students may experience some of the frustration that scientists and engineers often face as they struggle to perceive the patterns in their data, particularly when our simple equipment does not yield precise relationships but only trends when the data are compared in various ways. Students also may experience, however, the pleasures that scientists and engineers experience when finally recognizing and confirming both the conceptual and mathematical models developed.

C. Making connections to NGSS understandings about the nature of science

The *Next Generation Science Standards* recommends that students engage in three dimensions of learning science by using science and engineering practices and cross cutting concepts while learning disciplinary core ideas. In this unit, for example, students used the science and engineering practice of *analyzing and interpreting data* when they tracked and interpreted initial and final temperatures while mixing various amounts of hot and cold water. They became aware of the importance of the crosscutting concepts of *systems and system models* while attempting to minimize energy flowing into the cups and air by pouring the cold water into the hot water rather than the hot water into the cold. During these explorations of thermal phenomena, students learned disciplinary core ideas about *conservation of energy and energy transfer*.

This unit also has provided additional examples of understandings about the nature of science as articulated in Appendix H of the *Next Generation Science Standards* <https://www.nextgenscience.org/resources/ngss-appendices> . The learning progression

for the NGSS understanding that *science is a way of knowing*, for example, includes that middle school students should learn that *science is both a body of knowledge and the processes and practices used to add to that body of knowledge*. In this unit, for example, students observed that materials left for a long time in the same room have the same temperature even though the materials may feel warmer or colder when touched. From this, the students gained new knowledge about the role of an object's property, its *thermal conductivity*, in the rate at which such energy transfers occur. By mixing various amounts of hot and cold water, the students developed mathematical ways of tracking the flow of energy in a relatively simple system. This unit thus initiated explorations of such energy transfer processes. This focus continues in considering more complex energy transfer phenomena during local weather in Unit 3 and during global climate change in Unit 4.

XI. Exploring Physical Phenomena: Summary of Equipment and Supplies for Unit 2

Exploring Physical Phenomena: Summary of Equipment and Supplies for Unit 2

Exploring Physical Phenomena: Summary of Equipment and Supplies for Unit 2			
When used	For instructor and demonstrations	For each group of 3	For each student
<p align="center">Unit 2 Week 4 Day 7 Q2.1 – Q2.5 Exploring the difference between heat and temperature</p>		<p>4 blocks (aluminum, steel, wood, paper pad or Styrofoam), or kitchen items made of these materials; Regular bulb and tube liquid thermometer that reads between 0 and 100 degrees C</p>	<p>U2H2. Diagnostic Question about Temperature</p>
<p>Begin Day 7, Q2.6 – Q2.18 Exploring energy transfer in the context of mixing hot and cold water</p>	<p>If in a room without a sink:</p> <p>Big pot with cold water from tap</p> <p>Big pot with hot water from tap or on hot plate</p> <p>Dippers,</p> <p>hot pot holder</p> <p>Big bucket for waste water</p>	<p>Qualitative exploration:</p> <p>2 digital temperature probes and related software (We use vernier.com probes and Logger lite software) or 2 bulb and tube thermometers);</p> <p>cup for hot water (ceramic or Styrofoam),</p> <p>Plastic cup for cold water,</p> <p>1 clear plastic cup marked with 1,2,3,4 equal divisions,</p> <p>2 quart containers for hot and cold water,</p> <p>2 trays.</p> <p>(laptop computer, perhaps brought to class by group member if using digital temperature probes)</p> <p>U2H1 Exit Ticket</p>	

U2.Solving a Thermal Math Problem

1. **State the problem** in words
2. **Make a sketch** of the amounts to be mixed
3. **Review what you know** about the physics of this phenomenon: the relevant central ideas
4. **Draw a graph representing the problem** (use a ruler to make straight perpendicular axes, draw flat horizontal lines to represent temperatures that are not changing before the two substances are mixed and the shared equilibrium temperature after the mixing, draw the appropriately shaped graph for mixing more hot than cold or more cold than hot or equal amounts. Draw the graph to indicate whether the changes in temperature happen very quickly or are gradual processes.)
5. **Tell the 'story' of the graph** with the relevant central ideas to **explain** why the equilibrium temperature is likely to be where you have drawn it (nearer the temperature of the initial hot water, initial cold water, or in the middle.)
6. **Represent this scenario mathematically: State the equation in words** that relates the masses of the substances mixed and their changes in temperature. Use the experimental form of the equation derived from your exploration or the theoretical form based on the Conservation of Energy and **justify the use of the equal sign** accordingly.
7. **Define symbols, state equation in symbols**, and express how you are envisioning this equation.
8. **Solve for the unknown** in symbols
9. **Record given values and estimate any needed**
10. **Substitute values and calculate answer**
11. **Check answer:** why does the number you get from the calculation seem reasonable?

UNIT 3: CONSIDERING THE INFLUENCE OF LIGHT AND THERMAL PHENOMENA ON LOCAL WEATHER

Exploring Physical Phenomena: What happens when light from the Sun shines on the Earth?

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I. Introduction

The theme for this course is *what happens when light from the Sun shines on the Earth?* In this unit, you will be exploring the effect of light and thermal phenomena on local weather. While exploring these phenomena, you will be:

- **identifying resources** such as everyday knowledge about water and weather
- **developing central ideas based on evidence** about the water cycle
- **explaining intriguing phenomena** such as why, during a sunny day at the beach, the sand is hot, the water cool, and cloudy skies as well as sea breezes often appear in the afternoon
- **developing mathematical representations** of the transfer of energy in various contexts
- **using mathematical representations to estimate a quantity of interest** such as predictions about “will it rain tomorrow”? and
- **making connections to educational policy**, such as the Next Generation Science Standards (NGSS Lead States, 2013), the science standards adopted by many US departments of education.

This unit continues exploration of learning processes as well as of physical phenomena. Summarizing and reflecting upon these explorations will foster integrating science and literacy learning. This includes learning to speak clearly, listen closely, write coherently, read with comprehension, and create and critique media.

The main sections present questions with suggestions for exploring topics and for writing reflections about your findings. Text in gray font indicates that these are suggestions; you may think of other ways to explore the topic. Asking your own questions as well as those posed here will enhance learning both about physics and about learning. Check with your instructor if you choose to devise an alternative approach.

Keeping track of what one is doing and thinking is important. In this course, use a template for a physics notebook page on which to record your notes during class. The physics notebook page can help you remember your thoughts *before*, *during*, and *after* an exploration. An experienced elementary teacher, Adam Devitt, designed this notebook page to mirror the structure of *before*, *during*, and *after* reading strategies:

Before starting your exploration, think about and discuss with your group members

what you know already about the topic, how you plan to conduct the exploration, and what you think you might find out.

During your exploration, record what is happening, what you are observing, and what you are thinking about what you are observing. Include sketches of equipment and observations. Note any words that are new and their definitions.

After your exploration, record any central ideas that have emerged from your observations and discussions. Also note the evidence on which you have based these ideas. State explicitly how the evidence is relevant and supports the claims you are making in stating the ideas. Also explain why this result is important. Then write a reflection about whatever you want to remember about this experience. In addition, briefly state what you are still wondering in this context.

After class, use your physics notebook pages and any handouts to write a summary of your exploration and findings. Writing such a summary after every class is a good way to prepare for the midterm and final examinations.

Next, to be sure you have understood the physics involved, read this text and some examples of student work. The student authors first wrote drafts, received feedback for ways to enhance content and clarity, and submitted these final versions. Also read about some nuances to be aware of in these contexts.

You may also find helpful students' reflections about teaching friends and/or family members about what they had just learned in class, historical information about ways knowledge about the topic developed, and some relevant aspects of the nature of science in the context of the topic explored. These sections of the text may broaden your understanding of science and of science learning and teaching.

II. Identifying Student Resources

Water is a central player in weather phenomena. The Greek stem *hydro* for water has formed the basis of many English words related to water.

- If a plant wilts, what can you do to revive it?
- If you forget your water bottle on a long hike, what might happen to you?
- What chemical element is a component of water?
- Where is water found on a planet such as the planet Earth?

To *hydrate* something, for example, means to supply a liquid, usually water, to someone or something that needs it; to be *dehydrated* means to be lacking water. The element *hydrogen* is a component of water: two atoms of hydrogen and one of oxygen form one molecule of water (H₂O). The word *hydrosphere* refers to everywhere that water is found on Earth.

A. Connecting to relevant knowledge and experiences

Thinking about how water moves from place to place can provide resources on which to build in learning about weather phenomena.

Question 3.1 What do you already know about water and weather?

- Look at a cup of water. How did the water get there?

Trace the paths that the water in this cup might have followed to get to where you can see it here.



FIG. 3.1 Cup of water.

Discuss with your group members what you already know about water and how it moves from place to place. Record some of your ideas before reading an example of student work.

1. *Example of student work about early experiences with water and weather*

A student wrote:

In my early years of elementary school, I was taught about the water cycle. When I learned about the water cycle, I learned about the terms that could apply to experiences that I have previously had as a child. For example, I learned that condensation describes the droplets that form on the outside of my water bottle that has ice water in it. Next, I learned that precipitation is what describes that rain that I have seen falling from the sky. Finally, I learned that evaporation is what describes when a water puddle that I see is gone a few hours later. These are experiences that are likely to occur for students, so it is important that they understand what they observe.

Physics student, Spring 2016

Understanding the physical phenomena underlying the water cycle is the focus of this section.

B. Documenting initial ideas about changes in states of matter

Water exists on Earth in several *states of matter*: solid, liquid, and gas. These also are called *phases of matter*. Ice cubes melting, a puddle of water evaporating, water vapor condensing into fog, and the surface of a pond freezing over are examples of *changes in state* or *phase changes*.

1. Diagnostic Question about States of Matter

Responding to the diagnostic question below will document some of your initial knowledge about the roles of heat and temperature during changes in states of matter.

Name_____ Date_____

Diagnostic Question about Changes in States of Matter

Some ice and a little liquid water are placed in a pot on a hot plate.

When the hot plate is turned on, the ice begins to melt.

After the ice melts, the liquid water warms, and eventually begins to boil.

The boiling continues until the pot is removed from the hot plate.

How does the temperature change during this process?

Illustrate your prediction with a graph of temperature versus time.

III. Developing Central Ideas Based on Evidence

Unit 2 developed the central idea that heat and temperature are different ideas. The next exploration extends this development to examine what happens when water changes state.

A. Exploring the difference between heat and temperature during changes in states of matter

Question 3.2 What happens when ice melts, liquid water warms, and the liquid water eventually boils?

To explore changes in state, assemble:

- rice cooker, pot and hot plate, or pot and stove
- tray of ice cubes
- thermometer
- large binder clip.

We use a digital temperature probe connected to a computer that displays a continuous record of the temperature (see <https://www.vernier.com/product-category/?category=temperature-sensors>). We set the duration for 1800 seconds (30 minutes) and the temperature scale from about -5°C to about 110°C . A regular bulb and tube thermometer will work if the scale ranges at least from a little below the freezing point of water (0°C) to a little above its boiling point (100°C).

- In the **Before** section of your physics notebook page, predict how the temperature of water changes when a container of ice cubes is placed on a heater and the ice melts, the liquid water warms, and then boils. Sketch a graph of temperature versus time to

illustrate your prediction.

- What happens when ice melts, warms, and boils?
- Put some ice and a little water in a container that can be heated such as in a rice cooker or in a pot on a hot plate or on a stove. Place a thermometer or temperature probe connected to a computer in the ice.
- You can use a binder clip to keep the temperature probe or thermometer from touching the bottom of the pan as shown in Fig. 3.2.



FIG. 3.2 Pot with thermometer

- Turn on the heat source and keep its level constant throughout this exploration so that energy is flowing at a constant rate into the system of the pot and the frozen, liquid, and finally boiling water.
- In the ***During*** section of your physics notebook page, record what happens to the temperature when you heat the ice.
- Be sure that the thermometer or temperature probe is not resting on the bottom of the rice cooker or pot but is measuring the temperature of the melting mixture of ice and liquid water. Stir occasionally to be sure the temperature is the same throughout the mixture of melting ice and liquid water.
- Leave the temperature probe in the warming liquid water, stirring as needed, and for a few minutes while the liquid water is boiling.
- If using a regular thermometer, stir and record the temperature repeatedly while the ice melts, the liquid water is warming, and boils.